IN SEARCH OF CHAOS:

‘THE CASE OF ISTANBUL STOCK EXCHANGE’

FATİH KİRAZ

BOĞAZİÇİ UNIVERSITY

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Fatih Kiraz

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Thesis Abstract

Fatih Kiraz, “In Search of Chaos: The Case of Istanbul Stock Exchange”

There is increasing interest and an ongoing debate on the return behavior of financial markets. The problem is that there is no sufficient evidence about successful modeling of stock returns yet. In fact, the possibility of a successful prediction model itself is still open for discussion. Financial time series are complex, noisy, and random-looking. Linear modeling attempts have always failed whereas nonlinear ones have achieved only little. At this point, ‘chaos theory’ may provide some, if not all, answers we have been looking for. Finding a deterministic structure in a system implies that a successful prediction model is theoretically possible. Furthermore, being able to identify that structure’s characteristics, e.g. fractal dimension, means that such a model is practically possible as well.

This thesis examines the return behavior of Istanbul Stock Exchange index (ISE100) in the light of ‘chaos theory’, which is almost totally missing in the current literature. The time period covered is the last eleven years, from 01.01.1998 to 16.12.2008. The main return series were created by adding one index level at every tenth second and then by calculating the logarithmic differences of the consecutive values.

As a summary of the findings, there is yet no reason that prevents us from imagining the stock returns as different weather conditions. Successful short term predictions are *theoretically* possible but it becomes impossible to speak thoroughly about the long term. However, to become the true ‘meteorologist’ of the financial markets, one first has to develop an effective nonlinear noise filtering method which does not distort the original data and is still capable of thoroughly capturing the hidden signal in it. In the absence of such a good filtering method, the true ‘meteorologist’ becomes an ordinary ‘fisherman’ who has to rely on his/her luck at some point!

Tez Özeti

Fatih Kiraz, “Kaosun Peşinde: İstanbul Menkul Kıymetler Borsası Vakası”

Finansal piyasalardaki getiri davranışlarına olan ilgi ve konuyla ilgili tartışmalar gittikçe artmaktadır. Sorun başarılı bir getiri modeli için elimizde henüz yeterli delil olmamasından kaynaklanmaktadır. Aslında başarılı bir tahmin modelinin olası olup olmadığı da tartışmaya açık bir konudur. Finansal zaman serileri karmaşık, gürültülü ve rasgele görünümlüdür. Doğrusal modelleme çabaları herzaman başarısız olurken, doğrusal olmayan modelleme çabaları çok az sonuç verebilmiştir. Bu noktada, ‘kaos teorisi’ aradığımız cevapların hepsini değilse bile bir kısmını bize verebilir. Bir sistemin içinde deterministik bir yapı bulmak demek, aynı zamanda o sistem için başarılı bir modelin teorik olarak mümkün olması demektir. Dahası bulunan yapının karakteristikleri de belirlenebilirse, mesela fraktal boyutu gibi, bu başarılı bir modelin pratik olarak da mümkün olduğu anlamına gelir.

Bu tez İstanbul Menkul Kıymetler Borsası 100 endeksinin (IMKB100) getiri davranışını kaos teorisi ışığında incelemektedir ve mevcut literatürde bir benzeri yoktur. Kapsanan zaman periyodu 01.01.1998 den 16.12.2008 e kadar, yani geçtiğimiz onbir senedir. Asıl getiri serileri, her on saniyede bir alınan endeks seviyelerinin logaritmik farkları alınarak oluşturulmuştur.

Tüm bulguların özeti olarak, hisse getirilerini farklı hava koşulları gibi hayal etmemizi engelleyen geçerli bir sebep bulunamadığı söylenebilir. Başarılı kısa dönem tahminleri yapmak teorik olarak mümkün gözükürken, uzun dönem hakkında çok tutarlı konuşabilmek imkansız gibidir. Fakat finansal piyasalarin gerçek ‘meteoroloğu’ olabilmek isteyen kişi önce, orjinal veriyi çok değiştirmeyen ama yine de içerde saklı sinyali doğru bir şekilde yakalayabilecek kadar etkili bir doğrusal olmayan gürültü filtreleme metodu geliştirmek zorundadır. Böyle bir filtreleme metodunun yokluğunda, gerçek ‘meteorolog’, bir noktada şansına güvenmek zorunda kalacak olan sıradan bir ‘balıkçı’ya dönüşür!

CURRICULUM VITAE

NAME of AUTHOR: Fatih Kiraz
PLACE of BIRTH: Kütahya, Turkey
DATE of BIRTH: 02 August 1978

GRADUATE and UNDERGRADUATE SCHOOLS ATTENDED:
Yale University, August 2007 - June 2008, visiting scholar
Boğaziçi University
Dokuz Eylül University

DEGREES AWARDED:
Doctor of Philosophy in Finance, 2009, Boğaziçi University
MBA, 2002, Boğaziçi University
B Sc. in Management, 2000, Dokuz Eylül University

AREAS of SPECIAL INTEREST:
Time series analysis and Human mind

PROFESSIONAL EXPERIENCE:
Research Assistant, Department of Management, Boğaziçi University, 2000-2009.

AWARDS and HONORS:
Yale University, Fox International Fellowship, 2007-2008.

PUBLICATIONS:
Performance Analysis of Foreign Portfolio Investors in the Istanbul Stock Exchange (with Çetin Ali Dönmez and Abdülmecit Karataş), *İşletme ve Finans*, 225, Dec. 2004.

An Empirical Analysis of Foreign Traders and Their Portfolio Performances and the Effects of Foreign Capital Flow on Market Dynamics in the Istanbul Stock Exchange Equity Market. (with Çetin Ali Dönmez), *ERC/METU International Conference in Economics VII, September, 2003*.

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As may be understood, football has a great role in my life so I have to thank my football teammates for providing me with at least one excellent hour in every week and Beşiktaş JK for all the excitement and the double cups they gifted me!

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Finally and most importantly, all my thanks go to my family for everything…

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CHAPTER I

INTRODUCTION

There is increasing interest and an ongoing debate on the return behavior of financial markets. The problem is that there is no sufficient evidence about successful modeling of stock returns yet. In fact, the possibility of a successful prediction model itself is still open for discussion. Financial time series are complex, noisy, and random-looking. Linear modeling attempts have always failed whereas nonlinear ones have achieved only little. At this point, ‘chaos theory’ may provide some, if not all, answers we have been looking for. Random-looking and unpredictable behavior of deterministic systems has been called ‘chaos’ and it has already been called the next great revolution in science by some scientists.

This thesis is an attempt to provide new and significant evidence for the resolution of the debate on return series modeling. It truly examines the return behavior of Istanbul Stock Exchange index (ISE100) in the light of ‘chaos theory’, which is almost totally missing in the current literature. The time period covered is the last eleven years, from 01.01.1998 to 16.12.2008. The return series were created by adding one index level every tenth second (tick data) and then by calculating the logarithmic differences of the consecutive values. This is the common practice in the current literature, without any exception. However, when creating samples from this raw return series, the studies in the area differ among themselves. Some earlier studies (Brock & Sayers, 1988; Scheinkman

& LeBaron, 1989) utilize daily/weekly closing returns whereas more recent ones (Peters, 1994; Brock et al., 1996) focus more on tick data but at different frequencies generally ranging from one second to ten minutes and for different time periods generally ranging from one day to one month. This thesis has four main sample categories for tick data (one day, three days, five days, and ten days) and two main categories for the closing levels (daily and half daily) for 11 years. This sampling procedure aims to match, in general, the sample characteristics of the existing studies, enabling a sound comparison of the findings.

This thesis attempts to reveal if there are exploitable deterministic structures in ISE100 return series or not. Finding a deterministic structure in a system implies that a successful prediction model is theoretically possible. Furthermore, being able to identify that structure's characteristics, e.g. fractal dimension, means that such a model is practically possible as well. There are hundreds of serious attempts in the current literature, searching for random-looking deterministic behavior in financial time series. Although they differ in their findings and/or their own interpretations to some degree, they agree on some points; that is, the necessary conditions for random-looking deterministic behavior, some well-defined procedures to test these conditions (Barnett et al., 1997; Barnett & Serletis, 2000)**,** and the presence of some strange nonlinearity in financial time series (Hsieh, 1991). Initial research, including Frank and Stengos (1988), Scheinkman and LeBaron (1989), Hsieh (1991), Peters (1991), Mayfield and Mizrach (1992), Mandelbrot (1999), and many others, often found strong evidence for nonlinear dependence, but evidence for chaotic dynamics was somewhat inconclusive (Barnett & Serletis, 2000). On the other hand, Malliaris and Stein (1999) and Barnett and Serletis

(2000) stressed the exploitability of deterministic dynamics in financial time series and provided evidence against the initial findings above. It is almost certain that financial time series contain a combination of deterministic and stochastic behavior (Tino et al., 2001). In searching for the traces of that determinism, this thesis utilizes a unique ‘path’, a semi-auto ‘chaos detection algorithm’, developed by the author by editing and bridging some well-defined tests; BDS, WNN, TNN, Runs, Average Mutual Information, False Nearest Neighbors, Space Time Separation, Kaplan’s Determinism, Jacobian based Lyapunov Spectrum, and Correlation Dimension.

As a summary, this thesis basically shares the motivations and the sampling procedures of the studies in the area. However, its scope is broader**.** Instead of focusing on one type of sample category as most of other studies did, it analyses different possible sample categories at the same time, enabling a direct comparison between categories. On the other hand, it is significantly different from the existing studies for two reasons; first, it provides new and direct evidence from an emerging market (ISE) which is missing in the current literature and second, it introduces a new approach by providing a unique path which is a logical combination of the well-defined procedures in the area.

The next chapter provides a detailed background about both financial series and the theory of chaos. The third chapter presents a literature survey including the evolution of theory in; general, economics, and finance. Concluding part of this chapter truly summarizes the author’s feelings and intuitions towards the subject. The fourth chapter discusses the methodology of the thesis in detail. It is presented as three main headings which are; ‘data’, ‘tests of nonlinearity and chaos’, and ‘path’. This chapter also describes the semi-auto algorithm developed for the purposes of this thesis. Findings of the thesis are presented and interpreted in the fifth chapter. The sixth and final chapter concludes the thesis by summarizing the central mission, major findings, main contributions, limitations, and future research directions.

CHAPTER II

BACKGROUND

Finance

Financial Time Series

The last decades have witnessed very significant changes in finance, both as an academic subject and as a professional field. As thoroughly stated by Bouchaud (2002), finance is becoming an empirical science and so the financial engineers have an increasingly important role to play in the financial industry. There are obvious reasons for this change, but the most important must surely be the availability of the data, and the possibility to access and process this data very quickly. Everyone has now direct access to high-frequency data for stocks and for even more exotic markets such as option markets, weather derivatives, etc. This fact implies that any statistical model or theoretical idea can be tested.

Financial time series, where a trace of human activity is recorded and stored almost every second in a quantitative way, provide us with an excellent laboratory. They are more detailed than even the historical records of Heredot or the travel book of Evliya Çelebi. One can use those series to find answers to a lot of questions: What are the statistical features of a financial time series? Can physics based statistical tools and models help in developing a good model for price changes? If we can develop such a model, what is it useful for? Do we understand the basic mechanisms that are responsible for the observed anomalies and their universality across markets and time periods? These are just some of the questions awaiting answers. The scope of this thesis is more limited. The first thing this thesis does is to answer the question “is it worth dealing with chaos theory in order to understand ISE better?”. As the second thing, it provides a valid and fast tool for chaos detection.

Stylized facts about Financial Time Series

The study of return behaviors on many different assets has revealed a number of strong features which are often called ‘stylized facts’. Cont (2001) and Bouchaud (2002) discuss them in more detail. Most relevant ones are summarized below:

• Relative price changes (logreturns) are uncorrelated beyond a time scale of the order of tens of minutes on liquid markets. This means that the square of the logreturn grows linearly with time.

• The distribution of logreturns is strongly non-gaussian: these distributions can be characterized by Pareto (power-law) tails with an exponent close to 3 for rather liquid markets (Gopikrishnan et al, 1999; Lux, 1996; Dacorogna et al, 1993; Guillaume et al, 1997; Longin et al, 1996). Emerging markets have even more extreme tails, with an exponent that can be less than two, in which case the volatility is infinite.

• Localized outbursts of volatility, known as volatility clustering (Ding et al, 1993; Mantegna et al, 1999), is very similar to the intermittent fluctuations in turbulent flows (Frisch, 1997). The temporal correlation function of the daily volatility can be fitted by an inverse power of the lag, with a small exponent in the range 0.1 – 0.3 (Potters et al, 1998; Liu et al, 1997; Cizeau et al, 1997; Muzy et al, 2000). This slow decay of the volatility correlation function leads to a multifractal-like behavior of price changes (Schmitt et al, 1998; Bouchaud et al, 1999)

• Past price changes and future volatilities are negatively correlated (leverage effect). This correlation is most visible on stock indices and is characterized by a time scale of the order of 10 days (Bouchaud et al, 2001). This leverage effect leads to an abnormal negative skew in return distribution as a function of time (Fouque, 2000).

• There is an apparent increase of inter-stock correlations in volatile periods (Longin et al, 1999). Most of the eigenvalues of the full correlation matrix are well accounted for by a random matrix theory (Laloux et al, 1999; Plerou et al, 1999).

• Interest rates with different maturities evolve in an interesting correlated manner, similar to the motion of an elastic string subject to noise (Bouchaud et al, 1999; Matacz et al, 2000). It means that prices behave very differently from the simple geometric Brownian motion which is often assumed in mathematical finance. In other words, extreme events are much more probable.

Chaos

Brief History of Chaos Theory

Chaos can simply be defined as ‘random-looking deterministic behavior‘ which exhibits ‘extreme sensitivity to initial conditions‘. Its most surprising and attractive gift may be showing us the fact that even some very simple expressions can lead to amazingly complicated and beautiful outcomes.

The story starts in 1889 with Henri Poincaré (1854-1912). He produced a paper on the three-body problem. That was the first glimpse of Chaos Theory. He used the concept of a ‘phase space‘ which is a velocity versus position plot. But plotting the whole phase space was an enormous task, so Poincaré looked at a ’slice’ of the phase space. This slice is now called a ‘Poincaré Section‘. Such a section contains lots of points and by observing these; one can detect trends or chaotic behavior. His 200-page paper was error-ridden and had to be recalled for correction. It was during this second look that he identified a pattern in the Poincaré Section. Some regions became dense with points, while others had not. Analyzing in detail, it soon became clear that there is no way to predict the orbit of the bodies far into the future. The resulting 270 pages were published in Acta Mathematica. However, his work was not recognized as an indication of a new science until the 1970s.

An object important to chaos theory is the ‘attractor’. An attractor can be thought of as the final state of a phenomenon. The simplest example of an attractor is a point. If you release a ball from the brim of an inverted cone, the ball will settle at the vertex of the cone. The vertex is thus an attractor. Another common attractor is termed a ’limit cycle’. An example of this is our heartbeat. After exercise, our heartbeat starts slowing down until it settles into a regular beat. If there are two limit cycles, we get a ‘torus‘ instead of a circle. These attractors are well-behaved, i.e. they don’t give rise to chaos. However, there is at least one more type called ‘strange attractors‘ which is an integral part of chaos.

In 1961, meteorologist Edward Lorenz (1917- 2008) found that extremely small changes in initial conditions had a significant effect on the weather. He plotted phase space and observed that it looked like the wings of a butterfly. Although he did not name it a strange attractor, this is the first time such an attractor is observed. He had discovered the famous ‘Lorenz Attractor‘ or ‘Lorenz Butterfly‘.

Another strange attractor, Hénon Attractor, was discovered in 1962 by French astronomer Michel Hénon (1931-). He discovered this 'banana-shaped' attractor while working on star clusters. But until 1976, he did not realize he had got a strange attractor. Stephen Smale (1930-) showed, in the 1960s, that the phenomenon of strange attractors can be explained by a topological transformation in phase space. In the 1970s, Australian Robert May (1936-) used the logistic equation to show that biological populations could become chaotic. He discovered the ‘logistic attractor‘ as a result. His attractor is produced via a bifurcation process, in which a state splits into two other states. Mathematician Mitchell Feigenbaum (1945-) followed up on May's study and found there is order in the chaos that May had observed. The route to chaos was universal and there is a universal constant (approx. 4.66920160) involved. A few years later, naturally and finally, the relationship between chaos theory and fractal theory was set. All strange attractors are fractals which can be defined as the signatures of chaos.

Just to highlight the range of the chaos theory applications, consider the problem: why does a flock of birds seem to fly in harmony; why don't they collide with each other? In 1997, zoologist Frank H. Heppner used chaos theory to explain this phenomenon. He modeled the flocking behavior by 4 simple rules:

1. The birds are attracted to a focal point or roost.
2. They are attracted to each other.
3. They want to maintain a fixed speed.
4. Flight paths are altered by random occurrences, such as gusts of wind.

By varying the intensity of these rules, he could make his triangles (which represent the birds) flock in a bird-like fashion. This model could well be applied to other similar phenomena, such as the movement of a school of fish or a buffalo stampede. One can find similar examples in literature regarding many other disciplines. It seems that chaos theory has become an ‘attractor‘ itself among others.

Sensitivity to Initial Conditions

In the early 1960’s, Edward Lorenz, a meteorologist at the Massachusetts Institute of Technology, was developing models of atmospheric convection. His primitive computer took about one second per iteration. One day, he restarted one of his computer runs, using numbers rounded to three digits rather than the six significant figures demanded by the computer. He did so, because he was late for his coffee break! When he returned from the break, his views of globe and remaining life began to change dramatically.

For some time, the solutions of computer runs revealed the same graph but after a while they began to differ enormously until they became not related at all. At first glance, computer malfunction seemed to be the logical reason for the great discrepancy. But, in fact, he had discovered the ‘sensitivity to initial conditions‘ which is the most remarkable feature of chaos. He began working on his equations to determine the minimum conditions necessary for this strange behavior. The result is the now famous ‘Lorenz equations‘ which lead to the first example of a strange attractor (Figure 1):

$$X' = s(Y - X)$$

$$Y' = -XZ + rX - Y$$

$$Z' = XY - bZ$$

where s r and b are constants that Lorenz took to be s = 10 r = 28 and b = 8/3. Lorenz published his findings in 1963 in the Journal of the Atmospheric Sciences where they went largely unnoticed for the next decade. The title of his paper "Deterministic Non-periodic Flow" is an apt description of what we now call ‘chaos‘ (Sprott, 1993).

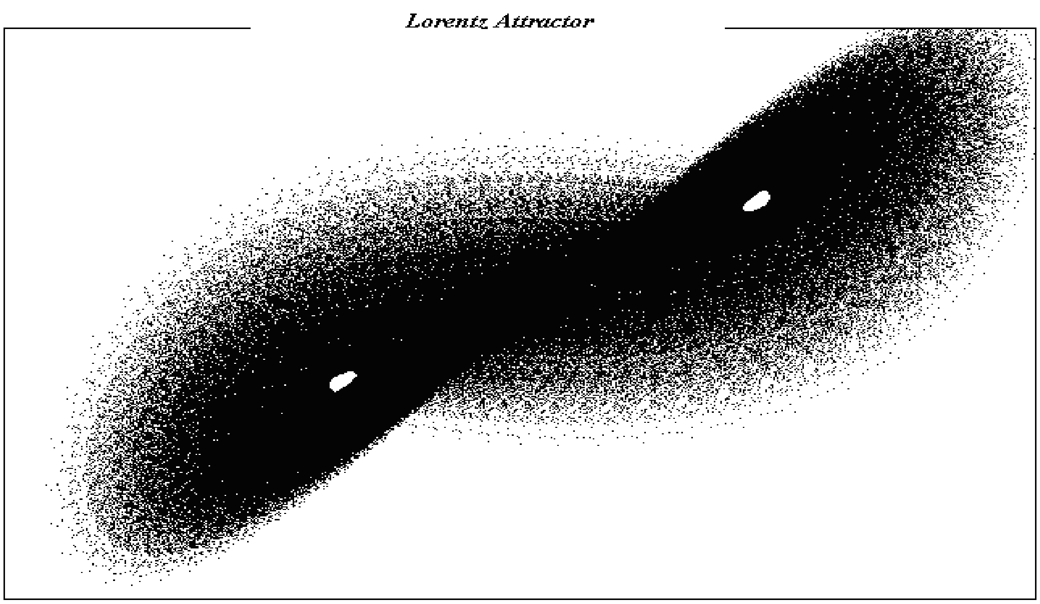

Figure 1 Lorenz attractor

Logistic Route to Chaos

Chaotic processes exhibit extreme sensitivity to initial conditions whereas in regular processes, different starting points usually converge to several points on a simple attractor. This fact can be exploited to develop a simple test for chaotic behavior in systems.

Think of two initial values of ‘r’ that differ by only a tiny amount. If successive iterates are attracted to a fixed point, the difference between the two solutions must get smaller and smaller as the fixed point is approached. This is also valid for a limit cycle.

If the solution is chaotic, the successive iterates for the two cases will not initially diverge much but then the difference will grow exponentially. If that difference, on average, doubles with each iteration, the ‘Lyapunov exponent‘ equals one. If it reduces to half, the Lyapunov exponent is minus one. Thus, the difference between the values changes by 2^L for each iteration. If L is negative, the solutions approach one another; if L is positive, it means sensitivity to initial conditions and hence chaos (Sprott, 1993). The Lyapunov exponent is the rate at which information is lost when a map is iterated.

N-dimensional Chaos

For two-dimensional chaotic iterated maps, the situation is more complicated than for one-dimensional maps. Let ‘p’ and ‘r’ be two initial points on a circle in a two-dimensional plane. After first iteration, the points would have a new position and they would form a region called an ellipse. The circle would contract in one direction and expand in the other. Therefore, two-dimensional chaotic maps do not have a single Lyapunov exponent but two, a positive one corresponding to the direction of expansion and a negative one corresponding to the direction of contraction. This can be generalized such that there is exactly one exponent for each of the ‘N’ possible dimensions in a system. All the exponents calculated are then reordered from the biggest (largest positive exponent) to the smallest (the negative exponent with the highest magnitude) to obtain the 'Lyapunov spectrum', a concept which is crucial for this thesis. It is discussed thoroughly in the 'methodology' and 'findings' chapters.

As Sprott (1993) nicely state: The signature of chaos is the existence of at least one positive Lyapunov exponent. Furthermore, the magnitude of the negative exponent has to be greater than the positive one so that initial conditions scattered throughout the basin of attraction contract onto an attractor that occupies a negligible portion of the plane. The area of the ellipse continually decreases even as it stretches to an infinite length (p. 54-55).

Predictability and Uncertainty

If nature is deterministic, then there is no room for free will. This implies perfect predictability which is boring and not much interesting. One possible escape was found in the early decades of the twentieth century; quantum mechanical laws are apparently probabilistic. This means that one can only predict the probability that something will happen. This was surprising because those laws are purely deterministic. After decades, we now know for certain that deterministic is not the same as predictable! Maybe the best example is the weather. The weather is governed by the atmosphere which obeys deterministic physical laws. However, although the observations are more detailed and the computers are more powerful now, accurate long-term weather predictions are still impossible. This phenomenon can be simply explained by revisiting the 'sensitivity to initial conditions' issue discussed earlier.

Chaos theory reconciles our intuitive sense of free will with the deterministic laws of nature. However it has an even deeper philosophical consequence. We have freedom to control our actions. Moreover, even our insignificant acts can dramatically change the future for better or for worse. The results of our behaviors are amplified day by day, finally resulting in a completely different world than would have existed in our absence.

CHAPTER III

LITERATURE SURVEY

Research on Nonlinear Dynamics

Research on nonlinear dynamics is concerned with the behavior of deterministic and stochastic nonlinear systems which have an implicit discrete or continuous time dimension. The field originates in the pioneering work of the mathematician Henri Poincaré at the turn of the century on the stability of the solar system (Poincaré, 1892 and 1899). Applied and theoretical research has flourished over the past two decades, mainly as a consequence of the widespread availability of cost-efficient computer power, and nonlinear dynamics are now playing an increasingly important role in the making of science and decisions. The applications are highly diverse across areas such as: Mathematics (Katok & Hasselblatt, 1995), Statistics (Chatterjee & Yilmaz, 1992), Bifurcation and Chaos theory (Shilnikov, 1997), Physics (Schuster et al., 1996), Biology (Dennis et al., 2001), Time Series Analysis (Tong, 1990; Kantz & Schreiber, 1997), Econometrics (Granger, 1995), Forecasting (LeBaron, 1994), Economics (Lorenz, 1989; Medio, 1993) and Finance (Scheinkman & LeBaron, 1989; Hsieh, 1991 and 1994). There are hundreds of other studies which could be listed here. These studies are just typical examples of their areas.

Meanings of the term nonlinear dynamics are different across scientific disciplines, sub-disciplines and time periods. This diversity of meanings is mainly due to the fact that no formal and thoroughly complete mathematical definition of chaotic systems exists. Broadly speaking, chaos is the mathematical condition whereby a simple nonlinear dynamical system produces very complex (random-like) behavior (Mills & Markellos, 2008). Even though these systems are deterministic (finite phase space dimension), they are completely unpredictable in the long-run. The reason is 'sensitive dependence on initial conditions' (or Lyapunov instability). Another characteristic of chaotic systems is that they invariably exhibit power-law behavior (continuous, broadband and power-law declining spectral density) and have 'fractal' (self-similar) pictorial representations. From a mathematical point of view, chaos theory can be broadly grouped along with Thom's catastrophe theory under the field of bifurcation theory.

Nonlinear Dynamics in Economics

Theoretically and practically, chaos is a natural choice for explaining complex empirical behavior for many nonlinear systems in the natural sciences which are low-dimensional. This is because in deterministic systems, the standard types of dynamic behavior are limited to fixed-point equilibria and limit cycles, and hence complexity has a limited and rather exotic role to play in the context of stochastic linear and nonlinear dynamics (Mills & Markellos, 2008). Thus, it is perhaps not surprising that applications of chaos theory in economics have been less limited and successful than in the natural sciences. Nevertheless, the amount of literature that has been concerned with chaos in economics is considerable and interest continues to persist. Most of this attention was motivated by the ability of chaotic systems to produce complicated behavior without resorting to exogenous stochastic factors and shocks.

Regarding chaos in the literature of economics, there are two main categories: The first is called ‘mathematical chaos’, demonstrating that specific economic model configurations can produce chaotic behavior (Boldrin & Woodford, 1988). However, because of the underlying strong assumptions of those models, the studies in this category are highly questionable (Granger et al., 1995). The studies in the second category are model-free and use nonparametric procedures to test actual economic time series for signs of chaotic behavior (Scheinkman & LeBaron, 1989). Although some studies claimed to have found ‘empirical’ chaos, much of these evidence are not considered as fully conclusive since the testing procedures used are susceptible to severe problems with respect to: autocorrelation (Theiler & Eubank, 1993), sample size (Chappell et al., 1996), nonstationarities (Brock et al., 1991), aggregation (Barnett et al., 1992), microstructures (Krämer & Runde, 1997), mean reversion and seasonalities (Brock, 1988), and long-run dependencies (Theiler, 1991).

In summary, for the economics case, there is an ongoing debate between two distinct groups; the ones who are against the utilization of chaos theory, in its current form, to macro-economic variables and the ones who are for it. The latter group seems to be losing the game. They need some innovations or at least clever modifications. However, for the case of finance, the game is still much more balanced and undecided.

The Case of Finance

When compared to economic data which are generally short and nonstationary, financial series provide potentially longer and cleaner series (for making predictions and performing out of sample testing). Also, the obvious potential monetary gains to forecasting stock price and foreign exchange rate series have drawn a lot of interest (LeBaron, 1994). Initial researches, including Frank & Stengos (1988), Hsieh (1991), Mayfield & Mizrach (1992), Peters (1991), Scheinkman & LeBaron (1989), Mandelbrot (1999), and many others, utilized the tests similar to those used for macroeconomic time series. The results often found strong evidence for nonlinear dependence, but evidence for chaotic dynamics was somewhat inconclusive (Barnett & Serletis, 2000). On the other hand, Malliaris & Stein (1999) and Barnett & Serletis (2000) stressed the exploitability of deterministic dynamics in financial time series and provided evidence against the initial findings above.

Hsieh (1991), after a detailed analysis of nonlinearity in financial indices, found evidence that stock market log-returns are not independent and identically distributed (i.i.d.), and also the deterministic fluctuation in volatility could be predicted. Logically, if anyone wants to exploit nonlinear determinism in financial time series, he/she should first be able to demonstrate that it exists. In an effort to quantify determinism, predictability and nonlinearity, a variety of nonlinear measures have been applied to a vast range of economic and financial time series. Typical examples include the estimation of Lyapunov exponents (Dechert & Gencay, 1992); the correlation dimension (Hsieh, 1991); the closely related BDS statistic (Brock et al., 1996); mutual information and other complexity measures (Darbellay & Wuertz 2000); and nonlinear predictability (Yao & Tong, 1994). The rationale of each of these reports is to apply a measure of nonlinearity or "chaoticity" to the supposed logreturns. Deviation from the expected statistic values for an i.i.d. process is then taken as a sign of nonlinearity (Small & Tse, 2003).

Definitions and details of the tests, mentioned above (and some other tests), are available in the second section of chapter IV.

It is almost certain that financial time series contain a combination of deterministic and stochastic behavior (Schittenkopf, Dorffner, & Dockner, 2001). The problem is that researchers do not usually have access to the expected distribution of statistic values for a noise process. Within the context of determinism, whether or not utilizing noise reduction procedures (and if yes, which methods are appropriate and when) is a pretty much undecided subject. Some studies claim that linear filtering procedures either remove deterministic nonlinearity (Theiler & Eubank, 1993) or introduce spurious determinism in a time series (Mees & Judd, 1993). Traditional linear filters are based on the assumption that signal and noise components can be distinguished in the spectrum. For coarsely sampled signals from nonlinear systems, this poses a problem since the signal itself can have a broad-band spectrum. In this case one has to apply nonlinear methods in order not to distort the signal (Schreiber, 1993). Surely, the alternative is not applying any filter, which is also a not very attractive choice. This issue is discussed in more detail in the methodology part of the thesis.

As a summary of the findings mentioned up to now in this finance section, it can be claimed that the evidence presented, as a whole, can be classified neither in favor of chaos theory nor against it. A possible idea that ‘we have already gained some direct benefits from this theory’ may be still open for debate but it would be improper to ignore the indirect benefits already gained:

Firstly, from a philosophical point of view (see also Chatterjee and Yilmaz, 1992), chaos has its place along a long list of undecidable propositions that includes

Gödel's undecidability proposition in logic, Heisenberg's uncertainty principle in quantum mechanics and Arrow's impossibility theorem on definitions of social rationality and numerical investigation of smooth three-dimensional systems.

Secondly, one must keep in mind that a complicated mathematical problem, such as chaos, may not have apparent practical implications yet but it will almost always inspire the creation of powerful mathematics, a prime example being the research on Fermat's Last Theorem. Invariably and eventually, the new mathematics finds important applications across several different fields and has strong multiplier effects. Some examples include; topological time series analysis (Tong, 1990), nearest neighbor modeling (Satchell & Timmermann, 1995), singular value decomposition (Lisi & Medio, 1997), BDS test of serial independence (Brock et al., 1996). The complex and rich behavior of chaotic systems have also inspired the creation of many innovative *theoretical conceptualizations* that involve nonlinearities and discontinuities.

Finally, as noted by Tsay (1992), "Chaos theory is an 'eye-opener' for statisticians and probabilists. It points out loudly and clearly the need to explore nonlinearity and to develop statistical methods and tools that can adequately analyze nonlinear models. The linear world is very limited" (p.113).

The reader may rightfully think that this is an unusual and brief survey for a thesis. However, there is at least one good reason to write it this way; *not to confuse the reader's mind more than necessary*. Within the context of social sciences, the subject on hand is like a cute baby who just began to talk and could complete only several sentences. So, almost everyone in the field is attracted to it (either as a developer or an observer) and a lot of them have their own beliefs, thoughts, and/or suggestions about it.

There are still hundreds of studies which could well be mentioned here. However, most of them can't be thoroughly compared to the rest (due to differences in; the goals, the tests proposed, the data sets, the periods, the sample sizes and frequencies…etc.), making it impossible to draw a desirably clear picture. It may really be an a bit exaggerated expression but *it is like we all are throwing different paint balls through an almost empty canvas*! However, this is usually how science evolves. Inclusion of many smart and (almost) unbiased people always enables not only many creative innovations but also the quick rejections of time wasting ones. Keeping this fact and the philosophical appeal of the chaos in mind, I am almost sure that we will get a master piece on that canvas in the end and that time is not very far away! The baby will become a child soon…

CHAPTER IV

METHODOLOGY

This chapter is presented as three main headings which are; ‘data’ (description of data, sample creation and noise reduction processes), ‘tests of nonlinearity and chaos’ (discussion of the available tests and the author’s choices among them), and ‘path’ (a list of procedures applied in this thesis in its logical order).

To perform every step of this analysis (even at one run, if desired) and to make it as fast and accurate as possible, a semi-auto (which requires user input at several points) chaos detection code was written by the author in R language. All parts of this code were created, bridged and used in the Rproject which is a free software environment for statistical computing and graphics. It compiles and runs on a wide variety of UNIX platforms, Windows and MacOS. More importantly, Rproject has a great scientific community. A lot of users, from all disciplines one can imagine, have been freely sharing their thoughts, problems, methods, and editable codes in every scientific area. So, the science has no chance but to evolve!

For the purposes of this thesis, some already written functions (or edited versions of them) by some other authors as well were embedded in the code and these authors’ names and sources for their functions are supplied whenever and wherever it is appropriate to do so.

Finally, the code could be full automatic for practical reasons but it was written intentionally as semi-auto. The simple reason for this can be explained by the intuition that a noisy chaotic system is like a big amazing forest in which we want to live although we can easily lose our way since we do not know it well enough yet. Thus, sometimes we have to stop and check how far we ran in it, if we want to enjoy it at the end.

Data

The data employed in this study are index levels (tick data) of Istanbul Stock Exchange 100 index. The period covered is 05.01.1998 - 16.12.2008. One corresponding price level is available for every ten seconds. This makes up a huge data set (roughly=1x6x60x5x250x11); an ocean to explore but a lot of pools to be drawn. In order to avoid the latter, a sound strategy is needed, which is described and discussed after this point of the thesis.

The first data series consist of first differences (returns) of logarithmic adjusted index levels. Return series of ISE100 index are calculated as:

$$x = r_t = \ln (ISE_t / ISE_{t-1})$$

This is the most commonly used method in similar studies in the field and it is necessary as the stationarity of the time series on hand is a requirement for most of the current tests available, since we are not perfectly comfortable with nonstationarity yet.

Next step is to create noise filtered series by a nonlinear noise reduction algorithm which is discussed in the next subsection.

Nonlinear Noise Reduction

Filtering of signals from nonlinear systems requires the use of special methods since the usual spectral or other linear filters may interact unfavorably with the nonlinear structure. Nonlinear noise reduction does not rely on frequency information in order to define the distinction between signal and noise. It takes into account that nonlinear signals will form curved structures in delay space. In particular, noisy deterministic signals form smeared-out lower dimensional manifolds. Nonlinear phase space filtering seeks to identify such structures and project onto them in order to reduce noise (Hegger, Kantz, & Schreiber, 1999). Among several similar approaches (see Kostelich & Schreiber, 1993), one was selected (the one that requires less computing time for the same job) and applied. The following subsection describes this approach.

Simple nonlinear noise reduction

This noise reduction scheme is implemented quite easily. First an embedding has to be chosen. Except for extremely oversampled data, it is advantageous to choose a short time delay. The embedding dimension m should be chosen somewhat higher than that required by the embedding theorems. Then for each embedding vector $\{s_n\}$, a neighborhood $U_{\in}^{(n)}$ is formed in phase space containing all points $\{s_{n^I}\}$ such that $\left\|\{s_n\}-\{s_{n^I}\}\right\| \prec \in$. The radius of the neighborhoods $\in$ should be taken large enough in order to cover the noise extent, but still smaller than a typical curvature radius. A corrected middle coordinate $\hat{s}_{n-m/2}$ is computed for each embedding vector by averaging over the neighborhood $U_{\epsilon}^{(n)}$:

$$\hat{s}_{n-m/2} = \frac{1}{\left|U_{\epsilon}^{(n)}\right|} \sum_{s_{n'} \in U_{\epsilon}^{(n)}} s_{n'-m/2}$$

After one complete sweep through the time series, all measurements s_n are replaced by the corrected values $\hat{s}_n$ (Hegger, Kantz, & Schreiber, 1999). The R code for this algorithm can be found in 'RTisean package version 3.0.10'.

By utilizing this algorithm, a new noise filtered series was produced for logreturns (nfax[1]). Each embedded point was replaced by the average vector calculated in its neighborhood with a given size.

Performing iterations by using smaller and smaller neighborhood size values (ϵ) is completely safe in this procedure. This can be explained by the fact that the neighborhood of each point at least contains the point itself. A small enough value of neighborhood may cause the original point to be the only member in its neighborhood and in that case the average value calculated above ($\hat{s}_{n-m/2}$) is simply the uncorrected measurement and no change is made by the algorithm. Experiments performed with ISE data showed that utilizing the 'multiple steps' approach described here is sometimes better sometimes even, but never worse than choosing only one single reasonable value for ϵ and then proceeding with it until the end. A simple Rcode was written (including RTisean package's "nrlazy" function) and used for this thesis (see Appendix B1 for the

[1] Noise Filtering All method Logreturns

code). Below are two representative graphs (Figure 2 and Figure 3) to give some insight about the possible outcomes of the algorithm:

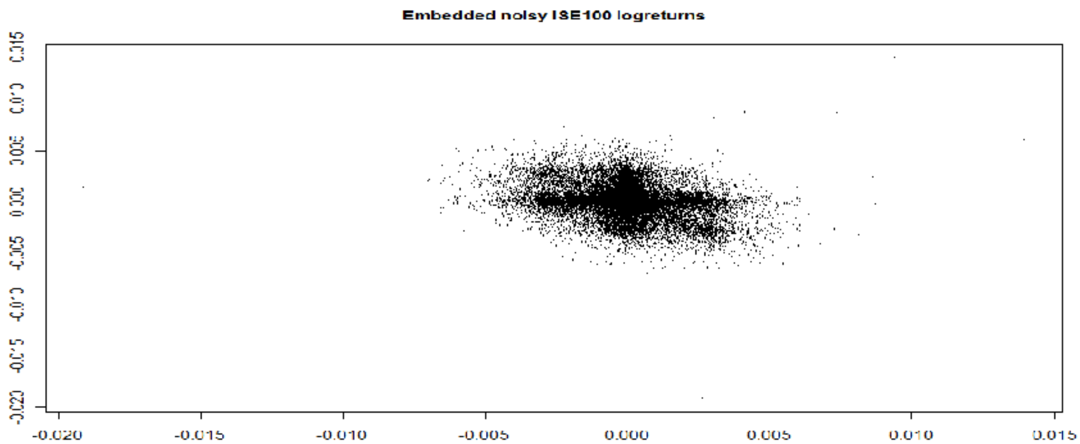

Figure 2 Embedded noisy ISE100 logreturns before noise filtering

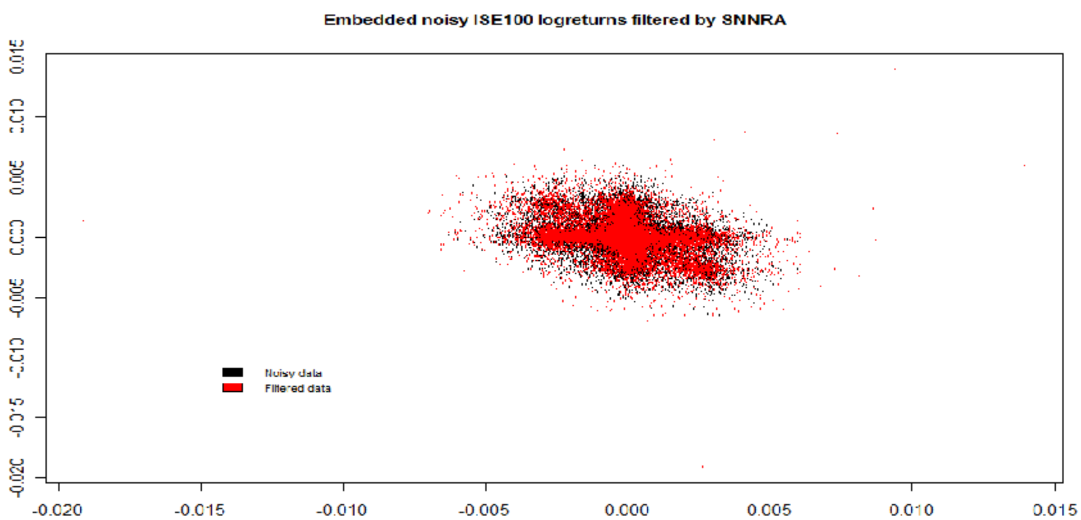

Figure 3 Embedded noisy ISE100 logreturns filtered by SNNRA[2]

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[2] Simple Nonlinear Noise Reduction Algorithm

With the aid of these illustrative graphs, the reader should be aware of two important facts:

- original data seems to keep all important properties after noise reduction process
- outliers (fat tails) remain exactly the same after noise reduction process

Both issues are vital in any successful separation of a true signal from the noise that hides it. However, one should always keep in mind that it is not an easy task that one can be always sure to succeed, especially in social science studies. The effects of noise reduction process, mainly within the special context of ISE100 index, are discussed in the findings part.

Before proceeding to the discussion of the tests applied in this thesis, the sampling procedures are given in the next subsection.

Sampling

This subsection describes the sampling procedures of the thesis under two headings; main samples and additional samples.

Main samples:

Main sampling categories for this study are;

a- 1 day (tick data),

b- 3 consecutive trading days (tick data), c- 5 consecutive trading days (tick data),

d- 10 consecutive trading days (tick data),

e- 11 years (daily closing returns),

f- 11 years (closing returns at the end of each day's morning and afternoon slots) .

The motivation for the latter two categories is not any scientific expectation of the author but his willingness to provide scientific answers to all the questions, although it is impossible, that may come to reader's mind.

Table 1 Summary of Sample Categories

	Sample Categories					
	a	b	c	d	e	f
No of samples theoretically possible	2750	2728	2706	2662	1	1
No of samples analyzed	44	44	44	44	1	1
	a	b	c	d	e	f
No of observations	1800	5400	9000	18000	2329	4658

As seen in the table, if one wants to examine and record all samples and possible findings, he/she has to run the semi-auto algorithm (including at least 10 main individual procedures tied to each other) twice (logreturns and their noise filtered versions) for each of the 10,848 possible samples. This task requires enormous time and processor and storage capacity. Fortunately, it is not necessary for the purposes of the thesis. Instead, a bootstrap-like method was developed to choose random samples (in accordance with their categories) from the original data set of eleven years. The only condition was to ensure an almost uniform distribution, avoiding concentration on any months and years (Rcode written for this method also enables one to choose user-specified periods of interest which will be discussed later). This method can be trusted for two reasons;

- it eliminates any bias that the researcher may have
- the important characteristics of financial series do not change radically and remain changed for long periods of time (which will be discussed in detail throughout the findings part)

Additional samples:

Additional sample categories are:

- X (user-defined number) consecutive trading days' logreturn series to fully cover some special periods of interest (tick data)
- Lorenz deterministic time series
- Randomly generated time series
- Randomized versions of main samples

The first category is self explanatory. It enables the user of the algorithm to include some periods which are not fully captured by the semi-random bootstrap-like method, where the changes in the index levels are surprisingly high and/or low.

The second and third categories were created in order to assess the validity of the Rcode written for this thesis. For this semi-auto algorithm to be trusted (at least to a certain degree, since 100% validation is theoretically impossible in this context) it should detect chaotic behavior accurately in the famous ‘lorenz time series’ and it should reject the existence of chaotic behavior at certain stages of the analysis when supplied with random data.

The last category helps to assess validity of both the Rcode and any possible finding claiming to find chaotic behavior in a period in ISE100 index. However, it should be noted that utilizing this category alone can’t be decisive and it can only strengthen or weaken the probability of validity in both issues. Randomization of actual series was performed by changing the time order of returns in a completely random way. Additionally, all returns had the chance of not being present in the new randomized series. In such a probable case, their gap was filled by other original returns, resulting in a very slightly overrepresentation of some returns. The word ‘slightly’ is of some importance here. Original and the randomized versions have to be similar regarding their basic statistics. Distortion rates were checked and they have never passed the critical 5% level for the series used in this study.

The next section discusses the available tests related to chaos theory. Most of them were edited and bridged to construct the Rcode of this thesis. The experienced reader familiar with these tests may skip this subsection (although the alternative is strongly recommended by the author) to proceed with the final section of the methodology part which describes the full path of this thesis’ fast algorithm for chaos detection.

Tests of nonlinearity and chaos

Experimental evidence and casual introspection suggest that investors' attitudes towards risk and expected return are nonlinear. The terms of many financial contracts such as options and other derivative securities are nonlinear. The strategic interactions among market participants, the process by which information is incorporated into security prices and the dynamics of economy-wide fluctuations are all inherently nonlinear. Therefore, a natural frontier for financial econometrics is the modeling of nonlinear phenomena. It is for such reasons that interest in deterministic nonlinear chaotic processes has in the recent past experienced a tremendous rate of development. Besides its obvious intellectual appeal, chaos is interesting because of its ability to generate output that mimics the output of stochastic systems, thereby offering an alternative explanation for the behavior of asset prices. Clearly then, an important area for potentially productive research is to test for chaos and (in the event that it exists) to identify the nonlinear deterministic system that generates it. Below are the short discussions of several univariate statistical tests for independence, nonlinearity and chaos, motivated by the mathematics of deterministic nonlinear dynamical systems.

The Correlation Dimension Test

Grassberger and Procaccia (1983) suggested the 'correlation dimension' test for chaos. To briefly discuss this test, they start with the one-dimensional series, $\{x_t\}_{t=1}^{n}$, which can be embedded into a series of m-dimensional vectors $X_t = (x_t, x_{t-1}, ..., x_{t-m+1})'$ giving the series $\left\{x_t\right\}_{t=m}^{n}$. The selected value of m is called the 'embedding dimension' and each X_t is known as an 'm-history' of the series $\left\{x_t\right\}_{t=1}^{n}$. This converts the series of scalars into a slightly shorter series of (m-dimensional) vectors with overlapping entries -- in particular, from the sample size n, $N = n - m + 1$ m-histories can be made. Assuming that the true, but unknown, system which generated $\left\{x_t\right\}_{t=1}^{n}$ is ∂-dimensional and provided that $m \geq 2\partial + 1$, then the N m-histories recreate the dynamics of the data generation process and can be used to analyze the dynamics of the system. (see Takens, 1981).

The correlation dimension test is based on the 'correlation function' (or 'correlation integral'), C(N, m, ε), which for a given embedding dimension m is given by

$$C(N,m,\varepsilon) = \frac{1}{N(N-1)} \sum_{m \leq t \neq s \leq n} H(\|X_t - X_s\|),$$

where ε is a sufficiently small number, H(z) is the Heavside function (which maps positive arguments into 1 and nonpositive arguments into 0), and $\|.\|$ denotes the distance induced by the selected norm (the maximum norm being the type used most often). In other words, the correlation integral is the number of pairs (t, s) such that each corresponding component of Xt and Xs are near to each other, nearness being measured in terms of distance being less than ε. Intuitively, C(N,m, ε) measures the probability that the distance between any two m-histories is less than ε. If C(N,m, ε) is large (which means close to 1) for a very small ε, then the data is very well correlated.

The correlation dimension can be defined as

$$D_c^m = \lim_{\varepsilon \to 0} \frac{\log C(N, m, \varepsilon)}{\log \varepsilon},$$

that is by the slope of the regression of logC(N,m, ε) versus log ε for small values of ε, and depends on the embedding dimension, m. As a practical matter one investigates the estimated value of D_c^m as m is increased. If as m increases, D_c^m continues to rise, then the system is stochastic. However, if the data are generated by a deterministic process (consistent with chaotic behavior), then D_c^m reaches a finite saturation limit beyond some relatively small m. The correlation dimension can therefore be used to distinguish true stochastic processes from deterministic chaos (which may be low-dimensional or high-dimensional).

While the correlation dimension measure is therefore potentially very useful in testing for chaos, the sampling properties of the correlation dimension are, however, unknown. As Barnett et al. (1995) claimed: "[i]f the only source of stochasticity is [observational] noise in the data, and if that noise is slight, then it is possible to filter the noise out of the data and use the correlation dimension test deterministically. However, if the economic structure that generated the data contains a stochastic disturbance within its equations, the correlation dimension is stochastic and its derived distribution is important in producing reliable inference" (p. 306).

Moreover, if the correlation dimension is very large as in the case of high-dimensional chaos, it will be very difficult to estimate it without an enormous amount of data. In this regard, Ruelle (1990) argues that a chaotic series can only be distinguished if it has a correlation dimension well below $2\log_{10} N$, where N is the size of the data set, suggesting that with economic time series, the correlation dimension can only distinguish low-dimensional chaos from high-dimensional stochastic processes. Grassberger and Procaccia (1983) discuss this issue in more details.

In the light of these definitions and discussions, this thesis does not consider correlation dimension test as a direct evidence for chaos. Instead, for already found deterministic systems, this test and its variations (information dimension and entropy estimation) may well be utilized to estimate the fractal dimension which would be very helpful in successful nonlinear modeling aiming future prediction.

The BDS Test

To deal with the problems of using the correlation dimension test, Brock et al. (1996) devised a new statistical test which is known as the BDS test. The BDS tests the null hypothesis of whiteness (independent and identically distributed observations) against an unspecified alternative, using a nonparametric technique.

The BDS test is based on the Grassberger and Procaccia (1983) correlation integral as the test statistic. In particular, under the null hypothesis of whiteness, the BDS statistic is

$$W(N,m,\varepsilon)=\sqrt{N}\,\frac{C(N,m,\varepsilon)-C(N,1,\varepsilon)^{m}}{\widehat{\sigma}(N,m,\varepsilon)}$$

where $\widehat{\sigma}(N,m,\varepsilon)$ is an estimate of the asymptotic standard deviation of $C(N,m,\varepsilon)-C(N,1,\varepsilon)^m$. The BDS statistic is asymptotically standard normal under the whiteness null hypothesis.

The intuition behind the BDS statistic is as follows. C(N,m,ε) is an estimate of the probability that the distance between any two m-histories, Xt and Xs of the series X_t is less than ε. If X_t were independent then for $t \neq s$ the probability of this joint event equals the product of the individual probabilities. Moreover, if $\{x_t\}$ were also identically distributed then all of the m probabilities under the product sign are the same. The BDS statistic therefore tests the null hypothesis that $C(N,m,\varepsilon)=C(N,1,\varepsilon)^m$ (the null hypothesis of whiteness).

Since the asymptotic distribution of the BDS test statistic is known under the null hypothesis of whiteness, the BDS test provides a direct (formal) statistical test for whiteness against general dependence, which includes both nonwhite linear and nonwhite nonlinear dependence. Hence, the BDS test does not provide a direct test for nonlinearity or for chaos, since the sampling distribution of the test statistic is not known (either in finite samples or asymptotically) under the null hypothesis of nonlinearity, linearity, or chaos. It is, however, possible to use the BDS test to produce indirect evidence about nonlinear dependence (may be chaotic or stochastic), which is necessary but not sufficient for chaos. Barnett et al. (1997) and Barnett and Hinich (1992) discuss these issues.

For the purposes of this thesis, the BDS test was applied, at the beginning, as a strong filter to detect and eliminate samples that can not exhibit any dependence which is one of the requirements for chaotic behavior.

Nonlinearity tests

The bispectrum in the frequency domain is easier to interpret than the multiplicity of third order moments $\{C_{xxx}(r,s): s \le r, r = 0,1,2...\}$ in the time domain (Hinich, 1982). For frequencies ω_1 and ω_2 in the principal domain given by:

$$\Omega = \{(\omega_1, \omega_2): 0 < \omega_1 < 0.5, \omega_2 < \omega_1, 2\omega_1 + \omega_2 < 1\},$$

the bispectrum, $B_{xxx}(\omega_1, \omega_2)$, is defined by:

$$B_{xxx}(\omega_1, \omega_2) = \sum_{r=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} C_{xxx}(r,s) \exp[-i2\pi(\omega_1 r + \omega_2 s)]$$

The bispectrum is the double Fourier transformation of the third-order moments function and is the third-order polyspectrum. The regular power spectrum is the second-order polyspectrum and is a function of only one frequency.

The skewness function $\Gamma\ (\omega_1, \omega_2)$ is defined in terms of the bispectrum as follows:

$$\Gamma^2(\omega_1, \omega_2) = \frac{|B_{xxx}(\omega_1, \omega_2)|^2}{S_{xxx}(\omega_1) S_{xxx}(\omega_2) S_{xxx}(\omega_1 + \omega_2)}$$

where $S_{xxx}(\omega)$ is the (ordinary power) spectrum of $x(t)$ at frequency ω. Since the bispectrum is complex valued, the absolute value (vertical lines) in the equation above designates modulus. Brillinger (1965) proves that the skewness function $\Gamma\ (\omega_1, \omega_2)$ is constant over all frequencies $(\omega_1, \omega_2) \in \Omega$ if $\{x(t)\}$ is linear; while is $\Gamma\ (\omega_1, \omega_2)$ flat at zero over all frequencies if $\{x(t)\}$ is Gaussian. Linearity and Gaussianity can be tested using a sample estimator of the skewness function. But those flatness conditions are necessary but not sufficient for general linearity and Gaussianity, respectively. On the other hand, flatness of the skewness function is necessary and sufficient for third order nonlinear dependence. The Hinich (1982) 'linearity test' tests the null hypothesis that the skewness function is flat, and hence is a test of lack of third order nonlinear dependence. For details of the test, see Hinich (1982).

This thesis utilizes 'Teraesvirta Neural Network Test' and 'White Neural Network Test' which are two similar approaches (to Hinich test) with the same null hypothesis of linearity in the mean. Teraesvirta et al. (1993) and Lee, White, and Granger (1993) can be checked for details.

In White's (1989) test, the time series is fitted by a single hidden-layer feed-forward neural network, which is used to determine whether any nonlinear structure remains in the residuals of an autoregressive (AR) process fitted to the same time series. The rationale for White's test can be summarized as follows: under the null hypothesis of linearity in the mean, the residuals obtained by applying a linear filter to the process should not be correlated with any measurable function of the history of the process. White's test uses a fitted neural net to produce the measurable function of the process's history and an AR process as the linear filter. White's method then tests the hypothesis that the fitted function does not correlate with the residuals of the AR process. The resulting test statistic has an asymptotic x^2 distribution under the null of linearity in the mean.

What makes both tests attractive to the author is the fact that they both use neural nets for proper estimation of the required F-statistic and/or Chi-Squared statistic. They together formed a two-folded strong barrier and never resulted in a disagreement for the samples tested. An editable version of these tests is available in 'FNonlinear package' (authors: Adrian Trapletti, Blake LeBaron, Diethelm Wuertz) of Rproject. The same package also contains 'runs test' which searches for randomness in the observed data series x by examining the frequency of runs. A 'run' is defined as a series of similar responses. Siegel & Castellan (1988) is recommended for the details of this simple and effective procedure. The motivation to add this test to the thesis' algorithm is twofold; to eliminate the random processes at the beginning of the way to chaos and to validate the thesis' sample randomization methods which were discussed under the section heading 'additional samples'.

Determinism test

Kaplan (1994) used the fact that solution paths in phase space reveal deterministic structure that is not evident in a plot of x_t versus t, to produce a test statistic which has a strictly positive lower bound for a stochastic process, but not for a deterministic solution path. By computing the test statistic from an adequately large number of linear processes that plausibly might have produced the data, the approach can be used to test for linearity against the alternative of noisy nonlinear dynamics. The procedure involves producing linear stochastic process surrogates for the data and determining whether the surrogates or a noisy continuous nonlinear dynamical solution path better describe the data. Linearity is rejected, if the value of the test statistic from the surrogates is never small enough relative to the value of the statistic computed from the data (Kaplan, 1994; Barnett et al., 1997).

An editable ready-to-use function is available in the ‘fractal’ package at Rproject website. It calculates the so-called delta-epsilon test for detecting deterministic structure in a time series by exploiting (possible) continuity of orbits comprising a phase space topology created by a time-delayed embedding of the original time series. This phase space continuity is non-existent for stochastic white noise processes. The delta-epsilon test can be summarized in three main steps:

1- creating randomized versions of original data (surrogate data)

2- calculating the phase space statistic (E-statistic) for both the time-delayed embedding of the original time series and the ensemble of surrogates

3- comparing the E-statistic for the original series and the ensemble of surrogate data

If there is a separation of the original E-statistic from that of the ensemble, it implies the existence of deterministic structure in the original time series. Reverse version is also true. In other words, an overlap of E-statstics means that the original series cannot be discriminated from the ensemble of randomized surrogates and thus it can be claimed that the original series is a realization of a random process.

As discussed in more detail in the ‘fractal’ package, the discriminating E-statistic is calculated as:

$$delta(j,k) = |z(j) - z(k)|$$

$$epsilon(j,k) = |z(j + kappa) - z(k + kappa)|$$

$$e(r) = mean(epsilon(j,k)) \text{ over scales } r <= delta(j,k) < r + dr$$

where *delta(j,k)* is the Euclidean distance (using an infinity-norm metric) between phase space points *z(j)* and *z(k)*, and *epsilon(j,k)* is the corresponding separation distance between the points at a times *kappa* points in the future along their respective orbits. These future points are referred to as *images* of the original pair. The variable *kappa* is referred to as the orbital lag. The increment *dr* is the width of a specified Euclidean bin size. Given *dr*, the distance *delta(j,k)* is used solely to identify the proper bin in which to store the image distance *epsilon(j,k)*. The average of each bin forms the *e(r)* statistic. Finally, the E-statistic is formed by calculating a cumulative summation over the the *e(r)* statistic, i.e.,

$$E(r) = cumsum(e(r))$$

As stressed earlier, if there is a distinct separation of the E-statstics for the original time series and the ensemble of surrogate data, it implies that the signal is deterministic. The orbital lag *kappa* should be chosen large enough to sufficiently decorrelate the points evaluated along a given orbit. For optimal orbital lag determination, space time separation plot function (from the same package) was edited and used as a visual tool. Kantz and Schreiber (1997) is recommended for details. The findings part of the thesis includes example plots and the corresponding decisions. The determined orbital lag for each sample was also used later as a user-defined (forced) input for the Lyapunov spectrum function which is described in the coming subsection.

Lyapunov Test

The distinctive feature of chaotic systems is sensitive dependence to initial conditions, which means exponential divergence of trajectories with similar initial conditions. The most important tool for diagnosing the presence of sensitive dependence to initial conditions (and thereby of chaos) is provided by the dominant Lyapunov exponent, λ. This exponent measures average exponential divergence or convergence between trajectories that differ only in having an 'infinitesimally small' difference in their initial conditions and remains well defined for noisy systems. A bounded system with a positive Lyapunov exponent is one operational definition of chaotic behavior.

One early method for calculating the dominant Lyapunov exponent is that proposed by Wolf et al., (1985). This method, however, requires long data series and is sensitive to dynamic noise, so inflated estimates of the dominant Lyapunov exponent are obtained. Nychka et al. (1992) have proposed a regression method, involving the use of neural network models, to test for positivity of the dominant Lyapunov exponent. The Nychka et al. (1992) Lyapunov exponent estimator (NEGM) is a regression (or Jacobian) method, unlike the Wolf et al. (1985) direct method which, as Brock and Sayers (1988) have found, requires long data series and is sensitive to dynamic noise.

Assume that the data $\{x_t\}$ are real valued and are generated by a nonlinear autoregressive model of the form

$$x_t = f(x_{t-L}, x_{t-2L}, ..., x_{t-mL}) + e_t$$

for $1 \le t \le N$, where L is the time-delay parameter and m is the length of the autoregression. Here f is a smooth unknown function, and $\{e_t\}$ is a sequence of independent random variables with zero mean and unknown constant variance. The Nychka et al. (1992) approach to estimation of the maximum Lyapunov exponent involves producing a state-space representation of

$$X_t = F(X_{t-L}) + E_t$$

where $X_t = (x_t, x_{t-L}, ..., x_{t-mL+L})'$, $F(X_{t-L}) = (f(x_{t-L}, ..., x_{t-mL}), x_{t-L}, ..., x_{t-mL+L})'$, and $E_t = (e_t, 0, ..., 0)'$, and using a Jacobian-based method to estimate λ through the intermediate stop of estimating the individual jacobian matrices

$$J_t = \frac{\partial F(X_t)}{\partial X'}$$

After using several nonparametric methods, McCaffrey et al. (1992) recommend using either thin plate splines or neural nets to estimate J_t. Estimation based on neural nets involves the use of a neural net with q units in the hidden layer

$$f(X_{t-L}, \theta) = \beta_0 + \sum_{j=1}^{q} \beta_j \psi(\gamma_{0j} + \sum_{i=1}^{m} \gamma_{ij} x_{t-iL}),$$

where ψ is a known (hidden) nonlinear 'activation function' (usually the logistic distribution function $\psi(u) = 1/(1+\exp(-u))$).

Another very promising approach to the estimation of Lyapunov exponents (that is similar in some respects to the Nychka et al., 1992, approach) has been proposed by Gencay and Dechert (1992). This involves estimating all Lyapunov exponents of an unknown dynamical system. The estimation is carried out, as in Nychka et al. (1992), by a multivariate feedforward network estimation technique. Gencay and Dechert (1992) discusses the details.

Estimation of only the largest Lyapunov exponent and then taking it alone as a sign of presence (or lack) of chaos in a system may be a too dangerous approach for a real scientist who is always after clear and unquestionable truth. However, as also noted previously, estimating all the Lyapunov spectrum is a less risky alternative. Although it may fail to say the final words in some cases and it may require more effort, it is worth trying because it always tells more of the story than its alternative. Bryant et al., (1990) and Abarbanel et al., (1993) also discuss this issues and propose a method to calculate the needed spectrum. This method is included in the Rcode written for the thesis. It is similar to other Jacobian methods (not the direct ones) discussed above and is described in the 'fractal' package of Rproject with an editable Rcode for it. For a global Lyapunov spectrum determination, each local spectrum is obtained by estimating the eigenvalues of the so-called Oseledec matrix, which is formed through a matrix product of successive local Jacobians with the transpose of the Jacobians. The number of Jacobians in the product is equivalent to the scale which is represented by a vector of integers defining the scales over which the local Lyapunov exponents are to be estimated. As this scale increases, one expects the local Lyapunov exponent estimates to converge towards the global estimates. For the purposes of this thesis, the scale vector used was (1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048) whenever the sample size exceeded 2048. This vector length proved to be good enough for proper stabilization of exponents for all samples.

Each Jacobian is formed by fitting a local neighborhood of points (relative to some user-supplied reference point) with a multidimensional polynomial order. The number of neighbors found for each reference point in the embedding is chosen to be twice the polynomial order for numerical stability. To further stabilize the results, a local Lyapunov spectrum is formed for each local reference point.

For this and any other similar methods to be trustworthy and successful, determining the proper values of ‘delay time’, ‘orbital lag’, and ‘embedding dimension’ for each individual sample is crucial. The case of ‘orbital lag’ was already discussed in the ‘determinism test’ part.

One sound strategy to choose the ‘delay time’ is to use the ‘average mutual information’ criterion which was suggested by Fraser and Swinney (1986). Since it also takes into account nonlinear correlations, it can be regarded as better than its possible alternatives such as autocorrelation function. There exist good arguments that if the time delayed mutual information exhibits a marked minimum at a certain value of observation time, then this is a good candidate for a reasonable time delay. Hegger et al., (1999) are recommended for details. The ‘tseriesChaos’ package of Fabio Di Narzo Antonio includes an editable version of this function which was embedded into the semi-auto code of chaos detection written for this thesis.

For the final important parameter ‘embedding dimension’, the ‘false nearest neighbors’ method (Kennel, Brown, & Abarbanel, 1992) was used. The idea of the method is quite intuitive. Suppose that m_0 is the minimal embedding dimension for a given time series. This implies that in an m_0-dimensional delay space, the reconstructed attractor is a one-to-one image of the attractor in the original phase space (so, the neighbors of a given point are mapped onto neighbors in the delay space). Due to the assumed smoothness of the dynamics, neighborhoods of the points are mapped onto neighborhoods again. The shape and the diameter of the neighborhoods are changed according to the Lyapunov exponents. However, if you embed in an m-dimensional space with $m < m_0$, the topological structure is no longer preserved. Points are projected into neighborhoods of other points to which they would not belong in higher dimensions. These points are called ‘false neighbors’.

The algorithm applied in this thesis calculates ‘fraction of false neighbors’ for a range of proper embedding dimension candidates which are supplied by the user. The default value for max embedding dimension is ‘5’ for most of the cases to save computer time but ‘10’ was chosen as default here to be on the safer side (see ‘fractal’ package for details). The m value for which the fraction of false neighbors drops to zero is considered as the minimal sufficient embedding dimension.

Since all the tests discussed up to now do not all have the same null hypothesis, differences among them are not due solely to differences in power against alternatives. So, one could consider using some of them sequentially in an attempt to narrow down the inference on the nature of the process. For example, the Teraesvirta Neural Network Test and the White Neural Network Test could be used initially to find out whether the process lacks third order nonlinear dependence and is linear in the mean. If either test rejects its null, one could try to narrow down the nature of the nonlinearity further by running the Lyapunov test to see if there is evidence of chaos. Alternatively, if the Teraesvirta Neural Network and White Neural Network tests both lead to acceptance of the null, one could run the BDS or Kaplan (Determinism) test to see if the process appears to be fully linear. If the data leads to rejection of full linearity but acceptance of linearity in the mean, then the data may exhibit stochastic volatility of the ARCH or GARCH type.

In short, the available tests provide useful information, and such comparisons of other tests could help further to narrow down alternatives. Excluding the sampling procedures (already discussed at the beginning of this chapter), the next section itemizes the steps of the full analysis of this thesis in a logical order, providing the reader an easy-to-use map to follow the chapter of findings.

Path

1- BDS test for i.i.d. returns

2- Teraesvirta Neural Network test for linearity in the mean

3- White Neural Network test for linearity in the mean

4- Runs test for randomness

5- Average Mutual Information index and plot for delay determination

6- Space-time plot for orbital lag determination

7- False Nearest Neighbors index and plot for embedding dimension determination

8- Determinism test

9- Lyapunov spectrum calculation

10- Fractal dimension calculation (if the spectrum allows)

From now on, the procedure is quite straightforward. If we fail to reject the null hypothesis of any of the first four tests (BDS, tnn, wnn, and runs) for a sample, that sample can't proceed to further steps. If we can reject the nulls of all four tests, it joins steps 5 through 7 to get a combination of triple values (tlag=delay, olag=orbital lag, m=embedding dimension) for itself. For that successful sample, step 8 represents the final barrier before it is considered to be a valid enough exhibitor of chaotic behavior. In this step, we should be able to clearly separate original and surrogate data from each other (to 100% level). In the case that we can't eliminate a sample in the first eight steps, a Lyapunov spectrum is calculated for it. Interpretation of it in the last part (fractal dimension) concludes the cycle. This cycle's (and the code's) validity was assessed by supplying it with a very well known chaotic series (Lorenz time series), randomized real financial logreturns, and a totally artificial random series. The code has truly identified the chaotic behavior in the Lorenz system and quickly eliminated the random processes from the analysis.

CHAPTER V

FINDINGS

Design of this chapter is as described step by step in the ‘path’ section of the previous part. Results (average, where applicable) of tests applied to the eligible samples of each step and corresponding illustrative plots (where applicable) are given and interpreted in the following sections.

As discussed in the sampling part, there are a huge number of theoretically possible samples and actually used samples. For the sake of keeping the clarity, this chapter is not designed a bundle of figures calculated for every sample analyzed. Instead, one summary table including all sample types is present in ‘filter’ steps (in a ‘competition of samples’ style) enabling the reader to get the crucial message as fast and accurate as possible. For ‘parameter calculation’ steps, illustrative plots are presented for Lorenz deterministic series and an author-defined sample for ISE100logreturns. Since, noise filtered logreturns have not changed the results in any step, they were discarded from the sample categories. The reason for this may be the author’s choice of conservative parameters for noise filtering process because of his hesitation to harm the yet unknown dynamic structure (as discussed earlier). However, assigning bigger values, for experimental reasons, did not produce trustable results. Any future research on a valid and local/global nonlinear noise reduction method would contribute well to the existing literature.

The reader may feel completely free in demanding the code and any test result for any sample type for any period (including the samples not selected from the original data set in this thesis) from the author. For the curious and interested reader, another alternative is to make some experiments and get the result himself/herself since the codes and definitions are available (the only small problem may be to get the same data set since the author of this thesis can not share the data for any reason because of a signed trust agreement with IMKB). The latter is also highly appreciated, since finding every chaotic or interesting region in a specified era in a single market is not the scope of this thesis. However, with a collective effort in finding strange periods with some unchanged similarities, we may be able to progress one more step towards a good enough global nonlinear model for prediction of complicated nonlinear systems, especially financial series (or towards the proof that such a model can not exist).

Before proceeding to the analysis' steps, it would be useful to remember the letters used to represent the sample categories described in the previous chapter:

a- 1 day (tick data),

b- 3 consecutive trading days (tick data),

c- 5 consecutive trading days (tick data),

d- 10 consecutive trading days (tick data),

e- 11 years (daily closing returns),

f- 11 years (closing returns at the end of each day's morning and afternoon slots) ,

g- Lorenz deterministic time series

h- Randomly generated time series

i- Randomized versions of main samples

BDS Results

Table 2 shows the results of the BDS Test which represents the first filter in the chaos detection code. As discussed in the introduction part of this chapter, average statistics and success ratios for all categories are presented.

Table 2 BDS Test Results for All Sample Categories

	BDS Results								
	a	b	c	d	e	f	g	h	i
No of samples analyzed	44	44	44	44	1	1	1	1	44
Average Statistics	a	b	c	d	e	f	g	h	i
Size	1800	5400	9000	18000	2329	4658	2001	18000	18000
Statistic	3.6017	11.4281	19.401	27.5597	8.0293	12.6246	536.4977	0.5006	0.2538
p value	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	0.6166	0.7996
Test Result	a	b	c	d	e	f	g	h	i
Pass	44	44	44	44	1	1	1	0	0
Fail	0	0	0	0	0	0	0	1	44
% success	100	100	100	100	100	100	100	0	0

As expected, Lorenz series and all original return samples passed this filter whereas the random categories were clearly rejected to pass through to the parameter estimation steps (five, six and seven).

Teraesvirta Neural Network Test Results

The TNN test is the second filter of the thesis' code. Only c (5 day logreturns), d (10 day logreturns), and g (Lorenz series) succeeded in passing through. In other words, samples of these categories are not linear in the mean. Table 3 summarizes the results.

Table 3 TNN Test Results for All Sample Categories

	TNN Results								
	a	b	c	d	e	f	g	h	i
No of samples analyzed	44	44	44	44	1	1	1	1	44
Average Statistics	a	b	c	d	e	f	g	h	i
Size	1800	5400	9000	18000	2329	4658	2001	18000	18000
Chi-squared	0.2605	3.208	360.4394	52.4332	2.5818	4.3405	25.9495	1.2688	2.2366
p value	0.8779	0.2011	<0.01	<0.01	0.275	0.1141	<0.01	0.5302	0.3268
Test Result	a	b	c	d	e	f	g	h	i
Pass	0	0	44	44	0	0	1	0	0
Fail	44	44	0	0	1	1	0	1	44
% success	0	0	100	100	0	0	100	0	0

White Neural Network Test Results

WNN test was utilized (Table 4) to double-check the ‘linearity in the mean’ issue for all samples. It simply confirms TNN test’s findings.

Table 4 WNN Test Results for All Sample Categories

	WNN Results								
	a	b	c	d	e	f	g	h	i
No of samples analyzed	44	44	44	44	1	1	1	1	44
Average Statistics	a	b	c	d	e	f	g	h	i
Size	1800	5400	9000	18000	2329	4658	2001	18000	18000
Chi-squared	0.2605	3.219	360.7945	170.135	2.5838	4.3402	5.9943	1.2694	2.2364
p value	0.8779	0.2002	<0.01	<0.01	0.2747	0.1142	<0.05	0.5301	0.3269
Test Result	a	b	c	d	e	f	g	h	i
Pass	0	0	44	44	0	0	1	0	0
Fail	44	44	0	0	1	1	0	1	44
% success	0	0	100	100	0	0	100	0	0

Runs Results

Table 5 summarizes the results for Runs test which is the last element of the first filter group of the code. Lorenz filter series (g) excluded from this section since the test requires dichotomous data to calculate its statistic. The p values calculated imply that ISE100 logreturns (not closing levels, only tick data) are far enough from being regarded as random. On the other hand, by checking the p values of h (random series) and i (randomized logreturns), one can claim that this test is clearly capable of identifying random processes.

Table 5 Runs Test Results for All Sample Categories

	Runs Results								
	a	b	c	d	e	f	g	h	I
No of samples analyzed	44	44	44	44	1	1		1	44
Average Statistics	a	b	c	d	e	f		h	I
Size	1800	5400	9000	18000	2329	4658		18000	18000
Statistic	2.6043	5.7334	4.5572	20.4128	0.1044	0.275		0.7181	0.2517
p value	<0.01	<0.01	<0.01	<0.01	0.9169	0.7833		0.4727	0.8013
Test Result	a	b	c	d	e	f		h	i
Pass	44	44	44	44	0	0		0	0
Fail	0	0	0	0	1	1		1	44
% success	100	100	100	100	0	0		0	0

Delay Determination

Figure 4 shows the average mutual information (ami) values for different possible lags for Lorenz series. Choosing the delay time where ami exhibits its first minimum is a sound idea. For the case below, appropriate delay time is three. To confirm it by another visual inspection, drawing a 3d plot can be useful, with the delay time specified above (see Figure 5).

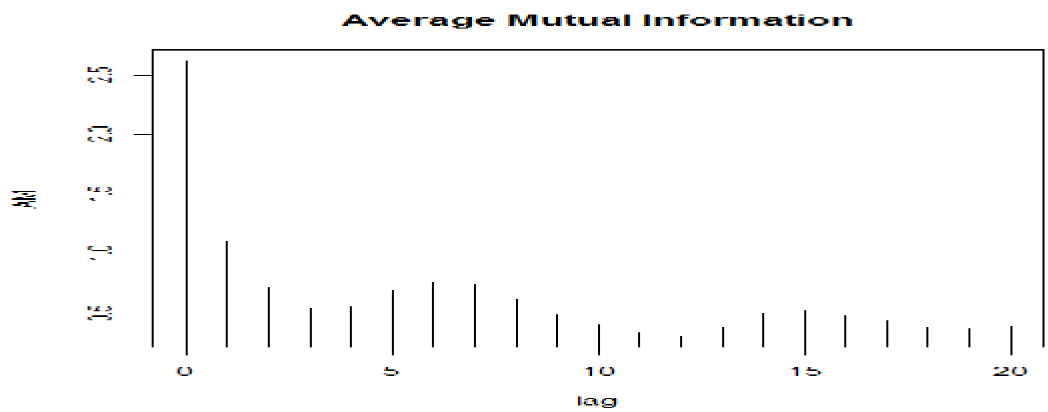

Figure 4 Average Mutual Information plot for Lorenz series (n=2001)

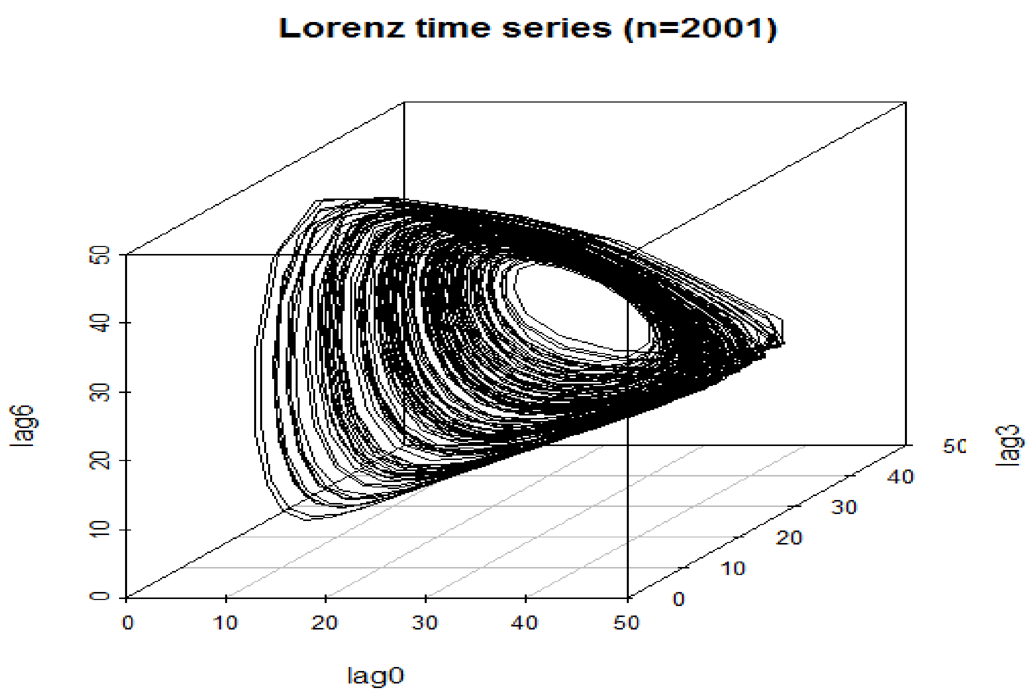

Figure 5 Delayed Chaotic Attractor for Lorenz series (n=2001)

Another example, from the original logreturn series of ISE100, is illustrated in Figure 6.

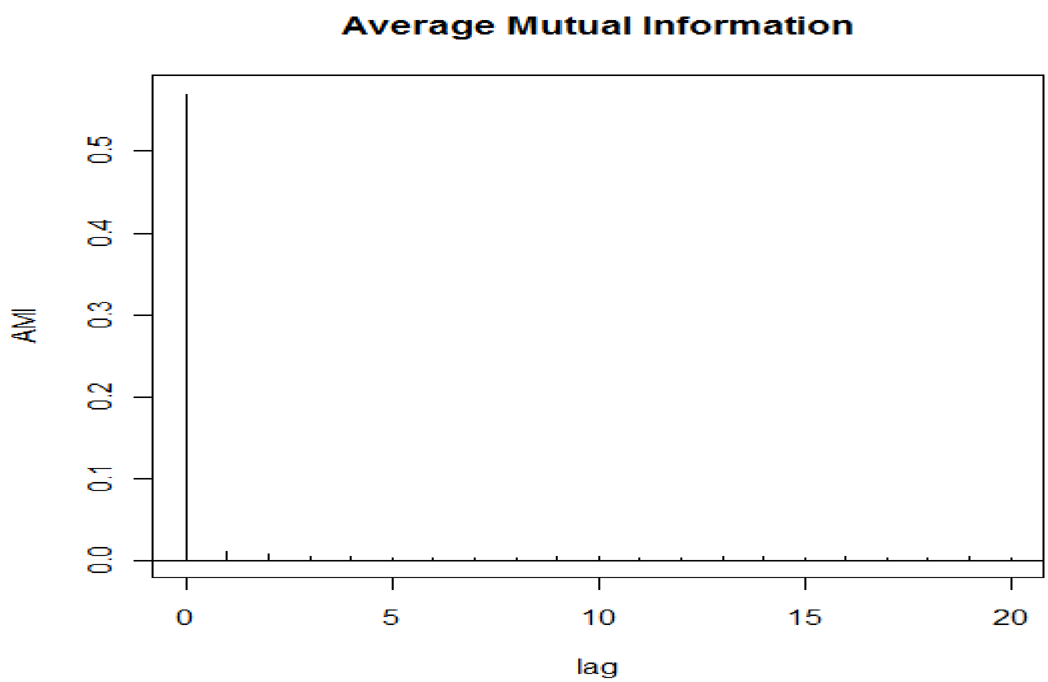

Figure 6 Average Mutual Information plot for ISE100 logreturns

Since the calculated 'ami' values are too low beginning with the first lag, this plot is not very useful (however, it implies one significant fact that this sample is not 'delay time sensitive'). In such a case, one can simply check the calculated 'ami' vector to find the first minimum, which is (0.56961167661986, 0.0110150308621269, 0.00813288116136834, 0.00518962138301493, 0.00500163485330907, 0.00330130383550209, 0.0032185076547242, 0.00264436403730639, 0.00284485899045794, 0.0040223011237881, …) for this sample. Starting from zero, the first minimum occurs at lag seven. However, when the values are so close to zero (they actually become zero for the famous Henon map after d=17 or 18), choosing d as 1 or 2 is not an uncommon practice. In this situation, some nonlinear decorrelation is at work, but not too much. We will revisit this problem at the end.

For 3d visual inspection, please see Figure 7.

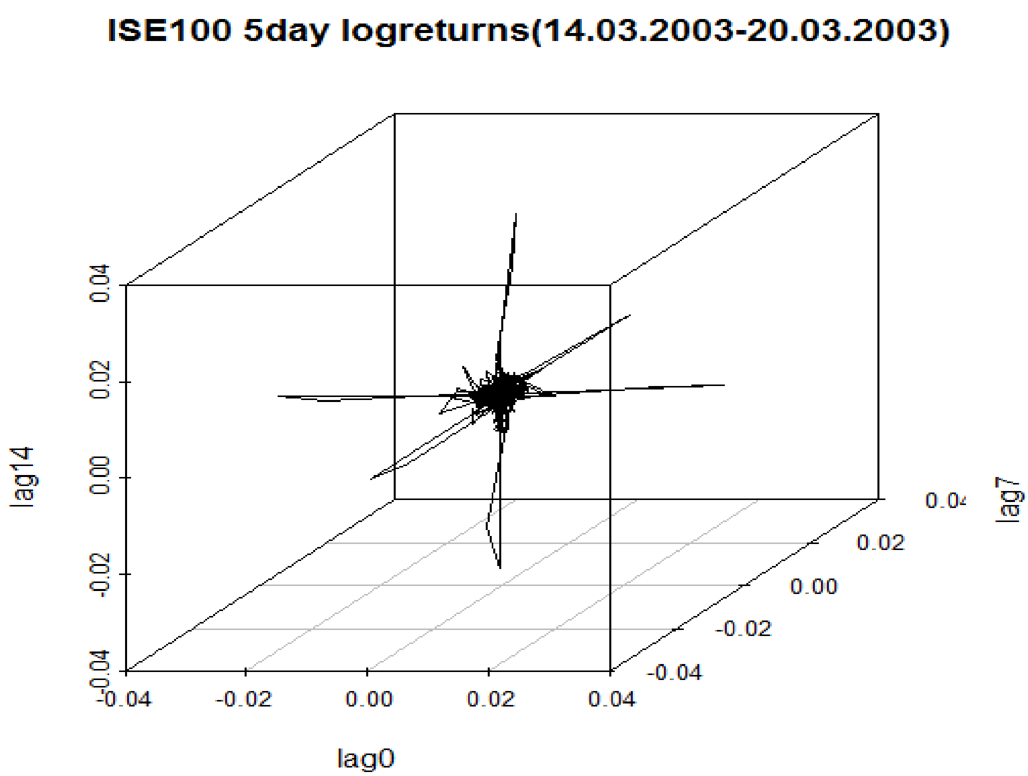

Figure 7 Delayed Attractor for ISE100 Logreturns

Delay times calculation for the logreturn samples consistently produced the same type of ‘ami’ plots leading to similar optimal values for different samples.

Orbital Lag Determination

Two example outputs for the calculated 'orbital lags' for Lorenz series and ISE100 logreturns series are shown in Figure 8 and Figure 9.

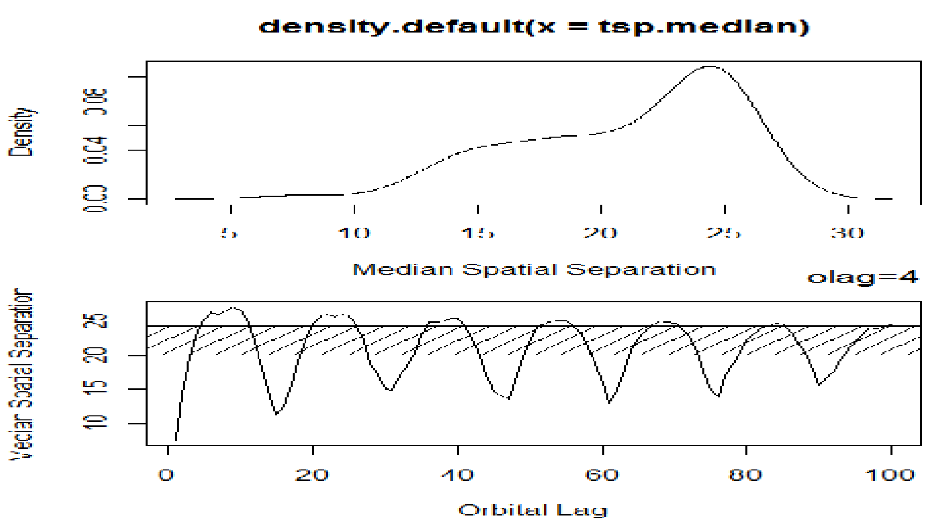

Figure 8 Orbital lag for Lorenz time series

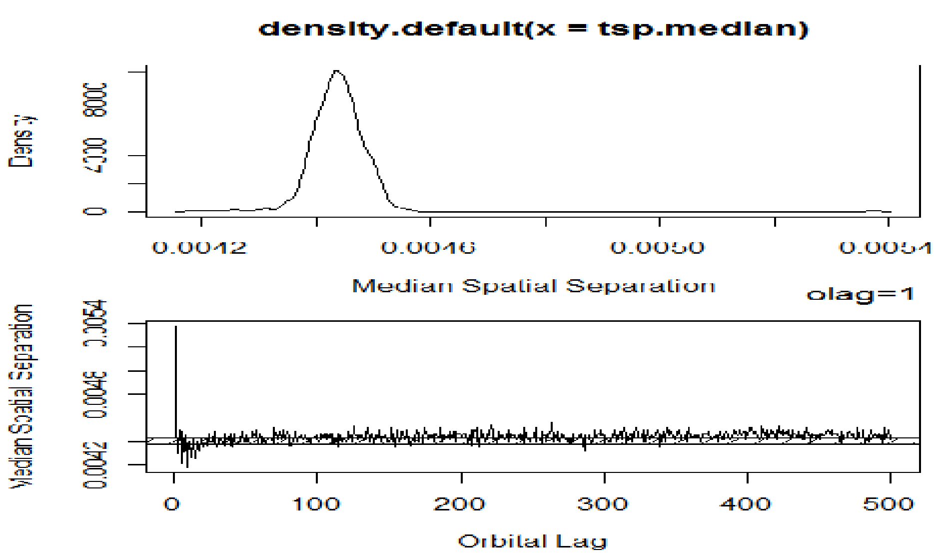

Figure 9 Orbital lag for ISE100 logreturns

The distribution of the orbital lags calculated for the logreturn samples which passed the first four filters concentrated around values one and two.

Embedding Dimension Determination

Figure 10 and 11 show the output of the ‘minimum embedding dimension procedure’ for Lorenz series and ISE100 logreturns series.

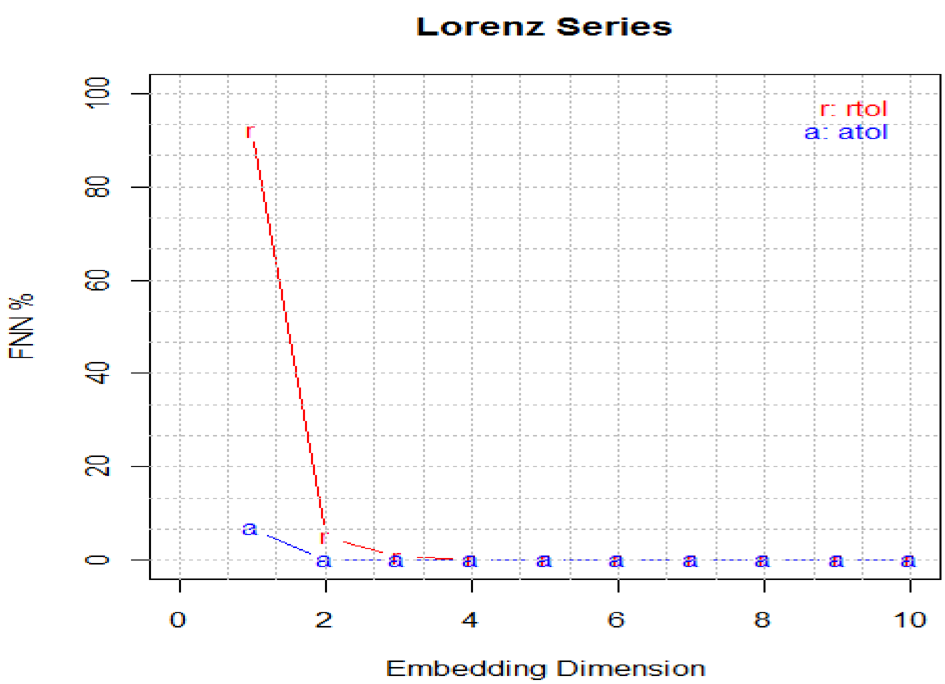

Figure 10 False Nearest Neighbors Plot for Lorenz Series

For this deterministic time series, ‘false nearest neighborhood test’ suggests three as the minimum embedding dimension, since before this value the number of false neighbors calculated is more than zero. Thus, calculating the Lyapunov spectrum for this sample with m=1 or 2 (as an input) would result in meaningless exponents.

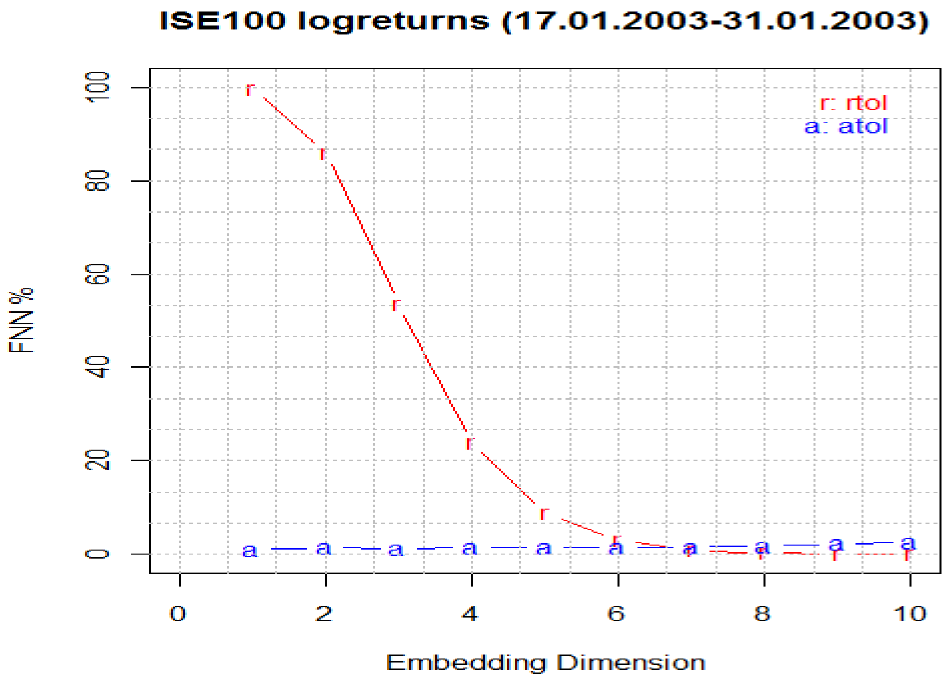

Figure 11 False Nearest Neighbors Plot for ISE100 logreturns

Looking at the plot (or checking the fnn vector calculated), optimal choice is seven for this sample of logreturns. This value consistently remained in the range of five to seven (with a clear bias towards six and seven) for all logreturn samples which could pass the first four filters and joined this step. This finding, together with the others in the previous two steps, may imply that there is a structure in the IMKB, preserved for long time. However, before jumping to this conclusion, we should complete our 10-step cycle (ending at fractal dimension calculation) appropriately.

Determinism test is the final barrier for the sample categories (c and d) which are still in our ‘competition’ in the route to chaos. Three individual output examples of determinism test are given in this section to discuss the subject.

Figure 12 exhibits the case for Lorenz deterministic series, Figure 13 does the same for a random series, and finally Figure 14 does it for ISE100 Logreturns. The dotted line and the continuous one should be clearly separated from each other for accepting that there is a deterministic structure in a given time series. For Lorenz series and ISE100 logreturns, one can observe the separation clearly. This is also valid for every sample in the competition. However, for random series, there is no separation at all. Thus, altogether with other findings up to now, it can be claimed that a deterministic structure (whether exploitable or not) is hidden in ISE100 index. The issue of ‘exploitability’ is discussed at the end of this chapter. We are finally ready now to calculate the Lyapunov spectrums.

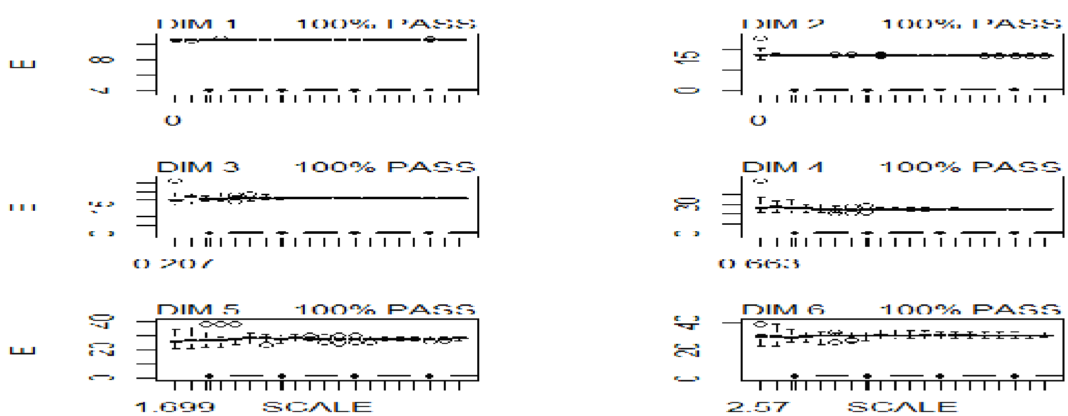

Figure 12 Determinism Test Output for Lorenz Deterministic Series

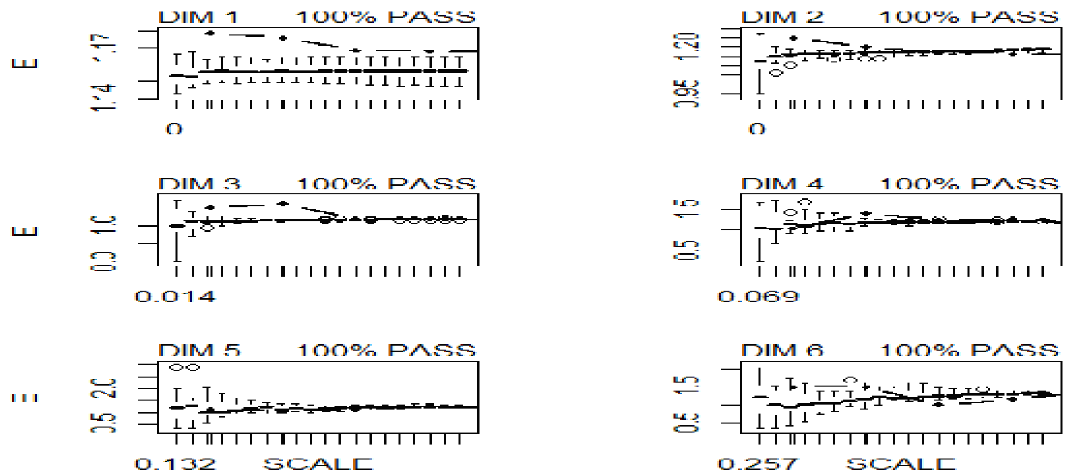

Figure 13 Determinism Test Output for a Random Series

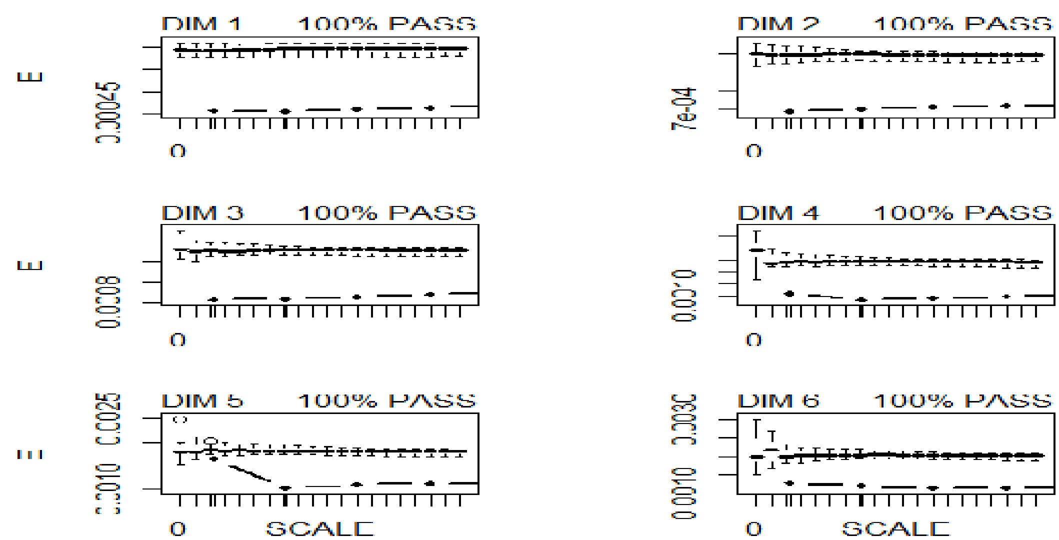

Figure 14 Determinism Test Output for ISE100 Logreturns

Lyapunov Spectrum

For the discussion of this final step of the code, calculated Lyapunov spectrum of Lorenz chaotic series is given first both as a graph (Figure 15) and a table (Table 6) and then the same thing is done for the successful sample categories of ISE100 logreturns (5-days and 10-days) in Figure 16 and Table 7.

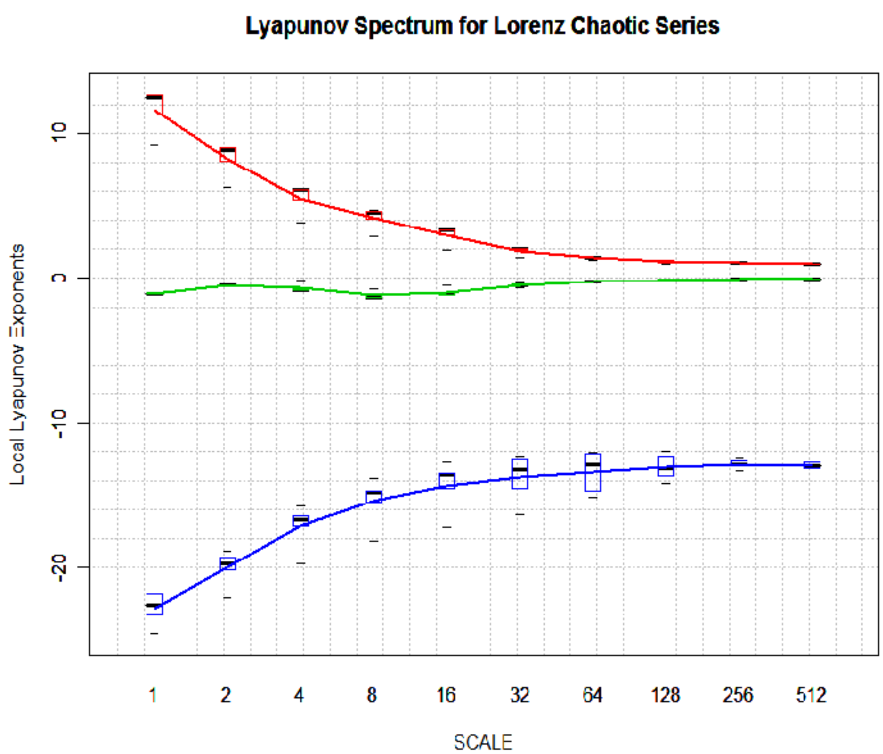

Figure 15 Lyapunov Spectrum for Lorenz Series

Table 6 Mean Values of Estimated Lyapunov Spectrum for Lorenz Series

Scales	1	2	3
1	11.67724	-1.06639	-22.8125
2	8.291879	-0.50122	-20.0019
4	5.515114	-0.67663	-17.1281
8	4.183925	-1.17977	-15.365
16	3.009664	-0.98569	-14.3039
32	1.893946	-0.47363	-13.7792
64	1.399365	-0.23106	-13.396
128	1.14341	-0.10071	-13.056
256	1.046987	-0.05823	-12.8418
512	0.969096	-0.02283	-12.9252

In such a spectrum, if all exponents are less than zero the attractor is a stable fixed point and has dimension zero. If the only non-negative exponents are zero, the attractor is a limit cycle. Multiple null exponents correspond to the number of incommensurate frequencies in a quasiperiodic system, which is also the system's dimension. When at least one exponent is positive and the last negative one is higher (in absolute value) than the biggest positive exponent, there is deterministic chaos. As the scale gets larger, the 'local' estimations of exponents are stabilized to a level which represents the 'global' Lyapunov exponents of the system. By using that global spectrum, one can calculate the fractal dimension which is invariant and then use it for a successful nonlinear modeling.

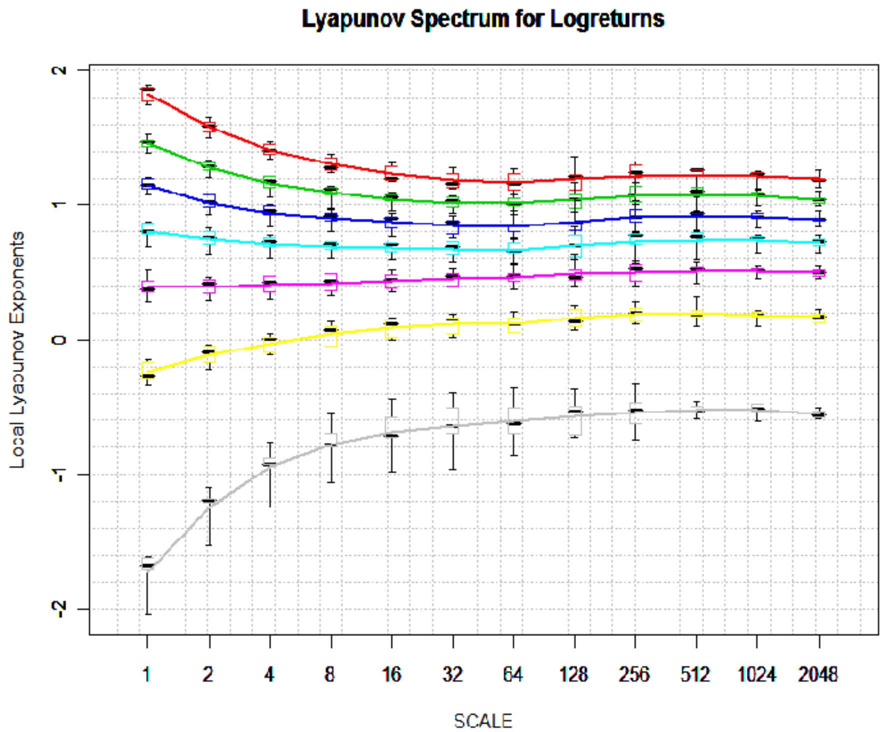

Figure 16 Lyapunov Spectrum for ISE100 Logreturns

Table 7 Mean Values of Estimated Lyapunov Spectrum for ISE100 Logreturns

Scales	1	2	3	4	5	6	7
1	1.829353	1.459226	1.152774	0.803852	0.391903	-0.2404	-1.7297
2	1.578494	1.279714	1.017133	0.747414	0.393771	-0.11176	-1.24115
4	1.409695	1.160589	0.94479	0.711041	0.40779	-0.02632	-0.94716
8	1.302204	1.091385	0.902502	0.69021	0.422702	0.043035	-0.77655
16	1.233608	1.044963	0.873101	0.680817	0.438522	0.09085	-0.68771
32	1.18608	1.016758	0.847142	0.67241	0.455555	0.114267	-0.63977
64	1.169246	1.009942	0.841919	0.670367	0.467687	0.120378	-0.60948
128	1.197458	1.041973	0.875228	0.704074	0.496215	0.159183	-0.56632
256	1.218552	1.066128	0.910308	0.727826	0.5008	0.192743	-0.53914
512	1.213191	1.076178	0.914535	0.736179	0.512993	0.199989	-0.52426
1024	1.214476	1.070714	0.909209	0.738306	0.512545	0.176934	-0.52103
2048	1.193911	1.040923	0.895385	0.720255	0.498412	0.168038	-0.55074

In the logreturns case, one can observe reasonable positive exponents but something is missing. There is no large enough negative exponent in the spectrum. The magnitude of the negative exponent has to be greater than the positive so that initial conditions scattered throughout the basin of attraction contract onto an attractor that occupies a negligible portion of the plane. This finding is valid for all samples joined this step. However, choosing delay time as “1” produces some favorable spectrums (like Lorenz’s) enabling the calculation of the required invariant for nonlinear prediction models which is beyond the scope of this thesis. Furthermore, remembering the discussion in ‘delay time selection with average mutual information procedure’, the author hesitates to claim these findings are decisive. On the other hand, the author strongly claims that there is a deterministic structure in ISE100 index and researchers in the field have every reason to continue their studies.

CHAPTER VI

CONCLUSION

The focus of this thesis is on assessing the possibility of a successful prediction model for the financial markets, in the light of chaos theory, by providing new and significant evidence. The ‘evidence’ mentioned here are provided in two main steps. The first step is a thorough discussion on whether there are any deterministic structures in Istanbul Stock Exchange 100 Index (ISE100) return series or not. The second step is on exploitability of the structures in ISE100 for prediction by calculating their fractal dimensions. To be able to discuss these steps in a scientific manner, a unique solution path which is a semi-auto algorithm was developed. Since it turned out to be fast and accurate, it may well be utilized by both the existing researchers in the area and the newcomers.

For the first step above, the evidence implies the presence of determinism in ISE100. All of the samples in five days and ten days categories have passed significantly all of the required tests with 99% of confidence level. In other words, independent and identically distributed (i.i.d.), linearity, and randomness hypotheses have been rejected significantly ($p<0.01$) for all cases. Even the presence of high noise level is unable to hide it from any careful eye in these periods for the last eleven years. These findings are consistent with the current literature (Malliaris & Stein, 1999; Barnett & Serletis, 2000; Schittenkopf et al., 2001). However, for the daily closing prices and one to three days sample categories, the evidence is contrary. The hypotheses mentioned above could not be rejected even at 95% confidence level. For the first group, daily closing levels categories, this is not surprising since one has to include all years to reach an acceptable sample size and it would be unwise to expect a persistent deterministic structure without any regime shifts for such a long period. For the latter group, one day and three days categories, determinism is again not decisive in the literature. There are two possible reasons for the inability to claim determinism for the ISE case. First, sample sizes are small when compared to five and ten days categories and this of course does not help in anyway to overcome the problems that the high noise level present in ISE100 return series creates. Noise level in ISE may be even higher than other markets, especially the developed ones. Second, the algorithm of this thesis is really conservative in the sense that it eliminates any sample which can not pass 100% of the strict tests, without any doubt.

As a summary of the first step, the findings altogether are consistent with the findings of all other studies mentioned above and these findings imply that a successful prediction model is theoretically possible for ISE100. Thus, there is enough reason to keep on working on the subject.

For the second step, which is on the exploitability of deterministic structure for successful prediction models, the evidence is mixed. Consistent with most of the studies in the current literature, the findings of this thesis reveal that the Lyapunov spectrums calculated do not allow reliable estimations of the ‘fractal dimension’ which are needed as inputs for a good predictive model. In theory, the magnitude of the negative exponent has to be greater than the positive one so that initial conditions scattered throughout the basin of attraction contract onto an attractor that occupies a negligible portion of the plane (Abarbanel et al., 1993). There are always positive exponents for ISE100 returns but large negative exponents are missing in most of the cases. On the other hand, for some samples with high trading activity and sharp return movements, the Lyapunov spectrums have been more favorable to estimate the dimensions. More specifically, the magnitudes of the last negative exponents in the spectrums are greater than the biggest positive exponents. One can claim that chaotic behavior dominates those periods, enabling accurate predictions of the near future. However, this might be a little bit improvident for two reasons. First, optimal delay times for those samples are not very clear and different delay times alter the resulting spectrum. In other words, a nonlinear decorrelation is at work there but we do not know how much and whether it is enough or not. This makes it harder to trust those spectrums and the dimension estimations extracted from them. Second, the present high noise level should arouse some suspicion about the accuracy of the estimated dimension figure. Therefore, for ISE, it seems it is still early to talk about a good prediction model in practice.

To summarize all, there is yet no reason that prevents us from imagining the stock returns as different weather conditions. Successful short term predictions are theoretically possible but it becomes impossible to speak thoroughly about the long term. However, to become the true ‘meteorologist’ of the financial markets, one first has to develop an effective nonlinear noise filtering method which does not distort the original data and is still capable of thoroughly capturing the hidden signal in it. In the absence of such a good filtering method, the true ‘meteorologist’ becomes an ordinary ‘fisherman’ who has to rely on his/her luck at some point!

This thesis has three main contributions. First, it has provided significant evidence from an important but almost totally unexploited, in terms of new techniques and chaos theory, emerging market. Second, it develops a fast, accurate, and easy-to-use algorithm for chaos detection in time series. Lastly, it aims to form a basis on which new researchers can start/continue their studies on the subject.

The basic limitation of this thesis is that its findings can not be generalized. They may not be valid for other financial markets. Exactly the same methodology should be applied to them before reaching any conclusion. Moreover, there are lots of theoretically possible samples within ISE100 dataset, which could be included in this analysis. The future studies may focus on them to check further the validity of the findings presented. However, there is one much more crucial issue that the future researches should certainly deal with. They should focus on developing a valid, trustworthy and established nonlinear noise reduction method for financial time series. At least, it must be proven that such a method can’t exist any time within the current circumstances. Any study performing one or the other would be very helpful in nonlinear modeling area.

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