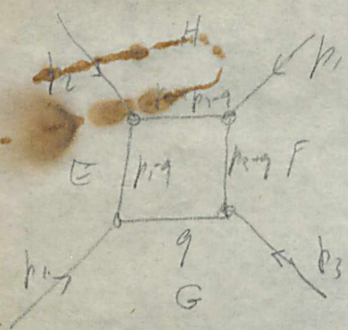
Blankenbecler — Mandelstam rep. in 4th order pert. Th.



$$s = -(P_1+P_2)^2 \qquad t = -(P_1+P_3)^2$$

propagators

$$A(s,t) = \frac{ig^4}{(2\pi)^4}\int d^4q\; E F G H$$

assume $t<0$ and want disp. rel. in s.

Combine E and F and write as Feynman form

$$\frac{1}{[(p_1-q)^2+M^2][(p_3+q)^2+M^2]} = \int_0^1 dx\, \frac{1}{\tilde{q}^2 - 2x(1-x)(M^2-P_1\cdot P_3)}$$

where $\tilde{q} = q - (xP_1 - (1-x)P_3)$

also $2(M^2 - P_1\cdot P_3) = t$

E and F cannot vanish in the physical region

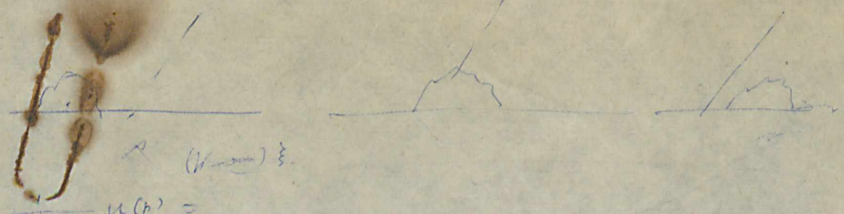
For H and G we will have s instead of t

but G and H can vanish for s

Phys. Rev. 111, short proof by ? Greco.

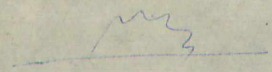
$\mathrm{Im}\, A \sim \int d\Omega\, (EF)$ — product of 2 Born approximations

for fixed negative t. write dispersion relations in s. Then show that Im A is analytic in t.



$(\gamma_\mu - m)\,\xi$

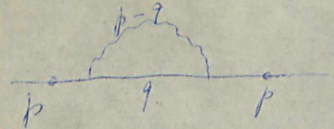
$\frac{1}{\not p + m}\, u(p) =$



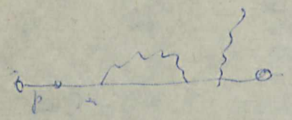
$u(p)$

$\Sigma(p)$

$\overset{M}{S}(p) = \bar u(p) \left(\int \frac{d^4 q}{(p-q)^2} \frac{1}{\not q + m} \right) u(p)$



$\Lambda_\mu(0) \left[\bar u(p) \gamma_\mu u(p) + \bar u(p+\epsilon) \Sigma(p+\epsilon) u \right]$

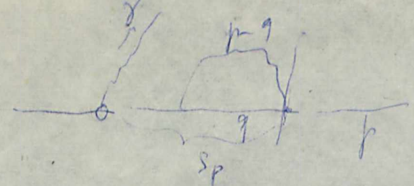


$u(\epsilon) \quad \bar u(p) \gamma_\mu S(p+\epsilon) \Sigma(p+\epsilon) u(p+\epsilon)$

$+ c.c.$

$\bar u(p) S(p) \Sigma(p) S(p) u(p)$

$\frac{\partial}{\partial p_\mu} u(p) \quad \frac{\partial}{\partial p_\mu}(\gamma_\mu p_\mu + m)\,\xi = \gamma_\mu u(p)$



$a_\mu(0) \left[\bar u \gamma_\mu S(p) u + \bar u S(p) \gamma_\mu u + \bar u D(p-q) \gamma_\mu D(p-q) S^0 \right.$

$\frac{\partial \bar u}{\partial p_\mu} S u + \bar u S \frac{\partial u}{\partial p_\mu} +$

$\bar u \frac{\partial S}{\partial p_\mu}$

$\frac{\partial S}{\partial p_\mu} = \frac{1}{(p-q)^2} \frac{\gamma_\mu}{\not q + m} +$

$\not p_\mu$

∂_μ

$\gamma_\mu + 2$

$S(p) = \bar u\, \Sigma(p)\, u$

$\frac{\partial S(p)}{\partial p_\mu} = \bar u \gamma_\mu \Sigma u + \bar u \Sigma \gamma_\mu u + \bar u \frac{\partial \Sigma}{\partial p_\mu} u$

$\frac{\partial S}{\partial p_\mu} = S \Gamma_\mu S$

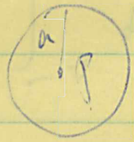
$\Gamma_\mu = S^{-1} \frac{\partial S}{\partial p_\mu} S^{-1}$

$S(p) = \bar u \not p u + \bar u \Sigma(p) u$

$a(q) \bar u \not\partial_\mu u =$

$\Gamma_\mu =$

Ruderman



Interaction vanishes outside radius a.

$$[\Box - \mu^2]\varphi = j$$

r'

$$\langle p | [j, j^\dagger] | p \rangle = 0 .$$

Limiting values of coupling constants.

1 – Potential case.

Asymptotic form $\varphi_k(r) = \dfrac{1}{(2\pi)^{3/2}} \dfrac{e^{-ikr} - S(k)\, e^{ikr}}{2kr}$ $\qquad S(k) = e^{2i\delta}$

bound state $\dfrac{C e^{-\alpha r}}{r}$. $\qquad$ for $r > a$.

Completeness of the functions (Stone)

$$4\pi \int_0^\infty k^2 \varphi_k(r)\, \varphi_k^*(r') = \frac{\delta(r-r')}{4\pi r r'} .$$

$$+ \frac{|C|^2 e^{-\alpha(r+r')}}{r r'}$$

bound state

Turning on the interaction we also add the bound state. $\qquad S(k) S^*(k) = 1.$

Def. $S(-k) = S^*(k)$
k real.

$$\int_{-\infty}^{\infty} dk \left[S(k) - 1 \right] e^{2iak} e^{2ik\rho} = 8\pi^2 |C|^2 e^{-2\alpha a} e^{-2\alpha\rho}$$

$\rho \geq 0$.

Analyticity:

$f(k)$: scattering amplitude $\frac{S(k)-1}{2ik}$

$$e^{2iak} \left\{ f(k) + \frac{4\pi C^2}{k^2+\alpha^2} \right\} = R(k) \text{ is regular in}$$

the upper half plane

$f(k)$ is the Schwinger amplitude
e^{2iak} is analytic.

$\alpha = \sqrt{2M \epsilon_B}$
binding energy.

Since $|C|^2 \leq \frac{\alpha}{2\pi} e^{2a\alpha}$ because the wave for $\varphi_B(r) = \frac{Ce^{-\alpha r}}{r}$ is normalized.

for Thomas one gets $|C| \leq \dfrac{\frac{\alpha}{2\pi} e^{2a\alpha}}{1 + \frac{1}{a\alpha}}$

Set of wave fns

$$\begin{bmatrix} e^{-ika} - S(k) e^{ika} \\ e^{-ik'r} \\ e^{-ik''r} \end{bmatrix} \qquad \begin{bmatrix} e^{-ikr} \\ e^{-ikr} - S(k') e^{ik'r} \\ \vdots \end{bmatrix}$$

$f(k) = f^*(-k)$

+ analyticity
gives restriction on $|C|^2$

Define $F(k) = \frac{k - i\alpha}{k + i\alpha} e^{2iak} f(k)$

$F(k) = F^*(-k)$ for k real.

then $F(k) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{F(k')\, dk'}{k' - k}$

$$F(i\alpha) = \frac{2}{\pi} \int_0^\infty \frac{k' \operatorname{Im} F(k')}{k'^2 + \alpha^2}$$

$$|F(i\alpha)|^2 \leq \frac{2}{\pi} \int |k'| \cdots$$

$$F_2(k) = \frac{k - i\alpha}{k + i\alpha} \frac{k + i\beta}{k - i\beta} e^{2iak} \left[f(k) + \frac{1}{2ik} \right] - \frac{1}{2ik}$$

$\beta = \frac{\alpha}{1 + a\alpha}$ $\quad F_2(k) = F_2^*(-k)$

$F_2(0) = 0$. $\quad$ for $K \leq 0$

$$F_2(i\alpha) = \frac{K}{-\alpha^2 + \beta^2} + \frac{2}{\pi} \int_0^\infty \frac{k' \operatorname{Im} F_2(k')}{k'^2 + \alpha^2} \geq 0.$$

K = residue at $\alpha = i\beta$.

$$-\pi \frac{C_p^2}{\alpha^2} e^{-2\alpha a} \left(1 + \frac{2}{a\alpha}\right) + \frac{1}{2\alpha} \geq 0.$$

Field theory with extended source.

n^- p scattering $\omega^2 - k^2 + \mu^2$.

$$M^+(\omega, \vec{k}, \vec{k}') = -i \int dt \int d\vec{x} \int d\vec{x}' \, e^{i\vec{k}\cdot\vec{x} - i\vec{k}'\cdot\vec{x}'} e^{i\omega t} \langle I | \theta(t) [j(\vec{x}, t) , j(\vec{x}', 0)] | I \rangle$$

has branch point $k = i\mu$.

$$+ \delta(t) \langle I [j, \dot{\varphi}] I \rangle .$$

$$M^-(\omega, \vec{k}, \vec{k}')^* = M^+(-\omega, -k, -k')$$

crossing relation.

$$f^+ f. \quad \frac{f^+(\omega) + f^-(\omega)}{2} \text{ has no branch points}$$

$$\frac{\mu}{2\omega} (f(\omega) - f^*(\omega)) \text{ also has no branch points}$$

Insert complete state $|m\rangle\langle m|$ of Hamiltonian and get the Low form.

$$f^+(\omega) = \sum_m \frac{\overbrace{\langle I | J^*(k) | m \rangle \langle m | J(k) | I \rangle}^{A}}{E_m - E_I - \omega - i\varepsilon} + \frac{\overbrace{\langle I | J | m \rangle \langle m | J^* | I \rangle}^{B}}{E_m - E_I + \omega + i\varepsilon}$$

$$f^-(\omega) = \sum_m \frac{B}{E_m - E_I - \omega - i\varepsilon} + \frac{A}{E_m - E_I + \omega + \varepsilon}$$

$$e^{2i\delta k} \frac{f^+(\omega) + f^-(\omega)}{2} = \sum \frac{(E_m - E_I)(A + B)}{(E_m - E_I)^2 - \mu^2 - k^2} e^{2i\delta k}$$

coefficient of the poles in complex conjugate

$$e^{2i\delta k} \frac{f^+(\omega) - f^-(\omega)}{2\omega} = \Big(\qquad \Big)$$

Result:

$$e^{2iak}\left[\frac{f^{+}(\omega)+f^{-}(\omega)}{2} \underset{\substack{\uparrow \\ \text{p wave}}}{\overset{\text{Swave}}{\mp}} \frac{\Delta M \times 2g^2}{(\Delta M)^2-\mu^2-k^2}\right] = R(k) \text{ is regular}$$

$$e^{2iak}\left[\frac{f^{+}(\omega)\pm f^{-}(\omega)}{2\omega} \pm \frac{2\mu g^2}{k^2+\mu^2-(\Delta M)^2}\right] = \bar{R}(k) \begin{array}{l}\text{also}\\ \text{is regular}\end{array}$$

ΔM = mass diff. between states of scatterer ($p-n$ mass difference).

$$(\Delta m) g^2 \le \frac{\alpha e^{2\alpha a}}{1+2/\alpha a}$$

$$(\mu+\Delta M)^2 g^2 \ge \frac{\alpha e^{2\alpha a}}{1+\frac{2}{\alpha a}}$$

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other method.

$$[\Pi^{*}(r), \varphi(r')] = i\delta(\vec{r}-\vec{r}').$$

for $S_{+}(k)+S_{-}(k)$

$$\frac{S_{+}(k)-S_{-}(k)}{\omega}$$

$$\frac{S_{+}(k)-S_{-}(k)}{2\omega}.$$

$$[\varphi \quad \varphi'] = 0$$
$$[\Pi \quad \Pi'] = 0.$$

still satisfied outside range

$$\varphi = \sum \frac{a_k e^{ikr}}{\sqrt{2\omega}}.$$

corresponds to

$$\alpha = \sqrt{\mu^2-(E_m-E_Z)^2}$$

$\left(\vec{\xi}\cdot\frac{\vec{\xi}\cdot\vec{\sigma}}{2}\right)$

$$\omega = \frac{3\,\vec{\xi}\cdot\vec{\sigma} - 1}{2}$$

$\frac{1+\varepsilon}{2}\;\vec{\sigma}\;\vec{n}\;\omega\;\vec{\sigma}\cdot\vec{n}\;\frac{1+\varepsilon}{2}$ $\qquad (\vec{\xi}\cdot\vec{n})(\vec{\sigma}\cdot\vec{n}) - \frac{1}{3}\,\vec{\xi}\cdot\vec{\sigma}\,n^2$

$n^+ \qquad \frac{3n^0}{2\sqrt{2}} \qquad \varepsilon\left(\varepsilon - \frac{1}{3}\right) = 1 - \frac{1}{3}\varepsilon)$

$\vec{\sigma}\cdot\vec{n}\quad \vec{\xi}\cdot\vec{\sigma}\quad \vec{\sigma}\cdot\vec{n} \qquad \xi_3\sigma_3 + \xi_1\sigma_1 + \xi_2\sigma_2$

$$\vec{\xi}\cdot\vec{\sigma} = \begin{pmatrix} \sigma_3 & \sigma_1 + i\sigma_2 \\ \sigma_1 - i\sigma_2 & -\sigma_3 \end{pmatrix}$$

$$\begin{pmatrix} \frac{n^0}{\sqrt{2}} & n^+ \\ n^- & -\frac{n^0}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{n^0}{\sqrt{2}} & n^+ \\ -n^- & \frac{n^0}{\sqrt{2}} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{n_0^2}{2} - n^-n^+ & \sqrt{2}\,n^0 n^+ \\ \sqrt{2}\,n^0 n^- & -\frac{n_0^2}{2} + n^-n^+ \end{pmatrix}$$

$$\vec{\xi}\cdot\vec{\sigma} - \frac{1}{3} = \begin{pmatrix} \sigma_3 - \frac{1}{3} & \sigma_1 + i\sigma_2 \\ \sigma_1 - i\sigma_2 & -\sigma_3 - \frac{1}{3} \end{pmatrix}$$

$n^2 = \frac{n_0^2}{2} + n^-n^+$

$\frac{1}{3}n^2 =$

$\frac{n_0^2}{2} - \frac{n_0^2}{6} = \frac{n_0^2}{3}$

$$\frac{1+\varepsilon}{2}\begin{pmatrix} n\sigma_3 n - \frac{1}{3}n^2 & n(\sigma_1 + i\sigma_2)n \\ n(\sigma_1 - i\sigma_2)n & -n\sigma_3 n - \frac{1}{3}n^2 \end{pmatrix}\frac{1+\varepsilon}{2}$$

$n_\sigma\,\omega\,n_\sigma + n_\xi\,\omega\,n_\xi + n_\xi\,\varepsilon\,\omega\,n_\sigma + n_\sigma\,\omega\,\varepsilon\,n_\xi$

$$\begin{pmatrix} K^+ \\ K^0 \\ \bar{K}^0 \\ K^- \end{pmatrix}$$

$K\,\xi_3\,\bar{K} = (K_1^0 + \vec{K})\,\xi_3\,(K_4^0 - \vec{K})$

$= K_1^0 + \vec{K}\,(-\xi_3 K_1^0 - \xi_3\vec{K})$

$= \xi_3 K_1^{0\,2} - K_1^0\,\xi_3\,\vec{K} + \vec{K}\,\xi_3 K_1^0 + \vec{K}\,\xi_3\,\vec{K}$

$\dot{\varphi}\cdot n\quad \xi_3\,\xi\cdot n\,\sigma_3 = \ldots$

N_1

$e^{i\xi\ldots}\;e^{i\tau\omega}\begin{pmatrix} p \\ n \\ \Xi^0 \\ \Xi^- \end{pmatrix}(\bar{\Sigma}^+\;\bar{Y}^0\;\bar{Z}^0\;\bar{\Sigma}^-)\,e^{i\tau\omega}\,e^{-i\tau\cdot\nu}$

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$\frac{\rho_{\ell}\sigma_m + \rho_m\sigma_\ell}{2} - \frac{1}{3}\rho_m\sigma_m$ $\quad n_m n_m - \frac{1}{3}n^2$

$\vec{\sigma} - i\frac{1}{2}\vec{\rho}\times\vec{\sigma}$.

$$\vec{\rho}\times\vec{\sigma}\cdot\vec{n} = \begin{pmatrix} -i(\sigma^- n^+ - n^-\sigma^+) & i(\sigma^- n_3 - \sigma_3 n^- \\ -i(\sigma_3 n^+ - \sigma^+ n_3) & i(\sigma^- n^+ - n^-\sigma^+) \end{pmatrix}$$

$$(\vec{\sigma} - i\frac{1}{2}\vec{\rho}\cdot\vec{\sigma})\vec{n} = \begin{pmatrix} \vec{\sigma}\cdot\vec{n} + \frac{i}{2}(\sigma^- n^+ - n^-\sigma^+) & \frac{\sigma^- n_3 - \sigma_3 n^-}{2} \\ -\frac{\sigma_3 n^+ - \sigma^+ n_3}{2} & \vec{\sigma}\cdot\vec{n} + \frac{1}{2}(\sigma^- n^+ - n^-\sigma^+) \end{pmatrix}$$

$n_+ i n_- \cdot n n^+$

$\sqrt{2} - \frac{1}{\sqrt{2}}$ $\quad \sqrt{2} + \frac{2}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

$$\begin{pmatrix} n_3 & \sqrt{2}n^+ \\ \sqrt{2}n^- & -n_3 \end{pmatrix} = \begin{pmatrix} 0 & \frac{n^+}{\sqrt{2}} \\ -\frac{n^-}{\sqrt{2}} & \end{pmatrix} = \begin{pmatrix} n_3 & \frac{3}{\sqrt{2}}n^+ \\ \frac{1}{\sqrt{2}}n^- & -n_3 \end{pmatrix}$$

$\vec{\sigma} + \frac{i}{2}\vec{\rho}\times\vec{\sigma}$

$1, \rho, \sigma, \vec{\rho}+\vec{\sigma}, \frac{\rho_\ell\sigma_m + \rho_m\sigma_\ell}{2} - \frac{1}{3}\rho_m\sigma_m$

$1, \rho, \sigma - \frac{i}{2}\vec{\rho}\times\vec{\sigma}, (\vec{\sigma} - \frac{i}{2}\vec{\rho}\times\vec{\sigma}), \frac{\rho_m\sigma_m - \rho_n\sigma_m}{2} - \frac{1}{3}\rho_m\sigma_m$.

$$\begin{pmatrix} -n^+ & 0 \\ n_3 & n^- \\ n_3 & \frac{3}{\sqrt{2}}n^+ \\ \frac{1}{\sqrt{2}}n^- & -n_3 \end{pmatrix}$$

$(\vec{\sigma} - \frac{i}{2}\vec{\rho}\times\vec{\sigma})\cdot(\rho - \frac{i}{2}\vec{\rho}\cdot\vec{\sigma})$

$(\sigma_p + i\frac{\rho_m\sigma_n - \rho_n\sigma_m}{2})(\rho_q + i\frac{\rho_k\sigma_\ell - \rho_\ell\sigma_k}{2})$

$= \sigma_p\rho_q +$

$$\gamma_\mu \partial_\mu \psi = m U' \psi' \qquad U' = U(\gamma_5 \vec{\tau}\cdot\vec{\phi}) \qquad \underline{UU^\dagger = 1}$$

$$U = U(-\vec{\tau}\cdot\vec{\phi})$$

$$\begin{cases} (\partial_0 + \vec{\sigma}\cdot\vec{\nabla})\chi = m U \zeta \\ (\partial_0 - \vec{\sigma}\cdot\vec{\nabla})\zeta = m U^\dagger \chi \end{cases}$$

Require invariance under the local group

$$\chi \to e^{i\vec{\tau}\cdot\vec{a}}\chi \qquad \vec{a} = \text{function of } t, x, y, z.$$

$$\left(\partial_0 + \vec{\sigma}\cdot\vec{\nabla} + \vec{B}_0^{(1)} + \vec{\sigma}\cdot\vec{B}^{(1)}\right)\chi = m U \zeta.$$

Generalize combination

$$\vec{\tau}\cdot\vec{B}_\mu \to e^{i\vec{\tau}\cdot\vec{a}}\left(\vec{\tau}\cdot\vec{B}_\mu - e^{-i\vec{\tau}\cdot\vec{a}}\partial_\mu e^{+i\vec{\tau}\cdot\vec{a}}\right)e^{-i\vec{\tau}\cdot\vec{a}}$$

$$U\partial_\mu U^\dagger \to e^{i\vec{\tau}\cdot\vec{a}}\left(U\partial_\mu U^\dagger - e^{-i\vec{\tau}\cdot\vec{a}}\partial_\mu e^{+i\vec{\tau}\cdot\vec{a}}\right)e^{-i\vec{\tau}\cdot\vec{a}}$$

$$\left(\vec{\tau}\cdot\vec{B}_\mu^{(1)} - U\partial_\mu U^\dagger\right) \to e^{i\vec{\tau}\cdot\vec{a}}\left(\vec{\tau}\cdot\vec{B}_\mu^{(1)} - U\partial_\mu U^\dagger\right)e^{-i\vec{\tau}\cdot\vec{a}}$$

in the same way

$$\left(\vec{\tau}\cdot\vec{B}_\mu^{(2)} - U^\dagger\partial_\mu U\right) \to e^{i\vec{\tau}\cdot\vec{b}}\left(\vec{\tau}\cdot\vec{B}_\mu^{(2)} - U^\dagger\partial_\mu U\right)e^{-i\vec{\tau}\cdot\vec{b}}.$$

$$\left(\vec{\tau}\cdot\vec{B}_\mu^{(1)} - U\partial_\mu U^\dagger\right)^2 + \left(\vec{\tau}\cdot\vec{B}_\mu^{(2)} - U^\dagger\partial_\mu U\right)^2 \text{ is invariant}$$

This gives $\vec{B}_\mu^{(1)}\vec{B}_\mu^{(1)} + \vec{B}_\mu^{(2)}\vec{B}_\mu^{(2)}$ terms

also $(U\partial_\mu U^\dagger)^2 + (U^\dagger\partial_\mu U)^2 = (\partial_\mu\vec{\phi})^2 + \dots(\vec{\phi}\times\partial_\mu\vec{\phi})^2$

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$$U \to e^{i\vec{\tau}\cdot\vec{a}}\, U\, e^{-i\vec{\tau}\cdot\vec{b}}$$

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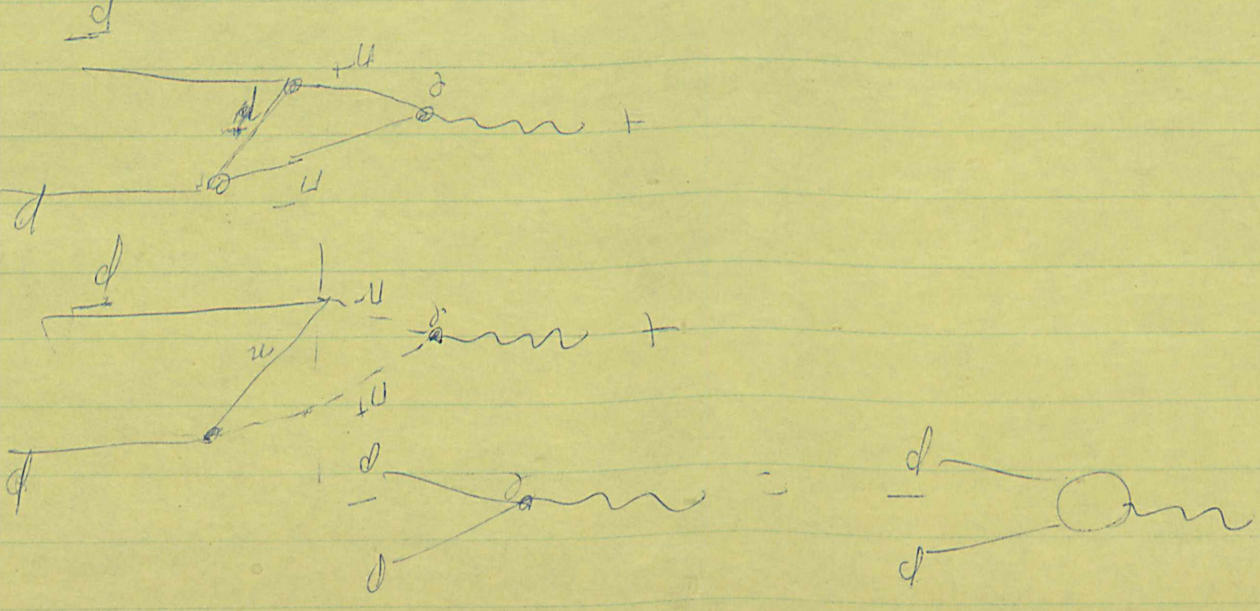
1 – if $\partial_\mu f_\mu = 0$ – only Col.mag.) :



(Abstract for New York Meeting of A.P.S. January 29,-February 1, 1957)

On Generalized Gauge Transformations.* F. GÜRSEY, Brookhaven National Laboratory. - Attention is drawn to a 4-dimensional symmetry of the pseudoscalar meson-nucleon system. This is best studied by analyzing the nucleon wave function (p,n) into two 4-spinors χ and ζ defined by $\chi = \frac{1}{2}(1-\gamma_5)p + \frac{1}{2}i(1+\gamma_5)p^c$ and $\zeta = \frac{1}{2}(1-\gamma_5)p - \frac{1}{2}i(1+\gamma_5)p^c$. It is shown that when χ and ζ undergo Paulis'(1) generalized gauge transformations $\chi \rightarrow a\chi + b\gamma_5\chi^c$ and $\zeta \rightarrow a'\zeta + b'\gamma_5\zeta^c$ $(|a|^2+|b|^2 = |a'|^2+|b'|^2 = 1)$, we obtain a 4-dimensional rotation group which leaves the quadratic form $m^2 + g^2\vec{\pi}^2$ invariant (m is the nucleon mass and $\vec{\pi}$ denotes the meson field). Isotopic spin rotations correspond to $a' = a$, $b' = b$ while transformations of the Dyson-Foldy type follow from the special case $a' = a^*$, $b' = -b$. Under such transformations p and n are mixed, but not χ and ζ. However parity (P) and charge conjugation (C) mix χ and ζ. Parity conserving terms in the Lagrangian contain χ and ζ in a symmetrical manner while interaction terms involving only χ (or ζ) destroy both C and P invariance. Such weak interactions may be introduced by the B field technique of Yang and Mills in the special case $a' = 1$, $b' = 0$, a and b being arbitrary functions of space-time variables. The behavior of strange particles under this group will be discussed.

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* Work performed under the auspices of the U.S. Atomic Energy Commission and the International Cooperation Administration.

(1) W. Pauli, Nuovo Cimento 6, 204 (1957).

$$\begin{array}{l} T^2_0 \\ T^2_{\pm1} \\ T^2_{\pm2} \end{array}\left\{\begin{array}{ll} \frac{1}{2}\ (2z^2-x^2-y^2) & = \frac{1}{2}(2z^2-x^2-y^2) = \\ \mp\frac{1}{2}\sqrt{\frac{3}{2}}\ z(x\pm iy) & = \mp\frac{\sqrt{3}}{2}\ z\left(\frac{x\pm iy}{\sqrt{2}}\right) \\ +\frac{1}{4}\sqrt{\frac{3}{2}}\ (x\pm iy)^2 & = \frac{1}{2}\sqrt{\frac{3}{2}}\left(\frac{x\pm iy}{\sqrt{2}}\right)^2 \end{array}\right.$$

$$2z^2-x^2-y^2 \qquad \sqrt{\frac{2}{3}}\,(2z^2-x^2-y^2) = 2\sqrt{\frac{2}{3}}\,(z^2-a^+a^-$$

$$\mp\sqrt{3}\ z\left(\frac{x\pm iy}{\sqrt{2}}\right) \qquad \mp\sqrt{2}\ z\,\frac{x\pm iy}{\sqrt{2}}$$

$$\sqrt{\frac{3}{2}}\ \left(\frac{x\pm iy}{\sqrt{2}}\right)^2 \qquad \left(\frac{x\pm iy}{\sqrt{2}}\right)^2$$

$$x^2+y^2 = (x+iy)(x-iy) = 2\,\frac{x+iy}{\sqrt{2}}\,\frac{x-iy}{\sqrt{2}}$$

$$2\sqrt{\frac{2}{3}}\left(n_0^2 - n^+n^-\right) = \sqrt{\frac{2}{3}}\,(2n_0^2 - 2n^+n^-)$$

$$\mp\sqrt{2}\ \ n_0 n^\pm$$

$$n^{+2}$$

$$n_m n_n = g_{mn}$$

$$\begin{array}{l} y_0 = \\ y_{\pm1} \\ y_{\pm2} \end{array}\left\{\begin{array}{ll} \sqrt{\frac{2}{3}}\left(2n_0^2 - n^+n^- - n^-n^+\right) & = \sqrt{\frac{2}{3}}\left(2n_0^2 - n_1^2 - n_2^2\right) \\ \mp\sqrt{2}\ \ n_0 n^\pm & = \mp\ n_0(n_1 + in_2) \\ n^{+2} & \ \ \frac{1}{2}(n_1^2 - n_2^2 \pm 2in_1n_2) \end{array}\right.$$

$$y_0 = \sqrt{\frac{2}{3}}\ \left(2g_{33} - g_{11} - g_{22}\right) =$$

$$y_{\pm1} = \mp\ (\ g_{13} \pm i g_{23})$$

$$y_{\pm2} = \frac{1}{2}\ (g_{11} - g_{22} \pm 2ig_{12}) \quad = n^{+2}$$

This Lagrangian contains a direct pion-lepton interaction term with a coupling constant $\sqrt{2}\,G/2f$ corresponding to the limit $\mu \to 0$. When the finite pion mass μ is taken into account, this coupling constant will be a function of μ^2, say $C(\mu^2)$. If $C(\mu^2)$ is a slowly varying function of μ^2 then

$$(2.6) \qquad C(\mu^2) \cong C(0) = \sqrt{2}\,G/2f$$

so that ~~C~~ $C(0)$ may be regarded as a good approximation to the actual π-lepton coupling constant insofar as the 4-dimensional group holds for the complete Lagrangian (including the pion mass), since it can be expected that non renormalization of $C(0)$ by strong interactions will follow from the vector character of $\vec{j}_\lambda$ under the $\vec{\mu}$ rotations. Nambu ( ), Feynman, Gell-Mann and Lévy ( ) and Gell-Mann, Bernstein and Michel ( ) have independently arrived at the same conjecture by assuming that the axial vector current ~~is~~ conserved in the limit $\mu \to 0$.

$$\begin{pmatrix} p \\ n \end{pmatrix} \qquad \begin{pmatrix} -n^c \\ p^c \end{pmatrix}$$

$$\begin{pmatrix} p & -n^c \\ n + p^c \end{pmatrix} \to e^{i\epsilon \cdot \omega} \begin{pmatrix} \quad \end{pmatrix}$$

$\psi^G \quad G^2 = -1$

$i\gamma_5 G$

$$\begin{pmatrix} p & -n^* \\ n + p^* \end{pmatrix} \qquad \psi = \begin{pmatrix} p \\ n \end{pmatrix} \qquad \psi^G = \begin{pmatrix} -n^* \\ p^* \end{pmatrix}$$

$i\gamma_5$ is real $\qquad\qquad = (i\tau_2)\psi^*$

$$\psi^{G'} = i\gamma_5 \psi^G = \begin{pmatrix} -i\gamma_5 n^* \\ i\gamma_5 p^* \end{pmatrix}$$

$i\gamma_5$ commutes with G

$(i\gamma_5) \qquad G'\psi = \psi' \qquad G'^2\psi' = \psi'$

$\gamma_\mu \partial_\mu \psi = m\psi$

$\gamma_\mu \partial_\mu \psi^G = m\psi^G$

$\gamma_\mu \partial_\mu \psi^{G'} = -m\psi^{G'}$

$\gamma_\mu \partial_\mu \psi^* = m\psi^*$

$\gamma_\mu \partial_\mu i\tau_2\psi^* = m\, i\tau_2\psi^*$

$\gamma_\mu \partial_\mu i\gamma_5 i\tau_2\psi^* = -m\, i\gamma_5 i\tau_2 \psi^*$

$\gamma_\mu \partial_\mu (\psi + \psi^{G'}) = m(\psi - \psi^{G'})$

$\gamma_\mu \partial_\mu (\psi - \psi^{G'}) = +m(\psi + \psi^{G'})$

Let $\psi = \psi^{G'}$ then

$\gamma_\mu \partial_\mu \psi = 0$

for $\gamma_\mu \partial_\mu \psi = m\psi$

If $\psi = \psi^{G'}$ the rest mass is zero.

$\psi = \tau_2 \gamma_5 \psi^C$ implies zero rest mass.

Let

then

$\gamma_\mu \partial_\mu \psi = m\psi^{G'}$

$\gamma_\mu \partial_\mu \psi^{G'} = -m\psi$

$\gamma_\mu \partial_\mu (\psi + \psi^{G'}) = m(\psi - \psi^{G'})$

Let

$\gamma_\mu \partial_\mu \psi = m\, \Phi\, \psi^{G'}$

$\gamma_\mu \partial_\mu \psi^{G'} = -m\, \Phi\, i\gamma_5\, \Phi\, \psi^{G'})$

$\phi_1 \quad \phi_2 \quad \phi_3 \qquad \phi_1 \quad \phi_2$

$(\sigma + \vec{e}\cdot\vec{\phi}) \qquad \sigma^2 + \phi^2 = 1$

$(\cos\alpha + e_3 \sin\alpha)(\sqrt{1-\phi^2} + \vec{e}\cdot\vec{\phi})$

$= \sqrt{1-\phi'^2} + \vec{e}\cdot\vec{\phi}' \qquad = (\quad) + \vec{e}\cdot\vec{\phi}\cos\alpha + e_3 \sin\alpha\sqrt{1-\phi^2} + (e_3 \times \vec{e})(\sin\alpha)\vec{\phi}$

$\vec{\phi}' = \vec{\phi}\cos\alpha + e_3$

$\sigma' = \sigma\cos\alpha \pm \phi_3 \sin\alpha$

$$\begin{cases} \phi_3' = \phi_3\cos\alpha + \sqrt{1-\phi^2}\sin\alpha + \\ \phi_1' = \phi_1\cos\alpha - \phi_2 \sin\alpha \\ \phi_2' = \phi_2\cos\alpha + \phi_1 \sin\alpha \end{cases}$$

when $\alpha = \pi \to$

if $\pi = \alpha \qquad \vec{\phi}' = -\vec{\phi}$

$\frac{1+\phi'}{1-\phi'} = e^{e_3\alpha}\frac{1+\phi}{1-\phi}$

$$\boxed{\vec{\phi}' = \frac{2(\cos\alpha)\vec{\phi} + e_3\sin\alpha(1-\phi^2)}{1+\phi^2+\cos\alpha(1-\phi^2) + 2\sin\alpha(e_3\cdot\vec{\phi})}}$$

for e_3 to $\Phi = e^{e_3\alpha}\Phi$

Let $\alpha = \pi$ then $\cos\alpha = \cos\pi = -1$

$\sin\alpha = 0$

$$\vec{\phi}' = \frac{-2\vec{\phi}}{1+\phi^2-1+\phi^2} = \frac{-\vec{\phi}}{\phi^2}$$

inversion

Define $\sqrt{1-\phi^2}$ so that when $\phi \to -\phi$ $\sqrt{1-\phi^2} \to -\sqrt{1-\phi^2}$

$e^{e_3\alpha}(\sqrt{1-\phi^2} + \vec{e}\cdot\vec{\Phi})$

$e^{e_3\alpha}$

$\psi \to \gamma_5\psi$

$\Phi \to -\Phi$

$\sqrt{1-\phi^2} \to -\sqrt{1-\phi^2}$

reflection in isospace

$\gamma_\mu \partial_\mu \psi = m\,g(\sqrt{1-\phi^2} + i f\phi)\psi$

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if $\Phi = \dfrac{1 + i \vec{e}\cdot\vec{\phi}}{\sqrt{1+\phi^2}}$ $(\cos\alpha + e_3 \sin\alpha)(1 + \vec{e}\cdot\vec{\phi})$

then $\dfrac{1 + i\,\vec{e}\cdot\vec{\phi}'}{\sqrt{1+\phi'^2}} = e^{e_3\alpha}\dfrac{1+i\vec{e}\cdot\vec{\phi}}{\sqrt{1+\phi^2}} = \dfrac{\cos\alpha - \phi_3\sin\alpha + e_3\sin\alpha + \vec{e}\cdot\vec{\phi}\cos\alpha + e_3\times\vec{e}\cdot\vec{\phi}/\sin\alpha}{\sqrt{\ }}$

$$1 + i\vec{e}\cdot\vec{\phi}' = \frac{1 + \left(\dfrac{\vec{e}\cdot\vec{\phi}\cos\alpha + e_3\sin\alpha + \vec{e}_3\times\vec{e}\cdot\vec{\phi}}{\cos\alpha - \phi_3\sin\alpha}\right)^2}{\sqrt{1+(\qquad)}}$$

Hence

$$\vec{\phi}' = \frac{\vec{e}\cdot\vec{\phi}\cos\alpha + e_3\sin\alpha + \vec{e}_3\times\vec{e}\cdot\vec{\phi}\sin\alpha}{\cos\alpha - \phi_3\sin\alpha}$$

$0 \quad 0 \quad 1 \quad 0 \quad 0$
$\phi_1 \quad \phi_2 \quad \phi_3 \quad \phi_1 \quad \phi_2$

or $\phi_3' = \dfrac{\phi_3\cos\alpha + \sin\alpha}{\cos\alpha - \phi_3\sin\alpha}$

$\phi_1' = \dfrac{\phi_1\cos\alpha - \phi_2\sin\alpha}{\cos\alpha - \phi_3\sin\alpha}$

$\phi_3' = \dfrac{\phi_2\cos\alpha + \phi_1\sin\alpha}{\cos\alpha - \phi_3\sin\alpha}$ $\qquad \dfrac{-1 - \vec{e}\cdot\vec{\phi}'}{\sqrt{1+\phi'^2}} = \dfrac{1+\vec{e}\cdot\vec{\phi}}{\sqrt{1+\phi^2}}$

if $\alpha = \pi$ $\qquad \dfrac{-1}{\sqrt{1+\phi'^2}} = \dfrac{1}{\sqrt{1+\phi^2}}$

$\phi_3' = \dfrac{-\phi_3}{-1}$

$1 + \phi'^2 = 4\phi^2 \qquad \phi' = \pm\phi$

$\dfrac{-\vec{\phi}'}{\sqrt{1+\phi'^2}} = \dfrac{\vec{\phi}}{\sqrt{1+\phi^2}}$

$\dfrac{\phi'^2}{1+\phi'^2} = \dfrac{\phi^2}{1+\phi^2}$ $\qquad \dfrac{1+\phi'^2}{\phi'^2} = \dfrac{1+\phi^2}{\phi^2}$

$a + \vec{b} = \dfrac{a + \vec{b}}{\sqrt{a^2+b^2}}$ $\quad a^2 + b^2 = 1$ $\qquad \dfrac{1}{\phi'^2} = \dfrac{1}{\phi^2}$

$\phi'^2(1+\phi^2) = \phi^2(1+\phi'^2)$ $\qquad \phi'^2 = \phi^2 \qquad \phi' = \pm\phi$

$= \dfrac{1+\vec{b}}{\sqrt{1+\left(\frac{b}{a}\right)^2}}$ $\qquad \phi'^2(1-\phi^2) =$

$$\gamma_\mu \partial_\mu \psi = m\, \Phi\, i\gamma_5 \psi^G = m\, \Phi\, i\gamma_5\, i\tau_2\, \psi^*$$

$$\gamma_\mu \partial_\mu \psi^* = m\, \Phi^*\, i\gamma_5\, i\tau_2\, \psi$$

$$-\gamma_\mu \partial_\mu (i\gamma_5 \psi^*) = m\, i\gamma_5\, \Phi^*\, i\gamma_5\, i\tau_2\, \psi$$

$$-\gamma_\mu \partial_\mu (i\tau_2 i\gamma_5 \psi^*) = m\, i\tau_2\, i\gamma_5\, \Phi^*\, i\tau_2\, i\gamma_5\, \psi$$

Let Φ commute with γ_5

$\gamma_\mu \partial_\mu$

$\gamma_\mu \partial_\mu \psi = im\, \psi'$

$\gamma_\mu \partial_\mu \psi^* = -im\, \psi'^*$

$$+\gamma_\mu \partial_\mu\, i\gamma_5 \psi^* = m\, i\tau_2\, \Phi^*\, i\tau_2\, \psi$$

$$\gamma_\mu \partial_\mu \psi = m\, \Phi\, \psi^{G'}$$

$$\gamma_\mu \partial_\mu \psi^{G'} = m\, i\tau_2\, \Phi^*\, i\tau_2\, \psi$$

$\gamma_\mu \partial_\mu \psi = \cancel{m}\, i\psi^G$

$\gamma_\mu \partial_\mu \psi^* = -im(i\tau_2)\psi$

$\gamma_\mu \partial_\mu i\tau_2 \psi^* = -im(i\tau_2)^2\psi$

$\gamma_\mu \partial_\mu \psi^G = im\, \psi$

$$(\gamma_\mu \partial_\mu)^2 \psi = m\, \Phi\, \gamma_\mu \partial_\mu \psi^{G'} = m^2\, \Phi\, (i\tau_2\, \Phi^*\, i\tau_2)\, \psi$$

$$= m^2\, \Phi\, (i\tau_2\, \Phi^{\dagger T}\, i\tau_2)\, \psi$$

$$\text{Let}\quad \Phi = e^{i\gamma_5 \vec\tau\cdot\vec\phi} \qquad i\gamma_5 \text{ is real}$$

$$\Phi^* = e^{i\gamma_5 \tau_3 \phi_3 + i\gamma_5 \tau_1 \phi_1 - i\gamma_5 \tau_2 \phi_2}$$

$$i\tau_2\, \Phi^*\, i\tau_2 = (i\tau_2)^2\, e^{-i\gamma_5 \vec\tau\cdot\vec\phi} = -e^{-i\gamma_5 \vec\tau\cdot\vec\phi}$$

$$\Phi = i\gamma_5\, e^{i\gamma_5 \vec\tau\cdot\vec\phi}$$

ψ

$\gamma_\mu \partial_\mu \psi = m\psi^G = m\, i\tau_2 \psi^*$

$\gamma_\mu \partial_\mu \psi^* = m\, i\tau_2\, \psi$

$\gamma_\mu \partial_\mu i\tau_2 \psi^* = m(i\tau_2)^2 \psi$

$= -m\,\psi$

$\gamma_\mu \partial_\mu \psi^G =$

$$\gamma_\mu \partial_\mu \psi = m\, \Phi\, \psi' \qquad \psi' = i\gamma_5\, i\tau_2\, \psi^*$$

$\psi^G = i\tau_2 \psi^*$

$(\psi^G)^G = i\tau_2\, (i\tau_2 \psi^*)^*$

$= (i\tau_2)^2\, \psi$

$$\psi' = i\gamma_5\, i\tau_2\, \psi^G = i\gamma_5\, i\tau_2\, \psi^*$$

$(i\gamma_5\, i\tau_2)^2 = +1.$

$$(\gamma_\mu \partial_\mu)^2 \psi = m\, \gamma_\mu \partial_\mu (\Phi \psi')$$

want

$(\gamma_\mu \partial_\mu)^2 \psi = m^2 \psi$

$\gamma_\mu \partial_\mu \psi = m\, \psi'$

$\gamma_\mu \partial_\mu \psi' = +m\, \psi$

$\gamma_\mu \psi^G = m\, \psi'^G = -m\, \psi.$

$\psi \quad \gamma_\mu \partial_\mu \psi = m\, \psi'$

$\gamma_\mu \partial_\mu \psi' = -m\, \psi$

$\gamma_\mu \partial_\mu \psi = m\, i\gamma_5\, i\tau_2\, \psi^*$

$\gamma_\mu \partial_\mu \psi^* = m\, i\gamma_5\, i\tau_2\, \psi$

$\gamma_\mu \partial_\mu i\tau_2 \psi^* = -m\, i\gamma_5\, \psi$

$\gamma_\mu \partial_\mu (i\gamma_5\, i\tau_2 \psi^*) = m(i\tau_2)^2 \psi$

$\gamma_\mu \partial_\mu \chi = m\, \xi$ we can have χ and ξ real

$\gamma_\mu \partial_\mu \xi = m\, \chi$ $\gamma_\mu \partial_\mu \psi = m \tau_1 \psi$

$\gamma_\mu \partial_\mu (\chi + \xi) = m (\chi + \xi)$

$\gamma_\mu \partial_\mu (\chi - \xi) = -m (\chi - \xi)$

$\gamma_\mu \partial_\mu\, i\gamma_5 (\chi - \xi) = m\, [i\gamma_5 (\chi - \xi)$.

Let $\psi = \begin{pmatrix} \xi \\ \varphi^* \end{pmatrix}$ $i\gamma_5\, i\tau_2\, \psi = \begin{pmatrix} i\gamma_5 \varphi \\ -i\gamma_5 \xi \end{pmatrix}$

If $\psi = i\gamma_5\, i\tau_2\, \psi$ $\begin{cases} \xi = i\gamma_5 \varphi \\ \varphi = -i\gamma_5 \xi . \end{cases}$

then $\psi = \begin{pmatrix} \xi \\ -i\gamma_5 \xi \end{pmatrix}$ $\tau_1 \psi = \begin{pmatrix} -i\gamma_5 \xi \\ \xi \end{pmatrix}$

then $\gamma_\mu \partial_\mu \xi = -m i\gamma_5 \xi$

$(\gamma_\mu \partial_\mu)^2 \xi = -im \gamma_\mu \partial_\mu \gamma_5 \xi = +im \gamma_5 \gamma_\mu \partial_\mu \xi = im\gamma_5 (-im \gamma_5) \xi$

$= m^2 \gamma_5^2 \xi = m^2 \xi$.

$\gamma_\mu \partial_\mu \xi = m e^{i\gamma_5 \alpha} \xi$.

$(\gamma_\mu \partial_\mu)^2 \xi = m\, \gamma_\mu \partial_\mu (e^{i\gamma_5\alpha} \xi) = m e^{-i\gamma_5 \alpha} \gamma_\mu \partial_\mu \xi = m^2 e^{-i\gamma_5\alpha} e^{i\gamma_5 \alpha} \xi$

General Majorana equation.

$$\boxed{\gamma_\mu \partial_\mu \psi = m e^{i\gamma_5 \alpha} \psi}$$ leads to $(\gamma_\mu \partial_\mu)^2 \psi = m^2 \psi$.

$\psi = \psi'$

If ψ is complex it has Touschek group.

$\chi = \chi'$
$\xi = \xi'$

$\gamma_\mu \partial_\mu \chi = m\, e^{i\gamma_5 \alpha} \xi$ $(\gamma_\mu \partial_\mu)^2 \chi = m e^{-i\gamma_5 \alpha} \gamma_\mu \partial_\mu \xi = m^2 \chi$

$\gamma_\mu \partial_\mu \xi = m\, e^{i\gamma_5 \alpha} \chi$

$(\gamma_\mu \partial_\mu)^2 \chi = m\chi$.

$\gamma_\mu \partial_\mu \chi = m\, e^{i\gamma_5\alpha} \zeta$

$\gamma_\mu \partial_\mu \zeta = m\, e^{i\gamma_5\alpha} \chi$.

or $\gamma_\mu \partial_\mu \begin{pmatrix} \chi \\ \zeta \end{pmatrix} = m\, e^{i\gamma_5\alpha} \tau_1 \begin{pmatrix} \chi \\ \zeta \end{pmatrix}$

$\gamma_\mu \partial_\mu (1+\tau_1)\psi = m\, e^{i\gamma_5\alpha} (1+\tau_1)\psi$.

$\gamma_\mu \partial_\mu (1+\tau_3)\psi = m\, e^{i\gamma_5 \tau_1} (1-\tau_3)\psi$.

$\phi = \begin{pmatrix} a & -b^* \\ b & a^* \end{pmatrix}$

$\bar\phi = \begin{pmatrix} a^* & b^* \\ -b & a \end{pmatrix}$

Let χ and ζ be complex.

$|a|^2 + |b|^2 = 1$

$$\begin{cases} \gamma_\mu \partial_\mu \chi = m\, e^{i\gamma_5\alpha} (a\psi + b\gamma_5 \psi^*) \\ \gamma_\mu \partial_\mu \psi = m\, e^{i\gamma_5\alpha} (a^*\chi + b\gamma_5 \chi^*) \end{cases}$$

$(\gamma_\mu \partial_\mu)^2 \chi = m\, e^{-i\gamma_5\alpha} (a\gamma_\mu\partial_\mu\psi \pm b\gamma_5\gamma_\mu\partial_\mu\psi^*)$

$= m^2 \chi$

Proof:

$a^*\chi - b\gamma_5\chi^*$

$$\begin{cases} a\gamma_\mu \partial_\mu \psi - b\gamma_5 \gamma_\mu\partial_\mu \psi^* = m\, e^{i\gamma_5\alpha} \chi \\ a^*\gamma_\mu \partial_\mu \psi^* + b^*\gamma_5 \gamma_\mu \partial_\mu \psi = m\, e^{i\gamma_5\alpha} \chi^* \end{cases}$$

$|a|^2 \gamma_\mu \partial_\mu \psi - a^* b \gamma_5 \gamma_\mu \partial_\mu \psi^* = m\, e^{i\gamma_5\alpha} a^*\chi$

$+ b a^* \gamma_5 \gamma_\mu \partial_\mu \psi^* + |b|^2 \gamma_\mu \partial_\mu \psi = + m\, e^{i\gamma_5} b\gamma_5 \chi^*$

$\gamma_\mu \partial_\mu \psi = m\, e^{i\gamma_5\alpha} (a^*\chi + b\gamma_5 \chi^*)$

$\gamma_\mu \partial_\mu \psi^* = m\, e^{i\gamma_5\alpha} (a\chi^* - b^*\gamma_5 \chi^{**})$

$(\gamma_\mu \partial_\mu)^2 \chi = m\, e^{-i\gamma_5\alpha} \left[a\gamma_\mu \partial_\mu \psi - b\gamma_5 \gamma_\mu \partial_\mu \psi^* \right]$

$= m\, e^{-i\gamma_5\alpha} \left[a m\, e^{i\gamma_5\alpha} (|a|^2\chi + a^* b \gamma_5 \chi^*) + m\, e^{i\gamma_5\alpha} \gamma_5 (-ba\chi^* + |b|^2 \gamma_5 \chi) \right.$

$= m^2 \left[(|a|^2 + |b|^2)\chi + (a^* b\gamma_5 - ba\gamma_5)\chi^* \right] = m^2\chi$

$$\begin{cases} \gamma_\mu \partial_\mu \chi = m\, e^{i\gamma_5\alpha} (a\psi + b\gamma_5 \psi^*) \\ \gamma_\mu \partial_\mu \psi = m\, e^{i\gamma_5\alpha} (a^*\chi + b\gamma_5 \chi^*) \end{cases}$$

$\gamma_\mu \partial_\mu \psi^* = m\, e^{i\gamma_5\alpha} (a\chi^* - b^*\gamma_5 \chi)$

$$\gamma_\mu \partial_\mu \chi = m(\lambda_1 \chi + \lambda_2$$

$$\gamma_\mu \partial_\mu \chi = m\left((\lambda + i\gamma_5\mu)\chi + (a + i\gamma_5 b)\xi\right)$$

$$\cancel{\gamma_\mu \partial_\mu \xi = m(}$$

$$(\gamma_\mu \partial_\mu)^2 \chi = m\left[(\lambda - i\gamma_5\mu)\gamma_\mu\partial_\mu \chi + (a - i\gamma_5 b)\gamma_\mu \partial_\mu \xi\right] = m^2 \chi$$

$$\rightarrow (\lambda - i\gamma_5\mu)\gamma_\mu\partial_\mu\chi + (a - i\gamma_5 b)\gamma_\mu\partial_\mu \xi = m\chi$$

Let $\gamma_\mu \partial_\mu \xi = m\left[(\lambda' + i\gamma_5\mu')\xi + (a' + i\gamma_5 b')\chi\right]$

$$\cancel{\gamma_\mu \partial_\mu \xi}$$

$$(\lambda - i\gamma_5\mu)\gamma_\mu\partial_\mu\chi + m(a - i\gamma_5 b)(\lambda' + i\gamma_5\mu')\xi + \cancel{m(a' + i\gamma_5 b')}$$

$$m(a - i\gamma_5 b)(a' + i\gamma_5 b')\chi = m\chi.$$

$$m(\lambda - i\gamma_5\mu)(\lambda + i\gamma_5\mu)\chi + m(\lambda - i\gamma_5\mu)(a + i\gamma_5 b)\xi.$$

coef. of χ

$$(\lambda - i\gamma_5\mu)(\lambda + i\gamma_5\mu) + (a - i\gamma_5 b)(a' + i\gamma_5 b') = 1$$

$$(\lambda - i\gamma_5\mu)(a + i\gamma_5 b) + (a - i\gamma_5 b)(\lambda' + i\gamma_5\mu') = 0$$

$$\lambda + i\gamma_5\mu - \quad \frac{\lambda - i\gamma_5\mu}{a - i\gamma_5 b} + \frac{a' + i\gamma_5 b'}{\lambda - i\gamma_5\mu} = 1 \qquad \frac{\lambda - i\gamma_5\mu}{a - i\gamma_5 b} + \frac{\lambda' + i\gamma_5\mu'}{a + i\gamma_5 b} = 0$$

$$\lambda^2 + \mu^2 + (a + i\gamma_5 b)(a' - i\gamma_5 b') = 1$$

$$\lambda \qquad \frac{\lambda' + i\gamma_5\mu'}{a + i\gamma_5 b} + \frac{a' + i\gamma_5 b'}{\lambda + i\gamma_5\mu} = 1.$$

$$D\psi^{x}_{\bar{a}} = m\,\bar{\psi}^{\bar{a}b} R_{ab} \bar{R}\bar{a}$$

$$D\psi^x = m\psi\,\vec{\Xi}$$

$$D\psi^x = m\psi(a e_5 + (b_i \ldots))$$

2 Majorana spinors → 1 Dirac spinor $\quad \gamma_\mu\partial_\mu\psi = m$

$$\partial_5 + i\vec{s}\cdot\vec{\sigma}$$

$$\gamma_\mu \partial_\mu \qquad \partial_0 \psi \vec{a} + \vec{e}\cdot\vec{\nabla}\psi = m\psi$$

$$\partial_0(\partial_0\psi\vec{a} + \vec{e}\cdot\vec{\nabla}\psi)\vec{a} - \vec{e}\cdot\vec{\nabla}(\partial_0\psi\vec{a} + \vec{e}\cdot\vec{\nabla}\psi) = m\,\partial_0\psi\vec{a} - \vec{e}\cdot\vec{\nabla}\psi$$

$$\gamma_\mu \partial_\mu \psi = [(\lambda + i\gamma_5\mu) + (\lambda' + i\gamma_5\mu')\tau_3]\psi + [(a + i\gamma_5 b) + (a' + i\gamma_5 b')\tau_3]\tau_1\psi$$

$$(\gamma_\mu \partial_\mu)^2\psi = [(\lambda - i\gamma_5\mu)$$

$a = \lambda + i\gamma_5\mu$, $a^* = \lambda - i\gamma_5\mu$

$$\gamma_\mu \partial_\mu \psi = (a + b\tau_3)\psi + (c + d\tau_3)\tau_1\psi$$

$$(\gamma_\mu \partial_\mu)^2\psi = (a^* + b^*\tau_3)\gamma_\mu\partial_\mu\psi + (c^* + d^*\tau_3)\tau_1\gamma_\mu\partial_\mu\psi$$

$$= \underbrace{[(a^* + b^*\tau_3) + (c^* + d^*\tau_3)\tau_1][(a + b\tau_3) + (c + d\tau_3)\tau_1]}_{1}\psi$$

$$(a^* + b^*\tau_3)(a + b\tau_3) + (c^* + d^*\tau_3)(c - d\tau_3)$$

$$+ (a^* + b^*\tau_3)(c + d\tau_3)\tau_1 + (c^* + d^*\tau_3)(a - b\tau_3)\tau_1 = 1$$

$$|a|^2 + |b|^2 + (a^* b + a b^*)\tau_3$$

$$+ |c|^2 - |d|^2 + (d^* c - c^* d)\tau_3$$

$$+ [a^* c + b^* d + (a^* d + c b^*)\tau_3]\tau_1$$

$$+ [c^* a - d^* b + (d^* a - c^* b)\tau_3]\tau_1$$

$a^* c + b^* d = 0$

$a^* c - b^* d =$

$$0 = [(a^* c + c^* a + b^* d - d^* b) + (a^* d + d^* a + c b^* - c^* b)\tau_3]\tau_1$$

$$\begin{cases} a^* c + c^* a = 0 \\ b^* d - d^* b = 0 \\ a^* d + d^* a = 0 \\ c b^* - c^* b = 0 \\ |a|^2 + |b|^2 + |c|^2 - |d|^2 = 1 \\ a^* b + a b^* = 0 \\ d^* c - c^* d = 0 \end{cases}$$

$\frac{a^*}{a} = -\frac{c^*}{c}$

$a = a_1 e^{i\alpha}$

$\frac{a^*}{a} = e^{-2i\alpha}$

$\frac{c^*}{c} = e^{-2i\alpha + i\pi}$

$c = c_1 e^{-i\alpha + i\frac{\pi}{2}}$

$a\ b\ c\ d$: 4 fns. of real parameters. 7 equations

$i\beta = -i\alpha + i\frac{\pi}{2}$

$\beta = -\alpha + \frac{\pi}{2}$

$\alpha = \frac{\pi}{2} - \beta$

$a = a_1 e^{i(\frac{\pi}{2} - \beta)}$

$b = b_1 e^{i\beta}$

$c = c_1 e^{i\beta}$

$d = d_1 e^{i\beta}$

$a_1^2 + b_1^2 + c_1^2 - d_1^2 = 1$

$\beta = \frac{\pi}{2}$, and

$\begin{cases} a = a_1 \\ b = b_1 i\gamma_5 \\ c = c_1 i\gamma_5 \\ d = d_1 i\gamma_5 \end{cases}$

$a = a_1 e^{i\alpha}$

$c = c_1 e^{-i\alpha + i\frac{\pi}{2}}$

$d = d_1 e^{-i\alpha + i\frac{\pi}{2}}$

$b = b_1 e^{i\beta}$

$d = d_1 e^{i\beta}$

$c = c_1 e^{i\beta}$

$b = b_1 e^{-i\alpha + i\frac{\pi}{2}}$

$c_1 e^{i\beta}$