THE RELATIONSHIP AMONG METACOGNITIVE KNOWLEDGE, METACOGNITIVE CALIBRATION ACCURACY AND MATHEMATICAL PROBLEM SOLVING PERFORMANCE

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2015

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Thesis submitted to the

Institute for Graduate Studies in the Social Sciences

in partial fulfillment of the requirements for the degree of

Master of Arts

in

Primary Education

by

Birce Başol

Boğaziçi University
2015

ACKNOWLEDGEMENTS

I would like to express my sincere appreciation to my thesis advisor Assist. Prof. Engin Ader for his patience, support, encouragement and guidance throughout this study. When some obstacles got my energy down from time to time, his enthusiasm and encouragement motivated me to work hard. His feedbacks and valuable suggestions made this study more valuable.

I would like to thank my committee members Prof. Emine Erktin and Assist. Prof. Günizi Kartal for their support and constructive critiques. Their extensive feedback has contributed greatly to the quality of this study.

I also owe thanks to Assist. Prof. Nalan Babür for her encouragement and guidance which makes me more excited to attend this master program. I will always remember her warm attitude while sharing of knowledge and expertise.

I am really lucky that I have great friends. I could not share their names one by one here. They were always so generous in sharing love, joy and support. Thank you all.

I wish to express my profound gratitude to my parents Recebiye and Muammer Başol. You taught me to go after my passions and to stand strongly wherever I am throughout my life. Thank you for your endless love, support and trust. I am grateful to you. And my brother Alikemal, you are my lifelong friend and joy. Thank you for your love, support and encouragement.

I wish to express my deepest thanks to my nearest and dearest friend and love, Levent, for providing me with unfailing support and continuous encouragement and love throughout my years of study. Without his endless trust in me, this study could not have been accomplished. Thank you for being in my life.

Dedicated to
my beloved family...

Thesis Abstract

Birce Başol, "The Relationship among Metacognitive Knowledge, Metacognitive Calibration Accuracy and Mathematical Problem Solving Performance"

The purpose of this research was to investigate relationships among metacognitive knowledge, metacognitive calibration accuracy and mathematical problem solving performance. In order to measure metacognition more holistically, metacognitive knowledge and metacognitive calibration (both prospective and retrospective) were taken into consideration together. Mathematical problem solving performance was assessed through three mathematical word problems. In the analyses, judgment bias and different levels of performance of students were taken into consideration. There were 200 participants in the study obtained from seventh grade students from public (N=90) and private (N=110) schools. The convenient sampling method was used in the data collection process of the study.

Results demonstrated a significant relationship between prospective and retrospective monitoring accuracy. Another significant relationship was found between problem solving performance and metacognitive monitoring calibration. In terms of judgment bias, students tended to be overconfident in their prospective judgments compared to retrospective ones. Moreover, there is a significant difference between overconfident and underconfident students' performances. High performers tend to be underconfident while low performers are generally overconfident. Lastly, metacognitive knowledge was a differentiating factor for low performers in prospective judgments not in retrospective judgments.

Tez Özeti

Birce Başol, "Üstbilişsel Bilgi, Üstblişsel Kalibrasyon Duyarlığı ve Matematiksel Problem Çözme Becerisi Arasındaki İlişki"

Bu çalışmanın amacı üstbilişsel bilgi, üstblişsel kalibrasyon duyarlığı ve matematiksel problem çözme becerisi arasındaki ilişkilerin incelenmesidir.
Üstbilişsel bilgi envanteri ve üstbilişsel kalibrasyon üstbilişi bir bütün olarak ölçmek amacıyla birlikte ele alınmıştır. Matematiksel problem çözme becerisi üç matematik problemi ile ölçülmüştür. Öğrencilerin kendi performansları hakkındaki yargılarının yönü ve farklı performans seviyeleri analizler de dikkate alınmıştır. Çalışmaya devlet (N=90) ve özel (N=110) okullardan toplam 200 yedinci sınıf öğrencisi katılmıştır. Kolay ulaşılabilir örneklem seçme yönteminden faydalanılmıştır.
Sonuçlar ileriye (prospektif) ve geriye (retrospektif) yönelik üstbilişsel izleme duyarlığı arasında anlamlı bir ilişki olduğunu ortaya koymuştur. Problem çözme becerisi ve üstbilişsel izleme kalibrasyonu arasında da anlamlı bir ilişki bulunmuştur. Öğrencilerin ileriye yönelik (prospektif) yargılarında geriye (retrospektif) yönelik yargılarına kıyasla fazla kendine güven (overconfidence) göstermeye eğilimli oldukları ortaya çıkmıştır. Fazla kendine güven (overconfidence) ve az kendine güven (underconfidence) gösteren öğrencilerin problem çözme performansları arasında anlamlı bir fark bulunmuştur. Bu sonucu takiben, düşük performanslı öğrenciler genellikle fazla kendine güvenli (overconfident) tutum sergilerken, yüksek performanslı öğrencilerin az kendine güvenli (underconfident) tutum sergilemeye yatkın oldukları da bir diğer önemli bulgudur. Son olarak, üstbilişsel bilginin düşük performanslı öğrenciler için kendi performansları hakkında ileriye yönelik (prospektif) yargılarında ayırıcı bir faktör olduğu saptanmıştır.

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CHAPTER 1

INTRODUCTION

Students are exposed to many mathematical questions and problems throughout their educational life. Almost all teachers report that their students have some difficulties with mathematics. Campione et al. (1989) claims that many students do not know what they are actually doing while they are solving mathematical problems even when they answer the problem correctly. Turkish students' math scores are below the average in international exams such as the Program for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS). According to the TIMSS 2011 results, mathematics results are below the average both in 4^{th} and 8^{th} grades. The 4^{th} grade average is 469 (standard deviation 4.7) and the 8^{th} grade average is 452 (standard deviation 3.9) while the mean of TIMMS is 500 (Mullis, Martin, Foy, & Arora, 2012). PISA 2012 is another cross-national exam which has been administered recently. Turkey has an average of 448 with a 62% variance while the PISA 2012 average of OECD countries is 494 with 37% variance (OECD, 2012). Having low performance averages and a high variance among the Turkish student scores caused a lot of concern. While low scores are interpreted as low overall achievement in mathematics, high variance indicates high differences in the quality of schools in Turkey.

Beside the cross national exams, standardized examinations are also conducted by the Ministry of Education for high school acceptance. Acceptance into competitive high schools is determined by a high-stakes test called the Secondary Education Entrance Examination (Ortaöğretim Kurumları Öğrenci Seçme ve

Yerleştirme Sınavı) OKS. The last announced scores (2007) indicate serious findings about students' performances in mathematics. The average score for mathematics was 3.35 (with a standard deviation of 5.2) in 25 questions (MEB, 2009). Thus, poor mathematics performance is also observed in national exams.

Mathematical problem solving is a general term used by educators to describe the problems with text information and numerical data which is used with the appropriate skills to arrive at correct responses. A student needs to use all obtained mathematical knowledge to solve the problems. These types of problems require multistep mathematical skills.

The new mathematics' curriculum was shifted to the constructivist approach as parallel to the new curriculum movement in 2005. And the most recently published mathematics teaching program emphasizes the importance of multistep mathematical skills, developing mathematical problem solving strategies and executing these strategies successfully (MEB, 2010). The new approach had an initiation to change the core of the instruction from teacher-centered to student-centered like Op't Eynde et al. (2007) mentioned as something happening by the students rather than something happening to the students. Students should regulate their own cognition, motivation and behavior for being effective learners (Zimmerman, 1989).

The new Turkish mathematics curriculum incorporated estimation, which is an important skill in mathematics, into its new approach. Estimation is not just a mathematics topic but also a lifelong ability. Mathematics is an important part of the everyday life, affecting job success, achievement in school programs, household management, and in situations which require solving problems and planning. Estimation skill is one of these abilities and it is often seen as one of the most important strategy that is needed for effective problem solving (Levin, 1981; O'Daffer, 1979). Self-regulated learners should be aware of their performance in order to monitor their learning; estimation is also important in that aspect of learning. Estimating a distance or estimating the correct number at the end of an arithmetic operation are not the only estimation skills. Estimating the correctness of a complex task, which is personally performed, is another example of an estimation skill. When the learners think about their thinking or predicting and evaluating their own performance to regulate their learning, this brings the issue into self-regulated learning area.

Much experimental work assessed the effects of self-regulated learning in mathematics (e.g. Schunk 1982; 1985; 1996), and they generally focus on discrete computational skills. However, for more than 25 years, mathematics education studies recommend deemphasizing computation but focus more on problem solving skills (Fuchs et al., 2003). Thus, in today's studies, self-regulated learning is seen as especially relevant for complex mathematical problem solving (e.g. De Corte, 2000; Fuchs et al., 2003).

Today, it is widely accepted that metacognition plays an important role in mathematical problem solving (Borkowski, Chan & Muthukrishna, 2000). Since metacognition is the awareness of one's own mental process and ability of self-regulate performance, an effective mathematical problem solver should regulate both his/her knowledge and the process of problem solving. In short, metacognition includes both knowledge about cognition and regulation of cognition together under one term.

Metacognitive experience is described as conscious cognitive and affective experiences emerging in any intellectual process (Flavell, 1979). Metacognitive experiences includes online metacognitive knowledge (also referred as online task specific knowledge) and metacognitive judgments (Efklides, 2001). Metacognitive experience in mathematical problem solving can be seen as one's reactions when faced with difficulties during problem solving. Individuals overcome the difficulties with the help of their previous experiences in similar problems. Individuals' self-esteem about their performance in a specific task or problem is important in terms of metacognitive experience. Therefore, the prediction (prospective) and evaluation (retrospective) skills of individuals are important elements in the assessment of metacognitive experience.

Metacognitive experiences are not something coming to the learners from their past; they are related to their actual judgments of learners on a processing task. These metacognitive judgments (prediction and evaluation) are important to determine the calibration of individuals in a specific concept. Calibration means the accuracy of learners' perceptions about their performance. Calibration is a skill of metacognitive monitoring (Pieschl, 2009). In order to observe this monitoring skill of learners during problem solving, their level of calibration should be considered. This can be done both prospectively and retrospectively.

The purpose of this research was to investigate relationships among metacognitive knowledge, metacognitive calibration accuracy and mathematical problem solving performance. In order to investigate these relationships, different levels of performance on mathematical word problem solving and students' metacognitive calibration level were examined.

CHAPTER 2

LITERATURE REVIEW

Metacognition

In 1976, Flavell introduced the term “metacognition”. Metacognition can be simply defined as “cognition about cognition” (Flavell, 1985). According to this definition, it is conscious regulation and control over one’s own cognitive processes such as thinking, remembering and problem solving (Flavell, 1987; Brown, 1987; Das, Naglieri & Kirby, 1994). As a basic definition, Metcalfe & Shimamura (1994) described metacognition as “what we know about what we know” (p. 11). O’Neil and Brown (1997) have defined metacognition as the active monitoring of cognitive processes in order to develop strategies to solve problems.

According to Brown (1987), cognitive and metacognitive functions are interchangeable in some contexts. For example in Flavell’s (1976) explanation, “Asking yourself questions about the chapter might function either to improve your knowledge (a cognitive function) or to monitor it (a metacognitive function)" (cited in Brown, 1987, p. 66). Jacobse and Harskamp (2012) also distinguish these two terms with an example of note taking. While decision of taking notes is metacognitive, the action itself is cognitive. In the light of these definitions and examples, metacognition can be defined as active monitoring and control over one’s own learning and thinking processes.

When Flavell (1976) first introduced the term metamemory, he defined metamemory as the knowledge of individuals about their memory processes. Later on, the definition was changed to metacognition with four classes: metacognitive knowledge, metacognitive experiences, goals and strategies (Flavell, 1979). Subsequent research (Brown et al., 1983; Schraw & Dennison, 1994; Schraw & Moshman, 1995) classified concepts into two categories: *knowledge about cognition* and *regulation of cognition.*

Metacognitive knowledge establishes a deeper understanding of cognitive processes and products (Flavell, 1976). In his theoretical framework, Flavell (1979) explains knowledge about cognition in three dimensions: person knowledge, task knowledge, and strategy knowledge. Person knowledge includes the individual's knowledge and beliefs about her/himself as a learner, and what s/he believes about other people's thinking processes. Task knowledge comprises perception of task difficulty, management of a task, information about the degree of success in that task, and information about possible mental resources necessary to complete that task. Strategy knowledge involves identifying goals and sub goals and selection of cognitive processes to use in task achievement (Flavell, 1979). Flavell (1979) also argued that these three dimensions have interactions and may be activated consciously or unconsciously by the learner. This type of knowledge can be summarized as "knowing what", "knowing how" and "knowing why and when" respectively (Brown, 1987). Hence, knowledge about cognition provides the reflective aspect of metacognition.

The second component of metacognition, regulation of cognition, includes active processes which enables control over cognition. These sub processes are planning, information management strategies, comprehension monitoring, debugging strategies and evaluation (Artz & Armour-Thomas, 1992; Baker, 1989; Schraw & Dennison, 1994; Veenman & Alexander, 2011).

Metacognitive processes are also described by Brown (1978, 1982) in four necessary capacities/skills: prediction, planning, monitoring and evaluation. Prediction is related to predicting one's own level of performance in a specific task; planning is the operations or procedures in achieving the goals of the task; monitoring refers the ability to recognize the strategies used to solve the task; finally evaluation refers one's own judgments about executed strategies. In short, learners execute these processes namely regulate their cognition by using metacognitive knowledge to control and modify the cognition.

Metacognition is experienced by a dialogue inside our brain. This can be happen while reading or solving a problem. We seek out the last word that we read in the sentence or try to understand what we need to solve a problem. These are all metacognitive skills when we experience a task. Flavell (1979) described metacognitive experience as conscious cognitive and affective experiences emerging in any intellectual process (Flavell, 1979). Metacognitive experience, which is part of Flavell's (1979) second component of metacognition, regulation of cognition, contains the subjective internal responses of an individual to her/his own metacognitive knowledge, goals, or strategies namely other categories of metacognition. These can occur before, during, or after a cognitive event. Similar to monitoring phenomena, these experiences can provide internal feedback about current progress or future expectations about the success of the task.

Self-Regulated Learning and Metacognitive Calibration

Dewey (1998) put forward the mastery of progressive education compared to traditional education as personal experience and experiment. So, the ideal goal of education is creation of power of self-control (Dewey, 1998). Self-regulated learning (SRL) is one of the widely researched topics today. Self-regulation of learning involves not only detailed knowledge of a skill, but also the self-awareness, self-motivation and behavioral skill to construct and use the knowledge appropriately (Zimmermann, 2002).

Self-regulated learning components can be categorized in four phases: task perception, goal setting and planning, enacting, and adaptation (Winne & Hadwin, 2008). Self-regulated learners are able to plan and check their work. They are aware of their thought processes and the strategies which are necessary to accomplish a task (Zimmermann, 2000). Moreover, highly self-regulated learners have powerful self-evaluation skills which enable them to recognize their strategy deficiencies or solution mistakes, compared to poorly self-regulated learners who attribute their poor performance to lack of ability (Zimmermann, 2000).

Individuals' self-evaluation about their performance in a specific task or problem is important in terms of metacognitive experience. Accuracy of this self-evaluation and real performance is called as *metacognitive calibration*. Pieschl (2009) expressed calibration as an aspect of metacognitive monitoring and also a component of self-regulated learning. In an educational context, metacognition strengthens effective control on learning (Metcalfe, 2009). Thus, if students judge their learning and performance accurately, then they can effectively manage their learning. Literature shows accurate metacognitive calibration as an important distinguishing factor between capable and less capable learners (Everson & Tobias, 1998). How metacognitive calibration differentiates the level of achievement should be investigated.

As a part of self-regulated learning, calibration means the accuracy of learners' perceptions about their performance. "It is a metacognitive skill (opposed to metacognitive knowledge; Veenman et al. 2006), more specifically a skill of metacognitive monitoring (opposed to metacognitive control; Nelson and Narens 1994)" (Pieschl, 2009, p. 4). Hence, calibration characterizes how individuals are aware of their own internal processes. Different terms are used in the literature to refer to this phenomenon: accuracy (Dunlosky and Hertzog, 2000; Nelson and Dunlosky, 1991), judgment bias (Schraw and Roedel, 1994), or illusion of knowing (Glenberg and Epstein 1985). However, calibration can be used as an umbrella term for all (Pieschl, 2009). This umbrella term "calibration" can be expressed as "extent of congruence between students' estimates of their capabilities [metacognitive judgment] and their actual performance [criterion task]" (Garavalia and Gredler, 2003).

Judgment accuracy can be analyzed in two ways. The first one is called *absolute accuracy,* which expresses "the degree to which the magnitude of the judgments is related to the actual magnitude of target performance" (Dunlosky & Thiede, 2013, p. 59). The second one does not reflect an absolute match between judgment magnitude and target performance. It seeks the "degree to which the judgments discriminate between different levels of performance across items" (Dunlosky & Thiede, 2013, p. 59). This type of accuracy is called *relative accuracy*.

In this study, absolute accuracy was used to define metacognitive calibration of participants since the study does not seek for different levels of performance. Absolute accuracy is calculated by finding the difference between actual performance and learner's judgment about the criterion task. The absolute value of this score shows how calibrated the learner is. Values closer to zero indicate greater accuracy. If absolute value is not taken, then the positivity or negativity of the difference gives the *bias index,* which means *calibration direction*. Positive values indicate overconfidence while negative ones indicate underconfidence. To be more specific, if the learner estimates his/her learning higher than his/her actual performance, then this means learner is overconfident. On the contrary, if s/he estimates his/her performance lower than his/her actual score, then this means s/he has underconfidence.

In order to examine the link between the bias index and academic performance, underestimation and overestimation terms describe judgments of competence to consider their ability as lower or higher compared to their actual ability (Schaefer, Williams, Goodie, & Campbell, 2003). The bias index has been shown to have an effect on learning and its indicators. Some studies show that individuals underestimate their competence in order to protect their self-esteem in any case of failure (Elliot & Church, 2003). Conversely, learners who underestimate their potential are negatively affected and show lower performance compared to the situations that they felt positively and accomplished more (Marsh & Hau, 2004). Overestimation also causes low performance when it leads to poor preparation (Stone & May, 2002; Vancouver & Kendall, 2006).

Calibration curves are used to represent over-and-under confidence on graphs. For such graphs, actual performance is plotted on the y-axis and estimated performance is put on the x-axis. The 45-degree line (identity line) represents perfect calibration where the actual scores fit estimated scores (see figure 1). Points that are under this line show overconfidence while the ones above show underconfidence. In order to illustrate this with an example, one could consider that if a learner estimated his/her performance as 92% while his/her actual score was 75% (point A on figure

1). This means the learner's score is under the identity line and s/he is overconfident. On the contrary, if s/he estimated his/her performance as 0% while his/her actual score was 56% (point B on figure 1), this means the learner's score is over the identity line and s/he is underconfident.

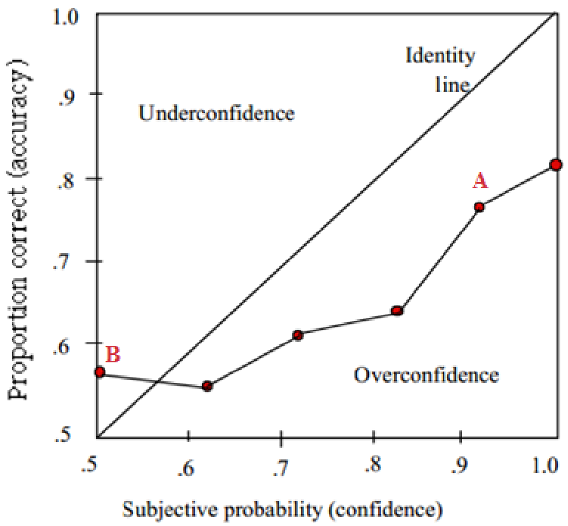


Figure 1: Example of a calibration curve with an identity line (adapted from Pullford, 1996)

Metacognitive calibration can be measured either prospectively or retrospectively. If learners judge their estimated performance before working on task, they are making a prediction. If they evaluate their performance after they worked on the task, they are making a postdiction. As Nelson and Narens (1994) expressed there are three stages of learning; acquisition, retention and retrieval. A prediction judgment is a monitoring judgment that comes after acquisition and retention but before retrieval; postdiction judgment follows retrieval. Therefore, Hacker, Bol and Keener (2008) argued that prediction can be thought of as a prospective monitoring judgment and conversely a postdiction can be thought of as a retrospective monitoring judgment.

Accuracy judgments differed according to depending on pre- or post evaluation. Studies generally report that learners have higher accuracy on retrospective judgments than prospective judgments (Hacker, Bol, Horgan & Rakow, 2000; Hacker, Bol & Keener, 2008; McCormick, 2003).

Important findings can be observed if the relationship between performance and calibration accuracy is analyzed. Researchers found out that high performers tend to be underconfident both in their predictions and postdictions while low performers tend to be overconfident in both (Pressley & Ghatala, 1990; Hacker et al., 2000; McCormick, 2003). Regardless of the type of measurement (i.e. prospective or retrospective), overestimated metacognitive self-confidence is generally observed in low performers and high performers tend to show underestimated metacognitive self-confidence (Chiu & Klassen, 2010). To summarize the relationship between calibration accuracy and performance in terms of bias score, it can be explained as high performers tend to show higher accuracy but underconfidence while low performers tend to show low accuracy but overconfidence (Hacker, Bol & Keener, 2008).

The relationship between type of measurement (i.e. prospective or retrospective) and performance level of students may show us how accuracy and bias scores vary. Furthermore, this study gives an opportunity to analyze these relationships.

Measuring Metacognition

Metacognition can be measured by various methods or scales. However, all of these can be grouped in two broad categories: off-line and on-line measures. Off-line and on-line measures are defined by various researchers working on this construct (Desoete, Roeyers & De Clercq, 2003; Veenman, 2005). On-line data are collected when the individuals are engaged with a specific task at the time of measurement. However, off-line measures assess metacognition either retrospectively or prospectively. Off-line measures usually collect data for measuring general metacognition while some off-line methods can be task specific (Saraç & Karakelle, 2012).

Think-aloud procedure and measures that assess metacognitive experience while engaging on a specific task are on-line measures of metacognition. Commonly used off-line measures are self-report questionnaires, interviews, and teacher ratings. According to Koriat (2000, 2008), off-line methods such as questionnaires and teacher ratings could be more sensitive to assess explicit and conscious processes of metacognition while on-line methods such as think-aloud protocols could be more sensitive to assess implicit and unconscious processes. Some on-line techniques can be applied also prospectively or retrospectively. This does not mean that learners comment on their metacognitive experience like in off-line measures, it still means that the learners are actually using metacognitive skills like in think-aloud protocols. Hence, metacognitive experience measures which are used during the task engagement can be classified as on-line measuring of metacognition.

Prospective and retrospective off-line metacognition measurements and prospective and retrospective on-line techniques to measure various components of metacognition should be dealt with separately to avoid confusion. For example, monitoring accuracy of learners can be measured with online techniques but can be named as prospective and retrospective since pre- and post-evaluations provide the data in calibration studies. Retrospective online measurement is considered when retrospective performance judgement is used with reference to the actual performance (i.e. calibration accuracy). On the other hand, retrospective offline measurements are utilized after a task is completed with an inference to what is happened during the solution process, but may not reveal what actually happened.

Relation studies among these measures stated that there is a significant relationship among on-line measures while measuring metacognition on a specific task (Saraç & Karakelle, 2012). However, off-line measures cannot show a significant relationship among themselves. Saraç and Karakelle (2012) declared that measuring an individual's metacognition within the same criterion task (e.g. problem solving) by using both on-line and off-line measure would enable more precise assessment about metacognition. Literature shows that metacognitive processes form a complex structure and should be assessed by using various methods. Namely, using both on-line and off-line measures would enable researchers to analyze the metacognition as a whole, opposed to studies which use only on-line or only off-line measures. The present study uses both on-line (metacognitive monitoring accuracy scale) and off-line (knowledge inventory) measures to assess metacognition as a whole, and investigate their relationship on both metacognitive judgment accuracy and mathematical problem solving performance. Off-line metacognitive knowledge scale will indicate a level of overall metacognitive knowledge with students' self report. On the other hand, on-line metacognitive monitoring accuracy scale will indicate task specific metacognitive monitoring accuracy.

Problem Solving in Mathematics

Mathematics skills are required in almost all professions in today’s society. Problem solving is an integral part of mathematics. Problem solving is a process used in many areas of mathematics. Thus, necessary skills and principles in mathematics can be learned in a problem solving environment. The National Council of Teachers of Mathematics (NCTM) in the United States emphasizes the importance of problem solving ability among five process standards of teaching mathematics. According to these standards of the NCTM, “students should build new mathematical knowledge through problem solving, solve problems that arise in mathematics and in other contexts, apply and adapt a variety of appropriate strategies to solve problems, and monitor and reflect on the process of mathematical problem solving” (NCTM, 2003, p. 52). Thus, problem solving is a key to constructing mathematical ideas. Moreover, having a high problem solving ability is important not only in mathematical problem solving but also in everyday life situations.

There are various important studies aimed at understanding the components of mathematical word problem solving. Mayer (1985, 1987) has introduced four cognitive processes of mathematical problem solving: translation, integration, planning and execution with an inspiration from Polya's (1957) four-phase description of problem solving activity. Translation can be described simply as finding the counterpart of each sentence in student’s mental representation. Integration is selecting and combining information in the problem within a logical representation of whole nature of problem. Planning is separating the problem into serial steps to solve. Finally, execution is implementing the planned steps and doing mathematical operations. Mathematics achievement tests, especially some National

Tests like the SBS and OSS in Turkey, assess the achievement of students in the execution process. Studies have showed that how students comprehend and represent problems (translation and integration) and monitor steps to solve the problem (planning and execution) is very important and prerequisite for each other (Stevenson, Lee & Stigler, 1986).

Problem solving can be separated into two aspects: problem solving comprehension and problem solving performance. Various studies imply that comprehending a problem is more difficult than the computation procedure necessary to solve the problem (Hegarty, Mayer & Monk, 1995). Problem solving comprehension is related to recognizing the key ideas and relationships in a problem. Brown and Walter (1993) has stated that understanding the problem, by considering key ideas and relationships, provides problem solvers with considering new situations and ideas while monitoring and controlling these ideas. Understanding the problem and using the ability to regulate, monitor and control the learning process have a crucial effect on implementing planned strategies (Stevenson, Azuma, & Hakuta, 1986; Akama & Yamauchi, 2004; Swanson, Hoskyn & Lee, 1999).

Previous studies have showed that comprehending the problem is a prerequisite to arrive at the solution. Assessing children's ability to mentally represent and understand an arithmetic word problem rather than to compute a numerical answer is essential (Mayer, Tajika & Stanley, 1991; Mayer et al., 1997). Sarver's study (2006) showed that good problem solvers focus on the structural features of a problem rather than its surface characteristics. In order to understand the structural facets of a mathematical word problem, students need to comprehend the problem and know what they are actually doing in order to implement planned strategies. Problem comprehension and regulating this understanding are all essential to monitor and regulate metacognitive skills which are necessary to be a good problem solver.

Metacognitive Skills in Mathematical Problem Solving

With the increase in metacognition studies, effects in various areas have begun to be investigated. These early studies showed that metacognition has been related to many cognitive abilities and aptitudes, such as intelligence (Borkowski, 1985), reading comprehension (Cross & Paris, 1988) abilities in mathematics (Schoenfeld, 1987), and memory (Pressley, Borkowski, & O'SulIivan, 1985). Many studies have described that metacognition and problem solving are theoretically connected by including components of each other (Hertzog & Robinson, 2005; Naglieri & Das, 2005; Sternberg, 2003; 2005; Sternberg & Ben-Zeev, 2001). The effect of metacognitive skills is significant also when compared to the effect of aptitude on problem solving performance. According to Swanson (1990), metacognitive skills are more effective in problem solving performance than aptitude is. In that sense, Swanson (1990) reported that children with high metacognitive knowledge but low aptitude outperform children with low metacognitive knowledge and higher aptitude scores.

Studies related to the link between metacognition and mathematical problem solving showed that using both cognitive and metacognitive strategies together increases students' mathematical word problem solving performance (Artz & Armour-Thomas, 1992; Carr & Jessup, 1997). Similarly, various studies have reported metacognition as essential to mathematical problem solving (Borkowski, 1992; Carr & Biddlecomb, 1998; De Corte, Verschaffel & Op'tEynde, 2000). It can be claimed that during the process of mathematical problem solving, metacognitive skills play a crucial role (Coutinho, Weimer-Hastings, Skowronski & Britt, 2005; Fuchs et al., 2003; Mayer, 2006; Swanson, 1990; Swanson, 1992).

Some metacognitive statements can also be used in order to assess metacognitive skills in mathematical problem solving. Four groups related to the metacognitive skills in mathematical problem solving were formed in a study by Fortunato, Hecht, Tittle and Alvarez (1991). These are similar to a combination of Mayer's mathematical problem solving steps (translation, integration, planning and execution) and Brown's metacognitive skills (prediction, planning, monitoring and evaluation): interpreting the problem and planning solution strategies, specific strategies or ways of working out the problem, monitoring of the solution processes, evaluation of the execution of the problem. Unfortunately, Polya's (1957) and Mayer's (1985) four-step problem solving conceptualization can express metacognitive activity only implicitly. Being successful in mathematical problem solving not only depends on these cognitive skills but also depends on metacognitive skills (Lester, 1994). Therefore, students' cognitive processes should be learned through their predictions and evaluations about a specific task.

Prediction and evaluation, metacognitive skills, are very important components of calibration. Calibration characterizes how individuals are aware of their own internal processes since it is the accuracy between actual score of a task and the learner's perception about the task. Winnie and Muis (2011) investigated calibration in several areas in order to figure out if the calibration concept was task specific. According to this study, students' calibration level on general ability and word recognition were similar while mathematics related calibration was lower than both. Therefore, calibration can be seen as task specific and for mathematics area it should be analyzed with its own tasks like ability in algebra or performance in problem solving.

While some researchers claimed that there was a moderate relationship between performance and calibration accuracy in the related literature (Glenberg & Epstein, 1985), others claimed that calibration and performance were barely correlated (Lin & Zabrucky, 1998). For investigating this construct in a mathematics context, some studies found a significant relationship between arithmetic performance and calibration (Desoete & Roeyers, 2006). Since Turkish students and their low mathematics scores are the issue in this study, it is important to consider the studies on calibration in Turkey. Calibration studies are very scarce in Turkey (e.g. Sarac & Tarhan, 2009, in comprehension calibration and Özsoy, 2012; Özsoy & Kuruyer, 2012, in mathematics related calibration). Findings related to mathematics education showed the existence of the significant relationship between algebra performance and metacognitive calibration (Özsoy, 2012). However, no significant relationship was found between mathematical problem solving and metacognitive calibration (Özsoy & Kuruyer, 2012).

Studies about calibration in Turkey related with mathematics have not been encountered frequently during the literature survey. Existing studies about the problem solving area of mathematics rarely found a significant relationship unlike algebra. It has been concluded that studies which aim to fill a gap on mathematics related calibration literature in Turkey was needed.

Another issue, which needs further studies is measuring calibration. Literature gives the results of calibration by looking at either pre-evaluations or post-evaluations of students. Generally post-evaluation procedure was used in mathematics related calibration research (e.g. Desoete & Roeyers, 2006; Nietfeld &

Schraw, 2002; Özsoy & Kuruyer, 2012). Few studies preferred pre-evaluations over post-evaluations for investigating calibration in mathematics (e.g. Özsoy, 2012). So, there is a need to implement both pre- and post evaluations in mathematics related calibration studies.

Metacognition is important with all of its components (prediction, planning, monitoring and evaluation) in mathematical problem solving. By assessing prediction and evaluation components of metacognition, average and above average problem solvers in mathematics can be differentiated (Deseote, 2006). The planning part of mathematical problem solving is also affected by metacognitive skills. Swanson (1990) has showed that students with high metacognitive skills need fewer steps in problem solution. Metacognitive skills are also important in execution step of mathematical problem solving. The same study (Swanson, 1990) has reported that students with high metacognitive skills were more efficient in solution execution than students with low metacognitive skills. Similarly, treatment studies showed that through metacognitive training, students' ability in overall to solve mathematics problems improves (Jacobse & Harskamp, 2012). In the light of all these findings from the literature, understanding the relationship between problem solving performance and components relating to metacognitive monitoring accuracy in problem solving situation is important. Moreover, investigating how students' performances differ according to their metacognitive knowledge quality can be a contribution to the existing literature. The current study offers notable relationship analyses on this issue in terms of problem solvers' calibration accuracy and metacognitive knowledge.

The Problem of the Study and Research Questions

The Problem of the Study

Mathematical problem solving has been considered as one of the most important issues in mathematics education. Since metacognition is the awareness of one's own mental process and ability of self-regulate performance, an effective mathematical problem solver is expected to regulate both his/her knowledge and the process of problem solving. From a metacognitive point of view, if students are able to judge their metacognitive experiences accurately, then their problem solving performances will be affected by such accurate calibration. Thus, the relationship between metacognitive calibration accuracy and performance needs to be analyzed.

Various research studies have been conducted to understand the relationship between mathematics achievement and metacognition. However, there is no consensus about their correlations. The present study aimed to examine for this relationship. Moreover, the effect of high metacognitive accuracy on problem solving performance level is another issue investigated in this study.

Although the literature indicates that using on-line and off-line measures together enables researchers to analyze the metacognition as a whole; this technique is not very commonly found in the literature. This study aims to apply both types of measures and figure out their relationship.

Lastly, the bias index of confidence judgments of students were investigated in some studies. However, those studies showed their evidences only prospectively or retrospectively. The current study aimed to used both pre- and post evaluations of online metacognitive calibration accuracy in order to understand their different implications.

The purpose of this research was to investigate relationships among metacognitive knowledge, metacognitive calibration accuracy and mathematical problem solving performance. In order to measure metacognition more holistically, a metacognitive knowledge inventory and metacognitive calibration (both prospective and retrospective) were taken into consideration together. Prospective metacognitive calibration was measured by asking students about their prediction about solution correctness. Retrospective metacognitive calibration was measured by students' evaluation about solution correctness. Mathematical problem solving performance was assessed through three word problems.

It is obvious that this study aimed to contribute to the literature in the context of education by investigating those relationships. In the light of these relationships, it might be possible to investigate the importance of metacognitive knowledge, metacognitive monitoring and calibration on high and low performance of problem solving.

Research Questions

This study focused on six main research questions.

1) What is the relationship between prospective and retrospective judgments of monitoring accuracy in a context of mathematical problem solving?

2) What is the relationship between metacognitive monitoring accuracy (prospectively and retrospectively) and mathematical problem solving performance?

3) What is the relationship between metacognitive knowledge and mathematical problem solving performance?

4) What is the relationship between offline measure of metacognitive monitoring accuracy and online measure of metacognitive knowledge?

5) Is there any significant difference between overconfident and underconfident learners in terms of mathematical problem solving performance?

6) Is there any significant difference between overconfident and underconfident learners in terms of metacognitive knowledge?

CHAPTER 3

METHOD

Sample

The sample was chosen from seventh grade students of four different (two public and two private) schools in İstanbul by using convenient sampling. There were 200 participants in the study and their ages were between 12 and 14. Fifty-two percent of the participants were female (n=104) and 48 percent were male (n=96). Fifty-five percent of the students (n=110) came from private school and rest, 45 percent (n=90), came from public schools. Students' distribution according to gender and school type for the study is shown in Table 1. There is no analysis according to the school type or gender. The aim was only to show the representativeness of groups inside the sample of the study. The study was conducted in the second semester of the 2012/13 academic year in Turkey.

Table 1. Gender and school type distribution of participants in main study

N_{total}=200		Participants	
		N	%
GENDER	Female	104	52
	Male	96	48
SCHOOL	Private	110	55
	Public	90	45

Instruments

Metacognitive Monitoring Accuracy Scale

The Metacognitive Monitoring Accuracy Scale was used to assess student's present metacognitive predictions and judgments on mathematics problems. It is adapted from Everson and Tobias' (1998) study. Students estimated/judged solution correctness both prospectively (Appendix A) and retrospectively (Appendix B). Students' metacognitive monitoring accuracy were measured by asking "How correctly do you think you can solve the problem?" for the prospective measure and "How correctly do you think you solved the problem?" for the retrospective measure. Students could judge solution correctness in a four-point Likert type format: Not at all (0), Little (1), Quite (2), Very (3).

Metacognitive Skills Inventory

Students' knowledge regarding their metacognitive skills was gathered by using the Metacognitive Skills Inventory (Çetinkaya, 2000) (see Appendix C). There were 32 items in the scale. The items required responses about the frequency of students' experiences and all items were four-point Likert-type, from 1 (never) to 4 (always). The scale comprised four dimensions: self-checking, awareness, cognitive strategy use and evaluation to score the knowledge of students on metacognitive skills quantitatively.

Çetinkaya and Erktin (2002) conducted the reliability analysis of the inventory with 111 sixth grade students of a private school and the Cronbach Alpha was calculated as 0.87. The internal consistency of the entire scale had been reported as acceptable. A principal component analysis and varimax rotation was also applied to see how the items grouped under each domain; overlapped domains and factors were shown as evidence of construct validity (Çetinkaya and Erktin, 2002).

Problem Solving Performance

Three mathematics problems were given to the students to assess their problem solving performance (see Appendix D). At the beginning many problems were found from several 7^{th} grade mathematics books, and then three of them were chosen by the researcher and a mathematics teacher. While choosing the problems, several criteria were considered, such as the difficulty level and being a multistep problem. Readability of the mathematics problems were also checked by an expert in the field of Turkish language training.

The first problem is about solving equations. Students can solve the first part of the problem by using the given information in the problem and forming an algebraic equation. Then, the second part of the problem can be solved by using the findings of the previous equation. In the second problem, students had to read the problem very carefully. With two steps of arithmetic operations, the problem could be solved. The third problem was related to the percentages concept. After students did the necessary reasoning and calculations about percentages, the rest of the problem could be solved by using arithmetic operations.

All problems were scored by using a Holistic Scoring Rubric (see Appendix E). It was adapted from a rubric originally developed by the Center for Research on Evaluation, Standards, and Student Testing (CRESST) in 1995 (Aschbacher et al., 1995 cited in Aşık, 2006). The rubric was modified for this study by changing the scoring interval from 0-4 to 0-3 in order to get a useful data for accuracy analysis since the other aspects in the study ranged from 0 to 3. Students' performances in each problem were scored as a whole to assign one score for each problem. To be consistent in each problem, the rubric describes criteria for each score.

The holistic scoring rubric of this study used a scale ranging from 0 to 3. Each score corresponded to indicators of students' performances as follows: 0 meant totally wrong answer or no answer at all; 1 meant an incomplete solution by selecting some of the appropriate strategies; 2 meant selecting appropriate strategies but solution is not totally correct; 3 meant totally a correct solution.

A mathematics teacher also scored problem solving performance scales of 20 participants who were chosen randomly among 200 participants. Scoring was done independently from the researcher's scores. The same holistic scoring rubric was used again. All three mathematics problems were scored separately and then inter-rater reliability was calculated. Inter-rater reliability was found as 98%. One different score between the two coders was discussed and resolved with consensus.

Procedure

All students were administered 3 tests. Firstly, they answered the questions for judgment of solution correctness and problem solving performance. Then, the metacognitive skills inventory was carried out.

Students' judgment of solution correctness is measured with the following procedure: After they saw all three problems but before being given time to solve them, they gave their prospective judgments for each problem. Then, they worked on each problem and solved them by showing every step of the solution on the paper. Finally, solution papers are taken and retrospective judgments are given by students for each problem.

Timing of the procedure for each instrument was determined and checked by the researcher in each class. Teachers observed students while they were answering the problems. In some of the schools, if more than one class took the tests at the same time, so the researcher checked the classes and the procedure periodically by visiting the classes.

The prospective measurement of judgment of solution correctness took 5-6 minutes for all three math problems. Students solve the problems in 15-20 minutes. Afterwards, the retrospective measurement of judgment of solution correctness took 5-6 minutes. In the last period of the first lesson, which was the time used by the researcher, the students filled out the Likert type scale for measuring their metacognitive knowledge.

Data Analysis

This study is mainly a relationship study. Thus, a bivariate correlational analysis was used to investigate the relationships between variables. For the different performance groups in the variables, and over/underconfident groups, an independent samples t-test was carried out. Graphs were used to present the data visually. All analyses were conducted by using statistical analysis software, Statistical Package for Social Studies (SPSS version 17.0).

CHAPTER 4

RESULTS

In this section, various descriptive measures - the range, mean and standard deviation - of the scores obtained from the instruments used to measure the variables are presented. Secondly, the correlation coefficients between the variables and significant differences between over/underconfident groups are calculated and presented. Results are presented in graphs when necessary.

Descriptive Characteristics of the Data

Descriptive statistics (range, mean and standard deviation) of variables are given in Table 2, where *pros_monitoring accuracy* and *retros_monitoring accuracy* indicate prospective metacognitive monitoring accuracy and retrospective monitoring accuracy, respectively. *MK* means the metacognitive knowledge score of students and *total_perf* shows the total score gained from all three mathematics problems.

Table 2. Descriptive statistics of variables

	N	Range	Mean	Std. Deviation	Variance
pros_monitoring accuracy	200	0-9	3.55	1.63	2.67
retros_monitoring accuracy	200	0-8	2.05	1.73	2.98
MK	200	18-97	66.91	15.52	240.75
total_perf	200	0-9	3.79	2.33	5.44
Valid N (listwise)	200				

Problem Difficulty

Problem difficulty was investigated by considering the number of correct answers from the participants in this study. Descriptive data showed that students struggled to solve the first problem, since 175 students out of 200 had 0 point from this problem. On the contrary, 119 students had 3 points (the highest score) for the second problem. Portion of correct answers for problem 3 was more than problem 1 and also less than problem 2 according to the overall results. The first problem can be named as a problem with low correctness while second problem can be named as problem with high correctness. All distributions for students' scores were given in Table 3. When high performers' scores were examined, they got 0 points only from problem with low correctness. Similarly, low performers generally got high points (2 or 3 points) from problem with high correctness.

Table 3. Descriptive statistics of students' scores for each problem

Scores \ Problem	Number of students for Problem-1		Number of students for Problem-2		Number of students for Problem-3	
0	175	96 low performers	32	32 low performers	89	89 low performers
		79 high performers		0 high performer		0 high performer
1	11	1 low performers	12	11 low performers	27	7 low performers
		10 high performers		1 high performers		20 high performers
2	5	1 low performers	37	14 low performers	12	0 low performer
		4 high performers		23 high performers		12 high performers
3	9	0 low performer	119	41 low performers	72	2 low performers
		9 high performers		78 high performers		70 high performers

Relationship between Prospective and Retrospective Judgments

In this study, metacognitive monitoring accuracy was measured both prospectively and retrospectively. Prospective and retrospective judgments were found to be positively correlated. The relationship coefficient was calculated as $r = .417$, $p < .01$, two-tailed.

This relationship differs across problems although they are all significantly positively correlated for all three problems. For the first problem, the relationship between prospective and retrospective judgment was found $r = .243$, $p < .01$, two-tailed. For the second problem, it was calculated as $r = .414$, $p < .01$, two-tailed. For the third problem, the relationship between prospective and retrospective judgment was found $r = .263$, $p < .01$, two-tailed.

Prospective and retrospective judgments were also analyzed. For all three mathematics problems, in total students were able to get a calibration score between -9 and +9, for both the prospective and retrospective calibrations. When the absolute value is taken, the range differs between 0 and 9. Students with a score of 0, 1, 2, 3 were classified as *calibrated.* On the other hand, students with the score of 4 or higher were classified as *not calibrated*. According to the frequency analyses, retrospective judgments are more accurate than prospective judgments. While 50.5 % of students (N=101) were prospectively calibrated, 82.5% of the students (N=165) were retrospectively calibrated (see Table 4 and Table 5).

Table 4. Prospective Monitoring Accuracy Frequency

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0	5	2.5	2.5	2.5
	1	9	4.5	4.5	7.0
	2	42	20.9	21.0	28.0
	3	45	22.4	22.5	50.5
	4	50	24.9	25.0	75.5
	5	29	14.4	14.5	90.0
	6	8	4.0	4.0	94.0
	7	9	4.5	4.5	98.5
	8	2	1.0	1.0	99.5
	9	1	.5	.5	100.0
	Total	200	99,5	100.0	
Missing	System	1	.5		
Total		201	100.0		

Table 5. Retrospective Monitoring Accuracy Frequency

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0	38	18.9	19.0	19.0
	1	52	25.9	26.0	45.0
	2	45	22.4	22.5	67.5
	3	30	14.9	15.0	82.5
	4	11	5.5	5.5	88.0
	5	14	7.0	7.0	95.0
	6	8	4.0	4.0	99.0
	7	1	.5	.5	99.5
	8	1	.5	.5	100.0
	Total	200	99.5	100.0	
Missing	System	1	.5		
Total		201	100.0		

Calibration and Performance Relation

Calibration between metacognitive monitoring accuracy and mathematical problem solving performance was calculated for both prospective and retrospective measurements. Prospective accuracy and performance scores were found to be negatively correlated, $r = -.452$, $p < .01$, two-tailed. This result indicates that high performance is positively correlated with greater accuracy according to prospective judgments since smaller values of difference scores indicate higher accuracy.

Retrospective metacognitive monitoring accuracy and performance scores were also found to be negatively correlated, $r = -.272$, $p < .01$, two-tailed. This correlation coefficient also indicates that high performance is positively correlated with greater accuracy according to retrospective judgments since smaller values of difference scores indicate higher accuracy. However, the relation between performance scores and retrospective judgments is weaker than the relation between performance scores and prospective judgments.

Calibration and Performance Relationship According to Performance Levels

Students' performance mean was found as 3.79. Students with scores under 3.79 were classified as low performers; those having scores above this level were classified as high performers.

Ninety-eight students were identified as low performers. There is no significant relationship between low performers' performance scores and their prospective metacognitive monitoring accuracy, $r = -.174$, $p = .087 > .01$, two-tailed. Moreover, there is no significant relationship between low performers' performance scores and their retrospective metacognitive monitoring accuracy, $r = -.058$, $p = .571 > .01$, two-tailed.

One hundred two students were identified as high performers. In contrast to low performers, there is a significant relationship between high performers' performance scores and their prospective metacognitive monitoring accuracy, r= -

.442, $p < .01$, two-tailed. However, there is no significant relationship between high performers' performance scores and their retrospective metacognitive monitoring accuracy, $r = -.091$, $p = .362 > .01$, two-tailed.

These results indicate that only high performers' prospective judgments show difference among the other relationships in terms of performance level and metacognitive monitoring accuracy.

Descriptive Characteristics of Calibration Direction

To calculate calibration, the difference between students' judgments about solution correctness and actual performance was calculated for each of the three problems. If the difference is negative, this means underconfidence; if it is positive, this indicates overconfidence. A score of 0 indicates perfect calibration. Scores ranged from -3 to +3.

Prospective Judgment Bias

In problem 1, only three students had underconfidence; 172 students had different levels of overconfidence; 25 students were well calibrated with 0 differences (Table 6a).

In problem 2, 71 students had underconfidence with different levels of negative scores. Sixty-one students had different levels of overconfidence while 68 students were perfectly calibrated (Table 6b).

In problem 3, 41 students had different levels of underconfidence; 104 students had different levels of overconfidence; 55 students were well calibrated with 0 differences (Table 6c).

Retrospective Judgment Bias

In problem 1, only seven students had underconfidence; 78 students had different levels of overconfidence; most of the students (115) were well calibrated with 0 differences (Table 7a).

In problem 2, 58 students had different levels of underconfidence; 43 students had difference levels of overconfidence; 99 students were well calibrated with 0 differences (Table 7b).

In problem 3, 35 students had different levels of underconfidence; 72 students had different levels of overconfidence; 93 students were well calibrated with 0 differences (Table 7c).

Table 6a. Prospective Judgment for P1

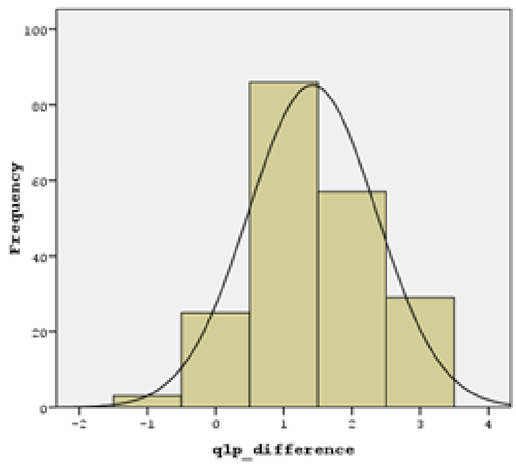


Table 6b. Prospective Judgment for P2

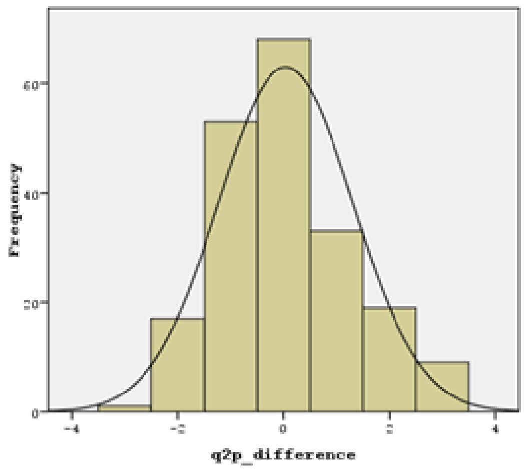


Table 6c. Prospective Judgment for P3

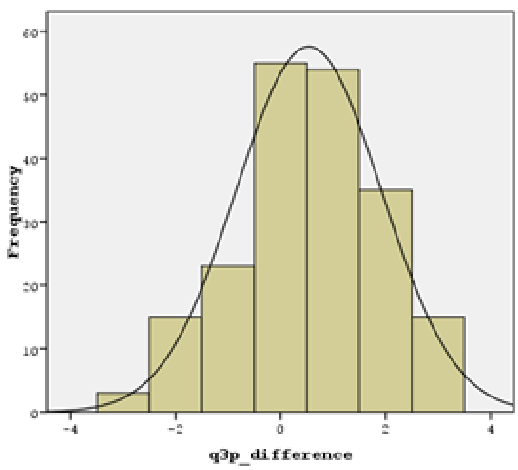


Table 7a. Retrospective Judgment for P1

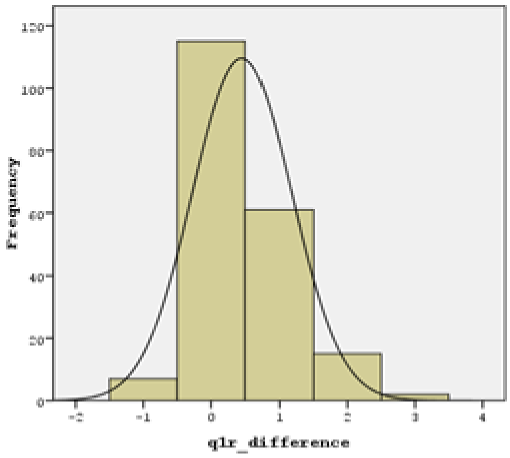


Table 7b. Retrospective Judgment for P2

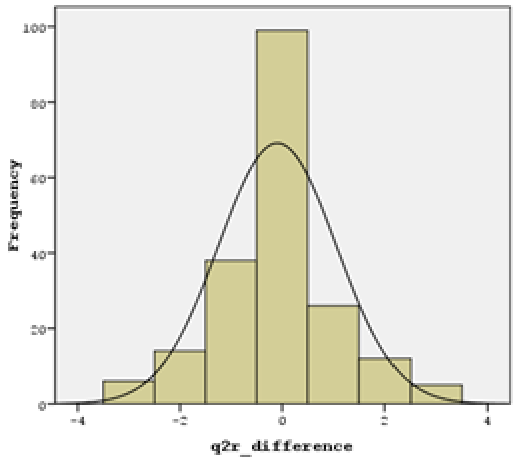
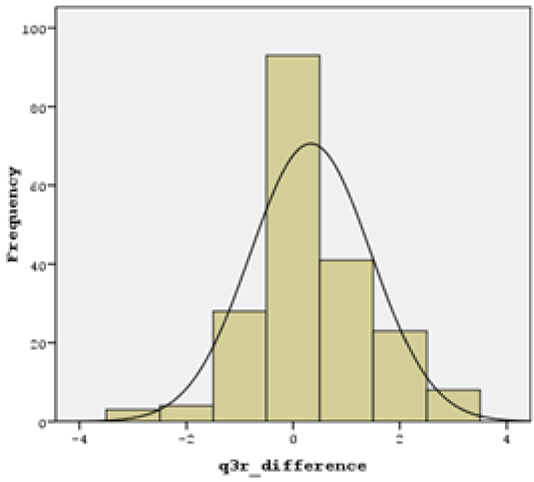


Table 7c. Retrospective Judgment for P3



Students' distributions in terms of calibration direction both prospectively and retrospectively are given in the following tables (See Table 8a, Table 8b, and Table 8c).

Table 8a. Distribution of students in terms of calibration direction for P1

1st problem	underconfident	well-calibrated	overconfident
prospective	3	25	172
retrospective	7	115	78

Table 8b. Distribution of students in terms of calibration direction for P2

2nd problem	Underconfident	well-calibrated	overconfident
prospective	71	68	61
retrospective	68	99	43

Table 8c. Distribution of students in terms of calibration direction for P3

3rd problem	Underconfident	well-calibrated	overconfident
prospective	41	55	104
retrospective	35	93	72

The Relation between Over/Underconfidence and Performance

Subjects were divided into 2-groups based on the difference between their performance judgments and actual performance for prospective and retrospective measures. If the difference was positive or 0 in all three problems, they were classified as overconfident. If the difference was negative or 0 in all three problems, they were classified as underconfident. After the data were separated, subjects with a score of 0 in differences for all three problems (namely perfect accuracy in all problems) were excluded from both prospective and retrospective judgments.

For prospective judgments, an independent samples t-test showed that performance scores of underconfident students (*M= 5.68, SD= 1.59)* were significantly higher than performance scores of overconfident students (*M= 2.97, SD= 2.47), t(122,667) = 7.814, p=.000*, *d= 1.56.* The effect size for this analysis (*d* = 1.56) was found to exceed Cohen's (1988) convention for a large effect (*d* = .80).

For retrospective judgments, an independent samples t-test showed that performance scores of underconfident students (*M= 5.18, SD= 2.049)* were significantly higher than performance scores of overconfident students (*M= 2.89, SD= 2.273), t(123) = 5.396, p=.000*< .001, *d= 1.06*. The effect size for this analysis (*d* = 1.06) was found to exceed Cohen's (1988) convention for a large effect (*d* = .80).

Metacognitive Knowledge and Performance Relation

The relationship between metacognitive knowledge level and mathematical problem solving performance was calculated. The metacognitive knowledge level of students was found to be positively correlated with mathematical problem solving performance, $r = .236$, $p < .01$, two-tailed. This result indicates that high metacognitive knowledge is positively correlated with greater performance in mathematical problem solving but the correlation coefficient is very small.

Relation Between Online and Offline Measures of Metacognition

Relationships between offline (Metacognitive Skills Inventory) and online measures (prospective and retrospective metacognitive monitoring accuracy) have been calculated.

There is no significant relationship between offline (Metacognitive Skills Inventory) and online prospective (prospective metacognitive monitoring accuracy) measures. The coefficient was found as $r = .056$, $p = .433 > .01$, two-tailed.

Offline (Metacognitive Skills Inventory) and online retrospective (retrospective metacognitive monitoring accuracy) measures were also found not to be significantly related. The relationship direction between two was negative, but they were not statistically significant: $r = -.058$, $p = .417 > .01$, two-tailed.

The Relationship between Online and Offline Measures of Metacognition according to Performance Levels

When no significant relationship was found between online and offline measures of metacognition for the whole group, the same relationships were calculated after participants were divided into two groups as low and high performers.

Offline (Metacognitive Skills Inventory) and online prospective (prospective metacognitive monitoring accuracy) measures were found to be significantly and positively correlated, $r= .292$, $p< .01$, two-tailed, for low performers while there was no significant relationship between them for high performing group ($p= .442 > .05$, two-tailed).

There is no significant relationship between offline (Metacognitive Skills Inventory) and online retrospective (retrospective metacognitive monitoring accuracy) measures neither for high performers ($p= .284 > .05$, two-tailed) nor for low performers ($p= .517 > .05$, two-tailed).

These results indicate that only low performers' relationship between offline (Metacognitive Skills Inventory) and online prospective (prospective metacognitive monitoring accuracy) measures showed difference among the others.

Over/Underconfidence and Metacognitive Knowledge Relation

In order to figure out whether there is a significant difference between metacognitive knowledge scores of overconfident and underconfident students or not, independent samples t-test was calculated both for prospective and retrospective judgments.

For prospective judgments, independent samples t-test showed that there is no significant difference between overconfident ($M= 68.09$, $SD= 16.407$) and underconfident ($M= 68.11$, $SD= 17.093$) students' metacognitive knowledge scores in terms of metacognitive knowledge scores ($t(123) = .005$, $p= 835>.000$).

For retrospective judgments, independent samples t-test showed that there is no significant difference between overconfident ($M= 67.89$, $SD= 15.572$) and underconfident ($M= 65.60$, $SD= 14.162$) students' metacognitive knowledge scores in terms of metacognitive knowledge scores ($t(123) = -.790$, $p= 245>.000$).

CHAPTER 5

DISCUSSION

The results of the study will be discussed and interpreted in the light of the literature. In the literature, moderate correlations were demonstrated between prospective and retrospective measures of metacognitive monitoring (Desoete, 2008; Veenman, 2003). The current study also showed that there was a moderate link between prospective and retrospective judgments. Although they are related at a level, it will be meaningful to further analyze their relations with the other variables separately as prospective and retrospective not as a whole metacognitive judgment since the relationship between them is not very strong. For both this study and further studies, analyzing prospective and retrospective judgments separately will help to interpret the internal dynamics of metacognition more clearly.

The present study showed that retrospective judgments are more accurate than prospective judgments. This result is consistent with many of the literature findings regarding higher accuracy on retrospective judgments compared to prospective judgments (Hacker, Bol, Horgan & Rakow, 2000; Hacker, Bol & Keener, 2008; McCormick, 2003). The findings of retrospective judgments are more difficult to interpret than prospective judgments because of the testing effect. Since the researchers do not know whether the learners are more experienced or more accurate in their retrospective judgments, this is a puzzle for now. However, the accuracy differences of prospective and retrospective judgments under the relationship with actual performance can be discussed.

The relationship between performance and calibration accuracy is also often researched in recent studies. While some researchers claimed that there was a moderate relationship between performance and calibration accuracy in the related literature (Glenberg & Epstein, 1985), others claimed calibration and performance were barely correlated (Lin & Zabrucky, 1998). While investigating this construct in a mathematics context, some studies found a significant relationship between mathematical problem-solving performance and calibration (Desoete & Roeyers, 2006). The findings of the present study also showed a significant relationship between mathematical problem-solving performance and metacognitive calibration accuracy both prospectively and retrospectively. However, the relationship for the prospective case was found to be stronger than the retrospective relationship. This result indicates that prospective judgments constitute a more relevant link in problem solving performance. As mentioned above, retrospective judgments were shown to be more accurate than prospective judgments. However, this does not indicate that the relationship between retrospective judgment and performance is necessarily strong.

This issue can be seen more clearly when we look at the relationship between performance and calibration accuracy in terms of performance levels. There is no significant relationship between performance and calibration accuracy neither prospectively nor retrospectively, for low performers. On the other hand, there is a significant relationship between performance and calibration accuracy prospectively for high performers. However, retrospective relationship was not found significant also for high performers. These results make the previous findings meaningful since prospective accuracy and performance relationship was stronger than retrospective accuracy and the performance relation. This difference comes from high performers' accurate judgments in prospective measure. It can be concluded that accurate prospective judgement differentiates high performers from low performers.

In calibration studies, judgment bias has been an indispensable issue. When the present study investigated over- and underconfidence judgments of participants for all three mathematics problems, prospective and retrospective differences were found in all three problems. The numbers of overconfident students decreased from prospective to retrospective judgment in all three problems. This result supports the previous findings of Nelson (1999) related to students' tendency to be grossly overconfident in their prospective judgments compared to retrospective ones.

Problem by problem analysis showed that numbers of well-calibrated students increased from prospective to retrospective judgment in all three problems. These numbers are particularly conspicuous in problem 1 with 25 well-calibrated students in prospective judgment and 115 well-calibrated students in retrospective judgment. This may result from the difficulty level of this problem. Descriptive analysis for the problem difficulty demonstrated that first problem has a small number of correct answers; second problem has a large number of correct answers; and third problem is between the two. The numbers of high performers were high in all three problems for high scores (2 and 3 points) in comparison with the numbers of low performers. Since low performers could not *predict* their performance accurately, the number of well-calibrated students increased in the retrospective measure while the number of overconfident students decreased. For example in the first problem, it can be observed that 172 overconfident students in prospective judgments generally made accurate judgments in retrospective measure of this problem. Hence, numbers of 25 well-calibrated students in prospective measure become 115 students in retrospective judgments.

In the other two problems, numbers of underconfident and overconfident students always decreased while the number of well-calibrated students increased from prospective to retrospective judgments. Low performance in the first problem (problem with *low correctness*) actually affects participants' judgments. 175 students got 0 points from this hard problem, and if they could not answer this problem they might judge themselves either accurately or with overconfidence. There was no other choice since they could not judge their own performance (which is 0 in that problem) below 0. Thus they cannot be classified as underconfident. And this explains why there are 172 overconfident and 25 well-calibrated students in prospective judgments while there are only 3 underconfident students.

The huge difference between the numbers of well-calibrated and overconfident students shows that low performers could not *predict* their performance accurately. These descriptive data and comments are similar to literature findings. Students show more accurate calibration on easy items compared to difficult items. Similar to findings from other studies, students display underconfidence on easy items but overconfidence on difficult items and researchers previously called this as the “hard-easy” effect (Hacker, Bol & Keener, in press). In terms of high and low performers, Hacker, Bol & Keener (in press) concluded that high performers were better predictors of what they know or do not know on a test, indicating better calibration accuracy. That also corresponds with the current study's discussions on the distribution of well-calibrated high and low performers according to level of correct answers for problems.

Differences between overconfident and underconfident students in terms of performance is worth investigating. The current study found a significant difference between overconfident and underconfident students' performances both prospectively and retrospectively. According to the results, underconfident students' performance scores are higher than overconfident students' performance scores. This result of the present study is consistent with the literature findings that higher-performing students tend to be underconfident both in their predictions and postdictions while lower performing students tend to be overconfident in both (Hacker et al., 2000). This important finding indicates that low performers are unaware of their deficiency and cannot predict their performances. Hence, they generally overestimate their future performance in problem solving situation. Problem by problem analyses have showed that, especially in relatively difficult problems, low performers exaggerate their future performances. These findings are very important for school teaching, specifically for mathematics teaching. Students' ability to judge how well they can monitor their performances while solving problems is an essential skill to perform better. This is because if students cannot produce accurate calibration, they may not notice possible mistakes during the problem solution or may not evaluate the solution accurately.

The other variable of the study was metacognitive knowledge. The relationship between metacognitive knowledge and mathematical problem solving performance has been found to be significantly related. This finding supports a widely accepted view in the literature which is that metacognitive knowledge influences mathematical problem solving (e.g. Borkowski, Chan, & Muthukrishna, 2000).

Another research question of the recent study was the relationship between offline and online measures of this study. No significant relationship was found between metacognitive knowledge scores (offline measure) and metacognitive calibration accuracy scores (online measure) either prospectively or retrospectively.

However, when the relationships were investigated in terms of performance level, low performers' prospective online metacognitive accuracy and metacognitive knowledge was found to be significantly related. Findings from related literature have been supported with this result since some studies suggested that students' metacognitive knowledge can influence the self-assessments of performance (Lin & Zabrucky, 1998; Nietfeld & Schraw, 2002). For the relationship between metacognitive knowledge and metacognitive calibration accuracy, prospective judgments and becoming a low performer are differential factors. For low performers, metacognitive knowledge plays an important role in accuracy of predicting problem solving performance. It can be deduced that low performers had either a lack of necessary metacognitive knowledge to predict their problem solving achievement or else did not recognize the relevance of their knowledge for mathematical problem solving performance, so they could not make an accurate prediction about their possible performance. This conclusion is also supported by the related literature on the issue of relationship between metacognitive knowledge and below-average students' problem solving performance (Baker, 1994). However, retrospective judgment is not a differential factor neither for low nor for high performers since students generally can *evaluate* their performance regardless of their performance level.

The last point related to this study which should be discussed is metacognitive knowledge differences between overconfident and underconfident students. There is no significant difference between overconfident and underconfident learners' metacognitive knowledge, neither for prospective nor for retrospective confidence scores. Metacognitive knowledge does not constitute a differentiating factor for the direction of bias scores. Hence, it can be concluded that a lack of necessary metacognitive knowledge makes a deviation in accurate judgment regardless of its direction, so that students with insufficient metacognitive knowledge become either overconfident or underconfident but not well-calibrated.

CHAPTER 6

CONCLUSION

Important results will be summarized in this section in order to emphasize the significance of the study and its implications in the light of the literature and findings. The present study was set out to investigate relationships among metacognitive knowledge, metacognitive calibration accuracy and mathematical problem solving performance. While investigating these relationships, different levels of performance on mathematical word problem solving and students' metacognitive calibration levels were important. The starting point was Turkish students' low mathematics performances both in national and international exams. Since high performers have powerful self-evaluation skills which enables recognizing possible strategy deficiencies or solution mistakes (Zimmermann, 2000), low performers' self-evaluation skills are needed to be analyzed like in this study. The importance of the study is based on the following points, which are highlighted in the related literature:

1) There is no consensus about the relationship between metacognitive calibration and mathematics performance.

2) Studies related to calibration direction usually showed their evidences only by pre- or post-evaluations but not at the same time on the same task.

This study reports the existence of a significant relationship between metacognitive calibration and mathematical problem solving performance. While there was no consensus about this relationship in related literature (Glenberg & Epstein, 1985; Lin & Zabrucky, 1998), no significant relationship has been previously found between calibration and problem solving performance in studies conducted in Turkey (Özsoy & Kuruyer, 2012).

One of the prominent findings of this study related to first point above was calibration differences between low and high performers. When the performances of low and high performing students were compared, this study showed that higher-performing students tend to be underconfident both in their predictions and postdictions while lower performing students tend to be overconfident in both. This result is consistent with the general literature (Hacker et al., 2000) and unique in Turkish literature in terms of problem solving area of mathematics. This result is remarkable since overconfidence and underconfidence may effect students' actions on problem solving. Overconfident problem solvers may be in a delusion of no need to fix their strategy and knowledge while underconfident problem solvers may direct their strategy unnecessarily since they see their strategy as inefficient. Turkish students' low scores both in national and cross national exams have been discussed in previous sections. In order to make a progress in the problem solving of these students, calibration levels of students' especially prospective calibration levels should be considered.

Another point which needed to be clarified was measuring mathematics related calibration of students. Literature gave the results of calibration by focusing on either pre-evaluations or post-evaluations of students. Generally post-evaluation procedure was used in mathematics related calibration studies (i.e. Desoete & Roeyers, 2006; Nietfeld & Schraw, 2002; Özsoy & Kuruyer, 2012). Few studies prefered pre-evaluations in calibration studies to post-evaluations (i.e. Özsoy, 2012). Özsoy' s (2012) study is about arithmetic achievement. The current study fills a gap in problem solving area of mathematics by implementing both pre- and post evaluations in mathematics related calibration studies. The current study showed the importance of prospective monitoring judgment accuracy on problem solving performance. Retrospective monitoring judgment accuracy could not predict students' performances in terms of being low or high performer. Therefore, measuring mathematics related calibration prospectively, namely by pre-evaluations, has been shown to be important especially for identifying high performers in mathematical word problem solving.

Implications

In the light the current study's results, it might be important to focus on calibration accuracy in mathematics education because accurate predictions about performance were related to being a good problem solver. Focusing on mathematics related calibration in teacher education programs might be beneficial because pre- and post-evaluation of performance might reinforce self-regulated mathematical problem solving. Since Zimmermann (2000) states that self-regulated learners are aware of their thought processes and the strategies which are necessary to accomplish a task, students' pre- and post-evaluation accuracy with their actual performance is very important to be a self-regulated learner.

The new curriculum movement in Turkey's educational system aims to create an educational environment where the learners have self-regulation abilities. Pre- and post-evaluation of performance is important in that context to be able to plan and check ongoing work. Calibration level may indicate the ability to recognize strategy deficiencies or solution mistakes in problem solving procedure. Hence, in order to make students detect their strategy deficiencies as self-regulated learners, curriculum should make room for improving students' calibration accuracy.

Calibration accuracy should be aimed in instruction. Mathematics teachers should consider calibration accuracy of students in order to make them think on their own performances. Mathematics teachers may carry out some practices to help students produce more accurate prospective judgments, so that, they may develop students' metacognitive monitoring skills and the mathematics problem solving performances.

Another implication for research studies is that using both prospective and retrospective calibration accuracy measurements is beneficial. Especially, usage of pre-evaluations in calibration studies are needed to increase since the current study indicated the importance of pre-evaluations compared to post-evaluations. For further studies, estimation problems in mathematics can be used while measuring mathematics related calibration level of students in problem solving context. Because of the estimation problems' nature, which is related to mathematical prediction, skill can indicate more powerful relationships among the calibration studies.

Limitations

One limitation of this study may be related to measuring metacognition. Today, measuring metacognition by offline and online measures is discussed in the literature. Concurrent measures like think-aloud protocols are suggested. However, concurrent measures are very accurate but time-consuming techniques. On the other hand, prospective and retrospective methods are less time-consuming and also can be taken into account as an online measure when they are applied in the process of problem solving and when they are taken into consideration together with actual performance – like calibration. Nevertheless, whether this method is online or not is still open for discussion.

Another limitation can be thought as the number of problems in this study to measure problem-solving performance. Since metacognitive microevaluation of problem solving performance, not macroevaluation, was measured in the present study, increasing the number of problems can be suggested for future studies intending to substantiate its findings regarding problem solving performance and its relationships with metacognition.

APPENDIX A

PROSPECTIVE METACOGNITIVE MONITORING ACCURACY SCALE

SORU-1:

Yıldız, Kaya ve Erdinç aileleri hep birlikte tatile giderler. Üç aile, 5 günlük tatil için toplam 2940 TL ödeme yapar. Bir çocuk için ödenen fiyat, bir yetişkin için ödenen fiyatın 2/3'sidir. Yıldız ailesi 5 yetişkin, 3 çocuktan, Kaya ailesi 3 yetişkin, 2 çocuktan ve Erdinç ailesi de 2 yetişkin, 1 çocuktan oluştuğuna göre;

a) Tatilin bir günü için, bir yetişkin kaç TL ödemiştir?

b) Her bir ailenin ödediği fiyatın toplam fiyata oranını bulunuz.

Problemi ne kadar doğru çözebileceğinizi düşünüyorsunuz?	Hiç ☐	Biraz ☐	Oldukça ☐	Çok ☐

SORU-2:

Gülsüm Hanım, yeni taşındığı evine kablolu TV bağlantısı için Türk Telekom'u arar ve bilgi alır. Bağlantı ücreti için 60 TL, sonraki her ay için ise 7 TL ücret alınmaktadır. Buna göre Gülsüm Hanım 1 yıllık abonelik karşılığında kaç TL ücret ödeyecektir?

Problemi ne kadar doğru çözebileceğinizi düşünüyorsunuz?	Hiç ☐	Biraz ☐	Oldukça ☐	Çok ☐

SORU-3:

Bir memur 1200 TL olan aylık maaşının %40'ını kiraya, %35'ini gıdaya, %15'ini diğer masraflar için harcayıp geriye kalanını biriktiriyor. Bu memur bir yıl boyunca biriktirdiği paralarla, fiyatı 580 TL olan bir televizyon alıp geri kalan parasıyla da tatile gitmek istiyor. Memurun, tatil için ne kadar parası kalır?

Problemi ne kadar doğru çözebileceğinizi düşünüyorsunuz?	Hiç ☐	Biraz ☐	Oldukça ☐	Çok ☐

APPENDIX B

RETROSPECTIVE METACOGNITIVE MONITORING ACCURACY SCALE

SORU-1:

Yıldız, Kaya ve Erdinç aileleri hep birlikte tatile giderler. Üç aile, 5 günlük tatil için toplam 2940 TL ödeme yapar. Bir çocuk için ödenen fiyat, bir yetişkin için ödenen fiyatın 2/3' sidir. Yıldız ailesi 5 yetişkin, 3 çocuktan, Kaya ailesi 3 yetişkin, 2 çocuktan ve Erdinç ailesi de 2 yetişkin, 1 çocuktan oluştuğuna göre;

a) Tatilin bir günü için, bir yetişkin kaç TL ödemiştir?

b) Her bir ailenin ödediği fiyatın toplam fiyata oranını bulunuz.

Doğru çözümü yaptığınızdan ne kadar eminsiniz?	Hiç ☐	Biraz ☐	Oldukça ☐	Çok ☐

SORU-2:

Gülsüm Hanım, yeni taşındığı evine kablolu TV bağlantısı için Türk Telekom'u arar ve bilgi alır. Bağlantı ücreti için 60 TL, sonraki her ay için ise 7 TL ücret alınmaktadır. Buna göre Gülsüm Hanım 1 yıllık abonelik karşılığında kaç TL ücret ödeyecektir?

Doğru çözümü yaptığınızdan ne kadar eminsiniz?	Hiç ☐	Biraz ☐	Oldukça ☐	Çok ☐

SORU-3:

Bir memur 1200 TL olan aylık maaşının %40'ını kiraya, %35'ini gıdaya, %15'ini diğer masraflar için harcayıp geriye kalanını biriktiriyor. Bu memur bir yıl boyunca biriktirdiği paralarla, fiyatı 580 TL olan bir televizyon alıp geri kalan parasıyla da tatile gitmek istiyor. Memurun, tatil için ne kadar parası kalır?

Doğru çözümü yaptığınızdan ne kadar eminsiniz?	Hiç ☐	Biraz ☐	Oldukça ☐	Çok ☐

APPENDIX C

METACOGNITIVE SKILLS INVENTORY

	HİÇ	BAZEN	SIK SIK	HER ZAMAN
Sınavda soruları cevaplarken, nasıl düşündüğümün farkındayım.	☐	☐	☐	☐
Bir soruyu cevaplarken, nasıl yaptığımı kontrol ederim.	☐	☐	☐	☐
Hangi düşünme biçimini, ne zaman kullanacağımı bilirim.	☐	☐	☐	☐
Sınavlarda hatalarımı fark eder, dönüp düzeltirim.	☐	☐	☐	☐
Sınav sorularının bildiğim konularla ilgili olup olmadığını anlamaya çalışırım.	☐	☐	☐	☐
Sınavlarda soruları cevaplamadan önce ne sorulduğunu anlamaya çalışırım.	☐	☐	☐	☐
Sınavlarda gerek görürsem, düşünme ve çözüm yollarımı değiştiririm.	☐	☐	☐	☐
Soruları cevaplarken doğru yapıp yapmadığımı kontrol ederim.	☐	☐	☐	☐
Hangi konuyu ne kadar anladığımı değerlendirebilirim.	☐	☐	☐	☐
Bir sınavdaki basarımı doğru olarak tahmin edebilirim.	☐	☐	☐	☐
Bir bilginin benim için önemli olup olmadığını anlar, dikkatimi ona yoğunlaştırırım.	☐	☐	☐	☐

	HİÇ	BAZEN	SIK SIK	HER ZAMAN
Hangi bilgiyi öğrenmemin daha önemli olduğunu bilirim.	☐	☐	☐	☐
Kafamdaki bilgileri kolay hatırlayabileceğim bir şekilde düzenlerim.	☐	☐	☐	☐
Bir sınavda soruları çözebilmek için belirli yöntemler kullandığımın farkındayım.	☐	☐	☐	☐
Fikir sahibi olduğum bir konuyu daha iyi öğrenirim.	☐	☐	☐	☐
Öğretmenin benden ne öğrenmemi beklediğini bilirim.	☐	☐	☐	☐
Duruma bağlı olarak farklı öğrenme yolları kullanırım.	☐	☐	☐	☐
Bir soruyu çözdükten sonra kendime, daha kolay bir çözüm yolu olup olmadığını sorarım.	☐	☐	☐	☐
Daha iyi öğrenip, öğrenememem bana bağlıdır.	☐	☐	☐	☐
Bir problemle karşılaştığımda bir sürü çözüm yolu düşünür, en iyisini seçerim.	☐	☐	☐	☐
Çalışırken hangi yöntemleri kullandığımın farkındayım.	☐	☐	☐	☐
Çalışırken kullandığım yöntemlerin ise yarayıp yaramadığını düşünürüm.	☐	☐	☐	☐
Bir konuyu anlayıp anlamadığımı bilirim.	☐	☐	☐	☐
Bir şeyi anlayıp anlamadığımı kontrol ederim.	☐	☐	☐	☐

	HİÇ	BAZEN	SIK SIK	HER ZAMAN
Hangi yöntemi, nerede kullanırsam daha etkili olacağımı bilirim.	☐	☐	☐	☐
Yeni öğrendiğim bir konuyu daha kolay anlayabileceğim bir hale getirmeye çalışırım.	☐	☐	☐	☐
Bir konuyu anlayamadığım zaman kullandığım yöntemi değiştiririm.	☐	☐	☐	☐
Sınavlarda soruları cevaplamak için gerekli olan süreyi bilir ve kendimi ona göre ayarlarım.	☐	☐	☐	☐
Sınavlara hazırlanırken, çalıştığım konuları bölümlere ayırırım.	☐	☐	☐	☐
Çalışmayı bitirdiğimde, öğrenebileceğim kadar öğrenip, öğrenmediğimi anlamaya çalışırım.	☐	☐	☐	☐
Tam olarak anlamadığım konuyu tekrar ederim.	☐	☐	☐	☐
Kafam karıştığı zaman durur ve tekrar okurum.	☐	☐	☐	☐

APPENDIX D

PROBLEM SOLVING PERFORMANCE TEST

SORU-1:

Yıldız, Kaya ve Erdinç aileleri hep birlikte tatile giderler. Üç aile, 5 günlük tatil için toplam 2940 TL ödeme yapar. Bir çocuk için ödenen fiyat, bir yetişkin için ödenen fiyatın 2/3' sidir. Yıldız ailesi 5 yetişkin, 3 çocuktan, Kaya ailesi 3 yetişkin, 2 çocuktan ve Erdinç ailesi de 2 yetişkin, 1 çocuktan oluştuğuna göre;

a) Tatilin bir günü için, bir yetişkin kaç TL ödemiştir?

b) Her bir ailenin ödediği fiyatın toplam fiyata oranını bulunuz.

SORU-2:

Gülsüm Hanım, yeni taşındığı evine kablolu TV bağlantısı için Türk Telekom'u arar ve bilgi alır. Bağlantı ücreti için 60 TL, sonraki her ay için ise 7 TL ücret alınmaktadır. Buna göre Gülsüm Hanım 1 yıllık abonelik karşılığında kaç TL ücret ödeyecektir?

SORU-3:

Bir memur 1200 TL olan aylık maaşının %40'ını kiraya, %35'ini gıdaya, %15'ini diğer masraflar için harcayıp geriye kalanını biriktiriyor. Bu memur bir yıl boyunca biriktirdiği paralarla, fiyatı 580 TL olan bir televizyon alıp geri kalan parasıyla da tatile gitmek istiyor. Memurun, tatil için ne kadar parası kalır?

APPENDIX E

THE HOLISTIC SCORING RUBRIC

This scale evaluates the process employed in response to a problem-solving task. It takes into consideration the level of student knowledge and understanding with respect to the given problem solving task; the selection and implementation of appropriate procedures and/or strategies; and the accuracy of the solution obtained.

3 - Response is characterized by all of the following:

• The student selects and implements relevant concepts and procedures/strategies needed to solve this problem.

• The student considers all constraints of the problem situation.

• The solution and all relevant work are correct; or, there is a mistake due to some minor computational or copying error.

2 - The student selects appropriate procedures/strategies to solve this problem; however, the response/solution is not correct because one or more of the following are:

• There is evidence that the student has several misconceptions or has failed to consider several relevant concepts needed to solve the problem correctly.

• The student fails to consider several constraints of the problem situation.

• The student has also considered several irrelevant variables or failed to consider several relevant variables.

• The student did not carry the procedures/strategies far enough to reach a solution.

• The response/solution is generally correct; however, there is no information showing how the student arrived at this response/solution.

1 - An incomplete and/or incorrect response/solution is provided evidencing an attempt to solve the problem. In addition, one or more of the following are apparent:

• The student did consider a constraint or variable of the problem situation.

• The student understands some concepts relevant to the problem task.

• The student selected a totally inappropriate procedure/strategy.

0 - Response is characterized by the following:

• It is blank.

• The student response only repeats information in the problem task.

• An incorrect solution/response is given and no other information is shown.

• The solution/response and supportive information is totally irrelevant to the problem task.

APPENDIX F

CONSENT FORM

Boğaziçi University Thesis Research

Research Title: Relationship among Metacognitive Knowledge, Metacognitive Calibration Accuracy and Mathematical Problem Solving Performance

Researcher: Birce Başol

Advisor: Engin Ader, Assist. Prof.

Introduction/Purpose:

The purpose of this study is to understand the relationship between metacognitive monitoring accuracy and mathematical problem solving performance of 7^{th} grade students.

Procedures:

Checklists and mathematical problems will be used in this study. Researcher will need 40 minutes (1 lesson hour) to apply all measures. Students' lesson teachers will be needed at the classroom during the application.

Risks:

Your students' participation in this study does not involve any physical, psychological, or emotional risks to them.

Benefits:

There may be no direct benefit to your students by their participation in this research study; however their participation in this study may aid in our understanding of their metacognitive monitoring accuracy and problem solving performance. These findings may lead to a better understanding of their relationship and possible benefits of this relationship.

Alternatives:

Your students have the alternative to choose not to participate in this research study.

Confidentiality:

Participation in this research study will not result in a loss of privacy, since all participants will be represented with a participant number. Only the researcher will know their real names. Your school name will not be declared, it is represented whether private or state school in the research sample.

Financial Information:

You will not be charged for any study-related procedures. You and your students will not be paid for participation in this study.

Contact Persons:

During or after the study, any questions you may have about this study may be directed to researcher, Birce Başol at birce.mat.boun@gmail.com

You may be directed to Assist. Prof. Engin Ader as the advisor of the researcher at ader@boun.edu.tr

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