DYNAMIC VOLUNTARY CONTRIBUTION UNDER TIME-INCONSISTENCY

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DYNAMIC VOLUNTARY CONTRIBUTION UNDER TIME-INCONSISTENCY

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DECLARATION OF ORIGINALITY

I, Ahhmet Altınok, certify that

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ABSTRACT

Dynamic Voluntary Contribution under Time-Inconsistency

I study the voluntary public good provision model and introduce time inconsistent agents who have $\beta - \delta$ preferences. There is a public project and finitely many agents where each agent is allowed to contribute any amount in any period before the project is completed. The agents have discontinuous and symmetric preferences over the total contribution with a jump when there is provision. There is complete information about the environment but imperfect information about others' individual actions: each period, each agent observes only the total contribution made, not the other agents' individual contributions. Assuming the agents are sophisticated, we characterize the set of equilibria. I compare the set of equilibria under sophisticated time-inconsistent agents to that under time-consistent agents with a discount factor equal to the average discount factor of a time-inconsistent agent. show that there are equilibria with time-inconsistent agents, which are not equilibria with time-consistent agents. We also show that there are some projects, which are completed by the sophisticated time-inconsistent agents earlier than the time-consistent agents complete.

ÖZET

Zaman Uyumsuz Ekonomide Dinamik Gönüllü Katkı

Bu çalışmada, hiperbol tercihlerin, bir kamu malı tedarik probleminin denge koşullarına etkisini inceledim. Alternatif olarak, tercihler bakımından zaman uyumlu bir ekonomide, kamu malı tedarik probleminin Nash dengesi, hiperbol tercihlerin olduğu ekonomide de bir Nash dengesidir. Bu çalışmadaki temel amaç bu iki ekonominin denge koşullarının benzerliklerini ve farklılıklarını incelemektir. Bu çalışmada kıstas model olarak Marx ve Matthews in 2000 yılında yayınladıkları makalenin dinamik, gönüllü katkı modelini kullandım. Temel olarak modelde bir kamu malı ve ölümsüz, sınırlı sayıda ajan vardır. Bu ajanlar kamu malı projesi kapsamında proje tamamlanmadan önce, istedikleri herhangi bir zaman diliminde istedikleri kadar katkı yapma yetkisine sahipler. Ajanların, toplam katkı miktarı üzerinden, kamu malının tedarik olduğu noktada bir sıçrama dolayısıyla süreksiz tercihleri vardır. Bu ekonomide ajanlar toplam katkı miktarını her zaman diliminde gözlemleyebiliyorlar, fakat bireysel katkıları gözlemleyemiyorlar. Ajanların, tercih bakımından sofistik zaman uyumsuz olduklarını farz ederek, bu ekonominin denge kümesini özelliklerini saptadım ve bunları yine tercih bakımından zaman uyumlu olan ajanların bulunduğu ekonominin denge kümesi ile karşılaştırdım. Bu karşılaştırma sonunda sofistik zaman uyumsuz ajanların denge kümesinin zaman uyumlu ajanların aynı ekonomide ki denge kümesini kapsadığını buldum. Ayrıca, tam tersi ilişkinin olmadığını gösteren örnekler türettim. Temel olarak beklenenin aksine, bu çalışmada; belirli koşullar altında, sofistik zaman uyumsuz ajanların belirli bir projeyi, zaman uyumlu ajanlara oranla daha erken bitirmek istedikleri sonucuna ulaştım.

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CONTENTS

CHAPTER 1

INTRODUCTION

Public projects such as building a library, a public recreational facility, a railroad or subway take a lot of time; generally for weeks, months and even years. Such a period of time is necessary for many voluntary contribution plans which finance the public project. This paper addresses such a joint project which is "socially desirable" in the sense that its total benefit which will be received by the agents joining the project, is more than the cost, where the classic free rider problem is present. A widely studied question, with the assumption of forming a contract over the contributions in advance is not possible, is whether the project will be completed and if it will be, what will be the contribution scheme for each agent.[1] However, when the nature of the problem has a dynamic component, then the agents' time-preferences become important. Evidence show that the agents may have time-preferences that change over time. In particular, an agent may have a higher discounting between the current and the next period than the discounting between any other two successive periods. Therefore, the question posed above is worth exploring under time-inconsistent preferences.

We consider the dynamic voluntary contribution model introduced in Marx and Matthews (2000). Each agent can contribute any amount at any period, and the contributions are non-refundable. At the end of each period, each agent learns the total amount of contribution at that period, but she does not observe the individual contributions of other agents. The cost of the public good is common knowledge. Each agent has a discontinuous benefit function, in the sense that there is a jump whenever the public project is completed. Also, each period the agents has a marginal benefit from the total contribution made in that period even when the public project is

[1]See Admati and Perry (1991) and Marx and Matthews (2000) for instance.

not completed.[2] Within this framework, I introduce time-inconsistent agents through $\beta - \delta$ preferences.[3] An agent with $\beta - \delta$ preferences has a discount factor $\beta\delta$ between the current and the next period, and a discount factor δ between any two future successive periods. If an agent is fully aware of her time-inconsistency, she is a sophisticated time-inconsistent agent, otherwise she is a naive agent. In this paper, I focus on sophisticated agent case and investigate the effect of agents' time-inconsistency on the set of equilibria and also on the number of periods to complete the project.

The Nash equilibria and the nearly efficient perfect Bayesian equilibria of the public good provision with the time consistent agents are characterized by Marx and Matthews (2000). One of their main results is that by allowing agents making contributions slowly over time, efficient outcomes can be constructed and the agents are willing to complete the project. Here, I first focus on the set of equilibria under both agent types and compare them. I find that there are strategy profiles which are equilibria for the sophisticated time-inconsistent agents, but not for the time-consistent agents.[4] We also compare the number of periods to complete the project under both agent types. Surprisingly, I find that there exist some projects which are completed by the time-inconsistent agents earlier than the corresponding

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[2]This marginal benefit can be zero or positive. Both cases are allowed in the model.

[3]$\beta - \delta$ preferences are first developed by Phelps and Pollak (1968) and then used in various settings. These include Laibson (1997), O'Donoghue and Rabin (1999a,1999b,2001) among others. Also see O'Donoghue and Rabin (2000) for various economics applications based on time-inconsistency captured with $\beta - \delta$ preferences.

[4]Here, I fix an equilibrium for the time-inconsistent agents and the number of periods, T, the project is completed in this equilibrium. Then, I consider the time-consistent agents with the discount factor equal to the average discount factor of the time-inconsistent agent, that is, for instance, for $T = 3$, I consider the time-consistent agents with $\hat{\delta}$ where $1+\beta\delta+\beta\delta^2 = 1+\hat{\delta}+\hat{\delta}^2$. I refer to the time-consistent agent with the discount factor $\hat{\delta}$ as the corresponding time-consistent agent. See Chade et.al (2008).

time-consistent agents. Intuitively, since the sophisticated agents are aware of their inconsistency, they tend to guard themselves against the future selves by contributing higher amounts in the earlier periods. Thus, they reach the total contribution needed for completion of the project earlier than the corresponding time-consistent agents.

The voluntary public good provision problem has been widely studied. Admati and Perry (1991) consider an alternating contribution game under both full-refunds and no refunds cases. They investigate whether efficient equilibria still exist if the contributions are divided into small sums over time. The answer is negative; each of the two players has an incentive to let the other player contribute in the future. Fresthman and Nitzan (1991) also consider a dynamic public good provision problem with flow benefits. Both papers have negative results which hinge on the fact that a player can sometimes increase the level of future contribution by lowering the current contribution and this is a potential incentive for her to free ride on the future contributions of other players. On the contrary, in our model, players are dissuaded from contributing too little in the current period since putting too little today results in no contribution by the other players in future periods. Time-inconsistent behavior is also widely studied in various contexts, including individual decision making (O'Donoghue and Rabin (1999a)) and contracting (O'Donoghue and Rabin (1999b), Yılmaz (2013)). However, to our best of knowledge time-inconsistent preferences have not been studied in a dynamic voluntary contribution problem. This paper contributes to the literature in this aspect as well.

CHAPTER 2

THE MODEL

The set of players is $N = \{1, 2, ..., n\}$ with $n \geq 2$ and a public good with a cost $\overline{X} > 0$. Each player decides how much to contribute for the public good in each period $t \geq 0$. Player i contributes $z_i(t)$ in period t and the contributions are non-refundable. Each player can observe her own past contributions and the aggregate contribution of the other players' past contributions in each period. Once the contributing horizon, $T^* \leq \infty$, is reached, no more contribution is allowed to be made. Agents are assumed to have infinite amount of private good to contribute, that is, the budget constraints are not binding. Hence, in any period $t \leq T^*$, any $z_i \geq 0$ is feasible, and in any period $t > T^*$, only $z_i = 0$ is feasible.

Let $z(t) \equiv (z_1(t), ..., z_n(t))$ be the contribution vector in period t and $z \equiv z(t)_{t=0}^{\infty}$ be the entire contribution sequence. Let $U_i(z)$ denote the payoff of player i. Let $Z(t) \equiv \sum\limits_{j \in N} z_j(t)$ be the total contribution made in the period t, and let $Z_i(t) \equiv Z(t) - z_i(t)$. Hence, the personal history in the beginning of the period *t* for the player *i* is

$$h_i^{t-1} \equiv (z_i(\tau), Z_i(\tau))_{\tau=0}^{t-1}$$

The player's strategy is to map each h_i^{t-1} into a contribution that is feasible in the following period. Thus, this is a contribution game with unobserved contributions. When $n = 2$ the game is a game with observed contributions.

Payoffs depend on the cumulative contribution. The cumulative contribution for an individual (say player i) at the end of the period t is $x_i(t) \equiv \sum\limits_{\tau \leq t} z_i(\tau)$. The aggregate cumulative contribution; *cumulation*, is $X(t) \equiv \sum_{j \in N} x_j(t)$. The total benefit player i receives from the project at any period t is $f_i(X(t))$, where f_i is the *benefit function* that is specified later. The cost function enters quasi-linearly. The contributions in a period are converted into the non-depreciating capital the project uses to generate benefit in that and subsequent periods. Hence, the cost of a contribution is borne when it is made. The total benefit over all periods, that the project generates for player i if the cumulation is forever fixed at X is still $f_i(X)$.

Players discount benefits and costs by the discounting scheme $(1, \delta\beta, \delta^2\beta, \delta^3\beta, ...)$. When $\beta = 1$, the model becomes identical to the one in Marx and Matthews (2000), which is, thus, a special case of our model.

In Marx and Matthews, for a given feasible contribution sequence, z, player i with a discount factor $\hat{\delta}$ has a present discounted overall net payoff given by[5]

$$U_i(z) \equiv \sum_{t=0}^{\infty} \hat{\delta}^t[(1-\hat{\delta})f_i(X(t)) - z_i(t)]$$

Here the benefit function, not the costs, within each period is scaled by $(1 - \hat{\delta})$, which then makes it possible to write the overall payoff in terms of period specific total contributions, rather than the cumulations.

For our comparison of time-inconsistent environment and the time-consistent environment to be reasonable, I also scale the benefit (not the cost) that a time-inconsistent agent gets in each period by the same scalar, $(1 - \hat{\delta})$, which is used in the payoff function of the time-consistent agent with discount factor $\hat{\delta}$. When $\beta < 1$, for a given feasible contribution sequence, z, player i's present discounted overall net payoff is

$$U_i(z) \equiv (1-\hat{\delta})f_i(X(0)) - z_i(0) + \sum_{t=1}^{\infty} \delta^t\beta[(1-\hat{\delta})f_i(X(t)) - z_i(t)]$$

[5]We will be comparing the environment with time-inconsistent agents to the environment with time-consistent agents who have a discount factor, $\hat{\delta}$, which is not necessarily equal to δ.

If $f_i(X(t)) > 0$, the project generates a benefit in period t, even if $t < T^*$. Alternatively, if $f_i(X(t)) = 0$ for all $t < T^*$, benefits are not generated until the project is completed. This is the case of a binary project such as the building of a bridge.

We consider the following benefit function.

$$f_i(X) = \begin{cases} \lambda_i X & X < \overline{X} \\ V_i & X \geq \overline{X} \end{cases}$$

where λ_i is the player's marginal benefit from a non-completing contribution, V_i is the benefit for player from the completed project, and b_i is the benefit jump at the completion, that is $b_i \equiv V_i - \lambda_i \bar{X}$ as shown in Figure 1.

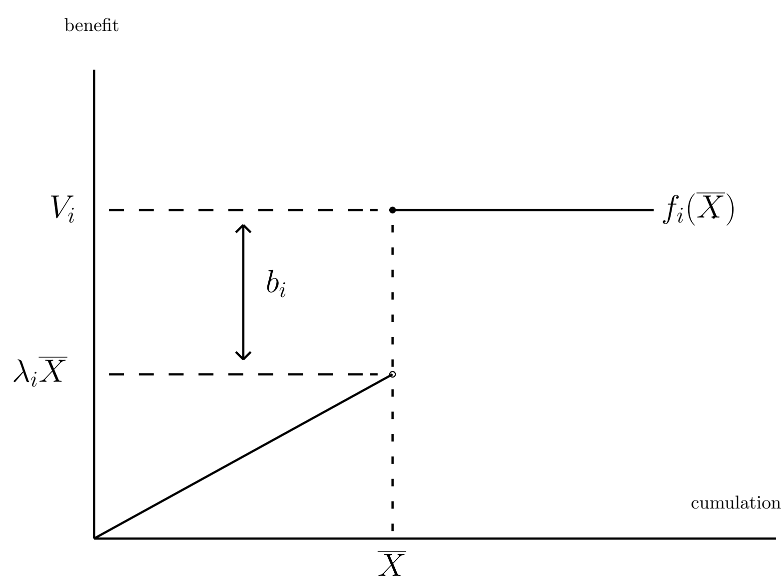

Fig. 1. The illustration of the benefit function.

We assume $\lambda_i \geq 0$, and $b_i \geq 0$, which implies

$$0 \leq \lambda_i \leq \frac{V_i}{\bar{X}} \ \ \forall i \in N.$$

Note that λ_i=0 yields the *binary benefit function* and b_i=0 yields the *continuous benefit function*. A positive b_i increases the incentive to finish the project because it represents strong increasing returns.

We focus on the case where no agent is willing to complete the project alone, but it's efficient to provide the public good. These are summarized in the following inequality:

$$V_i < \overline{X} < \sum_{j=1}^{n} V_j, \ \ \forall i \in N.$$

With the f_i specified above and the completion period of the project being T^*, this overall payoff, starting from period 0, can be written as

$$U_i(z,0) \equiv (\frac{1-\hat{\delta}}{1-\delta})(1-\delta(1-\beta))\lambda_i Z(0) - z_i(0) + \sum_{t=1}^{T^*} \delta^t \beta[(\frac{1-\hat{\delta}}{1-\delta})\lambda_i Z(t) - z_i(t)]$$

$$+\delta^{T^*}(\frac{1-\hat{\delta}}{1-\delta})\beta b_i$$

The discounted overall payoff, starting from a period $t < T^*$ is given by

$$U_i(z,t) \equiv (\frac{1-\hat{\delta}}{1-\delta})(1-\delta(1-\beta))\lambda_i X(t) - z_i(t) + \sum_{\tau=t+1}^{T^*(z)} \delta^\tau \beta[(\frac{1-\hat{\delta}}{1-\delta})\lambda_i Z(\tau) - z_i(\tau)]$$

$$+\delta^{T^*(z)}(\frac{1-\hat{\delta}}{1-\delta})\beta b_i$$

And the discounted overall payoff, starting from period $t = T^*(z)$ is given by

$$U_i(z,T^*) \equiv (\frac{1-\hat{\delta}}{1-\delta})(1-\delta(1-\beta))[\lambda_i \overline{X} + b_i] - z_i(T^*)$$

To ease notation, I will denote $K_{\hat{\delta}} = \frac{1-\hat{\delta}}{1-\delta}$, and $\Delta = 1 - \delta(1-\beta)$. Then, the payoff function will be

$$U_i(z,t) = \begin{cases} K_{\hat{\delta}}\Delta\lambda_i X(t) - z_i(t) + \sum_{\tau=t+1}^{T^*} \delta^\tau \beta[K_{\hat{\delta}}\lambda_i Z(\tau) - z_i(\tau)] + & \\ \cdots + \delta^{T^*} K_{\hat{\delta}}\beta b_i & 0 \leq t < T^* \\ K_{\hat{\delta}}\Delta[\lambda_i \overline{X} + b_i] - z_i(T^*) & t = T^* \end{cases} \tag{1}$$

CHAPTER 3

NASH EQUILIBRIA

As in Marx and Matthews (2000), I first look at the static version of the game which I use to construct the set of equilibria in the dynamic version. A strategy profile in the static game, $(z_1, z_2, ..., z_n)$, yields an aggregate contribution $Z = \sum_{i=1}^{n} z_i$, and player i receives payoff $f_i(Z) - z_i$. Denote the total contribution of other players by $Z_i = Z - z_i$. Player i's best response to $Z_i < \overline{X}$, is either finishing the project by contributing the rest of the amount required, that is, $\overline{X} - Z_i$; or contributing nothing. Intermediate amounts are dominated, because the marginal benefit is less than 1. The marginal benefit from completing the project is $f_i(\overline{X}) - f_i(Z_i) = V_i - \lambda_i Z_i$, and the marginal cost of doing so is $\overline{X} - Z_i$. The former exceeds the latter if and only if the completing amount is less than the contribution that I call *critical contribution*. It is the maximum amount that player i is willing to contribute, and specified below;

$$c_i^* \equiv \frac{V_i - \lambda_i \overline{X}}{1 - \lambda_i} = \frac{b_i}{1 - \lambda_i}$$

The reaction function of player i is; for $Z_i < \overline{X}$

$$z_i^R(Z_i) = \begin{cases} 0 & \overline{X} - Z_i > c_i^* \\ \overline{X} - Z_i & \overline{X} - Z_i \leq c_i^* \end{cases}$$

Whenever $b_i = 0$, that is, whenever there is no benefit jump, the critical contribution level is zero, $c_i^* = 0$. So, contributing nothing is a dominant strategy for each player.

In the dynamic version of the game, let $g = (g_1(t), ..., g_n(t))_{t=0}^{\infty}$ be a sequence of nonnegative contributions. Then the corresponding aggregate contribution in period t

is;

$$G(t) \equiv \sum_{i=1}^{n} g_i(t)$$

and the aggregate of all players' contributions but i's in period t is;

$$G_i(t) \equiv G(t) - g_i(t)$$

For g to be a candidate equilibrium outcome, it has to be *feasible* and *wasteless* i.e. $G(t) = 0$ for all $t > T^*$ where T^* is the period when the necessary amount for the project to be finished is reached.

A candidate outcome *g* is a *Nash equilibrium* if and only if no player wishes unilaterally to deviate from it when there is maximal punishment. The strategy profile in which all the other players never contribute imposes the maximal punishment on a unilateral deviator. This punishment is imposed by *grim-g strategy profile* in which *g* is played in each period unless $Z(t) \neq G(t)$ is observed. In that case, no player ever contributes again. Grim-g strategy profile is feasible, even though individual contributions are unobserved, because it is based on aggregates. Hence, *g* is a Nash equilibrium if and only if the grim-g profile is a Nash equilibrium.

Let's consider a contribution vector $z = (z_1, z_2, ..., z_n)$ that completes the project, but is not an equilibrium of the static game. Hence there is a player; say $i \in N$, if the others contribute Z_i, then player i prefers to contribute nothing rather than z_i:

$$\lambda_i Z_i > V_i - z_i \tag{2}$$

This implies that if the others contribute Z_i, the right side of (2) is still player i's payoff from contributing z_i, and the left side is a lower bound on her payoff if she deviates to zero - it is her payoff if no player contributes again after the deviation. Now consider an outcome corresponding to z that completes the project but the contributions are made in stages over multiple periods. In the no-discount case, g still gives player i payoff $V_i - z_i$. However her payoff from deviating to zero in the first period, given that it stops future contributions, is $\lambda_i G_i(0)$. The player will not deviate if the contributions of the others in the first period are so small that

$$\lambda_i G_i(0) < V_i - z_i \tag{3}$$

This demonstrates that a player will not be willing to deviate to zero in the first period if the others contribute only a small amount and shift the bulk of their contributions to the future to be made.

The generalization of equation 3 to other periods including the discount factor with sophisticated time-inconsistent players is

$$K_{\hat{\delta}}\Delta\lambda_i G_i(t) \leq \delta^{T^*-t} K_{\hat{\delta}}\beta b_i + K_{\hat{\delta}}\Delta\lambda_i G(t) - g_i(t) + \sum_{\tau=t+1}^{T^*} \delta^{\tau-t}\beta[K_{\hat{\delta}}\lambda_i G(\tau) - g_i(\tau)] \tag{4}$$

for all $i \in N$ and $t \leq T^*$. The left side is the current value of the player's payoff if she deviates to zero in period t and thereafter, dropping the terms due to previous contributions. The right side is her payoff if she does not deviate, dropping the same terms from previous periods and inconsistently discounting to period t. Given that the other players do not deviate (play grim-g strategies), player i prefers to contribute according to g in period t and thereafter, rather than to deviate to zero, if and only if (4) holds. Clearly, the gap between right hand side and left hand side increases if the players shift some of their current contributions to the future. Hence, by shifting, player i's incentive to contribute; to play according to grim-g strategy profile is increased. Rearrangement of (4) yields the constraint which deters downward deviations (free riding), the *under-contributing constraint*:

$$[1-\lambda_i \Delta K_{\hat{\delta}}]g_i(t) \leq \delta^{T^*-t}\beta K_{\hat{\delta}} b_i + \sum_{\tau=t+1}^{T^*} \delta^{\tau-t}\beta[\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau)] \tag{5}$$

for all $i \in N$ and $t \leq T^*$. The left-side of (5) is player i's net cost of contributing $g_i(t)$ and the right side is her continuation payoff if she does not deviate; given the grim-g strategies. The interpretation of the right side is the payoff she gives up by not contributing $g_i(t)$.

There is also another incentive for players to deviate which is to complete the project prematurely. Hence upward deviations are deterred by a second constraint, the *over-contributing constraint*:

$$(\lambda_i K_{\hat{\delta}}\Delta - 1)\left(\overline{X} - \sum_{\tau=0}^{t} G(\tau)\right) + \Delta K_{\hat{\delta}} b_i \leq \delta^{T^*-t}\beta K_{\hat{\delta}} b_i + \sum_{\tau=t+1}^{T^*} \delta^{\tau-t}\beta[\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau)] \tag{6}$$

for all $i \in N$ and $t \leq T^*$. This constraint provides that, the extra amount player i would have to contribute in period t to complete the project prematurely is so large that she will not want to contribute that amount. The right side of (6) is the same as in (5), the continuation payoff player i loses by deviating. The left side is the increase in her payoff in period t, over what it is if she contributes $g_i(t)$, if she completes the project then by contributing $g_i(t) + \bar{X} - \sum_{\tau=0}^{t} G(\tau)$. Even though the under-contributing and over-contributing constraints dissuade two kind of deviations from the grim-g strategies, they actually deter all deviations.

Theorem 1: A grim-g outcome g is a Nash equilibrium if and only if it satisfies over and under contributing constraints.

Proof. Let $T(g) = T^*$, that is, T^* is the completion period. The grim g profile gives player i payoff

$$U_i(g,0) \equiv (\frac{1-\hat{\delta}}{1-\delta})(1-\delta(1-\beta))\lambda_i G(0) - g_i(0) + \sum_{t=1}^{T^*} \delta^t \beta[(\frac{1-\hat{\delta}}{1-\delta})\lambda_i G(t) - g_i(t)]$$

$$+\delta^{T^*}(\frac{1-\hat{\delta}}{1-\delta})\beta b_i$$

Now, consider the following deviations.

Deviation 1: A contribution of non-completing $z_i \neq g_i(t)$ in period t and then never contributing again. Her payoff is then

$$U_i(g,t) \equiv (\frac{1-\hat{\delta}}{1-\delta})(1-\delta(1-\beta))\lambda_i G(0) - g_i(0) + \sum_{\tau=1}^{t-1} \delta^\tau \beta[(\frac{1-\hat{\delta}}{1-\delta})\lambda_i G(\tau) - g_i(\tau)]$$

$$+\underbrace{\delta^t \beta(\frac{1-\hat{\delta}}{1-\delta})(\lambda_i G_i(t) - (1-\lambda_i) z_i(t))}_{\text{net benefit at period t}}$$

Since $\lambda_i < 1$ and $z_i \geq 0$, one can see that $U_i^{d1}(z_i, t) < U_i^{d1}(0, t)$ which is less than $U_i(g)$ by construction.

Deviation 2: a contribution of the amount $\bar{z}_i$ exactly enough to finish the project immediately in period t. In this case the competing contribution is $\bar{z}_i \equiv g_i(t) + \overline{X} - \sum_{\tau=0}^{t} G(\tau)$. No more contributions will be made further hence her payoff is guaranteed not to exceed the equilibrium payoff by construction (The equilibrium outcome g satisfies both under and over contributing constraints.)

There is last kind of deviation for player i which is to contribute a non-completing amount $z_i \neq g_i(t)$ in period $t < T^*$, and also contribute later. Any such deviation is dominated by a deviation one of the deviations above. If $b_i \leq (1-\lambda_i)\bar{z}_i(t)$, the deviation is dominated by contributing zero in all periods $\tau \geq t$; otherwise it is dominated by contributing the competing amount $\bar{z}_i(t)$ in period t. ■

The over contributing constraint is often not a problem. For instance, if the total amount to be contributed in the completing period exceeds c_i^*, the over contributing constraint is implied by the under contributing constraint.

Corollary 1: Let g be a candidate outcome satisfying the under contributing constraint, and let $T = T(g)$. Then g is a Nash equilibrium if and only if $T = 0$ or $T < \infty$ and $g_i(T-1) + G(T) \geq c_i^*$ for all $i \in N$.

Proof. If $T^* = 0$, over-contributing constraint is vacuously satisfied. Let $T^* > 0$ and $t < T^*$. Then, condition (ii) and (iii) independently imply $g_i(t) + \sum_{\tau=t+1}^{T^*} G(\tau) \geq c_i^*$ for all i. Since, $\lambda_i < 1$, $c_i^* = \frac{b_i}{1-\lambda_i}$ and $\overline{X} \geq \sum_{\tau=0}^{T^*} G(\tau)$, we get

$$(\lambda_i - 1)(\overline{X} - \sum_{\tau=0}^{t} G(\tau)) + b_i \leq (1-\lambda_i)g_i(t)$$

This, together with the under-contributing constraint,

$$(1-\delta(1-\beta))(1-\lambda_i)g_i(t) \leq \delta^{T^*-t}\beta b_i + \sum_{\tau=t+1}^{T^*} \delta^{\tau-t}\beta[(\lambda_i G(\tau) - g_i(\tau)]$$

implies

$$(1-\delta(1-\beta))\left[(\lambda_i - 1)\left(\overline{X} - \sum_{\tau=0}^{t} G(\tau)\right) + b_i\right] \leq \delta^{T^*-t}\beta b_i$$

$$+\sum_{\tau=t+1}^{T^*}\delta^{\tau-t}\beta[\lambda_i G(\tau)-g_i(\tau)]$$

which is the over-contributing constraint. ■

CHAPTER 4

COMPARING THE EQUILIBRIUM

I restrict our attention to the projects completed in finite time. For such an equilibrium, players must have a discontinuous benefit function. In the equilibrium I construct, the players with positive benefit jumps complete the project immediately once the cumulation is close enough to $barX$. Before then, the threat of stopping future contributions keeps the players contributing.We construct the equilibrium profile recursively; as in Marx and Matthews, starting with the completion period T^*, in which the under-contributing constraint is $g_i(T^*) \leq \frac{\Delta K_{\hat{\delta}}}{1-\lambda_i \Delta K_{\hat{\delta}}} b_i = c_i^*$ for player i. Define $c_i(0) \equiv c_i^*$. Given that $(c_1(0), ..., c_n(0))$ is contributed in period T^*, then binding under-contributing constraints give us a sequence $c_i(k)_{k=0}^{\infty}$ for each i where

$$c_i(0) = \frac{\Delta K_{\hat{\delta}}}{1 - \lambda_i \Delta K_{\hat{\delta}}} b_i$$

$$c_i(1) = \frac{\delta \beta}{\Delta} \left[\frac{\lambda_i \Delta K_{\hat{\delta}}}{1 - \lambda_i \Delta K_{\hat{\delta}}} \sum_{j \neq i} c_j(0) + \frac{1 - \Delta}{1 - \lambda_i \Delta K_{\hat{\delta}}} c_i(0) \right]$$

$$c_i(2) = \delta \beta \frac{\lambda_i K_{\hat{\delta}}}{1 - \lambda_i \Delta K_{\hat{\delta}}} \sum_{j \neq i} c_j(1) + \frac{\delta(1-\beta)[1-(1-\hat{\delta})\lambda_i]}{1 - \lambda_i \Delta K_{\hat{\delta}}} c_i(1)$$

and

$$c_i(k) = \delta \beta \frac{\lambda_i K_{\hat{\delta}}}{1 - \lambda_i \Delta K_{\hat{\delta}}} \sum_{j \neq i} c_j(k-1) + \frac{\delta(1-\beta)[1-(1-\hat{\delta})\lambda_i]}{1 - \lambda_i \Delta K_{\hat{\delta}}} c_i(k-1)$$

for all $k \geq 2$. In an equilibrium outcome g^*, in the very first period, t=0, each player i contributes a fraction of c_i^* and in every other period $t > 0$, each player i contributes the amount $c_i(T^* - t)$.[6] Thus,

$$g_i^*(t) \equiv \begin{cases} \frac{\overline{X} - R(T^*-1)}{R(T^*) - R(T^*-1)} c_i(T^*) & \text{for } t = 0 \\ c_i(T^* - t) & \text{for } 0 < t \leq T^* \\ 0 & \text{for } t > T^* \end{cases}$$

where $R(k) = \sum_{\tau=0}^{k} \sum_{i \in N} c_i(\tau)$. Note that, $g^*(t)$ satisfies the under-contributing constraint by construction. If T^* is finite, then since $g_i^*(T^* - 1) + G^*(T^*) \geq c_i^* = c_i(0)$ for all i, Corollary 1 implies that $g^*(t)$ also satisfies the over-contributing constraint. Thus, it is a Nash equilibrium outcome.

Now I turn our attention to the time-consistent agent who has a discount factor that corresponds to the average discount factor of the sophisticated time-inconsistent agent. I follow the analysis in Chade et. al. (2008) to find the time-consistent agent who has a discount factor that corresponds to the average discount factor of the sophisticated time-inconsistent agent. That is, for a given number of periods t, I consider a time-consistent agent with a discount factor $\hat{\delta}_t$, such that

$$1 + \hat{\delta}_t + \hat{\delta}_t^{\,2} + ... + \hat{\delta}_t^{\,t} = 1 + \beta\delta + \beta\delta^2 + ... + \beta\delta^t$$

Thus, for a given δ and β, there is a discrete set of corresponding time-consistent agents with discount factor depending on the number of periods, $\{\hat{\delta}_t\}_{t=1}^{\infty}$. Note that

[6]Player i is willing contribute at most $c_i(k)$ in period $T^* - k$, provided that each player j contributes exactly $c_j(T^* - \tau)$ in period $\tau > t$. In the equilibrium I construct, player i contributes these critical amounts, $c_i(k)$, for every period $T^* - k$, except in the very first period. In the first period, period 0, players may need to contribute less. When each player contributes $c_i(\tau)$ in period $T^* - \tau$, the remaining total contribution to be made from $T^* - k$ on is given by $R(k) = \sum_{\tau=0}^{k} \sum_{i \in N} c_i(\tau)$. Thus, at $t = 0$, the remaining amount of contributions, $R(T^*)$, must be at least as big as $\overline{X}$. That is, T^* satisfies $R(T^* - 1) < \bar{X} \leq R(T^*)$.

when $t = \infty$, I have $\hat{\delta}_\infty = \frac{\delta\beta}{1-\delta+\delta\beta} = \frac{\delta\beta}{\Delta}$. And when $t = 2$, $\hat{\delta}_2 = \delta\beta$. Thus, for any $t > 2$ I have $\delta\beta < \hat{\delta}_t < \frac{\delta\beta}{\Delta}$, where $\Delta = 1 - \delta(1-\beta) < 1$.

Even in an equilibrium which completes the project in finitely many periods, V_i is received every period following the completion period. Thus, the relevant corresponding time-consistent agent seems to be the one with $\hat{\delta}_\infty$. Nevertheless, I carry out our comparison with respect to any possible corresponding time-consistent agent who has the discount factor $\hat{\delta}_t$ with $t = 2, ..., \infty$. Thus, I not only provide a comparison using $\hat{\delta}_\infty$, but also any possible $\hat{\delta}_t$.

The condition that is needed to ensure the existence of a completing equilibrium in the time-consistent environment is $\hat{\delta} > \delta^*$, where δ^* is the threshold discount factor provided in Marx and Matthews. To meet this condition I assume that δ and β are such that $\delta\beta \geq \delta^*$. This ensures that $\hat{\delta}_t > \delta^*$ for any $t > 2$.

Now, I prove two useful lemmas. Recall $K_{\hat{\delta}} = \frac{1-\hat{\delta}}{1-\delta}$.

Lemma 1: $\Delta K_{\hat{\delta}_\infty} = 1$.

Proof. This is straightforward.

$K_{\hat{\delta}_\infty} = \frac{1-\hat{\delta}_\infty}{1-\delta} = \frac{1}{1-\delta}(1 - \frac{\delta\beta}{1-\delta+\delta\beta}) = \frac{1}{1-\delta}\frac{1-\delta}{1-\delta+\delta\beta} = 1/\Delta$ ■

Lemma 2: $\hat{\delta}_t$ is strictly increasing in t.

Proof. First note that $1 + \hat{\delta}_t + \hat{\delta}_t^{\,2} + ... + \hat{\delta}_t^{\,t} = 1 + \beta\delta + \beta\delta^2 + ... + \beta\delta^t$ implies $\hat{\delta}_t(1 + \hat{\delta}_t + ... + \hat{\delta}_t^{\,t-1}) = \beta\delta(1 + \delta + ... + \delta^{t-1})$. Since, $\delta > \hat{\delta}_t$, I have $\hat{\delta}_t > \delta\beta$. Also, $\beta\delta^t > \hat{\delta}_t^{\,t}$. To see this, suppose otherwise, that is, assume $\hat{\delta}_t^{\,t} \geq \beta\delta^t$. Then, $\frac{\beta\delta^t}{\delta} < \frac{\hat{\delta}_t^t}{\hat{\delta}}$ since $\delta > \hat{\delta}_t$. Thus, $\hat{\delta}_t^{t-1} > \beta\delta^{t-1}$. Repeating this argument I get $\hat{\delta}_t^s > \beta\delta^s$ for all $s = 1, ..., t$, which implies $1 + \hat{\delta}_t + \hat{\delta}_t^{\,2} + ... + \hat{\delta}_t^{\,t} > 1 + \beta\delta + \beta\delta^2 + ... + \beta\delta^t$, which is a contradiction. Thus, I get $\hat{\delta}_t^{\,t} < \beta\delta^t$. Now, to see $\hat{\delta}_t$ is strictly increasing in t, add

$\hat{\delta}_t^{t+1}$ to the left hand side, and $\beta\delta^{t+1}$ to the right hand side of

$$1 + \hat{\delta}_t + \hat{\delta}_t^2 + ... + \hat{\delta}_t^t = 1 + \beta\delta + \beta\delta^2 + ... + \beta\delta^t$$

and get

$$1 + \hat{\delta}_t + \hat{\delta}_t^2 + ... + \hat{\delta}_t^t + \hat{\delta}_t^{t+1} < 1 + \beta\delta + \beta\delta^2 + ... + \beta\delta^t + \beta\delta^{t+1}$$

since $\hat{\delta}_t^{t+1} < \beta\delta^{t+1}$, which is because $\hat{\delta}_t < \delta$ and $\hat{\delta}_t^{\,t} < \beta\delta^t$. Thus, to get the equality

$$1 + \hat{\delta}_{t+1} + \hat{\delta}_{t+1}^2 + ... + \hat{\delta}_{t+1}^t + \hat{\delta}_{t+1}^{t+1} = 1 + \beta\delta + \beta\delta^2 + ... + \beta\delta^t + \beta\delta^{t+1}$$

we must have $\hat{\delta}_{t+1} > \hat{\delta}_t$. ■

These two lemmas imply

Lemma 3: For any $\hat{\delta}_t$ with $t < \infty$, $\Delta K_{\hat{\delta}_t} > 1$.

Proof. Note that $K_{\hat{\delta}_t} = \frac{1-\hat{\delta}_t}{1-\delta}$ is decreasing in $\hat{\delta}_t$, thus decreasing in t by Lemma 2. Thus, $K_{\hat{\delta}_t} > K_{\hat{\delta}_\infty} = 1/\Delta$ by Lemma 1. Thus, $\Delta K_{\hat{\delta}_t} > 1$. ■

I show that the set of Nash equilibria for the time inconsistent agents with any $\delta - \beta$ pairs is including the set of Nash equilibria for the time consistent agents with any of the corresponding discount factor to that given $\delta - \beta$ pair. However, these two sets are not same, that is to say, there are some Nash equilibrim outcomes with the time-inconsistent set-up such that they are not Nash equilibrium outcomes with the time-consistent set-up. Following propositions basically summarizes these results

Proposition 1: For all $\bar{X}$,δ,β and $\hat{\delta}_k$, any equilibrium outcome $g \equiv \{g_i(t)\}_{t=0}^{\infty}$ for the time-consistent agents with discount factor $\hat{\delta}_k$, is also an equilibrium outcome for the time-inconsistent agents.

Proof. Let $g \equiv \{g_i(t)\}_{t=0}^{\infty}$ be an arbitrary equilibrium outcome for a given project size $\bar{X}$ and $\hat{\delta}_k$, for time-consistent agents with discount factor $\hat{\delta}_k$. Suppose g completes the project in T^* periods. Note that, a contribution scheme g is a Nash equilibrium outcome if and only if the grim-g is a Nash equilibrium strategy profile. Thus, the grim-g is a Nash equilibrium strategy profile, thus, the under contributing constraints must be satisfied. That is,

$$(1-\lambda_i)g_i(t) \leq \hat{\delta}_k^{T^*-t} b_i + \sum_{\tau=t+1}^{T^*} \hat{\delta}_k^{\tau-t}(\lambda_i G(\tau) - g_i(\tau))$$

is satisfied for all $t \in \{0, 1, 2, .., T^* - 1\}$, and for period T^*, I have

$$g_i(T^*) \leq \frac{b_i}{1-\lambda_i}$$

Now, I check whether this sequence g satisfies the under contributing constraints of the time-inconsistent agents. For period T^*, the condition is $[1-\lambda_i \Delta K_{\hat{\delta}}]g_i(T^*) \leq \Delta K_{\hat{\delta}} b_i$ which is satisfied since $\frac{b_i}{1-\lambda_i} \leq \frac{\Delta K_{\hat{\delta}_t} b_i}{1-\lambda_i \Delta K_{\hat{\delta}_t}}$, which is because $\Delta K_{\hat{\delta}_t} \geq 1$ by Lemma 3.

For $t < T^* - 1$, I need to check

$$[1-\lambda_i \Delta K_{\hat{\delta}}]g_i(t) \leq \delta^{T^*-t} \beta K_{\hat{\delta}} b_i + \sum_{\tau=t+1}^{T^*} \delta^{\tau-t} \beta[(\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau))]$$

We have

$$
\begin{aligned}
[1-\lambda_i \Delta K_{\hat{\delta}}]g_i(t) &\leq (1-\lambda_i)g_i(t) \\
&\leq \hat{\delta}_k^{T^*-t} b_i + \sum_{\tau=t+1}^{T^*} \hat{\delta}_k^{\tau-t}(\lambda_i G(\tau) - g_i(\tau)) \\
&\leq \frac{\delta^{T^*-t}\beta}{\Delta} b_i + \sum_{\tau=t+1}^{T^*} \frac{\delta^{\tau-t}\beta}{\Delta}(\lambda_i G(\tau) - g_i(\tau)) \\
&\leq \delta^{T^*-t}\beta K_{\hat{\delta}} b_i + \sum_{\tau=t+1}^{T^*} \delta^{\tau-t}\beta[(\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau)]
\end{aligned}
$$

for all $t \in \{0, 1, 2, .., T^* - 1\}$. The first inequality follows from Lemma 3. The second inequality is simply the under-contributing constraint for the time-consistent agent. To see the third inequality, note that since $\hat{\delta}_t \leq \frac{\delta\beta}{1-\delta+\delta\beta}$ and $\hat{\delta}_t < \delta$ for any t. Thus, I have $\hat{\delta}_t^s \leq \frac{\delta^s\beta}{1-\delta+\delta\beta} = \frac{\delta^s\beta}{\Delta}$ for any $s \geq 1$, hence the third inequality. And the last inequality follows from Lemma 3 and from the fact that $1/\Delta \geq 1$.

That is, grim-g is also a Nash equilibrium strategy profile for the time-inconsistent agents, thus, g is a Nash equilibrium outcome for the time-inconsistent agents. ■

Proposition 2: For all $\bar{X}$,δ,β, there exists a sequence of nonnegative contributions, $g \equiv \{g_i(t)\}_{t=0}^{\infty}$, such that g is a Nash equilibrium outcome for the time-inconsistent agents while it is not a Nash equilibrium outcome for the corresponding time-consistent agents.

Proof. Let $g \equiv \{g_i(t)\}_{t=0}^{\infty}$ be an arbitrary equilibrium outcome for a given project size $\bar{X}$, for the inconsistent agents, where g completes the project in T^* periods, such that the under contributing constraint binds for some $t' \in \{0, 1, 2, ..., T^* - 2\}$. Since grim-$g$ strategy profile is a Nash equilibrium strategy profile, the under contributing constraint holds:

$$[1-\lambda_i \Delta K_{\hat{\delta}}]g_i(t) \leq \delta^{T^*-t}\beta K_{\hat{\delta}} b_i + \sum_{\tau=t+1}^{T^*} \delta^{\tau-t}\beta[(\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau)]$$

for all $t \in \{0,1,2,..,T^*\}$. Moreover for period t' I have,

$$[1-\lambda_i \Delta K_{\hat{\delta}}]g_i(t') = \delta^{T^*-t'}\beta K_{\hat{\delta}} b_i + \sum_{\tau=t'+1}^{T^*} \delta^{\tau-t'}\beta[(\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau)]$$

Now, I consider the corresponding time-consistent agents with $\hat{\delta}_k$ and check the under contributing constraint at period t' for these agents.

$$\begin{aligned}
(1-\lambda_i)g_i(t') &= \frac{1-\lambda_i}{1-\lambda_i \Delta K_{\hat{\delta}}}\left[\delta^{T^*-t'}\beta K_{\hat{\delta}} b_i + \sum_{\tau=t'+1}^{T^*} \delta^{\tau-t'}\beta[(\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau)]\right] \\
&\geq \delta^{T^*-t'}\beta K_{\hat{\delta}} b_i + \sum_{\tau=t'+1}^{T^*} \delta^{\tau-t'}\beta[(\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau)] \\
&= \frac{\delta^{T^*-t'}\beta}{\Delta} K_{\hat{\delta}} \Delta b_i + \sum_{\tau=t'+1}^{T^*} \frac{\delta^{\tau-t'}\beta}{\Delta}[(\lambda_i K_{\hat{\delta}} \Delta G(\tau) - \Delta g_i(\tau)] \\
&> \frac{\delta^{T^*-t'}\beta}{\Delta} b_i + \sum_{\tau=t'+1}^{T^*} \frac{\delta^{\tau-t'}\beta}{\Delta}[(\lambda_i G(\tau) - g_i(\tau)] \\
&> \hat{\delta}_k^{T^*-t'} b_i + \sum_{\tau=t'+1}^{T^*} \hat{\delta}_k^{\tau-t'}[(\lambda_i G(\tau) - g_i(\tau)]
\end{aligned}$$

where the first equality follows from the binding under contributing constraint for the time-inconsistent agent at period t'. The first inequality follows from the fact that $K_{\hat{\delta}_k}\Delta \geq 1$ by Lemma 3. The first strict inequality follows from both $K_{\hat{\delta}_k}\Delta \geq 1$ and that $\Delta < 1$. And the last strict inequality follows from the fact that $\hat{\delta}_k^s < \frac{\delta^s\beta}{\Delta}$ for all finite $s > 1$, which follows from $\hat{\delta}_k^s < \frac{\delta\beta}{\Delta}$ and $\hat{\delta}_k < \delta$. Thus, at period t', the under contributing constraint for the time-consistent agents does not hold. Thus, grim-g is not a Nash equilibrium strategy profile, hence not an equilibrium outcome, for the time-consistent agents. ■

When it comes to compare the inconsistent agents to the corresponding average consistent agents in the sense of completion period of a project, surprisingly; sophisticated time-inconsistent agents finish the project no later than the time-consistent agents do.

Let $T^*_{SO}(\bar{X})$ be the minimum number of periods that the sophisticated time-inconsistent agents finish the project of size $\bar{X}$. Let $T^*_{TC(\hat{\delta}_k)}(\bar{X})$ be the minimum number of periods that the corresponding time-consistent agents, with the discount factor $\hat{\delta}_k$, finish the project of size $\bar{X}$.

Theorem 2: For any $\bar{X}$, δ, β, $\hat{\delta}_k$, $\lambda_i < 1$, $b_i > 0$ and $n \geq 2$, with $\lambda_i < 1/(K_{\hat{\delta}_k}\Delta)$ for all i, we have $T^*_{SO}(\bar{X}) \leq T^*_{TC(\hat{\delta}_k)}(\bar{X})$.

Proof. First, I show that a grim-g strategy profile with binding critical values is the fastest, that is, $T^*_i(\bar{X})$, is the number of periods induced by the grim-g with binding critical values, where $i \in \{SO, TC(\hat{\delta}_k)\}$. To see this, suppose otherwise, that is, there is another equilibrium which finishes the project faster than the equilibrium profile with the binding critical values. Thus, at least one player at some period must be contributing more than her critical value for that period, violating the under contributing constraint. Thus, by Theorem 1, this profile is not a Nash equilibrium outcome. Thus, the grim-g equilibrium outcome with binding critical values is the fastest.

Now, I show that for any δ, β, $\hat{\delta}_k$ and for any given values of λ_i and n, I have

$$\sum_{t=0}^{T^*} c_i^{\delta\beta}(t) \geq \sum_{t=0}^{T^*} c_i^{\hat{\delta}_k}(t) \tag{7}$$

for any $T^* \geq 1$, where $c_i^{\delta\beta}(t)$ is the critical value of the time-inconsistent agent i in period t, and $c_i^{\hat{\delta}_k}(t)$ is the critical value of the corresponding time-consistent agent i, with the discount factor $\hat{\delta}_k$, in period t. To see this, I look at the critical values for the time-inconsistent agent.

$$c_i^{\delta\beta}(0) = \frac{\Delta K_{\hat{\delta}_k}}{1 - \lambda_i \Delta K_{\hat{\delta}_k}} b_i$$

$$c_i^{\delta\beta}(1) = \frac{\delta\beta}{\Delta} \frac{\lambda_i \Delta K_{\hat{\delta}_k}}{1 - \lambda_i \Delta K_{\hat{\delta}_k}} \left[\sum_{j \neq i} c_j(0) + \frac{1 - \Delta}{\lambda_i \Delta K_{\hat{\delta}_k}} c_i(0) \right]$$

$$c_i^{\delta\beta}(s) = \frac{\delta\beta}{\Delta} \frac{\lambda_i \Delta K_{\hat{\delta}_k}}{1 - \lambda_i \Delta K_{\hat{\delta}_k}} \left[\sum_{j \neq i} c_j(s-1) + \frac{1 - \Delta}{\beta} c_i(s-1) \right]$$

for all $s \geq 2$. The critical values of the corresponding time-consistent agent with a discount factor $\hat{\delta}_k$ are

$$c_i^{\hat{\delta}_k}(0) = \frac{b_i}{1 - \lambda_i}$$

$$c_i^{\hat{\delta}_k}(s) = \hat{\delta}_k \frac{\lambda_i}{1 - \lambda_i} \sum_{j \neq i} c_j^{\hat{\delta}_k}(s-1)$$

for all $s \geq 1$. Now, I compare the critical values. First note that, since $\Delta K_{\hat{\delta}_k} \geq 1$ for any k, we have

$$c_i^{\hat{\delta}_k}(0) = \frac{b_i}{1 - \lambda} \leq \frac{\Delta K_{\hat{\delta}_k}}{1 - \lambda_i \Delta K_{\hat{\delta}_k}} b_i = c_i^{\delta\beta}(0)$$

Also,

$$\begin{aligned} c_i^{\delta\beta}(1) &= \frac{\delta\beta}{\Delta}\frac{\lambda_i \Delta K_{\hat{\delta}_k}}{1-\lambda_i\Delta K_{\hat{\delta}_k}}\left[\sum_{j\neq i} c_j^{\delta\beta}(0) + \frac{1-\Delta}{\lambda_i\Delta K_{\hat{\delta}_k}} c_i^{\delta\beta}(0)\right] \\ &\geq \frac{\delta\beta}{\Delta}\frac{\lambda_i \Delta K_{\hat{\delta}_k}}{1-\lambda_i\Delta K_{\hat{\delta}_k}}\sum_{j\neq i} c_j^{\delta\beta}(0) \\ &\geq \hat{\delta}_k \frac{\lambda_i}{1-\lambda_i}\sum_{j\neq i} c_j^{\hat{\delta}_k}(0) = c_i^{\hat{\delta}_k}(1) \end{aligned}$$

which follows from the fact that $1-\Delta > 0$, $\frac{\delta\beta}{\Delta} \geq \hat{\delta}_k$ for any k, Lemma 3 and the fact that $c_i^{\hat{\delta}_k}(0) \leq c_i^{\delta\beta}(0)$. Similarly,

$$c_i(s) = \delta\beta\frac{\lambda_i K_{\hat{\delta}}}{1-\lambda_i\Delta K_{\hat{\delta}}}\sum_{j\neq i} c_j(s-1) + \frac{\delta(1-\beta)[1-(1-\hat{\delta})\lambda_i]}{1-\lambda_i\Delta K_{\hat{\delta}}} c_i(s-1)$$

$$> \frac{\delta\beta}{\Delta}\frac{\lambda_i \Delta K_{\hat{\delta}_k}}{1-\lambda_i\Delta K_{\hat{\delta}_k}}\sum_{j\neq i} c_j^{\delta\beta}(s-1)$$

$$\geq \hat{\delta}_k \frac{\lambda_i}{1-\lambda_i}\sum_{j\neq i} c_j^{\hat{\delta}_k}(s-1) = c_i^{\hat{\delta}_k}(s)$$

for all $s \geq 1$, where the last inequality follows from the fact that $c_j^{\delta\beta}(s-1) \geq c_j^{\hat{\delta}_k}(s-1)$ for all j and for all $s \geq 1$. Thus, we get $c_i^{\delta\beta}(t) \geq c_i^{\hat{\delta}_k}(t)$ for all i, t and k. Thus,

$$\sum_{t=0}^{T^*} c_i^{\delta\beta}(t) \geq \sum_{t=0}^{T^*} c_i^{\hat{\delta}_k}(t)$$

Thus, for the cumulative contributions we get

$$\sum_i \sum_{t=0}^{T^*} c_i^{\delta\beta}(t) \geq \sum_i \sum_{t=0}^{T^*} c_i^{\hat{\delta}_k}(t)$$

This implies that the time-inconsistent agents always achieve a weakly higher cumulative contribution at any period. Thus, they will always achieve a given $\bar{X}$ weakly earlier than any corresponding time-consistent agents, that is, $T^*_{SO}(\bar{X}) \leq T^*_{TC(\hat{\delta}_k)}(\bar{X})$. ■ A direct corollary to the theorem above is the following.

Corollary 2: For any $\bar{X}$, δ, β, $\lambda_i < 1$, $b_i > 0$ and $n \geq 2$, we have $T^*_{SO}(\bar{X}) \leq T^*_{TC(\hat{\delta}_\infty)}(\bar{X})$.

Proof. With $\hat{\delta}_\infty$ the condition $\lambda_i < 1/(K_{\hat{\delta}_k}\Delta)$ turns into $\lambda_i < 1$, since $K_{\hat{\delta}_\infty}\Delta = 1$ by Lemma 1, and the result directly follows from Theorem 2 above. ■

The intuition behind this result is that the time-inconsistent agents are sophisticated, thus they know that their future selves may tend to postpone contributions and cause the project be finished later. Thus, the current selves of time-inconsistent agents contribute more, relative to the time-consistent agents, in early periods in order to guard themselves against the future selves. Thus, with achieving higher cumulative contribution levels in the early periods, time-inconsistent agents manage to finish the project earlier than the time-consistent agents.

CHAPTER 5

CONCLUSION

The model of voluntary contribution, that I have used, lets the players contribute any amount in any period as they wish, by observing the aggregate contribution at each period. I introduced sophisticated time-inconsistent agents with linear discontinuous preferences to the model and characterized the Nash equilibria as well as the shortest time period in which the project is finished.

The main result is about the comparison of completion period of specific projects. I find that there are certain projects which are completed by the time-inconsistent agents earlier than the corresponding time-consistent agents. This is surprising because time inconsistent agents tend to be postpone costly actions. Intuitively, a time-inconsistent agent commits to higher contributions in the earlier periods in order to guard herself against the potentially low contribution levels of the future selves.

For the future research, I plan to look at the naive agent case, where the time-inconsistent agent is not aware of her time-inconsistency. I also plan to generalize the results to the heterogeneous agents case.

REFERENCES

Admati, A. R. & Perry, M. (1991). Joint projects without commitment. *The Review of Economic Studies*, 58(2), 259-276.

Chade, H., Prokopovych, P., & Smith, L. (2008). Repeated games with present-biased preferences. *Journal of Economic Theory*, 139, 157-175.

Freshtman, C. & Nitzan, S. (1991). Dynamic Voluntary Provision of public goods. *European Economic Review*, 35, 1057-1067.

Laibson, D. (1997). Golden eggs and hyperbolic discounting. *The Quarterly Journal of Economics*, 112(2), 443-477.

Marx, L. E. & Matthews, S. A. (2000). Dynamic voluntary contribution to a public project. *The Review of Economic Studies*, 67, 327-358.

O'Donoghue, T., & Rabin, M. (2001). Choice and procrastination. *The Quarterly Journal of Economics*, 116(1), 121-160.

O'Donoghue, T., & Rabin, M. (2000). Economics of immediate gratification. *Journal of Behavioral Decision Making*, 13(2), 233-250.

O'Donoghue, T., & Rabin, M. (1999a). Doing it now or later. *American Economic Review*, 89(1), 103-124.

O'Donoghue, T., & Rabin, M. (1999b). Incentives for procrastinators. *The Quarterly Journal of Economics*, 114(3), 769-816.

Phelps, E. S., & Pollak, R. A. (1968). On second-best national saving and game-equilibrium growth. *The Review of Economic Studies*, 35(2), 185-199.

Yılmaz, M. (2013). Repeated moral hazard with a time-inconsistent agent. *Journal of Economics Behavior and Organization*, 95, 70-89.