METHODS OF XENON STABILITY ANALYSIS IN NUCLEAR REACTORS

by

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Dedicated to

Nevin,

my fiancè

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ABSTRACT

The subject of this study is the construction of some sufficient conditions for Xenon stability in thermal reactors during operation, and the approximate time behaviour of the point kinetics system with feedback.

The same topic had been investigated by J. Chernick, G. Lellouche and W. Wollmann[7] in 1961. It had been shown that Xenon instability remains a serious concern in the presence of temperature damping. Later A. Z. Akçasu and P. Akhtar studied the problem in 1966[2]. They approached the problem as one of asymptotic stability in the large for point reactors with non-linear feedback; and gave a new criterion for boundedness of Xenon oscillations in the presence of temperature feedback.

In the first three chapters basic kinetic equations are derived for the point reactor model, mainly to emphasize the extent of careful work required to obtain the mean neutron generation time. Then global stability analysis of Xenon is examined and the region of asymptotic stability in the large in the plane of equilibrium flux vs. temperature coefficient is determined.

In chapter four linear stability analysis is considered and conditions for linear stability are determined with and without delayed neutrons; and the results are compared. In constructing the stability conditions, various approximations and combinations of parameters were utilized. Further, point kinetics equations are solved for certain reactor operating conditions and the time behaviour of the flux is

observed in order to assess some properties such as period and amplitude of oscillations in the region of stability and instability. The results are compared with that of other workers in the field[2].

Results, plots and the discussions are given in the last chapter. Computer programs used in this work are also provided in the appendices.

LIST OF SYMBOLS

Symbol	Definition
ℓ	Neutron generation time (sec.)
λ_I	decay constant of I^{135} (sec.$^{-1}$)
λ_x	decay constant of Xe^{135} (sec.$^{-1}$)
λ	average decay constant of delayed neutron precursors
ϕ_0	equilibrium value of flux (n/(cm^2.sec.))
γ	temperature reactivity coefficient
y_I	iodine yield (%)
y_x	xenon yield (%)
β	delayed neutron fraction
δ_0	initial reactivity of the clean reactor
σ, Σ	absorption cross sections (microscopic and macroscopic)
u	lethargy
$\underline{\Omega}$	unit vector denoting the direction of motion of neutron
σ_x	absorption cross section of Xenon (cm^2.)
σ_f	fission cross section (cm^2.)
D	delayed neutron precursor concentration
P	reactor power (watts)
n	neutron population
ν	mean number of neutrons per fission

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CHAPTER I

INTRODUCTION

I. BACKGROUND

The first controlled nuclear chain reaction was achieved in Chicago in 1942 in a reactor using natural uranium and graphite. The first nuclear reactor was designed and built without detailed knowledge of the products of fission of U^{235}. The power level of this first reactor and of the second reactor built in Oak Ridge in the follwing year was so low that the total quantity of fission products present in the reactor was not sufficient to noticably affect the reactivity of the system. It was not until the first Pulutonium production reactor(*) was built at Hanford in 1966 and operated at high power levels that the existence of fission products with high thermal neutron cross section was discowered. Serious loss of reactivity in this reactor at high power levels led to the postulate that a fission product with a high yield and high absorption cross section was providing the mechanism for reactivity changes. This fission product was found to be Xe^{135}.

(*) It was a LGR (Light water cooled, Graphite moderated Reactor).

The characteristics of this isotope are compared with those of U^{235} and Sm^{149} in table-I. Samarium-I49 is the only other fission product whose thermal cross section even approaches that of Xenon.

Isotope	Thermal absorption cross section, barns	Yield %
Xe^{135}	$3x10^{6}$	6.4
Sm^{149}	$5.3x10^{4}$	I.4
U^{235}	$6.7x10^{2}$	

Table-I Comparison of yields and cross sections of Xe^{135}, Sm^{149} and U^{235}.

Xenon-I35 absorption cross section can be considered constant because its variation with neutron energy is negligible about the theoretical value of $3x10^{6}$ barns. Figure-I shows this variation with neutron energy[I0].

Xenon-I35 is created in two ways: directly as a fission product (0.3 %) and as the grandoughter of fission product Te^{135}(6.0 %). The important characteristics of this decay chain are given below. Since the 2 min. half-life of I^{135}, we may assume that the Iodine is formed directly as a fission product.

$$Te^{135} \xrightarrow{2\ min.} I^{135} \xrightarrow{6.7\ hr.} Xe^{135} \xrightarrow{9.2\ hr.} Cs^{135} \xrightarrow{2\times10^{4}\ yr.} Ba^{135}\ (stable)$$

σ_a : (7b.) (7b.) (3xI0b.) (30b.) (Ib.)

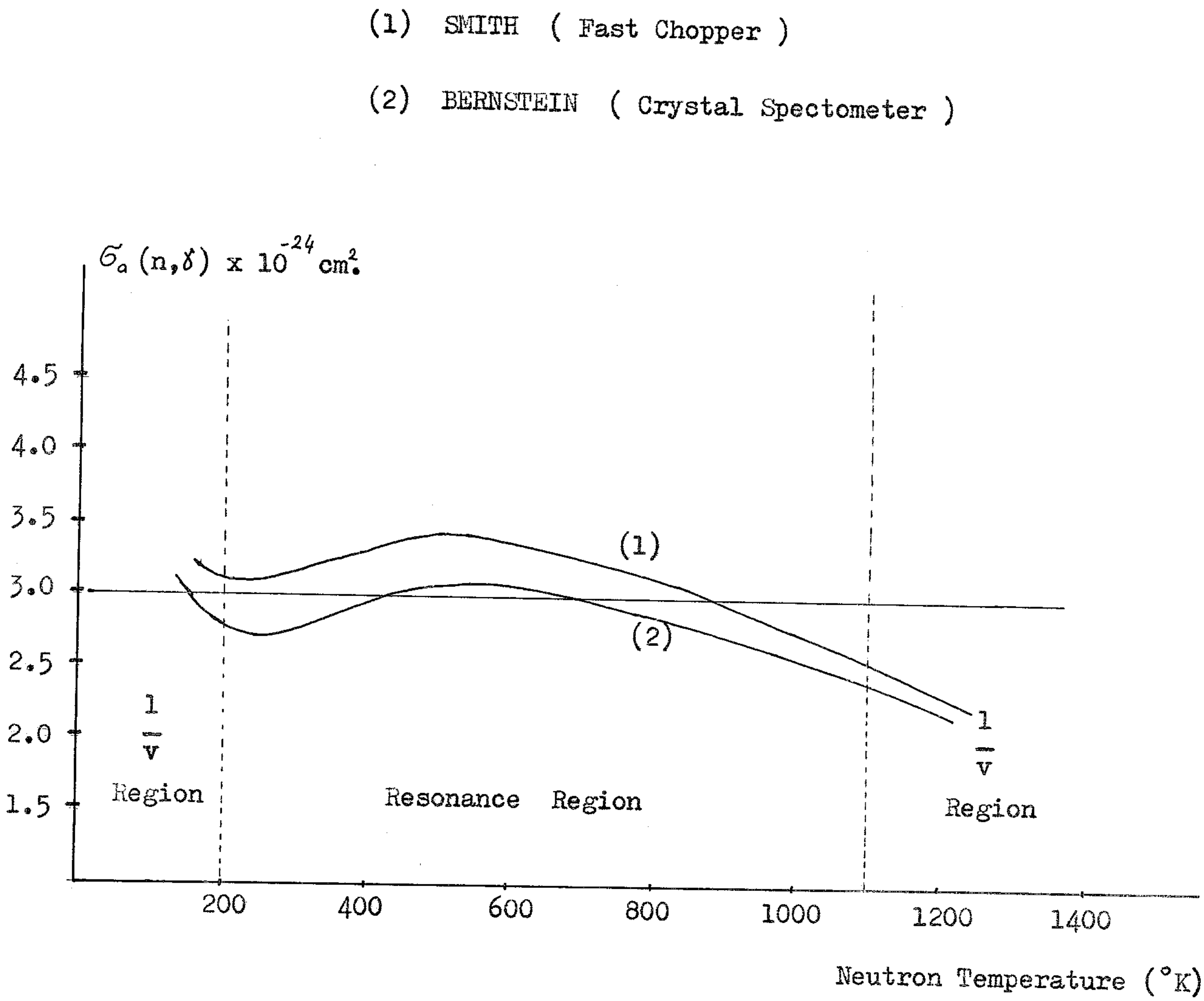


Figure - 1 Absorption Cross Section (n, γ) of Xenon as a function of neutron temperature.

The presence of Xenon in a thermal reactor gives rise to three serious control problems owing to its high absorption cross section for thermal neutrons. One control problem is caused by the build up of Xenon concentration due to Iodine decay after reactor shutdown. The peak in the Xenon concentration occurs about IO hrs. after shutdown. If one wanted to start-up the reactor at that time, enough excess reactivity would have to be incorparated into the control rods to compensate for this peak amount of neutron absorbing Xenon poison. This becomes a more serious problem for flux levels above 10^{13} n/(cm^2.sec.).

Another problem arises from the fact that during equilibrium operation the Xenon absorbs neutrons from the chain reaction. For this chain reaction to be sustained, enough excess neutrons have to be produced to compensate for the amount absorbed by equilibrium Xenon. The amount of thermal neutrons captured by equilibrium Xenon poison ranges from 0.7 % at a flux level of 10^{12} to 4.8 % at a flux level of 10^{15}.

The third problem is that of thermal flux instability due to a change in Xenon concentration from its equilibrium value. This problem of Xenon reactivity feedback during reactor operation is the topic of this study.

Delayed reactivity feedback can be defined as reactivity, created or destroyed at time $t=t_0$, whose effect on the system is not felt until a later time $t=t_1$. For the case of Xenon reactivity feedback, Iodine atoms are created at time $t=t_0$ and decay to Xenon atoms at time $t=t_1$; the time t_1 being determined by an exponential decay law. The Xenon atoms then absorb neutrons at some time $t \geq t_1$. Since the amount of Xenon produced by fission is much less than the amount of Iodine produced by fission, most of the Xenon present at a given time is due to the decay of Iodine.

To understand how flux instability due to Xenon reactivity feedback can occur, consider a steady-state reactor containing equilibrium amounts of Xenon and Iodine. A disturbance that slightly increases the flux will initially destroy Xenon through flux absorption and create Iodine and a small amount of Xenon through fission. If the initial amount of Xenon destroyed is greater than that created directly, the total amount is reduced below the equilibrium level, and the flux will tend to increase. If enough Xenon is produced through Iodine decay to replenish the equilibrium level, more neutrons will be absorbed, and the flux will decrease. Depending on the relative strength of the competing process, the flux will increase with time (unstable), return to the equilibrium level (stable), or oscillate continuously (neutrally stable).

The major part of this thesis will be devoted to deriving the relations between equilibrium flux and reactor parameters that must hold to ensure linear stability.

There are other feedback mechanisms in a reactor besides Xenon feedback. These are usually the result of temperature effects caused by changes in the power level, since all commercial reactors are designed to have negative reactivity feedback. This feedback tends to stabilize the system against power changes large enough to adversely affect the operation.

A negative reactivity coefficient in a thermal reactor may be caused by:

I) A decrease in the density of the moderator as the temperature increases.

2) A change in absorption cross section of fuel or moderator.

3) A change in leakage due to a change in internal geometry and reflector density or flux spectrum.

4) The Doppler effect in fuel, i.e., as the temperature increases, the resonances of U^{238} for absorption of neutrons broadens.

2. SCOPE

In the first part of the thesis theory of Nuclear Reactor Dynamics is given and equations describing the time behaviour of the reactor are derived. Application of these equations to Asymptotic Stability in the Large is given in chapter 3. In chapter 4 Linear Stability Analysis is presented for several cases and the stability conditions are derived. In chapter 5 point kinetics equations are solved using different techniques.

Results, plots and discussions are subsequently given.

CHAPTER II

THEORY

I. KINETICS EQUATIONS

Reactor Dynamics is concerned with the time behaviour of the neutron population in a reactor whose nuclear and geometric properties may vary in time. The first step in reactor dynamics is to introduce the macroscopic physical quantities and the dynamical variables that describe the medium and the neutron population in sufficient detail.

Angular density in terms of the lethargy u and the unit vector Ω, namely $n(\underline{r},u,\Omega,t)$, where u is a measure of the kinetic energy of the neutron in the lethargy scale ($u=\log(E_0/E)$, where E_0 is a reference energy such that there are no neutrons with $E>E_0$), and Ω is the unit vector denoting the direction of motion of the neutron ($\Omega = \underline{v}/v$).

Now we may write the transport equation in a multiplying medium;

$$\frac{\partial n(\underline{r},u,\underline{\Omega},t)}{\partial t} = -\underline{\Omega}\cdot\nabla v(u)n(\underline{r},u,\underline{\Omega},t) - \Sigma(\underline{r},u,t)\,v(u)n(\underline{r},u,\underline{\Omega},t) +$$

$$+\int du'\int d\Omega'\Big\{\sum_j\Big[(f_o^j(u)/4\pi)\Big]\nu^j(u')(I-\beta^j)\,\Sigma_f^j(\underline{r},u',t)$$

$$+\Sigma_s(\underline{r},u'\longrightarrow u,\underline{\Omega}'\longrightarrow\underline{\Omega},t)\Big\}\,v(u')n(\underline{r},u',\underline{\Omega}',t)$$

$$+\ S(\underline{r},u,\underline{\Omega},t) + \sum_{i=1}^{6}\lambda_i\Big[(f_i(u)/4\pi)\Big]C_i(\underline{r},t) \qquad (2.I)$$

where, $f_i(u)$ is the lethargy distribution of delayed neutrons of the i^{th} group; $f_o^j(u)$ the lethargy distribution of the prompt fission neutrons resulting from the j^{th} fissile nucleus and both are normalized to unity as,

$$\int_0^{\infty} du\ f_s^j(u) = I\ , \qquad s= 0,I,\ldots,6$$

$C_i(\underline{r},t)$ is the concentration of the delayed neutron precursors per unit volume at point $\underline{r}$, at time t which always decay by emitting a delayed neutron; $\nu^j(u)$ is the mean number of neutrons per fission in nucleus of type j induced by a neutron having lethargy u; $\Sigma_f^j(\underline{r},u',t)$ is the macroscopic fission cross section of j type nucleus for neutrons having lethargy u', at point $\underline{r}$, at time t; $\Sigma_s(\underline{r},u'\longrightarrow u,\underline{\Omega}'\longrightarrow\underline{\Omega},t)$ is the macroscopic scattering cross section of j^{th} type nucleus at point $\underline{r}$, for neutrons entering the collision with lethargy u', direction $\underline{\Omega}'$ and exiting with u, $\underline{\Omega}$. λ_i is the i^{th} kind delayed neutron precursor decay constant; and β^j is the number of precursors per fission of nucleus type j.

In this equation we have allowed the possibility of having more than one kind of fuel isotope, and distinguished them by the superscript j. Equation states neutron balance in an infinitesimal element of volume in the phase space $(\vec{r},\vec{v}) \equiv (\vec{r},u,\vec{\Omega})$.

The term $-\underline{\Omega}.\nabla v(u)n(\underline{r},u,\underline{\Omega},t)$ in eq. (2.I) denotes the removal of neutrons due to streaming, and is equal to the difference between the number of neutrons entering and emerging per second from the volume element $d^3r du d\vec{\Omega}$ at $(\underline{r},u,\underline{\Omega})$.

The second term is the number of neutrons in $d^3r du d\vec{\Omega}$ that suffer a collision of any kind per second.

The third term is the total number of fissionneutrons produced in $d^3r du d\vec{\Omega}$ per second by fission events in d^3r at $\underline{r}$ in the configuration space where the fissions are induced by neutrons of all energies.

The fourth term is equal to the number of neutrons that are scattered into $du d\vec{\Omega}$ at u per second in scattering events in configuration space at all energies.

The fifth term, $S(\underline{r},u,\underline{\Omega},t)d^3r du d\Omega$ denotes the number of neutrons introduced into d^3r at $\underline{r}$ and $du d\vec{\Omega}$ at u by external neutron sources.

Finally the last term is the number of delayed neutrons emitted per second in d^3r at $\underline{r}$ by the delayed neutron precursors of all types which are formed in fission events in d^3r in the past.

The second equation represents the balance relations for the precursors in an element of volume in the configuration space.

$$\frac{\partial C_i(\underline{r},t)}{\partial t} = -\lambda_i C_i(\underline{r},t) + \int du \left[\sum_j \beta_i^j \nu^j(u)\, v(u) \Sigma_f(\underline{r},u,t) n(\underline{r},u,t) \right] \qquad (2.2)$$

where we have defined,

$$n(\underline{r},u,t) \equiv \int d\vec{\Omega}\, n(\underline{r},u,\underline{\Omega},t)$$

which we refer to as the "scaler" neutron density.

2. REACTOR KINETICS EQUATIONS WITH FEEDBACK

Kinetics equations in operator form are,

$$\frac{\partial n}{\partial t} = H[n]\, n + \sum_{i=1}^{6} \lambda_i f_i C_i + S \qquad (2.3\ a)$$

$$\frac{\partial (f_i C_i)}{\partial t} = M_i[n]\, n - \lambda_i f_i C_i \quad , \qquad i=1,\ldots,6 \qquad (2.3\ b)$$

where, $H = L + M_o$

and

$$L = -\underline{\Omega}\cdot\nabla v(u) - \Sigma(\underline{r},u,t) v(u) + \int du' \int d\vec{\Omega}' \left[v(u') \Sigma_s(\underline{r},u' \rightarrow u, \underline{\Omega}\cdot\underline{\Omega}', t) \right]$$

$$M_o = \sum_j \left[\frac{f_o^j(u)}{4\pi} \int du' \int d\vec{\Omega}' \left[v(u') \nu^j(u') (1-\beta^j) \Sigma_f^j(\underline{r},u',t) \right] \right]$$

$$M_i = \sum_j \left[\frac{f_i^j(u)}{4\pi} \int du' \int d\vec{\Omega}' \left[\beta_i^j \nu^j(u') v(u') \Sigma_f^j(\underline{r},u',t) \right] \right]$$

The physical meaning of these operators can be deduced from their definitions : L describes the losses from the differential volume $d^3r du$ in phase space due to leakage, absorption and scattering and also the gains in $d^3r du$ as a result of scatterings from all other $u', \vec{\Omega}'$ into $\vec{\Omega}$,u at position $\underline{r}$. M_o determines the rate of production of prompt neutrons when it operates on the angular density; it can be called the prompt neutron production operator. Similarly, M_i can be called the delayed neutron precursor-production operator.

Finally H describes neutrons in a multiplying medium in the absence of delayed neutrons; it is called the Boltzmann operator. Note that the operators H and M_i depend on the composition of the medium, and describe the medium completely, so are functionals of n. They are, in general, time dependent because the cross sections are functions of time both due to changes in weighted microscopic cross sections resulting from spectral shifts in the assumed Maxwell-Boltzmann distribution of the nuclear velocities with temperature.

Now consider a stationary reference reactor supporting a neutron distribution characterized by $N_o(\underline{r},u,\underline{\Omega})$. Since a reactor is never truly stationary when the burn-up and build-up of the various nuclear species are included, we must either assume that the reference reactor is operated at zero power level, and hence free from all feedback effects, or ignore the long-term changes in the nuclear species due to burn-up and build-up of fission product isotopes by irradiation. In the first case, the reference reactor is critical in the absence of feedback effects, and represents a cold, clean reactor free from fission products.

In the second case, the reference reactor is critical in the presence of all the feedback effects except for those arising from the depletion of the fuel; the effects of the burnable poisons, such as Xe^{135}, are still included. It is more realistic to visualize the reference reactor as in the second case, because then the reference distribution $N_o(\underline{r},u,\underline{\Omega})$ can be chosen as the steady-state distribution in the actual reactor at the operating power level before the perturbations are introduced. Since the analysis based on the latter interpretation of $N_o(\underline{r},u,\underline{\Omega})$ is better justified than choosing it as the steady-state distribution in a reactor critical in the absence of feedback effects.

The steady-state distibution $N_o(\underline{r},u,\underline{\Omega})$ can be obtained in principle by solving the time independent set of coupled nonlinear integrodifferential equations derived. In operator form $N_o(\underline{r},u,\underline{\Omega})$ satisfies the following equation :

$$\mathcal{H}_o\left[N_o\right]N_o = 0 \tag{2.4}$$

which is obtained from (2.3) by setting the time derivatives equal to zero and eliminating $C_{io}(\underline{r})$. Here $\mathcal{H}_o$ is defined by

$$\mathcal{H}_o \equiv H + \sum_{i=1}^{6} M_i \equiv L + M \tag{2.5}$$

where M is the modified multiplication operator :

$$M \equiv \sum_{j} (f^j(u)/4\pi) \int du' \int d\underline{\Omega}' \left[\nu^j(u') \Sigma_f^j(\underline{r},u') v(u') \right] \tag{2.5 a}$$

in which $f^j(u)$ is defined by

$$f^j(u) \equiv (1-\beta^j)\ f_o^j(u) + \sum_{i=1}^{6} \beta_i^j f_i(u)$$

$$\mathcal{H}_o[N_o] \equiv -\underline{\Omega}\cdot\nabla v(u) - v(u)\Sigma(\underline{r},u,[N_o]) + \int du' \int d\Omega' v(u') \Big\{ \Sigma_s(\underline{r},u'\to u,\underline{\Omega}'\cdot\underline{\Omega};[N_o])$$

$$+\sum_j \left[f^j(u')/4\pi \right] \nu^j(u')\ \Sigma_f^j(\underline{r},u';[N_o]) \Big\} \qquad (2.6)$$

Note that the operator $\mathcal{H}[N_o]$ has the same structure as the steady-state Boltzmann operator. The presence of feedback modifies only the energy and space dependencies of the cross sections in the expression of $\mathcal{H}[N_o]$ but does not affect its form.

2.1 POINT KINETICS APPROXIMATION [3]

In order to obtain appropriate point kinetics equations with feedback, we partition the angular neutron density $n(\underline{r},u,\underline{\Omega},t)$ into a shape function $\phi(\underline{r},u,\underline{\Omega},t)$ and a time function $P(t)$ such that

$$n(\underline{r},u,\underline{\Omega},t) = P(t)\ \phi(\underline{r},u,\underline{\Omega},t) \qquad (2.10)$$

Assuming $N_o(\underline{r},u,\underline{\Omega})$ and $N_o^+(\underline{r},u,\underline{\Omega})$(*) to be known functions of $\underline{r}$,u

(*) Please see Appendix-2,I for the definition of and the method of solution for the adjoint or the importance function $N_o^+(\underline{r},u,\underline{\Omega})$.

and $\underline{\Omega}$; multiply (2.10) by N_o^+ and integrate over $\underline{r}$,u and $\underline{\Omega}$;

$$P(t)\,\frac{\partial \phi(\underline{r},u,\underline{\Omega},t)}{\partial t} + \phi(\underline{r},u,\underline{\Omega},t)\,\frac{\partial P(t)}{\partial t} = P(t)H\phi(\underline{r},u,\underline{\Omega},t) + \sum_{i=1}^{6}\lambda_i f_i C_i + S \qquad (2.11)$$

$$\frac{\partial (f_i C_i)}{\partial t} = P(t)\ M_i\phi(\underline{r},u,\underline{\Omega},t) - \lambda_i f_i C_i \qquad (2.12)$$

and, by using Dirac notation,

$$\langle N_o^+|\phi\rangle\frac{\partial P}{\partial t} + P\,\frac{\partial}{\partial t}\,\langle N_o^+|\phi\rangle = P\,\langle N_o^+|H|\phi\rangle + \sum_{i=1}^{6}\lambda_i\langle N_o^+|f_i C_i\rangle + \langle N_o^+|S\rangle \qquad (2.13)$$

$$\frac{\partial}{\partial t}\langle N_o^+|f_i C_i\rangle = P(t)\langle N_o^+|M_i|\phi\rangle - \lambda_i\langle N_o^+|f_i C_i\rangle \qquad (2.14)$$

We now impose a "normalization" condition on the shape function to ensure uniqueness, which we choose as,

$$\frac{d}{dt}\ \langle N_o^+|\phi\rangle = 0 \qquad (2.15)$$

Since $N_o^+(\underline{r},u,\underline{\Omega})$ is proportional to the importance of neutrons (see Appendix-2), $\langle N_o^+|\phi\rangle$ is the total importance of neutrons in the reference reactor with a distribution function $\phi(\underline{r},u,\underline{\Omega},t)$.

So, the shape function must be so chosen that the total importance in the reference reactor will remain constant in time even though $\phi(\underline{r},u,\underline{\Omega},t)$ itself may slowly vary in time. This assures us that when we start working with adjoint weighted neutron population in the form of P the multiplicative potential of total number of neutrons as measured by total importance is the same as in the actual core although we have no idea about the spatial distribution of neutron population.

Now we may interpret the physical meaning of time function. Multiplying both sides of (2.10) by N_o^+, we find that

$$P(t) = \langle N_o^+ | n \rangle \; / \; \langle N_o^+ | \phi \rangle \qquad (2.16)$$

which states that P(t) is the ratio of the total importance of neutrons with a distribution $n(\underline{r},u,\underline{\Omega},t)$ to the importance of those neutrons that have a distribution $\phi(\underline{r},u,\underline{\Omega},t)$. The denominator of (2.16) is constant in time, and can be scaled to unity.

Then P(t) becomes the instantaneous value of the total importance of the neutron population in the actual reactor which is necessary to sustain a chain reaction in the reference reactor. Note that P(t) is not the total number of neutrons in the reactor volume at time t. Then the equations become,

$$\langle N_o^+ | \phi \rangle \frac{\partial P}{\partial t} = P \langle N_o^+ | H | \phi \rangle + \sum_{i=1}^{6} \lambda_i \langle N_o^+ | f_i C_i \rangle + \langle N_o^+ | S \rangle \qquad (2.17\ a)$$

$$\frac{\partial}{\partial t}\left\langle N_o^{+}\middle| f_i C_i\right\rangle = P(t)\left\langle N_o^{+}\middle| M_i\middle|\phi\right\rangle - \lambda_i\left\langle N_o^{+}\middle| f_i C_i\right\rangle \qquad (2.17\text{ b})$$

Now we may introduce the concept of perturbation, i.e., the deviations of the reactor parameters of the actual reactor from those of the reference reactor, defining a perturbation operator $\delta\mathcal{H}[n]$ as

$$\delta\mathcal{H}[n] \equiv H[n] + \sum_{i=1}^{6} M_i[n] - \mathcal{H}_o \equiv L[n] + M[n] - \mathcal{H}_o$$

$$= \mathcal{H}[n] - \mathcal{H}_o[N_o]$$

or explicitly,

$$\delta\mathcal{H}[n] \equiv -v(u)\,\delta\Sigma(\underline{r},u,[n]) + \int du'\int d\underline{\Omega}'\left\{\Sigma_s(\underline{r},u'\rightarrow u,\underline{\Omega}\cdot\underline{\Omega}',[n])\right.$$

$$\left. + \sum_j\left[f^j(u)/4\pi\right]\nu^j(u')\Sigma_f^j(\underline{r},u',[n])\right\} v(u') \qquad (2.18)$$

where $\delta\Sigma_j(\underline{r},u,[n])$ measures the variations of the cross sections about their reference values, i.e.,

$$\delta\Sigma_j(\underline{r},u,[n]) \equiv \Sigma_j(\underline{r},u,[n]) - \Sigma_{jo}(\underline{r},u,n_o)$$

where the subscript j denotes a,f or s.

Substituting $H[n]$ from (2.18) into (2.17),

$$\frac{\partial P}{\partial t}\left\langle N_o^{+}\middle|\phi\right\rangle = P\left\langle N_o^{+}\middle|\delta\mathcal{H}\middle|\phi\right\rangle + P\left\langle N_o^{+}\middle|\mathcal{H}_o\middle|\phi\right\rangle - P\sum_{i=1}^{6}\left\langle N_o^{+}\middle|M_i\middle|\phi\right\rangle$$

$$+ \sum_{i=1}^{6}\lambda_i\left\langle N_o^{+}\middle| f_i C_i\right\rangle + \left\langle N_o^{+}\middle| S\right\rangle \qquad (2.19)$$

Recalling that $\langle N_o^+ | \mathcal{H}_o | \phi \rangle = \langle \mathcal{H}_o^+ N_o^+ | \phi \rangle = 0$ for any function ϕ with the proper boundary conditions,

$$\frac{\partial P}{\partial t} = \left\{ \frac{\langle N_o^+ | \delta\mathcal{H} | \phi \rangle}{\langle N_o^+ | \phi \rangle} - \frac{\sum_{i=1}^{6} \langle N_o^+ | M_i | \phi \rangle}{\langle N_o^+ | \phi \rangle} \right\} P(t) + \sum_{i=1}^{6} \lambda_i \frac{\langle N_o^+ | f_i C_i \rangle}{\langle N_o^+ | \phi \rangle} + \frac{\langle N_o^+ | S \rangle}{\langle N_o^+ | \phi \rangle} \qquad (2.20)$$

Now the desired form of the kinetics equations become,

$$dP \,/\, dt = \left[(\rho(t) - \beta) \,/\, \ell \right] P(t) + \sum_{i=1}^{6} \lambda_i \bar{C}_i(t) + \bar{S}(t) \qquad (2.21\ a)$$

$$d\bar{C}_i \,/\, dt = (\bar{\beta}_i / \ell)\, P(t) - \lambda_i \bar{C}_i(t) \qquad (2.21\ b)$$

with the following definitions :

Reactivity : $\rho / \ell \equiv \langle N_o^+ \delta\mathcal{H}[n] | \phi \rangle \,/\, \langle N_o^+ | \phi \rangle$ (2.22 a)

Effective delayed neutron fraction :

$$\bar{\beta}_i / \ell \equiv \langle N_o^+ | M_i[n] | \phi \rangle \,/\, \langle N_o^+ | \phi \rangle \qquad (2.22\ b)$$

$$\bar{\beta} = \sum_{i=1}^{6} \bar{\beta}_i$$

Effective concentration of delayed neutron precursors :

$$\bar{C}_i \equiv \langle N_o^+ | f_i C_i \rangle \,/\, \langle N_o^+ | \phi \rangle \qquad (2.22\ c)$$

Effective source :

$$\bar{S} \equiv \langle N_o^+ | S \rangle / \langle N_o^+ | \phi \rangle \qquad (2.22\ d)$$

Mean prompt neutron generation time :

$$\ell = \langle N_o^+ | \phi \rangle / \langle N_o^+ | M | \phi \rangle \qquad (2.22\ e)$$

M(t) was defined in eq. (2.5 a)

Equation (2.18) represents the difference between the nuclear properties of the reference reactor at steady-state and those of the actual reactor at time t_1 with feedback effects being included in both cases. this difference may be due to the changes in the cross sections resulting from feedback effects, or due to the changes introduced externally in the atomic composition of the reactor, e.g., by moving the control rods. So we can separate the reactivity into three parts :

$$\rho / \ell = (\delta\rho_{ext} / \ell) + (\delta\rho_c / \ell) + (\delta\rho_T / \ell) \qquad (2.23)$$

where,

$$\delta\rho_{ext} / \ell \equiv \langle N_o^+ | \sum_i \delta N_i^{ext} (\partial \mathcal{H}_o / \partial N_{io}^{ext}) | \phi \rangle / \langle N_o^+ | \phi \rangle \qquad (2.23\ a)$$

$$\delta\rho_c / \ell \equiv \langle N_o^+ | \sum_i \delta N_i^c (\partial \mathcal{H}_o / \partial N_{io}^c) | \phi \rangle / \langle N_o^+ | \phi \rangle \qquad (2.23\ b)$$

$$\delta\rho_T / \ell \equiv \langle N_o^+ | \ \delta T (\partial \mathcal{H}_o / \partial T_o) | \phi \rangle / \langle N_o^+ | \phi \rangle \qquad (2.23\ c)$$

where $N_{io}(\underline{r})$ and $T_o(\underline{r})$ are the equilibrium concentration of the i th nucleus and the local temperature at $\underline{r}$, respectively, and $\delta\mathcal{H}_o$ is defined in equation (2.18). the terms in (2.23) represent, respectively, the external reactivity changes, reactivity feedback due to changes in atomic concentrations, and reactivity feedback due to temperature variations.

Now the problem is to choose the shape function $\phi(\underline{r},u,\underline{\Omega})$ appropriately. We use the first-order perturbation approximation[3]. This the crudest, and the simplest, approximation which assumes the shape to be proportional to the steady-state distribution $N_o(\underline{r},u,\underline{\Omega})$ in the critical reference reactor. If we denote the proportionality constant by $(1/P_o)$, this approximation implies,

$$n(\underline{r},u,\underline{\Omega},t) \approx \frac{P(t)}{P_o} N_o(\underline{r},u,\underline{\Omega}) \qquad (2.24)$$

where $N_o(\underline{r},u,\underline{\Omega})$ is the angular neutron density at equilibrium. Thus the normalization condition $(d/dt)\langle N_o^+\phi\rangle = 0$ is automatically satisfied.

It is to be noted that the reactivity ρ can be expressed as the superposition of the external and feedback reactivities only in the first-order perturbation theory.

The point kinetics equations in the presence of feedback can be written as,

$$\mathring{P}(t) = \left[\left(\delta\rho_{ext}(t) + \delta\rho_f[P] - \bar{\beta}\right)/\ell\right] P(t) + \sum_{i=1}^{6} \lambda_i \bar{C}_i(t) + S(t) \qquad (2.25\ a)$$

$$\mathring{\bar{C}}_i(t) = (\bar{\beta}_i/\ell)\, P(t) - \lambda_i \bar{C}_i \qquad i=1,\ldots,6 \qquad (2.25\ b)$$

Defining $C_i = (\ell/\beta)\,\bar{C}_i$; and recalling $a_i = \beta_i/\beta$

$$(\ell/\beta)\,\dot{\bar{C}}_i(t) = (\beta_i/\beta)\,P(t) - \lambda_i(\ell/\beta)\,\bar{C}_i \qquad (2.26\ a)$$

$$\dot{\bar{C}}_i(t) = a_i P(t) - \lambda_i C_i \qquad i=1,\ldots,6 \qquad (2.26\ b)$$

3. FEEDBACK MODELS

3.1 DESCRIPTION OF FEEDBACK

A reactor with feedback can be represented, in the absence of external sources, by a block diagram,

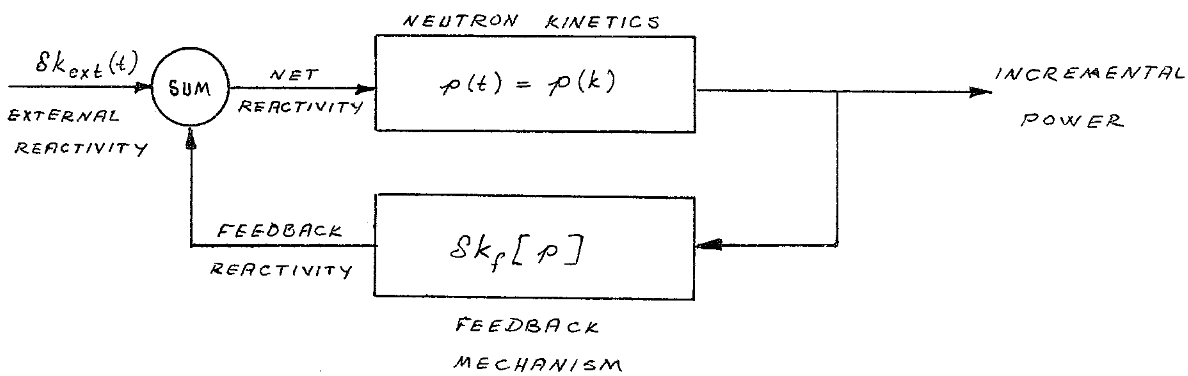


Figure-2 Block diagram of a reactor with feedback.

Here $p(t)$ is the incremental power, i.e., $p(t) = P(t) - P_o$ and $\delta k_f[p]$ is the feedback functional expressed in terms of the incremental power so that $\delta k_f(o) = 0$.

Reactivity feedback can be represented by,

$$k[P] = k[P_o + p] = k_o + k_f(P_o) + \delta k_f[p] + \delta k_{ext} \tag{2.26}$$

where $\delta k_f[p]$ measures the feedback reactivity from its value at equilibrium, and $\delta k_{ext}(t)$ measures the incremental reactivity from its constant positive value k_o at equilibrium. k_o just compensates the equilibrium feedback reactivity.

$$k_o + k_f(P_o) = 0 \tag{2.27}$$

Now it is to be noted that the reactivity can be expressed as the superposition of the external and feedback reactivities only in the first-order perturbation theory. In general, an external change in atomic concentration will affect not only the external reactivity but also the feedback reactivity as a result of the changes in the shape function. Hence the input reactivity is given by,

$$\rho / \beta = k(t) = \delta k_{ext}(t) + \delta k_f[p] \tag{2.28}$$

To find the output, i.e., incremental power $p(t) = p[k]$ which is a functional, one has to solve the point kinetics equations.

$$(\ell/\beta)\,\dot{P} = (k-1)\,P + \sum_{i=1}^{6} \lambda_i C_i \qquad (2.29\text{ a})$$

$$\dot{C}_i = a_i P - \lambda_i C_i \qquad i=1,\ldots,6 \qquad (2.29\text{ b})$$

Equilibrium values can be found as,

$$\dot{P} = 0 \quad ; \qquad (1-k)\,P_o = \sum_{i=1}^{6} \lambda_i C_{io} \qquad (2.30)$$

$$\dot{C}_i = 0 \quad ; \qquad a_i P_o = \lambda_i C_{io}$$

Since $\sum_{i=1}^{6} a_i = 1$; $P_o = \sum_{i=1}^{6} \lambda_i C_{io}$

Then equation (2.30) becomes

$(1-k)\,P_o = P_o$ which implies $k=0$

Writing departures from equilibrium,

$$P = P_o + p \quad , \qquad C_i = C_{io} + c_i$$

Point kinetics equations becomes

$$(\ell/\beta)\,\dot{p} = (k-1)\,(P_o + p) + \sum_{i=1}^{6} \lambda_i C_{io} + \sum_{i=1}^{6} \lambda_i c_i \qquad (2.31)$$

$$(\ell/\beta)\,\dot{p} = k(P_o + p) - P_o - p + P_o + \sum_{i=1}^{6} \lambda_i c_i$$

$$(\ell/\beta)\,\dot{p} = k(t)\,(P_o + p) - p + \sum_{i=1}^{6} \lambda_i c_i$$

On the other hand, the equation for the deviation of the delayed neutron precursor concentration becomes,

$$\dot{c}_i = a_i p - \lambda_i c_i \qquad i=1,..,6 \qquad (2.32)$$

$$\dot{c}_i + \lambda_i c_i = a_i p$$

Multiplying both sides with the integration factor $\exp(\lambda_i t)$ we obtain

$$\frac{d}{dt}\left(e^{\lambda_i t} c_i \right) = e^{\lambda_i t} a_i p$$

integrating over 0 to t,

$$e^{\lambda_i t} c_i - c_{io} = \int_0^t a_i p(t') e^{\lambda_i t'} dt'$$

Since the deviations from equilibrium $c_{io} = 0$ for $t=0$

$$c_i(t) = \int_0^t a_i p(t) e^{-\lambda_i (t-t')} dt' \qquad (2.33)$$

Going with this equation back to the differential equation describing power

$$(\ell/\beta)\,\dot{p} = k(t)\,(P_o + p) - p(t) + \sum_{i=1}^{6} \lambda_i a_i \int_0^t e^{-\lambda_i (t-t')}\, p(t')\, dt'$$

or

$$(\ell/\beta)\,\dot{p} = k(t)\,(P_o + p) + \int_0^t \sum_{i=1}^{6} \lambda_i a_i e^{-\lambda_i (t-t')} p(t')\, dt' - p(t) \qquad (2.34)$$

let $t-t'=u$, $t'=t-u$ and $dt'=-du$

$$(\ell/\beta)\,\dot{p} = k(t)\,(P + p) + \int_0^t \sum_{i=1}^{6} \lambda_i a_i e^{-\lambda_i u}\, p(t-u)\, du - p(t)$$

If we introduce $D(u) = \sum_{i=1}^{6} \lambda_i a_i \, e^{-\lambda_i u}$ (2.35)

Since $\int_0^{\infty} D(u)\,du = 1$

$$(\ell/\beta)\,\dot{p} = k(t)\,(P_\circ + p) + \int_0^t D(u)\,p(t-u)\,du - p(t)\int_0^{\infty} D(u)\,du$$

$$(\ell/\beta)\,\dot{p} = k(t)\,(P_\circ + p) + \int_0^{\infty} D(u)\left[p(t-u) - p(t)\right]du \qquad (2.36)$$

because $p(t-u) = 0$ for $u > t$.

3.2 FEEDBACK FUNCTIONAL

3.2.1 MATHEMATICAL PROPERTIES OF FUNCTIONALS

Recall that the input reactivity is

$$k(t) = \delta k_{ext}(t) + \delta k_f[p] \qquad (2.37)$$

There are three important properties of feedback functionals [4] :

a) Invariance : $\delta k_f[p]$ is invariant under a time translation when the feedback parameters are not explicit functions of time. Mathematically,

$$\delta k\,(t-t_\circ) = \delta k_f\left[p(t-t_\circ)\right] \qquad (2.38)$$

b) Causality : The feedback reactivity $\delta k_f(t)$ at a time t is uniquely determined if p(t) is known only in the interval $(-\infty, t)$.

c) Stability : The feedback reactivity is bounded for any bounded input.

Time invariant, causal functional may be represented by a power series as follows [4]:

$$\delta k_f[p] = \sum_{i=1}^{n} \int_{-\infty}^{t} du_1 \int_{-\infty}^{t} du_2 \ \ldots \int_{-\infty}^{t} du_n \ G_n(t-u_1,\ldots,t-u_n)p(u_1)\ldots p(u_n) \qquad (2.39)$$

Since only the "analytic functions" can be represented by a power series, we shall assume that the feedback functionals are analytic.

When the power variations are sufficiently small, the functional power-series expansion can be terminated after the first term, i.e.,

$$\delta k_f[p] = \int_{-\infty}^{t} du\ G(t-u)\ p(u) = \int_{0}^{\infty} du\ G(u)\ p(t-u) \qquad (2.40)$$

This kind of functional is called linear functional and the corresponding feedback mechanism is called linear, so the function $G(t)$ is referred to as the " linear feedback kernel " .

Physically $G(t)$ is the reactivity at $t>0$ due to a unit energy released at $t=0$, when the feedback is linear. When the stability and causality conditions are applied

$$G(t) = 0 \qquad \text{for} \qquad t<0 \qquad \text{and}$$

$$\int_0^{\infty} |G(t)|\ dt < \infty \qquad (2.40\ a)$$

Then in the case of a linear feedback the point kinetics equations are given by

$$(\ell/\beta)\,\dot{P} = \left\{\delta k_{ext} + \int_0^{\infty} du\; G(u)\left[P(t-u) - P_o\right]\right\} P$$

$$+ \int_0^{\infty}\left[P(t-u) - P(t)\right] D(u) \;+\; S\,(\ell/\beta) \qquad (2.41)$$

In order to understand the physical implication of $G(u)$, suppose that we operate the reactor at a constant power level P_o until t=0, in the absence of external sources. At time t=0 we introduce a constant reactivity $\delta k_{ext}(t) = \delta k_o$ and reactor power increases to another constant power level P_o'.

Then from equation (2.41) with $\dot{p} = 0$ gives

$$\delta k_{ext} + \int_0^{\infty} du\;\; G(u)\left[P_o'(t-u) - P_o\right] = 0$$

or $$\delta k_o = -\gamma(\,P_o' - P_o\,) \qquad (2.42)$$

where we have introduced

$$\gamma \equiv \int_0^{\infty} G(u)\;\; du \qquad (2.43)$$

Thus the incremental change in the steady-state power level is proportional to the incremental change in the external reactivity. The proportionality constant γ is called the " power " or " temperature coefficient " of reactivity.

This point kinetics functional relates the reactor power $p(t)$ to the reactivity insertion $k(t)$. If we specify the reactivity insertion in a reactor, we can find the output, incremental power $p(t)$. Reactivity insertion as can be achieved externally by moving control rods, also can be caused by poison or temperature feedback.

3.3 TEMPERATURE FEEDBACK

The behaviour of the reactor is governed by both the temperature feedback and the build-up and burn-up of higher cross section fission product poisons, e.g. Xe^{135}, in time intervals of the order of hours. Since the thermal time constants are much less than those of Xe^{135} and I^{135} (9.2 and 6.7 hr., respectively), the temperature feedback can be treated in the prompt power coefficient of reactivity , γ. Stability considerations require γ to have negative sign.

$$\delta k_T[p] = \int_0^\infty du \ \ p(t-u)\ G(u) \ \cong \ p(t) \int_0^\infty du\ G(u)$$

$$= \ \gamma \ p(t) \qquad (2.44)$$

3.4 XENON FEEDBACK

In order to establish the functional relationship between Xe and p(t), we need the equations desribing the time behaviour of I^{135} and Xe^{135};

$$\partial I / \partial t = -\lambda_I I + y_I \sigma_f(\underline{r})\phi_o(\underline{r},t) \qquad (2.45)$$

$$\partial Xe / \partial t = \lambda_I I + y_x \sigma_f(\underline{r})\phi_o(\underline{r},t) - \lambda_x Xe(\underline{r}) - Xe(\underline{r})\,\sigma_{Xe}(\underline{r})\phi_o(\underline{r},t) \qquad (2.46)$$

where Xe and I are the Xe^{135} and I^{135} concentrations per fuel atom, y_x and y_I their yields, λ_x and λ_I their decay constants.

In order to use space-independent model we integrate these equations over the reactor volume, and introduce

$$Xe(t) = (\ 1/V\ ) \int_R d^3r\ Xe(\underline{r},t) \qquad (2.47)$$

$$I(t) = (\ 1/V\ ) \int_R d^3r\ I(\underline{r},t) \qquad (2.48)$$

$$\sigma_f = \int_R d^3r\ \sigma_f(\underline{r})\ \phi_o(\underline{r}) \Big/ \int_R d^3r\ \phi_o(\underline{r}) \qquad (2.49)$$

$$\sigma_{Xe} = V \int_R d^3r\ \sigma_{Xe}(\underline{r})\ Xe_o(\underline{r})\ \phi_o(\underline{r}) \Big/ \left\{ \left[\int_R d^3r\ Xe_o(\underline{r}) \right] \left[\int_R d^3r\ \phi_o(\underline{r}) \right] \right\} \qquad (2.50)$$

It proves convinient to choose P_o in $\phi_o(\underline{r},t) \approx [P(t)/P_o]\ \phi(\underline{r})$ as the average flux ϕ_o defined by,

$$\phi_o \equiv (\ 1/V\ ) \int_R d^3r\ \phi_o(\underline{r}) \qquad (2.51)$$

with this choise, P(t) has the dimensions of flux.
Using equations 47,48,49,50 and 51 we obtain the following lumped-parameter description,

$$dI(t)/dt = -\lambda_I I(t) + y_I \sigma_f P(t) \tag{2.52}$$

$$dXe(t)/dt = \lambda_I I(t) + y_x \sigma_f P(t) - \lambda_x Xe(t) - Xe(t)\,\sigma_{Xe} P(t) \tag{2.53}$$

In obtaining the last term in 53, we have assumed that $Xe(\underline{r},t)$ as well as $\phi_o(\underline{r},t)$ is separable in time and space, that is

$$(1/V) \int_R d^3r\, Xe(\underline{r},t)\, \sigma_{Xe}(\underline{r})\, \phi_o(\underline{r},t) \cong \sigma_{Xe} Xe(t)\, P(t) \tag{2.54}$$

Now we may express the Xenon feedback functional as,

$$\delta k_{Xe}[p] = \alpha_{Xe}\, \delta Xe(t) \tag{2.55}$$

where α_{Xe} is the average Xenon reactivity coefficient defined by,

$$\alpha_{Xe} = -\sigma_{Xe} / (\beta c \sigma_f) \tag{2.56}$$

where c is a number converting the local Xenon absorption per fission to overall reactivity, β the fraction of delayed neutrons.

As a result, the equations describing the time behaviour of a reactor in the presence of Xenon feedback are compiled below(*).

(*) Power coefficient of reactivity is defined as $-\gamma$.

$$(\ell / \beta)\, \dot{P} = \left[\delta k_{ext}(t) - \sigma_{Xe} Xe / (c\, \sigma_f \beta) - \gamma P \right] P - P + \sum_{i=1}^{6} \lambda_i C_i + S(\ell / \beta) \qquad (2.57\ a)$$

$$\dot{C}_i = a_i P - \lambda_i C_i \qquad (2.57\ b)$$

$$\dot{Xe} = y_x\, \sigma_f P - (\lambda_x + \sigma_x P)\, Xe + \lambda_I I \qquad (2.57\ c)$$

$$\dot{I} = y_I\, \sigma_f P - \lambda_I I \qquad (2.57\ d)$$

where the various parameters are as defined before.

Since the reactor power is proportional to flux $\phi(t)$ in the critical reactor and they are of the same dimensions in our equations, it is more convenient to consider the symbol $\phi(t)$ instead of $P(t)$ just for simplicity. The kinetic equations with some manipulations becomes,

$$\ell \dot{\phi} = \left[\delta_o - \beta - \sigma_x Xe / (c\, \sigma_f) - \gamma \phi \right] \phi + \beta \sum_{i=1}^{6} \lambda_i C_i + S \ell$$

Introducing a new variable,

$$D_i = \beta C_i = \ell \bar{C}_i$$

the precursor concentration equation becomes, noting $a_i = \beta_i / \beta$;

$$\dot{D}_i = \beta_i \phi - \lambda_i D_i \qquad (2.58)$$

Now the kinetics equations for future reference, without external sources are

$$\ell \dot{\bar{\phi}} = \left[\delta_o - \beta - \sigma_x \, Xe / (c \, \sigma_f) - \gamma \bar{\phi} \right] \bar{\phi} + \sum_{i=1}^{6} \lambda_i D_i \qquad (2.59\ a)$$

$$\dot{D}_i = \beta_i \bar{\phi} - \lambda_i D_i \qquad (2.59\ b)$$

$$\dot{Xe} = y_x \sigma_f \bar{\phi} - (\lambda_x + \sigma_x \bar{\phi}) \, Xe + \lambda_I I \qquad (2.59c)$$

$$\dot{I} = y_I \sigma_f \bar{\phi} - \lambda_I I \qquad (2.59\ d)$$

Also note the integro-differential form of the point reactor kinetics equation for future reference,

$$\ell \dot{\phi} = \rho_f \left[\phi(t) \right] \; (\bar{\phi}_o + \phi) + \int_0^{\infty} \left[\phi(t-u) - \phi(t) \right] D(u) \, du \qquad (2.60)$$

CHAPTER III

ASYMPTOTIC STABILITY ANALYSIS

1. DEFINITION OF ASYMPTOTIC STABILITY IN THE LARGE

In this section we will investigate the region of linear stability in which the Xenon oscillations are always damped for any initial perturbation. We are thus interested in criteria sufficient for asymptotic stability in the large (A.S.L.).

Assume that the reactor becomes autonomous at t=0. The behaviour of the flux for $t > 0$ is described by the kinetic equation of a stationary point reactor with an arbitrary feedback. Recalling,

$$\ell \, \dot{\phi} = \rho_f \left[\phi(t) \right] (\phi_o + \phi) + \int_0^\infty \left[\phi(t-u) - \phi(t) \right] D(u) \, du \qquad (3.1)$$

where ℓ is the prompt neutron generation time, $D(t)$ the delayed neutron distribution kernel, i.e. $D(t) = \sum_{i=1}^{6} \lambda_i \beta_i \left[\exp(-\lambda_i t) \right]$, where β_i and λ_i are the delayed neutron fractions and the decay constants, ϕ_o and $\phi(t)$ the equilibrium, and incremental flux, and finally $\rho_f \left[\phi(t) \right]$ the feedback functional representing the incremental feedback reactivity satisfying $\rho_f \left[0 \right] = 0$.

Thus it is assumed that the reactor is critical at time t=0, and then an arbitrary perturbation is applied i.e. some reactivity is inserted into the reactor; and the conditions for decaying incremental flux $\phi(t)$ are investigated for subsequent times. The behaviour of $\phi(t)$ for $t > 0$ and for $t \rightarrow \infty$ in particular depends on the entire past history of the reactor due to feedback functional $\rho_f \left[\phi(t)\right]$.

2. GOVERNING EQUATIONS

The equations describing the time behaviour of a reactor in the presence of Xenon feedback are compiled below, neglecting delayed neutrons and defining the power coefficient of reactivity as $-\gamma$.

$$\ell \dot{\phi}' = \left(\delta_o - \frac{\sigma_x X'}{c \ \sigma_f} - \gamma \phi' \right) \phi' \tag{3.2}$$

$$\dot{X}' = y_x \sigma_f \phi' - \lambda_x X' - \sigma_x X' \phi' + \lambda_I I' \tag{3.3}$$

$$\dot{I}' = y_I \sigma_f \phi' - \lambda_I I' \tag{3.4}$$

Equilibrium values can be found as follows :

$$\dot{\phi}' = 0 \ ; \qquad \delta_o - \frac{\sigma_x X_o'}{c \ \sigma_f} - \gamma \phi_o' = 0 \tag{3.5}$$

$$\dot{X}' = 0 \ ; \qquad y_x \sigma_f \phi_o' - (\lambda_x + \sigma_x \phi_o') X_o' + \lambda_I I_o' = 0 \tag{3.6}$$

$$\dot{I}' = 0 \ ; \qquad y_I \sigma_f \phi_o' - \lambda_I I_o' = 0 \tag{3.7}$$

Eq. (3.7) gives $$I_o' = \frac{y_I \sigma_f \phi_o'}{\lambda_I} \tag{3.8}$$

Inserting this into eq. (3.6) and solving for X_o'

$$X_o' = \frac{(y_I + y_x)\, \sigma_f \,\phi_o'}{\lambda_x + \sigma_x \phi_o'} = \frac{y\ \sigma_f}{\sigma_x}\ Y \tag{3.9}$$

where Y is defined as,

$$Y = \sigma_x \phi_o' \ / \ (\lambda_x + \sigma_x \phi_o') \qquad \text{and} \qquad y = y_I + y_x \tag{3.10}$$

Finally inserting X_o' into eq. (3.5)

$$\mathcal{S}_o = y\ Y \ / \ c + \gamma\ \phi_o' \tag{3.11}$$

On the other hand the equality for Y gives ,

$$\phi_o' = \lambda_x Y \ / \ (\ \sigma_x\ (\ 1\text{-}Y\ )) \tag{3.12}$$

Now dividing equations 5,6,7 by ϕ_o', X_o', I_o', respectively and defining

$I'/I_o' = I$, $X'/X_o' = X$, $\phi'/\phi_o' = \phi$ gives,

$$\dot{I} = y_I \sigma_f \phi' \ / \ I_o - \lambda_I I = \lambda_I\ (\ \phi - I\ ) \tag{3.13 a}$$

where we inserted for I_o.

$$\dot{X} = -\lambda_x X + y_x \sigma_f \phi'/X_o' - \sigma_x X\ \phi\ \phi_o' + \lambda_I I\ I_o' \ / \ X_o' \tag{3.13 b}$$

where we assumed $y_x \cong 0$.

Here the third term, after inserting for ϕ_o' , becomes

$$\sigma_x \phi_o' X \phi = \lambda_x Y X \phi / (1 - Y)$$

and the fourth term, after inserting for $I_o' = y_I \sigma_f \phi_o' / \lambda_I$ and $X_o' = y \sigma_f Y / \sigma_x \cong y_I \sigma_f Y / \sigma_x$ becomes,

$$\frac{\lambda_I I I_o'}{X_o'} = \lambda_I I \left[\frac{y_I \sigma_f \phi_o'}{\lambda_I}\right] \Big/ \left[\frac{y_I \sigma_f Y}{\sigma_x}\right] = \frac{I \sigma_x}{Y} \phi_o'$$

and using $\phi_o' = \lambda_x Y / (\sigma_x (1 - Y))$ gives,

$$\lambda_I I I_o' / X_o' = \lambda_x I / (1 - Y)$$

Hence,

$$\dot{X} = - \lambda_x X - \frac{\lambda_x Y}{1-Y} X \phi + \frac{\lambda_x I}{1-Y} = \frac{\lambda_x}{1-Y} \left\{ I - X \left[Y (\phi - 1) + 1 \right] \right\} \qquad (3.14)$$

$$\ell \dot{\phi} = (\delta_o - \sigma_x X X_o' / (c \sigma_f) - \gamma \phi \phi_o') \phi$$

simply inserting for X_o' gives,

$$\ell \dot{\phi} = (\delta_o - y Y X / c - \gamma \phi \phi_o') \phi$$

$$= (y Y / c) \left[- X + \delta_o c / y Y - c \gamma \phi \phi_o' / (y Y) \right] \phi$$

$$= \frac{y Y}{c} \left[- X + 1 + (\frac{\delta_o c}{y Y} - 1) - \frac{c \gamma}{y Y} \phi \phi_o' \right] \phi$$

Now if we denote $c\,\gamma\,\phi_o' / (y\,Y) = R$ then from eq. (3.12)

$$\delta_o = y\,Y / c + \gamma\,\phi_o'$$

$$\frac{\delta_o\,c}{y\,Y} - 1 = \frac{c\,\gamma}{y\,Y}\,\phi_o' = R$$

Thus,

$$\ell\,\dot{\phi} = (y\,Y / c)(1 - X + R - R\,\phi)\,\phi$$

$$\ell\,\dot{\phi} = (y\,Y / c)(1 - X + R(1 - \phi))\,\phi$$

Restating the unit equilibrium equations,

$$\dot{I} = \lambda_I(\phi - I) \qquad (3.15)$$

$$\dot{X} = \frac{\lambda_x}{1 - Y}\left\{1 - X\left[Y(\phi - 1) + 1\right]\right\} \qquad (3.16)$$

$$\ell\,\dot{\phi} = \frac{y\,Y}{c}\left[1 - X + R(1 - \phi)\right]\phi \qquad (3.17)$$

with equilibrium $X_o = I_o = \phi_o = 1$.

On the other hand the kinetics equations with a temperature reactivity coefficient γ are; without delayed neutrons,

$$\ell\,\dot{\phi} = (\delta_o - \sigma_x X / (c\,\sigma_f) - \gamma\,\phi)\,\phi \qquad (3.18)$$

$$\dot{X} = y_x\,\sigma_f\,\phi - \lambda_x X - \sigma_x X\,\phi + \lambda_I I \qquad (3.19)$$

$$\dot{I} = y_I \sigma_f \phi - \lambda_I I \qquad (3.20)$$

The following transformation, given by Smets's [2] , casts these equations in a more compact form which is often preferred in the stability analysis of Xenon-controlled nuclear reactors.

$$Z \equiv \ell \phi - \frac{X}{c \;\sigma_f} - \frac{\lambda_I \; I}{c \;\sigma_f (\lambda_I - \lambda_x)}$$

Differentiate Z and eliminate $\dot{\phi}$, $\dot{X}$ and $\dot{I}$

$$\dot{Z} = \ell \dot{\phi} - \frac{\dot{X}}{c \;\sigma_f} - \frac{\lambda_I \; \dot{I}}{c \;\sigma_f (\lambda_I - \lambda_x)}$$

$$\dot{Z} = \left[\delta_o - \frac{\sigma_x X}{c \;\sigma_f} - \gamma \phi \right] \phi - \frac{1}{c \;\sigma_f} \Big[y_x \sigma_f \phi - \lambda_x X - \sigma_x X \phi$$

$$+ \lambda_I I \Big] - \frac{\lambda_I}{c \;\sigma_f (\lambda_I - \lambda_x)} \left[y_I \sigma_f \phi - \lambda_I I \right]$$

$$\dot{Z} = \left[\delta_o - \frac{\sigma_x X}{c \;\sigma_f} - \frac{y_x}{c} + \frac{\sigma_x X}{c \;\sigma_f} - \frac{y_I \; \lambda_I \; \sigma_f}{c \;\sigma_f (\lambda_I - \lambda_x)} \right] \phi$$

$$+ \frac{\lambda_x X}{c \;\sigma_f} - \frac{\lambda_I \; I}{c \;\sigma_f} + \frac{\lambda_I^2 \; I}{c \;\sigma_f (\lambda_I - \lambda_x)} - \gamma \phi^2$$

put the value of $X / (c \sigma_f) = \ell \phi - \lambda_I I \,/\, (c \;\sigma_f (\lambda_I - \lambda_x)) - Z$

$$\dot{Z} = \left[\delta_o - \frac{y_x}{c} - \frac{y_I \; \lambda_I}{c \; (\lambda_I - \lambda_x)} + \ell \lambda_x \right] \phi - \gamma \phi^2$$

$$- \frac{\lambda_I I}{c \;\sigma_f} \left[1 + \frac{\lambda_x}{\lambda_I - \lambda_x} - \frac{\lambda_I}{\lambda_I - \lambda_x} \right] - \lambda_x Z$$

Hence, $$\dot{Z} = a_1\bar{\phi} - \lambda_x Z - \gamma\,\bar{\phi}^2 \qquad (3.21)$$

where $$a_1 = \delta_o + \ell\,\lambda_x - \frac{y_x}{c} - \frac{\lambda_I\ y_I}{c\ (\lambda_I - \lambda_x)}$$

and $$\ell\,\dot{\bar{\phi}} = (\delta_o - \sigma_x\,\ell\,\bar{\phi} + \frac{\sigma_x\ \lambda_I\ I}{c\ \sigma_f\,(\lambda_I - \lambda_x)} + Z\,\sigma_x - \gamma\,\bar{\phi}\ )\ \bar{\phi}$$

$$\ell\,\dot{\bar{\phi}} = (\delta_o + \sigma_x Z + \alpha_I I - \alpha_f\bar{\phi}\ )\ \bar{\phi} \qquad (3.22)$$

where $\alpha_f = \gamma + \sigma_x\,\ell$; $\alpha_I = \lambda_I\,\sigma_x\ /\ (\ c\ \sigma_f\,(\lambda_I - \lambda_x))$

Equilibrium values are ;

$$\ell\,\dot{\phi} = 0 \quad ; \quad \delta_o + \sigma_x Z_o + \alpha_I I_o - \alpha_f\bar{\phi}_o = 0$$

$$\dot{Z} = 0 \quad ; \quad a_1\bar{\phi}_o - \lambda_x Z_o - \gamma\,\bar{\phi}_o^2 = 0$$

Expand equations (21) and (22) about equilibrium as follows,

$$\bar{\phi} = \bar{\phi}_o + \phi \quad ; \quad Z = Z_o + z \quad ; \quad I = I_o + y$$

$$\ell\ (\ \dot{\bar{\phi}}_o + \dot{\phi}\ ) = (\ \delta_o + \sigma_x Z_o + \sigma_x z + \alpha_I I_o + \alpha_I y - \alpha_f\bar{\phi}_o - \alpha_f\phi\ )(\ \bar{\phi}_o + \phi\ )$$

$$\ell\ \dot{\phi} = (\ \sigma_x\ z + \alpha_I y - \alpha_f\phi\ )\ (\ \bar{\phi}_o + \phi\ ) \qquad (3.23)$$

$$(\ \dot{Z}_o + \dot{z}\ ) = a_1\bar{\phi}_o + a_1\phi - \lambda_x Z_o - \lambda_x z - \gamma\,(\ \bar{\phi}_o^2 + 2\,\bar{\phi}_o\,\phi + \phi^2)$$

substitute the equilibrium values,

$$\dot{z} = a_2 \phi - \lambda_x z - \gamma \phi^2 \qquad (3.24)$$

where $a_2 = a_1 - 2\gamma \phi_o$

and $$(\dot{I}_o + \dot{y}) = y_I \sigma_f \phi_o + y_I \sigma_f \phi - \lambda_I I_o - \lambda_I y$$

$$\dot{y} = y_I \sigma_f \phi - \lambda_I y \qquad (3.25)$$

Hence ; $\ell \dot{\phi} = \rho_f [\phi(t)] \quad (\phi_o + \phi) \qquad ; \qquad \rho_f [\phi(t)] = \sigma_x z + \alpha_I y - \alpha_f \phi$

with $$\dot{z} + \lambda_x z = a_2 \phi - \gamma \phi^2$$

$$z(t) e^{\lambda_x t} - z_o = \int_{-\infty}^{t} \left[a_2 \phi(u) - \gamma \phi^2(u) \right] e^{\lambda_x u} du$$

$z(t) = 0$ for $t < 0$ because $Z(t) = Z_o$

$$z(t) = \int_{-\infty}^{0} \left[a_2 \phi(u) - \gamma \phi^2(u) \right] \exp\left[- \lambda_x (t-u) \right] du \qquad (3.26)$$

$$y(t) = \int_{-\infty}^{0} y_I \sigma_f \phi(u) \exp - \left[\lambda_I (t-u) \right] du \qquad (3.27)$$

so that,

$$\rho_f [\phi(t)] = \int_{-\infty}^{t} \left[\sigma_x a_2 \exp\left[- \lambda_x (t-u) \right] + \alpha_I y_I \sigma_f \exp\left[- \lambda_I (t-u) \right] - \alpha_f \delta(t) \right] \phi(u) du$$

$$- \int_{-\infty}^{t} \sigma_x \gamma \exp\left[- \lambda_x (t-u) \right] \phi^2(u) du$$

Now we can state a sufficient condition for the Asymptotic Stability in the Large obtained by AKÇASU and DALFES [17].

If
$$I \equiv \int_{-\infty}^{t} \rho_f \left[\phi(t')\right] \phi(t')\, dt' \leqslant 0$$

is satisfied for all t and for $\phi(t)$, then the equilibrium state $\phi(t) = 0$ of the stationary reactor, which is assumed to be unique, is asymptotically stable.

Substituting $\rho_f\left[\phi(t)\right]$ from above,

$$I \equiv \int_{-\infty}^{t} du\left[\int_{-\infty}^{u}\phi(v)dv\, K(u-v)\phi(u) - \alpha_f\int_{-\infty}^{u} \phi^2(u)\, du\right] - \sigma_x \gamma \int_{-\infty}^{t} du\, \phi(u) \int_{-\infty}^{u} dv\, \phi^2(v)\exp\left[-\lambda_x(u-v)\right] \quad (3.28)$$

where $K(t) = \sigma_x a_2 \exp(-\lambda_x t) + \alpha_I y_I \sigma_f \exp(-\lambda_I t)$ (3.29)

Our task is now to determine sufficient conditions under which (3.28) will be non-positive for all $t \geqslant 0$ and for all $\phi(t)$. When $\gamma < 0$ there are two equilibrium states although only one equilibrium state exists when $\gamma > 0$, as shown by Chernick [7]. Since global asymptotic stability requires a unique equilibrium state as a necessary condition, we shall consider only the case of $\gamma > 0$.

In eq. (3.28) the third term, recalling $-\phi(t) \leqslant \bar{\phi}_0$ at all times

$$-\sigma_x \gamma \int_{-\infty}^{t} \phi(u)\, du \int_{-\infty}^{u} e^{-\lambda_x(u-v)} \phi^2(v)\, dv \leqslant \sigma_x \gamma \bar{\phi}_0 \int_{-\infty}^{t} du \int_{-\infty}^{u} e^{-\lambda_x(u-v)} \phi(v)\, dv$$

let $u - v = v'$

$$= \sigma_x \gamma \bar{\phi}_o \int_{-\infty}^{t} du \int_0^{\infty} e^{-\lambda_x v'} \phi^2(u-v')\, dv'$$

change the order of integration and let $u - v' = u'$

$$= - \sigma_x \gamma \bar{\phi}_o \int_0^{\infty} dv' \; e^{-\lambda_x v'} \int_{-\infty}^{t-v'} du' \;\; \phi^2(u')$$

$$\leqslant - \sigma_x \gamma \bar{\phi}_o \int_0^{\infty} dv' \; e^{-\lambda_x v} \int_{-\infty}^{t} du \;\; \phi^2(u)$$

$$= \frac{\sigma_x \gamma \bar{\phi}_o}{\lambda_x} \int_{-\infty}^{t} du \;\; \phi^2(u)$$

using this to replace the third term in (3.28), we find

$$I < \int_{-\infty}^{t} du \int_{-\infty}^{u} dv\, K(u-v)\; \phi(u)\; \phi(v) - \left(\alpha_f - \frac{\bar{\phi}_o \sigma_x \gamma}{\lambda_x}\right) \int_{-\infty}^{t} \phi^2(u)\; du$$

or

$$I < \int_{-\infty}^{t} du \int_{-\infty}^{u} dv \phi(u)\; \phi(v) \left\{ K(u-v) - \left(\alpha_f - \frac{\bar{\phi}_o \sigma_x \gamma}{\lambda_x}\right) \;\; \delta\,(u-v) \right\} \qquad (3.31)$$

introduce here unit step function $h(t)$,

$$= \int_{-\infty}^{t} du \int_{-\infty}^{t} dv\; \phi(u)\; \phi(v) \left\{ h(u-v) \left[K(u-v) - \left(\alpha_f - \frac{\bar{\phi}_o \sigma_x \gamma}{\lambda_x}\right) \delta\,(u-v) \right] \right\}$$

let $$g(t) = h(t) \left[K(t) - \left(\alpha_f - \frac{\bar{\phi}_o \sigma_x \gamma}{\lambda_x}\right) \delta\,(t) \right] \qquad (3.32)$$

and $$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(iw)\; e^{iwt}\; dw$$

where $G(iw)$ is the Fourier transform of $g(t)$ i.e.,

$$G(iw) = \int_{-\infty}^{\infty} g(t)\, e^{-iwt}\, dt$$

$$= \int_{-\infty}^{\infty} h(t)\, K'(t)\, e^{-iwt}\, dt = \int_{0}^{\infty} K'(t)\, e^{-iwt}\, dt$$

$$= \overline{K}'(iw)$$

$\overline{K}'$ is the one sided Laplace transform of $K'(t)$.

$$I < \int_{-\infty}^{t} du \int_{-\infty}^{t} dv\ \phi(u)\, \phi(v)\, \frac{h(u-v)}{2\pi} \int_{-\infty}^{\infty} \overline{K}'(iw)\, e^{iw(u-v)}\, dw$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{K}'(iw) \left[\int_{-\infty}^{t} du\ \phi(u)\, e^{iwu} \right] \left[\int_{-\infty}^{t} dv\ \phi(v)\, e^{-iw} \right] dw$$

$$= \frac{1}{\pi} \int_{0}^{\infty} \mathrm{Re}\left[\overline{K}'(iw)\right] \left| \int_{-\infty}^{t} du\ \phi(u)\, e^{-iwu} \right|^2 \leqslant 0 \qquad (3.33)$$

This condition will hold if $\mathrm{Re}\left[\overline{K}'(iw)\right] \leqslant 0$ for all w or, since

$$\overline{K}(s) = \frac{\sigma_x\, a_2}{\lambda_x + s} + \frac{\alpha_I\, y_I\, \sigma_f}{\lambda_I + s} \qquad \text{from eq. (3.29)}$$

where $a_2 = a_1 - 2\gamma \bar{\phi}_o = \delta_o + \ell\lambda_x - \dfrac{y_x}{c} - \dfrac{\lambda_I\, y_I}{c\,(\lambda_I - \lambda_x)} - 2\gamma\bar{\phi}_o$

neglecting prompt neutron lifetime as $\ell \cong 0$; $y_x = o$.

$$a_2 = \delta_o - \frac{\lambda_I\, y_I}{c\,(\lambda_I - \lambda_x)} - 2\gamma\bar{\phi}_o \quad ; \text{ and } \quad \alpha_I = \frac{\lambda_I\, \sigma_x}{c\, \sigma_f\, (\lambda_I - \lambda_x)}$$

so, $$\overline{K}(s) = \frac{\sigma_x}{\lambda_x + s}\left[\delta_o - \frac{y_I\, \lambda_I}{c(\lambda_I - \lambda_x)} - 2\gamma\Phi_o\right] + \frac{\lambda_I\, y_I\, \sigma_x}{c\,(\lambda_I - \lambda_x)} \cdot \frac{1}{\lambda_I + s} \qquad (3.34)$$

inserting the value δ_o from eq. (3.11) and noting $\gamma \bar{\phi}_o = y\,Y\,R / c$

$$\bar{K}(s) = \sigma_x \left[\frac{y\,Y}{c} + \gamma\bar{\phi}_o - \frac{y_I\,\lambda_I}{c(\lambda_I - \lambda_x)} - 2\gamma\bar{\phi}_o \right] \frac{1}{\lambda_x + s} + \frac{\lambda_I\,\sigma_x\,y_I}{c\,(\lambda_I - \lambda_x)} \frac{1}{\lambda_I + s}$$

$$= \sigma_x \frac{y}{c} \left\{ \left[Y(1-R) - \frac{\lambda_I}{c\,(\lambda_I - \lambda_x)} \right] \frac{1}{\lambda_x + s} + \frac{\lambda_I}{(\lambda_I - \lambda_x)} \frac{1}{\lambda_I + s} \right.$$

$$= \sigma_x \frac{y}{c} \left\{ \frac{Y\,(1-R)}{\lambda_x + s} - \frac{\lambda_I}{\lambda_I - \lambda_x} \left(\frac{1}{\lambda_I + s} - \frac{1}{\lambda_x + s} \right) \right\}$$

$$= \sigma_x \frac{y}{c} \left[\frac{Y\,(1-R)}{\lambda_x + s} - \frac{\lambda_I}{(\lambda_I + s)(\lambda_x + s)} \right] \qquad (3.35)$$

$$\bar{K}'(s) = \bar{K}(s) - \alpha_f + \bar{\phi}_o\,\sigma_x\,\gamma / \lambda_x$$

multiplying both sides with $c\,\lambda_x / y\,\sigma_x$

$$\bar{K}'(s) \frac{c\,\lambda_x}{y\,\sigma_x} = \lambda_x \left[\frac{Y\,(1-R)}{\lambda_x + s} - \frac{\lambda_I}{(\lambda_I + s)(\lambda_x + s)} \right] - \left(\alpha_f - \frac{\bar{\phi}_o\,\sigma_x\,\gamma}{\lambda_x} \right) \frac{c\,\lambda_x}{y\,\sigma_x} \qquad (3.36)$$

now $\alpha_f \cong \gamma$ and $\dfrac{\gamma\,c\,\lambda_x}{y\,\sigma_x} = R\,(1 - Y)$ from definitions of $\bar{\phi}_o$ and R.

also, $$\frac{\bar{\phi}_o\,c\,\gamma}{y} = R\,Y$$

so, $$K'(s) \frac{c\,\lambda_x}{y\,\sigma_x} = \lambda_x \left[\frac{Y\,(1-R)}{\lambda_x + s} - \frac{\lambda_I}{(\lambda_I + s)(\lambda_x + s)} \right] - R\,(1-2Y)$$

$$= \frac{-a\,s^2 + B\,s - \lambda_I\,\lambda_x\,C}{s^2 + (\lambda_I + \lambda_x)\,s + \lambda_I\,\lambda_x} \qquad (3.37)$$

where $a = R(1-2Y)$, $b = Y(1-R)$

$B = \lambda_x b - (\lambda_I + \lambda_x) a$, $C = 1 + a - b = (1-Y)(1+R)$

The condition for positivity $\mathrm{Re}\left[K'(iw)\right] \leqslant 0$ leads to

$$(aw^2 - \lambda_I\lambda_x C)(-w^2 + \lambda_I\lambda_x) + w^2 B(\lambda_I + \lambda_x) \leqslant 0 \qquad (3.38)$$

or, $$aw^4 - \left[\lambda_I\lambda_x a + \lambda_I\lambda_x C + (\lambda_I+\lambda_x)(\lambda_x b - (\lambda_I+\lambda_x)a)\right]w^2 + \lambda_I^2\lambda_x^2 C \geqslant 0$$

replacing b by 1+a-C in the coefficient of w^2;

$$= \lambda_I\lambda_x a + \lambda_I\lambda_x C + (\lambda_I+\lambda_x)\lambda_x + (\lambda_I+\lambda_x)\lambda_x a - (\lambda_I+\lambda_x)\lambda_x C - (\lambda_I+\lambda_x)^2 a$$

$$= -\left[\lambda_I^2 a + \lambda_x^2 C - \lambda_x(\lambda_I+\lambda_x)\right]$$

so, $$aw^4 + \left[\lambda_I^2 a + \lambda_x^2 C - \lambda_x(\lambda_I+\lambda_x)\right] w^2 + \lambda_I^2\lambda_x^2 C \geqslant 0 \qquad \text{for all } w$$

This inequality is satisfied if $a > 0$, $C > 0$ and

$$\left[\lambda_I^2 a + \lambda_x^2 C - \lambda_x(\lambda_I+\lambda_x)\right]^2 \leqslant 4\lambda_I^2\lambda_x^2 a C \qquad (3.39)$$

1) $C = (1-Y)(1+R) \quad 0$ for all allowed values of Y,R

since $0 \leqslant \emptyset_o \leqslant \infty$ hence, $0 \leqslant Y \leqslant 1$ and $R \geqslant -1$

2) $a = R(1-2Y) \geqslant 0$ if $0 \leqslant Y \leqslant 1/2$

3) $\lambda_I^2 a + \lambda_x^2 C - \lambda_x(\lambda_I+\lambda_x) \leqslant 2\lambda_I\lambda_x\sqrt{aC}$

$$\lambda_I^2 a + \lambda_x^2 C - 2\lambda_I \lambda_x \sqrt{aC} \leqslant \lambda_x(\lambda_I + \lambda_x)$$

$$\lambda_I \sqrt{a} + \lambda_x \sqrt{C} \geqslant \sqrt{\lambda_x(\lambda_I + \lambda_x)}$$

$$\lambda_I \sqrt{R\ (1-2Y)} + \lambda_x \sqrt{(1-Y)(1+R)} \geqslant \sqrt{\lambda_x(\lambda_I + \lambda_x)} \qquad (3.40)$$

Then there are no real roots and there is a double root for the equality.

Here the physical quantities are given as follows :

$y = 6.4 \ 10^{-2}$

$\lambda_I = 2.87 \ 10^{-5}$

$\lambda_x = 2.09 \ 10^{-5}$

$\sigma_x = 3.0 \ 10^{-18}$

$c = 1.5$

So we can plot ϕ_o versus γ ,(Figure 3).

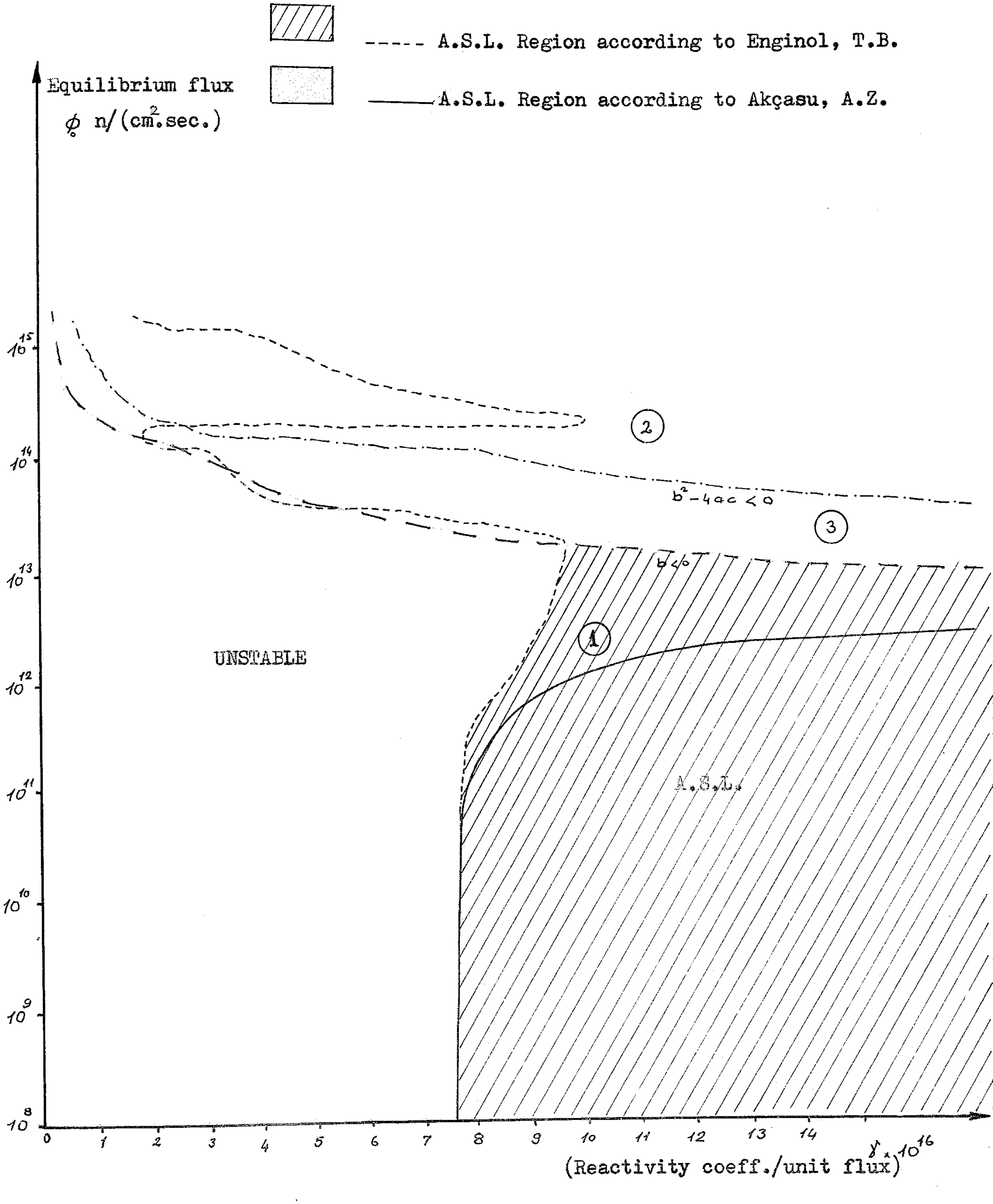


Figure - 3 Asymptotic Stability Regions according to two different criteria.

3. APPLICATION OF THE CRITERION PROPOSED BY ENGİNOL, T.B. [1]

The criterion proposed by Enginol, T.B. for the asymptotic stability of nuclear reactors is as follows : [1]

$$\mathrm{Re}\Big[K(iw)\Big] - \gamma - \frac{2\,\sigma_x\,\gamma\,\lambda_x\,\phi_o}{\lambda_x^2 + w^2} \leqslant 0 \qquad (3.41)$$

where the various parameters are as defined before.

$$\mathrm{Re}\Big[K(iw)\Big] = \mathrm{Re}\left\{\frac{\sigma_x\, y}{c}\left[\frac{Y\,(1-R)}{\lambda_x^2 + iw} - \frac{\lambda_I}{(\lambda_I+iw)(\lambda_x+iw)}\right]\right\}$$

$$= \frac{\sigma_x\, y}{c}\left[\frac{Y\,(1-R)\,\lambda_x}{\lambda_x^2 + w^2} - \frac{\lambda_I(\lambda_I\lambda_x - w^2)}{(\lambda_x^2+w^2)(\lambda_I^2+w^2)}\right] \qquad (3.42)$$

where $Y = \sigma_x\phi_o\,/\,(\lambda_x + \sigma_x\,\phi_o)$, $R = c\,\gamma\,\phi_o\,/\,(\,yY\,)$

Inserting this into eq. (3.41) and multiplying with $c\,\lambda_x/\,(\,y\,\sigma_x)$ gives,

$$\frac{Y\,(1-R)\lambda_x^2}{\lambda_x^2 + w^2} - \frac{\lambda_I\lambda_x(\lambda_I\lambda_x - w^2)}{(\lambda_I^2+w^2)(\lambda_x^2+w^2)} - \frac{c\,\gamma\,\lambda_x}{y\,\sigma_x} - \frac{2\,\gamma\,c\,\lambda_x^2\phi}{(\lambda_x^2+w^2)\,y} \leqslant 0$$

Noting that $\dfrac{\gamma\,c\,\lambda_x}{y\,\sigma_x} = R\,(1-Y)$ and $\dfrac{\gamma\,c\,\phi_o}{y} = R\,Y$

We obtain,

$$\frac{Y(1-R)\lambda_x^2}{\lambda_x^2 + w^2} - \frac{\lambda_I^2\lambda_x^2 - \lambda_I\lambda_x w^2}{(\lambda_I^2+w^2)(\lambda_x^2+w^2)} - R(1-Y) - \frac{2\lambda_x^2 R\;\;Y}{(\lambda_x^2+w^2)} \leqslant 0$$

$$Y(1-R)\lambda_x^2(\lambda_I^2+w^2)-\lambda_I^2\lambda_x^2+\lambda_I\lambda_x w^2-R(1-Y)(\lambda_I^2+w^2)(\lambda_x^2+w^2)-(\lambda_I^2+w^2)\,2\lambda_x^2 R\,Y \leqslant 0$$

$$R\,(1-Y)\,w^4+\left\{\left[\lambda_I^2+\lambda_x^2-Y\,(\lambda_I^2-2\lambda_x^2)\right]R-Y\lambda_x^2-\lambda_I\lambda_x\right\}w^2$$
$$+\left\{\left[2\,Y+1\right]\lambda_I^2\lambda_x^2 R+(\,1-Y\,)\lambda_I^2\lambda_x^2\right\}\geqslant 0 \qquad (3.43)$$

The form of which is $\quad a\,w^4+b\,w^2+c \geqslant 0$.

It is clear that the satisfaction of this inequality is assured by the imposition of the following conditions :

1)	$a \geqslant 0$	is satisfied already
2)	$c \geqslant 0$	is satisfied already
3)	$b \leqslant 0$	must be satisfied
4)	$b^2-4ac \leqslant 0$	must be satisfied.

The third condition is equivalent to,

$$\left[\lambda_I^2+\lambda_x^2-Y\,(\lambda_I^2-2\,\lambda_x^2)\right]\frac{c\,\gamma\,\phi_o}{y\,Y}-Y\lambda_x^2-\lambda_I\lambda_x \leqslant 0$$

or,

$$\gamma_1 \leqslant \frac{(\,Y\,\lambda_x^2+\lambda_I\lambda_x)\,y\,Y}{\left[\lambda_x^2+\lambda_I^2-Y\,(\lambda_I^2-2\,\lambda_x^2)\right]c\,\phi_o}$$

and the fourth condition gives,

$$\left\{\left[\lambda_I^2+\lambda_x^2-Y\,(\lambda_I^2-2\,\lambda_x^2)\right]R-Y\,\lambda_x^2-\lambda_I\lambda_x\right\}^2$$
$$-\left\{\left[2\,Y\,\lambda_I^2\lambda_x^2+\lambda_I^2\lambda_x^2\right]R+(\,1-Y\,)\lambda_I\lambda_x^2\right\}4\,R\,(\,1-Y\,) \geqslant 0$$

This stability criterion is also plotted in figure - 3.

4. DISCUSSION

It is seen from the plot of asymptotic stability of Xenon and temperature controlled point reactors that, a reactor is asymptotically stable against any arbitrary perturbation below the flux level of 10^{13} n./(cm^2.sec.), and for the temperature reactivity coefficient, γ greater than about $-7.5\ 10^{-16}$.

Now one may ask whether or not a point reactor could not be asymptotically stable outside this region. A positive answer to this question is possible. Recall that we examined the problem with the assumption that the delayed neutrons are produced " instantaneously " with respect to Xenon, since time decay constants of delayed neutrons are much shorter than that of I^{135} and Xe^{135}. So we did not considered them with a time delay. W. Baran and K. Meyer [12] studied the effect of delayed neutrons on the stability of a nuclear power reactor. They give an example showing that stability without delayed neutrons does not necessarily imply stability with delayed neutrons.

A sufficient condition for asymptotic stability of nuclear reactors with arbitrary feedback is proposed by T.B. Enginol [1]. This criterion leads to determination of three distinct regions ; the first one is (1) a region of asymptotic stability in the large, another one in which the system certainly is not asymptotically in the large(2), and no such conclusion can be derived for the third region(3).

The stability criterion given by Enginol is found to be more general than some previously proposed criteria. If the criterion proposed by Akçasu and Dalfes[17] is compared with the criterion proposed by Enginol, it is seen that the stability regions are different partly due to the fact that delayed neutrons are considered by the latter. Omitting the delayed neutrons, the two criteria become somewhat similar[1].

Akçasu and Dalfes' criterion to define the region of global asymptotic stability for equations (3.2),(3.3) and (3.4) has shown that there are large areas in parameter space ($\gamma - \phi_o$) which are known to be linearly stable. But outside this region of A.S.L. criterion given by Enginol can penetrate into this parameter region and suggest that the perturbations may have to be quite large for the system to show linear instability. This possibility was investigated by L.M. Shotkin[14], who gives a general method for determining the bounds on allowable disturbances in linearly stable systems, for which the system remains asymptotically stable. It is based on transforming a set of non-linear differential equations to a single equation that is valid within a given region of equilibrium. It is applicable to systems with a fairly general non-linear feedback as well as to systems that exhibit finite escape time.

One may refer to the paper by H.B. Smets[9] for asymptotic stability in the large with delayed neutrons in addition to analysis of Enginol[1]. According to Smets, if a linear reactor system is asymptotically stable when the delayed neutrons are neglected, then it is not necessarily asymptotically stable if the delayed neutrons are included in the model.

It should always be remembered that there is no " a priori " reason whatsoever to believe that the delayed neutrons have a stabilizing effect on this particular system. A converse generalization does not necessarily hold either. A linear numerical example showing that delayed neutrons may, in fact, destabilize a reactor has been given by Baran and Meyer [12].

CHAPTER IV

LINEAR STABILITY ANALYSIS

The stability of any equilibrium state may depend on the magnitude of the disturbance. An equilibrium state may be unstable for large perturbations even though it may be stable for small disturbances. In the latter case, the transients of the dynamical variables involve small departures from the original steady-state values, and can be adequately described by the linearized kinetic equations. The stability of a reactor for small disturbances is therefore treated by " linear " stability techniques.

1. CHARACTERISTIC FUNCTION AND LINEAR STABILITY

The question of stability of a physical system is associated with an equilibrium of an autonomous system.

A physical system is defined autonomous when the equations describing its temporal behaviour are invariant under a translation of the origin of time. In an autonomous system, all the changes take place automatically as a response to the changes in the past, and none of the parameters characterizing the system can depend on time explicitly. Hence, in an autonomous point reactor, the external reactivity and the external sources are constant in time.

Suppose that the reactor is operated at the equilibrium state P_o prior to t=0, and assume that an initial perturbation p(0) is introduced at t=0. The temporal behaviour of the reactor for $t > 0$ is governed by equation (2.60), i.e.,

$$(\ell/\beta)\,\dot{p} = \delta k_f\left[p(t)\right]\,(P_o+p) + \int_0^\infty du\left[p(t-u) - p(t)\right]D(u) \qquad (4.1)$$

neglecting p compared to P_o, and taking the value for $\delta k_f[p]$ from equation (2.40),

$$(\ell/\beta)\,\dot{p} = P_o\int_0^\infty p(t-u)\,G(u)\,du + \int_0^\infty p(t-u)\,D(u)\,du - p(t) \qquad (4.2)$$

Recalling that, $$D(t-u) = \sum_{i=1}^{6} a_i\lambda_i e^{-\lambda_i(t-u)}$$

$$(\ell/\beta)\,\dot{p}(t) = P_o\int_0^t p(t-u)\,G(u)\,du + \int_0^t du\sum_{i=1}^{6} a_i\lambda_i e^{-\lambda_i(t-u)}p(t) - p(t)$$

Taking the Laplace transform, we obtain

$$(\ell/\beta)\,s\,\bar{p}(s) - (\ell/\beta)\,p(o) = P_o\,\bar{p}(s)\,H(s) + \sum_{i=1}^{6}\frac{a_i\lambda_i}{s+\lambda_i}\,\bar{p}(s) - \bar{p}(s)$$

$$(\ell/\beta)\, s\, \bar{p}(s) - (\ell/\beta)\, p(o) = P_o \bar{p}(s)\, H(s) - s \sum_{i=1}^{6} \frac{a_i}{s + \lambda_i}\, \bar{p}(s)$$

where $H(s)$ is the Laplace transform of $G(t)$, i.e.,

$$H(s) = \int_0^{\infty} e^{-st}\, G(t)\, dt \qquad (4.3)$$

defining

$$\frac{1}{Z(s)} \equiv s \left[\frac{\ell}{\beta} + \sum_{i=1}^{6} \frac{a_i}{s + \lambda_i} \right]$$

$$p(o) = \bar{p}(s) \left[1 / Z(s) - P_o H(s) \right]$$

$$\bar{p}(s) / p(o) = Z(s) / (1 - P_o H(s)\, Z(s)) \qquad (4.4)$$

where $Z(s)$ is called zero-power transfer function and $H(s)$ is called the feedback transfer function which completely determines the linear feedback mechanism.

Since $G(t)$ must be of a stable linear system, eq. (2.40 a), it is absolutely integrable, and the integral in (4.3) converges for all $\text{Re } s \geqslant 0$. Thus $H(s)$ does not have any poles with positive or zero real parts. Note that $H(o) = \gamma^{\lambda}$, power coefficient of reactivity.

Equation (4.4) indicates that the behaviour of $p(t)$, $t > 0$, is determined by the singularities of $p(s)$ on the complex s plane. These singularities occur at the zeros of

$$Q(s) = 1 - P_o H(s)\, Z(s) \qquad (4.5)$$

which is called "characteristic " equation.

Thus the problem of linear stability of an equilibrium state is reduced to the problem of determining the sign of the real parts of the roots of the characteristic equation. If even one of these roots has a positive real part, then the reactor responce p(t) to an initial disturbance p(o), will increase exponentially with time, and hence the equilibrium state P_o will be unstable. We thus conclude : A reactor is linearly stable if the roots of the characteristic equation all have negative real parts.

In the following sections, we shall obtain the characteristic equation and discuss the necessary and sufficient conditions for all the roots of the characteristic equation to have negative real parts. These conditions are referred to as " linear stability criteria ", and enable one to investigate the question of stability of linear systems without explicitly solving the system equations.

2. LINEAR STABILITY ANALYSIS WITHOUT DELAYED NEUTRONS

2.1 CHARACTERISTIC EQUATION

Starting point kinetics equations are, as can be recalled from previous chapters, neglecting the delayed neutrons;

$$\ell \dot{\phi} = \left[\delta_o - (\sigma_x Xe \,/\, c\, \sigma_f) - \gamma \phi \right] \phi \qquad (4.6\ a)$$

$$\dot{Xe} = (y_x \sigma_f - \sigma_x Xe) \phi - \lambda_x Xe + \lambda_I I \qquad (4.6\ b)$$

$$\dot{I} = y_I \sigma_f \phi - \lambda_I I \qquad (4.6\ c)$$

The terms have the same interpretations as before.

Equilibrium values can be found as follows;

$$\dot{\phi} = 0 \quad ; \qquad \delta_o - (\sigma_x Xe_o/(c\, \sigma_f)) - \gamma \phi_o = 0 \qquad (4.7\ a)$$

$$\dot{Xe} = 0 \quad ; \qquad (y_x \sigma_f - \sigma_x Xe_o) \phi_o - \lambda_x Xe_o + \lambda_I I_o = 0 \qquad (4.7\ b)$$

$$\dot{I} = 0 \quad ; \qquad y_I \sigma_f \phi_o - \lambda_I I_o = 0 \qquad (4.7\ c)$$

From these equations equilibrium values are,

$$I_o = y_I \sigma_f \phi_o / \lambda_I \quad , \qquad Xe_o = (y_I + y_x) \sigma_f \phi_o / (\lambda_x + \sigma_x \phi_o) \qquad (4.8)$$

Initial reactivity may be determined (by control rod movement say) to define different equilibrium states ;

$$\delta_o = \sigma_x Xe_o / (c\, \sigma_f) + \gamma \phi_o \qquad (4.9)$$

Expand the equations (4.6) about equilibrium as follows ;

$$\bar{\phi} = \bar{\phi}_o + \phi \quad , \qquad Xe = Xe_o + \delta Xe \quad , \qquad I = I_o + \delta I$$

$$l\,(\dot{\bar{\phi}}_o + \dot{\phi}) = \left[\delta_o - (\sigma_x/(c\sigma_f))(Xe_o + \delta Xe) - \gamma(\bar{\phi}_o + \phi) \right] (\bar{\phi}_o + \phi) \qquad (4.10\ a)$$

$$\dot{Xe}_o + \delta \dot{Xe} = y_x \sigma_f (\bar{\phi}_o + \phi) - \sigma_x (Xe_o + \delta Xe)(\bar{\phi}_o + \phi) - \lambda_x (Xe_o + \delta Xe) + \lambda_I (I_o + \delta I) \qquad (4.1$$

$$\dot{I}_o + \delta \dot{I} = y_I \sigma_f (\bar{\phi}_o + \phi) - \lambda_I (I_o + \delta I) \qquad (4.10\ c)$$

$$l\ \dot{\phi} = \left[\delta_o - (\sigma_x Xe_o/(c\sigma_f)) - 2\gamma \bar{\phi}_o \right] \phi - \left[\sigma_x \phi_o/(c\, \sigma_f) \ \delta Xe \right. \qquad (4.11\ a)$$

$$\delta \dot{Xe} = (y_x \sigma_f - \sigma_x Xe_o) \phi - (\lambda_x + \sigma_x \phi_o) \delta Xe + \lambda_I \delta I \qquad (4.11\ b)$$

$$\delta \dot{I} = y_I \sigma_f \phi - \lambda_I \delta I \qquad (4.11\ c)$$

Taking the Laplace transforms,

$$l\ \ s\, \bar{\phi} = \left[\delta_o - \sigma_x Xe_o / (c\, \sigma_f) - 2\gamma \bar{\phi}_o \right] \bar{\phi} - (\sigma_x \phi_o / c\, \sigma_f) \ \overline{\delta Xe} \qquad (4.12\ a)$$

$$s\ \overline{\delta Xe} = (y_x \sigma_f - \sigma_x Xe_o) \bar{\phi} - (\lambda_x + \sigma_x \bar{\phi}_o) \ \overline{\delta Xe} + \lambda_I \overline{\delta I} \qquad (4.12\ b)$$

$$s\ \overline{\delta I} = y_I \sigma_f \bar{\phi} - \lambda_I \overline{\delta I} \qquad (4.13\ c)$$

Substituting eq.(4.12 b) and eq.(4.12 c) into eq.(4.12 a),

$$\ell s \bar{\phi} = \left[\delta_\circ - \sigma_x Xe_\circ / (c\,\sigma_f) - 2\gamma \bar{\phi}_\circ \right] \bar{\phi} - \frac{\sigma_x \bar{\phi}_\circ}{c\,\sigma_f (\, s + \lambda_x + \sigma_x \bar{\phi}_\circ)} \left[(y_x \sigma_f - \sigma_x Xe_\circ)\, \bar{\phi} + \frac{y_I \ \sigma_f \ \lambda_I}{s + \lambda_I} \ \bar{\phi} \right]$$

Substituting for   from eq.(4.9)

$$\ell s \bar{\phi} = - \gamma \bar{\phi}_\circ \bar{\phi} - \frac{\sigma_x \bar{\phi}_\circ}{c\,\sigma_f (s + \lambda_x + \sigma_x \bar{\phi}_\circ)} \left[(y_x \sigma_f - \sigma_x Xe_\circ)\, \bar{\phi} + \frac{y_I \ \sigma_f \ \lambda_I}{s + \lambda_I} \ \bar{\phi} \right] \tag{4.14}$$

Introducing some variables for simplicity in operations

$$y = y_I + y_x$$

$$U = \sigma_x Xe_\circ$$

$$R = y_x \sigma_f - \sigma_x Xe_\circ$$

$$PX = \sigma_x \bar{\phi}_\circ$$

$$ZX = \lambda_x + \sigma_x \bar{\phi}_\circ$$

$$T = c \ \sigma_f \ \ell$$

$$Z = \lambda_I + ZX$$

$$AF = \lambda_I PX \ (\ R + y_I \sigma_f)$$

We obtain finally,

$$s + \frac{\gamma \bar{\phi}_\circ}{\ell} + \frac{PX}{T \ (\ s + ZX \ )} \left[R + \frac{y_I \ \sigma_f \ \lambda_I}{s + \lambda_I} \right] = 0$$

$$T\ (s + \gamma \bar{\phi}_o / \ell)(s + ZX)(s + \lambda_I) + PX\ R\ (s + \lambda_I) + PX\ y_I\ \sigma_f\ \lambda_I = 0$$

$$s^3 + (Z + \gamma \bar{\phi}_o / \ell)\ s^2 + (\lambda_I ZX + Z\ \gamma \bar{\phi}_o / \ell + PX\ R / T)\ s + (\gamma \bar{\phi}_o \lambda_I ZX/\ell + AF/T) = 0 \qquad (4.15)$$

2.2 ROUTH - HURWITZ CRITERION

Routh - Hurwitz conditions are expressed in terms of the Hurwitz determinants, which are formed from the coefficients of the characteristic polynomials of the n th order as follows ; for polynomial

$$a_o s^n + a_1 s^{n-1} + \ldots\ldots\ldots + a_{n-1} s + a_n = 0 \qquad (4.16)$$

$$\Delta_n = \begin{vmatrix} a_1 & a_3 & a_5 & a_7 & \cdots \\ a_o & a_2 & a_4 & a_6 & \cdots \\ 0 & a_1 & a_3 & a_5 & \cdots \\ 0 & a_o & a_2 & a_4 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{vmatrix} \qquad (4.17)$$

We now state the Routh - Hurwitz stability criterion :
The roots of the characteristic equation all have negative real parts if, all the coefficients a are nonzero and positive, and if,

$$\Delta_0 = a_0 > 0$$

$$\Delta_1 = a_1 > 0$$

$$\Delta_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix} > 0$$

$$\vdots$$

$$\Delta_n = a_n \, \Delta_{n-1} > 0 \qquad (4.18)$$

are satisfied [2].

The conditions (4.18) are not independent of each other. In the case of a third-order system, these conditions are equivalent to $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1 a_2 > a_0 a_3$. We observe that there is only one additional condition in addition to the positiveness of all the coefficients. It is interesting to note that there is again only one condition in addition to the positiveness of all the coefficients in a fourth order system, i.e., $a_3 (a_1 a_2 - a_0 a_3) > a_4 a_1^2$. This observation is not true for high-order systems. For example, in a fifth-order system, there are two additional conditions[3]. In the general case of $n \geq 3$, the positiveness of the coefficients ensures only the negativeness of the real roots, but does not yield information about the sign of the real parts of the complex roots.

It is clear that, as more equations are added into the system description, the Routh-Hurwitz conditions are likely to be more restrictive.

2.2.1 APPLICATION OF ROUTH - HURWITZ CRITERION WITHOUT DELAYED NEUTRONS :

Characteristic equation being $a_o s^3 + a_1 s^2 + a_2 s + a_3 = 0$ where $a_o = 1$, $a_1 = Z + \gamma \phi_o / \ell$, $a_2 = \lambda_I ZX + Z \gamma \phi_o / \ell + PX\,R / T$, $a_3 = \gamma \phi_o \lambda_I ZX / \ell + AF / T$;

the stability conditions become,

1) $a_o > 0$ is satisfied already

2) $a_1 > 0$ is satisfied for all positive

3) $a_3 > 0$ gives,

$$\gamma \geqslant - \frac{\sigma_x \, (y_I \sigma_f - \sigma_x Xe_o)}{c \; \sigma_f \, (\lambda_x + \sigma_x \phi_o)} \qquad (4.19)$$

4) $a_1 a_2 > a_o a_3$ gives,

$$\left[Z + \gamma \phi_o / \ell\right] \left[\lambda_I ZX + Z \gamma \phi_o / \ell + PX\,R / T\right] > \left[\gamma \phi_o \lambda_I ZX / \ell + AF / T\right]$$

or

$$\left[\frac{Z \phi_o^2}{\ell^2}\right] \gamma^2 + \left[\frac{Z^2 \phi_o}{\ell} + \frac{PX\,R\,\phi_o}{T\,\ell}\right] \gamma + \left[Z \lambda_I ZX + \frac{Z\,PX\,R}{T} - \frac{AF}{T}\right] \geqslant 0 \qquad (4.20)$$

If eqs.(4.19) and (4.20) are solved for various equilibrium values of flux in the range $10^8 < \phi_o < 10^{15}$ we can find the stable values of prompt temperature reactivity coefficient γ. Results are plotted in figure - 4.

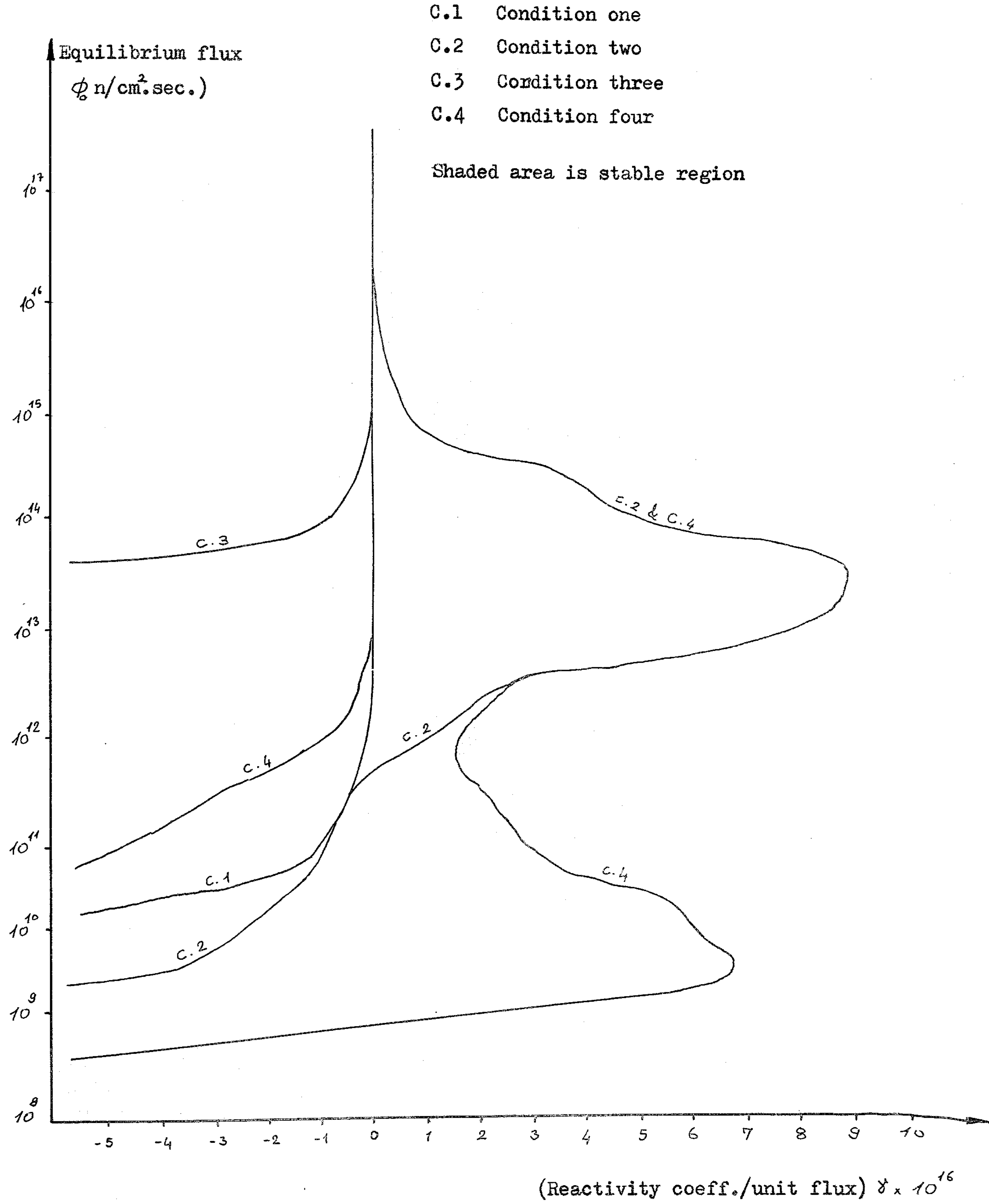


Figure - 4 Regions of Stability (shaded) and instability according to Routh-Hurwitz criterion without delayed Neutrons.

2.3 OTHER POSSIBILITIES FOR STABILITY WITHOUT DELAYED NEUTRONS

We stated that it is necessary to have roots with negative real parts of characteristic equation. This may be possible in two different sets of roots. Now we will consider these cases.

2.3.1 CASE I :

The roots of a third-order polynomial

$$a_0 s^3 + a_1 s^2 + a_2 s + a_3 = 0 \qquad (4.21)$$

may have the following form :

$$s_1 = 0 \quad , \qquad s_2 = -a + i\,b \quad , \qquad s_3 = -a - i\,b \qquad (4.22)$$

Characteristic equation can be written in terms of this set of roots.

$$(s - s_1)(s - s_2)(s - s_3) = 0 \qquad (4.23)$$

In which case this equation becomes

$$s (s + a + i\,b)(s + a - i\,b) = 0$$

$$s \left[(s + a)^2 + b^2 \right] = 0$$

$$s^3 + 2\,a\,s^2 + (a^2 + b^2)\,s = 0 \qquad (4.24)$$

This equation should have the same form as our characteristic equation (4.21). If we equate the coefficients, since $a_o = 1$

$$2\,a = a_1 \quad , \qquad a_2 = a^2 + b^2 \qquad \text{and} \qquad a_3 = 0 \tag{4.25}$$

Since a and b are positive, then those should be satisfied

$$a_1 > 0 \tag{4.26}$$

$$a_3 = 0 \tag{4.27}$$

$$b^2 = a_2 - a^2 > 0 \qquad \text{or} \qquad a_2 - a_1^2 / 4 > 0 \tag{4.28}$$

Let's write these 3 conditions more precisely recalling the terms of the coefficients from previous sections.

Condition 1) $a_1 > 0$ is satisfied for all positive temperature reactivity coefficient γ.

Condition 2) $\gamma \phi_o \lambda_I ZX / l + AF / T = 0$

$$\text{or} \qquad \gamma = \sigma_x \left(\sigma_x Xe - y\, \sigma_f \right) / \left[c\, \sigma_f (\lambda_x + \sigma_x \phi_o) \right] \tag{4.30}$$

Condition 3) $\lambda_I ZX + Z \gamma \phi_o / l + PX\,R / T \geq (Z + \gamma \phi_o / l)^2 / 4$

$$\text{or} \qquad \left[- \frac{\phi_o^2}{4 l^2} \right] \gamma^2 + \left[\frac{Z\, \phi_o}{2\, l} \right] \gamma + \left[\lambda_I ZX + \frac{PX\,R}{T} - \frac{Z^2}{4} \right] \geq 0 \tag{4.31}$$

If equations (4.30) and (4.31) are solved in the same range of equilibrium fluxes as before, we can find the stable values of γ. This can be seen in figure - 5.

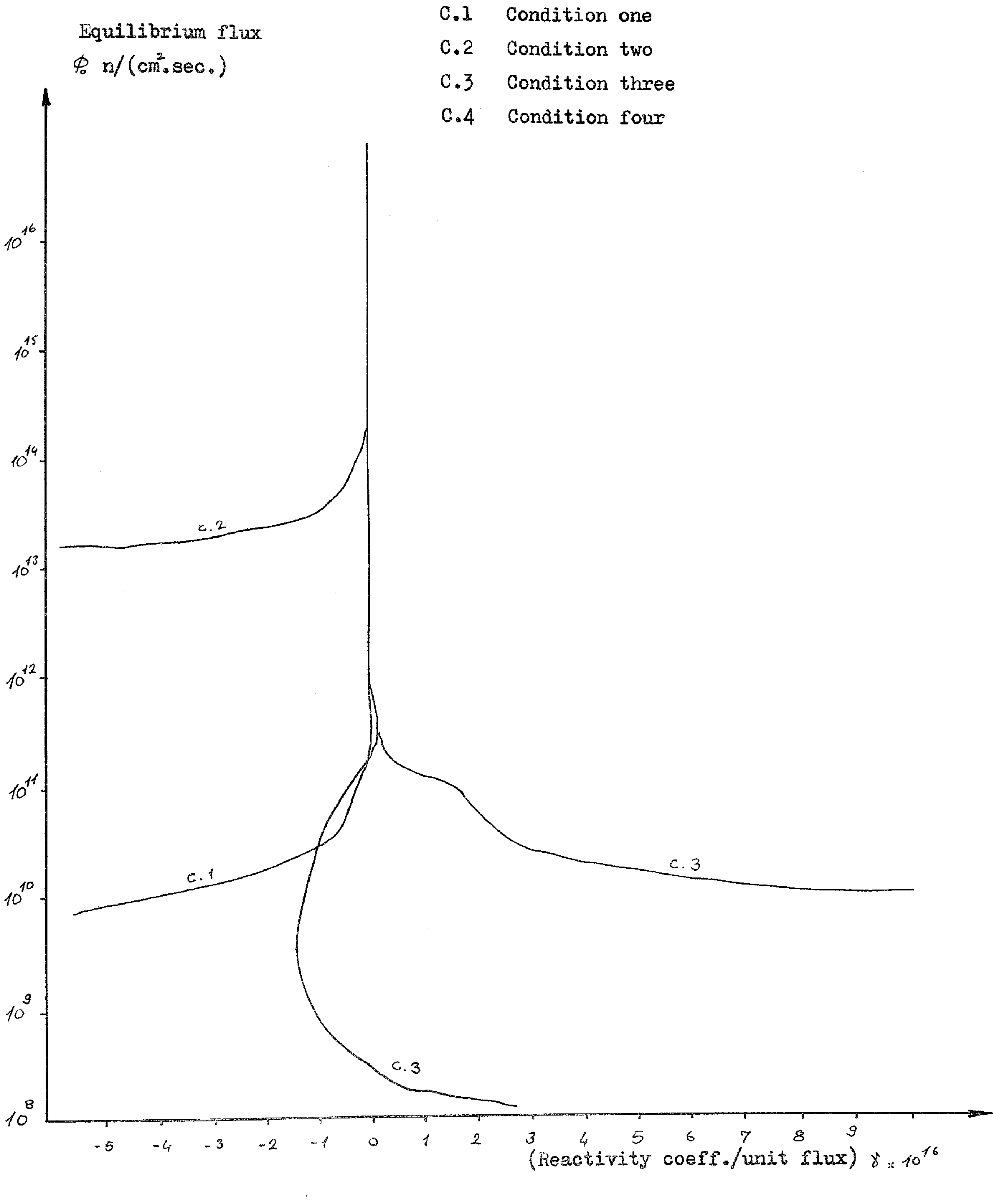


Figure - 5 Regions according to Case - I.

2.3.2. CASE II :

The roots of the third-order characteristic equation may also have the following form right before they enter the right half plane,

$$a_0 s^3 + a_1 s^2 + a_2 s + a_3 = 0 \tag{4.32}$$

$$s_1 = -a \quad , \quad s_2 = i\,b \quad , \quad s_3 = -i\,b \tag{4.33}$$

In order to force the roots of the characteristic equation to fit to this type, we should equate the coefficients of the characteristic equation to the coefficients of the following form :

$$(s - s_1)(s - s_2)(s - s_3) = 0$$

$$(s + a)(s - i\,b)(s + i\,b) = 0$$

$$s^3 + a\,s^2 + b^2 s + a\,b^2 = 0 \tag{4.34}$$

Since a and b are positive, then those should be satisfied.

$$a_1 > 0 \tag{4.35}$$

$$a_2 > 0 \tag{4.36}$$

$$a_1 a_2 = a_3 \tag{4.37}$$

It is obvious that first two conditions are the same as conditions (2) and (3) of Routh-Hurwitz criterion i.e., Condition 1) $a_1 > 0$ is satisfied for all positive .

Condition 2) $$\gamma \geqslant \left[-\frac{\sigma_x \phi}{c\ \sigma_f} (y_x \sigma_f - \sigma_x Xe) - \lambda_I (\lambda_x + \sigma_x \phi_o) \ell \right] \Big/ (\lambda_I + \lambda_x + \sigma_x \phi_o) \quad \phi_o$$

(4.38)

Condition 3) $a_1 a_2 = a_3$

$$\left[\frac{Z\ \phi_o^2}{\ell^2} \right] \gamma^2 + \left[\frac{Z^2 \phi_o}{\ell} + \frac{PX\ R\ \phi_o}{T\ \ell} \right] \gamma + \left[Z\ \lambda_I ZX + \frac{Z\ PX\ R}{T} - \frac{AF}{T} \right] = 0 \qquad (4.39)$$

which is a special case of condition (4) of Routh-Hurwitz criterion.

Again the region where these two conditions are satisfied is showed in figure - 6.

Total region of instability will be the union of these two cases. But considering the results obtained from the Routh-Hurwitz criterion, it may be concluded that roots of the characteristic equation can not pass to the right half plane in the form posited as case I.

Thus the resulting stability region is governed only by the second form of the roots, which is the same as Routh-Hurwitz criterion. This is shown in figure - 7. In this figure roots of the characteristic equation which give rise to instability are also shown.

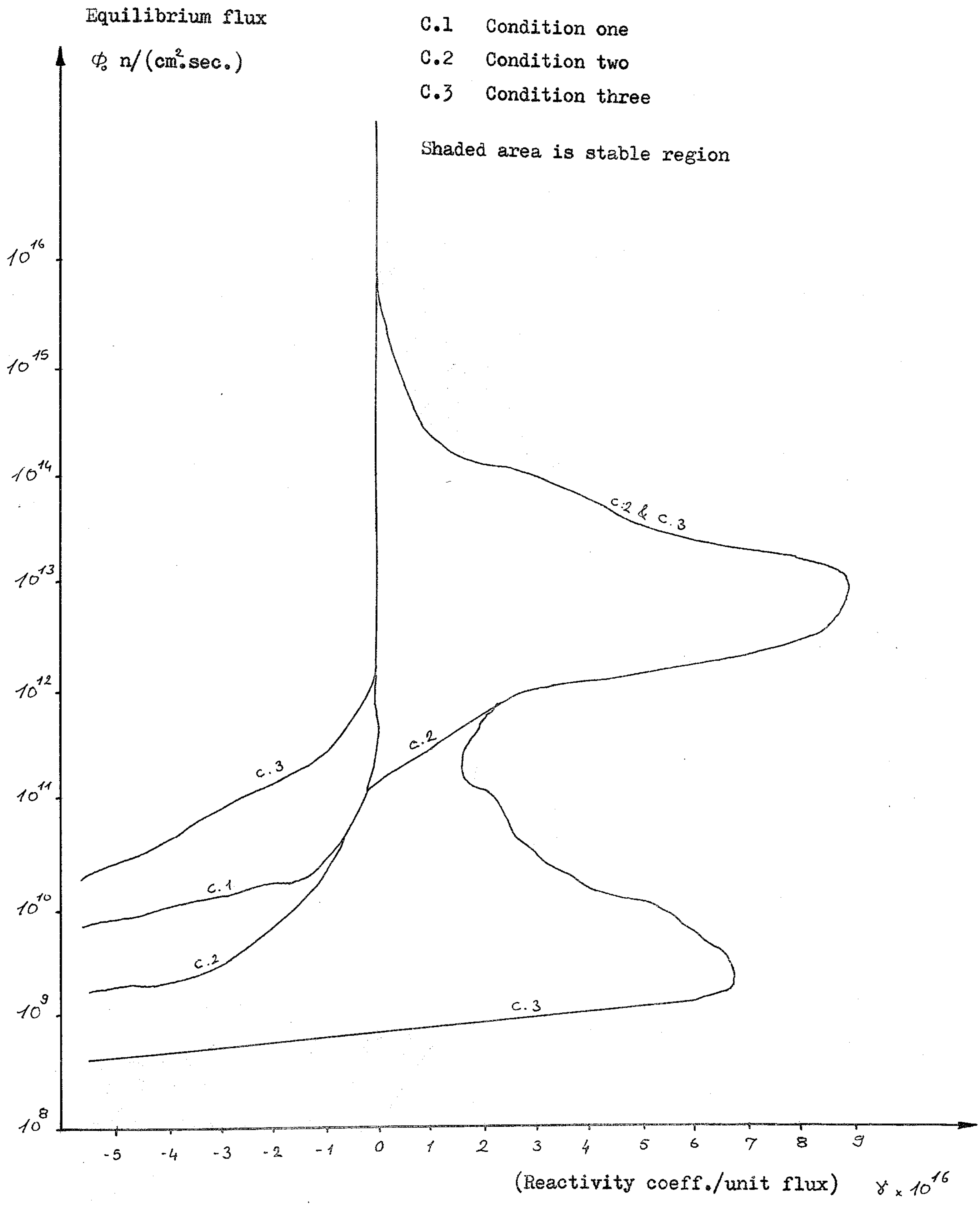


Figure - 6 Regions according to Case II.

2.4. DISCUSSION

In figure (7) one may notice that the principal feedback mechanism is the prompt temperature reactivity coefficient due to low flux values; so the reactor will be stable for any temperature reactivity coefficient below the flux level of 10^9 n/(cm^2.sec.). As the flux level is increased further from the value of 10^9 n/(cm^2.sec.) Xenon burnup begins to contribute to flux growth. For $\phi_o > 2\text{x}10^9$ n/(cm^2.sec.), the slope of the curve becomes steeper, showing that the stabilizing effect of the temperature reactivity feedback begins to be dominant and as ϕ_o increases, temperature feedback competes effectively with Xenon burnup so as to shrink the unstable region. However, when $\phi_o > 5\text{x}10^{11}$ n/(cm^2.sec.), the destabilizing effect of Xenon burnup begins to be felt, and as ϕ_o increases, this mechanism dominates the temperature feedback so that the curve bends again and the unstable region is enlarged.

It is clear that Xenon burnup is the dominant feedback effect in the flux range $2\text{x}10^{11} < \phi_o < 9\text{x}10^{12}$ n/(cm^2.sec.). As ϕ_o reaches 10^{13} n/(cm^2.sec.), the temperature reactivity feedback again becomes dominant, and finally stabilizes the reactor for $\phi_o > 10^{15}$ for almost any γ as all other reactor parameters are assumed fixed.

In order to check the validity of the unstable region, roots of the characteristic equation are found and worked out on the graph.

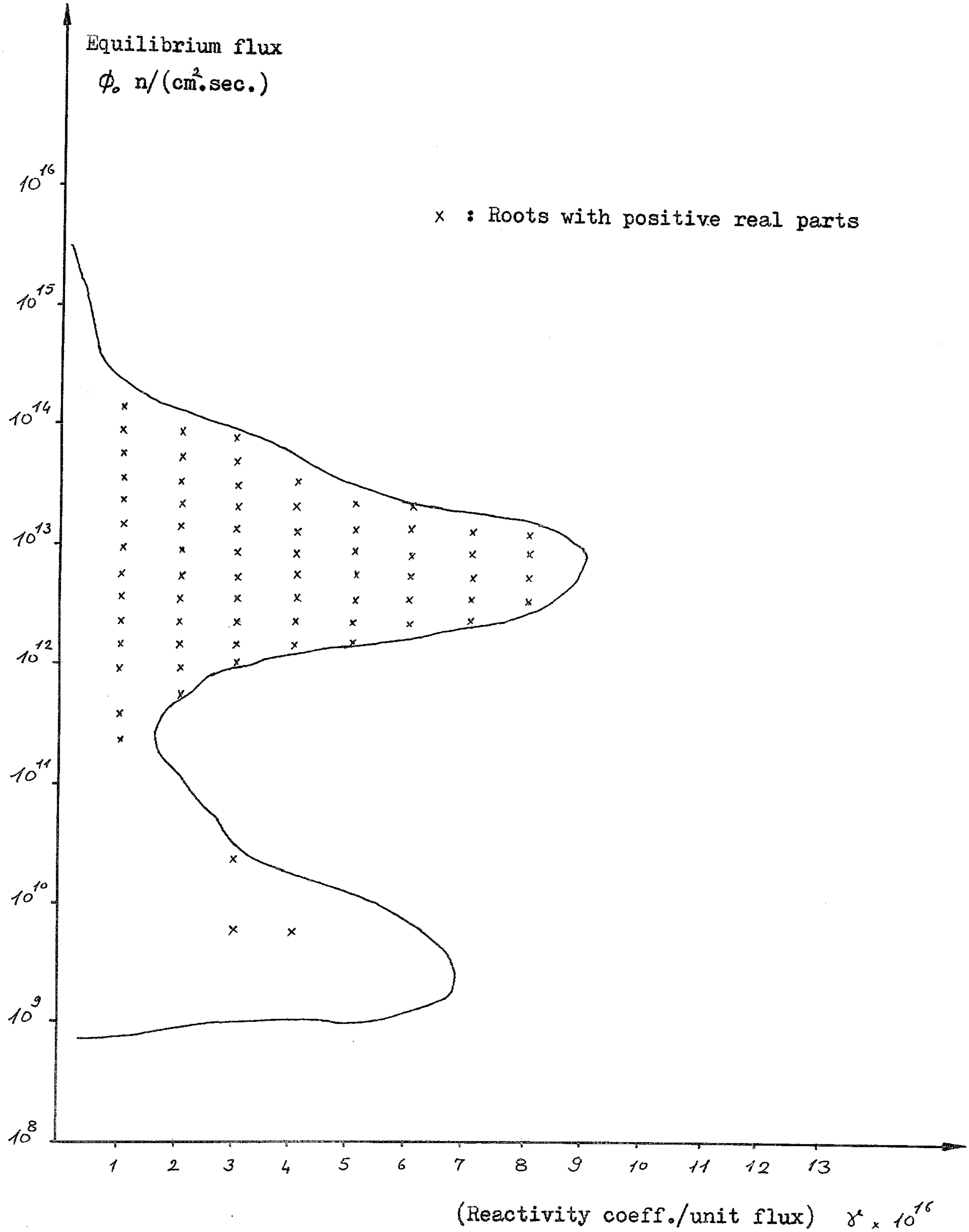


Figure - 7 Unstable region and roots with positive real parts of the characteristic equation.

Although the characteristic equation gives roots having positive real parts in each hump, it is interesting to note that points of instability is much denser in the upper hump.

Since the parameters are very small, in order to be sure about the validity of the roots of the characteristic equation, we applied a sensitivity analysis so as to determine the sensitivity of the roots of our third degree polynomial to its own coefficients via, the program (POLY) which computes the roots of an n order polynomial. It was observed that small changes in the coefficients of the polynomial did not lead to large changes in the roots, i.e., the roots of our characteristic equation is not very sensitive to the errors or approximations in the computations of its coefficients.

3. LINEAR STABILITY ANALYSIS WITH DELAYED NEUTRONS

3.1. CHARACTERISTIC EQUATION :

We begin our analysis by restating the point kinetics equations which can easily be recalled from previous chapters, considering one group of delayed neutrons as defining an average decay constant λ and total β .

$$\ell \dot{\phi} = \left[\delta_o - \beta - \sigma_x Xe / (c\,\sigma_f) - \gamma \phi \right] \phi + \lambda D \qquad (4.40\ a)$$

$$\dot{D} = \beta \phi - \lambda D \qquad (4.40\ b)$$

$$\dot{Xe} = (y_x \sigma_f - \sigma_x Xe)\,\phi - \lambda_x Xe + \lambda_I I \qquad (4.40\ c)$$

$$\dot{I} = y_I \sigma_f \phi - \lambda_I I \qquad (4.40\ d)$$

Equilibrium values can be found as follows :

$$\dot{\phi} = 0 \quad ; \quad \left[\delta_o - \beta - \sigma_x Xe_o / (c\,\sigma_f) - \gamma \phi_o \right] \phi_o = -\lambda D_o \qquad (4.41\ a)$$

$$\dot{D} = 0 \quad ; \quad \beta \phi_o - \lambda D_o = 0 \qquad (4.41\ b)$$

$$\dot{Xe} = 0 \quad ; \quad (y_x \sigma_f - \sigma_x Xe_o)\,\phi_o - \lambda_x Xe_o + \lambda_I I_o = 0 \qquad (4.41\ c)$$

$$\dot{I} = 0 \quad ; \quad y_I \sigma_f \phi_o - \lambda_I I_o = 0 \qquad (4.41\ d)$$

From these equations,

$$D_o = \beta \phi_o / \lambda \quad , \quad I_o = y_I \sigma_f \phi_o / \lambda_I \quad , \quad Xe_o = y\,\sigma_f \phi_o / (\lambda_x + \sigma_x \phi_o) \qquad (4.42)$$

Initial reactivity to compansate the other feedback is the same as before,

$$\delta_0 = \sigma_x Xe_0 / c\, \sigma_f + \gamma \bar{\phi}_0 \qquad (4.43)$$

Expanding the equations (4.40) as follows :

$$\bar{\phi} = \bar{\phi}_0 + \phi \quad , \qquad D = D_0 + \delta D \quad , \qquad Xe = Xe_0 + \delta Xe \quad , \qquad I = I_0 + \delta I$$

$$\ell (\dot{\bar{\phi}}_0 + \dot{\phi}) = \left[\delta_0 - \beta - \frac{\sigma_x (Xe_0 + \delta Xe)}{c\ \sigma_f} - \gamma (\bar{\phi}_0 + \phi) \right] (\bar{\phi}_0 + \phi) + \lambda (D_0 + \delta D) \qquad (4.44\ a)$$

$$\dot{D}_0 + \delta \dot{D} = \beta (\bar{\phi}_0 + \phi) - \lambda (D_0 + \delta D) \qquad (4.44\ b)$$

$$\dot{Xe}_0 + \delta \dot{Xe} = y_x \sigma_f (\bar{\phi}_0 + \phi) - \sigma_x (Xe_0 + \delta Xe)(\bar{\phi}_0 + \phi) - \lambda_x (Xe_0 + \delta Xe) + \lambda_I (I_0 + \delta I) \qquad (4.44\ c)$$

$$\dot{I}_0 + \delta \dot{I} = y_I \sigma_f (\bar{\phi}_0 + \phi) - \lambda_I (I_0 + \delta I) \qquad (4.44\ d)$$

Substituting the equilibrium values and neglecting the second order differentials, we obtain

$$\ell \dot{\phi} = \left[\delta_0 - \beta - \frac{\sigma_x\ Xe}{c\ \sigma_f} - 2\gamma \bar{\phi}_0 \right] \phi - \frac{\sigma_x\ \phi_0}{c\ \sigma_f}\ \delta Xe + \lambda\, \delta D \qquad (4.45\ a)$$

$$\delta \dot{D} = \beta \phi - \lambda\, \delta D \qquad (4.45\ b)$$

$$\delta \dot{Xe} = (y_x \sigma_f - \sigma_x Xe_0) \phi - (\lambda_x + \sigma_x \phi_0)\, \delta Xe + \lambda_I\, \delta I \qquad (4.45\ c)$$

$$\delta \dot{I} = y_I \sigma_f \phi - \lambda_I \delta I \qquad (4.45\ d)$$

Taking the Laplace transforms,

$$\ell s \bar{\phi} = \left[\delta_o - \beta - \frac{\sigma_x Xe_o}{c\ \sigma_f} - 2\gamma \bar{\phi}_o \right] \bar{\phi} - \frac{\sigma_x \phi_o}{c\ \sigma_f} \overline{\delta Xe} + \lambda \overline{\delta D} \qquad (4.46\ a)$$

$$s\ \overline{\delta D} = \beta \bar{\phi} - \lambda\ \overline{\delta D} \qquad (4.46\ b)$$

$$s\ \overline{\delta Xe} = (y_x \sigma_f - \sigma_x Xe_o)\ \bar{\phi} - (\lambda_x + \sigma_x \bar{\phi}_o)\ \overline{\delta Xe} + \lambda_I \overline{\delta I} \qquad (4.46\ c)$$

$$s\ \overline{\delta I} = y_I \sigma_f \bar{\phi} - \lambda_I\ \overline{\delta I} \qquad (4.46\ d)$$

Substituting the value for $\overline{\delta I}$ into eq.(4.46 c) $\overline{\delta Xe}$ becomes

$$\overline{\delta Xe} = \left[(y_x \sigma_f - \sigma_x Xe_o)\ \bar{\phi} + \frac{y_I \lambda_I \sigma_f \bar{\phi}}{s + \lambda_I} \right] \Big/ (s + \lambda_x + \sigma_x \phi_o)$$

Putting this and eq.(4.46 b) into eq.(4.46 a)

$$\ell s \bar{\phi} = \left[\delta_o - \beta - \frac{\sigma_x Xe}{c\ \sigma_f} - 2\gamma \bar{\phi}_o \right] \bar{\phi} + \frac{\beta \lambda \bar{\phi}}{s + \lambda}$$

$$- \frac{\sigma_x \bar{\phi}_o \bar{\phi}}{c\ \sigma_f (s + \lambda_x + \sigma_x \bar{\phi}_o)} \left[(y_x \sigma_f - \sigma_x Xe_o) + \frac{y_I \lambda_I \sigma_f}{s + \lambda_I} \right] \qquad (4.47)$$

Introducing some new variables in addition to those introduced in the previous section, for simplicity in operations.

$$BL = ZX\ \lambda\ \lambda_I$$

$$BG = \beta c\ \sigma_f\ \lambda$$

$$CL = \lambda + \lambda_I + ZX$$

$$P = \beta - \delta_o + 2\gamma \bar{\phi}_o = \beta - U / (c \sigma_f) + \gamma \bar{\phi}_o$$

$$E = P\ \ c\ \sigma_f + U\ =\ c\ \sigma_f (\beta - \gamma \bar{\phi}_o)$$

$$F = \lambda (\lambda_I + ZX) + \lambda_I ZX$$

so that,

$$\ell s + P + \frac{U}{c\ \sigma_f} + \frac{PX\ R}{c\ \sigma_f (s+ZX)} + \frac{PX\ y_I\ \lambda_I\ \sigma_f}{c\ \sigma_f (s+ZX)(s+\lambda_I)} - \frac{\beta\ \lambda}{s + \lambda} = 0 \qquad (4.48)$$

$$(\ell s + P)\ c\sigma_f (s + ZX)(s + \lambda_I)(s + \lambda) + U (s + ZX)(s + \lambda_I)(s + \lambda)$$

$$+ PX\ R (s + \lambda_I)(s + \lambda) + PX\ y_I\ \lambda_I \sigma_f (s + \lambda) - BG (s + ZX)(s + \lambda_I) = 0$$

Since $$(s + ZX)(s + \lambda_I)(s + \lambda) = s^3 + (\lambda + \lambda_I + ZX)\ s^2$$

$$+ \left[(\lambda + ZX) + \lambda_I ZX \right] s + \lambda \lambda_I ZX$$

$$= s^3 + CL\ s^2 + F\ s + BL$$

$$T \left[s^4 + CL\ s^3 + F\ s^2 + BL\ s \right] + E \left[s^3 + CL\ s^2 + F\ s + BL \right] + PX\ R \left[s^2 + (\lambda + \lambda_I)\ s \right.$$

$$\left. + \lambda \lambda_I \right] + PX\ y_I\ \lambda_I\ \sigma_f (s + \lambda) - BG \left[s + (\lambda_I + ZX)\ s + \lambda_I ZX \right] = 0$$

or,

$$T\ s^4 + (T\ CL + E)\ s^3 + (T\ F + E\ CL + PX\ R - BG)\ s^2 + \left[T\ BL + E\ F + AF \right.$$

$$\left. + PX\ R\ \lambda - BG\ (\lambda_I + ZX) \right] s + \left[E\ BL + \lambda\ AF - BG\ \lambda_I ZX \right] = 0$$

Defining, $$AK = T\ F + PX\ R - BG$$

$$EK = T\ BL + AF + PX\ R\ \lambda - BG\ Z$$

$$BK = \lambda\ AF - BG\ \lambda_I ZX$$

the characteristic equation in a simpler form is obtained as ;

$$T s^4 + (T\, CL + E) s^3 + (E\, CL + AK) s^2 + (E F + EK) s + (E\, BL + BK) = 0 \quad (4.49)$$

3.2. APPLICATION OF ROUTH-HURWITZ CRITERION WITH DELAYED NEUTRONS

When the stability conditions of the characteristic equation of the form $a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 = 0$ are applied we see that there is only one additional condition to the positiveness of the coefficients i.e., $\Delta_3 > 0$.

Condition 1) $a_0 > 0$, $T > 0$ is satisfied already

Condition 2) $a_1 > 0$, $T\, CL + E > 0$

$$c\, \sigma_f\, \ell\, CL + c\, \sigma_f\, (\beta - \gamma \bar{\phi}_0) > 0$$

$$\gamma_1 \gg (\beta + \ell\, CL) / \bar{\phi}_0 \quad (4.50)$$

Condition 3) $a_3 > 0$, $E F + AK > 0$

$$c\, \sigma_f (\beta - \gamma \bar{\phi}_0) F + AK > 0$$

$$\gamma_2 \gg \left[\beta + EK / (c\, \sigma_f E) \right] / \bar{\phi}_0 \quad (4.51)$$

Condition 4) $a_4 > 0$, $E\, BL + BK > 0$

$$c\,\sigma_f(\beta - \gamma\,\phi_o)\,BL + BK > 0$$

$$\gamma_3 \geqslant \left[\beta + BK\,/\,(c\,\sigma_f\,BL)\right]\,/\,\phi_o \qquad (4.52)$$

Condition 5) $\Delta_3 > 0$, $a_3(a_1 a_2 - a_o a_3) > a_1^2 a_4$

$$(E\,F + EK)\left[(E + T\,CL)(E\,CL + AK) - T\,(E\,F + EK)\right] > (T\,CL + E)^2(E\,BL + BK)$$

or,

$$E^3\left[F\,CL - BL\right] + E^2\left[F\,AK + F\,T\,(CL^2 - F) + CL\,(EK - 2\,T\,BL) - BK\right]$$

$$+ E\left[F\,T\,(CL\,AK - 2\,EK) + EK\,(AK + T\,CL^2) - T\,CL\,(T\,CL\,BL - 2\,BK)\right]$$

$$+ \left[EK\,T\,(AK\,CL - EK) - T^2 CL^2 BK\right] \geqslant 0 \qquad (4.53)$$

Condition - 5 is satisfied for all positives values of temperature reactivity coefficient, .

It is seen from the plot of the conditions that stability region (shaded) is much different from the previous results. This may be because of the order of the system under investigation increases Routh-Hurwitz criteria tend to give over restricted results. According to these considerations perhaps the delayed neutrons should be treated in a different way. For example, if the order of this characteristic equation can be reduced by one, reasonable results may be obtained.

In the next section we try to accomplish this with an approximation.

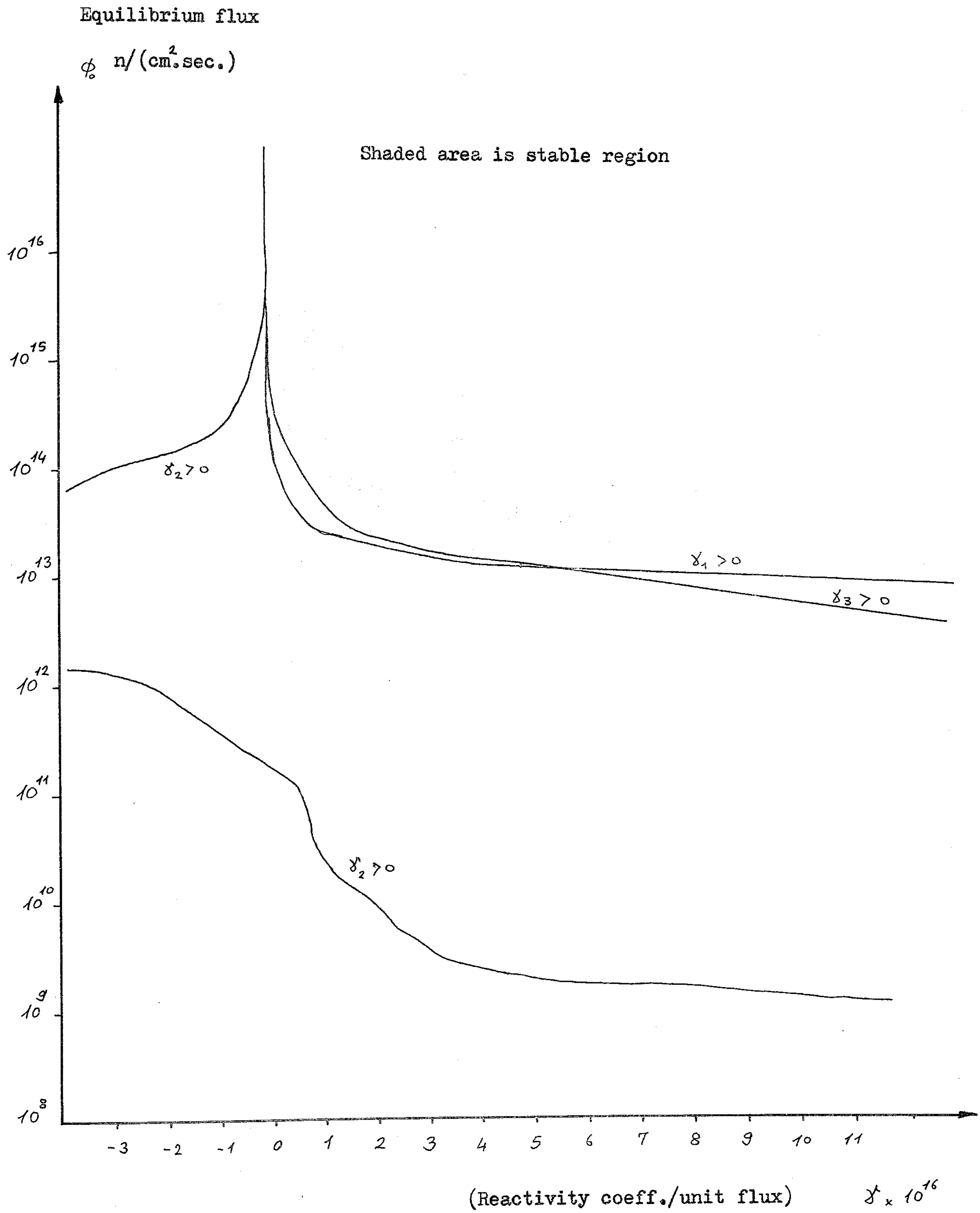


Figure - 8 Stability region (shaded) according to R - H criterion with delayed Neutrons.

3.3 A DIFFERENT APPROACH TO FIND THE CHARACTERISTIC EQUATION

In this section we will make some appoximations to the problem since we have faced some numerical difficulties in solving the fourth-order characteristic equation with delayed neutrons.

Restating the kinetics equations,

$$\ell \dot{\phi} = \left[\delta_o - \beta - \frac{\sigma_x \ Xe}{c \ \sigma_f} - \gamma \phi \right] \phi + \sum_{i=1}^{6} \lambda_i D_i \tag{4.54 a}$$

$$\dot{D}_i = \beta_i \phi - \lambda_i D_i \qquad i=1,\ldots,6 \tag{4.54 b}$$

$$\dot{I} = y_I \sigma_f \phi - \lambda_I I \tag{4.54 c}$$

$$\dot{Xe} + \lambda_x Xe = y_x \sigma_f \phi + \lambda_I I - \sigma_x Xe \phi \tag{4.54 d}$$

Equilibrium values are,

$$D_{io} = (\beta_i / \lambda_i)\, \phi_o \ , \qquad I_o = (y_I \sigma_f / \lambda_I)\, \phi_o$$

$$Xe_o = y \sigma_f \phi_o / (\lambda_x + \sigma_x \phi_o) \qquad \text{where} \quad y = y_I + y_x \tag{4.55}$$

In order to reduce the complexity of the system with its large number of parameters, we pass to the form where the dynamical variables are measured relative to their equilibrium values.

Define $Y = \sigma_x \phi_o / (\lambda_x + \sigma_x \phi_o)$ as before in chapter III.

$$Xe_o = y\, Y \frac{\sigma_f}{\sigma_x} \qquad (4.56)$$

$$\dot{\bar{\phi}} = 0 \quad ; \quad \left[\delta_o - \beta - \frac{\sigma_x\, Xe}{c\ \sigma_f} - \gamma\, \bar{\phi}_o\right]\bar{\phi}_o + \sum_{i=1}^{6} \lambda_i \frac{\beta_i}{\lambda_i} \phi_o = 0$$

recalling $\sum_{i=1}^{6} \beta_i = \beta$ and

$$\delta_o = \frac{\sigma_x\ Xe_o}{c\ \sigma_f} + \gamma\, \bar{\phi}_o \qquad (4.57)$$

On the other hand equality for Y gives,

$$\bar{\phi}_o = \frac{\lambda_x\ Y}{\sigma_x\,(1 - Y)} \qquad (4.58)$$

Define new variables as :

$$\phi = (\bar{\phi} - \phi_o) / \phi_o \quad , \quad \delta D_i = (D_i - D_{io}) / D_{io}$$

$$\delta Xe = (Xe - Xe_o) / Xe_o \quad , \quad \delta I = (I - I_o) / I_o$$

with these definitions equations (4.54) reduce to

$$\ell\, \dot{\phi}\, \phi_o = \left[\delta_o - \beta - \frac{\sigma_x\, Xe}{c\ \sigma_f}(1 + \delta Xe) - \gamma\, \phi_o (1 + \phi)\right](1+\phi)\, \phi_o + \sum_{i=1}^{6} \lambda_i D_{io} (1 + \delta D_i)$$

inserting the value for δ_o

$$\dot{\phi} = \left[-\beta - \frac{\sigma_x\, Xe}{c\, \sigma_f} \delta Xe - \gamma \phi_o \phi \right] (1 + \phi) + \sum_{i=1}^{6} \beta_i (1 + \delta \dot{D}_i) \qquad (4.59)$$

recalling $\sum_{i=1}^{6} \beta_i = \beta$ and replacing the value for Xe_o from eq.(4.56)

$$l \dot{\phi} = -\left[\beta + \frac{y\, Y \quad \sigma_x}{c \quad \sigma_f \quad \sigma_x} \sigma_f \delta Xe + \gamma \phi_o \phi \right] (1 + \phi) + \beta \sum_{i=1} (1 + \delta D_i)$$

Noting $\sum_{i=1}^{6} a_i = 1$ and neglecting the higher order terms

$$l \dot{\phi} = -\left[\frac{y\, Y}{c} \delta Xe + \gamma \phi_o \phi \right] + \beta \sum_{i=1}^{6} a_i (\delta D_i - \phi) \qquad (4.60)$$

$$\dot{D}_i = \beta_i \bar{\phi} - \lambda_i D_i$$

$$D_{io} \delta \dot{D}_i = \beta_i \bar{\phi}_o (1 + \phi) - \lambda_i D_{io} (1 + \delta D_i)$$

$$D_{io} \delta \dot{D}_i = \lambda_i D_{io} (1 + \phi) - \lambda_i D_{io} (1 + \delta D_i)$$

$$\delta \dot{D}_i = \lambda_i \left[\phi - \delta D_i \right] \qquad (4.61)$$

$$\dot{I} = y_I \sigma_f \bar{\phi} - \lambda_I I$$

$$I_o \delta \dot{I} = y_I \sigma_f \phi_o (1 + \phi) - \lambda_I I_o (1 + \delta I)$$

$$I_o \delta \dot{I} = \lambda_I I_o (1 + \phi) - \lambda_I I_o (1 + \delta I)$$

$$\delta \dot{I} = \lambda_I \left[\phi - \delta I \right] \qquad (4.62)$$

$$\dot{Xe} + \lambda_x Xe = y_x \sigma_f \bar{\phi} + \lambda_I I - \sigma_x Xe \bar{\phi}$$

$$Xe_o \, \delta\dot{Xe} + \lambda_x Xe (1 + \delta Xe) = y_x \sigma_f \phi_o (1 + \phi) + \lambda_I I_o (1 + \delta I) - \sigma_x Xe_o \phi_o (1 + \delta Xe)(1+\phi)$$

(4.63)

replacing the value for Xe_o and I_o and neglecting the second-order terms

$$\frac{y \sigma_f \phi_o}{\lambda_x + \sigma_x \phi_o} \left[\delta\dot{Xe} + \lambda_x + \lambda_x \delta Xe \right] = y_x \sigma_f \phi_o (1 + \phi) + y_I \sigma_f \phi (1 + \delta I) - \sigma_x \frac{y \, Y \, \sigma_f}{\sigma_x} \phi_o \left[1 + \phi + \delta Xe \right]$$

Define $Y_I = y_I / y$, $Y_x = y_x / y$ and recall $Y_I + Y_x = 1$

$$\delta\dot{Xe} + \lambda_x \delta Xe = \left[Y_x (1 + \phi) + Y_I (1 + \delta I) - Y(1+\phi) - Y \delta Xe \right] (\lambda_x + \sigma_x \phi_o) - \lambda_x$$

$$\delta\dot{Xe} + \lambda_x \delta Xe = (\lambda_x + \sigma_x \phi_o) \left[(Y_x - Y)(1+\phi) + Y_I (1+\delta I) - Y \delta Xe \right] - \lambda_x \qquad (4.64)$$

Taking the Laplace transforms,

$$s \, \delta\bar{D}_i = \lambda_i \left[\bar{\phi} - \delta\bar{D}_i \right]$$

$$\delta\bar{D}_i = \left[\lambda_i / (s + \lambda_i) \right] \bar{\phi} \qquad (4.65)$$

$$s \, \delta\bar{I} = \lambda_I \bar{\phi} - \lambda_I \delta\bar{I}$$

$$\delta\bar{I} = \left[\lambda_I / (s + \lambda_I) \right] \bar{\phi} \qquad (4.66)$$

$$s \; \delta\overline{Xe} + \lambda_x \; \delta\overline{Xe} = (\lambda_x + \sigma_x \phi_o) \left[(Y_x - Y)\,\bar{\phi} + Y_I \; \delta\bar{I} - Y \; \delta\overline{Xe} \right]$$

$$(s + \lambda_x) \; \delta\overline{Xe} = (\lambda_x + \sigma_x \phi_o) \left[(Y_x - Y)\,\bar{\phi} + \frac{Y_I \lambda_I}{s + \lambda_I}\,\bar{\phi} - Y \; \delta\overline{Xe} \right]$$

find the value for $(\lambda_x + \sigma_x \phi_o) = \dfrac{\sigma_x \phi_o'}{Y} = \dfrac{\sigma_x}{Y}\,\dfrac{\lambda_x}{\sigma_x}\,\dfrac{Y}{1 - Y} = \dfrac{\lambda_x}{1 - Y}$

$$(s + \lambda_x) \; \delta\overline{Xe} = \lambda_x \left(\frac{Y_x - Y}{1 - Y} \right) \bar{\phi} + \frac{Y_I \lambda_I \lambda_x \; \bar{\phi}}{(1-Y)(s + \lambda_I)} - \frac{Y \lambda_x}{1 - Y} \quad \delta\overline{Xe}$$

$$\delta\overline{Xe} = \frac{\lambda_x \bar{\phi}}{1 - Y} \left[(Y_x - Y) + \frac{Y_I \lambda_I}{(s + \lambda_I)} \right] \Big/ \left[s + \lambda_x + \frac{Y \lambda_x}{1 - Y} \right] \qquad (4.67)$$

putting this into the Laplace transformed form of the equation (4.60)

$$s \, l \, \bar{\phi} + \frac{y Y}{c} \; \frac{\lambda_x \bar{\phi}}{1 - Y} \; \frac{\left[(Y_x - Y) + \dfrac{Y_I \lambda_I}{s + \lambda_I} \right]}{\left[s + \lambda_x + \dfrac{Y \lambda_x}{1 - Y} \right]} + \gamma \phi_o \bar{\phi} - \beta \sum_{i=1}^{6} a_i \left(\frac{\lambda_i}{s + \lambda_i} - 1 \right) \bar{\phi}' = 0$$

$$s \left[l + \beta \sum_{i=1}^{6} \frac{a_i}{s + \lambda_i} \right] + \frac{y Y}{c} \; \frac{\lambda_x}{1 - Y} \; \frac{\left[(Y_x - Y)(s + \lambda_I) + Y_I \lambda_I \right] (1-Y)}{(s + \lambda_I) \left[(s + \lambda_x)(1-Y) + Y \lambda_x \right]} + \gamma \phi_o = 0 \qquad (4.68)$$

Since the decay constants λ_x and λ_I are much smaller than the decay constant λ of the delayed neutron emitters, one can ignore s as compared to λ_i in the discussion of Xenon oscillations [3].

$$|s| \ll \lambda_i$$

hence we can neglect it and define an average neutron generation time as

$$\ell^* = \ell + \beta \sum_{i=1}^{6} \frac{a_i}{\lambda_i} \tag{4.69}$$

$$s + \frac{y\,Y}{c\,\ell^*} \; \frac{\left[(Y_x - Y)\,s + \lambda_I(Y_I + Y_x) - Y \lambda_I\right]}{\left[(s + \lambda_I)(s + \lambda_x - s\,Y - \lambda_x Y + \lambda_x Y\right]} + \frac{\gamma\,\phi_o}{\ell^*} = 0$$

Defining $\omega_o = \dfrac{y\,Y}{c\,\ell^*}$ and recalling $Y_x + Y_I = 1$

$$\left[s + \frac{\gamma\,\phi}{\ell^*}\right]\left[s^2 + (\lambda_I + \lambda_x)s + \lambda_I\lambda_x - s^2 Y - s\,Y\lambda_I\right] + \omega_o\lambda_x\left[s(Y_x - Y) + \lambda_I(1-Y)\right] = 0$$

Define $\lambda = \lambda_I + \lambda_x$

$$\left[s + \frac{\gamma\,\phi_o}{\ell^*}\right]\left[s^2(1-Y) + (\lambda - \lambda_I Y)s + \lambda_I\lambda_x\right] + \omega_o\lambda_x\left[s(Y_x - Y) + \lambda_I(1-Y)\right] = 0$$

$$\left[s + \frac{\gamma\,\phi_o}{\ell^*}\right]\left[s^2 + \frac{\lambda - \lambda_I Y}{1 - Y}\,s + \frac{\lambda_I\lambda_x}{1 - Y}\right] + \frac{\omega_o\lambda_x(Y_x - Y)}{1 - Y}\,s + \omega_o\lambda_I\lambda_x = 0$$

$$s^3 + s^2\left[\frac{\lambda - \lambda_I Y}{1 - Y} + \frac{\gamma\,\phi_o}{\ell^*}\right] + s\left[\frac{\lambda_I\lambda_x}{1 - Y} + \frac{\gamma\,\phi_o}{\ell^*}\left(\frac{\lambda - \lambda_I Y}{1 - Y}\right)\right.$$

$$\left. + \frac{\omega_o\lambda_x}{1-Y}\,(Y_x - Y)\right] + \left[\frac{\lambda_I\lambda_x}{1-Y}\,\frac{\gamma\,\phi_o}{\ell^*} + \omega_o\lambda_I\lambda_x\right] = 0$$

$$s^3 + s^2 \left[\frac{\lambda - \lambda_I Y}{1 - Y} + \frac{\gamma \phi_o}{\ell^*} \right] + s \left[\lambda_I \lambda_x + \omega_o \lambda_x (Y_x - Y) + \frac{\gamma \phi_o}{\ell^*} (\lambda - \lambda_I Y) \right] \Big/ (1 - Y) + \lambda_I \lambda_x \left[\omega_o + \frac{\gamma \phi_o}{\ell^* (1-Y)} \right] = 0 \qquad (4.70)$$

3.3.1. APPLICATION OF THE ROUTH - HURWITZ CRITERION :

Taking into account the same considerations as in section 2.2.1 one may write the necessary conditions as follows :

Condition 1) $a_o > 0$ is satisfied automatically

Condition 2) $a_1 > 0$; $(\lambda - \lambda_I Y) / (1 - Y) + (\gamma \phi_o / \ell^*) > 0$

$$\gamma \geqslant (\ell^* / \phi_o)(Y - 1) / (\lambda - \lambda_I Y) \qquad (4.71)$$

Condition 3) $a_3 > 0$; $\lambda_I \lambda_x \left[\omega_o + \frac{\gamma \phi_o}{\ell^* (1 - Y)} \right] > 0$

$$\gamma \geqslant \omega_o \ell^* (Y - 1) / \phi_o \qquad (4.72)$$

Condition 4) $a_1 a_2 > a_3$

$$\left[\frac{\lambda - \lambda_I Y}{1 - Y} + \frac{\gamma \phi_\circ}{\ell^*}\right]\left[\lambda_I \lambda_x + \omega_\circ \lambda_x (Y_x - Y) + \frac{\gamma \phi_\circ}{\ell^*}(\lambda - \lambda_I Y)\right]\frac{1}{1-Y}$$

$$\geqslant \lambda_I \lambda_x \left[\omega_\circ + \frac{\gamma \phi_\circ}{\ell^*(1-Y)}\right]$$

$$\left[\frac{\phi_\circ^2}{\ell^{*2}}(\lambda - \lambda_I Y)\right]\gamma^2 + \frac{\phi_\circ}{\ell^*}\left[\frac{(\lambda - \lambda_I Y)^2}{1 - Y} + \omega_\circ \lambda_x (Y_x - Y)\right]\gamma$$

$$+\left[\frac{(\lambda - \lambda_I Y)\lambda_x}{1 - Y}\left[\lambda_I + \lambda_x \omega_\circ (Y_x - Y)\right] - \lambda_I \lambda_x \omega_\circ (1 - Y)\right] \geqslant 0 \qquad (4.73)$$

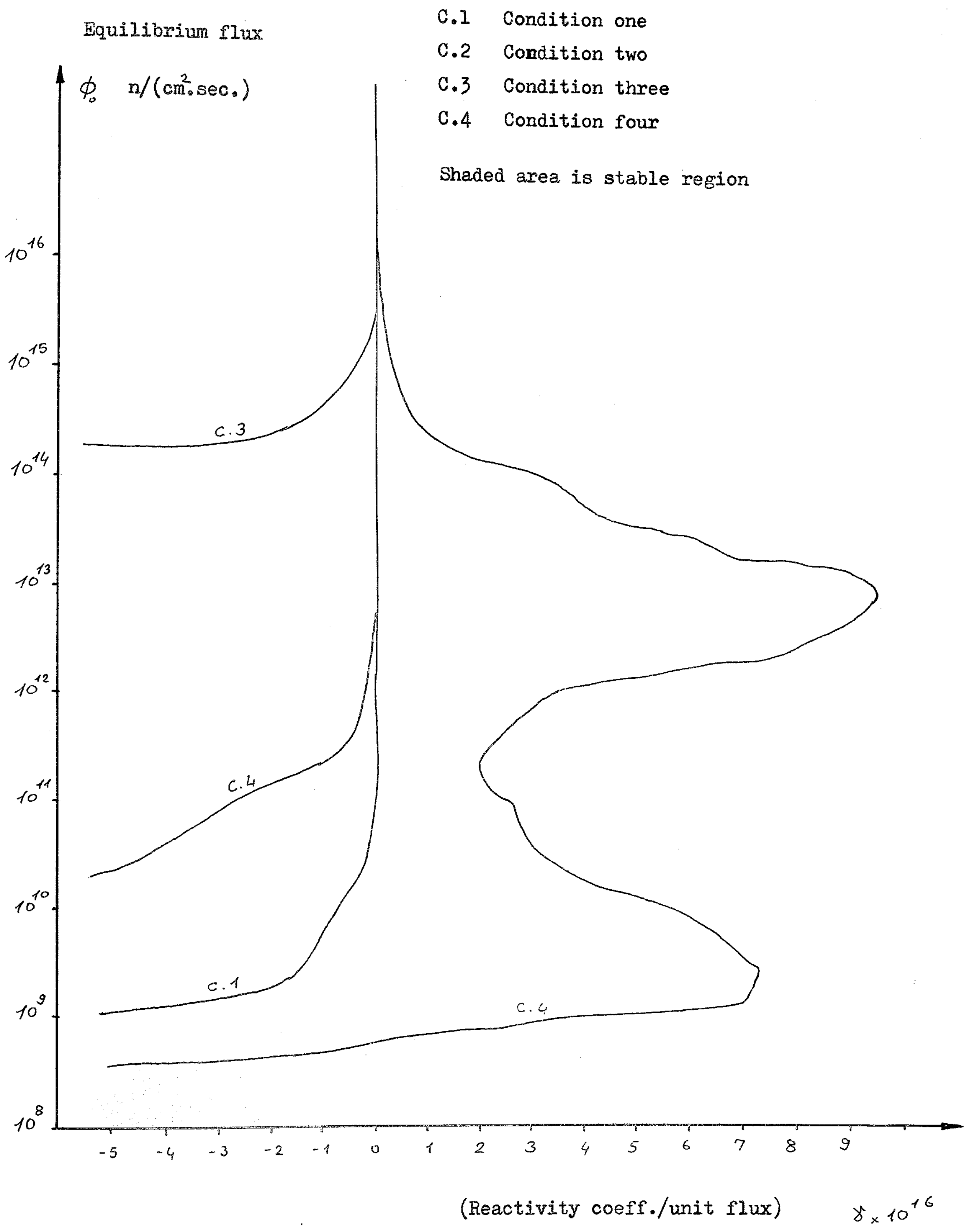


Figure - 9 Stability regions according to R-H criterion with delayed neutrons.

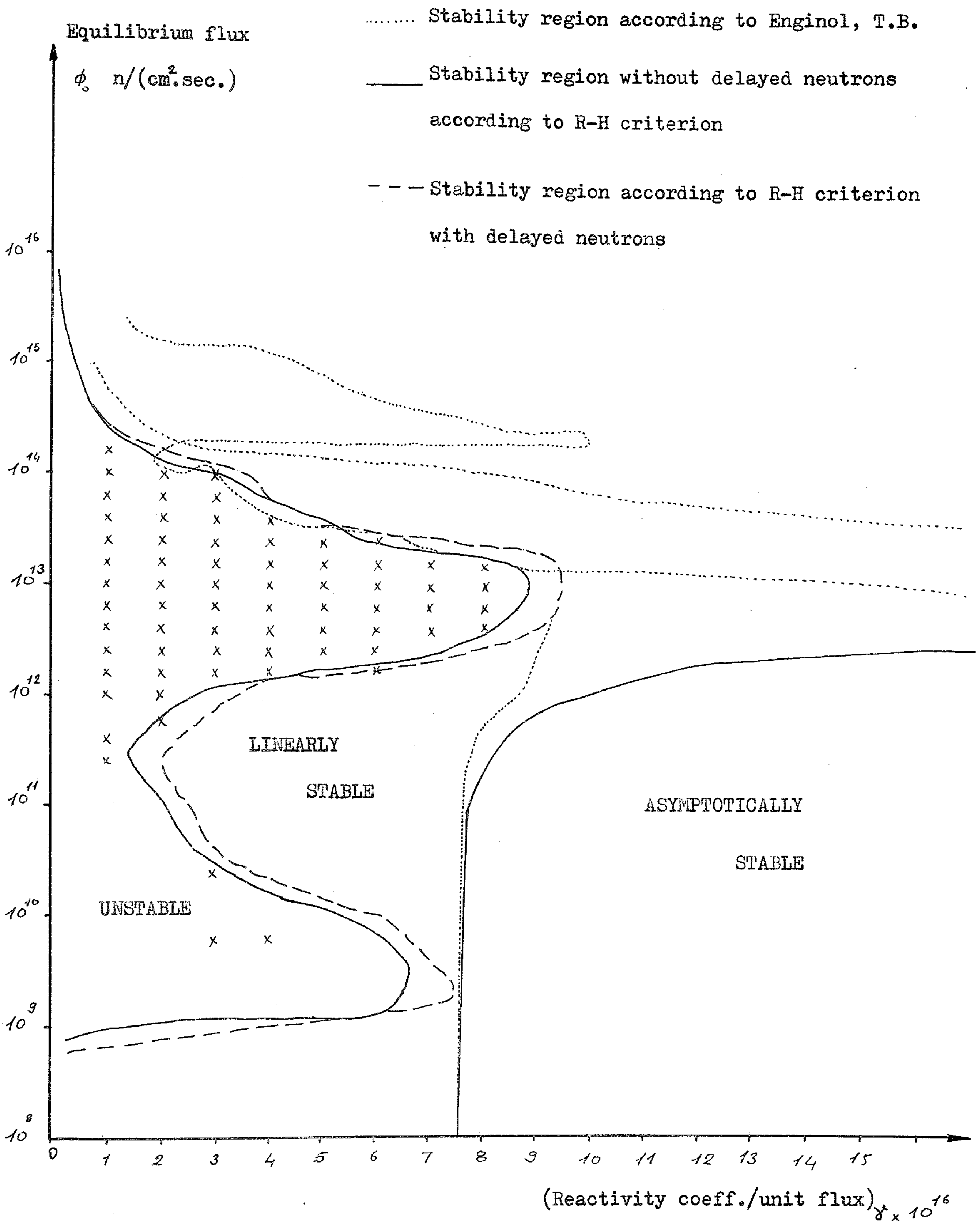


Figure - 10 Comparison of Regions with and without delayed Neutrons, and Roots with positive real parts of the characteristic equation.

3.4 DISCUSSION

In figure - 9 we obtain the double-humped curve again but this time the delayed neutron effects are considered in an average generation time, ℓ^*. It is realistic to consider the delayed neutrons to be produced " instantaneously " with respect to Xenon since time decay constants of delayed neutrons are much shorter than of I^{135} and Xe^{135}'s. Also for high flux levels, the quantity of Xenon produced is much larger than the quantity β of delayed neutron precursors.

As a matter of fact the validity of this approximation may be checked from the search of the roots of the characteristic equatio, i.e., all of the roots are always less than the lowest decay constant of delayed neutron precursors.

One can see from the comparison of this plot with figure - 7 that in some sections, region of instability is enlarged by the delayed neutrons. It can be concluded that the effect of delayed neutrons on the stability of the autonomous systems may be "destabilizing".

Actually their effect on the stability of the autonomous systems was not well understood until relatively recent times[3]. Smets[13] has reviewed the effect of delayed neutrons on the linear and non-linear stability of reactor systems under various conditions, and showed that the delayed neutrons do not always "improve" the stability of nuclear reactors at a given pover level and that a reactor may be unstable although it was stable when delayed neutrons are neglected. Later L.M. Shotkin, D.L. Hetrick and T.R. Schmidt[15] showed that delayed neutrons permit the existence of unstable limit cycles. They also concluded that for linearly stable systems the delayed neutrons can cause the system to become unbtable for large enough disturbances.

L.M. Shotkin[14] investigated the instability bounds in linearly stable systems and gave a general method for determining the bounds on allowable disturbances.

We also checked this in next chapter by solving the point kinetics equations for various perturbations at some selected operating points.

Chapter V

NUMERICAL SOLUTION METHODS

1. NUMERICAL SOLUTION BY USING FINITE DIFFERENCE METHOD [16]

This method to solve the kinetics equations, is based essentially on the definition of derivative. We begin to introduce the method by casting the equations in the general form

$$\frac{d\underline{\psi}(t)}{dt} = f(\underline{\psi}, t) \qquad (5.1)$$

The elemantary definition of the derivative,

$$\frac{d\underline{\psi}}{dt} = \lim_{\Delta t \to 0} \frac{\underline{\psi}(t+\Delta t) - \underline{\psi}(t)}{\Delta t} \qquad (5.2)$$

leads to a suitable numerical procedure. The limit is approximated by the so-called first divided differince :

$$\frac{\underline{\psi}(t+\Delta t) - \underline{\psi}(t)}{\Delta t} \approx f(\underline{\psi}, t) \qquad (5.3)$$

$$\underline{\Psi}_{n+1} = \underline{\Psi}_n + \Delta t \ f_n \tag{5.4}$$

This kind of solution can be satisfactory only when very small time intervals are considered due to definition of derivative; so we will examine the time intervals as 0.1 seconds since the neutron generation time is in this range. Casting these equations into matrix form :

$$\begin{bmatrix} \bar{\phi}_{n+1} \\ D_{n+1} \\ Xe_{n+1} \\ I_{n+1} \end{bmatrix} = \begin{bmatrix} \left(\delta_o - \beta - \dfrac{\sigma_x \, Xe}{c \ \sigma_f} - \gamma \bar{\phi}_n \right) \dfrac{\bar{\phi}_n}{\ell} + \dfrac{\lambda \ D}{\ell} \\ \beta \ \bar{\phi}_n - \lambda \ D_n \\ (y_x \sigma_f - \sigma_x Xe_n) \bar{\phi}_n - \lambda_x Xe_n + \lambda_I I_n \\ y_I \sigma_f \bar{\phi}_n - \lambda_n I_n \end{bmatrix} . \Delta t + \begin{bmatrix} \bar{\phi}_n \\ D_n \\ Xe_n \\ I_n \end{bmatrix} \tag{5.5}$$

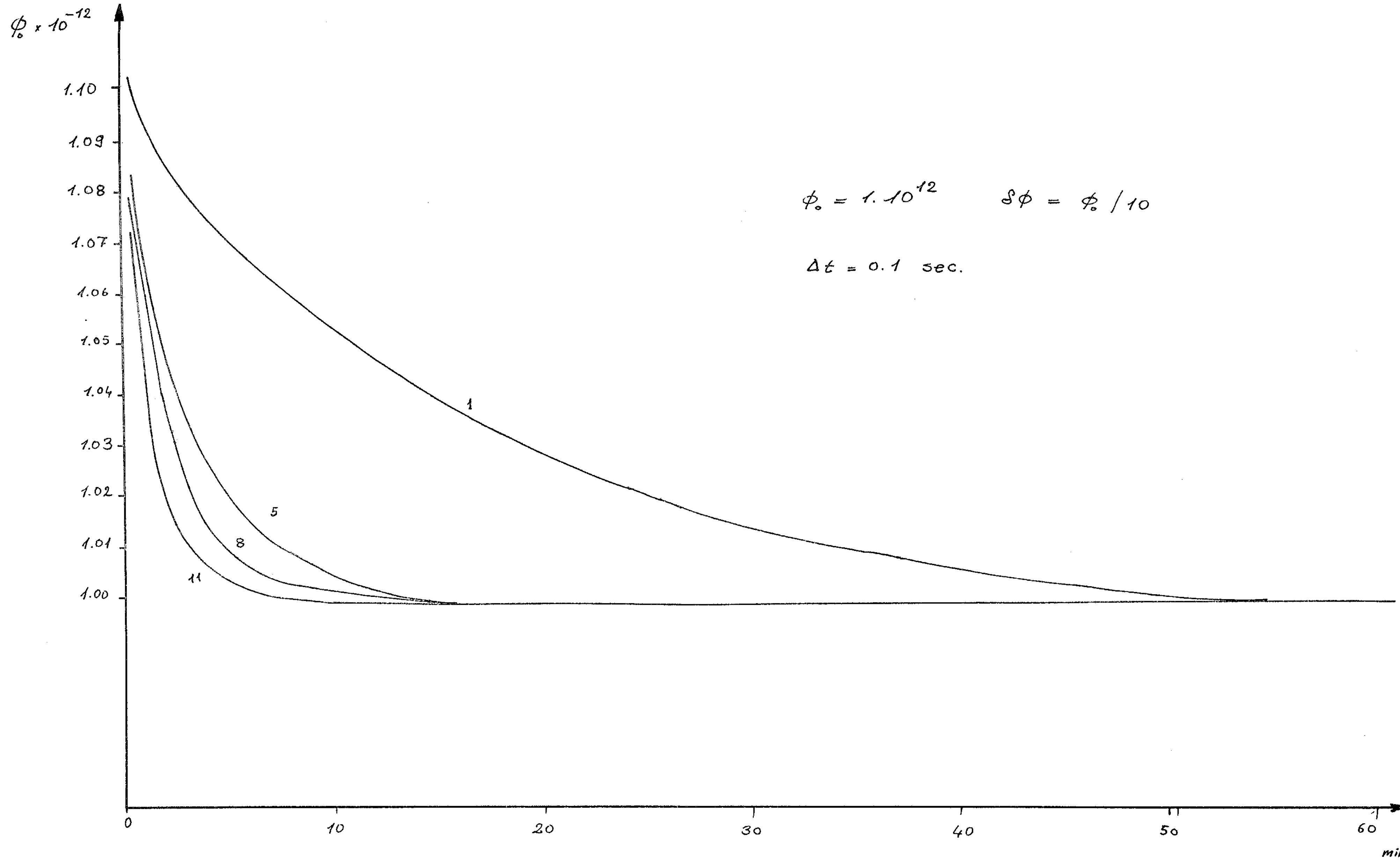
Beginning equations from equilibrium point with little perturbation are then ;

$$\begin{bmatrix} \bar{\phi}_1 \\ D_1 \\ Xe_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \left[\delta_o - \beta - \dfrac{\sigma_x \, Xe}{c \ \sigma_f} - \gamma (\bar{\phi}_o + \delta\phi) \right] \dfrac{(\bar{\phi}_o + \delta\phi)}{\ell} + \dfrac{\lambda \ D_o}{\ell} \\ \beta (\bar{\phi}_o + \delta \bar{\phi}) - \lambda \ D_o \\ (y_x \sigma_f - \sigma_x Xe_o)(\bar{\phi}_o + \delta\phi) - \lambda_x Xe + \lambda_I I_o \\ y_I \sigma_f (\bar{\phi}_o + \delta\phi) - \lambda_I I_o \end{bmatrix} . \Delta t + \begin{bmatrix} \bar{\phi}_o + \delta\phi \\ D_o \\ Xe_o \\ I_o \end{bmatrix}$$

Solution applied to computer and results are given in appendix - 6.

Departing from the definition of derivative, we assumed that the slope of the flux function remained constant at each time interval although it changes in time actually. In order to reduce this approximation error, the behaviour of the flux is observed at a time interval of average neutron generation time, ℓ^*, of prompt and delayed neutrons, i.e., 0.1 sec. So, the delayed neutrons are considered to be produced "instantaneously" due to the considerations stated before.

Flux behaviour was observed only for the first hour due to limitations of computing time. The accuracy of this method and the interpretation of the plots will be given after the second solution method is applied.



φ₀ x 10^-12
1.10
1.09
1.08
1.07
1.06
1.05
1.04
1.03
1.02
1.01
1.00
φ₀ = 1.10^12
δφ = φ₀/10
Δt = 0.1 sec.
1
5
8
11
0
10
20
30
40
50
60

Figure – 11

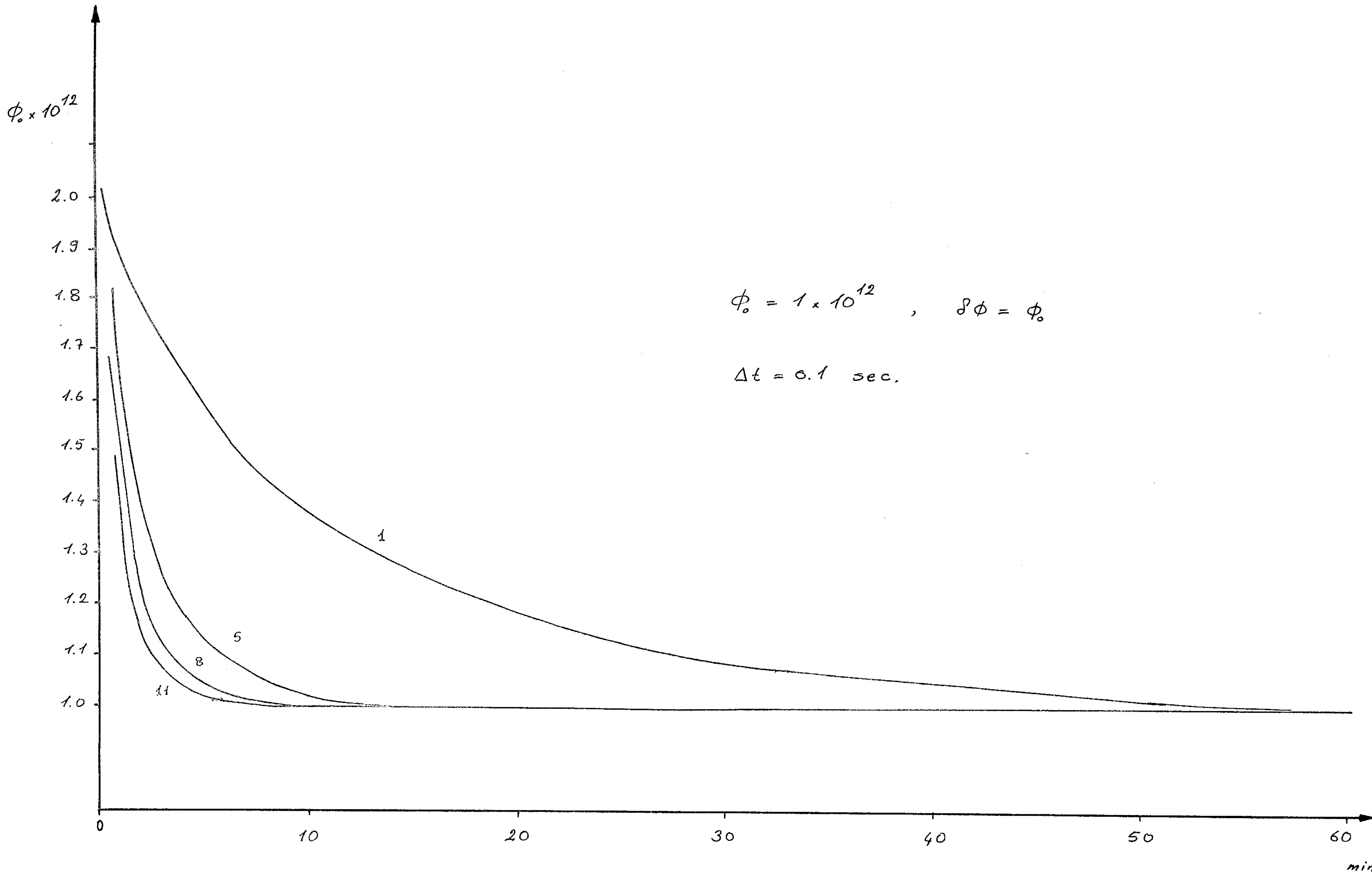

$\Phi_o \times 10^{12}$
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
1
5
8
11
$\Phi_o = 1 \times 10^{12}$, $\delta\Phi = \Phi_o$
$\Delta t = 0.1$ sec.
0
10
20
30
40
50
60
min

Figure - 12

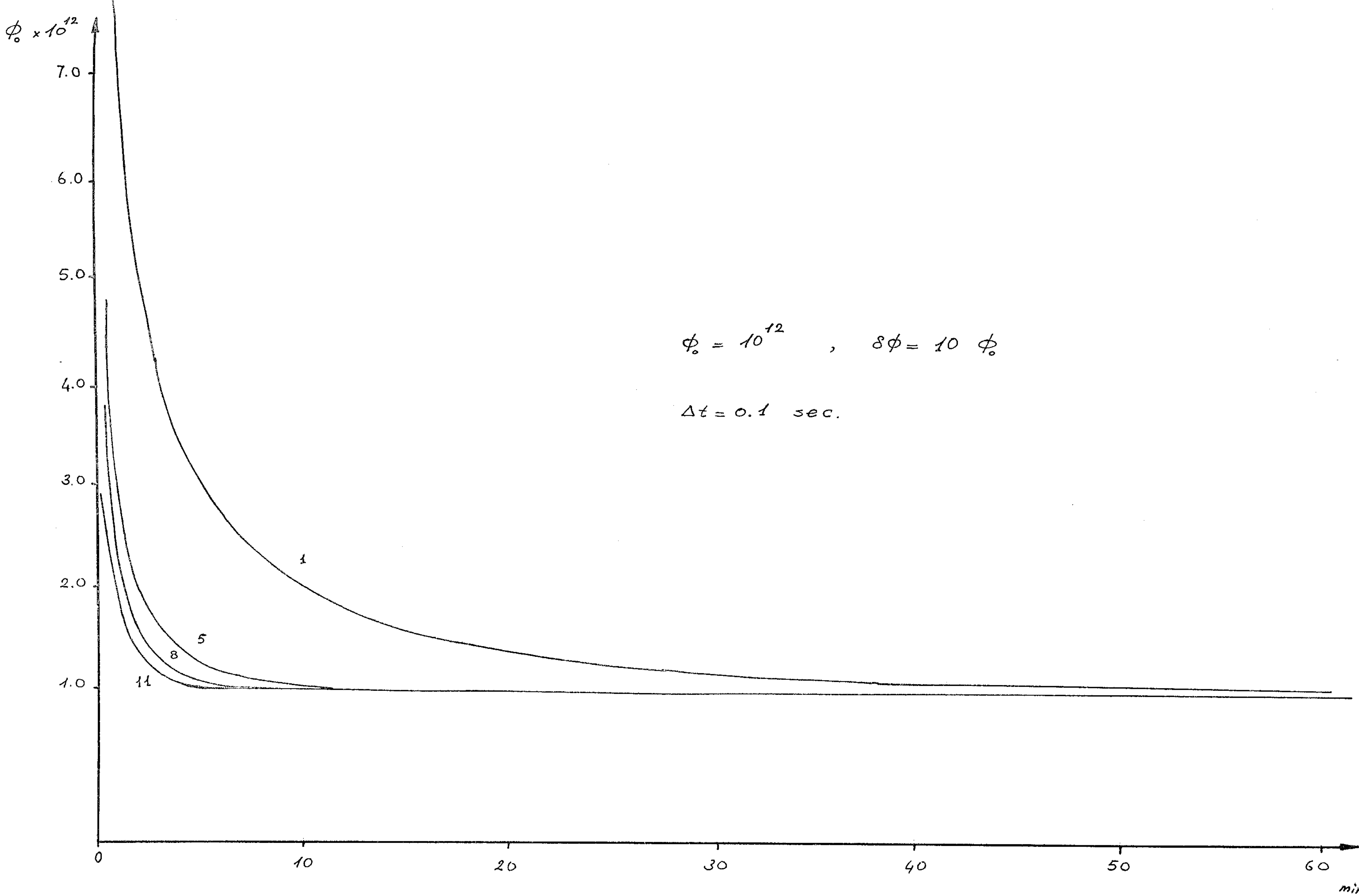

$\phi_o \times 10^{12}$
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
10
20
30
40
50
60
1
5
8
11
$\phi_o = 10^{12}$, $\delta\phi = 10\ \phi_o$
$\Delta t = 0.1$ sec.

Figure - 13

2. NUMERICAL SOLUTION OF THE POINT KINETICS EQUATIONS

HANSEN'S METHOD :

In this section we will try to solve the point kinetics equations starting with a specific equilibrium. We will use a modified form of the method proposed by Hansen[5].

The basic idea of Hansen's method is relatively simple. Writing the point kinetics equations in matrix form;

$$d\underline{\psi}(t) / dt = \underline{\underline{A}}\,\underline{\psi}(t) + \underline{C} \qquad (5.6)$$

where

$$\underline{\underline{A}} = \begin{bmatrix} (\delta_o - 2\gamma\phi_o)/l - \dfrac{U}{T} & -\dfrac{PX}{T} & 0 & \dfrac{\lambda}{l} \\ R & -ZX & \lambda_I & 0 \\ y_I\,\sigma_f & 0 & -\lambda_I & 0 \\ \beta & 0 & 0 & -\lambda \end{bmatrix} \qquad (5.7\text{ a})$$

$$C = \begin{bmatrix} \left(\delta_o - \dfrac{U}{c\,\sigma_f} - \gamma\phi_o\right)\dfrac{\phi_o}{l} \\ R\,\phi_o - \lambda_x Xe_o + \lambda_I I_o \\ y_I\,\sigma_f\,\phi_o - \lambda_I I_o \\ \beta\,\phi_o - \lambda\,D_o \end{bmatrix} \qquad \underline{\psi} = \begin{bmatrix} \phi(t) \\ \delta Xe(t) \\ \delta I(t) \\ \delta D(t) \end{bmatrix} \qquad (5.7\text{ b})$$

The matrix A can be decomposed into three matrices,

$$\underline{\underline{A}} = \underline{\underline{L}} + \underline{\underline{D}} + \underline{\underline{U}} \tag{5.8}$$

where $\underline{\underline{L}}$ is strictly lower triangular, $\underline{\underline{U}}$ strictly upper triangular, and $\underline{\underline{D}}$ diagonal. We assume $\underline{\underline{D}} \neq 0$. Equation (5.6) may be rewritten as,

$$d\underline{\psi}(t) / dt - \underline{\underline{D}}\,\underline{\psi}(t) = (\underline{\underline{L}} + \underline{\underline{U}})\ \underline{\psi}(t) + \underline{C} \tag{5.9}$$

The reason for splitting it up in this fashion is to develop an iteration procedure. We assume that we begin this calculation from a time t_o and advance to a time t_1, and t_2 and so on.

Let
$$h = t_1 - t_o = t_2 - t_1 = \ldots\ldots\ldots = t_{i+1} - t_i \tag{5.10}$$

Since $\underline{\underline{D}}$ is a diagonal matrix, an integrating factor for equation (5.9) is $\exp(- \underline{\underline{D}}t)$, if the reactivity does not change much during the time interval h. Therefore equation (5.9) becomes

$$e^{-\underline{\underline{D}}t}\,\dot{\underline{\psi}}(t) - e^{-\underline{\underline{D}}t}\,\underline{\underline{D}}\,\underline{\psi}(t) = e^{-\underline{\underline{D}}t} (\underline{\underline{L}} + \underline{\underline{U}})\ \underline{\psi}(t) + \underline{C}\ \ e^{-\underline{\underline{D}}t}$$

or

$$\frac{d}{dt}\left[e^{-\underline{\underline{D}}t}\ \underline{\psi}(t) \right] = e^{-\underline{\underline{D}}t} (\underline{\underline{L}} + \underline{\underline{U}})\ \underline{\psi}(t) + e^{-\underline{\underline{D}}t}\ \underline{C} \tag{5.11}$$

integrating between time intervals t_i and t_{i+1}

$$\int_{t_i}^{t_{i+1}} \frac{d}{dt}\left[e^{-\underline{\underline{D}}t}\ \underline{\psi}(t) \right] dt = \int_{t_i}^{t_{i+1}} (\underline{\underline{L}} + \underline{\underline{U}})\ e^{-\underline{\underline{D}}t}\ \underline{\psi}(t)\ dt + \int_{t_i}^{t_{i+1}} e^{-\underline{\underline{D}}t}\ \underline{C}\ dt \tag{5.12}$$

assuming $\underline{\psi}(t)$ remains constant in the time interval $h = t_{i+1} - t_i$ which has to be very short,

$$e^{-\underline{\underline{D}}t_{i+1}}\underline{\psi}(t_{i+1}) - e^{-\underline{\underline{D}}t}\ \underline{\psi}(t_i) = -\underline{\underline{D}}^{-1}\left[e^{-\underline{\underline{D}}t_{i+1}} - e^{-\underline{\underline{D}}t_i}\right](\underline{\underline{L}} + \underline{\underline{U}})\ \underline{\psi}(t_i)$$

$$- \underline{\underline{D}}^{-1}\left[e^{-\underline{\underline{D}}t_{i+1}} - e^{-\underline{\underline{D}}t_i}\right]\underline{C}$$

$$\underline{\psi}(t_{i+1}) = e^{\underline{\underline{D}}t_{i+1}}\left\{e^{-\underline{\underline{D}}t_i} - \underline{\underline{D}}^{-1}\left[e^{-\underline{\underline{D}}t_{i+1}} - e^{-\underline{\underline{D}}t_i}\right](\underline{\underline{L}} + \underline{\underline{U}})\right\}\ \underline{\psi}(t_i)$$

$$- D^{-1}\left[\underline{\underline{I}} - e^{\underline{\underline{D}}(t_{i+1} - t_i)}\right]\underline{C}$$

$$\underline{\psi}(t_{i+1}) = \left\{e^{\underline{\underline{D}}h} - \underline{\underline{D}}^{-1}\left[\underline{\underline{I}} - e^{\underline{\underline{D}}h}\right](\underline{\underline{L}} + \underline{\underline{U}})\right\}\underline{\psi}(t_i) - D^{-1}\left[\underline{\underline{I}} - e^{\underline{\underline{D}}h}\right]\underline{C} \qquad (5.13)$$

Here $\underline{\psi}(t_i)$ is the amount of perturbation applied to the system initially and $\underline{\psi}(t_{i+1})$ is the response of the reactor after a reasonable time step. $\underline{C}$ is a vector whose components have the values of the point about which linearization is made. This method of linearization at each point is more likely to approach the real behaviour of the reactor with the greater accuracy.

The components of vector $\underline{C}$ have the values characterizing the points at which the system is linearized. Clearly it will be zero for the first time step since we there linearize the system at the equilibrium point.

Yet it will not be zero for the second time step because, now the linearization point is not an equilibrium state but a perturbed value of the equilibrium flux. Similarly for the following points.

Mathematically it can be explained as follows ;

Kinetic equations having the form ;

$$\dot{x} = f(x) \tag{5.14}$$

right side can be expanded into Taylor series around any state x_L of linearization

$$\dot{x} = f(x_L) + \left.\frac{\partial f}{\partial x}\right|_{x=x_L} (x - x_L) + \underbrace{\text{Higher order terms}}_{\text{neglected}} \tag{5.15}$$

now pose $\quad x - x_L = \delta x^*$

it becomes $\quad \dfrac{d}{dt}(x_L + \delta x^*) = \left.\dfrac{\partial f}{\partial x}\right|_{x=x_L} \delta x^* + f(x_L)$

or

$$\delta \dot{x}^* = \left.\frac{\partial f}{\partial x}\right|_{x=x_L} \delta x^* + f(x_L) \tag{5.16}$$

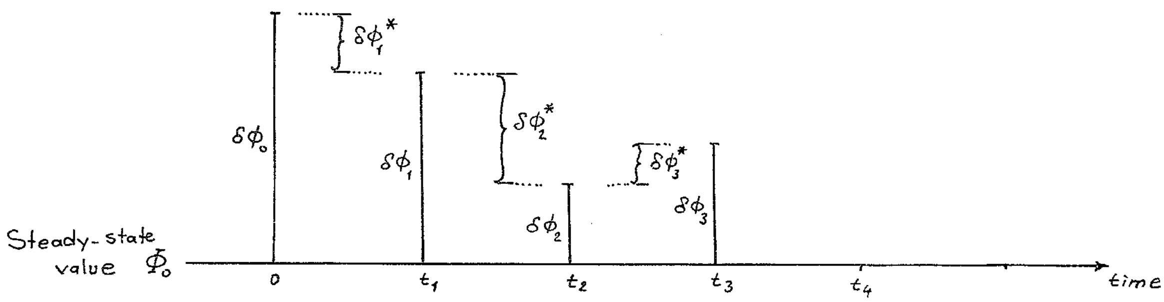
which is the form of equation (5.6)

The computed δx^* will give the difference between the current state and the previous point of linearization,

$$x = \delta x^* + x_L \tag{5.17}$$

For linearization at each time interval, question might arise about the amount of perturbation to be applied. The question of whether the perturbation will be applied from the initial steady-state operating condition or from the previous linearization point will be clear when the operations from the beginning is observed with the help of the following sketch .

We assumed that the reactor is operated at the steady-state flux level of $\bar{\phi}_o$ prior to t=0. So the equations describing the time behaviour of the system are linearized about this value. Then a small amount of perturbation $\delta\phi_o$ is applied at time t=0.



Resulting perturbation $\delta\phi_1$ is obtained by solving the perturbation equations. Now the linearization point is the first perturbed flux, i.e., $\phi_o + \delta\phi_o$ and the amount of perturbation will be applied is the difference, $\delta\phi_1^* = \delta\phi_o - \delta\phi_1$. Now the linearization point is $\phi_o + \delta\phi_1$, and the amount of perturbation is $\delta\phi_2^* = \delta\phi_1 - \delta\phi_2$.

This procedure is employed successively. Applied computer program is given in the appendix -4.

3. DISCUSSION :

We have solved the point kinetics equations without delayed neutrons using the proposed solution technique with a computer program. We were unable to examine the problem taking into account the long-term feedback effects of the delayed neutrons because the very short time response ℓ of the prompt neutrons is of the order of 10^{-4} sec. Whereas the delayed neutron time response is $1/\lambda$ or about 10 sec., a factor of 10^{5} greater. The implication of these facts is that in order to obtain the prompt response, very small time steps, of the order of 10^{-4}sec. are required. But then before the delayed neutron term comes into play, many time steps are required. For instance to examine the responce out to even one second about 10,000 steps of calculations would be required.

A method for solving the point reactor kinetics equations given by da Nòbrega[8] requires about 1.5×10^{6} steps in order to examine a one second span as can be seen from the table reproduced below from the work mentioned above[21].

TABLE II

Time (sec)		MOVER-I	
		$\epsilon = 10^{-4}$	$\epsilon = 10^{-3}$
0.0	T_1	1.0	1.0
	T_2	0.0	0.0
0.003	T_1	0.0373 (1)	0.0373 (2)
	T_2	0.5696	0.5700

(1) Took 44 time steps to get to $t = 0.002934$ sec.

(2) Took 19 time steps to get to $t = 0.003098$ sec.

In order to overcome this difficulty we considered the delayed neutrons as being produced " instantaneously ". So we examined the problem in 0.1 sec. time intervals, since the average prompt neutron generation time now is in this range.

β_i	a_i	λ_i	a_i/λ_i
0.2475×10^{-3}	0.033	0.0124	2.66
1.6425×10^{-3}	0.219	0.0305	7.18
1.47×10^{-3}	0.196	0.111	1.76
2.9625×10^{-3}	0.395	0.301	1.31
0.8625×10^{-3}	0.115	1.130	0.101
0.315×10^{-3}	0.042	3.00	0.014

Table - 2 Delayed neutron parameters for thermal fission in U^{235} [25].

$$\beta = \sum_{i=1}^{6} \beta_i = 0.0075$$

$$\sum_{i=1}^{6} (a_i/\lambda_i) = 13.03544$$

Average neutron generation time can be calculated as follows ;

$$\ell^{*} = \ell + \beta \sum_{i=1}^{6} (a_i / \lambda_i)$$

where ℓ is the prompt neutron generation time.

$$\ell^{*} = 10^{-4} + 0.0075 * 13.03544$$

$$= 0.09786$$

$$\ell^{*} \cong 0.1 \text{ sec.}$$

Thus we consider the feedback effects of the delayed neutrons, as being prompt.

First we select four specific points in $\phi_0 - \gamma$ plane. These points have the same equilibrium flux value but have different prompt temperature reactivity coefficients as, (1,5,8,11) x 10^{-16}. The aim of doing so is to observe how fast the flux behaviour will return to its equilibrium value or how fast it will diverge for different γ values.

We also applied to each point three different levels of perturbation (the one tenth of the equilibrium value of the flux, the same as the equilibrium value of the flux and ten times that of the equilibrium value) so as to examine the effect of the perturbation magnitude on the stability of this linear system.

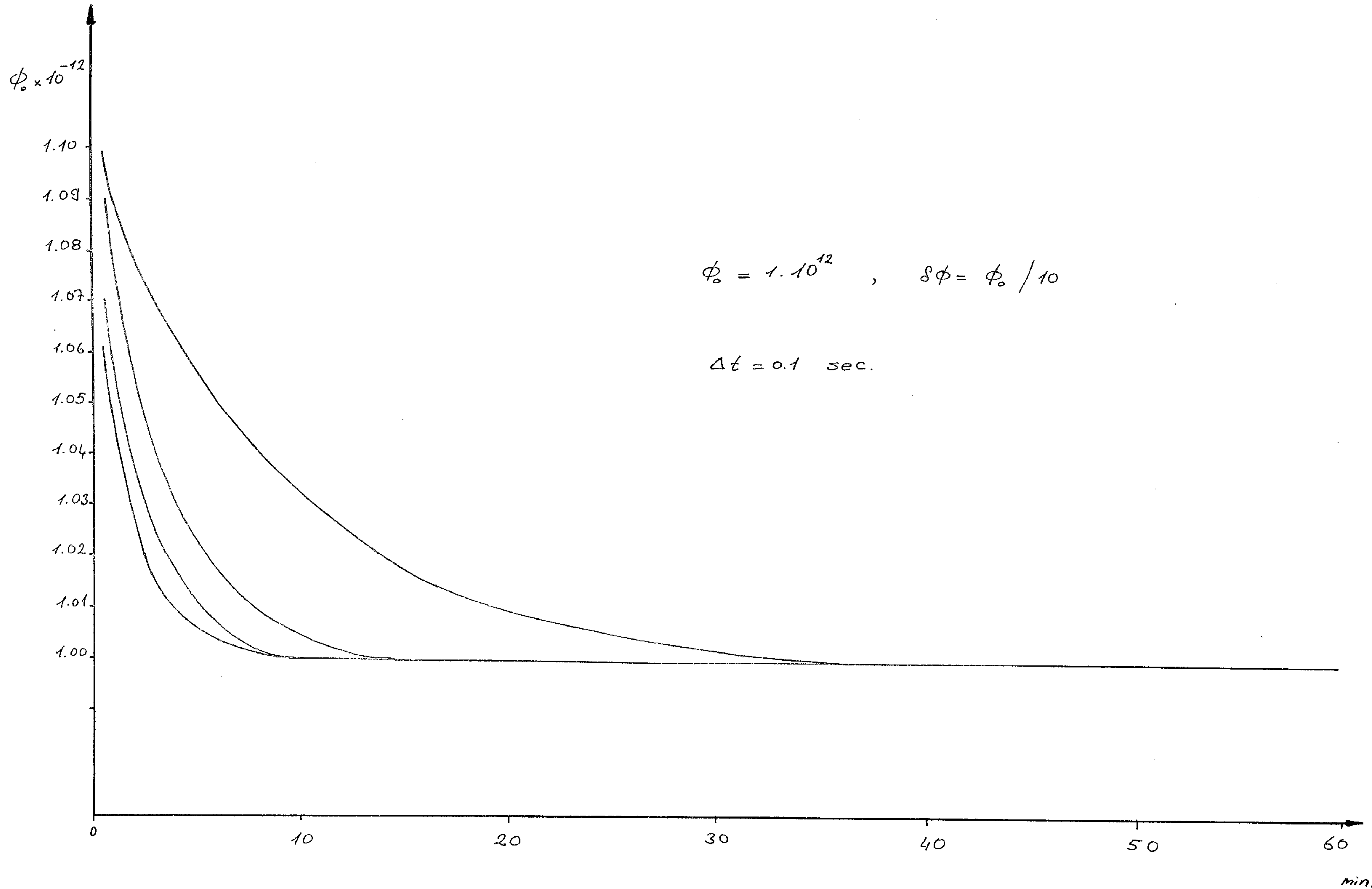
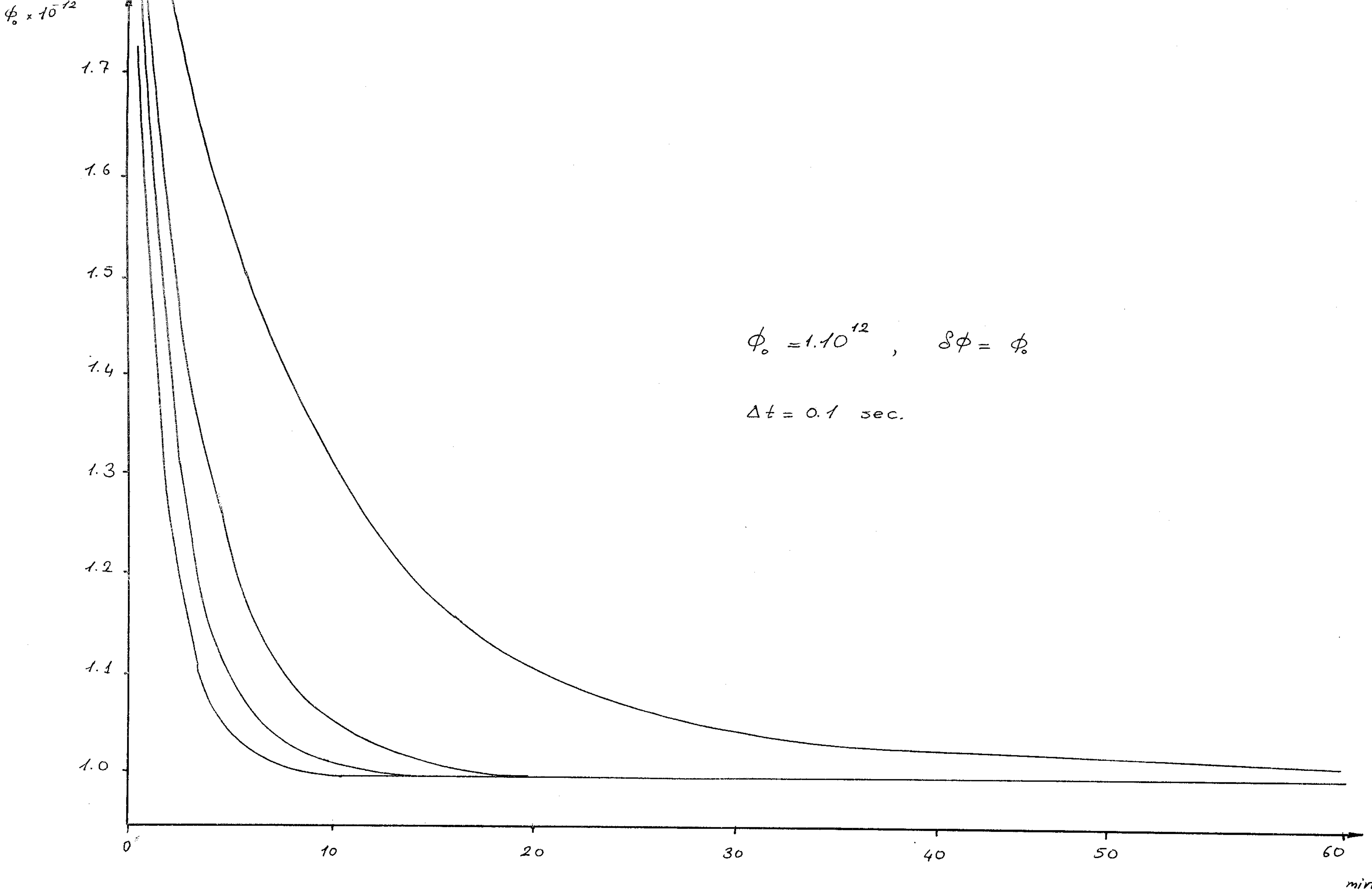
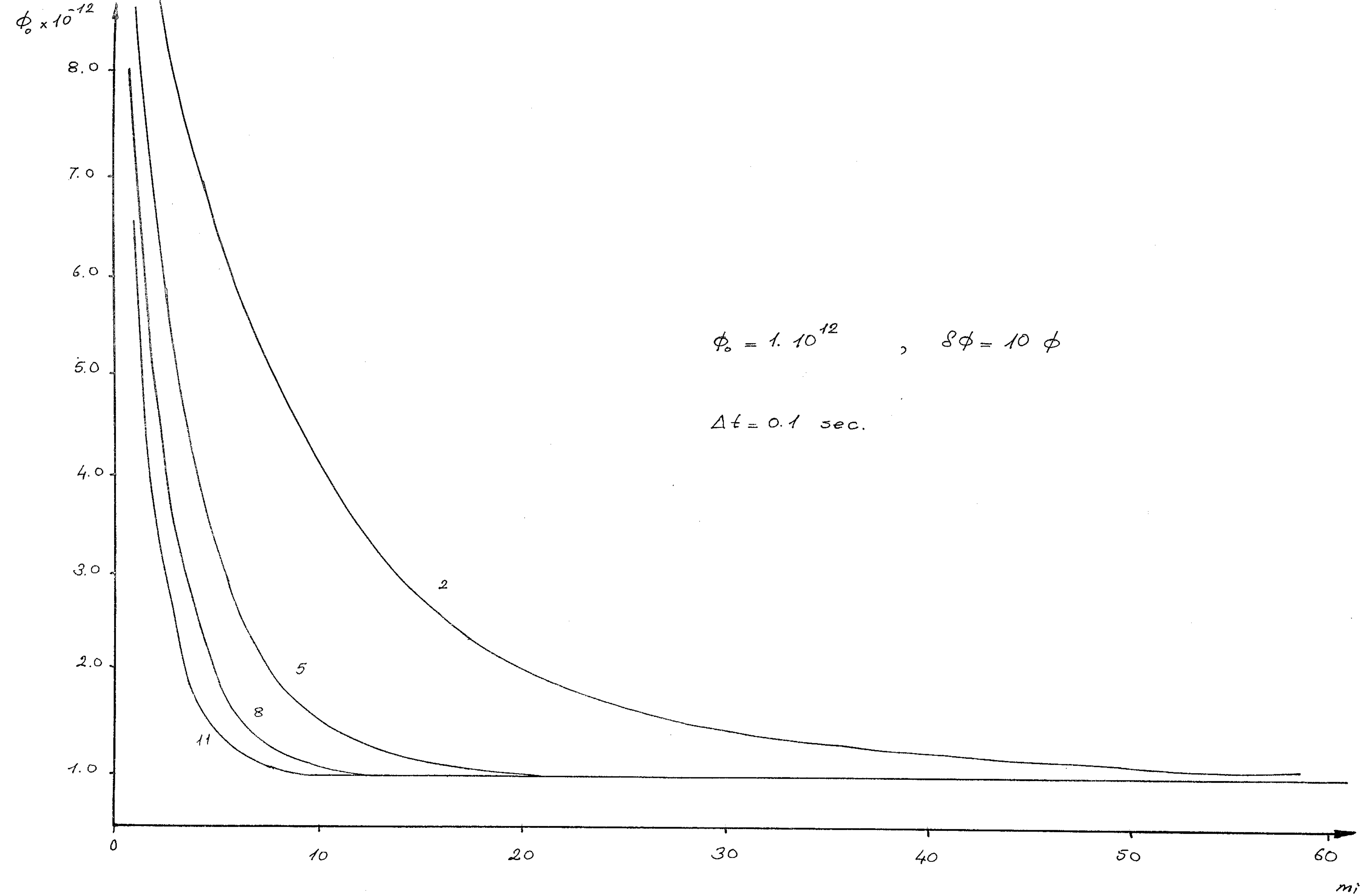

Φ₀ × 10⁻¹²
1.10
1.09
1.08
1.07
1.06
1.05
1.04
1.03
1.02
1.01
1.00
Φ₀ = 1. 10¹² , δΦ = Φ₀ / 10
Δt = 0.1 sec.
0
10
20
30
40
50
60
min.

Figure - 14



$\phi_o \times 10^{-12}$
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
$\phi_o = 1.10^{12}$, $\delta\phi = \phi_o$
$\Delta t = 0.1$ sec.
0
10
20
30
40
50
60
min

Figure - 15



Φo x 10^-12
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
10
20
30
40
50
60
mi
2
5
8
11
Φo = 1. 10^12 , δΦ = 10 Φ
Δt = 0.1 sec.

Figure – 16

Figures 14 to 16 show the behaviour of several equilibrium points under various perturbations. It can be seen from these plots that as the perturbation on the flux is increased the time required for the system to return to its equilibrium for the first time is decreased. This is expected because temperature reactivity feedback acts on the system promptly. As the perturbation on the flux is increased, temperature feedback behaves more efficiently, since it is proportional to this perturbation and generates a considerably large negative feedback.

Also as the temperature reactivity coefficient is increased for the same equilibrium point and perturbation, the slope of the flux becomes steeper, i.e., it returns to equilibrium point more rapidly. This is due to the fact that the temperature reactivity feedback is proportional to the operating value of the flux.

The question might arise about the behaviour of the flux returning to the equilibrium value at a point that was previously found to be unstable. This behaviour is reasonable because we can observe only the first hour of response due to the necessity of very short time steps used. Thus the long-term effect of Xenon poisoning could not be observed, but the effect of prompt temperature reactivity coefficient is active within the period investigated.

Chernick's observation of the flux behaviour at an operating condition near the line separating unstable and stable regions, is reproduced here for comparison of our results[7].

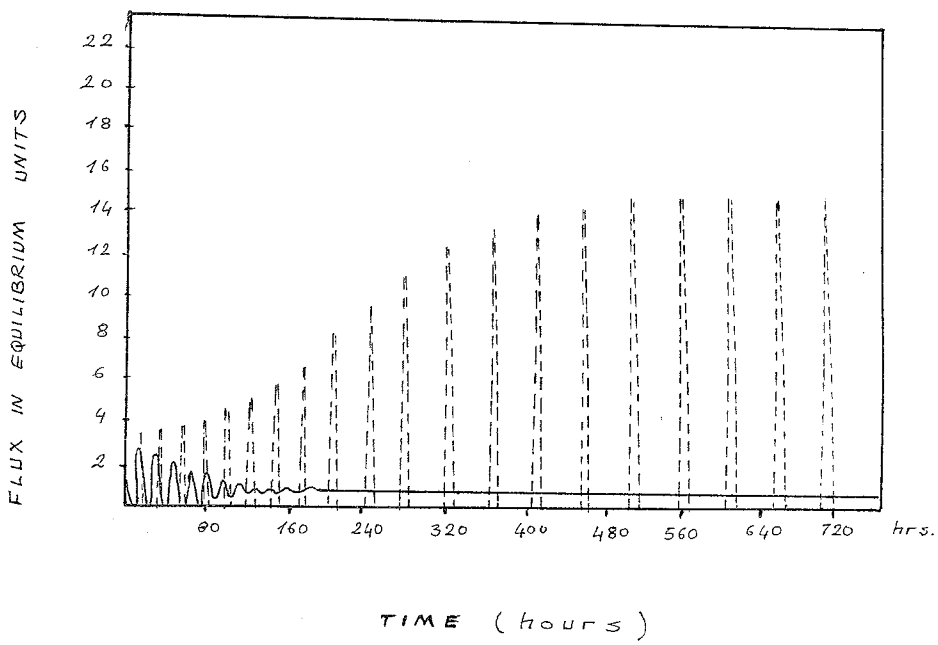


Figure - 17 Chernick's observation at the operating flux is

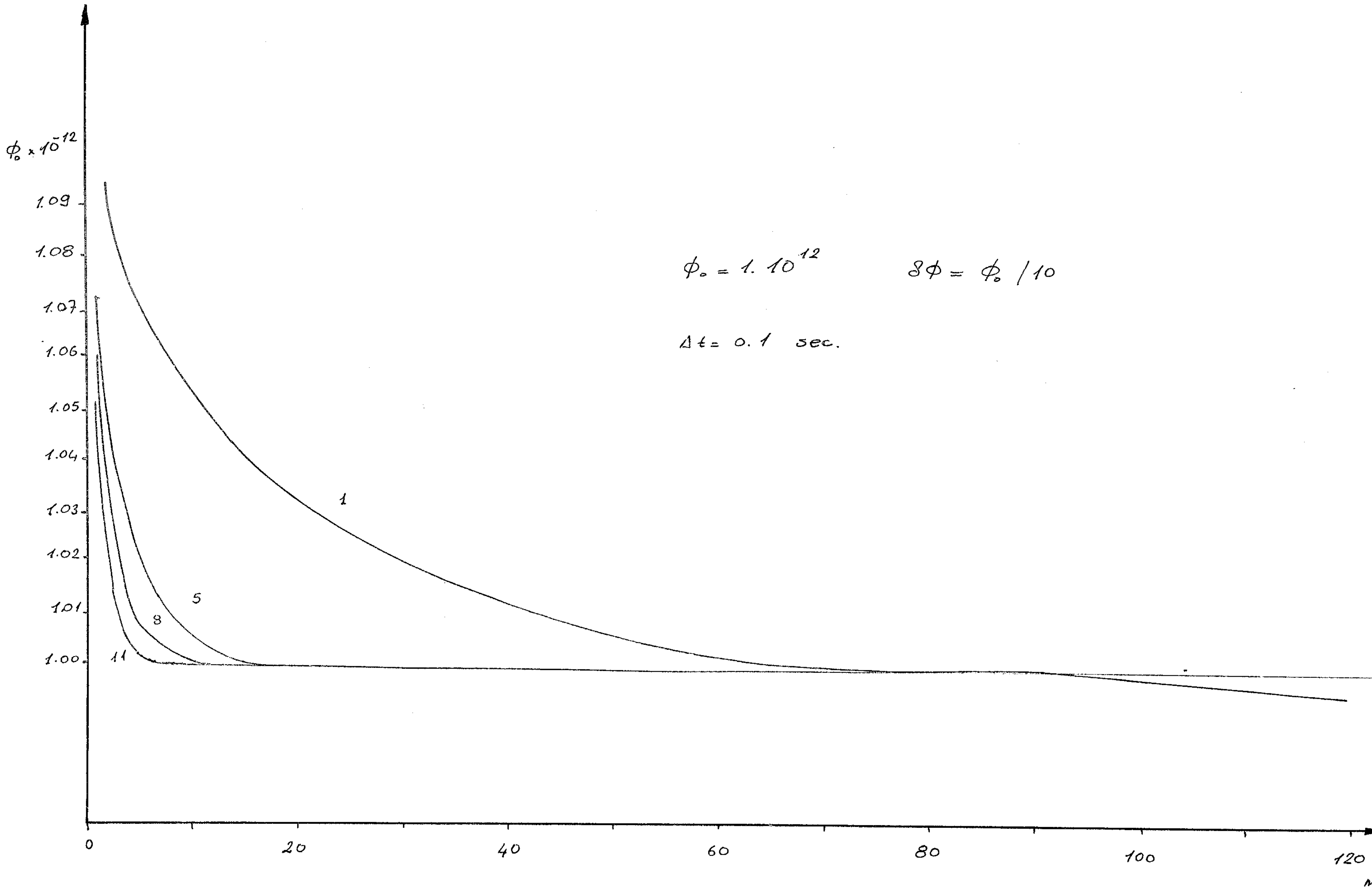
$\phi_o = 1.6\ 10^{11}$ n / (cm.sec.) and $\gamma = -3\ 10^{-16}$

—— Initial perturbation is $10^4 \times \phi_o$

---- Initial perturbation is $10^5 \times \phi_o$

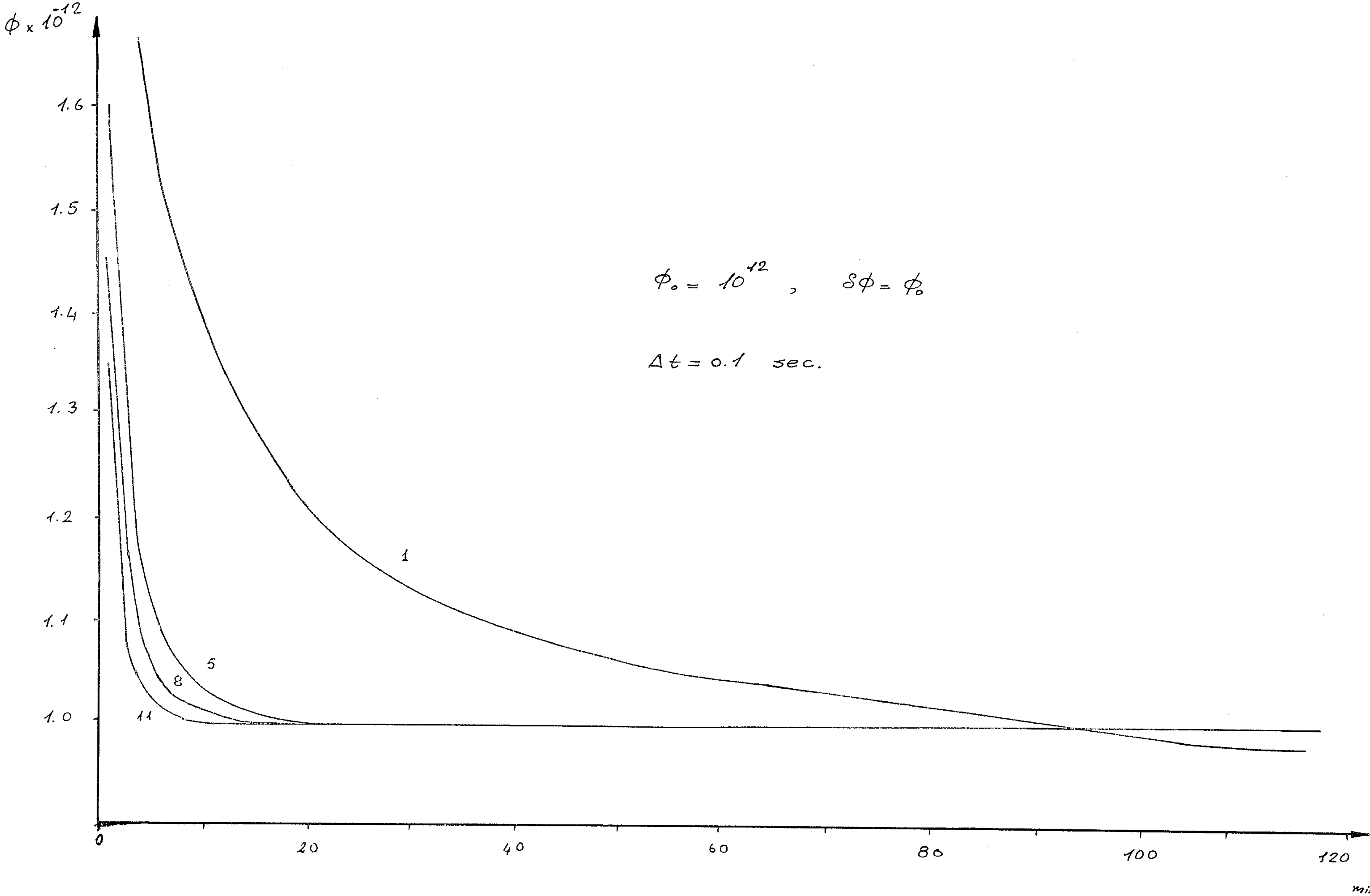
In order to see the flux oscillations one should solve the kinetics equations for 150 hrs., since the period of these oscillations is about 10 - 15 hrs.

In an attempt to obtain more accurate results we linearized the equations at each time step, i.e. 0.1 sec., and all the results are plotted in figures 18 to 20. Again as the prompt temperature reactivity coefficient is increased flux returns to its equilibrium point more rapidly.



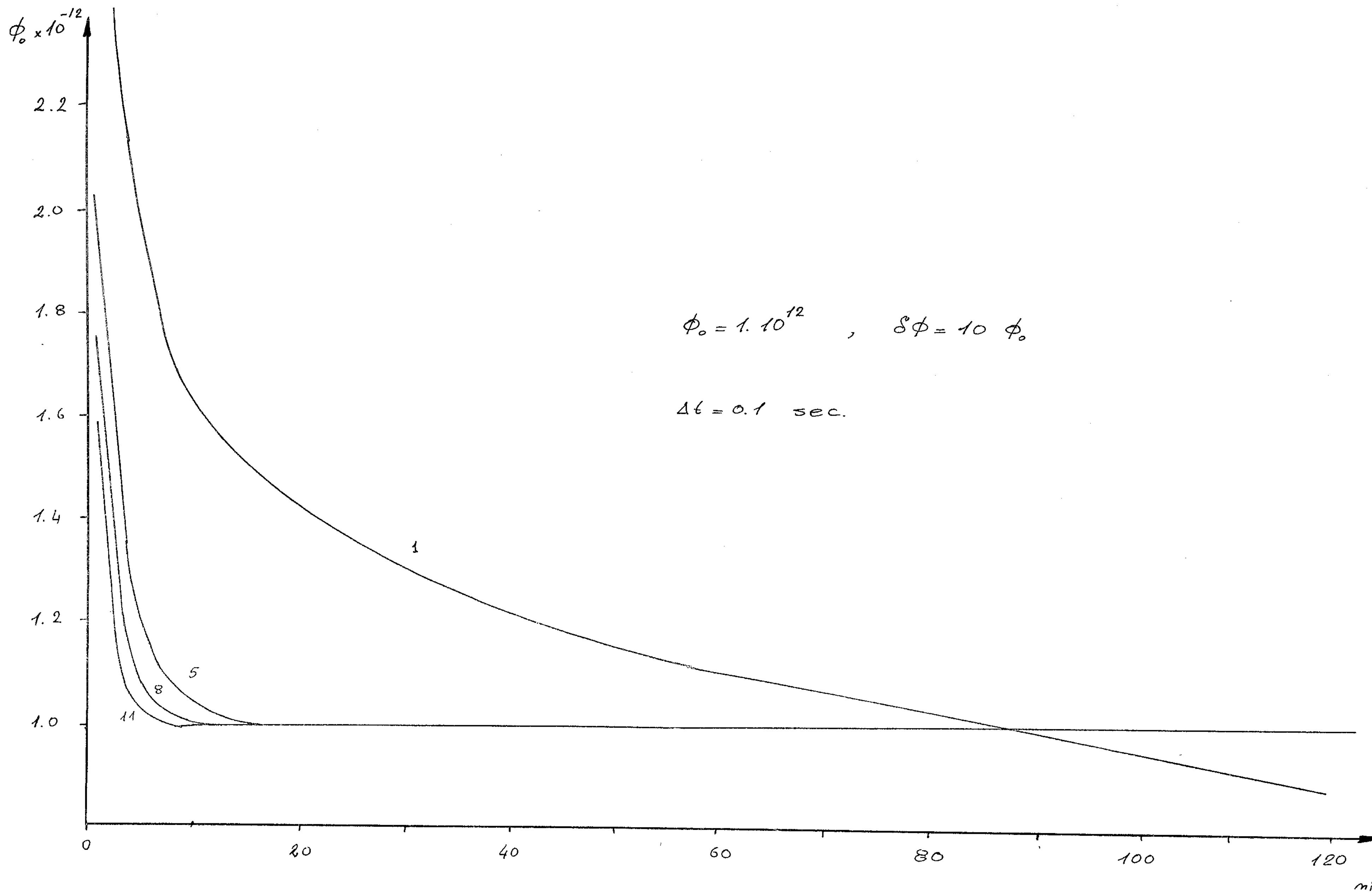
Φo x 10^-12
1.09
1.08
1.07
1.06
1.05
1.04
1.03
1.02
1.01
1.00
Φo = 1. 10^12
δΦ = Φo / 10
Δt = 0.1 sec.
1
5
8
11
0
20
40
60
80
100
120

Figure - 18



φ x 10^-12
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0
20
40
60
80
100
120
1
5
8
11
φ₀ = 10^12 , δφ = φ₀
Δt = 0.1 sec.

Figure – 19



$\phi_o \times 10^{-12}$
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
20
40
60
80
100
120
1
5
8
11
$\phi_o = 1.\ 10^{12}$, $\delta\phi = 10\ \phi_o$
$\Delta t = 0.1$ sec.

Figure – 20

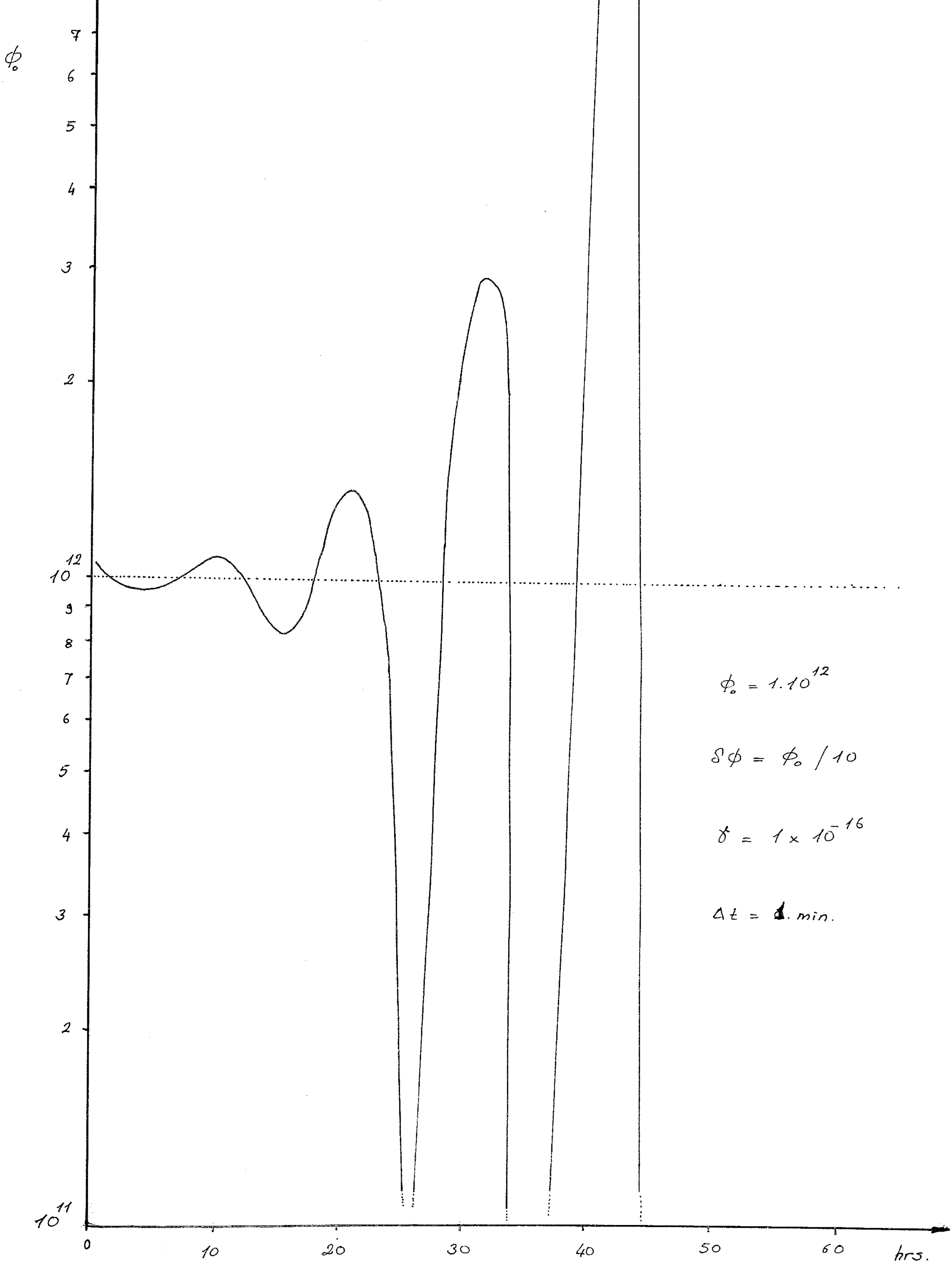
On the other hand, in order to see oscillations due to Xenon feedback one could solve the kinetics equations linearizing them only once at the initial equilibrium since the system behaviour indicated by the once linearization technique does not depend on the time step length chosen.

We observe the Xenon oscillations again at these four specific points during 120 hrs. It is interesting to note that, oscillations first begin with decreasing values of the flux due to prompt temperature reactivity feedback and then increases.

In the unstable region these oscillations increase more and more as the time flows, even though passing from the conditions which are called " shutdown " in nuclear reactor dynamics, i.e. solution is an analytic one not physically sustainable after the first shutdown. Figure - 21 shows this behaviour.

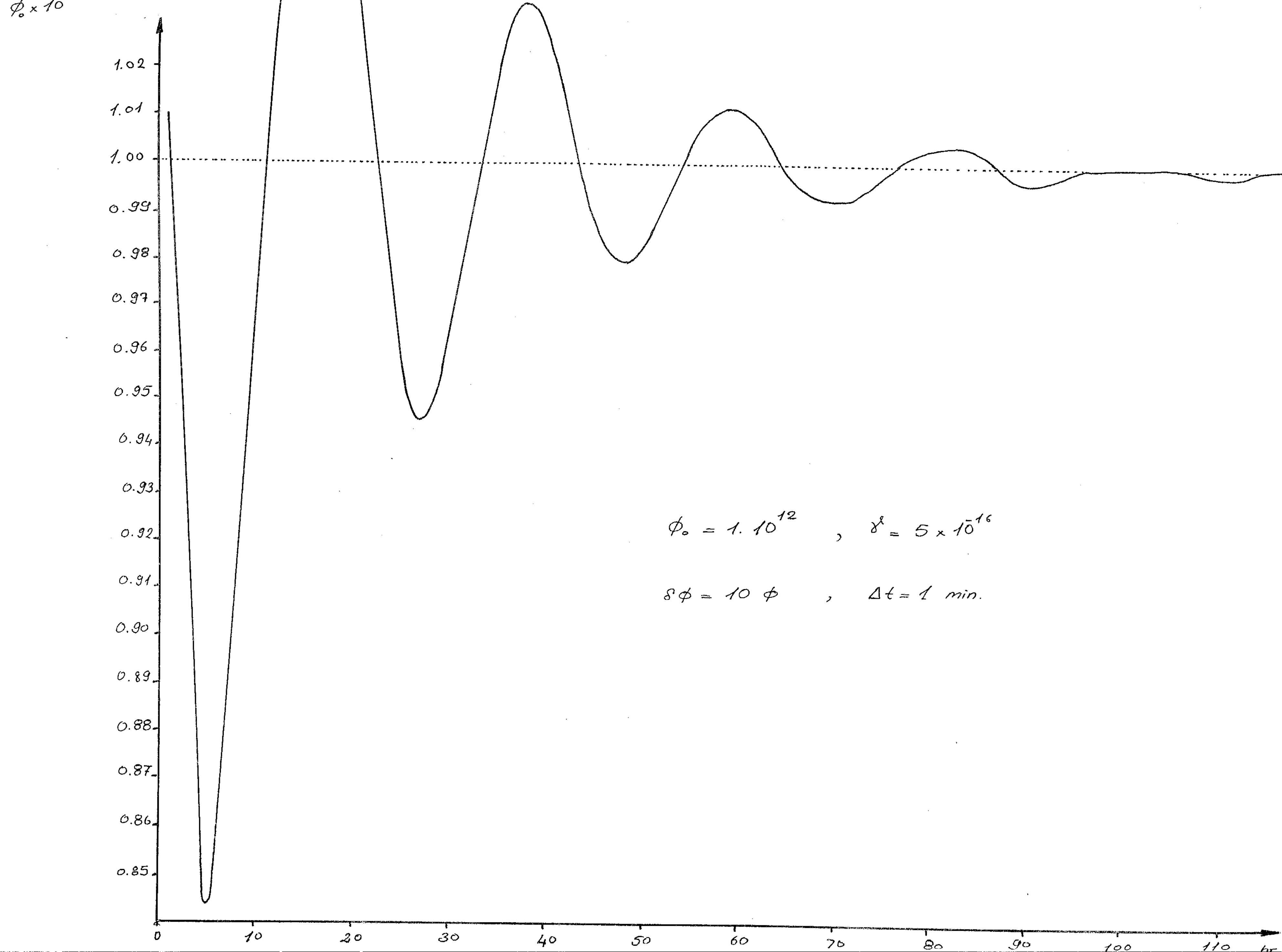
In linearly stable regions, the period and amplitude of these oscillations are damped as the prompt temperature reactivity coefficient is increased. This can be seen from figures 22 and 23.

In asymptotically stable region these oscillations die out in 50 hrs. and after that the flux remains constant at the initial equilibrium level (figure - 24) .



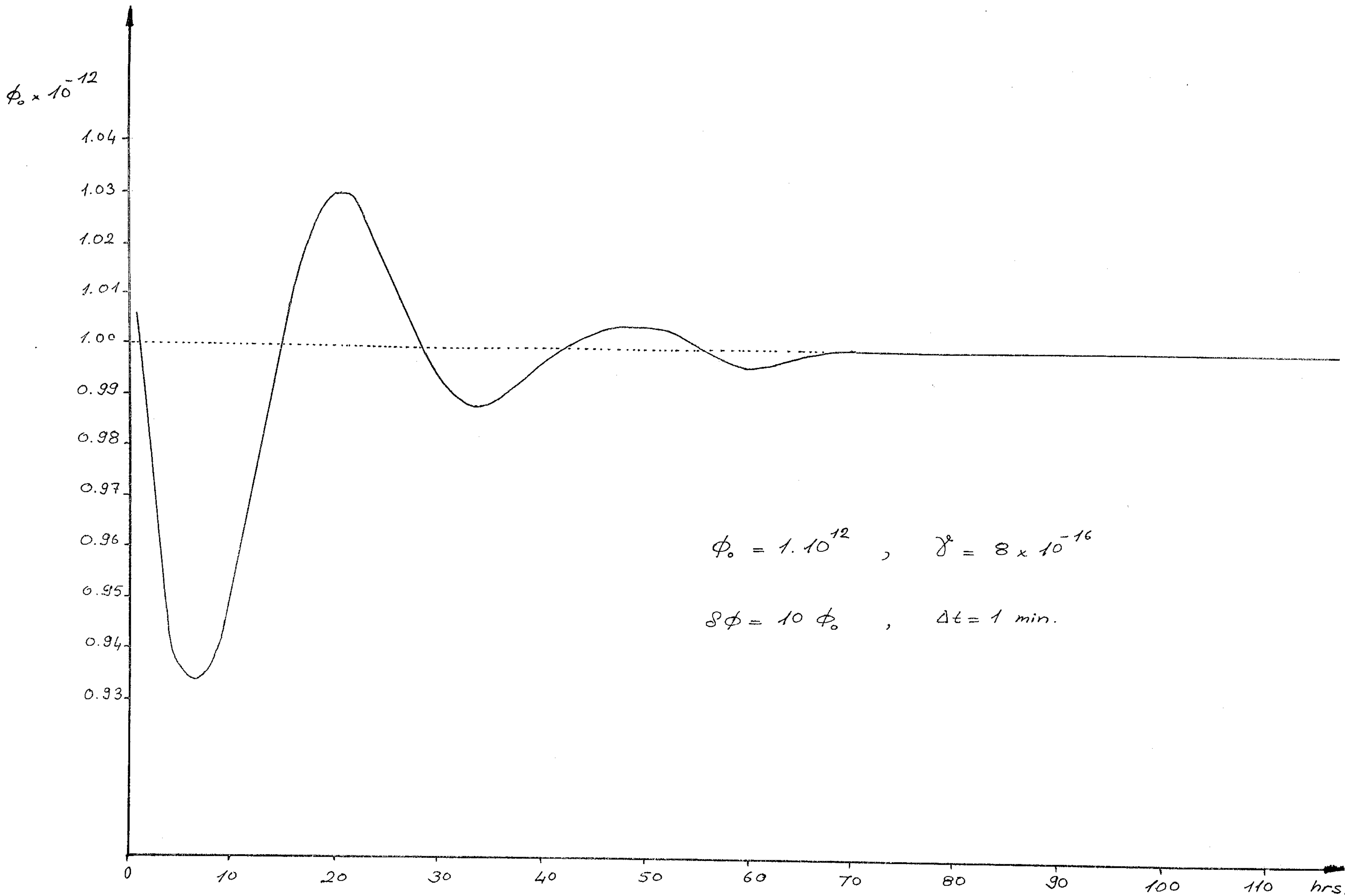
ϕ_o
7
6
5
4
3
2
10^{12}
9
8
7
6
5
4
3
2
10^{11}
0
10
20
30
40
50
60
hrs.
$\phi_o = 1.10^{12}$
$\delta\phi = \phi_o / 10$
$\gamma = 1 \times 10^{-16}$
$\Delta t = 1.$ min.

Figure - 21



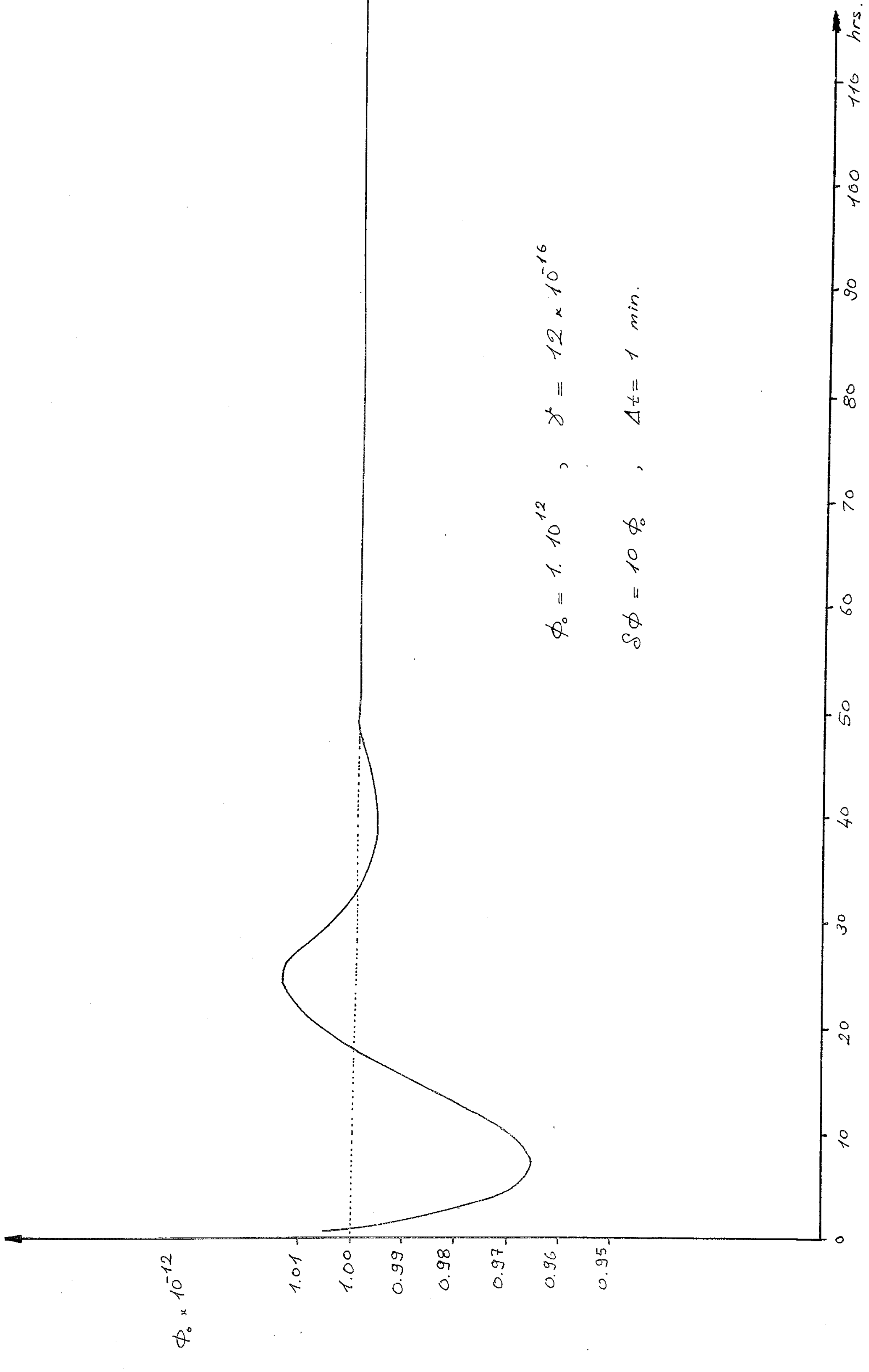
Φ₀ × 10
1.02
1.01
1.00
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
0.91
0.90
0.89
0.88
0.87
0.86
0.85
0
10
20
30
40
50
60
70
80
90
100
110
hr
$\phi_0 = 1.10^{12}$, $\gamma = 5 \times 10^{-16}$
$\delta\phi = 10\ \phi$, $\Delta t = 1$ min.

Figure – 22



Φ₀ × 10⁻¹²
1.04
1.03
1.02
1.01
1.00
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0
10
20
30
40
50
60
70
80
90
100
110
hrs.
Φ₀ = 1.10¹² , γ = 8 × 10⁻¹⁶
δΦ = 10 Φ₀ , Δt = 1 min.

Figure - 23



$\Phi_o \times 10^{-12}$
1.01
1.00
0.99
0.98
0.97
0.96
0.95
0
10
20
30
40
50
60
70
80
90
100
110
hrs.
$\Phi_o = 1.\ 10^{12}$, $\gamma = 12 \times 10^{-16}$
$\delta\Phi = 10\ \Phi_o$, $\Delta t = 1$ min.

Figure - 24

Chapter VI

CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

In this study we tried to construct sufficient conditions for Xenon and temperature controlled nuclear reactors to be stable against power excursions or inadvertent shutdowns. Later we solved the point kinetics equations using different techniques, for various operating conditions and under several perturbations.

For fixed values of flux and temperature reactivity coefficient, i.e. ϕ, γ; it is observed, for the first one hour that, as the perturbation on the flux is increased, return of the perturbed flux to its initial condition is speeded also. This is due to the prompt temperature reactivity feedback coming into play in proportion to the perturbed flux. This observation seems to be valid for all operating points in short time observation, i.e., before the Xenon feedback starts influencing the course of events.

According to linearized treatment, at an unstable point with fixed ϕ_0 and γ; as the perturbation on the flux is increased, amplitude of the divergent oscillations also increase. However in linearly stable region, system returns to its equilibrium condition more rapidly as the perturbation is increased although it becomes unstable for large enough disturbances. Yet, as the temperature reactivity coefficient is increased for a fixed value of the flux and the perturbation, the system returns to its equilibrium condition more rapidly. In other words, allowable limits for perturbations, in order not to destabilize the system, is increased in linearly stable region as the temperature reactivity coefficient is increased.

Computer calculations have shown that the Xenon problem with a prompt temperature reactivity coefficient posseses solutions that are asymptotically stable for bmall disturbances and depart from equilibrium when the disturbance is large enough. The parameter regions that exhibit this type of behaviour are near the boundary separating linearly stable and unstable regions.

It is interesting to note that, if one wants to represent the system in more detail, adding some more equations, e.g., for delayed neutrons, then Routh-Hurwitz conditions provide other degrees of freedom for escape from equilibrium. Hence the conditions tend to narrow the stability regions.

So it is realistic to consider the delayed neutrons to be produced " instantaneously " with respect to Xenon since time decay constants of delayed neutrons are much shorter than those of Xe^{135} and I^{135}.

If a linar reactor system is stable when the delayed neutrons are neglected, it is not necessarily stable if the delayed neutrons are included in the model. Stability regions can be enlarged in some parts while being narrowed in others.

It is realistic to lump all temperature feedback effects in a prompt reactivity coefficient, γ , since before enough Xenon is produced through decay from fission products to materially affect stability, the power generated by fission has time to be completely transferred to the coolant and the structural elements.

It has been shown that Xenon instability remains a serious concern in the presence of temperature damping. At flux levels above $\approx 1 \times 10^{13}$ n/(cm^2.sec.), the destabilizing factor is that of Xenon burnup. It is clear that Xenon instability is not a control problem for the large number of low power density research reactors which is operated at maximum flux levels below 1×10^{13} n/(cm^2.sec.), since their temperature reactivity coefficients are generally negative and sufficiently large for the reactor to be inherently stable against Xenon. On the other hand, economic considerations are deriving power reactor design in the direction of high-power density and hence efficient cooling, even for water moderated reactors with relatively large and negative temperature or void coefficients.

There are several directions in which the reactor designer can proceed : (1) by heavy fuel loading and poisoning of the reactor core which produces lower flux and long fuel burnup times but also high inventories and generally lower conversion ratios, (2) by increasing the reactor temperature reactivity coefficient sufficiently for inherent stability, (3) by adequate instrumentation and independent mechanical control of subdivisions of an inherently unstable reactor. However the latter would not be licenced in the current practice.

Finally, it has been shown that the simple theoretical model which neglects time lags in production of delayed neutrons and the time lag between flux and temperature is generally adequate and is recommended as a starting point in the investigation of more complex problems. Some of the limitations of the linearized equations have also been noted.

As extension of our study, long-term observation can be obtained with the same program, solving the point kinetics equations by linearizing them at each time step by repeated runs so as to synthesize solutions for sufficiently long periods of time (of the order of 100 hrs.) to allow the Xe oscillations effectively come into the picture.

Careful attention must be given to the calculation of parameters such as delayed neutron fraction, β , average decay constant of delayed neutron precursors, λ , and neutron generation time, ℓ , etc. It may be necessary or more accurate for these parameters to be calculated at each time step when the point kinetics equations are solved successively.

As a future work, the equations describing the system behaviour may be solved by modal separation according to their different time constants in different time intervals. For example, the equations for prompt and delayed neutrons are solved in fraction of a second time intervals for the first hour. At the end of the first hour we obtain the new values of flux, Xenon and Iodine concentrations. The equations representing the Xenon and Iodine feedback can be solved in time intervals of hours assuming that flux behaves promptly relative to Iodine and Xenon behaviour.

The effect of temperature with time delay may be introduced into this stability analysis, rather than treating the temperature feedback as being prompt. This can be accomplished by replacing $-\gamma P$ term with $-\alpha T$ and assuming the temperature to be related to the power through a Newton's Law of Cooling or another model. Thus point kinetics equation for prompt neutrons would have been replaced by the following two equations :

$$\frac{\ell}{\beta}\frac{dP(t)}{dt} = \left[\delta_o - \frac{\sigma_x Xe(t)}{c\,\sigma_f\,\beta} - \alpha T(t) \right] P(t) \qquad (6.1)$$

$$\frac{dT(t)}{dt} = \lambda_T \left[\frac{\gamma}{\alpha} P(t) - T(t) \right] \qquad (6.2)$$

where α is the temperature reactivity coefficient and λ_T the time delay constant for the temperature at zero power.

The time delay constant has to be chosen in this specific form so that, in the limit $\lambda_T \rightarrow \alpha$, we return to the prompt feedback model. We expect to observe that the longer the time delay, the more unstable the system is. The " temperature " T may be identified with fuel, moderator or coolant temperatures, steam void, volume, etc., depending on the variable which governs the reactivity.

Since the linear feedback model is only an idealization, it is also desirable to extend the theory of reactor stability to include the non-linearities in the feedback, and to obtain general stability criteria for temperature and Xenon controlled nuclear reactors. Clearly the time behaviour of a reactor can be described more realistically with a non-linear feedback model which contains the linear model as a special case.

It is clear that painless resolution of the Xenon stability problem will not be found and that satisfactory control will vary with the reactor type and purpose. That is unless the existing mathematical tools are enriched to the extent of allowing analytical or semi-analytical solutions to such non-linear systems of equations.

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APPENDICES

APPENDIX - I

ADJOINT OPERATOR

Adjoint operator $\mathcal{H}_o^+$ is defined to be the " adjoint " of $\mathcal{H}_o$ if,

$$\langle \mathcal{H}_o^+ N_o^+ | N_o \rangle = \langle N_o^+ | \mathcal{H}_o N_o \rangle \tag{1}$$

holds for any $N_o(\underline{r}, u, \underline{\Omega})$ and $N_o^+(\underline{r}, u, \underline{\Omega})$.

Note that the boundary conditions to be satisfied by $N_o^+(\underline{r}, u, \underline{\Omega})$ may have to be different from those of $N_o(\underline{r}, u, \underline{\Omega})$ and the former are referred to as the " adjoint boundary conditions ".

Using this definition we can derive the adjoint of the operator $\mathcal{H}_o$ as follows :

Consider the functions $\phi(\underline{r}, \underline{v})$ that satisfy the proper boundary condition, namely $\phi(\underline{r}, \underline{v}) = 0$ for $\hat{n}.\underline{v} < 0$ where $\underline{r}$ is on the outer surface of the reactor

a) Adjoint of the differential operator : $\underline{\Omega} \cdot \nabla$

$$\phi^+ | L\, \phi = \iint \phi^+ \underline{\Omega} \cdot \nabla\phi \; d^3v \; d^3r = \int_0^\infty v^2 \, dv \int_{\underline{\Omega}} d\underline{\Omega} \int_R d^3r \; \phi^+ \underline{\Omega} \cdot \nabla\phi$$

we can change the order of integration

$$= \int_0^\infty dv \, . \, v^2 \int_{\underline{\Omega}} d\underline{\Omega} \, . \, \underline{\Omega} \int_R d^3r \; \phi^+ \cdot \nabla\phi$$

using $$\phi^{+} \cdot \nabla \phi = \vec{\nabla} \cdot (\phi^{+} \phi) - \phi \cdot \nabla \phi^{+}$$

and by Green's theorem

$$\int_R d^3r \, \vec{\nabla} \cdot (\phi^{+} \phi) = \int_s d\vec{s} \cdot (\hat{n} \, \phi^{+} \phi)$$

where s is the outer surface of the reactor

$$= \int_0^\infty dv \, v^2 \int_{\underline{\Omega}} d\underline{\Omega} \, . \, \underline{\Omega} \left[\int_s d\vec{s} \cdot (\hat{n} \, \phi^{+} \phi) - \int_R d^3r \, \phi \cdot \nabla \phi^{+} \right]$$

If we choose the boundary conditions as

$$\phi \, (\underline{r},\underline{v}) = 0 \qquad \text{for} \qquad \hat{n} . \, \underline{v} < 0 \qquad \text{and}$$

$$\phi^{+} \, (\underline{r},\underline{v}) = 0 \qquad \text{for} \qquad \hat{n} . \, \underline{v} > 0 \quad , \qquad \underline{r} \in s$$

The surface integral vanishes because either ϕ or ϕ^{+} will always be zero on the surface. Then

$$= - \int_0^\infty dv \, v^2 \int_{\underline{\Omega}} d\Omega \int_R d^3r \, \phi \, \underline{\Omega} \cdot \nabla \phi^{+}$$

$$= \left\langle - \underline{\Omega} \cdot \nabla \phi^{+} \middle| \, \phi \right\rangle$$

thus the adjoint of $\underline{\Omega} \cdot \nabla$ is $- \underline{\Omega} \cdot \nabla$.

b) Adjoint of the integral operator :

$$\left\langle \phi^+ \middle| L\,\phi \right\rangle = \int_R d^3\underline{r} \int_{\underline{\Omega}} d\underline{\Omega} \int_0^\infty du\, \phi^+(\underline{r},u,\underline{\Omega}) \int_{\underline{\Omega}'} d\underline{\Omega}' \int_0^\infty du' \left[v \Sigma_s(\underline{r},u' \to u, \underline{\Omega}'.\underline{\Omega}) \right] \phi(\underline{r},u',\underline{\Omega}')$$

If the order of integration over u and $\underline{\Omega}$ is interchanged with u' and $\underline{\Omega}'$ the right-hand side becomes

$$= \int_R d^3\underline{r} \int_{\underline{\Omega}} d\underline{\Omega} \int_0^\infty du\, \phi(\underline{r},u,\underline{\Omega}) \int_{\underline{\Omega}'} d\underline{\Omega}' \int_0^\infty du' \left[v \Sigma_s(\underline{r},u \to u', \underline{\Omega}.\underline{\Omega}') \right] \phi^+(\underline{r},u',\underline{\Omega})$$

$$= \left\langle L^+ \phi^+ \middle| \phi \right\rangle$$

Thus the adjoint of $\int du' \int d\underline{\Omega}'\ v(u')\, \Sigma_s(\underline{r},u' \to u\ , \underline{\Omega}'.\underline{\Omega})$

is $\int_0^\infty du' \int_{\underline{\Omega}} d\underline{\Omega}'\ v(u)\, \Sigma_s(\underline{r},\ u \to u', \underline{\Omega}.\underline{\Omega}')$

It is clear from this example that the adjoint of

$$\int du' \int d\underline{\Omega}'\ v(u') \left[f(u)/4\pi \right]\ v(u')\Sigma_f(\underline{r},u')$$

is

$$\int du' \int d\underline{\Omega}'\ v(u\ ) \left[f(u')/4\pi \right]\ v(u)\Sigma_f(\underline{r},u)$$

Hence the adjoint of the operator $\mathcal{H}_o$ is

$$\mathcal{H}_o^+ \equiv \underline{\Omega} \cdot \nabla v(u) - \Sigma(\underline{r},u) v(u) + \int_0^\infty du' \int_{\underline{\Omega}} d\Omega' \Big\{ \Sigma_s(\underline{r}, u \to u', \underline{\Omega} \cdot \underline{\Omega}')$$

$$+ \sum_j \left[f^j(u') \,/\, 4\pi \right] \nu^j(u) \; \Sigma_f^j(\underline{r},u) \Big\} v(u) \qquad (2)$$

Using $\mathcal{H}_o^+[N_o]$, we define the adjoint angular density as the solution of

$$\mathcal{H}_o^+[N_o^+] \; N_o^+ = 0 \qquad (3)$$

with the adjoint boundary condition

$$N_o^+(\underline{r}, u, \underline{\Omega}) = 0 \qquad \text{for} \qquad \hat{n} \cdot \underline{\Omega} > 0, \qquad r \in s \; .$$

APPENDIX - II

NEUTRON IMPORTANCE

Suppose a neutron is injected into a critical reactor at $t=0$ at the space point $\underline{r}'$ with a velocity $\underline{v}'$, and assume that there are no neutrons in the reactor prior to $t=0$. We want to determine the time defendent angular density $n(\underline{r},u,\underline{\Omega},t)$ as a function of $\underline{r}$ and $\underline{v}$ for all subsequent times, and in particular as $t \to \infty$. For the time being we ignore the delayed neutrons for the sake of simplicity. Then $n(\underline{r},u,\underline{\Omega},t)$ satisfies

$$\frac{\partial n}{\partial t} = H\,n \qquad (1)$$

with the initial condition

$$n(\underline{r},u,\underline{\Omega},0) = \delta(\underline{r} - \underline{r}')\,\delta(u - u')\,\delta(\underline{\Omega} - \underline{\Omega}') \qquad (2)$$

In order to solve eq.(1), suppose it is possible to find the eigenfunctions of the operator H by solving the following equation.

$$H\,\phi_n = w_n \phi_n \qquad (3)$$

with the regular boundary conditions.

Since the Boltzmann operator is not self-adjoint we have to consider the adjoint eigenvalue problem also, i.e.,

$$H^{+} \phi_n^{+} = w_n^{*} \phi_n^{+} \tag{4}$$

so that $\{\phi_n\}$ and $\{\phi_n^{+}\}$ will form a complete biorthonormal set. Then we can expand the time-dependent angular density $n(\underline{r},u,\underline{\Omega},t)$ in the functions $\phi(\underline{r},u,\underline{\Omega})$ as

$$n(\underline{r},u,\underline{\Omega},t) = \sum_{n=0}^{\infty} a_n(\underline{r}',u',\underline{\Omega}',t)\, \phi_n(\underline{r},u,\underline{\Omega}) \tag{5}$$

where the expansion coefficients are of course given by

$$a_n = \langle \phi_n^{+} | n \rangle$$

Substituting eq.(5) into eq.(1) and using eq.(3) we obtain

$$\frac{\partial n}{\partial t} = \sum_{n=0}^{\infty} a_n\, w_n\, \phi_n$$

$$a_n(\underline{r}',u',\underline{\Omega}',t) = a_n(\underline{r}',u',\underline{\Omega}',0)\, e^{w_n t} \tag{6}$$

the initial values $a(\underline{r}',u',\underline{\Omega},0)$ must be determined by the initial condition on $n(\underline{r},u,\underline{\Omega},t)$

$$\delta(\underline{r}-\underline{r}')\,\delta(u-u')\,\delta(\underline{\Omega}-\underline{\Omega}') = \sum_{n=0}^{\infty} a_n(\underline{r}',u',\underline{\Omega}',0)\, \phi_n(\underline{r},u,\underline{\Omega})$$

multiplying both sides by $\phi_n^+(\underline{r},u,\underline{\Omega})$ and forming scaler products, we get $a_n(\underline{r}',u',\underline{\Omega}',0) = \phi_n^+(\underline{r}',u',\underline{\Omega}')$. Thus

$$n(\underline{r},u,\underline{\Omega},t) = \sum_{n=0}^{\infty} \phi_n^+(\underline{r}',u',\underline{\Omega})\, \phi_n(\underline{r},u,\underline{\Omega})\; e^{w_n t} \tag{7}$$

This equality follows from the fact that the reactor is critical, and hence $H\phi_o = 0$ has a unique nontrivial solution. It is also clear that the eigenfunction ϕ_o corresponds to $w_o = 0$ is the steady-state angular density $N_o(\underline{r},u,\underline{\Omega})$. Thus the coefficients of all the higher modes in eq.(7) decay exponentially in time, and asymptotic angular density is obtained as,

$$n_\infty(\underline{r}',u',\underline{\Omega}';\underline{r},u,\underline{\Omega}) = N_o^+(\underline{r}',u',\underline{\Omega}')\, N_o(\underline{r},u,\underline{\Omega}) \tag{8}$$

where we have shown the dependence of n_∞ on $\underline{r}',u',\underline{\Omega}'$ explicitly.

The " importance " of a neutron injected into a critical reactor at $\underline{r}'$ with a lethargy u' in the direction of $\underline{\Omega}'$ is the total number of fissions per second in the entire reactor at a long time following the injection of the neutron at $t=0$.

The importance function is readily obtained from eq.(8) by multiplying both sides by $\Sigma_f(\underline{r},u)\,\nu(u)$ and integrating over $\underline{r}$ and $\underline{v}$:

$$I(\underline{r}',u',\underline{\Omega}') = N_o^+(\underline{r}',u',\underline{\Omega}')\left\langle \nu\Sigma_f \,\middle|\, N_o\right\rangle$$

It is concluded from this result that the adjoint angular density $N_o^+(\underline{r}',u',\underline{\Omega}')$ is proportional to the importance of neutrons at $\underline{r}'$ moving with a lethargy u' in the direction of $\underline{\Omega}'$ in the reactor.

APPENDIX III

THE INHOUR EQUATION

Recall that the inhour equation

$$\left| \underline{\underline{A}} - s\,\underline{\underline{I}} \right| = 0$$

where matrix $\underline{\underline{A}}$ was defined before.

$$\left| \underline{\underline{A}} - s\,\underline{\underline{I}} \right| = \begin{vmatrix} a_{11}-s & a_{12} & 0 & a_{13} \\ a_{21} & a_{22}-s & a_{23} & 0 \\ a_{31} & 0 & a_{33}-s & 0 \\ a_{41} & 0 & 0 & a_{44}-s \end{vmatrix}$$

$$= s^4 - \left[a_{11} + a_{22} + a_{33} + a_{44} \right] s^3$$

$$+ \left[(a_{11} + a_{22})(a_{33} + a_{44}) - a_{11}a_{22} + a_{33}a_{44} - a_{12}a_{21} - a_{14}a_{41} \right] s^2$$

$$+ \left[(a_{33}+a_{44})(a_{12}a_{21}-a_{11}a_{22}) + a_{14}a_{41}(a_{22}+a_{33}) - (a_{11}+a_{22})a_{33}a_{44} \right.$$

$$\left. - a_{12}a_{31}a_{23} \right] s + a_{22}a_{33}(a_{11}a_{44}-a_{14}a_{41}) + a_{12}a_{44}(a_{31}a_{23}-a_{21}a_{33})$$

$$= 0$$

APPENDIX - IV

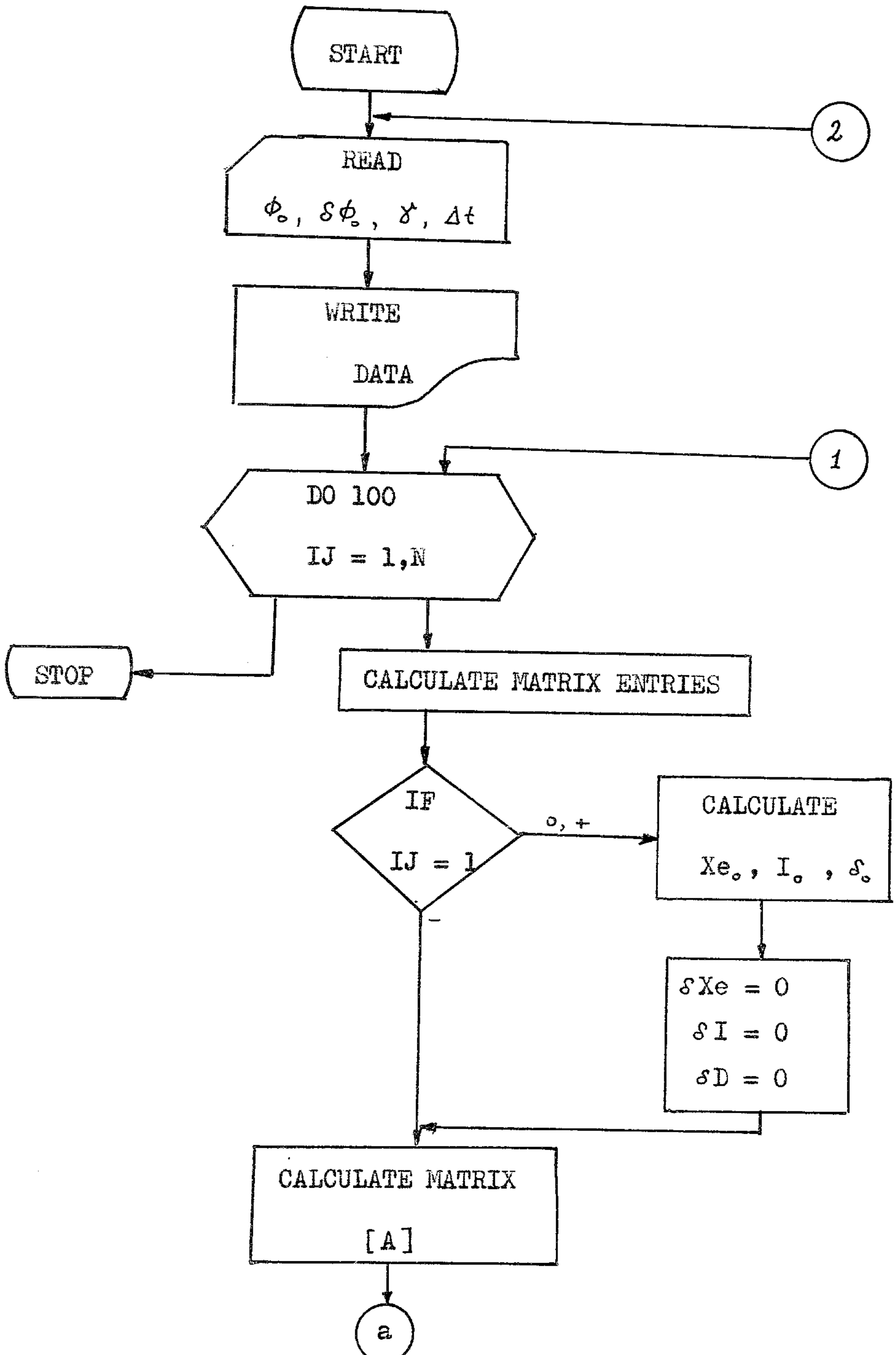
NOMENCLATURE

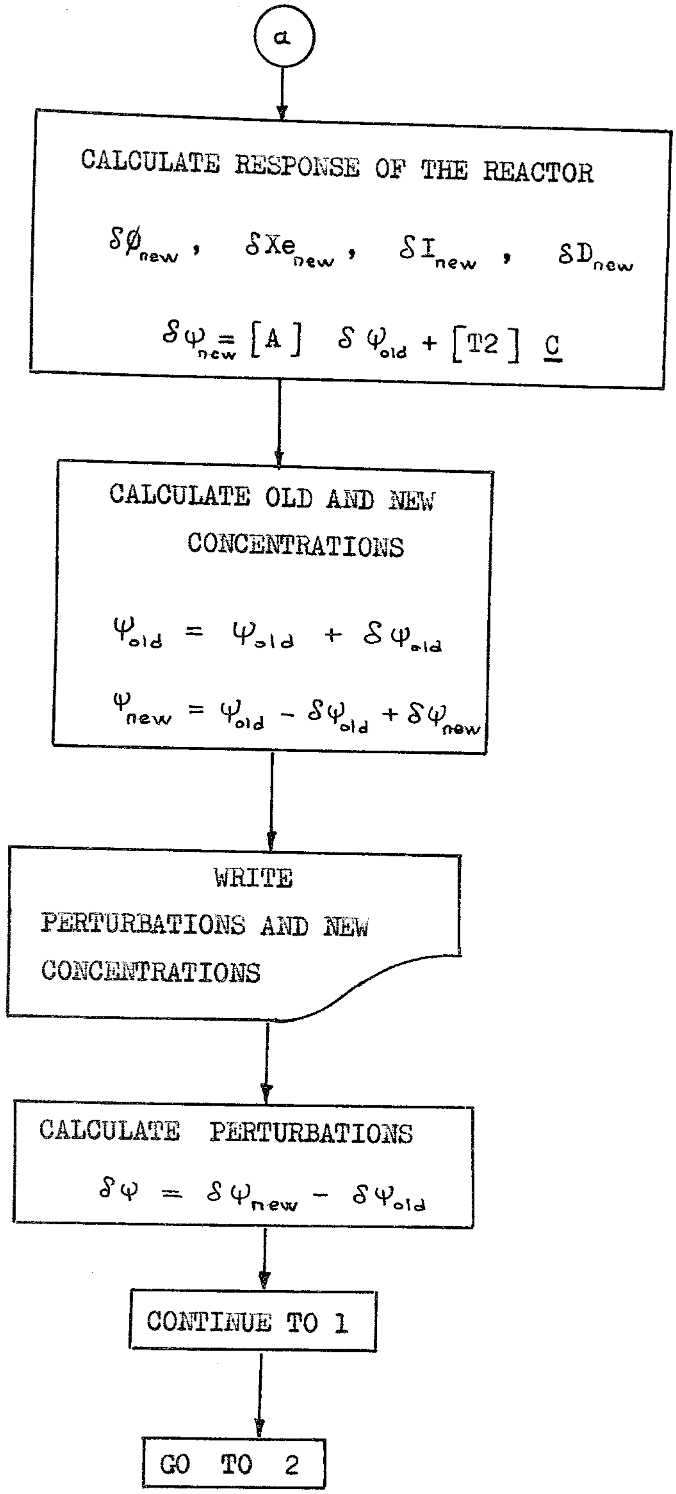
EQUATION	COMPUTER PROGRAM	MEANING
ℓ	L	Neutron generation time
λ_I	LI	Decay constant of I^{135}
λ_x	LX	Decay constant of Xe^{135}
λ	LAMDA	Average decay constant of Delayed Neutron precursors
ϕ_o	PHIO	Equilibrium value of flux
γ	GAMA	Temperature Reactivity Coefficient
$\delta\phi$	DPHIO	Perturbation to flux
Δt , h	TI	Time interval
y_I	YI	Iodine yield
y_x	YX	Xenon yield
y	Y	Total yield ($y_I + y_x$)
β	B	Delayed Neutron fraction

δ_o	DO	Initial Reactivity of the clean Reactor
σ_x	SIGX	Absorption cross section of Xe
σ_f	SIGF	Fission cross section
Xe_o	XEO	Equilibrium value of Xenon
I_o	IOO	Equilibrium value of Iodine
C	C	Delayed Neutron Precursor Concentration
c	CO	A factor converting the local Xenon absorption per fission to overall reactivity
δXe	DXEO	Increase in Xe concentration
δI	DIOO	Increase in Iodine concentration
δC	DCO	Increase in delayed Neutron Precursor concentration
C_o	COO	Equilibrium concentration of Delayed Neutron Precursors

APPENDIX - V

FLOW CHART OF THE PROGRAM

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CALCULATE RESPONSE OF THE REACTOR
$\delta\phi_{new}$, δXe_{new} , δI_{new} , δD_{new}
$\delta\psi_{new} = [A] \; \delta\psi_{old} + [T2] \; \underline{C}$
CALCULATE OLD AND NEW CONCENTRATIONS
$\psi_{old} = \psi_{old} + \delta\psi_{old}$
$\psi_{new} = \psi_{old} - \delta\psi_{old} + \delta\psi_{new}$
WRITE PERTURBATIONS AND NEW CONCENTRATIONS
CALCULATE PERTURBATIONS
$\delta\psi = \delta\psi_{new} - \delta\psi_{old}$
CONTINUE TO 1
GO TO 2

APPENDIX - VI

LISTING OF THE PROGRAMS

AND

NUMERICAL RESULTS

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42	[illegible]	[illegible]	[illegible]	[illegible]
43	[illegible]	[illegible]	[illegible]	[illegible]
44	[illegible]	[illegible]	[illegible]	[illegible]
45	[illegible]	[illegible]	[illegible]	[illegible]
46	[illegible]	[illegible]	[illegible]	[illegible]
47	[illegible]	[illegible]	[illegible]	[illegible]
48	[illegible]	[illegible]	[illegible]	[illegible]
49	[illegible]	[illegible]	[illegible]	[illegible]
50	[illegible]	[illegible]	[illegible]	[illegible]
51	[illegible]	[illegible]	[illegible]	[illegible]
52	[illegible]	[illegible]	[illegible]	[illegible]
53	[illegible]	[illegible]	[illegible]	[illegible]
54	[illegible]	[illegible]	[illegible]	[illegible]
55	[illegible]	[illegible]	[illegible]	[illegible]
56	[illegible]	[illegible]	[illegible]	[illegible]
57	[illegible]	[illegible]	[illegible]	[illegible]
58	[illegible]	[illegible]	[illegible]	[illegible]
59	[illegible]	[illegible]	[illegible]	[illegible]
60	[illegible]	[illegible]	[illegible]	[illegible]
61	[illegible]	[illegible]	[illegible]	[illegible]
62	[illegible]	[illegible]	[illegible]	[illegible]
63	[illegible]	[illegible]	[illegible]	[illegible]
64	[illegible]	[illegible]	[illegible]	[illegible]
65	[illegible]	[illegible]	[illegible]	[illegible]
66	[illegible]	[illegible]	[illegible]	[illegible]
67	[illegible]	[illegible]	[illegible]	[illegible]
68	[illegible]	[illegible]	[illegible]	[illegible]
69	[illegible]	[illegible]	[illegible]	[illegible]
70	[illegible]	[illegible]	[illegible]	[illegible]
71	[illegible]	[illegible]	[illegible]	[illegible]
72	[illegible]	[illegible]	[illegible]	[illegible]
73	[illegible]	[illegible]	[illegible]	[illegible]
74	[illegible]	[illegible]	[illegible]	[illegible]
75	[illegible]	[illegible]	[illegible]	[illegible]
76	[illegible]	[illegible]	[illegible]	[illegible]
77	[illegible]	[illegible]	[illegible]	[illegible]
78	[illegible]	[illegible]	[illegible]	[illegible]
79	[illegible]	[illegible]	[illegible]	[illegible]
80	[illegible]	[illegible]	[illegible]	[illegible]
81	[illegible]	[illegible]	[illegible]	[illegible]
82	[illegible]	[illegible]	[illegible]	[illegible]
83	[illegible]	[illegible]	[illegible]	[illegible]
84	[illegible]	[illegible]	[illegible]	[illegible]
85	[illegible]	[illegible]	[illegible]	[illegible]
86	[illegible]	[illegible]	[illegible]	[illegible]
87	[illegible]	[illegible]	[illegible]	[illegible]
88	[illegible]	[illegible]	[illegible]	[illegible]
89	[illegible]	[illegible]	[illegible]	[illegible]
90	[illegible]	[illegible]	[illegible]	[illegible]
91	[illegible]	[illegible]	[illegible]	[illegible]
92	[illegible]	[illegible]	[illegible]	[illegible]
93	[illegible]	[illegible]	[illegible]	[illegible]
94	[illegible]	[illegible]	[illegible]	[illegible]
95	[illegible]	[illegible]	[illegible]	[illegible]
96	[illegible]	[illegible]	[illegible]	[illegible]
97	[illegible]	[illegible]	[illegible]	[illegible]
98	[illegible]	[illegible]	[illegible]	[illegible]
99	[illegible]	[illegible]	[illegible]	[illegible]

100	.945+012	-.603+006	.155+00[illegible]	.2[illegible]
101	.945+012	-.665+006	.155+[illegible]	[illegible]
102	.948+.12	-.677+006	.155+00[illegible]	.2[illegible]
103	.950+012	-.686+006	.155+00[illegible]	.2[illegible]
104	.951+012	-.691+006	.155+[illegible]	.2[illegible]
105	.951+012	-.692+006	.155+00[illegible]	.2[illegible]
106	.952+012	-.692+006	.155+[illegible]	.2[illegible]
107	.957+012	-.696+006	.155+00[illegible]	.2[illegible]
108	.957+012	-.697+006	.154+[illegible]	.2[illegible]
109	.957+012	-.698+006	.155+[illegible]	.2[illegible]
110	.957+012	-.700+006	.155+[illegible]	.2[illegible]
111	.957+012	-.702+006	.155+[illegible]	.2[illegible]
112	.956+012	-.704+006	.155+[illegible]	.2[illegible]
113	.956+012	-.705+006	.155+[illegible]	.2[illegible]
114	.958+012	-.707+006	.155+[illegible]	.2[illegible]
115	.957+012	-.708+006	.155+[illegible]	.2[illegible]
116	.955+012	-.710+006	.155+[illegible]	.2[illegible]
117	.955+012	-.711+006	.155+[illegible]	.2[illegible]
118	.955+012	-.713+006	.155+[illegible]	.2[illegible]
119	.955+012	-.712+006	.155+[illegible]	.2[illegible]
120	.957+012	-.716+006	.155+[illegible]	.2[illegible]

```
.. TAHER      ACCT: [illegible]-17-2[illegible]      PROJECT: [illegible]
:     TOTAL: [illegible]            CUSUPS: [illegible]
      CPU:   [illegible]            I/O:    [illegible]
      CC/EMS: [illegible].933       CARDS: [illegible]
 USED:      [illegible]     SU'S REMAINING:    [illegible]
L CHARGE CALCULATED AT FOLLOWING RATES -
SUP              = 0.[illegible]
SO READ          = 0.[illegible]
.D PUNCHED       = [illegible]
.L PRINTED       = [illegible]
.L I/O MINUTE    = 1.[illegible]
CS  RE/O:    13[illegible]         PAGES:   7
T:      10:42:2[illegible] [illegible]       T[illegible]: [illegible]
```

```
TTTTTTTTTT          AAAAAAAA
TTTTTTTTTT         AAAAAAAAAA
    TT             AA      AA
[illegible]
C 1106 ** [illegible] UNIVERSITAET [illegible] COMPUTER [illegible]

DER            USER ID *                    PART [illegible]         INPUT [illegible]

C * PRODUCTABLE                    CREATED AT: [illegible]

567890123456789012345678901234567890123456789012345678901234567890[illegible]
R,111-[illegible],HCOTROA,5,50   *TABLE [illegible]*        [illegible]

781-15[illegible]

D: CODE(1) 00[illegible]; DATA(0) 00[illegible]; BLANK COMMON(2) [illegible]

ERENCES (BLOCK, NAME)

TR*
[illegible]
[illegible]
[illegible]

OFS

IGNMENT (BLOCK, [illegible], [illegible] LOCATION, [illegible])

774  20F     [illegible]    000145 2[illegible]    [illegible]
771  23F     [illegible]    00023[illegible] 2576  [illegible]
256  2545    [illegible]    000257 2576    [illegible]
035  30F     [illegible]    000354 3[illegible]    [illegible]
532  61C     0001    000563 [illegible]    [illegible]
010  A1      0000 0 000056 AL    [illegible]
530  C0      0000 0 00[illegible]20 [illegible]    [illegible]
063  D1C0    [illegible] 0 000450 [illegible]    [illegible]
711  DAE[illegible]  0000 0 000061 [illegible]    [illegible]
002  10N     0000 0 [illegible]    [illegible]
005  LI      [illegible] 0 [illegible]    [illegible]
054  RK      [illegible] 0 000715 [illegible]    [illegible]
010  S       [illegible] 0 [illegible]    [illegible]
[illegible]  T1      [illegible] 0 [illegible]    [illegible]
067  ALO     000[illegible] 0 [illegible]    [illegible]

      C      [illegible]
      C      SOLUTION OF POINT-KINETIC EQUATIONS [illegible]
      C
```

```
C        WITHOUT DELAYED NEUTRONS , LINEARIZATION AT EACH TIME STEP,
C        ****************************************************************
         IMPLICIT DOUBLE PRECISION (A,B,C,D,E,F,G,L,P,R,S,T,U,X,Y,Z)
         DOUBLE PRECISION I00,I0M
         REAL L,LI,LX,LAMDA
         DIMENSION A(4,4),AL(4,4),AD(4,4),AU(4,4),B(4,4),T1(4,4),T2(4,4),
        *T3(4,4),A(4,4),DIV(4,4),D(4,4),[illegible](4,4),C(4),[illegible](4))
         DATA L,C0,LI,LX,YI,YX,SIGF,SIGX,B/[illegible],[illegible],[illegible],2.0E
        .05,5.E-03,2.D0-03,677.D0-24,3.D0-13,0.7E-[illegible]-12/
         WRITE(6,21)
    21   FORMAT(1H1,10X,'NUMERICAL SOLUTION OF [illegible] [illegible] [illegible] IN
        .MATRIX FORM',///,1H,25X,'WITHOUT DELAYED NEUTRONS',//)
         WRITE(6,22)
    22   FORMAT(//1H,10X,'L',10X,'C0',10X,'LI',10X,'[illegible]',10X,'YI',10X,'[illegible]',
        .10X,'SIGF',8X,'SIGX',10X,'B',//)
         WRITE(6,23)L,C0,LI,LX,YI,YX,SIGF,SIGX,B
    23   FORMAT(1H,5X,9(E10.3,4X))
    99   READ(5,20,END=[illegible])PHI0,GAMA,OMEGA,TI
    20   FORMAT(4E10.3)
         IF(TI.LE.0.0) GO TO 99
    88   WRITE(6,24)PHI0,GAMA,TI,OMEGA
    24   FORMAT(1H,//,10X,'PHI0=',E10.3,10X,'GAMA=',[illegible],[illegible],'TI=',[illegible]
        .3X,'OMEGA=',E10.3,//)
    50   WRITE(6,25)
    25   FORMAT(//,1X,[illegible],'TIME',12X,'PHI',10X,'OMEGA',[illegible],'X',[illegible],'I',10
        .X,'[illegible]',10X,'[illegible]',//)
         N=1
         T=TI*[illegible]
         [illegible]=0.0
         [illegible]=[illegible]*[illegible]*PHI0
         [illegible]=[illegible]
         I10=0.
         [illegible]=0.
         [illegible]=YI*SIGF*PHI0/[illegible]
         I00=YI*SIGF*PHI0/LI
         [illegible]=[illegible]*PHI0*SIGF*[illegible]/([illegible])
    11   [illegible]
         [illegible]
         [illegible]
         [illegible]
         DO 1 I=1,4
         DO 1 J=1,4
         A(I,J)=0.
         AL(I,J)=0.
         AD(I,J)=0.
         AU(I,J)=0.
         B(I,J)=0.
         T1(I,J)=0.
         T2(I,J)=0.
         T3(I,J)=0.
         [illegible](I,J)=0.
         C(I)=0.
         [illegible](I,J)=0.
     1   CONTINUE
C        ****************************************************************
C        CALCULATION OF MATRIX
C        ****************************************************************
    10   AL(2,1)=0.
         AL(3,1)=[illegible]*SIGF
         [illegible](3,2)=[illegible]/T
         [illegible](2,3)=[illegible]
```

```
      AD(1,1)=(DJ*U/(CC*SIGF)*2.*[illegible]*PHI[illegible])/L
      AD(2,2)=-Y1
      AD(3,3)=-1
      DO 2 K=1,3
    2 A(K,K)=1.
      DO 3 M=1,3
      DIV(M,M)=1./AD(M,M)
      DH(M,M)=AD(M,M)*T1
    7 ZAD(M,M)=EXP(DH(M,M))
    3 [illegible](M,M)=ZAD(M,M)-A1(M,M)
      DO 4 I=1,3
      DO 4 J=1,3
    4 S(I,J)=[illegible](I,J)*AD(I,J)
      DO 5 I=1,3
    5 T2(M,N)=[illegible](K,N)*DIV(M,N)*T1(M,N)
      DO 6 [illegible]=1,3
      DO 6 [illegible]=1,3
      DO 6 [illegible]=1,3
    6 T3(M,L)=T3(M,L)+T2(M,K)*S(K,L)
      DO 7 [illegible]=1,3
      DO 7 [illegible]=1,3
    7 [illegible](M,N)=[illegible](M,K)*T3(K,N)
      C(1)=(DU*U/(CC*SIGF)-GAM*PHI0)/L [illegible]
      C(2)=[illegible]*PHI[illegible]-LX*A[illegible]+L[illegible].00
      C(3)=(1.*SIGF*PHI[illegible])*100
      [illegible](1,1)=T2(1,1)*C(1)
      [illegible](2,1)=T2(2,2)*C(2)
      [illegible](3,1)=T2(3,3)*C(3)
      PHI[illegible]=[illegible]*DPHI0
      [illegible]=A[illegible]+P[illegible]
      [illegible]=[illegible]+D[illegible]
      DO 100 [illegible]=1,36000
   51 [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      IF ([illegible]) GO TO 61
   60 PRINT [illegible]
   30 FORMAT ([illegible],5X,I3,10X,[illegible])
      [illegible]
   61 [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible] GO TO [illegible]
  100 CONTINUE
      [illegible]
   90 STOP
      END

COMPILATION [illegible] DIAGNOSTICS.
```

NUMERICAL SOLUTION OF POINT-KINETIC EQUATIONS IN [illegible]

WITHOUT DELAYED NEUTRONS

L	CO	L1	LX	Y1	[illegible]	[illegible]
.100+00	.150+01	.267-04	.209-04	.62[illegible]	[illegible]-02	.57[illegible]

PHI= .100+012 GAMA= .120-014 [illegible]= [illegible] [illegible]

TIME	PHI	PPHI	I	[illegible]
1	.971+013	.965+013	.17[illegible]	[illegible]
2	.941+013	.931+013	.176-004	[illegible]
3	.901+013	.89[illegible]+013	[illegible]	[illegible]
4	.87[illegible]+013	.866+013	[illegible]	[illegible]
5	.8[illegible]+013	.835+013	[illegible]	[illegible]
6	.816+013	.806+013	.176[illegible]	[illegible]
7	.787+013	.777+013	[illegible]	[illegible]
8	.760+013	.750+013	[illegible]	[illegible]
9	.733+013	.723+013	.17[illegible]	[illegible]
10	.707+013	.69[illegible]+013	[illegible]	[illegible]
11	.683+013	.673+013	[illegible]	[illegible]
12	.65[illegible]+013	.649+013	[illegible]	[illegible]
13	.63[illegible]+013	.62[illegible]+013	[illegible]	[illegible]
14	.6[illegible]+013	.60[illegible]+013	.177[illegible]	[illegible]
15	.5[illegible]+013	.583+013	[illegible]	[illegible]
16	.5[illegible]+013	.562+013	.177[illegible]	[illegible]
17	.5[illegible]+013	.542+013	[illegible]	[illegible]
18	.52[illegible]+013	.523+013	[illegible]	[illegible]
19	.5[illegible]+013	.504+013	[illegible]	[illegible]
20	.4[illegible]+013	.486+013	[illegible]	[illegible]
21	.4[illegible]+013	.46[illegible]+013	[illegible]	[illegible]
22	.4[illegible]+013	.45[illegible]+013	[illegible]	[illegible]
23	.4[illegible]+013	.436+013	[illegible]	[illegible]
24	.4[illegible]+013	.421+013	[illegible]	[illegible]
25	.4[illegible]+013	.406+013	[illegible]	[illegible]
26	.3[illegible]+013	.392+013	[illegible]	[illegible]
27	.3[illegible]+013	.37[illegible]+013	[illegible]	[illegible]
28	.3[illegible]+013	.36[illegible]+013	[illegible]	[illegible]
29	.3[illegible]+013	.35[illegible]+013	[illegible]	[illegible]
30	.3[illegible]+013	.33[illegible]+013	[illegible]	[illegible]
31	.3[illegible]+013	.32[illegible]+013	[illegible]	[illegible]
32	.3[illegible]+013	[illegible]	.10[illegible]	[illegible]
33	[illegible]	.306+013	[illegible]	[illegible]
34	.30[illegible]+013	.293+013	[illegible]	[illegible]
35	.29[illegible]+013	.283+013	.102[illegible]	[illegible]
36	.2[illegible]+013	.273+013	.102[illegible]	[illegible]
37	.2[illegible]+013	.263+013	.102[illegible]	[illegible]
38	.2[illegible]+013	.25[illegible]+013	.102[illegible]	[illegible]

39	.255+013	.245+013	.163-008	[illegible]
40	[illegible]	.236+013	.165-008	[illegible]
41	.237+013	.227+013	[illegible]	[illegible]
42	[illegible]	.219+013	[illegible]	[illegible]
43	[illegible]	.211+013	[illegible]	[illegible]
44	[illegible]	.204+013	[illegible]	[illegible]
45	[illegible]	.196+013	[illegible]	[illegible]
46	[illegible]	.189+013	[illegible]	[illegible]
47	[illegible]	.183+013	[illegible]	[illegible]
48	[illegible]	.176+013	[illegible]	[illegible]
49	[illegible]	.170+013	[illegible]	[illegible]
50	[illegible]	.164+013	[illegible]	[illegible]
51	[illegible]	.158+013	[illegible]	[illegible]
52	.162+013	.152+013	[illegible]	[illegible]
53	[illegible]	.147+013	[illegible]	[illegible]
54	[illegible]	.141+013	[illegible]	[illegible]
55	[illegible]	.136+013	[illegible]	[illegible]
56	[illegible]	.131+013	[illegible]	[illegible]
57	[illegible]	.126+013	[illegible]	[illegible]
58	[illegible]	.122+013	[illegible]	[illegible]
59	.127+013	.117+013	[illegible]	[illegible]
60	[illegible]	.113+013	[illegible]	[illegible]
61	[illegible]	[illegible]	[illegible]	[illegible]
62	[illegible]	.105+013	[illegible]	[illegible]
63	[illegible]	[illegible]	[illegible]	[illegible]
64	.107+013	[illegible]	[illegible]	[illegible]
65	[illegible]	[illegible]	[illegible]	[illegible]
66	[illegible]	[illegible]	[illegible]	[illegible]
67	[illegible]	.870+012	[illegible]	[illegible]
68	[illegible]	[illegible]	[illegible]	[illegible]
69	[illegible]	[illegible]	[illegible]	[illegible]
70	[illegible]	[illegible]	[illegible]	[illegible]
71	[illegible]	.747+012	[illegible]	[illegible]
72	[illegible]	[illegible]	[illegible]	[illegible]
73	[illegible]	.692+012	[illegible]	[illegible]
74	[illegible]	[illegible]	[illegible]	[illegible]
75	[illegible]	[illegible]	[illegible]	[illegible]
76	[illegible]	[illegible]	[illegible]	[illegible]
77	[illegible]	.593+012	[illegible]	[illegible]
78	[illegible]	[illegible]	[illegible]	[illegible]
79	[illegible]	.548+012	[illegible]	[illegible]
80	[illegible]	.527+012	[illegible]	[illegible]
81	[illegible]	[illegible]	[illegible]	[illegible]
82	[illegible]	.487+012	[illegible]	[illegible]
83	[illegible]	[illegible]	[illegible]	[illegible]
84	[illegible]	[illegible]	[illegible]	[illegible]
85	[illegible]	[illegible]	[illegible]	[illegible]
86	[illegible]	[illegible]	[illegible]	[illegible]
87	[illegible]	[illegible]	[illegible]	[illegible]
88	[illegible]	[illegible]	[illegible]	[illegible]
89	[illegible]	[illegible]	[illegible]	[illegible]
90	[illegible]	[illegible]	[illegible]	[illegible]
91	[illegible]	[illegible]	[illegible]	[illegible]
92	[illegible]	[illegible]	[illegible]	[illegible]
93	[illegible]	[illegible]	[illegible]	[illegible]
94	[illegible]	[illegible]	[illegible]	[illegible]
95	[illegible]	[illegible]	[illegible]	[illegible]
96	[illegible]	[illegible]	[illegible]	[illegible]
97	[illegible]	[illegible]	[illegible]	[illegible]
98	[illegible]	[illegible]	[illegible]	[illegible]
99	[illegible]	[illegible]	[illegible]	[illegible]

100	.326+012	.226+012	.157-00[illegible]	.22[illegible]-007
101	.316+012	.216+012	.197-[illegible]	.22[illegible]-007
102	.306+012	.206+012	.157-[illegible]	.22[illegible]-007
103	.2[illegible]6+012	.196+012	.[illegible]7-[illegible]	.231-007
104	.[illegible]7+012	.187+012	.1[illegible]-[illegible]6	.233-007
105	.2[illegible]+012	.178+012	.19[illegible]-[illegible]	.23[illegible]-007
106	.2[illegible]+012	.169+012	.[illegible]-[illegible]	.237-007
107	.261+012	.161+012	.1[illegible]-00[illegible]	.2[illegible]0-007
108	.253+01[illegible]	.153+012	.1[illegible]-00[illegible]	.2[illegible]7-007
109	.245+012	.145+012	.1[illegible]-[illegible]	.2[illegible]-007
110	.2[illegible]7+012	.137+012	[illegible]	.2[illegible]6-007
111	.230+012	.130+012	[illegible]	.2[illegible]-007
112	.22[illegible]+012	.123+012	[illegible]	.2[illegible]1-007
113	.21[illegible]+012	.116+012	[illegible]	.2[illegible]-007
114	.20[illegible]+012	.109+012	[illegible]	.2[illegible]-007
115	.2[illegible]+012	.103+012	[illegible]	.2[illegible]-007
116	.1[illegible]+012	.963+011	[illegible]	.2[illegible]-007
117	.19[illegible]+012	.903+011	[illegible]	.2[illegible]-007
118	.184+012	.849+011	.2[illegible]	.2[illegible]-007
119	.17[illegible]+012	.7[illegible]7+011	.2[illegible]	.2[illegible]-007
120	.1[illegible]+012	.7[illegible]2+011	.2[illegible]	.2[illegible]-007

```
TTTTTTTTTTTT        AAAAAAAA        MM          MM    EEEEEEEEEEE
TTTTTTTTTTTT       AAAAAAAAAA       MMM        MMM    EEEEEEEEEEE
     TT           AA        AA      MMMM      MMMM    EE
     TT           AA        AA      MMMMM    MMMMM    EE
     TT           AA        AA      MM MMMMMMM  MM    EE
     TT           AAAAAAAAAAAA      MM  MMMM    MM    EEEEEEEE
     TT           AAAAAAAAAAAA      MM   MM     MM    EEEEEEEE
     TT           AA        AA      MM          MM    EE
     TT           AA        AA      MM          MM    EE
     TT           AA        AA      MM          MM    EE
     TT           AA        AA      MM          MM    EEEEEEEEEEE
     TT           AA        AA      MM          MM    EEEEEEEEEEE
VAC 1106 -- BOGAZICI  UNIVERSITESI  KOMPUTER  MERKEZI --ISTANBUL  VER. 33R3/

TAMER          USER ID *                    PART NUMBER * 00      INPUT DEVICE *

AME * PR0000TAMER                      CREATED AT:    12:06:39 JUL 15,1980

3456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789
  TAMER,111-16-201,THESIS,5,50      .TAMER SENER.
POLY.
 RL72R1 07/15/80 12:06:41
Y.
LY.MAIN
15/80-12:06:49 (,0)

RAM

SED: CODE(1) 000407; DATA(0) 001152; BLANK COMMON(2) 000000


REFERENCES (BLOCK, NAME)

OLRT
INTR$
RDU$
IO2$
WDU$
IO1$
PRI
STOP$


SSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

00745 10F       0001   000403 100L     0001   000115 11L      0001   000271
00110 1746      0000   000776 200F     0000   001021 201F     0001   000126
00153 2146      0000   000750 30F      0000   001040 40F      0001   000002
01044 65F       0001   000322 70L      0000   001054 77F      0001   000334
00342 80L       0000   001075 85F      0000   001024 95F      0001   000347
00000 A         0000 R 000727 B        0000 R 000722 CO       0000 R 000743
00733 I         0000 I 000730 ID       0000 I 000744 IER      0000 R 000720
00732 ITAMER    0000 I 000734 J        0000 I 000736 K        0000 R 000715
00716 LI        0000 R 000717 LX       0000 R 000450 PHI      0000 D 000336
00725 SIGF      0000 R 000726 SIGX     0000 R 000741 T        0000 D 000112
00735 Y         0000 R 000723 YI       0000 R 000724 YX       0000 R 000737



*          DIMENSION A(37),W(37),ROOTR(37),ROOTI(37),PHI(10,15),G(15)
*          REAL L,LI,LX,IO,LAMDA
```

```
      DOUBLE PRECISION A,W,ROOTR,ROOTI
      DATA L,CO,LI,LX,YI,YX,SIGF,SIGX,B/1.0E-04,1.5,2.87E-05,2.09E-05,
     *6.2E-02,2.0E-03,577.0E-24,3.0E-18,7.5E-03/
    5 READ(5,10) ID,IORD,ITAMER
   10 FORMAT(1X,I4,3X,I2,I3)
      IF(ID+IORD)100,100,20
   20 WRITE(6,30) ID,IORD
   30 FORMAT(1H1,10X,'REAL AND COMPLEX ROOTS OF A POLYNOMIAL USING SUBR
     1UTINE POLRT',///,10X,'FOR POLYNOMIAL   ',I4,2X,'OF ORDER  ',I2,//
      WRITE(6,200)
  200 FORMAT(//1H,10X,'L',10X,'CO',10X,'LI',10X,'LX',10X,'YI',10X,'YX',
     *10X,'SIGF',8X,'SIGX',10X,'B',//)
      WRITE(6,201)L,CO,LI,LX,YI,YX,SIGF,SIGX,B
  201 FORMAT(1H,6X,9(E8.3,4X))
      WRITE(6,95)
   95 FORMAT(//1H,2X,'I',4X,'J',3X,4('REAL ROOT',2X,'IMAGINARY ROOT',2X
     *//)
      IF(ITAMER)9,9,11
    9 READ(5,40)(A(I),I=1,J)
   40 FORMAT(4E18.9)
      J=IORD+1
      WRITE(6,50)(A(I),I=1,J)
   50 FORMAT(6E16.7)
      GO TO 12
   11 Y=YI+YX
      LAMDA=LI+LX
      DO 2 K=1,15
      G(K)=K*1.0E-16
      DO 2 J=8,15
      DO 2 I=2,10,2
      PHI(I,J)=I*10.**J
      Z=LAMDA+SIGX*PHI(I,J)
      ZX=LX+SIGX*PHI(I,J)
      T=SIGX*PHI(I,J)/(CO*SIGF*L)
      IO=YI*SIGF*PHI(I,J)/LI
      XE=Y*SIGF*PHI(I,J)/ZX
      E=G(K)*PHI(I,J)/L
      A(1)=T*LI*(SIGF*Y-SIGX*XE)+E*LI*ZX
      A(2)=ZX*(LI+E)+LI*E+T*(YX*SIGF-SIGX*XE)
      A(3)=Z+E
      A(4)=1.
   12 CALL POLRT(A,W,IORD,ROOTR,ROOTI,IER)
      IF(IER-1)96,60,70
   60 WRITE(6,65)
   65 FORMAT(//1H,10X,'ORDER OF POLYNOMIAL LESS THAN ONE')
      GO TO 5
   70 IF(IER-3)75,80,78
   75 WRITE(6,77)
   77 FORMAT(//1H,10X,'ORDER OF POLYNOMIAL GREATER THAN 36')
      GO TO 5
   78 WRITE(6,79)
   79 FORMAT(//1H,10X,' HIGH ORDER COEFFICIENT IS ZERO')
      GO TO 5
   80 WRITE(6,85)
   85 FORMAT(//1H,10X,'UNABLE TO DETERMINE ROOT,THOSE ALREADY FOUND AR
     1)
   96 WRITE(6,97)I,J,ROOTR(1),ROOTI(1),ROOTR(2),ROOTI(2),ROOTR(3),ROOT
     *3),ROOTR(4),ROOTI(4)
   97 FORMAT(1H,1X,I2,3X,I2,8E13.5)
    2 CONTINUE
      GO TO 5
```

```
        100 STOP
            END
```

```
OF COMPILATION:          NO  DIAGNOSTICS.
OLY.POLRT
/15/80-12:07:04 (,0)
```

```
NE POLRT      ENTRY POINT 000527
```

```
USED: CODE(1) 000556; DATA(0) 000136; BLANK COMMON(2) 000000
```

```
 REFERENCES (BLOCK, NAME)

NERR3$
```

```
ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

000325 100L       0001     000361 110L       0001     000367 120L       0001     00041
000433 140L       0001     000457 155L       0001     000503 165L       0001     00016
000043 20L        0001     000333 240G       0001     000046 25L        0001     00005
000054 32L        0001     000110 45L        0001     000115 50L        0001     00013
000145 60L        0000 D 000044 ALPHA       0000 D 000036 DX          0000 D 00004
000060 I          0000 I 000057 ICT         0000 I 000046 IFIT        0000 I 00005
000062 ITEMP      0000 I 000053 KJ1         0000 I 000054 L           0000 I 00005
000050 NX         0000 I 000051 NXX         0000 I 000052 N2          0000 D 00003
000026 U          0000 D 000014 UX          0000 D 000016 UY          0000 D 00002
000010 XPR        0000 D 000024 XT          0000 D 000030 XT2         0000 D 00000
000012 YPR        0000 D 000022 YT          0000 D 000032 YT2         0000 D 00000
```

```
            SUBROUTINE POLRT(XCOF,COF,M,ROOTR,ROOTI,IER)
            DIMENSION XCOF(6),COF(6),ROOTR(6),ROOTI(6)
            DOUBLE PRECISION XCOF,COF,ROOTR,ROOTI
            DOUBLE PRECISION XO,YO,X,Y,XPR,YPR,UX,UY,V,YT,XT,U,XT2,YT2,SUMSQ,
           1DX,DY,TEMP,ALPHA
            IFIT=0
            N=M
            IER=0
            IF(XCOF(N+1))10,25,10
         10 IF(N)15,15,32
      C
      C         SET ERROR CODE TO 1
      C
         15 IER=1
         20 RETURN
      C
      C         SET ERROR CODE TO 4
      C
         25 IER=4
            GO TO 20
      C
      C         SET ERROR CODE TO 2
      C
         30 IER=2
```

```
      GO TO 20
   32 IF(N-36)35,35,30
   35 NX=M
      NXX=N+1
      N2=1
      KJ1=N+1
      DO 40 L=1,KJ1
      MT=KJ1-L+1
   40 COF(MT)=XCOF(L)
C
C        SET INITIAL VALUES
C
   45 X0=0.00500101
      Y0=0.01000101
C
C        ZERO INITIAL VALUE COUNTER
C
      IN=0
   50 X=X0
C
C        INCREMENT INITIAL VALUES AND COUNTER
C
      X0=-10.0*Y0
      Y0=-10.0*X
C
C        SET X AND Y TO CURRENT VALUE
C
      X=X0
      Y=Y0
      IN=IN+1
      GO TO 59
   55 IFIT=1
      XPR=X
      YPR=Y
C
   59 ICT=0
   60 UX=0.0
      UY=0.0
      V=0.0
      YT=0.0
      XT=1.0
      U=COF(N+1)
      IF(U)65,130,65
   65 DO 70 I=1,N
      L=N-I+1
      TEMP=COF(L)
      XT2=X*XT-Y*YT
      YT2=X*YT+Y*XT
      U=U+TEMP*XT2
      V=V+TEMP*YT2
      FI=I
      UX=UX+FI*XT*TEMP
      UY=UY-FI*YT*TEMP
      XT=XT2
   70 YT=YT2
      SUMSQ=UX*UX+UY*UY
      IF(SUMSQ)75,110,75
   75 DX=(V*UY-U*UX)/SUMSQ
      X=X+DX
      DY=-(U*UY+V*UX)/SUMSQ
      Y=Y+DY
```

```
      78 IF(DABS(DY)+DABS(DX)-1.0D-05)100,80,80
C
C        STEP ITERATION COUNTER
C
      80 ICT=ICT+1
         IF(ICT-500)60,85,85
      85 IF(IFIT)100,90,100
      90 IF(IN-5)50,95,95
C
C        SET ERROR CODE TO 3
C
      95 IER=3
         GO TO 20
     100 DO 105 L=1,NXX
         MT=KJ1-L+1
         TEMP=XCOF(MT)
         XCOF(MT)=COF(L)
     105 COF(L)=TEMP
         ITEMP=N
         N=NX
         NX=ITEMP
         IF(IFIT)120,55,120
     110 IF(IFIT)115,50,115
     115 X=XPR
         Y=YPR
     120 IFIT=0
     122 IF(DABS(Y)-1.0D-4*DABS(X))195,125,125
     125 ALPHA=X+X
         SUMSQ=X*X+Y*Y
         N=N-2
         GO TO 140
     130 X=0.0
         NX=NX-1
         NXX=NXX-1
     195 Y=0.0
         SUMSQ=0.0
         ALPHA=X
         N=N-1
     140 COF(2)=COF(2)+ALPHA*COF(1)
     145 DO 150 L=2,N
     150 COF(L+1)=COF(L+1)+ALPHA*COF(L)-SUMSQ*COF(L-1)
     155 ROOTI(N2)=Y
         ROOTR(N2)=X
         N2=N2+1
         IF(SUMSQ)160,165,160
     160 Y=-Y
         SUMSQ=0.0
         GO TO 155
     165 IF(N)20,20,45
GNOSTIC*  CONTROL CAN NEVER REACH THE NEXT STATEMENT
         RETURN
         END


OF COMPILATION:          1 DIAGNOSTICS.
Y.
  RL72R1 07/15/80 12:07:21

OLY.MAIN
IU1 07/15/80 12:07:25
 IN   POLY.MAIN
```

REAL AND COMPLEX ROOTS OF A POLYNOMIAL USING SUBROUTINE POLRT

FOR POLYNOMIAL 360 OF ORDER 3

L	CO	LI	LX	YI	YX
.100-03	.150+01	.287-04	.209-04	.620-01	.200-02

I	J	REAL ROOT	IMAGINARY ROOT	REAL ROOT	IMAGINARY ROOT	REAL ROOT	IMA
2	8	-.27912-03	.00000	.14759-04	-.16289-03	.14759-04	.
4	8	-.44344-03	.00000	-.30829-05	-.18349-03	-.30829-05	.
6	8	-.62029-03	.00000	-.14657-04	-.18946-03	-.14657-04	.
8	8	-.80604-03	.00000	-.21781-04	-.19125-03	-.21781-04	.
10	8	-.99687-03	.00000	-.26368-04	-.19171-03	-.26368-04	.
2	9	-.19782-02	.00000	-.35725-04	-.19095-03	-.35725-04	.
4	9	-.39694-02	.00000	-.40121-04	-.18972-03	-.40121-04	.
6	9	-.59668-02	.00000	-.41411-04	-.18920-03	-.41411-04	.
8	9	-.79657-02	.00000	-.41954-04	-.18893-03	-.41954-04	.
10	9	-.99652-02	.00000	-.42204-04	-.18877-03	-.42204-04	.
2	10	-.19965-01	.00000	-.42154-04	-.18847-03	-.42154-04	.
4	10	-.39968-01	.00000	-.40782-04	-.18840-03	-.40782-04	.
6	10	-.59971-01	.00000	-.39140-04	-.18845-03	-.39140-04	.
8	10	-.79975-01	.00000	-.37439-04	-.18851-03	-.37439-04	.
10	10	-.99978-01	.00000	-.35720-04	-.18856-03	-.35720-04	.
2	11	-.27151-04	.18866-03	-.27151-04	-.18866-03	-.20000+00	.
4	11	-.10605-04	-.18778-03	-.10605-04	.18778-03	-.40003+00	.
6	11	.50766-05	-.18559-03	.50766-05	.18559-03	-.60006+00	.
8	11	.19941-04	-.18226-03	.19941-04	.18226-03	-.80009+00	.
10	11	.34047-04	-.17790-03	.34047-04	.17790-03	-.10001+01	.
2	12	.94946-04	-.14246-03	.94946-04	.14246-03	-.20002+01	.
4	12	.87376-04	.00000	.27786-03	.00000	-.40004+01	.
6	12	.47724-04	.00000	.43692-03	.00000	-.60006+01	.
8	12	.34288-04	.00000	.53626-03	.00000	-.80006+01	.
10	12	.27230-04	.00000	.60755-03	.00000	-.10001+02	.
2	13	.15054-04	.00000	.78463-03	.00000	-.20001+02	.
4	13	.10916-04	.00000	.86960-03	.00000	-.40001+02	.
6	13	.11069-04	.00000	.86615-03	.00000	-.60001+02	.
8	13	.12486-04	.00000	.83536-03	.00000	-.80001+02	.
10	13	.14642-04	.00000	.79238-03	.00000	-.10000+03	.
2	14	.37370-04	.00000	.50994-03	.00000	-.20000+03	.
4	14	-.15756-04	-.18820-03	-.15756-04	.18820-03	-.40000+03	.
6	14	-.52369-03	.00000	-.10060-03	.00000	-.60000+03	.
8	14	-.11605-02	.00000	-.60144-04	.00000	-.80000+03	.
10	14	-.17693-02	.00000	-.49146-04	.00000	-.10000+04	.
2	15	-.47778-02	.00000	-.36194-04	.00000	-.20000+04	.
4	15	-.10780-01	.00000	-.32010-04	.00000	-.40000+04	.
6	15	-.16780-01	.00000	-.30824-04	.00000	-.60000+04	.
8	15	-.22780-01	.00000	-.30264-04	.00000	-.80000+04	.
10	15	-.28781-01	.00000	-.29938-04	.00000	-.10000+05	.
2	8	-.42472-03	.00000	-.12443-04	-.13307-03	-.12443-04	.
4	8	-.80308-03	.00000	-.23259-04	-.13547-03	-.23259-04	.
6	8	-.11952-02	.00000	-.27198-04	-.13529-03	-.27198-04	.
8	8	-.15913-02	.00000	-.29151-04	-.13498-03	-.29151-04	.

10	8	-.19890-02	.00000	-.30305-04	-.13473-03	-.30305-04	.1
2	9	-.39845-02	.00000	-.32532-04	-.13408-03	-.32532-04	.1
4	9	-.79825-02	.00000	-.33546-04	-.13369-03	-.33546-04	.1
6	9	-.11982-01	.00000	-.33819-04	-.13356-03	-.33819-04	.1
8	9	-.15982-01	.00000	-.33910-04	-.13349-03	-.33910-04	.1
10	9	-.19982-01	.00000	-.33928-04	-.13345-03	-.33928-04	.1
2	10	-.39982-01	.00000	-.33700-04	-.13338-03	-.33700-04	.1
4	10	-.79984-01	.00000	-.32926-04	-.13336-03	-.32926-04	.1
6	10	-.11999+00	.00000	-.32086-04	-.13338-03	-.32086-04	.1
8	10	-.31233-04	-.13339-03	-.31233-04	.13339-03	-.15999+00	.0
10	10	-.30378-04	.13340-03	-.30378-04	-.13340-03	-.19999+00	.0
2	11	-.26148-04	-.13337-03	-.26148-04	.13337-03	-.40000+00	.0
4	11	-.18013-04	-.13297-03	-.18013-04	.13297-03	-.80001+00	.0
6	11	-.10318-04	-.13212-03	-.10318-04	.13212-03	-.12000+01	.0
8	11	-.30334-05	-.13090-03	-.30334-05	.13090-03	-.16000+01	.0
10	11	.38713-05	-.12936-03	.38713-05	.12936-03	-.20001+01	.0
2	12	.33576-04	-.11796-03	.33576-04	.11796-03	-.40001+01	.0
4	12	.75916-04	-.82759-04	.75916-04	.82759-04	-.80002+01	.0
6	12	.10517-03	-.11669-04	.10517-03	.11669-04	-.12000+02	.0
8	12	.49422-04	.00000	.19907-03	.00000	-.16000+02	.0
10	12	.37494-04	.00000	.24011-03	.00000	-.20000+02	.0
2	13	.21866-04	.00000	.32319-03	.00000	-.40000+02	.0
4	13	.20187-04	.00000	.33528-03	.00000	-.80001+02	.0
6	13	.25755-04	.00000	.29806-03	.00000	-.12000+03	.0
8	13	.37002-04	.00000	.24213-03	.00000	-.16000+03	.0
10	13	.62669-04	.00000	.16605-03	.00000	-.20000+03	.0
2	14	-.25571-04	-.13336-03	-.25571-04	.13336-03	-.40000+03	.0
4	14	-.57955-03	.00000	-.61003-04	.00000	-.80000+03	.0
6	14	-.11930-02	.00000	-.43983-04	.00000	-.12000+04	.0
8	14	-.17964-02	.00000	-.38766-04	.00000	-.16000+04	.0
10	14	-.23978-02	.00000	-.36211-04	.00000	-.20000+04	.0
2	15	-.53998-02	.00000	-.32013-04	.00000	-.40000+04	.0
4	15	-.11400-01	.00000	-.30265-04	.00000	-.80000+04	.0
6	15	-.17401-01	.00000	-.29724-04	.00000	-.12000+05	.0
8	15	-.23401-01	.00000	-.29461-04	.00000	-.16000+05	.0
10	15	-.29401-01	.00000	-.29306-04	.00000	-.20000+05	.0
2	8	-.60735-03	.00000	-.21127-04	-.11065-03	-.21127-04	.1
4	8	-.11968-02	.00000	-.26409-04	-.11037-03	-.26409-04	.1
6	8	-.17933-02	.00000	-.28139-04	-.11000-03	-.28139-04	.1
8	8	-.23916-02	.00000	-.28984-04	-.10977-03	-.28984-04	.1
10	8	-.29906-02	.00000	-.29482-04	-.10961-03	-.29482-04	.1
2	9	-.59887-02	.00000	-.30443-04	-.10927-03	-.30443-04	.1
4	9	-.11988-01	.00000	-.30871-04	-.10908-03	-.30871-04	.1
6	9	-.17988-01	.00000	-.30973-04	-.10902-03	-.30973-04	.1
8	9	-.23988-01	.00000	-.30995-04	-.10898-03	-.30995-04	.1
10	9	-.29988-01	.00000	-.30985-04	-.10897-03	-.30985-04	.1
2	10	-.59988-01	.00000	-.30790-04	-.10893-03	-.30790-04	.1
4	10	-.11999+00	.00000	-.30261-04	-.10892-03	-.30261-04	.1
6	10	-.29703-04	-.10892-03	-.29703-04	.10892-03	-.17999+00	.0
8	10	-.29140-04	.10892-03	-.29140-04	-.10892-03	-.23999+00	.0
10	10	-.28578-04	-.10892-03	-.28578-04	.10892-03	-.29999+00	.0
2	11	-.25803-04	-.10888-03	-.25803-04	.10888-03	-.60000+00	.0
4	11	-.20478-04	-.10861-03	-.20478-04	.10861-03	-.12000+01	.0
6	11	-.15447-04	-.10811-03	-.15447-04	.10811-03	-.18000+01	.0
8	11	-.10690-04	-.10742-03	-.10690-04	.10742-03	-.24000+01	.0
10	11	-.61864-05	-.10656-03	-.61864-05	.10656-03	-.30000+01	.0
2	12	.13117-04	-.10057-03	.13117-04	.10057-03	-.60001+01	.0
4	12	.40345-04	-.84234-04	.40345-04	.84234-04	-.12000+02	.0
6	12	.58248-04	-.65595-04	.58248-04	.65595-04	-.18000+02	.0
8	12	.70565-04	-.44825-04	.70565-04	.44825-04	-.24000+02	.0
10	12	.79260-04	-.14311-04	.79260-04	.14311-04	-.30000+02	.0

2	13	.34497-04	.00000	.15901-03	.00000	-.60000+02	.0
4	13	.42474-04	.00000	.13797-03	.00000	-.12000+03	.0
6	13	.69673-04	-.46748-04	.69673-04	.46748-04	-.18000+03	.0
8	13	.44777-04	-.80398-04	.44777-04	.80398-04	-.24000+03	.0
0	13	.17972-04	-.98409-04	.17972-04	.98409-04	-.30000+03	.0
2	14	-.12531-03	-.50283-04	-.12531-03	.50283-04	-.60000+03	.0
4	14	-.79948-03	.00000	-.44090-04	.00000	-.12000+04	.0
6	14	-.14038-02	.00000	-.37327-04	.00000	-.18000+04	.0
8	14	-.20052-02	.00000	-.34702-04	.00000	-.24000+04	.0
0	14	-.26059-02	.00000	-.33303-04	.00000	-.30000+04	.0
2	15	-.56069-02	.00000	-.30827-04	.00000	-.60000+04	.0
4	15	-.11607-01	.00000	-.29725-04	.00000	-.12000+05	.0
6	15	-.17607-01	.00000	-.29375-04	.00000	-.18000+05	.0
8	15	-.23607-01	.00000	-.29203-04	.00000	-.24000+05	.0
0	15	-.29607-01	.00000	-.29101-04	.00000	-.30000+05	.0
2	8	-.80156-03	.00000	-.24021-04	-.95852-04	-.24021-04	.9
4	8	-.15956-02	.00000	-.26987-04	-.95299-04	-.26987-04	.9
6	8	-.23937-02	.00000	-.27942-04	-.95017-04	-.27942-04	.9
8	8	-.31928-02	.00000	-.28408-04	-.94858-04	-.28408-04	.9
0	8	-.39922-02	.00000	-.28684-04	-.94758-04	-.28684-04	.9
2	9	-.79912-02	.00000	-.29215-04	-.94545-04	-.29215-04	.9
4	9	-.15991-01	.00000	-.29444-04	-.94434-04	-.29444-04	.9
6	9	-.23991-01	.00000	-.29491-04	-.94396-04	-.29491-04	.9
8	9	-.31991-01	.00000	-.29493-04	-.94377-04	-.29493-04	.9
0	9	-.39991-01	.00000	-.29478-04	-.94366-04	-.29478-04	.9
2	10	-.79991-01	.00000	-.29318-04	-.94345-04	-.29318-04	.9
4	10	-.28920-04	-.94335-04	-.28920-04	.94335-04	-.15999+00	.0
6	10	-.28506-04	.94331-04	-.28506-04	-.94331-04	-.23999+00	.0
8	10	-.28090-04	-.94327-04	-.28090-04	.94327-04	-.31999+00	.0
10	10	-.27675-04	-.94322-04	-.27675-04	.94322-04	-.39999+00	.0
2	11	-.25629-04	-.94276-04	-.25629-04	.94276-04	-.80000+00	.0
4	11	-.21709-04	-.94065-04	-.21709-04	.94065-04	-.16000+01	.0
6	11	-.18011-04	-.93717-04	-.18011-04	.93717-04	-.24000+01	.0
8	11	-.14518-04	-.93253-04	-.14518-04	.93253-04	-.32000+01	.0
10	11	-.11215-04	-.92690-04	-.11215-04	.92690-04	-.40000+01	.0
2	12	.28882-05	-.88877-04	.28882-05	.88877-04	-.80001+01	.0
4	12	.22559-04	-.79180-04	.22559-04	.79180-04	-.16000+02	.0
6	12	.35236-04	-.69348-04	.35236-04	.69348-04	-.24000+02	.0
8	12	.43723-04	-.60430-04	.43723-04	.60430-04	-.32000+02	.0
10	12	.49502-04	-.52738-04	.49502-04	.52738-04	-.40000+02	.0
2	13	.58868-04	-.35069-04	.58868-04	.35069-04	-.80000+02	.0
4	13	.46467-04	-.56982-04	.46467-04	.56982-04	-.16000+03	.0
6	13	.23555-04	-.78527-04	.23555-04	.78527-04	-.24000+03	.0
8	13	-.26172-05	-.90646-04	-.26172-05	.90646-04	-.32000+03	.0
10	13	-.30220-04	-.94312-04	-.30220-04	.94312-04	-.40000+03	.0
2	14	-.28726-03	.00000	-.63110-04	.00000	-.80000+03	.0
4	14	-.90624-03	.00000	-.38839-04	.00000	-.16000+04	.0
6	14	-.15086-02	.00000	-.34712-04	.00000	-.24000+04	.0
8	14	-.21094-02	.00000	-.32976-04	.00000	-.32000+04	.0
10	14	-.27098-02	.00000	-.32018-04	.00000	-.40000+04	.
2	15	-.57104-02	.00000	-.30266-04	.00000	-.80000+04	.
4	15	-.11711-01	.00000	-.29462-04	.00000	-.16000+05	.
6	15	-.17711-01	.00000	-.29203-04	.00000	-.24000+05	.
8	15	-.23711-01	.00000	-.29076-04	.00000	-.32000+05	.
10	15	-.29711-01	.00000	-.29000-04	.00000	-.40000+05	.
2	8	-.99933-03	.00000	-.25137-04	-.85559-04	-.25137-04	.
4	8	-.19956-02	.00000	-.27012-04	-.85057-04	-.27012-04	.
6	8	-.29944-02	.00000	-.27615-04	-.84846-04	-.27615-04	.
8	8	-.39938-02	.00000	-.27910-04	-.84733-04	-.27910-04	.
10	8	-.49934-02	.00000	-.28084-04	-.84663-04	-.28084-04	.
2	9	-.99928-02	.00000	-.28419-04	-.84517-04	-.28419-04	.

4	9	-.19992-01	.00000	-.28560-04	-.84442-04	-.28560-04	.8
6	9	-.29992-01	.00000	-.28584-04	-.84417-04	-.28584-04	.8
8	9	-.39992-01	.00000	-.28579-04	-.84404-04	-.28579-04	.8
0	9	-.49993-01	.00000	-.28562-04	-.84396-04	-.28562-04	.8
2	10	-.99993-01	.00000	-.28429-04	-.84381-04	-.28429-04	.8
4	10	-.28112-04	.84371-04	-.28112-04	-.84371-04	-.19999+00	.0
6	10	-.27785-04	-.84366-04	-.27785-04	.84366-04	-.29999+00	.0
8	10	-.27458-04	-.84360-04	-.27458-04	.84360-04	-.39999+00	.0
0	10	-.27132-04	-.84354-04	-.27132-04	.84354-04	-.50000+00	.0
2	11	-.25524-04	-.84307-04	-.25524-04	.84307-04	-.10000+01	.0
4	11	-.22448-04	-.84134-04	-.22448-04	.84134-04	-.20000+01	.0
6	11	-.19549-04	-.83868-04	-.19549-04	.83868-04	-.30000+01	.0
8	11	-.16815-04	-.83525-04	-.16815-04	.83525-04	-.40000+01	.0
0	11	-.14232-04	-.83116-04	-.14232-04	.83116-04	-.50000+01	.0
2	12	-.32495-05	-.80435-04	-.32495-05	.80435-04	-.10000+02	.0
4	12	.11887-04	-.73961-04	.11887-04	.73961-04	-.20000+02	.0
6	12	.21429-04	-.67858-04	.21429-04	.67858-04	-.30000+02	.0
8	12	.27619-04	-.62816-04	.27619-04	.62816-04	-.40000+02	.0
0	12	.31642-04	-.58961-04	.31642-04	.58961-04	-.50000+02	.0
2	13	.36132-04	-.53985-04	.36132-04	.53985-04	-.10000+03	.0
4	13	.20213-04	-.68739-04	.20213-04	.68739-04	-.20000+03	.0
6	13	-.41161-05	-.80705-04	-.41161-05	.80705-04	-.30000+03	.0
8	13	-.31054-04	-.84333-04	-.31054-04	.84333-04	-.40000+03	.0
0	13	-.59137-04	-.78684-04	-.59137-04	.78684-04	-.50000+03	.0
2	14	-.36004-03	.00000	-.50182-04	.00000	-.10000+04	.0
4	14	-.96972-03	.00000	-.36264-04	.00000	-.20000+04	.0
6	14	-.15712-02	.00000	-.33314-04	.00000	-.30000+04	.0
8	14	-.21718-02	.00000	-.32021-04	.00000	-.40000+04	.0
0	14	-.27721-02	.00000	-.31294-04	.00000	-.50000+04	.0
2	15	-.57725-02	.00000	-.29939-04	.00000	-.10000+05	.0
4	15	-.11773-01	.00000	-.29306-04	.00000	-.20000+05	.0
6	15	-.17773-01	.00000	-.29101-04	.00000	-.30000+05	.0
8	15	-.23773-01	.00000	-.29000-04	.00000	-.40000+05	.0
0	15	-.29773-01	.00000	-.28939-04	.00000	-.50000+05	.0
2	8	-.11984-02	.00000	-.25610-04	-.77946-04	-.25610-04	.7
4	8	-.23958-02	.00000	-.26897-04	-.77527-04	-.26897-04	.7
6	8	-.35950-02	.00000	-.27312-04	-.77366-04	-.27312-04	.7
8	8	-.47946-02	.00000	-.27515-04	-.77281-04	-.27515-04	.7
10	8	-.59943-02	.00000	-.27635-04	-.77229-04	-.27635-04	.7
2	9	-.11994-01	.00000	-.27865-04	-.77123-04	-.27865-04	.7
4	9	-.23994-01	.00000	-.27958-04	-.77068-04	-.27958-04	.7
6	9	-.35994-01	.00000	-.27970-04	-.77049-04	-.27970-04	.7
8	9	-.47994-01	.00000	-.27963-04	-.77040-04	-.27963-04	.7
0	9	-.59994-01	.00000	-.27947-04	-.77034-04	-.27947-04	.7
2	10	-.11999+00	.00000	-.27834-04	-.77021-04	-.27834-04	.7
4	10	-.27572-04	.77012-04	-.27572-04	-.77012-04	-.23999+00	.0
6	10	-.27304-04	-.77006-04	-.27304-04	.77006-04	-.36000+00	.0
8	10	-.27036-04	-.77000-04	-.27036-04	.77000-04	-.48000+00	.0
10	10	-.26769-04	-.76993-04	-.26769-04	.76993-04	-.60000+00	.0
2	11	-.25454-04	-.76948-04	-.25454-04	.76948-04	-.12000+01	.0
4	11	-.22940-04	-.76800-04	-.22940-04	.76800-04	-.24000+01	.0
6	11	-.20574-04	-.76585-04	-.20574-04	.76585-04	-.36000+01	.0
8	11	-.18346-04	-.76316-04	-.18346-04	.76316-04	-.48000+01	.0
10	11	-.16244-04	-.76001-04	-.16244-04	.76001-04	-.60000+01	.0
2	12	-.73413-05	-.73994-04	-.73413-05	.73994-04	-.12000+02	.0
4	12	.47725-05	-.69361-04	.47725-05	.69361-04	-.24000+02	.0
6	12	.12224-04	-.65242-04	.12224-04	.65242-04	-.36000+02	.0
8	12	.16882-04	-.62077-04	.16882-04	.62077-04	-.48000+02	.0
10	12	.19735-04	-.59878-04	.19735-04	.59878-04	-.60000+02	.0
2	13	.20977-04	-.58852-04	.20977-04	.58852-04	-.12000+03	.0
4	13	.27112-05	-.70318-04	.27112-05	.70318-04	-.24000+03	.0

6	13	-.22563-04	-.76770-04	-.22563-04	.76770-04	-.36000+03	.0
8	13	-.50011-04	-.74008-04	-.50011-04	.74008-04	-.48000+03	.0
0	13	-.78414-04	-.58821-04	-.78414-04	.58821-04	-.60000+03	.0
2	14	-.40568-03	.00000	-.44434-04	.00000	-.12000+04	.0
4	14	-.10119-02	.00000	-.34733-04	.00000	-.24000+04	.0
6	14	-.16129-02	.00000	-.32444-04	.00000	-.36000+04	.0
8	14	-.22134-02	.00000	-.31415-04	.00000	-.48000+04	.0
0	14	-.28136-02	.00000	-.30830-04	.00000	-.60000+04	.0
2	15	-.58139-02	.00000	-.29725-04	.00000	-.12000+05	.0
4	15	-.11814-01	.00000	-.29203-04	.00000	-.24000+05	.0
6	15	-.17814-01	.00000	-.29033-04	.00000	-.36000+05	.0
8	15	-.23814-01	.00000	-.28949-04	.00000	-.48000+05	.0
0	15	-.29814-01	.00000	-.28899-04	.00000	-.60000+05	.0
2	8	-.13980-02	.00000	-.25815-04	-.72040-04	-.25815-04	.7
4	8	-.27961-02	.00000	-.26751-04	-.71694-04	-.26751-04	.7
6	8	-.41955-02	.00000	-.27054-04	-.71567-04	-.27054-04	.7
8	8	-.55952-02	.00000	-.27202-04	-.71501-04	-.27202-04	.7
0	8	-.69950-02	.00000	-.27290-04	-.71460-04	-.27290-04	.7
2	9	-.13995-01	.00000	-.27457-04	-.71378-04	-.27457-04	.7
4	9	-.27995-01	.00000	-.27522-04	-.71336-04	-.27522-04	.7
6	9	-.41995-01	.00000	-.27529-04	-.71322-04	-.27529-04	.7
8	9	-.55995-01	.00000	-.27520-04	-.71314-04	-.27520-04	.7
0	9	-.69995-01	.00000	-.27506-04	-.71310-04	-.27506-04	.7
2	10	-.27408-04	.71299-04	-.27408-04	-.71299-04	-.13999+00	.0
4	10	-.27186-04	-.71290-04	-.27186-04	.71290-04	-.28000+00	.0
6	10	-.26961-04	-.71284-04	-.26961-04	.71284-04	-.42000+00	.0
8	10	-.26735-04	-.71277-04	-.26735-04	.71277-04	-.56000+00	.0
0	10	-.26510-04	-.71270-04	-.26510-04	.71270-04	-.70000+00	.0
2	11	-.25404-04	-.71227-04	-.25404-04	.71227-04	-.14000+01	.0
4	11	-.23292-04	-.71097-04	-.23292-04	.71097-04	-.28000+01	.0
6	11	-.21307-04	-.70918-04	-.21307-04	.70918-04	-.42000+01	.0
8	11	-.19439-04	-.70698-04	-.19439-04	.70698-04	-.56000+01	.0
0	11	-.17680-04	-.70446-04	-.17680-04	.70446-04	-.70000+01	.0
2	12	-.10264-04	-.68878-04	-.10264-04	.68878-04	-.14000+02	.0
4	12	-.30928-06	-.65406-04	-.30928-06	.65406-04	-.28000+02	.0
6	12	.56492-05	-.62483-04	.56492-05	.62483-04	-.42000+02	.0
8	12	.92134-05	-.60387-04	.92134-05	.60387-04	-.56000+02	.0
0	12	.11230-04	-.59073-04	.11230-04	.59073-04	-.70000+02	.0
2	13	.10151-04	-.59788-04	.10151-04	.59788-04	-.14000+03	.0
4	13	-.97904-05	-.68749-04	-.97904-05	.68749-04	-.28000+03	.0
6	13	-.35740-04	-.70954-04	-.35740-04	.70954-04	-.42000+03	.0
8	13	-.63553-04	-.62204-04	-.63553-04	.62204-04	-.56000+03	.0
10	13	-.92183-04	-.32463-04	-.92183-04	.32463-04	-.70000+03	.0
2	14	-.43748-03	.00000	-.41137-04	.00000	-.14000+04	.0
4	14	-.10419-02	.00000	-.33718-04	.00000	-.28000+04	.0
6	14	-.16427-02	.00000	-.31850-04	.00000	-.42000+04	.0
8	14	-.22430-02	.00000	-.30996-04	.00000	-.56000+04	.0
10	14	-.28432-02	.00000	-.30506-04	.00000	-.70000+04	.0
2	15	-.58435-02	.00000	-.29574-04	.00000	-.14000+05	.0
4	15	-.11844-01	.00000	-.29130-04	.00000	-.28000+05	.0
6	15	-.17844-01	.00000	-.28985-04	.00000	-.42000+05	.0
8	15	-.23844-01	.00000	-.28913-04	.00000	-.56000+05	.0
10	15	-.29844-01	.00000	-.28871-04	.00000	-.70000+05	.0
2	8	-.15978-02	.00000	-.25896-04	-.67292-04	-.25896-04	.6
4	8	-.31964-02	.00000	-.26608-04	-.67004-04	-.26608-04	.6
6	8	-.47959-02	.00000	-.26838-04	-.66900-04	-.26838-04	.6
8	8	-.63957-02	.00000	-.26951-04	-.66847-04	-.26951-04	.6
10	8	-.79956-02	.00000	-.27018-04	-.66814-04	-.27018-04	.6
2	9	-.15995-01	.00000	-.27144-04	-.66749-04	-.27144-04	.6
4	9	-.31995-01	.00000	-.27192-04	-.66715-04	-.27192-04	.6
6	9	-.47995-01	.00000	-.27195-04	-.66703-04	-.27195-04	.6

8	9	-.63995-01	.00000	-.27186-04	-.66697-04	-.27186-04	.6
0	9	-.79995-01	.00000	-.27173-04	-.66694-04	-.27173-04	.6
2	10	-.27087-04	-.66685-04	-.27087-04	.66685-04	-.16000+00	.0
4	10	-.26897-04	-.66676-04	-.26897-04	.66676-04	-.32000+00	.0
6	10	-.26702-04	-.66669-04	-.26702-04	.66669-04	-.48000+00	.0
8	10	-.26508-04	-.66663-04	-.26508-04	.66663-04	-.64000+00	.0
0	10	-.26315-04	-.66656-04	-.26315-04	.66656-04	-.80000+00	.0
2	11	-.25366-04	-.66614-04	-.25366-04	.66614-04	-.16000+01	.0
4	11	-.23555-04	-.66499-04	-.23555-04	.66499-04	-.32000+01	.0
6	11	-.21856-04	-.66345-04	-.21856-04	.66345-04	-.48000+01	.0
8	11	-.20259-04	-.66161-04	-.20259-04	.66161-04	-.64000+01	.0
0	11	-.18758-04	-.65952-04	-.18758-04	.65952-04	-.80000+01	.0
2	12	-.12456-04	-.64689-04	-.12456-04	.64689-04	-.16000+02	.0
4	12	-.41206-05	-.62003-04	-.41206-05	.62003-04	-.32000+02	.0
6	12	.71802-06	-.59859-04	.71802-06	.59859-04	-.48000+02	.0
8	12	.34617-05	-.58431-04	.34617-05	.58431-04	-.64000+02	.0
0	12	.48513-05	-.57644-04	.48513-05	.57644-04	-.80000+02	.0
2	13	.20325-05	-.59195-04	.20325-05	.59195-04	-.16000+03	.0
4	13	-.19167-04	-.66012-04	-.19167-04	.66012-04	-.32000+03	.0
6	13	-.45623-04	-.64515-04	-.45623-04	.64515-04	-.48000+03	.0
8	13	-.73709-04	-.49221-04	-.73709-04	.49221-04	-.64000+03	.0
0	13	-.13412-03	.00000	-.70896-04	.00000	-.80000+03	.0
2	14	-.46100-03	.00000	-.38990-04	.00000	-.16000+04	.0
4	14	-.10643-02	.00000	-.32995-04	.00000	-.32000+04	.0
6	14	-.16650-02	.00000	-.31419-04	.00000	-.48000+04	.0
8	14	-.22653-02	.00000	-.30689-04	.00000	-.64000+04	.0
0	14	-.28654-02	.00000	-.30268-04	.00000	-.80000+04	.0
2	15	-.58657-02	.00000	-.29462-04	.00000	-.16000+05	.0
4	15	-.11866-01	.00000	-.29076-04	.00000	-.32000+05	.0
6	15	-.17866-01	.00000	-.28949-04	.00000	-.48000+05	.0
8	15	-.23866-01	.00000	-.28887-04	.00000	-.64000+05	.0
0	15	-.29866-01	.00000	-.28849-04	.00000	-.80000+05	.0
2	8	-.17978-02	.00000	-.25917-04	-.63369-04	-.25917-04	.6
4	8	-.35966-02	.00000	-.26476-04	-.63125-04	-.26476-04	.6
6	8	-.53963-02	.00000	-.26657-04	-.63039-04	-.26657-04	.6
8	8	-.71961-02	.00000	-.26746-04	-.62995-04	-.26746-04	.6
0	8	-.89960-02	.00000	-.26799-04	-.62968-04	-.26799-04	.6
2	9	-.17996-01	.00000	-.26898-04	-.62914-04	-.26898-04	.6
4	9	-.35996-01	.00000	-.26934-04	-.62887-04	-.26934-04	.6
6	9	-.53996-01	.00000	-.26934-04	-.62877-04	-.26934-04	.6
8	9	-.71996-01	.00000	-.26926-04	-.62872-04	-.26926-04	.6
0	9	-.89996-01	.00000	-.26914-04	-.62869-04	-.26914-04	.6
2	10	-.26838-04	-.62861-04	-.26838-04	.62861-04	-.18000+00	.0
4	10	-.26671-04	-.62853-04	-.26671-04	.62853-04	-.36000+00	.0
6	10	-.26501-04	-.62846-04	-.26501-04	.62846-04	-.54000+00	.0
8	10	-.26332-04	-.62839-04	-.26332-04	.62839-04	-.72000+00	.0
10	10	-.26164-04	-.62833-04	-.26164-04	.62833-04	-.90000+00	.0
2	11	-.25337-04	-.62793-04	-.25337-04	.62793-04	-.18000+01	.0
4	11	-.23760-04	-.62689-04	-.23760-04	.62689-04	-.36000+01	.0
6	11	-.22283-04	-.62554-04	-.22283-04	.62554-04	-.54000+01	.0
8	11	-.20897-04	-.62397-04	-.20897-04	.62397-04	-.72000+01	.0
10	11	-.19596-04	-.62220-04	-.19596-04	.62220-04	-.90000+01	.0
2	12	-.14161-04	-.61179-04	-.14161-04	.61179-04	-.18000+02	.0
4	12	-.70850-05	-.59051-04	-.70850-05	.59051-04	-.36000+02	.0
6	12	-.31173-05	-.57443-04	-.31173-05	.57443-04	-.54000+02	.0
8	12	-.10118-05	-.56459-04	-.10118-05	.56459-04	-.72000+02	.0
10	12	-.10992-06	-.56007-04	-.10992-06	.56007-04	-.90000+02	.0
2	13	-.42822-05	-.57948-04	-.42822-05	.57948-04	-.18000+03	.0
4	13	-.26459-04	-.62843-04	-.26459-04	.62843-04	-.36000+03	.0
6	13	-.53309-04	-.57867-04	-.53309-04	.57867-04	-.54000+03	.0
8	13	-.81600-04	-.33990-04	-.81600-04	.33990-04	-.72000+03	.0

0	13	-.16293-03	.00000	-.58160-04	.00000	-.90000+03	.0
2	14	-.47913-03	.00000	-.37479-04	.00000	-.18000+04	.0
4	14	-.10818-02	.00000	-.32455-04	.00000	-.36000+04	.0
6	14	-.16824-02	.00000	-.31091-04	.00000	-.54000+04	.0
8	14	-.22826-02	.00000	-.30454-04	.00000	-.72000+04	.0
0	14	-.28827-02	.00000	-.30085-04	.00000	-.90000+04	.0
2	15	-.58829-02	.00000	-.29375-04	.00000	-.18000+05	.0
4	15	-.11883-01	.00000	-.29034-04	.00000	-.36000+05	.0
6	15	-.17883-01	.00000	-.28921-04	.00000	-.54000+05	.0
8	15	-.23883-01	.00000	-.28866-04	.00000	-.72000+05	.0
0	15	-.29883-01	.00000	-.28832-04	.00000	-.90000+05	.0
2	8	-.19978-02	.00000	-.25908-04	-.60057-04	-.25908-04	.6
4	8	-.39969-02	.00000	-.26358-04	-.59849-04	-.26358-04	.5
6	8	-.59966-02	.00000	-.26504-04	-.59776-04	-.26504-04	.5
8	8	-.79965-02	.00000	-.26576-04	-.59739-04	-.26576-04	.5
0	8	-.99964-02	.00000	-.26618-04	-.59716-04	-.26618-04	.5
2	9	-.19996-01	.00000	-.26698-04	-.59671-04	-.26698-04	.5
4	9	-.39996-01	.00000	-.26726-04	-.59647-04	-.26726-04	.5
6	9	-.59996-01	.00000	-.26725-04	-.59639-04	-.26725-04	.5
8	9	-.79996-01	.00000	-.26717-04	-.59635-04	-.26717-04	.5
0	9	-.99996-01	.00000	-.26706-04	-.59632-04	-.26706-04	.5
2	10	-.26637-04	.59625-04	-.26637-04	-.59625-04	-.20000+00	.0
4	10	-.26491-04	-.59618-04	-.26491-04	.59618-04	-.40000+00	.0
6	10	-.26341-04	-.59611-04	-.26341-04	.59611-04	-.60000+00	.0
8	10	-.26191-04	-.59604-04	-.26191-04	.59604-04	-.80000+00	.0
0	10	-.26043-04	-.59597-04	-.26043-04	.59597-04	-.10000+01	.0
2	11	-.25313-04	-.59560-04	-.25313-04	.59560-04	-.20000+01	.0
4	11	-.23924-04	-.59464-04	-.23924-04	.59464-04	-.40000+01	.0
6	11	-.22625-04	-.59346-04	-.22625-04	.59346-04	-.60000+01	.0
8	11	-.21407-04	-.59208-04	-.21407-04	.59208-04	-.80000+01	.0
0	11	-.20266-04	-.59057-04	-.20266-04	.59057-04	-.10000+02	.0
2	12	-.15525-04	-.58183-04	-.15525-04	.58183-04	-.20000+02	.0
4	12	-.94565-05	-.56467-04	-.94565-05	.56467-04	-.40000+02	.0
6	12	-.61855-05	-.55244-04	-.61855-05	.55244-04	-.60000+02	.0
8	12	-.45906-05	-.54567-04	-.45906-05	.54567-04	-.80000+02	.0
10	12	-.40790-05	-.54338-04	-.40790-05	.54338-04	-.10000+03	.0
2	13	-.93339-05	-.56425-04	-.93339-05	.56425-04	-.20000+03	.0
4	13	-.32293-04	-.59547-04	-.32293-04	.59547-04	-.40000+03	.0
6	13	-.59458-04	-.51115-04	-.59458-04	.51115-04	-.60000+03	.0
8	13	-.86952-04	-.54629-05	-.86952-04	.54629-05	-.80000+03	.0
10	13	-.18203-03	.00000	-.51911-04	.00000	-.10000+04	.0
2	14	-.49355-03	.00000	-.36356-04	.00000	-.20000+04	.0
4	14	-.10958-02	.00000	-.32035-04	.00000	-.40000+04	.0
6	14	-.16962-02	.00000	-.30834-04	.00000	-.60000+04	.0
8	14	-.22964-02	.00000	-.30269-04	.00000	-.80000+04	.0
10	14	-.28965-02	.00000	-.29941-04	.00000	-.10000+05	.0
2	15	-.58967-02	.00000	-.29306-04	.00000	-.20000+05	.0
4	15	-.11897-01	.00000	-.29000-04	.00000	-.40000+05	.0
6	15	-.17897-01	.00000	-.28899-04	.00000	-.60000+05	.0
8	15	-.23897-01	.00000	-.28849-04	.00000	-.80000+05	.0
10	15	-.29897-01	.00000	-.28819-04	.00000	-.10000+06	.0
2	8	-.21978-02	.00000	-.25882-04	-.57213-04	-.25882-04	.5
4	8	-.43971-02	.00000	-.26253-04	-.57032-04	-.26253-04	.5
6	8	-.65969-02	.00000	-.26373-04	-.56970-04	-.26373-04	.5
8	8	-.87967-02	.00000	-.26433-04	-.56938-04	-.26433-04	.5
10	8	-.10997-01	.00000	-.26467-04	-.56919-04	-.26467-04	.5
2	9	-.21997-01	.00000	-.26533-04	-.56880-04	-.26533-04	.5
4	9	-.43997-01	.00000	-.26555-04	-.56860-04	-.26555-04	.5
6	9	-.65997-01	.00000	-.26553-04	-.56853-04	-.26553-04	.5
8	9	-.87997-01	.00000	-.26546-04	-.56849-04	-.26546-04	.5
10	9	-.11000+00	.00000	-.26535-04	-.56847-04	-.26535-04	.5

2	10	-.26474-04	.56840-04	-.26474-04	-.56840-04	-.22000+00	.00
4	10	-.26342-04	-.56832-04	-.26342-04	.56832-04	-.44000+00	.00
6	10	-.26209-04	-.56826-04	-.26209-04	.56826-04	-.66000+00	.00
8	10	-.26076-04	-.56820-04	-.26076-04	.56820-04	-.88000+00	.00
0	10	-.25943-04	-.56813-04	-.25943-04	.56813-04	-.11000+01	.00
2	11	-.25294-04	-.56778-04	-.25294-04	.56778-04	-.22000+01	.00
4	11	-.24059-04	-.56690-04	-.24059-04	.56690-04	-.44000+01	.00
6	11	-.22904-04	-.56584-04	-.22904-04	.56584-04	-.66000+01	.00
8	11	-.21825-04	-.56463-04	-.21825-04	.56463-04	-.88000+01	.00
0	11	-.20815-04	-.56330-04	-.20815-04	.56330-04	-.11000+02	.00
2	12	-.16641-04	-.55586-04	-.16641-04	.55586-04	-.22000+02	.00
4	12	-.11397-04	-.54184-04	-.11397-04	.54184-04	-.44000+02	.00
6	12	-.86960-05	-.53246-04	-.86960-05	.53246-04	-.66000+02	.00
8	12	-.75188-05	-.52788-04	-.75188-05	.52788-04	-.88000+02	.00
0	12	-.73264-05	-.52711-04	-.73264-05	.52711-04	-.11000+03	.00
2	13	-.13467-04	-.54802-04	-.13467-04	.54802-04	-.22000+03	.00
4	13	-.37067-04	-.56261-04	-.37067-04	.56261-04	-.44000+03	.00
6	13	-.64488-04	-.44208-04	-.64488-04	.44208-04	-.66000+03	.00
8	13	-.12329-03	.00000	-.62902-04	.00000	-.88000+03	.00
0	13	-.19647-03	.00000	-.47984-04	.00000	-.11000+04	.00
2	14	-.50530-03	.00000	-.35488-04	.00000	-.22000+04	.00
4	14	-.11072-02	.00000	-.31700-04	.00000	-.44000+04	.00
6	14	-.17076-02	.00000	-.30627-04	.00000	-.66000+04	.00
8	14	-.23078-02	.00000	-.30120-04	.00000	-.88000+04	.00
0	14	-.29079-02	.00000	-.29824-04	.00000	-.11000+05	.00
2	15	-.59080-02	.00000	-.29250-04	.00000	-.22000+05	.00
4	15	-.11908-01	.00000	-.28972-04	.00000	-.44000+05	.00
6	15	-.17908-01	.00000	-.28881-04	.00000	-.66000+05	.00
8	15	-.23908-01	.00000	-.28835-04	.00000	-.88000+05	.00
0	15	-.29908-01	.00000	-.28808-04	.00000	-.11000+06	.00
2	8	-.23979-02	.00000	-.25850-04	-.54737-04	-.25850-04	.5
4	8	-.47973-02	.00000	-.26160-04	-.54578-04	-.26160-04	.5
6	8	-.71971-02	.00000	-.26261-04	-.54523-04	-.26261-04	.5
8	8	-.95970-02	.00000	-.26311-04	-.54496-04	-.26311-04	.5
0	8	-.11997-01	.00000	-.26340-04	-.54479-04	-.26340-04	.5
2	9	-.23997-01	.00000	-.26394-04	-.54445-04	-.26394-04	.5
4	9	-.47997-01	.00000	-.26412-04	-.54428-04	-.26412-04	.5
6	9	-.71997-01	.00000	-.26410-04	-.54422-04	-.26410-04	.5
8	9	-.95997-01	.00000	-.26402-04	-.54418-04	-.26402-04	.5
0	9	-.12000+00	.00000	-.26393-04	-.54416-04	-.26393-04	.5
2	10	-.26338-04	.54410-04	-.26338-04	-.54410-04	-.24000+00	.0
4	10	-.26220-04	-.54404-04	-.26220-04	.54404-04	-.48000+00	.0
6	10	-.26099-04	-.54397-04	-.26099-04	.54397-04	-.72000+00	.0
8	10	-.25980-04	-.54391-04	-.25980-04	.54391-04	-.96000+00	.0
10	10	-.25861-04	-.54385-04	-.25861-04	.54385-04	-.12000+01	.0
2	11	-.25278-04	-.54351-04	-.25278-04	.54351-04	-.24000+01	.0
4	11	-.24170-04	-.54269-04	-.24170-04	.54269-04	-.48000+01	.0
6	11	-.23137-04	-.54173-04	-.23137-04	.54173-04	-.72000+01	.0
8	11	-.22173-04	-.54065-04	-.22173-04	.54065-04	-.96000+01	.0
10	11	-.21272-04	-.53949-04	-.21272-04	.53949-04	-.12000+02	.0
2	12	-.17571-04	-.53309-04	-.17571-04	.53309-04	-.24000+02	.0
4	12	-.13014-04	-.52150-04	-.13014-04	.52150-04	-.48000+02	.0
6	12	-.10788-04	-.51428-04	-.10788-04	.51428-04	-.72000+02	.0
8	12	-.99590-05	-.51132-04	-.99590-05	.51132-04	-.96000+02	.0
10	12	-.10033-04	-.51158-04	-.10033-04	.51158-04	-.12000+03	.0
2	13	-.16912-04	-.53167-04	-.16912-04	.53167-04	-.24000+03	.0
4	13	-.41044-04	-.53040-04	-.41044-04	.53040-04	-.48000+03	.0
6	13	-.68684-04	-.36979-04	-.68684-04	.36979-04	-.72000+03	.0
8	13	-.13930-03	.00000	-.55515-04	.00000	-.96000+03	.0
10	13	-.20797-03	.00000	-.45243-04	.00000	-.12000+04	.0
2	14	-.51506-03	.00000	-.34798-04	.00000	-.24000+04	.0

	14	-.11167-02	.00000	-.31426-04	.00000	-.48000+04	.00
	14	-.17170-02	.00000	-.30457-04	.00000	-.72000+04	.00
	14	-.23172-02	.00000	-.29996-04	.00000	-.96000+04	.00
	14	-.29173-02	.00000	-.29727-04	.00000	-.12000+05	.00
	15	-.59174-02	.00000	-.29204-04	.00000	-.24000+05	.00
	15	-.11918-01	.00000	-.28949-04	.00000	-.48000+05	.00
	15	-.17918-01	.00000	-.28866-04	.00000	-.72000+05	.00
	15	-.23918-01	.00000	-.28824-04	.00000	-.96000+05	.00
	15	-.29918-01	.00000	-.28799-04	.00000	-.12000+06	.00
2	8	-.25980-02	.00000	-.25814-04	-.52554-04	-.25814-04	.52
4	8	-.51974-02	.00000	-.26077-04	-.52414-04	-.26077-04	.52
6	8	-.77973-02	.00000	-.26163-04	-.52366-04	-.26163-04	.52
8	8	-.10397-01	.00000	-.26205-04	-.52341-04	-.26205-04	.52
0	8	-.12997-01	.00000	-.26230-04	-.52327-04	-.26230-04	.52
2	9	-.25997-01	.00000	-.26276-04	-.52297-04	-.26276-04	.52
4	9	-.51997-01	.00000	-.26291-04	-.52282-04	-.26291-04	.52
6	9	-.77997-01	.00000	-.26288-04	-.52276-04	-.26288-04	.52
8	9	-.10400+00	.00000	-.26281-04	-.52273-04	-.26281-04	.52
0	9	-.26274-04	-.52271-04	-.26274-04	.52271-04	-.13000+00	.00
2	10	-.26222-04	-.52266-04	-.26222-04	.52266-04	-.26000+00	.00
4	10	-.26114-04	-.52259-04	-.26114-04	.52259-04	-.52000+00	.00
6	10	-.26007-04	-.52253-04	-.26007-04	.52253-04	-.78000+00	.00
8	10	-.25898-04	-.52247-04	-.25898-04	.52247-04	-.10400+01	.00
0	10	-.25791-04	-.52241-04	-.25791-04	.52241-04	-.13000+01	.00
2	11	-.25264-04	-.52209-04	-.25264-04	.52209-04	-.26000+01	.00
4	11	-.24265-04	-.52133-04	-.24265-04	.52133-04	-.52000+01	.00
6	11	-.23335-04	-.52046-04	-.23335-04	.52046-04	-.78000+01	.00
8	11	-.22467-04	-.51949-04	-.22467-04	.51949-04	-.10400+02	.00
0	11	-.21659-04	-.51845-04	-.21659-04	.51845-04	-.13000+02	.00
2	12	-.18358-04	-.51289-04	-.18358-04	.51289-04	-.26000+02	.00
4	12	-.14382-04	-.50324-04	-.14382-04	.50324-04	-.52000+02	.00
6	12	-.12558-04	-.49769-04	-.12558-04	.49769-04	-.78000+02	.00
8	12	-.12024-04	-.49593-04	-.12024-04	.49593-04	-.10400+03	.00
0	12	-.12322-04	-.49692-04	-.12322-04	.49692-04	-.13000+03	.00
2	13	-.19826-04	-.51563-04	-.19826-04	.51563-04	-.26000+03	.00
4	13	-.44410-04	-.49907-04	-.44410-04	.49907-04	-.52000+03	.00
6	13	-.72229-04	-.29030-04	-.72229-04	.29030-04	-.78000+03	.00
8	13	-.15102-03	.00000	-.51080-04	.00000	-.10400+04	.0
0	13	-.21742-03	.00000	-.43206-04	.00000	-.13000+04	.0
2	14	-.52329-03	.00000	-.34235-04	.00000	-.26000+04	.0
4	14	-.11247-02	.00000	-.31198-04	.00000	-.52000+04	.0
6	14	-.17250-02	.00000	-.30314-04	.00000	-.78000+04	.0
8	14	-.23252-02	.00000	-.29892-04	.00000	-.10400+05	.0
0	14	-.29253-02	.00000	-.29645-04	.00000	-.13000+05	.0
2	15	-.59254-02	.00000	-.29164-04	.00000	-.26000+05	.0
4	15	-.11925-01	.00000	-.28930-04	.00000	-.52000+05	.0
6	15	-.17925-01	.00000	-.28853-04	.00000	-.78000+05	.0
8	15	-.23925-01	.00000	-.28815-04	.00000	-.10400+06	.0
0	15	-.29925-01	.00000	-.28792-04	.00000	-.13000+06	.0
2	8	-.27980-02	.00000	-.25777-04	-.50612-04	-.25777-04	.5
4	8	-.55976-02	.00000	-.26003-04	-.50487-04	-.26003-04	.5
6	8	-.83974-02	.00000	-.26077-04	-.50444-04	-.26077-04	.5
8	8	-.11197-01	.00000	-.26114-04	-.50423-04	-.26114-04	.5
10	8	-.13997-01	.00000	-.26135-04	-.50410-04	-.26135-04	.5
2	9	-.27997-01	.00000	-.26174-04	-.50383-04	-.26174-04	.5
4	9	-.55997-01	.00000	-.26186-04	-.50370-04	-.26186-04	.5
6	9	-.83997-01	.00000	-.26184-04	-.50365-04	-.26184-04	.5
8	9	-.11200+00	.00000	-.26177-04	-.50362-04	-.26177-04	.5
10	9	-.26169-04	.50360-04	-.26169-04	-.50360-04	-.14000+00	.0
2	10	-.26123-04	-.50356-04	-.26123-04	.50356-04	-.28000+00	.0
4	10	-.26026-04	-.50348-04	-.26026-04	.50348-04	-.56000+00	.0

.	10	-.25927-04	-.50343-04	-.25927-04	.50343-04	-.84000+00	.00
8	10	-.25828-04	-.50337-04	-.25828-04	.50337-04	-.11200+01	.00
0	10	-.25731-04	-.50331-04	-.25731-04	.50331-04	-.14000+01	.00
2	11	-.25252-04	-.50300-04	-.25252-04	.50300-04	-.28000+01	.00
4	11	-.24346-04	-.50230-04	-.24346-04	.50230-04	-.56000+01	.00
6	11	-.23504-04	-.50150-04	-.23504-04	.50150-04	-.84000+01	.00
8	11	-.22720-04	-.50062-04	-.22720-04	.50062-04	-.11200+02	.00
0	11	-.21990-04	-.49970-04	-.21990-04	.49970-04	-.14000+02	.00
2	12	-.19032-04	-.49483-04	-.19032-04	.49483-04	-.28000+02	.00
4	12	-.15555-04	-.48675-04	-.15555-04	.48675-04	-.56000+02	.00
6	12	-.14075-04	-.48251-04	-.14075-04	.48251-04	-.84000+02	.00
8	12	-.13793-04	-.48165-04	-.13793-04	.48165-04	-.11200+03	.00
0	12	-.14285-04	-.48314-04	-.14285-04	.48314-04	-.14000+03	.00
2	13	-.22324-04	-.50014-04	-.22324-04	.50014-04	-.28000+03	.00
4	13	-.47295-04	-.46864-04	-.47295-04	.46864-04	-.56000+03	.00
6	13	-.75272-04	-.19319-04	-.75272-04	.19319-04	-.84000+03	.00
8	13	-.16034-03	.00000	-.48010-04	.00000	-.11200+04	.00
0	13	-.22536-03	.00000	-.41626-04	.00000	-.14000+04	.00
2	14	-.53034-03	.00000	-.33767-04	.00000	-.28000+04	.00
4	14	-.11316-02	.00000	-.31005-04	.00000	-.56000+04	.00
6	14	-.17319-02	.00000	-.30192-04	.00000	-.84000+04	.00
8	14	-.23320-02	.00000	-.29804-04	.00000	-.11200+05	.00
0	14	-.29321-02	.00000	-.29576-04	.00000	-.14000+05	.00
2	15	-.59322-02	.00000	-.29131-04	.00000	-.28000+05	.00
4	15	-.11932-01	.00000	-.28914-04	.00000	-.56000+05	.00
6	15	-.17932-01	.00000	-.28842-04	.00000	-.84000+05	.00
8	15	-.23932-01	.00000	-.28806-04	.00000	-.11200+06	.00
0	15	-.29932-01	.00000	-.28785-04	.00000	-.14000+06	.00
2	8	-.29981-02	.00000	-.25740-04	-.48869-04	-.25740-04	.48
4	8	-.59977-02	.00000	-.25937-04	-.48757-04	-.25937-04	.48
6	8	-.89976-02	.00000	-.26002-04	-.48719-04	-.26002-04	.48
8	8	-.11998-01	.00000	-.26033-04	-.48699-04	-.26033-04	.48
0	8	-.14997-01	.00000	-.26052-04	-.48688-04	-.26052-04	.48
2	9	-.29997-01	.00000	-.26086-04	-.48664-04	-.26086-04	.48
4	9	-.59997-01	.00000	-.26096-04	-.48652-04	-.26096-04	.4
6	9	-.89997-01	.00000	-.26093-04	-.48648-04	-.26093-04	.4
8	9	-.12000+00	.00000	-.26087-04	-.48645-04	-.26087-04	.4
0	9	-.26079-04	.48643-04	-.26079-04	-.48643-04	-.15000+00	.0
2	10	-.26037-04	-.48638-04	-.26037-04	.48638-04	-.30000+00	.
4	10	-.25948-04	-.48633-04	-.25948-04	.48633-04	-.60000+00	.
6	10	-.25857-04	-.48626-04	-.25857-04	.48626-04	-.90000+00	
8	10	-.25768-04	-.48621-04	-.25768-04	.48621-04	-.12000+01	.0
0	10	-.25679-04	-.48615-04	-.25679-04	.48615-04	-.15000+01	.0
2	11	-.25243-04	-.48586-04	-.25243-04	.48586-04	-.30000+01	.0
4	11	-.24417-04	-.48520-04	-.24417-04	.48520-04	-.60000+01	.0
6	11	-.23650-04	-.48446-04	-.23650-04	.48446-04	-.90000+01	.0
8	11	-.22939-04	-.48367-04	-.22939-04	.48367-04	-.12000+02	.0
10	11	-.22278-04	-.48284-04	-.22278-04	.48284-04	-.15000+02	.0
2	12	-.19617-04	-.47855-04	-.19617-04	.47855-04	-.30000+02	.0
4	12	-.16571-04	-.47175-04	-.16571-04	.47175-04	-.60000+02	.0
6	12	-.15390-04	-.46855-04	-.15390-04	.46855-04	-.90000+02	.0
8	12	-.15327-04	-.46837-04	-.15327-04	.46837-04	-.12000+03	.0
10	12	-.15986-04	-.47020-04	-.15986-04	.47020-04	-.15000+03	.0
2	13	-.24489-04	-.48527-04	-.24489-04	.48527-04	-.30000+03	.0
4	13	-.49795-04	-.43903-04	-.49795-04	.43903-04	-.60000+03	.0
6	13	-.84779-04	-.34283-06	-.84779-04	.34283-06	-.90000+03	.0
8	13	-.16804-03	.00000	-.45727-04	.00000	-.12000+04	.0
10	13	-.23213-03	.00000	-.40363-04	.00000	-.15000+04	.0
2	14	-.53643-03	.00000	-.33373-04	.00000	-.30000+04	.0
4	14	-.11376-02	.00000	-.30840-04	.00000	-.60000+04	.0
6	14	-.17378-02	.00000	-.30068-04	.00000	-.90000+04	.0

14	-.23379-02	.00000	-.29727-04	.00000	-.12000+05	.00
14	-.29380-02	.00000	-.29515-04	.00000	-.15000+05	.00
15	-.59381-02	.00000	-.29101-04	.00000	-.30000+05	.00
15	-.11938-01	.00000	-.28899-04	.00000	-.60000+05	.00
15	-.17938-01	.00000	-.28832-04	.00000	-.90000+05	.00
15	-.23938-01	.00000	-.28799-04	.00000	-.12000+06	[illegible]
15	-.29938-01	.00000	-.28779-04	.00000	-.15000+06	[illegible]

```
UNID: TAMER    ACCT: 111-16-201      PROJECT: THESIS
IME:      TOTAL: 00:01:00.958      CBSUPS:  000002388
          CPU:   00:00:29.762      I/O:  00:00:13.748
          CC/ER: 00:00:17.447      WAIT: 00:00:00.150
UAS USED:      81.69TL    SUAS REMAINING:       0.00TL
BOVE CHARGE CALCULATED AT FOLLOWING RATES -
  CBSUP              = 0.02TL
  CARD READ          = 0.05TL
  CARD PUNCHED       = 0.40TL
  PAGE PRINTED       = 1.50TL
  TAPE I/O MINUTE    = 1.50TL
MAGES  READ:    215        PAGES:  18
TART:     12:06:39 JUL 15,1980    FIN:  12:09:22 JUL 15,1980
```