OPTIMUM ALLOCATION OF RESOURCES IN THE SÜMERBANK LEATHER AND SHOE FACTORY IN BEYKOZ

A Practical Application of Linear Programming
In Turkey

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CHAPTER I

INTRODUCTION

1.1.-OBJECTIVE OF THE STUDY

Linear programming was developed by economists as a deterministic approach for analyzing economic problems. However, in recent years it became an effective method in the field of Operations Research. Especially the Simplex Method developed into a practical tool for approaching industrial problems. Managers now make use of this technique in many different fields of business. Linear programming is successfully applied in determining the most profitable product mix, giving make or buy decisions rationally, scheduling the orders to the machine centers economically, establishing the best allocation of warehouses, and many other situations in business, where the resources are limited and the best use of the existing capacities is sought in order to attain a specific objective such as maximum profit, minimum cost or least time.

The application of linear programming, in the industrial field, is so rare in Turkey that one may safely say its importance and uses are not yet appreciated. On the other hand, the author of this study believes that there are considerable benefits in the use of linear programming in Turkish industry. Therefore, one of the objectives of this study is to show that linear programming may be a tool very useful to managers in Turkey.

The mathematical theory that underlies linear programming makes it seem as a tool that may not be used by managers who do not have an extensive knowledge in mathematics. The mathematical concepts and principles that underlie this technique have been proved valid by the mathematicians. Thus, managers may stress the application of the procedure and make use of it as a practical tool without getting into details of the mathematical analysis. The second objective of this study is to demonstrate the practical use of linear proramming in Turkey.

The literature on Linear Programming is very rich, especially in the Anglo-Saxon countries. Unfortunately, the articles and books that are written by Turks, on this subject are very few. Therefore, the third objective of this study is to make a contribution in this respect. This study is attempted as an introductory essay to serve Turkish students of business administration who may be interested in the subject.

Finally, the fourth objective of this study is to program the optimal allocation of resources in the Sümerbank Leather and Shoe Factory in Beykoz. This is the first attempt at applying linear programming in Turkish Shoe and Leather industry.

1.2.- SCOPE OF THE STUDY

There are different types of optimum value models, which may be listed as allocation problems, inventory problems waiting-line problems, replacement or maintenance problems, routing problems, search problems and competitive problems.

Generally, allocation problems occur more frequently than the other types in the field of industrial administration. These problems where linear programming is applicable consist of different models such as the Simplex Method, transportation problems and assignment problems, In this study, the scope is focused on the Simplex Method, which is known as the basic method in linear programming.

Allocation problem of the Sümerbank Leather and Shoe Factory in Beykoz is presented in Chapter II. First, brief information about the background of the factory, its present state, production system and aims are presented in order to provide the reader with the basic knowledge he may need for appreciating the problem. Later, the problem is stated as:

1.- Under the given resources and restrictions, how many pairs of each type of the footwear must be produced in the different lines in order to obtain the maximum profit?

2.- Under the given resources and restrictions, how many raw hides must the company purchase from the different sources in order to minimize the cost of raw materials?

Then, the organized data about the production and raw materials which are essential for programming are presented.

The concept of allocation problems is presented in

Chapter III. The meanings of "optimal outcomes" and "allocation of resources and productive capacities" are discussed as an introduction to the discussion of the linear programming.

Linear programming is discussed without mathematical explanations. First, brief explanations are given about its uses in management, and its development. Then, the fundamental concepts in linear programming are discussed. Following this, the basic points in building the model and the solution of linear programming problems are stated. The dual problem is mentioned very briefly in this Chapter. It is not discussed in detail, in order to keep the study within its scope. The value of linear programming and its shortcomings are the last topics discussed in Chapter III.

The Simplex Method is discussed in Chapter IV, based on the information given in Chapter III. The algebraic and geometric approaches are discussed first in order to give a better understanding of the procedure. Later, the essential requirements of programming a problem for a simplex solution, the essential steps before the arrangement of the first matrix, the computational routine, and degeneracy are explained in sequence. Again, the explanations are made with the intention of giving information about the practical application rather than presenting a mathematical discussion.

Optimum allocation of resources in the Leather and Shoe Factory are programmed in Chapters V and VI, based on the information given in the previous chapters. Optimum allocation of the raw materials are programmed in Chapter VI. The procedures are carried on until the arrangement of the first matrix in both cases. In Chapter V, a sample problem is developed and solved in order to show the computational routine of simplex problems explicitly.

1.3.- METHOD OF THE STUDY

The theoretical part of this study is written on the basis of the knowledge of the author -which he has attained in Robert College- and the research he has carried out in the books that have been written on this subject. The books used for the research work are listed in the bibliography.

The problems of the Sümerbank Leather and Shoe Factory have been programmed upon continuous research and work in the planning department of the factory. During this phase of the study, frequent consultations have been held with Mr. Metin Göker.

The allocation problems of the factory are programmed for a solution by the Simplex Method. Therefore, the descriptive explanations about the theory are made explicitly only on the points that are essential for understanding the simplex method, such as what the method is, what it means, where it comes from, how and where it is used, its value and shortcomings, etc.

1.4.- TIME OF THE STUDY

The work on this dissertation started in April, 1965. The research work on the problem and the theory ended in May, 1966. The arrangement of the presentation and typing of the first draft was over at the end of July, 1966. The study was read and approved by Mr. Metin Göker in August. The final copy was typed and presented as of September 15, 1966.

CHAPTER II

SÜMERBANK LEATHER AND SHOE FACTORY IN BEYKOZ AND ITS ALLOCATION PROBLEM

2.1.- THE GENERAL OUTLOOK OF THE FACTORY

When linear programming is applied in attaining optimum allocation of resources in a plant, it is useful to take a general look at some points in order to understand the problem easier. Such discussions may be an orientation to the reader in visualizing the procedures clearly. Therefore, the background of the Sümerbank Leather and Shoe Factory in Beykoz, its present capacity, organization goals, policies, and production system are stated briefly before the presentation of the problem.

2.1.1.- Background of the Factory

In the year 1800, new types of uniforms and boots were designed for the Ottoman army. While the governors were looking for means of meeting this new demand, Sultan Mahmut II saw a tannery shop during one of his visits to Tokat and Beykoz Palaces. This shop was owned by Hamza Efendi, an Ottoman citizen. Mahmut II wanted the government to purchase the shop and use it for the needs of the army. Upon his order Hamza Efendi sold his property but did not quit his job. He continued to work there as the consultant.

During 1820-1830 a small installation was added to the old ones, and the first-hand-made army boots were produced. Later, with further additions, these shops became a factory in the sense of those times. In 1842, a 40 HP steam engine and two steam tanks were installed. By 1870, capacity of the factory had increased so as to process 100 raw hides and produce 300 pairs of hand-made army boots in one day.

1900-1914 were the years of considerable mechanization of the factory. Before World War I, 600 raw hides were processed and 200 pairs of army boots were manufactured.

After the Turkish Revolution, the factory became a subsidiary organization of the Ministry of Defence. Its function was to meet the needs of the army. In 1925, the ownership passed by Law to the Industry and Mines Bank. Under its new status, the outlook of the factory became a big manufacturing unit, which could produce ordinary shoes also.

Sümerbank was established in 1933. However, the Beykoz Leather and Shoe Factory was not included among its establishments until 1939. Upon the liquidation of the Industry and Mines Bank, the factory was directed as a corporation for a short period. But, since 1939, it has been working as a subsidiary plant of Sümerbank.

2.1.2.- Capacity of the Factory

In its present state the Beykoz Leather and Shoe Factory is a modern plant in Beykoz,on the shore of Bosphorus. It can produce two million pairs of various types of shoes and boots per year, working in one shift. However, it works at 75% of its capacity and manufactures one and a half million pairs of footwear. About 250 office-men and 2046 laborers are employed in the factory.

2.1.3.- Organization, Goals and Policies of the Factory

Figure 2.1. presents a clear picture of the general organization in the Beykoz Leather and Shoe Factory. Besides, the Chart shows the line of authority in the plant.

Management of the factory is an executive organ rather than a decision organ. Departments execute their jobs in the name of the management. The executive board may decide on measures to be taken in the company, or new policies. However, these decisions must be approved by the general administration of Sümerbank in Ankara, which also has the right of imposing new measures or policies to the president for execution. The Sümerbank administration, at the same time, controls the performance of the factory on the basis of the reports which are submitted by inspectors and the president.

The present policies of the factory may be generalized in four major points:

1.- Supply the demand of the army

2.- Provide cheap and sturdy shoes to public

3.- Meet the demand for leather, shoe, saddlery and other leather products in Turkey

4.- Cut down costs and increase productivity in order to fulfill the above-mentioned purposes.

Thus the factory is in a position of serving public and working rationally.

In order to achieve the aforementioned goals, management is expected to perform its activities within the path of the following policies:

"1.- Take 'ability of making profit' as the yardstick for the performance of the factory. Thus, attain the maximum productivity within the existing resources, install new additions for increasing the production of leather, shoe and other products; make new investments for the creation of raw materials and production facilities; and carry out research for the purchases of the mentioned needs and the sales of finished products.

2.- Carry out research in increasing productivity and training the personnel in the best way.

3.- Offer more benefits to the personnel."(1)

2.1.4.- Production System of the Factory

There are three manufacturing units in the factory:

1.- Tannery -Leather production unit

2.- Shoe manufacturing unit

3.- Auxilliary manufacturing units -rubber bottom and heels manufacturing department, vulcanized rubber shoes manufacturing department, etc.

A lot of possibilities appear in different processes which are engaged in manufacturing different kinds of footwear from different hides. Our study attempts to find the best possibilities in this situation and does not deal with rubber works.

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(1) Organization Handbook, Sümerbank Leather and Shoe Factory in Beykoz, İstanbul, 1966.

Hides are processed into upper and/or bottom leather in the tannery. Then, they are sent to the shoe manufacturing unit, where there are four departments and seven production lines.

The departments of the shoe manufacturing unit are:

1.- Upper leather cutting department
2.- Bottom leather cutting department
3.- Sewing department
4.- Bottom manufacturing department

In the first two departments leather is cut by powered presses in separate units. Here, the productivity changes with the skill of the worker who handles the machine. Sewing department and bottom manufacturing department are assembly lines, where continuous manufacturing is carried out. Products are passed by conveyors from one operation to the other until they become finished.

The production lines of the shoe manufacturing unit are:

1.- Military production line I
2.- Military production line II
3.- Military production line III
4.- Good-year welted line I
5.- Good-year welted line II
6.- Staple welted line
7.- Mixed product line.

Different products are manufactured in different amounts in the mentioned production lines. These may be listed as follows:

1.- Army boots manufactured in the McKay system,

2.- Staple welted shoes,

3.- Ordinary shoes manufactured in the Good-year system,

4.- Welted shoes,

5.- Cemented shoes.

Specific information on the capabilities of the production lines as to products are given later under the heading "Production Data".

2.2. THE ALLOCATION PROBLEM OF THE FACTORY

Various types of shoes and army boots are produced in the Beykoz shoe factory of Sümerbank. Army boots are manufactured by the McKay system. The different types of shoes produced are classified as ordinary shoes manufactured by the McKay system, ordinary shoes manufactured in the Good-year system, welted shoes and ordinary shoes manufactured in the cemented system.

These different products are produced in different production lines. The capabilities and capacities of the production lines differ according to each product. The present production lines of the factory are: Military production line I, Military production line II, military production line III, Good-year welted line I, Good-year welted line II, Staple welted line and mixed product line.

Every product yields a different amount of profit. Now, the first problem of the company can be stated as:

"UNDER THE GIVEN RESOURCES AND RESTRICTIONS, HOW MANY PAIRS OF

EACH TYPE OF THE FOOTWEAR MUST BE PRODUCED IN THE DIFFERENT LINES, IN ORDER TO OBTAIN THE MAXIMUM PROFIT?" Another way of saying it is, "HOW MUST THE PRODUCTS BE ALLOCATED TO THE PRODUCTION LINES IN ORDER TO YIELD THE MAXIMUM PROFIT?"

While searching for the allocation that will yield the maximum profit, the company has to search for the policy that will minimize the costs of production also.

There are several possible ways to use the raw materials. Therefore, it becomes a vital problem to minimize the costs that are due to different uses of different raw materials.

For its different products the company needs butt, shoulder and neck, and bottom leather for welt to be used as bottom leather, and chrome tanned upper leather and vegetable tanned upper leather to be used as upper leather. These can be processed from the raw hides that are classified as Meat and Fishery Products Corporation (Et ve Balık Kurumu) hides, hides from the domestic private sector, medium weight hides of foreign origin, and heavy hides of foreign origin. It is necessary to produce certain quantities of the afore-mentioned leathers. The capabilities, yields, the maximum purchase quantities, and the prices of the raw hides that are purchased from the different sources are all different. Thus, the second problem is:
"UNDER THE GIVEN RESOURCES AND RESTRICTIONS, HOW MANY RAW HIDES MUST THE COMPANY PURCHASE FROM THE DIFFERENT SOURCES IN ORDER TO MINIMIZE THE COST OF RAW MATERIALS?" In other words, "HOW MUST THE PURCHASES BE ALLOCATED IN ORDER TO OBTAIN THE MINIMUM POSSIBLE COST?"

It can be deduced from the above discussion that the company now faces the problem of finding the best alternative of the product policy that will maximize the profits and the best alternative of raw material purchases that will minimize the costs. In order to solve this problem, it is necessary to decompose it into its elements and get a detailed exposition of the data about production and raw materials. This will be accomplished in the following paragraphs.

2.2.1.- Production Data

Every production line has restrictions as to type and amount of products that may be manufactured in it. Besides, the market conditions prohibit the production of every footwear beyond a certain point. Thus, the production program has to be set up by considering these limitations.

In military production line I, it is possible to produce McKay army boots and staple welted shoes, but it is not possible to produce the other products. If only army boots are produced, the capacity of the line is 1000 pairs/day; and if only staple welted shoes are produced, the capacity is 800 pairs/day. The same is also true for military production line II.

In military production line III all types ofproducts can be produced except the Good-year ordinary shoes. The maximum capacities of this line for each product are as follows:

McKay army boots	1000 pairs/day
Staple welted shoes	800 pairs/day
Welted shoes	750 pairs/day
Cemented shoes	750 pairs/day

Only Good-year ordinary shoes can be produced in the Good-year welted line I. Maximum capacity of this line is 850 pairs/day.

In staple welted line, it is not possible to produce ordinary Good-year shoes and cemented shoes. MAXimum capacity of this line, when it is assigned to produce only one of the other types, is as follows:

McKay army boots	1000 pairs /day
Staple welted shoes	800 pairs /day
Welted shoes	750 pairs /day

Every type of product, except ordinary Good-year shoes, can be produced in the mixed product line. The maximum capacities as to products are:

McKay army boots	1000 pairs/day
Staple welted shoes	800 pairs/day
Welted shoes	750 pairs/day
Cemented	750 pairs/day

Aside from the production line restrictions, market conditions also limit the number of pairs that can be sold. Maximum number of pairs that can be sold of each product are estimated, from yearly figures, to be as follows:

McKay army boots	4000 pairs/day
Staple welted shoes	2000 pairs/day
Good-year ordinary shoes	1500 pairs/day
Welted shoes	2000 pairs/day
Cemented shoes	1200 pairs/day

Data, given above, is tabulated in Table 2.1. It summarizes the information related to the products of Sümerbank Beykoz Factory.

Production Line	Boots	Shoes			
	McKay Army Boots	Staple Welted	Good-year ordinary	Welted	Cemented
Military ProductionLine I	1000	800	–	–	–
Military Production Line II	1000	800	–	–	–
Military Production Line III	1000	800	–	750	750
Good-year Welted Line I	–	–	850	–	–
Good-year Welted Line II	1000	–	850	–	–
Staple welted line	1000	800	–	750	–
Mixed Product line	1000	800	–	750	750
Maximum sales possibilities (converted from yearly figures)	4000	2000	1500	2000	1200

CAPACITIES (pairs / day)

Table 2.1
Product Data

2.3.- RAW MATERIALS

Leather is the major material used for footwear manufacturing. Different types of leather are processed from raw hides that are purchased from different sources, which have different prices, capabilities and yields. Thus, using the right raw hide for the right purpose will minimize the costs. The cost minimization due to different uses of the other materials is negligible. Therefore, it is not considered in the problem.

In order to understand the allocation problem due to raw materials it is essential to discuss the different types of leathers used in manufacturing, and the restrictions of the raw hides from different sources. Figure 2.2. may help the reader in following the information easily.

2.3.1.- Types of Leather

Types of leather can, generally, be classified as bottom leather and upper leather. Different types of these leathers which are used in the Beykoz Factory, are described in this section.

2.3.1.1.- Bottom Leather

Butt, belly and shoulder, and bottom leather for welt are the different types of bottom leather. The description and uses of these types will be the subject of the preceeding paragraphs.

2.3.1.1.1.- Butt: Butt is processed from the central part of the raw hide, which is the thickest and strongest part of the

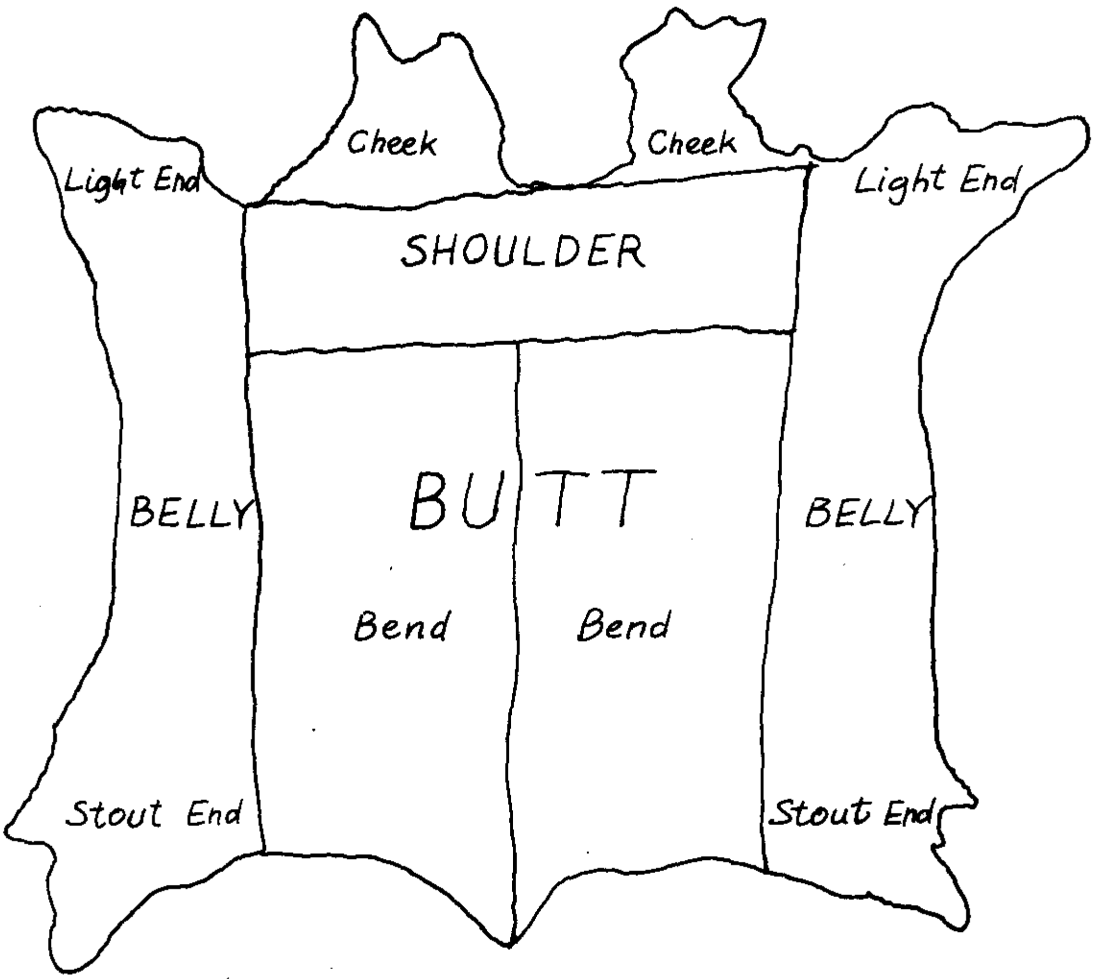

Figure 2.2
Cattle hide laid out flat
showing divisions

animal skin. It is normally used as sole leather because of its strength. The leftover parts may be used as arch supports and fillers.

2.3.1.1.2.- Belly and Shoulder: Shoulder is processed from the neck part of the raw hide, and belly is processed from the other two sides. Good-year welted inner soles, insoles for shoes manufactured in the McKay system, stiffeners, and other inner soles can be produced from shoulder. On the other hand, the products of belly can be listed as: Good-year welted inner soles, stiffeners, arch supports, other inner soles, fillers and upper leather. Use of belly in products manufactured in the Good-year welted system is rare because belly is too thin.

2.3.1.1.3.- Bottom Leather for Welt: The thin line of sole, that extends out of the upper leather, is called the welt. The type of leather that is used for welt is named "Bottom Leather for Welt".

2.3.1.2.- Upper Leather:

There are two types of upper leather: chrome-tanned upper leather and vegetable tanned upper leather. The former is used in shoe manufacturing and the latter in production of army boots.

2.3.1.3.- Annual Need of the Factory:

According to the production plan, the factory needs at least the following amounts of processed leathers.

Types of Leather	Amount Needed
Butt	254,660 kg.
Chrome tanned upper leather	16,070,000 dm^2
Vegetable tanned upper leather	26,800,000 dm^2
Shoulder, belly and substitute leathers	947,215 kg.
Bottom leather for welt	127,550 kg.

The maximum amounts that may be used of each type are as follows:

Types of Leather	Amount Needed
Butt	305,592 kg.
Chrome-tanned upper leather	19,284,000 dm^2
Vegetable tanned upper leather	32,160,000 dm^2
Shoulder, belly, and substitute leathers	1,136,658 kg.
Bottom leather for welt	153,060 kg.

2.3.2.-Sources of Raw Hide

The sources of the raw hides may be classified generally as domestic raw hides and foreign raw hides. Hides purchased from the Meat and Fishery Products Corporation (Et ve Balık Kurumu) and hides bought from the domestic private sector are the domestic raw hides. Hides imported from the foreign countries are classified as medium weight hides of foreign origin and heavy hides of foreign origin. Thus, there are four types of raw hides used in Beykoz Factory; they have different capabilities, yields, prices and purchase limitations. The typical characteristics of each source are discussed below.

2.3.2.1.- Hides of the Meat and Fishery Products Corporation:

Goats are not fed well in Turkey. Thus they do not get enough nourishment to have thick hides. Therefore, it is not possible to obtain butt from the hides of the Meat And Fishery Products Corporation.

It is possible to obtain raw leather -which yield processed chrome-tanned upper leather- only from 25% of each hide. The yield of 1 kg. of this raw leather is 14 dm^2 of processed chrome-tanned upper leather.

It is possible to obtain raw leather for vegetable tanned upper leather from all of the raw hide. Thus, the capability of thr raw hide for raw vegetable tanned upper leather is 100%. The yield of the raw leather is 14 dm^2 of processed leather from 1 kg.

Possibility of obtaining raw shoulder, belly and substitute leather is 15%. The yield is 0.60 kg. of processed leather from 1 kg.

Raw bottom leather for welt may be obtained only from 10% of the hide. 45% of this raw leather can be used as bottom leather for welt. The other 55% is not lost, or need not be thrown away; but it can be used as belly, shoulder and substitute leather. The yield of raw bottom leather for welt is 0.60 kg.

The maximum number of raw hides that can be purchased from the Meat and Fishery Products Corporation in one year is 100,000. This makes about 1,500,000 kg. of raw hide. The price of these hides is 3.60 TL/kg.

2.3.2.2.- Domestic Raw Hides from Private Sector:

Because of the aforementioned reason in the previous section, it is not possible to obtain butt from these hides.

The capability of these hides for raw chrome-tanned upper leather, is 40%. The yield is 12 dm^2 of processed chrome-tanned leather from 1 kg.

The possibility of obtaining raw vegetable tanned upper leather is 70%. It yields 12 dm^2 of processed leather from 1 kg.

It is possible to obtain raw belly, shoulder and substitute leather from 5% of the raw hide. The yield is 0.55kg of processed belly, shoulder and substitute leather from 1 kg.

The capability of domestic private raw hides for producing bottom leather for welt are the same as the capability of raw hides of the Meat and Fishery Products Corporation. The yield, in this case, is 0.55 kg.

It is possible to purchase 200,000 (3,000,000 kg) raw hides in one year from domestic private sector. However, prices go up, when more than 80,000 raw hides are purchased. Price is 3.75 TL/kg. for the first 80,000 (1,200,000 kg.) raw hides, and 4.25 TL/kg. for the next 120,000 (1,800,000 kg.) raw hides.

2.3.2.3.- Medium Weight Hides of Foreign Origin:

Medium weight hides of foreign origin are relatively thin. Therefore, it is not possible to obtain butt from these hides also.

The capability of these hides for raw chrome-tanned upper leather is 90%. The yield is 14 dm^2 of processed leather from 1 kg. of raw leather.

It is possible to use all of the raw hide for producing raw vegetable tanned upper leather, (capability 100%) The yield of the raw leather is 14 dm^2 of processed leather from 1 kg.

Capability of producing raw belly, shoulder and substitute leather from the raw hide is also 100%. The yield is 0.60 kg. of processed leather from 1 kg.

Bottom leather for welt can be produced from all of the hide. However, only 45% of this leather can be used as bottom leather for welt. The other 55% can be used as raw belly and shoulder. The yield is 0.60 kg.

2.3.2.4.- Heavy Hides of Foreign Origin:

Heavy hides of foreign origin are the only hides, which are useful for producing butt. It is possible to produce raw leather for butt from 45% of the hide. The remaining may be used as raw leather for producing belly, shoulder and substitute leather. The yield of the raw butt is 0.60 kg. of processed butt from 1 kg.

It is possible to obtain chrome-tanned upper leather from 80% of the raw hide. The yield of the raw leather is 10 dm^2 of processed leather from 1 kg.

The capability of these hides for vegetable tanned upper leathers is 100%. The yield is 10 dm^2 of processed leather from 1 kg.

The capability of heavy hides of foreign origin for belly, shoulder and substitute leather is also 100%. However, the capability is 55% when they are used for producing butt. The yield is 0.60 kg. of processed belly, shoulder and substitute leather from 1 kg.

It is not possible to produce welt from these hides. There is no limit on their purchase amounts. The price of heavy hides of foreign origin is 4.80 TL/kg.

The facts stated about leather so far, are summarized in Table II. This table gives compact information about the major raw materials used in footwear manufacturing in the Beykoz Leather and Shoe Factory.

Hides from the Meat and Fishery Prod. Corporation	can't be produced.	Capability: 25% Yield 1 kg: 14dm^2	Capability: 100% Yield: 1 kg: 14dm^2	Capability: 15% Yield: 1 kg: 0.6kg	Capability: 10% Yield: 45%, 0.60-The rest can be used as belly and shoulder	A:100,000 raw hide=1,500,000kg. Price:4.60 TL/kg.
Hides from domestic private sector	"	Capability: 40% Yield: 1kg: 12dm^2	Capability: 70% Yield: 1 kg:12 dm^2	Capability: 5% Yield: 1 kg: 0.55kg.	Capability: 10% Yield:45%, 0.55 the rest can be used as belly and shoul.	A:80,000 raw hides=1,200,000kg. price:3.75 TL/kg A:120,000 additional raw hides=1,800,000 kg. Price:4.25 TL/kg.
Medium weight hides from foreign orig.	"	Capability: 90% Yield: 1 kg:14 dm^2	Capability: 100% Yield: 1 kg:14 dm^2	Capability: 100% Yield: 1kg: 0.60kg.	Capability: 100% Yield:45%,0.60 The rest can be used as belly and shoulder	No limit on purchase amount Price: 5,20TL/kg.
Heavy hides of foreign origin	Capability: 45% Yield: 1kg: 0.6kg.	Capability: 80% Yield: 1kg: 10dm^2	Capability: 100% Yield: 1 kg:10dm^2	Capability: 100%(if only belly and shoulder produced) 55%(if butt is also produced) Yield: 1 kg: 0.60kg.	Can't be produced.	No limit on purchase amount Price: 4.80TL/kg.
The programmed amounts	254,660kg.	16,070,000 dm^2	26,800,000 dm^2	947,215 kg.	127,550 kg.	
Maximum Amounts	305,592 kg.	19,284,000 dm^2	32,160,000 dm^2	1,136,658 kg.	153,060 kg.	

Table 2.2.- DATA ABOUT LEATHER

CHAPTER III
THE ALLOCATION PROBLEM
3.1.- OPTIMAL OUTCOMES

In the modern world, attainment of optimal outcomes appears as a problem in many different subjects. Upon continuous developments in industrial field, it has become the vital question of managers also. As problems of industrial units have grown, managers have searched for using more deterministic techniques of overcoming them. Consequently, various optimum value models have been developed by people who have felt the necessity of developing more rational methods in order to keep on successfully in the new complex situations.

One method --perhaps the oldest-- that may be used for searching the optimal outcome is to express the problem as a mathematical equation and find the derivative of that equation. It is possible to determine the optimal strategy by equating the derivative to zero. For example, if the profit in a firm may be expressed in a quadratic equation as a function of sales price; then, every point along the curve which this function describes becomes a profit outcome of different price strategies. So, by taking the first derivative of this function, which is the slope of the curve at a point, one can measure the rate of change of profit as compared to the rate of change of price. If slopes are determined at various points, along the curve, it will be seen that somewhere the slope will change sign from plus to minus. This point is the maximum point of the curve, and here the slope is zero.

The best possible outcome may be determined in a less laborious way by making use of the second derivative. The first derivative gives the critical value (or values) when it is equated to zero. Then these values may be substituted in the second derivative in order to find the optimal strategy. Negative outcomes indicate the maximum, and positive outcomes show the minimum points of the curve.

We saw that optimal outcomes may be determined by expressing the situations in equations and making use of mathematical procedures. However, a lot of optimum value models are based on the solution of inequalities rather than equalities. The solutions of the problems expressed in equations lie somewhere on the curves; whereas the solutions of the equalities are found in the areas bounded by the curves. Thus we have a much larger number of alternative solutions for problems that are expressed in inequalities.

There are several types of problems considered under the topic of optimum value models. These problems may be listed as:

1.- Allocation problems
2.- Inventory problems
3.- Waiting line problems
4.- Replacement or maintenance problems
5.- Routing problems
6.- Search problems
7.- Competitive problems.

These problems keep recurring under different circumstances, but their techniques are quite general. The general optimum value model can be expressed as:

$$E = f(X,Y)$$

where X represents the controllable variables and Y represents the uncontrollable variables. The effectiveness depends on how one can combine these. Since the other problems are beyond the subject of this study, only allocation problems will be discussed in the following parts.

3-2.- ALLOCATION OF RESOURCES AND PRODUCTIVE FACILITIES

Allocation of resources and productive facilities, frequently, becomes a major decision problem in industrial plants. The problem arises when the resources are limited (as a consequence of which a conflict occurs between the activities) and when there are several alternatives to attain the best possible strategy. There would be no problem if the resources were sufficient enough to do everything in the best way. Therefore, the allocation problem arises when the limitation of resources requires that something must be done in less than the best way. This forces the decision maker to arrange the alternatives in such a way to attain the best strategy within the boundaries of the limitations. Thus opportunity costs arise in allocation problems and the decision maker faces the problem of minimizing or maximizing them.

It is common to have different departments in industrial plants. Different products may be produced in these departments and the capacities may be different for different products. In such a case it may be a decision problem to determine how many units to produce of each product in order to achieve the maximum profit. Similarly, the problem may be, "how much of the capacity of each department must be allocated to each product?" Also, the situation may be more complicated, where different kinds of raw materials may be needed for different products that are processed or produced in different departments. In such problems, the decision maker faces a great number of strategies and has to find the best one.

Fortunately, managers have a powerful tool available in finding the best strategy or strategies. This tool is the linear programming method, which formulates the allocation problems.

3.3.- LINEAR PROGRAMMING IN MANAGEMENT

Linear Programming is one of the most widely used of all Operations-Research models. In the field of management,

"it is a technique for specifying how to use limited resources or capacities of a business to obtain a particular objective, such as least cost, highest margin, or least time, when those resources have alternate uses. It is a technique that systemizes for certain conditions the process of selecting the most desirable course of action from a number of available courses of action, thereby giving management information for making a more effective decision about the resources under its control." (1)

Linear programming develops a different way of looking at the management problems.

"(1) Robert O. Ferguson and Lauren F. Sargent, Linear Programming, London, 1958, P-3.

3.3.1.- Development of Linear Programming

Linear programming appeared first in the field of econometrics and the research involving economic relationships.

"Its roots go back to 1874 and the works of the mathematical economist Leon Walras. In his work Elements d'Economie Politique (Elements of Political Economy) Walras showed that the price of any number of commodities at a single time can be determined by solving simultaneously the correct number of equations in terms of the number of unknowns for which a solution is sought. At the time, the concept was revolutionary, and today it is recognized as a contribution to economics. It was this first attempt to solve problems of scarcity by stating problem conditions in equation form that provides the connection between Walras and linear programming." (1)

However, linear programming, as it is thought of currently, is a method that has been developed from W.Leontief's input-output method of analysis. Recently, it has become an effective tool in the field of Operations Research. Development of the Simplex Method, by George B. Dantzig in 1947, led managers to attack industrial and other business problems from a rational approach.

3.3.2.- Fundamental Concepts in Linear Programming

The two words, linear and programming, indicate two of the important concepts of this mathematical model. Besides, linearity and programming, activity, proportionality, nonnegativity and additivity are the other concepts that must be understood before attempting to use linear programming as a managerial technique.

"(1) Ibid, P-6

3.3.2.1- Activity:

"The linear programming approach is to consider a system as decomposable into a number of elementary functions, the activities. An activity is thought as a kind of 'black box' into which flow tangible inputs, such as men, material, and equipment, and out of which may flow the products of manufacture, or trained crews of the military. What happens to the inputs inside the 'box' is the concern of the engineer or of the educator; to the programmer, only the rates of flow into and out of the activity are of interest. The various kinds of flow are called items. The quantity of each activity is called the activity level. To change the activity level it is necessary to change the flows in and out of the activity." (1) "An activity in the business sense is a specific method for carrying out a task or making a product." (2)

For example, production of a specific type of shoe in a shoe factory is an activity. The activities, in a linear programming problem, minimize or maximize the objective function as they satisfy all of the restrictions.

3-3-2.2.- Proportionality:

One of the characteristics of the linear programming model is the assumption of proportionality. It indicates that the quantities of flow of various items into and out of the activity are always proportional to the activity level. An attempt of doubling an activity level must be preceeded by measures of doubling all corresponding flows for the unit activity level.

"(1) George B. Dantzig, Linear Programming and Extensions, Princeton, New Jersey, 1963, P-32

"(2) Linear Programming, Op-Cit: p.8

3.3.2.3.- Nonnegativity:

Another characteristic assumption of the linear programming problems is nonnegativity. In these problems a negative amount of an activity has no meaning. It is assumed that an activity occurs at a positive or, at least, zero level. Thus, any positive multiple of an activity is possible, but negative quantities of activities are impossible.

3-3-2.4.- Additivity:

"For each item it is required that the total amount specified by the system as a whole equals the sum of the amounts flowing into the various activities minus the sum of the amounts flowing out. Thus, each item in our abstract system, is characterized by a material balance equation, the various items of which represent the flows into or out of the various activities." (1)

3-3-2-5.- Linearity:

Linearity means that the equations which make up the model have a linear characteristic. They do not indicate a quadratic, cubic, logarithmic, or other functional relation. Thus, every expression that shows a restriction is linear with regard to the activities they formulate. "Each additional unit of the activity must add a constant amount to the quantity being restricted. Similarly, the expression that shows the relation of the activities to the objective must be linear with regard to the activities. That is, returns from the activities must be to scale: twice as much of an activity produces twice as much profit if the objective is to maximize profit; or twice as much of an activity reduces the costs twice as much if the objective is to minimize the costs." (2)

"(1) Linear Programming and Extensions, OP-cit, P-33

"(2) Miller, David W. and Starr, Martin K., Executive Decisions And Operations Research, Prentice-Hall Inc.,Englewood Cliffs, N.J. 1960, P-402.

3-3-2.6.- Programming:

Programming, as Ferguson and Sargent define; "means calculating for a fixed time period and set of conditions the solution to a set of linear equations and inequations."(1)

It indicates the formulation of the problem in form of inequalities, which are later converted to equalities, in an order. These mathematical relations lead to the attainment of the maximum or minimum results.

3-3-2-7.- Alternative Solutions:

Another concept that must be considered, while discussing linear programming, is alternative solutions. When the resources are limited --as a result of which conflicts occur between the activities-- the decision maker faces several alternative solutions. He has to choose the best one in order to attain his objective. Linear programming is a technique that enables the executive to do this.

3-3-3.- Requirements for Building the Model

In order to use linear programming as a system for converting the available data and facts into usable information, it is necessary to meet several requirements. In other words, it is essential to make a systematic approach while collecting the mathematical relationships that characterize all of the feasible solutions of the system.

First of all it is necessary to state the problem as explicitly as possible in terms of an objective. Therefore,

(1) Linear Programming, Op.cit, P-9

as the first step, the programmermust define the activity and the item set. He must decompose the system in question into all its elementary functions, and choose a unit of measure for each activity. He must also choose a unit for measuring each item.

The second requirement is to determine the information that will be used in solving the problem. It is necessary to determine the input-output coefficients. In other words, the quantity of each item that will be consumed or produced by the operation of each activity, at its unit level, must be determined.

The third requirement, in a linear programming problem, is to state the information in terms of units of measure. The statements need not be specific. They may indicate that some quantity will not be more than -or less than- a certain value.

The fourth requirement is to determine the exogenous flows, that is, the net inputs or outputs of the items between the system and the outside.

Finally, the fifth requirement is to determine the material balance equations by expressing the statements as equations or inequations (equations express the specific statements in mathematical form, the inequations represent approximations). Here the programmer is to "assign unknown nonnegative activity levels X_1, X_2......... , to all activities, then for each item, write the material balance equation which asserts that the algebraic sum of the flows of that item into each activity (given as the product of the activity level by the appropriate input-output coefficient) is equal to the exogenous flow of the item". (1)

"(1) Ibid, p-20

3.3.4.- Method of Solution of Linear Programming Problems

Although there are different special types of linear programming problems, basic steps used in solving them are the same. "In general, limited resources indicate to the executive what he 'cannot do' - the point beyond which he cannot go. Under these circumstances, then, the executive wants to know what 'can be done' to use most effectively resources that he does have..., Each method, where it applies, permits the calculation of the most desirable 'can do' program within the 'cannot do' restrictions according to profit, cost, or other measures of desirability." (1)

Every method involves a formalized trial-and-error process and common elements which may be found in all algorithms. These common elements can be listed as follows:

1.- Start out with an objective

There must be a certain definable objective that will be optimized. This objective must be an operation, which may be expressed in a quantitative form.

2.- Collect the problem information and data

It is essential to use the most accurate data. When such data is not available, one has to make the most reliable estimates. In every case, it is required to use a convenient measure for expressing the information and data.

3.- Arrange the problem information in an order

Every problem contains certain restrictions on the

"(1) Ibid, P-20

attainment of the objective. Therefore, it is essential to define these restrictions. Then, it is useful to set up the information in a tabular form in order to clarify the relation ships and the conditions involved.

4.- Begin with a feasible solution

Every algorithm has a mean for developing a feasible or approximate solution. One needs to set up a feasible solution to a problem as the starting point.

5.- Test the program

Every method provides a procedure for testing the feasibility of each program.

6.- Improve the program with revisions

If the test shows that the first solution is feasible, improve it by revising according to the definite procedures, which the type of the problem requires.

7.- Repeat testing and revisions

Repeat the procedure of testing and revision until the test indicates that no further improvement is possible on the existing solution.

3-3-5.- The Dual Problems

An important characteristic of linear programming problems is that they come in pairs. To every linear programming maximization problem corresponds a linear programming minimization problem, and vice versa. Thus, another problem is associated with every linear programming problem. This problem is called the dual,and two problems paired as such are called dual problems.

The dual of a linear programming problem is formed from the data of the primal problem. A nearly symetric relation is obtained by writing the standard inequalities of the primal problem in a different form. The new order of inequalities are then solved by the simplex method, or by some other method. At the end of the solution, the value of the objective function of the dual comes out the same as the final value of the objective function in the primal problem.

The following passage from Andrew Vazsonyi may help the reader to visualize the concept of dual problems more clearly:

"Let us consider two linear programming problems.

Problem 1.- Determine the nonnegative solution to the system of inequalities:

$$A_{1,1}X_1 + A_{1,2}X_2 + \ldots\ldots\ldots + A_{1,n}X_n \geqq B_1$$

$$A_{2,1}X_1 + A_{2,2}X_2 + \ldots\ldots\ldots + A_{2,n}X_n \geqq b_2$$

" " " " "
" " "
"
" " " "

$$Am,_1X_1 + Am,_2X_2 + \ldots\ldots\ldots + Am,_nX_n \geqq b_m$$

which minimizes

$$C_1X_1 + C_2X_2 + \ldots\ldots\ldots + C_nX_n$$

Problem 2.- Determine the nonnegative solution to the system of inequalities:

$$A_{1,1}W_1 + A_{2,1}W_1 + \cdot \quad \cdot \quad \cdot + A_{m,1}W_m \leq C_1$$

$$A_{1,2}W_1 + A_{2,2}W_2 + \cdot \quad \cdot \quad \cdot + A_{m,2}W_m \leq C_2$$

$$\vdots$$

$$A_{1,n}W_1 + A_{2,n}W_2 + \cdot \quad \cdot \quad \cdot + A_{m,n}W_m \leq C_n$$

which maximizes:

$$b_1w_1 + b_2w_2 + \cdot \quad \cdot \quad \cdot + b_mw_m$$

These two linear programming problems are called duals to each other. Problem 1 has n unknowns and m equations. Problem 2 has m unknowns and n equations. The relationship between the two problems can be made clearer by showing the table of detached coefficients:

	X_1	X_2	.	.	.	X_n	
W_1	$A_{1,1}$	$A_{1,2}$	.	.	.	$A_{1,n}$	b_1
W_2	$A_{2,1}$	$A_{2,2}$	.	.	.	$A_{2,n}$	b_2
.	.	.				.	.
.	.	.				.	.
.	.	.				.	.
W_m	$A_{m,1}$	$A_{m,2}$	.	.	.	$A_{m,n}$	b_m
	C_1	C_2	.	.	.	C_n	

Problems 1 and 2 can now be stated in shorthand notation.

Problem 1.- Determine $X_1, X_2, \ldots, X_n$, so that

$$\left.\begin{array}{c} X_i \geq 0 \\ \sum_j A_{i,j} X_j \geq b_i \end{array}\right\} \quad i = 1, 2, \ldots, m$$

Minimize:

$$\sum_i c_j X_j$$

Problem 2.- Determine $W_1, W_2 \ldots, W_m$, so that

$$W_i \geq 0$$

$$\sum_i A_{i,j} W_i \geq c_j \qquad J = 1, 2, \ldots, m$$

Maximize:

$$\sum_i b_i W_i$$

Now that we have developed the concept of duality, let us turn our attention to the dual theorem of linear programming.

"Denote by Z_{min} the minimum value associated with the solution of Problem 1 and by Z_{max} the maximum value associated with the solution of Problem 2. The dual theorem of linear programming asserts that:

$$Z_{min} = Z_{max} \text{ " (1)}$$

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(1) Vazsonyi, Andrew, Scientific Programming in Business and Industry, John Wiley and Sons, Inc., New York, 1963, P-152.

The advantage of the dual theorem is that one may choose either of the problems to solve; often one of them is solved more easily than the other. Also, finding the result of the dual of a linear programming problem provides a solid dependable check for the programmer.

3.3.6.- Value of Linear Programming in Business

Linear programming models are of great value to the firms and their executives. Thus, it is possible to look at the advantages linear programming problems provide from the point of business operations and from the point of managerial activities.

3.3.6.1.- Value of L.P. in Business Operations:

As was stated previously, linear programming leads to ways of making the most effective use of existing facilities. It is possible to determine ways of increasing the profits, decreasing the costs, building up the most efficient organization, and delivery of the optimum services to the customers.

It is usual to meet with very large and complex problems in business. These problems may involve many different combinations of factors. It may be impossible to overcome these problems successfully unless a methodologic approach is followed. Linear programming is very effective in such cases.

After obtaining the solution of the problem, it becomes possible to compare the present status of the firm with what it should be. Thus, it may be possible to revise the policies of the firm. Also, if the future conditions are considered while building up the model, it may be possible to make anticipations for the future of the business.

3.3.6.2.- Value of L.P. in Managerial Activities:

It is hard to separate the business operations from managerial activities. Therefore, what has been said above and the points that will be mentioned now may be interrelated with each other.

Linear programming provides the executives and managers with information that enables them to take effective measures. Compared with information the executives may obtain from other sources, this information is quicker, more factual and more accurate.

As the executive programs the problem, he obtains a better perspective and a keener insight into the problem; for he is obliged to organize the data about the problem before he can apply it. "Actually, linear programming forces logical organization and study of information in the same way that the scientific approach to a problem does. This generally results in a clearer picture of the true problem which frequently is as valuable and revealing as the answer itself because it leads more surely to a dealing with causes rather than effects -solutions, rather than stop-gap expedients." (1)
While the executive tries to find the best solution to a problem, he inevitably considers all of the possible solutions which give him a wider scope of understanding the obstacles and remedies. He sees the true reflection of the limitations involved in the problem, which indicate to him the conditions he must work under and the penalty he may receive if he neglects to follow the optimum solution.

"51) Linear Programming, Op-cit, P-11

The solution of the problem may give the answers to important questions that the managers have in mind. It shows what to make, where to make it, how to make it, and how many to make. Besides, it indicates the most efficient level of the inventory and what the costs and profits should be. This information enables the manager to give the most effective orders and instructions necessary to the improvement of the present conditions.

"Once a basic plan is arrived at through linear programming, the basic plan can be reevaluated for changing conditions. Plans can be laid for several sets of conditions to find out how to prepare best for possible future changes. If conditions change when the plan is partly carried out, changes can be determined so as to adjust the remainder of the plan for best results." (1)

In addition to the above benefits, a manager may use the results as a control device. He may use them as the yardstick for efficient performance and determine the deviations of the actual performance and take measures when it becomes necessary.

"It is also apparent to date that the use of the new decision-making tools of which LP is one provides the executive with the following:

1.- More time to manage, by providing him with information quicker and permitting more delegation.

2.- Less pressure to get things done, because he can do more accurate planning and avoid pitfalls that create crisis situations.

3.- Better qualified personnel, as the techniques provide insight into management problems and decision making

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" (1) Ibid, P-12

that formerly would have required years of experience and practice to obtain.

Each andall of these factors will addto the ability of the manager to manage better, which, in turn will reflect the increased profitability for the firm." (1)

3.3.7.-The Shortcomings of Linear Programming

Mathematical models reflect the real life situations by mathematical symbols. However, although they present the problem explicitly, they never show the real life situations exactly. In order to reflect the actual conditions as accurately as possible one has to choose the right variables, make the right assumptions and eliminations. Therefore, the reliability of the results are correlated with the ability and value judgements of the programmer.

The results of the linear programming porblems indicate what would happen under rational procedures. The human factor is almost completely ignored in the programs. One must not forget that operations are performed by human beings and their behaviours have considerable effects on the outcomes. Because of the human factor, it may not be possible to reach at the optimum level or it may be necessary to make some modifications and operate by less efficient means. It is not possible to include such considerations, by all means, in the mathematical models, Also, there are limits for stretching the resources. Thus, some compromises may be necessary in actual life.

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(1) Ibid, P- 17

The adequacy and accuracy of the input information impose a limit on the reliability and the range of applicability of the linear programming problems. First of all, the costs are accepted deterministic in such models. It is not possible to deal with probabilistic costs in linear programming problems, although they reflect the actual conditions more closely in certain areas. Secondly, the extent the costs are accurately obtained, determines the extent of the reliability of the answers.

Finally, the linearity assumptions may limit the kinds of decision problems for which linear programming can be applied. If the nonlinearity aspects of a problem can not be remedied, then it becomes essential to use a nonlinear programming model.

CHAPTER IV

THE SIMPLEX METHOD

Although there are a number of different linear programming methods, the Simplex Method is considered to be the classical one. It is known as the fundamental linear programming method, from which the other special methods have been derived.

"The Simplex Method can best be described as a technique of matrix algebra, used to obtain the optimum values for variables related in a system of linear inequalities." (1)

The Simplex Method developed from the works of G.B. Danzig, Orden, Charnes, Cooper, Henderson, Lemke, Dorfman and their colleagues. The scope of the works of these people in general, lies in the field of mathematical economics. Thus, the origin of the method is mathematical rather than industrial.

Since the Simplex Method is used for programming the problem of the Sümerbank Beykoz Factory in this study, it will be discussed in detail. However, its presentation will be different from the way the aforementioned authors would present it. In order to accomplish the purpose of this thesis, the presentation will be relatively non-mathematical, because the derivation of the Simplex Method and proof of the mathematics that underlie it will be kept out of the discussion.

"(1) Robert W. Metzger, Elementary Mathematical Programming, John Wiley and Sons, Inc., New York, 1963, P-59

4.1.- GEOMETRIC AND ALGEBRAIC APPROACHES

Before starting the discussion on the basic Simplex Method, it is useful to go over the two limited solution methods of product-mix problems; the geometric, or graphic solution, and the algebraic solution. Both of the approaches are applicable in special cases. They are unpractical for problems that involve many unknowns.

4.1.1.- Geometric Solution

A problem may be solved graphically as long as the restrictions and objective can be expressed by lines that lie on the same plane. It is possible to solve problems in three dimensions, also, if restrictions and objective can be expressed as planes in a three dimensional space. However, it is impossible to express larger problems geometrically, because there is no way of showing multi-dimensional, on n-dimensional space. Besides this, graphical solution does not yield as much information as the Simplex solution. Also, the accuracy of the graphical solution depends on the accuracy of the geometric construction.

Linear programming problems may be visualized easily in geometrical terms. Therefore, it is useful, for further discussions, to produce a two dimensional problem, and analyze it on rectangular coordinates.

Suppose that there are two products, X and Y, and you are in a position of determining the most profitable product-mix under the following conditions:

1.- All of the products can be sold.

2.- There are three product lines, where X and Y can be produced.

3.- The profit margin is 10.00 T.L. for X and 15.00 T.L. for Y.

4.- In production line I, it is possible to produce at most 60 units of X , or 30 units of Y, or an equivalent mixture of the two products in one day.

5.- In ppoduction line II, it is possible to produce at most 15 units of Y in one day.

6.- In production line III, it is possible to produce at most 45 units of X or Y on an equivalent mixture of the two in one day.

The restriction of production line I can mathematically be formulized as:

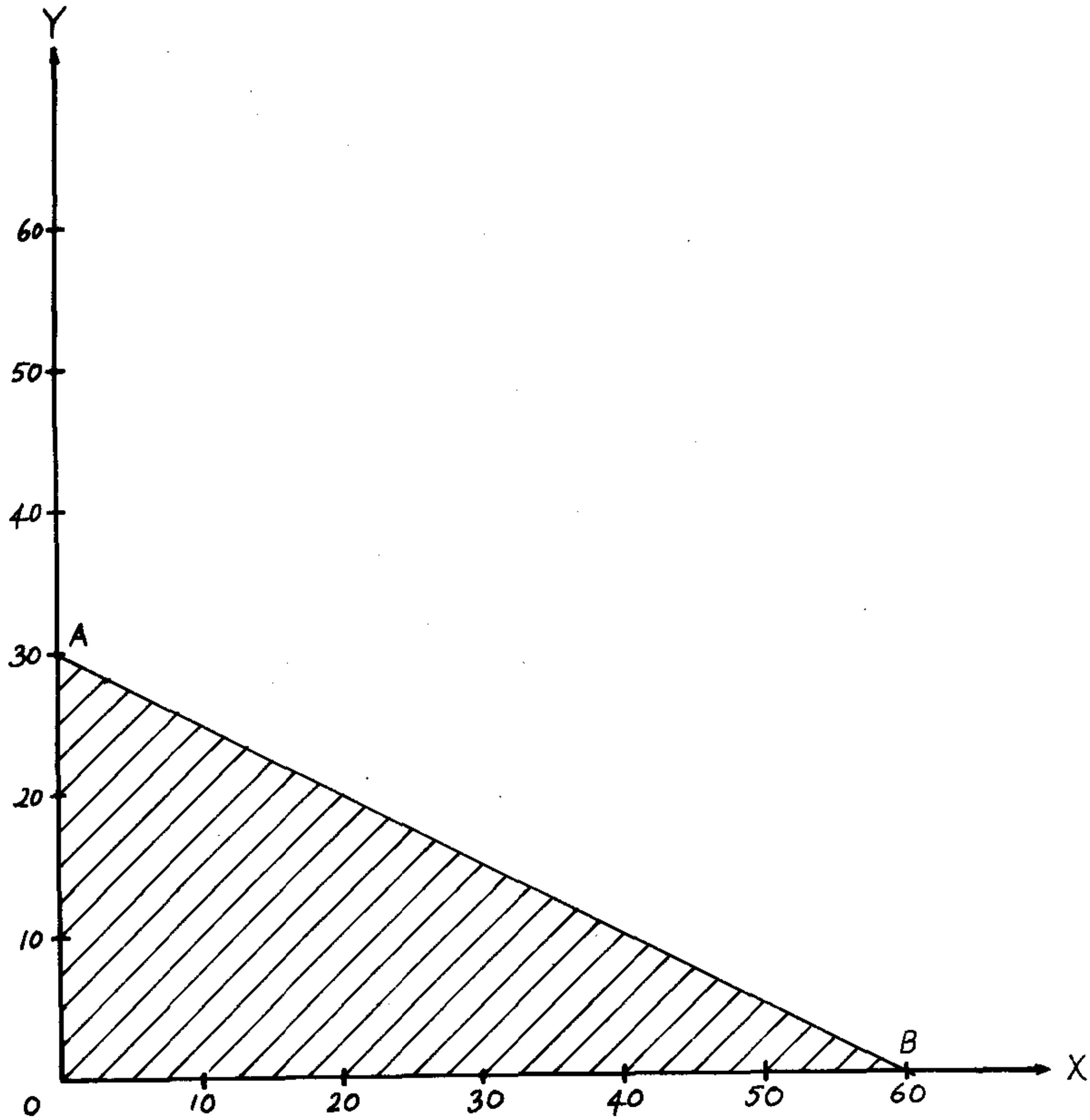
$$X + 2Y \leq 60$$

The graphical expression of this restriction is shown in graph 4.1. This graph indicates that the answer of the problem must lie somewhere in the shaded area under the line or on the line if the productive facilities are utilized at full capacity.

If the restriction would be, "in production line I, it is possible to produce at least 60 units of x, or 30 units of Y, or an equivalent mixture of the two products in one day" which would then be expressed as:

$$X + 2Y \geq 60$$

This expression would mean that the answer of the problem must lie at least on the line or somewhere in the shaded



GRAPH 4.1

GRAPHICAL EXPRESSION OF

$X+2Y \leq 60$

area above it, as it is shown in graph 4.2.

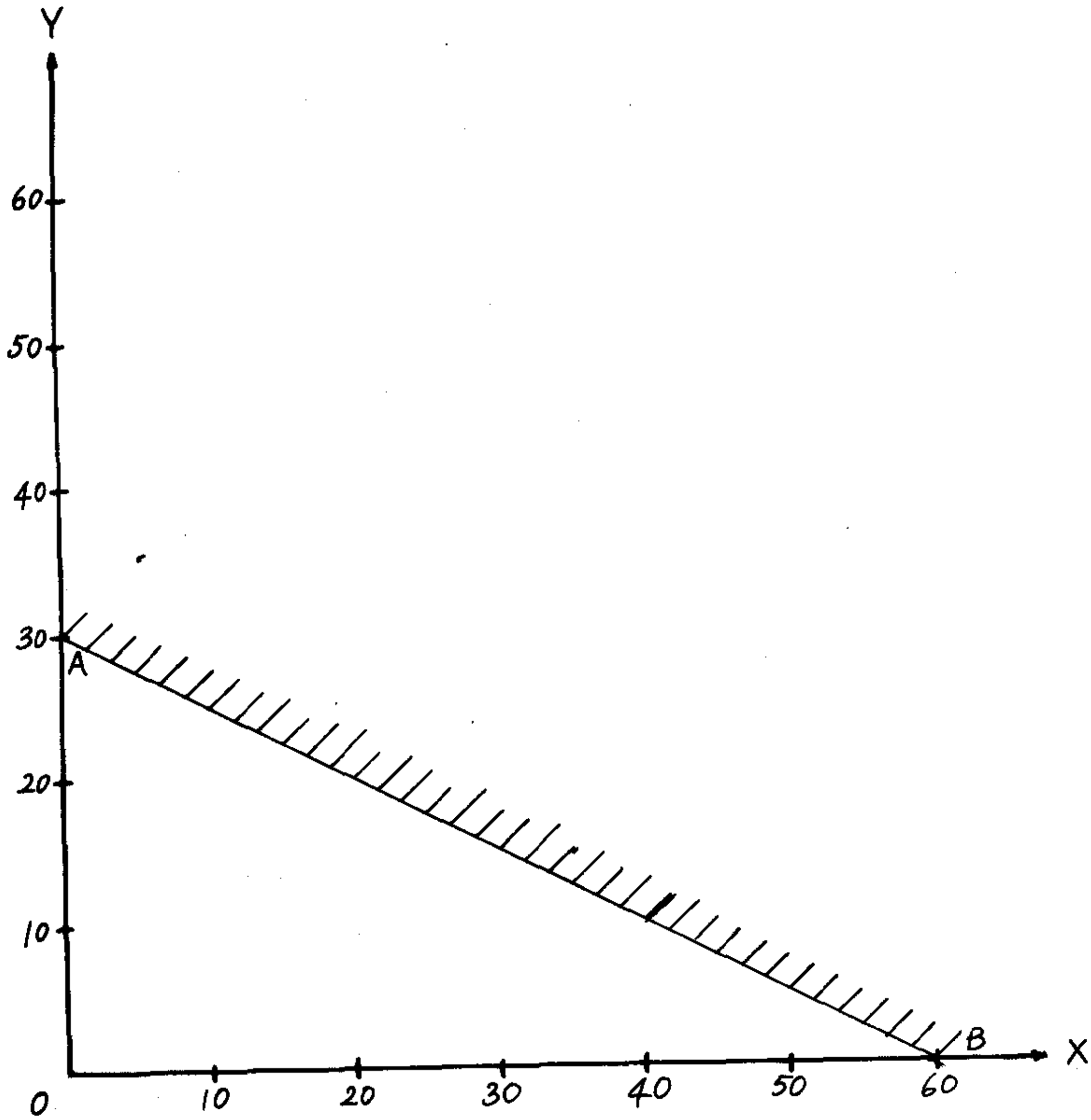
Now, the restrictions of production line II and production line III can be formulized as:

$$Y \leq 15$$
$$X + Y \leq 45$$

These expressions are geometrically presented in graph 4.3. with the first expression $(X + 2Y \leq 60)$. In this graph, line AWB is the locus of points that satisfy the equation $(X + 2Y = 60)$. The area ABO is the area of possible solutions to the inequality $(X + 2Y \leq 60)$, because this inequality expresses a "less than or equal to relationship. Similarly, line CWD is the locus of points that satisfy the equation $(Y = 15)$, and line EWF is the locus of points that satisfy the equation $(X + Y = 45)$. The area CDPO is the area of possible solutions to the inequality $(Y \leq 45)$ and the area EFO is the area of possible solutions to the inequality $(X + Y \leq 45)$.

The area that is common to all of the three inequalities is CWFO. It represents the locus of points that satisfy the three restrictions which the production lines create. Therefore, this area contains the possible solutions to the problem. In another way of saying, the answers to the problem, under the three restrictions, must lie somewhere in the shaded area that is shown in graph 4.3.

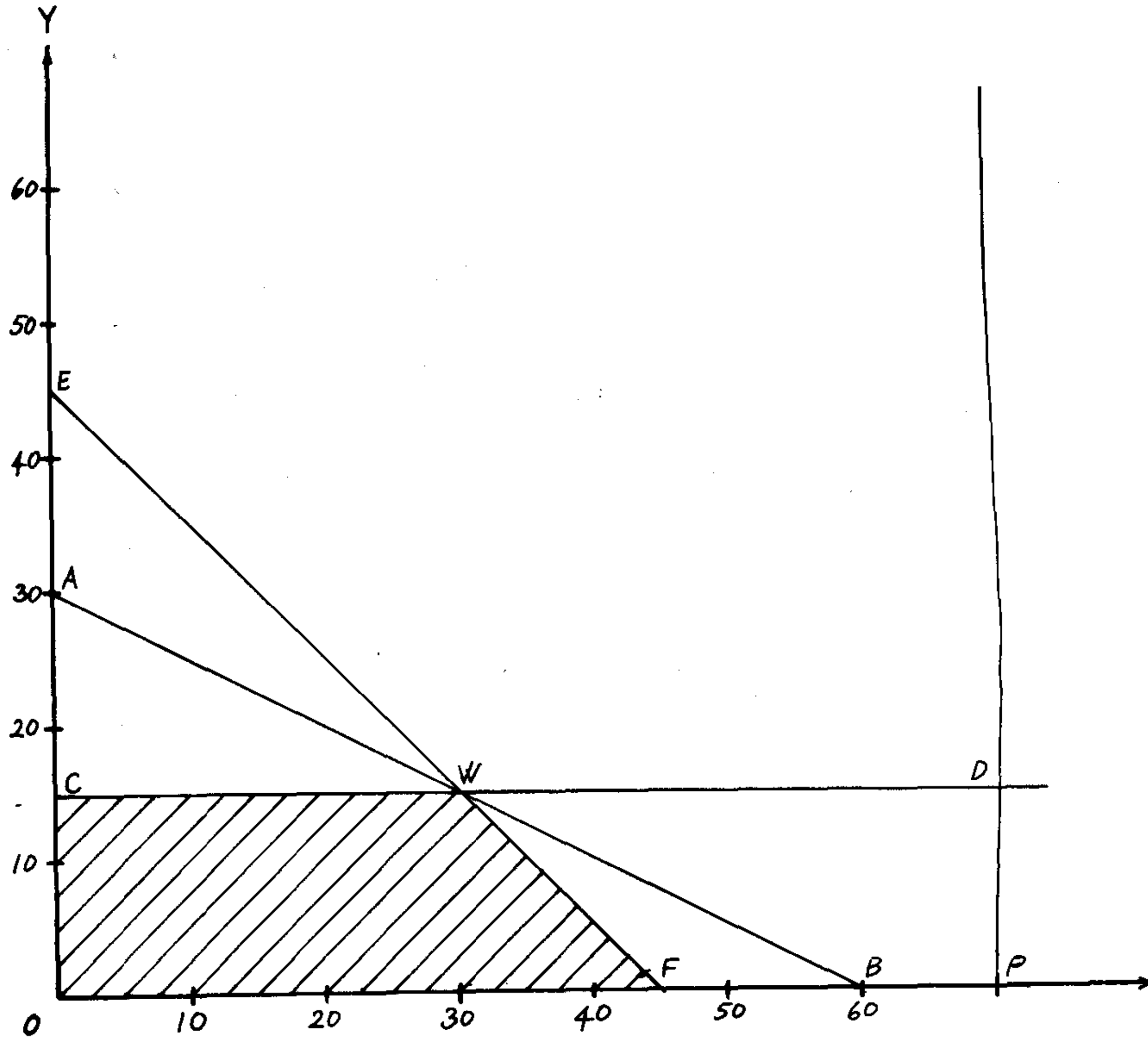
Although there are several solutions to such problems, the industrial executive is interested in finding the best one. Besides using the Simplex Method, he may do this graphically also. In this problem, the objective (profit) function can be formulized as:



GRAPH 4.2

GRAPHICAL EXPRESSION OF

$X + 2Y \geqq 60$



GRAPH 4.3

GRAPHICAL EXPRESSION OF

$$Y \leq 15,$$
$$X + Y \leq 45,$$
$$X + 2Y \leq 60.$$

$$10X + 15Y = \text{maximum}$$

At this point it is helpful to recall that a line is generally expressed by the equation

$$y = mx + b$$

where (x) and (y) are the variables, (m) is the slope of the line and (b) is the (y) intercept.

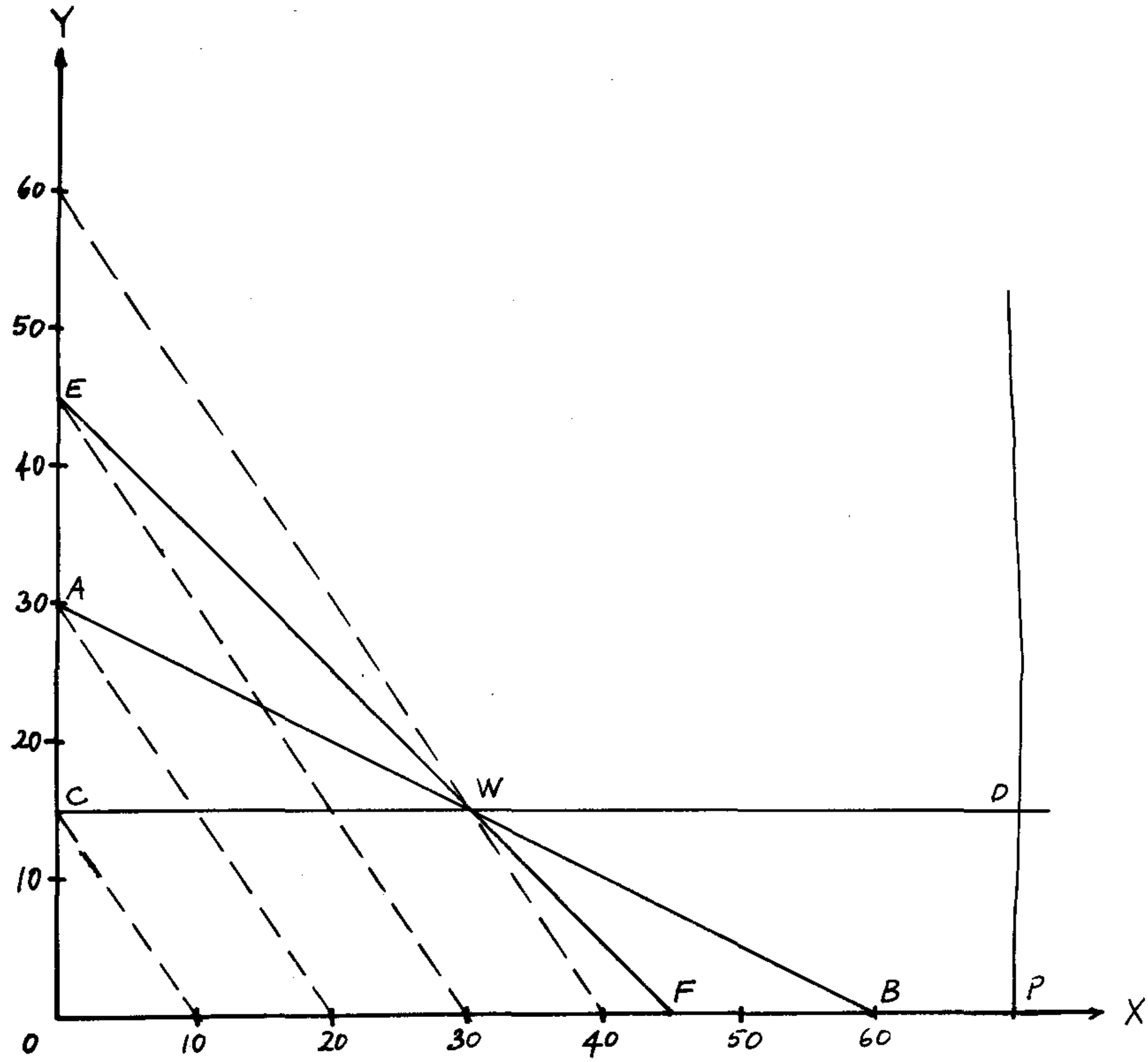
The objective function, that is written previously, may be placed into this general form as :

$$y = -\frac{10}{15}x + \frac{\text{maximum}}{15}$$

It is seen that the profit function may be represented by a family of parallel straight lines whose slopes are -2/3. These lines are shown in graph 4.4. The objective function, in the above form, indicates that the profit yields are greater as the lines move outward from the origin of the graph. Thus, it follows that the maximum solution to the problem is to produce 30 units of X and 15 units of Y for point W is the furthest point of the area CWFO from the origin. The objective lines that lie under the line, which passes through point W, also represent desirable and feasible solutions; but they indicate that there is capacity available for further production and profit.

4.1.2.- Algebraic Solution

The use of algebraic solutions is very rare in finding optimum solutions in industrial problems. However, a



GRAPH 4.4

GRAPHICAL SOLUTION OF
THE SAMPLE PROBLEM IN SECTION 4.1.1

short discussion on this subject is useful for demonstrating some of the features of the Simplex Method.

Now, let us take the problem, which was produced in the previous section, and solve it algebraicly. Assuming that there is no idle capacity, the restrictions can be expressed by the following equations:

$$X + 2Y = 60 \qquad (I)$$
$$Y = 15 \qquad (II)$$
$$X + Y = 45 \qquad (III)$$

Substitution of the value of Y, in equation II, to equation (I) or (III) gives the algebraic solution to the problem ($X = 30$, $Y = 15$). This is the same solution that was obtained graphically.

When there are more equations than the unknowns, one has to choose at least as many equations as there are unknowns in order to solve the problem. In industry, however, many problems arise, where the number of unknowns exceed the number of equations. In such cases, it becomes necessary to use the Simplex Method because it does not require as many equations as the unknowns in order to calculate an answer. Even in cases where the number of unknowns equal to the number of the equations, the large size of the problem may make it impractical to solve the equations simultaneously.

Another weakness of simultaneous solutions is that they are not always the optimum. The answer may not indicate the maximum profit, minimum cost etc. to the executive.

4.2.- THE BASIC SIMPLEX METHOD

4.2.1.- Essential Requirements of Programming a Problem

The requirements for building a model for a linear programming problem were discussed in the previous chapter. These requirements, of course, are essential in building up a problem for a solution by the Simplex Method. Therefore, the discussion in this section is made under the light of the previously mentioned points and is not in detail if the subject is already covered in chapter three.

The first requirement of programming a problem for a simplex solution is to have a known, definable objective. This objective may be maximizing profits, minimizing cost, or another criteria to be max imized or minimized. It must be eligible for algebraic expression with explicit coefficients, and it must be a function of the level of each of the activities that are engaged in the problem.

Secondly, "there must be known, definable restrictions or limitations on the amount or extent of the attainment of the objective. Such limitations might be machine capacity, material restrictions, sales committments, and storage limitations."(1) These limitations or restrictions must also be eligible for algebraic expression. The unknowns must be contained in the

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(1) Linear Programming, Op.Cit, p.77

algebraic expression of the objective. The problem is solved by finding the different levels of the various activities that maximize or minimize the objective function, as they satisfy all of the limitations, until the optimum level is obtained.

"Naturally, all the activities must occur either at a zero level or in some positive amount, since there is no meaning to a negative amount of an activity. It is usually the case that most of the restrictions in linear programming problems take the form of inequalities. In this event it is necessary to add one variable to each inequality to convert it to an equation. These variables are called slack variables and are activities just as much as the other activity variables in the equations." (1)

The slack variable will be discussed further in the following sections of this chapter.

Thirdly, the unknowns of the restrictions and the objective must be linear. Therefore, a change in the level of the activity must produce a proportional effect.

The fourth requirement is about the activities. The various activities that are involved in the problem must be identified and the level of each activity must be specific and measurable. Also, these activities must be interdependent.

"Activities are interdependent when they must share limited amounts of resources which they use in common, when one activity produces a commodity which another uses, or when several activities each produce a commodity used by another activity." (2)

Finally, the direct restrictions on the level of the activities must be stated in numerical terms.

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(1) Executive Decisions and Operations Research, Op-Cit.p.402

(2) Linear Programming, Op-Cit, P.79

4.2.2.- Essential Steps Before the Arrangement of the First Matrix

It is necessary to take some staps before putting the problem information in matrix. First of all it is essential to have comparable units of measure. Secondly, the inequations must be converted to equations. Finally, equation terms must be arranged so as to provide an easy convertison to matrix. These steps will be discussed in the following sections.

4.2.2.1.- Comparable Units of Measure

While programming a problem, one must notice to arrange the units so as to provide a logical relationship of problem information and answers. The units that are used in algebraic expressions need not be identical for all terms, but they must be comparable in order to run a valid computational process.

4.2.2.2.- Conversion of Inequalities to Equations

In the previous chapter it was mentioned that inequalities represent approximations, while equations express specific statements. The inequalities express the phrases "equal to or less than", or "equal to or greater than". Mathematically the former is indicated by the sign ($\leqq$) and the latter by ($\geqq$).

In order to carry out the Simplex calculations, it is essential to convert the inequalities into equalities by adding a slack variable , or a slack and an artificial variable. The addition of these variables may be explained clearly by examples.

Suppose that the restriction is: :"B-type of the products can be produced either in production line I or in production line II. If all the resources are given to production line I, it is possible to produce at most 2000 units of the B-type products. On the other hand, if all resources are given to production line II, the amount of products will be 1000 or less." This restriction can mathematically be expressed as:

$$X_1 + 2X_2 \leq 2000,$$

or,

$$1/2X_1 + X_2 \leq 1000$$

This means that maximum production may be attained ifthe corresponding productive facility can be utilized to its full capacity. Since it is probable to fail in obtaining the maximum utilization due to unused capacity, such as idle time, it becomes necessary to show it in the equation. This is done by adding a slack variable. This term can be regarded as an artificial product which will have a zero value when all of the capacity is used to produce (X) and (Y). It may also be regarded as the indicator of idle time when some capacity is not used while producing (X) and (Y). Therefore, slack variable has a rate of production of one and a profit of zero. Thus the above expressions become as follows when they are converted to equations;

$$X_1 + 2X_2 + X_3 = 2000,$$

or,

$$1/2X_1 + X_2 + X_3 = 1000.$$

Now, let us suppose that the restriction is as follows: "The capacities of the production lines I and II are 200 units and 300 units. The sum of the units of products that are produced in these lines must be at least 400." The mathematical expression of this statement is:

$$2X_1 + 3X_2 \geqq 400$$

In this case the value of the slack variable comes out negative. This can be seen more clearly if we assume that the other inequality signs point at the opposite way. "The rule is that all inequalities must be pointed in the same direction before they are converted to equations."(1) Therefore, it is necessary to divide or multiply both sides of the equation by minus one in order to change the direction of the inequality sign. Thus the above inequality becomes:

$$-2X_1 - 3X_2 \leqq -400$$

This inequality is converted to an equation by adding a slack variable as follows:

$$-2X_1 - 3X_2 + X_3 = -400$$

However, as it was mentioned previously, all the activities must occur at a zero level or in some positive amount. Therefore, we multiply or divide both sides of the equation by minus one. Then, the equation becomes:

$$2X_1 + 3X_2 - X_3 = 400$$

(1) Ibid, P.84

Here, it is seen that the value of slack variable is negative. However, it is the rule that the value of slack variable can not be negative. Therefore, an artificial variable, whose value is hypothetically very large and positive, is added in order to make the slack variable positive. Thus the above equation becomes:

$$2X_1 + 3X_2 - X_3 + X_4 = 400$$

where X_3 denotes slack variable and X_4 the artificial variable.

4.2.2.3.- Arrangement of the Terms of the Equations

In order to convert the information to matrix easily, it is useful to arrange the equations according to the form of matrix. In this study, constants are written on the right-hand side of the equations and slack variables are placed after the variables and their coefficients.

4.2.3.- Arrangement of the First Matrix

Another name for matrix is "Tableau". It is a form to which problem information is transferred from equations in order to compute the answer. Different forms have been developed by different authors. The form that is used in this study is explained in the following paragraphs.

The parts of matrix are shown in Figure 4.1.

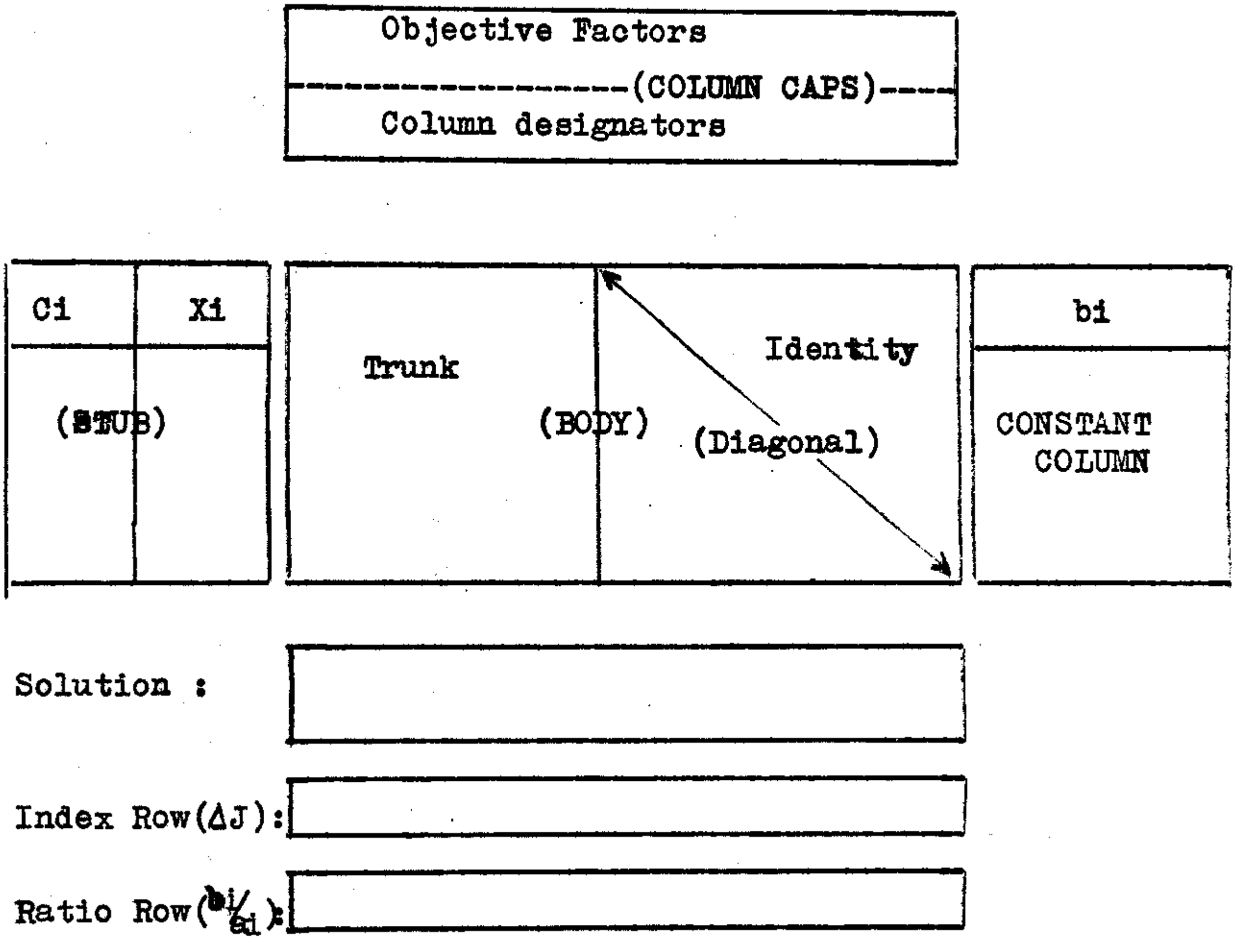

Fig. 4-1.- Parts of a Simplex Matrix.

Objective factors and column designators are placed at the top of the body of matrix and are called column caps. Objective factors are the coefficients of the variables in the objective equation. They are expressed as profits per unit, cost per unit and the like. Column designators, on the other hand, are the symbols which designate the unknowns, or variables, in each column of the matrix. They may be indicated by the different letters of the alphabet, such as X,Y,Z etc., or by one letter of the alphabet with numbered subscripts, such as X_1, X_2, X_3 etc.

The stub of the matrix consists of two columns. Column X_i designates the variables that are in operation. Upon each iteration of the matrix, one variable departs from this column and another one enters. Column C_i indicates the objective coefficients- the cost or the profit coefficients- of the variables in column X_i. Column caps of the identity and the stub are identical in the first matrix.

The body of the matrix consists of coefficients of the unknowns which express the restrictions. The trunk of the matrix contains the original coefficients of the unknown. They are the coefficients of the unknowns that were present before the inequations were converted to equations. Coefficients of the unknowns that are added for converting the inequalities to equations make up the identity.

"The identity must always be a square. That is, there must be as many rows as there are columns shown in the identity in order to be certain that the computations will work. This is a mathematical requirement, but it is obviously true if all unknowns in the identity are in the stub. The zero profit values of the identity provide the first zero profit program."(1)

The constant column consists of the constants of each equation. This is referred as the (bi) column in this study.

The solution row indicates the values of the unknowns at the end of each iteration. It is determined by looking at

(1) Ibid, P-83

the corresponding values that lie in the constant column to each unknown in the stub. The solution of each Tableau is modified by successive iterations until the optimum is obtained.

The index row of the matrix determines the successive improvements by use of the margins which indicate the points of improvement. This, and the calculation of bi/ai are processes, which are performed after the establishment of the first Tableau. Therefore, these parts of the matrix will be discussed in the following paragraphs.

Now, let us see the establishment of the first matrix by a simple example. Suppose that the following restrictions and the profit function are given: (1)

Restrictions:

$$2X_1 + 3X_2 \leq 8$$

$$2X_2 + 5X_3 \leq 10$$

$$3X_1 + 2X_2 + 4X_3 \leq 50$$

Profit Function:

$$Z = 3X_1 + 5X_2 + 4X_3$$

The restrictions are converted to equality expressions by addition of slack variables, and the profit function is expressed as follows:

Restrictions:

$$2X_1 + 3X_2 + 0X_3 + X_4 + 0X_5 + 0X_6 = 8$$

(1) Göker, Metin, Lecture on "Production Programming", Robert College, Oct. 14, 1964, İstanbul

$$0X_1 + 2X_2 + 5X_3 + 0X_4 + X_5 + 0X_6 = 10$$

$$3X_1 + 2X_2 + 4X_3 + 0X_4 + 0X_5 + X_6 = 15$$

Profit Function:

$$\text{Maximize } Z = 3X_1 + 5X_2 + 4X_3 + 0X_4 + 0X_5 + 0X_6$$

The above information is expressed in the first Tableau as follows:

Ci	Xi	3	5	4	0	0	0	bi
		X_1	X_2	X_3	X_4	X_5	X_6	
0	X_4	2	3	0	1	0	0	8
0	X_5	0	2	5	0	1	0	10
0	X_6	3	2	4	0	0	1	15
	Sol:	0	0	0	8	10	15	

Profit of the first program is zero; because, upon substitution of the values obtained at the solution to the profit function gives this result. The products that are contained in this program are listed in Xi column. Since these are the slack products, each has a profit of zero; and consequently, the total profit of the program comes out zero.

4.2.4.- The Computational Routine

Calculation of the index row (ΔJ) is the first step in the computational routine of the Simplex Method. IT is determined by subtracting the sum of the products, which are formed by multiplying each entry in each column by the corresponding value in the stub, from the objective factor of each column in the column cap. Since the objective factors indicate the optimum attainment, the positive numbers that come out after the above operation indicate margins of increase in profit which may be achieved by further modifications. Thus, in such cases, the variable that is indicated by the greatest margin is picked and put in the matrix in order to push the profit to the maximum value.

The index row (ΔJ) of the first Tableau is calculated as follows:

		3		5		4		0		0		0
		X_1		X_2		X_3		X_4		X_5		X_6
	0 x 2	0	0 x 3	0	0 x 0	0	0 x 1	0	0 x 0	0	0x 0	0
	0 x 0	0	0 x 2	0	0 x 5	0	0 x 0	0	0 x 1	0	0x 0	0
Total		0		0		0		0		0		0
Obj.		3		5		4		0		0		0
Total		-0		-0		-0		-0		-0		-0
Differ.		3		5		4		0		0		0

Thus, the index row comes out as:

3	5	4	0	0	0

The positive values indicate the possibility of further improvement. Three means that 3 units of profit can be added to the profit of the program for each unit of X_1 that is brought into the program. Five and four mean the same thing with respect to X_2 and X_3.

It is seen that, among the three variables that may enter into the next matrix, X_2 will add the most profit per unit. Therefore, it is included first in the next matrix. However, at this point, the operator must keep in mind that the addition of one unit of this product will require production time, which eventually is taken away from the time that could be allocated to other products, for the amount of production time is limited. Thus, he has to take out the variable which takes away the least productive capacity for the production of other products on that production line. The above discussion may be summarized as:

"Whenever a product is brought into the solution, two conditions have to be considered in order to make sure that a more profitable solution will be obtained. They are:

1.- The amount of profit per unit that will be added to the profit total by including the new product in the program.

2.- The effect of using the time required to produce that product since manufacturing capacity will be taken away from other products which can add to profits also." (1)

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(1) Linear Programming, Op.Cit., P-83

Above discussion shows that the second step in computational routine is to determine the entering variable and the third step is to find the departing variable. Entering variable is the product, which will add the most profit per unit to the current program. The column that lies under its column designator is called the key column. This column is the column that is above the most positive number in the index row. In our example, the entering variable is X_2 and the key column is the one that lies under it in the body of the matrix.

Departing variable is determined from the key row, which is always in the body of the matrix. Key row is selected by dividing each positive number in the key column into a positive number or zero in the constant column, which lie in the same row with the number in the key column. These ratios are written in the ratiorow (bi/ai), which is under the index row. Of course, negative ratios have no meaning. The smallest ratio shows the smallest capacity that will be lost by departure of that product from the current production program. In our example, the smallest ratio is 8/3; and thus the departing variable is X_4 and the key row is the first row in the body of the matrix.

" The number in the location at the intersection of the key column and key row is called the key number."[1] In our example, the key number is three in the first row of the body of the matrix.

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(1) Ibid, p.91

"The initial row, or first row, of the next matrix is formed by dividing each number in the key row (of the current matrix) by the key number and placing each value thus obtained in the initial row and the appropriate column of the new matrix, starting with the constant column." (1)

In our example, the values in the new matrix are 8/3, in the constant column, and 2/3,1,0,1/3,0,0 in the body of the matrix. The rest of the entries, in the body of the new matrix and the constant column are determined as follows:

"1.- Select a square in the new matrix tobe filled in
2.- Refer to the value occupying the corresponding square in the preceeding matrix.
3.- Subtract from that value the product formed by the ratio of the number at the intersection of the column containing the value and the key row and the key number, and by the number found in the same row as the original value selected but in the key column.
4.- Place the result of these computations in the square in the new matrix.
5.- Perform the same type of calculations for all remaining squares, including those in the index row, until all squares have been filled in." (2)

Stub of the new matrix is determined by replacing the departing variable and its coefficient, with the entering variable and its coefficient. The rest of the stub remains the same as the preceeding one.

Zero or negative values in the index row indicate no further improvements. Therefore, the best solution is reached when all of the numbers in the index row become zero or negative.

Under the light of the above discussion, the second

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(1) Ibid.

(2) Ibid.

matrix of the example problem is determined as follows:

Tableau : I

(Key number) (Key column) (Key row)

		"3	5	4	0	0	0	
Ci	Xi	X_1	X_2	X_3	X_4	X_5	X_6	bi
0	X_4	2	3	0	1	0	0	8
0	X_5	0	2	5	0	1	0	10
0	X_6	3	2	4	0	0	1	15
Sol:		0	0	0	8	10	15	
Δ J:		3	5	4	0	0	0	
bi/ai		-	-	-	8/3	10/2	15/2	

Tableau: II

Ci	X i	3	5	4	0	0	0	bi
		X_1	X_2	X_3	X_4	X_5	X_6	
5	X_2	2/3	1	0	1/3	0	0	8/3
0	X_5	-4/3	0	5	-2/3	1	0	14/3
0	X_6	5/4	0	4	-2/3	0	1	29/3

Sol:	0	8/3	0	0	14/3	29/3
Δ J:	-1/3	0	4	-5/3	0	0
bi/ai:	-	∞	-	-	14/15	29/12

Now, the computational routine may be summarized as follows:

1.- Calculate the index row of the first matrix.

2.- Determine the entering variable and the key column.

3.- Determine the departing variable and the key row.

4.- Determine the key number.

5.- Form the first row of the next matrix.

6.- Fill the rest of the rows in the new Tableau.

7.- Continue the process until all of the numbers in the index row are zero or negative.

4.2.5.- Degeneracy

" Degeneracy occurs either when the problem begins to cycle and never reach optimum or when the problem collapses (one of the variable disappears) before an optimum has been obtained. This can, of course, happen as a result of error, but it can also occur as a result of the problem itself." (1)

Degeneracy is predicted while selecting the key row. In some cases a tie occurs between two or more rows; that is, the ratios turn out to have the same value. Here, the problem may begin to cycle or continue indefinitely without reaching to an optimum solution if the operator selects a wrong row as the key row. Selection of wrong row arises the possibility that the variable in the stub of the other tied row may disappear permanently. Therefore, it becomes necessary to resolve the degeneracy in order to avoid the possibility of continuous cycles, which prevent the attainment of the optimum answer.

"The degeneracy is resolved as follows:

1.- Divide each element in the tied rows by its key column number.

2.- Compare the ratios so obtained term by term (column by column) from left to right first in the identity and then in the trunk.

3.- The tie is broken when the ratios are unequal.

4.- Select as the key row that row with the algebraically smaller ratio."(1)

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(1) Elementary Mathematical Programming, Op.Cit,p.88

(1) Ibid.

CHAPTER V

PROGRAMMING THE OPTIMUM ALLOCATION OF PRODUCTS IN THE BEYKOZ FACTORY IN ORDER TO REACH AT THE MAXIMUM PROFIT

5.1.- PROGRAM OF THE PROBLEM

In chapter II the first problem of the Sümerbank Leather and Shoe Factory is stated as: "UNDER THE GIVEN RESOURCES AND RESTRICTIONS, HOW MANY PAIRS OF EACH TYPE OF THE FOOT WEAR SHOULD BE PRODUCED IN THE DIFFERENT LINES IN ORDER TO OBTAIN THE MAXIMUM PROFIT? " In other words, " HOW MUST THE PRODUCTS BE ALLOCATED TO THE PRODUCTION LINES IN ORDER TO YIELD THE MAXIMUM PROFIT?"

The information collected on this problem is presented in Chapter II under the heading "Production Data" , Now let us refer to Table 2.1 and prepare the mathematical model of the problem.

5.1.1.- Mathematical Designation of the Variables

The variables in this problem are defined as follows:

X_1 : Number of pairs of McKay army boots manufactured in military production line I.

X_2 : Number of pairs of staple welted shoes manufactured in military production line I.

X_3 : Number of pairs of McKay army boots manufactured in military line II.

X_4 : Number of pairs of staple welted shoes manufactured in military production line II.

X_5 : Number of pairs of McKay army boots manufactured in military production line III.

X_6 : Number of pairs of staple welted shoes manufactured in military production line III.

X_7 : Number of pairs of welted shoes manufactured in military production line III.

X_8 : Number of pairs of cemented shoes manufactured in military production line III.

X_9 : Number of pairs of Good-year ordinary shoes manufactured in Good-year welted line I.

X_{10}: Number of pairs of McKay army boots manufactured in Good-year welted line II.

X_{11}: Number of pairs of Good-year ordinary shoes manufactured in Good-year welted line II.

X_{12}: Number of pairs of McKay army boots manufactured in staple welted line.

X_{13}: Number of pairs of staple welted shoes manufactured in staple welted line.

X_{14}: Number of pairs of welted shoes manufactured in staple welted line.

X_{15}: Number of pairs of McKay army boots manufactured in mixed product line.

X_{16}: Number of pairs of staple welted shoes manufactured in mixed product line.

X_{17}: Number of pairs of welted shoes manufactured in mixed product line.

X_{18}: Number of pairs of cemented shoes manufactured in mixed product line.

These variables are shown in tabular form on the following page.

5.1.2.- The Objective Function

The objective of the problem is to find the product mix that will yield the maximum profit. The profit per pair on each product are designated by the following symbols. The current figures for these symbols may be calculated from the data in the cost accounting department, if anybody is interested in solving the problem by a computer.

A: TL/pair on McKay army boots manufactured in military production line I.

B: TL/pair on staple welted shoes manufactured in military production line I.

C: TL/pair on McKay army boots manufactured in military production line II.

D: TL/pair on staple welted shoes manufactured in military production line II.

E: TL/pair on McKay army boots manufactured in military production line III.

F: TL/pair on staple welted shoes manufactured in military production line III.

G: TL/pair on welted shoes manufactured in military production line III.

H: TL/pair on cemented shoes manufactured in military produdtion line III.

	CAPACITIES (pairs/day)				
Production Line	Boots		Shoes		
	McKay Army Boots	Staple welted	Good-year ordinary	Welted	Cemented
Military Production Line I	X_1	X_2	-	-	-
Military Production Line II	X_3	X_4	-	-	-
Military Production Line III	X_5	X_6	-	X_7	X_8
Good-year welted line I	-	-	X_9	-	-
Good-year Welted line II	X_{10}	-	X_{11}	-	-
Staple welted line	X_{12}	X_{13}	-	X_{14}	-
Mixed Product line	X_{15}	X_{16}	-	X_{17}	X_{18}

Table 5.1

Variables of the First Problem

I: TL/pair on Good-year ordinary shoes manufactured in Good-year welted line I.

J: TL/pair on McKay army boots manufactured in Good-year welted line II.

K: TL/pair on Good-year ordinary shoes manufactured in Good-year welted line II.

L: TL/pair on MCKay army boots manufactured in staple welted line.

M: TL/pair on staple welted shoes manufactured in staple welted line.

N: TL/pair on welted shoes manufactured in staple welted line.

O: TL/pair on McKay army boots manufactured in mixed product line.

P: TL/pair on staple welted shoes manufactured in mixed product line.

R: TL/pair on welted shoes manufactured in mixed product line.

S: TL/pair on cemented shoes manufactured in mixed product line.

Now the objective can be expressed mathematically as follows:

$$\text{Maximize } Z = AX_1 + BX_2 + CX_3 + DX_4 + EX_5 + FX_6 + GX_7 + HX_8 + IX_9 + JX_{10} + KX_{11} + LX_{12} + MX_{13} + NX_{14} + OX_{15} + PX_{16} + RX_{17} + SX_{18}$$

5.1.3.- The Restrictions

The restrictions that are deduced from the product data are as follows:

1.- The number of pairs of McKay army boots that can be produced in military production line I is equal to or less than 1000 per day; and the number of pairs of staple welted shoes that can be produced in the same line is equal to or less than 800 per day.

2.- The number of pairs of McKay army boots that can be produced in military production line II is equal to or less than 1000 per day; and the number of pairs of staple welted shoes that can be produced in the same line is equal to or less than 800 per day.

3.- The number of pairs of McKay army boots that can be produced in military production line III is equal to or less than 1000 per day; the number of pairs of staple welted shoes that can be produced in the same line is equal to or less than 800 per day; the number of pairs of welted shoes that can be produced in the same line is equal to or less than 750 per day, and the number of pairs of cemented shoes that can be produced in the same line is equal to or less than 750 per day.

4.- The number of pairs of Good-year ordinary shoes that can be produced in Good-year welted line I is equal to or less than 850 per day.

5.- The number of pairs of McKay army boots that can be produced in Good-year welted line II is equal to or less than 1000 per day; and the number of pairs of Good-year ordinary shoes that can be produced in the same line is equal to or less than 850 per day.

6.- The number of pairs of McKay army boots that can be produced in staple welted line is equal to or less than 1000 per day; the number of pairs of staple welted shoes that can be produced in the same line is equal to or less than 800 per day; and the number of pairs of welted shoes that can be produced in the same line is equal to or less than 750 per day.

7.- The number of pairs of McKay army boots that can be produced in mixed product line is equal to or less than 1000 per day; the number of pairs of staple welted shoes that can be produced in the same line is equal to or less than 750 per day; and the number of pairs of cemented shoes that can be produced in the same line is equal to or less than 750 per day.

8.- The number of pairs of McKay army boots that can be sold in one day is equal to or less than 4000.

9.- The number of pairs of staple welted shoes that can be sold in one day is equal to or less than 2000.

10.- The number of pairs of Good-year ordinary shoes that can be sold in one day is equal to or less than 1500.

11.- The number of pairs of welted shoes that can be sold in one day is equal to or lesss than 2000.

12.- The number of pairs of cemented shoes that can be sold in one day is equal to or less than 1200.

The mathematical expressions of these restrictions, in equivalent units, are as follows:

1) $X_1 + \frac{5}{4} X_2 \leq 1000$

2) $X_3 + \frac{5}{4} X_4 \leq 1000$

3) $X_5 + \frac{5}{4} X_6 + \frac{4}{3} X_7 + \frac{4}{3} X_8 \leq 1000$

4) $X_9 \leq 850$

5) $X_{10} + \frac{20}{17} X_{11} \leq 1000$

6) $X_{12} + \frac{5}{4} X_{13} + \frac{4}{3} X_{14} \leq 1000$

7) $X_{15} + \frac{5}{4} X_{16} + \frac{4}{3} X_{17} + \frac{4}{3} X_{18} \leq 1000$

8) $X_1 + X_3 + X_5 + X_{10} + X_{12} + X_{15} \leq 4000$

9) $X_2 + X_4 + X_6 + X_{13} + X_{16} \leq 2000$

10) $X_9 + X_{11} \leq 1500$

11) $X_7 + X_{14} + X_{17} \leq 2000$

12) $X_8 + X_{18} \leq 1200$

5.1.4.- Conversion of Inequalities to Equalities

As mentioned in the previous chapter, it is essential to convert the inequalities to equalities in order to set up the first tableau. The expression of the restrictions in equalities (upon addition of slack variables) are presented in the following pages.

5.1.5.- The First Tableau

The first tableau of this problem is presented after the equations. The computational routine may be carried on by the people who are interested in the solution.

THE EQUATIONS

1) $X_1 + \frac{5}{4} X_2 + 0X_3 + 0X_4 + 0X_5 + 0X_6 + 0X_7 + 0X_8 + 0X_9 + 0X_{10} + 0X_{11}$
$+ 0X_{12} + 0X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17} + 0X_{18} + X_{19} + 0X_{20}$
$+ 0X_{21} + 0X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + 0X_{26} + 0X_{27} + 0X_{28} + 0X_{29}$
$+ 0X_{30} = 1000$

2) $0X_1 + 0X_2 + X_3 + \frac{5}{4} X_4 + 0X_5 + 0X_6 + 0X_7 + 0X_8 + 0X_9 + 0X_{10}$
$+ 0X_{11} + 0X_{12} + 0X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17} + 0X_{18}$
$+ 0X_{19} + X_{20} + 0X_{21} + 0X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + 0X_{26}$
$+ 0X_{27} + 0X_{28} + 0X_{29} + 0X_{30} = 1000$

3) $0X_1 + 0X_2 + 0X_3 + 0X_4 + X_5 + \frac{5}{4} X_6 + \frac{4}{3} X_7 + \frac{4}{3} X_8 + 0X_9 + 0X_{10}$
$+ 0X_{11} + 0X_{12} + 0X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17} + 0X_{18} + 0X_{19}$
$+ 0X_{20} + X_{21} + 0X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + 0X_{26} + 0X_{27} + 0X_{28}$
$+ 0X_{29} + 0X_{30} = 1000$

4) $0X_1 + 0X_2 + 0X_3 + 0X_4 + 0X_5 + 0X_6 + 0X_7 + 0X_8 + X_9 + 0X_{10} + 0X_{11}$
$+ 0X_{12} + 0X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17} + 0X_{18} + 0X_{19} + 0X_{20}$
$+ 0X_{21} + X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + 0X_{26} + 0X_{27} + 0X_{28} + 0X_{29}$
$+ 0X_{30} = 1000$

5) $0x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5 + 0x_6 + 0x_7 + 0x_8 + 0x_9 + x_{10} + \frac{20}{17} x_{11}$
$+ 0x_{12} + 0x_{13} + 0x_{14} + 0x_{15} + 0x_{16} + 0x_{17} + 0x_{18} + 0x_{19} + 0x_{20}$
$+ 0x_{21} + 0x_{22} + x_{23} + 0x_{24} + 0x_{25} + 0x_{26} + 0x_{27} + 0x_{28} + 0x_{29}$
$+ 0x_{30} = 1000$

6) $0x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5 + 0x_6 + 0x_7 + 0x_8 + 0x_9 + 0x_{10} + 0x_{11}$
$+ x_{12} + \frac{5}{4} x_{13} + \frac{4}{3} x_{14} + 0x_{15} + 0x_{16} + 0x_{17} + 0x_{18} + 0x_{19} + 0x_{20}$
$+ 0x_{21} + 0x_{22} + 0x_{23} + x_{24} + 0x_{25} + 0x_{26} + 0x_{27} + 0x_{28} + 0x_{29}$
$+ 0x_{30} = 1000$

7) $0x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5 + 0x_6 + 0x_7 + 0x_8 + 0x_9 + 0x_{10} + 0x_{11}$
$+ 0x_{12} + 0x_{13} + 0x_{14} + x_{15} + \frac{5}{4} x_{16} + \frac{4}{3} x_{17} + \frac{4}{3} x_{18} + 0x_{19} + 0x_{20}$
$+ 0x_{21} + 0x_{22} + 0x_{23} + 0x_{24} + x_{25} + 0x_{26} + 0x_{27} + 0x_{28} + 0x_{29}$
$+ 0x_{30} = 1000$

8) $x_1 + 0x_2 + x_3 + 0x_4 + x_5 + 0x_6 + 0x_7 + 0x_8 + 0x_9 + x_{10} + 0x_{11}$
$+ x_{12} + 0x_{13} + 0x_{14} + x_{15} + 0x_{16} + 0x_{17} + 0x_{18} + 0x_{19} + 0x_{20}$
$+ 0x_{21} + 0x_{22} + 0x_{23} + 0x_{24} + 0x_{25} + x_{26} + 0x_{27} + 0x_{28} + 0x_{29}$
$+ 0x_{30} = 4000$

9) $0x_1 + x_2 + 0x_3 + x_4 + 0x_5 + x_6 + 0x_7 + 0x_8 + 0x_9 + 0x_{10} + 0x_{11}$
$+ 0x_{12} + 0x_{13} + 0x_{14} + 0x_{15} + x_{16} + 0x_{17} + 0x_{18} + 0x_{19} + 0x_{20}$
$+ 0x_{21} + 0x_{22} + 0x_{23} + 0x_{24} + 0x_{25} + 0x_{26} + x_{27} + 0x_{28} + 0x_{29}$
$+ 0x_{30} = 2000$

10) $0X_1 + 0X_2 + 0X_3 + 0X_4 + 0X_5 + 0X_6 + 0X_7 + 0X_8 + X_9 + 0X_{10} + X_{11}$
$+ 0X_{12} + 0X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17} + 0X_{18} + 0X_{19} + 0X_{20}$
$+ 0X_{21} + 0X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + 0X_{26} + 0X_{27} + X_{28} + 0X_{29}$
$+ 0X_{30} = 1500$

11) $0X_1 + 0X_2 + 0X_3 + 0X_4 + 0X_5 + 0X_6 + X_7 + 0X_8 + 0X_9 + 0X_{10} + 0X_{11}$
$+ 0X_{12} + 0X_{13} + X_{14} + 0X_{15} + 0X_{16} + X_{17} + 0X_{18} + 0X_{19} + 0X_{20}$
$+ 0X_{21} + 0X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + 0X_{26} + 0X_{27} + 0X_{28} + X_{29}$
$+ 0X_{30} = 2000$

12) $0X_1 + 0X_2 + 0X_3 + 0X_4 + 0X_5 + 0X_6 + 0X_7 + X_8 + 0X_9 + 0X_{10} + 0X_{11}$
$+ 0X_{12} + 0X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17} + X_{18} + 0X_{19} + 0X_{20}$
$+ 0X_{21} + 0X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + 0X_{26} + 0X_{27} + 0X_{28} + 0X_{29}$
$+ X_{30} = 1200$

5.2.- SAMPLE PROBLEM

5.2.1.- The Product Data

Suppose that the following conditions prevail in the Sümerbank Leather and Shoe Factory: There are three production lines; military production line, Good-year welted line and staple welted line. It is possible to produce only 1000 pairs of army boots in one day in the military production line. In the Good-year welted line, it is possible to produce only 800 pairs of Good-year welted shoes; and in the staple welted line it is possible to produce either 1000 pairs of army boots or 750 pairs of welted shoes in one day.

The maximum amounts of pairs of each footwear that can be sold in one day are as follows:

Army boots	1800 pairs
Good-year welted shoes	800 pairs
Welted shoes	600 pairs

The unit profits of each product, with respect to the production lines they are produced in, are listed below:

Army boots manufactured in military production line	10TL/pair
Army boots manufactured in staple welted line	6TL/pair
Good-year welted shoes produced in Good-year welted line	12TL/pair
Welted shoes produced in staple welted line	8TL/pair

The product data are summarized in Table 5I.

CAPACITIES (pairs/day)			
Production Line	Army Boots	Good-year welted sh.	Welted Shoes
Military production line	1000	-	-
Good-year welted line	-	800	-
Staple welted line	1000	-	750
Maximum sales possibilities	1800	800	600

Table5.I

Product Date of the Sample Problem

5.2.2.-Mathematical Designation of the Variables

The variables of the problem may be defined as follows:

X_1 : Number of pairs of army boots manufactured in the military production line.

X_2 : Number of pairs of Good-year welted shoes manufactured in the Good-year welted line.

X_3 : Number of pairs of army boots manufactured in the staple welted line.

X_4 : Number of pairs of welted shoes manufactured in the staple welted line.

5.2.3.- The Objective Function

The objective of this problem is to find the product mix that will yield the maximum profit. The unit profits per pair on each product are indicated in the product data. From this, the objective can be expressed mathematically as follows:

Maximize $Z = 10X_1 + 12X_2 + 6X_3 + 8X_4$

5.2.4.- The Restrictions

The restrictions that are deduced from the product data are as follows:

1.- The number of pairs of army boots that can be produced in the military production line is equal to or less than 1000 per day.

2.- The number of pairs of Good-year welted shoes that can be produced in the Good-year welted line is equal to or less than 800 per day.

3.- The number of pairs of army boots that can be produced in the staple welted line is equal to or less than 1000 per day; and the number of pairs of welted shoes that can be produced in the same line is equal to or less than 750 per day.

4.- The number of pairs of army boots that can be sold in one day is equal to or less than 1800.

5.- The number of pairs of Good-year welted shoes that can be sold in one day is equal to or less than 800.

6.- The number of pairs of welted shoes that can be sold in one day is equal to or less than 600.

The mathematical expressions of these restrictions, in equivalent units, are as follows:

1) $X_1 \leq 1000$

2) $X_2 \leq 800$

3) $X_3 + \frac{4}{3} X_4 \leq 1000$

4) $X_1 + X_3 \leq 1800$

5) $X_2 \leq 800$

6) $X_4 \leq 600$

The inequalities 2 and 5 are the same. Therefore, restriction five is omitted for it is redundant.

5.2.5. Conversion of Inequalities to Equalities

The following equations are deduced from the inequalities that are listed above:

1) $X_1 + 0X_2 + 0X_3 + 0X_4 + X_5 + 0X_6 + 0X_7 + 0X_8 + 0X_9 = 1000$

2) $0X_1 + X_2 + 0X_3 + 0X_4 + 0X_5 + X_6 + 0X_7 + 0X_8 + 0X_9 = 800$

3) $0X_1 + 0X_2 + X_3 + \frac{4}{3} X_4 + 0X_5 + 0X_6 + X_7 + 0X_8 + 0X_9 = 1000$

4) $X_1 + 0X_2 + X_3 + 0X_4 + 0X_5 + 0X_6 + 0X_7 + X_8 + 0X_9 = 1800$

5) $0X_1 + 0X_2 + 0X_3 + X_4 + 0X_5 + 0X_6 + 0X_7 + 0X_8 + X_9 = 600$

5.2.6.- The Computational Routine

The complete solution of the problem is presented in the following pages. The solution shows that the maximum profit may be obtained by manufacturing 1000 pairs of army boots in the military production line, 800 pairs of Good-year welted shoes in the Good-year welted line, 200 pairs of army boots and 600 pairs of welted shoes in staple welted line.

This allocation satisfies all of the restrictions. It is seen when the values of the variables are substituted in the inequalities which describe the restrictions.

1) $X_1 \leqq 1000$ .... $1000 \leqq 1000$

2) $X_2 \leqq 800$ .... $800 \leqq 800$

3) $X_3 + \frac{4}{3} X_4 \leqq 1000$ .... $200 + \frac{4}{3}(600) \leqq 1000$

4) $X_1 + X_3 \leqq 1800$ .... $1000 + 200 \leqq 1800$

5) redundant

6) $X_4 \leqq 600$ .... $600 \leqq 600$

The maximum profit that may be attained by this allocation is 25,600 T.L.

Tableau: I

Ci	Xi	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	bi
		10	12	6	8	0	0	0	0	0	
0	X_5	1	0	0	0	1	0	0	0	0	1000
0	X_6	0	1	0	0	0	1	0	0	0	800
0	X_7	0	0	1	4/3	0	0	1	0	0	1000
0	X_8	1	0	1	0	0	0	0	1	0	1800
0	X_9	0	0	0	1	0	0	0	0	1	600
Sol:		0	0	0	0	1000	800	1000	1800	600	

Profit: $(10)(0) + (12)(0) + (6)(0) + (8)(0) = 0$

Δj:	10	12	6	8	0	0	0	0	0
bi/ai :	0	0	0	0	0	800	0	0	0

X_2 = IN , X_6 = OUT

Tableau : II

Ci	Xi	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	bi
		10	12	6	8	0	0	0	0	0	
0	X_5	1	0	0	0	1	0	0	0	0	1000
12	X_2	0	1	0	0	0	1	0	0	0	800
0	X_7	0	0	1	4/3	0	0	1	0	0	1000
0	X_8	1	0	1	0	0	0	0	1	0	1800
0	X_9	0	0	0	1	0	0	0	0	1	600
Sol. :		0	800	0	0	1000	0	1000	1800	600	

Profit: (10)(0) + (12) (800) + (6)(0) + (8)(0) = 9600 TL.

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
Δ_j :	10	0	6	8	0	-12	0	0	0
Bi/ai :	0	0	0	0	1000	0	0	1800	0

X_1 = IN , X_5 = OUT

Tableau: III

Ci	Xi	10 X_1	12 X_2	6 X_3	8 X_4	0 X_5	0 X_6	0 X_7	0 X_8	0 X_9	bi
10	X_1	1	0	0	0	1	0	0	0	0	1000
12	X_2	0	1	0	0	0	1	0	0	0	800
0	X_7	0	0	1	4/3	0	0	1	0	0	1000
0	X_8	0	0	0	1	0	0	0	1	0	800
0	X_9	0	0	0	1	0	0	0	0	1	600

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
Sol. :	1000	800	0	0	0	0	1000	800	600

Profit : (10)(1000) + (12) (800) + (6) (0) = 19,600 TL.

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
Δj :	0	0	6	8	-10	-12	0	0	0
bi/ai :	0	0	0	0	0	0	$\frac{4000}{3}$	0	600

X_4 = IN, X_6 = OUT

Tableau: IV

Ci	Xi	10 X_1	12 X_2	6 X_3	8 X_4	0 X_5	0 X_6	0 X_7	0 X_8	0 X_9	bi
10	X_1	1	0	0	0	1	0	0	0	0	1000
12	X_2	0	1	0	0	0	1	0	0	0	800
0	X_7	0	0	1	0	0	0	1	0	-4/3	200
0	X_8	0	0	1	0	-1	0	0	1	0	800
8	X_9	0	0	0	1	0	0	0	0	1	600

Sol. :	1000	800	0	600	0	0	200	800	0

Profit : (10)(1000) + (12)(800) + (6) (0) + (8) (600) = 24,400 TL.

Δ_j :	0	0	6	0	-10	-12	0	0	-8
bi/ai :	0	0	0	600	0	0	200	800	0

X_3 = IN , X_7 = OUT

Tableau : V

C_i	X_i	10	12	6	8	0	0	0	0	0	b_i
		X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	
10	X_1	1	0	0	0	1	0	0	0	0	1000
12	X_2	0	1	0	0	0	1	0	0	0	800
6	X_3	0	0	1	0	0	0	1	0	$-4/3$	200
0	X_8	0	0	0	0	-1	0	-1	1	$4/3$	600
8	X_4	0	0	0	1	0	0	0	0	1	600

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
Sol. :	1000	800	200	600	0	0	0	600	0

Profit : $(10)(1000) + (12)(800) + (6)(200) + (8)(600) \simeq 25,600$ TL.

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
Δ_j :	0	0	0	0	-10	-12	-6	0	0

		A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	R	S	0	0	0	0	0	0	0	0	0	0	0	0	
c_i	X_i	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}	X_{20}	X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	X_{26}	X_{27}	X_{28}	X_{29}	X_{30}	b_i
0	X_{19}	1	5/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1000
0	X_{20}	0	0	1	5/4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1000
0	X_{21}	0	0	0	0	1	5/4	4/3	4/3	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1000
0	X_{22}	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1000
0	X_{23}	0	0	0	0	0	0	0	0	0	1	20/17	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1000
0	X_{24}	0	0	0	0	0	0	0	0	0	0	0	1	5/4	4/3	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1000
0	X_{25}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	5/4	4/3	4/3	0	0	0	0	0	0	1	0	0	0	0	0	1000
0	X_{26}	1	0	1	0	1	0	0	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	4000
0	X_{27}	0	1	0	1	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2000
0	X_{28}	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1500
0	X_{29}	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	2000
0	X_{30}	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1200
	Sol:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1000	1000	1000	1000	1000	1000	1000	4000	2000	1500	2000	1200	
	Δj:																															
	b_i/a_i:																															

THE FIRST TABLEAU OF PROBLEM 1.

CHAPTER VI

PROGRAMMING THE OPTIMUM ALLOCATION OF RAW MATERIALS IN BEYKOZ FACTORY IN ORDER TO MINIMIZE THE COSTS

6.1.- PROGRAM OF THE PROBLEM

In chapter II the first problem of the Sümerbank Leather and Shoe Factory is stated as: " UNDER THE GIVEN RESOURCES AND RESTRICTIONS, HOW MANY RAW HIDES MUST THE COMPANY PURCHASE FROM THE DIFFERENT SOURCES IN ORDER TO MINIMIZE THE COSTS OF RAW MATERIALS?" In other words, "HOW MUST THE PURCHASES BE ALLOCATED IN ORDER TO OBTAIN THE MINIMUM POSSIBLE COST?"

The information collected on this problem in presented in chapter two, under the heading "Raw Materials". The problem is programmed referring to Table 2.2., which presents the whole data in tabular form.

6.1.1.- Mathematical Designation of the Variables

The variables of the problem may be defined as follow:

X_1 = The amount of raw chrome tanned upper leather that is obtained from the Meat and Fishery Products Corporation.

X_2 = The amount of raw vegetable tanned upper leather that is obtained from the Meat and Fishery Products Corporation.

X_3 = The amount of raw belly, shoulder and substitute leather that is obtained from the Meat and Fishery Products Corporation.

X_4 = The amount of raw bottom leather for welt that is obtained from the Meat and Fishery Products Corporation.

X_5 = The amount of raw chrome tanned upper leather that is obtained from the domestic private sector at 3.75 TL/kg.

X_6 = The amount of raw vegetable tanned upper leather that is obtained from the domestic private sector at TL. 3.75/kg.

X_7 = The amount of raw belly, shoulder and substitute leather that is obtained from the domestic private sector at TL. 3.75/Kg.

X_8 = The amount of raw bottom leather for welt that is obtained from the domestic private sector at TL. 3.75/Kg.

X_9 = The amount of raw chrome tanned upper leather that is obtained from the domestic private sector at TL. 4.25/Kg.

X_{10} = The amount of raw vegetable tanned upper leather that is obtained from the domestic private sector at TL.4.25/Kg.

X_{11} = The amount of raw belly, shoulder and substitute leather that is obtained from the domestic private sector at TL. 4.25/Kg.

X_{12} = The amount of raw bottom leather for welt that is obtained from the domestic private sector at TL. 4.25/Kg.

X_{13} = The amount of chrome tanned upper leather that is obtained from the medium weight hides of foreign origin.

X_{14} = The amount of raw vegetable tanned upper leather that is obtained from the medium weight hides of foreign origin.

X_{15} = The amount of raw belly, shoulder and substitute leather that is obtained from the medium weight hides of foreign origin.

X_{16} = The amount of raw bottom leather for welt that is obtained from the medium weight hides of foreign origin.

X_{17} = The amount of raw butt that is obtained from the heavy hides of foreign origin.

X_{18} = The amount of raw chrome tanned upper leather that is obtained from the heavy hides of foreign origin.

X_{19} = The amount of raw vegetable tanned upper leather that is obtained from the heavy hides of foreign origin.

X_{20} = The amount of raw belly, shoulder and substitute leather that is obtained from the heavy hides of foreign origin.

These variables are shown in tabular form on the following page.

6.1.2.- The Objective Function

The objective of this problem is to determine the best allocation of raw hides that are purchased from different sources in order to minimize the costs. The cost of processing each different leather consists of:

PURCHASE PRICE OF THE RAW HIDE (TL/Kg) + COST OF DIRECT LABOR FOR PROCESSING THE RAW HIDE (TL/Kg) + THE VARIABLE OVERHEAD COST(TL/Kg).

The fixed overhead cost is not included in the function because it is constant and has no alternative effect. However, it must be kept in mind and added to the minimum cost that will come out in the final solution.

Type \ Sources	Butt	Chr.tanned Upper leath.	Vegetable tanned upper leath.	Belly, shoulder and substitute leather	Bottom leather for welt
Hides from Meat and Fishery Products Corp.	-	X_1	X_2	X_3	X_4
Hides from domestic private sector at 3.75TL/kg.	-	X_5	X_6	X_7	X_8
Hides from domestic private sector at 4.25 TL/kg.	-	X_9	X_{10}	X_{11}	X_{12}
Medium weight hides of foreign origin.	-	X_{13}	X_{14}	X_{15}	X_{16}
Heavy hides of foreign origin	X_{17}	X_{18}	X_{19}	X_{20}	-

Table 6.1

Variables of the Second Problem

Besides the overhead cost, the programmer must pay attention to one other thing in this problem. It is not possible to utilize all of the leathers. After the processed leathers are cut for the products, little bits are left over. This excess amount may be used as fillers, etc., or they may be thrown away. This causes a relative gain or loss. Therefore, it must be considered in the minimum cost also. Referring to Table II in Chapter II and to the mathematical designation of the variables in this Chapter, the gain or loss comes out as follows:

$$\pm\left\{(.6)\ X_{17}\ \text{Kg} - 254{,}660\ \text{Kg})\ (\text{Market rate of processed butt})\right\}$$

$$\pm\left\{(14X_1\text{dm2} + 12X_5\text{dm2} + 12X_9\text{dm2} + 14X_{13}\text{dm2} + 10X_{18}\,\text{dm}^2 - 16{,}070{,}000\ \text{dm2})\right.$$
$$\left.(\text{market rate of processed chrome tanned upper leather})\right\}$$

$$\pm\left\{(14X_2\text{dm2} + 12X_6\text{dm2} + 12X_{10}\text{dm2} + 14X_{14}\text{dm2} + 10X_{19}\text{dm2} -\right.$$
$$\left.26{,}800{,}000\text{dm2})\ (\text{market rate of processed chrome tanned upper leather})\right\}$$

$$\pm\left\{(.6)X_3\text{Kg} + (.55)X_7\text{Kg} + (.55)X_{11}\text{Kg} + (.6)X_{15}\text{Kg} +\right.$$
$$(.6)X_{20}\text{Kg} + (.55)\ (.6)X_4\text{Kg} + (.55)\ (.55)\ X_8\text{Kg} + (.55)(.55)X_{12}\text{Kg} +$$
$$\left.(.55)(.6)X_{16}\text{Kg} - 947{,}215\ \text{Kg})\ (\text{market rate of processed belly and shoulder})\right\}$$

$$\pm\left\{(.45)X_4\text{Kg} + (.45)X_8 + (.45)X_{12} + (.45)X_{16}\text{Kg} -\right.$$
$$\left.12{,}550\ \text{Kg})\ (\text{market rate of bottom leather for welt})\right\}$$

The purchase prices of the raw hides are given in Chapter II. The cost of direct labor and the variable overhead costs may be determined from the data in the cost accounting department. Since the purpose of this study is to program the problem, it is convenient to indicate these costs by symbols such as $l_1, l_2, l_3, l_4 \ldots$ and $V_1, V_2, V_3, V_4 \ldots$ Then, the objective function may be expressed mathematically as follows:

$$
\begin{aligned}
\text{Minimize } Z = {} & (3.6 + l_1 + V_1)X_1 + (3.6 + l_2 + V_2)X_2 + (3.6 + l_3 + V_3)X_3 \\
& + (3.6 + l_4 + V_4)X_4 + (3.75 + l_5 + V_5)X_5 \\
& + (3.75 + l_6 + V_6)X_6 + (3.75 + l_7 + V_7)X_7 \\
& + (3.75 + l_8 + V_8)X_8 + (4.25 + l_9 + V_9)X_9 + \\
& (4.25 + l_{10} + V_{10})X_{10} + (4.25 + l_{11} + V_{11})X_{11} \\
& + (4.25 + l_{12} + V_{12})X_{12} + (5.2 + l_{13} + V_{13})X_{13} \\
& + (5.2 + l_{14} + V_{14})X_{14} + (5.2 + l_{15} + V_{15})X_{15} \\
& + (5.2 + l_{16} + V_{16})X_{16} + (4.8 + l_{17} + V_{17})X_{17} \\
& + (4.8 + l_{19} + V_{19})X_{19} + (4.8 + l_{19} + V_{19})X_{19} \\
& + (4.8 + l_{20} + V_{20})X_{20}
\end{aligned}
$$

6.1.3.- The Restrictions

The restrictions may be deduced from the information in Section 4, in Chapter II as follows:

1.- The amount of raw chrome tanned upper leather that is obtained from the hides of Meat and Fishery Products Corporation is equal to or less than 25% of the hides that can be obtained from this source.

2.- The amount of raw vegetable tanned upper leather that is obtained from the hides of Meat and Fishery Products Corporation is equal to or less than 100% of the hides than can be obtained from this source.

3.- The amount of raw belly, shoulder and substitute leather that is obtained from the hides of Meat and Fishery Products Corporation is equal to or less than 15% of the hides that can be obtained from this source and 55% of the raw bottom leather for welt that is obtained from the same source.

4.- The amount of raw bottom leather for welt that can be obtained from the hides of Meat and Fishery Products Corporation is equal to or less than 10% of the hides than can be obtained from this source.

5.- The total amount of raw chrome tanned upper leather, raw vegetable tanned upper leather, raw belly, shoulder and substitute leather and raw bottom leather for welt that are obtained from the Meat and Fishery Products Corporation is equal to or less than the amount of hides that can be obtained from this source.

6.- The amount of raw chrome tanned upper leather that is obtained from the hides of domestic private sector at TL.3.75/Kg. is equal to or less than 40% of the hides that can be obtained from this source with the stated price.

7.- The amount of raw vegetable tanned upper leather that is obtained from the hides of domestic private sector at TL. 3.75/Kg. is equal to or less than 70% of the hides that can be obtained from this source with the stated price.

8.- The amount of raw belly, shoulder and substitute leather that is obtained from the hides of domestic private sector at 3.75TL/Kg. is equal to or less than 5% of the hides that can be obtained from this source with the stated price and 55% of the raw bottom for welt that is obtained from the same source.

9.- The amount of raw bottom leather for welt that is obtained from the hides of the domestic private sector at 3.75 TL/Kg. is equal to or less than 10% of the hides that can be obtained from this source with the stated price.

10.- The total amount of raw chrome tanned upper leather, raw vegetable tanned upper leather, raw belly, shoulder and substitute leather and raw bottom leather for welt that are obtained from the hides of domestic private sector at 3.75 TL/Kg. is equal to or less than the amount of hides that can be obtained from this source with the stated price.

11.- The amount of raw chrome tanned upper leather that is obtained from the hides of domestic private sector at 4.25 TL/Kg. is equal to or less than 40% of the hides that can be obtained from this source with the stated price.

12.- The amount of raw vegetable tanned upper leather that is obtained from the hides of domestic private sector at 4.25 TL/Kg. is equal to or less than 70% of the hides that can be obtained from this source with the stated price.

13.- The amount of raw belly, shoulder and substitute leather that is obtained from the hides of domestic private sector at 4.25 TL/Kg. is equal to or less than 5% of the hides that can be obtained from this source at this price and 55% of the raw bottom leather for welt that is obtained from the same source.

14.- The amount of raw bottom leather for welt that is obtained from the hides of domestic private sector at 4.25 TL/Kg. is equal to or less than 10% of the hides that can be obtained from this source with the stated price.

15.- The total amount of raw chrome tanned upper leather, raw vegetable tanned upper leather, raw belly, shoulder and substitute leather and raw bottom leather for welt that are obtained from the hides of private domestic sector at 4.25 TL/Kg. is equal to or less than the amount of hides that can be obtained from this source with the stated price.

16.- The amount of processed butt that can be obtained from the heavy hides of foreign origin is equal to or less than the maximum amount of processed butt that can be utilized in the factory.

17.- The amount of processed chrome tanned upper leather that can be obtained from the hides of Meat and Fishery Products Corporation, domestic private sector, medium weight hides of foreign origin and heavy hides of foreign origin is qual to or less than the maximum amount of processed chrome tanned upper leather that can be utilized in the factory.

18.- The amount of processed vegetable tanned upper leather that can be obtained from the hides of Meat and Fishery Products corporation, domestic private sector, medium weight hides of foreign origin and heavy hides of foreign origin is equal to or less than the maximum amount of processed vegatable tanned upper leather that can be utilized in the factory.

19.- The amount of processed belly, shoulder and substitute leather that can be obtained from the hides of Meat and Fishery Products Corporation, domestic private sector, medium weight hides of foreign origin , heavy hides of foreign origin is equal to or less than the maximum amount of processed belly, shoulder and substitute leather that can be utilized in the factory.

20.- The amount of processed bottom leather that can be obtained from the hides of Meat and Fishery Products Corporation, domestic private sector and medium weight hides of foreign origin is equal to or less than the maximum amount of processed bottom leather for welt that can be utilized at the factory.

The mathematical expression of these restrictions, in equivalent units, are as follows:

1) $X_1 \leq (0.25)(1,500,000)$Kg.

2) $X_2 \leq 1,500,000$ Kg.

3) $X_3 + (0.55)X_4 \leq (0.15)(1,500,000) + (0.10)(0.55)(1,500,000)$Kg

4) $X_4 \leq (0.10)(1,500,000)$Kg.

5) $X_1 + X_2 + X_3 + X_4 \leq 1,500,000$ Kg.

6) $X_5 \leq (0.40)(1,200,000)$ Kg.

7) $X_6 \leq (0.70)(1,200,000)$ Kg.

8) $X_7 + (0.55)X_8 \leq (0.05)\ (1{,}200{,}000) + (0.10)(0.55)(1{,}200{,}000)$ Kg.

9) $X_8 \leq (0.10)(1{,}200{,}000)$ Kg.

10) $X_5 + X_6 + X_7 + X_8 \leq 1{,}200{,}000$ Kg.

11) $X_9 \leq (0.40)(1{,}800{,}000)$ Kg.

12) $X_{10} \leq (0.70)(1{,}800{,}000)$ Kg.

13) $X_{11} + (0.55)X_{12} \leq (0.05)(1{,}800{,}000) + (0.55)(1{,}800{,}000)$ Kg.

14) $X_{12} \leq (0.10)(1{,}800{,}000)$ Kg.

15) $X_9 + X_{10} + X_{11} + X_{12} \leq 1{,}800{,}000$ Kg.

16) $(0.45)(0.60)X_{17} \leq 305{,}592$ Kg.

17) $14X_1 + 12X_5 + 12X_9 + 14X_{13} + 10X_{18} \leq 19{,}284{,}000$ dm2

18) $14X_2 + 12X_6 + 12X_{10} + 14X_{14} + 10X_{19} \leq 32{,}160{,}000$ dm2

19) $(0.60)X_3 + (0.60)(0.55)X_4 + (0.55)X_7 + (0.55)(0.55)X_8$
$+ (0.55)X_{11} + (0.55)(0.55)X_{12} + (0.60)X_{15} + (0.60)(0.55)X_{16}$
$+ (0.60)(0.55)X_{20} \leq 1{,}136{,}658$ Kg.

20) $(0.60)(0.45)X_4 + (0.55)(0.45)X_8 + (0.55)\ (0.45)X_{12}$
$+ (0.60)(0.45)X_{16} \leq 153{,}060$ Kg.

6.1.4.- Conversion of Inequalities to Equalities

The above inequalities may be expressed as equations by adding slack variables to every inequality that describes a restriction. However, in this problem, the inequalities involve two different types of measure, namely, kilogram and decimeter square. Therefore, it is necessary to express the decimeter squares with the corresponding units in kilograms.

One kilogram of chrome tanned upper leather equals 30dm^2. One kilogram of vegetable tanned upper leather equals 40 dm^2. On these bases, it becomes possible to write all of the equations in the same units.

The equations, after the conversion, stand as they are presented in the following pages.

6.1.5. The First Tableau

The first tableau of this problem is presented on page 94 . After this point, the computational routine may be carried on, by the people who may be interested in the solution with the aid of a 1672 computer.

THE EQUATIONS

1) $X_1 + 0X_2 + 0X_3 + 0X_4 + 0X_5 + 0X_6 + 0X_7 + 0X_8 + 0X_9 + 0X_{10} + 0X_{11} + 0X_{12} + 0X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17} + 0X_{18} + 0X_{19} + 0X_{20} + X_{21} + 0X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + 0X_{26} + 0X_{27} + 0X_{28} + 0X_{29} + 0X_{30} + 0X_{31} + 0X_{32} + 0X_{33} + 0X_{34} + 0X_{35} + 0X_{36} + 0X_{37} + 0X_{38} + 0X_{39} + 0X_{40} = 375{,}000$

2) $0X_1 + X_2 + 0X_3 + 0X_4 + 0X_5 + 0X_6 + 0X_7 + 0X_8 + 0X_9 + 0X_{10} + 0X_{11} + 0X_{12} + 0X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17} + 0X_{18} + 0X_{19} + 0X_{20} + 0X_{21} + X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + 0X_{26} + 0X_{27} + 0X_{28} + 0X_{29} + 0X_{30} + 0X_{31} + 0X_{32} + 0X_{33} + 0X_{34} + 0X_{35} + 0X_{36} + 0X_{37} + 0X_{38} + 0X_{39} + 0X_{40} = 1{,}500{,}000$

3) $0X_1 + 0X_2 + X_3 + 0.55X_4 + 0X_5 + 0X_6 + 0X_7 + 0X_8 + 0X_9 + 0X_{10} + 0X_{11} + 0X_{12} + 0X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17} + 0X_{18} + 0X_{19} + 0X_{20} + 0X_{21} + 0X_{22} + X_{23} + 0X_{24} + 0X_{25} + 0X_{26} + 0X_{27} + 0X_{28} + 0X_{29} + 0X_{30} + 0X_{31} + 0X_{32} + 0X_{33} + 0X_{34} + 0X_{35} + 0X_{36} + 0X_{37} + 0X_{38} + 0X_{39} + 0X_{40} = 307{,}500$

4) $0X_1 + 0X_2 + 0X_3 + X_4 + 0X_5 + 0X_6 + 0X_7 + 0X_8 + 0X_9 + 0X_{10} + 0X_{11} + 0X_{12} + 0X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17} + 0X_{18} + 0X_{19} + 0X_{20} + 0X_{21} + 0X_{22} + 0X_{23} + X_{24} + 0X_{25} + 0X_{26} + 0X_{27} + 0X_{28} + 0X_{29} + 0X_{30} + 0X_{31} + 0X_{32} + 0X_{33} + 0X_{34} + 0X_{35} + 0X_{36} + 0X_{37} + 0X_{38} + 0X_{39} + 0X_{40} = 150{,}000$

5) $X_1 + X_2 + X_3 + X_4 + 0X_5 + 0X_6 + 0X_7 + 0X_8 + 0X_9 + 0X_{10} + 0X_{11} + 0X_{12} + 0X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17} + 0X_{18} + 0X_{19} + 0X_{20} + 0X_{21} + 0X_{22} + 0X_{23} + 0X_{24} + X_{25} + 0X_{26} + 0X_{27} + 0X_{28} + 0X_{29} + 0X_{30} + 0X_{31} + 0X_{32} + 0X_{33} + 0X_{34} + 0X_{35} + 0X_{36} + 0X_{37} + 0X_{38} + 0X_{39} + 0X_{40} = 1,500,000$

6) $0X_1 + 0X_2 + 0X_3 + 0X_4 + X_5 + 0X_6 + 0X_7 + 0X_8 + 0X_9 + 0X_{10} + 0X_{11} + 0X_{12} + 0X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17} + 0X_{18} + 0X_{19} + 0X_{20} + 0X_{21} + 0X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + X_{26} + 0X_{27} + 0X_{28} + 0X_{29} + 0X_{30} + 0X_{31} + 0X_{32} + 0X_{33} + 0X_{34} + 0X_{35} + 0X_{36} + 0X_{37} + 0X_{38} + 0X_{39} + 0X_{40} = 480,000$

7) $0X_1 + 0X_2 + 0X_3 + 0X_4 + 0X_5 + X_6 + 0X_7 + 0X_8 + 0X_9 + 0X_{10} + 0X_{11} + 0X_{12} + 0X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17} + 0X_{18} + 0X_{19} + 0X_{20} + 0X_{21} + 0X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + 0X_{26} + X_{27} + 0X_{28} + 0X_{29} + 0X_{30} + 0X_{31} + 0X_{32} + 0X_{33} + 0X_{34} + 0X_{35} + 0X_{36} + 0X_{37} + 0X_{38} + 0X_{39} + 0X_{40} = 840,000$

8) $0X_1 + 0X_2 + 0X_3 + 0X_4 + 0X_5 + 0X_6 + X_7 + (0.55)X_8 + 0X_9 + 0X_{10} + 0X_{11} + 0X_{12} + 0X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17} + 0X_{18} + 0X_{19} + 0X_{20} + 0X_{21} + 0X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + 0X_{26} + 0X_{27} + X_{28} + 0X_{29} + 0X_{30} + 0X_{31} + 0X_{32} + 0X_{33} + 0X_{34} + 0X_{35} + 0X_{36} + 0X_{37} + 0X_{38} + 0X_{39} + 0X_{40} = 126,000$

9) $0X_1 + 0X_2 + 0X_3 + 0X_4 + 0X_5 + 0X_6 + 0X_7 + X_8 + 0X_9 + 0X_{10} + 0X_{11}$
$0X_{12} + 0X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17} + 0X_{18} + 0X_{19} + 0X_{20} + 0X_{21}$
$+ 0X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + 0X_{26} + 0X_{27} + 0X_{28} + X_{29} + 0X_{30}$
$+ 0X_{31} + 0X_{32} + 0X_{33} + 0X_{34} + 0X_{35} + 0X_{36} + 0X_{37} + 0X_{38} + 0X_{39}$
$+ 0X_{40} = 120{,}000$

10) $0X_1 + 0X_2 + 0X_3 + 0X_4 + X_5 + X_6 + X_7 + X_8 + 0X_9 + 0X_{10} + 0X_{11} + 0X_{12}$
$+ 0X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17} + 0X_{18} + 0X_{19} + 0X_{20} + 0X_{21}$
$+ 0X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + 0X_{26} + 0X_{27} + 0X_{28} + 0X_{29} + X_{30}$
$+ 0X_{31} + 0X_{32} + 0X_{33} + 0X_{34} + 0X_{35} + 0X_{36} + 0X_{37} + 0X_{38} + 0X_{39}$
$+ 0X_{40} = 1{,}200{,}000$

11) $0X_1 + 0X_2 + 0X_3 + 0X_4 + 0X_5 + 0X_6 + 0X_7 + 0X_8 + X_9 + 0X_{10} + 0X_{11}$
$+ 0X_{12} + 0X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17} + 0X_{18} + 0X_{19} + 0X_{20}$
$+ 0X_{21} + 0X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + 0X_{26} + 0X_{27} + 0X_{28} + 0X_{29}$
$+ 0X_{30} + X_{31} + 0X_{32} + 0X_{33} + 0X_{34} + 0X_{35} + 0X_{36} + 0X_{37} + 0X_{38}$
$+ 0X_{39} + 0X_{40} = 720{,}000$

12) $0X_1 + 0X_2 + 0X_3 + 0X_4 + 0X_5 + 0X_6 + 0X_7 + 0X_8 + 0X_9 + X_{10} + 0X_{11}$
$+ 0X_{12} + 0X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17} + 0X_{18} + 0X_{19} + 0X_{20}$
$+ 0X_{21} + 0X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + 0X_{26} + 0X_{27} + 0X_{28} + 0X_{29}$
$+ 0X_{30} + 0X_{31} + X_{32} + 0X_{33} + 0X_{34} + 0X_{35} + 0X_{36} + 0X_{37} + 0X_{38}$
$+ 0X_{39} + 0X_{40} = 1{,}260{,}000$

13) $0X_1+0X_2+0X_3+0X_4+0X_5+0X_6+0X_7+0X_8+0X_9+0X_{10}+X_{11}+(0.55)X_{12}+0X_{13}+0X_{14}+0X_{15}+0X_{16}+0X_{17}+0X_{18}+0X_{19}+0X_{20}+0X_{21}+0X_{22}+0X_{23}+0X_{24}+0X_{25}+0X_{26}+0X_{27}+0X_{28}+0X_{29}+0X_{30}+0X_{31}+0X_{32}+X_{33}+0X_{34}+0X_{35}+0X_{36}+0X_{37}+0X_{38}+0X_{39}+0X_{40}=189{,}000$

14) $0X_1+0X_2+0X_3+0X_4+0X_5+0X_6+0X_7+0X_8+0X_9+0X_{10}+0X_{11}+X_{12}+0X_{13}+0X_{14}+0X_{15}+0X_{16}+0X_{17}+0X_{18}+0X_{19}+0X_{20}+0X_{21}+0X_{22}+0X_{23}+0X_{24}+0X_{25}+0X_{26}+0X_{27}+0X_{28}+0X_{29}+0X_{30}+0X_{31}+0X_{32}+0X_{33}+X_{34}+0X_{35}+0X_{36}+0X_{37}+0X_{38}+0X_{39}+0X_{40}=180{,}000$

15) $0X_1+0X_2+0X_3+0X_4+0X_5+0X_6+0X_7+0X_8+X_9+X_{10}+X_{11}+X_{12}+0X_{13}+0X_{14}+0X_{15}+0X_{16}+0X_{17}+0X_{18}+0X_{19}+0X_{20}+0X_{21}+0X_{22}+0X_{23}+0X_{24}+0X_{25}+0X_{26}+0X_{27}+0X_{28}+0X_{29}+0X_{30}+0X_{31}+0X_{32}+0X_{33}+0X_{34}+X_{35}+0X_{36}+0X_{37}+0X_{38}+0X_{39}+0X_{40}=1{,}800{,}000$

16) $0X_1+0X_2+0X_3+0X_4+0X_5+0X_6+0X_7+0X_8+0X_9+0X_{10}+0X_{11}+0X_{12}+0X_{13}+0X_{14}+0X_{15}+0X_{16}+(0.27)X_{17}+0X_{18}+0X_{19}+0X_{20}+0X_{21}+0X_{22}+0X_{23}+0X_{24}+0X_{25}+0X_{26}+0X_{27}+0X_{28}+0X_{29}+0X_{30}+0X_{31}+0X_{32}+0X_{33}+0X_{34}+0X_{35}+X_{36}+0X_{37}+0X_{38}+0X_{39}+0X_{40}=305{,}592$

17) $7/15X_1 + 0X_2 + 0X_3 + 0X_4 + 2/5X_5 + 0X_6 + 0X_7 + 0X_8 + 2/5X_9$
$+ 0X_{10} + 0X_{11} + 0X_{12} + 7/15X_{13} + 0X_{14} + 0X_{15} + 0X_{16} + 0X_{17}$
$+ 1/3X_{18} + 0X_{19} + 0X_{20} + 0X_{21} + 0X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + 0X_{26}$
$+ 0X_{27} + 0X_{28} + 0X_{29} + 0X_{30} + 0X_{31} + 0X_{32} + 0X_{33} + 0X_{34} + 0X_{35}$
$+ 0X_{36} + X_{37} + 0X_{38} + 0X_{39} + 0X_{40} = 642,800$

18) $0X_1 + 7/15X_2 + 0X_3 + 0X_4 + 0X_5 + 2/5X_6 + 0X_7 + 0X_8 + 0X_9$
$+ 2/5X_{10} + 0X_{11} + 0X_{12} + 0X_{13} + 7/15X_{14} + 0X_{15} + 0X_{16} + 0X_{17}$
$+ 0X_{18} + 1/3X_{19} + 0X_{20} + 0X_{21} + 0X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + 0X_{26}$
$+ 0X_{27} + 0X_{28} + 0X_{29} + 0X_{30} + 0X_{31} + 0X_{32} + 0X_{33} + 0X_{34} + 0X_{35}$
$+ 0X_{36} + 0X_{37} + X_{38} + 0X_{39} + 0X_{40} = 1,072,000$

19) $0X_1 + 0X_2 + .60X_3 + .33X_4 + 0X_5 + 0X_6 + .55X_7 + .33X_8 + 0X_9$
$+ 0X_{10} + .55X_{11} + .3025X_{12} + 0X_{13} + 0X_{14} + .60X_{15} + .33X_{16}$
$+ 0X_{17} + 0X_{18} + 0X_{19} + .33X_{20} + 0X_{21} + 0X_{22} + 0X_{23} + 0X_{24}$
$+ 0X_{25} + 0X_{26} + 0X_{27} + 0X_{28} + 0X_{29} + 0X_{30} + 0X_{31} + 0X_{32} + 0X_{33}$
$+ 0X_{34} + 0X_{35} + 0X_{36} + 0X_{37} + 0X_{38} + X_{39} + 0X_{40} = 1,136,658$

20) $0X_1 + 0X_2 + 0X_3 + .27X_4 + 0X_5 + 0X_6 + 0X_7 + .2475X_8 + 0X_9 + 0X_{10}$
$0X_{11} + .2475X_{12} + 0X_{13} + 0X_{14} + 0X_{15} + .27X_{16} + 0X_{17} + 0X_{18}$
$+ 0X_{19} + 0X_{20} + 0X_{21} + 0X_{22} + 0X_{23} + 0X_{24} + 0X_{25} + 0X_{26} + 0X_{27}$
$+ 0X_{28} + 0X_{29} + 0X_{30} + 0X_{31} + 0X_{32} + 0X_{33} + 0X_{34} + 0X_{35} + 0X_{36}$
$+ 0X_{37} + 0X_{38} + 0X_{39} + X_{40} = 153,060$

c_i	x_i	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	x_{28}	x_{29}	x_{30}	x_{31}	x_{32}	x_{33}	x_{34}	x_{36}	x_{37}	x_{38}	x_{39}	x_{40}	b_i
		$(3.6+l_1+v_1)$	$(3.6+l_2+v_2)$	$(2.6+l_3+v_3)$	$(3.6+l_4+v_4)$	$(3.75+l_5+v_5)$	$(3.75+l_6+v_6)$	$(3.75+l_7+v_7)$	$(3.75+l_8+v_8)$	$(4.25+l_9+v_9)$	$(4.25+l_{10}+v_{10})$	$(4.25+l_{11}+v_{11})$	$(4.25+l_{12}+v_{12})$	$(5.2+l_{13}+v_{13})$	$(5.2+l_{14}+v_{14})$	$(5.2+l_{15}+v_{15})$	$(5.2+l_{16}+v_{16})$	$(4.8+l_{17}+v_{17})$	$(4.8+l_{18}+v_{18})$	$(4.8+l_{19}+v_{19})$	$(4.8+l_{20}+v_{20})$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	x_{21}	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	375 000
0	x_{22}	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1 500 000
0	x_{23}	0	0	1	.55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	307 500
0	x_{24}	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	150 000
0	x_{25}	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1500000
0	x_{26}	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	480000
0	x_{27}	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	840000
0	x_{28}	0	0	0	0	0	0	1	.55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	126000
0	x_{29}	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	120 000
0	x_{30}	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1200000
0	x_{31}	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	720000
0	x_{32}	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1260000
0	x_{33}	0	0	0	0	0	0	0	0	0	0	1	.55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	189000
0	x_{34}	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	180000
0	x_{35}	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1800000
0	x_{36}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	305592
0	x_{37}	7/15	0	0	0	2/5	0	0	0	2/5	0	0	0	7/15	0	0	0	0	1/3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	642800
0	x_{38}	0	7/15	0	0	0	2/5	0	0	0	2/5	0	0	0	7/15	0	0	0	0	1/3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1072000
0	x_{39}	0	0	.60	.33	0	0	.55	.33	0	0	.55	.3025	0	0	.60	.33	0	0	0	.33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1136658
0	x_{40}	0	0	0	.27	0	0	0	.2475	0	0	0	.2475	0	0	0	.27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	153060
Sol:		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	375000	1500000	307500	150000	1500000	480000	840000	126000	120000	1200000	720000	1260000	189000	180000	305592	642800	1072000	1136658	153060	
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