



OPTIMAL SHORT-TERM FINANCING/INVESTMENT
DECISION UNDER UNCERTAINTY

BY

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ABSTRACT

OPTIMAL SHORT-TERM FINANCING/INVESTMENT DECISION UNDER
UNCERTAINTY

This thesis aims at developing a solution procedure to the firms' short-term financing and investment decisions under uncertainty. In doing this, realizing the fact that the decisions related to a specific function of the firm have impacts on the others, an integrative approach is conducted.

First, different demand quantity forecasts, depending on the environmental conditions the firm is operating, are generated for the current planning period.

Secondly, utilising a spreadsheet cash budget model cash requirements are determined as a result of collections, manpower, production, inventory, purchasing and payments planning for each demand forecast separately. Then, distributions of period cash requirements are obtained.

Thirdly, a linear programming model developed is used to obtain the optimal solutions to five different short-term financing/investment decision problems ranging from the most pessimistic to the most optimistic demand

forecasts representing the Management's attitude towards risk.

Opportunity costs and the optimality ranges of the cost coefficients and resource constants for conducting sensitivity analysis are also utilised as complementary parts of the optimal solutions.

Finally, the optimal solutions are interpreted in terms of decisions, opportunity costs and optimality ranges and some guidelines to Management are deducted.

In the study, the real-life data of a Turkish production firm is used.

ÖZET

BELİRSİZLİK ORTAMINDA OPTİMAL KISA VADELİ FİNANSMAN/YATIRIM KARARI

Bu tez, firmaların belirsizlik ortamında vereceği kısa vadeli finansman ve yatırım kararları için bir çözüm işlem dizisi geliştirmeyi amaçlamaktadır. Bunu yaparken, firmaların belirli bir işlevle ilgili aldıkları kararların diğerleri üzerinde de etkilerinin olması gerçeğinden hareketle bütünleştirici bir yaklaşım izlenmiştir.

Birinci aşamada, planlama yapılacak dönem için firmanın içinde bulunduğu çevresel koşullara bağlı olarak değişik talep tahminleri çıkarılmaktadır.

İkinci olarak, bir tablo nakit bütçesi modeli kullanarak her talep tahmini için ayrı ayrı tahsilat, işgücü, üretim, stok, satınalma ve borç ödeme planı yapılarak nakit gereksinimleri belirlenmektedir. Daha sonra, dönemlik nakit gereksinmelerinin olasılık dağılımları elde edilmektedir.

Üçüncü olarak, şirket yönetiminin riske olan eğilimini temsil eden en kötümserden en iyimsere doğru sıralanan talep tahminlerine dayanan beş değişik kısa vadeli finansman/yatırım kararı probleminin optimal çözümleri bulunmaktadır. Optimal çözümlerin tamamlayıcı unsurları olarak gölge maliyetler ve duyarlılık analizleri için maliyet sabit katsayıları ile kaynak sabitlerinin optimal aralıklarından yararlanılmaktadır.

Son olarak, optimal çözümler kararlar, gölge maliyetler ve optimal aralıklar açısından yorumlanmakta ve yönetim için bazı sonuçlar çıkarılmaktadır.

Çalışmada üretici bir Türk firmasının gerçek verileri kullanılmıştır.

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LIST OF SYMBOLS

- \bar{X} : Sample mean
 S : Sample standard deviation
 $MOCR_i$: Minimum operating cash requirement in period i
 MC : Predetermined minimum amount of cash that the firm wishes to have on hand anytime
 ND_{i+1} : Total cash receipts minus total operating cash disbursements in period $i+1$ (net drain)
 y : Percentage of net operating cash flow (ND) that the firm wishes to have on hand
 m : Total number of months in the planning period
 COH : Cash on hand at the beginning of planning period
 x_{ij} : Amount borrowed from source i at the beginning of period j
 y_{ij} : Amount voluntarily repaid to source i at the beginning of period j
 v_{ij} : Amount of mandatory repayment to source i at the beginning of period j
 $x_{i\ell j}$: Amount of investment made in investment alternative i to the instrument ℓ at the beginning of period j

- z_{ij} : Net cumulative amount borrowed from source i at the beginning of period j after borrowing and repayment for period j .
- r_i : Interest rate for alternative i
- a_{in} : n^{th} coefficient used in stating constraints on alternative i
- b_{in} : n^{th} constraint limit for alternative i
- S_{12j} : Additional amount borrowed to meet compensating balance requirement in the beginning of period j
- A_j : Amount of accounts receivable outstanding at the beginning of period j obtained from the cash budget
- x_{5jk} : Amount of domestic payables, due in period k , which is stretched in period j ; $k=j, j-1$ or $j-2$
- x'_{5jk} : Amount of foreign payables, due in period k , which is stretched in period j ; $k=j, j-1$ or $j-2$
- y_{5jk} : Amount of domestic payables, due in period k , which is actually paid in period j ; $k=j, j-1, j-2$ or $j-3$
- y'_{5jk} : Amount of foreign payables, due in period k , which is actually paid in period j ; $k=j, j-1, j-2$ or $j-3$
- P_j : Domestic purchases made before which are scheduled to be paid in the beginning of period j in the cash budget
- P'_j : Foreign purchases made before which are scheduled to be paid in the beginning of period j in the cash budget
- x_{6jk} : Amount of taxes payable, due in period k , which is stretched in period j ; $k=j, j-1$
- y_{6jk} : Amount of taxes payable, due in period k , which is actually paid in period j ; $k=j, j-1, j-2$
- T_j : Taxes which are scheduled to be paid in the beginning of period j in the cash budget

- I_{kj} : Total monthly interest expense of source k in period j
 E_j : Total explicit cost in period j
 q_{5k} : Implicit cost of ill will to domestic creditors when payments are stretched for k periods ; $k=1,2,3$
 q_{5k}^* : Implicit cost of ill will to foreign creditors when payments are stretched for k periods ; $k=1,2,3$
 q_{6k} : Implicit cost of ill will to tax authorities when payments are stretched for k periods ; $k=1,2$
 q_{71} : Implicit cost of the term loan
 S_j : Rate of implicit end costs for source j
 D_m : Total end condition implicit cost
 TRC : Total relevant cost
 z^* : Objective function value

CHAPTER I

INTRODUCTION

In this study, we try to construct an integrated solution procedure to the firms' short-term financing/investment decisions in an ever-changing environment under conditions of uncertainty.

The principal determinants of the cash requirements throughout the planning period are the sales figures which are the main sources of uncertainty for the firms. In the model, assuming the sales prices for each product being predetermined in the corresponding periods, the sales quantities are obtained from stochastic demand distributions. In order to obtain the cash requirements for the planning horizon a spreadsheet model including production, purchasing, inventory level and collections planning for each stochastic sales quantity mix is constructed with the required parameters such as inflation and currency rate expectations, sales prices, standard material requirements needed for unit production, beginning of planning period inventory levels both for finished goods and raw materials, costs of raw materials, payment terms of raw materials purchased and repayment schedule of outstanding debts as being given. Since sales quantities are

stochastic, the cash requirements will also be stochastic. Depending on the Top Management's being risk averse or risk seeker determined by their attitudes towards pessimistic or optimistic sales forecasts, a different short-term financing/investment plan, obtained by the linear programming model under the operating conditions and targets of the firm will be implemented.

The solution process of the optimal short-term financing/investment decision problem under uncertainty is summarized in the following stages:

- 1) Generation of period demand quantity forecasts for domestic and export market for each product line throughout the whole planning period.
- 2) Determination of period cash requirements by a spreadsheet cash budget model for each demand forecast for the whole planning period.
- 3) Defining risk levels with respect to the distribution of period cash requirements.
- 4) Obtaining the optimal solutions of the short-term financing/investment decision problem at each risk level by a linear programming model developed for financing the cash requirements in the periods with cash shortages and investing the excess cash in the periods with cash averages in order to minimize the total explicit and implicit costs relevant for the whole planning period.
- 5) Interpretation of the optimal solutions at each risk level with activity levels, opportunity costs of the resources and the optimality ranges for cost coefficients and resource constants.
- 6) Selection of the financial plan by the Management with respect to their risk perceptions and evaluations, and taking necessary precautions to implement the plan for the purpose of integrating the planning model in to the planning process.

CHAPTER II

REVIEW OF LITERATURE

In this chapter a review of the existing literature is undertaken. In Part 1, the evolution of corporate strategic planning models are examined, and in Part 2, linear programming and mathematical models developed to be used in short-term financial planning are investigated.

2.1. CORPORATE STRATEGIC PLANNING MODELS

The corporate strategic planning models were started to be developed by the second half of 1960's with the increasing importance given to formal planning. The first research on this subject was conducted by George W. Gershefski(1) in 1969. In his study, he found out that the major effort in corporate model building was started in 1966 when 13 companies began developing their first model.

Gershefski also pointed out that since corporate models were used primarily to project statements of Net Income, Capital Expenditures, Sources and Uses of Funds and Balance Sheet they were of greatest interest to the President or the Vice Presidents of Planning and Finance. These first

planning models were developed by major corporations and used for generating proforma financial statements.

In his article Thomas H. Naylor⁽²⁾ stated that during the decade of the 1970's three alternative approaches to strategic planning modelling evolved: 1) corporate simulation models, 2) analytical portfolio models, and 3) optimization portfolio models. Those corporate simulation models developed between 1965 and 1973 are defined as first generation models. The first generation models tended to be stand-alone financial models with limited marketing and production components. All of these models were "What If" models. There were no optimization models being used for overall corporate planning. All of the early models were deterministic.

The second generation corporate simulation models were developed between 1974 and 1979. This period in the evolution of corporate simulation models was characterized by 1) the development of integrated planning models, 2) attempts to integrate planning models into the planning process, 3) increased attention focused on the human aspects of corporate simulation modelling, 4) the introduction of a number of very powerful new corporate simulation languages such as EXPRESS, SIMPLAN, and XSIM*, and 5) a substantial increase in the use of econometric models to link corporate simulation models to product markets and to the national economy.

Most of the second generation models were strictly "What If?" models. However, there were a few optimization models.

* These software systems include: (1) a database, (2) a report generator, (3) graphics, (4) a security system, (5) the ability to solve linear, nonlinear, recursive, and simultaneous equations, (6) risk analysis, (7) time series forecasting, and (8) econometric modeling.

Analytical portfolio models emerged as the need for decision making for companies which have a collection or portfolio of businesses, products, or profit centers, and scarce financial resources to allocate across the portfolio has increased. As pointed out by Naylor in his article⁽²⁾ the most popular one among the analytical portfolio models is the Boston Consulting Group (BCG) model. The BCG approach is based on two concepts—the growth-share matrix and the experience curve.

The basic idea of the growth-share matrix is that a company can be divided into component products or businesses, each of which is separable from others. Specifically, each business is characterized as having a high or low market growth rate and either a high or low market share.

The experience curve concept is stated by the BCG as the unit cost (in real terms) of manufacturing a product declines approximately 20 to 30% each time accumulated experience is doubled.

In the model portfolio investment decisions are based on normative rules depending on the position of the business on the growth-share matrix.

Naylor pointed out that a second type of portfolio planning model is the PIMS (Profit Impact of Market Strategy) Program of the Strategic Planning Institute (SPI). Utilizing a database of extensive time series and cross-sectional data on finance, marketing and production operations for over two thousand product-line businesses, the objectives of the PIMS Program are to discover the general laws that determine what business strategy, in what kind of competitive environment, produces what profit

results and to produce reports for the managers of each business unit, which they can use as a basis for decision making.

Huber and McCann (3) state that for portfolio planning the PIMS model simulates the return on investment and cash flow implications of actions on different businesses. When aggregated the effect of different allocation decisions across businesses can be estimated.

Naylor⁽²⁾ stated in his article that the experience gained by firms in the 1970's with corporate simulation models and analytical portfolio models has laid the groundwork for the introduction of optimization models as strategic planning tools in the 1980's. One of these models is developed by the BCG named "Strategy Based Resource Allocation Model". The model is a linear programming model which helps management choose either a growth strategy or a cash strategy for each business in the company's portfolio, thus allocating as optimally as possible the company's scarce resources. For each business, the model indicates which strategic option maximizes the company's growth or long-run earnings subject to a set of financial constraints.

Another portfolio optimization model is the one published by Hamilton and Moses⁽⁴⁾ in 1973. The model includes a full range of financial decisions including internal capital budgeting, acquisitions, divestments, debt creation/repayment, stock issue/repurchase and dividend payout. The model employs mixed integer programming to select optimal investment and financing strategies over a multiperiod planning horizon.

In a recent paper by M.B. Coate⁽⁵⁾ a new approach to portfolio planning models is handled. In the model the firm chooses individual

investment levels within a strategy (growth, cash or mixed) for each business unit. The model can be solved to define the optimal strategies for variations in the time horizon, the discount rate, the cash flow constraint, growth in earnings per share and required assets.

All of the aforementioned portfolio optimization models are deterministic models. The capital asset pricing model (CAPM) extends the concept of risk analysis to portfolio optimization models. Naylor⁽²⁾ stated that two management consulting firms, Marakon Associates and Strategic Planning Associates, have proposed the possibility of employing the CAPM not only as a decision making tool for investors with a portfolio of financial assets, but also as a planning tool for corporations that manage a portfolio of businesses, divisions, strategic business units (tangible assets).

For the future of corporate planning models, Jae K. Shim and Randy Mc Glade⁽⁶⁾ stated that as tight economic conditions and intensified competition required managers to formulate more effective strategies, the advantages of modelling became more apparent. As modelling success becomes more common, managers of all-sized firms can be expected more readily to lend their support to in-house modelling projects or to the purchase of ready-made systems. Improving computer software facilities will also help them. Currently, planning and modelling languages (P.M.L.'s) have taken the place of general programming languages (G.P.L.'s) such as FORTRAN and BASIC. Today, over 70 P.M.L.'s are available at reasonable cost, including EMPRE, FINPLAN, VISICALC, BUDPLAN, MULTIPLAN, LOTUS 1-2-3 and SYMPHONY. P.M.L.'s bring flexibility and ease of use to all

managers even for those who are not familiar with the computer practices.

Naylor⁽²⁾ states that many of the existing planning and modelling software systems will adopt a modularized approach in the future. That is it will become possible to acquire the database management capabilities and report generation of a particular planning system without purchasing some of the more sophisticated modules including econometric modelling, risk analysis and optimization. He also states that third generation decision support systems will reflect the complementarity of corporate simulation models, analytical portfolio models and optimization portfolio models. Various linkages among these alternative approaches to planning modelling will be provided by third generation planning and modelling software systems.

2.2. SHORT-TERM FINANCIAL PLANNING MODELS

Short-term financial management decisions gained importance in Turkey in recent years because of high inflation rates, implementation of liberal economic policies and increasing uncertainty in the environmental conditions. This has forced firms to implement integrated credit, inventory, financing and investment policies.

As pointed out by Sartoris and Hill⁽⁷⁾ short-term financial management studies have developed along fragmented lines such as credit

policy and associated accounts receivable management as well as inventory management, but few attempts have been made to integrate credit policy and inventory management decisions. According to them, the reasons for this tendency are : 1) Since each element of short-term finance is managed by a separate entity with managers possibly separated by several organizational layers, they have learned to think of short-term finance problems as isolated decisions. 2) Since accounting conventions compartmentalize short-term assets and liabilities into packages, this has led some in the past to build decision models based not on sound financial principles but on these accounting constructs.

In the article, incorporating the interactions between the various working capital elements Sartoris and Hill⁽⁷⁾ extend the net present value concept to the short-term financial decisions focusing on the cash flow cycle not on the level of liquidity of working capital. The paper develops first a certainty model for working capital decisions. Uncertainty is then introduced and three methods are suggested for dealing with it: simulation, explicit pricing, and risk neutralization.

As stated by James Mao⁽⁸⁾, the application of linear programming to financial decisions had been limited to long-term financing and investment decisions. The potential use of the technique for short-term financial decisions was shown for the first time in an article by A.A. Robichek, D. Teichrow, and J.M. Jones⁽⁹⁾ published in 1965. In this paper, given the set of cash requirements, and the costs and constraints relating to alternative sources of cash, short-term financing problem under certainty is formulated as a mathematical model. Optimum solutions

are determined for a number of cases and the general form of the solution is discussed. The paper concentrates on the financing side of the short-term decisions not on the investment side. Also uncertainty is not incorporated into the model.

In his paper Mao⁽⁸⁾ applies the model developed by Robichek, Teichroew and Jones⁽⁹⁾ to a case study involving the short-term financing decision of a greeting card business. As a difference Mao formulates the model in terms of cumulative decision variables, and discusses opportunity costs and marginal analysis in an extensive way.

There is much more emphasis on short-term financial planning models under uncertainty since they reflect the facts of the real life. One of the first models in this matter discusses the application of the chance-constrained method on planning for liquidity in financial institutions by Charnes and Thore⁽¹⁰⁾. The method of chance-constrained programming has been developed to take care of the probabilistic elements both in the objective function and the constraints. In the paper, they apply this method to the process of financial planning in savings and loan associations. Given the needs for liquidity (withdrawals of savers and legal requirements) they build a model to choose between alternative sources and uses of funds in order to maximize net operating income over time.

Bühler and Gehring⁽¹¹⁾ model takes a different viewpoint against uncertainty in short-term financial planning. The model treats cash requirements as uncertain but it does not assume that the probability distributions of the uncertain cash requirements are known. Rather, it

is only presupposed that the financial officer has some idea about the cash requirements which permit qualitative probability statements such as the following: "It is no less probable that the cash requirement lies in an interval I_1 than in an interval I_2 ." However, the model is built on pure mathematics and much less practical for real-life application.

Kallberg, White and Ziemba⁽¹²⁾ developed a linear programming under uncertainty (LPUU) model to deal with uncertainties in short-term financial planning. In the formulation, forecasted cash requirements, liquidation and termination costs are all random variables. The objective is to minimize costs of the various sources of funds employed plus the expected penalty costs due to constraint violations over the planning period. The authors state that solving the LPUU model using a stochastic linear programming with simple recourse algorithm gives better results than solving the "Mean Model", that is the deterministic model obtained by replacing all random variables by their means.

All the models incorporating uncertainty in short-term financial planning utilize probability distributions of all the random variables independently, in general those at the right-hand side vector of the formulation, and try to reach to the optimal solution. However, this approach skips the interrelationships inherent in short-term financial planning, restricts incorporation of some short-term tools such as stretching payables and does not permit management to take action parallel to their attitudes against risk taking. Sales figures are the principal sources of uncertainty and production, purchasing, and collections have strong

interrelationships with them. These managerial functions and corresponding policies implemented are closely linked to each other. A short-term financial plan should be able to reflect the impact of the changes in any of these functions and policies to the whole system. This study is aimed at constructing a short-term financial planning model integrating all the functions and interrelated policies of a business firm implied by uncertain sales levels and offering alternative courses of action to choose from with respect to the attitude of Top Management against risk.

CHAPTER III

METHODOLOGY AND FINDINGS

The short-term financial planning process will be developed in three parts. In Part 1, generation of demand quantities for domestic and export markets; in Part 2, determination of monthly cash requirements by a spreadsheet cash budget model; and in Part 3, linear programming optimization model for solving short-term financial decision problem will be examined. The real-life data are obtained from a chemical firm established in 1964 manufacturing products in four main product-lines. The firm has entered Middle Eastern export market and has had troubles in taking short-term financial decisions in recent years.

3.1. GENERATION OF DEMAND QUANTITIES

Generation of demand quantities is handled separately for domestic and export markets. For the demand of domestic market, realized monthly sales figures¹ after 1980 are analysed since the firm investigated is deeply

¹See Appendix I for the realized monthly sales figures of each product line.

affected by the liberal economic measures that has been implemented since then. First, a regression study to reveal the impact of time over the realized demand quantities for each of the product lines are performed. This study indicated that time, being the independent variable, does not explain the variation in the realized sales quantities for all of the four product-lines in the period analysed (see Table 3.1.1). Therefore, no time trend is observed in sales.

TABLE 3.1.1. Adjusted R^2 Values of Four Product-Lines Obtained From the Time Trend Analysis

Product Line	1	2	3	4
Adjusted R^2	0.17933	0.00376	0.03606	0.01922

These results led to the conclusion that monthly sales quantities are random variables fitting to certain probability distributions. To determine the probability distributions Chi-square Goodness-of-Fit Tests are applied to the relevant data² for all of the four product lines. Chi-square tests revealed that sales quantities are normally distributed with corresponding means and standard deviations as shown in Table 3.1.2.

TABLE 3.1.2. Means (\bar{X}) and Standard Deviation(s) of Normally Distributed Monthly Demand Quantities of Four Product Lines

Product Line	1	2	3	4
\bar{X}	29,405	47,438	19,663	21,090
S	8,538	16,043	5,882	7,342

² See Appendix 2 for the results of Chi-square Goodness-of-Fit Tests.

In order to generate demand quantities for exports, discrete subjective probability distributions are used both for the quantity and timing of yearly exports of product lines 1 and 2. As to the timing of exports, the experts agreed that 20% of the total yearly exports will be realized in the first five months of the planning period and 80% of the exports in the remaining seven months where monthly exports are uniformly distributed both for product line 1 and 2. For the quantity of exports, experts have different forecasts categorized as pessimistic, normal and optimistic.

The subjective discrete probability distribution obtained for the yearly exports of product line 1 is :

$$P(X_1 = 530,000) = 0.30$$

$$P(X_1 = 1,000,000) = 0.50$$

$$P(X_1 = 1,500,000) = 0.20$$

where X_1 = discrete random variable for the yearly exports of product line 1.

The subjective discrete probability distribution obtained for the yearly exports of product line 2 is :

$$P(X_2 = 500,000) = 0.25$$

$$P(X_2 = 1,000,000) = 0.50$$

$$P(X_2 = 1,500,000) = 0.25$$

where x_2 = discrete random variable for the yearly exports of product line 2.

Utilizing these probabilistic demand distributions, thirty different monthly demand forecasts³ are generated for the 12-month planning period that will be used in the cash budget model to obtain the necessary inputs for the optimization model.

3.2. DETERMINATION OF CASH REQUIREMENTS

In order to determine the cash requirements for each period, a spreadsheet cash budget model is used. The model integrates sales, production, inventory, purchasing, payment functions and the corresponding policies of a business firm under the environmental conditions the firm is operating. Cash requirements are obtained as a result of detailed production, raw material requirements, inventory and purchasing planning, and payment schedule with respect to uncertain forecasted demand.

After the demand quantities are generated, for each forecast production planning is performed for every product in each of the product-lines over the whole planning period. To accomplish this, beginning-of-planning-period inventory and the forecasted monthly demand for each product should be determined. Production decisions are given by taking care of beginning inventory on hand, forecasted demand, production capacity, and the firm's policy of carrying finished goods inventory. The spreadsheet model of production planning is shown in Exhibit 3.2.1.

³See Appendix 3 for the generation of random variates.

Exhibit 3.2.1. Spreadsheet Production Planning Model.

<u>Name of the Product</u>	<u>Month1</u>	<u>Month2</u>	<u>Month3</u>
(1) Beginning-of-Period Inventory*	400	300	200
(2) Forecasted Sales	2,100	1,500	1,800
(3) Production Decision	2,000	1,400	1,800
(4) End-of-Period Inventory $\{=(1)-(2)+(3)\}$	300	200	200

Since the firm examined currently produces 18 different products this production planning is performed over the 12-month planning period for each of the 30 demand forecasts generated for all of these products.

After the production planning is performed then comes the material requirements planning (MRP). Standard material requirements including spoilage and wastage for unit production should be predetermined and included into the spreadsheet model. Total monthly requirement for a raw material is then calculated as a result of production planning performed depending upon which product(s) consume this material. Purchasing decision is then taken by considering beginning raw material inventory on hand, total material requirement, firm's policy of carrying raw material inventory, purchasing order quantities and lead time. The spreadsheet model of MRP is shown in Exhibit 3.2.2.

* For month 1, this figure is the actual inventory on hand at the beginning of planning period, for the remaining months it is equal to the end-of-period inventory of the previous month.

Exhibit 3.2.2. Spreadsheet Material Requirements Planning Model.

<u>Name of Raw Material</u>	<u>Month1</u>	<u>Month2</u>	<u>Month3</u>
(1) Beginning-of-Period Inventory *	700	600	650
(2) Total Material Requirement **	1,600	700	1,700
(3) Purchasing Decision ***	1,500	750	1,500
(4) End-of-Period Inventory $\{=(1)-(2)+(3)\}$	600	650	450

For each production planning performed with the corresponding demand quantity forecast, MRP including 21 imported and 21 domestic raw materials is realized over the 12-month planning period.

Demand forecasts and the associated production and raw material requirements planning provide the necessary inputs for the cash budget. With the predetermined prices and forecasted demands sales figures are obtained. Collections are determined by taking care of beginning-of-period accounts receivable, sales and timing of collections. Raw material payments are scheduled according to terms of payment by taking care of the inflation adjusted unit prices in each period, quantities purchased, transportation costs, and customs fees, insurance and freight for imported raw materials.

* For month1, this figure is the actual inventory on hand at the beginning of planning period, for the remaining months it is equal to the end-of-period inventory of the previous month.

**It is the total material requirement calculated using the unit production standard requirements and the production decisions for each product utilizing this raw material.

*** For the imported raw materials, purchasing decision is separated into two since certain amount of raw materials can be imported customs-tax free if they are consumed by the exported products.

For the collections of foreign accounts receivable and payments of imported raw materials as well as repayments of outstanding prefinancing credits estimates of foreigns currency rates are required. Payment schedules of outstanding debt, both bank credit and trade credit, are fixed when compared to those of the current planning period since they were incurred in the previous planning period.

After cash receipts and disbursements in each period are obtained, change in minimum operating cash requirement should be determined. Minimum operating cash requirement is the minimum amount of cash that the firm wishes to have on hand so that it can feel safe to pay its operating cash disbursements. Minimum operating cash requirement (MOCR) has two determinants : 1) Predetermined minimum amount of cash (MC) that the firm wishes to have on hand anytime. 2) A percentage of net drain (ND) for the succeeding period which is defined as the total cash receipts minus total operating cash disbursements. Then minimum operating cash requirement in period i ($MOCR_i$) is :

$$MOCR_i = \begin{cases} \text{Max } \{ MC, - (ND_{i+1} \times y) \} , & \text{for } i=1, \dots, m-1 \\ MC & , \text{for } i=m \end{cases}$$

where MC = Predetermined minimum amount of cash that the firm wishes to have on hand anytime.

ND_{i+1} = Total cash receipts minus total operating cash disbursements in period $i+1$.

y = Percentage of net operating cash flow (ND) that the firm wishes to have on hand.

m = Total number of months in the planning period.

Then, change in minimum operating cash requirement in period i (ΔMOCR_i) is defined as:

$$\Delta\text{MOCR}_i = \begin{cases} \text{MOCR}_1 - \text{COH} & , \text{ for } i=1, \\ \text{MOCR}_{i-1} - \text{MOCR}_i & , \text{ for } i=2, \dots, m \end{cases}$$

where COH = Cash on hand at the beginning of planning period.

m = Total number of months in the planning period.

Period cash requirement before additional financing and investment is obtained after adjusting net cash flow with the change in minimum operating cash requirement.⁴

Period cash requirements for each of the 30 demand forecasts are obtained by getting production and purchasing decisions for each forecast separately by utilizing the spreadsheet cash budget model and their probability distributions for each period are calculated⁵. While determining period cash requirements by the cash budget model, beginning-of-period accounts receivable⁶, amount of payments scheduled for domestic⁷ and

⁴ See Appendix 4 for the full format of Cash Budget.

⁵ See Appendix 5 for the period cash requirements of each demand forecast and their probability distributions.

⁶ See Appendix 6 for the beginning of period accounts receivable for each demand forecast and their probability distributions.

⁷ See Appendix 7 for the payment schedule of domestic purchases for each demand forecast and their probability distributions.

foreign⁸ purchases and their probability distributions for each period, which will be used in the optimization model, are also obtained.

3.3. LINEAR PROGRAMMING OPTIMIZATION MODEL

3.3.1. Short-Term Financing and Investment Alternatives

Before developing the mathematical formulation of the model, it is necessary to examine the short-term financing and investment alternatives open to the Financial Management. The basic assumption in the model is that all cash transactions, with no exception, take place at the beginning of a period. The only exception occurs in the collections on accounts receivable. A certain proportion of the accounts receivable which are pledged at the beginning of a period would normally be paid by the firm's customers during that period. Once they are paid the effective amount borrowed by the firm from the bank is decreased by the proportion of maximum loan to accounts receivable as of the payment. If these payments by the customers were treated as taken place at the beginning of the period, an artificially high amount of borrowing would be required. Therefore, it will be assumed that a certain proportion of the receivables outstanding at the beginning of a period are paid by the customers during that period.⁹

⁸ See Appendix 8 for the payment schedule of foreign purchases for each demand forecast and their probability distributions.

⁹ Robichek, Teichroew, and Jones Op. Cit.

The short-term financing alternatives available to the Financial Management in the model are:

- 1) Unsecured Line of Credit,
- 2) Pledging of Accounts Receivable,
- 3) Short-Term Bank Credit,
- 4) Issuing Commercial Paper,
- 5) Stretching of Accounts Payable,
- 6) Stretching of Taxes Payable,
- 7) Term Loan.

The short-term investment alternatives available to the Financial Management in the model are:

- 1) Investment in Term Deposits
 - with one-month maturity,
 - with three-months maturity,
 - with six-months maturity.
- 2) Investment in Marketable Securities
 - with K different types of instruments.

3.3.2. Mathematical Formulation of the Model

The following notation will be used¹⁰ in developing the linear programming formulation of the short-term financing/investment decision

¹⁰ Robichek, Teichroew, and Jones Op.Cit.

problem:

x_{ij} = amount borrowed from source i at the beginning of period j .

y_{ij} = amount voluntarily repaid to source i at the beginning of period j .

v_{ij} = amount of mandatory repayment to source i at the beginning of period j . ($i=1, \dots, 7$),
($j=1, \dots, m$ where m =total number of months in planning period)

x_{ilj} = amount of investment made in investment alternative i to the instrument l at the beginning of period j .
($i=8, 9$), ($l=1, \dots, K$ where K total number of instruments),
($j=1, \dots, m$ where m total # of months in the planning period)

z_{ij} = net cumulative amount borrowed from source i at the beginning of period j after borrowing and repayment for period j .

$$= \sum_{k=1}^i (x_{ik} - y_{ik} - v_{ik})$$

r_i = interest rate for alternative i .

a_{in} = n^{th} coefficient used in stating constraints on alternative i .

b_{in} = n^{th} constraint limit for alternative i .

The subscript i denotes the following alternatives :

$i = 1$: unsecured line of credit

2 : pledging of accounts receivable

3 : short-term bank credit

4 : issuing commercial paper

5 : stretching of accounts payable

6 : stretching of taxes payable

7 : term loan

8 : investment in term deposits

9 : investment in marketable securities

3.3.2.1. Constraints on Financing Alternatives

1) Line of Credit

1.1) To ensure that voluntary and mandatory repayments do not exceed the amount borrowed.

$$\sum_{k=1}^j (x_{1k} - y_{1k} - v_{1k}) \geq 0, \quad j=1, \dots, m$$

1.2) The amount of outstanding borrowing under the line of credit is limited.

$$\sum_{k=1}^j (x_{1k} - y_{1k} - v_{1k}) \leq b_{11j}, \quad j=1, \dots, m$$

where b_{11j} = max. outstanding balance under the line of credit in period j .

1.3) Min. and max. amount of borrowing in any period is bounded.

$$b_{13j} \leq x_{1j} \leq b_{12j}$$

where b_{12j} = max. amount that can be borrowed in period j .

b_{13j} = min. amount that can be borrowed in period j .

1.4) The bank requires a compensating balance of not less than a_{11j} percent of the amount borrowed in period j .

$$a_{11j} \sum_{k=1}^j (x_{1k} - y_{1k} - v_{1k}) - s_{12j} \leq \text{MOCR}_j, \quad j=1, \dots, m$$

where a_{11j} = proportion of loan which is required as a compensating balance in period j .

s_{12j} = additional amount borrowed in period j to meet compensating balance requirement.

$MOCR_j$ = minimum operating cash balance in period j obtained from the cash budget.

1.5) The bank requires a mandatory payment of not less than a_{12j} percent of the amount outstanding in the previous period.

i) $v_{11} = 0$ (no mandatory payment is required in the first month of the planning period)

ii) $v_{1j} = a_{12j} \sum_{k=1}^{j-1} (x_{1k} - y_{1k} - v_{1k})$, $j=2, \dots, m-1$

iii) $v_{1m} = \sum_{k=1}^{m-1} (x_{1k} - y_{1k} - v_{1k}) + x_{1m} - y_{1m}$

where a_{12j} = proportion of outstanding loan which is required to be paid in period j .

2) Pledging of Accounts Receivable

2.1) To ensure that voluntary and mandatory repayments do not exceed the amount borrowed.

$$\sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k}) \geq 0, \quad j = 1, \dots, m$$

2.2) Max. amount of outstanding borrowing under pledging of accounts receivable is limited.

$$\sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k}) \leq b_{21j}, \quad j = 1, \dots, m$$

where b_{21j} = max. outstanding balance under pledging of accounts receivable in period j .

2.3) The bank will lend up to a_{21j} percent of the face value of pledged accounts receivable.

$$\sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k}) \leq a_{21j} A_j, \quad j = 1, \dots, m$$

where a_{21j} = proportion of max. loan to accounts receivable in period j .

A_j = amount of accounts receivable at the beginning of period j obtained from the cash budget.

2.4) Min. and max. amount of borrowing in any period is bounded.

$$b_{23j} \leq x_{2j} \leq b_{22j}, \quad j = 1, \dots, m$$

where b_{22j} = max. amount that can be borrowed in period j .

b_{23j} = min. amount that can be borrowed in period j .

2.5) To decrease the outstanding balance the payments of customers during the period are taken as mandatory repayments.

$$i) \quad v_{21} = 0$$

$$ii) \quad v_{2j} = a_{22j} \sum_{k=1}^{j-1} (x_{2k} - y_{2k} - v_{2k}), \quad j = 2, \dots, m$$

where a_{22j} = proportion of accounts receivable that are collected during period j .

At the beginning of period j , the amount outstanding is z_{2j} , at the end of the period, the amount outstanding is $(1 - a_{22j}) z_{2j}$. Assuming that the payments from the customers are uniform throughout the period, the average amount outstanding throughout the period is,

$$\frac{1}{2} (z_{2j} + (1-a_{22j})z_{2j}) = (1-a_{22j}/2) \quad z_{2j} = a_{23j}z_{2j}$$

This average amount outstanding is used in the fulfillment of requirements constraints.

3) Short-Term Bank Credit (n-period maturity, $n < 12$)

3.1) To ensure that voluntary repayments do not exceed the amount borrowed.

$$\sum_{k=1}^j (x_{3k} - y_{3k}) \geq 0, \quad j = 1, \dots, m$$

3.2) Max. amount of outstanding borrowing under short-term bank credit is limited.

$$\sum_{k=1}^j (x_{3k} - y_{3k}) \leq b_{31j}, \quad j = 1, \dots, m$$

where b_{31j} = max. outstanding balance under short-term bank credit in period j .

3.3) Min. and max. amount of borrowing in any period is limited.

$$b_{32j} \leq x_{3j} \leq b_{33j}, \quad j = 1, \dots, m$$

where b_{32j} = min. amount that can be borrowed in period j .

b_{33j} = max. amount that can be borrowed in period j .

3.4) At the beginning of the $(n+1)^{st}$ period, at least the amount of the loan that was taken n periods ago should be repaid.

$$\sum_{k=j}^{j+n} y_{3k} - x_{3j} \geq 0, \quad j = 1, \dots, m-n$$

4.) Issuing Commercial Paper

The variables are defined as:

x_{41j} = amount of commercial paper with one-month maturity period issued in period j .

x_{42j} = amount of commercial paper with three-months maturity period issued in period j .

x_{43j} = amount of commercial paper with six-months maturity period issued in period j .

4.1) The outstanding amount of commercial papers in any period is limited.

$$b_{411} \leq x_{411} + x_{421} + x_{431} \leq b_{421}, \text{ for period 1}$$

$$b_{412} \leq x_{412} + x_{421} + x_{422} + x_{431} + x_{432} \leq b_{422}, \text{ for period 2}$$

$$b_{413} \leq x_{413} + x_{421} + x_{422} + x_{423} + x_{431} + x_{432} + x_{433} \leq b_{423}, \text{ for period 3}$$

$$b_{41j} \leq x_{41j} + \sum_{k=j-2}^j x_{42k} + \sum_{k=\ell}^j x_{43k} \leq b_{42j}, \text{ for } j=4, \dots, m$$

Where $\ell = 1$, for $j = 4, 5, 6$

$\ell = j-5$, for $j = 7, \dots, m$

b_{41j} = min. amount of outstanding commercial paper in period j .

b_{42j} = max. amount of outstanding commercial paper in period j .

4.2) Min. and max. amount of commercial paper that can be issued for all three categories in period j are limited.

$$b_{412j} \leq x_{41j} \leq b_{411j} \quad , j = 1, \dots, m$$

$$b_{422j} \leq x_{42j} \leq b_{421j} \quad , j = 1, \dots, m$$

$$b_{432j} \leq x_{43j} \leq b_{431j} \quad , j = 1, \dots, m$$

where $b_{4i2j} = \min.$ amount of commercial paper that can be issued from category i ($i=1,2,3$) in period j .

$b_{4i1j} = \max.$ amount of commercial paper that can be issued from category i ($i=1,2,3$) in period j .

4.3) The proportion of each category commercial paper to total amount of outstanding commercial papers is limited.

$$a_{4111} \leq \frac{x_{411}}{x_{411} + x_{421} + x_{431}} \leq a_{4121} \quad , \text{ for period 1}$$

$$a_{4132} \leq \frac{x_{412}}{x_{412} + x_{421} + x_{422} + x_{431} + x_{432}} \leq a_{4142} \quad , \text{ for period 2}$$

$$a_{4153} \leq \frac{x_{413}}{x_{413} + x_{421} + x_{422} + x_{431} + x_{432} + x_{433}} \leq a_{4163} \quad , \text{ for}$$

period 3

$$a_{41(z+1)j} \leq \frac{x_{41j}}{x_{41j} + \sum_{k=j-2}^j x_{42k} + \sum_{k=\ell}^j x_{43k}} \leq a_{41(z+2)j}, \text{ for } j=4, 5, \dots, m$$

where $\ell = 1$, for $j = 4, 5, 6$

$\ell = j-5$, for $j = 7, 8, \dots, m$

$z = 6, 7, \dots, 2m-2$

a_{41zj} = min. proportion of one-month maturity commercial paper
to total outstanding commercial papers in period j .

$a_{41(z+1)j}$ = max. proportion of one-month maturity commercial paper
to total outstanding commercial papers in period j .

Same set of constraints should also be added to the LP formulation for
the three-month maturity commercial paper.

5) Stretching of Accounts Payable

The firm is able to acquire cash by not paying its accounts
payable when they first come due according to the payment schedule made
in the cash budget.

An additional subscript here is required since payables may be
stretched for one or more periods and are separated into two groups :
i) those to domestic vendors, ii) those to foreign vendors. The variables
are defined as:

x_{5jk} = amount of domestic payables, due in period k , which is
stretched in period j ; $k=j$, $j-1$ or $j-2$.

x'_{5jk} = amount of foreign payables, due in period k , which is stretched in period j ; $k=j, j-1$ or $j-2$

y_{5jk} = amount of domestic payables, due in period k , which is stretched in period j ; $k=j, j-1, j-2$ or $j-3$

y'_{5jk} = amount of foreign payables, due in period k , which is actually paid in period j ; $k=j, j-1, j-2$ or $j-3$

5.1) The financial manager may stretch up to a_{51j} (a'_{51j}) percent of the payments due in the period in which they first come due, i.e. he must pay at least $(1-a_{51j})$ $\{1-a'_{51j}\}$ percent. (stretching first period)

$$x_{5jj} \leq a_{51j} P_j, \quad j = 1, \dots, m$$

$$x'_{5jj} \leq a'_{51j} P'_j, \quad j = 1, \dots, m$$

where a_{51j} = proportion of domestic accounts payable which can be stretched in the period in which they first become due.

a'_{51j} = proportion of foreign accounts payable which can be stretched in the period in which they first become due.

P_j = domestic purchases made before which are scheduled to be paid in period j in the cash budget.

P'_j = foreign purchases made before which are scheduled to be paid in period j in the cash budget.

5.2) If an amount x_{5jj} (x'_{5jj}) is stretched then the amount due in period $(j+1)$ is $(1+r_{51})(x_{5jj})$ $\{(1+r'_{51})(x'_{5jj})\}$

where r_{51} = cost of stretching domestic accounts payable for one period for the first time.

r'_{51} = cost of stretching foreign accounts payable for one period for the first time.

As in the first period in which payments are stretched, a certain proportion of the outstanding amount must be paid if the payment is stretched to the second period, i.e. only a certain portion can be stretched for another period.

$$x_{5jj-1} \leq a_{52j}(1+r_{51}) x_{5j-1j-1}, \quad j = 2, \dots, m$$

$$x'_{5jj-1} \leq a'_{52j}(1+r'_{51}) x'_{5j-1j-1}, \quad j = 2, \dots, m$$

where a_{52j} = proportion of domestic accounts payable stretched in the first period which can be stretched in the second period.

a'_{52j} = proportion of foreign accounts payable stretched in the first period which can be stretched in the second period.

5.3) If an amount $x_{5jj-1}(x'_{5jj-1})$ is stretched then the amount due in period $(j+1)$ is $(1+r_{52})(x_{5jj-1}) \{ (1+r'_{52})(x'_{5jj-1}) \}$

where r_{52} = cost of stretching domestic accounts payable for the second time

r'_{52} = cost of stretching foreign accounts payable for the second time

$$x_{5jj-2} \leq a_{53j}(1+r_{52}) x_{5j-1j-2}, \quad j = 3, \dots, m$$

$$x'_{5jj-2} \leq a'_{53j}(1+r'_{52}) x'_{5j-1j-2}, \quad j = 3, \dots, m$$

where a_{53j} = proportion of domestic accounts payable stretched in the second period which can be stretched in the third period.

a'_{53j} = proportion of foreign accounts payable stretched in the second period which can be stretched in the third period.

Mathematical Equalities Related to Payments of Accounts Payable

i) When an accounts payable payment comes due as scheduled, it is either paid immediately or stretched.

$$x_{5jj} + y_{5jj} = P_j, \quad j = 1, \dots, m$$

$$x'_{5jj} + y'_{5jj} = P'_j, \quad j = 1, \dots, m$$

ii) The amount that is not stretched in the second period must be paid.

$$y_{5jj-1} = (1+r_{51}) x_{5j-1j-1} - x_{5jj-1}, \quad j = 2, \dots, m$$

$$y'_{5jj-1} = (1+r'_{51}) x'_{5j-1j-1} - x'_{5jj-1}, \quad j = 2, \dots, m$$

iii) The amount that is not stretched in the third period must be paid.

$$y_{5jj-2} = (1+r_{52}) x_{5j-1j-2} - x_{5jj-2}, \quad j = 3, \dots, m$$

$$y'_{5jj-2} = (1+r'_{52}) x'_{5j-1j-2} - x'_{5jj-2}, \quad j = 3, \dots, m$$

iv) Any payables stretched in the third period must be paid in the fourth period.

$$y_{5jj-3} = (1+r_{53}) x_{5j-1j-3}, \quad j = 4, \dots, m$$

$$y'_{5jj-3} = (1+r'_{53}) x'_{5j-1j-3} \quad , \quad j=4, \dots, m$$

where r_{53} = cost of stretching domestic accounts payable for the third time,

r'_{53} = cost of stretching foreign accounts payable for the third time,

These mathematical equalities developed will be utilised in the Fulfillment of Requirements constraints.

6) Stretching of Taxes Payable

It is assumed that the tax payments can be stretched for two periods. The variables are defined as :

x_{6jk} = amount of taxes payable, due in period k, which is stretched in period j; $k=j, j-1$.

y_{6jk} = amount of taxes payable, due in period k, which is actually paid in period j; $k=j, j-1, j-2$

6.1) The financial manager may stretch up to a_{61j} percent of the tax payments due in the period in which they first become due. (stretching first period)

$$x_{6jj} \leq a_{61j} T_j \quad , \quad j = 1, \dots, m$$

where a_{61j} = proportion of taxes payable which can be stretched in

the period in which they first become due.

T_j = taxes which are scheduled to be paid in period j in the cash budget.

6.2) If an amount x_{6jj} is stretched then the amount due in period $(j+1)$ is

$$(1 + r_{61}) (x_{6jj})$$

where r_{61} = cost of stretching taxes payable for one period for the first time.

As in the first period in which tax payments are stretched, a certain proportion of the outstanding amount must be paid, if the payment is stretched to the second period. (Stretching second period)

$$x_{6jj-1} \leq a_{62j} (1 + r_{61}) x_{6j-1j-1}, \quad j=2, \dots, m$$

where a_{62j} = proportion of taxes payable stretched in the first period which can be stretched in the second period.

Mathematical Equalities Related to Payments of Taxes Payable

i) When a tax payment comes due as scheduled, it is either paid immediately or stretched.

$$x_{6jj} + y_{6jj} = T_j, \quad j=1, \dots, m$$

ii) The amount that is not stretched in the second period must be paid.

$$y_{6jj-1} = (1+r_{61})x_{6j-1j-1} - x_{6jj-1}, \quad j = 2, \dots, m$$

iii) Any payables stretched in the second period must be paid in the third period.

$$y_{6jj-2} = (1+r_{62})x_{6j-1j-2}, \quad j = 3, \dots, m$$

where r_{62} = cost of stretching taxes payable for the second time

These mathematical equalities developed will be utilised in the Fulfillment of Requirements constraints.

7) Term Loan

7.1) Min. and max. amount of borrowing by term loan in any period is limited.

$$b_{71j} \leq x_{7j} \leq b_{72j}, \quad j = \text{period(s) when term loan(s) can be taken.}$$

where b_{71j} = min. amount of term loan that can be taken in period j .

b_{72j} = max. amount of term loan that can be taken in period j .

7.2) The principal amount of the term loan must be repaid in equal installments. The first installment is due after z periods, in the beginning of $(z+1)^{st}$ period. No speed-up of payments is possible.

$$v_{7\ell} = a_{71}x_{7j}, \quad \ell = zk+j, \quad k = 1, 2, \dots, 1/a_{71} \\ (\ell \leq m)$$

j = period(s) when term loan(s) can be taken

where a_{71} = proportion of term loan principal repaid at each installment

$1/a_{71}$ = number of the installment payments of the term loan.

z = number of periods between consecutive installment payments.

7.3) The amount of outstanding term loan is limited.

$$\sum_j (x_{7j} - v_{7\ell}) \leq b_{73k}, \quad k = 1, \dots, m$$

where b_{73k} = max. outstanding balance under the term loan in period k

3.3.2.2. Constraints on Investment Alternatives

1) Investment In Term Deposits

The variables are defined as:

x_{81j} = amount of investment made in one-month maturity term deposits
in period j

x_{82j} = amount of investment made in three-months maturity term
deposits in period j

x_{83j} = amount of investment made in six-months maturity term deposits
in period j

1.1) The proportion of one-month maturity term deposits to total outstanding term deposits is limited.

$$a_{8111} \leq \frac{x_{811}}{x_{811} + x_{821} + x_{831}} \leq a_{8121}, \text{ for period 1}$$

$$a_{8112} \leq \frac{x_{812}}{x_{812} + x_{821} + x_{822} + x_{831} + x_{832}} \leq a_{8122}, \text{ for period 2}$$

$$a_{8113} \leq \frac{x_{813}}{x_{813} + x_{821} + x_{822} + x_{823} + x_{831} + x_{832} + x_{833}} \leq a_{8123},$$

for period 3

$$a_{811j} \leq \frac{x_{81j}}{x_{81j} + \sum_{k=j-2}^j x_{82k} + \sum_{k=\ell}^j x_{83k}} \leq a_{812j}, \text{ for } j = 4, \dots, m$$

where $\ell = 1$, for $j = 4, 5, 6$

$\ell = j - 5$, for $j = 7, 8, \dots, m$

a_{811j} = min. proportion of one-month maturity term deposits to total outstanding term deposits in period j .

a_{812j} = max. proportion of one-month maturity term deposits to total outstanding term deposits in period j .

Same set of constraints should also be added to the LP formulation for the three-month maturity term deposits.

1.2) The outstanding amount of term deposits in any period is limited.

$$b_{811} \leq x_{811} + x_{821} + x_{831} \leq b_{821} \quad , \quad \text{for period 1}$$

$$b_{812} \leq x_{812} + x_{821} + x_{822} + x_{831} + x_{832} \leq b_{822} \quad , \quad \text{for period 2}$$

$$b_{813} \leq x_{813} + x_{821} + x_{822} + x_{823} + x_{831} + x_{832} + x_{833} \leq b_{823} \quad , \quad \text{for period 3}$$

$$b_{81j} \leq x_{81j} + \sum_{k=j-2}^j x_{82k} + \sum_{k=\ell}^j x_{83k} \leq b_{82j} \quad , \quad \text{for } j=4, \dots, m$$

where $\ell = 1$, for $j = 4, 5, 6$

$\ell = j-5$, for $j = 7, \dots, m$

b_{81j} = min. amount of outstanding term deposits in period j .

b_{82j} = max. amount of outstanding term deposits in period j .

1.3) Min. and max. amount of investment in term deposits for all maturity periods is limited.

$$b_{81ij} \leq x_{8ij} \leq b_{82ij} \quad , \quad i = 1, 2, 3 \quad , \quad j = 1, \dots, m$$

where b_{81ij} = min. amount of investment to i^{th} type term deposits in period j .

b_{82ij} = max. amount of investment to i^{th} type term deposits in period j .

2) Investment In Marketable Securities

The variables are defined as:

x_{9ij} = amount of investment made in marketable securities from instrument i in period j . ($j = 1, \dots, m$) ($i = 1, \dots, K$ where K = total number of marketable security instruments)

Marketable securities bought in the beginning of period j should

be sold in the beginning of period $(j+1)$, i.e. all instruments are assumed to have one-month maturity period.

2.1) The proportion of investment in each instrument to total outstanding marketable securities is limited.

$$a_{91ij} \leq \frac{x_{9ij}}{\sum_{i=1}^K x_{9ij}} \leq a_{92ij} \quad , \quad j=1, \dots, m$$

$$i=1, \dots, K-1$$

where a_{91ij} = min. proportion of marketable securities from instrument i to total outstanding marketable securities in period j .

a_{92ij} = max. proportion of marketable securities from instrument i to total outstanding marketable securities in period j .

2.2) The outstanding amount of marketable security investments in any period is limited.

$$b_{91j} \leq \sum_{i=1}^K x_{9ij} \leq b_{92j} \quad , \quad j=1, \dots, m$$

where b_{91j} = min. amount of marketable security investments in period j .

b_{92j} = max. amount of marketable security investments in period j .

2.3) Min. and max. amount of investment in marketable securities for all instruments in any period is limited.

$$b_{91ij} \leq x_{9ij} \leq b_{92ij} \quad , \quad i=1, \dots, K, \quad j=1, \dots, m$$

where b_{91ij} = min. amount of investment to i^{th} instrument of marketable securities in period j .

b_{92ij} = max. amount of investment to i^{th} instrument of marketable securities in period j .

3.3.2.3 Fulfillment of Requirements Constraints

Cash requirements for each period, before interest and compensating balance requirements (R_j) are obtained from the cash budget. These requirements must be adjusted for the interest expense, cost of stretching payables, and any interest earned on investment of excess cash.

The requirements inequality for each period states that the adjusted requirements must be satisfied by the returns of investments, borrowings less repayments, plus any change in compensating balance requirements.

The fulfillment of requirements constraint can be written as :

$$\begin{aligned}
 R_j &\leq \text{requirements in period } j \text{ must be less than or equal to} \\
 &(x_{1j} + a_{23j} x_{2j} + x_{3j} + \\
 &x_{41j} + x_{42j} + x_{43j} + x_{5jj} + \\
 &x'_{5jj} + x_{6jj} + x_{7j}) \quad \text{new borrowing} \\
 &+ ((1+r_{81j-1})x_{81j-1} + (1+r_{82j-3}) x_{82j-3} + \\
 &(1+r_{83j-6})x_{83j-6}) \quad \text{+ maturing term deposits and their interest income} \\
 &+ \sum_{i=1}^K (1+r_{9ij-1})x_{9ij-1} \quad \text{+ maturing marketable securities and their returns}
 \end{aligned}$$

- $(y_{1j} + v_{1j} + a_{23j} (y_{2j} + v_{2j}) +$
 $y_{3j} + v_{3j} + (1+r_{41j-1})x_{41j-1} +$
 $(1+r_{42j-3})x_{42j-3} + (1+r_{43j-6})x_{43j-6} +$
 $y_{5jj-1} + y'_{5jj-1} + y_{5jj-2} + y'_{5jj-2} +$
 $y_{5jj-3} + y'_{5jj-3} + y_{6jj-1} + y_{6jj-2} + v_{7j})$ - repayments
 - $(r_{1j-1} z_{1j-1} + r_{2j-1} a_{23j-1} z_{2j-1} +$
 $r_{3j-1} z_{3j-1} + r_{7j-1} z_{7j-1})$ - interest expense in previous period
 - $(x_{81j} + x_{82j} + x_{83j})$ - investment in term deposits
 - $\sum_{i=1}^K x_{9ij}$ - investment in marketable securities
 - $(s_{12j} - s_{12j-1})$ - change in compensating balance requirement
- $j=7, \dots, m$

where r_{1j} = interest rate on line of credit in period j
 r_{2j} = interest rate on the pledged receivables in period j
 r_{3j} = interest rate on short-term bank credit in period j
 r_{7j} = interest rate on the term loan in period j
 r_{4ij} = interest rate on the i^{th} type commercial paper in period j
 r_{8ij} = interest rate on the i^{th} type term deposit in period j
 $(i=1, 2, 3)$
 r_{9ij} = return on the i^{th} marketable security instrument in period j
 $(i=1, \dots, K)$

For periods 1 through 6, the variables with subscript $j-6$ are deleted.

For period 3, the variables with subscript $j-3$ are deleted.

For period 2, the variables with subscripts $j-3$ and $j-2$ are deleted.

For period 1, the variables with subscripts $j-3$, $j-2$ and $j-1$ are deleted.

After substituting the previously developed mathematical equalities related to accounts and taxes payable, and rearranging the following Fulfillment of Requirements constraint is obtained:

$$\begin{aligned}
 R_j \leq & (x_{1j} - y_{1j} - v_{1j}) + a_{23j}(x_{2j} - y_{2j} - v_{2j}) + (x_{3j} - y_{3j} - v_{3j}) + \\
 & x_{41j} - (1 + r_{41j-1})x_{41j-1} + x_{42j} - (1 + r_{42j-3})x_{42j-3} + x_{43j} - (1 + r_{43j-6})x_{43j-6} \\
 & x_{5jj} + x_{5jj-1} - (1 + r_{51})x_{5j-1j-1} + x'_{5jj} + x'_{5jj-1} - (1 + r'_{51})x'_{5j-1j-1} + \\
 & x_{5jj-2} - (1 + r_{52})x_{5j-1j-2} + x'_{5jj-2} - (1 + r'_{52})x'_{5j-1j-2} - (1 + r_{53})x_{5j-1j-3} - \\
 & (1 + r'_{53})x'_{5j-1j-3} + x_{6jj} + x_{6jj-1} - (1 + r_{61})x_{6j-1j-1} - (1 + r_{62})x_{6j-1j-2} + \\
 & (x_{7j} - v_{7j}) + (1 + r_{81j-1})x_{81j-1} + (1 + r_{82j-3})x_{82j-3} + (1 + r_{83j-6})x_{83j-6} - \\
 & (x_{81j} + x_{82j} + x_{83j}) + \left(\sum_{i=1}^K (1 + r_{9ij-1})x_{9ij-1} - \sum_{i=1}^K x_{9ij}\right) - \\
 & (r_{1j-1}z_{1j-1} + r_{2j-1}a_{23j-1}z_{2j-1} + r_{3j-1}z_{3j-1} + r_{7j-1}z_{7j-1}) - \\
 & (s_{12j} - s_{12j-1}) \quad j = 7, \dots, m
 \end{aligned}$$

3.3.2.4. Financial Policy Constraints

Financial policy constraints represent the target performances and desired interrelations among various decision variables of financial sources and investment choices. Including these constraints into the model are at the discretion of the financial management. Some examples to these kind of constraints are stated below.

1) The proportion of the outstanding i^{th} type bank source to total

outstanding bank sources is limited.

$$a_{1011j} \leq \frac{\sum_{k=1}^j (x_{ik} - y_{ik} - v_{ik})}{\sum_{k=1}^j (x_{1k} - y_{1k} - v_{1k}) + \sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k}) + \sum_{k=1}^j (x_{3k} - y_{3k}) + \sum_{k=1}^j (x_{7k} - v_{7k})} \leq a_{1012j}$$

$i = 1, 2, 3 \text{ or } 7$, $j = 1, \dots, m$ or any period(s).

where a_{1011j} = min. proportion of the outstanding i^{th} type source to total outstanding bank sources in period j

a_{1012j} = max. proportion of the outstanding i^{th} type source to total outstanding bank sources in period j

2) Min. and max. amount of total bank sources that can be used are limited.

$$b_{101j} \leq \sum_{k=1}^j (x_{1k} - y_{1k} - v_{1k}) + \sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k}) + \sum_{k=1}^j (x_{3k} - y_{3k}) + \sum_{k=1}^j (x_{7k} - v_{7k}) \leq b_{102j}$$

$j = 1, \dots, m$ or any period(s)

where b_{101j} = min. amount of total bank sources that can be used in period j

b_{102j} = max. amount of total bank sources that can be used in period j

3) The proportion of the total outstanding non-bank sources to total outstanding bank sources is limited.

$$a_{1021j} \leq \frac{x_{41j} + \sum_{k=j-2}^j x_{42k} + \sum_{k=1}^j x_{43k} + x_{5jj} + x'_{5jj} + x_{5jj-1} + x'_{5jj-1} + x_{5jj-2} + x'_{5jj-2} + x_{6jj} + x_{6j}}{\sum_{k=1}^j (x_{1k} - y_{1k} - v_{1k}) + \sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k}) + \sum_{k=1}^j (x_{3k} - y_{3k}) + \sum_{k=1}^j (x_{7k} - v_{7k})} \leq a_{1022j}$$

$j = 3, \dots, m$

where

a_{1021j} = min. proportion of the total outstanding non-bank sources
to that of bank sources

a_{1022j} = max. proportion of the total outstanding non-bank sources
to that of bank sources.

4) The proportion of the total outstanding investment in term deposits
to total outstanding investment in marketable securities is limited.

$$a_{1031j} \leq \frac{x_{81j} + \sum_{k=j-2}^j x_{82k} + \sum_{k=\ell}^j x_{83k}}{\sum_{i=1}^K x_{9ij}} \leq a_{1032j}, \quad j=3, \dots, m$$

where $\ell=1$, for $j=4,5,6$

$\ell=j-5$, for $j=7, \dots, m$

a_{1031j} = min. proportion of the total outstanding non-bank
sources to that of bank sources

a_{1032j} = max. proportion of the total outstanding non-bank
sources to that of bank sources.

3.3.2.5. Objective (Cost) Function

The cost components of the objective function is divided into two
parts: 1) Explicit costs, 2) Implicit costs.

1) Explicit costs are defined as the costs that can be measured objectively
and quantitatively as the interest expense of a financial resource or
the interest income (return) of an investment. These costs depend on the

outstanding amount of resources and investments, and the required interest rates for them. Total monthly interest expense (I_{1j}) for line of credit, pledging of accounts receivable, short-term bank credit, stretching of accounts and taxes payable and term loan is written as:

$$I_{1j} = r_{1j}z_{1j} + r_{2j}a_{23j}z_{2j} + r_{3j}z_{3j} + r_{51}x_{5jj} + r'_{51}x'_{5jj} + r_{52}x_{5jj-1} + r'_{52}x'_{5jj-1} + r_{53}x_{5jj-2} + r'_{53}x'_{5jj-2} + r_{61}x_{6jj} + r_{62}x_{6jj-1} + r_{7j}z_{7j}$$

$$j=3, \dots, m$$

For period 2, the variables with subscript $j-2$ are deleted.

For period 1, the variables with subscript $j-1$ and $j-2$ are deleted.

The interest expense for one-month maturity period commercial paper (I_{2j}) is:

$$I_{2j} = r_{41j} x_{41j} \quad , \quad j=1, \dots, m$$

The interest expense for three-month maturity period commercial paper (I_{3j}) is:

$$I_{3j} = \frac{1}{3} r_{421} x_{421} \quad , \quad \text{for period 1}$$

$$= \frac{2}{3} r_{421} x_{421} + \frac{1}{3} r_{422} x_{422} \quad , \quad \text{for period 2}$$

$$= \frac{1}{3} \left(\sum_{k=j-2}^j (j+1-k) r_{42k} x_{42k} \right) \quad , \quad \text{for } j=3, \dots, m$$

The interest expense for six-month maturity period commercial paper (I_{4j}) is:

$$I_{4j} = \frac{1}{6} r_{431} x_{421}, \text{ for period 1}$$

$$= \frac{1}{6} (2r_{431} x_{431} + r_{432} x_{432}), \text{ for period 2}$$

$$= \frac{1}{6} (3r_{431} x_{431} + 2r_{432} x_{432} + r_{433} x_{433}), \text{ for period 3}$$

$$= \frac{1}{6} (4r_{431} x_{431} + 3r_{432} x_{432} + 2r_{433} x_{433} + r_{434} x_{434}), \text{ for period 4}$$

$$= \frac{1}{6} (5r_{431} x_{431} + 4r_{432} x_{432} + 3r_{433} x_{433} + 2r_{434} x_{434} + r_{435} x_{435}), \text{ for period 5}$$

$$= \frac{1}{6} \sum_{k=j-5}^j (j+1-k) r_{43k} x_{43k}, \quad j = 6, \dots, m$$

The interest income for one-month maturity period term deposits (I_{5j}) is:

$$I_{5j} = r_{81j} x_{81j}, \quad j = 1, \dots, m$$

The interest income for three-month maturity period term deposits (I_{6j}) is:

$$I_{6j} = \frac{1}{3} r_{821} x_{821}, \text{ for period 1}$$

$$= \frac{2}{3} r_{821} x_{821} + \frac{1}{3} r_{822} x_{822} \quad , \quad \text{for period 2}$$

$$= \frac{1}{3} \sum_{k=j-2}^j (j+1-k) r_{82k} x_{82k} \quad , \quad \text{for } j = 3, \dots, m$$

The interest income for six month maturity period term deposits (I_{7j}) is:

$$I_{7j} = \frac{1}{6} r_{831} x_{831} \quad , \quad \text{for period 1}$$

$$= \frac{1}{6} (2r_{831} x_{831} + r_{832} x_{832}) \quad , \quad \text{for period 2}$$

$$= \frac{1}{6} (3r_{831} x_{831} + 2r_{832} x_{832} + r_{833} x_{833}) \quad , \quad \text{for period 3}$$

$$= \frac{1}{6} (4r_{831} x_{831} + 3r_{832} x_{832} + 2r_{833} x_{833} + r_{834} x_{834}) \quad , \quad \text{for}$$

period 4

$$= \frac{1}{6} (5r_{831} x_{831} + 4r_{832} x_{832} + 3r_{833} x_{833} + 2r_{834} x_{834} + r_{835} x_{835}) \quad ,$$

for period 5

$$= \frac{1}{6} \sum_{k=j-5}^j (j+1-k) r_{83k} x_{83k} \quad , \quad \text{for } j = 6, \dots, m$$

The interest income for marketable securities (I_{8j}) is:

$$I_{8j} = \sum_{i=1}^K r_{9ij} x_{9ij} \quad , \quad j = 1, \dots, m$$

Then, total explicit cost (E_j) in period j is:

$$E_j = I_{1j} + I_{2j} + I_{3j} + I_{4j} - I_{5j} - I_{6j} - I_{7j} - I_{8j}, \quad j=1, \dots, m$$

2) Implicit Costs: The qualitative considerations in the stretching of payables (cost of ill will to creditors, tax authorities) and the term loan (restrictions on the company operations, such as officers' salaries, dividend payments, and capital expenditures) will be incorporated by assigning implicit costs. These costs will be assumed to be proportional to the amount of money borrowed and hence can be specified as rates per period. Let,

q_{51} = implicit cost of ill will to domestic creditors when payments are stretched for one period.

q_{52} = implicit cost of ill will to domestic creditors when payments are stretched for two periods.

q_{53} = implicit cost of ill will to domestic creditors when payments are stretched for three periods.

q'_{51} = implicit cost of ill will to foreign creditors when payments are stretched for one period.

q'_{52} = implicit cost of ill will to foreign creditors when payments are stretched for two periods.

q'_{53} = implicit cost of ill will to foreign creditors when payments are stretched for three periods.

q_{61} = implicit cost of ill will to tax authorities when payments are stretched for one period.

q_{62} = implicit cost of ill will to tax authorities when payments are stretched for two periods.

q_{71} = implicit cost of the term loan.

The total implicit cost (I_j) for the j^{th} period is:

$$I_j = q_{51}x_{5jj} + q'_{51}x'_{5jj} + q_{52}x_{5jj-1} + q'_{52}x'_{5jj-1} + q_{53}x_{5jj-2} +$$

$$q'_{53}x'_{5jj-2} + q_{61}x_{6jj} + q_{62}x_{6jj-1} + q_{71}z_{7j}, \quad j=3, \dots, m$$

For period 2, the variables with subscript $j-2$ are deleted.

For period 1, the variables with subscript $j-1$ and $j-2$ are deleted.

An adjustment can also be made for terminating the model after m periods. If the cash budget covers a complete seasonal cycle, any difference between the financial conditions at the end of the last budget period and those at the beginning of the initial period must be taken into account in the making of the financial decision. In the formulation, it will be assumed that the conditions at the end of the last period under consideration need not necessarily be the same as they were at the beginning of the initial period. An "end condition implicit cost or credit" is assigned where the beginning and ending conditions are not the same. This consists of a one-time cost on any outstanding loans at the end of the period and a one-time credit for any marketable securities or term deposits available.

The rate for implicit end costs are defined as:

S_1 = for line of credit

S_2 = for pledging of accounts receivable

S_3 = for short-term bank credit

S_4 = for commercial paper

S_5 = for stretching of domestic accounts payable

S_5' = for stretching of foreign accounts payable

S_6 = for stretching of taxes payable

S_7 = for term loan

S_8 = for term deposits

S_9 = for marketable securities

The total end condition implicit cost (D_m) is:

$$\begin{aligned}
 D_m = & S_1 z_{1m} + S_2 z_{2m} + S_3 z_{3m} + S_4 (x_{41m} + x_{42m} + x_{42m-1} + x_{42m-2} + x_{43m} + \\
 & x_{43m-1} + x_{43m-2} + x_{43m-3} + x_{43m-4} + x_{43m-5}) + S_5 (x_{5mm} + x_{5mm-1} + \\
 & x_{5mm-2}) + S_5' (x_{5mm}' + x_{5mm-1}' + x_{5mm-2}') + S_6 (x_{6mm} + x_{6mm-1}) + \\
 & S_7 z_{7m} - S_8 (x_{81m} + x_{82m} + x_{82m-1} + x_{82m-2} + x_{83m} + x_{83m-1} + \\
 & x_{83m-2} + x_{83m-3} + x_{83m-4} + x_{83m-5}) - S_9 \sum_{i=1}^K x_{9im}
 \end{aligned}$$

Total relevant cost (TRC) is the sum of all explicit and implicit costs :

$$TRC = \sum_{j=1}^m E_j + \sum_{j=1}^m I_j + D_m$$

The objective is to minimize TRC subject to previously defined constraints.

3.3.3. Data of the Model

The data of the model will be analysed in two subsections. First, data of the cash budget model and then data of the optimization model will be investigated.

3.3.3.1. Data of the Cash Budget Model

Cash budget model provides all the necessary inputs to be utilised by the optimization model. For this reason, the data and the parameters of this model should be realistic and reliable.

The financial planning period of the model includes 12 months starting from Jan. 1986 ending in Dec. 1986.

Since the company imports 80% of the raw materials it utilizes in the production process and highly depends on exports earnings, foreign currency rate expectations are crucially important in financial planning. Imports are realized by Swiss Frank (SF) and German Mark (DM), and exports by U.S.\$. Monthly foreign currency rate expectations for the whole planning period are depicted in Table 3.3.3.1.1.

Foreign currency rate expectations are used in determining the TL. equivalent of the payment requirements of imported raw materials (for SF and DM) and earnings from exports (for U.S.\$).

TABLE 3.3.3.1.1. Monthly Foreign Currency Rate Expectations For
Swiss Frank (SF), German Mark (DM) and U.S.\$.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
\$	580.0	596.8	614.1	631.9	650.3	669.1	685.5	708.5	729.0	750.2	771.9	793.7
SF	332.5	336.6	346.3	356.4	366.7	377.4	388.3	399.6	411.1	423.1	435.3	447.6
DM	280.6	284.1	292.3	300.8	309.5	318.5	327.7	337.7	347.0	357.1	367.4	377.8

Expectation of domestic inflation rate is another environmental condition that is incorporated into the model. In the periods of high inflation, the cash requirements of the firms are deeply affected from rapidly increasing inflation rates. In the model, the impacts of inflation are observed in the prices of domestic raw materials and transportation costs. The inflation rate throughout the 12-month planning period is estimated to be 38% being uniformly distributed among months.

As a hedge against inflation, the domestic selling prices of products are predetermined to be increased by 10% once in every three months the first one being in the fourth month. This represents a price increase of 33% in the whole planning period which is under the expected inflation rate. This is explained by the firm's policy of targeting to increase its domestic market share.

In the domestic market, one product from each of the Product Lines I, III, IV and ten products from Product Line II are marketed. Four of the products marketed from Product Line II constituted 79% of total sales quantity of this category. Once demand quantity is

generated for Product Line II, the sales mix among ten different products are determined by the previous year's sales proportions of this category.

For the outstanding export orders, previously negotiated export prices will be valid. However, since the value of U.S.\$ has lost its value recently against SF and DM which are the currencies used in the imports of the firm, the management decided to increase export prices by 20% on the average for the new orders starting from the fourth month of the planning period.

In the export market, one product from Product line I and four different products from Product Line II which are not demanded by domestic customers are marketed. One of the products for the export market from Product Line II constitutes 70% of total export quantity of this category. The sales mix for Product Line II is determined by taking care of both previous year's exports and the experts' opinions for the current planning period.

For the exports market, the firm does not carry any finished goods inventory. The orders are taken as batches. For each order, production planning is performed separately and goods are dispatched immediately after the production process. For the domestic market, the firm's policy is to carry a finished goods inventory of 5% of the current monthly sales quantity.

The firm has to carry a certain amount of inventory for imported raw materials because of predetermined order quantities and lead times.

For domestic raw materials, the policy is to have a safety stock of 5% of the total requirement of monthly production.

The customs fees and freight expenses are paid in advance when the raw materials are imported. The payment to foreign vendors are scheduled to be made after three months the importation have been realized. Payments of some of the domestic raw materials are made in advance, however, generally payment terms are one month.

Payments of energy expenses are realized with one month lag. The energy expense amounting to TL. 20 Million is constant and the variable portion is equal to 1.5% of the total sales figure.

The payments of factory overhead expenses are made in advance being equal to 4% of the total sales.

Selling and administrative expenses are equal to 3% of the domestic sales and paid in advance. As a provision, other expenses are taken to be 5% of the domestic sales.

In the beginning of the planning period, the firm's outstanding debt to foreign vendors amounts to SF 921,199. SF 215,200 of this accounts payable are scheduled to be paid in the first month, SF 121,894 in the second month and the remaining SF 584,105 in the third month of the planning period.

The firm's outstanding debt to domestic vendors amounts to TL.

30 Million of which TL. 20 Million will be paid in the first month and the remaining TL. 10 Million in the second month.

The firm will pay TL. 118 Million in the third month and TL. 334.5 Million in the sixth month of its outstanding short-term bank credits and in the third month it will also pay the accrued interest of its bank loans amounting to TL. 44.5 Million.

The firm has made the agreement with the bank previously of taking prefinancing credit amounting to TL. 336.6 Million in the third month and repaying it including the interest as TL. 426.573 Million in the 9th month of the current planning period.

The payment schedules of outstanding debts are obligatory, they cannot be changed.

The firm collects 25% of its monthly domestic sales in advance, 43% with one month lag and 32% with two months lag. The exports are collected immediately after the sales are incurred.

The average tax rebate on exports is 15% and this amount is collected after two months the exports have been realized.

In the beginning of the planning period the firm's outstanding accounts receivable amounts to TL. 160 Million. 60% of this amount is expected to be collected in the first month and the remaining 40% in the second month.

The minimum amount of cash (MC) that the firm wishes to have on hand anytime is TL. 20 Million. The proportion of net operating cash flow or net drain (y) that the firm desires is 25%. The beginning of period cash on hand is TL. 160 Million.

3.3.3.2. Data of the Optimization Model

In the optimization model, the short-term financing alternatives available are : i) Pledging of Accounts Receivable, ii) Short-term Bank Credit, iii) Stretching of Accounts Payable, iv) Stretching of Taxes Payable, and v) Term Loan.

In the "Pledging of Accounts Receivable" alternative, the bank lends up to 80% of the face value of the beginning-of-period accounts receivable in all the periods. Thus,

$$a_{21j} = 0.8 \quad , \quad j = 1, \dots, 12$$

The distribution of the amount of accounts receivable at the beginning of period j is obtained from the cash budget model by successive planning processes with respect to the different demand forecasts generated.¹¹ For each risk level, namely being in the left or right of the average value of Period Cash Requirement by 1 or 2 standard deviations this amount varies and gives differing opportunities to the firm in using this credit.

The proportion of accounts receivable that are collected during any period is 60%. That is,

¹¹ See Appendix 9 for the distribution of the beginning of period accounts receivable revised with the bank's maximum lending proportion. $(0.80 \times A_j)$

$$a_{22j} = 0.6, \quad j = 1, \dots, 12$$

This means that during any period the firm collects 60% of its outstanding beginning of period accounts receivable balance from its customers.

The max. amount that can be borrowed during any period by pledging of accounts receivable is TL. 150 Million. Thus,

$$b_{22j} = 150, \quad j = 1, \dots, 12$$

In the "Short-Term Bank Credit" alternative, the maturity period is 6 months. However, the Management can repay all or part of its loan before the maturity period if it has the opportunity to do so. Therefore, once taken the firm is not obliged to carry the debt burden until it matures.

In order to fulfill the requirements, the max. amount of loan that can be raised in any period and the max. outstanding balance under short-term bank credit change with respect to the risk levels. If the Management is risk-averse, desiring to feel safe all the time in terms of satisfying its cash needs, it would require a greater limit on borrowing. Risk levels and the management's being risk-averse or risk seeker is shown in Exhibit 3.3.3.2.1

Risk-averse management would be pessimistic, unlike risk-seeker management's being optimistic, in terms of cash requirements resulting from lower expected sales quantities and would take the necessary precautions accordingly.

Exhibit 3.3.3.2.1. Risk Levels and the Meaning of the Management's Being Risk-Averse or Risk-Seeker.

Risk Level	I	II	III	IV	V
Period Cash Requirements	$\bar{X}-2s$	$\bar{X}-1s$	\bar{X}	$\bar{X}+1s$	$\bar{X}+2s$
Degree of Risk-Taking					
	Risk-Seeker		Risk-Averse		

The max. amount of required outstanding balance (b_{31j}) and the max. amount that can be borrowed (b_{33j}) under Short-term bank credit in each risk level is shown in Table 3.3.3.2.1

TABLE 3.3.3.2.1. The Max. Amount of Required Outstanding Balance (b_{31j}) And the Max. Amount That Can Be Borrowed (b_{33j}) Under Short-Term Bank Credit In Each Risk Level.

<u>Risk Level</u>	<u>$b_{31j}(j = 1, \dots, 12)$</u>	<u>$b_{33j}(j = 1, \dots, 12)$</u>
I	1,300	300
II	1,150	300
III	1,000	250
IV	650	200
V	500	200

In the "Stretching of Accounts Payable" alternative, the financial management may stretch up to 60% of the foreign accounts payable and 75% of the domestic accounts payable in the period in which they first

come due. Thus,

$$\begin{aligned} a_{51j} &= 0.75 & , & \quad j = 1, \dots, 12 \\ a'_{51j} &= 0.60 & , & \quad j = 1, \dots, 12 \end{aligned}$$

The distribution of the payment schedules of both foreign and domestic payables are obtained from the cash budget model and was shown previously in Appendix 7 and Appendix 8, respectively.¹²

The proportion of domestic accounts payable stretched in the first period which can be stretched in the second period is 50% and that of foreign accounts payable is 30%. That is,

$$\begin{aligned} a_{52j} &= 0.5 & , & \quad j = 1, \dots, 12 \\ a'_{52j} &= 0.3 & , & \quad j = 1, \dots, 12 \end{aligned}$$

The explicit cost of stretching accounts payable one month in the period in which they first come due for domestic vendors is 3% and for foreign vendors is 4%. That is,

$$\begin{aligned} r_{51} &= 0.03 \\ r'_{51} &= 0.04 \end{aligned}$$

Since the constraints of stretching accounts payable for the second month are of the type:

¹²See Appendix 10 and Appendix 11 for the distributions of the amount of domestic and foreign payables that can be stretched in the first period respectively after they are adjusted with the stretching proportion for the first month, namely 75% for domestic payables ($0.75 \times P_j$) and 60% for foreign payables ($0.60 \times P_j$).

$$x_{5jj-1} \leq a_{52j}(1+r_{51})x_{5j-1j-1} \quad , \quad j = 2, \dots, m$$

$$x'_{5jj-1} \leq a'_{52j}(1+r'_{51})x'_{5j-1j-1} \quad , \quad j = 2, \dots, m$$

the constraint sets used in the model are:

$$x_{5jj-1} \leq 0.515x_{5j-1j-1} \quad , \quad j = 2, \dots, 12$$

$$x'_{5jj-1} \leq 0.312x'_{5j-1j-1} \quad , \quad j = 2, \dots, 12$$

The financial management's policy is to stretch both domestic and foreign payables at most two months after they first come due, i.e. the amount stretched in the second month should be paid in the third month with the second period's cost of stretching.

In the "Stretching of Taxes Payable" alternative, the financial management may stretch up to 80% of the taxes payable in the period in which they first come due. That is,

$$a_{61j} = 0.8 \quad , \quad j = 1, \dots, 6$$

As a policy, the firm plans to benefit from this source, only in the first six months of the planning period.

The payment schedule of taxes payable is obtained from the cash budget model after the manpower planning process and is shown in Table 3.3.3.2.2.

TABLE 3.3.3.2.2. Payment Schedule of Taxes Payable Throughout the Planning Period.

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Scheduled Tax Payments	12,5	18	18	12,5	37,5	12,5	43	12,5	18	12,5	43	18

The proportion of taxes payable stretched in the first period which can be stretched in the second period is 100%. That is,

$$a_{62j} = 1.0 \quad , \quad j = 1, \dots, 6$$

The explicit cost of stretching taxes payable one month in the period in which they first come due is 11%. That is,

$$r_{61} = 0.11$$

Since the constraints of stretching taxes payable for the second month are of the type:

$$x_{6jj-1} \leq a_{62j}(1+r_{61})x_{6j-1j-1} \quad , \quad j = 2, \dots, m$$

the constraint set used in the model is:

$$x_{6jj-1} \leq 1.1x_{6j-1j-1} \quad , \quad j = 2, \dots, 6$$

The firm's policy in stretching of taxes payable is at most two months, i.e. the amount stretched in the second month should be paid in the third month with the second period's cost of stretching.

The financial management has decided to raise term loan, if necessary, only in the third month of the planning period. The principal will be repaid in eight equal installments once in every six months. Thus, the first installment will be in the 9th month if the term loan is taken equaling 12.5% of the amount borrowed. The accrued interest of the outstanding term will be paid together with the principal payments. The explicit cost of the term loan is 60% per year. No speed-up of payments is possible. The max. limit on the amount of borrowing by term loan depends on which risk level the management is. If it is risk-averse this limit is higher, if not it is lower because of the changing level of each requirements.

The firm has three different kinds of investment alternatives:

- i) One-month maturity term deposits, ii) Three-months maturity term deposits, iii) Government securities.

As a policy, the proportion of one-month maturity term deposits to total outstanding term deposits should be at least 75%. That is,

$$a_{811j} = 0.75 \quad , \quad j = 1, \dots, 12$$

The constraint set is:

$$\frac{x_{81j}}{x_{81j} + \sum_{k=j-2}^j x_{82k}} \geq 0.75, \quad j = 3, \dots, 12$$

The max. amount of investment that can be made in one-month maturity term deposits is TL. 150 Million, three-months maturity term deposits is TL. 80 Million and government securities is TL. 90 Million for each month in the planning period. That is,

$$b_{821j} = 150, \quad j = 1, \dots, 12$$

$$b_{822j} = 80, \quad j = 1, \dots, 12$$

$$b_{921j} = 90, \quad j = 1, \dots, 12$$

The accrued interest of pledged account receivables and short-term bank credits is paid once in every three months, namely in the third, sixth, ninth and twelveth months. Consequently, the interest payments are included to the fulfillment of requirements constraints accordingly.

The firm has two financial policies with respect to its short-term investment portfolio. Management desires the proportion of government security investments to total one-month maturity investments to be at most 30% for each month. That is,

$$\frac{x_{91j}}{x_{91j} + x_{81j}} \leq 0.30, \quad j = 1, \dots, 12$$

The firm sets the objective of lowering the proportion of one-month maturity investments to total investments to at most 60% gradually starting from 75% in the first period and 70% in the second period.

The resulting constraint set is:

$$\frac{x_{811} + x_{911}}{x_{811} + x_{911} + x_{821}} \geq 0.75, \text{ for period 1}$$

$$\frac{x_{812} + x_{912}}{x_{812} + x_{912} + x_{821} + x_{822}} \geq 0.70, \text{ for period 2}$$

$$\frac{x_{81j} + x_{91j}}{x_{81j} + x_{91j} + \sum_{k=j-2}^j x_{82k}} \geq 0.60, \quad j = 3, \dots, 12$$

For the objective function, the cost coefficients of all the alternative financial sources and the returns (negative costs) of the investment alternatives should be determined. The cost of pledging of accounts receivable alternative is 70% per year and that of short-term bank credit is 80% per year.

The explicit cost of stretching domestic accounts payable one month when they first come due is 3% per month and that of foreign accounts payable is 4% per month throughout the planning period. However in order to incorporate the cost of ill will to creditors an implicit cost of 5% is added to the explicit cost of both of the alternatives. Thus,

The total cost of stretching domestic accounts payable one month when they first come due = 0.08

The total cost of stretching foreign accounts payable one month when they first come due = 0.09

The cost of stretching accounts payable for the second month is calculated as follows:

For domestic accounts payable: $(1.08)^2 - 1.08 = 0.0864$

For foreign accounts payable : $(1.09)^2 - 1.09 = 0.0981$

The cost of stretching taxes payable when they first come due is 11% per month. The cost of stretching for the second month is calculated as follows :

$$(1.11)(1.07) - 1.11 = 0.0777$$

since the second month's cost of stretching is 7%.

The explicit cost of borrowing by term loan is 60% per year and the implicit cost is taken to be 60% per year. Therefore, the total cost is 120% per year.

In the fulfillment of requirements constraints only the explicit costs are relevant since they represent the real monetary values.

The monthly return is 3.33% (40% per year) for government securities and 2.625% (31.5% per year) for one-month maturity term deposits. The three-monthly return for three-month maturity period term deposits is

10.125% (40.5% per year).

In the optimization model, there are 167 variables and 245 constraints. All the variables are continuous and greater than or equal to zero. Among the constraints, 25 are equality, 160 are less than or equal to and 60 are greater than or equal to constraints.¹³

3.3.4. Findings and Interpretation of the Results

In Section 1, the optimal solution of the short-term financing/investment decision problem obtained for each risk level will be analysed. In Section 2, the opportunity costs (shadow prices) of the optimal solutions will be interpreted. In Section 3, sensitivity analyses with respect to cost coefficients and right-hand side resource constants will be conducted.

3.3.4.1. Optimal Solution of the Short-Term Financing/Investment Decision Problem At Each Risk Level

The optimal solution of the short-term financing/investment decision problem at each of the five risk levels is obtained with the execution of MPOS (Multi-Purpose Optimization System) Version 4.1¹⁴ package program which is available to the CDC-Cyber users at the Boğaziçi University by the relevant data.

As previously described, Risk Level I represents the most pessimistic

¹³ See Appendix 12 for the complete linear programming formulation of the short-term financing/investment decision problem to be executed by MPOS

¹⁴ See "A Guide to MPOS Version 4" a publication of Northwestern University Vogelback Computing Center which is available at Boğaziçi University Computer Center

view of the Management in terms of demand forecasts resulting in the largest cash requirements ($\bar{x}-2s$) for each month in the planning period. As demand forecasts become more optimistic period cash requirements decrease reaching to a minimum at Risk Level V ($\bar{x}+2s$).

In the model, there are three bank sources for financing the cash requirements; i) pledging of accounts receivable, ii) short-term bank credit, and iii) term loan. The amount of available resource that can be used by the pledging of accounts receivable alternative depends on the outstanding balance of beginning-of-period accounts receivable revised with the bank's maximum lending proportion (80%), i.e. the firm's outstanding borrowing from this source can be at most 80% times beginning-of-period accounts receivable. The outstanding balance of beginning of period accounts receivable increase as demand forecasts become more optimistic¹⁵ since sales level will consequently rise collection proportions remaining constant. For this reason, the Management can benefit from this source more as their risk level gets closer to Risk Level V. This outcome can be easily observed in Table 3.3.4.1.1.

In some periods, even though the firm's beginning-of-period accounts receivable level is sufficient Management cannot borrow more because of the bank's max. limit of lending which is equivalent to TL 150 million.

This source is relatively restricted when compared to short-term bank credits since it depends on the firm's account receivables.

¹⁵See Appendix-9.

TABLE 3.3.4.1.1 The Amount of Pledged Accounts Receivables At Each Risk Level in the Optimal Solution

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.	TOTAL
Risk Credit Raised				118.46	72.347	53.845	85.774	43.478	97.009	91.405	88.193	94.356	744.867
Level I Voluntary Repayment					71.076	71.839	61.043	75.881	56.439	80.781	87.155	87.778	591.992
Level I Mandatory Repayment				118.46	119.731	101.737	126.468	94.065	134.635	145.259	146.297	152.875	
Outstanding Balance													
Risk Credit Raised			125.821	86.93	90.078	91.069	88.899	102.212	150	104.06	99.458	111.241	1,049.768
Level II Voluntary Repayment									53.692				53.692
Level II Mandatory Repayment				75.493	82.355	86.989	89.114	96.973	96.973	96.574	101.065	100.101	818.101
Outstanding Balance			125.821	137.258	144.981	149.061	148.523	161.621	160.956	168.442	166.835	177.975	
Risk Credit Raised			150	96.056	107.81	102.806	102.216	115.417	150	116.714	150	128.126	1,219.145
Level III Voluntary Repayment									36.182		39.277		75.459
Level III Mandatory Repayment				90	93.634	102.139	102.539	102.346	110.188	112.366	114.975	112.424	940.611
Outstanding Balance			150	156.056	170.232	170.899	170.576	183.647	187.277	191.625	187.373	203.075	
Risk Credit Raised			150	114.854	150	114.543	150	128.621	150	129.369	121.988	145.011	1,354.386
Level IV Voluntary Repayment					24.459		34.464		18.671				77.594
Level IV Mandatory Repayment				90	104.912	117.29	115.642	115.578	123.404	128.159	128.885	124.747	1,048.617
Outstanding Balance			150	174.854	195.483	192.736	192.63	205.673	213.598	214.808	207.911	228.175	
Risk Credit Raised			150	133.653	143.273	126.28	128.853	150	148.839	142.022	133.253	150	1,406.173
Level V Voluntary Repayment								18.173					8.173
Level V Mandatory Repayment				90	116.192	132.44	128.744	128.81	136.62	143.951	142.794	137.069	1,156.62
Outstanding Balance			150	193.653	220.734	214.574	214.683	227.7	239.919	237.99	228.449	241.38	

However, short-term bank credits alternative can be utilised by the Management in a more relaxed way. The max. amount of required outstanding balance under short-term bank credit alternative falls down to TL. 500 million at Risk Level V starting from TL. 1,300 million at Risk Level I.¹⁶ This implies that the firm's policy is to use the pledged accounts receivable source as much as possible as the demand gets higher. The max. limit is attained at the end of the planning period in the first four risk levels. At Risk Level V, the firm never reaches the max. required outstanding balance of TL. 500 million attaining at most to TL. 491.398 million in the 8th month. In the first three risk levels the firm reaches these required limits by continuously borrowing at each month. Thus, the Management should take the necessary precautions in order to raise these short-term bank credits whose amounts are shown in Table 3.3.4.1.2, depending on their risk level and the corresponding monthly borrowing limit.¹⁷

Since the cost of pledging of accounts receivable (70% per year) is cheaper than that of short-term bank credits (80% per year) the firm will borrow from the former alternative until the upper limit is reached and then the latter will be utilised.

The third bank source is the term loan. The Management decides to raise a term loan, if necessary, only in the third month of the planning period because of excessive foreign payments, bank loans and interest

¹⁶ See Table 3.4.

¹⁷ See Table 3.4.

TABLE 3.3.4.1.2 The Amount of Short-Term Bank Credits At Each Level In the Optimal Solution

		Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.	TOTAL
Risk	Credit Raised				60.95	194.32	194.32	300	47.085	300	168.839		228.807	1,494.321
Level I	Credit Repaid						78.71				97.851	17.76		194.321
	Outstanding Balance				60.95	255.27	370.88	670.88	719.965	1,017.965	1,088.953	1,071.193	1,300	
Risk	Credit Raised			78.953	174.274	178.283	113.402	300		300	97.739		172.472	1,415.123
Level II	Credit Repaid								88.53		85.744	90.849		265.123
	Outstanding Balance			78.953	253.227	431.51	544.912	844.912	756.382	1,056.382	1,068.377	977.527	1,150	
Risk	Credit Raised			250	168.376	136.021	250	250		233.665		23.136	48.999	1,360.197
Level III	Credit Repaid			74.921			138.446		74.694	62.832	9.304			360.197
	Outstanding Balance			175.079	343.455	479.476	591.03	841.03	766.336	937.169	927.865	951.001	1,000	
Risk	Credit Raised			124.539	200	106.872	200	200		112.103	200		31.61	1,175.124
Level IV	Credit Repaid				67.166		123.498		103.088		230.602	0.77		525.124
	Outstanding Balance			124.539	257.373	364.245	440.747	640.747	537.659	649.762	619.16	618.39	650	
Risk	Credit Raised			74.001	96.292	103.741	16.364	200		56.797			0.264	548.459
Level V	Credit Repaid								182.979		51.9	24.678		259.557
	Outstanding Balance			74.001	161.293	275.034	291.398	491.398	308.419	365.216	313.316	288.638	288.902	

payments scheduled to that period. Raising term loan is required only in the first two risk levels. In the optimal solution. Term loan required to be raised amounts to TL. 452.629 million in Risk Level I and TL. 163.589 million in Risk Level II. In the other risk levels, financial sources other than term loan which is the most expensive one (120% per year) are sufficient to meet requirements. Therefore, if the Management is in one of these risk levels they should immediately start negotiating term loan agreement with the bank.

The non-bank financial sources to the firm are stretching of accounts payable. For domestic and foreign accounts payable, the amounts that can be stretched are obtained from the cash budget as the payment schedule of purchases. Taxes payable are also obtained from the cash budget as a result of manpower planning. Domestic and foreign accounts payable increase as the demand forecasts become more optimistic, i.e. as risk level gets closer to V, since increased demand means increased production and purchasing. The amounts of accounts payable that can be stretched after being adjusted with the stretching proportion for the first month are shown in Appendix-10 and Appendix-11, respectively. The firm aims at using these sources after all of the bank sources have been utilised in order not to suffer from the ill will to creditors. For this reason, the Management assigns 5% implicit cost per month to these sources.

In the first two risk levels these sources are used to a great extend in the last two months of the planning period since bank borrowing limits have been fully utilised at these periods as seen in Table 3.3.4.1.3.

TABLE 3.3.4.1.3 The Amount of Stretched Accounts Payable At Each Risk Level in the Optimal Solution

[illegible]

At the other risk levels, stretching accounts payable alternative is used generally for domestic payables in months 3,4,5,7 and 12 which represent the periods of relatively more cash requirements.

Stretching of taxes payable alternative is not used in any of the risk levels since it is the most expensive financing alternative and other sources are sufficient to meet the requirements.

The payments of accrued interest expenses of pledging account receivables and short-term bank credits are made once in every three months. These payments and the payments of cost of stretching account payable at each risk level in the optimal solution are shown in Table 3.3.4.1.4.

There are three short-term investment alternatives in the model:

i) one-month maturity term deposits, ii) three-months maturity term deposits, and iii) one-month maturity government securities.

Short-term investments are realized in the first two months since the firm has excess cash on hand in the beginning of the planning period because of a short-term bank credit just raised. Investments are made in one-month maturity alternatives since the firm does not have excess cash on hand in three consecutive periods and cost of financing is much higher than the return of investment. The amount of short-term investments and the cash receipts of the corresponding investments' interest income at each risk level is depicted in Table 3.3.4.1.5.

TABLE 3.3.4.1.4 The Interest Payments (I) of Pledged Account Receivables and Short-Term Bank Credits and the Payments of Cost of Stretching Account Payables At Each Risk Level In the Optimal Solution

		Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.	TOTAL
Risk Level I	Int.Exp.Stret.Dom.Pay.(1 st Per.)			0.306		0.67	0.749		1.227		1.098		1.047	5.097
	Int.Exp.Stret.For.Pay.(1 st Per.)												2.153	2,153
	Int.Exp.Stret.Dom.Pay.(2 nd Per.)						0.355							0.355
	Int.Exp.Stret.For.Pay.(2 nd Per.)													
	Interest Payments of Bank Loans						28.738			136.701			225.661	391.1
Risk Level II	Int.Exp.Stret.Dom.Pay.(1 st Per.)				0.741	0.881	0.951		1.598	0.638			1.6	7.676
	Int.Exp.Stret.For.Pay.(1 st Per.)												3.382	3.382
	Int.Exp.Stret.Dom.Pay.(2 nd Per.)					0.393	0.467							0.86
	Int.Exp.Stret.For.Pay.(2 nd Per.)													
	Interest Payments of Bank Loans						64.037			171.483			222.852	458.372
Risk Level III	Int.Exp.Stret.Dom.Pay.(1 st Per.)				0.98	1.093	1.152		1.014					4.239
	Int.Exp.Stret.For.Pay.(1 st Per.)													
	Int.Exp.Stret.Dom.Pay.(2 nd Per.)						0.58							0.58
	Int.Exp.Strat.For.Pay.(2 nd Per.)													
	Interest Payments of Bank Loans						81.857			179.24			206.007	467.104
Risk Level IV	Int.Exp.Stret.Dom.Pay.(1 st Per.)				1.22	1.304	1.353		1.39					5.267
	Int.Exp.Stret.For.Pay.(1 st Per.)													
	Int.Exp.Stret.Dom.Pay.(2 nd Per.)						0.672							0.672
	Int.Exp.Stret.For.Pay.(2 nd Per.)													
	Interest Payments of Bank Loans						66.472		144.414				146.31	357.196
Risk Level V	Int.Exp.Stret.Dom.Pay.(1 st Per.)				1.459	1.516	1.554		0.266					4.795
	Int.Exp.Stret.For.Pay.(1 st Per.)													
	Int.Exp.Stret.Dom.Pay.(2 nd Per.)													
	Int.Exp.Stret.For.Pay.(2 nd Per.)													
	Interest Payments of Bank Loans						52.823		113.01				87.184	253,017

Investment levels increase as the demand forecast becomes more optimistic since cash requirements decrease and more are met by internally generated funds.

The complete tables of sources and uses of cash to fulfill the requirements at each risk level at the optimal solution are presented in Appendix-13.

TABLE 3.3.4.1.6. Objective Function Value (z^*) and the Rate of Change Between Risk levels.

<u>Risk Level</u>	<u>I</u>	<u>II</u>	<u>III</u>	<u>IV</u>	<u>V</u>
z^*	899.306	720.95	558.447	421.749	285.601
Rate of Change(%)		-19.83	-22.5	-24.48	-32.28

The objective function value ranges between TL. 899.306 million and TL. 285.601 million among Risk Level I through V as observed in Table 3.3.4.1.6. The objective function value of Risk Level I is more than three times than that of Risk Level V. This shows that the Management's attitude towards risk under the impact of environmental conditions extensively affects the firm's performance. This big dispersion should make the Management realize that they must force the environmental conditions as much as possible in order to attain optimistic demands.

The analysis of the results reveals that the Management's main

difficulty is to decide whether they are in Risk Level I or II because this distinction makes the largest change in the policies to be implemented as justified by raising term loan. If they agree that they are in Risk Level I or II than they observe that short-term sources are not sufficient for the firm to meet the requirements. They must refer to long-term sources which will take away some portion of their flexibility in future years. However, if the Management agrees that they are in Risk Level III, IV or V than short-term financing sources will be sufficient for the current planning period.

3.3.4.2 Opportunity Costs of the Resources At Each Risk Level In the Optimal Solution

The solution of a linear programming problem does not only produce optimal activity levels for the corresponding variables but also provides other valuable information for further analysis of the model. The linear programming problems typically can be interpreted as allocating resources to activities. Because there may be some latitude in the amounts that will be made available, information on the economic contribution of the resources would be extremely useful. This information is provided in the form of shadow prices (opportunity costs) for the respective resources.¹⁸ The shadow price for a resource measures the marginal value of this resource, that is, the rate at which objective function value could be increased by slightly increasing the amount of this resource being made available. The increase in the amount of resource must be sufficiently

¹⁸ See "Introduction to Operations Research" by Frederick S. Hillier and Gerald J. Lieberman, pp. 40-41, 1980

small that the current set of basic variables remains optimal, since this rate (marginal value) changes if the set of basic variables changes. If the shadow price of a resource is zero then this means that this resource is oversupplied not being totally consumed in the model, consequently, having no economic price.

In the pledged accounts receivable alternative, credit source is restricted by the bank's max. lending proportion and the firm's outstanding beginning-of-period accounts receivable. When this source is totally utilised, namely, maximum credit limit has been reached, the corresponding opportunity costs (marginal values) become negative as seen in Table 3.3.4.2.1. implying the objective function value will improve by that rate for a unit increase in the level of pledged account receivables. In these periods, since the outstanding account receivables are predetermined by the sales forecast the Management should try to increase the bank's max. lending proportion if the cost of this effort is less than the corresponding marginal values in order to improve the objective function value.

In Risk Level I and II, opportunity costs are present starting from the first month that the pledged accounts receivable source is used, namely, month 4 and 3 respectively, indicating that this source is fully utilised in the first period it is referred. However, in Risk Level III, IV and V opportunity costs are present starting from the 4th month, one month after this source is utilised for the first time. The reason for this is that in month 3 the bank's maximum lending limit of TL. 150 million is effective i.e. since the firm's adjusted beginning-of-period

TABLE 3.3.4.2.1. Opportunity Costs of the Resources Related to the Pledged Accounts Receivable
Alternative In Each Risk Level At the Optimal Solution.

RISK LEVEL	CONSTRAINT	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
	$\sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k}) \leq a_{21j} A_j$		-.0076	-.0076		-.0071		-.0223	-.0093	-.0091	-.1226
I	$x_{2j} \leq b_{21j}$										
	$v_{2j} = a_{22j} \sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k})$		-.016	-.016	-.016	-.016	-.016				
	$\sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k}) \leq a_{21j} A_j$	-.008	-.0079	-.008	-.0034	-.0044	-.0105	-.0099	-.0085	-.0084	-.0855
II	$x_{2j} \leq b_{21j}$										
	$v_{2j} = a_{22j} \sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k})$	-.0134	-.0134	-.0134	-.0134	-.0134	-.0134				
	$\sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k}) \leq a_{21j} A_j$		-.0082	-.0082	-.0012	-.0077	-.0102	-.0072	-.0072	-.0070	-.0222
III	$x_{2j} \leq b_{21j}$	-.0083									
	$v_{2j} = a_{22j} \sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k})$.0393	-.0099	-.0099	-.0099	-.0099	-.0099				
	$\sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k}) \leq a_{21j} A_j$		-.0082	-.0082	-.0010	-.0080	-.01	-.0071	-.0071	-.0069	-.0152
IV	$x_{2j} \leq b_{21j}$	-.0083									
	$v_{2j} = a_{22j} \sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k})$.0389	-.0095	-.0095	-.0095	-.0095	-.0095				
	$\sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k}) \leq a_{21j} A_j$		-.0082	-.0082	-.0007	-.0085	-.0098	-.0069	-.0069	-.0090	
V	$x_{2j} \leq b_{21j}$	-.0083									-.0059
	$a_{2j} = a_{22j} \sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k})$.0384	-.009	-.009	-.009	-.009	-.009				-.0059

account receivables are greater than the firm's maximum borrowing limit borrowing constraints for the period become binding. This is evidenced, by the opportunity costs of pledged accounts receivable credit raised (0.0083) in month 3 for Risk Level III, IV and V showing if the maximum lending limit is increased by one unit the objective function value will improve by the amount 0.0083. In Risk Level I, the highest marginal values in maximum outstanding credit constraints are in months 12 and 9, respectively, and in Risk Level II they are in months 12, 8 and 9, respectively. Thus, the Management should try hard especially in these periods in order to increase the bank's max. lending proportion so as to improve the objective function value. In Table 3.3.4.2.1 it is also observed that, in general, the opportunity costs of mandatory repayment constraints are greater than the maximum outstanding borrowing constraints for each period in each risk level. This can be explained with the increased flexibility obtained by regulating this amount with respect to the requirements. Thus, increasing the proportion of account receivables that are collected during any period (currently 60%) help improve the objective function value more than increasing the maximum outstanding credit limit since the amounts of mandatory repayments are directly proportional to this rate in the model.

Short-term bank credit source can be used by the firm more freely and easily than the pledged account receivable alternative. Credit limits are larger indicating that this source is much easier to find, however, it has a higher cost. For this reason, opportunity costs in maximum borrowing limit are present only in month 7, 9 and 12 for various risk

levels as shown in Table 3.3.4.2.2 indicating that the upper limits are reached. Therefore, depending on which risk level the Management is, it should try to raise these limits in the corresponding periods.

TABLE 3.3.4.2.2 Opportunity Costs of the Resources Related to the Short-Term Bank Credit Alternative In Each Risk Level At the Optimal Solution.

RISK LEVEL	CONSTRAINT	July	Sept.	Dec.
I	$\sum_{k=1}^j (x_{3k} - y_{3k}) \leq b_{31j}$ $x_{3j} \leq b_{33j}$	-.0051	-.0186	-.1667
II	$\sum_{k=1}^j (x_{3k} - y_{3k}) \leq b_{31j}$ $x_{3j} \leq b_{33j}$	-.0007	-.002	-.1137
III	$\sum_{k=1}^j (x_{3k} - y_{3k}) \leq b_{31j}$ $x_{3j} \leq b_{33j}$	-.0046		-.0233
IV	$\sum_{k=1}^j (x_{3k} - y_{3k}) \leq b_{31j}$ $x_{3j} \leq b_{33j}$	-.005		-.0133
V	$\sum_{k=1}^j (x_{3k} - y_{3k}) \leq b_{31j}$ $x_{3j} \leq b_{33j}$	-.0056		

The constraints to ensure that voluntary and mandatory repayments do not exceed the amount borrowed have positive opportunity costs in the first three periods as shown in Table 3.3.4.2.3 for pledged accounts receivable and short-term bank credit sources in Risk Level I and II indicating that the objective function value will get worse if there is any outstanding bank credit in the first two periods. This justifies that in RiskLevel I raising term loan in the third period for the whole cash requirement by not using any other bank credit in spite of its relatively higher cost is more profitable to the firm and in Risk Level II since only a portion of the cash requirement is met by the term loan utilising other bank sources have no effect on the objective function value as evidenced by zero opportunity costs.

Since the cost of short-term bank credit is higher than that of pledged accounts receivable utilising short-term bank credit source makes the objective function value get worse more when compared to pledged accounts receivable alternative.

Stretching of accounts payable alternatives are limited sources when compared to bank credits since they depend on purchases made which are derived from sales forecast, their payment schedule and the firm's policy of stretching. Stretching of domestic and foreign accounts payable sources are more important in Risk Level I and II because of their higher cash requirements and insufficient bank sources with respect to the other risk levels. The highest opportunity costs are in months 4, 5, 11 and 12 as shown in Table 3.3.4.2.4. This means that if the Management relaxes

its policy of stretching accounts payable in these periods, namely, if it raises the proportion of accounts payable that can be stretched when they become due, the objective function value will improve.

TABLE 3.3.4.2.3 Opportunity Costs of the Resources to Ensure That Voluntary and Mandatory Repayments Do Not Exceed the Amount Borrowed For Pledged Accounts Receivable and Short-Term Bank Credit Sources In Risk Level I and II At the Optimal Solution.

RISK LEVEL	CONSTRAINT	JAN	FEB.	MAR.
I	$\sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k}) \geq 0$	0 ⁽¹⁾	0 ⁽¹⁾	.0354
	$\sum_{k=1}^j (x_{3k} - y_{3k}) \geq 0$.0763	.0127	.0165
II	$\sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k}) \geq 0$.0037	.0418	0
	$\sum_{k=1}^j (x_{3k} - y_{3k}) \geq 0$	0 ⁽¹⁾	.0781	0
(1) Since the slack variable is basic (equaling to zero) at the optimal solution, the opportunity cost is zero.				

In the model, the proportion of government security investments to total one-month maturity period investments in each period is restricted to be at most 30%. However, since the return of government securities is greater than that of one-month maturity investments, this

Table 3.3.4.2.4 Opportunity Costs of the Resources of Stretching Domestic and Foreign Accounts Payable In Risk Levels I and II At the Optimal Solution

[illegible]

policy of the firm affects the objective function value negatively in the periods when the firm has excess cash on hand for investment as observed from the opportunity cost of the related constraint set in Table 3.3.4.2.5. Increasing this proportion will help improve the objective function value. This change in the firm's policy will be more useful in Risk Levels I and II when compared to the others since term loans are raised in these levels and having more return from investments will cause raising less amount of term loans.

TABLE 3.3.4.2.5 Opportunity Costs of Resources Related To the Proportion of Government Security Investments to Total One- Month Maturity Period Investments To Be At Most 30% At Each Risk Level In the Optimal Solution.

RISK LEVEL	JAN.	FEB.	MAR.
I	-.0154		-.0142
II	-.0148	-.0144	
III	-.0137	-.0133	
IV	-.0135	-.0132	
V	-.0134	-.0130	

3.3.4.3 Sensitivity Analysis of the Optimal Linear Programming Solution At Each Risk Level

Conducting sensitivity analysis after the optimal linear programming solution is obtained enables analyst to have valuable information about the range for cost coefficients and right-hand side constants within which the current optimal basic solution will remain the same and the impact of the changes in the range on the objective function value.

In this section, first the optimality range for cost coefficients (only basic variables) and then the optimality range for right-hand-side constants will be analysed.

The theoretical basis of sensitivity analysis is presented in Appendix-14.

3.3.4.3.1 Optimality Range For Cost Coefficients (Only Basic Variables)

Optimality range for cost coefficients determines the minimum and the maximum level within which the optimal solution will remain the same, and the lowest and the highest values of the objective function value in this range. In order the optimal solution to remain the same, the cost coefficients of the basic variables should be in such a range that the reduced costs of all the nonbasic variables are greater than or equal to zero so that they do not enter into the basis.

In the pledging of accounts receivable alternative, the minimum

level of the cost coefficients representing the amount of credit raised are, in general, equal to the original cost coefficients¹⁹ indicating that the optimal solution will change when the cost of this credit decreases in any period. Thus, the financial model in any of the risk levels are very sensitive to the decrease in the cost of this source. The upper limits of the range for each basic variable in all risk levels are greater than the original cost coefficients. This means that not every increase in the cost of pledged accounts receivable source requires a change in the optimal basic solution. There is always a margin indicating that the current optimal solution is still valid for a certain amount of cost increase. The widest margin occurs in the 12th period at each risk level representing 300.4%, 209.5%, 54.4%, 37.3% and 14.5% increase in the original cost coefficients from Risk Level I to V, respectively. This means that the firm should refer to this source even though the costs get unexpectedly higher in the 12th period because of its cash requirements and no access to the other sources. However, this range narrows continuously to the fifth risk level because of decreasing requirements and increasing alternative sources.

The cost coefficients of credit repayment variables are negative. So, an increase in these coefficients cause a decrease in the objective function value since the objective is cost minimization. The decrease in cost coefficients of mandatory repayment variables are not, in general, sensitive as observed in Appendix-15 indicating that the optimal solution will not change as the cost coefficients decrease since these are mandatory repayments depending on the outstanding balance of the previous period.

¹⁹ See Appendix-15

However, the model is very sensitive to the increases of these coefficients requiring changes in the optimal solution in case this source becomes expensive.

As in the pledging of accounts receivable alternative, the minimum level of the cost coefficients representing the amount of short-term bank credit raised are, in general, very sensitive to the decreases in the cost of this source. This is evidenced by the level of minimum cost coefficients being equal to the original cost coefficients.²⁰ However, in the periods when the maximum credit raising limit is reached - e.g. 7th period in all risk levels and 9th period in Risk Levels I and II- decreases in the cost of the source do not require any change in the current optimal solution, in turn, increases within the range makes the objective function value get worse more than the increases in the other periods since the model has to utilise this credit completely to meet the requirements. In period 9 and 12, even though the cost of short-term bank credit gets much higher when compared to the current cost of 80% per year the optimal solution does not change.

The tolerable credit costs get as high as 96% in period 12 at Risk Levels I and V as shown in Table 3.3.4.3.1.1. Thus, increased cost of short-term bank credit in these periods only changes the objective function value without causing any change in the financing policy of the firm.

²⁰ See Appendix-16

TABLE 3.3.4.3.1.1. Maximum Cost of Short-Term Bank Credit in Periods 9 and 12 That Does Not Change The Current Optimal Solution.

PER.	RISK LEVEL				
	I	II	III	IV	V
9	86%	81%	80%	84%	84%
12	96%	92%	92%	82%	96%

The third bank credit source is the term loan being utilised only at Risk Levels I and II. The yearly cost of the term loan is taken to be 120% in the model. The tolerable decrease in this cost without changing the current optimal solution is 117% both for Risk Levels I and II.²¹ However, the tolerable increase is up to 130% for Risk Level I and 123% for Risk Level II. The difference in the maximum cost coefficient limits reveals that the firm is more dependent on the term loan in the current financing composition at Risk Level I unlike Risk Level II where an increase in the cost over 123% per year causes a change in the financing policy.

The decreases in the costs of stretching accounts payable variables are not, in general, effective in the model meaning that the current optimal solution does not change for each risk level how much they decrease.²¹ The reason for this is that whenever this source is utilised, it is used up to the upper limit. So, a decrease

²¹ See Appendix-18

in the cost can not increase its utilisation by causing a change in the optimal basis. The maximum cost coefficient limits get closer to original cost coefficients going from Risk Level I to V. For example, the maximum cost coefficient limit of stretching domestic accounts payable for the first period variable in period 4 at Risk Level I is 9.6% per month whereas that of Risk Level V is 8.3% original cost coefficient being 8%. This is because requirements decrease and consequently, financing alternatives increase as we go to Risk Level V.

The highest maximum cost coefficient limits are in periods 4 and 5 at all risk levels and periods 11 and 12 at Risk Levels I and II for stretching domestic account payables for the first period variables. These are the periods when this source mostly required. In period 12, the optimal solution does not change even when the cost of this source gets as high as 23% per month at Risk Level I and 18% per month at Risk Level II justifying the necessity of this source. The range between maximum and original cost coefficients gets narrower as more expensive stretching of accounts payable sources are used indicating that the model is more sensitive to the increases in the costs of expensive sources.

The cost coefficients of investment variables have wide optimality ranges indicating that they are not sensitive to the changes in the rates of returns of the investments.²² For one-month maturity term deposit investments, the optimal solution does not change even when the return becomes negative and/or increases up to 48% from the

²² See Appendix-18

current level of 31.5% at all risk levels. For government security investments, the maximum tolerable decrease in the rate of return without causing a basis change is 24% at all risk levels. Since such big fluctuations in the rates of returns of investment alternatives are not expected in the first three months of the planning period it can be deduced that changes in the returns of the investments will not effect the current optimal basis.

3.3.4.3.2. Optimality Range For Right-Hand-Side Constants (Non-Slack Resources Only)

Optimality range for the right-hand side constants determines the minimum and the maximum level for the resources within which the optimal basis will not change, and the lowest and the highest values of the objective function value in this range. Increasing or decreasing the resource constant beyond the optimality range requires a basis change indicating that the current optimal solution is not valid anymore. On the other hand, fluctuations within the range changes the activity level of the basic variables without causing a basis change.

Changes in the objective function value within the optimality range are directly related to the opportunity costs of the corresponding resources. For example, in the maximum outstanding credit constraints of the pledging of accounts receivable source, the optimality range for the resource is between 36.012 and 150, original resource being 118.46 and the objective function value fluctuates between 899.93 and 899.07 in period 4.²³ The opportunity cost of this resource is

²³ See Appendix-19

(-0.0076) .²⁴ This means that if the resource is increased by one unit the objective function value will improve by 0.0076 unit within the optimality range and vice versa. Since the maximum level is 150, the additional amount of resource that can be used without changing the current optimal solution is $31.54(150-118.46)$. The corresponding improvement in the objective function value is $0.24(0.0076 \times 31.54)$. Since the objective is to minimize improvement means decrease in the objective function value. So, the revised objective function value is $899.07(899.31-0.24)$. Same logic holds true for the decrease in the resource level within the optimality range, however, this time causing an increase in the objective function value. In the above example, the minimum level that the resource can fall without changing the optimal basis is 36.012. The amount of decrease in the resource level is $82.448(118.46-36.012)$ and the decrease in the objective function value is $0.62(0.0076 \times 82.448)$. This makes the revised objective function value to be $889.93(899.31 + 0.62)$.

In the maximum outstanding credit constraints of the pledging of accounts receivable alternative, the optimality ranges get narrow going from Risk Level I to V.²⁵ This shows that the current optimal solutions become more sensitive to the changes in the beginning-of-period accounts receivable levels as demand forecasts are more optimistic giving more flexibility in financing the requirements to the Management. When the demand forecasts are pessimistic the Management does not have

²⁴ See Table 3.3.4.2.1

²⁵ See Appendix-19

to watch out the changes in the accounts receivable levels carefully since they do not imply changes at the current financial plan up to the limits in the range. The greatest dispersion in the objective function value caused by the fluctuations within the range among the periods in the same risk level occurs at the 12th period. For instance, in Risk Level I the original resource level is 152.88 and the objective function value is 899.31, but if the resource level could be increased to 208.52 the objective function value becomes 892.49. Thus, in the 12th period, especially in the first three risk levels Management should try hard to increase the bank's maximum lending proportion which is currently 80% in order to obtain the greatest improvement in the objective function value without changing the optimal basis.

In the maximum borrowing constraints of the pledging of accounts receivable source, there are no maximum resource limits in some periods in each risk level other than the first one. The reason for this is that in these periods the maximum borrowing limit of 150 is reached and because of excessive cash requirements the basis will not change how much this limit increases by improving the objective function value. So, in these periods the Management should try to increase the bank's maximum lending limit. This interpretation does not hold in Risk Level I since the amount of credit raised can not reach to maximum borrowing limit because of insufficient beginning-of-period accounts receivable levels.

In periods when there are voluntary repayments together with

the credits raised the minimum resource levels become the net amount of credits raised after voluntary repayments are deducted without changing the objective function value- e.g. periods 5,7 and 9 in Risk Level IV.

The minimum resource levels of the mandatory repayment constraints of the pledging of accounts receivable alternative are negative²⁶ in all risk levels, original resource being zero, indicating that increasing repayments within these limits will make the objective function value get worse. On the contrary, positive maximum resource levels imply that decreasing repayments will improve the objective function value.

The optimality ranges for maximum outstanding short-term bank credits are present in the 12th period in all risk levels other than the fifth one since the slack variables of the resources are zero only in this period.²⁶ In period 9 at Risk Level III, there is an optimality range for the maximum outstanding credit resource but fluctuations within this range do not have any effect on the objective function value. Thus, the Management does not have to worry about the changes of this resource within the optimality range, moreover, it gives them discretion in giving up decisions about this resource implying there are multiple optimal solutions.

In the maximum borrowing constraints of short-term bank credit alternative at periods when credits raised reach to the bank's maximum lending limit-e.g. periods 3 and 6 in Risk Level III and periods 4,6

²⁶See Appendix-20

and 10 in Risk Level IV- there are no upper limits in the optimality range indicating that indefinite increase in the credit limits will not imply any basis change improving the objective function value.

In stretching of accounts payable alternatives the largest dispersion in the objective function values within the optimality range occur in the 11th and 12th periods in the first three levels.²⁷ Therefore, increasing the amount of account payables that can be stretched in these periods by loosening the firm's stretching policy will make the largest improvement in the objective function value than increasing any other financing resource without changing the optimal basis. For example, in stretching of domestic accounts payable constraint in period 12 at Risk Level I, if the original resource equal 35.05 can be increased to 98.753 the objective function value falls down to 889.53 from its original level of 899.31 whereas if it is decreased down to zero the objective function value rises up to 904.68. In stretching of domestic accounts payable for the second period constraints, the optimal solution is more sensitive to the decreases in the resource levels which imply tightening firm's stretching policy. If there is a decline in the resource level evidenced by negative minimum resource the objective function value will get worse. On the other hand, in stretching of foreign accounts payable for the second period constraints just the opposite occur-increases in the resource level require basis change.²⁷

The fulfillment of requirements constraints are, in fact, the cash flow constraints which are subject to drastic fluctuations due

²⁷ See Appendix-21

to the uncertainties in the environmental conditions. For this reason, they should be watched out very carefully to guarantee that the current optimal solution is still valid. Changes of the resource constants within the optimality range may have great effects in the objective function value.²⁸

The improvements in the objective function value at Risk Levels I and II are shown in Table 3.3.4.3.2.1. In Risk Level I, if the requirement can be decreased by TL. 61 million at period 3 without increasing the other periods' requirements the objective function value improves by 7.2%. In Risk Level II, the highest improvement occurs at period 7 provided that the requirement is decreased by TL. 10.6 million. Thus, in order to obtain the largest improvement in the objective function value without changing the current optimal solution the Management should try to decrease the requirement by 15.8% at period 3 in Risk Level I and by 5.5% at period 7 in Risk Level II.

TABLE 3.3.4.3.2.1. The Decrease In the Objective Function Value At the Current Optimal Solution With Respect To the Changes In the Requirements At Risk Levels I and II.

Risk Level	Period	Min.Resource	Original Resource	Obj.Fun.Value
		z-LOWER	z-ORIGINAL	Improvement Rate
I	3	326.29	387.38	7.2%
		833.67	899.31	
II	7	303.33	320.90	1.5%
		710.30	720.95	

²⁸ See Appendix-22

TABLE 3.3.4.3.2.2. The Highest Increase In the Objective Function Value
At the Current Optimal Solution With Respect To
the Changes In the Requirements At Risk Levels
I and II.

Risk Level	Period	Original Resource	Max.Resource	Obj.Func.Value
		z-ORIGINAL	z-UPPER	Increase Rate
I	3	387.38 899.31	445.71 961.98	7%
II	3	375.69 720.95	456.10 804.60	11.6%

The highest increases in the objective function value as a result of the increases in the requirements occur at period 3 both in Risk Levels I and II by 7% and 11.6%, respectively as seen in Table 3.3.4.3.2.2. This increase corresponds to the requirement rise of 15% and 21.4% for Risk Levels I and II, respectively meaning that the Management should not suspect whether it is still operating optimally with the current decision policies or not if the requirements at period 3 increase by TL. 58.3 million in Risk Level I and TL. 80.4 million in Risk Level II.

CHAPTER IV

SUMMARY AND CONCLUSIONS

This study aims at developing a solution procedure to the firms' short-term financing and investment decisions under conditions of uncertainty by integrating various operating functions within a company and making use of optimization techniques.

Different demand forecasts with respect to the environmental conditions are taken to be the determinants of uncertainty. For this purpose, thirty different demand forecasts which are based on previous years' sales quantities for domestic sales and subjective estimations of the experts for export sales are generated. Through a spreadsheet cash budget model, monthly cash requirements corresponding to each demand forecast are determined for the whole planning period. Then, normal probability distributions of the period cash requirements are obtained. These requirements are categorized in five different risk levels by means of their averages and standard deviations—Risk Level I representing average values less two standard deviations, Risk Level V

representing average values plus two standard deviations-where being in Risk Level I means anticipating the most pessimistic demand and correspondingly the highest cash requirements, on the contrary, being in Risk Level V means expecting the most optimistic demand resulting in the lowest cash requirements to the Management for the planning period. Finding optimal solutions for each risk level in financing the requirements in periods of cash shortages and investing excess cash in periods of cash surpluses among various alternatives are realized by utilising a linear programming model developed for this purpose. Optimal solutions do not only provide activity levels for the alternative variables but also the shadow prices of the resources and the optimality ranges for the cost coefficients and resources with which the optimal basis will not change. Through interpretations of all these valuable information, the firm's Management decides on which risk level they are in, what their decisions, new policies and precautions should be, what marginal values their resources possess and in which range the fluctuations of cost coefficients and resource constants guarantee that the firm is operating optimally in that risk level.

The decisions related to the different functions of the firm like marketing, production, finance can not be separated from each other definitely. They are all interrelated and integrated. Each decision thought to be taken for a different function has various impacts on the others. Attempts to optimize each function of the firm within itself do not yield optimal operation as a whole since distinct boundaries can not be defined easily and since this results in

uncoordinated actions and unnecessary overusage of scarce resources. For these reasons, decision models developed should not be concentrated on a unique function but try to perform a means of balance among competing functions, provide smooth operation and integration by allocating the resources optimally which bring maximum yield to the firm as a whole.

In today's ever-changing environmental conditions decision models should incorporate uncertainty. Since nothing is certain decisions based on simple deterministic models are unrealistic and can be misleading. Such decisions need to be reviewed and revised continuously. However, models incorporating uncertainty for some important stochastic variables produce more reliable solutions by taking care of all states of nature prevailing in the environment.

Optimization models are more difficult to understand and visualize than the simulation or "what if" models for the Managers. However, they are the most efficient tools in the hands of knowledgeable modellers which evaluate all the alternative courses of action automatically to reach an optimal solution by observing the future effects of the current decisions that is hard to examine in "what if" models for multi-period decisions. Moreover, optimization models provide managers valuable information on the economic marginal values of the resources which will force them to reevaluate their decisions.

It should also be emphasized that the success of any optimization model depends on the reliability of the data it utilizes. This necessity forces Management to revise current accounting routines, operational

procedures and functional relations in order to be able to access to the most up-to-date and reliable data which will bring discipline and control to the whole firm by means of planning effort.

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A P P E N D I C E S

APPENDIX I. Realized Monthly Sales Figures

Prod. Line Period	I	II	III	IV
1981	9,465	30,025	12,839	15,944
	18,795	61,307	20,016	21,266
	24,897	24,365	12,772	27,240
	31,573	16,667	14,920	15,000
	23,244	53,560	13,520	23,826
	24,217	69,320	25,105	12,560
	23,576	86,800	22,353	13,462
	24,571	66,160	18,953	20,920
	31,698	53,910	25,296	9,921
	16,559	42,550	19,000	15,960
	38,410	57,300	16,627	7,791
	26,814	54,219	28,908	21,650
	20,618	52,260	23,834	21,540
	24,702	61,851	21,447	5,480
1982	39,326	54,423	25,617	21,080
	31,633	26,627	21,832	29,460
	31,423	21,101	16,630	27,520
	22,901	54,376	16,440	31,150
	16,771	34,355	18,818	24,180
	21,715	42,408	10,198	26,600
	24,290	44,365	10,589	28,880
	23,834	42,250	15,880	16,245
	29,029	49,921	19,964	24,420
	19,691	65,029	23,997	28,080
1983	22,678	71,715	14,409	15,960
	20,817	26,354	15,950	7,880
	26,816	57,527	14,355	37,320
	44,032	45,787	19,313	25,110
	33,857	46,248	18,341	22,120
	28,816	56,790	10,240	31,380
	15,236	10,101	28,695	33,685
	31,621	36,940	20,098	19,206
	32,501	27,115	22,021	9,400
	37,393	74,255	25,411	13,060
	39,765	58,112	31,581	19,760
	43,743	60,672	31,910	25,780

Appendix 1 continued.

	I	II	III	IV
1984	33,250	43,826		19,280
	31,019	40,615		16,520
	30,939	58,346		21,100
	32,927	69,131		13,200
	50,237	59,436		25,200
	21,059	54,713		25,660
	28,609	59,197		24,560
	27,479	68,070		14,960
	32,775	56,343		25,920
	33,618	45,367		21,360
	37,869	44,901		30,980
	33,090	46,285		22,760
	22,815	48,703		
	39,873	29,939		
1985	38,929	27,495		
	35,759	26,015		
	35,349	27,780		
	16,694	30,179		
	31,396	54,465		
	21,592	25,654		
	32,570	40,775		
	28,684	38,901		
	48,604	54,014		
	42,162	59,346		

APPENDIX 2. Chi-Square Goodness-of-Fit Tests to Validate the Assumption That Sales Quantities Are Normally Distributed.

Product Line 1		$X < \bar{X} - 1S$	$\bar{X} - 1S \leq X \leq \bar{X}$	$\bar{X} \leq X \leq \bar{X} + 1S$	$X > \bar{X} + 1S$
$\bar{X}_1 = 29,405$	Observed Frequencies (o_i)	9	21	20	10
$S_1 = 8,538$	Expected Frequencies (e_i)	9.522	20.478	20.478	9.522
# of observations=60	$(o_i - e_i)^2/e_i$	0.029	0.013	0.011	0.024
Product Line 2					
$\bar{X}_2 = 47,438$	Observed Frequencies (o_i)	13	16	23	8
$S_2 = 16,043$	Expected Frequencies (e_i)	9.522	20.478	20.478	9.522
# of observations=60	$(o_i - e_i)^2/e_i$	1.270	0.979	0.311	0.243
Product Line 3					
$\bar{X}_3 = 19,663$	Observed Frequencies (o_i)	8	11	11	6
$S_3 = 5,882$	Expected Frequencies (e_i)	5.713	12.287	12.287	5.713
# of observations=36	$(o_i - e_i)^2/e_i$	0.916	0.135	0.135	0.014
Product Line 4					
$\bar{X}_4 = 21,090$	Observed Frequencies (o_i)	9	12	20	7
$S_4 = 7,342$	Expected Frequencies (e_i)	7.618	16.382	16.382	7.618
# of observations=48	$(o_i - e_i)^2/e_i$	0.251	1.172	0.799	0.050

Product Line	$\chi^2_{\text{calc.}}$	α	$\chi^2_{1,\alpha}(\text{tab})$
1	0.077	99 %	6.635
2	2.803	97.5 %	5.024
3	1.2	95 %	3.841
4	2.272		

Since $\chi^2_{\text{calc.}}$ for all product lines are less than $\chi^2_{\text{tab.}}$ we can conclude that data for each product line represent a sample from a normal distribution with corresponding means and standard deviations.

APPENDIX 3. Generation of Random Variates

The following method is used in generating a normal random variate.

Let RN_1, RN_2, \dots, RN_n be n uniformly distributed $(0,1)$ independent random variables. Define, Y statistics such that,

$$Y = \sum_{i=1}^n RN_i$$

Statistics Y is normally distributed with,

$$E(Y) = E\left[\sum_{i=1}^n RN_i\right] = nE[RN_i] = n\left(\frac{a+b}{2}\right)$$

$$\text{Var}(Y) = \text{Var}\left[\sum_{i=1}^n RN_i\right] = n \text{Var}[RN_i] = n\left(\frac{a-b}{12}\right)^2$$

Take $n=12$. Since RN_i are uniformly distributed random variables between 0 and 1,

$$E(Y) = 12 \cdot \frac{1}{2} = 6$$

$$\text{Var}(Y) = 12(0-1)^2/12 = 1$$

Standard normal random variate, z , is

$$z = \frac{Y - E(Y)}{\sqrt{\text{Var}(Y)}} = \frac{Y - 6}{1} = Y - 6$$

So, to generate a standard normal random variate, z , obtain 12 uniformly distributed random variables, sum them up and subtract 6.

To generate a random variate, X_1 , which is normally distributed with sample mean \bar{X} and standard deviation \hat{S} , using z obtained by the previous method:

$$z = \frac{X - \bar{X}}{\hat{S}} \Rightarrow X = z \cdot \hat{S} + \bar{X}$$

The following method is used in generating a random variate from a discrete probability distribution.

Let the discrete probability distribution of random variable X be defined as:

$$P(X=500) = 0.3$$

$$P(X=1000) = 0.5$$

$$P(X=1500) = 0.2$$

A random number, RN , from a uniformly distributed probability distribution between 0 and 1 is generated. Then, if

$$RN < 0.3 \quad \text{then} \quad X = 500$$

$$0.3 \leq RN \leq 0.8 \quad \text{then} \quad X = 1000$$

$$RN > 0.8 \quad \text{then} \quad X = 1500$$

APPENDIX 4. Cash Budget Format

CASH RECEIPTSACCOUNTS RECEIVABLE AND COLLECTIONS

BEG. OF PERIOD ACCTS. REC.

Domestic Customers

Foreign Customers

Tax Rebate on Exports

Other

SALES IN PERIOD

Domestic Market

Exports

Tax Rebate on Exports (with avg. TRE %)

Other

COLLECTIONS ON ACCOUNTS RECEIVABLE

Domestic Market (from expected sales collection %)

- Cash Sales

- One Month Lag

- Two Months Lag

Exports (from expected sales collection %)

- Cash Sales

- One Month Lag

- Two Months Lag

Tax Rebate on Exports

Other

END OF PERIOD ACCOUNTS RECEIVABLE

Domestic Customers

Foreign Customers

Tax Rebate on Exports

Other

OTHER CASH RECEIPTS

Vade Farkı

Miscellaneous

TOTAL CASH RECEIPTS

Collections on Accounts Receivable

Other Cash Receipts

CASH DISBURSEMENTSACCOUNTS PAYABLE AND PAYMENTS OF PURCHASES

BEG. OF PERIOD ACCOUNTS PAYABLE

Domestic Payables

- Raw Materials
- Packaging Materials
- Energy
- Miscellaneous

Foreign Payables

- Raw Materials
- Miscellaneous

PURCHASES IN PERIOD

Domestic Purchases

- Raw Materials
- Packaging Materials
- Energy

- Miscellaneous

Foreign Purchases

- Raw Materials

- Miscellaneous

SCHEDULED MANDATORY PAYMENTS OF OUTSTANDING DEBT

Domestic Payments

- Raw Materials

- Packaging Materials

- Energy

- Miscellaneous

Foreign Payments

- Raw Materials

- Miscellaneous

PAYMENT SCHEDULE OF IN-PERIOD PURCHASES

Domestic Payments

- Raw Materials

- Packaging Materials

- Energy

- Miscellaneous

Foreign Payments

- Raw Materials

- Miscellaneous

ACCOUNTS PAYABLE AFTER SCHEDULED PAYMENTS

Domestic Payables

- Raw Materials

- Packaging Materials

- Energy

- Miscellaneous

Foreign Payables

- Raw Materials

- Miscellaneous

IN-PERIOD MANDATORY DISBURSEMENTS

DISBURSEMENTS RELATED TO RAW MATERIAL PURCHASES

Customs Fees

Freight

Letter of Credit - Net

Transportation Fees

DISBURSEMENTS RELATED TO FACTORY OPERATIONS

Wages and Salaries

Factory Overhead

DISBURSEMENTS RELATED TO SELLING AND ADMINISTRATIVE EXPENSES

Wages and Salaries

Selling Expenses

Sales Commissions

Administrative Expenses

DISBURSEMENTS RELATED TO TAX PAYMENTS

Factory Wages and Salaries Tax Provision

Administrative Wages and Salaries Tax Provision

(+) Value Added Tax to Purchases

(-) Value Added Tax From Sales

Payment Schedule of the Previous Operating Period's Accrued Taxes

MISCELLANEOUS DISBURSEMENTS

Dividend Payments

Fixed Investment to Be Realized By Short-Term Sources

Other

TOTAL IN-PERIOD MANDATORY DISBURSEMENTS

OUTSTANDING FINANCIAL DISBURSEMENTS

MATURING CREDITS

Pledging of Accounts Receivable

Prefinancing

Other Short-Term Bank Credits

Short-Term Maturity of Long-Term Debt

BONDS

INTEREST PAYMENTS

Pledging of Accounts Receivable

Prefinancing

Other Short-Term Bank Credits

Short-Term Maturity of Long-Term Debt

BONDS

NET OUTSTANDING FINANCIAL DISBURSEMENTSOTHER FINANCIAL ACTIVITIES

Bank Commissions

(-) Maturing of Outstanding Marketable Securities and Their Returns

(-) Bank Credits Previously Decided to Be Raised

(-) Bond Issue

Interest Payments Previously Decided Bank Credits and Bonds Issued

NET OTHER FINANCIAL ACTIVITIESTOTAL CASH DISBURSEMENTS

Payments From Accounts Payable

Total In-Period Mandatory Disbursements

Net Outstanding Financial Disbursements

Net Other Financial Activities

CASH RECEIPTS - CASH DISBURSEMENTS

CHANGE IN MINIMUM OPERATING CASH REQUIREMENT

PERIOD CASH REQUIREMENT BEFORE ADDITIONAL FINANCING AND INVESTMENT

CUMULATIVE CASH REQUIREMENT BEFORE ADDITIONAL FINANCING AND INVESTMENT

APPENDIX 5. Period Cash Requirements of Each Demand Forecast and Their Probability Distributions

JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEP.	OCT.	NOV.	DEC.	
80,198	(51,524)	(377,486)	(210,380)	(141,849)	(9,311)	(336,378)	21,761	15,128	(16,641)	208	(6,073)	
101,493	(79,155)	(380,198)	(197,800)	(126,021)	28,727	(300,868)	59,898	19,145	17,860	(27,273)	37,957	
121,094	(32,938)	(366,511)	(165,042)	(144,849)	23,797	(193,831)	73,724	11,366	53,135	(56,202)	31,163	
79,503	(64,714)	(360,691)	(182,529)	(157,017)	(13,117)	(350,590)	50,790	17,994	13,201	(4,289)	17,587	
121,854	(60,977)	(346,272)	(156,140)	(192,099)	60,994	(226,037)	140,309	(30,561)	(1,949)	40,685	(32,422)	
132,949	(48,040)	(361,317)	(162,114)	(168,023)	71,047	(225,275)	157,087	8,437	13,020	38,364	11,496	
106,979	(60,727)	(358,776)	(168,769)	(209,227)	37,129	(248,735)	122,557	(4,476)	29,100	24,089	(10,224)	
102,578	(37,951)	(359,244)	(195,218)	(180,503)	20,552	(290,732)	141,740	13,806	(40,820)	3,207	31,744	
106,987	(70,372)	(366,310)	(164,965)	(195,871)	28,021	(253,096)	116,256	(4,237)	402	(17,711)	(20,716)	
104,050	(39,768)	(358,950)	(189,873)	(182,956)	21,985	(282,341)	114,497	(27,287)	26,460	(18,560)	(18,148)	
102,571	(50,563)	(354,810)	(191,402)	(163,527)	46,817	(305,209)	110,351	(23,796)	(12,963)	(15,433)	19,908	
114,997	(50,900)	(342,719)	(135,517)	(159,201)	46,839	(326,330)	169,308	(3,230)	12,372	18,969	16,313	
118,063	(56,732)	(363,305)	(157,080)	(140,215)	68,051	(283,233)	109,336	10,953	16,699	(15,675)	44,188	
91,377	(67,810)	(368,109)	(165,986)	(183,311)	11,807	(316,864)	91,730	29,973	(25,807)	(29,225)	50,674	
91,394	(66,485)	(369,248)	(186,475)	(162,193)	12,014	(288,558)	35,333	(13,571)	27,862	(64,364)	84,061	
95,882	(87,467)	(390,243)	(202,677)	(131,389)	16,318	(315,576)	95,127	27,689	19,360	(32,001)	35,440	
92,309	(64,128)	(369,713)	(211,488)	(154,363)	18,682	(315,341)	89,166	21,762	(17,542)	(44,174)	59,187	
122,853	(49,638)	(361,526)	(175,101)	(175,002)	85,920	(255,256)	143,531	(24,163)	3,834	(40,327)	31,330	
121,421	(46,759)	(351,315)	(175,835)	(158,059)	85,227	(259,036)	151,485	9,771	144	(3,168)	56,479	
95,845	(89,052)	(370,254)	(139,777)	(160,496)	(15,956)	(323,450)	79,234	(3,971)	(7,274)	(6,918)	49,981	
102,610	(41,247)	(351,391)	(155,767)	(177,625)	26,148	(299,018)	140,406	29,324	(4,388)	(20,855)	27,123	
95,960	(46,799)	(376,165)	(208,687)	(148,026)	1,238	(285,205)	122,328	(26,995)	1,085	(32,561)	3,010	
119,304	(64,728)	(373,242)	(174,299)	(120,729)	69,485	(277,158)	115,377	5,549	50,633	(37,713)	32,625	
115,121	(50,169)	(353,104)	(151,468)	(186,344)	39,723	(286,503)	89,929	60,894	48,894	(49,247)	28,857	
102,026	(47,050)	(367,379)	(202,864)	(168,054)	34,504	(281,151)	63,167	(3,583)	(22,819)	(7,724)	31,922	
106,475	(72,312)	(367,806)	(144,837)	(176,199)	936	(269,881)	58,556	9,560	35,373	(30,577)	34,936	
103,360	(56,170)	(369,002)	(190,066)	(166,051)	39,306	(266,501)	48,706	22,331	10,569	(38,367)	50,184	
84,341	(75,484)	(377,125)	(174,604)	(165,860)	(39,628)	(318,498)	76,192	10,033	16,931	(48,343)	25,904	
81,217	(68,924)	(369,899)	(196,430)	(186,859)	(22,801)	(303,527)	56,585	22,058	2,451	(41,617)	10,444	
129,428	(39,984)	(337,738)	(127,915)	(185,369)	67,347	(223,160)	166,526	(3,915)	(14,114)	2,183	34,174	
104,808	(57,956)	(363,995)	(175,379)	(165,576)	28,713	(283,578)	100,363	5,866	6,260	(20,160)	25,637	AVG. VAL.
14,843	14,750	11,692	23,555	21,557	32,706	37,317	41,320	20,897	23,495	25,758	26,116	STD. DEV.
75,122	(87,456)	(387,379)	(222,489)	(208,691)	(39,698)	(358,211)	17,724	(35,928)	(40,729)	(71,677)	(26,596)	AV-2 STD.
89,965	(72,706)	(375,687)	(198,934)	(187,134)	(3,992)	(320,895)	59,043	(15,031)	(17,235)	(45,918)	(479)	AV-1 STD.
104,808	(57,956)	(363,995)	(175,379)	(165,576)	28,713	(283,578)	100,363	5,866	6,260	(20,160)	25,637	AV.
119,651	(43,205)	(352,303)	(151,824)	(144,019)	61,419	(246,261)	141,683	26,753	29,755	5,598	51,753	AV+1 STD.
134,494	(28,455)	(340,611)	(128,270)	(122,462)	94,125	(208,945)	183,002	47,660	53,250	31,357	77,869	AV+2 STD.

Chi-Square Goodness-of-Fit Test to Validate the Assumption That Period Cash Requirements Are Normally Distributed.

Interval Range	I	II	III	IV	I	II	III	IV	χ^2_{calc}
	$X < \bar{X} - 1s$	$\bar{X} - 1s \leq X \leq \bar{X}$	$\bar{X} < X \leq \bar{X} + 1s$	$X > \bar{X} + 1s$					
Expected Frequencies(e_i)	4.761	10.236	10.239	4.761					
Observed Frequencies(o_i)					$(o_i - e_i)^2 / e_i$	$(o_i - e_i)^2 / e_i$	$(o_i - e_i)^2 / e_i$	$(o_i - e_i)^2 / e_i$	
January	4	13	7	6	0.1216	0.7445	1.0246	0.3224	2.2131
February	5	9	11	5	0.012	0.1499	0.0566	0.012	0.2305
March	5	12	8	5	0.012	0.3029	0.3896	0.012	0.8165
April	5	9	11	5	0.012	0.1499	0.0566	0.012	0.2305
May	3	13	9	5	0.6514	0.7445	0.1499	0.012	1.5578
June	5	11	8	6	0.012	0.0566	0.4896	0.3224	0.8806
July	4	12	10	4	0.1216	0.3029	0.0056	0.1216	0.5517
August	6	9	9	6	0.3224	0.1499	0.1499	0.3224	0.9446
September	5	8	13	4	0.012	0.4896	0.7445	0.1216	1.3677
October	4	13	9	4	0.1216	0.7445	0.1499	0.1216	1.1376
November	4	12	11	3	0.1216	0.3029	0.0566	0.6514	1.1324
December	5	7	15	3	0.012	1.0246	2.1138	0.6514	3.8018

For all months χ^2_{calc} are less than $\chi^2_{1,95\%}$ (3.841). So, data represent a sample from a normal distribution with corresponding sample means and standard deviations.

APPENDIX 6. Beginning of Period Accounts Receivable for Each Demand Forecast and Their Probability Dist.

JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEP.	OCT.	NOV.	DEC.	
160,000	222,119	175,444	186,338	245,765	228,552	196,167	255,575	214,315	214,026	187,553	180,511	
160,000	161,510	143,086	153,541	263,737	229,607	211,709	232,351	284,927	283,645	251,595	243,979	
160,000	185,071	142,149	211,384	194,320	248,047	258,111	196,321	248,264	260,808	247,254	272,592	
160,000	255,014	213,713	191,939	235,017	216,286	202,114	216,856	242,417	275,081	240,590	280,540	
160,000	171,405	191,481	171,869	177,573	231,672	222,439	219,743	202,687	278,013	225,630	249,932	
160,000	182,370	137,870	171,646	162,370	206,043	222,320	245,375	221,994	274,488	292,494	279,407	
160,000	182,834	227,253	240,093	231,842	173,736	183,986	242,517	194,390	245,389	268,196	300,345	
160,000	230,270	239,540	232,992	250,966	190,961	214,572	274,899	247,042	242,437	234,722	303,090	
160,000	203,338	158,939	193,037	231,195	203,109	236,026	221,617	211,674	242,924	217,245	203,266	
160,000	162,863	147,950	208,947	251,625	266,730	230,814	206,566	259,615	235,874	237,673	265,633	
160,000	199,322	235,404	214,461	173,954	187,078	133,203	214,987	306,041	252,576	217,633	264,928	
160,000	188,227	208,717	158,861	171,106	216,587	254,492	252,187	281,315	286,402	254,300	245,154	
160,000	230,967	192,271	178,098	177,657	159,853	178,569	273,922	218,451	228,830	257,187	274,328	
160,000	164,910	180,314	193,399	189,987	216,488	236,753	241,292	233,146	253,072	183,973	216,180	
160,000	205,829	191,346	170,979	203,254	180,202	221,389	222,465	201,743	266,855	202,036	215,969	
160,000	223,849	205,232	174,298	212,440	193,983	179,772	201,464	197,203	188,071	219,265	230,959	
160,000	225,431	262,030	214,768	237,577	186,315	187,549	260,919	239,387	237,571	252,956	247,042	
160,000	191,593	205,226	213,514	222,658	240,917	205,640	200,732	201,984	204,616	247,215	271,450	
160,000	211,792	214,784	205,792	225,556	234,023	185,384	260,564	248,808	218,625	240,531	284,923	
160,000	187,203	197,447	221,256	235,683	222,371	210,436	179,045	205,841	188,482	230,406	246,163	
160,000	138,903	143,115	210,050	240,581	236,195	239,603	286,332	286,427	252,138	256,053	269,638	
160,000	223,560	168,871	186,434	235,363	212,104	215,212	223,582	239,124	188,498	221,245	280,233	
160,000	172,147	170,067	169,741	247,966	228,230	217,179	190,935	210,207	221,664	209,694	239,191	
160,000	200,562	181,788	186,435	249,064	262,650	244,304	226,050	240,224	262,571	290,658	264,241	
160,000	229,534	167,215	165,596	197,360	192,899	191,756	236,167	179,600	201,710	223,162	219,898	
160,000	191,466	215,648	220,569	195,561	205,751	200,366	189,564	263,171	235,993	218,926	220,005	
160,000	204,605	173,315	179,954	191,191	254,870	204,531	214,313	247,754	236,476	218,621	234,713	
160,000	172,466	193,197	229,373	197,560	182,348	228,906	246,972	229,980	251,282	224,054	271,114	
160,000	191,648	189,883	189,068	155,566	205,969	231,005	217,137	274,661	247,645	221,462	231,362	
160,000	225,232	248,336	207,676	179,215	195,118	252,275	236,298	190,501	210,161	234,155	308,538	
160,000	197,868	190,721	195,070	212,790	213,623	213,220	229,558	234,096	239,531	234,216	253,844	AVG. VAL.
0	26,884	33,445	23,498	31,563	27,297	27,567	27,533	32,901	28,979	25,673	31,375	STD. DEV.
160,000	144,101	123,832	148,075	149,664	159,029	158,087	174,492	168,294	181,573	182,871	191,094	AV-2 STD.
160,000	170,984	157,276	171,572	181,227	186,326	185,654	202,025	201,195	210,552	208,544	222,469	AV-1 STD.
160,000	197,868	190,721	195,070	212,790	213,623	213,220	229,558	234,096	239,531	234,216	253,844	AV.
160,000	224,752	224,166	218,568	244,354	240,920	240,787	257,091	266,998	268,509	259,889	285,219	AV+1 STD.
160,000	251,635	257,610	242,066	275,917	268,218	268,354	284,624	299,899	297,488	285,561	316,594	AV+2 STD.

Chi-Square Goodness-of-Fit Test to Validate the Assumption That Beginning of Period Accounts Receivable Are Normally Distributed.

Interval Range	I	II	III	IV	I	II	III	IV	χ^2_{calc}
	$X < \bar{X} - 1s$	$\bar{X} - 1s \leq X \leq \bar{X}$	$\bar{X} < X \leq \bar{X} + 1s$	$X > \bar{X} + 1s$					
Expected Frequencies (e_i)	4.761	10.239	10.239	4.761					
Observed Frequencies (o_i)					$(o_i - e_i)^2 / e_i$	$(o_i - e_i)^2 / e_i$	$(o_i - e_i)^2 / e_i$	$(o_i - e_i)^2 / e_i$	
February	4	11	9	6	0.1216	0.0566	0.1499	0.3224	0.6505
March	5	9	11	5	0.012	0.1499	0.0555	0.012	0.2305
April	5	12	8	5	0.012	0.3029	0.4896	0.012	0.8165
May	7	8	9	6	1.053	0.4896	0.1499	0.3224	2.0149
June	5	10	11	4	0.012	0.0056	0.0566	0.1216	0.1958
July	5	9	12	4	0.012	0.1499	0.3029	0.1216	0.5864
August	6	10	9	5	0.3224	0.0056	0.1499	0.012	0.4899
September	4	11	10	5	0.1216	0.0566	0.0056	0.012	0.1958
October	6	8	11	5	0.3224	0.4896	0.0566	0.012	0.8806
November	3	13	11	3	0.6514	0.7445	0.0566	0.6514	2.1039
December	6	9	12	3	0.3224	0.1499	0.3029	0.6514	1.4266

For all months χ^2_{calc} are less than $\chi^2_{1,95\%}(3.841)$. So, data represent a sample from a normal distribution with corresponding sample means and standard deviations.

APPENDIX 7.-Payment Schedule of Domestic Purchases for Each Demand Forecast and Their Probability Dist.

JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEP.	OCT.	NOV.	DEC.	
21,680	30,096	35,766	33,106	49,205	44,267	62,204	74,643	67,121	60,799	44,119	50,575	
21,174	40,306	30,391	32,050	58,291	36,651	54,385	49,501	76,121	70,800	64,841	58,406	
21,174	28,657	31,296	42,540	40,189	61,640	77,082	85,290	75,147	84,984	106,456	95,185	
22,228	55,486	34,842	27,688	43,432	40,436	77,782	59,423	66,269	76,988	59,635	63,885	
21,538	38,916	28,407	50,405	36,742	65,396	79,760	82,868	93,539	113,316	77,707	109,194	
21,797	51,577	40,029	42,279	53,710	64,434	118,223	99,726	107,308	123,777	134,529	136,559	
21,322	46,156	56,254	48,508	50,581	36,922	78,773	89,742	62,843	92,547	93,255	91,796	
22,012	35,350	50,852	53,858	64,926	37,886	97,298	95,518	75,079	87,306	76,866	109,986	
21,571	23,441	37,632	54,818	46,905	40,553	66,302	49,057	61,896	62,568	66,512	62,159	
21,447	39,947	45,815	56,807	58,240	73,187	101,227	105,630	116,674	90,054	127,172	123,784	
21,640	41,357	61,381	61,536	50,866	55,839	84,948	119,257	119,797	107,711	111,953	140,388	
21,559	33,728	46,840	38,359	57,403	45,345	94,266	76,225	102,633	91,855	92,265	108,594	
22,053	39,526	45,181	50,917	53,089	36,513	84,744	88,111	81,761	101,920	99,891	96,213	
21,240	29,757	37,281	43,474	55,744	50,215	91,027	83,914	99,646	92,357	70,627	94,885	
21,588	22,528	39,337	44,584	49,706	47,730	88,483	89,470	74,924	95,586	80,187	86,618	
21,965	45,098	43,706	46,553	41,364	41,092	83,395	76,942	87,640	84,095	88,235	85,121	
22,111	54,090	60,546	49,339	50,616	55,360	105,394	112,137	121,454	101,108	126,086	119,478	
21,728	47,663	63,213	57,574	50,140	65,991	101,978	95,986	101,214	113,909	128,987	123,253	
21,699	42,546	50,626	39,972	67,116	52,008	87,816	71,155	89,441	84,094	103,177	102,102	
21,354	27,032	38,733	66,972	54,982	45,943	85,869	86,321	98,264	76,962	126,699	64,379	
20,699	23,045	42,223	59,536	42,200	51,175	89,303	93,598	93,679	76,264	96,419	105,469	
21,801	48,205	35,454	60,256	61,807	52,282	90,908	76,570	82,985	79,503	87,737	109,646	
21,426	27,678	40,721	41,407	64,667	36,556	87,394	55,529	97,124	78,787	97,205	97,493	
21,893	39,761	32,418	53,212	51,918	56,966	107,974	69,658	86,360	104,332	105,931	104,284	
22,029	45,959	41,414	52,617	55,596	41,533	89,948	111,625	96,322	112,844	112,117	102,050	
21,549	45,694	62,073	55,309	55,253	74,295	92,843	103,050	126,300	111,668	109,180	112,751	
21,705	40,980	43,716	54,761	53,176	70,372	95,657	105,455	110,611	113,991	119,286	105,546	
21,013	18,063	38,902	49,634	37,757	33,669	65,268	51,758	48,383	60,528	82,854	64,361	
21,259	22,347	30,234	40,503	28,703	44,415	59,133	47,418	60,328	61,894	57,545	65,568	
22,290	60,735	61,882	48,310	51,235	56,829	124,857	93,858	103,984	111,082	125,035	147,533	
21,638	38,190	43,575	48,563	51,185	50,517	87,475	83,315	89,495	90,784	95,750	97,909	AVG. VAL.
368	11,245	10,641	9,399	8,940	12,101	16,471	20,432	20,349	18,482	24,619	25,588	STD. DEV.
20,901	15,700	22,288	29,765	33,305	26,316	54,532	42,450	48,797	53,820	46,513	46,733	AV-2 STD.
21,270	26,945	32,929	39,164	42,245	38,416	71,003	62,882	69,146	72,302	71,132	72,351	AV-1 STD.
21,638	38,190	43,571	48,563	51,185	50,517	87,475	83,315	89,495	90,784	95,750	97,909	AV.
22,006	49,435	54,212	57,962	60,126	62,618	103,946	103,747	109,844	109,266	120,369	123,497	AV 1+STD.
22,375	60,680	64,854	67,630	69,066	74,718	120,417	124,179	130,193	127,749	144,987	149,084	AV 2+STD.

Chi-Square Goodness-of-Fit Test To Validate the Assumption That Payment Schedule of Domestic Purchases Are Normally Distributed.

Interval Range	I	II	III	IV	I	II	III	IV	χ^2_{calc}
	$X < \bar{X} - 1s$	$\bar{X} - 1s \leq X \leq \bar{X}$	$\bar{X} < X \leq \bar{X} + 1s$	$X > \bar{X} + 1s$					
	Expected Frequencies (e_i)	Expected Frequencies (e_i)	Expected Frequencies (e_i)	Expected Frequencies (e_i)	$(o_i - e_i)^2 / e_i$	$(o_i - e_i)^2 / e_i$	$(o_i - e_i)^2 / e_i$	$(o_i - e_i)^2 / e_i$	
Observed Frequencies (o_i)	4.761	10.239	10.239	4.761					
January	5	9	10	6	0.012	0.1499	0.0056	0.3224	0.4899
February	5	7	14	4	0.012	1.0246	1.3815	0.1216	2.5397
March	5	12	7	6	0.012	0.3029	1.0246	0.3224	1.6619
April	4	10	12	4	0.1216	0.0056	0.3029	0.1216	0.5517
May	6	8	12	4	0.3224	0.4896	0.3029	0.1216	1.2365
June	6	10	8	6	0.3224	0.0056	0.4896	0.3224	1.14
July	5	9	12	4	0.012	0.1499	0.3029	0.1216	0.5864
August	6	8	11	5	0.3224	0.4896	0.0566	0.012	0.8806
September	6	9	10	5	0.3224	0.1499	0.0056	0.012	0.4899
October	5	10	8	7	0.012	0.0056	0.4896	1.053	1.5602
November	6	8	10	6	0.3224	0.4896	0.0056	0.3224	1.14
December	7	7	12	4	1.053	1.0246	0.3029	0.1216	2.5021

For all months χ^2_{calc} are less than $\chi^2_{1,95\%}$ (3.841). So, data represent a sample from a normal distribution with corresponding sample means and standard deviations.

APPENDIX-8 Payment Schedule of Foreign Purchases For Each Demand Forecast and Their Probability Dist.

JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEP.	OCT.	NOV.	DEC.	
71,545	41,030	209,223	64,215	68,465	79,226	65,438	108,240	114,837	134,100	90,349	117,468	
71,545	41,030	209,223	46,181	65,948	92,593	90,981	191,651	195,221	200,351	191,550	174,774	
71,545	41,030	209,223	83,678	104,258	147,184	95,504	161,920	273,947	243,973	288,167	273,882	
71,545	41,030	209,223	63,983	89,750	77,252	71,303	69,098	114,976	130,743	109,602	135,536	
71,545	41,030	209,223	75,270	147,124	97,862	93,468	121,132	268,627	273,668	221,669	304,504	
71,545	41,030	209,223	78,241	112,959	108,391	88,424	144,377	273,596	282,558	255,215	316,981	
71,545	41,030	209,223	42,173	112,959	80,934	64,122	62,925	161,340	141,219	148,107	147,276	
71,545	41,030	209,223	83,651	144,502	100,030	86,296	71,978	202,943	208,820	217,657	215,220	
71,545	41,030	209,223	62,646	107,392	84,712	82,391	70,316	206,820	177,292	192,175	174,310	
71,545	41,030	209,223	60,207	107,392	97,411	124,943	86,924	230,362	298,752	227,445	229,425	
71,545	41,030	209,223	78,241	131,514	91,682	84,409	85,647	211,528	197,402	249,471	206,777	
71,545	41,030	209,223	65,617	128,457	72,586	68,675	98,486	225,712	208,092	227,088	219,479	
71,545	41,030	209,223	83,651	109,902	78,315	65,980	78,266	204,911	214,514	188,255	203,626	
71,545	41,030	209,223	65,617	128,457	91,682	65,980	92,420	204,911	208,092	203,673	203,626	
71,545	41,030	209,223	78,241	128,457	78,315	85,628	78,266	225,712	186,683	203,673	226,274	
71,545	41,030	209,223	83,651	122,891	72,586	91,522	78,266	211,151	186,683	188,255	203,626	
71,545	41,030	209,223	78,241	144,502	92,886	123,704	65,427	233,645	218,811	190,001	229,425	
71,545	41,030	209,223	65,617	112,959	92,886	109,951	99,801	211,528	203,825	212,027	213,571	
71,545	41,030	209,223	60,207	106,222	77,760	53,682	76,939	142,506	162,627	139,131	132,282	
71,545	41,030	209,223	60,207	106,222	77,760	53,682	76,939	140,538	104,823	139,131	109,634	
71,545	41,030	209,223	42,173	93,234	72,031	87,084	70,873	161,340	147,641	163,240	131,624	
71,545	41,030	209,223	60,207	93,234	58,663	67,436	56,719	146,779	141,219	154,549	107,297	
71,545	41,030	209,223	79,378	144,502	113,234	107,971	120,570	270,595	258,060	268,152	248,898	
71,545	41,030	209,223	86,493	144,502	113,234	113,865	119,293	270,595	258,060	274,760	248,898	
71,545	41,030	209,223	65,617	93,234	72,586	56,478	99,596	166,308	151,352	146,290	152,504	
71,545	41,030	209,223	47,583	111,789	78,315	55,239	79,377	166,308	132,848	174,924	136,650	
71,545	41,030	209,223	60,207	93,234	72,586	55,239	99,596	166,308	154,256	166,570	136,650	
71,545	41,030	209,223	45,136	95,317	70,386	64,405	68,096	140,051	138,752	132,079	128,356	
71,545	41,030	209,223	57,760	95,317	51,289	64,405	68,096	140,051	134,354	145,415	121,562	
71,545	41,030	209,223	97,412	144,502	114,573	95,774	139,338	278,806	278,891	254,366	286,791	
71,545	41,030	209,223	67,383	112,972	86,965	81,133	92,686	198,732	189,282	192,100	191,231	AVG. VAL.
0	0	0	14,164	21,949	18,939	20,599	26,677	49,448	48,867	49,489	59,444	STD. DEV.
71,545	41,030	209,223	39,056	69,075	49,086	39,935	39,333	99,835	91,549	93,122	72,344	AV-2 STD.
71,545	41,030	209,223	53,220	91,023	68,026	60,534	66,009	149,283	140,415	142,611	131,787	AV-1 STD.
71,545	41,030	209,223	67,383	112,972	86,965	81,133	92,686	198,732	189,282	192,100	191,231	AV.
71,545	41,030	209,223	81,547	134,921	105,904	101,732	119,362	248,180	238,149	241,588	250,675	AV+1 STD.
71,545	41,030	209,223	95,711	156,870	124,844	122,330	146,039	297,628	287,015	291,077	310,118	AV+2 STD.

Chi-Square Goodness-of-Fit Test To Validate the Assumption That Payment Schedule of Foreign Purchases Are Normally Distributed

Interval Range	I	II	III	IV	I	II	III	IV	
	$X < \bar{X} - 1s$	$\bar{X} - 1s \leq X \leq \bar{X}$	$\bar{X} < X \leq \bar{X} + 1s$	$X > \bar{X} + 1s$					
Expected Frequencies (e_i)	4.761	10.239	10.239	4.761					
Observed Frequencies (o_i)					$(o_i - e_i)^2 / e_i$	$(o_i - e_i)^2 / e_i$	$(o_i - e_i)^2 / e_i$	$(o_i - e_i)^2 / e_i$	χ^2_{calc}
April	5	13	6	6	0.012	0.7445	1.755	0.3224	2.8339
May	3	13	8	6	0.6514	0.7445	0.4896	0.3224	2.2079
June	3	14	8	5	0.6514	1.3815	0.4896	0.012	2.5345
July	5	9	11	5	0.012	0.1499	0.0566	0.012	0.2305
August	3	13	8	6	0.6514	0.7445	0.4896	0.3224	2.2079
September	7	6	11	6	1.053	1.755	0.0566	0.3224	3.187
October	5	10	9	6	0.012	0.0056	0.1499	0.3224	0.4899
November	5	11	8	6	0.013	0.0566	0.4896	0.3224	0.8806
December	5	9	12	4	0.012	0.1499	0.3029	0.1216	0.5864

For all months χ^2_{calc} are less than $\chi^2_{1,95\%}$ (3.841). So, data represent a sample from a normal distribution with corresponding sample means and standard deviations.

APPENDIX 9. The Distribution of the Beginning of Period Accounts Receivable Adjusted With the Bank's Maximum Lending Proportion ($a_{22j} \times A_j$)

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
$\bar{X} - 2S$	128	115.28	99.065	118.46	119.731	127.223	126.469	139.594	134.635	145.259	146.297	152.875
$\bar{X} - 1S$	128	136.787	125.821	137.258	144.981	149.061	148.523	161.621	160.956	168.442	166.835	177.975
\bar{X}	128	158.294	152.577	156.056	170.232	170.899	170.576	183.647	187.277	191.625	187.373	203.075
$\bar{X} + 1S$	128	179.801	179.333	174.854	195.483	192.736	192.63	205.673	213.593	214.808	207.911	228.175
$\bar{X} + S$	128	201.308	206.088	193.653	220.734	214.574	214.683	227.700	239.919	237.990	228.449	253.275
S	0	21.507	26.756	18.798	25.251	21.838	22.053	22.026	26.321	23.183	20.538	25.100

APPENDIX 10. The Distribution of Domestic Accounts Payable That Can Be Stretched In the First Period Adjusted
With the Stretching Proportion For the First Month ($a_{51j} \times P_j$)

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
$\bar{X} - 2s$	15.676	11.775	16.716	22.324	24.978	19.737	40.899	31.838	36.598	40.365	34.885	35.050
$\bar{X} - 1s$	15.952	20.209	24.697	29.373	31.684	28.812	53.253	47.162	51.860	54.227	53.348	54.241
\bar{X}	16.228	28.642	32.678	36.422	38.389	37.888	65.606	62.486	67.121	68.088	71.813	73.432
$\bar{X} + 1s$	16.505	37.076	40.659	43.471	45.094	46.963	77.959	77.810	82.383	81.950	90.277	92.622
$\bar{X} + 2s$	16.781	45.510	48.640	50.520	51.800	56.039	93.134	93.134	97.644	95.811	108.741	111.813
S	0.276	8.434	7.981	7.049	6.705	9.075	12.353	15.324	15.262	13.862	18.464	19.191

APPENDIX 11. The Distribution of Foreign Accounts Payable That Can Be Stretched In the First Period Adjusted
With the Stretching Proportion For the First Month ($a'_{51j} \times P'_j$)

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
$\bar{X} - 2S$	42.927	24.618	125.534	22.848	40.537	28.668	23.109	22.496	57.855	52.907	53.825	40.946
$\bar{X} - 1S$	42.927	24.618	125.534	31.639	54.160	40.424	35.894	39.054	88.547	83.238	84.543	77.842
\bar{X}	42.927	24.618	125.534	40.430	67.783	52.179	48.680	55.611	119.239	113.569	115.260	114.739
$\bar{X} + 1S$	42.927	24.618	125.534	49.221	81.407	63.934	61.465	72.169	149.931	143.900	145.977	151.635
$\bar{X} + 2S$	42.927	24.618	125.534	58.012	95.030	75.690	74.251	88.727	180.623	174.231	176.694	188.531
S	0	0	0	8.791	13.623	11.755	12.786	16.558	30.692	30.331	30.717	36.896

APPENDIX - 12

THE COMPLETE LINEAR PROGRAMMING FORMULATION OF
THE SHORT-TERM FINANCING/INVESTMENT DECISION PROBLEM
TO BE EXECUTED BY MPOS

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*               M P O S               *
*                                     *
*               VERSION 4.1           *
*                                     *
*  MULTI-PURPOSE OPTIMIZATION SYSTEM  *
*                                     *
*****

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***** PROBLEM NUMBER 1 *****

PREVISED

TITLE

HAKAN MAT -MBA THESIS - 'OPTIMAL SHORT-TERM FINANCING/INVESTMENT DECISION'
 * ALL PARAMETERS ARE AT THEIR AVG.+ 2 STD.DEV. VALUES.

VARIABLES

XAR1 TO XAR12 YAR1 TO YAR12 VAR1 TO VAR12

XP1 TO XP12 YP1 TO YP12

XDP11 XFP11 XDP21 XFP21 XDP22 XFP22 XDP32 XFP32 XDP33 XFP33 XDP43 XFP43
 XDP44 XFP44 XDP54 XFP54 XDP55 XFP55 XDP65 XFP65 XDP66 XFP66 XDP76 XFP76
 XDP77 XFP77 XDP87 XFP87 XDP88 XFP88 XDP98 XFP98 XDP99 XFP99 XDP109 XFP109
 XDP110 XFP110 XDP111 XFP111 XDP121 XFP121
 XDP1212 XFP1212 I1 TO I12
 XTP11 XTP22 XTP21 XTP33 XTP32 XTP44 XTP43 XTP55 XTP54 XTP66 XTP65

XTL3 VTL9

T1D1 TO T1D12 T3D1 TO T3D12

MS1 TO MS12

* O B J E C T I V E F U N C T I O N

MINIMIZE

.49XAR1-.49YAR1+.4492XAR2-.4492YAR2+.4492VAR2+.4083XAR3-.4083YAR3-
 .4083VAR3+.3675XAR4-.3675YAR4-.3675VAR4+.3266XAR5-.3266YAR5-.3266VAR5+
 .2853XAR6-.2853YAR6-.2853VAR6+.245XAR7-.245YAR7-.245VAR7+.2042XAR8-
 .2042YAR8-.2042VAR8+.1633XAR9-.1633YAR9-.1633VAR9+.1225XAR10-.1225YAR10-
 .1225VAR10+.0817XAR11-.0817YAR11-.0817VAR11+.0408XAR12-.0408YAR12-.0408VAR12+
 .8XP1-.8YP1+.7333XP2-.7333YP2+.6667XP3-.6667YP3+.5XP4-.6YP4+
 .5333XP5-.5333YP5+.4667XP6-.4667YP6+.4XP7-.4YP7+.3333XP8-.3333YP8+
 .2667XP9-.2667YP9+.2XP10-.2YP10+.1333XP11-.1333YP11+.0667XP12-
 .0667YP12+.03XDP11+.03XDP22+.03XDP33+.03XDP44+.03XDP55+.03XDP66+.03XDP77+
 .03XDP88+.03XDP99+.03XDP101+.03XDP111+.03XDP1212+
 .09XFP11+.09XFP22+.09XFP33+.09XFP44+.09XFP55+.09XFP66+.09XFP77+.09XFP88+
 .09XFP99+.09XFP101+.09XFP111+.09XFP1212+
 .0364XDP21+.0364XDP32+.0364XDP43+.0364XDP54+.0364XDP65+.0364XDP76+
 .0364XDP87+.0364XDP98+.0364XDP109+.0364XDP1110+.0364XDP1211+
 .0981XFP21+.0981XFP32+.0981XFP43+.0981XFP54+.0981XFP65+.0981XFP76+
 .0981XFP87+.0981XFP98+.0981XFP109+.0981XFP1110+.0981XFP1211+
 .11XTP11+.11XTP22+.11XTP33+.11XTP44+.11XTP55+.11XTP66+
 .0777XTP21+.0777XTP32+.0777XTP43+
 .0777XTP54+.0777XTP65+.9XTL3-.3VTL9-
 .0333MS1-.0333MS2-.0333MS3-.0333MS4-.0333MS5-.0333MS6-.0333MS7-
 .0333MS8-.0333MS9-.0333MS10-.0333MS11-.0333MS12-
 .02625T1D1-.02625T1D2-.02625T1D3-.02625T1D4-.02625T1D5-.02625T1D6-
 .02625T1D7-.02625T1D8-.02625T1D9-.02625T1D10-.02625T1D11-.02625T1D12-
 .10125T3D1-.10125T3D2-.10125T3D3-.10125T3D4-.10125T3D5-.10125T3D6-
 .10125T3D7-.10125T3D8-.10125T3D9-.10125T3D10-.10125T3D11-.10125T3D12

CONSTRAINTS

* FORMULATION OF FINANCING ALTERNATIVES

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*****
*                                     *
*               M P O S               *
*                                     *
*               VERSION 4.1           *
*                                     *
*  MULTI-PURPOSE OPTIMIZATION SYSTEM  *
*                                     *
*****

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***** PROBLEM NUMBER 1 *****

PREVISED

TITLE

HAKAN MAT -MBA THESIS - 'OPTIMAL SHORT-TERM FINANCING/INVESTMENT DECISION'

* ALL PARAMETERS ARE AT THEIR AVG.+ 2 STD.DEV. VALUES.

VARIABLES

XAR1 TO XAR12 YAR1 TO YAR12 VAR1 TO VAR12

XP1 TO XP12 YP1 TO YP12

XDP11 XFP11 XDP21 XFP21 XDP22 XFP22 XDP32 XFP32 XDP33 XFP33 XDP43 XFP43

XDP44 XFP44 XDP54 XFP54 XDP55 XFP55 XDP56 XFP56 XDP76 XFP76

XDP77 XFP77 XDP37 XFP37 XDP38 XFP38 XDP98 XFP98 XDP99 XFP99 XDP109 XFP109

XDP1010 XFP1010 XDP1110 XFP1110 XDP1111 XFP1111 XDP1211 XFP1211

XDP1212 XFP1212 I1 TO I12

XTP11 XTP22 XTP21 XTP33 XTP32 XTP44 XTP43 XTP55 XTP54 XTP66 XTP65

XTL3 VTL9

T1D1 TO T1D12 T3D1 TO T3D12

MS1 TO MS12

* O B J E C T I V E F U N C T I O N

MINIMIZE

.49XAR1-.49YAR1-.49VAR1+.4492XAR2-.4492YAR2-.4492VAR2+.4083XAR3-.4083YAR3-
.4083VAR3+.3675XAR4-.3675YAR4-.3675VAR4+.3265XAR5-.3265YAR5-.3265VAR5+
.2853XAR6-.2853YAR6-.2853VAR6+.245XAR7-.245YAR7-.245VAR7+.2042XAR8-
.2042YAR8-.2042VAR8+.1633XAR9-.1633YAR9-.1633VAR9+.1225XAR10-.1225YAR10-
.1225VAR10+.0817XAR11-.0817YAR11-.0817VAR11+.0408XAR12-.0408YAR12+.
.0408VAR12+.0817XP1-.0817YP1+.0817XDP11-.0817YDP11+.0817XDP12-.0817YDP12+
.0817XDP13-.0817YDP13+.0817XDP14-.0817YDP14+.0817XDP15-.0817YDP15+.0817XDP16-
.0817YDP16+.0817XDP17-.0817YDP17+.0817XDP18-.0817YDP18+.0817XDP19-.0817YDP19+
.0817XDP20-.0817YDP20+.0817XDP21-.0817YDP21+.0817XDP22-.0817YDP22+.0817XDP23-
.0817YDP23+.0817XDP24-.0817YDP24+.0817XDP25-.0817YDP25+.0817XDP26-.0817YDP26+
.0817XDP27-.0817YDP27+.0817XDP28-.0817YDP28+.0817XDP29-.0817YDP29+.0817XDP30-
.0817YDP30+.0817XDP31-.0817YDP31+.0817XDP32-.0817YDP32+.0817XDP33-.0817YDP33+
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.0817XDP48-.0817YDP48+.0817XDP49-.0817YDP49+.0817XDP50-.0817YDP50+.0817XDP51-
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.0817YDP478+.0817XDP479-.0817YDP479+.0817XDP480-.0817YDP480+.0817XDP481-
.0817YDP481+.0817XDP482-.0817YDP482+.0817XDP483-.0817YDP483+.0817XDP484-
.0817YDP484+.0817XDP485-.0817YDP485+.0817XDP486-.0817YDP486+.0817XDP487-
.0817YDP487+.0817XDP488-.0817YDP488+.0817XDP489-.0817YDP489+.0817XDP490-
.0817YDP490+.0817XDP491-.0817YDP491+.0817XDP492-.0817YDP492+.0817XDP493-
.0817YDP493+.0817XDP494-.0817YDP494+.0817XDP495-.0817YDP495+.0817XDP496-
.0817YDP496+.0817XDP497-.0817YDP497+.0817XDP498-.0817YDP498+.0817XDP499-
.0817YDP499+.0817XDP500-.0817YDP500+.0817XDP501-.0817YDP501+.0817XDP502-
.0817YDP502+.0817XDP503-.0817YDP503+.0817XDP504-.0817YDP504+.0817XDP505-
.0817YDP505+.0817XDP506-.0817YDP506+.0817XDP507-.0817YDP507+.0817XDP508-
.0817YDP508+.0817XDP509-.0817YDP509+.0817XDP510-.0817YDP510+.0817XDP511-
.0817YDP511+.0817XDP512-.0817YDP512+.0817XDP513-.0817YDP513+.0817XDP514-
.0817YDP514+.0817XDP515-.0817YDP515+.0817XDP516-.

 * PROBLEM NUMBER 1 *

JSING PREVIEWED

HACAN MAT -MBA THESIS - 'OPTIMAL SHORT-TERM FINANCING/INVESTMENT DECISION'

- * 1)PLEDGING OF ACCOUNTS RECEIVABLE
- * 1.1)TO ENSURE THAT VOLUNTARY & MANDATORY REPAYMENTS DO NOT EXCEED THE AMOUNT BORROWED.
- * 1.2)MAX. AMOUNT OF OUTSTANDING BORROWING UNDER PLEDGING OF ACCOUNTS RECEIVABLE IS BOUNDED FROM ABOVE.
- * 1.3)THE BANK WILL LEND UP TO 80% OF THE FACE VALUE OF PLEDGED ACCOUNTS RECEIVABLE.

1. XAR1-YAR1-VAR1 .GE. 0
2. XAR1-YAR1-VAR1 .LE. 128
3. XAR1+XAR2-YAR1-YAR2-VAR1-VAR2 .GE. 0
4. XAR1+XAR2-YAR1-YAR2-VAR1-VAR2 .LE. 201.393
5. XAR1+XAR2+XAR3-YAR1-YAR2-YAR3-VAR1-VAR2-VAR3 .GE. 0
6. XAR1+XAR2+XAR3-YAR1-YAR2-YAR3-VAR1-VAR2-VAR3 .LE. 206.088
7. XAR1+XAR2+XAR3+XAR4-YAR1-YAR2-YAR3-YAR4-VAR1-VAR2-VAR3-VAR4 .GE. 0
8. XAR1+XAR2+XAR3+XAR4-YAR1-YAR2-YAR3-YAR4-VAR1-VAR2-VAR3-VAR4 .LE. 193.553
9. XAR1+XAR2+XAR3+XAR4+XAR5-YAR1-YAR2-YAR3-YAR4-YAR5-VAR1-VAR2-VAR3-VAR4-VAR5 .LE. 220.734
10. XAR1+XAR2+XAR3+XAR4+XAR5-YAR1-YAR2-YAR3-YAR4-YAR5-VAR1-VAR2-VAR3-VAR4-VAR5 .GE. 0
11. XAR1+XAR2+XAR3+XAR4+XAR5+XAR6-YAR1-YAR2-YAR3-YAR4-YAR5-YAR6-VAR1-VAR2-VAR3-VAR4-VAR5-VAR6 .LE. 214.574
12. XAR1+XAR2+XAR3+XAR4+XAR5+XAR6-YAR1-YAR2-YAR3-YAR4-YAR5-YAR6-VAR1-VAR2-VAR3-VAR4-VAR5-VAR6 .GE. 0
13. XAR1+XAR2+XAR3+XAR4+XAR5+XAR6+XAR7-YAR1-YAR2-YAR3-YAR4-YAR5-YAR6-VAR1-VAR2-VAR3-VAR4-VAR5-VAR6-VAR7 .LE. 214.683
14. XAR1+XAR2+XAR3+XAR4+XAR5+XAR6+XAR7-YAR1-YAR2-YAR3-YAR4-YAR5-YAR6-VAR1-VAR2-VAR3-VAR4-VAR5-VAR6-VAR7 .GE. 0
15. XAR1+XAR2+XAR3+XAR4+XAR5+XAR6+XAR7+XAR8-YAR1-YAR2-YAR3-YAR4-YAR5-YAR6-YAR7-YAR8-VAR1-VAR2-VAR3-VAR4-VAR5-VAR6-VAR7-VAR8 .LE. 227.700
16. XAR1+XAR2+XAR3+XAR4+XAR5+XAR6+XAR7+XAR8-YAR1-YAR2-YAR3-YAR4-YAR5-YAR6-YAR7-YAR8-VAR1-VAR2-VAR3-VAR4-VAR5-VAR6-VAR7-VAR8 .GE. 0
17. XAR1+XAR2+XAR3+XAR4+XAR5+XAR6+XAR7+XAR8+XAR9-YAR1-YAR2-YAR3-YAR4-YAR5-YAR6-YAR7-YAR8-YAR9-VAR1-VAR2-VAR3-VAR4-VAR5-VAR6-VAR7-VAR8-VAR9 .LE. 239.919
18. XAR1+XAR2+XAR3+XAR4+XAR5+XAR6+XAR7+XAR8+XAR9-YAR1-YAR2-YAR3-YAR4-YAR5-YAR6-YAR7-YAR8-YAR9-VAR1-VAR2-VAR3-VAR4-VAR5-VAR6-VAR7-VAR8-VAR9 .GE. 0
19. XAR1+XAR2+XAR3+XAR4+XAR5+XAR6+XAR7+XAR8+XAR9+XAR10-YAR1-YAR2-YAR3-YAR4-YAR5-YAR6-YAR7-YAR8-YAR9-VAR1-VAR2-VAR3-VAR4-VAR5-VAR6-VAR7-VAR8-VAR9-VAR10 .LE. 237.990
20. XAR1+XAR2+XAR3+XAR4+XAR5+XAR6+XAR7+XAR8+XAR9+XAR10-YAR1-YAR2-YAR3-YAR4-YAR5-YAR6-YAR7-YAR8-YAR9-VAR1-VAR2-VAR3-VAR4-VAR5-VAR6-VAR7-VAR8-VAR9-VAR10 .GE. 0
21. XAR1+XAR2+XAR3+XAR4+XAR5+XAR6+XAR7+XAR8+XAR9+XAR10+XAR11-YAR1-YAR2-YAR3-YAR4-YAR5-YAR6-YAR7-YAR8-YAR9-YAR10-VAR1-VAR2-VAR3-VAR4-VAR5-VAR6-VAR7-VAR8-VAR9-VAR10-VAR11 .LE. 228.449
22. XAR1+XAR2+XAR3+XAR4+XAR5+XAR6+XAR7+XAR8+XAR9+XAR10+XAR11-YAR1-YAR2-YAR3-YAR4-YAR5-YAR6-YAR7-YAR8-YAR9-YAR10-VAR1-VAR2-VAR3-VAR4-VAR5-VAR6-VAR7-VAR8-VAR9-VAR10-VAR11 .GE. 0
23. XAR1+XAR2+XAR3+XAR4+XAR5+XAR6+XAR7+XAR8+XAR9+XAR10+XAR11+XAR12-YAR1-YAR2-YAR3-YAR4-YAR5-YAR6-YAR7-YAR8-YAR9-YAR10-YAR11-VAR1-VAR2-VAR3-VAR4-VAR5-VAR6-VAR7-VAR8-VAR9-VAR10-VAR11-VAR12 .LE. 253.275
24. XAR1+XAR2+XAR3+XAR4+XAR5+XAR6+XAR7+XAR8+XAR9+XAR10+XAR11+XAR12-YAR1-YAR2-YAR3-YAR4-YAR5-YAR6-YAR7-YAR8-YAR9-YAR10-YAR11-VAR1-VAR2-VAR3-VAR4-VAR5-VAR6-VAR7-VAR8-VAR9-VAR10-VAR11-VAR12 .GE. 0

 * PROBLEM NUMBER 1 *

JSING PREVIEWED

HACAN MAT -MBA THESIS - 'OPTIMAL SHORT-TERM FINANCING/INVESTMENT DECISION'

YAR2-YAR3-YAR4-YAR5-YAR6-YAR7-YAR8-YAR9-YAR10-YAR11-YAR12-VAR1-VAR2-
 VAR3-VAR4-VAR5-VAR6-VAR7-VAR8-VAR9-VAR10-VAR11-VAR12 .GE. 0
 * 1.4) MAX. AMOUNT OF BORROWING BY PLEDGING OF ACCOUNTS RECEIVABLE IN ANY
 * PERIOD IS BOUNDED FROM ABOVE.

25. XAR1 .LE. 150

26. XAR2 .LE. 150

27. XAR3 .LE. 150

28. XAR4 .LE. 150

29. XAR5 .LE. 150

30. XAR6 .LE. 150

31. XAR7 .LE. 150

32. XAR8 .LE. 150

33. XAR9 .LE. 150

34. XAR10 .LE. 150

35. XAR11 .LE. 150

36. XAR12 .LE. 150

* 1.5) TO DETERMINE MANDATORY REPAYMENTS RESULTING FROM THE COLLECTION OF
 * ACCOUNTS RECEIVABLE BY THE BANK DURING THE PERIOD TO DECREASE THE
 * OUTSTANDING AMOUNT.

37. VAR1 = 0

38. .5XAR1-.6YAR1-.5VAR1-VAR2 = 0

39. .5XAR1+.6XAR2-.5YAR1-.6YAR2-.5VAR1-.5VAR2-VAR3 = 0

40. .5XAR1+.5XAR2+.5XAR3-.6YAR1-.5YAR2-.6YAR3-.5VAR1-.5VAR2-.6VAR3-VAR4 = 0

41. .5XAR1+.6XAR2+.5XAR3+.6XAR4-.5YAR1-.6YAR2-.5YAR3-.5YAR4-.6VAR1-.5VAR2-
 .5VAR3-.6VAR4-VAR5 = 0

42. .5XAR1+.6XAR2+.5XAR3+.6XAR4+.5XAR5-.6YAR1-.5YAR2-.6YAR3-.6YAR4-.6YAR5-
 .5VAR1-.6VAR2-.5VAR3-.6VAR4-.6VAR5-VAR6 = 0

43. .5XAR1+.6XAR2+.5XAR3+.6XAR4+.5XAR5+.6XAR6-.6YAR1-.6YAR2-.5YAR3-.6YAR4-
 .6YAR5-.6YAR6-.6VAR1-.6VAR2-.5VAR3-.6VAR4-.5VAR5-.6VAR6-VAR7 = 0

44. .5XAR1+.6XAR2+.5XAR3+.6XAR4+.5XAR5+.6XAR6+.5XAR7-.5YAR1-.6YAR2-.6YAR3-
 .6YAR4-.6YAR5-.6YAR6-.6YAR7-.6VAR1-.6VAR2-.5VAR3-.5VAR4-.5VAR5-.6VAR6-
 .5VAR7-VAR8 = 0

45. .6XAR1+.6XAR2+.5XAR3+.6XAR4+.5XAR5+.6XAR6+.5XAR7+.6XAR8-.6YAR1-.6YAR2-
 .5YAR3-.6YAR4-.5YAR5-.6YAR6-.6YAR7-.6YAR8-.5VAR1-.6VAR2-.5VAR3-.6VAR4-
 .5VAR5-.6VAR6-.6VAR7-.6VAR8-VAR9 = 0

46. .5XAR1+.6XAR2+.5XAR3+.6XAR4+.5XAR5+.6XAR6+.5XAR7+.6XAR8+.5XAR9-.6YAR1-
 .6YAR2-.6YAR3-.5YAR4-.6YAR5-.6YAR6-.6YAR7-.5YAR8-.6YAR9-.6VAR1-.6VAR2-
 .5VAR3-.6VAR4-.5VAR5-.6VAR6-.6VAR7-.6VAR8-.5VAR9-VAR10 = 0

47. .5XAR1+.6XAR2+.5XAR3+.6XAR4+.5XAR5+.6XAR6+.5XAR7+.6XAR8+.6XAR9+.6XAR10-
 .6YAR1-.6YAR2-.5YAR3-.6YAR4-.5YAR5-.6YAR6-.6YAR7-.6YAR8-.5YAR9-.6YAR10-
 .6VAR1-.6VAR2-.5VAR3-.6VAR4-.5VAR5-.6VAR6-.6VAR7-.6VAR8-.6VAR9-.6VAR10-
 VAR11 = 0

48. .5XAR1+.6XAR2+.5XAR3+.6XAR4+.5XAR5+.6XAR6+.5XAR7+.6XAR8+.5XAR9+.6XAR10+
 .5XAR11-.6YAR1-.6YAR2-.5YAR3-.6YAR4-.5YAR5-.6YAR6-.6YAR7-.6YAR8-.5YAR9-
 .6YAR10-.6YAR11-.5VAR1-.6VAR2-.5VAR3-.6VAR4-.6VAR5-.6VAR6-.6VAR7-.6VAR8-
 .5VAR9-.6VAR10-.6VAR11-VAR12 = 0

* SHORT-TERM BANK CREDIT (6-MONTH MATURITY PERIOD)

* 2.1) TO ENSURE THAT VOLUNTARY & MANDATORY REPAYMENTS DO NOT EXCEED THE
 * AMOUNT BORROWED.

49. XP1-YP1 .GE. 0

50. XP1+XP2-YP1-YP2 .GE. 0

 * PROBLEM NUMBER 1 *

JSING PREVIEWED

HAKAN MAT -MBA THESIS - 'OPTIMAL SHORT-TERM FINANCING/INVESTMENT DECISION'

51. $XP1+XP2+XP3-YP1-YP2-YP3 \geq 0$
 52. $XP1+XP2+XP3+XP4-YP1-YP2-YP3-YP4 \geq 0$
 53. $XP1+XP2+XP3+XP4+XP5-YP1-YP2-YP3-YP4-YP5 \geq 0$
 54. $XP1+XP2+XP3+XP4+XP5+XP6-YP1-YP2-YP3-YP4-YP5-YP6 \geq 0$
 55. $XP1+XP2+XP3+XP4+XP5+XP6+XP7-YP1-YP2-YP3-YP4-YP5-YP6-YP7 \geq 0$
 56. $XP1+XP2+XP3+XP4+XP5+XP6+XP7+XP8-YP1-YP2-YP3-YP4-YP5-YP6-YP7-YP8 \geq 0$
 57. $XP1+XP2+XP3+XP4+XP5+XP6+XP7+XP8+XP9-YP1-YP2-YP3-YP4-YP5-YP6-YP7-YP8-YP9 \geq 0$
 58. $XP1+XP2+XP3+XP4+XP5+XP6+XP7+XP8+XP9+XP10-YP1-YP2-YP3-YP4-YP5-YP6-YP7-YP8-YP9-YP10 \geq 0$
 59. $XP1+XP2+XP3+XP4+XP5+XP6+XP7+XP8+XP9+XP10+XP11-YP1-YP2-YP3-YP4-YP5-YP6-YP7-YP8-YP9-YP10-YP11 \geq 0$
 60. $XP1+XP2+XP3+XP4+XP5+XP6+XP7+XP8+XP9+XP10+XP11+XP12-YP1-YP2-YP3-YP4-YP5-YP6-YP7-YP8-YP9-YP10-YP11-YP12 \geq 0$
 * 2.2) MAX. AMOUNT OF BORROWING BY S-T BANK CREDIT IN ANY PERIOD IS BOUNDED FROM ABOVE.
 61. $XP1 \leq 200$
 62. $XP2 \leq 200$
 63. $XP3 \leq 200$
 64. $XP4 \leq 200$
 65. $XP5 \leq 200$
 66. $XP6 \leq 200$
 67. $XP7 \leq 200$
 68. $XP8 \leq 200$
 69. $XP9 \leq 200$
 70. $XP10 \leq 200$
 71. $XP11 \leq 200$
 72. $XP12 \leq 200$
 * 2.3) AT LEAST THE AMOUNT OF THE LOAN THAT HAS TAKEN SHOULD BE REPAYED AFTER 6 PERIODS.
 73. $XP1-YP1-YP2-YP3-YP4-YP5-YP6-YP7 \leq 0$
 74. $XP2-YP2-YP3-YP4-YP5-YP6-YP7-YP8 \leq 0$
 75. $XP3-YP3-YP4-YP5-YP6-YP7-YP8-YP9 \leq 0$
 76. $XP4-YP4-YP5-YP6-YP7-YP8-YP9-YP10 \leq 0$
 77. $XP5-YP5-YP6-YP7-YP8-YP9-YP10-YP11 \leq 0$
 78. $XP6-YP6-YP7-YP8-YP9-YP10-YP11-YP12 \leq 0$
 * 2.4) MAX. AMOUNT OF OUTSTANDING BORROWING UNDER S-T BANK CREDIT IS BOUNDED FROM ABOVE.
 79. $XP1-YP1 \leq 500$
 80. $XP1+XP2-YP1-YP2 \leq 500$
 81. $XP1+XP2+XP3-YP1-YP2-YP3 \leq 500$
 82. $XP1+XP2+XP3+XP4-YP1-YP2-YP3-YP4 \leq 500$
 83. $XP1+XP2+XP3+XP4+XP5-YP1-YP2-YP3-YP4-YP5 \leq 500$
 84. $XP1+XP2+XP3+XP4+XP5+XP6-YP1-YP2-YP3-YP4-YP5-YP6 \leq 500$
 85. $XP1+XP2+XP3+XP4+XP5+XP6+XP7-YP1-YP2-YP3-YP4-YP5-YP6-YP7 \leq 500$
 86. $XP1+XP2+XP3+XP4+XP5+XP6+XP7+XP8-YP1-YP2-YP3-YP4-YP5-YP6-YP7-YP8 \leq 500$
 87. $XP1+XP2+XP3+XP4+XP5+XP6+XP7+XP8+XP9-YP1-YP2-YP3-YP4-YP5-YP6-YP7-YP8-YP9 \leq 500$
 88. $XP1+XP2+XP3+XP4+XP5+XP6+XP7+XP8+XP9+XP10-YP1-YP2-YP3-YP4-YP5-YP6-YP7-YP8-YP9-YP10 \leq 500$
 89. $XP1+XP2+XP3+XP4+XP5+XP6+XP7+XP8+XP9+XP10+XP11-YP1-YP2-YP3-YP4-YP5-YP6-YP7-YP8-YP9-YP10-YP11 \leq 500$

 * PROBLEM NUMBER 1 *

JSING PREVIEWED

HAKAN MAT -MBA THESIS - 'OPTIMAL SHORT-TERM FINANCING/INVESTMENT DECISION'

90. YP7-YP8-YP9-YP10-YP11 .LE. 500
 XP1+XP2+XP3+XP4+XP5+XP6+XP7+XP8+XP9+XP10+XP11+XP12-YP1-YP2-YP3-YP4-YP5-
 YP6-YP7-YP8-YP9-YP10-YP11-YP12 .LE. 500
 * 3)STRETCHING ACCOUNTS PAYABLE
 * 3.1)THE FINANCIAL MANAGER MAY STRETCH UP TO 75% OF THE FOREIGN PAYMENTS AND
 * 60% OF THE DOMESTIC PAYMENTS DUE
 * IN THE PERIOD IN WHICH THEY FIRST COME DUE.(STRETCHING 1. PERIOD)

91. XDP11 .LE. 16.731
 92. XFP11 .LE. 42.927
 93. XDP22 .LE. 45.510
 94. XFP22 .LE. 24.618
 95. XDP33 .LE. 43.640
 96. XFP33 .LE. 125.534
 97. XDP44 .LE. 50.520
 98. XFP44 .LE. 58.012
 99. XDP55 .LE. 51.800
 100. XFP55 .LE. 95.030
 101. XDP66 .LE. 56.039
 102. XFP66 .LE. 75.690
 103. XDP77 .LE. 90.313
 104. XFP77 .LE. 74.251
 105. XDP88 .LE. 93.134
 106. XFP88 .LE. 83.727
 107. XDP99 .LE. 97.644
 108. XFP99 .LE. 180.523
 109. XDP1010 .LE. 95.311
 110. XFP1010 .LE. 174.231
 111. XDP1111 .LE. 103.741
 112. XFP1111 .LE. 175.594
 113. XDP1212 .LE. 111.813
 114. XFP1212 .LE. 183.531
 * 3.2)STRETCHING 2. PERIOD

115. XDP21-.515XDP11 .LE. 0
 116. XFP21-.312XFP11 .LE. 0
 117. XDP32-.515XDP22 .LE. 0
 118. XFP32-.312XFP22 .LE. 0
 119. XDP43-.515XDP33 .LE. 0
 120. XFP43-.312XFP33 .LE. 0
 121. XDP54-.515XDP44 .LE. 0
 122. XFP54-.312XFP44 .LE. 0
 123. XDP65-.515XDP55 .LE. 0
 124. XFP65-.312XFP55 .LE. 0
 125. XDP76-.515XDP66 .LE. 0
 126. XFP76-.312XFP66 .LE. 0
 127. XDP87-.515XDP77 .LE. 0
 128. XFP87-.312XFP77 .LE. 0
 129. XDP98-.515XDP88 .LE. 0
 130. XFP98-.312XFP88 .LE. 0
 131. XDP109-.515XDP99 .LE. 0
 132. XFP109-.312XFP99 .LE. 0
 133. XDP1110-.515XDP1010 .LE. 0

 * PROBLEM NUMBER 1 *

JSING PREVIEWED

HAKAN MAT -MBA THESIS - 'OPTIMAL SHORT-TERM FINANCING/INVESTMENT DECISION'

134. XFP1110-.312XFP1010 .LE. 0
 135. XDP1211-.515XDP1111 .LE. 0
 136. XFP1211-.312XFP1111 .LE. 0
 * 4)STRETCHING TAXES PAYABLE
 * 4.1)THE FINANCIAL MANAGER MAY STRETCH UP TO 80% OF THE TAX PAYMENTS DUE
 * IN THE PERIOD IN WHICH THEY FIRST BECOME DUE.(STRETCHING 1.PERIOD)
 137. XTP11 .LE. 10
 138. XTP22 .LE. 14.400
 139. XTP33 .LE. 14.400
 140. XTP44 .LE. 10
 141. XTP55 .LE. 30
 142. XTP66 .LE. 10
 *STRETCHING 2.PERIOD
 143. XTP21-1.11XTP11 .LE. 0
 144. XTP32-1.11XTP22 .LE. 0
 145. XTP43-1.11XTP33 .LE. 0
 146. XTP54-1.11XTP44 .LE. 0
 147. XTP65-1.11XTP55 .LE. 0
 * 5)TERM LOAN (INSTALLMENT PAYMENTS ARE ONCE IN EVERY 6 MONTHS.)
 * 5.1)TERM LOAN CAN BE TAKEN ONLY IN THE THIRD MONTH.
 *THE AMOUNT OF BORROWING BY TERM LOAN IS LIMITED FROM BELOW.
 148. XTL3 .LE. 400
 * 5.2)THE PRINCIPAL AMOUNT MUST BE REPAYED IN EQUAL INSTALLMENTS.
 149. VTL9-.125XTL3 = 0
 * 6)INTEREST EXPENSES FROM PLEDGING OF ACCTS. REC. AND SHORT-TERM BANK CREDIT.
 *
 150. .J321XAR1-.J321YAR1-.J321VAR1+.J667XP1-.J667YP1-I1 = 0
 151. .J321XAR1-.J321YAR1-.J321VAR1+.J321XAR2-.J321YAR2-.J321VAR2+
 .J667XP1+.J667XP2-.J667YP1-.J667YP2-I2 = 0
 152. .J321XAR1+.J321XAR2+.J321XAR3-.J321YAR1-.J321YAR2-.J321YAR3-.J321VAR1-
 .J321VAR2-.J321VAR3+.J667XP1+.J667XP2+.J667XP3-.J667YP3-.J667YP2-.J667YP1-I3 = 0
 153. .J321XAR1+.J321XAR2+.J321XAR3+.J321XAR4-.J321YAR1-.J321YAR2-
 .J321YAR3-.J321YAR4-.J321VAR1-.J321VAR2-.J321VAR3-.J321VAR4+
 .J667XP1+.J667XP2+.J667XP3+.J667XP4-.J667YP1-.J667YP2-.J667YP3-.J667YP4-I4 = 0
 154. .J321XAR1+.J321XAR2+.J321XAR3+.J321XAR4+.J321XAR5-.J321YAR1-.J321YAR2-.J321YAR3-
 .J321YAR4-.J321YAR5-.J321VAR1-.J321VAR2-.J321VAR3-.J321VAR4-.J321VAR5+
 .J667XP1+.J667XP2+.J667XP3+.J667XP4+.J667XP5-.J667YP1-.J667YP2-
 .J667YP3-.J667YP4-.J667YP5-I5 = 0
 155. .J321XAR1+.J321XAR2+.J321XAR3+.J321XAR4+.J321XAR5+.J321XAR6-
 .J321YAR1-.J321YAR2-.J321YAR3-.J321YAR4-.J321YAR5-.J321YAR6-
 .J321VAR1-.J321VAR2-.J321VAR3-.J321VAR4-.J321VAR5-.J321VAR6+
 .J667XP1+.J667XP2+.J667XP3+.J667XP4+.J667XP5+.J667XP6-
 .J667YP1-.J667YP2-.J667YP3-.J667YP4-.J667YP5-I6 = 0
 156. .J321XAR1+.J321XAR2+.J321XAR3+.J321XAR4+.J321XAR5+.J321XAR6+.J321XAR7-
 .J321YAR1-.J321YAR2-.J321YAR3-.J321YAR4-.J321YAR5-.J321YAR6-.J321YAR7-
 .J321VAR1-.J321VAR2-.J321VAR3-.J321VAR4-.J321VAR5-.J321VAR6-.J321VAR7+
 .J667XP1+.J667XP2+.J667XP3+.J667XP4+.J667XP5+.J667XP6+.J667XP7-
 .J667YP1-.J667YP2-.J667YP3-.J667YP4-.J667YP5-.J667YP6-.J667YP7-I7 = 0
 157. .J321XAR1+.J321XAR2+.J321XAR3+.J321XAR4+.J321XAR5+.J321XAR6+
 .J321XAR7+.J321XAR8-.J321YAR1-.J321YAR2-.J321YAR3-.J321YAR4-.J321YAR5-

 * PROBLEM NUMBER 1 *

USING PREVIOUS
 HAKAN MAT -MBA THESIS - 'OPTIMAL SHORT-TERM FINANCING/INVESTMENT DECISION'

158. .J321YAR6-.J321YAR7-.J321YAR8+
 .J667XP1+.J667XP2+.J667XP3+.J667XP4+.J667XP5+.J667XP6+.J667XP7+.J667XP8-
 .J667YP1-.J667YP2-.J667YP3-.J667YP4-.J667YP5-.J667YP6-
 .J667YP7-.J667YP8-18 = 0
 .J321XAR1+.J321XAR2+.J321XAR3+.J321XAR4+.J321XAR5+.J321XAR6+.J321XAR7+
 .J321XAR8+.J321XAR9-.J321YAR1-.J321YAR2-.J321YAR3-.J321YAR4-.J321YAR5-
 .J321YAR6-.J321YAR7-.J321YAR8-.J321YAR9-.J321VAR1-.J321VAR2-.J321VAR3-
 .J321VAR4-.J321VAR5-.J321VAR6-.J321VAR7-.J321VAR8-.J321VAR9+
 .J667XP1+.J667XP2+.J667XP3+.J667XP4+.J667XP5+.J667XP6+.J667XP7+
 .J667XP8+.J667XP9-.J667YP1-.J667YP2-.J667YP3-.J667YP4-
 .J667YP5-.J667YP6-.J667YP7-.J667YP8-.J667YP9-19 = 0
 159. .J321XAR1+.J321XAR2+.J321XAR3+.J321XAR4+.J321XAR5+.J321XAR6+.J321XAR7+
 .J321XAR8+.J321XAR9+.J321XAR10-.J321YAR1-.J321YAR2-.J321YAR3-.J321YAR4-
 .J321YAR5-.J321YAR6-.J321YAR7-.J321YAR8-.J321YAR9-.J321YAR10-.J321VAR1-
 .J321VAR2-.J321VAR3-.J321VAR4-.J321VAR5-.J321VAR6-.J321VAR7-.J321VAR8-
 .J321VAR9-.J321VAR10+.J667XP1+.J667XP2+.J667XP3+.J667XP4+.J667XP5+
 .J667XP6+.J667XP7+.J667XP8+.J667XP9+.J667XP10-.J667YP1-.J667YP2-
 .J667YP3-.J667YP4-.J667YP5-.J667YP6-.J667YP7-.J667YP8-
 .J667YP9-.J667YP10-113 = 0
 160. .J321XAR1+.J321XAR2+.J321XAR3+.J321XAR4+.J321XAR5+.J321XAR6+.J321XAR7+
 .J321XAR8+.J321XAR9+.J321XAR10+.J321XAR11-.J321YAR1-.J321YAR2-
 .J321YAR3-.J321YAR4-.J321YAR5-.J321YAR6-.J321YAR7-.J321YAR8-.J321YAR9-
 .J321YAR10-.J321YAR11-.J321VAR1-.J321VAR2-.J321VAR3-.J321VAR4-.J321VAR5-
 .J321VAR6-.J321VAR7-.J321VAR8-.J321VAR9-.J321VAR10-.J321VAR11+
 .J667XP1+.J667XP2+.J667XP3+.J667XP4+.J667XP5+.J667XP6+.J667XP7+
 .J667XP8+.J667XP9+.J667XP10+.J667XP11-.J667YP1-.J667YP2-.J667YP3-
 .J667YP4-.J667YP5-.J667YP6-.J667YP7-.J667YP8-.J667YP9-
 .J667YP10-.J667YP11-111 = 0
 161. .J321XAR1+.J321XAR2+.J321XAR3+.J321XAR4+.J321XAR5+.J321XAR6+.J321XAR7+
 .J321XAR8+.J321XAR9+.J321XAR10+.J321XAR11+.J321XAR12-.J321YAR1-
 .J321YAR2-.J321YAR3-.J321YAR4-.J321YAR5-.J321YAR6-.J321YAR7-.J321YAR8-
 .J321YAR9-.J321YAR10-.J321YAR11-.J321YAR12-.J321VAR1-.J321VAR2-
 .J321VAR3-.J321VAR4-.J321VAR5-.J321VAR6-.J321VAR7-.J321VAR8-.J321VAR9-
 .J321VAR10-.J321VAR11-.J321VAR12+.J667XP1+.J667XP2+.J667XP3+
 .J667XP4+.J667XP5+.J667XP6+.J667XP7+.J667XP8+.J667XP9+.J667XP10+
 .J667XP11+.J667XP12-.J667YP1-.J667YP2-.J667YP3-.J667YP4-.J667YP5-
 .J667YP6-.J667YP7-.J667YP8-.J667YP9-.J667YP10-
 .J667YP11-.J667YP12-112 = 0
 * 7) POLICY CONSTRAINTS
 * 7.1) THE PROPORTION OF ONE-MONTH MATURITY INVESTMENTS TO TOTAL INVESTMENTS
 * IS BOUNDED FROM BELOW.
 162. .25T101+.25MS1-.75T301 .GE. 0
 163. .30T102+.30MS2-.70T301-.70T302 .GE. 0
 164. .4T103+.4MS3-.6T301-.5T302-.6T303 .GE. 0
 165. .4T104+.4MS4-.6T302-.5T303-.6T304 .GE. 0
 166. .4T105+.4MS5-.6T303-.5T304-.6T305 .GE. 0
 167. .4T106+.4MS6-.6T304-.5T305-.6T306 .GE. 0
 168. .4T107+.4MS7-.6T305-.6T306-.6T307 .GE. 0
 169. .4T108+.4MS8-.6T306-.5T307-.6T308 .GE. 0
 170. .4T109+.4MS9-.6T307-.5T308-.6T309 .GE. 0
 171. .4T110+.4MS10-.6T308-.5T309-.6T310 .GE. 0

* PROBLEM NUMBER 1 *

USING PREVIOUS

HACAN YAT -MBA THESIS - 'OPTIMAL SHORT-TERM FINANCING/INVESTMENT DECISION'

172. .4T1011+.4MS11-.6T309-.6T3010-.6T3011 .GE. 0
173. .4T1012+.4MS12-.6T3010-.6T3011-.6T3012 .GE. 0
* 7.2)THE PROPORTION OF MARKETABLE SECURITIES INVESTMENTS TO TOTAL ONE-MONTH
* MATJORITY INVESTMENTS IS BOUNDED FROM ABOVE.
174. .7MS1-.3T101 .LE. 0
175. .7MS2-.3T102 .LE. 0
176. .7MS3-.3T103 .LE. 0
177. .7MS4-.3T104 .LE. 0
178. .7MS5-.3T105 .LE. 0
179. .7MS6-.3T106 .LE. 0
180. .7MS7-.3T107 .LE. 0
181. .7MS8-.3T108 .LE. 0
182. .7MS9-.3T109 .LE. 0
183. .7MS10-.3T1010 .LE. 0
184. .7MS11-.3T1011 .LE. 0
185. .7MS12-.3T1012 .LE. 0

* I N V E S T M E N T A L T E R N A T I V E S

* 1)TERM DEPOSITS A)ONE-MONTH MATURITY B)THREE-MONTHS MATURITY
* 1.1)THE PROPORTION OF ONE-MONTH MATJORITY TERM DEPOSITS TO TOTAL OUTSTANDING
* TERM DEPOSITS IS BOUNDED FROM BELOW.

186. .25T101-.75T301 .GE. 0
187. .25T102-.75T301-.75T302 .GE. 0
188. .25T103-.75T301-.75T302-.75T303 .GE. 0
189. .25T104-.75T302-.75T303-.75T304 .GE. 0
190. .25T105-.75T303-.75T304-.75T305 .GE. 0
191. .25T106-.75T304-.75T305-.75T306 .GE. 0
192. .25T107-.75T305-.75T306-.75T307 .GE. 0
193. .25T108-.75T306-.75T307-.75T308 .GE. 0
194. .25T109-.75T307-.75T308-.75T309 .GE. 0
195. .25T1010-.75T308-.75T309-.75T3010 .GE. 0
196. .25T1011-.75T309-.75T3010-.75T3011 .GE. 0
197. .25T1012-.75T3010-.75T3011-.75T3012 .GE. 0
198. T101 .LE. 150
199. T102 .LE. 150
200. T103 .LE. 150
201. T104 .LE. 150
202. T105 .LE. 150
203. T106 .LE. 150
204. T107 .LE. 150
205. T108 .LE. 150
206. T109 .LE. 150
207. T1010 .LE. 150
208. T1011 .LE. 150
209. T1012 .LE. 150
210. T301 .LE. 80
211. T302 .LE. 80
212. T303 .LE. 80
213. T304 .LE. 80
214. T305 .LE. 80

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USING PREVIEWED

HACAN MAT -MBA THESIS - 'OPTIMAL SHORT-TERM FINANCING/INVESTMENT DECISION'

215. T306 .LE. 80
 216. T307 .LE. 80
 217. T308 .LE. 80
 218. T309 .LE. 80
 219. T3010 .LE. 80
 220. T3011 .LE. 80
 221. T3012 .LE. 80

* 2)MARKETABLE SECURITIES

222. MS1 .LE. 90
 223. MS2 .LE. 90
 224. MS3 .LE. 90
 225. MS4 .LE. 90
 226. MS5 .LE. 90
 227. MS6 .LE. 90
 228. MS7 .LE. 90
 229. MS8 .LE. 90
 230. MS9 .LE. 90
 231. MS10 .LE. 90
 232. MS11 .LE. 90
 233. MS12 .LE. 90

* F U L F I L M E N T O F R E Q U I R E M E N T S

234. .70XAR1-.70YAR1-.70VAR1+XP1-YP1+XDP11+XFP11+XTP11-T101-T301-
 MS1 .GE. -134.474
 235. .70XAR2-.70YAR2-.70VAR2+XP2-YP2+XDP22+XFP22+XDP21+XFP21-1.03XDP11-1.04XFP11+
 XTP22+XTP21-1.11XTP11-T102-T302-MS2+1.02625T101+1.0333MS1 .GE. 28.455
 236. .70XAR3-.70YAR3-.70VAR3+XP3-YP3+XDP33+XFP33+XDP32+XFP32-1.03XDP22-1.04XFP22-
 1.0309XDP21-1.0415XFP21+XTP33+XTP32+XTL3-1.11XTP22-1.0777XTP21-T103-T303-MS3+
 1.02625T102+1.0333MS2-I1-I2 .GE. 340.611
 237. .70XAR4-.70YAR4-.70VAR4+XP4-YP4+XDP44+XFP44+XDP43+XFP43-1.03XDP33-1.04XFP33-
 1.0309XDP32-1.0415XFP32+XTP44+XTP43-1.11XTP33-1.0777XTP32-T104-T304-MS4+
 1.02625T103+1.0333MS3+1.10125T301 .GE. 123.270
 238. .70XAR5-.70YAR5-.70VAR5+XP5-YP5+XDP55+XFP55+XDP54+XFP54-1.03XDP44-1.04XFP44-
 1.0309XDP43-1.0415XFP43+XTP55+XTP54-1.11XTP44-1.0777XTP43-T105-T305-MS5+
 1.02625T104+1.0333MS4+1.10125T302 .GE. 122.462
 239. .70XAR6-.70YAR6-.70VAR6+XP6-YP6+XDP66+XFP66+XDP65+XFP65-1.03XDP55-1.04XFP55-
 1.0309XDP54-1.0415XFP54+XTP66+XTP65-1.11XTP55-1.0777XTP54-T106-T306-MS6+
 1.02625T105+1.0333MS5+1.10125T303-I3-I4-I5 .GE. -74.125
 240. .70XAR7-.70YAR7-.70VAR7+XP7-YP7+XDP77+XFP77+XDP76+XFP76-1.03XDP66-1.04XFP66-
 1.0309XDP65-1.0415XFP65-1.11XTP66-1.0777XTP65-T107-
 T307-MS7+1.02625T106+1.0333MS6+1.10125T304 .GE. 208.745
 241. .70XAR8-.70YAR8-.70VAR8+XP8-YP8+XDP88+XFP88+XDP87+XFP87-1.03XDP77-1.04XFP77-
 1.0309XDP76-1.0415XFP76-T108-
 T308-MS8+1.02625T107+1.0333MS7+1.10125T305 .GE. -183.002
 242. .70XAR9-.70YAR9-.70VAR9+XP9-YP9+XDP99+XFP99+XDP98+XFP98-1.03XDP88-1.04XFP88-
 1.0309XDP97-1.0415XFP97-.3XTL3-VTL9-T109-
 T309-MS9+1.02625T108+1.0333MS8+1.10125T306-I6-I7-I8 .GE. -47.560
 243. .70XAR10-.70YAR10-.70VAR10+XP10-YP10+XDP100+XFP100+XDP109+XFP109-
 1.03XDP99-1.04XFP99-1.0309XDP98-1.0415XFP98-
 T1010-T3010-MS10+1.02625T109+1.0333MS9+

* PROBLEM NUMBER 1 *

JSING PREVIEWED

HAKAN MAT -MBA THESIS - 'OPTIMAL SHORT-TERM FINANCING/INVESTMENT DECISION'

1.10125T3D7 .GE. -53.250
244. .70XAR11-.70YAR11-.70VAR11+XP11-YP11+XDP1111+XFP1111+XDP1113+XFP1110-
1.03XDP1010-1.04XFP1010-1.0309XDP109-1.0416XFP109-
T1D11-T3D11-MS11+1.02625T1D10+
1.0333MS10+1.10125T3D3 .GE. -31.357
245. .70XAR12-.70YAR12-.70VAR12+XP12-YP12+XDP1212+XFP1212+XDP1211+XFP1211-
1.03XDP1111-1.04XFP1111-1.0309XDP1110-1.0415XFP1110-
T1D12-T3D12-MS12+1.02625T1D11+
1.0333MS11+1.10125T3D9-I9-I10-I11 .GE. -77.869
RNGOBJ
RNGRHS
OPTIMIZE

APPENDIX - 13

THE TABLES OF SOURCES AND USES OF
CASH TO FULFILL THE REQUIREMENTS AT
EACH RISK LEVEL IN THE OPTIMAL SOLUTION

RISK LEVEL I	JANUARY	FEBRUARY	MARCH	APRIL	MAY	JUNE	JULY	AUGUST	SEPTEMBER	OCTOBER	NOVEMBER	DECEMBER	TOTAL
REQUIREMENT	-75.122	87.456	387.379	222.489	208.691	36.698	358.211	-17.724	35.928	40.729	71.677	26.596	1,383.008
SOURCES OF CASH													
Pled.Acct.Rec.x0.70				82.922	50.643	37.692	60.042	30.435	67.906	63.984	61.735	66.049	521.407
Short-Term Bank Credit				60.95	194.32	194.32	300	47.085	300	168.839	228.807	228.807	1,494.321
Term Loan			452.629										452.629
Stret.Dom.Pay.(1 st Per.)		10.203		22.324	24.978		40.899		36.598		34.885	35.05	204.937
Stret.For.Pay.(1 st Per.)											53.825	40.946	94.771
Stret.Dom.Pay.(2 nd Per.)					11.497							17.966	29.463
Stret.For.Pay.(2 nd Per.)												16.793	16.793
Maturing One-Month T.D.		52.585		38.318									90.903
Maturing Government Sec.		22.537		16.422									38.959
Maturing Three-Month T.D.													
Int.Income One-Month T.D.		1.38		1.01									2.39
Int.Income Gov.Sec.		0.75		0.547									1.297
Int.Income Three-Month T.D.													
TOTAL SOURCES		87.456	452.629	222.489	281.438	232.012	400.941	77.52	404.941	232.823	150.445		2,947.87
USES OF CASH													
Vol.Pay.Pled.Acct.Rec.x0.70													
Man.Repay.Pled.Acct.Rec.x0.70					49.753	50.287	42.73	53.117	39.507	56.547	61.009	61.445	414.394
Repay Short-Term Bank Credit						78.71				97.851	17.76		194.321
Repayment of Term Loan									56.579				56.579
Repay.Stret.Dom.Pay.(1 st Per.)		10.203		22.324	24.978		40.899		36.598			34.885	169.887
Repay.Stret.For.Pay.(1 st Per.)												53.825	53.825
Repay.Stret.Dom.Pay.(2 nd Per.)					11.497								11.497
Repay.Stret.For.Pay.(2 nd Per.)													
Investment One-Month T.D.	52.585		38.318										90.903
Investment Gov. Sec.	22.537		16.422										38.959
Investment Three-Month T.D.													
Int.Exp.Stret.Dom.Pay.(1 st Per.)			0.306		0.67	0.749		1.227		1.098		1.047	5.097
Int.Exp.Stret.For.Pay.(2 nd Per.)												2.153	2.153
Int.Exp.Stret.Dom.Pay.(2 nd Per.)						0.355							0.355
Int.Exp.Stret.For.Pay.(2 nd Per.)													
Interest Payment Term Loan									135.789				135.789
Interest Payment Bank Loans						28.738			136.701			225.661	391.1
TOTAL USES	75.122		65.249		72.747	195.314	42.73	95.243	368.576	192.094	78.769	379.016	1,564.86

RISK LEVEL II	JANUARY	FEBRUARY	MARCH	APRIL	MAY	JUNE	JULY	AUGUST	SEPTEMBER	OCTOBER	NOVEMBER	DECEMBER	TOTAL
REQUIREMENT	-89.965	72.706	375.687	198.934	187.134	3.992	320.895	-59.043	15.031	17.235	45.918	0.479	1,089,003
<u>SOURCES OF CASH</u>													
Pled.Acct.Rec.x0.70			88.075	60.851	63.055	63.748	62.229	71.548	105	72.842	69.621	77.869	734.838
Short-Term Bank Credit			78.953	174.274	178.283	113.402	300		300	97.739		172.472	1,415.123
Term Loan			163.589										163.589
Stret.Dom.Pay.(1 st Per.)			24.697	29.373	31.684		21.272	42.228			53.349	54.349	250.844
Stret.For.Pay.(1 st Per.)											84.543	77.842	162.385
Stret.Dom.Pay.(2 nd Per.)			12.719	15.127								27.475	55.321
Stret.For.Pay.(2 nd Per.)												26.377	26.377
Maturing One-Month T.D.		62.976	13.868										76.844
Maturing Gov. Sec.		26.989	5.943										32.932
Maturing Three-Month T.D.													
Int.Income One-Month T.D.		1.653	0.364										2.017
Int.Income Gov. Sec.		0.899	0.198										1.097
Int.Income Three-Month T.D.													
TOTAL SOURCES		92.517	375.687	277.216	288.149	177.15	383.50	113.776	405	170.581	207.513	436.276	2,927.367
<u>USES OF CASH</u>													
Vol.Pay.Pled.Acct.Rec.x0.70									37.584				37.584
Man.Repay.Pled.Acct.Rec.x0.70				52.845	57.649	60.892	62.606	62.38	67.881	67.602	70.746	70.071	572.671
Repay.Short-Term Bank Credit								88.53		85.744	90.849		265.123
Repayment of Term Loan									20.449				20.449
Repay.Stret.Dom.Pay.(1 st Per.)				24.697	29.373	31.684		21.272	42.228			53.349	202.603
Repay.Stret.For.Pay.(1 st Per.)												84.543	84.543
Repay.Stret.Dom.Pay.(2 nd Per.)					12.719	15.127							27.846
Repay.Stret.For.Pay.(2 nd Per.)													
Investment One-Month T.D.	62.976	13.868											76.844
Investment Gov. Sec.	26.983	5.943											32.932
Investment Three-Month T.D.													
Int.Exp.Stret.Dom.Pay.(1 st Per.)				0.741	0.881	0.951		1.598	0.638			1.6	7.676
Int.Exp.Stret.For.Pay.(1 st Per.)												3.382	3.382
Int.Exp.Stret.Dom.Pay.(2 nd Per.)					0.393	0.467							0.86
Int.Exp.Stret.For.Pay.(2 nd Per.)													
Interest Payment Term Loan									49.077				
Interest Payment Bank Loans						64.037			171.483			222.853	458.372
TOTAL USES	89.965	19.811		78.283	101.015	173.158	62.606	173.78	389.34	153.346	161.595	435.797	1,838.696

RISK LEVEL III	JANUARY	FEBRUARY	MAR	APRIL	MAY	JUNE	JULY	AUGUST	SEPTEMBER	OCTOBER	NOVEMBER	DECEMBER	TOTAL
REQUIREMENT	-104.808	57.956	363.995	175.379	165.576	-28.713	283.578	-100.363	-5.866	-6.26	20.16	-25.637	794.997
<u>SOURCES OF CASH</u>													
Pled.Acct.Rec.x0.70			105	67.239	75.467	71.964	71.551	80.792	105	81.7	105	89.688	853.402
Short-Term Bank Credit			250	168.376	136.021	250	250		233.665		23.136	48.999	1.360.197
Term Loan													
Stret.Dom.Pay.(First Per.)			32.678	36.422	38.389		33.804					73.432	214.725
Stret.For.Pay.(First Per.)												46.947	46.947
Stret.Dom.Pay.(Second Per.)					18.757								18.757
Stret.For.Pay.(Second Per.)													
Maturing Three-Month T.D.		73.366	34.877										108.243
Maturing Government Sec.		31.442	14.947										46.389
Maturing Three-Month T.D.													
Int.Income One-Month T.D.		1.926	0.916										2.842
Int.Income Gov.Sec.		1.047	0.498										1.545
Int.Income Three-Month T.D.													
TOTAL SOURCES		107.781	438.916	272.037	268.634	321.964	355.355	80.792	338.665	81.7	128.136	259.066	2.653.046
<u>USES OF CASH</u>													
Vol.Pay.Pled.Acct.Rec.x0.70									25.327		27.494		52.821
Man.Repay.Pled.Acct.Rec.x0.7				63	65.544	71.497	71.777	71.642	77.132	78.656	80.483	78.697	658.429
Repay.Short-Term Bank Credit			74.921			138.446		74.694	62.832	9.304			360.197
Repayment of Term Loan													
Repay.Stret.Dom.Pay.(1 st per.)				32.678	36.422	38.389		33.804					141.293
Repay.Stret.For.Pay.(1 st per.)													
Repay.Stret.Dom.Pay.(2 nd per.)						18.757							18.757
Repay.Stret.For.Pay.(2 nd per.)													
Investment One-Month Term D.	73.366	34.877											108.243
Investment Gov.Sec.	31.442	14.947											46.389
Investment Three-Month T.D.													
Int.Exp.Stret.Dom.Pay(1 st Per)				0.98	1.093	1.152		1.014					4.239
Int.Exp.Stret.For.Pay(1 st Per.)													
Int.Exp.Stret.Dom.Pay.(2 nd Per.)						0.58							0.58
Int.Exp.Stret.For.Pay.(2 nd Per.)													
Interest Payment Term Loan													
Interest Payment Bank Loans						81.857			179.24			206.007	467.104
TOTAL USES	104.808	49.824	74.921	96.658	103.059	350.678	71.777	181.154	344.531	87.96	107.977	284.704	1.858.051

RISK LEVEL IV	JANUARY	FEBRUARY	MARCH	APRIL	MAY	JUNE	JULY	AUGUST	SEPTEMBER	OCTOBER	NOVEMBER	DECEMBER	TOTAL
REQUIREMENT	-119.651	43.205	352.303	151.824	144.019	-61.419	246.261	-141.683	-26.763	-29.755	-5.598	-51.753	501
<u>SOURCES OF CASH</u>													
Pled.Acct. Rec.x0.70			105	80.398	105	0.	105	90.035	105	90.558	85.392	101.508	948.07
Short-Term Bank Credit			124.539	200	106.872	200	200		112.103	200		31.61	1,175.124
Term Loan													
Stret.Dom.Pay.(First Per.)			40.659	43.471	45.094		46.335					48.762	224.321
Stret.For.Pay.(First Per.)													
Stret.Dom.Pay.(Second Per.)					22.388								22.388
Stret.For.Pay.(Second Per.)													
Maturing One Month Term D.		83.756	55.888										139.644
Maturing Government Sec.		35.895	23.952										59.847
Maturing Three-Month T.D.													
Int.Income One-Month T.D.		2.199	1.467										3.666
Int.Income Gov.Sec.		1.195	0.798										1.993
Int.Income Three-Month T.D.													
<u>TOTAL SOURCES</u>		123.045	352.303	323.869	279.354	280.18	351.335	90.035	217.103	290.558	85.392	181.88	2,575.054
													54.316
<u>USES OF CASH</u>													
Vol.Pay.Pled.Acct.Rec.x0.70					17.121		24.125		13.07				54.316
Man.Repay.Pled.Acct.Rec.x0.7				63	73.438	82.103	80.949	80.905	86.383	89.711	90.22	87.323	734.032
Repay Short-Term Bank Credit.				67.166		123.498		103.088		230.602	0.77		525.124
Repayment of Term Loan													
Repay.Stret.Dom.Pay.(1 st per.)				40.659	43.471	45.094		46.335					175.559
Repay Stret.For.Pay.(1 st per)													
Repay Stret.Dom.Pay.(2 nd per)						22.388							22.388
Repay Stret.For.Pay.(2 nd per)													
Investment One-Month Term D.	83.756	55.888											139.644
Investment Gov.Sec.	35.895	23.952											59.847
Investment Three-Month T.D.													
Int.Exp.Stret.Dom.Pay.(1 st per.)				1.22	1.304	1.353		1.39					5.267
Int.Exp.Stret.For.Pay(1 st per)													
Int.Exp.Stret Dom.Pay(2 nd per)						0.672							0.672
Int.Exp.Stret.For.Pay(2 nd per)													
Interest Payment Term-Loan													
Interest Payment Bank Loans						66.472			144.414			146.31	357.196
<u>TOTAL USES</u>	119.651	79.84		172.045	135.334	341.58	105.074	231.718	243.867	320.313	90.99	233.633	2,074.045

RISK LEVEL V	JANUARY	FEBRUARY	MARCH	APRIL	MAY	JUNE	JULY	AUGUST	SEPTEMBER	OCTABER	NOVEMBER	DECEMBER	TOTAL
REQUIREMENT	-134.494	28.455	340.611	128.27	122.462	-94.125	208.945	-183.002	-47.66	-53.25	-31.357	-77.869	206.986
<u>SOURCES OF CASH</u>			105	93.557	100.291	88.396	90.197	105	104.187	99.415	93.277	105	984.321
Pled. Acct. Rec.x0.70													
Short-Term Bank Credit			74.001	97.292	103.741	16.364	200		56.797			0.264	548.459
Term Loan													
Stret.Dom.Pay.(First Per.)			48.64	50.52	51.8		8.869						159.829
Stret.For.Pay.(First Per.)													
Stret.Dom.Pay.(Second Per.)													
Stret.For.Pay.(Second Per.)													
Maturing One-Month Term D.		94.146	76.898										
Maturing Government Sec.		40.348	32.956										
Maturing Three-Month T.D.													
Int.Income One-Month T.D.		2.471	2.019										3.666
Int.Income Gdv.Sec.		1.344	1.097										1.993
Int.Income Three-Month T.D.													
TOTAL SOURCES		138.309	340.611	241.369	255.832	104.76	299.066	105	160.984	99.415	93.277	105.264	1,943.887
<u>USES OF CASH</u>													
Vol.Pay.Pled.Acct.Rec.x0.70								5.721					5.721
Man.Pay.Pled.Acct.Rec.x0.7				63	81.334	92.708	90.121	90.167	95.634	100.766	99.956	95.948	809.634
Repay Short-Term Bank Cr.								182.979		51.9	24.678		259.557
Repayment of Term Loan													
Repay.Stret.Dom.Pay.(1 st per.)				48.64	50.52	51.8		8.869					159.829
Repay.Stret.For.Pay.(1 st per.)													
Repay.Stret.Dom.Pay.(2 nd per.)													
Repay.Stret.For.Pay.(2 nd per.)													
Investment One-Month Term D.	94.146	76.898											171.044
Investment Gov.Sec.	40.348	32.956											73.304
Investment Three-Month T.D.													
Int.Exp.Stret.Dom.Pay.(1 st per)				1.459	1.516	1.554		0.266					4.795
Int.Exp.Stret.For.Pay.(1 st per)													
Int.Exp.stret.Dom.Pay.(2 nd per)													
Int.Exp.Stret.For.Pay.(2 nd per)													
Interest Payment Term Loan													
Interest Payment Bank Loans						52.823			113.01			87.184	253.017
TOTAL USES	134.494	109.854		113.099	133.37	198.885	90.121	288.002	208.644	152.666	124.634	183.132	1,736.901

APPENDIX 14. Theoretical Basis Of Sensitivity Analysis

1. Matrix Form of Simplex Method

Consider the following L.P. problem

$$P^0 \quad \min cx = z$$

$$Ax = b \quad \text{or in tableau form}$$

$$x \geq 0$$

	$x_1 x_2 \dots x_n$	z
b	A	0 0 \vdots
0	c	-1

Let B' be a particular basis of the coefficient matrix A . Rearrange the columns of A and renumber the components of x and c such that

$$x = (x_1, x_2, \dots, x_m, x_{m+1}, \dots, x_n) = (x_B, x_D)$$

where

$x_B = (x_1, \dots, x_m)$ are the basic variables with respect to B'

$x_D = (x_{m+1}, x_{m+2}, \dots, x_n)$ are the nonbasic variables with respect to B'

$$A = (B, D)$$

where

$$B = (A_1^T, \dots, A_m^T) \quad A_i^T \in B' \quad i = 1, \dots, m$$

$$D = (A_{m+1}^T, \dots, A_n^T) \quad A_i^T \notin B' \quad i = m+1, \dots, n$$

$$C = (C_1, C_2, \dots, C_m, C_{m+1}, \dots, C_n) \quad (C_B, C_D)$$

where

$$C_B = (C_1, \dots, C_m)$$

$$C_D = (C_{m+1}, \dots, C_n)$$

Then problem P^0 can be expressed as

$$\min C_B x_B + C_D x_D = z$$

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$$P^0 \quad \min cx = z$$

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$$x \geq 0$$

	$x_1 x_2 \dots x_n$	z
b	A	0 0 \vdots
0	c	-1

Let B' be a particular basis of the coefficient matrix A . Rearrange the columns of A and renumber the components of x and c such that

$$x = (x_1, x_2, \dots, x_m, x_{m+1}, \dots, x_n) = (x_B, x_D)$$

where

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$$A = (B, D)$$

where

$$B = (A_1^T, \dots, A_m^T) \quad A_i^T \in B' \quad i = 1, \dots, m$$

$$D = (A_{m+1}^T, \dots, A_n^T) \quad A_i^T \notin B' \quad i = m+1, \dots, n$$

$$C = (C_1, C_2, \dots, C_m, C_{m+1}, \dots, C_n) \quad (C_B, C_D)$$

where

$$C_B = (C_1, \dots, C_m)$$

$$C_D = (C_{m+1}, \dots, C_n)$$

Then problem P^0 can be expressed as

$$\min C_B x_B + C_D x_D = z$$

$$\begin{aligned}
 P^1 \quad & \text{s.t. } Bx_B + Dx_D = b \\
 & x_B \geq 0, x_D \geq 0
 \end{aligned}$$

The constraints in problem P^1 can be written as

$$Ix_B + B^{-1}Dx_D = B^{-1}b \quad (I \text{ is an } m \times m \text{ identity matrix})$$

and the vector variable x_B in the objective function can be replaced by

$$x_B = B^{-1}b - B^{-1}Dx_D$$

so the objective function becomes

$$z = C_B B^{-1}b + (C_D - C_B B^{-1}D)x_D$$

Thus problem P^1 is equivalent to

$$\min (C_D - C_B B^{-1}D)x_D = z - C_B B^{-1}b$$

$$\begin{aligned}
 P^2 \quad & \text{s.t. } Ix_B + B^{-1}Dx_D = B^{-1}b \\
 & x_B \geq 0, x_D \geq 0
 \end{aligned}$$

or in tableau form

	1	2	...	m	m+1	...	n
1	$B^{-1}b$			I		$B^{-1}D$	0
\vdots							\vdots
m							0
m+1	$-C_B B^{-1}b$			0		$C_D - C_B B^{-1}D$	-1

If $B^{-1}b \geq 0$, a basic feasible solution (with respect to the basis B') is available, namely;

$$\begin{aligned}\bar{x}_D &= 0 \\ \bar{x}_B &= B^{-1}b \\ \text{and } \bar{z} &= C_B B^{-1}b\end{aligned}$$

At the optimal solution, $C_D - C_B B^{-1}D \geq 0$

2. Sensitivity Analysis By Changing the Resource Level (b)

Let the original resource level be b and at the optimal solution we have B^*, x^* and z^* . Now, let $b \longrightarrow \bar{b} = b + \Delta b$

This change affects the vectors $(B^{-1}b)$ and $(-C_B B^{-1}b)$ as seen in the matrix tableau above. If $B^{-1}\bar{b} \geq 0$, then B^* is still optimal under \bar{b} , no basis change is required. However, optimal solution and optimal objective function value changes as:

$$\begin{aligned}\bar{x}_B^* &= B^{-1}\bar{b} \\ \bar{z}^* &= C_B B^{-1}\bar{b}\end{aligned}$$

If $B^{-1}\bar{b} < 0$, then original optimal basis B^* is not feasible. Basis should be changed by dual simplex algorithm.

3. Sensitivity Analysis By Changing the Cost Coefficients (c)

Let the original cost vector be c and at the optimal solution we have B^* , x^* and z^* . Now, let $C \rightarrow \bar{C} = C + \Delta C$

This change affects the vectors $(C_D - C_B B^{-1} D)$ and $(-C_B B^{-1} b)$

If $\bar{C}_D - \bar{C}_B B^{-1} D \geq 0$, then B^* is still optimal under \bar{C} , no basis change is required. Only the objective function value changes if the cost coefficient of a basic variable is changed;

$$\bar{x}^* = x^* = B^{-1} b$$

$$\bar{z}^* = \bar{C}_B B^{-1} b$$

If $\bar{C}_D - \bar{C}_B B^{-1} D < 0$, then original optimal basis B^* is no longer optimal.

A new basis should be obtained by simplex algorithm.

[illegible]

APPENDIX-15 Continued

		I			II			III			IV			V		
RISK LEVEL		MINIMUM	ORIGINAL	MAXIMUM	MINIMUM	ORIGINAL	MAXIMUM	MINIMUM	ORIGINAL	MAXIMUM	MINIMUM	ORIGINAL	MAXIMUM	MINIMUM	ORIGINAL	MAXIMUM
PERIOD		C.COEF	COEF	C.COEF	C.COEF	C.COEF	C.COEF	C.COEF	C.COEF	C.COEF	C.COEF	C.COEF	C.COEF	C.COEF	C.COEF	C.COEF
		z-LOWER	z=899.31	z-UPPER	z-LOWER	z=720.95	z-UPPER	z-LOWER	z=558.45	z-UPPER	z-LOWER	z=421.75	z-UPPER	z-LOWER	z=285.6	z-UPPER
MANDATORY REPAYMENT	4	-.42648 899.31	-.3675	INF	INF	-.3675	-.35409 721.96	-.51118 545.52	-.3675	-.35372 559.69	-.5093 408.99	-.3675	-.35368 422.99	-.5068 273.06	-.3675	-.35364 286.8
	5	INF	-.3266	-.31399 900.2	INF	-.3266	-.31336 722.04	INF	-.3266	-.31299 559.72	INF	-.3266	-.31295 423.18	INF	-.3266	-.3129 287.19
	6	INF	-.2858	-.27314 900.22	INF	-.2858	-.31336 722.04	INF	-.2858	-.27214 559.84	INF	-.2858	-.2721 423.36	INF	-.2858	-.27205 287.42
	7	-.25451 898.73	-.245	-.24415 899.36	INF	-.245	-.23928 721.46	INF	-.245	-.24296 558.66	INF	-.245	-.24335 421.94	INF	-.245	-.24387 285.75
	8	INF	-.2042	-.19233 900.21	INF	-.2042	-.1968 721.61	INF	-.2042	-.1914 559.76	INF	-.2042	-.19082 423.3	INF	-.2042	-.19005 287.42
	9	-.16906 898.98	-.1633	-.16088 899.44	INF	-.1633	-.14587 722.64	INF	-.1633	-.14628 560.32	INF	-.1633	-.14657 423.81	INF	-.1633	-.14696 287.83
	10	INF	-.1225	-.08535 902.31	INF	-.1225	-.10601 722.54	INF	-.1225	-.11049 559.8	INF	-.1225	-.11074 423.26	INF	-.1225	-.11106 287.25
	11	INF	-.0817	-.06621 900.66	INF	-.0817	-.0675 722.39	INF	-.0817	-.0697 559.83	INF	-.0817	-.06994 423.26	INF	-.0817	-.07026 287.23
	12	INF	-.0408	-.02559 900.64	INF	-.0408	-.0269 722.34	INF	-.0408	-.02907 559.77	INF	-.0408	-.02932 423.1	INF	-.0408	-.02572 287.67

APPENDIX-16 Optimality Range For the Cost Coefficients of Short-Term Bank Credit Variables.

RISK LEVEL		I			II			III			IV			V		
		MINIMUM	ORIGINAL	MAXIMUM	MINIMUM	ORIGINAL	MAXIMUM	MINIMUM	ORIGINAL	MAXIMUM	MINIMUM	ORIGINAL	MAXIMUM	MINIMUM	ORIGINAL	MAXIMUM
		C.COEf	C.COEf	C.COEf	C.COEf	C.COEf	C.COEf	C.COEf	C.COEf	C.COEf	C.COEf	C.COEf	C.COEf	C.COEf	C.COEf	C.COEf
PERIOD		z-LOWER	z=899.31	z-UPPER	z-LOWER	z=720.95	z-UPPER	z-LOWER	z=558.45	z-UPPER	z-LOWER	z=421.75	z-UPPER	z-LOWER	z=285.6	z-UPPER
CREDIT	3				.6667		.6684			.6667	.6667		.67293	.6667		.6667
RAISED	4	.6		.60745	.60		.60914	.60		.60129	.60		.6	.6		.60022
	5	.52636	.5333	.53588	.52803	.5333	.53511	.53271	.5333	.54149	.53322	.5333	.54108	.52974	.5333	.53391
	6	.4667	.4667	.4667	.4667	.4667	.4667	.4667	.4667	.4667	.4667	.4667	.4667	.4667	.4667	.47321
	7	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31
	8	.3333	.3333	.33405	.3333	.3333	.33405	.3333	.3333	.33405	.3333	.3333	.33405	.3333	.3333	.33405
	9	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20
	10	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31
	11															
	12	.0667	.0667	.07997	.0667	.0667	.07658	.0667	.0667	.07675	.0667	.0667	.068505	.0667	.0667	.080
		.899.31	.899.31	.902.34	.899.31	.899.31	.902.34	.899.31	.899.31	.902.34	.899.31	.899.31	.902.34	.899.31	.899.31	.902.34
CREDIT	3							-.67247		-.6667						
RAISED	4							558.01	-.6667	558.45						
	5															
	6	-.4667	-.4667	-.4667	-.4667	-.4667	-.4667	-.46732	-.4667	-.4667	-.46678	-.4667	-.4667	-.4667	-.4667	-.4667
	7	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31
	8				-.3333	-.3333	-.33255	-.3333	-.3333	-.32882	-.3333	-.3333	-.32842	-.3333	-.3333	-.32788
	9							558.45	-.3333	558.78	421.75	-.3333	422.25	285.6	-.3333	286.59
	10	-.2	-.2	-.2	-.2	-.2	-.2	557.55	-.2667	558.45						
	11	.899.31	.899.31	.899.31	.899.31	.899.31	.899.31	558.45	-.20	558.54	419.42	-.20	421.75	285.6	-.20	286.11
	12	-.1333	-.1333	-.12631	-.1333	-.1333	-.12621				-.1333	-.1333	-.12346	-.1333	-.1333	-.12069
		.899.31	.899.31	.899.43	.899.31	.899.31	.899.43	.899.31	.899.31	.899.43	.899.31	.899.31	.899.43	.899.31	.899.31	.899.43

APPENDIX-19 Optimality Range For the Right-Hand-Side Constants of Pledging of Accounts Receivable Constraints

RISK LEVEL		I			II			III			IV			V		
CONSTRAINT	PERIOD	MINIMUM RESOURCE	ORIGINAL RESOURCE	MAXIMUM RESOURCE	MINIMUM RESOURCE	ORIGINAL RESOURCE	MAXIMUM RESOURCE	MINIMUM RESOURCE	ORIGINAL RESOURCE	MAXIMUM RESOURCE	MINIMUM RESOURCE	ORIGINAL RESOURCE	MAXIMUM RESOURCE	MINIMUM RESOURCE	ORIGINAL RESOURCE	MAXIMUM RESOURCE
		z-LOWER	z=899.31	z-UPPER	z-LOWER	z=720.95	z-UPPER	z-LOWER	z=558.45	z-UPPER	z-LOWER	z=421.75	z-UPPER	z-LOWER	z=285.6	z-UPPER
$\sum_{k=1}^j (x_{2k} - y_{2k} - v_{2k})$	3				111.84 721.06	125.82 720.95	150 720.76									
$\leq a_{21j} A_j$	4	36.012 899.93	118.46	150 899.07	50.328 721.64	137.26 720.95	199.29 720.46	60 559.23	156.06	210 558.01	113.17 422.25	174.85	210 421.46	176.84 285.74	193.65	210 285.47
	5	47.384 899.86	119.73	176.56 898.87	54.903 721.67	144.98	204.90 720.47	62.422 559.33	170.23	212.42 558.10	106.84 422.48	109.48	219.94 421.55	196.86 285.80	220.73	227.46 285.55
	6				118.67 721.06	149.06	162.82 720.90	122.61 558.51	170.90	216.33 558.39	180.64 421.76	192.74	22.819 421.71	202.29 285.61	214.57	237.95 285.59
	7	72.624 899.69	126.47	151.95 899.12	102.84 721.15	148.52	178.91 720.82	125.14 558.80	170.58	218.36 558.08	147.45 422.11	192.63	200.82 421.68	194.25 285.77	214.68	227.35 285.49
	8				154.71 721.02	161.62	209.41 720.45	134.28 558.95	183.65	206.98 558.21	189.41 421.91	205.67	227.05 421.53	224.80 285.63	227.70	235.87 285.52
	9	134.64 899.31	134.64	134.64 899.31	147.19 721.09	160.96	167.68 720.88	104.06 559.05	187.28	200.57 558.35	213.26 421.75	213.60	232.27 421.62	219.98 285.74	239.92	24.108 285.59
	10	53.854 900.16	145.26	170.63 899.07	64.382 721.84	168.44	214.38 720.56	178.33 558.54	191.63	224.91 558.21	170.75 422.06	214.81	215.91 421.74	196.12 285.89	237.99	245.97 285.55
	11	120.93 899.54	146.30	208.10 898.74	69.938 721.76	166.84	217.38 720.53	132.69 558.83	187.37	220.43 558.21	206.81 421.76	207.91	235.92 421.56	227.80 285.61	228.45	245.20 285.45
	12	58.519 910.87	152.88	208.52 892.49	66.734 730.46	198.46 177.98	198.46 719.20	106.23 560.60	203.08	224.95 557.96	165.52 422.70	228.18	233.16 421.67			
$x_{2j} \leq b_{21j} (=150)$	3							42.97 559.33	150	152.58 558.43	62.135 422.48	150	179.33 421.51	109.13 285.94	150	206.09 285.13
	5										125.54 421.75	150	INF			
	7										115.54 421.75	150	INF			
	8													141.83 285.60	150	INF
	9				96.308 720.95	150	INF	113.82 558.45	150	INF	131.33 421.75	150	INF			
	11							110.72 558.45	150	INF						
	12													0 286.48	150	150.38 285.60

APPENDIX-20 Optimality Range For the Right-Hand-Side Constants of Pledging of Accounts Receivable and Short-Term Credit Constraints

CONSTRAINT	RISK LEVEL PERIOD	I			II			III			IV			V		
		MINIMUM	ORIGINAL	MAXIMUM	MINIMUM	ORIGINAL	MAXIMUM	MINIMUM	ORIGINAL	MAXIMUM	MINIMUM	ORIGINAL	MAXIMUM	MINIMUM	ORIGINAL	MAXIMUM
		RESOURCE	RESOURCE	RESOURCE	RESOURCE	RESOURCE	RESOURCE	RESOURCE	RESOURCE	RESOURCE	RESOURCE	RESOURCE	RESOURCE	RESOURCE	RESOURCE	RESOURCE
		z-LOWER	z=899.31	z-UPPER	z-LOWER	z=720.95	z-UPPER	z-LOWER	z=558.45	z-UPPER	z-LOWER	z=421.75	z-UPPER	z-LOWER	z=285.60	z-UPPER
$\sum_{k=1}^{j-1} (x_{2k} - y_{2k} - v_{2k}) - v_{2j} = 0$	3				-24.179 721.28	0	720.95	0 558.45		1.5462 558.51	0 421.75		7.3429 422.03	0 285.6		33.653 286.89
	4	-31.54 899.81	0	899.31	-63.070 721.80	0	719.94	-53.944 558.98		90 557.56	-7.3908 421.82	0	90 420.89	-16.347 285.75	0	41.067 285.23
	5	-77.653 900.55	0	898.17	-59.922 721.76	0	719.85	-42.19 558.86		93.634 557.52	-7.3908 421.82	0	104.91 420.75	-6.7272 285.66	0	41.067 285.23
	6	-96.155 900.85	0	898.44	-58.931 721.74	0	719.78	-47.194 558.91		102.14 557.44	-7.3908 421.82	0	114.54 420.66	-23.72 285.81	0	41.067 285.23
	7	-64.226 900.33	0	898.33	-61.101 721.77	0	719.76	-47.784 558.92		102.22 557.44	-7.3908 421.82	0	115.64 420.65	-21.147 285.79	0	41.067 285.23
	8	-112.88 901.11	0	898.57	-47.788 721.59	0	719.75	-34.583 558.79		102.35 557.44	-7.3908 421.82	0	115.58 420.65	-8.1732 285.67	0	41.067 285.23
	9	-52.991 899.31	0	899.31	-53.692 720.95	0	720.95	-36.182 558.45		110.19 558.45	-18.671 421.75	0	123.40 421.75	-1.161 285.6	0	136.62 285.6
	10	-58.595 899.31	0	899.31	-45.94 720.95	0	720.95	-33.286 558.45		112.37 558.45	-20.631 421.75	0	128.16 421.75	-7.9776 285.6	0	142.02 285.6
	11	-61.807 899.31	0	899.31	-50.542 720.95	0	720.95	-39.277 558.45		114.98 558.45	-28.012 421.75	0	121.99 421.75	-16.747 285.6	0	133.25 285.6
	12	-55.644 899.31	0	899.31	-38.759 720.95	0	720.95	-21.874 558.45		112.42 558.45	-4.9894 421.75	0	124.75 421.75	-241.38 287.02	0	37683 285.6
$\sum_{k=1}^j (x_{3k} - y_{3k}) \leq b_{31j}$	9							964.72 558.45		1016.3 558.45						
	12	1231.0 910.80	1300	1363.7 888.69	1054.5 731.81	1150	1164.3 719.32	951.0 559.59	1000 1000	1046.9 557.35	618.39 422.17	650	698.76 421.10			
$x_{3j} \leq b_{33j}$	3							175.08 558.45	250	INF						
	4										132.83 421.75	200	INF			
	6							149.11 558.45	250	INF	76.502 421.75	200	INF			
	7	262.31 899.50	300	317.84 899.22	290.35 720.96	300	321.27 720.94	218.20 558.59	250	283.80 558.29	168.38 421.91	200	206.84 421.71	162.01 285.81	200	208.60 285.55
	9	270.32 899.86	300	331.07 898.73	290.50 720.97	300	304.72 720.94									
	10										0 421.75	200	INF			

RISK LEVEL		I			II			III			IV					
		MINIMUM RESOURCE	ORIGINAL RESOURCE	MAXIMUM RESOURCE	MINIMUM RESOURCE	ORIGINAL RESOURCE	MAXIMUM RESOURCE	MINIMUM RESOURCE	ORIGINAL RESOURCE	MAXIMUM RESOURCE	MINIMUM RESOURCE	ORIGINAL RESOURCE	MAXIMUM RESOURCE	MINIMUM RESOURCE	ORIGINAL RESOURCE	MAXIMUM RESOURCE
CONSTRAINT	PERIOD	Z-LOWER	Z-899.31	Z-UPPER	Z-LOWER	Z-720.95	Z-UPPER	Z-LOWER	Z-558.45	Z-UPPER	Z-LOWER	Z-421.75	Z-UPPER	Z-LOWER	Z-285.6	Z-UPPER
STRET.DOM.	3				14.742		108.69	0		111.92	34.613		105.87	0		82.234
					721.02	24.697	720.40	558.49	32.678	558.34	421.75	40.359	421.69	285.61	48.64	285.59
	4	0		87.711	0		86.693	0		132.26	39.751		224.30	0		82.176
		899.66	22.324	898.26	721.3	29.373	720.28	558.62	36.422	557.99	421.76	43.171	421.02	285.77	50.52	285.5
	5	0		64.941	0		187.76	0		76.764	39.707		151.97	34.813		81.73
		899.69	24.978	898.70	721.34	31.684	719.03	558.74	38.389	558.16	421.79	45.194	421.0	285.71	51.8	285.41
	6															
	7	3.2074		58.739												
		899.59	40.899	899.17												
	8															
	9	7.0812		67.507												
		899.57	36.598	899.03												
	10															
	11	17.125		112.61	0	53.349	79.339									
		900.56	34.885	893.80	723.17	720.95	719.87									
	12	0		98.753	0		68.58	5.6402		120.38						
		904.68	35.05	889.53	726.4	54.241	719.15	559.13	73.432	557.98						
STRET. FOR.	11	36.065		142.80	0		126.88									
		899.75	53.825	897.08	721.55	84.543	720.65									
PAY.(1 st PER.)	12	0		104.65	0		92.181									
-		905.18	40.946	890.17	727.99	77.842	719.66									
STRET.DOM.	4	0		63.694	-12.719		42.759									
		899.31	0	899.02	727.99	0	720.87									
PAY.(2 nd PER.)	5	-11.497		39.948	-15.127		155.97	-18.757		38.292	-5.522		108.87			
		899.40	0	898.98	721.03	0	720.13	558.46	0	558.42	421.75	0	421.74			
	7	0		17.84												
		899.31	0	899.29												
	9	0		30.914				-8.4128		0						
		899.31	0	899.24				558.48	0	558.45						

APPENDIX-22 Optimality Range For the Right-Hand-Side Constants of Stretching Foreign Payables (2nd Period), Proportion of Government Security Inv.

To Total One-Month Maturity Period Investments and Fulfillment of Requirements Constraints.

CONSTRAINT	PERIOD	I RISK LEVEL			II			III			IV					
		MINIMUM RESOURCE	ORIGINAL RESOURCE	MAXIMUM RESOURCE	MINIMUM RESOURCE	ORIGINAL RESOURCE	MAXIMUM RESOURCE	MINIMUM RESOURCE	ORIGINAL RESOURCE	MAXIMUM RESOURCE	MINIMUM RESOURCE	ORIGINAL RESOURCE	MAXIMUM RESOURCE	MINIMUM RESOURCE	ORIGINAL RESOURCE	MAXIMUM RESOURCE
		z-LOWER	z=899.31	z-UPPER	z-LOWER	z=720.95	z-UPPER	z-LOWER	z=558.45	z-UPPER	z-LOWER	z=421.75	z-UPPER	z-LOWER	z=285.6	z-UPPER
STRET.FOR. PAY.(2 nd PER.)	5				-9.8714 721.24	0	0 720.95									
	6	-12.416 899.41	0	0 899.31	-16.898 721.21	0	0 720.95				-25.399 422.45	0	0 421.75			
	10	-9.6615 899.46	0	0 899.31							-34.976 424.77	0	0 421.75			
	12	-16.793 901.58	0	0 890.69	-26.377 723.12	0	14.339 719.77									
GOV. SEC. INV./ TOT.ONE-MON. INV. .30	1	-22.537 899.65	0	52.585 898.50	-26.989 721.35	0	62.975 720.02	-31.442 558.88	0	58.558 557.65	-22.721 422.06	0	54.105 421.02	-40.348 286.14	0	49.652 284.94
	2				-5.9433 721.04	0	13.868 720.75	-14.947 558.65	0	34.877 557.98	-23.365 422.06	0	55.888 421.01	-32.956 286.03	0	57.044 284.86
	3	-16.471 899.54	0	38.054 898.76												
FULFILLMENT OF REQUIREMENTS	1	-76.65 901.21	-75.122 899.31	-65.200 886.92	-109.23 743.25	-89.965	-81.081 710.67	-125.14 578.62	-104.81	-91.937 545.68	-119.81 421.90	-119.65	-92.974 395.77	-141.27 292.04	-134.49	-133.63 284.78
	2	77.253 887.2	87.456	89.028 901.17	63.57 710.92	72.906	92.517 742.71	44.720 546.04	57.956	78.868 578.04	15.771 396.53	43.205	43.365 421.90	27.565 284.80	28.455	35.425 291.84
	3	326.29 833.67	387.38	445.71 961.38	366.29 711.18	375.69	456.10 804.60	350.38 546.42	364.0	385.50 577.45	324.09 397.31	352.30	352.47 421.89	339.70 294.83	340.61	347.78 291.65
	4	183.60 859.75	222.49	282.48 960.33	188.99 711.73	198.93	257.09 774.85	160.97 547.22	175.38	199.0 576.84	121.95 398.97	151.82	152.0 421.88	127.30 284.88	128.27	135.86 291.22
	5	167.37 862.18	208.69	253.43 939.50	176.56 712.35	187.13	248.92 771.22	150.26 548.12	165.58	191.77 576.11	112.28 400.84	144.02	144.20 421.87	121.43 284.94	122.46	130.53 290.75
	6	-7.3766 864.93	36.698	84.418 936.52	-7.2828 713.06	3.992	69.902 767.10	-44.747 567.58	-28.713	-12.378 549.14	-61.617 421.86	-61.419	-27.562 402.95	-102.73 290.21	-94.125	-93.027 285.01
	7	340.37 887.09	358.21	395.90 925.12	303.33 710.30	320.90	352.88 740.34	267.72 550.72	283.58	300.15 566.52	209.28 404.23	246.26	246.48 421.85	207.75 285.05	208.95	290.39 322.76
	8	-71.531 930.51	-17.724	18.950 878.04	-104.17 744.01	-59.043	-40.954 711.71	-117.43 565.19	-100.36	-84.028 551.99	-141.91 421.83	-141.68	-103.59 407.18	-291.98 325.41	-183.0	-181.77 285.15
	9	9.966 886.36	35.928	60.721 911.67	11.038 719.28	15.031	49.205 735.25	-41.142 569.31	-5.866	33.253 546.40	-27.0 421.82	-26.763	13.869 409.72	-182.44 323.35	-47.66	-46.342 285.23
	10	-15.476 876.95	40.729	101.58 923.52	4.5841 716.68	17.235	101.52 749.42	-15.564 560.64	-6.26	35.162 548.70	-60.594 428.66	-29.755	13.268 412.11	-105.15 296.44	53.25	-51.273 285.19
	11	12.164 880.53	71.677	89.437 904.91	32.476 717.47	45.918	132.76 743.45	-2.9764 554.69	20.160	69.159 566.41	-6.3681 421.87	-5.598	40.115 414.80	-56.035 289.0	-31.357	-27.402 285.06
	12	-27.107 884.44	26.596	95.568 915.40	-13.86 718.36	.479	96.008 738.18	-93.429 564.55	-25.637	21.310 554.22	-95.613 425.26	-51.753	-2.9905 417.85	-277.61 298.92	-77.869	-77.605 285.58