PRESERVICE MATHEMATICS TEACHERS SOLVING WORD PROBLEMS: VISUAL-SPATIAL ABILITIES, USE OF REPRESENTATIONS, AND TYPES OF MATHEMATICAL THINKING

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DECLARATION OF ORIGINALITY

I, Beyza Olgun, certify that

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ABSTRACT

Preservice Mathematics Teachers Solving Word Problems: Visual-Spatial Abilities, Use of Representations, and Types of Mathematical Thinking

The main purpose of this study was to examine preservice teachers' types of mathematical thinking, use of visual-spatial representations, visual-spatial abilities, and mathematical problem solving performances and to investigate the relationships between these variables. The sample of the study consisted of 113 preservice mathematics teachers in a private and four public universities in Istanbul and Ankara. In order to investigate the research questions two instruments were used. Firstly, preservice teachers' types of mathematical thinking were determined. Although problem solving performances were similar for each type of mathematical thinking, preservice teachers who adopted harmonic and geometric types of mathematical thinking preferred to use schematic representations more than analytic thinkers in their problem solving processes. Use of visual-spatial representations was related with problem solving performance and schematic representations were associated more strongly with correct solutions in comparison with pictorial representations. The participants' visual-spatial abilities had a significant relationship only with their use of schematic representations. The findings provided an insight about preservice teachers' preferences for visual approach and their implications for teacher education programs. Preservice teachers should have an opportunity to learn how a schematic representation can be created and teacher education programs should include visual approaches with consideration of their efficacies.

ÖZET

Matematik Öğretmeni Adaylarının Sözel Problemleri Çözümü: Görsel-Uzamsal Yetenekler, Temsil Kullanımı ve Matematiksel Düşünme Yapıları

Bu çalışmanın temel amacı öğretmen adaylarının matematiksel düşünme yapılarını, görsel-uzamsal temsilleri kullanımlarını, görsel-uzamsal yeteneklerini ve sözel matematik problemlerini çözme performanslarını incelemek ve bu değişkenler arasındaki ilişkileri araştırmaktır. İstanbul ve Ankara illerinde bulunan, bir özel ve dört devlet üniversitesinde öğrenimlerine devam eden 113 öğretmen adayı çalışmanın örneklemi olarak seçilmiştir. Araştırma sorularını incelemek amacıyla iki ölçek kullanılmıştır. Öncelikle öğretmen adaylarının matematiksel düşünme yapıları belirlenmiştir. Ardından düşünce gruplarının temsil kullanımı, görsel-uzamsal yetenek seviyeleri ve sözel matematik problemlerini çözme performansları açısından farklılıkları incelenmiştir. Farklı matematiksel düşünme yapılarını benimseyen öğretmen adaylarının problem çözme performansları birbirine benzer olduğu halde, harmonik ve geometrik düşünenler problem çözme süreçlerinde analitik düşünenlere göre şematik temsili daha fazla kullanmışlardır. Farklı tipteki temsil kullanımının problem çözme performansını etkilediği ve şematik temsilin kullanıldığı sorulardaki doğru cevap oranının resimsel temsilin kullanıldığı sorulara göre daha yüksek olduğu bulunmuştur. Öğretmen adaylarının görsel-uzamsal yeteneklerinin yalnızca şematik temsil kullanımı ile anlamlı pozitif bir ilişkisi olduğu görülmüştür. Sonuçlar öğretmen adaylarının görsel yaklaşımları için tercihlerini ortaya koymuştur. Bulguların öğretmen eğitimine ne şekilde ışık tutabileceği tartışılmıştır. Bu bağlamda öğretmen yetiştirme programlarının görsel yaklaşımların üzerinde durmalarının önemine vurgu yapılmıştır.

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Dedicated to the my beloved mother and father,

and my dearest sisters and brother,

and my cutest nieces

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CHAPTER 1

INTRODUCTION

Problem solving has an important role in mathematics and lies in the focus of almost every math curriculum (Van De Walle, Karp, & Bay-Williams, 2010). According to the American National Council of Teachers of Mathematics (National Council of Teachers of Mathematics [NCTM], 2000), “one of the most significant aims of mathematics teaching and learning is to develop students’ problem solving ability” (Deliyianni, Monoyiou, Elia, Georgiou, & Zannettou, 2009, p. 96). Understanding of problem solving process includes identifying, exploring, implementing, and using visual images is related with visualization (Deliyianni et al., 2009).

With the rise of constructivism, the importance of the role of visualization in the learning process was emphasized more and more. Visual representations are accepted as assistance for both mathematical learning and problem solving. Many researchers investigated the role of visualization within the mathematical problem solving process (Campbell, Collis, & Watson, 1995; Deliyianni et al., 2009; Hegarty & Kozhevnikov, 1999; Presmeg, 1986a, 1986b; Presmeg & Balderas-Cañas, 2001; van Garderen 2006; van Garderen & Montague, 2003). Their common finding was that visual representation had an effect on the problem solving processes. There are different types of representations used by people and their effects could be supportive, misleading or facilitating in problem solving (Campbell et al., 1995). Schematic representations have a positive correlation with problem solving performances while pictorial representations have a negative influence on problem solving performance. People’s visual-spatial abilities and their learning experiences have been claimed to have an influence on their use of supportive visual-spatial representations (van Garderen, 2006). Exploring which type of representations people use in the problem solving processes, and how and when they use them could give an insight about their preferences for visualization. All these point not only to a need to consider learners' performance and preferences, but also a focus on teachers' preferences and the teaching and learning experiences.

Studies on visualization in mathematics often linked it with mathematical thinking. There are various definitions of mathematical thinking. Although there is no consensus about what mathematical thinking is (Sternberg, 1996) all kinds of mathematical thinking have common features such as operations, processes and dynamics (Burton, 1984). With the development of mathematical thinking, people's mathematical skills and abilities can change and as a result of these changes their mathematical achievement can be affected (Krutetskii, 1976). There are different styles of mathematical thinking and these differences can affect individuals in various aspects. In a problem solving context, three types of mathematical thinking were suggested according to disposition of visualization (Krutetskii, 1976): the analytic type, the harmonic type, and the geometric type. Previous studies showed that most of the students and teachers were not geometric thinkers. Analytic thinkers and harmonic thinkers were more successful in problem solving than geometric thinkers.

The types of mathematical thinking have an influence on teachers' teaching styles and methods (Presmeg, 1986b). Teachers who adopt the geometric type of mathematical thinking can use visuality more effectively in their teaching. They include different kinds of activities, which provide the transition between mathematics and the real world, whereas teachers who are in the groups of the analytic type often use lecturing as a teaching style (Presmeg, 1986a, 1986b). Since teachers' approaches in mathematical thinking have a relationship with their teaching (Presmeg, 1986b), it is important to investigate which approach, analytic, geometric or harmonic, is adopted by preservice teachers.

Teachers' mathematical beliefs and learning experiences affect their mathematical thinking and visual approaches (Presmeg & Balderas-Cañas, 2001). Their thinking styles and use of visuality have an impact on their teaching (Presmeg, 1986b). Therefore preservice teachers' approaches towards different types of mathematical thinking and visual-spatial representations could be an important component of teacher education programs. This necessitates a careful study of the interrelations among teachers' visualization, mathematical thinking and problem solving performance before focusing on how these can be supported through teacher education programs. The study was conducted to investigate preservice teachers' visual approaches in a problem solving context and their relations with the performance and abilities.

1.1 Significance of the study

Visualization is one of the key elements of the transition between concrete and abstract modes of thinking (Ben-Chaim, Lappen, & Houang, 1989). In this sense, it is important to elicit preservice teachers' analytic, geometric or harmonic thinking since the mode of thinking adopted may influence their teaching. Their preferences of problem solving strategies may be transferred to their students, when they start working as teachers.

There are studies investigating the role of visualization in students' mathematical problem solving processes (Booth & Thomas, 2000; Goldin, 1998; Hegarty & Kozhevnikov, 1999; Kozhevnikov, Kosslyn, & Shephard, 2005; Presmeg, 1985, 1986a, 1986b; Presmeg & Balderas-Cañas, 2001; Stylianou, 2011; Stylianou & Silver, 2004, 2009; van Garderen, 2006; van Garderen & Montague, 2003). However, there are a limited number of studies that examine preservice mathematics teachers' preferences of the use of visual components in solving word problems (Hacıömeroğlu & Haciömeroğlu, 2014; Presmeg, 1995; Taşova, 2011). Teachers who are inclined towards the visual approach can use visuality in a more effective way that gives them the opportunity of making transitions between mathematics and the real world (Presmeg, 1986b). However previous studies showed that teachers use nonvisual methods more than visual methods in problem solving (Sağlam & Bülbül, 2012).

There are contradictory findings for the influence of the types of mathematical thinking on visual-spatial abilities and mathematical problem solving. While some studies pointed out positive or negative relationships of mathematical thinking with problem solving performances or visual-spatial abilities, others suggested different styles of mathematical thinking did not have a significant relationship with these variables. Visual-spatial representations have different kind of impacts on problem solving processes based on which type of representation was used. Therefore it is important to investigate preservice teachers' mathematical thinking approaches, preferences for visual representations, visual-spatial abilities, and the influence of all of these on mathematical problem solving. With the help of these findings the study may provide implications for preservice teacher education and suggestion for further studies.

1.2 The purpose of the study

The main purpose of the study was to investigate preservice teachers' preferences of problem solving strategies and how their mathematical thinking (analytic, harmonic or geometric types) might affect the visualization process in mathematical word problem solving. This study aimed to focus on preservice teachers' use of visual-spatial representations and their influences on mathematical word problem solving processes. Whether there is a relationship among preservice teachers' visual-spatial abilities, types of mathematical thinking, use of visual-spatial representations, and mathematical problem solving performances was also investigated.

1.3 Research questions

Considering the purpose of the current study, there are seven main research questions that this study has tried to answer. The research questions were as follows:

Research Question 1: Which structure of mathematical thinking, analytic, geometric or harmonic types, is adopted most frequently by preservice teachers?

Research Question 2: Is there a significant difference in preservice teachers' mathematical word problem solving performance according to their types of mathematical thinking?

Research Question 3: Is there a significant difference in preservice teachers' use of schematic representations, pictorial representations, and visual-spatial representations based on their types of mathematical thinking?

Research Question 4: Is there a significant difference in preservice teachers' levels of visual-spatial abilities based on their types of mathematical thinking?

Research Question 5: Is there an association between the use of schematic, pictorial, visual-spatial representations, and mathematical word problem solving performance?

Research Question 6: Is there a relationship between preservice teachers' mathematical problem solving performance and levels of visual-spatial abilities?

Research Question 7: Is there a relationship between preservice teachers' use of schematic representations and levels of visual-spatial abilities?

Visual representations of the research questions are presented in Fig. 1, Fig. 2, and Fig. 3.

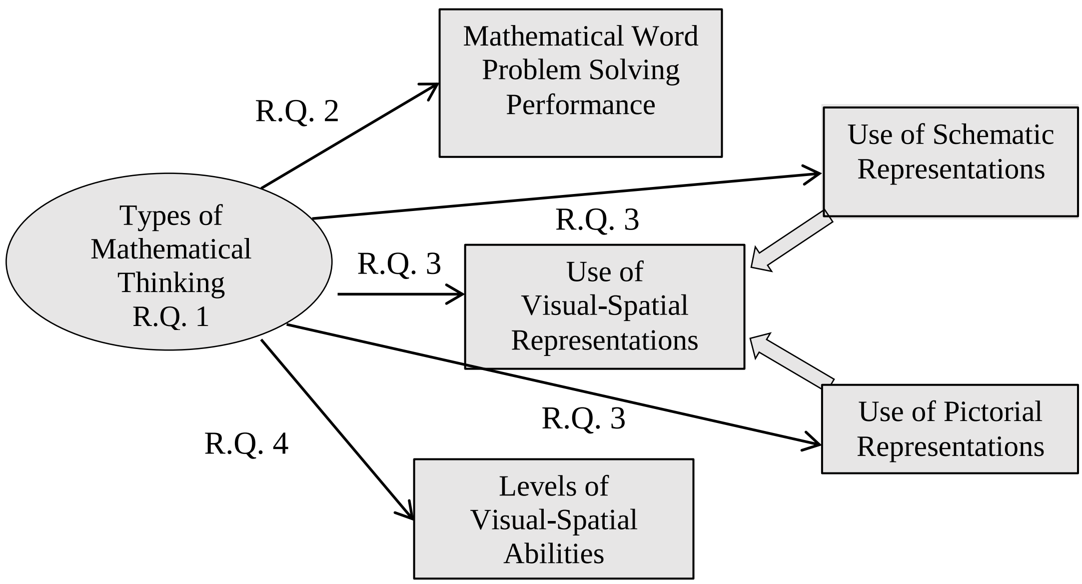


Fig. 1 Visual representations of first, second, third and fourth research questions

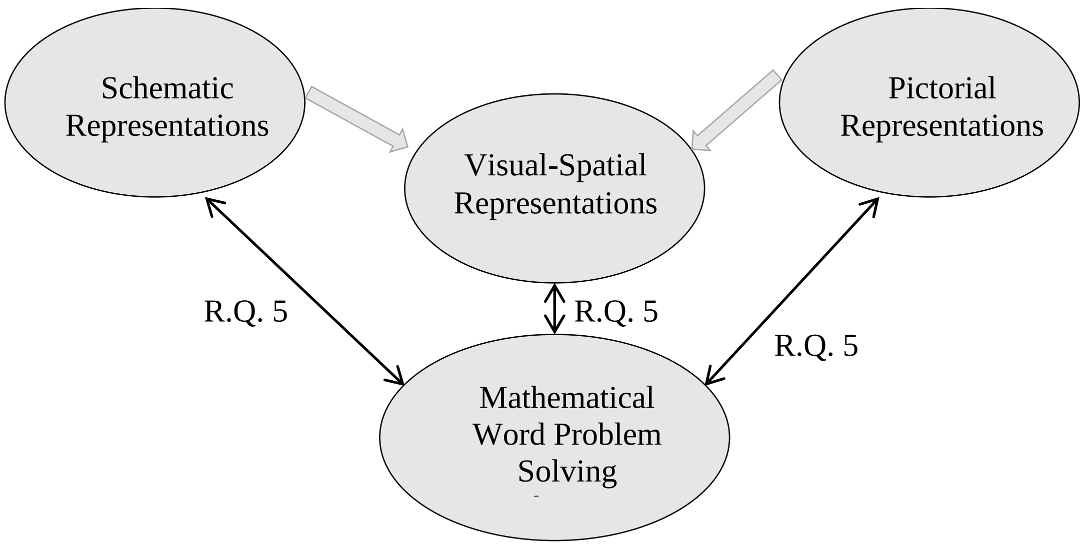


Fig. 2 Visual representation of fifth research question

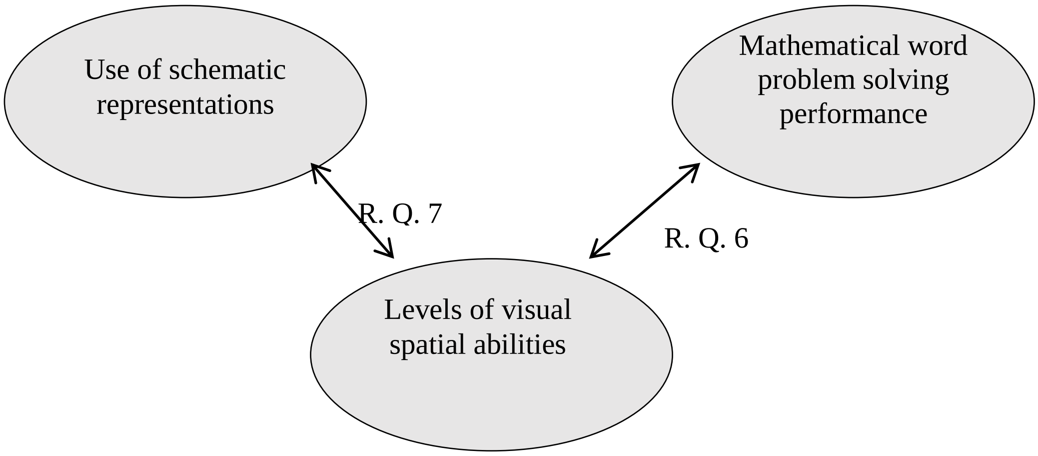


Fig. 3 Visual representations of sixth and seventh research questions

The next chapter is aimed at presenting the review of the literature to reveal the theoretical background of the study and the findings of the related research.

CHAPTER 2

LITERATURE REVIEW

In the literature, the role of visualization in problem solving context was investigated through three main constructs. These were mathematical thinking, visual-spatial abilities and visual-spatial representations. In this section firstly the definition of visualization was given. Then each construct was presented with the help of previous studies. Finally the relationships of types of mathematical thinking, visual-spatial representations, and visual-spatial abilities with mathematical problem solving were discussed.

2.1 Visualization

Visualization is a noun that means:

- "formation of mental visual images"
- "the act or process of interpreting in visual terms or of putting into visible form" (Visualization, 2015).

Visualization includes not only mental processes to form visual images but also an act of interpretations of these mental visual images. To act in these interpretations individuals should have some skills or abilities. According to Presmeg (2006),

> …when a person creates a spatial arrangement (including a mathematical inscription) there is a visual image in the person's mind, guiding this creation. Thus visualization is taken to include processes of constructing and transforming both visual mental imagery and all of the inscriptions of a spatial nature that may be implicated in doing mathematics (p. 206).

Although visualization is a term that is often used in empirical research, there is no certain consensus about its definition and the components of forming in the literature. Phillips, Norris, and Macnab (2010) listed 28 explicit definitions in chronological order provided in research literature (See Appendix A, Table 9). They made a classification of the essential postulations of “visualization”, such as an ability, a tool, a strategy or a cognitive skill. Gutierrez (1996) also claimed that the same concept is used with different meanings. Therefore it is important to justify the perspective that is adopted in the study.

Among the definitions on the list that Phillips et al. (2010) were provided, Zazkis, Dubinsky, and Dautermann’s definition (1996) is very explanatory:

> Visualization is an act in which an individual establishes a strong connection between an internal construct and something to which access is gained through the senses. Such a connection can be made in either of two directions. An act of visualization may consists of any mental construction of objects or processes that an individual associates with objects or events perceived by her or him as external. Alternatively, an act of visualization may consist of the construction, on some external medium such as paper, chalkboard or computer screen, of objects or events that the individual identifies with object(s) or process (es) in her or his mind (p. 441).

While it defines what visualization is and how visualization occurs, it involves different processes about visualization (Taşova, 2011). Since this study tries to investigate visualization in the context of mathematical problem solving, it will consider that visualization is related with “the understanding of the problem with the construction and/or the use of a diagram or a picture to help obtain a solution” (Deliyianni et al, 2009, p. 97).

In the literature, the role of visualization in mathematical problem solving is investigated in three main constructs. These are mathematical thinking in terms of predisposition of visualization in problem solving, the use of visual-spatial representations and visual-spatial abilities. As the current study was interested with all of these three fields, a review of each construct is presented in the following part.

2.2 Visual-spatial representations, visual images and visual imagery

In mathematics education literature, there were various definitions and classifications for representations (Zazkis & Liljedahl, 2004). Mesquita (1998) suggested that mathematical context could be one the factors for the various meanings and interpretations. The place where the representation as in mind or on a paper or screen is formed, the reason why the representation is used, and the activities which were included in the studies were some of the parameters for various distinctions about the nature of representations.

Janvier (1987) suggested conceptualization of external and internal representations. He explained internal representations as informed in mind and external representations as exposed in a form. External representations could be a symbol, a schema or a graph. His classification started an argument about internal and external representation. Therefore many researchers discussed about the discrimination as internal and external, and the existence of an internal representation (Goldin, 2001; Goldin, 2003; Goldin & Shteingold, 2001; Haciömeroğlu, Aspinwall, & Presmeg, 2010; Zhang, 1997). Since accessing and measuring what was going on in a person's mind is difficult, the current study used the general term representation instead of external representation to avoid such controversies.

Researchers used various classifications according to nature and modes of representations (Mainali, 2014). Janvier (1987) suggested four modes of representations: verbal descriptive as text, symbols, and sentences, tabular as tables, graphic as images and figures, and formulaic as formulas and equations. Vergnaud (1998) proposed representations as dynamic processes related to mathematical activities and mind. Although there were different categories that were classified for representations, the common feature of representations was its important role in learning and teaching mathematics. Dufour-Janvier, Bednarz, and Balange (1987) suggested that the representations were one of the inner parts of mathematics and helped students to see more attractive and interesting mathematics.

In visualization and problem solving context, some researchers related the representations with visual images and visual imagery (Hegarty & Kozhevnikov, 1999; van Garderen, 2006; van Garderen & Montegue, 2003). Hegarty and Kozhevnikov (1999) investigated representations in the mathematical problem solving context and associated representations with visual images and visual imagery. They defined visual-spatial representations that were classified as pictorial or schematic.

Visual images are mental constructs that demonstrate spatial or visual information (Presmeg, 1992) and visual imagery is the ability to form these visual images and to manipulate them in mind (Kosslyn, 1995). In the research literature different types of visual imagery has been identified. Presmeg (1986a, 1986b) categorized five types of visual imagery:

- Concrete imagery is the picture or prototype of an object in mind,
- Kinesthetic imagery is about physical movements of the objects,
- Dynamic imagery is the image itself moved or transformed,

- Memory images of the formulas,
- Pattern imagery is pure relationships between the object and its environment.

She suggested that kinesthetic imagery, dynamic imagery, memory images of the formulas and pattern imagery could play a positive role in problem solving process but concrete imagery had a negative effect and it kept students' attention in irrelevant details.

On the other hand Dörfler embraced the idea that "meaning is viewed to be induced by concrete 'mental images' as opposed to propositional approaches" (as cited in Presmeg, 2006, p. 208) and hypothesized mental image schemata: Figurative is purely perspective, operative operates with the carrier, relational is the transformation of the concrete carrier and symbolic image schemata is formulas with symbols and spatial relations. When compared with Presmeg's types of imagery, they can be matched as follows: figurative ~ concrete, operative ~ kinesthetic, relational ~ dynamic, symbolic image schemata~ memory images of formula.

Most research studies chose two out of these broader categories that are discussed above and considered two types of visual imagery: pictorial and schematic imagery. According to Hegarty and Kozhevnikov (1999):

> Pictorial imagery is constructing detailed and vivid visual images on the other hand schematic imagery is constructing spatial relationships between objects and imagining spatial performance (p. 685).

Their common findings were similar to Presmeg's (1985, 1986a, 1986b) studies that schematic imagery guided students for better performance in solving mathematical problems but pictorial imagery was associated with poor performance because it took students' attention from the main meaning of the problem to irrelevant things. Moore and Carlson (2012) suggested that students' image and its structure could be static or dynamic in the study done with 9 undergraduate pre-calculus students. Whereas static images can cause a hindrance to obtain a solution, dynamic images include mathematical relationships and helps students to understand the information of a problem and transitions between variables (Moore & Carlson, 2012). In summary, researchers' classification and definitions may be different from each other still the similarities in findings that a person's image can be an obstacle or guidance in problem solving process are discovered.

2.3 Visual-spatial abilities

Like visualization and representations, the researchers used different definitions for visual-spatial abilities. Elliot and Smith (1983) suggested spatial abilities as keeping in the mind, understanding, using with skills, and organizing of visual images. Lord (1985) defined visual-spatial abilities as the forming an image in the mind and the controlling this image. Stockdale and Possin (1998) approached with a different context and identified them as people's ability to understand the spatial relationships between them and the environment or the objects other than their own. Considering these definitions, it should be realized that visual-spatial abilities were accepted as the combination of different abilities rather than a single ability (Pellegrino & Hunt, 1991).

Tartre (1990) explained visual-spatial abilities as a mental ability including understanding, changing, using, renovating and expressing the relationships visually. Hegarty and Kozhevnikov (1999) suggested two types of ability for visualization: "visual imagery ability refers to representations of the visual appearance of an object like its color and shape, and spatial imagery ability is representation of the spatial relationships of the parts of the object and its location or movements" (p. 685).

McGee claimed that spatial abilities had two prominent features: visualization and orientation (as cited in Yolcu, 2008) and Clemets (1998) classified two important abilities that individuals should have to get the spatial intelligence: spatial orientation ability and spatial visualization ability.

Spatial orientation ability is “the ability to perceive spatial patterns or to maintain orientation with respect to objects in space” (Ekstrom, French, Harman, & Derman, 1976, p. 149). It requires from individuals to understand and compare an object and its location with others objects (Yolcu, 2008). It includes mental pictures of an object from another perspective and variation of individuals’ viewpoints (Taşova, 2011)

Spatial Visualization Ability is “the ability to manipulate or transform the image of spatial patterns into other arrangements” (Ekstrom et al., 1976, p. 173). It requires from individuals to reverse, fold, turn and change a visual object or a part of its (Tartre, 1990). It includes to form mental images of two-dimensional or three-dimensional objects and to rotate these images in mind.

The main difference between the spatial orientation abilities and spatial visualization abilities is the motion of the object. If a situation requires a mental motion including all parts of the object, it was concerned by spatial visualization abilities Taşova, 2011).

2.4 Styles of mathematical thinking

Skills have an effect on achievement in science, like it happens in art. If individual skills are investigated, typologically differences can be seen. These differences can also be seen in mathematics and the skills, which can influence mathematical achievement, vary from person to person (Krutetskii, 1976). Therefore a learning environment that provides opportunities to develop mathematical skills can influence positively students' achievement. While mathematical skills have a changing structure with the development of mathematical thinking by learning experiences, the styles of mathematical thinking are directly related to person's education and development (Taşova, 2011). Individual differences depending on education and development make it natural to have different approaches to the same phenomenon or events. Therefore the differences of the styles of mathematical thinking show that various aspects of the individuals can come to the forefront (Alkan & Bukova Güzel, 2005). For example some people can understand better the concepts with the help of diagrams and figures, whereas others try to learn the content, algorithms and connections of the concept.

Since understanding various features of mathematical thinking is one of the major aims of mathematics education research, there are several research studies that exposed the features of mathematical thinking. According to Burton (1984), mathematical thinking includes the way of thinking in particular operations, mathematical processes, and functions besides including subject matter knowledge of mathematics. Mathematical thinking embodies both procedural and conceptual understanding of mathematics and learning process of mathematics (Barwell, 2009).

Tall (2004) argues that mathematical thinking operates in three worlds: embodied, symbolic and formal. These three worlds represent how individuals enhance their conceptions through making concepts thinkable. The first world of the mathematics indicates the object that individual has interactions and how individuals perceive this world and attach internal meanings to this mental perception (Stewart &Thomas, 2009). This world mainly deals with visual-spatial abilities. The symbolic world is the world where embodied actions such as counting, adding, taking away, and sharing are realized and symbolized (Tall, 2009). In the formal world objects are defined and represented with their properties. In this world individuals can deduce new properties of the objects by formal proof (Stewart &Thomas, 2009).

Ferri (2003) classified the styles of mathematical thinking in three categories:

- Visual Style (Thinking in graphs, diagrams, figures and pictures)
- Analytic Style (Thinking symbolically, formalistically)
- Conceptual Style (Thinking in ideas, classifying)

The differences of the styles of mathematical thinking do not dictate that individuals use only one of them. Individuals can use two styles or all of them together if they deem necessary. The existence of different styles of mathematical thinking does not only arise from individual differences, also it is derived from the requirements of different areas of mathematics.

The tendency for visual approaches varies among individuals in mathematical problem solving (Krutetskii, 1976). Krutetskii (1976) classified students in three groups: analytic thinkers, geometric thinkers and harmonic thinkers. Analytic thinking involves strong verbal-logical components and poor visual-pictorial components. Students who embrace the analytic thinking style do not feel the need to benefit from visual supports and also they do not have enough strength for the use of visual components. For geometric thinkers it is the contrary. They make use of visual-pictorial components and verbal-logical components have poor influence on their reasoning. The reasoning of harmonic thinker students includes both verbal-logical components and visual-pictorial components and their preferences can change according to the problems that they face.

Likewise classification of Krutetskii, Clements (1982) suggested three groups for styles of mathematical thinking according to personal traits of students:

visualizers, verbalizers, and mixers (as cited in Zazkis et al., 1996). In some studies (Krutetskii, 1976; Presmeg 1985, 1986b) these classifications were applied at school level and their findings showed that most of the successful students were not in the group “visualizers”. These kind of results may form an impression that the use of visualization has a negative influence on the achievement in mathematics but Presmeg (1986b) suggests that the reason of these results is that nonvisual methods are emphasized acutely by curriculums, course books, and teaching practices. This situation is not accordant with visualizers and therefore to bring students who are not in tendency of visualization to a successful conclusion can be expected.

Studies that investigated the role of visualization in the context of mathematical thinking on mathematical problem solving process used the predisposition of visual - nonvisual or visual – analytical problem solving methods (Avcu & Avcu, 2010; Avcu, 2012; Haciömeroğlu & Haciömeroğlu, 2014; Presmeg, 1986b; Sağlam & Bülbül, 2012; Sağlam, 2014). According to Presmeg (1986a),

A visual method of solution is one, which involves visual imagery, with or without a diagram, as an essential part of the method of solution, even if reasoning or algebraic methods are also employed. A nonvisual method of solution is one, which involves no visual imagery as an essential part of the method of solution (p. 298).

The findings of Taşova’s research (2011), which is conducted with 75 preservice mathematic teachers, revealed that visual problem solving methods were used dramatically less than nonvisual problem solving methods. According to Hacıömeroğlu and Hacıömeroğlu (2014), preservice teachers’ preferences in problem solving processes are affected by the task difficulty. The more difficult the problem is, the more preservice teachers tend to use analytic problem solving strategies. The result of their study is similar to Sevimli and Delice’s study (2012), which suggests that students often use nonvisual methods in difficult integral problems. Sağlam (2014) also supports this idea and suggests that preservice teachers apply numerical and analytical operations for calculus. These studies showed that most preservice teachers are in the nonvisual group rather than the visual group like the results of previous studies (Krutetskii, 1976; Presmeg 1985, 1986b).

Presmeg (1986b), in her study with 13 high school mathematics teachers and their students, analyzed teaching styles of teachers for eight months. The results indicated that teachers who used visuality effectively in their teaching could make transition between mathematics, the real world and other disciplines. Presmeg categorized teachers as visual and nonvisual groups with respect of the Mathematical Processing Instrument score and the visuality of their teaching Although the nonvisual group of teachers adopted lecturing and formally teaching, the visual group of teachers included the activities and events that would reveal the creativity in their teaching (Presmeg, 1986b).

Overall, teachers' mathematical beliefs and their previous experiments influence their mathematical thinking styles (Presmeg & Balderas-Cañas, 2001; Sağlam & Bülbül, 2012). Preservice teachers use analytical methods more than visual methods in problem solving process because their lecturers also use analytical methods (Sağlam & Bülbül, 2012). This might be pointing to a vicious circle, which can be further investigated; the interaction of teachers and students keeps a considerable role in the predisposition of visualization in the context of mathematical thinking.

2.4.1 The styles of Krutetskii's mathematical thinking

Krutetskii (1976) classified three groups according to the students' predisposition of visualization in mathematical problem solving:

- the analytic type: who tends to think in verbal-logical terms
- the geometric type: who tends to think in visual-pictorial terms
- the harmonic type: who combines characteristics of the other two (as cited in Siswono, 2005, p. 193)

Krutetskii (1976) detected distinctive features of students in his study adopting that students' mental activities include verbal-logical components and visual-pictorial components. The relationship between the use of verbal-logical components and visual-pictorial components determines which group the student is in.

2.4.1.1 The analytic type

Verbal-logical components show quite high level of development in the mathematical thinking of the students who are in this group and they show obvious predominance over visual-pictorial components. Students who have an analytic cast of mind do not feel the need to use visual supports. They can study easily the mathematical relations of the problems in the abstract form. Although the solution of the problem could be easier with the help of graphs, diagrams, and figures, they would use more difficult and complex logical-analytical solutions. For example, they would not prefer the easy way that is rotating the object in their imagination for a rotation problem. In Krutetskii's study (1976), a student gave the following complex answer for a rotation problem that was asked, about the final image after the rotation of a right triangle on its one edge that is not the hypotenuse:

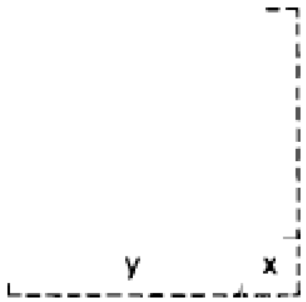
> "A right triangle is rotated about the leg? Now I'm thinking. . . . The upper point will not be rotated — it is on the leg. The points on the other leg will be rotated at a different distance from the axis, but each will move an equal distance. Since it is an equal distance, each will describe a circumference, and all together — a circle. That means, a circle is below, and a point on top. And the hypotenuse, when rotated, connects them. A cone is obtained, right?" (Krutetskii, 1976, p. 319).

On the other hand students who are the members of the geometric type classified the problem as "childish". They could easily see the rotated shape and gave simple answers: "Here I picture the way it is rotated, and it is obvious that a cone is obtained" (Krutetskii, 1976).

2.4.1.2 The geometric type

The representatives of this type have very well developed visual-pictorial components in their thinking. They need the visual interpretations of abstract mathematical relationships and they are excelled at making these visual interpretations. If they cannot create the necessary diagrams to solve a problem, the solution of the problem will become more difficult. Although to follow a path that involves the use of verbal-logical components can be easier for the solution of the problems than the use of visual cues, they insist on the use of visual diagrams.

Students whose mathematical thinking is the geometric type are well developed on spatial concepts. They can smoothly perform on the analyses of graphs, diagrams, figures and tables. They can draw the visual-schematic representations that are required for the solution of the problem with ease despite the fact that they have difficulty in analytical procedures related to concepts and definitions. For instance, “Each side of a square was increased by 3 cm and therefore its area was increased by 39 cm^2. Find the side of the resulting square.” (Krutetskii, 1976), this problem can be solved in an effortless way by using “$(x+3)^2 - x^2 = 39$” equation. Students who are in the group of the geometric type follow a more complex way (see Fig. 4) instead of answering like the representatives of the analytic type.



“This [x] has to be a square, and it has to have a side of 3, that is, its area is 9 cm”. Then, two of these rectangles [y] must be 30 cm^2, and 15 cm^2 each. One side is 3, and so the other is 5 cm. Then it was 5 and it became 8 cm."

Fig. 4 The solution of a geometric type student for “Each side of a square was increased by 3 cm and therefore its area was increased by 39 cm^2. Find the side of the resulting square.”
Adapted from “The Psychology of Mathematical Abilities in School Children,” by Krutetskii, 1976, p. 322.

Presmeg’s study (1986b) showed that the teachers who have the geometric type of mathematical thinking could use different teaching methods more than teachers who have the analytic type of mathematical thinking. The representative teachers of the geometric type can establish relationships between learned new concepts, students’ background, and the real world.

2.4.1.3 The harmonic type

The mathematical thinking of students who are in this type is characterized by a balanced manner of well-developed verbal-logical components and visual-pictorial components. The development of the spatial concepts is high leveled in this type.

The representatives of the harmonic type can successfully carry out analytical analyses as much as they are good at visual reviews of abstract relationships. They are successful at performing both analytical and visual approaches. To illustrate, in Krutetskii's study (1976), the following problem was asked:

"$a^2 + b^2 = c^2; a, b, c > 0$. What can be said about the relation between the first powers of these numbers?". Harmonic thinkers used both analytical approaches and visual approaches as following examples in their solutions:

"1. $a^2 + b^2 = c^2;\ a^2 + b^2 + 2ab = c^2 2ab;\ (a + b)^2 = c^2 + 2ab;$

$$(a + b)^2 > c^2; (a + b) > c$$

2. a, b, and c here are the sides of a right triangle, and therefore $c < a + b$." (p. 327).

2.5 Problem solving in mathematics

Problem solving plays a fundamental role in learning mathematics (Erbaş & Okur, 2012; Krulik & Rudnick, 2003; NCTM, 2000; OECD, 2003; Polya, 1973; Van De Walle et al., 2010). By analyzing and synthesizing the knowledge, it helps to deepen understanding of mathematical concepts (Erbaş & Okur, 2012). Problem solving is a teaching method which helps students to explore, develop, and apply understanding of a mathematical concept as well as being a scientific research method (Avcu & Avcu, 2010; Charles, Lester, & O'Daffer, 1987; Wilson, Fernandez, & Hadaway, 1993). Problem solving is also used in understanding and communication with other disciplines (Wilson et al., 1993). It can increase students' intrinsic motivation by stimulating interest and enthusiasm (Wilson et al., 1993). Therefore developing students' problem solving skills is one of the general aims of the mathematics curriculum, (MEB, 2013).

In problem solving process, Johnson and Rising (1967) suggested that students could learn “new concepts, practice computational skills and transfer these concepts and skills to new situations” (as cited in Orton & Frobisher, 1996, p. 22). To develop students’ problem solving skills, problem solving strategies can help students succeed to apply problem solving steps and, especially for challenging problems, make progress (Erbaş & Okur, 2012; Hatfield, Edwards, Bitter, & Morrow, 2007). According to Suydam (1987) “if teachers teach problem solving as an approach, where teachers must think and can apply anything that works, then students are likely to be less rigid” (p. 104*)*. Problem solving approaches have their own interventions, strategies and assumption (d’Estree, 2008). Therefore visual approaches require its specific frames for teaching and learning.

2.6 Visualization and mathematical problem solving

In this section the role of visualization in problem solving was presented by regarding three constructs mentioned above. Most of the studies investigated collectively the relationships of these three constructs with problem solving. Therefore the role of visual-spatial abilities, visual-spatial representations, and types of mathematical thinking in mathematical problem solving was collected under the same heading.

In the school when students begin to learn problem solving with didactical contract they just start to focus on linguistic structure of problems and numbers while ignoring the real meaning and follow the rules that they learned in the mathematics classrooms (Verschaffel, Greer, & Corte, 2000). Didactical contract is “a set of partly explicit and mainly implicit rules that determine the relationships between the teacher, the pupil and the mathematical knowledge” (Deliyianni et al., 2009, p. 99).

Students usually follow the procedure where they select an operation, perform arithmetical operations and find the result (Greer, 1997) with the influence of didactical contract. Although imitating the teacher's solution can help students to succeed in standard practice for problematic problems, they can have difficulties with memorized strategies (Erbaş & Okur, 2012; Harskamp & Suhre, 2007; Posamentier & Krulik, 1998). Several studies (Ehlinger & Pritchard, 1994; Gay & White, 2002; Halpern & Halpern, 2006; Kembitz, 2009; Kresse, 1984) suggested that the problem solving strategies, which receive support from illustrations, diagrams, tables, charts, graffiti, and etc., help students solve word problems. Furthermore, use of visualization in word problems prompts to "improve of understanding of the problem" (Kresse, 1984; as cited in Friedland, McMillen, & Hill, 2011, p. 60).

A study compared kindergarteners' and first grade students' visual representation and mathematical problem solving process in terms of didactical contract (Deliyianni et al., 2009). It showed kindergarteners often used pictorial representation while first graders used symbolic representations and sometimes they added a picture in their solution (Deliyianni et al., 2009). The significant result was first graders solved problematic problems, which involve a question that not actually related with the information of the problems, with symbolic representation even though they were suspecting their solutions. Their focus was to answer the question without thinking about the real meaning in the question. On the other hand kindergarteners did not give an answer or their answers were related to their real life. For them, visualization was an important factor to connect the answers with their meanings. This change on the students' behaviors seemed to start with school thus it might be said that it is important to support students' visual development while teaching the numerical operations. For creating such an instructional environment on realistic mathematical modeling and reducing the didactical contract impact, the investigation of teachers' tendencies of visualization in their mathematical thinking and their preferences of visual methods may give some hints about implications for teacher education.

To investigate the role of visual-spatial representations in mathematical word problem solving many studies were conducted with students in various levels of education such as kindergartens, primary schools, elementary schools, high schools and colleges. They exposed a significant relationship between visual representations and mathematical problem solving performance. Their common finding was that visual representations could affect students' performance in mathematical word problem solving in varied ways. Use of schematic imagery was positively correlated with mathematical problem solving and use of pictorial imagery was negatively correlated with mathematical problem solving (Blatto-Valle, Kelly, Gaustad, Porter & Fonzi, 2007; Hegarty & Kozhevnikov, 1999; Presmeg, 1986a, 1986b; van Garderen, 2006).

The preference for using different types of visual representations is correlated with achievement in mathematics because studies showed that students who performed better in mathematics used more schematic representations in mathematical problem solving. In order to investigate the different relationships of types of representations with achievement, some researchers compared gifted and learning disabled students or experts and novices. Some researchers (van Garderen, 2006; van Garderen & Montague, 2003) studied with learning disabled students, average achievers and gifted students. The results of their studies showed significant differences between students. Gifted and average students generally used schematic representations, which were more sophisticated and led to correct solutions, while the students with learning disabilities used more often pictorial representations, which offered solutions ending with the incorrect answers. Blatto-Vallee and his colleagues (2007) also explored the differences between deaf and hearing students. Their findings showed that hearing students performed significantly better in solving correctly mathematical problems and used schematic representations more extent than deaf students. Stylianou and Silver (2004) compared experts and novices and they found that experts used more frequently visual representations. However, their main finding was related to the richness and functionality of their visual representations. Experts had rich structures that helped them to recognize meaningful patterns in their constructed diagrams. On the other hand novices could not use visual representations functionally and efficiently. Although they attempted to draw diagrams, they could not operate them as useful tools.

Students' perceptions about the links between the use of visual images and problem solving have also been studied. Researchers suggested that visual images could have different kinds of impacts on problem solving processes. In order to investigate these impacts, Campbell et al. (1995) asked students whether visual images had a relationship with problem solutions. Answers provided three different elaborations:

- supportive: it motivated students but did not affect their problem solving process
- misleading: it guided students in misleading way but did not influence problem solutions
- facilitating: "everyday" reasoning could mislead to facilitate problem solving.

According to Presmeg and Balderas-Cañas (2001), students' experiences and previous knowledge were related with the use of visual imagery. Their study

(Presmeg & Balderas-Cañas, 2001) that investigated whether graduate students used visualization in their mathematical problem solving and when, why, and how, showed diagrams were used commonly in the preparation phase and to make sense of relations.

The role of visual-spatial representations in problem solving was investigated and different impacts of types of representations on problem solving were confirmed in many studies. However for the role of visual-spatial abilities and types of mathematical thinking in problem solving previous studies showed different results. While some studies found a significant relationship between the variables, others presented no association. Lean and Clements (1981) claimed that nonvisual problem solving strategies were more effective than visual problem solving strategies. Nonvisualizers showed better performance in problem solving than visualizers. On the other hand Moses (1977, 1980) suggested that students who adopted visual approaches in their mathematical thinking performed better than others. Hence many studies did not show a significant difference among types of mathematical thinking in terms of problem solving performances (Kolloffel, 2012; Pitta-Pantazi & Christou, 2009; Suwarsano, 1982). For these contradictory findings Presmeg (1986a, 1986b) suggested that there were external or internal factors, which could affect people's preferences for visual approaches and they could make a group superior to others in terms of problem solving performance. If textbooks, curriculums, and teaching styles emphasize one kind of methods, this could give the opportunity to outperform to a specific group in terms of mathematical thinking.

Analyzing whether individuals' visual-spatial abilities have an influence on problem solving performance is a key area explored by researchers. Many studies pointed out a positive correlation between visual-spatial abilities and problem solving performance (Battista, 1990; Clements & Battista, 1992; van Garderen, 2006; van Garderen & Montague, 2003). People's high level of visual-spatial abilities was related with better problem solving performance. Van Garderen (2006) also claimed that gifted students had quite successful in visual-spatial ability tests and also problem solving whereas average achievers and learning disable students poorly performed in both of them.

Some studies investigated the relationship between visual-spatial abilities and the use of visual representations in mathematical problem solving process (Booth & Thomas, 2000; Goldin, 1998; Hegarty &Kozhevnikov, 1999; Kozhevnikov, Kosslyn & Shephard, 2005; van Garderen, 2006; van Garderen & Montague, 2003). They claimed that schematic imagery was positively correlated with spatial visualization ability whereas pictorial imagery was negatively correlated with spatial visualization ability (Booth & Thomas, 2000; Hegarty & Kozhevnikov, 1999; van Garderen, 2006; van Garderen & Montague, 2003). While types of representations showed a significant relationship with visual-spatial abilities, people's types of mathematical thinking were not related with their levels of visual-spatial abilities. Many studies claimed that there was no significant relationship between people's preferences for visual and nonvisual strategies and their visual-spatial abilities (Haciomeroglu et al., 2013; Hagarty & Kozhevnikov, 1999; Kozhevnikov, Hagarty, & Mayer, 2002; Lean & Clements, 1981; Moses, 1977; Suwarsono, 1982).

In summary, visualization can have an effect on own mathematical problem solving performance and teachers' opinions and preferences about visualization also have an influence on their students' performance. While many studies pointed out various significant relationships among these variables, there were contradictory findings. Therefore exploring all visual approaches as mathematical thinking, the use of visual-spatial representations, and visual-spatial abilities in problem solving is essential for better understanding the association among the variables. Regarding this purpose, this study aimed to investigate these constructs in a mathematical word problem solving context.

CHAPTER 3

METHODOLOGY

This chapter provides detailed information about the methodology of the study. Sample, study context, research design, data collection procedure, definition of key terms and variables, instruments and the data analyses of the study are described.

3.1 Sample

In the study, participants were selected by convenient sampling. The study was conducted with senior preservice teachers in the Primary Education Department and the Secondary Education Department of five universities in İstanbul and Ankara, Turkey. One private and four state universities were included in the study. The students involved in the study were enrolled in Primary Mathematics Education (n = 91) and Secondary School Mathematics Education (n = 32). The target population of the study was 226 students and the data were collected from a sample of 116 participants.

Before the data analysis, missing data and outliers were determined. Three extreme outliers were detected and removed from the study (see Appendix B, Fig. 9, Fig. 10, Fig. 11, Fig. 12, Fig. 13, and Fig. 14). As a result, the sample of the main study was 113, where %50 of the target population participated in the study. During the data collection process, 12 of the participants from Primary Mathematics Education Department at Marmara University could not complete all the scales in the study due to their limited time and their missing data was handled in data analyzing process.

3.2 Research design

This quantitative study adopted a correlational-relational research design. It is interested in the relationships between two or more variables and determining the directions, magnitudes and forms such relationships. The data were collected with the implementations of two instruments; the Mathematical Processing Instrument (MPI) and the Spatial Ability Tests (SAT). Parametric or non-parametric inferential and correlational analyses were conducted to investigate the research questions.

3.3 Procedure of the study

Before the study was conducted, approval from the Ethics Committee of Boğaziçi University (see Appendix C, Fig. 15). The participants of the study were informed and their consent was taken before participating in the study (see Appendix D). Data were collected during the second semester of 2015-2016 academic year. Implementations of the instruments took about 1 hour and 45 minutes and the researcher was present during data collection.

3.4 Definitions of key terms and variables

In the study, there were four variables related with the research questions. These were types of mathematical thinking, visual-spatial representations (schematic or pictorial), visual-spatial abilities, and mathematical word problem solving performances. Preservice teachers' types of mathematical thinking, use of visual-spatial representations, and mathematical word problem solving performances were measured by the MPI. The SAT was used to measure preservice teachers' visual-spatial abilities.

In order to determine preservice teachers’ styles of mathematical thinking (analytic, harmonic, or geometric), preservice teachers’ visualizing mathematical scores were used. Visualizing mathematical score is the extent of a preservice teacher’s attempts to use visual methods in mathematical problem solving processes. The score is calculated by a preservice teacher’s answers to the questionnaire of the MPI. If a solution involves visual imagery, which plays an essential role in the solution method even though the algebraic methods also are employed, it is accepted to be a visual solution method. On the other hand a nonvisual solution method does not involve a visual imagery in the solution processes (Presmeg, 1986a, 1986b). Preservice teachers’ mathematical word problem solving performances and use of visual-spatial representations were determined according their problem solutions. A person’s total number of correct answers to the problems was accepted as his or her mathematical word problem solving performance. “A reported or drawn image of objects or persons referred to in the problem” (van Garderen & Montague, 2003, p. 248) was coded as a pictorial representation. “A drawn diagram showing the spatial relations between objects in a problem, or a reported spatial image of the relations expressed in the problem” (van Garderen & Montague, 2003, p. 248) was coded as a schematic representation. After the coding three scores were generated:

- Pictorial representation score: The total number of pictorial representations used by a person in his or her responses to the MPI test.
- Schematic representation score: The total number of schematic representations used by a person in his or her responses to the MPI test.
- Visual-spatial representation score: The total number of visual-spatial representations (pictorial or schematic) used by a person in his or her responses to the MPI test.

In order to determine preservice teachers' levels of visual-spatial abilities, their SAT scores were used. A person's SAT score was the summation of his or her spatial orientation test score and spatial visualization test score. The spatial orientation ability was defined as "the ability to perceive spatial patterns or to maintain orientation with respect to objects in space" (Ekstrom et al., 1976, p. 149). A person's spatial orientation test score was obtained from the Card Rotation Test and the Cube Comparison Test. The spatial visualization ability was "the ability to manipulate or transform the image of spatial patterns into other arrangements" (Ekstrom et al., 1976, p. 173). A person's spatial visualization test score was obtained from the Paper Folding Test and the Surface Development Test.

3.5 Instruments

3.5.1 The MPI

In this study, the MPI developed by Presmeg (1985) and adapted to Turkish by Taşova (2011) (see Appendix E and Appendix F) was used for measuring preservice teachers' types of mathematical thinking, use of representations, and mathematical word problem solving performances. The MPI was developed for the first time by Krutetskii (1976) to measure students' preferences of the use of visual methods. Then Suwarsano (1982) designed the instrument with the same name for elementary school students. According to Presmeg (1995), the instrument which was designed by Suwarsano (1982) was not convenient for teachers. Thus she arranged the instrument for three sections according to fieldworks in which both students and teachers participated. With the new arrangement, the instrument took its final form

(See Table 1). In this study, since participants were pre-service teachers, Section B and Section C of the MPI was used.

Table 1**.** The Sections of Mathematical Processing Instrument

	Number of problems	Designed for	Level of difficulty
Section A	6	students	easy
Section B	12	students and teachers	intermediate
Section C	6	teachers	difficult

Not: From "Preference for visual methods: An international study" by Presmeg, 1995, *Proceedings of the 19th Annual Meeting of the International Group for the Psychology of Mathematics Education, 9*, p. 69

The MPI includes a test and a questionnaire. Each problem of the MPI can be solved with visual or nonvisual methods. The questionnaire has at least 3 at most 6 possible solutions, which include visual or nonvisual problem solving strategies, for each problem. When the MPI is administered, participants are first asked to solve the problems in the test. Then, they are given the questionnaire and from the given list, they chose one or more solutions that they think are similar to their solutions. If the participants do not find a similar solution, they have the option of indicating that their solution is not included in the questionnaire.

For the Turkish version, Sağlam and Bülbül (2010) adapted section B of the MPI. They found that the reliability coefficient as .96 by using split-half method and as .80 by using test-retest method. They conducted clinical interviews for construct validity. Taşova (2011) also studied for the adaptation of section B and section C. He was conducted a pilot study and calculated the reliability coefficient as .89 by using the split-half method. For the section C, acknowledged experts examined the instrument for face and content validity. The items are checked for incoherencies and translation errors and they were revised according to experts' comments and findings of the pilot study.

According to participants' responses on the questionnaire of the MPI, visualizing mathematical scores were generated. In this score, without taking into consideration whether the students solved the problem correctly, if the student chose only visual problem solving strategy for a problem, 2 points were given. For the responses that did not include visual problem solving strategy 0 points were given. For the responses including both visual and nonvisual strategies, 1 point was given. Therefore the possible minimum and maximum scores for preservice mathematical teachers' visualizing mathematical scores were respectively 0 and 36.

In order to group preservice teachers based on their mathematical thinking, participants' visualizing mathematical scores were used. In the literature, there are different methods suggested for classification of analytic, harmonic and geometric thinking. Richardson (1977) determined the groups according to percentages. The first 15% segment is analytic type, the last 15% segment is geometric type and others are harmonic type. Galindo-Morales (1994) determined the groups according to prearranged visualizing mathematical scores. Such as who has 22 points and above is a geometric thinker. In Presmeg and Taşova's studies $\frac{\text{the range of visualizing mathematical score}}{\text{the number of groups (3)}}$ was used to determine maximum and minimum of the each group. In this study, the standard deviation and mean of the participants' visualizing mathematical scores were used for deciding the group intervals considering the distributions of the data. The limits of the type of harmonic thinking was determined by the half of the standard deviation of the participants' visualizing mathematical scores around the mean of the participants' visualizing mathematical scores.

According to participants' responses on the test section of MPI, four different scores were generated. The first score was the total number of problems solved correctly. 1 point for correct answers and 0 points for incorrect answers were given for each problem, and possible minimum and maximum mathematical word problem solving scores were 0 and 18. Some problems in the MPI include two different questions and if the participant solved only one of the questions correctly, 0.5 points were gives as a score. The second was the total number of times students reported using a visual-spatial representation. Each representation used by participants was counted as 1 point. If a participant used two schematic representations for one problem, 2 points were given for the schematic representation score. The third and fourth scores were similar to the studies of Hegarty and Kozhevnikov (1999), van Garderen and Montague (2003), and van Garderen (2002, 2006) with respect to the number of pictorial or schematic visual representations. The same coding system used in these four studies was followed.

If preservice teachers reported or drew an image of objects or persons referred to in the problem, a visual-spatial representation was scored as primarily pictorial. If they drew a diagram, showed the spatial relations between objects in a problem, or reported a spatial image of the relations expressed in the problem a visual-spatial representation was scored as primarily schematic (See Fig. 5).

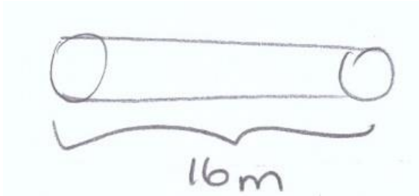
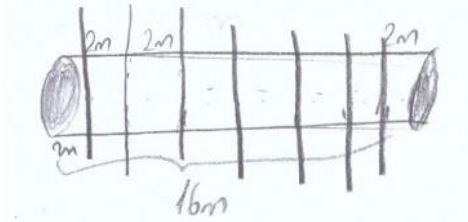
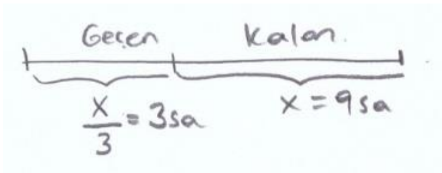
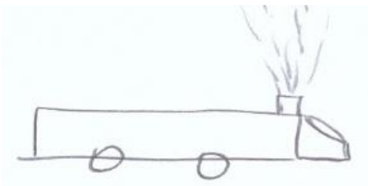
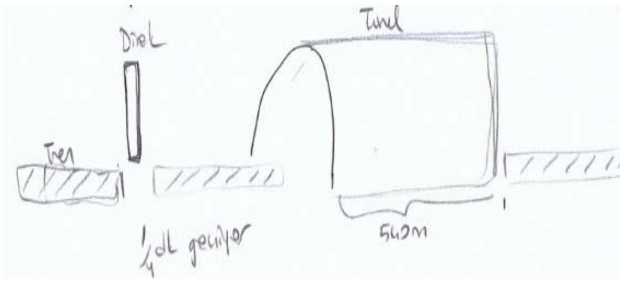
Examples of pictorial representations	Examples of schematic representations
B-7: A saw in a sawmill saws long logs, each 16 m long, into short logs, each 2 m long. If each cut takes two minutes, how long will it take for the saw to produce eight short logs from one long log?	
	
C-2: If the elapsed time since noon (12:00) is accounted for 1 in 3 of the remaining time to midnight, what time is it now?	
	
C-6: A train goes through a telegraph pole in $\frac{1}{4}$ minutes and goes through exactly $\frac{3}{4}$ minutes in the 540 m long tunnel. What is the speed of the train in per minute and how many meters is the length of the train?	
	

Fig. 5 Examples for preservice teachers' pictorial and schematic representations from this study

3.5.2 The SAT

In order to measure preservice teachers' levels of visual-spatial abilities, The SAT developed by Ekstrom et al. (1976) and adapted to Turkish by Delialioğlu (1996) were used. These tests were previously used by several researchers (Linn & Petersen, 1985; Lord, 1985; Delialioğlu, 1996; Bulut & Köroğlu, 2000; Kayhan, 2005; Tekin, 2007; Taşova, 2011). Since the reported reliability coefficients were high (see Table 2) the instruments were used considered as appropriate for the study.

The SAT involves spatial orientation and spatial visualization tests. The spatial orientation ability is established with the Card Rotation Test (CRT) (see Appendix G, Fig. 16, Fig. 17, and Fig.18) and the Cube Comparison Test (CCT) (see Appendix H, Fig. 19, Fig. 20, and Fig. 21), and the spatial visualization ability is established with the Paper Folding Test (PFT) (see Appendix I, Fig. 22, Fig. 23, and Fig. 24) and the Surface Development Test (SDT) (see Appendix J, Fig. 25, Fig. 26, Fig. 27, Fig. 28, and Fig. 29).

Table 2**.** The Reliability Coefficients of the Spatial Ability Tests

	The names of tests	Reliability Coefficients
The Spatial Orientation (Rotation) Tests	The Card Rotation Test (CRT)	0.80
	The Cube Comparison Test (CCT)	0.84
The Spatial Visualization Tests	The Paper Folding Test (PFT)	0.84
	The Surface Development Test (SDT)	0.82

Each item of the CRT included "a drawing of a card cut into irregular shape" and "eight other drawings of the same card" (Ekstrom et al., 1976, p. 150) which were the shapes rotated or turned over of the same card. In the CRT, students should determine whether or not the eight drawings were turned over for each item in 6 minutes. The test had two parts and each part included 10 items. The score of each item was 8 points and the possible maximum score of the CRT was 160 points.

Each item of the CCT included two cubes that the surfaces of the each cube had represented with different symbols. It was expected to determine whether the given cubes are the same or different by participants. The test had two parts and each part included 21 items. The score of each item was 1 point and the maximum score of the CCT was 42 points. The duration of the test was 6 minutes.

Each item of the PFT included figures, which represented the folding of a square paper. The last figure of the folding showed where the paper was punched. The task required determining which one of the five given choices was the correct image of the completely unfolded paper. The test had two parts, each consisting of 10 items. The score of each item was 1 point and the maximum score of the CRT was 20 points. The duration of the test was 6 minutes.

Each item of the SDT included a diagram, which consisted of five pieces that could form a three-dimensional fissure and its solid form. A piece of the drawing and its formed surface in the solid form were marked as X. The task requested participants “to indicate which lettered edge of the solid form correspond to five numbered edges or dotted lines in the diagram” (Ekstrom et al., 1976, p. 174). The test had two parts and each part had 6 items. The score of each item was 5 points and the total score of the SDT was 60 points. The duration of the test was 12 minutes.

3.6 Data analysis

In this section the statistical analyses that were conducted to test the research questions were presented. For the variables that were not normally distributed non-parametric statistic tests were used.

- The range, means and standard deviations of the scores from the scales were used to present descriptive characteristics of data. The results of descriptive statistics for visualizing mathematical scores were used for determining the groups for types of mathematical thinking.

- Kruskall-Wallis H test was used to investigate whether there were any differences in mathematical word problem solving performance and use of visual-spatial representations between three groups of preservice teachers having different types of mathematical thinking.
- One-way ANOVA test was used to investigate any differences between the SAT scores of groups of preservice teachers with different types of mathematical thinking.
- Spearman’s correlation analyses were conducted to determine the strength and direction of relationships between preservice teachers’ use of visual-spatial representations, mathematical word problem solving performance, and visual-spatial abilities.
- Chi-square test was used to investigate whether there were any association between use of pictorial or schematic representations and correct solutions.

CHAPTER 4

RESULTS

The results chapter of the study consists of three main sections. Firstly, descriptive statistics about preservice teachers' types of mathematical thinking, used visual representations and visual-spatial abilities are presented. Secondly, group differences among variables are revealed. Finally the relationships between variables are introduced.

4.1 Descriptive statistics

This section provides description of the data through means, standard deviations, and range for each variable as measured by the instruments.

4.1.1 Descriptive statistics for the variables measured by the MPI

The descriptive statistics about mathematical word problem solving performance, use of schematic representations, use of pictorial representations, use of visual-spatial representations, and visualizing mathematical score as measured by the MPI are presented in Table 3.

The results showed that the mean of participants' scores for mathematical word problem solving performance was 14.89 (SD = 2.28) (see Table 3). In particular, 97.3% of the participants solved half of the problems. 61% of the participants scored over the mean. All of the participants solved first problem correctly and more than 93% of the participants gave a right answer for seven problems from the Section B. Approximately 45% of the participants could not performed successfully on problems numbered C3, C5, and C6. The 9th problem of the Section B also was the last but one based on performance.

Table 3. Descriptive Statistics Results for the Variables Measured by the MPI

N_{total} = 113	Range	Mean	Std. Deviation
Mathematical Word Problem Solving Performance	7 - 18	14.89	2.28
Schematic Representations Score	2 - 19	8.07	3.23
Pictorial Representations Score	0 - 3	.95	.90
Visual-Spatial Representations Score	2 - 20	9.02	3.29
Visualizing Mathematical Score	5 - 28	13.97	4.73

In total participants used 1047 visual-spatial representations, of which 925 were schematic and 122 were pictorial. Descriptive statistics related to the preservice teachers' use of visual-spatial representations in problem solving process showed that the mean of participants' scores for use of schematic representations was 8.07 (SD = 3.23), the mean of participants' scores for use of pictorial representations was .95 (SD = .90), and the mean of participants' scores for use of visual-spatial representations was 13.97 (SD = 3.29) (see Table 3). The participant who scored at minimum for the use of representations applied to schematic representation at least for the solution of two problems. 58% of the participants was used a representation for the half of the problems. Especially 93% of the participants for the problem numbered B4 and 85% of the participants for the problems numbered B7 and B9 used a representation in problem solving process. Although all participants used schematic representations in their problem solving processes, pictorial representations were rarely used by preservice teachers. 40% of the participants used schematic representations for more than nine of the problems. Thirty-five percentages of the participants did not use any pictorial representation and 43% of participants used only one pictorial representation. Only 8 participants used three pictorial representations and there was no participant who is represented more than three pictorial representations.

The results showed that preservice teachers' visualizing mathematical scores had a mean of 13.97 (SD = 4.73), the minimum score was 5, and the maximum score was 28 (see Table 3). Fifty-three percentages of the participants rated below the mean and 91% of the participants' visualizing mathematical scores were below 20 although possible maximum score was 36 (see Table 4).

Table 4. Frequencies for Visualizing Mathematical Score

Visualizing Mathematical Score	Frequency	Percent	Valid Percent	Cumulative Percent
5	1	.9	.9	.9
6	5	4.4	4.4	5.3
7	4	3.5	3.5	8.8
8	4	3.5	3.5	12.4
9	4	3.5	3.5	15.9
10	8	7.1	7.1	23.0
11	8	7.1	7.1	30.1
12	10	8.8	8.8	38.9
13	10	8.8	8.8	47.8
14	13	11.5	11.5	59.3
15	7	6.2	6.2	65.5
16	8	7.1	7.1	72.6
17	5	4.4	4.4	77.0
18	12	10.6	10.6	87.6
19	4	3.5	3.5	91.2
20	2	1.8	1.8	92.9
21	1	.9	.9	93.8
22	1	.9	.9	94.7
23	1	.9	.9	95.6
26	4	3.5	3.5	99.1
28	1	.9	.9	100.0
Total	113	100.0	100.0	

4.1.2 Preservice teachers' types of mathematical thinking

Research Question 1: Which structure of mathematical thinking, analytic, geometric or harmonic types is adopted most frequently by preservice teachers?

According to participants' visualizing mathematical scores, participants were divided into three groups. In this study a different method was used for the classification of types of mathematical thinking compared to other studies. The results showed that for 5 problems (B4, B5, B6, B11, and C4) preservice teachers did not tend to use any representation and also they did not select a visual solution in the questionnaire section (see Fig. 6).

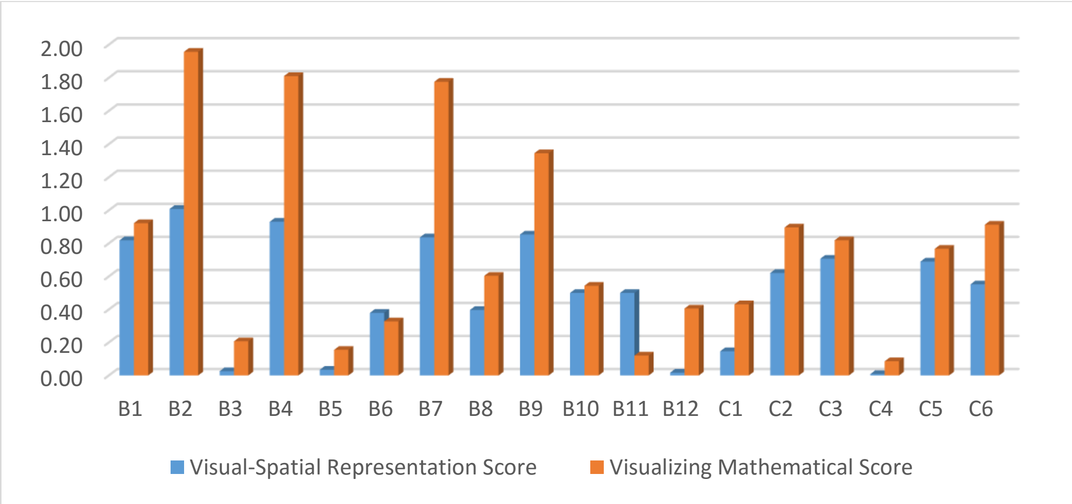


Fig. 6 The means of the visual-spatial representation score and the visualizing mathematical score for each problem of the MPI

Therefore a participant with a visualizing mathematical score of 18, which is the half of the maximum score, preferred a visual method in at least 9 of the remaining 13 problems. Under these circumstances such a participant who preferred visual methods more than nonvisual methods in approximately 70% of the remaining problems was considered as a geometric thinker.

Due to the considerations mentioned in the previous paragraph, groupings were not obtained by dividing the range into equal chunks. Instead the mean score was used while deciding on the center of the interval for the harmonic type and the intervals for all three types were found by taking the standard deviation of the scores into consideration. The minimum and maximum scores of the type of harmonic thinking were assigned by the half of the standard deviation of the preservice teachers' visualizing mathematical score around its mean. The intervals for the groups were 5-11 for the analytic type, 12-16 for the harmonic type and 17-28 for the geometric type. According to this classification, the number of people grouped for each type of mathematical thinking was 34 (30%) for the analytic type, 48 (43%) for the harmonic type and 31 (27%) for the geometric type (see Table 5).

Table 5. Frequencies for Types of Mathematical Thinking

Types of Mathematical Thinking	Frequency	Percent	The range of visualizing mathematical score
The analytic type	34	30.1	5 – 11
The harmonic type	48	42.5	12 – 16
The geometric type	31	27.4	17 – 28
Total	113	100.0	5 – 28

4.1.3 Descriptive statistics for the variables measured by the SAT

The descriptive statistics of the spatial visualization tests, the spatial orientation tests, and the SAT scores are presented in Table 6. Thirteen participants could not complete all of the spatial ability tests; hence their data were accepted as missing data and excluded from the data analysis process related to visual-spatial abilities.

Descriptive statistics related to the preservice teachers' visual-spatial abilities showed that the means of participants' scores for the spatial orientation tests, the spatial visualization tests, and the SAT were respectively 127.6 (SD = 36.83), 45.2 (SD = 14.5), and 172.8 (SD = 48.22) (see Table 6). Although the possible maximum scores of the spatial orientation tests, the spatial visualization tests, and the SAT scores were respectively 202, 80, and 282 points, the participants' maximum scores were respectively 190, 73, and 260 points.

Table 6. Descriptive Statistics Results for the Variables Measured by the SAT

N = 100	Range	Mean	Std. Deviation
The spatial orientation tests	39 - 190	127.60	36.83
The spatial visualization tests	15 - 73	45.20	14.50
The SAT Scores	81 - 260	172.80	48.22

4.2 The Investigation of group differences

4.2.1 Non-parametric analyses

When the parametric tests assumptions are violated the non-parametric tests can be used. Because of some variables from the data could not fit the normality assumption of the one-way ANOVA, which uses the means of the groups to investigate the differences among the dispersion in the sample (Leard Statistics, 2015; Tabachnick & Fidell, 2006), non-parametric tests were used to investigate group difference. Kruskal-Wallis H test is a non-parametric statistical technique that uses ranking order to reveal whether there are significant differences between two or more groups of an in dependent variable on a dependent variable (Leard Statistics, 2015). Kruskal-Wallis H test was used to clarify whether there were any statistically differences in preservice teachers' mathematical word problem solving performances, use of schematic representations, use of pictorial representations, use of visual-spatial representations, and visual-spatial abilities between groups of participants having different types of mathematical thinking.

4.2.1.1 Comparisons of mathematical word problem solving performances based on types of mathematical thinking

Research Question 2: Is there a significant difference in preservice teachers' mathematical word problem solving performance according to their types of mathematical thinking?

Mathematical word problem solving performance was the continuous dependent variable and types of mathematical thinking were the independent variable that consists of three categorical independent groups having different participants. Preliminary analyses showed that the variables were not normally distributed, as assessed by the Shapiro-Wilk test ($p < .05$) and there was homogeneity of variances as assessed by Levene's test for equality of variances ($p = .76$).

A Kruskal-Wallis H test was run to determine whether there were any differences in mathematical word problem solving performance between three groups of preservice teachers having different types of mathematical thinking: analytic type ($n =34$), harmonic type ($n = 48$) and geometric type ($n = 31$). The results revealed that the distribution of mathematical word problem solving performance scores for each group with different types of mathematical thinking was similar. The medians of mathematical word problem solving scores were not significantly different among the analytic type (mean rank = 15.5), the harmonic type (mean rank = 15), and the geometric type (mean rank = 15), χ^2 (2) = 14.468, $p = .24$.

4.2.1.2 Comparisons of use of schematic, pictorial and visual-spatial representations based on types of mathematical thinking

Research Question 3: Is there a significant difference in preservice teachers' use of schematic representations, pictorial representations, and visual-spatial representations based on their types of mathematical thinking?

Use of schematic representations, use of pictorial representations, and use of visual-spatial representations were the continuous dependent variables and the types of mathematical thinking was the independent variable that consisted of three categorical independent groups. Preliminary analyses showed that the variables were not normally distributed, as assessed by the Shapiro-Wilk test ($p < .05$).

In order to determine whether there were any differences in the use of schematic representations between three groups for types of mathematical thinking: the analytic type ($n =34$), the harmonic type ($n = 48$) and the geometric type ($n = 31$) a Kruskal-Wallis H test was run. The results revealed that mean ranks of schematic representation scores were statistically significantly different between the groups ($\chi 2$ $(2) = 13.435$, $p = .01$).

Subsequently, pairwise comparisons were performed using Dunn's (1964) procedure. A Bonferroni correction multiple comparisons was made and significant differences were found ($p < .05$). As a result of post hoc analysis, it was discovered there were statistically significant differences in preservice teachers' schematic representation scores between the analytic type (mean rank = 6) and the harmonic type (mean rank = 8) ($p = .01$) and the analytic type and the geometric type (mean rank = 9) ($p = .01$). On the other hand, there were no significantly differences in schematic representation scores between the geometric type and the harmonic type ($p = .1$).

A Kruskal-Wallis test was run to determine whether there were any differences in the use of pictorial representations between three groups for types of mathematical thinking: the analytic type (n =34), the harmonic type (n = 48) and the geometric type (n = 31).. The results revealed that mean ranks of pictorial representation scores between the analytic type (mean rank = .5), the harmonic type (mean rank = 1), and the geometric type (mean rank = 1) were not statistically significantly different (χ^2 (2) = 2.281, p = .32).

A Kruskal-Wallis test was run to identify whether there were any differences in the use of visual-spatial representations between three groups for types of mathematical thinking: the analytic type (n =34), the harmonic type (n = 48) and the geometric type (n = 31). The results revealed that median scores of visual-spatial representation scores increased from the analytic type (mean rank = 6.5), to the harmonic type (mean rank = 9), to the geometric type (mean rank = 10) and the group differences were statistically significantly different (χ^2 (2) = 11.575, p = .01)

By using Dunn's (1964) procedure, pairwise comparisons were done. A Bonferroni correction multiple comparisons was made and the significant differences were found ($p < .01$). Results of the post hoc analysis showed that there were statistically significant differences in schematic representation scores between the analytic type and the harmonic type (p = .01), and the analytic type and the geometric type (p = .01). On the other hand there were no significantly differences in visual-spatial representation scores between the geometric type and the harmonic type ($p > .05$).

4.2.2 Analysis of variance

Research Question 4: Is there a significant difference in preservice teachers' levels of visual-spatial abilities based on their types of mathematical thinking?

Sixth research question was tested by the analysis of variance. One-way ANOVA is a statistical test that compares the means of two or more groups to investigate whether there are significantly differences between the groups (Tabachnick & Fidell, 2006). It also provides post-hoc tests to present the difference between the each group.

Before running the analysis, the assumptions of one-way ANOVA were checked. Preservice teachers' levels of visual-spatial abilities were the continuous dependent variable and the types of mathematical thinking were the independent variable. One-way ANOVA analysis requires testing outliers in the data set. To determine these outliers with the boxplots were used. As shown in the Fig. 7 there were not any value 3 box-lengths away from the edge of the box. Therefore the data set did not include any outlier.

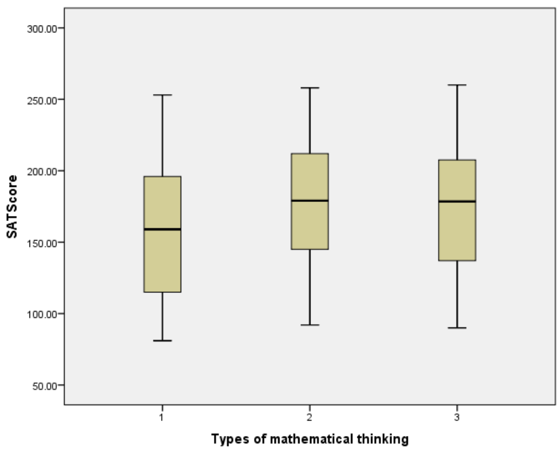


Fig. 7 Boxplots results for the SAT score of each type of mathematical thinking

Shapiro-Wilk test used to determine for the distribution of normality and the results showed participants' SAT scores were normally distributed ($p > .05$).

Levene's Test of Homogeneity of Variance was used to investigate the homogeneity of the variances. A homogeneity of variances was discovered ($p > .05$).

One-way ANOVA test was run to investigate any differences between the SAT scores of groups of participants with different types of mathematical thinking. Participants were classified as three groups: the analytic type ($n = 29$), the harmonic type ($n = 43$) and the geometric type ($n = 28$). The SAT scores from the three groups, the analytic type ($M = 161.83$, $SD = 50.1$), geometric type ($M = 179.02$, $SD = 43.62$) and harmonic type ($M = 174.57$, $SD = 45.57$) did not differ significantly ($F(2, 97) = 1.233$, $p = .30$) (see Table 7).

Table 7. One-Way ANOVA Results for the SAT Scores of Types of Mathematical Thinking

The SAT Scores	df	F	Sig.
Between Groups	2	1.233	.30
Within Groups	97		
Total	99		

4.3 Association analyses

4.3.1 Association between the use of representations and mathematical word problem solving performance

Research Question 5: Is there an association between the use of schematic, pictorial, and visual-spatial representations and mathematical word problem solving performance?

Correlational analysis was conducted to investigate the association between the use of schematic, pictorial, and visual-spatial representations and mathematical word problem solving performance. Firstly Spearman’s correlation was used to determine whether there is an association between the variables. Secondly chi-square test was run to investigate whether use of pictorial or schematic representation and problem solving performance were independent of one another.

4.3.1.1 Spearman's correlation analyses

The assumptions of Spearman's correlational analysis were checked before testing the hypothesis. The variables of the schematic representation score, the pictorial representation score, the visual-spatial representation score, and mathematical word problem solving performance were continuous variables. Preliminary analyses showed that the variables were not normally distributed, as assessed by the Shapiro-Wilk test ($p < .05$) and the relationships between variables were slightly monotonic.

Spearman's Correlation Analyses were run to determine the strength and direction of similarly monotonic relationships between participants' use of representations and mathematical word problem solving performance. There was no statistically significant relationship between variables ($p > .05$) except between the schematic representation scores and mathematical word problem solving performances (see Table 8). However, this significant correlation was a weak positive one, ($r_s\,(111) = .18$, $p < .05$).

Table 8. Spearman's Correlation Coefficient Between the Variables

	Mathematical Word Problem Solving Performance
Schematic representation score	.18*
Pictorial representation score	-.05
Visual-Spatial Representation Score	.17

Correlations are Spearman rho coefficients
* The level of significance ($p < .05$)

4.3.1.2 Chi-square test analysis

In order to investigate the association between the types of representations and problem solving performance chi-square test was used. Before running the analysis, participants' representations were classified as schematic or pictorial for the items they used representations. Each item was coded as correct or incorrect. The items without any used representations or not answered accurately (previously coded as .5 points) were not included in the analysis. Before testing the research question, firstly assumptions of the chi-square test were checked.

The variables both of the used representations as "schematic" or "pictorial" and the mathematical word problem solving performance as "correct" or "incorrect" answer were categorical variables. The expected counts for each categorical variable included more than 5 cases.

For the items that the participants used pictorial representations, approximately half of them were solved correctly, whereas the percentage of correct solutions from the items, for which schematic representations were used, was 86% (see Fig. 8).

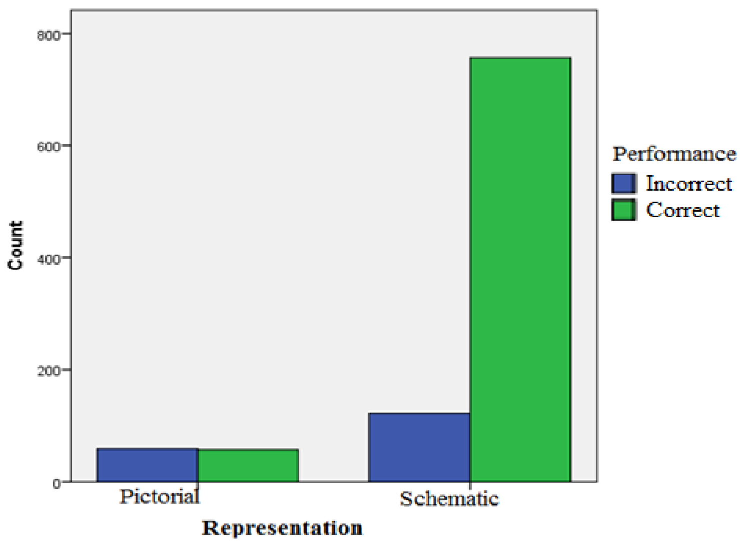


Fig. 8 Bar chart for performance results according to representation types

A chi-square test was conducted to investigate the association between mathematical word problem solving performance and the use of visual-spatial representations as pictorial or schematic. There was a statistically significant association between performance and preference of pictorial or schematic representations (χ^2 (1) = 94.181, p = .01). Results showed a moderately strong association between types of representations and problem solving performance (φ = 0.31, p = .01).

4.3.2 Association between mathematical word problem solving performance and visual-spatial abilities

Research Question 6: Is there a relationship between preservice teachers' mathematical problem solving performance and levels of visual-spatial abilities?

Spearman's correlational analysis was used whether mathematical word problem solving performance and visual-spatial abilities were independent of one another. The variables of the mathematical word problem solving performance and the SAT scores were continuous variables. Preliminary analyses showed that the variables were not normally distributed, as assessed by the Shapiro-Wilk test ($p < .05$) and the relationships between variables were slightly monotonic. The results of Spearman's correlation analyses showed there was no statistically significant relationship between the variables ($p > .05$).

4.3.3 Association between the use of schematic representations and visual-spatial abilities

Research question 7: Is there a relationship between preservice teachers' use of schematic representations and levels of visual-spatial abilities?

A Spearman's correlation analyses were run to determine the strength and direction of similarly monotonic relationships between participants' use of representations and levels of visual-spatial abilities. The assumptions of Spearman's correlational analysis were checked before testing the hypothesis. The variables of the schematic representation scores and the SAT scores were continuous variables. Preliminary analyses showed that the variables were not normally distributed, as assessed by the Shapiro-Wilk test ($p < .05$) and the relationships between variables were slightly monotonic. There was a significant week positively correlation between the use of schematic representations and levels of visual-spatial abilities ($r_s (98) = .21$, $p = .01$).

CHAPTER 5

DISCUSSION

In this chapter, the results of the study are discussed in the light of the literature. Firstly, to provide an insight into the whole study the summary of the study is given. The following sections include the discussion of the results for each research question. Finally, the limitations and implications of the study are explained.

5.1 Summary of the study

The current study was conducted in a correlational research design with 113 participants from a private and four public universities in Istanbul and Ankara. The aim was to investigate preservice teachers' types of mathematical thinking, use of visual-spatial representations: schematic or pictorial, and visual-spatial abilities in mathematical word problem solving process. Two instruments were used for the data collection: The Mathematical Processing Instrument and the Spatial Ability Tests. Data were collected in the 2016 spring semester. In the data analyses process, descriptive, correlational and inferential, parametric and non-parametric statistics, analyses were conducted to test the research questions.

The findings of the study revealed that 43% of the preservice teachers were analytic thinkers, 30% of the preservice teachers were harmonic thinkers, and 27% of the preservice teachers were geometric thinkers. There was no significant difference in the use of pictorial representations, mathematical word problem solving performance, and levels of visual-spatial abilities among the groups of each type of mathematical thinking. The only difference in group comparisons for thinking type was found in the use of schematic representations. Preservice teachers who adopted harmonic or geometric mathematical thinking tended to use more schematic representations in their mathematical problem solving processes than analytic thinkers.

The results of correlational analysis showed no significant relationship between mathematical word problem solving performance and visual-spatial abilities. Preservice teachers' visual-spatial abilities only had a weak relationship with use of schematic representations among all the variables. The investigation of the factors having a correlation with mathematical word problem solving performance showed that only use of visual-spatial representations had a relationship with performance. While the use of schematic representations was associated with correct solutions, the use of pictorial representations did not show a positive relationship with the problem solving performances.

5.2 Discussion of the results

In this section, firstly the results of the descriptive statistics and then the type of mathematical thinking adopted by preservice teachers, analytic, harmonic or geometric, are discussed with the findings of the previous studies from the literature. Secondly, a discussion of the group differences among types of mathematical thinking according to mathematical word problem solving performance, the use of visual-spatial representations, and levels of visual-spatial abilities arepresented. Finally the factors influencing mathematical word problem solving performance are discussed.

5.2.1 The results of descriptive statistics

The results showed that preservice teachers were quite successful in the MPI. The mean score was 14.89 (SD = 2.28) while the maximum possible score was 18. The MPI was initially designed for 6th - 8th grade students. After that some researchers brought about new arrangements as they applied the instrument to college students and teachers. Presmeg (1995) argued that not all of the problems in the MPI were appropriate for teachers and she arranged the instrument for three sections based on the difficulty of the problems. Section B, which included intermediate level problems, and Section C, which included difficult level problems, were suggested for teachers. Since the participants were preservice teachers, these two sections were applied in the current study. Whereas preservice teachers showed high performance in Section B, they could not show the same success for Section C. The findings of the study supported Presmeg's classification. It was observed that this arrangement of the MPI was appropriate for preservice teachers.

All participants used representations in their solutions. More than half of the participants preferred to use a representation for half of the solutions. Descriptive analyses showed that the mean score of participants on using visual-spatial representations was 9.02, SD = 3.29. These findings were similar with the results of van Garderen and Montague's study (2003). They also reported that sixth grade students used a visual-spatial representation for more than half of the problems

In the current study, schematic representations were seen more frequently compared to pictorial representations. These findings were similar to a study that was conducted with gifted students. Van Garderen (2006) conducted a study with sixth grade students who were classified into three categories as "gifted students", "average achievers", and "learning disabilities" and investigated the relationships between visual-spatial abilities and use of representations. His study exposed that the frequency of the use of these two representations were very similar in problem solving processes. However gifted students used most of the time schematic representations, whereas average achievers and learning disabled students preferred pictorial representations more. In the current study while schematic representations were used approximately in half of the problem solutions, preservice teachers rarely used pictorial representations. Their preferences for use of schematic or pictorial representations were similar with gifted students. The reason of this similarity could be participants' competence in mathematics and strong relationship with mathematics education.

Preservice teachers' spatial orientation tests scores, spatial visualization tests scores and SAT scores were normally distributed. The mean scores and the median scores were quietly approximate. The findings exposed the mean scores of preservice teachers' were M = 127.6 (SD = 36.83) for spatial orientation tests and M = 45.2 (SD = 14.5) for spatial visualization tests. In total the mean of the SAT were M = 172.8 (SD = 48.22). The findings regarding to preservice teachers' levels of visual-spatial abilities were observed lower than Taşova's study (2011). Taşova (2011) conducted a study to investigate the influence of preservice teachers' visual-spatial abilities and types of mathematical thinking on their modeling activities and performances. In his findings, the mean scores for participants' visual-spatial abilities were higher than the current study. Although the findings of this study were observed to be lower, the distribution of preservice teachers' levels of visual-spatial abilities showed a normal distribution and the results were significant.

5.2.2 The structure of mathematical thinking; analytic, geometric or harmonic types adopted by preservice teachers

Preservice teachers' types of mathematical thinking were determined according to their visualizing mathematical scores. Visualizing mathematical scores were obtained by preservice teachers' preferences for visual or nonvisual problem solving methods. The current study used a different classification method to determine each type of mathematical thinking based on visualizing mathematical score than most of the studies in the literature. For five specific problems from the MPI, participants did not tend to use a visual method for the solutions. The maximum visualizing mathematical score obtained by a participant was 28 points and this particular participant also did not choose a visual method at least for five problems. In Taşova's study (2011), a participant who scored 18 points was accepted as a harmonic thinker. However in the current study, a participant with this score preferred a visual problem solving method at least 70% of all but five problems mentioned above. With this consideration the limit values of all types of mathematical thinking were calculated with the mean and standard deviation of participants' visualizing mathematical scores.

The findings showed that 30% of the preservice teachers were analytic type, 43% of preservice teachers were harmonic type, and 27% of preservice teachers were geometric type. The slightly high proportion of the trend for the harmonic type was similar with the literature. Many studies suggested that teachers and college students prefer a solution that includes both visual and nonvisual problem solving methods. Hacıömeroğlu & Hacıömeroğlu (2014) pointed out that most of preservice teachers adopted the harmonic type of mathematical thinking. Taşova's findings (2011) supported that the harmonic type of thinking was the most adopted by preservice teachers whereas the least percent of the preservice teachers were geometric thinkers. In the current study, these differences were not clearly seen and the classification method could be the reason for it. Hacıömeroğlu and Hacıömeroğlu (2014) found that senior preservice teachers used visual methods more than juniors. Therefore they related this difference with seniors' experiences through teaching mathematics and practicum courses. Since participants of the current study were also seniors and the data collection was done close to end of the second term, their final year experiences may have an impact on their preferences.

The findings of the study did not show any relationship between preservice teachers' preferences for visual or nonvisual methods and the degree of difficulty of problems. Previous studies suggested that there was an association between these variables. Hacıömeroğlu and Hacıömeroğlu (2013) suggested that the more the difficulty of problems increased the more nonvisual solution methods were used. On the other hand Lowrie and Kay (2001) and Hacıömeroğlu (2012) found that as the difficulty of the problems increased, visual methods were chosen significantly more than nonvisual methods. Although the MPI used in the current study had two sections as intermediate and difficult by Presmeg (1995), the findings did not expose any tendency towards visual methods for both these two sections. This result was similar with the findings of Lowrie's study (2001). Lowrie (2001) also suggested that there were no significant relationship between preferences for visual or nonvisual methods and the task difficulty.

For these controversial findings there could be two main reasons: the instrument and sample selection. Hacıömeroğlu and Hacıömeroğlu (2013) used Suvarsano's MPI. On the other hand Hacımömeroğlu (2012) used 20 different problems rather than the MPI. While Lowrie (2001) studied with middle school students other studies were conducted with college level students. In the current study, Presmeg's MPI was used. In Presmeg's study (1985) participants were selected from both students and mathematics teachers, and she especially arranged the MPI according to task difficulty. Although the previous results revealed a relationship between the task difficulty and preferences for visual or nonvisual methods, controversial findings suggest there could be other reasons that may have an influence on preferences. It could be person's education, developments, and habits. Preservice mathematics teachers have many opportunities for experiencing different problem solving methods during their education life. They could prefer either an easier and quicker solution or a solution that they deem more appropriate for teaching.

5.2.3 Group differences for the variables of the study

In this section, whether there were any differences among types of mathematical thinking based on performance, use of representations and visual-spatial abilities are discussed. The findings of the current study are compared with the results of the previous studies.

5.2.3.1 Mathematical word problem solving performance according to types of mathematical thinking

Results showed that there was no significant difference among groups with analytic, harmonic, and geometric types of mathematical thinking in terms of problem solving performance. While the findings were supported by various studies (Kolloffel, 2012; Pitta-Pantazi & Christou, 2009; Suwarsano, 1982) there were some conflicts in the literature.

There were many studies (Kolloffel, 2012; Lean & Clements, 1981; Moses, 1977, 1980; Pitta-Pantazi & Christou, 2009; Suwarsano, 1982; Webb, 1979) that investigated the relationship between individuals' preferences for visual and nonvisual methods and mathematical problem solving performance and the results of these studies presented a controversy. While Lean and Clements (1981) suggested that the preference had a significant effect on performance and students who preferred nonvisual strategies outperformed visualizers, Moses (1977; 1980) and Webb (1979) claimed that visual solution methods guide college students to more effective solutions. On the other hand Suwarsano (1982), Pitta-Pantazi and Christou (2009), and Kolloffel (2012) revealed that a person's mathematical thinking did not have a significant influence on mathematical problem solving performance. The current study also showed there were no statistically significant differences on mathematical word problem solving performances among the groups: the analytic type, the harmonic type and the geometric type.

These controversial findings in the literature might be caused by sample selection. The studies applied the same instrument with some adjustments to different groups such as elementary school students, college students, and teachers. The participants' individual differences like how they were taught, grade level, courses they were enrolled also could be factors influencing this relationship. In terms of performance, Presmeg (1986a) suggested that there were internal and external factors, which could make a group superior compared to others. She conducted a study with both teachers and their students. In this study (Presmeg, 1986b) she investigated efficacy of visual approaches and effects of teaching styles in terms of visuality. She discussed that textbooks and teachers' teaching styles emphasized nonvisual methods. Therefore this situation could favor for analytic thinkers. However with the educational developments the role of visualization and its importance in problem solving was recognized (Deliyianni et al., 2009). Visual approaches were included in both teacher education programs and curriculums. Therefore preservice teachers could be experienced both visual and nonvisual approaches during their method courses and school practices. It could be also that school exams might constraint students for using visual methods, which could take more time for solutions (Presmeg, 1986a). In Turkey, school entrance exams also have an influence on students' preferences for problem solving methods. Although preservice teachers that participated in this study had similar experiences, their learning experiences through university life reduce the influence of these internal or external factors on performance.

5.2.3.2 Use of visual-spatial representations according to types of mathematical thinking

In the literature, many studies used the MPI investigating visualization through use of representations or the structure of mathematical thinking. This study focused on the relationship between these two fields by investigating how preservice teachers' practice on paper and the claims they make about their preference of visual and nonvisual methods during word problem solving were related each other. The results showed significant differences in the use of schematic representations and visual-spatial representations among groups with different types of mathematical thinking while no difference was found in the use of pictorial representations. Preservice teachers did not tend to use pictorial representations as much as elementary or high school level students did as the previous studies suggested (van Garderen, 2006; van Garderen & Montegue, 2003). The frequency and the variance of preservice teachers’ pictorial representations scores were very low. This made it impossible to run a statistical significance analysis. The rare use of pictorial representations by participants may be one reason for not observing significant differences between the groups.

The preference for use of schematic representations and in general visual-spatial representations was significantly different among the groups of mathematical thinking. Further investigation revealed that although there was not a significant difference between the harmonic type and the geometric type, analytic type of mathematical thinkers used visibly less schematic representations than others. Harmonic thinkers used representations in problem solving as frequently as geometric thinkers.

In the current study harmonic thinkers and geometric thinkers had similar preferences for use of representations in problem solving whereas analytic thinkers separated from others by using fewer representations. These findings were different from Sevimli and Delice’s study (2011). They conducted a study investigating the relationships between college students’ preferences for problem solving solution methods and use of representations. They found that analytic thinkers and harmonic thinkers had similar preference for use of representations and their use of representations were significantly less frequent than geometric thinkers. There might be two reasons for the differences in these findings. One of them was the mathematical context of the studies. Sevimi and Delice (2011) carried out their study on a specific topic: definite integral. They discussed that in calculus courses students were mainly taught nonvisual methods and algebraic expressions. The context of definite integral and how it is taught can lead to students for using algebraic solutions. On the other hand word problems that were used in this study promote preservice teachers more for using representations in solutions.

The second reason could be that the participants did not express all problem solving procedures in their mind on the paper in the current study. A geometric thinker could have used internal representations during the problem solving procedure. For definite integral context although representations were not preferred by preservice teachers during the problem solving processes, when they used it might be a difficult procedure to operate representations in mind. The context requires specific graphical representations that include complex processes (Sevimli & Delice, 2011) and they could push the preservice teachers for operation on paper. However the representations that used in solutions of word problems could be built in mind. They did not have a complex structure as much as graphical representations that used in integral context. Further studies could be done for different mathematical contexts. Researchers might prefer interviews in data collection processes to detect representations that people construct in their mind.

5.2.3.3 Levels of visual-spatial abilities according to types of mathematical thinking

Results revealed that there was no significant difference on preservice teachers' levels of visual-spatial abilities according to type of mathematical thinking. The preference for visual and nonvisual solution methods in problem solving did not have an association with participants' visual-spatial abilities. On the other hand the correlational analysis that was conducted between the use of schematic representation and visual-spatial abilities presented a weak positive relationship and as discussed before, the use of schematic representations differed among the mathematical thinking groups. Although participants' visual-spatial abilities might have a slight indirect influence on the preferring for solution strategies, the findings did not favor any type of mathematical thinking for preservice teachers' levels of visual-spatial abilities.

The findings of the current study did not show statistically significant difference for preservice teachers' visual-spatial abilities in terms of types of mathematical thinking. Taşova (2011) suggested that geometric thinkers were more successful in visual-spatial ability tests than analytic or harmonic thinkers. However he did not run a statistical analysis to compare the groups for types of mathematical thinking in terms of their levels of visual-spatial abilities. The findings of this study also revealed a slight increase for levels of visual-spatial abilities from the analytic type to geometric type. Still these differences were not statistically significant. Many studies also did not find a significant relationship between people's visual-spatial abilities and their preferences for visual or nonvisual methods (Haciomeroglu et al., 2013; Hagarty & Kozhevnikov, 1999; Kozhevnikov, Hagarty, & Mayer, 2002; Moses, 1977; Lean & Clements, 1981; Suwarsono, 1982). Krutetskii (1976) suggested that there were many other factors, which effects people's preferences like learning experiences. Therefore further studies could research what these factors are rather than focusing on peoples' visual-spatial abilities.

5.2.4 The factors influencing mathematical word problem solving performance

5.2.4.1 The association between use of representations and mathematical word problem solving performance

The results showed use of visual-spatial representations did not have a significant influence on preservice teachers' mathematical problem solving performance.

Participants' pictorial representation scores and performance scores were also not associated. However, use of schematic representations showed a weak positive correlation with problem solving performance. These results were not compatible with many other studies in the literature (Barratt, 1953; Campbell et al., 1995; Hegarty & Kozhevnikov, 1999; Kozhevnikov, Hegarty, & Mayer, 2002; Lean & Clements, 1981; Presmeg, 1986a, 1986b; Sevimli & Delice, 2011; van Garderen, 2006; van Garderen & Montegue, 2003). Many researchers investigated use of representations and problem solving performance in mathematics education literature and their common findings was a significant relationship between these variables.

In the current study, as mentioned before, preservice teachers rarely used pictorial representations in problem solving processes. It was thought that they could be constructing these representations in their minds without representing them on paper. Hence only drawn representations could be detected by the researcher. In order to investigate the association of used representations with problem solving performance a Chi-square test was run. Reducing the data for items including only problems that representations were used in their solutions was appropriate to understand deeply the relationship between use of representations and problem solving performance. Each item for which participants used a type of representation was determined. And these items were categorized according to use of representation, schematic or pictorial, and correctness of the solution, correct or incorrect. The results of the analysis showed a moderately strong association between the variables, $\varphi = 0.31$, $p = .01$.

The findings of the study showed that for the items where schematic representations were used the ratio of the correct responses was higher than the items where pictorial representations were used. Many studies (Barratt, 1953; Campbell et al., 1995; Lean & Clements, 1981) found significant relationship between use of visual-spatial representations and mathematical problem solving performance. Hegarty and Kozhevnikov (1999) investigated the relationship between sixth grade students' visual-spatial representations and their problem solving performances. They found that use of schematic representations was positively correlated with problem solving performance but pictorial representations had a negative influence on problem solving performance. Stylianou and Silver (2004) suggested in order to succeed in problem solving use of schematic representations was essential. Similarly other studies (Kozhevnikov et al., 2002; van Garderen, 2006; van Garderen & Montegue, 2003) reported that students who preferred using schematic representations were more successful than students who chose using pictorial representations. Although in this study the use of pictorial or schematic representations had no significant or a weak relationship with problem solving performance, the influence of representations on performance significantly differed in terms of the types of representations.

The findings supported that this classification for different types of representations was reliable (Hegarty & Kozhevnikov, 1999). These types differently related with problem solving performance. According to the American National Council of Teachers of Mathematics (NCTM, 2000), a student should use representations for organizing and communicating mathematical ideas, and modeling and interpreting a mathematical phenomenon. The explanation was clearly referring to schematic representation itself. As many researchers emphasized the importance of schematic representations, further studies could investigate how schematic representations are related with the stages of problem solving processes.

5.2.4.2 The association between visual-spatial abilities and mathematical word problem solving performance

The findings did not show a significant correlation between preservice teachers' visual-spatial abilities and their mathematical word problem solving performances. This result was not similar with the findings of previous studies. Various research studies explored the relationship between visual-spatial abilities and mathematical performance and they revealed a positive correlation between visual-spatial abilities and problem solving performance (Battista, 1990; Clements & Battista, 1992; Haciomeroglu et al., 2013; Hegarty & Kozhevnikov, 1999; van Garderen, 2006).

For the singular findings of this study, sample selection and different visual-ability tests that were used in the studies might be the reasons. Van Garderen (2006) investigated the relationship between visual-spatial abilities and problem solving performance. He selected participants from three different groups. These groups were determined to be "students with learning disabilities", "average achievers", and "gifted students" according to their problem solving abilities and an intelligence test scores. Gifted students showed highly successful performance on the MPI and also visual-spatial ability tests while learning disabled students showed poor performance on both of them. Besides these group differences he also found that high level visual-spatial abilities were associated with high level of mathematical problem solving performance for the entire group. Sample selection could be an effective factor for the significant relationships between visual-spatial abilities and problem solving performance for van Garderen's study. Van Garderen (2006) selected the sample with instruments measuring calculation ability, math fluency, and an IQ. His sample may not include the participants who have high levels of abilities and poorly perform in mathematics or who are successful in problem solving and have lower levels of abilities. This might influence the association between variables and lead to a positive correlation.

In the current study, the limited varience of preservice teachers' mathematical word problem solving performances might have prevented the detection of the relationship between visual-spatial abilities and problem solving performance. Besides word problems could be an area that does not involve visual-spatial abilities as strongly as other mathematical areas such as geometry. Krutetskii (1976) pointed out, that visual-spatial abilities alone could not determine students' mathematical performance. Rather than visual-spatial abilities there could be other variables that may affect problem solving performances. The findings of the study may lead us to ask different questions as to what these variables could be. Further studies should be conducted to investigate the other factors that may have an influence on problem solving.

5.2.4.3 The association between visual-spatial abilities and use of schematic representations

The results showed that there was a weak correlation between visual-spatial abilities and use of schematic representations. As Krutetskii (1976) and Presmeg (1985) suggested although a student who had high level of visual-spatial abilities could solve the problems with using representations, he or she might prefer analytic solutions without using representations. Preservice teachers' learning experiences and aims for teaching might be more effective on their preferences. In order to solve problems as quickly as possible they could abstain themselves from time-consuming actions like the drawing. Furthermore many studies also revealed visual-spatial abilities did not have an influence on people' preferences for problem solving methods (Haciomeroglu, Chicken, & Dixon, 2013; Hagarty & Kozhevnikov, 1999; Kozhevnikov, Hagarty, & Mayer, 2002; Moses, 1977; Lean & Clements, 1981; Suwarsono, 1982).

By taking into account all of these findings, a weak positive relationship between use of schematic representations and visual-spatial abilities that was found in this study corroborates what has been previously claimed in the literature. Although preservice teachers who had high levels of visual-spatial abilities could use schematic representations in problem solving process they might have preferred to solve problems by algebraic operations and equations without using representations. It could also be that they prefer to operate the representations in their mind during the solution processes. These factors might have affected the correlation between use of representations and visual-spatial abilities.

The representations that preservice teachers use in the solutions of the word problems may rarely involve a cognitive action that requires them to use their visual-spatial abilities as actively as other areas in mathematics. Rather than visual-spatial abilities there could be other factors that may lead people to prefer a visual solution method. While examining the representations in preservice teachers' solutions it was noticed that these representations might be used more frequently in certain stages of problem solving such as defining and understanding the problem or developing alternatives for solutions. How the subjects comprehend the word problems may be an issue. According to the difficulties experienced in comprehending the problems preservice teachers' preferneces for visual solutions or nonvisual solutions might vary. This should be examined with a further study investigating how representations have a role in the different stages of problem solving.

5.3 Implications of the study

Many studies have investigated the role of visualization in a problem solving context and they emphasize the importance of visualization (Campbell et al., 1995; Deliyianni et al., 2009; Haciomeroglu et al., 2013; Hagarty & Kozhevnikov, 1999; Kozhevnikov, Hagarty, & Mayer, 2002; Krutetskii, 1976; Moses, 1977; Lean & Clements, 198; Presmeg, 1985, 1986a, 1986b; Presmeg & Balderas-Cañas, 2001; Suwarsono, 1982; van Garderen 2006; van Garderen & Montague, 2003). These studies have been interested in different sub fields of visualization: preferences for visual and nonvisual problem solving methods use of representations and visual-spatial abilities. Although there have been some controversial findings, some common issues are pointed out by most of them.

The influence of types of mathematical thinking on problem solving performance has been a debated issue. While some studies have revealed the analytic thinkers performed better than other types of thinkers (Lean & Clements, 1981), in most of the studies no significant relationship has been expressed (Kolloffel, 2012; Pitta-Pantazi & Christou, 2009; Suwarsano, 1982). On the other hand there were researchers who claimed that visual solutions could be more effective for a correct solution (Moses, 1977, 1980; Webb; 1979). Among these controversial results, the current study suggested that adopted type of mathematical thinking is only related to the use of schematic representations, which showed a positive association with problem solving performance. Types of mathematical thinking may not be a predictor for the problem solving performance.

Previous studies showed that the use of representations have a strong association with problem solving performance (Kozhevnikov et al., 2002; van Garderen, 2006; van Garderen & Montegue, 2003). Moreover this association has been reported to vary according to types of representations (van Garderen, 2006; van Garderen & Montegue, 2003). The current study also showed that types of representations had a different impact on problem solving performance. Pictorial representations may hinder students' achievement but schematic representations are positively associated with correct solutions (van Garderen 2006; van Garderen & Montague, 2003). Considering the importance of schematic representations that many researchers emphasized, preservice teachers should have an opportunity to learn how a schematic representation can be created and be used efficiently. Therefore teacher education programs should introduce schematic representations and how they help in organizing and communicating mathematical ideas, and modeling and interpreting a mathematical phenomenon (NCTM, 2000).

In this current study, no significant relationships between visual-spatial abilities and other variables except use of schematic representations were found. While previous studies have suggested a positive correlation between visual-spatial abilities and problem solving performance (Battista, 1990; Clements & Battista, 1992, van Garderen, 2006; van Garderen & Montegue, 2003) the current findings did not support a significant relationship between them. Further studies might investigate other variables requiring use of representations.

Many studies have found that visual-spatial abilities had a weak or no influence on preferences for visual or nonvisual methods (Haciomeroglu et al., 2013; Hagarty & Kozhevnikov, 1999; Kozhevnikov, Hagarty, & Mayer, 2002; Moses, 1977; Lean & Clements, 1981; Suwarsono, 1982). Visual-spatial abilities are not alone a predictive factor for preferences of visual approaches. In this sense, a person's learning experiences, courses taken, and teachers' influence can be more efficient factors.

Teachers who adopted visual approaches use visuality more effectively in their teaching (Presmeg, 1986b). Moreover they can relate easily real world situations with mathematical phenomena (Presmeg, 1986b). Since new educational approaches and learning theories such as constructivism have been incorporated into curriculums and educational programs the role of visual approaches have increased. The relationship between teachers and preservice teachers' own learning experiences and practices and their teaching approaches can be also important. In a further research this relationship should be also investigated. In summary, teachers play an important role in the learning process and teacher education programs should include visual approaches with the consideration of their efficacies.

5.4 Limitations and suggestions for further research

In this section, the limitations of the study are presented and some possible suggestions are revealed. The first limitation concerns generalizability. Since in order to determine the participants of the study convenient sampling was, the results may not be generalized to all preservice teachers. In particular, the study was limited to one private and four public universities in Istanbul and Ankara. For generalizing the results for all preservice teachers, a further study can be conducted with participants from a larger sample in other universities.

Second concern was the amount of time spent in data collection. Implementation of the instruments took approximately one hour and 45 minutes. The procedure might be tiring for some of the participants. For further studies the implementations of two instruments can be conducted at different times.

The other limitation concerned participants' problem solving process. Preservice teachers' all solution process that they had in mind might not have been represented on paper. A small part of the responses did not include any expression or they included only a number as an answer. In other words participants might not reflect their actual cognitive actions in terms of use of visual-spatial representations. Further studies can be conducted with interviews, which help researchers to ask questions to explore problem solutions.

APPENDIX A

EXPLICIT DEFINITIONS OF VISUALIZATION

Table 9. Explicit Definitions of “Visualization” in Chronological Order Provided in Research Literature

Year	Author(s)	Explicit definition
1974	Paivio	“. . .the conception of imagery as a dynamic symbolic system capable of organizing and transforming the perceptual information that we receive” (p. 6)
1982	Hortin	“visual literacy is the ability to understand and use images and to think and learn in terms of images, i.e., to think visually” (p. 262)
1983	Nelson	“Visualization is an effective technique for determining just what a problem is asking you to find. If you can picture in your mind’s eye what facts are present and which are missing, it is easier to decide what steps to take to find the missing facts” (p. 54)
1985	Sharma	“Visualization (mental imagery) serves as a kind of ‘mental blackboard’ on which ideas can be developed and their implications explored” (p. 1)
1986	Presmeg	“. . .a visual image was defined as a mental scheme depicting visual or spatial information” (p. 297)
1989	Ben-Chaim, Lappan, & Houang	“Visualization is a central component of many processes for making transitions from the concrete to the abstract modes of thinking. It is a tool to represent mathematical ideas and information, and it is used extensively in the middle grades” (p. 50)
1989	Bishop	“Visual processing ability was defined as follows: ‘This ability involves visualization and the translation of abstract relationships and non-figural information into visual terms. It also includes the manipulation and transformation of visual representations and visual imagery. It is an ability of process and does not relate to the form of the stimulus material presented’ (Bishop, 1983)” (p. 11)
1989	DeFanti, Brown, & McCormick	“Visualization is a form of communication that transcends application and technological boundaries” (p. 12)
1991	Arnheim	“Visualization refers to the cognitive functions in visual perception. In visualization, pictures combine aspects of naturalistic representation with more formal shapes to enhance cognitive understanding” (p. 2)
1994	Lanzing & Stanchev	“Presenting information in visual, non-textual form is what is meant when we speak of visualization. The non-textual symbols, pictures, graphs, images and so on conveying the information will be called visuals” (p. 69)

Year	Author(s)	Explicit definition
1995	Rieber	"Visualization is defined as representations of information consisting of spatial, nonarbitrary (i.e. 'picture-like' qualities resembling actual objects or events), and continuous (i.e. an 'all-in-oneness' quality) characteristics (see Paivio, 1990). Visualization includes both internal (for example, mental imagery) and external representations (for example, real objects, printed pictures and graphs, video, film, animation)" (p. 45)
1996	Zazkis, Dubinsky, & Dautermann	"Visualization is an act in which an individual establishes a strong connection between an internal construct and something to which access is gained through the senses. Such a connection can be made in either of two directions. An act of visualization may consists of any mental construction of objects or processes that an individual associates with objects or events perceived by her or him as external. Alternatively, an act of visualization may consist of the construction, on some external medium such as paper, chalkboard or computer screen, of objects or events that the individual identifies with object(s) or process(es) in her or his mind" (p. 441)
1999	Antonietti	"Imagery is a kind of mental representation which can represent objects, persons, scenes, situations, words, discourses, concepts, argumentations, and so on in a visuospatial format. Mental images can refer to entities that a person: (a) is perceiving at present, (b) has perceived previously, or (c) has never perceived. Mental images can represent either concrete or abstract, either real or imaginary entities and may be either like photographs or motion-pictures or like diagrams, schemas, sketches, symbols. Finally, mental images either may be static or may represent movements and transformations" (p. 413)
1999	Habre	"Visualization is the process of using geometry to illustrate mathematical concepts" (p. 3)
1999	Mathewson	"Visualization retains its usual meanings in cognitive science, but also has been arrogated by science and technology to mean computer-generated displays of data or numerical models" (p. 3 footnote)
1999	Liu, Salvendy, & Kuczek	"Visualization is the graphical representation of underlying data. It is also the process of transforming information into a perceptual form so that the resulting display make[s] visible the underlying relation in the data. The definition by McCormick, DeFanti, and Brown (1987) of visualization is 'the study of mechanisms in computers and humans which allow them in concert to perceive, use and communicate visual information (p. 63)'" (pp. 289–290)
2001	Presmeg & Balderas-Canas	"The use of visual imagery with or without drawing diagrams is called visualization" (p. 2)
2001	Strong & Smith	". . .spatial visualization is the ability to manipulate an object in an imaginary 3-D space and create a representation of the object from a new viewpoint" (p. 2)
2002	Schnotz	"Visual displays are considered tools for communication, thinking, and learning that require specific individual prerequisites (especially prior knowledge and cognitive skills) in order to be used effectively" (p. 102). "Representations are objects or events that stand for something else (Peterson, 1996)." (p. 102)

Year	Author(s)	Explicit definition
2002	Schnotz	"Texts and visual displays are external representations. These external representations are understood when a reader or observer constructs internal mental representations of the content described in the text or shown in the picture" (p. 102)
2002	Stokes	". . .visual literacy defined as the ability to interpret images as well as to generate images for communicating ideas and concepts" (p. 1)
2003	Linn	"Visualization for the purposes of this paper refers to any representation of a scientific phenomena in two dimensions, three dimensions, or with an animation ". "Visualizations. . .test ideas and reveal underspecified aspects of the scientific phenomena. . .display new insights and help investigators compare one conjecture with another. . .illustrate an idea that words cannot describe" (p. 743)
2004	Zaraycki	". . . visualization is the process of using geometrical illustrations of mathematical concepts. Visualization is one of the most common techniques used in teaching mathematics" (p. 108)
2005	Piburn et al.	"visualization. . .('the ability to manipulate or transform the image of spatial patterns into other arrangements')" (p. 514)
2007	Garmendia, Guisasola, & Sierra	"Part visualization is understood to be the skill to study the views of an object and to form a mental image of it, meaning, to visualize its three-dimensional shape (Giesecke et al., 2001). . ..visualization is mental comprehension of visual information" (p. 315)
2008	Gilbert, Reiner, & Nakhleh	"Visualization is concerned with External Representation, the systematic and focused public display of information in the form of pictures, diagrams, tables, and the like (Tufte, 1983). It is also concerned with Internal Representation, the mental production, storage and use of an image that often (but not always. . .) is the result of external representation" (p. 4). "A visualization can be thought of as the mental outcome of a visual display that depicts an object or event" (p. 30)
2009	Deliyianni, Monoyiou, Elia, Georgiou, & Zannettou	"Particularly, in the context of mathematical problem solving, visualization refers to the understanding of the problem with the construction and/or the use of a diagram or a picture to help obtain a solution (Bishop, 1989)" (p. 97)
2009	Korakakis, Pavlatou, Palyvos, & Spyrellis	"'Spatial visualization', the ability to understand accurately three-dimensional (3D) objects from their two-dimensional (2D) representation" (p. 391)
2009	Mathai & Ramadas	"Visualisation is defined in terms of understanding transformations on structure and relating these with function" (p. 439)

Not: From "*Visualization in Mathematics, Reading and Science Education*" by Phillips et al., 2010, p. 23

APPENDIX B

BOXPLOT RESULTS

Boxplot, Normal Q-Q Plot, and DE trended Q-Q Plot for mathematical word problem solving performance and pictorial representation from the MPI are presented below to show the distribution of the data and to detect outliers:

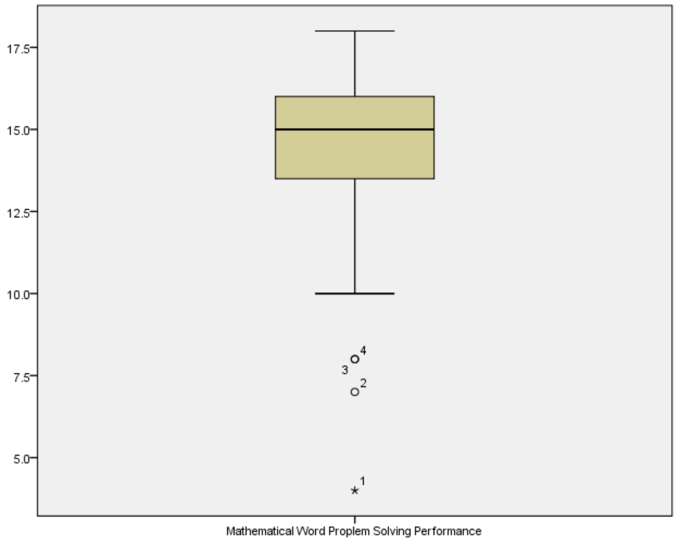


Fig. 9 Boxplots results for mathematical word problem solving performance

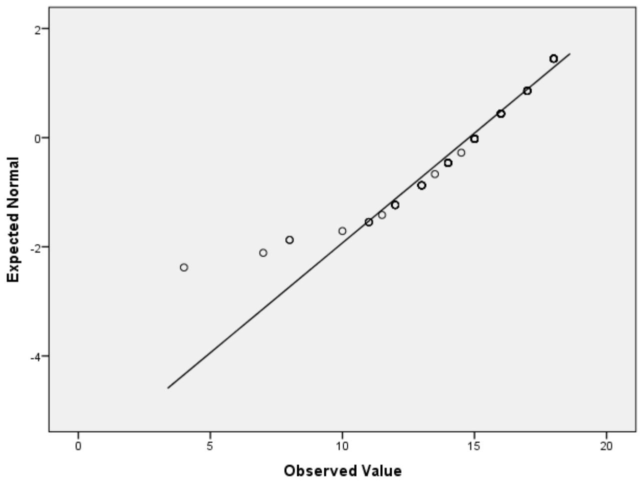


Fig. 10 Normal Q-Q Plot results for mathematical word problem solving performance

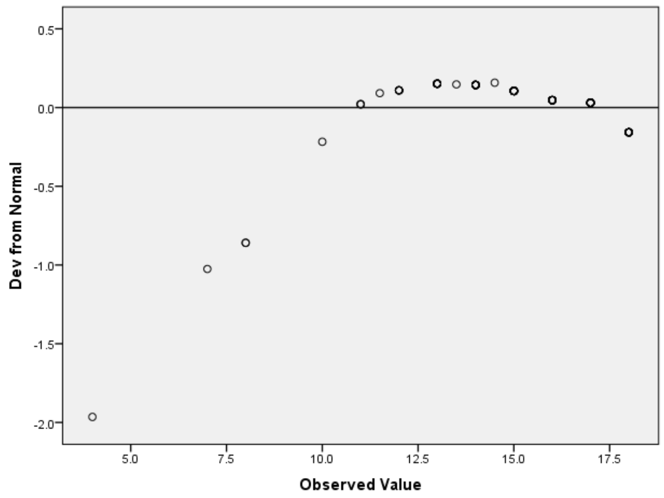


Fig. 11 DE trended Normal Q-Q Plot results for mathematical word problem solving performance

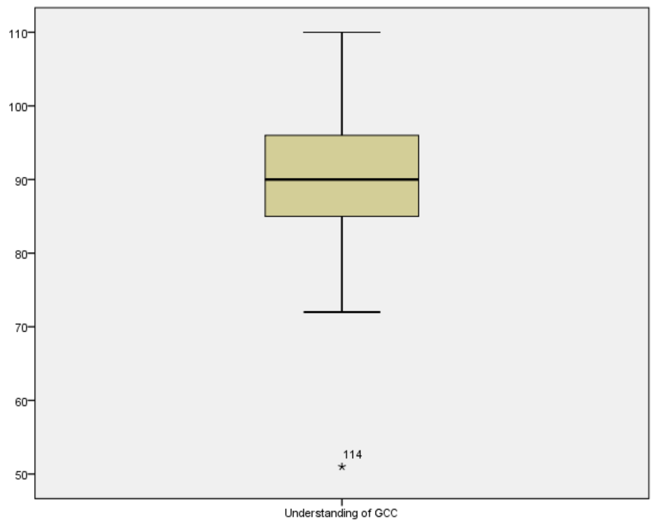


Fig. 12 Boxplots results for pictorial representation scores

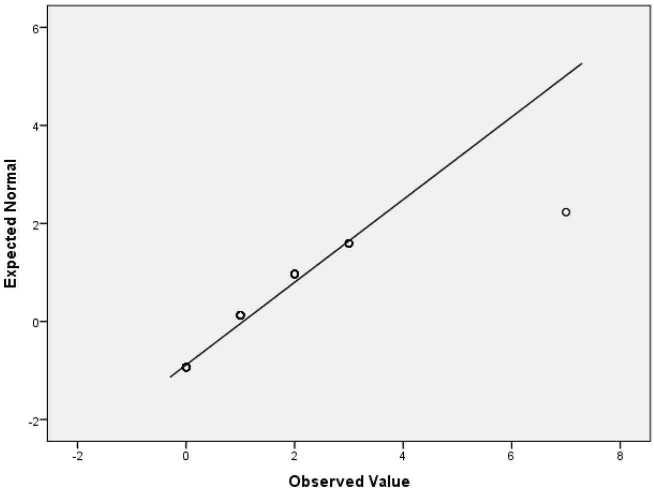


Fig. 13 Normal Q-Q Plot results for pictorial representation scores

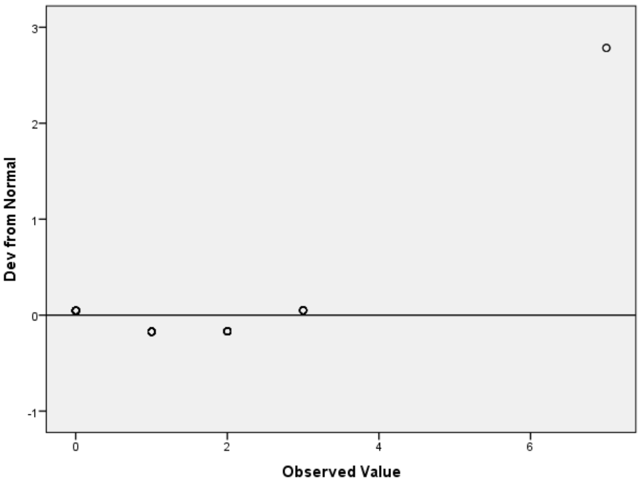


Fig. 14 DE trended Normal Q-Q Plot results for pictorial representation scores

APPENDIX C

ETHICS COMMITTEE RESULTS

T.C.
BOĞAZİÇİ ÜNİVERSİTESİ
İnsan Araştırmaları Kurumsal Değerlendirme Alt Kurulu

28 Nisan 2016

Sayı: 2016/3

Beyza Olgun

İlköğretim Bölümü

Eğitim Fakültesi

Sayın Araştırmacı,

"Sözel Matematik Problemlerinin Çözümünde Öğretmen Adaylarının Temsil Kullanımı, Matematiksel Düşünme Yapıları ve Görsel-Uzamsal Yeteneklerinin İncelenmesi" başlıklı projeniz ile ilgili olarak yaptığınız SBB-EAK 2016/14 sayılı başvurunuz İnsan Araştırmaları Kurumsal Değerlendirme Alt Kurulu tarafından 28 Nisan 2016 tarihli toplantıda incelenmiş ve uygun bulunmuştur.

Saygılarımızla,

İnsan Araştırmaları Kurumsal Değerlendirme Alt Kurulu

Doç. Dr. Ebru Kaya

Doç. Dr. Mehmet Yiğit Gürdal

Yrd. Doç. Dr. Gül Sosay

Yrd. Doç. Dr. Mehmet Nafi Artemel

Dr. Nur Yeniçeri

Fig. 15 The ethics committee results

APPENDIX D

CONSENT FORM

KATILIMCI BİLGİ VE ONAM FORMU

Araştırmayı destekleyen kurum: Boğaziçi Üniversitesi

Araştırmanın adı: Matematik Öğretmen Adaylarının Görsel Temsilleri Kullanımı, Matematiksel Düşünme Yapıları Ve Uzamsal Yeteneklerinin Sözel Matematik Problemlerinin Çözümü Sürecinde İncelenmesi

Proje Yürütücüsü: Yrd. Doç. Dr. Engin Ader

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Telefon: 0212 359 67 96

Sayın öğretmen adayı,

Boğaziçi Üniversitesi Eğitim Fakültesi İlköğretim Bölümü yüksek lisans öğrencisi Arş. Gör. Beyza Olgun, "Matematik öğretmen adaylarının görsel temsilleri kullanımı, matematiksel düşünme yapıları ve uzamsal yeteneklerinin sözel matematik problemlerinin çözümü sürecinde incelenmesi" adlı bilimsel bir araştırma yürütmektedir. Siz matematik öğretmen adaylarını bu araştırmaya katılmaya davet ediyoruz. Kabul ettiğiniz takdirde sizlerden:

- ➢ Biri yaklaşık 30 dakika diğeri ise yaklaşık 60 dakika sürecek olan iki görüşmeye katılmanız,
- ➢ Bu görüşmeler sırasında araştırmacının size yönelttiği iki farklı ölçeği açık cevaplar vererek doldurmanız beklenmektedir.

Onay Bildirimi:

- Bu araştırmada toplanan veriler gizli tutulacaktır.
- Araştırmanın sonuçları akademik amaçlar için kullanılacaktır ve verdiğim cevapların notlarım üzerinde herhangi bir etkisi olmayacaktır.
- Toplanan bilgiler şahsi bilgilerim paylaşılmadan, araştırma sonuçlarını yorumlamada ve bu araştırma kapsamında düzenlenecek olan çalışmalarda kullanılacaktır.
- Araştırmanın amaçlarını ve prosedürleri daha iyi anlamak için sorular sorabilirim.
- Araştırmadan istediğim zaman ayrılabilirim.

- Araştırmaya katılmak istemezsem veya araştırmadan ayrılırsam bana ait bilgiler imha edilecektir.
- Araştırmaya katıldığım için bir ücret ödenmeyecektir.

Araştırmanın amacı konusunda bilgilendirildim ve gönüllü olarak katılmayı kabul ediyorum.

Katılımcının Adı-Soyadı: ____

İmza: ____

Tarih: ____

APPENDIX E

MATHEMATICAL PROCESSING INSTRUMENT – FIRST SECTION

Matematiksel Süreç Aracı- 1. Bölüm

Adı — Soyadı:

<u>BÖLÜM-B</u>

B-l: Bir atletizm yarış parkuru eşit olmayan üç bölüme ayrılıyor. Parkurun tüm uzunluğu 450m. Birinci ve ikinci bölümlerin uzunlukları toplamı 350m, ikinci ve üçüncü bölümlerin uzunlukları toplamı 250m'dir. Buna göre her bir bölüm ne kadar uzunluktadır?

B-2: Bir balon bulunduğu yerden 200m yüksekliğe çıkıyor ve 100m doğuya hareket ettikten sonra 100m alçalıyor. Daha sonra 50m daha doğuya hareket ediyor ve son olarak dümdüz yere iniyor. Bu balon başlangıç noktasına ne kadar uzaklıktadır?

B-3: Bir anne kızının yaşının yedi katı yaşındadır. Anne ile kızının yaşları farkı 24 olduğuna göre, annenin ve kızının yaşı nedir?

B-4: Bir atletizm yarışında Enes, Mustafa'nın 10 m önündedir. Yusuf, Burak'ın 4 m önünde ve Burak, Mustafa'nın 3 m önündedir. Buna göre Enes, Yusuf'un kaç metre önündedir?

B-5: En başta 1kg şekerin fiyatı 1kg tuzun fiyatının 3 katıdır. Daha sonra, tuzun 1 kilogramının fiyatı önceki fiyatının yarısı kadar arttırılırken şekerin fiyatı değiştirilmiyor. Tuzun kilogramının şuan ki fiyatı 30 Krş olduğuna göre şekerin kilogramı ne kadardır?

B-6: İki ağaçta aynı sayıda serçe bulunmaktadır. Birinci ağaçtan kalkan 2 serçe ikinci ağaca konmuştur. Buna göre, ikinci ağaçtaki serçe sayısı birinci ağaçtakinden kaç fazladır?

B-7: Bir kerestecide, her biri 16m uzunlukta olan kütükler 2m uzunluğunda eşit boylarda testereler yardımıyla kesilmektedir. Eğer her bir kesme işlemi 2 dakika sürüyorsa uzun kütükleri 8 eşit parçaya ayırmak ne kadar sürer?

B-8: Tamamı gazyağı ile dolu olan bir cam şişe, toplam 8kg ağırlığındadır. Gazyağının yarısı döküldükten sonra, cam şişenin ağırlığı içindekiyle birlikte 4,5 kilogramdır. Buna göre, cam şişenin ağırlığı nedir?

B-9: Yolculuğunun yarısını tamamladıktan sonra uykuya dalan bir yolcu, uyandığında uyurken ki aldığı yolun yarısı kadar daha yol gitmesi gerektiğini görüyor. Buna göre, yolculuğunun ne kadarlık kısmını uyuyarak geçirmiştir?

B-10: Terazinin bir kefesine bir tam dilim peynir, diğer kefesine de 3 tane çeyrek dilim peynir ve $\frac{3}{4}$ kg ağırlık konursa terazinin kefeleri dengede kalmaktadır. Buna göre, bir tam dilim peynirin ağırlığı nedir?

B-11: Biri diğerinin iki katı kadar süt bulunduran süt tanklarının ikisinden de 20 litre süt dökülüyor. Son durumda, tanklarda kalan süt miktarı biri diğerinin 3 katı olacak şekildedir. Buna göre ilk başta, tanklardaki süt miktarı ne kadardı?

B-12: 10 tane eriğin ağırlığı, 3 kayısı ve 1 mangonun ağırlığı kadardır, 6 erik ve 1 kayısı, 1 mangonun ağırlığına eşittir. Buna göre, kaç tane erik 1 mangoyu terazide dengede tutar?

BÖLÜM-C

C-l: Bir turistin trenle aldığı mesafe, vapurla aldığı mesafeden 150 km, yürüyerek aldığı mesafeden ise 750 km daha uzundur. Yürüyerek aldığı mesafe, vapurla aldığı mesafenin $\frac{1}{3}$'i olduğu biliniyorsa, seyahatin toplam uzunluğunu hesaplayınız.

C-2: Öğleden (12:00) beri geçen süre, gece yarısına (00:00) kalan sürenin 3'te 1'ini oluşturuyorsa, şimdi saat kaçtır?

C-3: Bir çocuk, evinden okula 30 dk'da yürüyorken, kardeşi 40 dk'da yürüyor. Kardeşi, abisinin çıktığı saatten 5 dakika erken çıkarsa; çocuk kardeşini kaç dakika sonra yakalar?

C-4: Ağabey, kardeşine "Bana 8 tane ceviz ver ki, senin cevizlerinin 2 katına sahip olayım" diyor. Fakat kardeşi ona "Sen bana 8 ceviz verirsen, eşit sayıda cevizimiz olacak" diyor. O hâlde her birinin kaç tane cevizi vardır?

C-5: Farklı uzunluk ve kalınlığa sahip iki mumdan, uzun olan mum $3\frac{1}{2}$ saat yanarken, kısa olan 5 saat yanabiliyor. 2 saat yandıktan sonra, mumlar eşit uzunluğa eriştiklerine göre; kısa mumun, uzun muma göre ilk baştaki uzunluğunun oranı nedir?

C-6: Bir tren, bir telgraf direğini $\frac{1}{4}$ dakikada geçiyor ve 540 m uzunluğundaki tünelden tam olarak $\frac{3}{4}$ dakikada geçiyor. Trenin dakikadaki hızı ve trenin uzunluğu kaç metredir?

APPENDIX F

MATHEMATICAL PROCESSING INSTRUMENT – SECOND SECTION

Matematiksel Süreç Aracı - 2. Bölüm

Cevap Kâğıdı

Adı – Soyadı:

	ÇÖZÜM						
	1	2	3	4	5	6	Hiçbiri
B-1							
B-2							
B-3							
B-4							
B-5							
B-6							
B-7							
B-8							
B-9							
B-10							
B-11							
B-12							

	ÇÖZÜM						
	1	2	3	4	5	6	Hiçbiri
C-1							
C-2							
C-3							
C-4							
C-5							
C-6							

Matematiksel Süreç Aracı - 2. Bölüm

• Bu ankette sizden matematiksel süreç aracı 1. Bölüm'de yer alan problemlere nasıl yanıt verdiğinizi düşünmeniz istenmektedir. Her problemin üç veya daha fazla çözümü vardır.

• Problemi ilk çözüşünüzde kullandığınız yolla aynı veya çok benzer olanı aşağıda verilen çözüm yöntemleri arasından seçerek cevap kâğıdına işaretleyiniz. Problemi tamamlayıp tamamlamamış olmanız veya yanıtınızın doğru olup olmaması önemli değildir.

• Çözüm yolunuz verilen seçeneklerden ikisine benziyorsa bu iki çözüm yollarını da işaretleyebilirsiniz.

• Problemlerden herhangi biri için verilen çözüm yollarından hiçbiri sizin çözüm yolunuzla aynı veya çok benzer değilse "Hiçbiri" şıkkını işaretleyiniz.

ÇÖZÜMLER

BÖLÜM-B

B-l

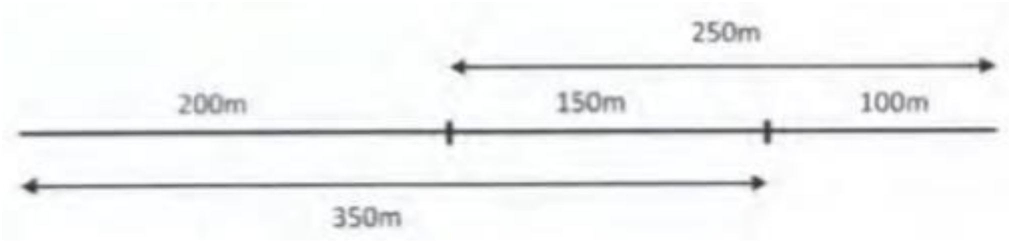
B-l. **<u>Çözüm 1:</u>** Bu problemi yarış pistini hayal ederek çözdüm ve her bir bölümün uzunluğunu hesapladım.

Üçüncü bölümün uzunluğu = 450- 350 = 100 m.

Birinci bölümün uzunluğu = 450- 250 = 200 m.

Ve böylece ikinci bölümün uzunluğu = 150 m.

B-1. **<u>Çözüm 2:</u>** Yarış pistini temsilen bir diyagram çizdim ve her bir bölümün uzunluğunu böyle hesapladım.



İlk bölümün uzunluğu 200 m, ikinci bölümün uzunluğu 150 m ve üçüncü bölümün uzunluğu 100 m'dir.

B-1. **Çözüm 3:** Bu problemi çözmek için, verilenlerden yola çıkarak (cebirsel veya cebirsel olmayan) bir sonuca ulaştım ve herhangi bir resim hayal edip çizmedim.

Parkurun tüm uzunluğu 450m. $x + y + z = 450$

Birinci ve ikinci bölümlerin uzunlukları toplamı 350 m'dir. $x + y = 350$

Sonuç: Üçüncü bölümün uzunluğu = 450 - 350 = 100 m.

$z = 100$

İkinci ve üçüncü bölümlerin uzunlukları toplamı 250 m'dir. $y + z = 250$

Sonuç: Birinci bölümün uzunluğu = 450- 250 = 200 m.

$x = 200$

Böylece ikici bölümün uzunluğu = 450- 200- 100 = 150m olur.

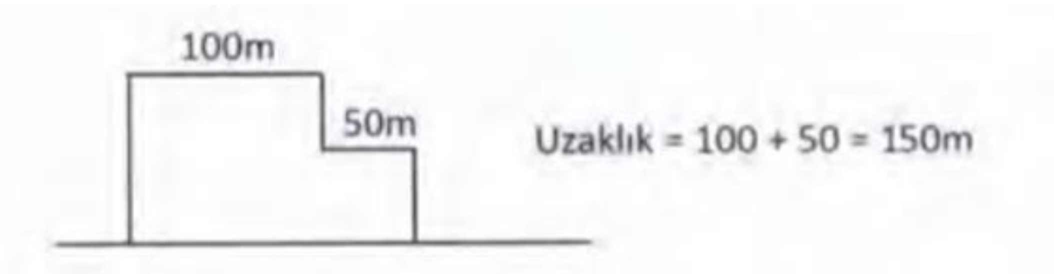
$y = 150$

B-2

B-2. **Çözüm 1:** Balon tarafından alınan yolu hayal ederek başlangıç ve bitiş noktaları arasındaki mesafeyi hesapladım.

Mesafenin 100 + 50 = 150 m olacağını buldum.

B-2. **Çözüm 2:** Balon tarafından alınan yolu temsilen bir diyagram çizdim ve başlangıç ve bitiş noktaları arasındaki mesafeyi buldum.



B-2. <u>Çözüm 3:</u> Bu soruyu çözmek için, çözüm için önemli olan bilgilere dikkat ettim (balonun aldığı yolu hayal etmeden). Böylelikle başlangıç ve varış noktaları arasındaki mesafe 100+50= 150 m'dir.

B-3

B-3. <u>Çözüm 1:</u> Bu soruyu deneme yanılma yoluyla çözdüm.

Kızın Yaşı	Annenin yaşı	
2	26	Hayır
3	27	Hayır
4	28	Evet

Böylece, anne 28, kızı 4 yaşındadır.

B-3. <u>Çözüm 2:</u> Bu soruyu, sembol ve eşitlik kullanarak çözdüm.

Mesela kızın yaşı x otsun. Buradan anne $7x$ yaşındadır. Yaşlarının farkı $6x$ yıldır.

Bundan dolayı $6x = 24$ ve $x = 4$ olur.

Böylece kız 4 yaşındadır ve anne 28 yaşındadır.

B-3. <u>Çözüm 3:</u> Bu soruyu, yaşları temsil eden diyagram çizerek çözdüm.

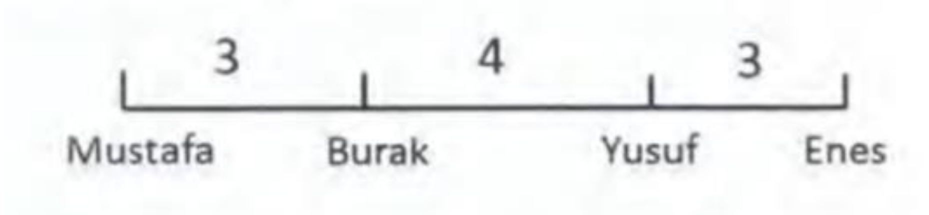
Diyagramdan, yaşları arasındaki fark 6 parçadır. Bu fark 24 yıla eşittir. Bundan dolayı her bir parça 4 yılı temsil etmektedir, böylece kız 4 yaşında ve anne 28 yaşındadır.

B-3. **<u>Çözüm 4:</u>** Çözüm 3'teki gibi bir diyagram hayal ettim ve 6 parçanın 24 yılı temsil ettiği sonucuna ulaştım, dolayısıyla bir parça 4 yılı temsil eder. Böylece, kızın yaşı 4, annenin yaşı 28'dir.

B-4

B-4. **<u>Çözüm 1:</u>** Dört kişi hayal ederek, Enes ve Yusuf un arasındaki mesafeyi hesapladım. Enes, Yusuf'un 3m önündedir.

B-4. **<u>Çözüm 2:</u>** Dört kişiyi temsil eden bir diyagram çizerek. Enes ve Yusuf arasındaki mesafeyi hesapladım.



Enes, Yusuf un 3 m önündedir.

B-4. **<u>Çözüm 3:</u>** Bu problemi, sadece soruda geçen cümlelerden yola çıkarak çözdüm:

Yusuf, Burak'ın 4m önünde ve Burak, Mustafa'nın 3m önündedir. Sonuç: Yusuf Mustafa'nın 7m önündedir,

Enes, Mustafa'nın 10 m önündedir.

Sonuç: Enes, Yusuf un 3m önündedir.

B-5

B-5. <u>Çözüm 1:</u> Bu problemi, şekerin ve tuzun fiyatlarını temsil eden bir diyagram çizerek çözdüm.

Diyagramdan da görülebileceği üzere, tuzun fiyatı arttırıldıktan sonra 1kg şekerin fiyatı 1kg tuzun fiyatının iki katıdır (şu an 30 Krş). Böylece 1kg şekerin fiyatı 60 Krş'tur.

B-5. <u>Çüzüm 2:</u> Birinci çözümdeki yöntemi kullanarak çözdüm, fakat diyagramı "zihnimde" canlandırdım, (kağıt üzerine çizmedim)

B-5. <u>Çüzüm 3:</u> Soruyu muhakeme ederek çözdüm. 1kg tuz şu an 30 krş. Bu, bir önceki fiyatının $1\frac{1}{2}$ katı olduğuna göre bir önceki kg fiyatı 20 Krş'tur. Böylelikle şekerin kg fiyatı 3 x 20'dir, yani 60 Krş.

B-5. <u>Çözüm 4:</u> Soruyu, sembol ve eşitlik kullanarak çözdüm. Örneğin, tuzun bir Önceki kg fiyatının *x* kuruş olduğunu farz edersek, şekerin kg fiyatı 3x kuruştur. Artıştan sonra tuzun kg fiyatı $1\frac{1}{2}$x Krş'tur. Şekerin kg fiyatı şu an ki tuz fiyatının iki katı olduğuna göre şekerin kg fiyatı 60 Krş'tur.

B-6

B-6. <u>Çözüm 1:</u> Soruyu muhakeme yoluyla çözdüm. İki serçe birinci ağaçtan uçup ikinci ağaca konduklarında, birinci ağaçtaki serçe sayısı öncekine göre 2 tane azalırken, ikinci ağaçta öncekine göre 2 tane artmıştır. Böylelikle İkinci ağaçta birinci ağaca göre 4 tane daha fazla serçe vardır.

B-6. <u>Çözüm 2:</u> Bir diyagram çizdim.

İkinci ağaçta birinciye göre 4 tane daha fazla serçe vardır.

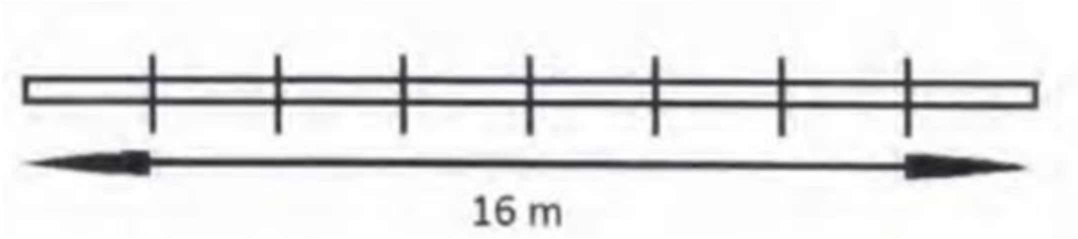
B-6. <u>Çözüm 3:</u> İkinci çözümdeki yöntemi kullandım, fakat diyagramı “zihnimde” canlandırdım, (kâğıt üstüne çizmedim)

B-6. <u>Çözüm 4:</u> Bu soruyu bir örnek kullanarak çözdüm. Örneğin; her iki ağaçta 8 tane serçe olsun. 2 tane serçe birinci ağaçtan ikinci ağaca uçtuktan sonra, birinci ağaçta 6 tane, İkinci ağaçta 10 tane serçe vardır. Buradan; ikinci ağaçta birinciye göre 4 tane daha fazla serçe vardır.

B-6. <u>Çözüm 5:</u> Bu soruyu semboller kullanarak çözdüm. En başında, her iki ağaçta bulunan serçe sayısına x diyelim. 2 tane serçe birinci ağaçtan ikinci ağaca uçtuktan sonra; birinci ağaçta $x — 2$, ikinci ağaçta $x + 2$ tane serçe bulunur. Serçe sayıları arasındaki fark $(x + 2) – (x-2) = 4$’tür.

B-7

B-7. Çözüm 1: Soruyu çözmek için, kısa parçalara kesilecek uzun kütüğü temsilen bir diyagram çizdim.



Diyagramdan, 8 tane kısa kütüğü üretmek için 7 kere kesme işlemi gerekmektedir. Buradan gereken süre 7x2 = 14 dakikadır.

B-7. Çözüm 2: Birinci çözümle aynı yöntemi kullandım, fakat diyagramı kafamda canlandırdım.

B-7. Çözüm 3: Soruyu muhakeme yoluyla çözdüm:

Eğer uzun kütükler 16 m'den uzun olsaydı, 8 tane kısa kütük elde etmek için 8 kesme işlemi gerekirdi. Fakat son kesme işlemi gereksizdir, yani 7 kesme işlemi yeterlidir. Geçen süre 7x2 = 14 dakikadır.

B-8

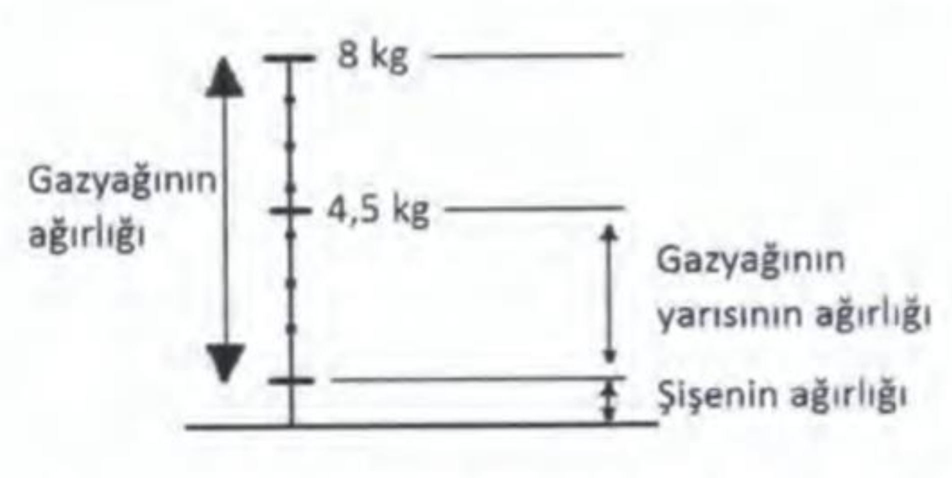
B-8. Çözüm 1: Bu soruyu sembol ve eşitlik kullanarak çözdüm. Örneğin; şişenin ağırlığının x kg olduğunu varsayalım.

Buradan gaz yağının ağırlığı $(8 — x)$ kg'dır.

Yani gaz yağının yarısının ağırlığı $\frac{1}{2}(8 — x)$ kg'dır.

Buradan $x + \frac{1}{2}(8 - x) = 4\frac{1}{2} \Rightarrow x = 1$

Böylelikle şişenin ağırlığı 1 kg'dır.



B-8. Çözüm 2: Sırasıyla ağırlıkları temsil eden bir diyagram çizdim.

Diyagramdan yarım gaz yağının ağırlığı = 8 - 4,5 = 3,5 kg

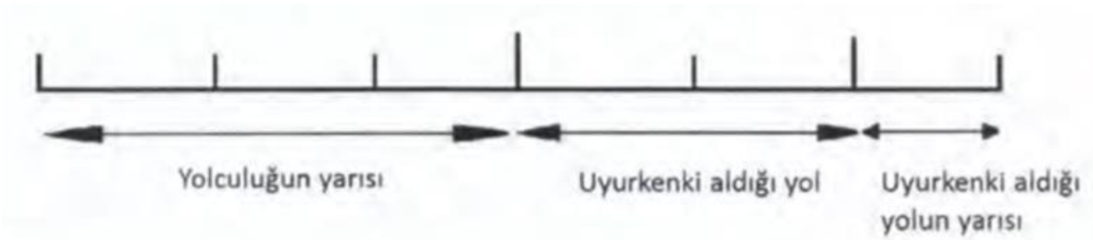
Böylece gaz yağının ağırlığı 7 kg'dır. Ve şişenin ağırlığı 1 kg'dır, (Ya da doğrudan: şişenin ağırlığı = 4,5 - 3,5 = 1 kg)

B-8. Çözüm 3: İkinci çözümdeki gibi, fakat diyagramı zihnimde "canlandırdım".

B-8. Çözüm 4: İkinci çözümdeki gibi, fakat herhangi bir diyagram veya benzetme kullanmadan.

B-9

B-9. Çözüm 1: Yolculuğun tamamını temsilen bir diyagram çizdim.



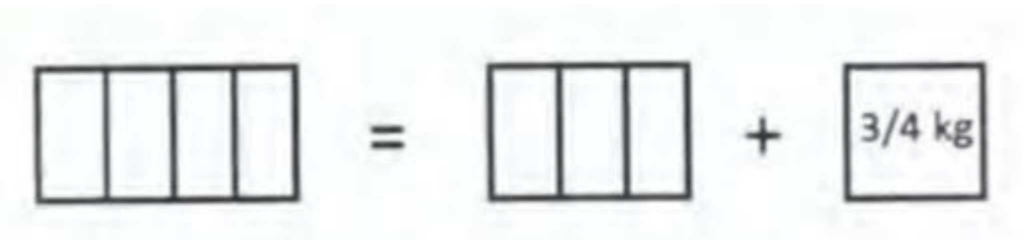
Diyagramdan: yolculuğu tamamı 6 parçadan oluşursa, iki parçalık kısmında uyumuştur, yani yolculuğun $\frac{1}{3}$'i kadarında uyumuştur.

B-9. Çözüm 2: Birinci çözümdeki gibi, fakat diyagramı zihnimde "canlandırdım".

B-9. <u>Çözüm 3:</u> Bu soruyu sembol ve eşitlik kullanarak çözdüm, örneğin; uyuyarak geçirdiği mesafeye *x* birim diyelim. Uyandığında kalan mesafe $\frac{1}{2}$x birim olacaktır. Buradan (x+$\frac{1}{2}$x) birim yolculuğun yarısını oluşturmaktadır. Yani yolculuğun tamamı 2(x+$\frac{1}{2}$x) = 3x birimdir.

Böylelikle, yolculuğun $\frac{1}{3}$'i kadarında uyumuştur.

B-10



B-10. <u>Çözüm 1:</u> Bu soruyu nesneleri temsil eden diyagram çizerek çözdüm.

Her iki kefeden de 3 çeyrek dilim peynir çıkartılırsa, bir çeyrek dilim peynir $\frac{3}{4}$ kg ile dengede kalır. Buradan bir tam peynirin ağırlığı 4 x $\frac{3}{4}$ kg, yani 3 kg'dır.

B-10. <u>Çözüm 2:</u> Birinci çözümdeki gibi, fakat diyagramı zihnimde "canlandırdım".

B-10. <u>Çözüm 3:</u> Bu soruyu, sembol ve eşitlik kullanarak çözdüm, örneğin; bir tam dilim peynirin ağılığına *x* kg diyelim.

Buradan x = $\frac{3}{4}x + \frac{3}{4}$, dolayısıyla x = 3

Böylece, bir tam dilim peynirin ağırlığı 3 kg'dır.

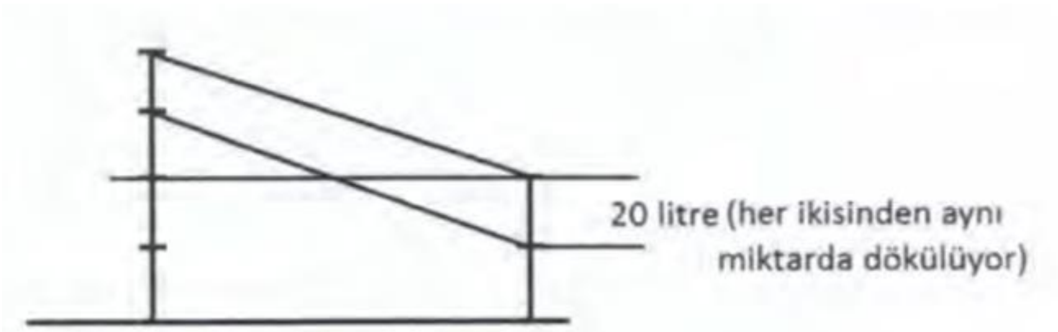
B-10. <u>Çözüm 4</u>: $\frac{1}{4}$ peynirin ağırlığı $\frac{3}{4}$ kg'dır. Buradan bir tam dilim peynir 3 kg'dır. (herhangi bir diagram veya benzetme kullanmadan)

B-11

B-ll. <u>Çözüm 1:</u> Bu soruyu sembol ve eşitlik kullanarak çözdüm, örneğin; ilk başta tanklarda bulunan süt miktarlarına *x* litre ve *2x* litre diyelim.

Daha sonra $3(x - 20) = 2x - 20$, böylece $x = 40$.

Buradan, en baştaki süt miktarları 40 litre ve 80 litre'dir.



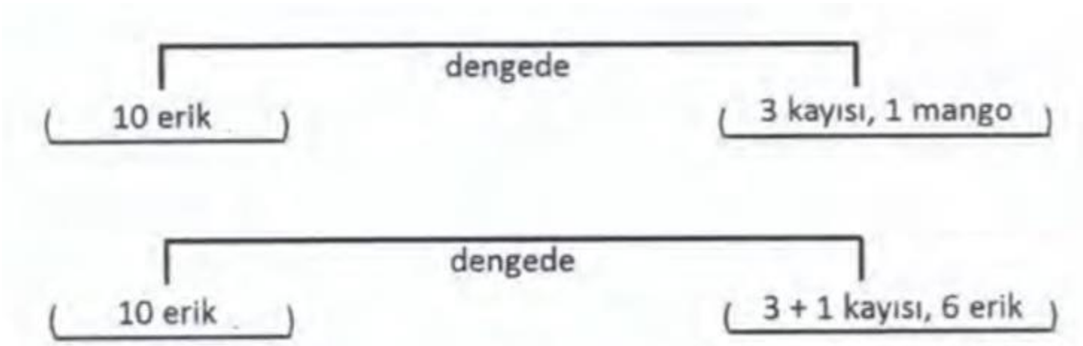
B-ll. <u>Çözüm 2:</u> Sütlerin miktarını temsilen bir diyagram çizdim.

Diyagramdan, her bir tanktan süt boşaltıldıktan sonra biri diğerinden 3 katı kadar daha fazla süt bulundurması için, ikinci tankta 20 litre süt kalması gerekmektedir.

Böylece, en başta 40 litre ve 80 litre süt bulunmaktadır.

B-ll. <u>Çözüm 3:</u> İkinci çözümdeki gibi, fakat diyagramı zihnimde "canlandırdım".

B-12



B-12. <u>Çözüm 1:</u> Sembol ve eşitlik kullandım, örneğin; bir eriğin ağırlığı *x* birim ve bir kayısının ağırlığı y birim olsun.

Buradan bir mango (6x + y) birimdir.

Böylece, 10x = 3y + (6x + y), yani x = y'dir.

Buradan, mangonun ağırlığı 6x + x, yani *7x* birimdir. Böylece, 7 erik 1 mangoyu terazide dengede tutar.

B-12. <u>Çözüm 2:</u> Bu problemi, ağırlıkları temsil eden bir diyagram çizerek çözdüm. Terazinin her kefesinden 6 erik alırsak, 4 erik ile 4 kayısı dengede kalır. Yani 1 erik 1 kayısıyla eşit ağırlıktadır. 1 mango, 6 erik ve 1 kayısı ile dengelenmektedir. Buradan 7 erik 1 mangoyu dengede tutabilir.

B-12. <u>Çözüm 3:</u> ikinci çözümdeki gibi, fakat diyagramı kafamda canlandırdım.

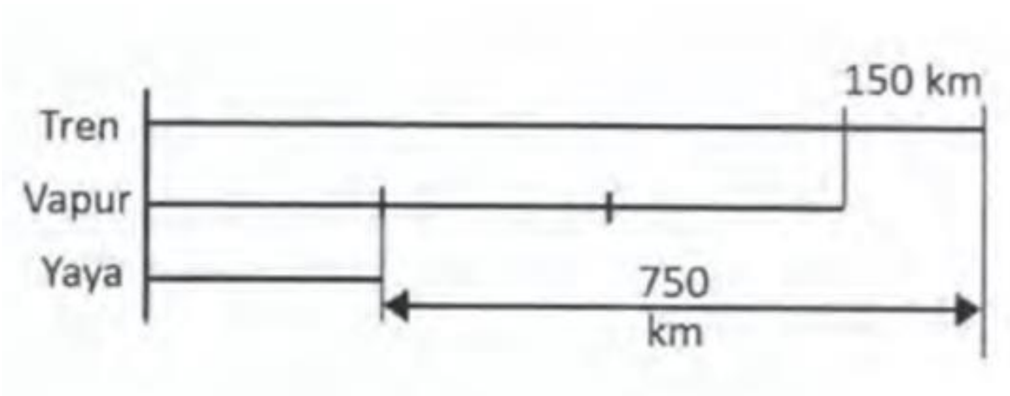
B-12. <u>Çözüm 4:</u> Bu soruyu muhakeme yoluyla çözdüm, (her hangi bir resim hayal etmeden)

1 mango, 6 erik ve 1 kayısı ile dengede kalabilmektedir, buradan 3 kayısı + 6 erik + 1 kayısı İle 10 erik dengede kalabilmektedir. Yani 4 erik, 4 kayısıyı dengelemektedir. Böylece 1 mango, 7 erik ile dengelenmektedir.

BÖLÜM – C

C-l

C-1. <u>Çözüm 1:</u> Uzunlukları temsilen bir diyagram çizdim.



Diyagramdan görüleceği üzere, vapurla gerçekleştirilen seyahatin 3’te 2’lik kısmı

=750-150 = 600 km.

Böylece, vapurla seyahatin uzunluğu 900 km, trenle 1050 km ve yürüyerek 300 km’dir, bundan dolayı bütün seyahatin uzunluğu 2250 km’dir.

C-l. Çözüm 2: Çözüm 1 ‘de olduğu gibi, fakat diyagramı hayal ettim.

C-l. Çözüm 3: Soruyu, sembol ve denklem kullanarak çözdüm.

Örneğin; Yürüyerek alınan yola *x* km diyelim.

Bundan dolayı vapurla alınan yol 3x km ve trenle alınan yol (x + 750) km’dir.

Böylece 3x + 150 = (x + 750) ve x = 300

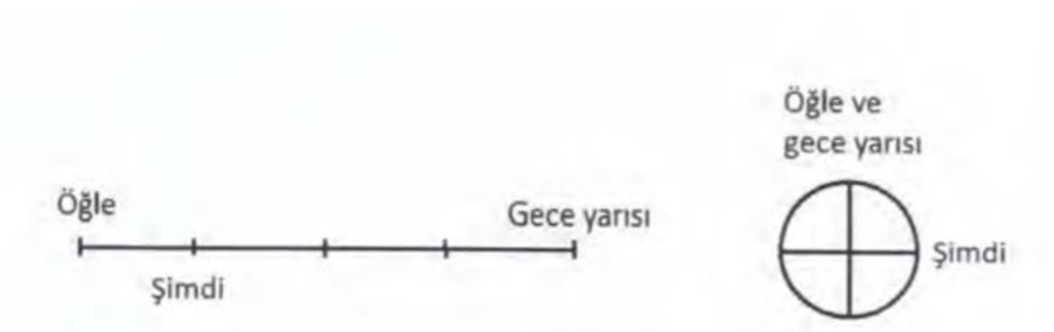
Demek ki yürüyerek alınan yol 300 km, vapurla 900 km ve trenle 1050 km; böylece bütün seyahatin uzunluğu 2250 km’dir.

C-2

C-2. Çözüm 1: Sembol ve denklem kullandım. Örneğin,

Öğleden beri geçen süreye x saat diyelim. Gece yarısına kadar kalan süre de (12 — x) saat olur. Böylece x = $\frac{1}{3}$ (12 — x) ve x = 3 Bu nedenle şu anda saat öğlen 3’tür.

C-2. Çözüm *2:* Zamanı temsilen bir diyagram çizdim, (çizgi veya saat kadranı)

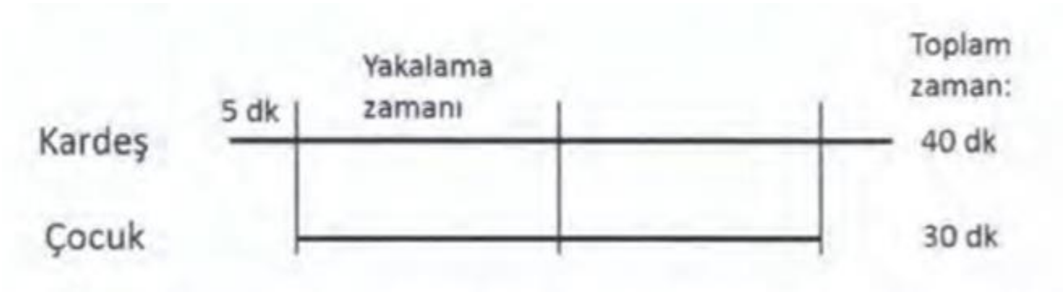


Diyagramdan, saat öğleden sonra 3’tür.

C-2. Çözüm 3: Çözüm 2’de olduğu gibi, fakat diyagramı hayal ettim.

C-2. <u>Çözüm 4:</u> Herhangi bir şekil veya diyagram kullanmadan, akıl yürüterek öğlen ve gece yarısı arasındaki sürenin $\frac{1}{4}$'inin geçtiğini anladım, böylece saat öğleden sonra 3'tür.

C-3



C-3. <u>Çözüm 1:</u> Zamanları temsilen bir diyagram çizdim.

Diyagramdan: Çocuk, kardeşinden 5 dk önce okula ulaşacaktır, böylece şeklin iki yarısı simetrik olmalı, bundan dolayı çocuk kardeşini yarı yolda yakalayacaktır, yani 15 dk sonra.

C-3. <u>Çözüm *2:*</u> Çözüm 1 'de olduğu gibi, fakat diyagramı hayal ettim.

C-3. <u>Çözüm 3:</u> Sembol ve denklem kullandım, örneğin:

Okula olan mesafenin *d* birim olduğunu ve kardeşini *x* dakikada yakaladığını varsayalım.

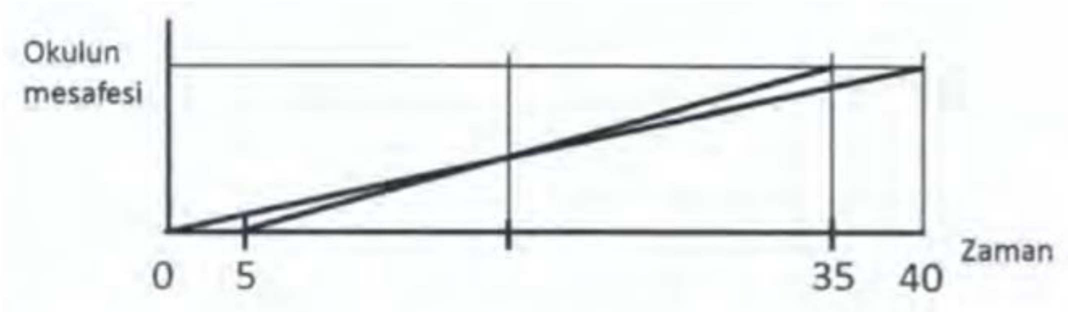
Buradan kardeşinin yürüdüğü süre $(x + 5)$ dakika olur.

Çocuğun hızı dakikada $\frac{d}{30}$ birim, kardeşinin ki ise $\frac{d}{40}$ birimdir.

Çocuk, kardeşini yakaladığı zaman; ikisi de aynı mesafeyi gitmiş olur. Böylece $\frac{d}{30}x=\frac{d}{40}(x+5)$ ve buradan $x= 15$. Çocuk kardeşini 15 dakikada yakalar.

C-3. <u>Çözüm 4:</u> Bu problemi, çocuk ve kardeşinin yarı yola ulaşma sürelerini hesaplayarak çözdüm.

Bu süre, çocuk için 15 dakika ve kardeşi için de 20 dakikadır. Fakat kardeşi yola 5 dakika erken çıkmıştır, böylece yarı yola aynı anda ulaşacaklardır. Çocuk, kardeşini 15 dakikada yakalar.



C-3. **<u>Çözüm 5:</u>** Grafik çizdim.

Simetriden grafikler orta noktada kesişir. Bu yüzden çocuk kardeşini 15 dakikada yakalar.

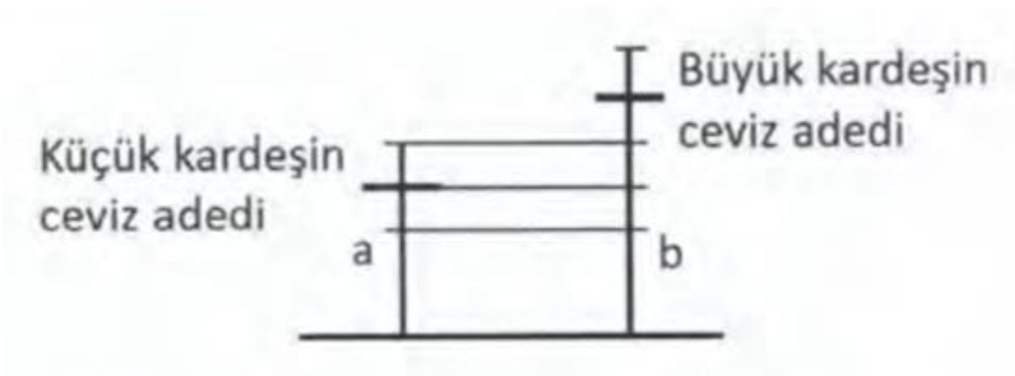
C-4

C-4. **<u>Çözüm 1:</u>** Sembol ve denklem kullandım. Örneğin; küçük kardeşte *x* ceviz olduğunu, büyüğündeyse *y* ceviz olduğunu varsayalım.

$y + 8 = 2(x — 8)$ ve $y - 8 = x + 8$

Denklemler aynı anda çözülürse: $x = 40$ ve $y = 56$. Küçük kardeşin 40, ağabeyin ise 56 cevizi vardır.

C-4. **<u>Çözüm 2:</u>** Cevizlerin adedini temsilen bir diyagram çizdim.



Sorudaki durumlardan, b çizgisinin üst yarısı, her biri 8 cevizi temsil eden 4 eşit parçaya bölünürse, bu bölmelerin 7 tanesi ağabeyin ceviz sayısını gösterirken, 5 tanesi küçük kardeşinkileri gösterir. Bu yüzden kardeş 40 adet, ağabeyi 56 adet cevize sahiptir.

C-4. **<u>Çözüm 3:</u>** Çözüm 2'de olduğu gibi, fakat diyagramı hayal ettim.

C-5

C-5. **<u>Çözüm 1:</u>** Verilen bilgi üzerinden akıl yürüttüm.

2 saatten sonra, uzun olan mumun 7'de 4'ü tükenir, bu yüzden 7'de 3'lük kısmı geriye kalır. Bu arada, kısa mumun 5'te 2'lik kısmı tükenir, geriye 5'te 3'lük kısmı kalır.

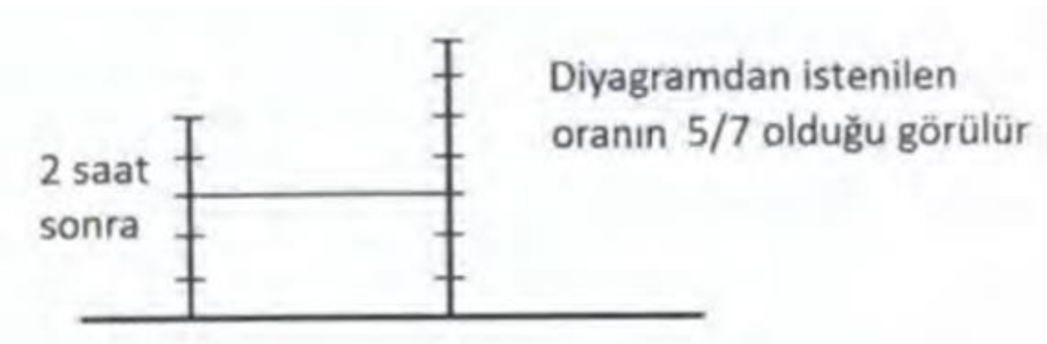
Fakat geriye kalan uzunlukları eşittir.

Bu yüzden $\frac{3}{7}$= uzun mumun uzunluğu. $\frac{3}{5}$= kısa mumun uzunluğu.

Buna bağlı olarak gerekli orantı = $\frac{5}{7}$

C-5. **<u>Çözüm 2:</u>** Çözüm 1'deki gibi akıl yürüttüm, fakat matematikse (cebirsel) denklem ve semboller kullandım.

C-5. **<u>Çözüm 3:</u>** Mumların uzunluklarını temsilen diyagram çizdim. 2 saat geçtiğini



düşündükten sonra; uzun mumun 7'de 4'ü, kısa mumun 5'te 2'si tükenir.

C-5. **<u>Çözüm 4:</u>** Çözüm 3'te olduğu gibi, fakat diyagramı hayal ettim.

C-5. <u>Çözüm 5:</u> Çözüm 3’te olduğu gibi bir diyagram çizdim veya hayal ettim ve aşağıdaki sonuca ulaştım;

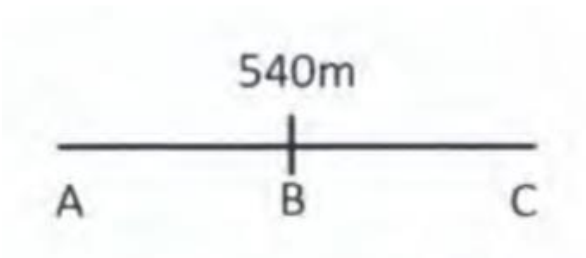
2 saat sonra, küçük olan mumun tamamen yanması için 3 saati ve uzun mumun tamamen yanması için 1,5 saati vardır. Boylan eşit olduğuna göre, küçük mumun kalınlığı uzun mumun kalınlığının iki katıdır.

Buradan; istenilen orantı $\frac{5}{3\frac{1}{2}\times 2}$, yani $\frac{5}{7}$

C-5. <u>Çözüm 6:</u> Çözüm 5’ teki gibi düşündüm, fakat herhangi bir şekil çizmedim veya hayal etmedim.

C-6

C-6. <u>Çözüm 1:</u> Tüneli temsilen bir diyagram çizdim.



B, doğru parçasının orta noktasıdır. Trenin, A noktasını geçmesi $\frac{1}{4}$ dakika alıyor ve trenin ön tarafı B noktasına ulaşıyor. Diğer $\frac{1}{4}$’lük dakikada trenin ön tarafı C noktasına ulaşıyor ve bir sonraki $\frac{1}{4}$’lük dakikada tren tünelden tamamıyla çıkıyor. Böylece; trenin uzunluğu = $540 \div 2 = 270$m’dir. Ve trenin hızı = $4 \times 270 =$ 1080m/dk’dır.

C-6. <u>Çözüm 2:</u> Çözüm 1 ‘de olduğu gibi, fakat diyagramı hayal ettim.

C-6. <u>Çözüm 3:</u> Her hangi bir şekil veya diyagram kullanmadan, sembol ve denklem kullandım; örneğin:

Trenin uzunluğuna *x* metre diyelim. Buradan trenin hızı x ÷ $\frac{1}{4}$ = 4x m/dk olur.

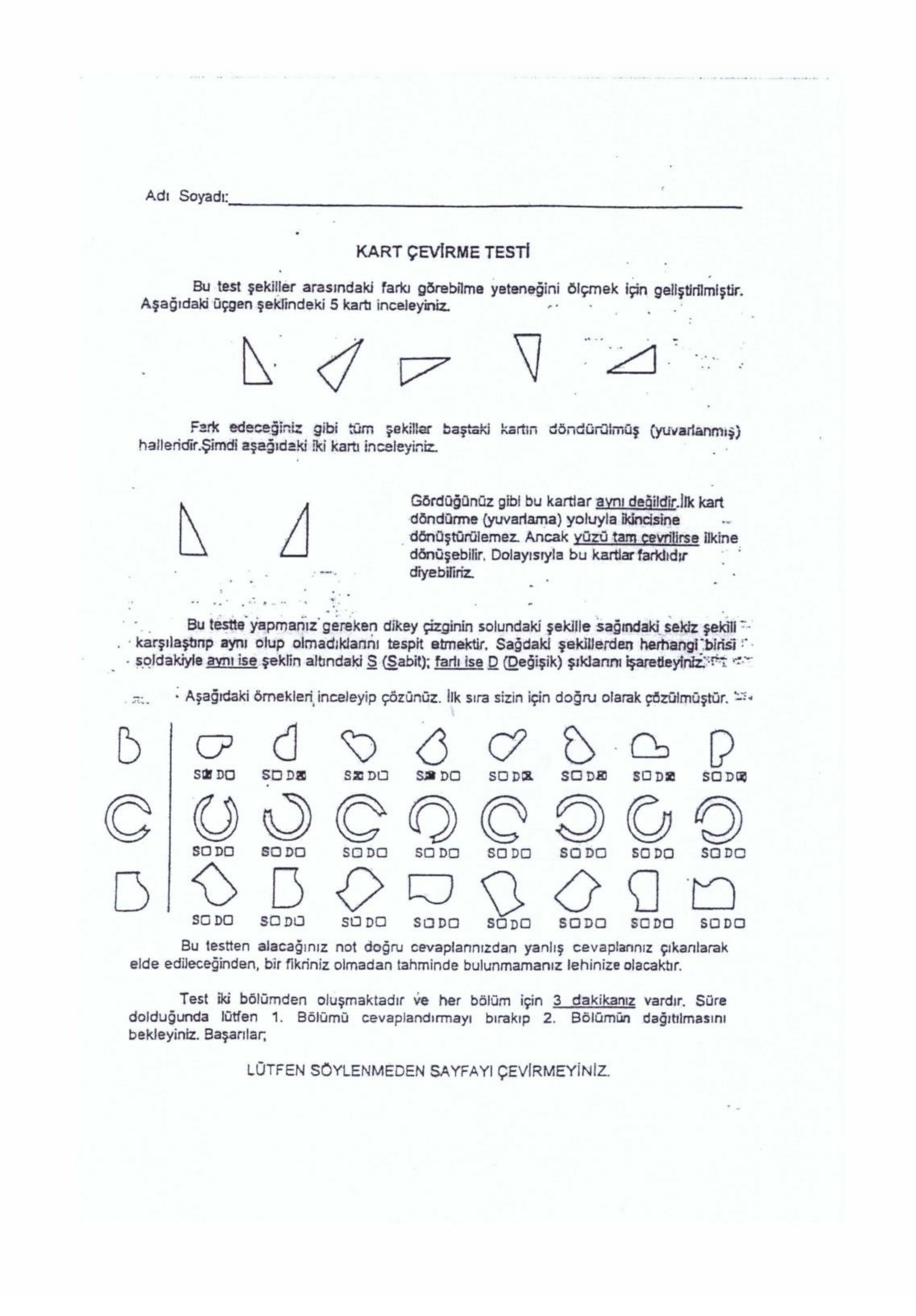
Mademki trenin tünele tamamen girmesi $\frac{1}{4}$ dakika sürüyor, tüneli tamamıyla çıkması için $\frac{3}{4}$ -$\frac{1}{4}$ = $\frac{1}{2}$ dakika geçer.

Buradan, 4x ‘in $\frac{1}{2}$ ile çarpımı 540 ise, *x* = 270 olur.

Trenin uzunluğu 270 m’dir ve hızı dakikada 1080 m’dir.

APPENDIX G

THE CARD ROTATION TEST



Adı Soyadı:______________________________

KART ÇEVİRME TESTİ

Bu test şekiller arasındaki farkı görebilme yeteneğini ölçmek için geliştirilmiştir. Aşağıdaki üçgen şeklindeki 5 kartı inceleyiniz.

Fark edeceğiniz gibi tüm şekiller baştaki kartın döndürülmüş (yuvarlanmış) halleridir.Şimdi aşağıdaki iki kartı inceleyiniz.

Gördüğünüz gibi bu kartlar aynı değildir.İlk kart döndürme (yuvarlama) yoluyla ikincisine dönüştürülemez. Ancak yüzü tam çevrilirse ilkine dönüşebilir. Dolayısıyla bu kartlar farklıdır diyebiliriz.

Bu testte yapmanız gereken dikey çizginin solundaki şekille sağındaki sekiz şekili karşılaştırıp aynı olup olmadıklarını tespit etmektir. Sağdaki şekillerden herhangi birisi soldakiyle aynı ise şeklin altındaki S (Sabit); farlı ise D (Değişik) şıklarını işaretleyiniz.

Aşağıdaki örnekleri inceleyip çözünüz. İlk sıra sizin için doğru olarak çözülmüştür.

S☒ D☐ S☐ D☒ S☒ D☐ S☒ D☐ S☐ D☒ S☐ D☒ S☐ D☒ S☐ D☒

S☐ D☐ S☐ D☐ S☐ D☐ S☐ D☐ S☐ D☐ S☐ D☐ S☐ D☐ S☐ D☐

S☐ D☐ S☐ D☐ S☐ D☐ S☐ D☐ S☐ D☐ S☐ D☐ S☐ D☐ S☐ D☐

Bu testten alacağınız not doğru cevaplarınızdan yanlış cevaplarınız çıkarılarak elde edileceğinden, bir fikriniz olmadan tahminde bulunmamanız lehinize olacaktır.

Test iki bölümden oluşmaktadır ve her bölüm için 3 dakikanız vardır. Süre dolduğunda lütfen 1. Bölümü cevaplandırmayı bırakıp 2. Bölümün dağıtılmasını bekleyiniz. Başarılar;

LÜTFEN SÖYLENMEDEN SAYFAYI ÇEVİRMEYİNİZ.

Fig. 16 1st page of the card rotation test

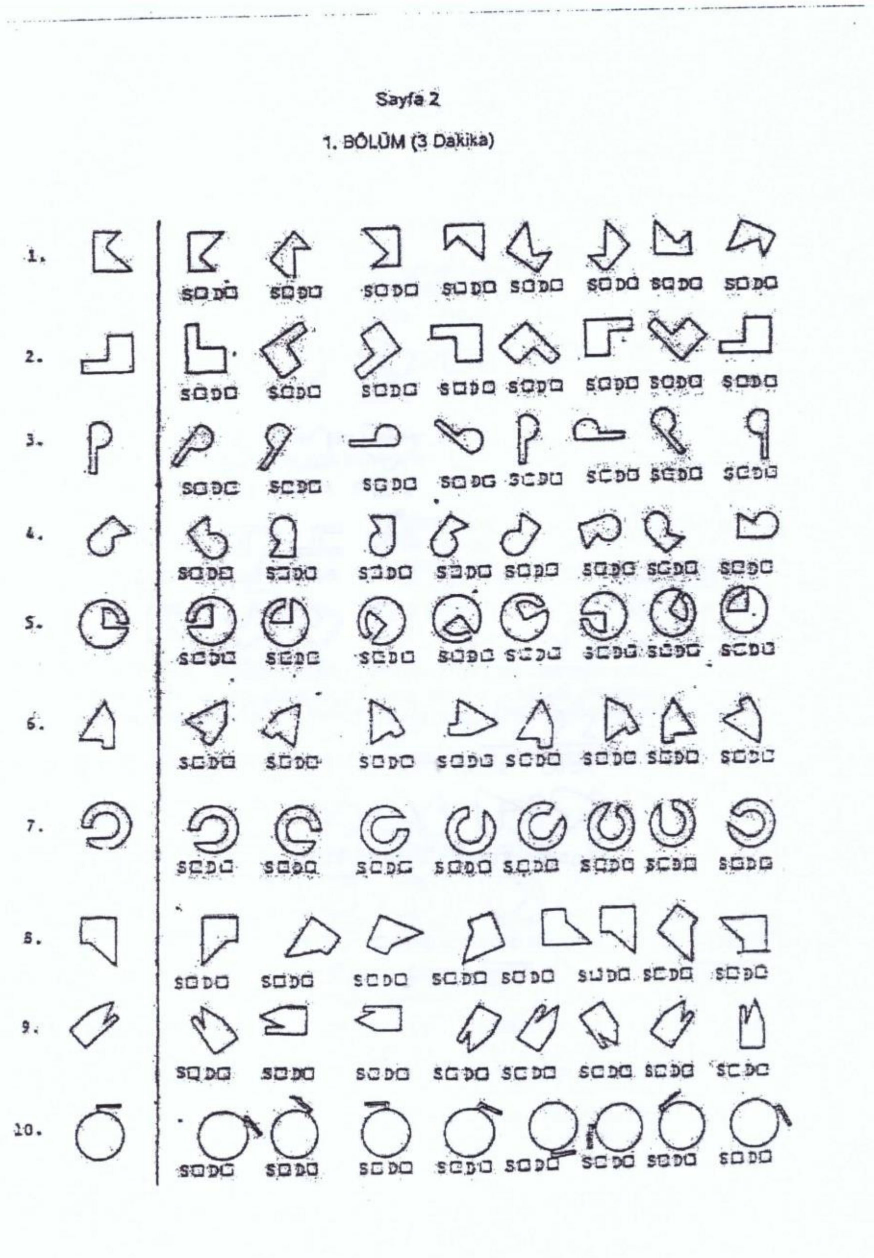


Fig. 17 2nd page of the card rotation test

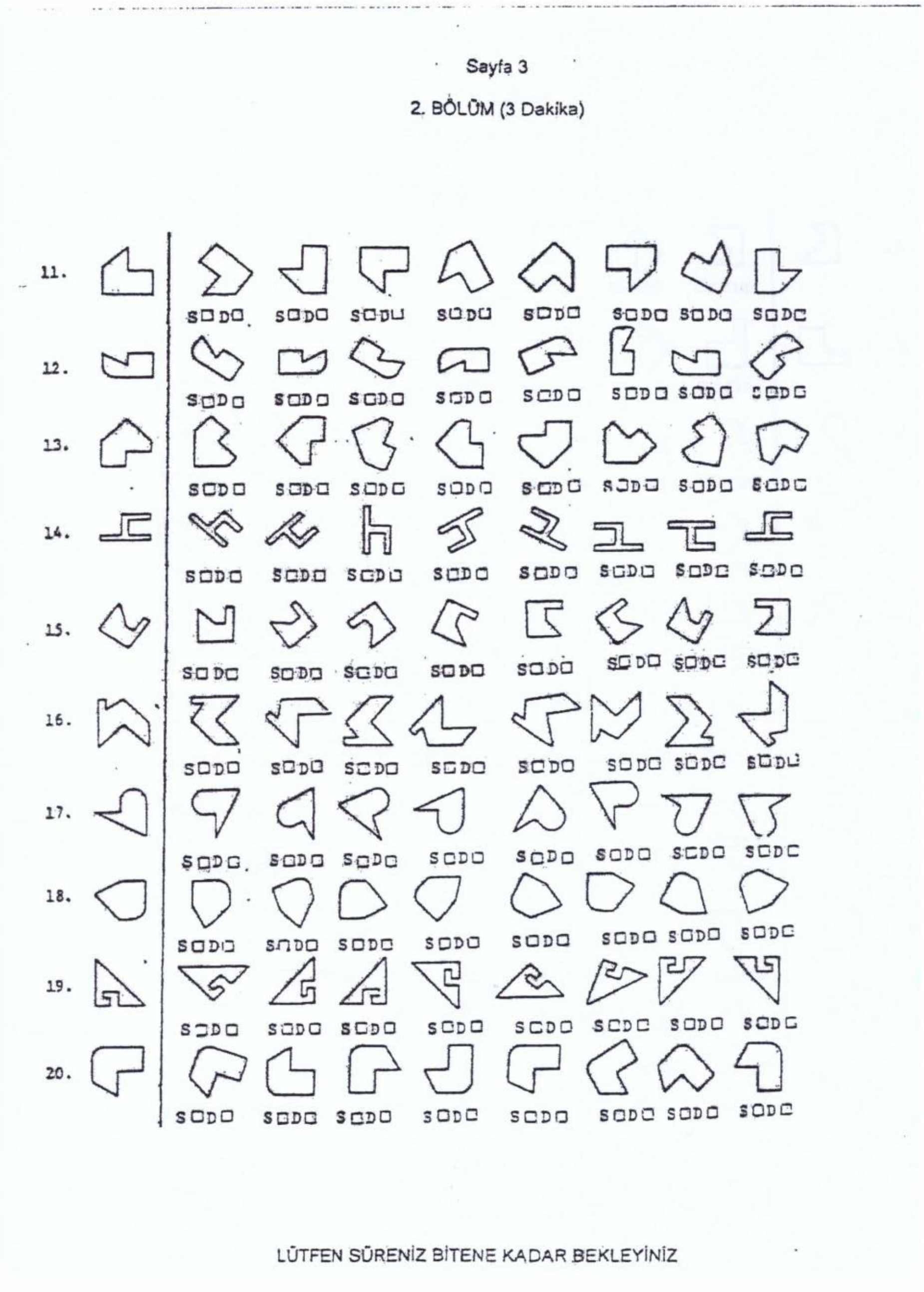
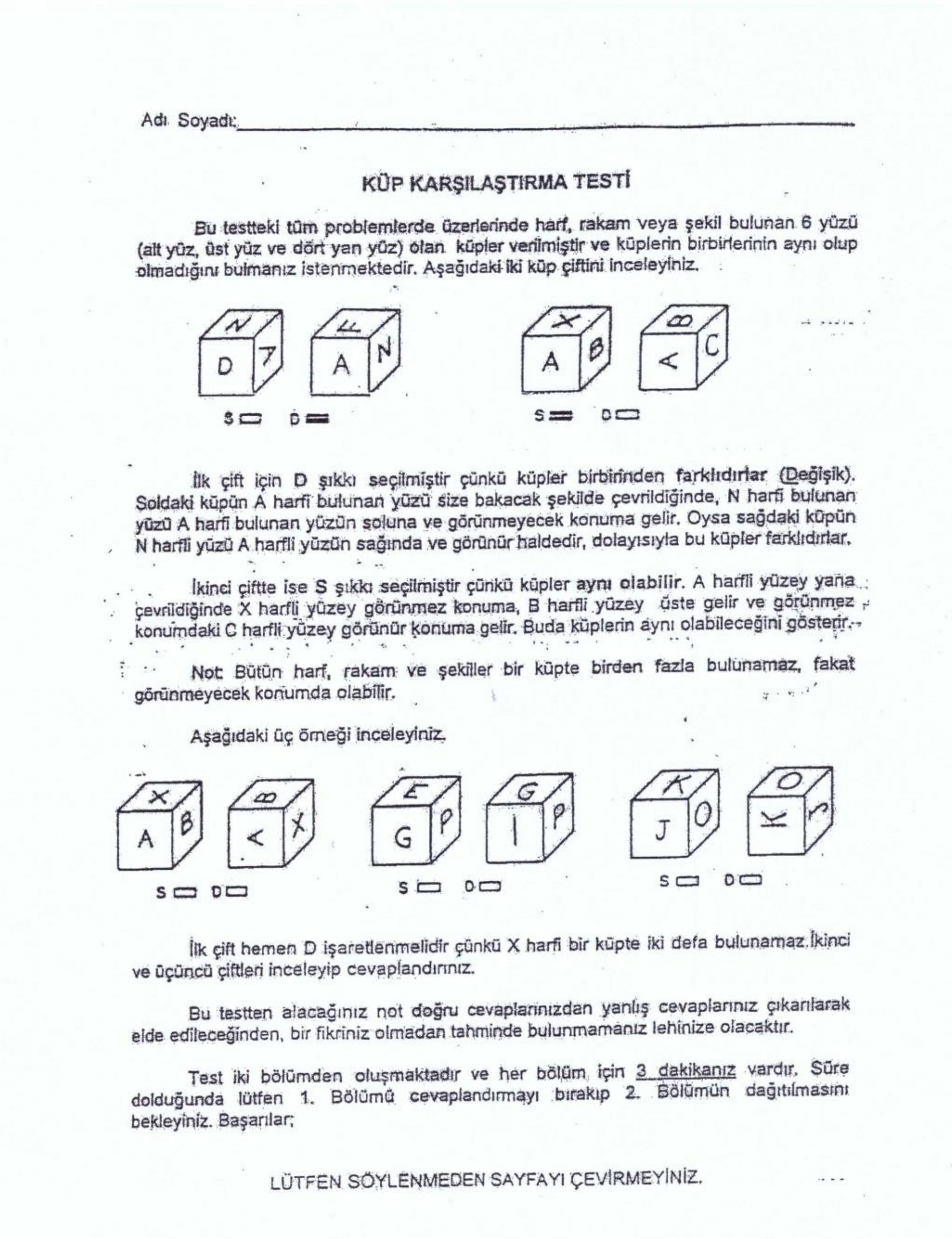


Fig. 18 3rd page of the card rotation test

APPENDIX H

THE CUBE COMPARISON TEST



Adı Soyadı:______________________________

KÜP KARŞILAŞTIRMA TESTİ

Bu testteki tüm problemlerde üzerlerinde harf, rakam veya şekil bulunan 6 yüzü (alt yüz, üst yüz ve dört yan yüz) olan küpler verilmiştir ve küplerin birbirlerinin aynı olup olmadığını bulmanız istenmektedir. Aşağıdaki iki küp çiftini inceleyiniz.

S ▭ D ▬ S ▬ D ▭

İlk çift için D şıkkı seçilmiştir çünkü küpler birbirinden farklıdırlar (Değişik). Soldaki küpün A harfi bulunan yüzü size bakacak şekilde çevrildiğinde, N harfi bulunan yüzü A harfi bulunan yüzün soluna ve görünmeyecek konuma gelir. Oysa sağdaki küpün N harfli yüzü A harfli yüzün sağında ve görünür haldedir, dolayısıyla bu küpler farklıdırlar.

İkinci çiftte ise S şıkkı seçilmiştir çünkü küpler aynı olabilir. A harfli yüzey yana çevrildiğinde X harfli yüzey görünmez konuma, B harfli yüzey üste gelir ve görünmez konumdaki C harfli yüzey görünür konuma gelir. Buda küplerin aynı olabileceğini gösterir.

Not: Bütün harf, rakam ve şekiller bir küpte birden fazla bulunamaz, fakat görünmeyecek konumda olabilir.

Aşağıdaki üç örneği inceleyiniz.

S ▭ D ▭ S ▭ D ▭ S ▭ D ▭

İlk çift hemen D işaretlenmelidir çünkü X harfi bir küpte iki defa bulunamaz. İkinci ve üçüncü çiftleri inceleyip cevaplandırınız.

Bu testten alacağınız not doğru cevaplarınızdan yanlış cevaplarınız çıkarılarak elde edileceğinden, bir fikriniz olmadan tahminde bulunmamanız lehinize olacaktır.

Test iki bölümden oluşmaktadır ve her bölüm için 3 dakikanız vardır. Süre dolduğunda lütfen 1. Bölümü cevaplandırmayı bırakıp 2. Bölümün dağıtılmasını bekleyiniz. Başarılar;

LÜTFEN SÖYLENMEDEN SAYFAYI ÇEVİRMEYİNİZ.

Fig. 19 1st page of the cube comparison test

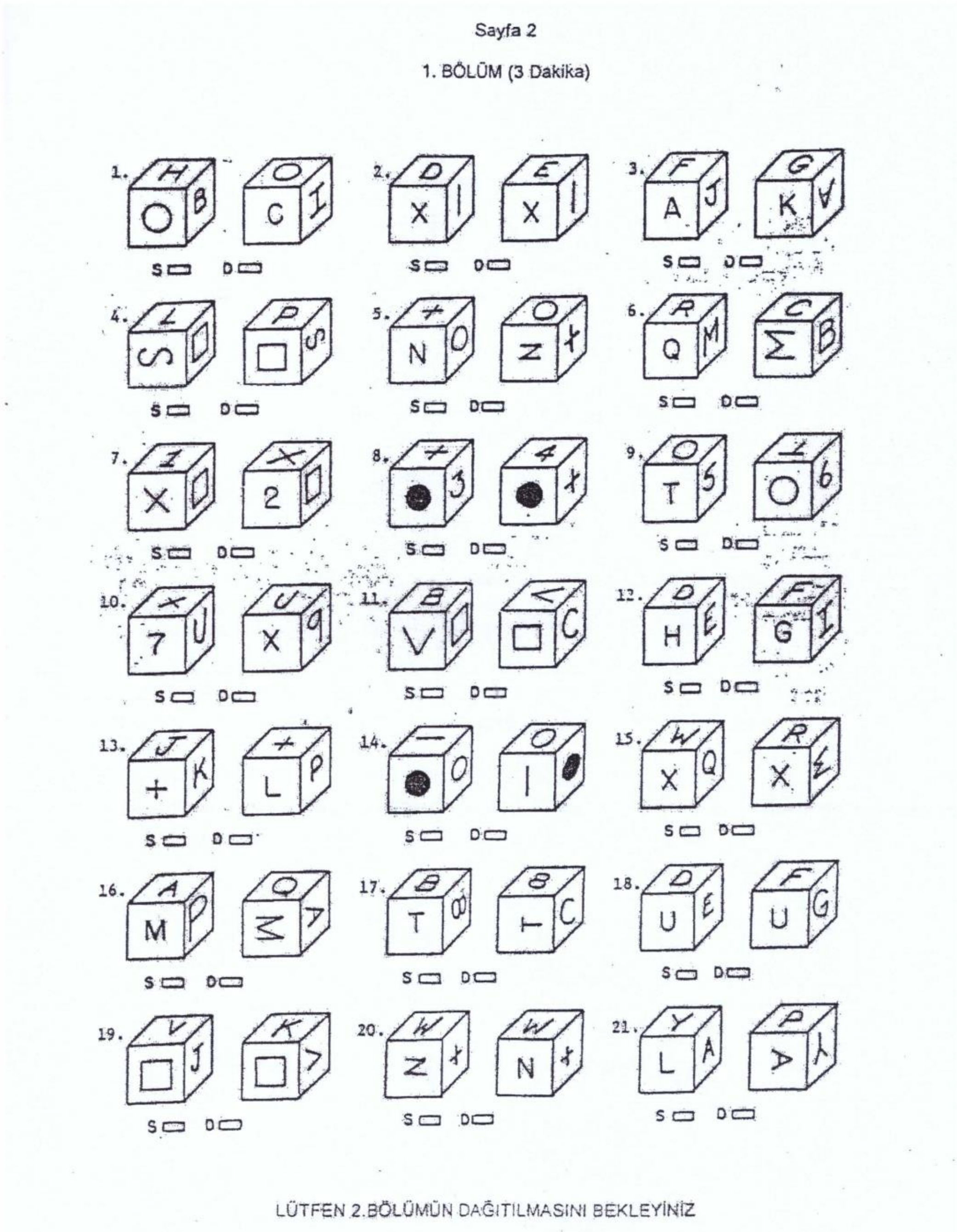


Fig. 20 2nd page of the cube comparison test

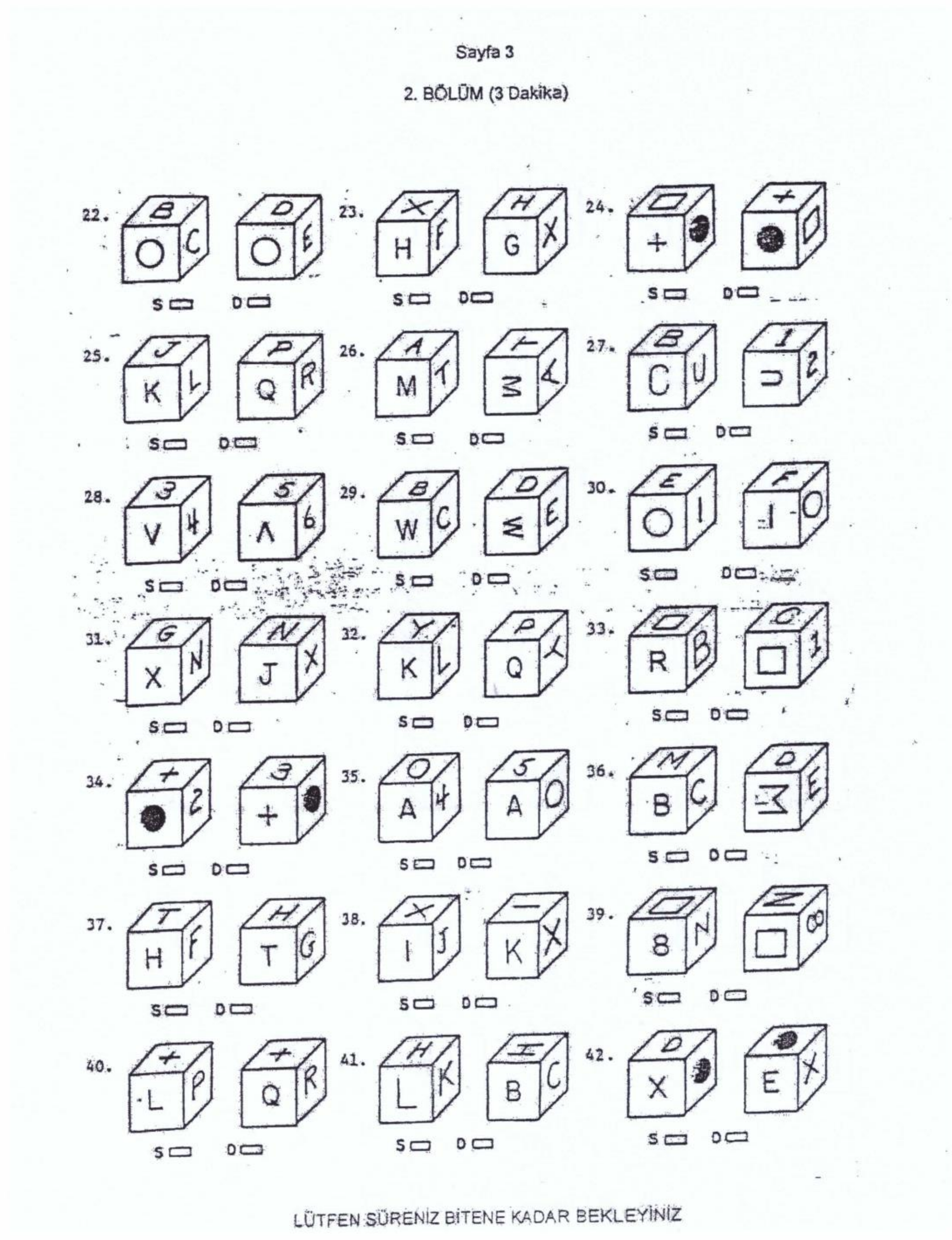


Fig. 21 3rd page of the cube comparison test

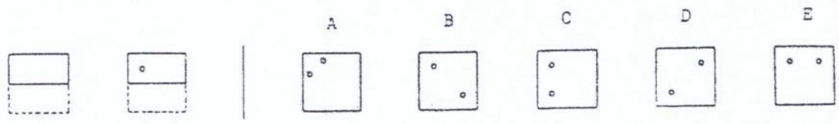
APPENDIX I

THE PAPER FOLDING TEST

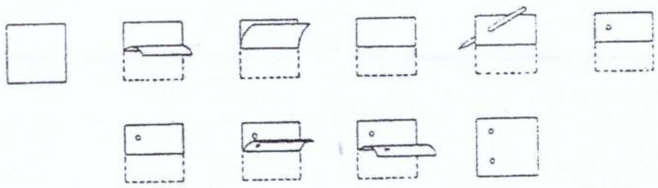
Adı Soyadı:____________________________________

KAĞIT KATLAMA TESTİ

Bu testte bir parça kağıdın katlanıp açılmasını hayal etmeniz gerekmektedir. Aşağıdaki örnekte dikey çizginin solundaki kare şeklindeki kağıt, katlanıp bir noktadan deliniyor. Kağıt açıldıktan sonra sağdaki şekillerden hangisinin oluşacağını bulunuz.



Yukardaki örnekte doğru cevap C şıkkıdır. Kağıdın nasıl katlandığını ve doğru cevabın neden C şıkkı olduğunu gösteren şekilleri inceleyiniz.



Tüm problemlerde katlamalar dikey çizginin solunda yapılmaktadır. Ayrıca kağıt hiç bir yöne çevrilmemekte sadece katlanmaktadır. Doğru cevabın kağıdın tamamen açıldıktan sonraki deliklerin yerini gösteren seçenek olduğunu unutmayınız.

Bu testten alacağınız not doğru cevaplarınızdan yanlış cevaplarınız çıkarılarak elde edileceğinden, birkaç seçeneği bertaraf etmeden tahminde bulunmamanız lehinize olacaktır.

Test iki bölümden oluşmaktadır ve her bölüm için 3 dakikanız vardır. Süre dolduğunda lütfen 1. Bölümü cevaplandırmayı bırakıp 2. Bölümün dağıtılmasını bekleyiniz. Başarılar;

LÜTFEN SÖYLENMEDEN SAYFAYI ÇEVİRMEYİNİZ.

Fig. 22 1st page of the paper folding test

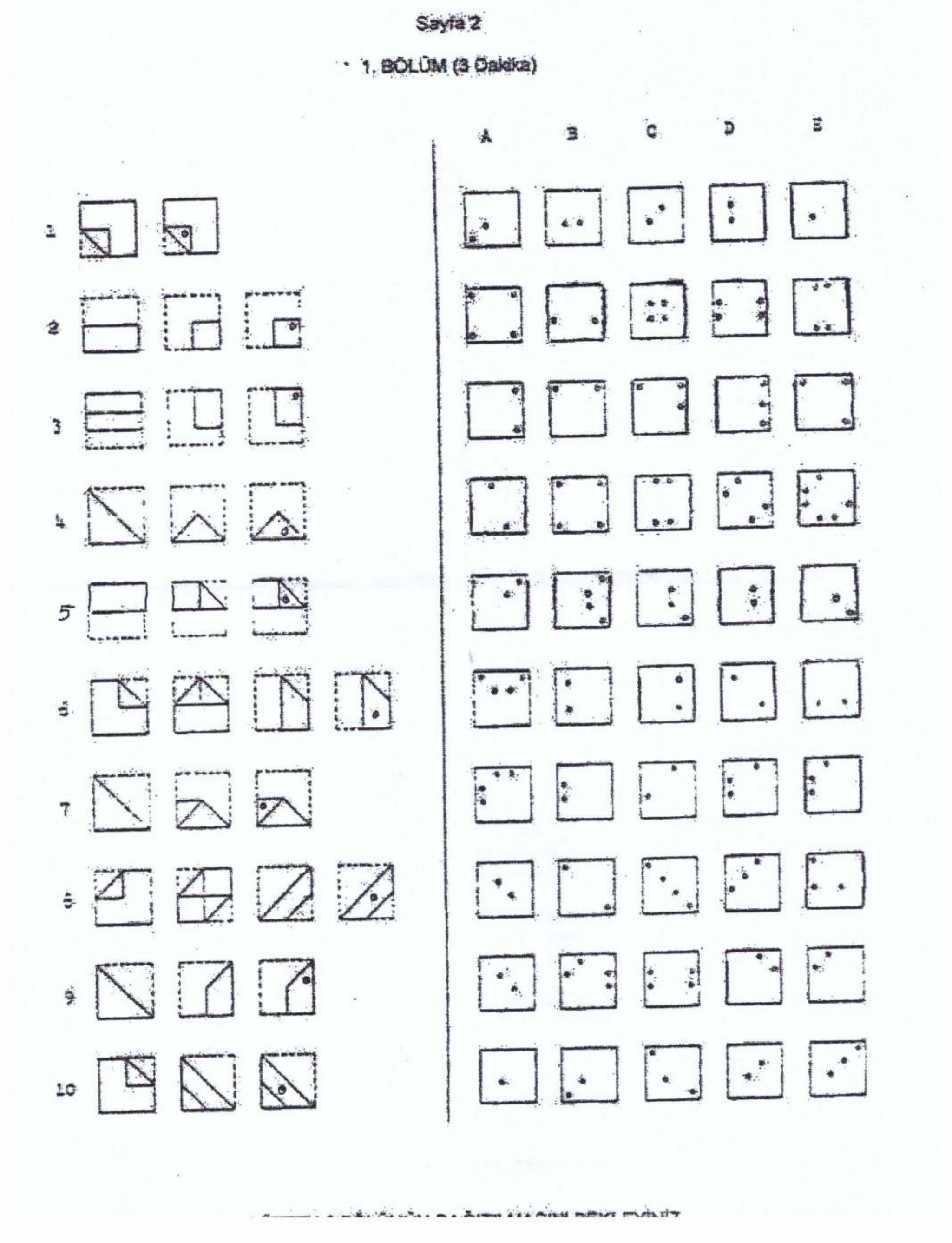


Fig. 23 2nd page of the paper folding test

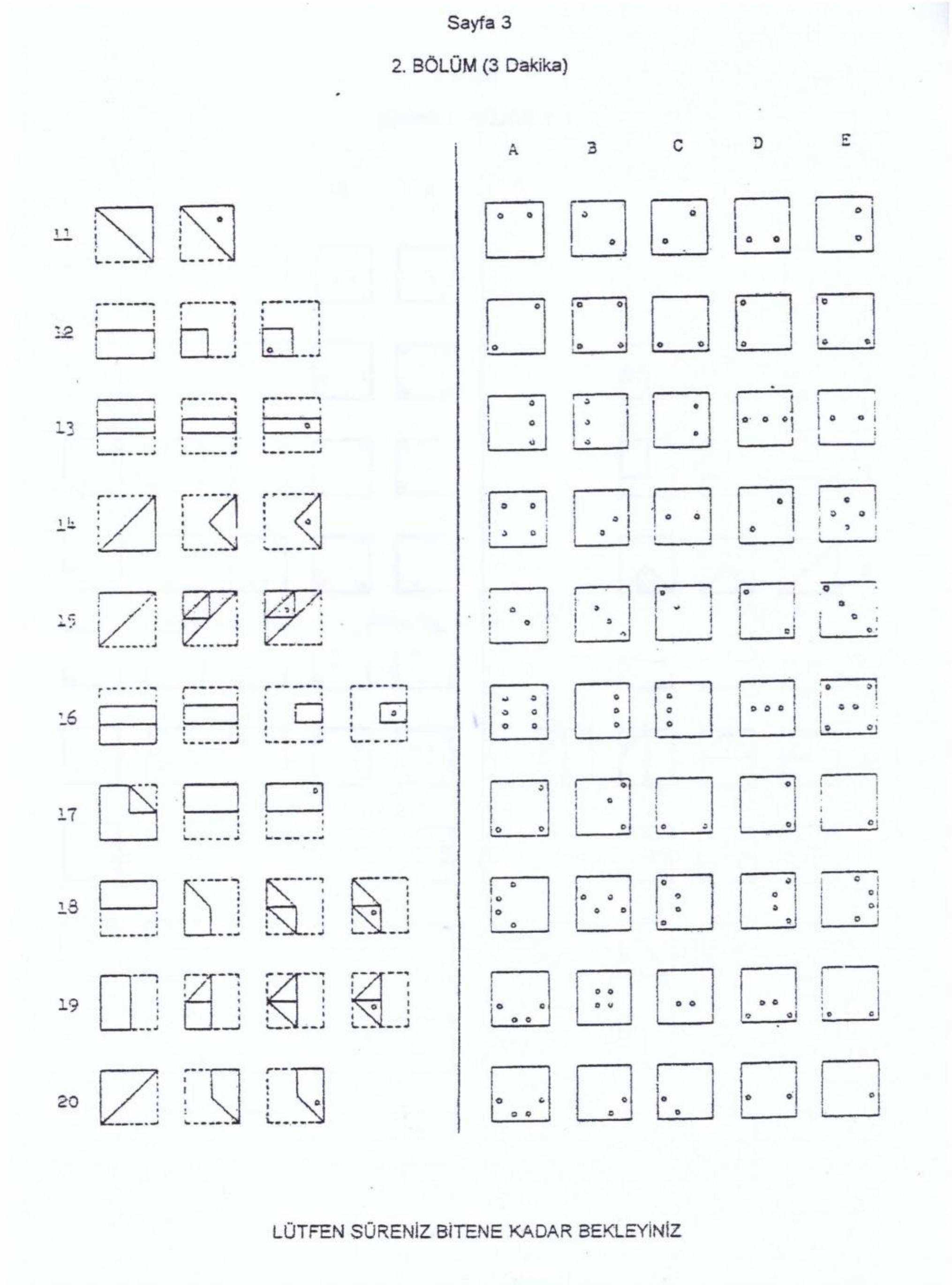


Fig. 24 3rd page of the paper folding test

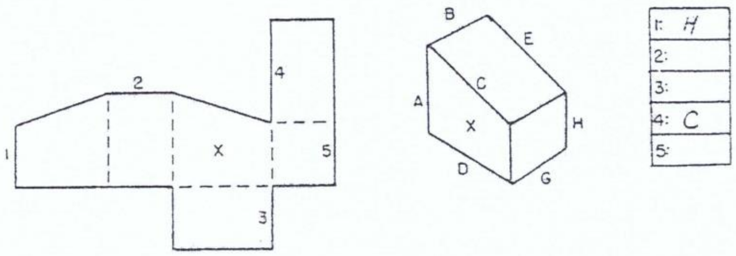
APPENDIX J

THE SURFACE DEVELOPMENT TEST

Adı Soyadı:______________________________________

YÜZEY OLUŞTURMA TESTİ

Bu testte bir parça kağıdı katlayarak değişik cisimler hayal etmeniz istenmektedir. Aşağıdaki şekillerden soldaki şekil noktalı çizgili yerlerden katlandığında sağdaki cisim oluşmaktadır. Katlamayı hayal ederek numaralı köşelerin hangi harflere denk geldiğini bulunuz ve en sağdaki kutunun içine yazınız. 1 ve 4 sizin için doldurulmuştur.



Not: Düz parçadaki X ile işaretlenmiş yüzey katlandıktan sonra oluşan cisimdeki X yüzeyiyle aynıdır. Dolayısıyla kağıt her zaman X yüzeyi cismin dış yüzünde olacak şekilde katlanmalıdır.

Yukardaki problemde, 1 köşeli yüzey cismin arka yüzünü oluşturmak için arkaya katlandığında, 1 köşesi H köşesiyle aynı olur. 5 köşeli yüzey arkaya katlandığında, 4 köşeli yüzey aşağı katlanır ve C köşesiyle aynı olur. Diğer cevaplarsa şöyledir: 2 B olur; 3 G olur; 5 H olur. İki cevabın aynı olabileceğine dikkat ediniz.

Bu testten alacağınız not doğru cevaplarınızdan yanlış cevaplarınız çıkarılarak elde edileceğinden, birkaç seçeneği bertaraf etmeden tahminde bulunmamanız lehinize olacaktır.

Test iki bölümden oluşmaktadır ve her bölüm için <u>6 dakikanız</u> vardır. Süre dolduğunda lütfen 1. Bölümü cevaplandırmayı bırakıp 2. Bölümün dağıtılmasını bekleyiniz. Başarılar:

LÜTFEN SÖYLENMEDEN SAYFAYI ÇEVİRMEYİNİZ.

Fig. 25 1st page of the surface development test

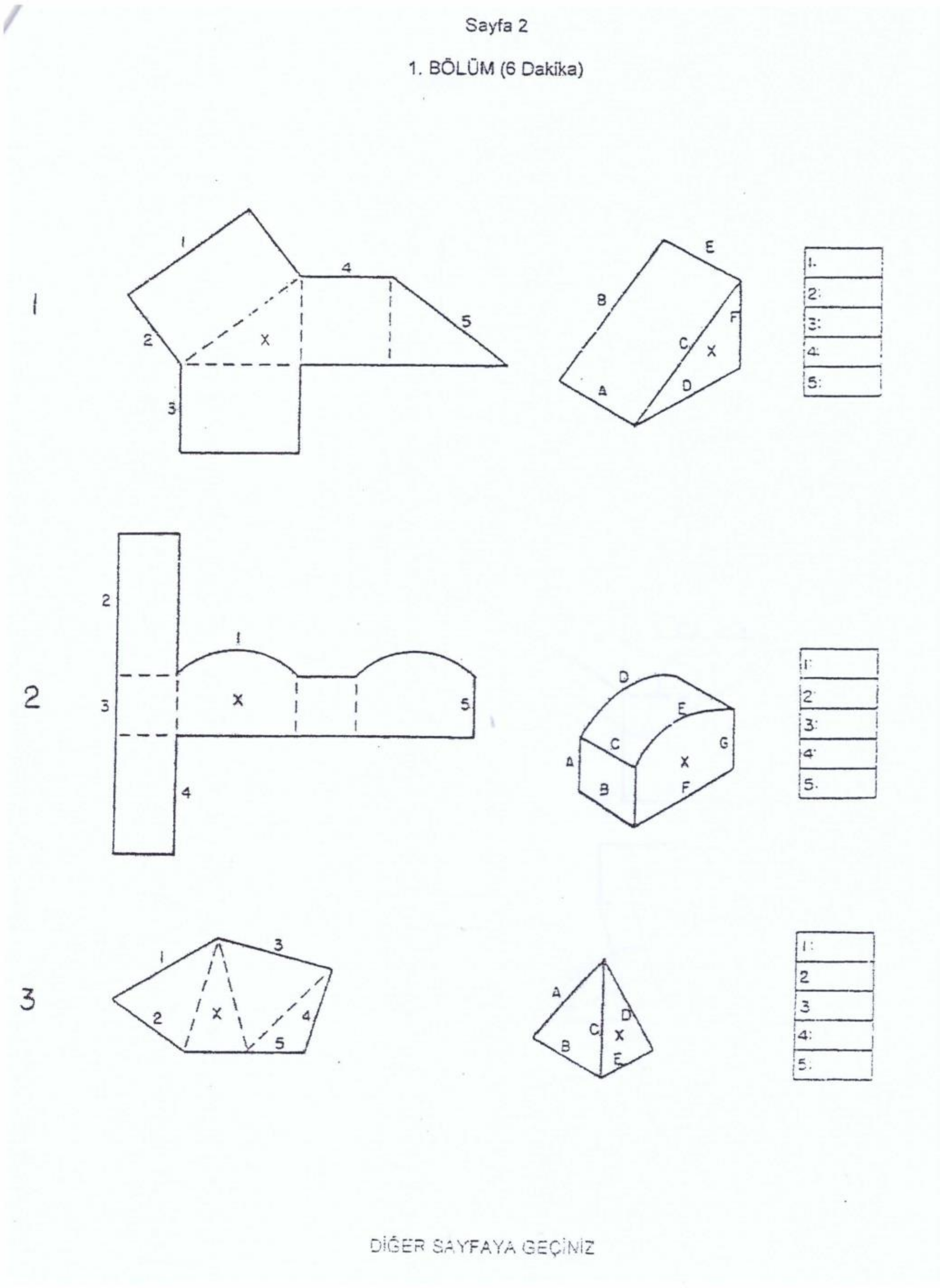


Fig. 26 2nd page of the surface development test

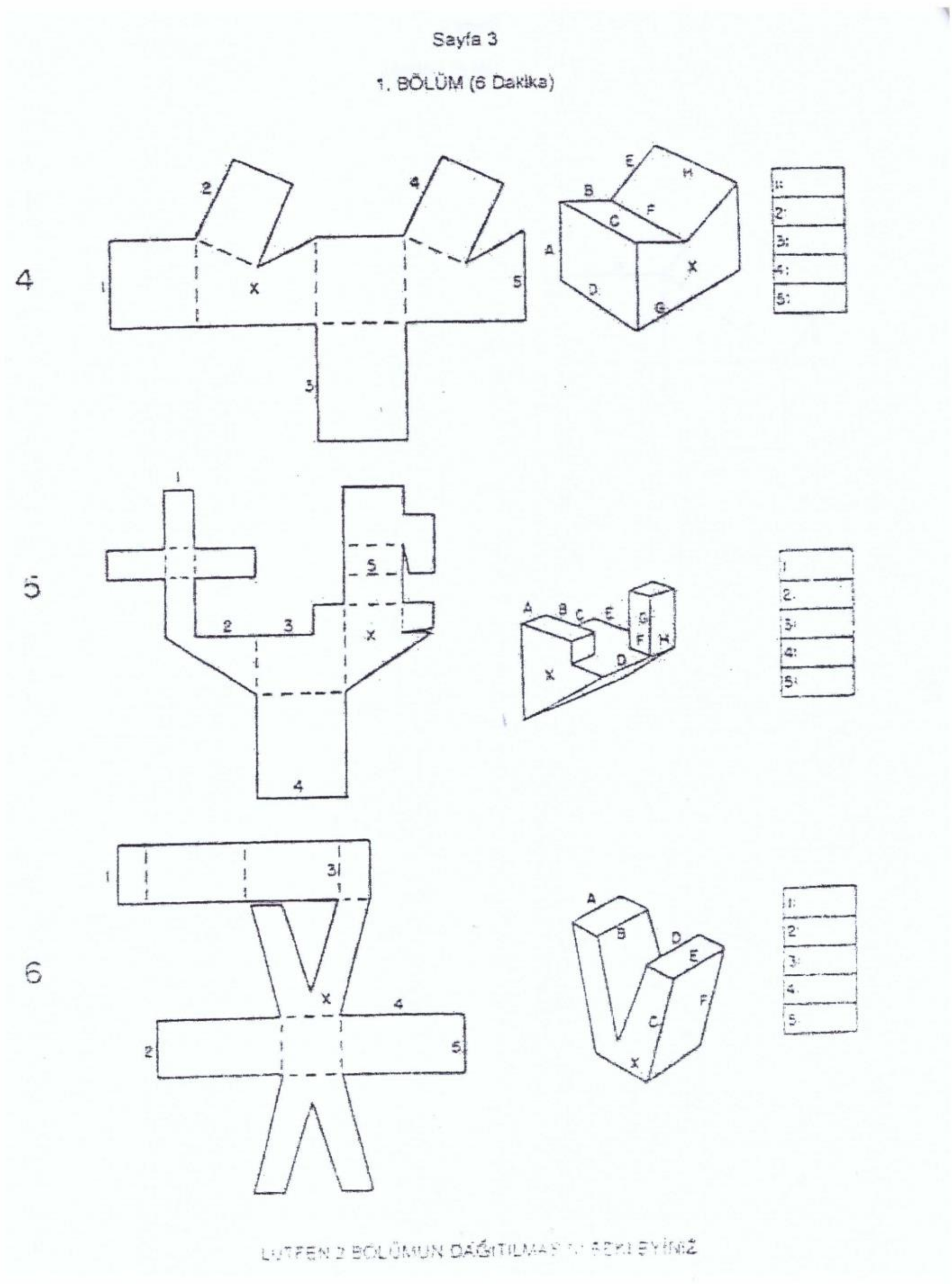


Fig. 27 3rd page of the surface development test

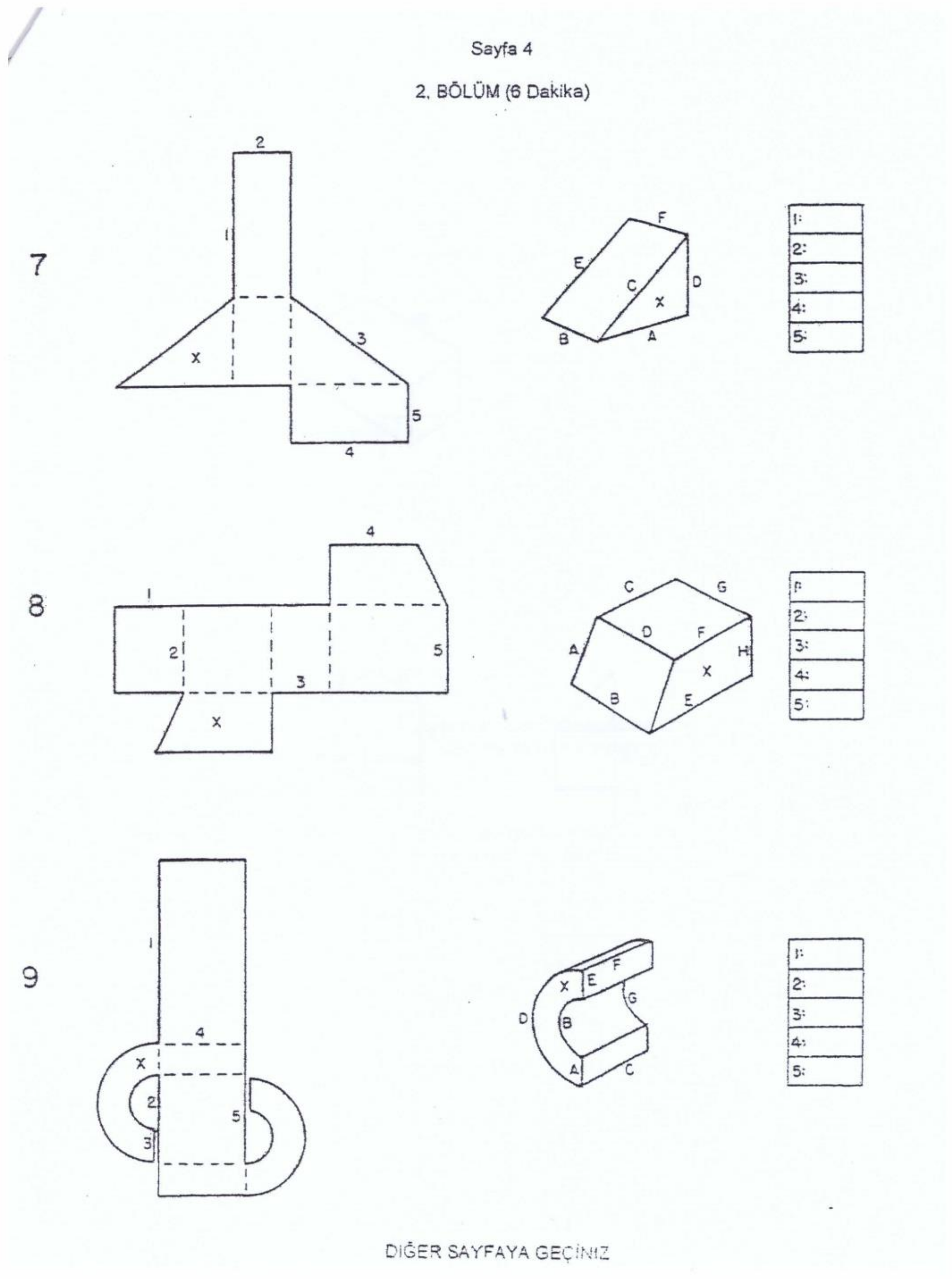


Fig. 28 4th page of the surface development test

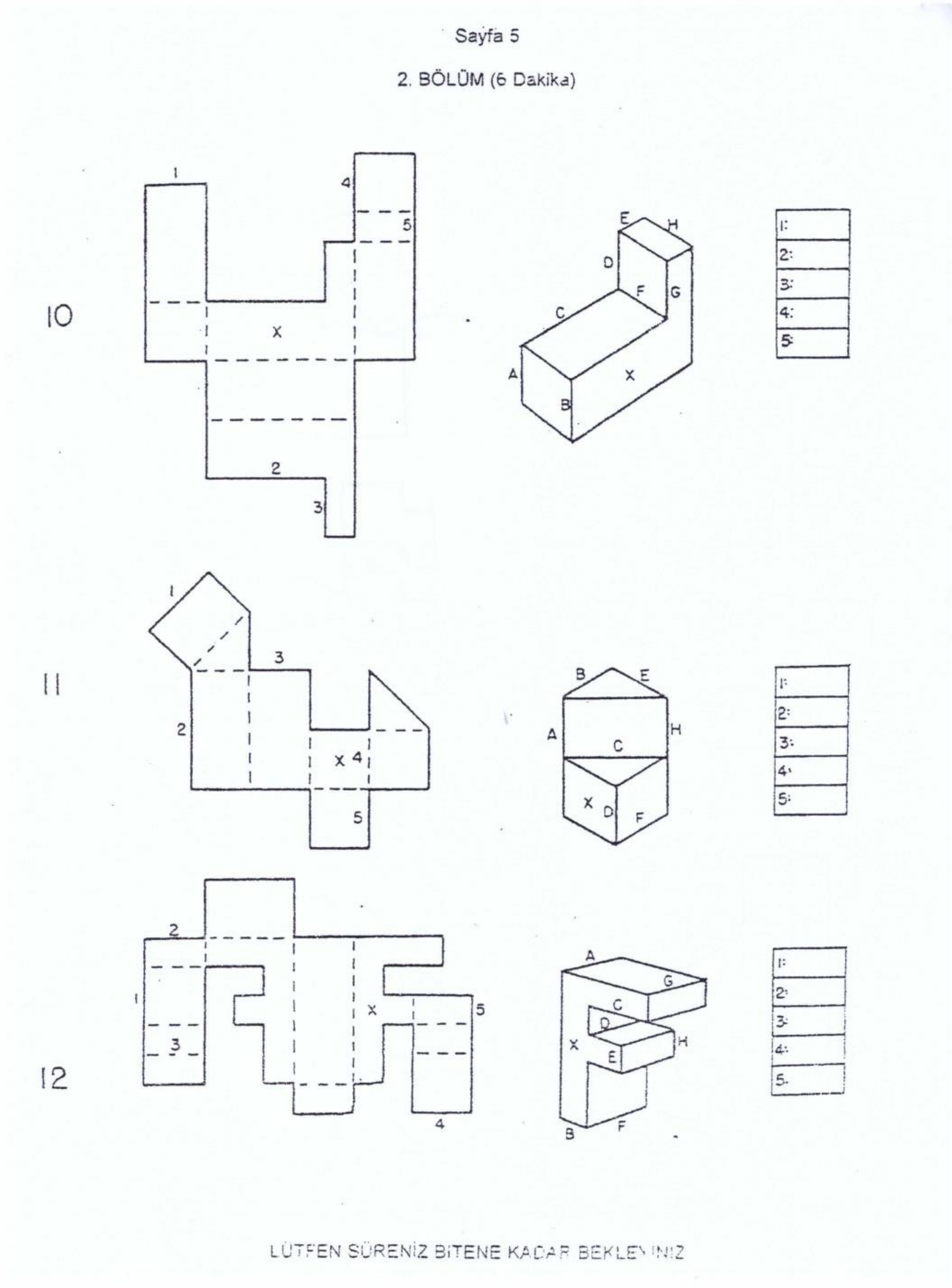


Fig. 29 5th page of the surface development test

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