THE PROPORTIONAL REASONING ABILITY OF PRESERVICE MATHEMATICS TEACHERS: A MIXED METHOD STUDY

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DECLARATION OF ORIGINALITY

I, Hayriye Sinem Boyacı, certify that

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ABSTRACT

The Proportional Reasoning Ability of Preservice Mathematics Teachers: A Mixed Method Study

The aim of the current study was to investigate preservice mathematics teachers' (PSMTs) proportional reasoning. The study was conducted in five public universities in Turkey, during the spring semester of 2018-2019 academic year. A total of 261 junior and senior university students from middle and secondary school mathematics teaching programs participated in the study. Proportional Reasoning Instrument (PRI) which is prepared by the researcher based on six components among characteristics of proportional reasoners outlined by Lamon (2005, 2007) and task-based interviews with selected participants were used as data sources. Quantitative analysis results revealed that although PSMTs had relatively high scores on PRI, they experienced difficulties in reasoning about the multiplicative relationships in both direct and inverse proportions, realizing and understanding the invariance and covariance structures of proportional relationships, evaluating students' alternative strategies and developing proper language for ratio and proportion. Results from the qualitative analysis showed that highest scorers on PRI provide conceptual explanations about the answers more than average and lowest scorers. It was also concluded that all PSMTs in the clinical interviews regardless of their performances on PRI, had difficulty in proper use of ratio and proportion language and overgeneralized use of cross-multiplication algorithm even for non-proportional and inverse proportion situations. While highest scorers overused the algorithm for inverse proportion situations, others used the algorithm frequently for both situations.

ÖZET

Matematik Öğretmen Adaylarının Orantısal Akıl Yürütme Becerisi:

Karma Yöntem Çalışması

Bu çalışmanın amacı matematik öğretmen adaylarının orantısal akıl yürütme becerisini incelemektir. Çalışma, Türkiye’de yer alan beş farklı üniversitede 2018-2019 akademik yılı ikinci yarısında 3. ve 4. sınıf düzeyinde ilköğretim ve ortaöğretim matematik öğretmenliği programlarında öğrenim görmekte olan 261 öğretmen adayının katılımı ile gerçekleştirilmiştir. Katılımcılara 13 sorudan oluşan ve araştırmacı tarafından geliştirilen “Orantısal Akıl Yürütme Ölçeği” uygulanmıştır. Ölçek uygulaması sonrasında belirlenen öğretmen adaylarıyla aynı ya da paralel sorular üzerinden klinik görüşmeler yapılmıştır. Nicel araştırma sonuçlarına göre öğretmen adaylarının yüksek puan elde etmesine rağmen doğru ve ters orantıdaki çarpımsal ilişkiyi anlamlandırma, orantısal ilişkilerde değişen ve değişmeyen yapıları fark etme ve kavrama, öğrencilerin alternative çözümlerini değerlendirme ve oran ve orantı kavramlarına ilişkin dili doğru kullanma/geliştirme gibi noktalarda zorluk yaşadıkları belirlenmiştir. Nitel araştırma sonuçları ise ölçekte alınan puanlara göre en yüksek puanları alan öğretmen adaylarının, düşük ve ortalama puan alan öğretmen adaylarına göre çözümlerine yönelik daha fazla kavramsal açıklama yapabildiklerini; fakat kavramlara ilişkin hatalı dil kullanımının ve çapraz çarpım algoritmasının tüm durumlarda kullanılma eğiliminin tüm öğretmen adayları için problem teşkil ettiğini göstermektedir. Çapraz çarpım algoritması ölçekten yüksek puan alan öğretmen adayları tarafından sadece ters orantı içeren durumlarda yanlış kullanılırken, ölçekten ortalama ve en düşük puanları alan öğretmen adayları her iki durumda da (ters orantılı ve orantısal olmayan durumlar) bu algoritmayı kullanma eğilimi göstermiştir.

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Dedicated to my lovely parents, Adnan and Dilek Boyacı

and to my awesome brother, Semih Boyacı

for all their love and support

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ABBREVIATIONS

CCK	Common Content Knowledge
HCK	Horizon Content Knowledge
KCC	Knowledge of Content and Curriculum
KCS	Knowledge of Content and Students
KCT	Knowledge of Content and Students
MCK	Mathematical Content Knowledge
MKT	Mathematical Knowledge for Teaching
PCK	Pedagogical Content Knowledge
PRI	Proportional Reasoning Instrument
PSMT	Preservice Mathematics Teachers
SCK	Specialized Content Knowledge
SMK	Subject Matter Knowledge

CHAPTER 1

INTRODUCTION

People often encounter in their daily lives situations that entail thinking proportionally in contexts ranging from economics to geography, from architecture to daily life routines such as adjusting recipes, comparing prices of products in different sizes. Due to its diverse applications in mathematics, science and everyday life, proportional reasoning is at the forefront of topics that are emphasized and it is one of the most researched topics in the literature (Cramer, Post & Currier, 1993; Harel, Behr, Post & Lesh, 1991; Karplus, Pulos & Stage, 1983; Lamon, 2005; Lesh, Post & Behr, 1988; Orrill & Cohen, 2016; Pelen & Dinç Artut, 2015; Tatto, Peck, Schwillie, Bankov, Senk, Rodriguez, … & Rowley, 2012; Toluk Uçar & Bozkus, 2016; Vergnaud, 1983; Yenilmez & Kavuncu, 2017). Its role as a basis for advanced mathematical topics within algebra (e.g., slope), geometry (e.g., congruence and similarity), trigonometry, probability, and measurement has also increased the importance of proportional reasoning in the mathematics curriculum. Such a central role of proportional reasoning causes it to be described as a "cornerstone" for these topics (Lamon, 2005).

The defining characteristic of proportional reasoning which is as “critical to mathematical and scientific thinking” (Lamon, 2007, p.637), is mainly stated as the ability to discern and interpret the multiplicative relationship between quantities (Cramer et al., 1993). In other words, proportional reasoning ability requires considering relative changes, rather than absolute changes, during comparison of quantities. Van de Walle, Karp, and Bay-Williams (2013) emphasize the difficulty of expressing proportional reasoning within a single definition and state that proportional reasoning includes both quantitative and qualitative processes. Likewise, Post, Behr, and Lesh, (1988) state that the essence of relative comparison which is the base for proportional reasoning does not depend on the specific numerical values and emphasize the necessity qualitative thinking which is an essential initial step to a better understanding of ratio and proportion concepts. For example, the question "If Nicki ran fewer laps in more time that she did yesterday, would her running speed be faster, slower, the same, or can't tell?" can provide a great initial opportunity for students to make sense of relative comparison before proceeding the algorithmic calculations (Post et al., 1988, p. 80).

Moreover, it has been emphasized that the ability to distinguish between proportional and non-proportional situations plays a vital role in the development of proportional reasoning (Cramer et al., 1993; Lim, 2009). Langrall and Swafford (2000) state that the distinction between these two situations is closely related to recognizing the difference in absolute (additive) and relative (multiplicative) change. National Council of Teachers of Mathematics (NTCM), an influential stakeholder aiming to improve quality of mathematics education in the US, also points out the importance of different representations such as tables, graphs, and algebraic expressions while denoting the multiplicative relationship between quantities (National Council of Teachers of Mathematics, 2000). The same emphasis is also expressed in the mathematics curriculum by Rebuplic of Turkey Ministry of National Education (MEB) (MEB-TTKB, 2008).

Studies conducted to measure students' proportional reasoning ability have shown that many students experience difficulties on problems requiring proportional reasoning. In a study with students from fourth grade to eighth grades, Van Dooren, Bock, Hessels, Janssens and Verschaffel (2005) found that students whose mathematics curriculum involves ratio and proportion in their grade level were prone to using traditional proportion algorithm improperly when solving an additive problem. Additionally, in studies with primary and middle school students (Çelik & Özdemir, 2014; Toluk Uçar & Bozkuş, 2016), the results indicated that students incorrectly use additive and multiplicative strategies in problems due to lack of ability to distinguish proportional from non-proportional situations. These misused strategies stem from a lack of understanding of multiplicative and additive relationships embedded in situations.

Another difficulty that students experience while reasoning proportionally is to comprehend what direct and inverse proportion mean and to solve problems that include especially inverse proportion. Yenilmez and Kavuncu (2017), in their study aimed at examining difficulties of students in ratio and proportion, concluded that students were able to answer a maximum of nearly half of the questions correctly. Although it had been observed that the success rate of students in direct proportion problems is higher than inverse proportional problems, their findings showed that students had trouble in deciding whether two quantities are directly or inversely proportional due to lack of conceptual understanding regarding these concepts. Similar results had been also obtained in other studies: direct and inverse proportional problems were difficult to distinguish for students, and direct proportion problems were solved with a high success rate when compared with inverse proportion problems (Dinç Artut & Pelen, 2015; Irfan, Nusantara, Subanji & Sisworo, 2018).

The limited number of studies on teachers' proportional reasoning ability has shown that not only students but also preservice and in-service teachers have difficulties in problems requiring proportional reasoning (Post, Cramer, Behr, Lesh

& Harel, 1993; Orrill & Cohen, 2016; Riley, 2010; Tatto et al., 2012). In a study by Orrill and Cohen (2016), it was seen that several secondary school teachers experience difficulty when distinguishing non-proportional situations from proportional ones. According to the TEDS-M study carried out with the participation of elementary and secondary school preservice mathematics teachers from many countries, it has been pointed out that proportional reasoning is an issue for preservice teachers (Tatto et al., 2012). In the study, it was concluded that the preservice teachers do not have conceptual knowledge about the subject even though they reached the right results with the traditional proportion algorithm. These findings are parallel to the notion of Post et al. (1993) that preservice teachers have operational knowledge rather than conceptual knowledge on proportional reasoning.

How teacher knowledge is related to the mathematical achievement of students has been a subject drawing attention in the educational field. Ma (1999) observed in her study, teachers who do not have enough conceptual knowledge remain incapable of providing sufficient support in understanding the mathematical concepts and comprehending the mathematical subjects for students. Considering that students struggle with proportional reasoning that is the basis for many subjects, it is essential to evaluate the knowledge of preservice and in-service teachers about the topic. Studies carried out on teacher's proportional reasoning has often been confined to qualitative studies, and lack of quantitative measures of proportional reasoning ability has become one of the limiting factors for conducting large scale studies.

While there is no instrument about proportional reasoning ability for preservice and in-service teachers in Turkey, there are several studies across the world (Ekawati, Lin & Yang, 2015a; Hill, Schilling & Ball, 2004). A scale was developed by Hill and colleagues (2004) as a part of a project called Learning Mathematics for Teaching aimed at measuring teachers' knowledge of several mathematical topics. Although the instrument developed in the scope of the project has been accepted internationally, access is limited by annual fees, which can be a restrictive factor in its use. On the other hand, the instrument developed by Ekawati and her colleagues (2015a) has not been adopted widely for use. From this point of view, the present study aims to investigate the proportional reasoning ability of preservice mathematics teachers.

1.1 Purpose of the study

This study aims to examine the proportional reasoning ability of preservice mathematics teachers with a particular focus on the measurement process to fill this gap in Turkey. In addition to providing an account of PSMTs' proportional reasoning ability, this study can also be characterized as a first step that can guide further studies to develop a proportional reasoning instrument.

1.2 Significance of the study

Despite the importance of the proportional reasoning in the middle school mathematics curriculum and its teaching, studies about proportional reasoning has been generally restricted to studies conducted with students (Aladağ & Dinç Artut, 2014; Bright, Joyner & Wallis, 2003; Pelen & Dinç Artut, 2015; Toluk Uçar & Bozkuş, 2016; Van Dooren, De Bock, Hessels, Janssens & Verschaffel, 2005; Yenilmez & Kavuncu, 2017). Considering that teachers should have a coherent and robust understanding of subjects that they teach and potential effects of a teacher's knowledge and ability on students' success, it is crucial to examine teachers' understanding (Hill, Rowan & Ball, 2005). In the case of proportional reasoning, Parker (1999) emphasized the importance of teachers having flexible ways of thinking about proportional relationships and providing a variety of representations to these relationships during the student's transition in developing proportional reasoning. Although preservice mathematics teachers' proportional reasoning ability have been researched with both quantitative and qualitative studies in various countries (Lobato, Orrill, Druken & Jacobson 2011, Ekawati, Lin & Yan, 2015b, Tattoo et al. 2015), there is a limited number of studies available on teacher's proportional reasoning ability in Turkey (Akkuş Çıkla & Duatepe 2002).

There is also a need for making joint use of quantitative and qualitative methods to thoroughly investigate this construct. Therefore, this study, in its scope aims to open a new window for the body of research focusing on the issue of teachers' proportional reasoning in Turkey. The study is expected to contribute to determining the current situation of preservice mathematics teachers' proportional reasoning ability and have implications for making regulations in teacher training programs through the accumulation of findings from future studies.

CHAPTER 2

LITERATURE REVIEW

In this chapter, literature related to proportional reasoning and teacher's knowledge will be reviewed. It aims to provide a substantial basis for the instrument by focusing on essential elements of proportional reasoning and mathematics teachers' knowledge. Studies previously conducted on this topic, and instruments that were constructed to measure students' and teachers' proportional reasoning ability will be discussed. It consists of six main sections respectively: i) ratio and Proportion, ii) Defining Elements of Proportional Reasoning, iii) Problem Types for Measuring Proportional Reasoning, iv) Strategies used for Proportional Reasoning Problems, v) Mathematics Teacher' Knowledge and vi) Studies about Teachers' Proportional Reasoning.

2.1 Ratio and proportion

Ratio and proportion have an essential role in proportional reasoning. Therefore, it is critical to describe and clarify these terms for understanding proportional reasoning.

According to Ben-Chaim, Keret and Ilany (2012), ratio is described as "the quantification of a multiplicative relationship that is calculated by dividing (or multiplying) one quantity by another" (p. 25). Ratio that is defined as "the multiplicative relationship between two quantities" (Smith, 2002, p. 4) is represented as a:b or $\frac{a}{b}$, when b $\neq$ 0.

Lamon (2007) emphasizes the difference between ratio and rate and describes ratio as "a comparison between like quantities (e.g., pounds : pounds)" and rate as "comparison of unlike quantities (e.g., distance : time)" (p. 634). In other words, ratio is the comparison of the same units while the other is comparison of the different units. The result of the comparison of different units "produces a unique, new concept with its own entity" (Ben-Chaim et al., 2012). For example, the ratio of the velocity to time produces the physical concept of accelaration.

On the other hand, Van de Walle et al. (2013) identified rate as a type of ratio in which part-whole ratios and part-part ratios are the other types of the ratio. Part-whole ratios simply refer to the comparison of a part with the whole. The ratio of girls to whole class can be given as an example of part-whole ratios. Fractions includes the comparison of the same type of objects such "one of the seven part" (Hoffer, 1988) Therefore, fractions, percentage and probability are considered as part-whole ratios because all represents the relationship between a part and the whole (Van de Walle et al., 2013). Part-part ratios specify the relationship between one part to another part of the whole. The ratio of the length to width is an example of part-part ratios.

Van de Walle et al. (2013) define proportion, other key concept in proportional reasoning, as the expression of the equivalence of two ratios. Ben-Claim et al. (2012) claim that "proportion is a direct and indirect linear relationship between two variable quantities" and explain the definition by stating "corresponding elements of two sets are in proportion when there is a constant ratio (either direct or indirect) between them" (p. 34). Corresponding elements of two sets can be in direct proportion or inverse proportion. While direct proportion is that "where the multiplicative relationship is expressed as a constant quotient between the two values", inverse proportion states "the multiplicative relationship is expressed by the constant product between the two values" (Ben-Chaim et al., 2012, p.35). Invariance structures of inverse and direct proportions are preserved to keep the relationship between them constant when both of the quantities covary.

2.2 Defining elements of proportional reasoning

Proportional reasoning is a type of mathematical reasoning that requires an understanding of multiplicative relationships between quantities (Lesh et al., 1988). It has been identified as a cognitively challenging mathematical topic that is the most difficult to teach and develop as well as the most essential to develop to enable robust understanding of higher mathematics (Lamon, 2005). Although proportional reasoning is mostly associated with topics such as ratio, fraction, percentage, and proportion, it virtually penetrates all branches of mathematics. It is accepted as a "watershed" concept because of being a "cornerstone" for higher-level mathematics and a "capstone" for primary school mathematics (Lesh et al., 1988). Besides its critical role in mathematics, other fields depending on the use of mathematics, such as science, economy, and geography, also require competence in proportional reasoning.

Its role as foundational knowledge for advanced topics in mathematics and widespread application in other fields has led proportional reasoning to gain importance both in the curriculum and in academic studies. NCTM (2000) promotes its importance by stating that proportional reasoning "merits whatever time and effort must be expanded to assure its careful development" (p. 82). In addition to NCTM, Common Core State Standards Initiatives (CCSSI) which provides vision to mathematics education, also classified ratio and proportional relationships as a common core state standard for mathematics that needs to be emphasized (The Common Core Standards Writing Team, 2011).

Throughout the literature of mathematics education, the definition of proportional reasoning has been a controversial issue. Researchers have stated that proportional reasoning is difficult to describe completely within several sentences and clarified mainly the essential characteristics of proportional reasoning in detail rather than defining it (Cramer et al., 1993; Karplus et al., 1983; Lamon, 2005; Lesh et al., 1988; Van de Walle, Karp & Bay Williams, 2013). As highlighted in the literature, properties and components of proportional reasoning has been explained to provide a better understanding of what proportional reasoning means in this study.

Ability to recognize and understand mathematical relationships which are multiplicatively embedded in proportional situations is one of the essential elements for proportional reasoning (Cramer et al., 1993). Multiplicative relationships between quantities are expressed mathematically as $f(x) = xa,\ a \neq 0$ and graphically on the coordinate plane as a straight line going through the origin, whereas additive relationship is expressed as $f(x) = x \pm b,\ b \neq 0$ (Cramer et al., 1993; Van Dooren, De Bock & Verschaffel, 2010). In brief, the focus is the invariant difference in additive thinking, whereas the invariant ratio in multiplicative thinking (Van Dooren et al., 2010). Multiplicative and additive thinking are also termed as relative and absolute thinking in the literature, respectively (Lamon, 2005).

Siemon, Breed, and Virgona (2005) identified multiplicative thinking as “a capacity to work flexibly with the concepts, strategies, and representations of multiplication (and division) as they occur in a wide range of contexts” (p. 2). It means that a comparison between two quantities is defined in multiplicative terms rather than additive. Following example can be used for elaboration on the distinction between multiplicative and additive comparisons: When numerical comparisons are to be made between 24 girls and eight boys in the school, there are several ways to compare the number of girls and boys:

- Number of girls is 16 more than the number of boys
- Number of boys is 16 less than the number of girls.
- Number of boys is one-third of the number of girls.
- Number of girls is three times the number of boys.

While the first two comparisons focus on the differences between the number of girls and boys and are examples of additive thinking, the remaining two statements are examples of multiplicative thinking in which the number of girls and boys can be expressed as multiples of each other (Cramer et al., 1993). In that point, what is critical in proportional reasoning is being able to make comparisons between quantities in both additive and multiplicative terms and adjusting properly according to context (Dole, Wright, Clarke & Hilton, 2007).

However, the transition from additive to multiplicative thinking, which has a critical role in the development of proportional reasoning is not always straightforward (Robichaux-Davis, 2017). Studies done with different age groups have showed that students have difficulty to think multiplicatively and tend to use additive strategies when solving ratio and proportion problems especially in the presence of non-integer ratios (Cramer et al., 1993; Karplus et al., 1983; Toluk Uçar & Bozkus, 2016; Tourniaire & Pulos, 1985; Van Dooren, De Bock, Vleugels & Verschaffel, 2010).

A study conducted by Duatepe, Çıkla and Kayhan (2005) with middle school students on the purpose of specifying their strategies in ratio and proportion problems and distribution of these strategies by class level showed that students' tendency to approach ratio and proportion problems additively decreased as the grade level increased. Like the findings of the study conducted by Duatepe and colleagues (2005), in a study conducted with primary and secondary students, Fernández, Llinares, Van Dooren, De Bock, and Verschaffel (2012) found that use of additive strategies in proportional problems decreased with increasing grade levels. These findings seem to confirm the idea that proportional reasoning is an ongoing process that evolves through experience (Cramer et al., 1993; Lamon, 2005; Lesh et al., 1988; Tournaire & Pulos, 1985).

Within the use of multiplicative thinking, one essential ability is to differentiate proportional situations from non-proportional ones which requires recognizing situations which involve absolute (additive) and relative (multiplicative) change (Cramer et al., 1993; Duatepe, Çıkla & Kayhan, 2015; Langrall and Swafford, 2000). One must differentiate proportional situations from non-proportional ones to be a competent proportional reasoner with a conceptual understanding of ratio and proportion (Lim, 2009).

Studies have indicated that many middle school students have difficulty in distinguishing proportional and non-proportional situations and use proportional strategies excessively even in inappropriate problems (Duatepe et al., 2015; Van Dooren et al., 2010). Van Dooren et al. (2005) conducted a study with students from different grades in middle school on the purpose of examining the effects of problem types and age on the topic of proportionality. At the end of the study, they concluded that sixth and seventh-grade students tend to misuse proportional strategies in solving constant-difference running problem: “Ellen and Kim are running around a track. They run equally fast but Ellen started later. When Ellen has run five rounds, Kim has run 15 rounds. When Ellen has run 30 rounds, how many has Kim run?” (Van Dooren et al., 2010, p. 65)

Aladağ and Dinç Artut (2014) constructed a study with sixth, seventh and eighth-grade students to examine how students responded to realistic problems which look like proportional problems even though there is neither a multiplicative nor an additive relationship between quantities. The results of the study revealed that most of the students solve realistic problems erroneously by setting up proportions with little or no relation to the real world due to their tendency to use proportional strategies in all superficially resembling problems requiring proportional reasoning. Hence, the crucial point of proportional reasoning is that understanding and interpreting of proportional and non-proportional relationships are rather than just setting up proportion mechanically (Lamon, 2005). Cramer et al. (1993) also highlight the situation with the following words: “we cannot define a proportional reasoner as simply one who knows how to set up and solve a proportion” (p. 159). It can be the case even in situations where anyone solves proportional reasoning problems correctly by using cross multiplication algorithm. Even though one reached a correct answer through traditional proportion algorithm, there is no guarantee that he/she can think proportionally.

Indeed, in a study conducted by Lobato, Ellis and Zbiek (2010) with a student, Bonita, displayed the issue raised by Cramer’s et al. (1993) while working on proportional tasks during an interview. Lobato and colleagues (2010), in their study, gave the following problem and asked her to solve it by using paper and pencils: “If leaky faucet dripped 6 ounces of water in 8 minutes, how much water does it drip in 4 minutes at the same rate?” (p. 32)

Bonita set up proportion and reached a correct answer by using cross-multiplication rule in this problem. However, she did not answer the problem even though interviewer changed the number from 40 to 16, which is easy to solve by doubling eight mentally, when a student was asked to find out how much water dripped in 40 minutes without using cross multiplication rule. At the end of the study, it was concluded that being able to set up and solve proportions is not sufficient for conceptual understanding of proportional reasoning. Therefore, researchers suggest that students should be encouraged to develop and use more than one solution strategy in order to assist the development of their proportional reasoning (Lamon, 2005).

Ben-Chaim, Keret and Ilany (2012) stated that being able to discern the relationship involving direct and inverse proportions and solve these problems is another essential element of proportional reasoning. If two quantities are proportional to each other, it means they change by a related amount (Large, 2006). Two quantities can be proportional in two ways: directly or inversely. Two quantities being directly proportional means values of quantities increase or decrease together at a constant rate. In other words, if "x is directly proportional to y" or "x varies directly as y" then the equation is expressed as $\mathrm{x} = \mathrm{k} \times \mathrm{y}, \mathrm{k} \neq 0$ in which k is a constant of proportionality (Lamon, 2005, p.6). What is important here is that the increase or decrease occurs in proportion to each other. Therefore, the increase or decrease at the same time is not sufficient for the condition of being directly proportional. On the other hand, two quantities are considered to be inversely proportional if "one quantity varies with other, but in the opposite direction" (Lamon, 2005, p.107). In other words, if x is inversely proportional to y, then the equation is expressed as $x = \frac{k}{y}$ in which k is a constant of proportionality. In this situation, the vital point is the invariance of the product of two quantities. So, in order to describe this relationship, it is not sufficient to state that one increases as the other decreases or similarly, one decreases as the other increases. For both proportionality type, expressing and understanding multiplicative relationship in algebraic form as given above, are considered as indicators of proportional reasoning and have an important role (Lamon, 2005).

Although the importance of realizing the relationship in which involving direct and inverse proportion and solving these problems has been emphasized, researchers have found that not only students but also preservice teachers have difficulty in solving these problems especially in inverse proportion problems (Riley, 2010; Tatto et al., 2012). Research results showed that preservice elementary mathematics teachers misused cross-multiplication algorithm on solving inverse proportion problems (Riley, 2010). Researchers assert that preservice teachers' inappropriate use of cross multiplication algorithm in inverse proportion problems has arisen from overemphasizing of this traditional algorithm in instruction without understanding the underlying concept of strategy. In addition to this claim, this situation may result from teachers' lack of thinking about invariance and covariance structure of proportional relationships.

Lamon (2005) defined comprehension about what remains unchanged in proportional relationships as a crucial part of proportional reasoning. Such a comprehension not only assists learners in solving problems involving a different kind of proportional reasoning also but adequately provides a conceptual basis for proportional reasoning strategies. Consistent with ideas of Lamon (2005), invariance structure of proportional relationships is also identified as a big idea underlying ratio, proportion and proportional reasoning in teaching mathematics (Lobato et al., 2010).

Despite that, the covariance of quantities is apparent in proportional relations, what has an essential role for the development of proportional reasoning is the invariance structure embedded inherently in a proportional relationship (Olson,

Olson & Slovin, 2015). However, what is invariant varies depending on whether the proportion is direct or inverse. What remains unchanged in the process is explained as "the ratio of the two quantities in the case when quantities are directly proportional" and "the product of the quantities in the case when the quantities are related in an inversely proportional way" (Lamon, 2005, p. 7).

Besides stating the main defining elements of proportional reasoning, Lamon (2005, 2007) identified several characteristics of proportional reasoners through these elements. Six of them among these characteristics which are emphasized in the literature will be used as base within the scope of study. According to Lamon (2005, 2007), a proportional reasoner is able to:

- Solve proportional problems in a wide range of context from slope to similarity or proportional problems involving numerical complexities e.g. non-integer ratios, fractions or decimals.
- Develop and use different strategies in solving problems requiring reasoning proportionally rather than using only traditional proportion algorithm.
- Distinguish proportional situations from non-proportional ones.
- Understand the multiplicative relationships both in direct and inverse proportions
- Realize and understand the invariance and covariance structure of the proportional relationships.
- Develop and use the language for ratio and proportions

As seen above, Lamon (2005, 2007) have gathered all key components which underpin proportional reasoning mentioned by other researchers under the same roof of a component proportional reasoner. Therefore, in this study, preservice mathematics teachers' proportional reasoning will be investigated in the light of these six components of proportional reasoning.

2.3 Type of problems used for assessing proportional reasoning ability

Different researchers in the literature have defined a variety of mathematics problems related to proportional reasoning. Cramer et al. (1993) categorize these problems used to measure proportional reasoning ability as missing value problems, comparison problems, qualitative prediction problems, and qualitative comparison problems. While a missing value problem is a kind of question that requires finding the unknown value (d) according to given three pieces of information (a, b and c), a comparison problem is a kind of question that requires comparing the magnitudes of two ratios ($\frac{\mathrm{a}}{\mathrm{b}}$ and $\frac{\mathrm{c}}{\mathrm{d}}$) according to given four pieces of the information (a, b, c and d). Mixture problems are examples of comparison problems that were introduced to the relevant literature by Noelting (1980a). On the other hand, kind of questions requiring an interpretation of two ratios without giving numerical values are described as qualitative prediction and qualitative comparison problems (Post, Behr & Lesh, 1988). Table 1 shows examples of each type of problem. Lamon (1993) takes a slightly different approach to categorize proportional reasoning problems by focusing on their mathematical and semantic characteristics. Lamon classified proportional reasoning problems into four categories in terms of their semantic type: Well-chunked measure, part-part-whole, associated sets, and stretchers and shrinkers. The first category of problems, well-chunked measure problem, is a kind of problem that includes a third well-known quantity (rate) resulting from the comparison of two extensive measures (Lamon, 1993). Speed ($\frac{distance}{time}$) and density ($\frac{\mathrm{mass}}{\mathrm{volume}}$) problems are examples of this kind of problems. Part-part-whole problems are described as problems in which “the extensive measure (cardinality) of a single subset of a whole is given in terms of the cardinalities of two sub-subsets of which it is composed” (Lamon, 1993, p.42). Percentages and probabilities can be given as examples of part-part-whole problems. Lamon (1993) identified that associate sets problems as a kind of problem that requires analyzing the relationship in which a connection between two elements is not commonly known due to the explicit expression on the problem. Problems involving sharing pizzas among a different number of people constitutes a typical example of this category. For the last category, stretchers and shrinkers problems refer to situations involving scaling up or down within a fixed ratio. Similarity problems are included in this kind of semantic problems.

Table 1. Examples of Missing Value and Comparison Problems

Type of Problems	Examples
Missing Value Problem	3 U.S. dollars can be exchanged for 2 British pounds. How many pounds for 21 U.S. dollars? (p.159)
Comparison Problem	Richard bought three pieces of gum for 12 cents. Susan bought five pieces of gum for 20 cents. Who bought the cheaper gum or were the prices equal? (p.222)
Qualitative Comparison Problem	Mary ran more laps than Greg. Mary ran for less time than Greg. Who was the faster runner? (a) Mary, (b) Greg, (c) same, (d) not enough information to tell. (p.166)
Qualitative Prediction Problem	If Devan ran fewer laps in more time than she did yesterday, would her running speed be (a) faster, (b) slower, (c) the same, (d) not enough information to tell. (p.166)

Note: From Cramer et al. (1993) and Karplus et al. (1983)

On the other hand, Lesh et al. (1988) who have influenced and shaped these two researchers specified seven types of problems related to proportional reasoning:

missing value problems, comparison problems, transformation problems, mean value problems, proportions involving conversions from ratios to rates or fractions, proportions involving unit labels as well as numbers and between-mode translation problems. While two of them, missing value and comparison problems, have the same definition as cited in the study of Cramer et al. (1993), rest of them provided a different perspective on the literature of proportional reasoning.

As cited in past research, there are different kinds of proportional problems that students may encounter. However, problems related to proportional reasoning have been substantially limited to missing value problems and comparison problems in the books, instructions and relevant research (Cramer et al., 1993; Lesh et al., 1988). Within the scope of this study as well, these two kinds of questions will be mainly focused on.

2.4 Strategies used for proportional reasoning problems

Existing research has shown that there are a variety of strategies defined when solving proportional reasoning problems. These are; unit rate strategy, factor of change, cross-multiplication algorithm, equivalent fractions, equivalent class, build up strategy and additive strategy (Bart, Post, Behr & Lesh, 1994; Cramer et al., 1993). These strategies can be explained through a sample proportional missing value problem:

If two bags of rice weigh 40 kilograms, how much will eight identical bags weigh?

Unit Rate**:** A student using unit rate strategy for this problem firstly calculates the weight for a bag of rice, which is a single unit in this problem. After realizing that one bag has 20 kilograms of rice, student multiplies 20 by 8 in order to find the weight of eight bags, 160 kilograms. Weight per bag; the unit rate is the constant factor that relates to weight and bags (Cramer et al., 1993).

Factor of Change*:* This strategy is also defined as “how many times greater” approach by researchers (Cramer et al., 1993). Therefore, a student using a factor of change strategy tries to find the multiplicative relationship between quantities within measure field, in this situation between several bags and weights and applies this relationship to another measure field (Post et al., 1988). In this problem, a student using this strategy reaches an answer by reasoning as follows: “2 is 1/4 of 8, so 40 must be 1/4 of the required total weight. Then the total weight is 4 x 40, which is 160 kilograms”.

Cross-Multiplication Algorithm: A student using cross multiplication algorithm, which is a traditional proportion strategy, firstly sets up a proportion and then solves an equation. For this problem, the following proportion is set up and solved.

2 bags 40 kilograms

8 bags ? kilograms

8 bags × 40 kilograms = 2 bags × ?, (8 bags × 40 kilograms) / 2 bags = 160 kilograms

Equivalent Fraction: A student using equivalent fraction strategy perceives the ratio as a fraction and finds the equivalent fraction to get a result. For this problem, the ratio is set between the weight and the number of bags as $\frac{2\ \text{bags}}{40\ \text{kilograms}}$ and then multiplication rule applied to this ratio in order to solve the problem as $\frac{2 \times 4}{40 \times 4} = \frac{8\ \text{bags}}{160\ \text{kilograms}}$. Although it is useful to reach a correct solution, it is considered that this strategy is applied without regarding the context of the problem (Cramer et al., 1993).

Equivalent Class: A student using equivalent class strategy tries to reach the desired ratio by using more than one equivalent class of a given ratio (Duatepe et al., 2005). For this problem, firstly ratio is constructed as $\frac{2\text{ bags}}{40\text{ kilograms}}$ (within ratio) or $\frac{2}{8}$ (between ratio) and then equivalent classes are used until the desired ratio is obtained as $\frac{2\text{ bags}}{40\text{ kilograms}} = \frac{4\text{ bags}}{80\text{ kilograms}} = \frac{8\text{ bags}}{160\text{ kilograms}}$ or $\frac{2}{8} = \frac{4}{16} = \frac{20}{80} = \frac{40}{160}$.

Build-Up Strategy*:* A student using build-up strategy, also referred to as "intuitive strategy" by Lamon (2005), constructs a ratio and then extends it by using addition until reaching the desired one. For this problem, the student using this strategy solves with the following way:

2 bags 40 kilograms + 2 bags 40 kilograms = 4 bags 80 kilograms

4 bags 80 kilograms + 2 bags 40 kilograms = 6 bags 120 kilograms

6 bags 120 kilograms + 2 bags 40 kilograms = 8 bags 160 kilograms

Additive Strategy: A student using additive strategy does not realize the multiplicative relationships between quantities and focuses on the difference between them. For this problem, student erroneously thinks that eight bags are six more than two bags so the weight for eight bags also must be six more than 40 kilograms, 46 = 40 + 6.

The available research on proportional reasoning has shown that strategies used for proportional problems are categorized differently. Tournaire and Pulos (1985), in their review of the literature on proportional reasoning, categorized strategies into two main parts as correct and erroneous strategies. According to this categorization, multiplicative strategies and building up are defined as correct strategies while ignoring one part of data and additive strategy are listed as erroneous strategies. Tournaire and Pulos (1985) also defined full-back strategies as strategies in which students use simple strategies in more complicated problems even if they are able to other strategies (Karplus et al., 1983).

In addition to categorizing students' strategies used as correct and erroneous, Chapin and Anderson (2003) classified solutions of missing value problems as scalar and functional methods. It is stated that the scalar method is a method used when focusing on the multiplicative relationship within ratios while the latter is a method used when focusing on the relationship between ratios (Chapin & Anderson, 2003). It is possible to understand better what they mean with an example of mixture problem which asked the number lemons required for making lemonade with 600 ml of water to obtain the same taste of lemonade which has four lemons for every 30 ml of water.

Shield and Dole (2008) explained that it could be solved in two ways by using measure fields. This conversion problem has two measurement fields: milliliters of water and the number of lemons. In scalar strategy, unknown is found by keeping the relationship same within one measure field. To find x, the number of lemons must be multiplied by 20 because the relationship between the quantities in the second field relied on the multiplication by 20. Whereas, unknown is found by considering the multiplicative change across the two fields in functional strategy, to find x, it is required to divide 600 mL of water by $\frac{15}{2}$ because there is one lemon for 7.5 mL of water (Shield & Dole, 2008). Both types of strategies can be seen in Figure 1.

As compared the example above, scalar and functional methods mentioned by Shield and Dole (2008) are compatible with the unit ratio (Cramer et al., 1993) and factor of change (Post et al. 1988) strategies in terms of keeping relationship same within a ratio and considering multiplicative change across the ratios.

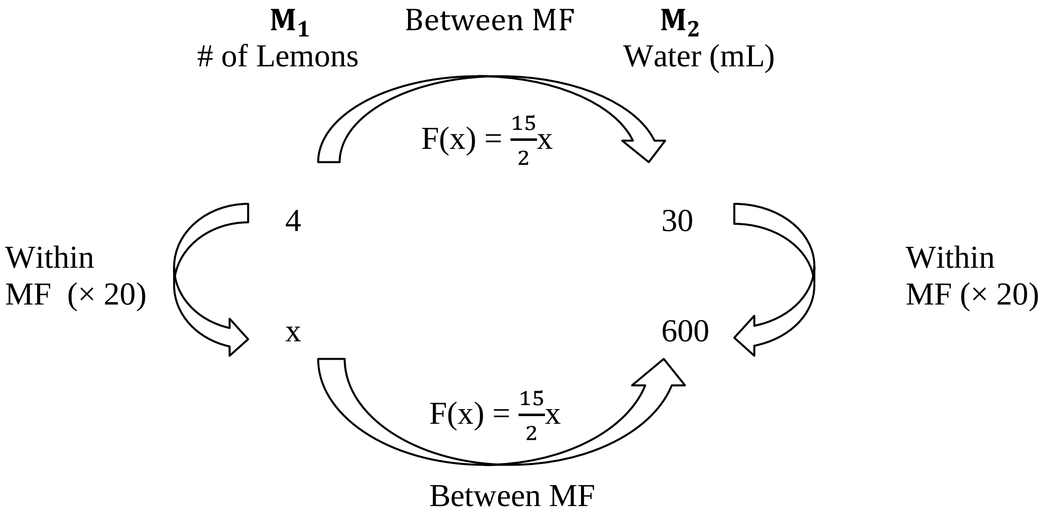


Figure 1. Scalar (within) and functional (between) strategies

Within and between ratios have been approached differently in the literature. For example, solution strategy given in Figure 1 was given in the case that within ratios refer that quantities come from the same measure fields (lemons in their own) and between ratios that quantities come from the different measure fields (lemons and water). By contrast, some researchers have used within ratios as quantities come from the different measurement fields, and between ratios as quantities come from the same measurement fields (Heinz, 2000; Karagöz Akar, 2007; Noelting, 1980b). Although the latter case of within and between ratios was used in mixture problem by Noelting (1980b), he highlighted that these concepts could be used in any other problems as within state ratios which include quantities from different measurement fields and as between state ratios which include quantities from same measurement field. Lamon (2007) alternatively offer using the terminology “within or between systems” or “within or between measure spaces” to eliminate the confusion easily.

In the current study, within ratios and between ratios concepts were used as in the latter case. To be more precise, the ratio of lemon to water (or the direct opposite) will refer to within ratios, and the ratio of lemons in one situation to another will refer to between ratios in the study.

2.5 Mathematics teacher's knowledge

For many years, several studies have been conducted on both teachers and what they need to have for teaching and researchers, in their studies, have tried to identify and model the components of teacher knowledge (Ball, Thames & Phelps 2008; Grossman, 1990; Shulman, 1986; Tatto et al., 2008). Among these studies, Shulman's study is considered as a distinctive study with its influence in nature on other teacher knowledge approaches. In particular, the term pedagogical content knowledge introduced by Shulman (1986) have shaped the further studies concerning developing their models.

Shulman (1986) proposed a model of teachers' content knowledge, which involves subject matter knowledge (SMK), pedagogical content knowledge (PCK), and curricular knowledge. SMK is the type of knowledge that related to a domain taught by a teacher (e.g., geography, mathematics). It entails knowing the underlying elements of facts, procedures, and concepts as "going beyond the knowledge of the facts or concepts of a domain" (Shulman, 1986, p.9). On the other hand, pedagogical content knowledge was defined as a "particular form of content knowledge that embodies the aspects of content most germane to its teachability" (Shulman, 1986, p.9). It implies that to make the subject more comprehensible to students, teacher know and use alternative forms of representations, give proper examples and explanations related to the subject and discern students' misconceptions (Ertaş & Aslan-Tutak, 2017). The last type of teachers' content knowledge, curricular knowledge, includes a teacher’s ability to use curriculum materials efficiently and to connect the content of the subject to others.

In the field of mathematics education, several studies have been conducted to conceptualize mathematics teachers’ knowledge based on Shulman’s model of teacher knowledge (Ball et al., 2008; Grossman, 1990; Tatto et al., 2008). The most well-known and internationally accepted one among these models is the Mathematics Knowledge for Teaching model (MKT), which is empirically based refinement of Shulman’s models. Ball and a group of researchers (2008) have developed the MKT model of teacher content knowledge within a project (Learning Mathematics for Teaching) aims to examine mathematical knowledge in teaching settings. They also made significant contributions to the literature by providing instruments to measure such knowledge they modeled in the scope of the project (Hill et al., 2004). Both MKT model and instrument has been developed based on analyses of classroom lessons.

In the MKT model, there are six domains of content knowledge which are settled under the Shulman’s initial categories of SMK and PCK. While subject matter knowledge consists of common content knowledge (CCK), specialized content knowledge (SCK) and horizon content knowledge (HCC), pedagogical content knowledge (PCK) includes knowledge of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of content and curriculum (KCC) as can be seen Figure 2. It can be said that there are commonalities between Shulman’s model and MKT; however, the latter one is a more detailed and enhanced version of the former. Also different from Shulman’s model, Ball et al. placed curricular knowledge within pedagogical content knowledge while this domain of knowledge is addressed in a separate section in the first model.

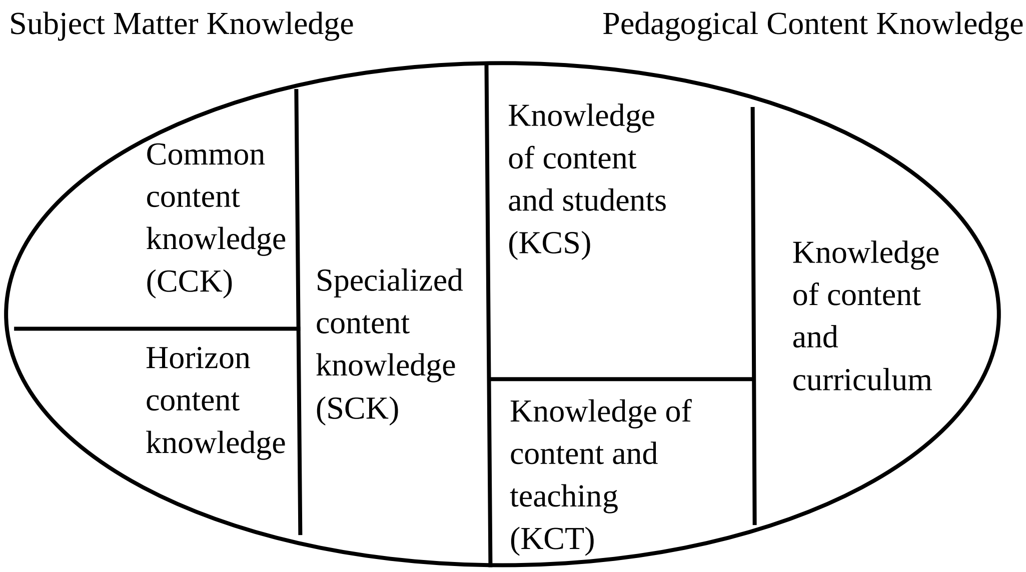


Figure 2. Mathematical knowledge for teaching model

Source: [Ball et al., 2008, p.403]

According to MKT, common content knowledge (CCK) is "mathematical knowledge that teachers are responsible for developing in students" (Hill, Sleep, Lewis & Ball, 2007). For teachers, it refers to be able to solve mathematics problems directed at students, use mathematical terms, and discern the incorrect students' answers and inaccurate definitions. Knowing how to divide $\frac{4}{5}$ by $\frac{1}{2}$ refers to common content knowledge. However, such knowledge, not unique to work of teaching, is the kind of knowledge that any educated person is expected to have (Ball et al., 2008). Therefore, it can be said that it is parallel with Shulman's subject matter knowledge.

Specialized content knowledge (SCK) refers to mathematical knowledge that teachers need to use in teaching (Ball et al., 2008). In contrast to common content knowledge, it is unique to teaching and is "beyond what other educated adults know" (Hill et al., 2007). SCK is not mathematical knowledge that should be directly taught to students (Hill et al., 2007). It includes being able to express mathematical terms accurately, use a different kind of representations efficiently, know and explain underlying elements of facts, procedures, and algorithms. The most common and comprehensible example is given over the division of the fractions (Aslan-Tutak & Köklü, 2016). In the division of fractions, teachers need to carry out the procedure correctly as the first of teaching. However, it is not sufficient in teaching fractions to students. In the teaching process, teachers are expected to not only solve such a problem and know how to perform an invert-and-multiply algorithm but also provide a conceptually-based justification why and how this algorithm works (Hill et al., 2007). In this example, being able to divide fractions and use algorithm correctly is within common content knowledge while providing a conceptually-based justification and judging students' solutions whether they are valid or not are included in the scope of specialized content knowledge.

In the scope of specialized content knowledge, analyzing alternative students' solutions also has an important role (Ball et al., 2008). Teachers need to analyze these alternative solutions mathematically and use them in teaching effectively. It is important to note that skillful teaching requires not only identifying students' errors but also being able to plumb the source of these errors. Table 2 shows mathematical tasks of teaching placed in SCK which listed by Ball et al. (2008). It is emphasized that these tasks which teachers regularly do in their classrooms require mathematical understanding and reasoning beyond the tacit knowledge needed by most people (Ball et al., 2008).

The third category of subject matter knowledge, horizon content knowledge, was described as "an awareness of how mathematics topics are related over the span of mathematics included in the curriculum" (Ball et al., 2008, p. 403). It means that this type of knowledge requires to relate mathematical topics taught topics both in the past and further grades. Therefore, teachers should be able to understand mathematics in a holistic way and reflect it in their teaching.

Table 2. Mathematical Tasks of Teaching

Presenting mathematical ideas
Responding to students "why" questions
Finding an example to make a specific mathematical point
Recognizing what is involved in using a particular presentation
Linking representations to underlying ideas and to other representations
Connecting a topic being taught to topics from prior or future years
Explaining mathematical goals and purposes to parents
Appraising and adapting the mathematical content and textbooks
Modifying tasks to be easier to harder
Evaluating the plausibility of students' claims
Giving or evaluating mathematical explanations
Choosing and developing usable definitions
Using mathematical notation and critiquing its use
Asking productive mathematical questions
Selecting representations for particular purposes
Inspecting equivalences

Source: [Ball et al. 2008, p. 400]

Ball and colleagues placed three subcategories of PCK within the model by emphasizing that just content knowledge is not sufficient for teaching, as Shulman mentioned (1986). First domain under the pedagogical content knowledge, KCS, is defined as a combination of both knowledge of students and knowledge of content (Ball et al., 2008). KCS involves knowledge about which topics students are likely to find confounding, which examples they are likely to find interesting, which type of problems they are likely to have difficulty and which errors they are likely to make for teachers. Knowledge of content and teaching is another domain of PCK in the model. KCT includes both knowledge of teaching and knowledge of mathematics and requires being able to design proper instruction of a mathematical topic. For effective teaching, teachers need to be able to choose the most appropriate models and representations related to mathematical topics. For example, using Cuisenaire rods for comparison or addition of fractions as a model can be involved in KCT. The last domain of the PCK is knowledge of content and curriculum. It requires teachers to know which topics are related in the curriculum with each other, what instructional materials are available in the curriculum for a specific topic, and how effective they are.

Mathematical Knowledge for Teaching model and its instrument developed by Ball and colleagues has been widely accepted and used in the mathematics education field. The instrument aims to measure teachers' mathematics knowledge of teaching within the Learning Mathematics for Teaching project involve multiple choice items about three mathematical topics: geometry; number and operations; pattern, functions, and algebra. Through developed instrument, researchers have been tried to measure teachers' ability in these areas and to investigate how their ability affect the students' learning (Hill et al., 2005; Hill & Lubienski, 2007).

For questions in this study, the focus was on two subdomains of teacher knowledge, common content knowledge and specialized content knowledge under the subject matter knowledge which are associated with the six components of proportional reasoning outlined by Lamon (2005, 2007).

2.6 Studies about teachers' proportional reasoning

Over the past century, researchers have mentioned the need for teachers' broad and conceptual understanding of subjects that they teach (Ball & Cohen, 1999; Ma, 1999; Shulman 1986; Shulman, 1987). In particular, it is emphasized that teachers need to have a comprehensive understanding of the underlying causes as well as knowing what something is (Shulman, 1986). Indeed, teachers must have more than applying procedures mechanically. Hence, it is crucial for a teacher to have a more in-depth and conceptual understanding of proportional reasoning to provide their students with richer opportunities in their own classrooms. Also, it is described as a way of thinking and beyond just an algorithm to be used in solutions (Thompson & Bush, 2003). Therefore, being proportional reasoners themselves for teachers can be accepted as a prerequisite to facilitate students' improvement of proportional reasoning. In light of these ideas, several researchers have been focused on the understanding of preservice and in-service teachers' proportional reasoning around the world.

One of these researchers, Riley (2010), constructed a study with elementary preservice mathematics teachers to examine these teachers' understanding of proportional reasoning and concentrate on just teachers' ability to solve problems that require proportional reasoning, which appropriate within common content knowledge, not unique to work of teaching. In order to achieve this aim, students were asked to open-ended items whose types change from missing value problem to a mixture problem, from inverse proportion problem to additive thinking problem. Findings of the study demonstrated that more than half of the preservice teachers were not able to respond to inverse proportion problems correctly and had difficulty in differentiating multiplicative from additive thinking.

There are also relatively few studies focused on preservice teachers' proportional reasoning in terms of knowledge of mathematics and knowledge of mathematics teaching. In a qualitative study conducted with in-service middle school mathematics teachers, Lobato, Orrill, and Druken (2011) aimed to investigate both teachers' challenges about proportional reasoning and abilities to understand and build on students' thinking by taking MKT as a starting point. For this purpose, they conducted several interviews with 13 teachers and asked them to solve and talk about a few tasks about proportional reasoning. According to the results of the study, teachers' reasoning classified into three categories, namely employing rules without reasoning, applying mathematical structural reasoning and quantitative reasoning. While mathematical structural reasoning was defined as mainly relied on the use of mathematical properties, quantitative reasoning was defined as mainly relied on the explanation of the multiplicative relationship between quantities. Researchers concluded that both types of reasoning include conceptual underpinnings, but they differ in terms of groundings.

Other research focuses on both content, and pedagogical knowledge is the study conducted by Ekawati et al. (2015b). In the scope of the research, Ekawati et al. developed instruments to assess teachers' knowledge, including both content and pedagogical content knowledge for teaching ratio and proportions. The research conducted with in-service primary teachers in Indonesia from different educational backgrounds. Items of instruments consist of multiple choice, complex multiple choice, and open-ended problems. While items designed to measure mathematics content knowledge involves missing value and comparisons problems, items designed to measure mathematics pedagogical content knowledge involves identifying and analyzing students' solutions, choosing appropriate teaching methods and students' misconceptions. Although the instruments were checked in terms of validity and reliability factors, they have been used rarely in international research.

Apart from the international studies, preservice or in-service teachers' proportional reasoning has not been gained attention adequately in Turkey. Although there are several studies about proportional reasoning conducted with students, there is just one qualitative study aimed at teachers' understanding of the same topic. In the qualitative study, Akkuş Çıkla and Duatepe (2002) interviewed with elementary preservice teachers about several ratio and proportion problems through open-ended questions which designed for students by Miller, Lincoln, and James (2000). Their results showed that teachers have difficulty in providing conceptual justifications for their solutions and using appropriate language even though they solved questions via cross-multiplication rule. For example, they used the ratio and proportion terms interchangeably.

Available research in Turkey suggests that there is a need to determine the current level of preservice mathematics teachers' proportional reasoning. Because there is only one study conducted with preservice teachers and its participants were not preservice mathematics teachers who are responsible for teaching this topic to students. This study aims to serve the purpose of determining the current level of preservice mathematics teachers' proportional reasoning.

Also, it is revealed that teachers who have a limited conceptual understanding about subjects to teach do not provide adequate support for students on making sense of mathematical concepts and comprehending subjects (Ma, 1999) and students have difficulty in proportional reasoning which is the basis for many topics related to mathematics and science. Thus, considering all of these factors, investigating proportional reasoning ability of preservice mathematics teachers who will enter upon “teaching” career has importance. In this context, the purpose of the current study is to investigate their proportional reasoning ability with a particular focus on the measurement process to fill this gap in Turkey. So, the answers to the following research questions are sought:

- How do preservice mathematics teachers perform on questions requiring proportional reasoning ability?
- What are the similarities and differences between PSMTs’ performances on questions requiring proportional ability on the task-based interviews and on the PRI?

CHAPTER 3

METHOD

This study focused on the preservice mathematics teachers' proportional reasoning through the PRI and interviews followed by the implementation of instrument. A sequential explanatory mixed methods design was used to gain insight into preservice mathematics teachers' proportional reasoning ability in the current study.

In the explanatory sequential design whereby both quantitative and qualitative data are collected, analyzed and integrated into a single study, both methods are combined to get benefit from the strengths of each and to compensate weaknesses of these two methods (Creswell & Plano Clark, 2011). In that sense, quantitative data obtained from the instrument provided a general information about preservice mathematics teachers' proportional reasoning. As Creswell (2014) stated, qualitative method is best suited for analyzing participants´ understanding and attitudes in more depth. Therefore, qualitative data obtained from the task-based interviews also was used to explain and elaborate the initial quantitative results and to get a more thorough understanding of PSMTs' proportional reasoning ability (Creswell & Plano Clark, 2011). The reason for collecting both qualitative and quantitative data in the current study was to combine elements of both methods to provide more in-depth and complete understanding of research questions than only one method (Creswell & Plano Clark, 2011). After the analysis of the data from the instrument, quantitative findings were supported and detailed through analyses of PSMTs' responses to the interview questions in a more elaborated manner (Creswell, 2014). Thus, more persuasive investigation and interpretation of PSMTs' proportional reasoning ability became possible with the use of qualitative method in this study.

3.1 Sample

This study has been carried out within three phases in the second semester of the 2018-2019 academic year. In the first phase, a prepared instrument was tested in a pilot study. Sixty-nine junior and senior preservice mathematics teachers middle and secondary level teaching mathematics programs in the two public universities of Turkey took part in the pilot study through convenience sampling.

The main study was performed as the second phase of the study. The participants were chosen from junior and senior preservice mathematics teachers at middle and secondary levels studying at five public universities located in different parts of Turkey by convenience sampling method. Easy access to the participants was the main reason behind selection of this sampling approach (Gay, Mills & Airasian, 2009). 263 junior and senior preservice teachers (213 female, 50 male) from five public universities were contacted and they volunteered to take part in the study. These five universities are considered among the successful universities in Turkey since middle and secondary level mathematics teacher education programs of four out five of these universities are ranked in the top ten according to the scores of the admitted students in the university entrance exams (Yüksek Öğretim Kurumu, nd). Participants in both the pilot and main study had completed a majority of their courses related to mathematics teaching and teaching methods. The elimination of two participants who withdrew from the study during the implementation left 261 participants in the sample for analyses.

There are suggestions about minimum sample size for questionnaires based on the number of questions. Bryman and Cramer (2005) stated that sample size should be five times larger than the number of questions in the instrument. In this study, the instrument consists of 13 questions. So, it has been deemed sufficient to have 261 participants which is more than 65, a sufficient number for the current study to achieve valid and reliable results.

As a third phase of the study, a qualitative part has been designed with interviews aiming to validate the findings from the data collected by the instrument and getting an in-dept account of teachers' proportional reasoning. Task-based interviews have been conducted with seven preservice mathematics teachers who accepted to participate in the qualitative part of the study voluntarily. They were purposively selected according to their performances on the instrument (PRI) from three different universities in İstanbul. It should be noted that the aim of this selection is not divide all participants into three groups. Rather, selection of participants was done with the aim of making comparisons among the participants who got the highest, average and lowest scores on PRI in order to investigate their differences and similarities on proportional reasoning ability. Information about the participants of the interviews can be found in Table 3.

Table 3. Information about Participants of Interview

Participants	Gender	Scores on PRI
Preservice Mathematics Teacher 1	Male	Highest (37.5)
Preservice Mathematics Teacher 2	Female	Highest (36.32)
Preservice Mathematics Teacher 3	Female	Average (30.29)
Preservice Mathematics Teacher 4	Male	Average (29.82)
Preservice Mathematics Teacher 5	Male	Average (28.96)
Preservice Mathematics Teacher 6	Male	Lowest (21.21)
Preservice Mathematics Teacher 7	Female	Lowest (21.08)

3.2 Instrument

PRI was developed by the researcher within the scope of the study. It aims to measure preservice mathematics teachers' proportional reasoning through questions about ratio and proportions that reflects preservice mathematics teachers' both common and specialized content knowledge. Instrument's questions were shaped around components of proportional reasoning and were intended to focus on the potential challenges that can be faced by preservice teachers or in-service teachers on proportional reasoning. Due to the limited number of studies done with both preservice and in-service teachers, questions in the instrument requiring proportional reasoning were developed based on six characteristics of proportional reasoner as Lamon (2005, 2007) identified in light of the related literature as seen in Table 4.

Table 4. PRI Questions and Components of Proportional Reasoning

	Comp (1)	Comp (2)	Comp (3)	Comp (4)	Comp (5)	Comp (6)
Q1	✓	✓		✓	✓	
Q2			✓	✓		
Q3				✓	✓	✓
Q4	✓	✓		✓		
Q5			✓	✓	✓	
Q6		✓		✓	✓	
Q7	✓					✓
Q8	✓			✓		
Q9				✓	✓	
Q10	✓	✓		✓	✓	
Q11	✓	✓		✓	✓	✓
Q12				✓	✓	✓
Q13	✓			✓	✓	✓

Note: Six components of proportional reasoning identified by Lamon (2005, 2007) are as follows: (1) Solve proportional problems in a wide range of context from slope to similarity or proportional problems involving number complexities e.g., non-integer ratios, fractions or decimals (2) Develop and use different strategies in solving problems requiring reasoning proportionally rather than using only traditional proportion algorithm (3) Distinguish proportional situations from non-proportional ones (4) Understand the multiplicative relationships both in direct and inverse proportions (5) Realize and understand the invariance and covariance structure of the proportional relationships (6) Develop and use the language for ratio and proportions

Six components of proportional reasoning also reflects both common content knowledge and specialized content knowledge. While solving a wide range of problems, distinguishing proportional situations from non-proportional ones, realizing invariance and covariance structures of proportional relationships can be associated with common content knowledge, components relating to proper use of language and being able to know and explain underlying elements of facts, procedures, and algorithms such as understanding multiplicative relationships both in direct and inverse proportions, developing different solution strategies, can be associated with specialized content knowledge.

Questions based on the six components of proportional reasoning stated by Lamon (2005, 2007) in the developed instrument were designed in the light of related literature. As outlined before, these were solving proportional problems in a wide range of contexts and in numerical complexieties, differentiating between proportional and non-proportional situations, developing and evaluating different strategies to proportional problems, understanding multiplicative relatioships both in direct and inverse proportions, using language including algebraic representations properly, realizing and understanding the invariance and covariance structure of the proportional relationships. While some questions were adopted from previous studies (e.g., Cramer et al., 1993; Ball et al., 2008; Ekawati et al., 2015a), some questions were developed for the current study using similar questions in the literature (e.g., Aladağ & Dinç Artut, 2014; Bright, et al., 2003; Van de Walle et al., 2013; Yenilmez & Kavuncu, 2017) and others were written by the researcher [see Appendix A (for the English version of the instrument) and Appendix B (for the Turkish version of the instrument)].

Questions which aim to elicit teachers' ability to use ratio and proportion as they would in practice as in the study done by Hill et al. (2004), were designed in selected response format and constructed response format. Both question formats are scored dichotomously which automatically means that they are scored as correct or incorrect. Questions with constructed response format including short answers were grounded in common tasks of instruction on ratio and proportion. While questions with both selected response and constructed response format contribute towards the PRI scoring, questions with constructed response format were also used for getting a more in-depth understanding about future teachers' proportional reasoning. Also, the instrument includes a section that gathers some demographic information such as gender, grade, and university from the participants.

Questions in the developed instrument were selected/developed and adapted by considering both task variables and essential components regarded in the literature, as mentioned above. Information about the questions, their links with proportional reasoning ability, the overview of expectations and description about the question format can be found in Appendix C, Table C1. However, giving detail on how the questions in the instrument were selected and prepared by the researcher is necessary to better understanding. The rationale behind how questions were selected/developed and adapted was the following:

Studies have shown that students have difficulty mostly in the shrinkers and stretchers questions among context variables (Ben-Chaim, Fay, Fitzgerald, Benedetto & Miller, 1998; Kaput & Maxwell-West, 1994). Considering the results that many elementary and middle school teachers also have the same difficulties and misconceptions as students have (Cramer et al., 1993), two questions were designed as in the context of stretchers and shrinkers. While one of them focusing on common content knowledge was related to making sense of invariance and covariance structure of proportional situations, the other one asking preservice teachers to evaluate each of the three different students' answers to enlargement problem focused on specialized content knowledge. The question focusing on specialized content knowledge was adapted from the study constructed by Ekawati et al. (2015a). Non-integer ratios were used for whole missing and comparison problems because it is well known that type of ratio affects which strategy (additive or multiplicative) students use in the way that students tended to use more additive strategy when the ratios are non-integer (Cramer et al., 1993; Karplus et al., 1983; Toluk Uçar & Bozkus, 2016; Tourniaire & Pulos, 1985; Van Dooren et al., 2010).

Considering that differentiating proportional situations from non-proportional ones has a critical role in the development of proportional reasoning (Cramer et al., 1993; Lim, 2009), Question two (Q2) that elicits preservice teachers' ability to distinguish these two situations from each other was constructed. It consisted of seven sub-questions: four of them include non-proportional situations, and three of them include proportional situations. In the construction process, integrating different proportional and non-proportional situations into Q2 to better evaluation through several different situations was a primary focus: While the context of proportional situations ranged from area unit conversion and similarity to the velocity problem; non-proportional situations were designed in the context of linear (parking problem), constant (laundry problem) and additive structure (age problem). The context of age and parking problem were adapted from the questions in Ekawati and colleagues' study (2015a); the running track problem was taken from Cramer and colleagues' study (1993); the laundry problem which is a constant problem was adapted from the study constructed by Van Dooren and colleagues' (2010). All sub-questions were designed to include integer ratios because it is expected that preservice teachers are able to differentiate the situations without being affected by number structures compared to the students' difficulties in number structures (Çelik & Özdemir, 2014; Toluk Uçar & Bozkuş, 2016).

Another question directed to specialized content knowledge is question three (Q3). It consists of four sub-questions. While two of them are about definitions of ratio and inverse ratio, others are significant statements about ratio and the relationship of ratio and fraction. Preservice teachers were asked to decide whether the statement is correct or not and then to justify their decisions in the context of Q3, which was designed for the current study. Constructed-response items were used in the aim of revealing different information about preservice teachers' proportional reasoning (Bright, et al., 2003).

Question four (Q4), which is the comparison problem, was adapted from the instrument used in the study of Bright and colleagues (2003). It was designed in two parts, including a multiple choice question and two related sub-questions in terms of a given answer. Numbers in Q4 were selected as the differences between length and width within each photograph, and differences between lengths and widths of rectangles are equal two. As Bright and colleagues (2003) offered in their article, participants who reasoning additively tend to compare the differences absolutely rather than relatively. So, an option that all photographs have the same shape was added to question. While the first part of the question, which is directed to common content knowledge, is about comparing photographs in terms of their squareness, the second part of the question is about evaluating alternative students' solutions. In the second part, preservice teachers were told that they should evaluate relevant two solutions according to their answers of multiple choice question.

The graphs of the proportional relationships which are expressed mathematically as $f(x) = xa,\ a \neq 0$ goes through the origin as a straight line while the graphs of additive relationships which are expressed mathematically as $f(x) = xa + b,\ a \neq 0$ are also straight lines but do not pass through the origin (Cramer et al., 1993). In order to evaluate preservice teachers' ability to differentiate proportional from non-proportional situations expressed with algebraic equations and graphs, and also examine how they associate linearity with proportionality, four sub-questions were adapted and developed in question five (Q5): two of them were graphical representations, the other two were algebraic representations. Both graphical and algebraic representations consist of one multiplicative relationship and one additive relationship. Graphs were adapted from the study done by Ekawati et al. (2015a). Because it is not unique to teaching, it is a question directed to common content knowledge.

Question six (Q6) and Question nine (Q9) were questions developed within the scope of this study. While Q9 aims preservice teachers to only decide which quantities in the equations are directly or inversely proportional, Q6 aims them to evaluate students' explanations why given quantities in the table were proportional. In other words former is directed to common content knowledge, the latter is directed to specialized content knowledge.

Considering that both students and preservice teachers have difficulty on performing inverse proportion problems (Riley, 2010; Yenilmez & Kavuncu, 2017), question eight (Q8), which is a missing value problem, was prepared for PRI. Rather than the worker problem which is a typical inverse proportion problem familiar to students because of the emphasis in the Turkish mathematics curriculum, the problem was designed in the context of a bicycle journey by using the fact that number of laps and radius of the wheel are inversely proportional because the distance is equal.

Based on the idea of Lamon (2005) that a competent proportional reasoner should solve a variety of problems related to ratio, and slope is a significant ratio which not only identifies a steepness of the line but also indicates the change factor of one variable in terms of another (Van de Walle et al., 2013) question thirteen (Q13) was designed. It focused on slope through stairs. In this question, preservice teachers were expected to know and make decisions about what the slope is and what the invariant is in stairs problem without any given numbers.

For assessing proportional reasoning, mixture problems can be considered as typical problems many researchers have studied (Kaput & Maxwell-West, 1994; Heinz, 2000; Karagöz Akar, 2007). In the scope of the current study, the mixture problem (Q10) was designed as evaluating different students' solutions, including equivalent fractions and unit ratio. Preservice teachers were asked to determine whether reasoning through equivalent fractions and unit ratio solutions is valid in the context of the flavor problem.

Lastly, preservice teachers was aimed to determine invariance and covariance structures of direct proportion which was expressed with three different representations in question twelve (Q12), and to recognize all representations (percentage, fraction, and decimals) that were appropriate for expressing proportional situations in question seven (Q7).

As explained, the instrument consists of 13 questions. Sub-questions were scored such that each question would have a total of three points. Therefore, a total score obtained from the instrument is 39, as seen in Table 5. Selected response format questions were scored as correct or incorrect. For the constructed response part of the third question, which requires justifications about the accuracy of the statements, preservice teachers' explanations were classified as sufficient, insufficient, or incorrect. While preservice teachers with insufficient and incorrect explanations did not get any point from the constructed part of the relevant sub-question, preservice teachers who were able to provide sufficient explanation got full point from the constructed part of the relevant sub-question. As appropriate examples and correct mathematical statements were categorized as sufficient, incorrect mathematical statements (e.g., using the terms ratio and proportion interchangeably in the first subquestion) were categorized as incorrect. Answers which are blank and not properly explained were incorporated into an insufficient category. Appendix C, Table C2 shows the preservice teachers' explanations categorized as sufficient, insufficient, and incorrect for each sub-questions in the third question. Those who both replied the sub-question correctly and made sufficient explanation got a full point in Q3. Those who replied the sub-question correctly but explained incorrectly did not get the point from this sub-question. Those who replied the sub-question correctly but did not justify their reasoning or justified insufficiently got half points from the related sub-question. Those who replied the sub-question incorrectly but made correct explanation also got half points from the related sub-question. Such a situation was encountered only in the fourth sub-question: Some preservice teachers explained that the statement is valid as long as it is at the same rate for being inversely proportional. Because they clearly expressed the necessary condition which is not stated in the fourth statement, they got the full point. For the second, third, and fourth questions, there were no incorrect answers in the PRI: all answers were insufficient or sufficient.

Table 5. Scoring Guide for PRI

Questions	Scoring	Total
Q1	There are three sub-questions. Each sub-question has one point.	Three points
Q2	There are seven sub-questions. Each sub-question has 3/7 points.	Three points
Q3	There are four sub-questions and their explanations. Each sub-question and its sufficient explanation have 3/8 points.	Three points
Q4	There is one multiple-choice question and two sub-questions for each option. Multiple question has one and half points and each sub-question has 0.75 points.	Three points
Q5	There are four sub-questions. Each sub-question has 3/4 points.	Three points
Q6	There are three sub-questions. Each sub-question has one point.	Three points
Q7	There are three sub-questions. Each sub-question has one point.	Three points
Q8	There is one multiple-choice question.	Three points
Q9	There are five sub-questions. Each sub-question has 3/5 points.	Three points
Q10	There is one multiple-choice question.	Three points
Q11	There is one multiple-choice question.	Three points
Q12	There are three sub-questions. Each sub-question has one point.	Three points
Q13	There are three sub-questions. Each sub-question has one point.	Three points

Interviews consisted of several questions focusing on proportional reasoning from the instrument and additional questions that some of them developed, and some were taken from the existing literature (Karagöz Akar, 2007). The process behind how the decision was made was the following: firstly, the questions in the instruments which were mostly answered incorrectly were identified. In this way, examining how these questions were correctly or incorrectly answered and the reasoning behind them and thus gathering more detailed information about preservice teachers' proportional reasoning ability was aimed. Although only Q5b is one of the items that were most frequently answered incorrectly, both graphs in Q5b and Q5c were included in the interviews to understand preservice teachers' reasoning and justifications in comparisons relating to:

- How preservice teachers distinguish between these two linear equation graphs
- How they explain the way these two differ from each other in terms of proportionality
- How they make conclusions about linearity and proportionality

Q3d in the instrument was designed as a comparison problem that was about detecting the missing part in the definition of inverse proportion, which is a common misconception. In order to gather data about the reason behind answering this question incorrectly, the question was constructed: is it because participants failed to notice the lack of word in the definition, namely inattention or is it because they supposed that a situation in which one quantity decreases while the other decreases is sufficient for being inversely proportional. So, rather than asking its definition directly, "inverse proportion - table representation" question which includes a table where values of x increased while values of y decreased and gave a hypothetical wrong solution of a student made by considering x and y values in the table was prepared. Through this question, researcher tried to take an opportunity to discuss with participants what is invariant or covariant in inverse proportions.

Same examples of Q9 that took place in the instrument as were used an interview problem to understand participants' reasoning when deciding whether quantities are directly or inversely proportional. It was realized that many PSMTs assigned numbers to the quantities when deciding which are directly or inversely proportional. Thereby, this question was used to understand the rationale of participants' decision-making mechanism and whether they explain the reasons in situations numbers were not assigned to the quantities. Also "enlargement of rectangle" question's stem in the instrument was used with the aim of deciding whether preservice mathematics teachers use different strategies in solving problems requiring proportional reasoning rather than using only traditional proportion algorithm and whether they can explain these solutions with appropriate mathematical expressions. These are are essential skills for a proportional reasoner (Lamon, 2005). Another part of this question including students' different solutions was the same as the question in the instrument.

In the task-based interviews, "inverse proportion - bicycle" question was used with an open-ended format. This question was included because it was a commonly incorrectly answered question among participant with low scores. The aim was to understand why and how participants with high and average scores solved it correctly, and participants with low scores did not. Also, "inverse proportion - faucet" question was constructed in order to gain insight into participants' thinking about the invariance and covariance structure of the proportional relationships specific to faucet problem which was considered as in the context of inverse proportion. The numbers were selected with non-integer multiples as in the instrument questions.

Lastly, in order to see how preservice teachers reason between-ratios strategies and make sense of change factor in between ratios strategies and multiplicative relationship in within ratios strategies, a mosaic problem which was developed by Karagöz Akar (2007) was used. The reason behind the selection of the mosaic problem is to understand their thinking about the invariance and covariance structure of proportional relationships, in addition to their evaluations for students' solution. It was also selected because its aim was parallel to that of Q11 located in the instrument. The order of the problems in the interview was similar to the order of the problems in the instrument.

For all items in the interview, questions which expect participants to explain their solutions, justifications, and thinking processes were asked, as seen in Appendix C, Table C3. Through the preparation process, follow-up questions suggested by Hunting (1997) were used for guidance. Questions that should be asked during the interview, were substantially ensured after the pilot interview. However, additional questions based on the preservice teachers' level in some situations were also asked according to the dynamics of the interviews. For example, a participant who got a high score in the instrument was asked whether or not he can give an example for the graph which is not linear but still proportional at the end of the question related to linearity and proportionality. During the interview, the main aim is making sense of preservice teachers' both common content knowledge by inspecting their solution strategies and its accuracy and specialized content knowledge by looking for conceptually-based justifications for proportion algorithms, number of alternative solutions for questions and evaluations of students' strategies.

As in the PRI questions, task-based interview questions which can be seen in Appendix D (for the English version of interview questions) and E (for the Turkish version of interview questions) were also based on characteristics of proportional reasoners. Table 6 shows which components of proportional reasoning are related to the task-based interview questions.

Table 6. Task-Based Interview Questions and Components of Proportional Reasoning

	Comp (1)	Comp (2)	Comp (3)	Comp (4)	Comp (5)	Comp(6)
Q1	✓	✓		✓	✓	
Q2			✓	✓		
Q3		✓		✓	✓	✓
Q4			✓	✓	✓	
Q5				✓	✓	
Q6	✓	✓		✓	✓	✓
Q7	✓			✓	✓	
Q8	✓			✓		

Note: Six components of proportional reasoning identified by Lamon (2005, 2007) are as follows: (1) Solve proportional problems in a wide range of context from slope to similarity or proportional problems involving number complexities e.g., non-integer ratios, fractions or decimals (2) Develop and use different strategies in solving problems requiring reasoning proportionally rather than using only traditional proportion algorithm (3) Distinguish proportional situations from non-proportional ones (4) Understand the multiplicative relationships both in direct and inverse proportions (5) Realize and understand the invariance and covariance structure of the proportional relationships (6) Develop and use the language for ratio and proportions

3.3 Procedure

In order to examine preservice teachers' proportional reasoning, work towards instrument development was initiated. During the development of the PRI, eight steps identified by De Vellis (2003) were followed. As a first step, what was aimed to measure and its components in the related literature were clearly decided and as a second step a pool of items was constructed according to these components. During the item pool construction, the researcher also attended three teaching methods courses as an observer, in the topic of proportional reasoning in order to specify possible misconceptions of preservice teachers. For example it was observed that preservice teachers had difficulties in providing alternative solutions in addition to traditional algorithm when solving proportional questions and in differentiating non-proportional situations from proportional ones especially in which there is a constant difference between quantities.

As the third and fourth steps, question formats were confirmed, and experts' opinions were taken for the initial item pool before the pilot study. Content and face validity was ensured through experts' opinions about content and language appropriateness that play an essential role in the validity process. Four academics, who specialize in areas ranging from mathematics education to measurement and evaluation at different universities were consulted. In this process, also one preservice and two in-service teachers were asked to solve questions in the instrument for determining the duration of the instrument and for feedback whether questions are clear and comprehensible. According to the feedback of preservice and in-service teachers, the duration of implementation was decided as 45 minutes.

The instrument was then administered to a pilot sample from two different universities. Participants were asked first to read and sign the informed consent form, and to respond to the items in the instrument. Data of the pilot study were analyzed to document construct validy as the fifth step in the process. The findings informed small changes on the instrument and the main study was then implemented.

As a sixth step, the main implementation that was improved by the pilot study and experts' opinions was applied to volunteering preservice teachers enrolled in the elementary and secondary school mathematics teacher education programs as a second phase of the study. The obtained data were transferred to the computer environment. Reliability of the instrument was computed, and items were analyzed in terms of item discrimination and item difficulty in the SPSS 25.0 as the seventh step of the instrument development. The reliability estimate of the main study was calculated as 0.627. According to Field (2009), the instrument with the values of alpha above 0.7 has acceptable reliability and the instrument with the values of alpha above 0.8 has good reliability. Hence, it can be said that the instrument had a low but marginially acceptable reliability. A low alpha value could be due to a low number of questions in the instrument and this effect was possibly further exacerbated by the fact that for some items the variance was very limited. As the last step, the number of questions in the instrument were found to be adequate in terms of obtained reliability coefficient.

The qualitative component of the study was then constructed and preservice teachers who were purposively selected in terms of their performances were interviewed. For the interviews, preservice teachers having highest (in top ten), average (in average ten) and lowest (in last ten) scores from PRI who were located in İstanbul were identified and contacted. Seven participants volunteered to participate in the interview process. Among them, PSMT1 and PSMT2 who were among top ten participants on PRI performances were called as highest scorers, PSMT3, PSMT4 and PSMT5 who were among the ten participants having scores closest to the mean were called as average scorers and PSMT6 and PSMT7 who were among the last ten participants according to PRI scores were called as lowest scorers during the study. Before starting the task-based interviews, pilot intervies was conducted with a preservice teacher not participating in the study. This pilot interview helped construct the final version of the interview questions. For example, the statement “preserving ratio” in the enlargement of the rectangle problem was changed because it directed the participant in the pilot interview to solve the problem with using ratios. So, the statement preserving “the construction of shape” was used rather than “ratio” as said by one of the mathematics educators. It was also noticed that the interviewee felt uncomfortable about explaining her reasoning while solving questions simultaneously during the pilot interview session. So, giving enough time to interviewees to solve the questions and then asking them to explain their solutions and thinking was preferred.

All interview sessions consisted of only an interviewee and the interviewer who were interacting in relation to questions (Goldin, 2000). The interviews were audiotaped and videotaped. Before the interviews, permissions for records were asked from all participants, and they signed audio and video record consent forms. The duration of the interviews ranged from 32 minutes to 90 minutes.

3.4 Statistical analysis

In this study, the data sources consist of preservice teachers’ results of proportional reasoning instrument, transcripts of the interviews, their written responses in the third question related to definitions and features of ratio and proportion and written artifacts from the interviews. Statistical analysis comprises validity, reliability, and item analysis of the instrument and content analysis of task-based interviews.

3.4.1 Item analysis, validity, and reliability of instrument

As a measure of the validity of the current instrument, content, face, and construct validity were assessed. Analysis process for quantitative part took place within two stages. Judgments of the field experts, including mathematics educators, mathematics teachers, and mathematics specialists, were used for providing content and face validity. After getting experts’ judgments and revising in line with their feedback, the pilot study was done. According to the item analysis and reliability results of this study obtained from SPSS, the instrument took its final shape. The pilot study and its analysis have a crucial role in ensuring construct validity.

In the analysis process for the main study, several steps were executed for the instrument that had been previously checked for face and content validity. Item analysis was applied to the instrument's items to examine the item performance. Item analysis is used for evaluating the instrument quality by using the items/questions in the instrument (McCowan & McCowan, 1999). Item analysis provides information about how powerful an item is in discriminating participants and difficulty level of an item. Item difficulty and item discrimination can be considered as the most prevalent statistics reported in the item analysis. Therefore, item difficulty based on average item score and item discrimination obtained by calculating the Pearson Product Moment Correlation, which refers to the correlation between the score on the question and total score on the instrument, were analyzed in the item analysis process of this study.

Reliability coefficient alpha for the measure of internal consistency related to "homogeneity of the items within a scale" (De Vellis, 2003) was calculated. In addition to reliability, validity and item analysis, performances of preservice mathematics teachers on proportional reasoning instrument were examined. Preservice mathematics teachers' answers were evaluated based on answers' accuracy, and the total point was calculated for each participant.

After quantitative analysis of the instrument, explanations done by preservice teachers for the third question were begun to analyze in the instrument. The analysis of these explanations aimed to gain insight into how preservice teachers justify both correct and incorrect statements related to ratio and proportion. In the analysis process, firstly the participants' explanations were transferred to the computer, and these explanations of correct answers were classified as sufficient and insufficient. Before starting the analysis, several key points for each statement were defined, and these were diversified from giving relevant examples to algebraic explanations as read the preservice teachers' explanations. For the second statement "because the ratio is a mathematical notation of a quantity, it is always given in the part-whole relationship", giving examples that represent part-part relationships was also considered as sufficient explanations as well as saying it could be given within the part-part relationships. These explanations also provided additional information on students' proportional reasoning when making conclusions. Also, it gave an idea during the preparation of problems in the interview.

3.4.2 Content analysis of task-based interviews

I began the data analysis in the qualitative part by reading transcripts of all task-based interviews several times to specify overall performances of preservice mathematics teachers on interview questions. For each question, I evaluated their performances in terms of accuracy of answers, providing alternative solutions, justifying answers, and using proper language related to topics ratio and proportion. After evaluating overall performances of participants during the interviews, I reread all transcripts and identified segments which were mostly related to the main research question: "How do preservice mathematics teachers perform on questions requiring proportional reasoning ability?" During the identifying segments, I utilized the previous research studies related to proportional reasoning. I decided to analyze qualitative data through six components of proportional reasoning as Lamon (2005) mentioned. After specified segments as significant in the proportional reasoning, I analyzed the data obtained from all seven participants' written transcripts line by line according to six components of proportional reasoning. So, I investigated their proportional reasoning ability for each question by defining and highlighting relevant parts. Initially, I classified the students' answers and excerpts in terms of these segments on excel. Then I compared them by focusing on similarities and differences in terms of their proportional reasoning level obtained from the instrument and made inferences.

For the second research question "What are the similarities and differences between PSMTs' performances on questions requiring proportional reasoning ability on the task-based interviews and on the PRI?", I constructed a table on excel that includes data about whether participants replied each question correctly or not. After completing tabulation, I focused on the questions in which participants' answers were different in the instrument and task-based intervies and tried to analyze how the answers differ and show similiarities.

Validity and reliability issues are components of good qualitative research as much as of good quantitative research (Creswell & Plano Clark, 2011). Creswell and Plano Clark (2011) emphasize that validity has a major role in the qualitative studies compared to reliability and state its importance to ensure. To establish qualitative validation, triangulation of the data from several sources or several individuals is one approach (Creswell & Plano Clark, 2011). In the scope of the study, the triangulation method was used by collecting data from several sources, such as task-based interviews and written artifacts. Peer debriefing was also conducted by another mathematics education researcher in order to check the trustworthiness of the qualitative analysis.

CHAPTER 4

RESULTS

In this section, the findings in the quantitative and qualitative parts of the current study will be presented. The findings for the quantitative part will be provided in two sections as item analysis of PRI questions and preservice mathematics teachers' performances on PRI. For the qualitative part of the study, results will be given under the three titles including preservice teachers' proportional reasoning, and common difficulties on proportional reasoning, contradictory situations between interview and PRI results.

4.1 The item analysis results of PRI

Within the scope of the current study, reliability and item analyses were conducted for PRI questions both in the pilot and the main study. The reliability coefficient for the pilot study with 69 participants was 0.640. Item difficulties ranged from 1.6 to 2.75, and discrimination indices ranged from 0.10 to 0.66 for the pilot study. Item discrimination is defined as the measurement of how well the item differentiates among participants based on their overall performance on the instrument (McCowan & McCowan, 1999). The discrimination values above or equal to 0.4 are considered as high while the discrimination values below or equal to 0.2 are considered as low (Ebel, 1954). Two questions had item discrimination indices less than 0.4: Q1 (0.105) and Q12 (0.389). Ebel (1954) stated that the power of discrimination is high and the function of an item is satisfactory if the item discrimination calculated by using Pearson product moment correlation is greater than or equal to 0.4. Based on this, Question one (Q1) which has an item discrimination index far less than 0.4 was improved by using more clear statements in the main question (using the statement "for all stretch and shrink questions including direct proportion" rather than "all ratio and proportion questions"). Question 12 was reviewed for improvement; however, no changes were made in Question 12 because its item discrimination index (0.389) was very close to 0.4. This may be due to the low number of participants in the pilot study (Crocker & Algina, 2008).

In the scope of item analysis, item difficulty and item discrimination were computed. For each question in PRI, item means and variances were computed. It was revealed that Q3 which is related to evaluating the definitions and statements about the ratio and proportion, Q4 which is related to deciding squareness of the photographs and Q11 which is related to invariance and covariance structures of the proportional reasoning specific to enlargement of a rectangle were moderately difficult questions (p = 1.23, p = 1.95 and p = 1.70 respectively) among the questions in the PRI. Questions in the PRI except these three (Q1, Q2, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13) were moderately easy (*i.e. having p values* 2.35; 2.33; 2.56; 2.40; 2.87; 2.54; 2.87; 2.49; 2.28; 2.52 respectively) for preservice mathematics teachers as seen Table 7. It was deduced that the overall instrument was moderately easy (p_{mean} = 2.316) for preservice mathematics teachers participating in the main study.

Questions in the PRI except for Q3, Q5, Q6, Q7, and Q9 were good discriminators, as seen in Table 7. Discrimination indices of those questions that range between 0.2 and 0.4 (Q3, Q5, and Q6) were moderate, while the discrimination indices of those questions below the value 0.2 (Q7 and Q9) were low. Considering both item difficulties for Q7 and Q9 (p = 2.879; 2.878 respectively), it can be regarded that both questions were easy for almost all participants and probably for that reason their abilities to discriminate between preservice teachers who have high scores and those who have low scores on PRI were low (r = 0.144 and r = 0.106). Indeed, Q9 is a common question type related to proportions that the preservice teachers have faced since the primary education, and Q7 is a question that requires knowing about different representations (percentages, fractions, and rational numbers) of ratios relationships of which are emphasized in the Turkish mathematics curriculum (MEB-TTKB, 2009).

Table 7. The Item Difficulty and Discrimination Indices for PRI

Question	Item Discrimination	Item Difficulty
Q1	0.403	2.35
Q2	0.528	2.33
Q3	0.343	1.23
Q4	0.576	1.95
Q5	0.293	2.56
Q6	0.256	2.40
Q7	0.141	2.87
Q8	0.481	2.54
Q9	0.102	2.87
Q10	0.424	2.49
Q11	0.589	1.70
Q12	0.405	2.28
Q13	0.466	2.52

Consequently, it was concluded that proportional reasoning instrument was moderately easy for preservice mathematics teachers in the study and its items had moderately high discrimination.

4.2 Preservice mathematics teachers' proportional reasoning performances

The aim of the current study is measuring the proportional reasoning ability of preservice mathematics teachers. In this section, preservice mathematics teachers' proportional reasoning performances will be given within two sections. For the performances of PSMTs in two sections: overall performances of PSMTs on PRI and PSMTs performances for each question on PRI, descriptive statistics, including frequency tables and graphs, measures of central tendency, and variability related to participants' proportional reasoning ability as measured by their PRI performances, will be given.

4.2.1 Analysis of overall performances of PSMTs on PRI

The results related to their PRI performances were demonstrated in Table 8. It is shown that the average score of preservice mathematics teachers was 30.17 (SD = 4.32), and the scores range from 16.41 to 37.87. Considering that the possible total scores from the instrument ranges between 0 and 39, it can be inferred that preservice mathematics teachers' performances on PRI were high as seen in Figure 3.

Table 8. Proportional Reasoning Performances of Preservice Mathematics Teachers

	N	M	Min	Max	SD
Proportional Reasoning Instrument (PRI)	261	30.17	16.41	37.87	4.32

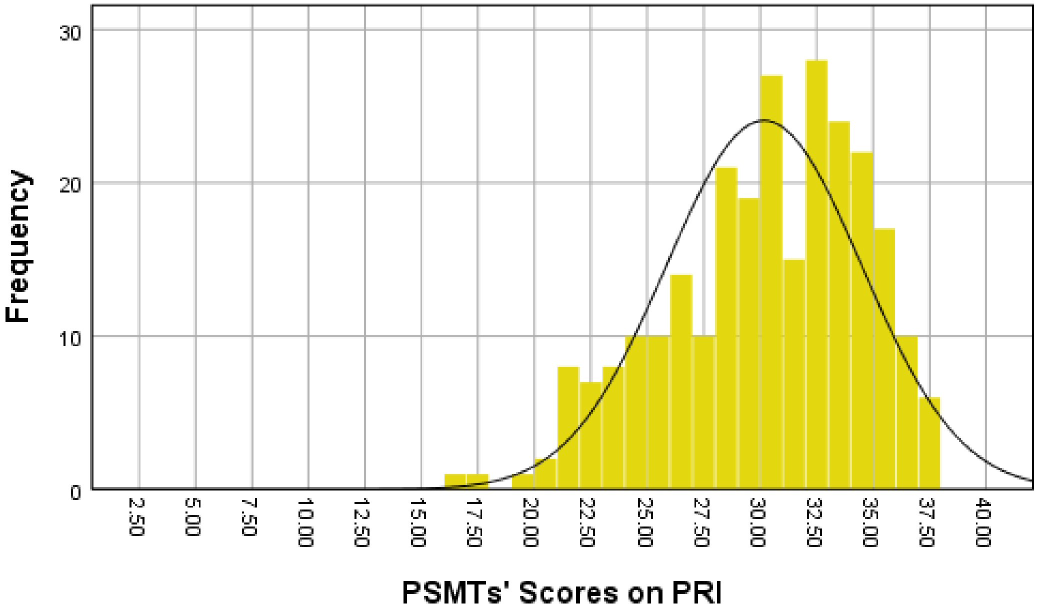


Figure 3. Distribution of PSMTs' scores on PRI

4.2.2 Analysis of PSMTs' performances for each question on PRI

Besides the total scores of the participants, a distribution of correct and incorrect answers for each question in PRI provides information about the overall strengths and weaknesses in preservice teachers' proportional reasoning ability. Figure 4 shows the distribution of PSMTs' answers as correct, partial and incorrect according to the question in the PRI. Answer labelled as correct in the figure state that all sub-questions in the questions are answered correctly while answer labelled as incorrect means that none of the sub-questions are correctly answered. So, partial answers occur when at least one of the sub-questions was correctly answered. There is no partial answer for multiple choice questions (Q8, Q10, Q11). The only exception among multiple choice questions is Q4, which has sub-questions and these sub-questions cause it to have partially correct answers.

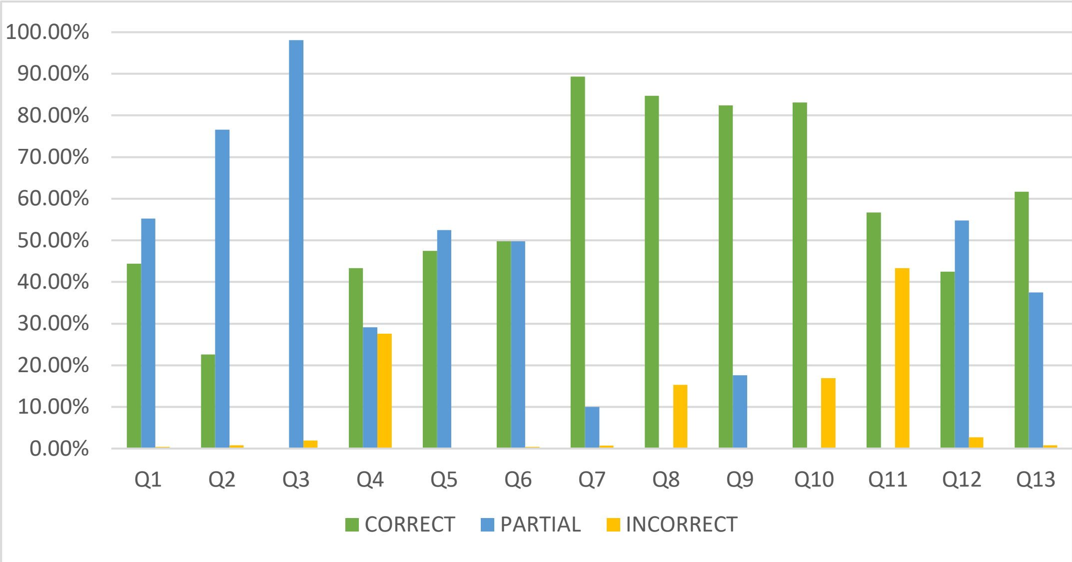


Figure 4. PSMTs' answers for each question in the PRI

Q1 (see Appendix C, Table C1) requires ability to evaluate three alternative students' solutions to an enlargement problem. According to Table 9, 43% of participants correctly decided whether each of the three alternative solutions are valid or not. While the unit strategy and cross-multiplication algorithm were easily recognized as valid solutions by preservice teachers, approximately half of the participants (47% of them) were not able to make the right decision for the solution of student B, which was built on use of within ratios.

Table 9. Scores Obtained from Q1

Scores	Frequency	Percent	Cumulative Percent
0.00	1	0.4	0.4
1.00	19	7.3	7.7
2.00	128	49.0	56.7
3.00	113	43.3	100.0

In Q2, preservice teachers were expected to categorize seven problem situations as proportional and non-proportional. Although the differentiation the proportional situations from non-proportional ones varies in terms of problem context among participants, 77% of preservice teachers were able to reach the right decisions in at least five out of seven different problem contexts as indicated in Table 10. Problems based on the running track and laundry drying were the most challenging situations for teacher candidates to distinguish: nearly half of the participants for each question (respectively 38% and 47%) interpreted these questions wrongly as proportional situations. However, the preservice teachers with a high percentage (87%; 97% and 93% of participants respectively) appropriately identified the age problem as a non-proportional situation; speed and similarity problems as proportional situations to which they were very familiar due to the emphasis on these contexts in the Turkish mathematics curriculum.

Table 10. Scores Obtained from Q2

Scores	Frequency	Percent	Cumulative Percent
0.43	2	0.8	0.8
0.86	5	1.9	2.7
1.29	18	6.9	9.6
1.71	34	13.0	22.6
2.14	49	18.8	41.4
2.57	94	36.0	77.4
3.00	59	22.6	100.0
Total	261	100.0	

Q3, which was about evaluating and explaining statements related to ratio and proportion, is one of the questions with the lowest success rate in the PRI. As shown in Table 11, no one had three points from this question. Three points would mean that they could decide whether the statements were accurate and they could justify their reasons clearly. Although preservice teachers were able to reach correct answers for the statements, most of them were not able to justify their answers. Majority of the participants (80% of preservice mathematics teachers) answered incorrectly the last statement of the question related to inverse proportion. They ignored the necessity of a constant ratio of change in the opposite direction and found it sufficient to state that one increases as the other decreases when describing the inversely proportional relationships among quantities. Some examples from students' answers are as follows:

- "Increasing and decreasing are opposite to each other. That is why they are inversely proportional".
- "Because an increase occurred in one and decrease occurred in another and these are opposite relationships".
- "Because of the definition of the inverse proportion".
- "Because we have learned like that".

And also a moderate percentage of the preservice teachers (33%) thought that ratios should be given within the part-whole relationship by ignoring ratio as a multiplicative comparison of any quantities.

Table 11. Scores Obtained from Q3

Scores	Frequency	Percent	Cumulative Percent
0.00	5	1.9	1.9
0.375	25	9.6	11.5
0.75	71	27.2	38.7
1.125	48	18.4	57.1
1.50	51	19.5	76.6
1.875	27	10.3	87.0
2.25	26	10.0	96.9
2.625	8	3.1	100.0
3.00	0	0	100.0
Total	261	100.0	

For the question, Q4, approximately 28% of preservice teachers were not able to answer correctly as seen in Table 12. To give an example, some of the preservice teachers who responded that both photos have the same shape tried to determine their squareness by focusing on the differences between the sides within each photograph. Therefore, they were not able to realize the multiplicative structures of the ratio and failed to recognize “the squareness as a ratio of one to one” (Johnson, 2013, p. 57). In other words, they reasoned additively as suggested by Bright et al. (2003). Those who were able to realize the multiplicative structures of the ratio had difficulty in evaluating alternative solutions of students especially in the second part: Indeed, only six participants were not able to correctly label the first answer as a valid solution while 70 participants were not able to choose the second answer as a valid solution. So, results point out that preservice teachers had less difficulty in the comparison of within-ratios for each photograph considering the ratio of edges should be close to one for more squareness while they did not make sense of the second solution which includes comparing the change factors within each photograph.

Table 12. Scores Obtained from Q4

Scores	Frequency	Percent	Cumulative Percent
0.00	72	27.6	27.6
2.25	76	29.1	56.7
3.00	113	43.3	100.0
Total	261	100.0	

In Q5, preservice teachers were asked to decide whether given linear equations and graphs have proportional relationships or not. About half of the preservice teachers (47.5%) performed all four sub-questions successfully, including two graphs and two equations as illustrated in Table 13. The graph which does not pass through the origin (third sub-question) was the most incorrectly answered sub-question among preservice teachers in this question (p = 0.55). This showed that those preservice teachers were not able to internalize the idea that if the equation of a linear function involves both addition and multiplication operations, then the variables involved are not proportional. In other words, nearly half of the PSMTs misidentified non-proportional linear situation presented in graphical representation as proportional and visually associated the linearity with the proportionality without paying attention to the multiplicative relationships between quantities. On the other hand, other sub-questions were answered correctly with a high percentage (98.8%, 97.3%, and 89.6% respectively).

Table 13. Scores Obtained from Q5

Scores	Frequency	Percent	Cumulative Percent
0.75	2	0.8	0.8
1.50	13	5.0	5.7
2.25	122	46.7	52.5
3.00	124	47.5	100.0
Total	261	100.0	

In Q6, preservice teachers were asked to classify three students' explanations about whether x and y values in the given table are proportional or not, as valid and invalid. Table 14 indicates that majority of preservice teachers (90.8%) identified given answers correctly in terms of their validity. While preservice teachers performed successfully in deciding solutions including scale factor and equivalent fractions as valid solutions (p = 0.977 and p = 858), they experienced difficulties mainly in deciding on the solution focusing on the differences between values of the variable as an invalid solution (p = 0.567).

Table 14. Scores Obtained from Q6

Scores	Frequency	Percent	Cumulative Percent
0.00	1	0.4	0.4
1.00	23	8.8	9.2
2.00	107	41.0	50.2
3.00	130	49.8	100.0
Total	261	100.0	

Q7, which is about different representations of ratio, is the most correctly answered question. Preservice mathematics teachers performed successfully with high percentages on all three sub-questions (p = 0.938; 0.969 and 0.961 respectively). Table 15 displays their partial scores obtained from the question and indicates that vast majority of preservice teachers (89.3%) got a full point from the question by realizing all representations (percentage, ratio, and decimals) were appropriate to find and express the best 2-point shooting performance.

Table 15. Scores Obtained from Q7

Scores	Frequency	Percent	Cumulative Percent
0.00	2	0.7	0.7
1.00	2	0.8	1.5
2.00	24	9.2	10.7
3.00	233	89.3	100.0
Total	261	100.0	

Majority of the participants (84.7%) reached the correct answer in the multiple-choice question (Q8) which is related to inversely proportional relationships. Although it is concluded that the general performance of preservice teachers was good on this question, it is remarkable that 34 out of 40 participants who made the question wrong were in the lowest scorers' group.

Another question which has a high percentage of correct answers among the preservice mathematics teachers following the seventh question is the ninth question, which requires categorization of mathematical statements involving direct or inverse proportionality. At least four out of five mathematical statements were performed successfully by a vast majority of participants (97%) as seen in Table 16.

Table 16. Scores Obtained from Q9

Scores	Frequency	Percent	Cumulative Percent
0.60	1	0.4	0.4
1.20	1	0.4	0.8
1.80	6	2.3	3.1
2.40	38	14.6	17.6
3.00	215	82.4	100.0
Total	261	100.0	

Majority of preservice teachers performed successfully in the question requiring evaluation of different solutions (unit stategy in within and between situations and equivalence fractions) to the lemonade taste question. Indeed, the ratio of the correct response to this question is high (83%). It means that majority of the preservice teachers were able to recognize unit strategy in within and between situations and equivalence fractions and identify them as valid solutions for comparison problems.

Table 17 shows that about half of the participants (43.3%) had difficulty in the stretchers problem focusing on the proportionality constant, and invariance and covariance structure of the proportional relationships, as addressed by Q11. Indeed, they were not able to realize what remains constant in changing situations and what changes simultaneously within quantities in the case of stretchers problem.

Table 17. Scores Obtained from Q11

Scores	Frequency	Percent	Cumulative Percent
0.00	113	43.3	43.3
3.00	148	56.7	100.0
Total	261	100.0	

Q12 requiring the reasoning about invariance and covariance structures of proportional relationships in algebraic expressions and as seen in Table 18, about half of the participants (42.1%) performed successfully to decide all whether algebraic equations are valid or not. In particular, 45% of participants did not get the correct answer for the sub-question of $\frac{x_5}{x_6} = \frac{y_1}{y_2}$ which means that they got confused about what is invariant in proportional reasoning and were not be able to realize the ratio is constant within related x and y's.

Table 18. Scores Obtained from Q12

Scores	Frequency	Percent	Cumulative Percent
0.00	7	2.7	2.7
1.00	23	8.8	11.5
2.00	121	46.4	57.9
3.00	110	42.1	100.0
Total	261	100.0	

Table 19 shows the scores preservice teachers got from the Q13, which is about proportional relationships unique to the slope (mentioned as steepness in PRI). Majority of preservice teachers (90.8%) evaluated accurately at least two out of three statements presented in the context of the slope. When the percentage of correct answers were analyzed for each sub-question, it can be concluded that all statements were performed successfully with a high percentage (p = 0.79; 0,77 and 0.96).

Table 19. Scores Obtained from Q13

Scores	Frequency	Percent	Cumulative Percent
0.00	2	0.8	0.8
1.00	22	8.4	9.2
2.00	76	29.1	38.3
3.00	161	61.7	100.0
Total	261	100.0	

As seen in the quantitive analysis results, the majority of PSMTs obtained high scores on PRI. They performed well (i.e. a high percentage of correct answers) in the questions focused on employing rules but without necessarily reasoning. In other words, many PSMTs were successful on questions that can be solved by applying procedures or assigning numbers to the variables like the bicycle problem (Q8), problems requiring decisions about whether quantities are directly or inversely proportional (Q9), problems related to different representations of the ratio (Q7) and sub-questions or solution strategies that were so familiar to them from the middle school years (age problem, speed problem, unit strategy, cross-multiplication rule).

On the other hand, they have difficulty in particular questions that requires more explanation and reasoning in addition to employing rules in solution process. In other words, PSMTs performed unsatisfactorily in the questions which are mainly related to understanding the invariance and covariance structures of the proportional relationships (Q4), realizing multiplicative and additive relationships in the graphical representations (graphs which represent the non-proportional linear situation), and deciding and justifying whether the definitions and properties of ratio and proportion are correct (Q3).

Consequently, quantitative analysis results indicate that PSMTs' have more difficulty in the components of proportional reasoning with more emphasis on SCK than in the components of proportional reasoning with more emphasis on CCK.

4.3 Results from the interviews

In order to reach an in-depth understanding for preservice teachers' proportional reasoning and answer the question: "What are the similarities and differences between PSMTs' performances on questions requiring proportional reasoning ability on the task-based interviews and on the PRI?", a qualitative part of the study was conducted with seven preservice mathematics teachers. Results of data obtained during the task-based interviews will be presented within three sections: i) Preservice mathematics teachers' proportional reasoning, ii) Common difficulties among preservice mathematics teachers and iii) Contradictions between preservice mathematics teachers' proportional reasoning as manifested by PRI and task-based interviews.

4.3.1 Preservice mathematics teachers' proportional reasoning

In this section, preservice teachers' proportional reasoning will be presented within three parts as highest, average, and lowest level reasoners based on their scores obtained from PRI. Their proportional reasoning is considered in detail according to six components of proportional reasoning, as Lamon (2005, 2007) mentioned

It is important to clarify that these components consist of both procedural and conceptual competence in proportional reasoning. While understanding and explaining the invariance and covariance structures of proportional relationships are associated with conceptual competence, solving these proportional problems with an algorithm is mainly associated with procedural competence. In other words, they complement each other in the proportional reasoning process. Therefore, these characteristics can be considered as sub-domains, which are indicative of preservice mathematics teachers' proportional reasoning in terms of procedural competence as well as conceptual competence.

4.3.1.1 Proportional reasoning of highest scorers on PRI

In this section, proportional reasoning ability of two highest scorers in PRI will be presented with illustrative examples. These scorers were Preservice Mathematics Teacher 1 (PSMT1) who obtained 37.5 out of 39 points on PRI and Preservice Mathematics Teacher 2 (PSMT2) who obtained 36.32 out of 39 points on PRI. In parallel with their performances on PRI, PSMT1 and PSMT2 were able to provide alternative solutions for proportional problems, use terms "ratio and proportion" properly, make conceptual explanations as well as they answered nearly 90% of interview questions as seen in detail in Appendix F, Table F1.

PSMT1 used a variety of solution strategies for several problems during the interview. This was an indicator of the flexible use of proportional reasoning. For the enlargement problem, he provided two additional strategies as well as cross multiplication algorithm. He explained his first solution strategy by stating:

> First of all, I find the ratio of width and length. If the shape of the rectangle will be preserved, the ratio is also preserved in the same way, a width-length ratio. So, the ratio of width to length is like this (pointing the ratio $\frac{6}{7}$), I applied it to the second rectangle.

The above excerpt indicates that he was able to recognize the invariant structure in the problem context, which is the multiplicative relationship between the sides, and he correctly used this idea during his thinking about the question. He also solved the enlargement problem by using a scale factor when I asked for an alternative solution.

He emphasized that width should also enlarge with the same ratio in the second rectangle as the length enlarged, by stating that $\frac{20}{7}$ represents the ratio at which the length of the first rectangle enlarged. His work on this problem showed clearly that he was able to recognize what is invariant in both within and between ratios as well as providing a variety of solution strategies fed by the flexible use of proportional reasoning. However, although he explained in his solution that $\frac{6}{7}$ represents the ratio of width to length and it should be applied to the second rectangle, he was not able to make sense of Student B's solution including the operation $\frac{6}{7} \times 20$. He defined this solution as mathematically deficient.

In addition to the enlargement of the rectangle problem, he was also able to provide additional solution strategies for the faucet problem involving inverse proportionality. He got the correct answer by setting a proportion and using the invariance of inverse proportions, which is the multiplication of quantities comes from different measure fields. When I asked him whether he could solve it in another way, he explained the strategy in the following way:

> …If 11 faucets fill the pool with water in 13 hours, I thought how many hours it would take one faucet to fill the pool. It lasts as slow as 11 times, so the time for it is 11×13 hours. Moreover, then with the same logic, seven faucets fill the pool as fast as seven times if one faucet fills the pool 11×13 hours. It is because I divided that time by seven.

The excerpt shows that PSMT1 was able to solve the inverse proportion problems with unit strategy as well as he was able to set and solve an inverse proportion algorithm. It is also shown in the excerpt that PSMT1 internalized the invariance structure of inversely proportional relationships and applied the invariance to an alternative solution without confusion. When I asked him what 11×13, obtained as a result of the multiplication of the number of faucets and the number of hours, represents, he responded: "it is the water in the full pool". As seen in PSMT1's responses, what 11×13 means was interpreted as hours for filling the water in the first answer and as the water in the pool in the second. These two different replies imply two different interpretations: He may have reasoned proportionally and used these terms (hours and the water in the pool) consciously. Thefore, it can be considered as indicative of PSMT1's capability about making sense of the invariance structure of inversely proportional relationships within the problem context and expressing it. On the other hand, he may have used these terms unconsciously. He might not be even aware that these two different units are same within the context of the problem. Had I been a better interviewer I would have asked a further question including what he meant for the clarification of his reasoning or what the differences were between them.

PSMT2, who has the second highest score on PRI among the interviews' participants, had manifestations of proportional reasoning involving conceptually strong explanations of procedures she used and the flexible use of a variety of strategies. She solved the first question with an alternative solution in addition to cross multiplication algorithm, and expressed the solution strategy as follows:

> In one solution, I found the ratios of sides for each rectangle and then applied cross multiplication ($\frac{6}{7} = \frac{a}{20}$). In another, the length is seven and the width is six, the relationship between them is that seven is multiplied by $\frac{6}{7}$. Therefore, 20 also should be multiplied by $\frac{6}{7}$ to preserve the same ratio.

According to her explanation above and written artifact for the related question, it can be claimed that for the second solution, she used the multiplicative relationship within a rectangle and applied it to the enlarged rectangle considering that this multiplication remains constant at all time.

When I asked her to explain what $\frac{6}{7}$ refers to, she explained that it referred to "the relationship between the length and width, … in other words, it is about how six was obtained from seven" and added, "ratio must be constant because it refers to the change of two quantities relative to each other". Although she provided only one alternative solution apart from the cross-multiplication algorithm, she was able to apprehend the ratio as multiplicative relationships between length and width and the necessity to apply this relationship to the other rectangle for the same structure of the shape. Such an emphasis by PSMT2 on the multiplicative relationship between quantities and the change relative to each other can be considered as indicative of both the accurate usage of ratio and proportion language and the recognition of the relative change idea, which are underpinnings of proportional reasoning.

She was also able to make alternative explanations for deciding whether quantities are directly or inversely proportional rather than giving numerical values for each quantity to make such decisions. When I asked her to decide on the type of proportionality in the fifth question (see Appendix D & E) without assigning any numerical values to quantities, she reasoned in the following way:

> Without giving numbers, I compare $\frac{a}{b}$ values. If I obtain the mathematical statement as $\frac{a}{b}$ then I can say that a directly proportional to b. Because $\frac{a}{b}$ refers to the multiplicative relationship between the quantities and as a increases and b increases by the same ratio. However, I can not obtain $\frac{a}{b}$ in the last statement in the question. Always I get a×b. Being two quantities in multiplication and resulting in a constant value shows the necessity of one increasing and the other decreasing.

As seen in the answers of PSMT1 and PSMT2, both preservice teachers provided conceptual explanations to the problems. A conceptual explanation for this context means that it involves statements focusing on the meaning of ratio that is constructed with quantities coming from the different measure spaces and focusing on the concepts and their properties rather than just explaining how to perform the algorithm.

Both preservice teachers with high scores from PRI, differentiated proportional situations from non-proportional ones in graphical representations as well as in word problems. They were able to recognize the multiplicative and additive relationships between quantities even though they are linear functions that were represented graphically and explained their solutions precisely by emphasizing the constant ratio:

> This is a graph of y = -x. We can see $\frac{y}{x}$ = -1. However, this, I do not know exactly but, it is a kind of y = ax+b…This does not pass from the origin (referring to the first equation); in other words, it is not something like that y = x, y = ax. There is extra, +b. In this case $\frac{x}{y}$ does not give a constant ratio (PSMT1).
>
> I wrote the equations for each line. In terms of their equations, the first one is y = 3x+6. If this is y = 3x (showing the y = 3x+6), I would say there is a proportional relationship. Because for every x, I obtained y as three times x. However, +6 broke the proportionality. The second is a graph of y = -x. For every x, I obtained y as negative x. The ratio of x to y is -1. So, the first one is not proportional but the second one is… It goes through origin…If for every value of x, there is y that is multiple of x, in other words, if there is no extra addition or subtraction operations in the equation, we can say there is a proportional relationship (PSMT2).

This episode shows that both students have an understanding that having both multiplication/division and addition/subtraction operations in the equation, violate the proportionality for linear equations as in the first graph. When I asked them to make generalizations about proportionality based on the two graphs, they both were aware that for a line, passing from the origin and being linear were sufficient for proportionality of the variables involved.

For the mosaic problem, preservice teachers were asked to make sense of the student's solution in which between ratios are used. Both participants (PSMT1 and PSMT2) were able to recognize that the student reached an answer by using the factor of change within the same measure fields, called between ratios. When I asked them how they solved the mosaic problem, they also replied that they prefer solving this question by finding an area per minute and multiply it with 36 minutes. It means that both prefer using the within ratios in the mosaic problem. Making sense of the student's solution, which includes between ratios and solving the problem with within ratios are considered as a sign for flexible thinking in proportional problems.

4.3.1.2 Proportional reasoning of average scorers on PRI

PSMT3, PSMT4, and PSMT5 are preservice mathematics teachers whose scores were very close to the mean of the group; i.e. 30.29, 29.82 and 28.96 on PRI respectively. On the contrary to highest scorers' performances during the interview, they were able provide conceptual explanations to the several questions. Although they reached the correct solutions for the problems, their justifications remained limited to the procedurally-based explanations most of the time. They had difficulty to distinguish proportional situations from some non-proportional ones. They mostly used the terms "ratio" and "proportion" interchangeably or used the terms together as if they are the same constructs. Table F2 (see Appendix F) provides detailed information about their performances during the interviews.

Considering their ability to solve problems with different strategies, it can be said that they were able to solve the enlargement problem with at least two different strategies. However, their solutions mainly relied on the cross-multiplication algorithm, which was based on firstly setting up a proportion and then solving the equation. Although PSMT3 and PSMT5 were aware of the ratio between lengths in different rectangles referring to scale factor and that it must be held to remain same for the other measure field by stating that "I found the enlargement ratio between lengths and then I multiplied this ratio by the width" (PSMT3) and "…because the increase from seven to 20. So the length of the rectangle changed from seven to 20. It means the enlargement occurred with a particular ratio" (PSMT5), they were not able to comprehend the value of $\frac{6}{7}$ in the student's solution as the multiplicative relationships between side lengths. The following interaction can be considered as a manifestation of a preservice teacher's inability to make sense of student's reasoning:

> R: Are there any solutions like your solutions? Are they valid?
> PSMT5: …(Pointing at student B's solution) $\frac{6}{7}$ is the ratio but student directly multiplied it with 20. This is not like my solutions.
> R: What did this student do?
> PSMT5: Firstly, the student found the ratio width to length in the first shape and then multiplied this value with 20. Why did the student multiply by 20?
> R: Is this a valid solution?
> PSMT5: I think this is not a valid solution. S/he maybe thinks like that: the ratio in the first rectangle is $\frac{6}{7}$ and s/he wants to reach the same ratio for the second rectangle. And then multiply the ratio by 20 by using length. However, I think this is not reasonable.

Although PSMT4 set up the proportions for both within and between measure fields, he did not pay attention to the meaning of the ratios and could not provide any alternative explanations apart from the ratios having to be equal:

> PSMT4: Because these two rectangles are similar, the ratio of lengths must be equal to the ratio of widths. Then I found a is equal to $\frac{120}{7}$ by using these ratios. For the first one, because the ratio of width to length is not changed, I mean, the ratio of width to length in the first rectangle is same to the ratio of width to length in the second one, the result is the same as my other solution.
> R: …For example, what does $\frac{20}{7}$ refer to?
> PSMT4: The ratio of lengths.

He also had difficulty like the other two preservice teachers who had average scores on PRI in deciding whether the second student's solution was a valid solution or not, because he was not able to apprehend that the value $\frac{6}{7}$ represented the multiplicative relationship between side lengths within a rectangle. Nevertheless, preservice teachers with average scores still used the cross-multiplication algorithm correctly and this can be as an indicator of part of understanding that they know how and when it is applied (Lamon, 2007).

In addition to the enlargement problem, they also had difficulty in providing alternative explanations to the problem about whether quantities are directly or inversely proportional. Almost all participants attending the interviews determined the type of proportionality by assigning numbers to the unknowns. When I asked them to decide whether quantities represented in algebraic notation are directly or inversely proportional, PSMT3 responded by saying “without giving numbers; I can not understand their change together”. PSMT4 also replied to me, “most probably I cannot do” and then I wanted him to think about it for a couple of minutes. After a while, he explained his reasoning as:

> I am not sure whether it is correct, but for example, when a and b are on opposite sides of the equation, this is a generally direct proportion. However, when they are on the same side as the product, this is an inverse proportion.

This excerpt reveals that this preservice teacher was not able to detect the invariance structures of direct and inverse proportion, but rather focused on superficial features of algebraic expressions relating to these proportional contexts. He reached the alternative explanation through patterns in the equations. So it can be concluded that he was not able to associate this change with the invariance structures of the proportional situations although he was able to recognize that the related quantities changed together.

For this group of preservice teachers, it can be said that they had difficulty in differentiating non-proportional relationships from proportional ones. Indeed, all three preservice teachers were not able to recognize the additive nature of the running track problem. Even though they explained their solutions verbally during the interview, they did not realize the additive structure and solved the problem erroneously by cross multiplication algorithm. Although they had a tendency to reason multiplicatively in non-proportional problems, they did not tend to reason additively for the area conversion, and all of them accurately decided the area conversion as a proportional situation. However, the tendency to the use of proportional algorithm did not occur in the age problem, which is an additive problem as the running track problem. As Van Dooren et al. (2005) stated, the difference may stem from the unfamiliar context of the running track problem. Because preservice teachers were familiar with the context of age problem from their everyday lives, they were able to easily recognize the additive structure in the age problem compared with the running track problem. In addition to unfamiliarity, running track problem requires participants to pay more attention to particular situations such as running at the same speed.

For the third non-proportional situation, PSMT3 was able to realize that the constant factor, the entrance fee for a car distorts the proportional situations: "multiplying number of people by the fee is proportional, but because there is also the car's fee, the proportional situation is distorted". On the other hand, PSMT4 decided its non-proportionality through numbers rather than realizing the constant factor and did not give any conceptual explanation: "for five people, 20 TL was paid, but for ten people 30 TL was paid…If we obtained 40 TL rather than 30 TL, we could say it is a proportional situation". PSMT5 had difficulty in deciding whether it is proportional or non-proportional because he was not sure that a car's fee distorts the proportionality. However, he ensured that it is a non-proportional situation by comparing ratios of the number of people to the total fee with given numbers.

These explanations were also parallel to the preservice teachers' explanations for the question about linearity and proportionality. PSMT3 was able to recognize that x and y's are not proportional in the first graph by using linear equation and she stated: "…but here this constant (pointing at the constant value in the equation) violates the proportional relationship". On the other hand, PSMT4 and PSMT5 were able to explain their answer by assigning numbers for variables like in the parking problem. While PSMT4 used a table for comparing x and y, PSMT5 wrote the related line equation. When I asked them to provide alternative solutions, PSMT4 was not able to provide any explanation. On the other hand, PSMT5 was able to make an explanation focusing on the constant of proportionality as the following:

> …I can do like that: dividing by x for each side I can obtain $\frac{y}{x} = 3$. It means the ratio of y to x is three. However, there is a constant here. Because of that, there is no proportional situation.

Based on their explanations above, it can be concluded that preservice teachers whose scores were in close proximity of the mean score of the participants, provided a limited explanation at some points for proportionality of quantities in the problems and graphs compared to the highest scorers on PRI. They mainly focused on the numbers assigned to the variables and made a decision through these numbers rather than the multiplicative relationship between quantities, namely ratios.

For the mosaic problem, PSMT3 was not able to recognize the student's solution by using between ratios, 36:16 = 2.25 2.25×40 = 90, as a valid solution. She was not able to explain correctly the meaning of 2.25 and interpreted this solution as a different way of using cross-multiplication algorithm that was performed by a student:

> PSMT3: Student's solution is correct. There is a direct proportion here: They worked together, there is no change in the workers, no change in their speed. If the minutes increase, then the area they paint also is increased…Hmm, the student divided minutes each other, I mean, s/he divided 36 by 16. If s/he answered because s/he thinks to use cross multiplication algorithm by setting a direct proportion, as a result, s/he may have done something like this. Because there is no necessity to multiply 40 by 36 firstly, s/he can divide initially and then multiply.
> R: This value, 2.25, is this a meaningful value? What does it mean?
> PSMT3: Hmm, actually it will be better if s/he writes it as $\frac{36}{16}$. I express the ratio with a fraction. It is okay because I set the proportion, but if s/he

> represent the ratio as fraction notation, it will be more meaningful…We might think that it is a little memorization. Because when I found the ratio, $\frac{36}{16}$, I do not mean I want to divide 36 by 16. Actually, I show the ratio between them. What did s/he do? Division and the solution. It is like a slightly result oriented. So, s/he recited if s/he knows the algorithm. Erm…she may not know what the ratio is, she found the result directly.

The excerpt above indicates that PSMT3 ignored the meaning of the ratio $\frac{36}{16}$ in its context and appraised it only as of the fraction notation. As she explained the scale factor in her own solution to the enlargement problem as an enlargement ratio, not the quotient of quantities in the same measure fields, she was not able to reason about the ratio $\frac{36}{16}$ consequently 2.25, as the multiplicative change in the minutes. Although she knew that these quantities both coming from the same measure fields, time, she was not able to use and make sense of the quotitive division. In other words, she was not able to interpret the ratio $\frac{36}{16}$ as how many times 16 goes into 36, which is 2.25 times. Instead, she interpreted the student's solution as a different way of applying the cross-multiplication algorithm after setting the proportion as $\frac{36}{16} = \frac{?}{40}$.

On the other hand, PSMT5 was able to recognize the ratio $\frac{36}{16}$ as a multiplicative change in the amount of time and explain that the area, 40 cm^2 needs to be multiplied by the same number, 2.25, by saying "…when we divided 36 by 16, it is 2.25. It means 36 is 2.25 times 16. So, the area also enlarged with 2.25". However, he was confused in reasoning about the proportion between areas and minutes during the interview. Then he concretized the problem by thinking about the unit tiles (10 cm^2 as 10 unit tiles) and made sense of the problem. It suggests that he was not able to apprehend that ratio is a multiplicative comparison of any quantities without considering its dimension.

Like PSMT5, PSMT3 also was not able to think about the meaning of the ratio $\frac{36}{16}$ as a quantity,or how many times minutes increased, and that the value of 2.25 represented the factor of change. She was not able to make sense of the result of the division $\frac{36}{16}$, even though she was able to comprehend and explain 2.5 cm^2 as the work done per minute in his solution. This was an indicator of his difficulties to make sense of contexts having proportionally related quantities in a flexible way by using multiple approaches or strategies.

In inverse proportion problems, especially problems that presented a context, e.g. faucet and bicycle problems, PSMT4 and PSMT5 solved problems by using their knowledge about the product of the measures being invariant. They explained the invariance structures and identified the distance for bicycle problem and the amount of water to fill the pool for the faucet problem as being invariant. PSMT3 solved the bicycle problem considering that the number of turns and the radius of the wheels are inversely proportional because the distance for both wheels is the same. However, she solved the faucet problem by applying the inverse proportion algorithm mechanically as seen in Figure 5 without any recognition of what is invariant within the context.

$$\left.\begin{array}{ccc} \text{Faucet} & & \text{Time} \\ 11 & \longrightarrow & 13 \\ 7 & \longrightarrow & x \end{array}\right] \qquad \begin{array}{c} 11 \times 13 = 7 \times x \\ x = \dfrac{143}{7} \end{array}$$

Figure 5. PSMT3's solution for the faucet problem

Although she knows that time and number of faucets covary in the opposite direction, she did not have an understanding that change in quantities occurs at the same ratio as shown in the excerpt below:

> I know that the time to fill the pool increases as the number of faucets decreases. Errrrm, the situation that as one increases the other decreases does not occur in direct proportion. So, I thought that this is an inverse proportion. However, multiplication of those is something like that we code some procedures: such as for direct proportion do this, for inverse proportion do that. I solved it directly from there, but that was my way of thinking.

When I asked her to solve this question with any other solution upon her disclosure on following some guidelines about when to use which procedure, she tried to think about and reach the solution through the unit strategy – thinking about one faucet. However, she was confused when following the unit strategy and erroneously set up a direct proportion:

> PSMT3: I can find the time for one faucet (setting direct proportion). But not again, then it would be directly proportional…If I found one faucet, I said I found it, if it asks for seven then I have to multiply by seven. This time it does not match (pointing to the previous answer) Either how I am thinking now is wrong, or this is wrong.
> R: Which solution is valid? Which solution are you sure of?
> PSMT3: I seem to be sure of my solution (inverse proportion algorithm) because we're used to it. But if I look for a different solution, I think if I go over it, I don't know.

As can be seen, PSMT3 applied the inverse proportion algorithm by only considering that she uses that algorithm for similar questions and provided no explanation with a conceptual basis. A limited understanding of what is invariant in inverse proportions may have also caused her to be confused in the solution based on thinking how long it would take for the pool to be filled by one faucet.

4.3.1.3 Proportional reasoning of lowest scorers on PRI

PSMT6 and PSMT7 are preservice mathematics teachers whose scores were close to each other, 21.21 and 21.08 respectively. PSMT6 and PSMT7 were two of the preservice teachers who obtained the lowest scores on PRI. Both preservice teachers mainly solved questions in a procedural way. It means that they reached a result by setting and solving proportions and they had difficulties in making explanations about its conceptual base. They also underperformed on developing and reasoning with alternative strategies. In contrast to other PSMTs, they were not able to differentiate non-proportional situations from proportional ones. They tended to change frequently their answers because they were not sure about their understanding of ratio and proportion. This situation also led to an increase in the duration of interviews which were conducted with these PSMTs. Interviews with lowest scorers lasted approximately thirty minutes longer than those conducted with highest scorers. Their improper use of language about ratio and proportion also led them to get confused and to change their answers at times (see Appendix F, Table F3 for an overview of their overall performances on each question in the interview).

Although both PSMTs were able to set the proportions and solve them, they had difficulties in providing sufficient explanations. The most conspicuous example of it, was observed in the enlargement of the rectangle problem. Both PSMTs solved the enlargement problem with at least two solution strategies. These strategies are based on mainly setting proportions between quantities in both the same and different measure fields and applying cross multiplication. However, they were not able to clarify their solutions by focusing on the meaning of the ratios. The excerpt from PSMT7's interview can be a good example of this situation:

> PSMT7: For the first solution, I think that there is a relation between sides because the rectangle is enlarged with preserving its shape. So, I constructed a ratio between the lengths of rectangles. Then I obtained a result by constructing a ratio between the widths of the rectangles. (applied cross multiplication). This is the result…For the second one, actually it comes to the same conclusion, but there is a difference: we construct a ratio between sides of rectangles in the first solution…But here I used the ratio of length to width and then the ratio of length and width. Again we got the same result.

As seen above, PSMT7 was able to recognize the multiplicative relationships between sides of lengths and set the proportions accurately. However, when I asked her what $\frac{7}{6}$ refers to in her solution, she responded as follows:

> What I obtain here is the ratio of length to width, $\frac{7}{6}$. In the other solution, the length and widths are enlarged in the same way. For this solution, as the numbers increase the ratio also increases. So, I applied the same thing in this solution. $\frac{7}{6}$ is the ratio between sides in the same rectangle and I thought that this ratio increases and equal to $\frac{20}{a}$.

It can be seen that she was able to recognize that the change in side lengths between rectangles occurred in the same way in the explanation of the first solution. However, for the second one, within ratios, she asserted that the ratio increases as the related numbers increase even though she set the proportion. It is a striking result because her solution is built on proportion concept, which is based on the equivalence of ratios. Consequently, it can be concluded that she has a limited understanding of the meaning of the ratio and proportion and the underlying concept of the cross-multiplication algorithm. She managed to apply the rule and the algorithm but with limited understanding.

PSMT6 also solved the enlargement problem with three solution strategies, including setting up proportions for between and within ratios. But two of the solutions are built on the same proportion, with the only difference being the order of the quantities in the ratios: $\frac{6}{a} = \frac{7}{20}$ and $\frac{a}{6} = \frac{20}{7}$. Differently from PSMT7, he was able to recognize that the ratio remains constant and emphasized the equivalence of the ratios, $\frac{20}{7} = \frac{a}{6}$, by stating, "I thought these are equivalent because the question stated that the rectangle is enlarged with preserving the structure of the shape, so the ratio must be preserved". However, he was not able to make sense of between ratios as a change factor, and he was not able to define $\frac{20}{7}$ as anything other than the ratio of length in a large rectangle to the length of the small rectangle when I asked him to explain what it refers to.

In differentiating proportional situations from non-proportional ones, both preservice teachers had difficulties. Like the average scorers, PSMT6 and PSMT7 were not able to realize the additive structure of the problem. They set the proportion and performed the traditional proportion algorithm improperly in the running track problem. Indeed, PSMT7 explained her solution as “I set the proportion because any change in velocities and in situations like Mehmet gave a break was mentioned”. She made the same explanations in the area conversion problem. When I asked her why she set the proportion in the area conversion while not setting proportion in the parking problem, she emphasized the idea of “same conditions" by stating:

> Because again, I deal with the same ground. I think the point is the same part. So, as long as the conditions do not change, I can construct the ratio. When the conditions changed, I did not construct the ratio. This is the situation here.

PSMT7 associated proportionality with the *same* contextual conditions in the problem. She reasoned that if there is no change in the context, a proportion, can still be set up which led her to incorrect solutions. In parallel with this explanation, she categorized the laundry problem as the proportional situation and replied it as 80 minutes by emphasizing “the same weather conditions”. Additionally, she also solved this problem with the unit ratio, which is not applicable in such a context: “…for each minute, the fourth one of the laundry dried. So, 20 : 0.25 = 80 minutes”. It strongly suggests that she ignored the critical aspects of the real-life problems and applied proportional methods without thinking about the conceptual underpinnings.

In comparison to PSMT7, PSMT6 categorized the problems as proportional and non-proportional accurately and explained the solutions clearly except the running track problem. For example, for the laundry problem, he was able to evaluate the problem within the real-life context and to realize the relationship between a number of clothes and time to dry: “within the same weather conditions, the required time to dry for 20 pieces of laundry is again 20 minutes. Increase or decrease in the amount of laundry does not change the time because there are the same weather conditions, same laundry, and enough space”. He also decided on the proportionality of the parking problems through quantities rather than realizing that the constant factor, the fee for the car distorts the assumption of the proportionality: “because five people were paid 10 TL considering two TL for each person, and there is a car fee, 10 TL, in total 20 TL…10 people were paid 20 TL and 10 TL for a car fee, total amount 30TL. I can not reach 40 TL. So, this is not a proportional situation”. As he did for the word problems, he also determined through numerical values whether x and y values are proportional or not in the graph. He used the slope and similarity between triangles to find the values of x and y in the first graph, $y = 3x+6$. After finding several values for x and y, he constructed a table seen in Figure 6 and compared the change in corresponding values of x and y. He concluded that the graph is not proportional because the change in x and the corresponding change in y are not the same (i.e., 2 times $\neq$ 1.5 times and 3 times $\neq$ 2 times). As in the graph $y = 3x+6$, he was able to reason about the proportionality of the $y = -x$ graph. Again, he found the values x and y by using the slope and similarities of triangles. After identifying the points, he compared the values and defined the graph $y = -x$ as proportional.

x	y
-2	0
0	6
2	12
4	18
6	24

3 times

2 times

1.5 times

2 times

Figure 6. PSMT6's solution for the graph y = 3x+6

In both word and graphical problems, he did not realize the constant factor which is mentioned in the word problems and represented in the graphical problems explicitly, and reached an answer through numerical values. When I asked him at the end of the question of how he decides the proportionality of the graph, he replied to me in parallel with his solutions: " … After I look at the slope of the graph, I identified several values of x and y and compared these values. If there is a constant ratio between them, I can say that the graph is proportional".

In deciding on the proportionality of graphs, PSMT7 preferred starting with the graph of y = -x, which is easy to determine for her. She explained her decision clearly by emphasizing that the ratio between x and y, $\frac{y}{x}$ = -1, is constant along the line. However, when she focused on the graph y = 3x+6, she got confused about what is necessary for proportionality even though she clearly stated a few minutes ago, "the relationship between x and y is -1. So it preserves the ratio all the time. So there is a proportional relationship…the ratio of -1 is always provided". She firstly wrote the line equation then gave numbers for x and corresponding y and considered its proportionality. When comparing values, she expressed that there is no constant ratio. But also she realized the relationship between x and y: as x changes, y also changes as 3x+6. The fact that changes in values of y depended on values of x confused her and she pondered over the meaning of proportional relationships:

> PSMT7: From the beginning of the interview I defined proportional relationships as what I found here (referring to the ratio $\frac{y}{x}$ = -1) is always constant. So far, only one result occurred. For now, changes in y depending on x also seems like a proportional relationship … I felt like I thought I was missing from the beginning. I considered that there is a constant proportional relationship in the second graph (y = -x) and a variant proportional relationship in the first one (y = -3x+6).
> R: Well, what makes you think there's a proportional relationship here?
> PSMT7: … I want to do like, butterfly (theorem) we can write ratios. Alphas (referring to angle) are common. So I found most of the things by ratio. In this graph (y = 3x+6) also I can find values of x and y by using ratios … So, there is a relationship.

As seen above, she changed the perception of the proportional relationship from quantities varying together with the same ratio to any relationship providing y changes as based on x. She did not take care of the multiplicative relationships between ratios and erroneously assumed that any relationship in which x and y vary together is proportional. The unstability and radical change in her definition of the proportional relationships may be due to her limited understanding about the ratio and proportion and her inability to make sense of the associated concepts can be considered as a source of confusion and stability in her explanations.

PSMT7's perception of proportionality also affected her decision in evaluating a hypothetical solution that claims, according to the values provided in a table, x and are inversely proportional. At the beginning of question 3, she stated:

> … for the first time I read the question, I thought it is valid. However, when I read the question the second time I thought like that: When we say the proportional relationship, relationships related to the "times" came to my mind…For example, as one quantity is doubled, the other is halved. The inverse proportion sounds like something like this to me.

Although she was not able to make clear and conceptually coherent explanations of why x and y are not inversely proportional, she relates proportionality to multiplicative relationships. However, such an understanding is not sufficient for her to reach an answer. When I asked her what needed to change so that x and y would be inversely proportional, she erroneously set the direct proportion between the values of x and y. Because she was not sure in her solution, she stated that she wanted to think about this question later. At the end of the interview, she returned the question to consider and emphasized again her concerns by stating, "…here, there is something wrong. It does not make sense to me, something is missing here…(reading the text given in the question) one increases as the other decreases. It does not make sense". The excerpt indicates that she was not able to make sense of the invariant structure in inverse proportion even though she comprehends that the change in quantities are in opposite directions. In order to decide whether values of x and y are inversely proportional, she wrote the linear equation, $y = -x+14$, according to the values in the table and she tried to make a decision by considering the increase and decrease in the values of x and y on the graph. She could not be sure whether there is a direct or inverse proportion for a while. In order to be sure she selected some points on the graph and decided these values were inversely proportional by reasoning upon the changes between corresponding values of x and y:

> In the graph, as x increases, y really decreases…It is more meaningful to me because you can see what the students say on the graph: as one increases by one, the other decreases by one. this solution seemed more convincing to me.

In PSMT7's solution, there are two prominent points. First, she was not able to recognize that the graphs of variables having an inverse proportional relationship is not linear. So, any linear graph cannot be identified as a graph involving an inverse proportional relationship. The second and most crucial point is that she does not know that the change occurs in inverse proportion in ways that the product of quantities remains constant. Since she ignored the change that occurs by the same ratio in inverse proportion, she was not able to decide precisely. On the other hand, she was able to interpret the inverse proportion between the number of faucets and the number of hours in the "faucet" problem. Although she solved the problem by setting a proportion and applying inverse proportion algorithm, she was able to explain what the multiplication 11 by 13 refers to in the problem context as "water in the full pool". It suggests that she was able to decide more easily and accurately whether quantities are inversely proportional or not when the problem is given in a context. Indeed, she was able to make sense of the invariant product of the quantities that are inversely proportional by stating that "it is the same pool, so the amount of the water is also the same".

On the other hand, PSMT6 knows that quantities should vary in the opposite direction at the same ratio for being inversely proportional: "x increases from five to nine so it increased by four not times four…So it is not inverse proportion…But it does not decrease at the same ratio; I think these are not inversely proportional". However, he was not able to provide a sufficient explanation, including the invariance structure of the inverse proportion. Also, he had difficulty with numbers:

> For example, we consider six and eight. x increases six from to eight here; it means the increase is at the ratio of $\frac{4}{3}$, For seven, there is a decrease in the ratio of $\frac{4}{3}$, sorry a decrease at the ratio of $\frac{3}{4}$, right? No, no decrease at the ratio of $\frac{3}{4}$. Is there a decrease in the ratio of $\frac{4}{3}$ again? This is definitely not an inverse proportion, but I could not tell exactly.

As seen above, he failed to interpret the constant of proportionality even though he realized change does not occur at the same ratio. It might be due to the fact that he was not able to internalize that being inversely proportional simply means that as one quantity is multiplied by a number, the other is divided by the same number.

In parallel with PSMT7's results, PSMT6 was able to identify inversely proportional quantities both in the bicycle and faucet problems. He also explained clearly that the distance wheels cover and the amount of water to fill the pool remain constant for the whole situation, which is the invariant structure of the inverse proportion. It can be explained that preservice teachers are familiar to these problems in a way that they can easily make sense of from their experiences.

4.3.2 Common difficulties among preservice mathematics teachers

Although performances of preservice teachers during the interview have varied, as mentioned in the previous section, there are several key issues that stood out as difficulties faced by all participants regardless of their scores on PRI. One of these is an appropriate use of ratio and proportion language, which is one of the features that a competent proportional reasoner has.

Preservice teachers sometimes used additive terms when expressing the multiplicative relationships within measure fields as seen in as discussed for lowest scorers. For example, PSMT2 explained whether the quantities in the equation $a \times b = 8$ are directly or inversely proportional by stating that “when one of the quantities increased twice, the other decreased twice, I mean the other reduced by half”. Such misuse of language leads to the emphasis of the additive relationship between quantities rather than the multiplicative one. Instead of using words that emphasize the difference between quantities, words referring to the multiplicative relationships should be used in proportional contexts. The proper language use such as “when one of the quantities is doubled, the other is halved” for this situation, also prevents the confusion above: decreasing twice and reducing half. Like PSMT2, PSMT1 also misused proportion language when comparing the quantities in the faucet problem by stating “one faucet will fill the pool 11 times slower than 11 faucets” and “seven faucets will fill the pool seven times faster than one faucet”. Instead, “one faucet will fill the pool 11 times as slow as 11 faucets” and “seven faucets will fill the pool seven times as fast as one faucet" are more accurate expressions to state multiplicative relationships. Additionally, PSMT4 used the following statement when interpreting the proportionality of y = -x graph: "When this increases by four units, the other decreases by four units. When this increased eight units, the other decreases eight units again". In Table 20, various examples of improper use of ratio and proportion language are presented.

Table 20. Examples of Improper Use in Ratio and Proportion Language

Participants	Examples of Improper Use in Ratio and Proportion Language
PSMT1	Seven faucets will fill the pool seven times faster than one faucet.
PSMT2	When one of the quantities increased twice, the other decreased twice, I mean the other reduced by half.
PSMT3	If I said 2, it would be 4. While this increased twice, I can say that decreased twice.
PSMT4	When this increases by 4 unit, the other decreases by 4 unit. When this increased 8 unit, the other decreases 8 unit again.
PSMT5	There is a "linear proportion" between them. I mean, if increases, for example, I said 10, I did generally by assigning numbers (for $\frac{a}{b} = 2$), and b is five. And I said a is 20, then b is 10. So if a is increased twice, then b is needed to be increased twice.
PSMT6	For example, if this (referring circumference) increased twice and the length will be 20 and width will be ten.
PSMT7	I said there is a "constant" proportional reasoning. For that, I said there is a "variant" proportional relationship.

It was observed that preservice teachers used inaccurate and imprecise language related to the ratio and proportion concepts: For example they produced terms that do not exist in mathematics. One of them is a "linear proportion" stated by PSMT5 which is used on behalf of the direct proportion, as seen in Appendix F, Table F3.

Using the terms ratio and proportion interchangeably is one of the inappropriate language usages that was also observed during the interviews. PSMT5, also, misused the term ratio when expressing his thinking in the running problem despite the fact that he meant the proportion: “If Mehmet finished the six laps and she finished four, I said how many laps Ayşe finished when he finished 12 laps. And I found an answer eight 8. So, there is a ratio”. Additionally, PSMT6 used the terms “ratio and proportion” together as an expression without considering their differences by stating: “there is nothing that is related to the ratio and proportion” and “it is a ratio and proportion question”. He also used the expression “set a ratio” for setting up an equality of ratios for crossmultiplication, which was interpreted as another indicator that he did not have proper terminology for ratio and proportion concepts. Although this type of improper language use was rarely observed during the interviews, it was more frequently observed in the explanations of Q4 especially for the statement - ratio is the multiplicative comparison of two quantities rather than additive comparison. Several preservice mathematics teachers misinterpreted the word “multiplicative comparison” by thinking proportion, the equivalence of the two ratios $\frac{A}{B} = \frac{C}{D}$, as $A \times B = C \times D$.

Another situation that is problematic among preservice mathematics teachers is the tendency to use direct proportional strategies to non-proportional or inverse proportional problems. This tendency occurred among all participants, from the highest to lowest scorers on PRI without any exception. For example, the tendency to use cross-multiplication algorithm occurred in the faucet problem, which is related to inverse proportion for both highest scorers:

> R: How did you solve the problem?
> PSMT1: The number of faucets is decreased. But we know that the ratio remains the same. So, I multiply the ratio of a number of the faucets of to the

> number of hours, which is the time to need for one faucet, by seven and I found the time.
> R: How can you be sure that your answer is true?
> PSMT1: So, the number of faucets decreased. I know the time will be increased. By the way, I solved incorrectly. Anyway, I understand that time should be increased. If the time decreased, it means there is a problem … I reached the wrong answer. Can I calculate again? ... If 11 faucets fill the pool in 13 hours, I have 11×13 amount of water. If I want to obtain this amount of water with seven faucets, I divided the amount of water by seven faucets.

As seen above, he used the cross-multiplication algorithm for the inverse proportion problem before starting the explanation even though he realized the mistake in his thinking when I asked him to evaluate the accuracy of his solution. However, differently from PSMT1, PSMT2 was not able to notice the improper use of cross-multiplication algorithm in this question even though when I asked how she was sure of the answer. Both PSMT1 and PSMT2 used inappropriate direct proportion algorithm in only inverse proportion problems. They were able to think about non-proportional problems in their context and reason appropriately.

The tendency of using cross multiplication algorithms among average and lowest scorers on PRI is seen both in the problems including inverse proportions and non-proportional situations, differently from highest scorers. All participants applied the cross-multiplication algorithm on the running track problem erroneously, as mentioned before. Additionally, PSMT7 applied the algorithm even on the laundry problem, which can be reasoned by thinking about real-life experiences. Like PSMT1, PSMT7 applied the cross-multiplication algorithm to the faucet problem in the beginning and changed her solution after realizing that they are inversely proportional during the explanation. Use of direct proportion algorithm also occurred among these groups when they tried to provide alternative solutions to the inverse problems. For example, although PSMT3 easily reached a correct answer with inverse proportion algorithm, she could not maintain the idea that these quantities are inversely proportional when reasoning by unit strategy, which is the amount of water per faucet, and reached the incorrect answer by using cross-multiplication.

4.3.3 Contradictions between preservice mathematics teachers' proportional reasoning as manifested by PRI and task-based interviews

Qualitative analysis results as mentioned before reveal that performances of PSMTs during the interview are parallel with their scores on PRI. In other words, highest scorers on PRI performed better than average and lowest scorers in having flexible ways of thinking about proportional relationships and providing more conceptual explanations for problems. In the same way, average scorers were more successful than lowest scorers in making sense of context of proportionality and providing explanations even though these explanations were mainly based on procedural understanding. Although these results give an idea about the relationship between the proportional reasoning scores obtained from the measurement tool and the data obtained from task-based interviews in terms of consistency in differentiating the participants, contradictory situations are explained also in this section in order to answer the second research question in detail.

Three different contradictory situations between the answers in the interview and PRI occurred throughout the current study. The first includes situations in which although PSMTs answered correctly the question in the instrument, they gave incorrect answer to the same question during the interviews. Even though such situations rarely occurred in the study, they need to be addressed and explicitly discussed. A salient example is PSMT7's response to the question related to deciding proportionality and non-proportionality in graphical representations as presented previously. Conversations about the graph of $y = 3x+6$ which is confusing for her during the interview, reveal her limited and uncertain understanding in proportionality. Such an issue was not observed only in lowest scorers, it was also observed in highest and average scorers. For example, PSMT1 correctly evaluated the student B's solution in PRI. However, during the interview, he was not able to make sense of the student's solution and he explained that the solution is insufficient mathematically by stating "when comparing the student C's solution ($\frac{6}{7} = \frac{a}{20}$), one of them (student C) set the proportion, and one of them is as if s/he had written more verbal expression. I mean, it's like s/he's got it in his head" and he added "I think, it is deficient mathematically. But the result is correct. lf we asked him what he did, he would explain his thinking behind the solution". The fact that PSMT1 was not able to make sense of $\frac{6}{7}$ as a multiplicative relationship between sides which needs to be preserved also in enlarged rectangle, was elicited through the interview on the contrary to the answer in PRI.

The second incompatibility between students' answers on PRI and interviews which was more frequently observed than the first one includes situations in which although PSMTs answered the question incorrectly in the instrument, they gave a correct answer to the corresponding question during the interviews. An example of such a situation was observed in the problems focusing on inverse proportion (Q3d in the instrument and Q3 in the interview). Although three PSMTs decided erroneously that one increases as the other decreases is sufficient for being inversely proportional in the instrument, they made conceptual explanations as to why it is not sufficient during the interview such as "an increase in the value of x leads to a proportional decrease in the value of y and vice versa" or "whenever the values of one quantity increase, then the value of another quantity increase in such a way that product of the quantities remains same". Similarly, in problems requiring differentiating proportional situations from non-proportional ones, four PSMTs realized mistakes in their thinking when justifying their decisions about why these situations are proportional or non-proportional. These examples imply that being active in thinking and questioning through interview questions allows participants to think about more and so to notice their mistakes.

The third and the most frequently observed one includes cases in which although PSMTs answered the questions correctly either in the instrument or in the interview, they did not provide accurate explanations to these questions during the interview. For example, although PSMT4 categorized the parking question as involving a non-proportional relationship both in PRI and in the interview, he had a wrong way of thinking: "…For five people, 20 tl was paid. For ten people, 30 tl. But this is non-proportional. Because the product of five and twenty is not equal to the product of 10 and 30". As seen in this excerpt, he considered that these quantities might be inversely proportional and because of that, he compared their product. Although his decision about the proportionality is correct, he was not able to realize that either quantities increase or decrease at the same time so the proportionality of the situation should be considered as direct proportion.

A slightly different example was observed in another sub-question of the problems focusing on differentiating proportional relationships from non-proportional ones. PSMT7 categorized an area conversion as involving proportionality during the interview although she was not able to this sub-question in the instrument. In the beginning of the conversation, she was still confused as to whether it was proportional or non-proportional even though she marked it as proportional situation in the interview question paper:

> I am actually undecided between two options: proportional or non-proportional. It is more confusing one for me. I considered that the

> conversion is proportional or not but I had same conclusion again…Because I make operations with the same land. I think, there is something related to the word “same”…No change. I mentioned the area of same land. I stated it with smaller units rather than km^2 and so there is ratio between them.

The excerpt above shows that although she mentioned there is a ratio between units, she mainly did not focus on the ratio and she mostly thought about the word “same” which she realized during the interview. So, it can be concluded that she did not reach the answer by understanding the conceptual underpinnings although she answered correctly.

Considering all of contradictory cases of PSMTs performance in the instrument and interviews, it can be said that both lowest and average scorers separated from the highest scorers during the interview especially in developing and using different strategies, understanding and explaining the multiplicative relationships both in direct and inverse proportions, and realizing and understanding the invariance and covariance structure of the proportional relationships, although these participants obtained at least half of the total scores in the instrument.

CHAPTER 5

DISCUSSION

This chapter consists of discussion for research findings, limitations of the study and recommendations for future studies based on the discussion of findings. Findings will be discussed within the context of research questions and relevant literature in the first part of the chapter. Limitations of the current study and several recommendations for the further studies will follow.

5.1 Discussion of research findings

The purpose of the study was to investigate preservice mathematics teachers' proportional reasoning. To achieve this aim, PRI that is developed by the researcher and task-based interviews were conducted with participants. The study also explored the relationship between proportional reasoning scores obtained from PRI and the data obtained from task-based interviews, in other words how consistent scores on PRI and the data obtained from task-based interviews were and how the two sets of data, together, enable making sense of preservice mathematics teachers' proportioning reasoning ability.

In the scope of the study, PSMTs' proportional reasoning was investigated through the instrument first. According to the quantitative analysis results, PSMTs' average score on PRI was found as 30.17 out of 39. Considering the high average scores obtained by PSMTs, it is concluded that majority of the PSMTs answered at least half of the questions correctly. The reason for this result can be explained by the fact that four of the five universities in which the quantitative data were collected from are in the top ten in terms of the student rankings in the related departments

(YÖK, nd). In other words, these preservice teachers were successful in highly competitive university entry exams. The questions in the exam have multiple choice format and do not require to any explanations. In Turkey, preservice teachers are not accustomed to making explanation from the primary school years. Rather they have received an education which often guides them to reach only correct results without focusing on the explanations.

Quantitative analysis results also indicate that PSMTs were relatively proficient at correctly solving questions requiring answers that can be reached by following procedures because they have seen them earlier. Realizing different representations of ratio: decimal, fraction and percentage (Q7), solving the bicycle problem (Q8), correctly identifying variables in the mathematical statements as directly or inversely proportional (Q9), and evaluating solution strategies to the lemonade (mixture) problem (Q10) are examples of these type of questions.

On the other hand, quantitative analysis results also reveal that PSMTs have difficulty in particular questions, e.g. Q1, Q3, Q4, Q5, Q11, Q12. Difficulties that PSMTs experienced also are parallel with the studies in the literature. Preservice teachers had difficulty mostly in the stretchers and shrinkers problem focusing on the proportionality constant, and invariance and covariance structure of the proportional relationships just as reported in studies carried out with students (Ben-Chaim et al., 1998; Kaput & Maxwell-West, 1994).

All of these questions that PSMTs had difficulty seems to be related because the results are overlapping. These are mainly related to understanding the invariance and covariance structures of the proportional relationships, realizing multiplicative relationships and deciding and justifying the properties and statements of ratio and proportion. So, rather than requiring use of procedures such as applying directly the algorithm or assigning numbers to the variables, these questions require conceptual engagement: answers with explanations and reasoning. Considering these questions require understanding and explaining the underlying reason of statements and strategies, these findings are not surprising and consistent with the notion of Post et al. (1993) that preservice teachers have operational knowledge rather than conceptual knowledge on proportional reasoning.

To investigate PSMTs' proportional reasoning in detail, task-based interviews including questions constructed with similar purposes to those in the instrument were conducted with participants in each level, lowest, average and highest, according to their scores form PRI. Qualitative data were analyzed according to six components of proportional reasoning. Through the results of qualitative analysis, the differences in proportional reasoning of participants having different levels of scores from PRI, were observed more clearly.

In the first component of proportional reasoning, highest scorers solved proportional problems independent from context and numerical complexities and made more conceptual explanations by focusing on the meaning of ratio and proportion in comparison with average and lowest scorers. The lack of understanding in ratio and proportion concepts led lowest and average scorers to not being able to make sense and solve the questions given in unfamiliar contexts or in a rather mathematically abstract fashion without context. Providing learning experiences with questions in different contexts and explicitly inviting students to reason proportionally can be important in fostering proportional reasoning ability (Dole, 2008). In this respect, teachers might focus on the structure of the relationships between variables in the questions and encourage discussion process about these questions in order to create opportunities for their students to develop this ability.

When considering the development and use of different strategies, results show that highest scorers developed and made sense of different strategies in addition to cross-multiplication and explained these strategies in a more sophisticated way by highlighting the concepts ratio and proportion. Although average scorers provided different kind of solution strategies, they tended to give more superficial explanations to these solutions. Different from both of these groups, lowest scorers' solution strategies were more restricted to the cross-multiplication algorithm and they had inability to explain understanding underneath the algorithm. The reason for the tendency to use cross multiplication algorithm may be stemming from its overemphasis in the mathematics curriculum and consequently overshadowing other strategies. On the other hand, all PSMTs, without regarding their scores on PRI, were able to use and provide solution strategies in within and between ratios for several questions. However, some of them were not able to make sense of solution strategy involving between ratios even though they were able to comprehend and explain the solution strategy built on within ratios. Karagöz Akar (2007) has previously discussed that development of an understanding between ratios is independent of the development of an understanding within ratios. Therefore, it is important to discuss both concepts explicitly in the classroom environment in order to make students develop flexible and different reasoning ability.

Study results in this component contradict the categorization of proportional reasoning proposed by Langrall and Swafford (2000) in which the use of the cross multiplication strategy is considered as the highest level that can be reached for proportional reasoning. However, one should interpret their claims with care when reaching conclusions. Within the study, it was revealed that being able to apply the algorithm did not mean that one had high levels of proportional reasoning because participants of this study were not able to articulate on the reasons underlying the algorithm in contrast to the participants of the study conducted by Langrall and Swafford (2000). Rather, being able to apply the algorithm meant that one performed well in setting proportions and applying the algorithm mechanically within the current study. In order to prevent blindly applying the algorithm, teachers might focus on the "development of the meaning by postponing efficient procedures until understanding is internalized by students" in their classrooms (Cramer et al., 1993, p. 165).

In differentiating proportional situations from non-proportional situatons, it was pointed out that highest scorers performed better and provided more conceptual explanations in both word and graphical problems compared to average and lowest scorers. Almost all average and lowest scorers solved the running track problem, additive problem, with cross multiplication algorithm. It points the lack of understanding in relative and absolute change, and is consistent with other studies' results (Duatepe et al., 2015; Ekawati et al., 2015a). Solving the non-proportional problems with an inappropriate multiplicative strategy was more frequently observed in lowest scorers. Inability to distinguish multiplicative relationships from additive or no relationships may cause them to overuse this algorithm. Another reason for overuse of the algorithm may be the number structure that cross-multiplication is used when questions involve integer ratios as cited in studies conducted with students (Çelik & Özdemir, 2014; Toluk Uçar & Bozkuş, 2016).

In understanding multiplicative relationships in direct and inverse proportions, the multiplicative relationship in inverse proportion was more difficult to recognize for lowest and average scorers. Highest scorers generally were able to both understand and express the multiplicative relationships in direct and inverse proportions regardless of context. It is compatible with the notion that "proportional reasoner should overcome the effects of unfamiliar settings and cumbersome numbers" (Cramer et al., 1993, p. 171). On the contrary, lowest scorers had difficulty in explaining the relationship between variables when using non-integer ratios and "exhibited a kind of fraction avoidance" in their explanations of solutions (Lamon, 2015, p. 108).

Like the previous component of proportional reasoning, average and lowest scorers experienced more difficulty in realizing and explaining invariance and covariance structures of proportional relationships including quantities which are inversely proportional than directly proportional. It is consistent with the study done with preservice teachers which concluded that they struggled with inverse proportion problems (Riley, 2010). These results suggest that transformations of quantities in such a way that some underlying structure remains invariant, should be focused on by emphasizing their meanings within the context.

Considering the development and use of proper language for ratio and proportion, the last component of proportional reasoning, it is revealed that all PSMTs, without any level difference, have deficiencies. This is in line with the results of the study conducted by Akkuş Çıkla and Duatepe (2002). These deficiencies, misuse of language in multiplicative relationships, usage of nonexistent terms, and usage of ratio and proportion interchangeably, differ in terms of frequency and variety among highest, average and lowest scorers. Lowest scorers are those who use additive language in proportional situations most and it confirms the idea that "being able to describe proportional situations using multiplicative language is an indicator of proportional reasoning" (Dole, 2008, p. 19). These results related to use of proper language suggest that this issue needs to be explicitly addressed in mathematics classrooms to support healthy development of proportional reasoning ability.

In brief, quantitative and qualitative analysis results of the study indicated that although PSMTs obtained relatively high scores from the instrument, majority of PSMTs were not able to provide explanations about the underlying concepts of the ratio and proportion. Their understanding are mainly based on procedures rather than concepts. Also solution strategies they used and made sense of were limited.

All in all, the study has contributions in terms of shedding light on what the current level of preservice mathematics teachers' proportional reasoning in Turkey is. In addition to providing insights about PSMTs' proportional reasoning, this study informs about the difficulties PSMTs experienced, particularly in inverse proportional contexts.

5.2 Limitations of the study and recommendations for further studies

This study also has limitations and these limitations may have affected the results of the study and consequently the conclusions drawn. First of all, this study is limited to five public universities in Turkey which are ranked in the top ten according to the scores of the admitted students in the university entrance exams. Conducting this study in a variety of public and private universities could lead to different results. Therefore, this study represents the situation from a particular sample and these results may not be generalized to all preservice mathematics teachers in Turkey. In order to get more representative results, further studies can be conducted in a large scale. Additionally, participants may be chosen from in-service teachers who have the most essential role in students' understanding of these important concepts and their proportional reasoning may be investigated with instruments, interviews and observations.

The second limitation is related to the number of participants who attended the task-based interviews. Only seven PSMTs participated in the qualitative part of the study. Although increasing the number of participants would enrich the qualitative data, it was not possible because of limited number of positive returns from participants and the time limitations of the researcher.

The final limitation concerns how to determine preservice mathematics teachers' proportional reasoning. In this study, determining PSMTs' proportional reasoning was restricted to the concepts and competencies based on six characteristics of proportional reasoners which are outlined by Lamon (2005, 2007). Additionally, expansion of interview questions could also lead some differences PSMTs' answers and influence the results of the study. Therefore, future studies can combine different components of proportional reasoning and different interview questions to address the current situation for PSMTs' proportional reasoning more comprehensively.

APPENDIX A

PROPORTIONAL REASONING INSTRUMENT

Dear Teacher Candidates,

This instrument has been improved to determine your common content and specialized content knowledge about ratio and proportion. Therefore, it is very important that you write your e-mail address so that you can share your thoughts and information with us and we can get your opinions about the test.

During the test;

- Please, read each material carefully.

You have 45 minutes to complete the test.

Student Number:

Male ☐ Female ☐ Others ☐

University:

Term:

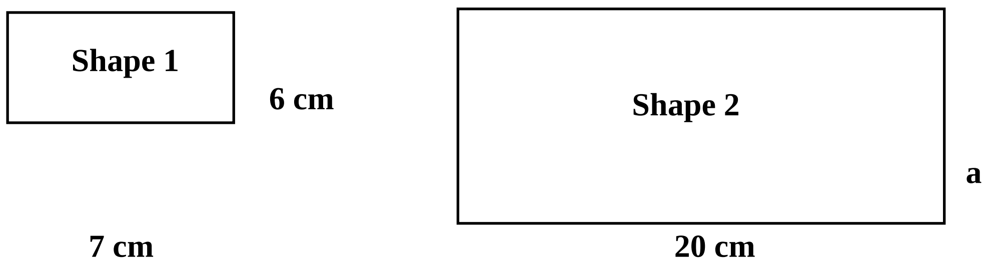
GPA (Grade Point Average):

Math Courses taken as electives:

E-mail:

1) One of the questions that Nalan teacher asked during her class as is as below:

The dimension of the first shape, which has lenght of 7 cm and width of 6 cm is extented by keeping the ratio constant and the second shape is obtained. According to this, what is the width (a) of X shape in cm?



The solutions used by some students for this question are given as below:

Student A	Student B	Student C
According to 7 cm $\rightarrow$ 20 cm proportionally 1 cm is $\frac{20}{7}$ 6 cm $\times \frac{20}{7} = \frac{\mathbf{120}}{\mathbf{7}}$ **cm**	Ratio of length to width of the first shape $= \frac{6}{7}$ 20 cm $\times \frac{6}{7} = \frac{\mathbf{120}}{\mathbf{7}}$ **cm**	$\frac{6}{7} = \frac{a}{20}$ $6 \times 20 = 7a$ $a = \frac{\mathbf{120}}{\mathbf{7}}$ **cm**

Which of the method or methods used by the students will lead the students to the correct result in finding the unknown value in all stretchers and shrinkers problems containing the direct proportion? Please mark it.

	Leading to the correct result in all questions	Leading to the correct result in some questions
The method used by Student A	☐	☐
The method used by Student B	☐	☐
The method used by Student C	☐	☐

2) Please, decide whether the questions are the proportional situation. Mark the proper choice for each question.

I. Ayşe and Mehmet are running at the same speed on the running track. When Ayşe finished 4th tour, Mehmet finished 6th tour. According to this, which tour does Ayşe finish when Mehmet has finished 12th tour at the same running track?

Proportional Situation ☐ **Non-proportional situation** ☐

II. When Ayşe is 10 years old, her sister is 5 years old. When Ayşe become 30, how old is her sister?

Proportional Situation ☐ **Non-proportional Situation** ☐

III. How many hours does a vehicle, which travels 225 km in 3 hours, spend to take 300 km at the same speed?

Proportional Situation ☐ **Non-proportional Situation** ☐

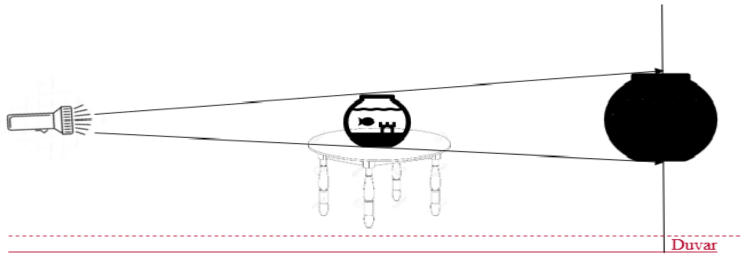
IV. If 20 minutes are required to dry 5 pieces of clothes in the open air, how much time is required to dry 20 pieces of clothes under the same conditions?

Proportional Situation ☐ **Non-proportional Situation** ☐

V. 2 TL per person and 10 TL for a vehicle are paid for the entrance of Kuşadası National Park. 5 people, who enters into National Park by their 10 seater vehicle, pay 20 TL for the entrance. How much is required to pay when 10 people want to enter into the park by the same vehicle?

Proportional Situation ☐ **Non-proportional Situation** ☐

VI. The light is shed from 40 cm distance to a 10 cm aquarium which stands on the table in a dark setting. How many cm is the shade of the aquarium being on the wall at 100 cm distance from the light source?



Proportional Situation ☐ **Non-proportional Situation** ☐

VII. Please, find the value of a land, the area of which is 10 km^2, in cm^2.

Proportional Situation ☐ **Non-proportional Situation** ☐

3) While Baran teacher is preparing his syllabus about ratio and proportion topic, he realizes that the more emphasis is placed on ratio and proportion topic in the books as opposed to the past. It is asked to decide the accuracy of some statements related to ratio and proportion in the following questions that he come across in one of the books.

According to this, Which of the statement or statements below are **always** true?

Please, mark true and false for each choice below and explain the reason why you have chosen it.

	TRUE	FALSE
I. Ratio is not the additive comparison of two quantites but the multiplicative comparison of two quantites. Explanation:	☐	☐
II. Ratio is always given in part-whole relationship as it is the notation of an amount. Explanation:	☐	☐
III. All fractions are ratio. Explanation:	☐	☐
IV. If one of two quantities increases as the other decreases, these quantities are inversely proportional Explanation:	☐	☐

4) Dilek took a 3 x 5 cm^2 photo of Fairy Tale Castle during Eskişehir trip that she went with her school. She took it to the photoshop and wanted it to be edited as 7×9 cm^2 thanks to a programme. According to this, which photo has more squareness? Mark it. Indicate your methods for the result on the paper.

A) 3×5 cm^2 photo

B) 7 x 9 cm^2 photo

C) Both photos have the same shape. There is no way to have the other one more squareness.

On the following page, you are required to answer the questions related to the answer that you give for the question above (Question 4)

Please, answer the explations **just** related to the answer that you have given on the previous page. Do not answer all questions on this page.

If your answer is **A** for the question 4**,** please mark whether each explanation below is TRUE or FALSE.

- 3×5 cm² photo has more squareness since the fact that 3×5 cm² photo has shorter width leads it to be seen as smaller. The fact that 7×9 cm² photo has longer length makes it resemble to a rectangle.

TRUE ☐ FALSE ☐

- 3×5 cm² photo has more squareness because 3 x 5 cm² photo needs 10 squares and 7x9 cm² photo needs 18 squares to make a square.

TRUE ☐ FALSE ☐

If your answer is **B** for the question 4**,** please mark whether each explanation below is TRUE or FALSE.

- 7 x 9 cm² photo has more squareness because $\frac{7}{9}$ is closer to 1 than $\frac{3}{5}$

TRUE ☐ FALSE ☐

- 7 x 9 cm² photo has more squareness because the growth rate is less in this photo. In other words, the length in the edited photo became 9 by growing in the ratio of 2/7 according to 7 x 7 cm² square (which is required one), the length in the original photo became 5 by growing in the ratio of 2/3 according to 3 x 3 cm² square (which is required one).

TRUE ☐ FALSE ☐

If your answer is **C** for the question 4**,** please mark whether each explanation below is TRUE or FALSE.

- There is just 2 cm difference between the original and the edited one. Therefore, both have the same shape. There is no way to have the other one more squareness.

TRUE ☐ FALSE ☐

- The photo is edited by increasing both the width and length of the original photo for 4 cm. Therefore, both have the same shape. There is no way to have the other one more squareness.

TRUE ☐ FALSE ☐

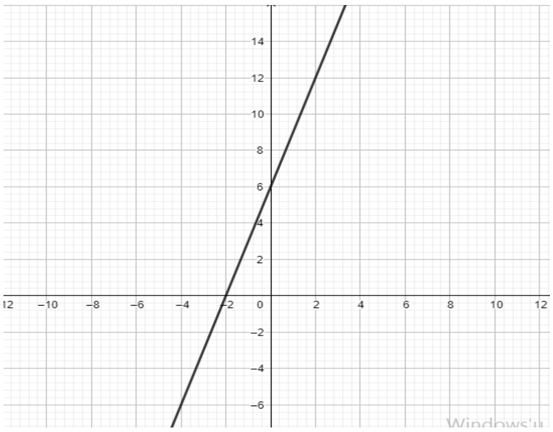
5) In which of the equations or the graphs below is there proportional relationship between x and y?

I. $y = 2x$

There is a proportional situation. ☐

There is no proportional situation. ☐

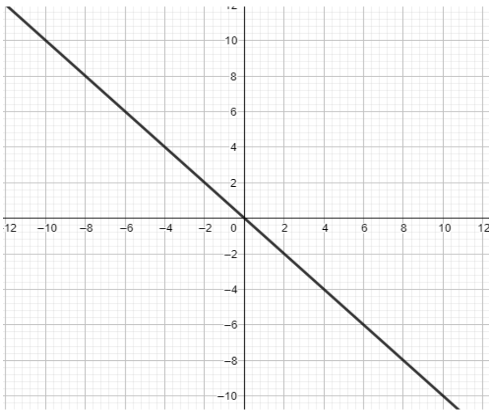
II.



There is a proportional situation. ☐

There is no proportional situation. ☐

III.



There is a proportional situation. ☐

There is no proportional situation. ☐

y = 1

There is a proportional situation. ☐

There is no proportional situation. ☐

6) The explanations, which are got by Aygül teacher who wants her students to explain whether x and y variables placed in the table are proportional, are as below.

x	9	15	21	27
y	12	20	28	36

Ali: x and y's in the chart are proportional because the increase of 6 unities in the value of x is equal to each increase of 8 unities in the value of y.

Sezen: x and y are proportional because y in any unit is 4/3 times of x.

Zehra: x and y are proportional because the ratio of any two pairs (x,y) in the table are equal.

Evaluate whether these explanations, which are given for the solution of this question by students, are valid.

	Valid	Invalid
Ali's method	☐	☐
Sezen's method	☐	☐
Zehra's method	☐	☐

7) One of the questions that Zeynep teacher asked for the math exam is as below.

Player	Shot on target (two-shot)	Total used shot (two-shot)
Dusko Savanovic (DS)	12	23
Hidayet Türkoğlu (HT)	10	18
Marko Keselj (MK)	8	20
Ömer Onan (ÖO)	9	21

It is given on the chart that the total number for the two-shot used by 4 players during Turkey- Serbia semi-final match in 2010 and the information of how many of these shot on target are. According to this, which player has the best performance on two-shot? (The success on shot performance in basketball is determined by the ratio of shots on target to the total used shots.)

Which answer or answers below shoul be accepted as true by Zeynep teacher? Mark TRUE and FALSE in all questions for each choice below.

	TRUE	FALSE
Sevde: Hidayet Türkoğlu's shot percentage is more when compared to the other shot percentages. Therefore, his performance is the best one. (HT: % 55.6 > DS: % 52.1 > ÖO: % 42 > MK:% 40)	☐	☐
Vuslat: Hidayet Türkoğlu has the best ratio when we compare all shots on target with the attempted shots. Therefore, his performance is the best one. $(\frac{10}{18} > \frac{12}{23} > \frac{9}{21} > \frac{8}{20})$	☐	☐
Mustafa: Hidayet Türkoğlu has the biggest number when the performances of players are expressed by decimal notation. Therefore, he has the best performance. (HT: 0.556 > DS: 0.521 > ÖO: 0.428 > MK: 0.40)	☐	☐

8) Granbi bicycles on the picture, which have differrent front and back wheel in dimension, are invented. These bicycles are designed for increasing the speed of bicycles that has a pedal attached to the front wheel in France in the early 1870s.

The perimeter of Granbi's front wheel is 405 cm and its radius is three times of radius of the back wheel.

According to this, how many meters does the bicycle go when the back wheel turns for 45 tour?

A) 40.50 m

B) 60.75 m

C) 63.25 m

D) 81 m

E) 121.5 m

9) Please, decide whether the quantites in following expressions are direct proportional or inverse proportional. Mark DIRECTLY PROPORTIONAL or INVERSELY PROPORTIONAL choice for each one.

	DIRECTLY PROPORTIONAL	INVERSELY PROPORTIONAL
a) $\frac{a}{4} = \frac{b}{5}$	☐	☐
b) $\frac{a}{b} = 2$	☐	☐
c) $a \times b = 4$	☐	☐
d) $2a = 3b$	☐	☐
e) $\frac{a}{1} = \frac{8}{b}$	☐	☐

10) The methods for the solutions of the problems below that the students used are below. According to this, which method or methods are valid for the solution?

Sinem's mother and father prepared different lemonade in 2 decanters for the birthday party in the evening. They used equal lemons, glasses and decanters in size for the lemonade. According to this, what can be said about the taste of lemonade prepared by Sinem's mother and father?

I. If her mother puts 4 glasses of water for 3 lemons, she puts 4/3 glasses of water for 1 lemon. If her father puts 5 glasses of water for 4 lemons, he puts 5/4 glasses of water for 1 lemon. Her mother's lemonade contains more lemon juice thane her father's lemonade because her mother puts more water for 1 lemon.

II. If her mother uses 3 lemons for 4 glasses of water, she uses 3/4 of lemon in 1 glass. If her father uses 4 lemons for 5 glasses of water, he uses 4/5 of lemon in 1 glass.In this case, her father's lemonade contains more lemon juice because he uses more lemons for 1 glass of lemonade that he prepared.

III. They think that they should use the same number of lemons to make comparison with each other. While her mother uses 16 glasses of water for 12 lemons to get the same taste with the lemonade for which she adds 4 glasses of water for 3 lemons, her father will use 15 glasses of water to get the same taste with the lemonade for which he adds 5 glasses of water for 4 lemons. In this case, her father's lemonade contains more lemon juice because he uses less water for the same number of lemons.

A) I and II

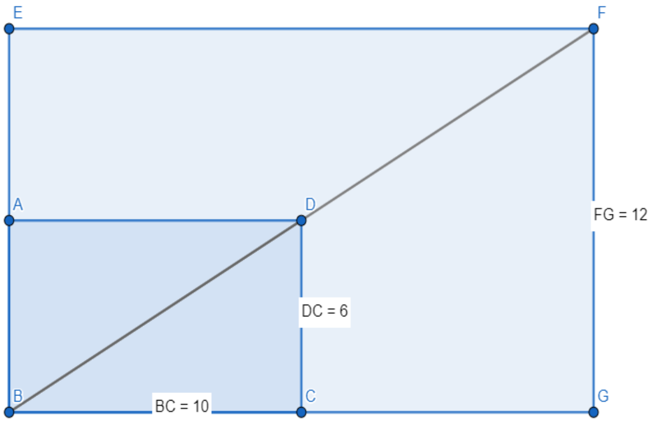
B) I and III

C) Just III

D) II and III

E) I, II and III

11)



A ABCD rectangle, which has length of 10 cm and width of 6 cm, can be streched and shrank in the direction of BD diagonal by holding from D corner via a computer programme. What is the length of EBGF rectangle formed like this.

This problem above is solved with two different solution strategies.

i. $\frac{5}{3} \times 12 = 20$

ii. $10 \times \frac{12}{6} = 20$

Which expressions below related to the solution strategies are false?

A) $\frac{5}{3}$ in the first solution method expresses multiplicative relationship between sides.This multiplicative relationship is used to find [BG] side length in case of stretching the rectangle from D corner.

B) $\frac{5}{3}$ value got in the first solution method should be kept in case of shrinking of ABCD rectangle.

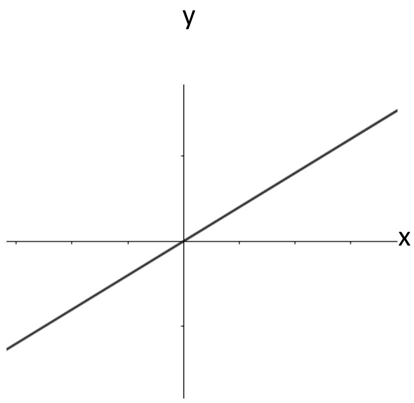
C) $\frac{12}{6}$ value in the second solution expresses change factor between two shapes. This coeffiecient is used to find [BG] side length.

D) $\frac{12}{6}$ value got in second solution should be kept in case of shrinking of ABCD rectangle.

12) Three different representation of the same function is as below.

y = kx

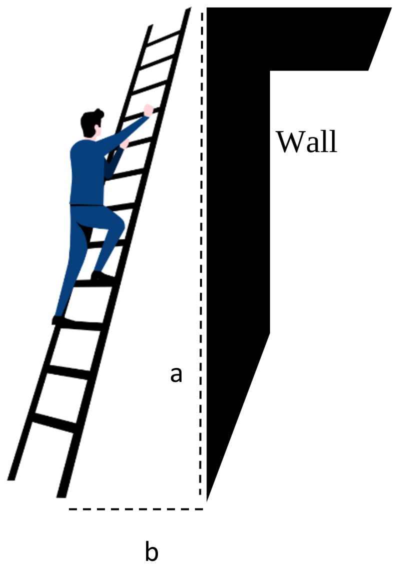
x	y
0	0
x_1	y_1
x_2	y_2
.	.
.	.
x_n	y_n



I. Representation	II. Representation	III. Representation

According to this, which one below is <u>always</u> valid expression except for x = 0 for the function? Mark it.

	Valid	Invalid
1. $x_1 + x_2 = y_1 + y_2$	☐	☐
2. $\frac{x_5}{x_6} = \frac{y_1}{y_2}$	☐	☐
3. $\frac{y_n}{x_n} = k$	☐	☐



13) In the picture, workman Ali and the ladder, steps of which are immobilized on the ground in case of slipping, are depicted.

According to this, which expression is true?

I. The stepness of the ladder indicates the difference in 1st and 5th step of the ladder.

True ☐ False ☐

II. The stepness of the ladder is expressed by numerical value of vertical heigth (a).

True ☐ False ☐

III. There is a relative change of vertical height (a) according to horizontal distance (b) in the stepness of the ladder.

True ☐ False ☐

APPENDIX B

PROPORTIONAL REASONING INSTRUMENT (TURKISH)

Sevgili öğretmen adayları,

Bu test sizin oran orantı konusundaki genel alan ve uzman alan bilginizi ölçmek amacıyla geliştirilmiştir. Bu yüzden sizin düşüncelerinizi ve bilgilerinizi bizimle paylaşmanız ve testle ilgili görüşlerinizi alabilmemiz adına mail adresinizi yazmanız çok önemlidir.

Test süresince;

-Lütfen her bir maddeyi dikkatlice okuyunuz.

Testi tamamlamak için süreniz yaklaşık 45 dakikadır.

Öğrenci Numarası:

Erkek ☐ Kadın ☐ Diğer ☐

Üniversite:

Dönem:

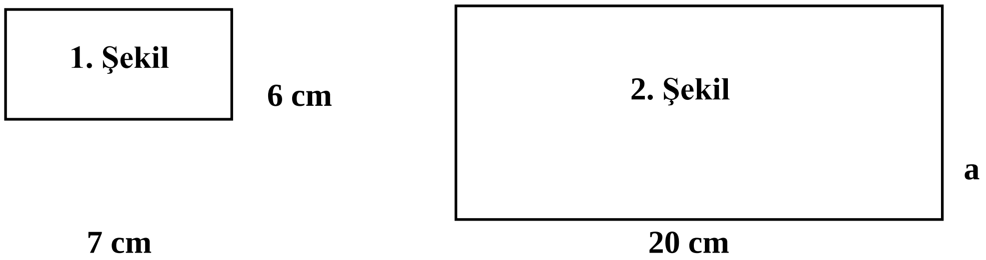
GNO (Genel Not Ortalaması):

Seçmeli Olarak Alınan Matematik Dersleri:

E-posta:

1) Nalan öğretmenin dersinde sorduğu sorulardan birisi aşağıdaki gibidir:

Kısa kenarı 6 cm ve uzun kenarı 7 cm olan 1. şeklin boyutları oran sabit tutularak genişletilip 2. şekil elde ediliyor. Buna göre X şeklinin kısa kenarı (a) kaç cm'dir?



Aşağıda bazı öğrencilerin bu soru için kullandıkları çözümler verilmiştir.

A Öğrencisi	B Öğrencisi	C Öğrencisi
7 cm $\rightarrow$ 20 cm olduğuna göre 1 cm $\rightarrow \frac{20}{7}$ katına çıkmış 6 cm $\times \frac{20}{7} = \frac{\mathbf{120}}{\mathbf{7}}$ **cm**	1. şeklin kenarları oranı $= \frac{6}{7}$ 20 cm $\times \frac{6}{7} = \frac{\mathbf{120}}{\mathbf{7}}$ **cm**	$\frac{6}{7} = \frac{a}{20}$ $6 \times 20 = 7a$ $a = \frac{\mathbf{120}}{\mathbf{7}}$ **cm**

Öğrencilerin kullandıkları yöntem ya da yöntemlerden hangisi <u>tüm doğru orantı içeren büyültme-küçültme sorularında</u> bilinmeyen değeri bulmada öğrencileri doğru sonuca ulaştıracaktır? İşaretleyiniz.

	Tüm sorularda doğru sonuca ulaştırır	Bazı sorularda doğru sonuca ulaştırır
A Öğrencisinin kullandığı yöntem	☐	☐
B Öğrencisinin kullandığı yöntem	☐	☐
C Öğrencisinin kullandığı yöntem	☐	☐

2) Aşağıdaki soruların orantısal bir durum olup olmama durumuna karar veriniz. Her soru için uygun seçeneği işaretleyiniz.

I. Ayşe ve Mehmet bir koşu parkurunda eşit hızlarla koşmaktadır. Mehmet'in koşmaya daha önce başladığı bilinmektedir. Ayşe 4. turu bitirdiğinde Mehmet 6. turu bitirmiştir. Buna göre aynı koşu parkurunda Mehmet 12. turu bitirdiğinde Ayşe kaçıncı turu bitirir?

Orantısal Durum ☐ **Orantısal Olmayan Durum** ☐

II. Ayşe 10 yaşındayken kardeşi 5 yaşındadır. Ayşe 30 yaşına geldiğinde kardeşi kaç yaşında olur?

Orantısal Durum ☐ **Orantısal Olmayan Durum** ☐

III. Sabit hızla 3 saatte 225 km yol alan bir araç, aynı hızla 300 kilometre yolu kaç saatte alır?

Orantısal Durum ☐ **Orantısal Olmayan Durum** ☐

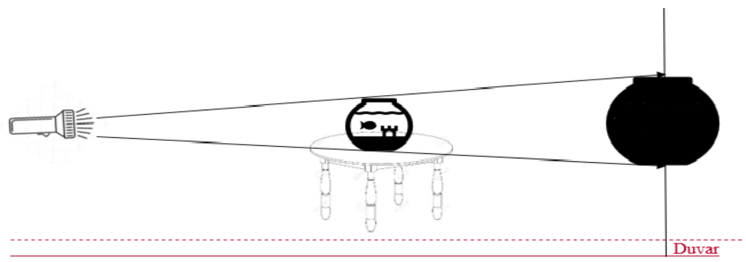
IV. Açık havada 5 parça çamaşırın kuruması için gerekli süre 20 dakika ise aynı hava şartlarında 20 parça çamaşırın kuruması için gereken süre ne kadardır?

Orantısal Durum ☐ **Orantısal Olmayan Durum** ☐

V. Kuşadası Milli Parkına giriş için kişi başı 2 TL, yanı sıra araba parası olarak da 10 TL ödenmektedir. 10 kişilik arabasıyla Milli Parka giriş yapan 5 kişi giriş için 20 TL ödemektedir. Aynı araba ile 10 kişi girmek istendiğinde ne kadar ücret ödenmelidir?

Orantısal Durum ☐ **Orantısal Olmayan Durum** ☐

VI. Karanlık bir ortamda sehpanın üzerinde duran 10 cm boyundaki bir akvaryuma 40 cm uzaklıktan ışık tutuluyor. Işık kaynağından 100 cm uzaklıktaki duvarda oluşan akvaryumun gölgesi kaç santimetre boyundadır?



Orantısal Durum ☐ **Orantısal Olmayan Durum** ☐

VII. Alanı 10 km² olan bir arazinin cm² cinsinden değerini bulunuz.

Orantısal Durum ☐ **Orantısal Olmayan Durum** ☐

3) Baran öğretmen işleyeceği oran-orantı konusu ile ilgili ders programını hazırlarken kaynaklarda oran-orantı konusuna eskisine göre **daha fazla** vurgu yapıldığını fark ediyor. Kaynakların birinde karşılaştığı aşağıdaki soruda, oran orantı ile ilgili bazı ifadelerin doğruluğuna karar verilmesi isteniyor.

Buna göre bu soruda yer alan aşağıdaki ifadelerden hangisi ya da hangileri **<u>her zaman</u>** doğrudur?

Aşağıdaki her şık için DOĞRU ya da YANLIŞ'ı işaretleyip sebebini açıklayınız.

	DOĞRU	YANLIŞ
I. Oran, iki çokluğun toplamsal değil, çarpımsal karşılaştırılmasıdır. Açıklama:	☐	☐
II. Oran, bir miktarın matematiksel gösterimi olduğu için her zaman parça-bütün ilişkisi içerisinde verilir. Açıklama:	☐	☐
III. Bütün kesirler orandır. Açıklama:	☐	☐
IV. İki çokluktan birisi artarken diğeri azalıyorsa bu iki çokluk ters orantılıdır. Açıklama:	☐	☐

4) Dilek, okul ile gittiği Eskişehir gezisinde Masal Şatosunun 3 x 5 cm² lik bir fotoğrafını çekmiştir. Bu fotoğrafı fotoğrafçıya götürüp bir program sayesinde 7 × 9 cm² olacak şekilde düzenlemesini istemiştir. Buna göre orijinal fotoğraf mı yoksa düzenlenen fotoğraf mı daha "kareseldir"? İşaretleyiniz, sonuca yönelik adımlarınızı kağıt üzerinde gösteriniz.

A) 3 × 5 cm²'lik fotoğraf

B) 7 x 9 cm²'lik fotoğraf

C) İki fotoğraf da aynı şekle sahiptir. Birisinin daha karesel olma durumu yoktur.

Bir sonraki sayfada, yukarıdaki soruya (4. soru) verdiğiniz cevaba ilişkin soruları cevaplamanız gerekmektedir.

Bir önceki sayfada hangi cevabı verdiyseniz bu sayfada yer alan sorulardan **sadece** o cevaba ilişkin açıklamaların olduğu bölümü cevaplayınız**.** Bu sayfadaki soruların hepsini cevaplamayınız.

4. soruya **cevabınız A ise** aşağıda verilen her açıklamanın DOĞRU ya da YANLIŞ olma durumunu işaretleyiniz.

- 3×5 cm²'lik fotoğraf daha kareseldir çünkü 3×5 cm²'lik fotoğrafın daha kısa kenarlara sahip olması onun daha küçük görünmesine neden olur. 7×9 cm²'lik fotoğrafın daha uzun kenarlara sahip olması ise onun dikdörtgene benzemesini sağlar.

DOĞRU ☐ YANLIŞ ☐

- 3 x 5 cm²'lik fotoğraf daha kareseldir çünkü 3 x 5 cm²'lik fotoğrafın kare oluşturmak için 10 kareye 7x9 cm²'lik fotoğrafın 18 kareye daha ihtiyacı vardır.

DOĞRU ☐ YANLIŞ ☐

4. soruya **cevabınız B ise** aşağıda verilen her açıklamanın DOĞRU ya da YANLIŞ olma durumunu işaretleyiniz.

- 7 x 9 cm²'lik fotoğraf daha kareseldir çünkü $\frac{7}{9}$, $\frac{3}{5}$ 'e göre 1'e daha yakındır.

DOĞRU ☐ YANLIŞ ☐

- 7 x 9 cm²'lik fotoğraf daha kareseldir çünkü bu fotoğrafta büyüme oranı daha azdır. Yani,
düzenlenen fotoğrafta uzunluk, 7 x 7 cm²'lik kareye (olması gereken kareye) göre 2/7'lik bir oranda büyüyüp 9 olurken; orjinal fotoğrafta uzunluk, 3 x 3 cm²'lik kareye (olması gereken kareye) göre 2/3'lük bir oradan büyüyüp 5 olmuştur.

DOĞRU ☐ YANLIŞ ☐

4. soruya **cevabınız C ise** aşağıda verilen her açıklamanın DOĞRU ya da YANLIŞ olma durumunu işaretleyiniz.

- Asıl ve büyütülen fotoğrafta da kenarlar arasında sadece iki cm'lik fark vardır. Bu yüzden ikisi de aynı şekle sahiptir. Birisinin daha karesel olma durumu yoktur.

DOĞRU ☐ YANLIŞ ☐

- Asıl fotoğrafın kısa kenarı da uzun kenarı da 4'er cm arttırılarak fotoğraf düzenlenmiştir. Dolayısıyla iki fotoğraf da aynı şekle sahiptir. Birisinin daha karesel olma durumu yoktur.

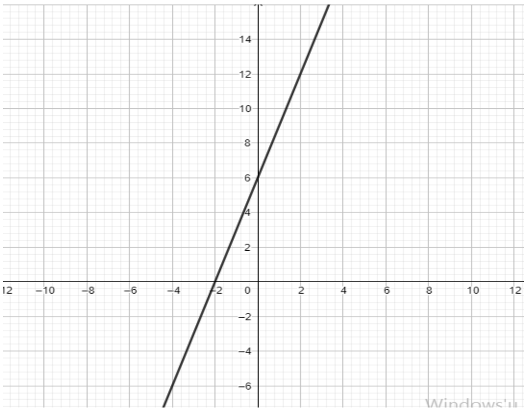
DOĞRU ☐ YANLIŞ ☐

5) Aşağıda verilen denklem ya da grafiklerden hangisi ya da hangilerinde x ve y arasında orantısal ilişki vardır?

I. $y = 2x$

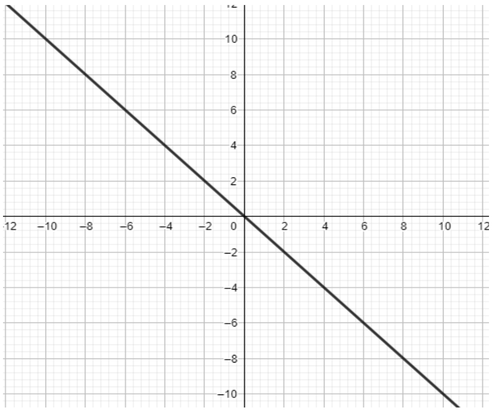
Orantısal ilişki vardır ☐ Orantısal ilişki yoktur ☐

II.



Orantısal ilişki vardır ☐ Orantısal ilişki yoktur ☐

III.



Orantısal ilişki vardır ☐ Orantısal ilişki yoktur ☐

IV. y = 1

	Orantısal ilişki vardır	Orantısal ilişki yoktur
	☐	☐

6) Öğrencilerinden yandaki tabloda yer alan x ve y değişkenlerinin orantılı olup olmadığını açıklamalarını isteyen Aygül öğretmenin aldığı açıklamalardan bazıları aşağıdaki gibidir.

x	9	15	21	27
y	12	20	28	36

Ali: x'in değerindeki her 6 birimlik artış y'nin değerindeki her 8 birimlik artışa denk geldiği için tablodaki x,y ikilileri birbiriyle orantılıdır.

Sezen: Herhangi bir adımdaki y sayısı ilgili x sayısının 4/3 katı olduğu için tablodaki x,y ikilileri birbiriyle orantılıdır.

Zehra: Tablodaki herhangi iki (x,y) ikilisinin oranları denk olduğu için tablodaki x, y ikilileri birbiriyle orantılıdır.

Öğrencilerin bu sorunun çözümüne yönelik yaptıkları açıklamaların geçerli bir açıklama olup olmadığını değerlendiriniz.

	Geçerli	Geçerli değil
Ali'nin kullandığı yöntem	☐	☐
Sezen'in kullandığı yöntem	☐	☐
Zehra'nın kullandığı yöntem	☐	☐

7) Zeynep öğretmenin matematik sınavında sorduğu sorulardan birisi aşağıdaki gibidir.

Oyuncu	İsabetli Atış (2'lik Atış)	Kullanılan Toplam Atış (2'lik Atış)
Dusko Savanovic (DS)	12	23
Hidayet Türkoğlu (HT)	10	18
Marko Keselj (MK)	8	20
Ömer Onan (ÖO)	9	21

Tabloda 2010 yılında oynanan Türkiye-Sırbistan yarı final maçındaki dört oyuncuya ait kullanılan toplam 2 sayılık atış sayıları ve bunlardan kaç tanesinin isabetli olduğu bilgisi verilmiştir. Buna göre hangi oyuncunun 2 sayılık atış performansı en iyidir? (Basketbolda atış performansında başarı, isabetli atışların kullanılan toplam atışa oranı ile belirlenmektedir)

Zeynep öğretmen aşağıdaki öğrenci cevaplarından hangisi ya da hangilerini doğru cevap olarak kabul etmelidir? Aşağıdaki her şık için DOĞRU ya da YANLIŞ'ı tüm sorularda işaretleyiniz.

	DOĞRU	YANLIŞ
Sevde: Atış yüzdeleri karşılaştırıldığında diğerlerine göre Hidayet Türkoğlunun atış yüzdesi daha fazladır. O yüzden atış performansı en iyi olan odur. (HT: % 55.6 > DS: % 52.1 > ÖO: % 42 > MK:% 40)	☐	☐
Vuslat: Tüm oyuncular için isabetli atışları teşebbüs edilen atışlar ile karşılaştırdığımızda en büyük oran Hidayet Türkoğlu'na aittir. Bu yüzden en iyi atış performansı ona aittir. ($\frac{10}{18} > \frac{12}{23} > \frac{9}{21} > \frac{8}{20}$)	☐	☐
Mustafa: Oyuncuların atış performansları ondalık gösterim ile ifade edildiğinde en büyük sayı Hidayet Türkoğluna aittir. Bu yüzden en iyi performans onundur. (HT: 0.556 > DS: 0.521 > ÖO: 0.428 > MK: 0.40)	☐	☐

8) Yandaki resimde görülen ön ve arka tekerlek boyutlarının birbirinden farklı olduğu Granbi isimli bisikletler icat edilmiştir. Bu bisikletler 1870'lerin başında Fransa'da ön tekere bağlı pedala sahip bisikletlerin hızını arttırmak amacıyla tasarlanmıştır.

Granbilerin ön tekerleğinin çevresi 405 cm ve yarıçapı arka tekerleğinin yarıçapının 3 katıdır.

Bu bilgilere göre arka tekerlek 45 tur döndüğünde bisiklet kaç metre ilerler?

A) 40.50 m

B) 60.75 m

C) 63.25 m

D) 81 m

E) 121.5 m

9) Aşağıdaki ifadelerdeki çoklukların birbiriyle doğru ya da ters orantılı olma durumuna karar veriniz. Her şık için DOĞRU ORANTILI ya da TERS ORANTILI şıkkını işaretleyiniz.

	DOĞRU ORANTILI	TERS ORANTILI
a) $\frac{a}{4} = \frac{b}{5}$	☐	☐
b) $\frac{a}{b} = 2$	☐	☐
c) $a \times b = 4$	☐	☐
d) $2a = 3b$	☐	☐
e) $\frac{a}{1} = \frac{8}{b}$	☐	☐

10) Aşağıdaki probleminin çözümü için öğrencilerin kullandığı çözüm yöntemleri aşağıda verilmiştir. Buna göre aşağıdakilerden hangisi ya da hangileri geçerli bir çözüm yöntemidir?

Sinem'in annesi ve babası akşamki doğum günü partisi için 2 farklı sürahide limonata hazırlamıştır. Hazırlanan limonata için eşit büyüklükteki limonları, su bardaklarını ve sürahileri kullanmışlardır. Annesi, hazırladığı limonatada 4 bardak suya 3 limon kullanırken; babası, 5 bardak suya 4 limon kullanmıştır. Buna göre Sinem'in anne ve babasının hazırladığı sürahilerdeki limon tadı ile ilgili ne söylenebilir?

I. Annesi 3 limona 4 bardak su koymuş ise 1 limon için $4/3$ bardak su; Babası 4 limona 5 bardak su koymuş ise 1 limon için $5/4$ bardak su koymuş olur. Annesi bir limon için daha fazla su koyduğu için babasının limonatasının tadı daha limonludur.

II. Annesi 4 bardak su için 3 limon kullanmış ise 1 bardakta limonun $3/4$ nü, babası 5 bardak su için 4 limon kullanmış ise 1 bardakta limonun $4/5$ ni kullanmış olur. Bu durumda babasının hazırladığı limonatanın 1 bardağında daha çok limon kullanıldığı için onun hazırladığı limonatanın tadı daha limonludur.

III. Karşılaştırma yapabilmek için aynı sayıda limon kullanılmasını düşünmüşlerdir. Annesi 3 limona 4 bardak su ilave ettiği limonatanın tadını elde etmek için 12 limona 16 bardak su kullanırken babası 4 limona 5 bardak su ilave ettiği limonatanın tadını elde etmek için 15 bardak su kullanacaktır. Bu durumda aynı sayıdaki limon için daha az su kullandığı için babasının hazırladığı limonatanın tadı daha limonludur.

A) I ve II

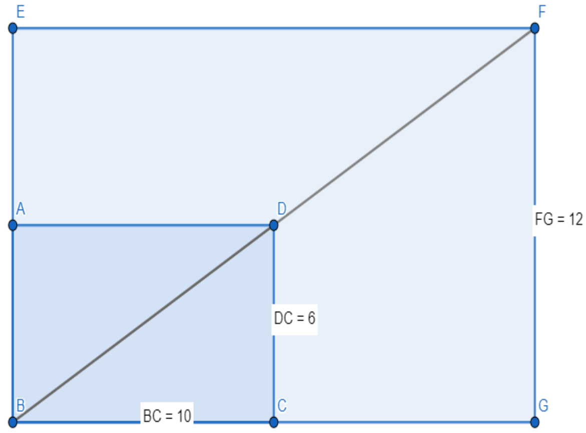
B) I ve III

C) Yalnız III

D) II ve III

E) I, II ve III

11)



Bir bilgisayar programı ile kısa kenarı 6 cm, uzun kenarı 10 cm olan bir ABCD dikdörtgeni D köşesinden tutularak BD köşegeni doğrultusunda büyütülüp küçültülebilmektedir. Bu şekilde oluşturulan EBGF dikdörtgenin uzun kenarı kaç cm'dir?

Yukarıdaki problem iki farklı yöntem ile çözülmüştür.

i. $\frac{5}{3} \times 12 = 20$

ii. $10 \times \frac{12}{6} = 20$

Bu çözüm yöntemleri ile ilgili aşağıda verilen ifadelerden hangisi yanlıştır?

A) Birinci çözüm yönteminde $\frac{5}{3}$ kenarlar arasındaki çarpımsal ilişkiyi ifade etmektedir. Bu çarpımsal ilişki dikdörtgenin D köşesinden genişletilmesi durumunda [BG] kenar uzunluğunun bulunmasında kullanılmıştır.

B) Birinci çözüm yönteminde elde edilen $\frac{5}{3}$ değeri ABCD dikdörtgeninin küçültüldüğü durumlarda da korunmalıdır.

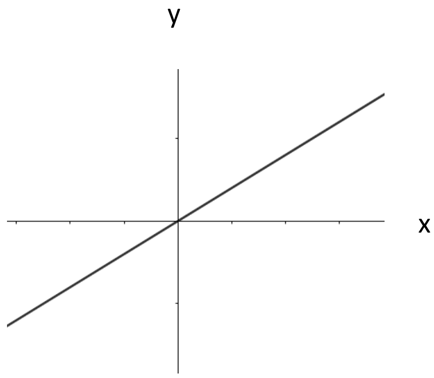
C) İkinci yöntemde $\frac{12}{6}$ değeri iki şekil arasındaki değişiklik katsayısını ifade etmektedir. Bu katsayı [BG] kenar uzunluğunun bulunmasında kullanılmıştır.

D) İkinci yöntemde elde edilen $\frac{12}{6}$ değeri ABCD dikdörtgeninin küçültüldüğü durumlarda da korunmalıdır.

12) Aşağıda aynı fonksiyona ait üç farklı gösterim verilmiştir.

$y = kx$

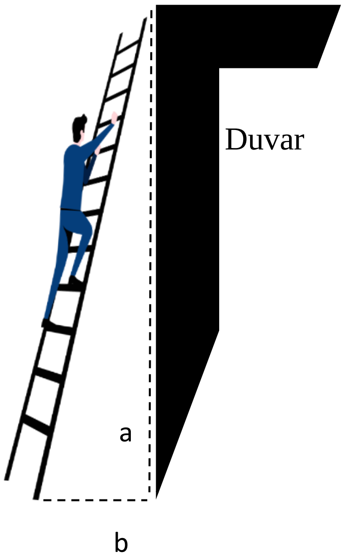
x	y
0	0
x_1	y_1
x_2	y_2
.	.
.	.
.	.
x_n	y_n



I. Gösterim	II. Gösterim	III. Gösterim

Buna göre aşağıdakilerden hangisi belirtilen fonksiyon için x = 0 hariç <u>her zaman</u> geçerli bir ifadedir? İşaretleyiniz.

	Geçerli	Geçerli Değil
1. $x_1 + x_2 = y_1 + y_2$	☐	☐
2. $\frac{x_5}{x_6} = \frac{y_1}{y_2}$	☐	☐
3. $\frac{y_n}{x_n} = k$	☐	☐



13) Yanda verilen resimde duvara yaslanmış ve kayma ihtimaline karşı yere sabitlenmiş basamaklarının arası eşit mesafedeki bir merdiven ve bu merdivene tırmanan Ali Usta resmedilmiştir.

Buna göre aşağıdakilerden hangisi doğru bir ifadedir?

I. Merdivenin dikliği, merdivenin 1. ve 5. basamaklarında farklılık gösterir.

Doğru ☐ Yanlış ☐

II. Merdivenin dikliği dikey yüksekliğin (a) sayısal değeri ile ifade edilir.

Doğru ☐ Yanlış ☐

III. Merdivenin dikliğinde dikey yüksekliğin (a) yatay mesafeye (b) göre bağıl değişimi söz konusudur.

Doğru ☐ Yanlış ☐

APPENDIX C

OVERVIEW OF QUESTIONS IN PROPORTIONAL REASONING INSTRUMENT AND TASK-BASED INTERVIEWS

Table C1. Overview of Questions in PRI

Label	Construct	Components of PR	Overview	Sub-questions	Task type
Q1	Develop and use different strategies for proportional problems	1, 2, 4 & 5	Students' answers to missing value problems about the enlargement of the rectangle with non-integer number structure	Three sub-questions (students' answers)	Includes missing value problem
Q2	Differentiate proportional situatios from non-proportional ones	3 & 4	Proportional and non-proportional situations	Seven sub-questions	Includes missing value problems
Q3	Evaluate and explain the definitions and properties of ratio and proportion	4, 5 & 6	Definitions and statements of concepts related to ratio and proportions	Four sub-questions	Includes definitions or statements related to ratio and proportion
Q4	Realize the multiplicative relationship in proportional situations	1, 2 & 4	Squareness problem and students' solutions to the given problem	One question and two related questions	Ratio Comparison Problem
Q5	Distinguish proportional situatios from non-proportional ones in graphical and algebraic expressions	3, 4, & 5	Relationship between proportionality and linearity in line graphs and algebraic equations	Four sub-questions	Ratio Comparison Problems

Q6	Understand the invariance and covariance structures of proportional relationships	2, 4 & 5	Students' explanations about the reasons for being proportional	Three sub-questions (students' answers)	Ratio Comparison Problem
Q7	Recognize different representations of ratio	1 & 6	Different representations of proportional situations (percentage, fraction, decimal)	Three sub-questions	Ratio comparison problem
Q8	Solve problems including inversely proportional variables	1 & 4	Inverse proportion problem in the context of bicycle	One question	Missing value problems
Q9	Determine the proportionality type of variables in the equation	4 & 5	Inverse or direct proportion	Five sub-questions	Includes five equations
Q10	Develop and use different strategies for proportional problems	1, 2, 4 & 5	Different students' solutions for mixture problem (unit ratio, equivalent fractions)	One question	Ratio comparison problem
Q11	Understand the invariance and covariance structures of proportional relationships	1, 2, 4, 5 & 6	Change factor and multiplicative relationships between quantities (coming from same and different measure fields)	One question	Includes missing value problem
Q12	Recognize invariance and covariance structures of proportional relationships in algebraic expressions	4, 5 & 6	Invariance and covariance structures of direct proportion	Three sub-questions	Evaluation of three statements

Q13	Understand the invariance and covariance structures of proportional relationships in the context of slope	1, 4, 5 & 6	Definition of slope and its invariance structure in stairs problem	Three sub-questions	Evaluation of three statements

Note: Six components of proportional reasoning identified by Lamon (2005, 2007) are as follows: (1) Solve proportional problems in a wide range of context from slope to similarity or proportional problems involving number complexities e.g., non-integer ratios, fractions or decimals (2) Develop and use different strategies in solving problems requiring reasoning proportionally rather than using only traditional proportion algorithm (3) Distinguish proportional situations from non-proportional ones (4) Understand the multiplicative relationships both in direct and inverse proportions (5) Realize and understand the invariance and covariance structure of the proportional relationships (6) Develop and use the language for ratio and proportions

Table C2. Scoring Guide for Sub-Questions in Q3

Sub-Question Descriptor	Sub-Question	Sufficient Examples	Insufficient Examples	Incorrect Examples
The definition of ratio	The ratio is a multiplicative comparison of two quantities rather than an additive comparison of them	Multiplicative means multiplying ratio constant (k) with quantities Because when we say ratio, we mention about the multiplies/times	From the definition of ratio Ratio includes multiplicative operations	Issues such an age, the ratio of two people cannot be possible. Because there is a difference In A/B = C/D equation if denominators were made equal, A × C = B × D
Ratio and Part-Whole	The ratio is always given within the part-whole relationship because it is the mathematical notation of quantity	It can part-part comparison, too Values that are independent of each other also can be compared	Giving within part-whole relationship is not compulsory 2/1 is also the ratio	---------------------
Ratio and Fraction	All fractions are the ratio	There is a part-whole relationship within fractions. It can be explained by the ratio The relationship between numerator and denominator was constant when extending and simplifying	A fraction is the division of any two numbers. Division can be considered as a particular case of multiplication. Because the ratio is shown as a/b.	----------------------

Inverse Proportion	If one of two quantities increases as the other decreases, these quantities are inversely proportional	It can also decrease as an additive Their decrease and increase must be at the same ratio	It should be a constant condition For example, if five people completed work in two days, ten people completed work in one day. When one increases, the other decreases	-------------------

Table C3. Interview Questions for Problems

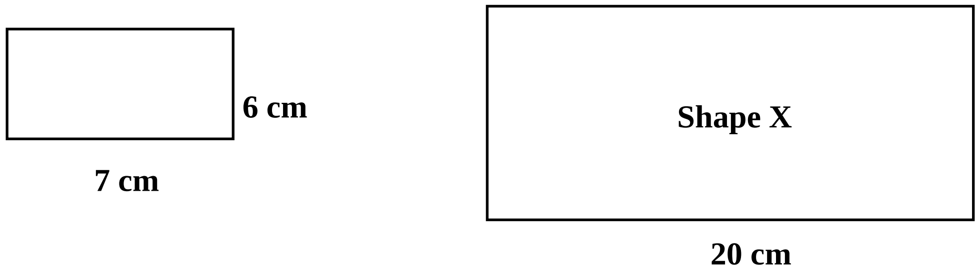
Problems in the interview	Interview Questions asked by Researcher
Q1 Enlargement of Rectangle	Can you explain how you solve the question? What do you think about students' answers?
Q1 Evaluation of Students' Answers to the enlargement question	What are the ways/strategies used by students? Are they similar to your solutions? What kind of similarity or difference is between these solutions? How did you decide whether or not both students and your own solutions are valid for whole scaling situations?
Q2 Proportional and Non-proportional Situations	How did you decide whether these situations proportional or non-proportional? Can you explain in detail?
Q3 Inverse Proportion Table Representation	Is student explanation correct or not? Why? Is there any alternative justification/explanations you provide for x and y not being inversely proportional to each other? If you are asked the definition of the inverse proportion, how do you define it?
Q4 Linearity and Proportionality Graphs Representation	Which stuff do you focus when deciding whether x and y in related graphs are proportional? How can you be sure your result and reasoning are correct? How can you conclude for being proportionality taking into consideration of two graphs? What are the common points for graphs that x and y are proportional?

Q5 Direct and Inverse Proportion	Can you explain your answers? How did you decide whether quantities are inversely or directly proportional? Can you decide whether quantities are inversely or directly proportional without giving numbers for a and b? How can you?
Q6 Mosaic Problem (Between Ratios)	Can you explain the student's solution? What does 2.25 represent? Is it meaningful to multiply this value with 40? Why? How would you solve it?
Q7 Faucet Problem (Inverse Proportion)	Can you explain your answers? How can you be sure your result and reasoning are correct? How did you decide that these quantities are inversely proportional? Is there any alternative way to solve this problem? What does the value that was obtained when multiply 11 by 13 mean? Why did you multiply quantities in horizontal lines in inverse proportion although multiplying quantities in vertical lines in direct proportion?
Q8 Bicycle Problem (Inverse Proportion)	Can you explain your answers? How can you be sure your result and reasoning are correct? How did you decide that these quantities are inversely proportional?

APPENDIX D

INTERVIEW QUESTIONS

1. The dimension of the first shape, which has lenght of 7 cm and width of 6 cm is extented by keeping the ratio constant and the second shape is obtained. According to this, what is the width (a) of X shape in cm?



Please, try to solve the problem by using as different solutions as possible

Student A	Student B	Student C
According to 7 cm $\rightarrow$ 20 cm 1 cm is proportionally $\rightarrow \frac{20}{7}$ $6\text{ cm} \times \frac{20}{7} = \mathbf{\frac{120}{7}\ cm}$	Ratio of length to width of the first shape: $\frac{6}{7}$ $20\text{ cm} \times \frac{6}{7} = \mathbf{\frac{120}{7}\ cm}$	$\frac{6}{7} = \frac{a}{20}$ $6 \times 20 = 7a$ $a = \mathbf{\frac{120}{7}\ cm}$

2. Please, decide whether the situations in the questions are the proportional or non-proportional. Mark the proper choice for each question.

Ayşe and Mehmet are running at the same speed on the running track. When Ayşe finished 4th tour, Mehmet finished 6th tour. According to this, which tour does Ayşe finish when Mehmet has finished 12th tour at the same running track?

Proportional Situation ☐ **Non-proportional Situation** ☐

If 20 minutes are required to dry 5 pieces of clothes in the open air, how much time is required to dry 20 pieces of clothes under the same conditions?

Proportional Situation ☐ **Non-proportional Situation** ☐

2 TL per person and 10 TL for a vehicle are paid for the entrance of Kuşadası National Park. 5 people, who enters into National Park by their 10 seater vehicle, pay 20 TL for the entrance. How much is required to pay when 10 people want to enter into the park by the same vehicle?

Proportional Situation ☐ **Non-proportional Situation** ☐

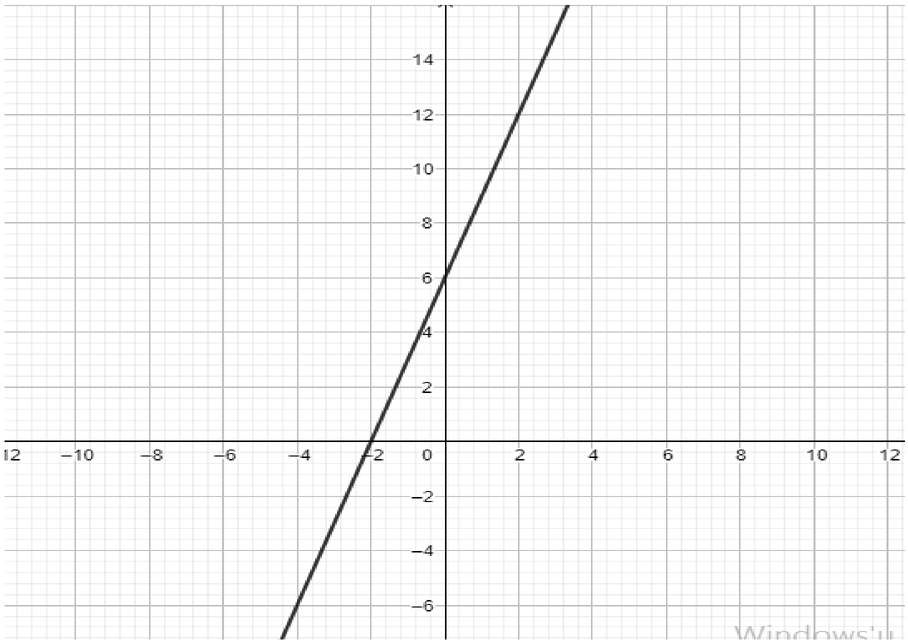
Please, find the value of a land , the area of which is 10 km², in cm².

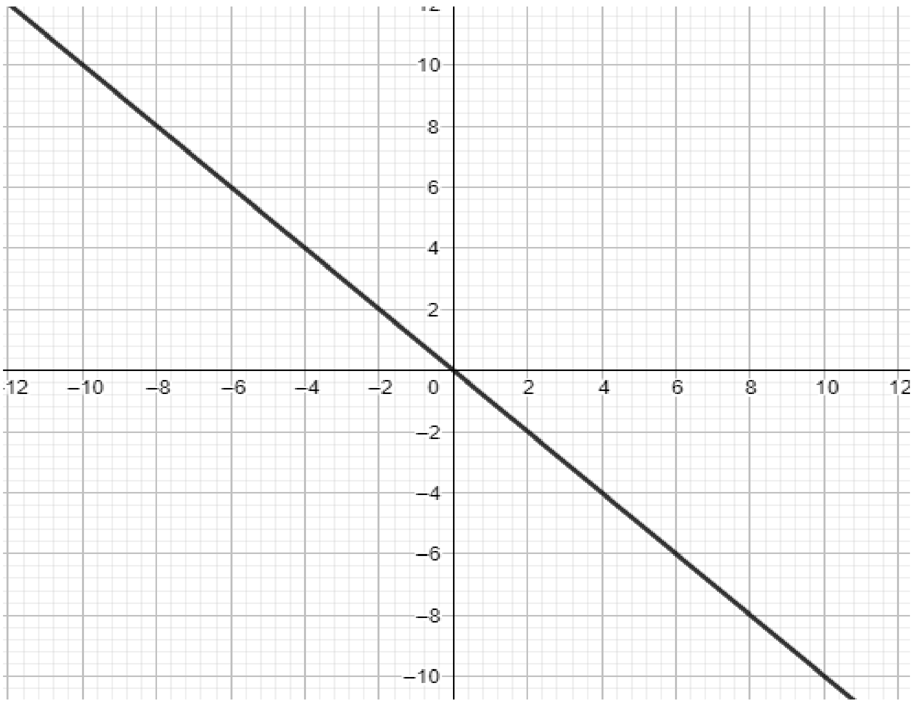
Proportional Situation ☐ **Non-proportional Situation** ☐

3. A student who interprets the quantities in the chart below indicates that (x,y) pairs are inversely proportional because y decreases for 1 unit as x increases for 1 unit.

X	5	6	7	8	9
Y	9	8	7	6	5

Please, evaluate the explanation of the student.





4. In which graph or graphs is there a proportional situation between x and y?

5. Please, decide whether the quantities are directly proportional or inversely proportional with each other.

$$\frac{a}{4} = \frac{b}{5} \quad .$$

$$\frac{a}{b} = 2$$

$$\frac{a}{1} = \frac{8}{b}$$

6. Two siblings, Ayşe and Serkan, decided to create a mosaic design on their table top from broken pieces of tile. They worked for sometime, and then they had a break. When they had the break they realized that, it took 16 minutes to finish an area of 40 cm square. If they worked at the same rate, how much of the table top could they finish in 36 minutes?

The solution for the problem is as below:
We divide 36 into 16, 36:16 = 2.25, then, we multiply it with 40. 2.25×40 = 9

Please, evaluate the solution.

7. If the identical 11 faucets can fill a storage in 13 hours, how many hours does 7 same faucets spend to fill the same storage?

8. Granbi bicycles on the next picture, which have differrent front and back wheel in dimension, are invented. These bicycles are designed for increasing the speed of bicycles that has a pedal attached to the front wheel in France in the early 1870s.

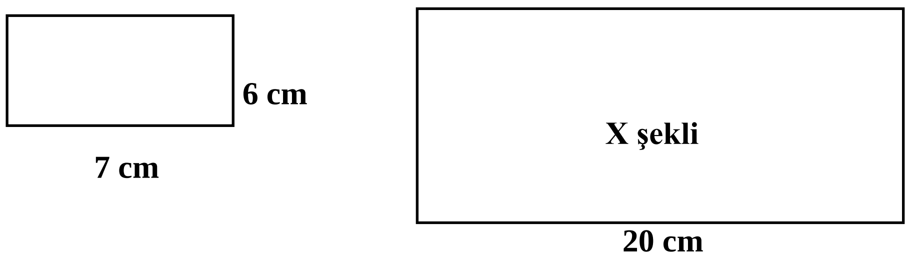
The perimeter of Granbi's front wheel is 405 cm and its radius is three times of radius of the back wheel.

According to this, how many meters does the bicycle go when the back wheel turns for 45 tour?

APPENDIX E

INTERVIEW QUESTIONS (TURKISH)

1. Boyutları 6 cm ve 7 cm olan bir dikdörtgenin boyutları şeklin yapısı korunarak genişletilip X şekli elde ediliyor. Buna göre X şeklinin kısa kenarı (a) kaç cm'dir?



Soruyu olabildiğinde farklı çözüm yöntemi kullanarak çözmeye çalışınız.

A Öğrencisi	B Öğrencisi	C Öğrencisi
7 cm → 20 cm olduğuna göre 2 cm → $\frac{20}{7}$ katına çıkmış 6 cm $\times \frac{20}{7} = \frac{\mathbf{120}}{\mathbf{7}}$ **cm**	Birinci şekilde kenarlar oranı = $\frac{6}{7}$ 20 cm $\times \frac{6}{7} = \frac{\mathbf{120}}{\mathbf{7}}$ **cm**	$\frac{6}{7} = \frac{a}{20}$ $6 \times 20 = 7a$ $a = \frac{\mathbf{120}}{\mathbf{7}}$ **cm**

2. Aşağıdaki soruların orantısal olup olmama durumuna karar veriniz. Her soru için uygun seçeneği işaretleyiniz.

Ayşe ve Mehmet bir koşu parkurunda eşit hızlarla koşmaktadır. Mehmet'in koşmaya daha önce başladığı bilinmektedir. Ayşe 4. turu bitirdiğinde Mehmet 6. turu bitirmiştir. Buna göre aynı koşu parkurunda Mehmet 12. turu bitirdiğinde Ayşe kaçıncı turu bitirir?

Orantısal Durum	**Orantısal Olmayan Durum**
☐	☐

Açık havada 5 parça çamaşırın kuruması için gerekli süre 20 dakika ise aynı hava şartlarında 20 parça çamaşırın kuruması için gereken süre ne kadardır?

Orantısal Durum	**Orantısal Olmayan Durum**
☐	☐

Kuşadası Milli Parkına giriş için kişi başı 2 TL, yanı sıra araba parası olarak da 10 TL ödenmektedir. 10 kişilik arabasıyla Milli Parka giriş yapan 5 kişi giriş için 20 TL ödemektedir. Aynı araba ile 10 kişi girmek istendiğinde ne kadar ücret ödenmelidir?

Orantısal Durum	**Orantısal Olmayan Durum**
☐	☐

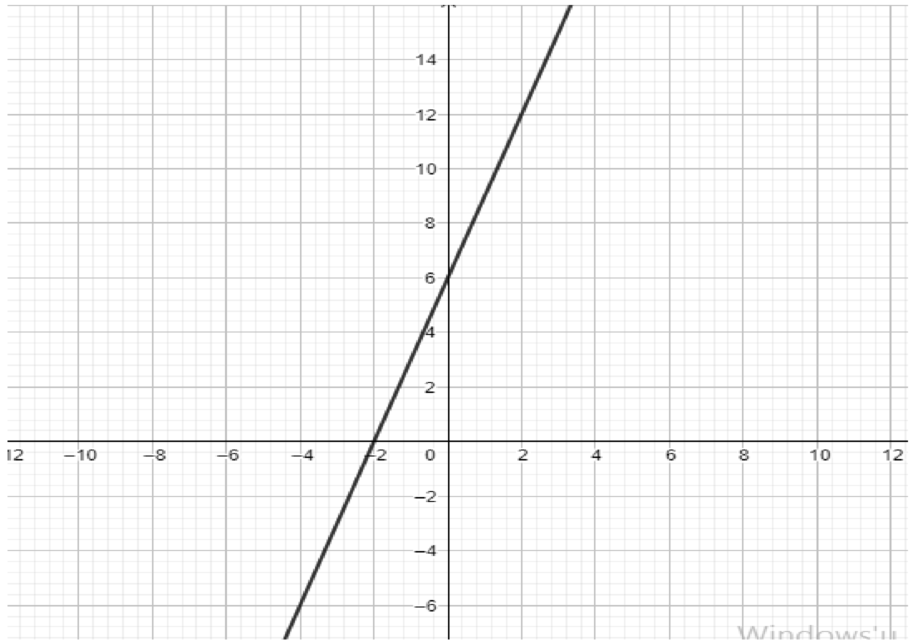
Alanı 10 km^2 olan bir arazinin cm^2 cinsinden değerini bulunuz.

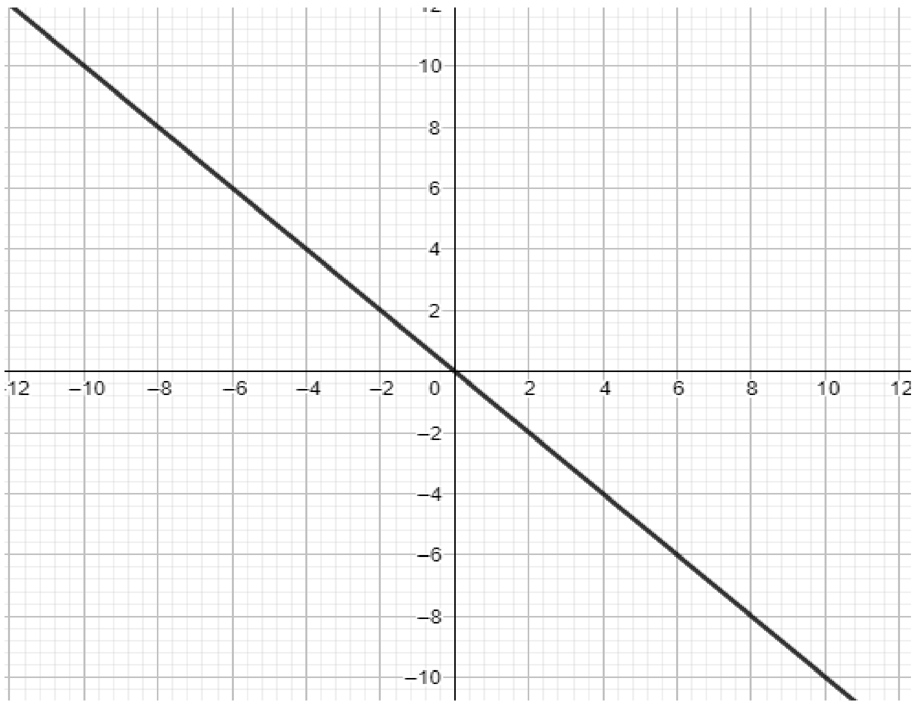
Orantısal Durum	**Orantısal Olmayan Durum**
☐	☐

3. Aşağıdaki tabloyu yorumlayan bir öğrenci, (x,y) ikililerinin ters orantılı olduğunu, çünkü x’in her 1 birimlik atışında y’nin 1 birim azaldığını belirtmiştir.

X	5	6	7	8	9
Y	9	8	7	6	5

Öğrencinin açıklamasını değerlendiriniz.





4. Verilen grafiklerden hangisi ya da hangilerinde x ve y arasında orantısal ilişki vardır?

5. Aşağıda yer alan ifadelerdeki çoklukların birbiriyle doğru ya da ters orantılı olma durumuna karar veriniz.

$$\frac{a}{4} = \frac{b}{5} \quad .$$

$$\frac{a}{b} = 2$$

$$\frac{a}{1} = \frac{8}{b}$$

6. İki kardeş, Ayşe ve Serkan, kırık kiremit parçaları ile masalarının üzerine bir mozaik desen yapmaya karar vermişlerdir. Yapım aşamasında belirli bir zaman birlikte çalıştıktan sonra ara vermişlerdir. Kardeşler, ara verdikleri anda 40 cm^2'lik alanı 16 dakikada bitirdiklerini fark etmişlerdir. Aynı hızla çalışmaya devam ederlerse, 36 dakika içerisinde ne kadarlık alanı kaplayabilirler?

Aşağıda bu probleme ilişkin bir çözüm verilmiştir:
36'yı 16'ya böleriz, 36:16 = 2.25 sonra da 40 ile çarparız. 2.25×40 = 9

Çözümü değerlendiriniz.

7. Özdeş 11 musluk bir depoyu 13 saatte doldurabildiğine göre, bu musluklardan 7 tanesi aynı depoyu kaç saatte doldurur?

8. Yandaki resimde görülen ön ve arka tekerlek boyutlarının birbirinden farklı olduğu Granbi isimli bisikletler icat edilmiştir. Bu bisikletler 1870'lerin başında Fransa'da ön tekere bağlı pedala sahip bisikletlerin hızını arttırmak amacıyla tasarlanmıştır.

Granbilerin ön tekerleğinin çevresi 405 cm ve yarıçapı arka tekerleğinin yarıçapının 3 katıdır.

Bu bilgilere göre arka tekerlek 45 tur döndüğünde bisiklet kaç metre ilerler?

APPENDIX F

PROPORTIONAL REASONING PERFORMANCES OF PARTICIPANTS DURING THE INTERVIEW

Table F1. Proportional Reasoning of Participants with Highest Scores on PRI During the Interview

QUESTIONS	PSMT1	PSMT2
Q1a: Enlargement of the Rectangle (alternative solutions)	Three different solution strategies: Change factor (between ratios), cross-multiplication algorithm, within ratios. Conceptual explanations were provided: He mentioned the term “change factor” between rectangles and explained that the ratio width to length should be preserved among rectangles.	Two different solution strategies: Cross-multiplication and within ratios (multiplicative relationship). Conceptual explanations provided: The ratio was defined as the relationship between side lengths and she explained it must be preserved.
Q1b: Enlargement of the Rectangle (evaluation of students’ solutions)	Solutions of Student B and C were perceived as the similar solutions. The operation of $\frac{6}{7} \times 20$ was not made sense by him and Student B solution was considered as mathematically deficient.	Three student’s solution were correctly identified as a valid solution. The ratio of $\frac{6}{7}$ was defined as the multiplicative relationship between width and length.
Q2: Distinguish proportional situations from non-proportional ones.	All four situations were categorized accurately. For all situations, he made explanations linked with concepts.	All four situations were categorized and explained accurately. The first situation was incorrectly defined as proportional initially, but then she corrected it.
Q3: Inverse Proportion (Table Representation)	He explained that their products are not same, which would be necessary for being inversely proportional. He also stated that their increase and decreases were not with the same ratio.	She explained that the values of x and y are not inversely proportional and emphasized the multiplicative relationship between values and the invariance of the product.

Q4: Proportional or Non-proportional (y = mx and y = mx+n graphs)	He identified y = -x as proportional because of the constant ratio of $\frac{y}{x}$. He identified the first graph y = 3x+6 as non-proportional and explained that the term +6 prevents the proportionality.	She identified y = -x as proportional because there is a constant ratio, -1. She also identified y = 3x+6 as non-proportional and explained that the value, +6, prevents the proportionality.
Q5: Inverse proportion or direct proportion	He identified correctly whether a and b in three statements are inversely or directly proportional by stating the constant of proportionality.	She identified correctly whether a and b in three statements are inversely or directly proportional by assigning number to a and b. She made explanations about the invariance structures too.
Q6: Mosaic Problem (Between Ratio)	He was able to recognize what 2.25 refers to. He explained that 16 served as a new unit and so multiplying 40 by 2.25 is meaningful. He solved this problem by using within ratio.	She was able to recognize what 2.25 refers to. She explained that relationship (change factor) between minutes must be preserved in areas. She solved this problem by using within ratios.
Q7: Faucet Problem (Inverse Proportion)	He first set a direct proportion between quantities. However, in the explanation part he noticed that they are inversely proportional. Then he solved the problem by using constant pool capacity and unit strategy.	She incorrectly solved the faucet question by setting a direct proportion between quantities. She was not able to recognize her mistake even when asked to reflect on it.
Q8: Bicycle Problem (Inverse Proportion)	He solved the problem by emphasizing the inverse proportion between number of turns and radius of wheels since the total distance is the same for both wheels.	She solved the problem by emphasizing the inverse proportion between number of turns and radius of wheels since the total distance is the same for both wheels.

Table F2. Proportional Reasoning of Participants with Average Scores on PRI During the Interview

QUESTIONS	PSMT3	PSMT4	PSMT5
Q1a: Enlargement of the Rectangle (alternative solutions)	Three different solution strategies: setting proportions with between ratios and within ratios and factor of the change strategy. Explanations were provided: She mentioned the term "change factor" and also the ratio must be preserved in other situations (within and between ratios).	Two different solution strategies: setting proportions with between ratios and within ratios. However, the meaning of these ratios was not explained (change factor or multiplicative relationship). She only emphasized that the ratio between them must be preserved because the rectangle is enlarged by keeping the structure of shape constant.	Two different solution strategies: setting proportions with between ratios and within ratios. He solved the question by referring to the change factor. He also explained other solution by stating that the ratio, $\frac{6}{7}$, between them must be preserved because the rectangle is enlarged by keeping the structure of shape constant.
Q1b: Enlargement of the Rectangle (evaluation of students' solutions)	She was able to explain the first and third solution strategies as unit and cross-multiplication strategy. However, she considered that the second one is a different version of the traditional algorithm so that student B unconsciously solved the question like that.	He had difficulty in making sense of a student's unit strategy. He was not able to recognize the ratio, $\frac{6}{7}$, the as multiplicative relationship between width to length. However, this solution was considered valid only because the result is correct. He also stated that the second solution is a different version of the third one, cross-multiplication strategy.	He recognized Student A and C's solutions as a unit and cross-multiplication strategies respectively. However, he was not able to comprehend the ratio, $\frac{6}{7}$, as the relationship between width and length. So, he did not find the second solution meaningful. He emphasized the second and third solution were similar procedures: the second was a more practical one.

Q2: Distinguish proportional situations from non-proportional ones	She solved the running track question by setting proportion and considered it as proportional erroneously. Although she decided incorrectly first the fourth question, all three were identified correctly when explaining answers.	He identified situations correctly as proportional or non-proportional except the running-track problem. He solved this problem by setting and solving proportions. Although he correctly identified the parking problem as non-proportional, he confused the direct proportion with inverse proportion.	He identified situations correctly as proportional or non-proportional except the running-track problem. He solved this problem by setting and solving proportions.
Q3: Inverse Proportion (Table Representation)	She decided the values x and y's in the table are inversely proportional. Although she knew that the product of quantities must be equal for inverse proportion, she was not able to integrate and use this knowledge in the question.	He decided whether or not x and y's in the table were inversely proportional by using the invariance structure of inverse proportion, a product of quantities. However, he did not mention when x increasesi y will decrease by an amount such that $x \times y$ remains the same.	He associated ratio with multiplication and division. So, he was able to recognize that increase and decrease in the same amount is not related to inverse proportion. He explained why they were not inversely proportional by using the invariance structure, product of quantities.
Q5: Inverse proportion or direct proportion	She identified correctly whether a and b in three statements are inversely or directly proportional by assigning a number to a and b. She was not able to provide any explanations	He identified correctly whether a and b in three statements are inversely or directly proportional by assigning a number to a and b. When I asked alternative solutions apart from assigning numbers, he looked patterns and concluded that the notion $\frac{a}{b}$ is related to direct, the notion of a $\times$ b to an inverse proportion.	He identified correctly whether a and b in three statements are inversely or directly proportional by assigning numbers to a and b. When I asked for alternative solutions apart from assigning numbers, he looked for patterns and associated direct proportion as division and inverse proportion as multiplication.

Q6: Mosaic Problem (Between Ratio)	She considered the student's solution as valid only by looking at the result. She was not able to comprehend the meaning of the value of 2.25. Rather, she explained that student solved the problem in a way that she firstly divide 36 by 16 and then 40 when setting direct proportion (within ratios) rather than first multiply 40 with 36 and then the resulting divide by 16.	He did not consider student solution as a valid solution. He tried to make sense of 2.25 by using the equivalence of between ratios, $\frac{36}{16}=\frac{x}{40}$. He identified 2.25 as the ratio of 36 to 16 as seen in the proportion above. Although he associated this situation with the ratio of the length of the small rectangle to the length of the enlarged rectangle, he decided it as an invalid solution.	He stated that 2.25 refers to the scale factor between minutes and this value also must be preserved between the amount of works. Although he was able to recognize and explain the student solution, he got confused in comparing area and time. He also solves the problem with unit strategy.
Q7: Faucet Problem (Inverse Proportion)	She solved the problem with inverse proportion algorithm. When I asked her about alternative solution strategies, she tried to use unit strategy however she was not able to. Also she could not provide any conceptual explanations about the horizontal multiplication in the algorithm.	He solved the problem with inverse proportion algorithm. When I asked him about alternative solution strategies, he solved the problem by emphasizing the capacity of the pool is the same all the time. He also explained that 13 is needed to be multiplied with $\frac{11}{7}$, however he was not able to explain the underlying reason.	He applied the inverse proportion algorithm to find the time needed to fill a pool for a faucet. He also clearly told that the product of 11×13 refers to the capacity of pool and it remains constant even though the number of faucets changes.
Q8: Bicycle Problem (Inverse Proportion)	She solved the problem by emphasizing the inverse proportion between the number of turns and radius of wheels because of the distance is the same for both wheels	He solved the problem by emphasizing the inverse proportion between the number of turns and radius of wheels because of the distance is the same for both wheels.	He solved the problem by emphasizing the inverse proportion between the number of turns and radius of wheels because of the distance is the same for both wheels.

Table F3. Proportional Reasoning of Participants with Lowest Scores on PRI During the Interview

QUESTIONS	PSMT6	PSMT7
Q1a: Enlargement of the Rectangle (alternative solutions)	Three different solution strategies: Setting the proportions with between and within ratios. Although first and third solutions are the same solutions, the only difference was the order of sides. The meaning of ratios, scale factor or multiplicative relationships were not mentioned. He only emphasized the ratios must be the same.	Two different solution strategies: setting proportions with between ratios and within ratios. However, the meaning of these ratios was not explained (change factor or multiplicative relationship). She only emphasized that the ratio between them must be preserved. She also emphasized the similarity between rectangles.
Q1b: Enlargement of the Rectangle (evaluation of students' solutions)	He was able to recognize and understand the solutions with unit and cross-multiplication strategies. He considered the meaning of the value $\frac{6}{7}$ as the ratio width to length and explained that second and third solutions are different representations of the same procedures.	She was making sense of and explained unit and cross-multiplication strategies. Like PSMT6, she also stated that second and third solution strategies are similar. The only difference is that the second solution is a more practical and abbreviated version.
Q2: Distinguish proportional situations from non-proportional ones	He had difficulty in differentiating non-proportional situations from proportional ones. Area conversion and laundry problem were categorized correctly. However, parking and running track problems were incorrectly categorized as proportional.	She incorrectly defined the running track, laundry problems as proportional. She decided correctly park problem as non-proportional by using numerical values and area conversion as proportional. However, she made an incorrect explanation for area conversion.
Q3: Inverse Proportion (Table Representation)	He realized that x and y's in the given table are not inversely proportional. He stated that increase and decrease must be at the same ratio. However, his improper language use caused him to be confused.	She was satisfied with the student's explanation about whether x and y's are inversely proportional. However, she was not able to provide any explanation. She showed by drawing a graph by using x and y values.

Q4: Proportional or Non-proportional (y = mx and y = mx+n graphs)	He decided the proportionality of corresponding points on the graphs correctly by selecting several points and looking at the ratios. However, he emphasized the slope of the lines in his explanations.	Although she stated that x and y’s on the second graph is proportional because the ratio of y to x is contant, she got confused in the first graph. She changed the perception of proportionality and she decided the first graph as proportional because y changes depending on x.
Q5: Inverse proportion or direct proportion	He decided whether quantities inversely or directly proportional by assigning numbers to them. For alternative solutions, he stated while the $\frac{a}{b}$ format is related to a direct proportion, $a \times b$ format is related to an inverse proportion.	She decided whether quantities inversely or directly proportional by assigning numbers to them. When asked for an alternative approach, she made insufficient and wrong explanations.
Q6: Mosaic Problem (Between Ratio)	First, he identified 2.25 as the amount of work per minute, which he corrected later.. During the interview, he was not able explain the value of 2.25 After realizing the mistake, he found the solution as valid because it’s a different version of traditional algorithm.	She found the solution as valid. She was able to recognize 2.25 refers how many times 16 into 36. She explained the solution that student may consider 16 as a unit and try to find the work done in 16 minutes. She also solved the question bu using the amount of work per minute.
Q7: Faucet Problem (Inverse Proportion)	He solved the question with inverse proportion algorithm after deciding they are inversely proportional. However, he was not able to explain this solution within the context by linking to what remains constant.	First she solved the question with cross-multiplication algorithm. Then she reached a correct solution with inverse proportion algorithm. She explained unsurely that 11×13 refers the the amount of water so needs to be constant for the other situation.
Q8: Bicycle Problem (Inverse Proportion)	Although he realized that number of turns and radius of wheels are inversely proportional and found correctly that front wheel turned 15 rounds, he replied incorrectly by associating turns with distance.	She solved the problem by using cross-multiplication algorithm after finding the number of turns that the front wheel has.

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