URBANIZATION AND STRUCTURAL CHANGE IN AN ENDOGENOUS GROWTH SETTING

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URBANIZATION AND STRUCTURAL CHANGE IN AN ENDOGENOUS GROWTH SETTING

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Thesis Abstract

Burak Türkgülü, "Urbanization and Structural Change in an Endogenous Growth Setting"

This thesis studies the implications of a Romer-type endogenous technological growth on urbanization and structural change using a two-sector, continuous-time growth model. We make use of the tight link between urbanization and structural change by assuming that industrial and agricultural sectors are located in urban and rural areas respectively.

We present two models. First model assumes perfect labor mobility between rural and urban sectors such that wage rates are equalized between the two sectors. The implication of such an assumption together with the existence of an externality from urban production is that there are multiple equilibria and no transitional dynamics. Main aggregate variables grow at the same rate at the steady state but the model can account for both increasing and decreasing patterns of prices. For the plausible case, we find that urbanization level is related positively with the taste in urban goods, negatively with the share of labor in rural sector, positively with the share of labor in urban sector and related negatively with the discount factor.

Second model we present features imperfect mobility of labor between the two sectors, where migration function is increasing in the wage gap between urban and rural sectors. The implication is that there are transitional dynamics on a single equilibrium path, where both the urbanization level and the urban consumption to capital ratio increase, for a given initial level of urbanization.

Tez Özeti

Burak Türkgülü, “Endojen Büyüme Bağlamında Kentleşme ve Yapısal Değişim”

Bu tez Romer tipi endojen teknolojik büyümenin kentleşme ve sektörel değişim üzerine etkilerini iki sektörlü ve sürekli zamanlı bir büyüme modeli kullanarak incelemektedir. Bu tezde kentleşme ve sektörel değişim arasındaki sıkı bağ kullanılarak sanayi ve tarım sektörlerinin sırasıyla kentsel ve kırsal alanlarda bulundukları varsayılmaktadır.

Bu tezde iki ayrı model sunmaktayız. İlk model kırsal ve kentsel sektörlerde kusursuz işgücü akışkanlığı varsayarak iki sektördeki fiyatların eşitlendiği bir modeldir. Böyle bir varsayım kentsel sektörden kaynaklanan olumlu dışsallıklarla beraber çoklu dengeye sebebiyet vermekte ve geçişsel dinamiklerden yoksun bir model ortaya çıkarmaktadır. Kararlı durumda modelin temel toplam değişkenlerinin aynı hızla büyümekte; ancak modelin hem artan hem de azalan tarımsal ürün fiyatlarıyla tutarlı olduğu görülmektedir. Ampirik olarak desteklenebilecek olan denge noktasında şehirleşme seviyesinin kentsel ürünlere olan beğeniyle pozitif, kırsal sektördeki işgücünün payıyla negatif, kentsel sektördeki işgücünün payıyla pozitif ve iskonto faktörüyle negatif ilişkili olduğu gösterilmektedir.

Sunduğumuz ikinci modelde, iki sektör arasında kusursuz işgücü akışkanlığı olduğu varsayımı kentsel ve kırsal sektörler arasındaki ücret farkında artan bir göç fonksiyonuyla değiştirilmektedir. Bu varsayım değişikliği, herhangi bir başlangıç kentleşme seviyesi için hem kentleşme hem de kentsel ürün tüketiminin sermayeye oranının arttığı geçişsel tek bir yolun varlığının gösterilebilmesini sağlamaktadır.

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CHAPTER 1
INTRODUCTION

Urbanization and structural change have accompanied the growth experiences of almost all developed and developing countries. In England, which can be considered as the first country that started developing in the modern sense, urban population made up 13 to 16 percent of the whole population and 60 to 70 percent of the population was employed in agriculture before the industrial revolution, when modern growth is considered to have begun. After the industrial revolution, only 9 percent of the work force was employed in agriculture in 1901 and the level of urbanization was 67.6 percent in 1900 (Bairoch, 1985/1988; Kuznets, 1966).

Urbanization can be defined as the process whereby composition of population shifts in favor of the urban areas. According to the general understanding an urban area, or a city, is the geographic area where population density is relatively higher and whose borders are set by public authorities. A more economic definition is that it is the area which covers "the entire local labor market" and all the activities such as manufacturing, services or residence dependent on it (Henderson, 2005: 1548). Throughout the thesis, whenever we mention "urban area" or "city" it will be in the economic sense.

However, this definition poses a challenge in determining which agglomeration is a city, which affects the estimates of urbanization. Bairoch (1985/1988) considers areas where more than 5,000 people live as a city and bases his estimates on this threshold. He also reports estimates using a threshold of 2,000 but the differences between the estimates using different thresholds do not turn out to significant.

Historically, urbanization almost as a rule involved people moving from rural areas to urban areas (Hohenberg & Lees, 1995). Observe that the definition given above does not logically imply that this has to be so; it could very well be the case that net birth rates in urban areas were higher than the rural areas, which would again lead to an increase in the proportion of urban population. However, during the development phase, net birth rates in the cities have almost always been less than the net birth rates in the rural areas historically (Bairoch 1985/1988).

On the other hand, structural change is the process through which resources of an economy are reallocated across different sectors. The historical accounts show that all of today's developed countries have gone through a phase of structural change, where percentage of labor force working in agriculture has declined and percentage of labor force employed in the industry has increased (Kuznets, 1966). Kuznets (1966) also found some weak evidence that reproducible capital was reallocated towards industry from agriculture. In most countries, the current trend is towards an increase in the service sector's labor share while the shares of other sectors are declining or stagnating. Kongsamut, Rebelo and Xie (2001) call these facts the "Kuznets facts" as they are evidenced by Kuznets and relating them to the Kaldor facts as sub-processes behind the growth process.

Although they are usually taken almost as proxies for one another, from the definitions given in the discussion above, there is no reason for urbanization and structural change to imply each other. To understand the relationship between these two processes better, let us look at two examples, one from arguably the most developed region in the world and the other is from a developing country.

Figure 1 shows the evolution of urbanization level in North America from 1800 to 2000. The data for the period between 1800 and 1980 comes from Bairoch

(1985/1988) and we used the data from United Nations (2006) to update the series until 2000. Observe that both data sets are comparable as the trends match for the period they overlap although the United Nations data apparently has a lower threshold for their measure of urbanization level. Observe that urbanization level is increasing for North America and it seems to have leveled off since 1960s.

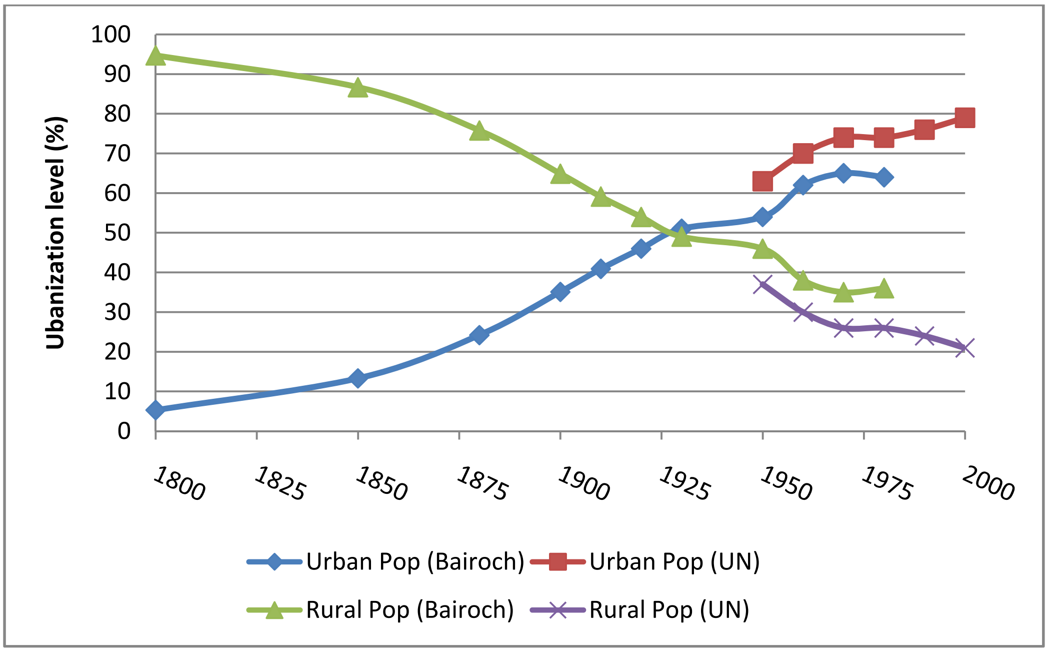

Figure 1. Urbanization level in North America, 1800 – 2000.

Figure 2 shows the evolution of employment shares in the United States from 1800 to 2000. The figure is taken from Acemoglu (2009: 698). It can be observed that the share of agriculture has declined constantly from above 80 percent in 1800 to below 10 percent approaching 2000. Observe that the employment share of agricultural sector has leveled off at the same time as the urbanization level. Additionally, the shares of manufacturing and services increased until around 1970. However, after that, the growth rate of the share of service sector picked up and the employment share of manufacturing sector has decreased since. Both Figure 1 and Figure 2 are typical for the experiences of developed countries.

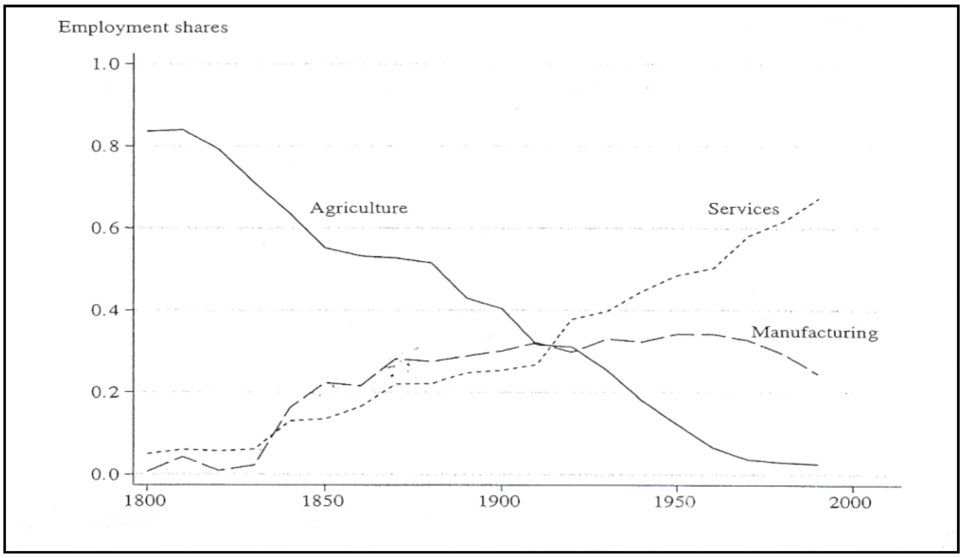

Figure 2. Employment shares of agricultural, manufacturing and service sectors in the US, 1800 – 2000 (from Acemoglu, 2009: 698).

The same series for Turkey are shown in Figure 3 and Figure 4 respectively. The data are taken from Turkish Statistical Institute (2010). Figure 3 shows that the urbanization took off around 1950s and it still has an upward trend as of 2007. Figure 4 shows that employment shares in Turkey follow a similar path to the case of early development in the United States as agriculture's share is decreasing and manufacturing and service sectors' shares are both increasing. Observe that the time Turkey started to urbanize coincides with the time that the employment share of agriculture started to decline sharply.

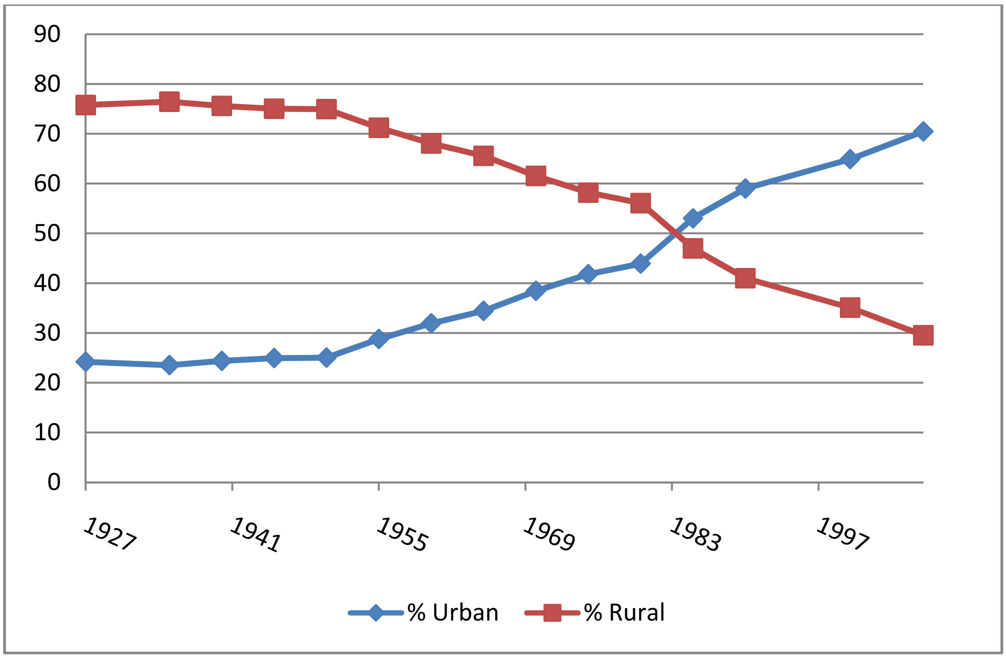

Figure 3. Percentage of urban population and rural population in Turkey, 1927 - 2007

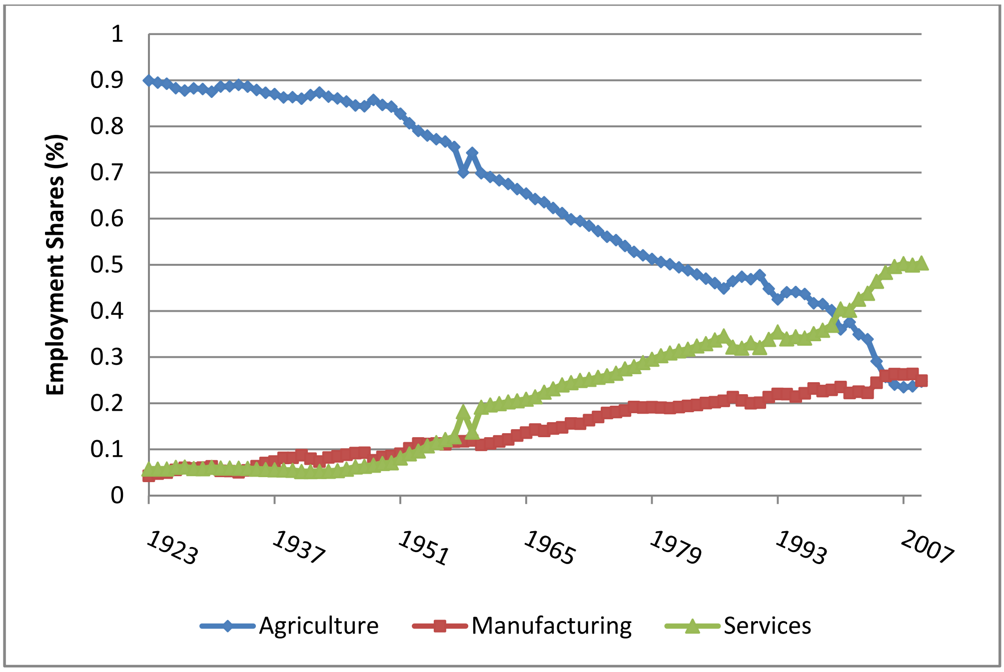

Figure 4. Employment shares of the three major sectors in Turkey, 1923 – 2009.

The experiences of both North America and Turkey show that there is a tight link between urbanization and structural change. The employment share of agricultural sector declines with industrialization. This is actually hardly surprising since while agricultural activities are almost exclusive to the rural areas, modern sectors like manufacturing and services are concentrated in urban areas. Thus, if for some reason it becomes more attractive to work in sectors other than agriculture, the laborer has to migrate to an urban area.

However, Bairoch (1985/1988) warns against taking the implications of this link too far. In the United States, agricultural employment is around 4 percent in metropolitan areas with population between 200,000 and 300,000 but lower in other developed countries. In some developing countries of 1970s, it was estimated that about 20 to 25 percent of the population were employed in agriculture in urban areas with population between 20,000 and 50,000. Thus, existence of a tight link between urbanization and structural change might be viewed with a bit more skepticism in today's developing world while it is more relevant for the developing experiences of today's developed countries.

As hinted in the beginning of the chapter, there seems to be a relationship between urbanization and level of income. Figure 5 shows that there is a positive relationship between domestic income and urbanization. The figure is an updated version of the one given in Annez and Buckley (2009: 3) using World Bank's World Development Indicators dataset.

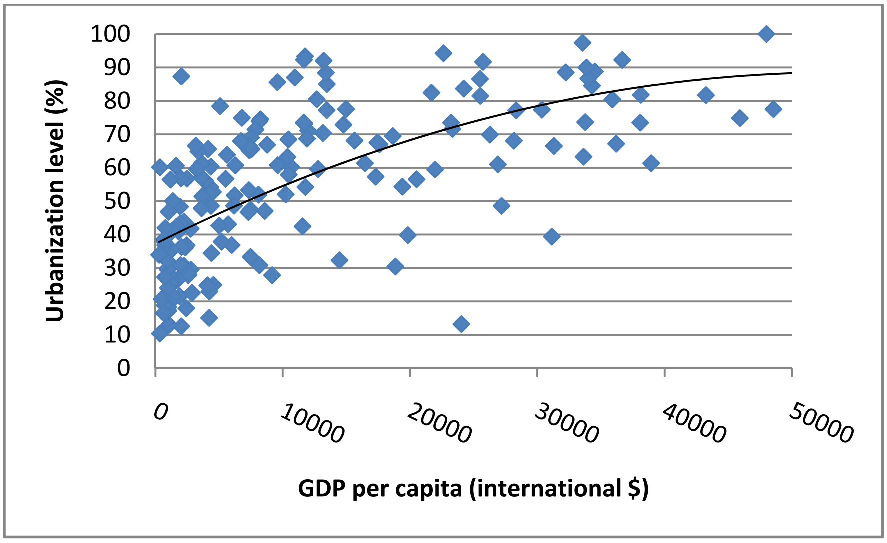

Figure 5. Urbanization level vs. GDP per capita in 2008, PPP-adjusted, 2005 constant prices.

Other than levels, growth seems to be also positively correlated with structural change, and hence with urbanization. In United Nation's World Economic and Social Survey 2006 report it is firmly stated that "economic growth requires structural change" (United Nations, 2006: 30). Their data show that between 1970 and 2003, the countries that performed better were the ones that could actually achieve some structural change. For example, fast growth in China and South Asia is accompanied by a rapid decline in employment share of agricultural sector.

The evidence above shows that there is a correlation between urbanization (or structural change), and growth and income. However, they do not necessarily imply causation in any direction.

Urbanization and structural change might just be a consequence of economic growth, which is determined at the aggregate. There are at least three different arguments for this line of reasoning. First, as economy grows, people's incomes increase. This increase itself will change the composition of demand through what is called the Engel's Law. According to Engel's Law, since demand for agricultural goods is inelastic with respect to the output of industrial sectors, demand share of agricultural goods decrease as demand share of industrial goods increase. This can lead to a shift in resources out of agriculture to industrial sectors. Kuznets (1966) mentions this is probably the case.

Second, as the economy grows through exogenous technological progress, the sectors where productivity increases faster can produce the same amount using relatively less labor. Thus, if there is low substitutability among goods, then labor would flow into the sector that is stagnant (Baumol, 1967). Then, for example, if agricultural productivity grows faster than the productivity of manufacturing, labor would be reallocated to the manufacturing sector.

Third, economy grows through innovations so that when new innovations occur, new sectors, new types of work emerge. Then, labor moves into these sectors as these sectors often have the highest productivity. However, this entails structural change by definition. If we assume that innovation occurs in the modern, urban sectors, then labor would migrate to cities and the urban sectors.

The arguments for the reverse causality, from urbanization to economic growth, are more controversial from the macroeconomist's point of view since they often involve some kind of labor market failure or positive externality from the urban sectors. First is what is dubbed as "Smithian growth." It is based on the idea that if there are many firms, which can cater to different needs, in proximity to each other, firms can specialize and through Adam Smith's idea of division of labor, efficiency would increase. This in turn would create higher growth and wealth (Quigley, 2009; Mokyr, 1995).

Second argument is that urbanization comes with agglomeration benefits. It reduces the transaction costs for both final and intermediate goods markets and for searching and matching the labor that is needed. This in turn means that as structural change occurs and urbanization level gets higher, economy becomes more efficient and higher growth can be achieved (Henderson, 2005; Quigley, 2009). Henderson (2005) and Annez and Buckley (2009) argue that agglomeration effects can be seen as the rationale behind the existence of urban areas in the first place. But the effect is reinforced by growth of urban areas.

The third and probably the most important aspect of how urban areas can create growth is that they encourage technological innovations. There are at least four different arguments on how this can be. First, by higher density of population in the urban areas, more contacts are formed and this accelerates the flow of information (Bairoch, 1985/1988). However, the mere existence of contacts is not sufficient; they must have a productivity enhancing capacity. For that to occur, more contacts should be accompanied by higher levels of economic activity in the cities so that higher level of contacts actually make more people aware of the technical need that arises and it matches the need with the person who can actually solve it through innovations. Urban areas are also hubs where people from different cultures come together for trade and migration, which might lead to new productivity enhancing contacts.

Second, urban areas through higher economic activity and contacts as mentioned above also lead to knowledge spillovers across firms or urban sectors (Bairoch, 1985/1988; Quigley, 2009). Technology that has already been adopted by one firm can be adopted by another firm which has more chance to come into contact with it due to proximity. Furthermore, a technology, which might even be traditional in one sector, might lead to an important technological advance in another when it is adopted in this new sector. Thus, problems can be solved by borrowed ideas from other sectors. Jacobs (1969) also has a similar idea. She suggests that what city does to advance the technology is that it adds new types of work to others through ideas adopted from existing works. Thus, the economy grows through this continuous addition of new works to older ones.

Simmie (2001) proposes a third way through which urban areas can encourage growth as he adopts an entrepreneurial way of thinking. If research and development activities can be considered as investment decisions, then an entrepreneur would engage in innovation if she foresees sufficient gains from it. Then, through the agglomeration effect more potential demand is there to undertake such investment and hence, leads to higher growth.

Fourth, urban economic activity has a spillover effect on agricultural sector. Bairoch (1985/1988) gives the example that in 1800s in France as iron production became more and more productive, its price fell down, which improved the terms of trade for food producers. He also suggests that the improvements in transportation networks and invention of artificial fertilizers were developments that were made possible by the urban activity and had enormous positive effects on agricultural productivity. Bairoch (1985/1988) also contends that the productivity increase in agriculture was higher than any other sector during major growth periods of nations and he links this to the spillover effects stemming from urban industrial activities.

One last concern about the nature of urbanization and structural change is if urbanization is a transitory process or a balanced growth phenomenon. Henderson (2005) argues that it is a transitory phenomenon based on the evidence from both developed and developing countries. The level of urbanization in the developed countries is settled at some level between 60 and 90 percent but in the developing countries urbanization levels are increasing. Figure 6 compares paths of urbanization levels between developed and developing countries. The data is from World Bank Development Indicators. Observe that urbanization level in developed countries leveled off while the urbanization levels of developing countries are growing.

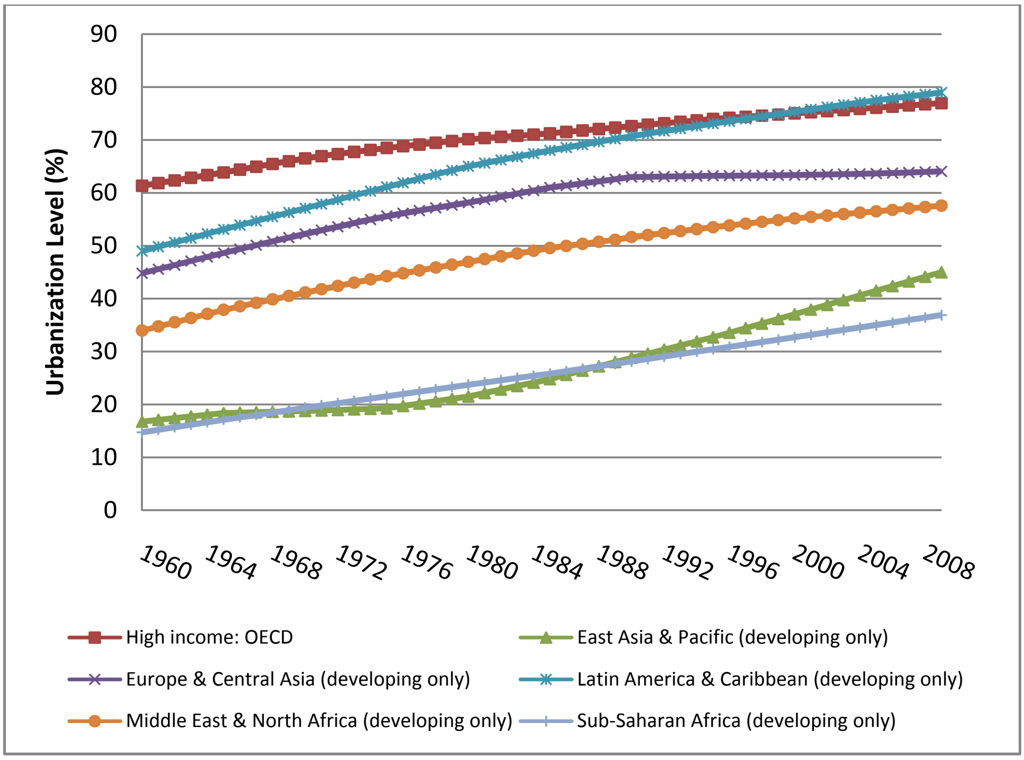

Figure 6. Urbanization level in developed and developing countries in different regions, 1960 - 2009

In this thesis, we analyze urbanization and structural change using a specific continuous-time endogenous growth model. The model we construct is based on assumptions that are consistent with the observations above. In the model economy there are two sectors, one of which is located in the urban area and the other is located in the rural area. To keep the model manageable, we make use of the tight link between urbanization and structural change such that they mean the same thing in the model.

Another important aspect of the model is that we assume a closed economy such that prices and population movements are endogenously determined inside the confines of the interactions of the two sectors we define. However, this assumption restricts the applicability of our model to the currently developed countries, during whose development stages openness to trade was more restricted with trade partners that are not disproportionately large with respect to themselves. Thus, using this model for currently developing countries might be problematic in predicting patterns but the model should yield some insights for developing countries since we contend that the same processes are also important in these countries.

In the causality debate, we take the side that urbanization and structural change causes growth. We lump all the productivity enhancing properties of urbanization into a positive externality from the urban production level on the level of productivity in both the urban and the rural sectors. Observe that the rationale behind making the externality depend on production rather than urbanization itself is based on the innovation and efficiency gains arguments given in this chapter. Almost all of the processes through which urbanization can cause growth are dependent on there to be high levels of urban production, not merely large amounts of people.

The last consideration that can be mapped from this chapter to our model is that urbanization is treated as a transitory phenomenon, which is constant at the steady state based on the argument that as countries develop, urbanization levels off. In Chapter 2, we see that most models that investigate structural change fail to recognize that fact.

The thesis is structured as follows: In Chapter 2, a literature review of the relevant theoretical literature is provided. In Chapter 3, we introduce the model with perfect labor mobility and in Chapter 4 the model is extended to the case of imperfect labor mobility. Chapter 5 consists of our conclusions from the thesis.

CHAPTER 2
LITERATURE REVIEW

We can divide the relevant theoretical literature into two major threads. First one emphasizes structural change and the second emphasizes urbanization. The difference between the two threads can be traced back to our discussion in Chapter 1 on whether economic growth causes structural change or structural change causes economic growth. The literature emphasizing structural change treats the process as almost a by-product of the growth process while the literature emphasizing urbanization makes use of market failures or positive externalities in one of the sectors in its modeling effort. In this chapter, we will conduct a review of both literatures.

Structural Change Literature

Articles in the structural change literature focus on the growth properties of the models that they construct. As is the usual case, as the economy grows, based on some assumptions structural change occurs by the fact that incomes are increasing or technological progress is not even across all sectors. Thus, economic growth is the major process while structural change accompanies it rather than affecting it.

The main aim of these studies is to be able to match the patterns of structural change, called the Kuznets facts as discussed in Chapter 1, while being consistent with the Kaldor facts. Thus, most effort spent in this thread is specifically confined to the balanced growth path where the Kaldor facts are bound to hold.

The literature mainly makes use of three different assumptions that enable the shift of labor across sectors as the economy grows: Engel's Law, different productivity growth rates at different sectors and different factor intensities across sectors.

Engel's Law

As mentioned in Chapter 1, Engel's Law is the fact that the share of agricultural goods in total demand goes down as income increases. It entails a low income elasticity of substitution for agricultural goods relative to the industrial goods. Since demand for the industrial goods increase relative to agricultural goods, percentage of labor employed in industry increases to cater to the increasing demand.

The major study that incorporates Engel's Law to its formulation is by Kongsamut et al. (2001). They build a continuous growth model where there are three sectors, agricultural, manufacturing and services, with identical production functions and the household's utility function is of Stone-Geary type. As is the case in almost all models in the structural change thread, there is perfect labor mobility. There is exogenous labor-augmenting productivity growth, which is at the same rate for all sectors. They restrict their attention to the "generalized balanced growth path," which they define as the path where real interest rate is constant.

Using a Stone-Geary utility function is one of the most popular ways to have varying levels of income elasticity of demand for different goods. In the formulation of Kongsamut et al. (2001), there is a subsistence level of agricultural consumption, "negative" amount of subsistence level of service good consumption, which can be interpreted as an endowment, and no such effects on manufacturing consumption. Hence, at low levels of income, consumption share of agricultural goods is high since the household can barely afford the subsistence level. But as incomes grow with the constant exogenous productivity growth on the balanced growth path, household starts consuming more of the other goods and labor moves towards sectors for which demand is increasing in a smooth way. Their findings indicate that the employment share of agriculture decreases as the employment share of services increase and manufacturing's stay the same, which are mostly consistent with the empirical evidence except that the employment share of manufacturing is hardly constant over time as we showed in Chapter 1. The major caveat of their model is that their findings depend on a knife-edge condition ensuring the existence of a balanced growth path.

Earlier models by Echevarria (1997) and Laitner (2000) also use different income elasticities across different consumption goods. Echevarria (1997) tries to explain Lucas' (1988) observation that middle income countries grow faster than both the poorest and the richest countries using computational methods in a discrete-time setup. She reaches the conclusion she desires by the assumption that productivity growth in manufacturing is higher since by Engel's Law middle income countries are intensively engaged in manufacturing.

In an overlapping-generations setup, Laitner (2000) explains the fact that a country's average propensity to save endogenously rises as the economy industrializes. The model incorporates Engel's Law in a crude way, such that households only want to consume a fixed amount of agricultural product and spend rest of their income in industrial goods and capital is only used in the industrial sector. Thus, as the economy grows and people start demanding industrial goods, savings rates increase to enable capital formation.

Engel's Law has also been used to explain the importance of agricultural productivity for development by Matsuyama (1992) and Gollin, Parente and Rogerson (2002). The idea is that when agricultural productivity is high, the subsistence level of agricultural consumption can be produced more easily, which "frees" labor from agriculture so that they can work in the industry where productivity is assumed to be higher. In Matsuyama's (1992) model utility function is of Stone-Geary type and there is a learning-by-doing effect in manufacturing. However, there is no capital in the model; hence, there are no dynamics in equilibrium. He finds that the fraction of labor that is employed in manufacturing increases for a country with higher agricultural productivity, which means that equilibrium growth rate is higher for those countries also through the learning-by-doing effect. The major issue with the model is that there are no labor dynamics, i.e. employment shares are fixed on the equilibrium path. This is in contrast with the fact that structural change takes some time to occur.

Different Productivity Growth Rates

Another way to induce structural change is to assume that productivity growth rates in different sectors are different from each other. Then, if elasticity of substitution across final goods is low, factors of production move from the sectors with fast productivity growth towards stagnant sectors. The idea is usually attributed to Baumol (1967) as mentioned in Chapter 1.

Ngai and Pissarides (2007) is a rigorous analysis of the conditions when both structural change and balanced growth can occur together. They specify a multi-sector economy where total factor productivity growth rates of different sectors are different from each other. They assume production functions are identical apart from that. Their results show that for the existence of a balanced growth path, intertemporal elasticity of substitution has to be equal to one and the elasticity of substitution should be different from one. If the elasticity of substitution across goods is less than unity, factors shift towards the more stagnant sector and vice versa. Thus, if we assume low substitutability and that agriculture has the highest productivity growth, on the balanced growth path, labor moves from agriculture to slower growing manufacturing sector. The major caveat of this argument is that it depends on low substitutability, which is not necessarily true.

Different Factor Intensities

It is plausible to think that factor intensities in different sectors are distinct. For example, it is more reasonable to assume that manufacturing is more capital intensive than agriculture. Considering the case of Cobb-Douglas production function, in manufacturing capital's share would be higher than its share in agriculture. The effect of this difference, again if we assume low elasticity of substitution across goods, is that labor moves towards the less capital intensive sector. As more capital is accumulated, the more capital intensive sector grows faster than the less capital intensive sector. But if the substitution between goods is low, then composition of demand does not change as much. Thus, more labor shifts to the less capital intensive sector to be able to cater to the demand.

Acemoglu and Guerrieri (2008) incorporate both different factor intensities and different total factor productivity growth rates in a two-sector growth model. Again, concentrating on the balanced growth path, which they call the constant growth path, they find that given a condition that takes both different productivity growth rates and different factor intensities into account, they find that relatively less capital intensive and slower growing sector swallows all factors asymptotically. Still, the other sector will have a higher growth rate but the overall growth rate becomes dominated by the less capital intensive sector.

Urbanization Literature

The second thread that looks at the same problem focuses on urbanization. There is an emphasis on externalities and market failures based on different labor markets in rural and urban areas and the questions they deal with are more about how economic performance can be better rather than matching the growth and sectoral share patterns. Thus, the focus is more on development in these models rather than growth.

In this literature review, we will consider two major strands of research in this thread: Dual economy models and human capital externalities.

Dual Economy Models

The main idea behind the dual economy models is that labor is stuck in a stagnant mode of production, in agriculture, while there is a more productive sector is available, manufacturing or services. However, somehow labor cannot be allocated to the sector with higher productivity.

Lewis (1954) is the pioneer of this approach. He starts by assuming that there exists an unlimited labor source in the countryside. Hence, their marginal product is zero. However, he argues that in agriculture, the farmer earns his average product since he contends that a farmer would not leave his family farm unless he earns what he was already earning there. The wages in the capitalist sector, or in the city, are constant at a level that is somewhat above the subsistence wages earned in agriculture. In this case, although wages are constant, people in the rural areas migrate to the cities to work in the capitalist sector. This process goes on until labor becomes scarce such that wages in the rural area exceed subsistence wages.

Ranis and Fei (1961) develops Lewis' (1954) ideas further. They analyze what happens when labor ceases to be unlimited anymore. They imagine three phases of development process in their analysis. First phase is where Lewis' analysis is relevant with unlimited labor supply being paid subsistence level wages and constant and higher-than-substance wages which are institutionally set in the capitalist sector. In the second stage labor starts to become scarce but still not scarce enough to derive up the institutionally set price. Third phase represents the developed stage, where there is no more surplus or underemployed labor and marginal productivity in the rural area has increased above the institutionally set wage.

Harris and Todaro (1970) further formalizes the argument by explicitly formulating the economic agents and the migration decision of a rural laborer. Based on an expectation-based migration decision function, they find that urban unemployment can occur in this model.

However, models above have their roots in the classical school of economics, where factors are paid subsistence wages or their average products, or there are institutionally set prices. Mas-Colell and Razin (1973) brought this analysis to a neoclassical framework where factors are paid their marginal products, there is full employment and markets clear. The main differences from the models in the structural transformation literature are that labor is imperfectly mobile between the sectors and there is a constant savings rate. By treating urbanization as transitory and analyzing phase planes, they find that the urbanization level increases towards a steady state from a given initial level while capital also grows but its growth rate first increases and then declines.

Human Capital Externalities

The introduction of human capital to such urbanization models is due to Lucas (1988). According to his view human capital is mainly productive in the urban sector and that is what actually makes this sector modern compared to the agricultural sector. Capasso and Carillo (2009) also state that explaining structural change using dual economies entails incorporating human capital accumulation externalities into these models.

The relationship is more explicitly modeled in Lucas (2004). In this model, there are two sectors, urban and rural, which produce perfectly substitutable goods. Urban sector uses labor and human capital while rural sector only uses labor. There are many infinitely-lived households, who can decide to migrate at any instant until life-time earnings from rural and urban sectors equalize. They can spend some of their time endowment for education in the city to increase their human capital. However, there is an externality from the average human capital of the urban producers. Thus, migrating to the city becomes more and more attractive as time goes by through this externality and there will be a continuous increase in the level of urbanization. The economy continues to grow through human capital investments.

Critique and Our Approach

The main problem with structural change literature is that their assumptions determine the dynamics to the extent that structural change is exogenously determined in these models. Also, the assumption on preferences that there is low substitutability between different goods in models explaining structural change through differences in technology does not seem plausible and is not justified by the authors.

Another problem with structural change literature is that the analysis of changes in factor shares in different sectors has been restricted to the balanced growth path, where main aggregates grow at the same rate but employment shares change. However, this is empirically problematic. As argued in Chapter 1, urbanization and hence, structural change that involves migration, is a transitory phenomenon rather than a steady state behavior.

The early dual economy modeling is quite dated and hard to pursue in today's neoclassical economics discipline. However, Mas-Colell and Razin's (1973) article is a pioneering work that formalizes the dual economy models in a neoclassical framework. The interesting issue is that this article has not been cited in the structural change or urbanization literatures although it definitely lays down the backbone of most structural change models.

Our approach is to borrow the basic structure of most structural change models in a two-sector framework but not to borrow the assumptions that externally derive structural change. We try to explain urbanization without using Engel's Law or any external technological assumption. But we assume that there is a positive externality on both the urban and the rural sectors from the urban production level. Thus, we also borrow from the urbanization literature that externalities play an important role in how structural change occurs and that labor mobility might not be perfect. First, we develop a model where labor can move instantaneously and then we extend it to account for the limited mobility across labor markets.

CHAPTER 3
THE MODEL UNDER PERFECT LABOR MOBILITY

The basic model which we use to understand the phenomenon of urbanization is a variation on the models in structural change literature with two sectors, where on each sector, there is a positive externality, which stems from one of the sectors. Thus, viewing the model in a different way, it is an extension of the endogenous growth model posed by Romer (1986) to two sectors. Although the model is based on structural change models, we will not ex-ante assume that structural change occur on a balanced growth path and urbanization will be treated as occurring in transition.

Definition of the Environment

There are two sectors in the economy: rural and urban. The main distinction between the two sectors is the type of capital that they use. While rural sector only utilizes a fixed type of capital that we call "land," urban sector utilizes a reproducible type of capital that we call simply "capital." The reason why only the urban sector uses reproducible capital is that the "modern" sector, which is a product of industrial revolution, is productive mainly under the agglomeration effects in the urban sphere as discussed in Chapter 1.

Each sector produces a distinct product and capital can only be produced by the urban sector. There is labor-augmenting technology in both sectors. Both sectors are subject to technological change stemming from the same process. The technology level is governed by an externality from the aggregate urban production level, which enters into both sectors' production functions with different weights. There are infinitely-many firms and we assume perfect competition among firms.

For reasons of tractability, we use Cobb-Douglas specification where shares of production factors are between 0 and 1, i.e.. Hence, rural and urban production functions for producer j in the rural sector and producer l in the urban sector are specified respectively as

$$Y_{Rj}(t) = D_j(t)^{\alpha_R} \left(Y_U(t)^{\psi} L_{Rj}(t)\right)^{1-\alpha_R} \tag{1}$$

$$Y_{Ul}(t) = K_l(t)^{\alpha_U} \left(Y_U(t) L_{Ul}(t)\right)^{1-\alpha_U} \tag{2}$$

To clarify notation, Y denotes production level, D denotes amount of land, K denotes amount of capital, L denotes amount of labor used in production. Subscript R indicates rural and subscript U indicates urban counterpart of a variable, and subscript j and l denote the choice variable of the relevant producer. Observe that the externality from urban production enters into the urban production equation linearly and with a power ψ into the rural production function. The role of ψ is to weight the effect of externality on rural production relative to its effect on the urban sector.

On the other side of the economy, infinitely- lived households decide how much of each type of good to consume, how much to save and how much labor to allocate to each sector. Each household has one unit of labor endowment. Basic assumption in this section is that labor is perfectly mobile, which means that households can change their labor allocations immediately and without any cost. We assume that there is no population growth. Utility function of a representative household is of the following form:

$$\int_0^{\infty} e^{-\rho t} \left(\beta_R \ln c_R(t) + \beta_U \ln c_U(t)\right) dt,$$

where c_i indicates consumption levels of good i, which is produced in sector $i = R,U$ and ρ is the discount rate at which household discounts future utility. Instantaneous utility function is of Constant Intertemporal Elasticity of Substitution type with unit intertemporal elasticity of substitution and the consumption aggregator is of Cobb-Douglas specification with share β_i for consumption of good i. Observe that Engel's Law, which is widely used in structural change models, is not incorporated into this model. So, in this model any change in the urbanization level cannot be attributed to different income eleasticities of substitution in different sectors. Constraint of the household is the capital accumulation condition:

$$\dot{k}(t) = \omega_U(t)\theta(t) + \omega_R(t)(1-\theta(t)) + r_D(t)d(t) + r(t)k(t) - p(t)c_R(t) - c_U(t) - \delta k(t)$$

where ω_i indicates wage rate in sector i, r indicates interest rate obtained from renting out capital, r_D indicates rental rate of land, p indicates the relative price of rural good in terms of urban good and δ indicates the depreciation rate of capital. All factors are paid in terms of the urban good in this model. θ is the percentage of labor endowment allocated to the urban sector so that it earns wages that are offered in the urban sector and the remaining earns wages in the rural sector. This variable will be the main focus of attention in our model and we will call it the "urbanization level." This specification is based on the tight link between structural change and urbanization as shown in Chapter 1. People employed in capital intensive sector earn income in urban areas and the others in rural areas. In the model there is no unemployment or underemployment in contrast to the dual economy models considered in Chapter 2.

The model adopts continuous time, so we drop the time scripts from now on to make the exposition clearer. Also, capital letters indicate aggregate levels and small case letters indicate per-capita levels as they often do in the growth literature.

Representative Firms' Problems

There are two representative firms, one for each sector. As usual in the neoclassical literature, factors are paid their marginal products. Thus the problem of the rural producer j is

$$\max_{D_j, L_{Rj}} pD_j^{\alpha_R}\left(Y_U^{\psi}L_{Rj}\right)^{1-\alpha_R} - r_D D_j - \omega_R L_{Rj}$$

The solution implies the following factor prices in the rural sector:

$$r_D = p\alpha_R D_j^{\alpha_R - 1}\left(Y_U^{\psi}L_{Rj}\right)^{1-\alpha_R} \tag{3}$$

$$\omega_R = p(1-\alpha_R)D_j^{\alpha_R}Y_U^{\psi(1-\alpha_R)}L_{Rj}^{-\alpha_R} = p\frac{(1-\alpha_R)}{L_{Rj}}Y_{Rj} \tag{4}$$

The problem of the urban producer l is:

$$\max_{K_l, L_{Ul}} K_l^{\alpha_U}\left(Y_U L_{Ul}\right)^{1-\alpha_U} - rK_l - \omega_U L_{Ul}$$

The solution implies the following factor prices in the urban sector:

$$r = \alpha_U K_l^{\alpha_U - 1}\left(Y_U L_{Ul}\right)^{1-\alpha_U} \tag{5}$$

$$\omega_U = (1-\alpha_U)K_l^{\alpha_U}Y_U^{1-\alpha_U}L_{Ul}^{-\alpha_U} = \frac{(1-\alpha_U)}{L_{Ul}}Y_{Ul} \tag{6}$$

Representative Household's Problem

The representative household maximizes her lifetime utility subject to her budget constraint. That is, she solves the following optimal control problem:

$$\max_{c_R, c_U, \theta, k} \int_0^\infty e^{-\rho t} \left(\beta_R \ln c_R + \beta_U \ln c_U \right) dt$$

subject to

$$\dot{k} = \omega_U \theta + \omega_R (1-\theta) + r_D d + rk - pc_R - c_U - \delta k \tag{7}$$

The first order conditions and the Euler equation imply the following relationships (see Appendix A for the derivation):

$$\frac{c_U}{\beta_U} = \frac{pc_R}{\beta_R} \tag{8}$$

$$\omega_U = \omega_R \tag{9}$$

$$\frac{\dot{c}_U}{c_U} = r - \delta - \rho \tag{10}$$

Equation (8) is the intratemporal condition, which determines the ratio of two goods at any instant of time given the price of the rural good in terms of the urban good. This is a direct implication of the fact that the aggregator is of Cobb-Douglas specification.

Equation (9) is the direct consequence of the assumption of perfect mobility of labor. If there are no costs associated with labor's move from one sector to the other, then the returns from both options of allocation have to be the same for labor to be allocated in nonzero amounts to both sectors. Although (9) is derived from the household's utility maximization problem, it does not include any choice variables from the household's point of view. It is actually a condition for there to be an interior solution to the household's problem. Any case where wages across the sectors are not equal imply a corner solution to the household's problem, where the household allocates all of her labor to the sector in which wage rate is higher.

However, any such case is not interesting since equilibrium does not exist under those cases.

Equation (10) is the regular intertemporal consumption equation, which determines the growth rate of demand for the urban good given the net interest rate $r-\delta$ and time preference of the household, which is represented by the discount rate ρ. In this case it can be specified in terms of only the urban good since investment induces an intertemporal trade-off between consuming urban good and investing into capital, which can be produced only by the urban sector. So, as usual growth rate of consumption depends on the relative sizes of net interest rate and the subjective discount factor.

Equations (7), (8), and (10) determine the solution to the household's problem given (9), the initial amount of capital that household holds, k_0 and the regular transversality condition, which in this case is (see any textbook in economic growth such as Barro & Sala-i Martin (2004) for derivation)

$$\lim_{t\to\infty} k(t)\cdot \exp\left\{-\int_0^t (r(s)-\delta)ds\right\}=0 \tag{11}$$

Characterization of the Equilibrium

As a model in the neoclassical framework, we assume that markets clear in the model. There are 4 markets in this model: rural labor market, urban labor market, rural goods market and urban goods market, which imply the following market clearing conditions in equilibrium:

$$(1-\theta)L = L_R \tag{12}$$

$$\theta L = L_U \tag{13}$$

$$Lc_R = C_R = Y_R \tag{14}$$

$$Lc_U + L\dot{k} + \delta Lk = C_U + \dot{K} + \delta K = Y_U \tag{15}$$

Observe that (15) can be derived using (3), (4), (5), (6), (7), (9),(12), (13) and (14) which would be a demonstration of the Walras' Law for this case.

Also in equilibrium, for the model to be consistent with representative agent framework aggregate production is equal to the production of the representative firm, i.e. $Y_{Rj} = Y_R$ and $Y_{Ul} = Y_U$. So, using the latter equality and (2), we can derive Y_U in equilibrium as

$$Y_U = K\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}}, \tag{16}$$

which is linear in K. The implications of the linearity of (16) is a well-investigated phenomenon in one-sector endogenous models. It generally allows for a balanced growth in the steady state without resorting to an exogenous technological growth assumption.

In equilibrium, (9) implies that (4) equals (6). Inserting (16) into the wages and imposing the market clearing conditions for labor markets, (9) becomes

$$\frac{(1-\alpha_U)}{\theta L} K\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}} = p\frac{(1-\alpha_R)}{(1-\theta)L} Y_R \tag{17}$$

By (14), production of rural goods is equal to their consumption in equilibrium. Substituting (8) into (17), we can write C_U in terms of K and θ:

$$C_U = \frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)} K\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}} \frac{1-\theta}{\theta} \tag{18}$$

Using (16) and (18), we can rewrite (15) as

$$\frac{\dot{K}}{K} = \left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}} - \frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}} \frac{1-\theta}{\theta} - \delta \tag{19}$$

In equilibrium, r becomes $\alpha_U \left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}}$ by inserting (16) into (5). As a result we can write (10) as

$$\frac{\dot{C}_U}{C_U} = \alpha_U \left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}} - \delta - \rho \tag{20}$$

Using (18), (19) and (20), we can derive the change in urbanization level, θ (see Appendix B for the derivation):

$$\dot{\theta} = \frac{\alpha_U \theta(1-\theta)}{\alpha_U - (1-\alpha_U)(1-\theta)} \left[\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}} \left(1-\alpha_U - \frac{\beta_U}{\beta_R} \frac{(1-\alpha_U)}{(1-\alpha_R)} \frac{1-\theta}{\theta} \right) + \rho \right] \tag{21}$$

Then, (21) and (19) characterize the system given K_0 and the transversality condition, which becomes (see Appendix B for its derivation using (11) and (19)):

$$\lim_{t\to\infty} K(0) \cdot \exp\left\{ \int_0^t \left(\theta(s) L\right)^{\frac{1-\alpha_U}{\alpha_U}} \left(1-\alpha_U - \frac{\beta_U}{\beta_R} \frac{(1-\alpha_U)}{(1-\alpha_R)} \frac{1-\theta(s)}{\theta(s)} \right) ds \right\} = 0 \tag{22}$$

Behavior of Urbanization Level in Equilibrium

We can observe from (21) that change in urbanization level only depends on the current urbanization level. So, we can characterize the path of θ without considering the paths of other endogenous variables, which simplifies the analysis quite a bit. Hence, we can draw the vector field of θ by analyzing only (21). It can easily be seen that $\dot{\theta} = 0$ when $\theta = 0$ or $\theta = 1$ or

$$\Gamma(\theta) = \left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}} \left(1-\alpha_U - \frac{\beta_U}{\beta_R} \frac{(1-\alpha_U)}{(1-\alpha_R)} \frac{1-\theta}{\theta} \right) + \rho = 0 \tag{23}$$

<u>Proposition 1</u>: Assuming $\alpha_U < 1/2$ and $\left(\theta_c L\right)^{\frac{1-\alpha_U}{\alpha_U}} \left(1-\alpha_U + \frac{\beta_U}{\beta_R} \frac{(1-\alpha_U)}{(1-\alpha_R)} \right) > \rho$, where $\theta_c = \{\theta \mid \partial\Gamma(\theta)/\partial\theta = 0\}$, $\Gamma(\theta) = 0$ has two roots.

Proof is in Appendix F. The condition implicitly defined in the proposition is explicitly driven in the proof and skipped here for clarity of exposition.

Let us denote the roots referred in Proposition 1 as θ_{ss}^{0} and θ_{ss} in order of magnitude. Computational examples with plausible parameter values give incredibly low values for θ_{ss}^{0} in the order of 10^{-10}. Nevertheless, it is a root of the equation and hence a steady state.

Characterization of the rest of the vector field involves some algebra and some mild assumptions on parameter values, the system can be represented by the vector field represented by Figure 7 (see Appendix C for the details of its derivation).

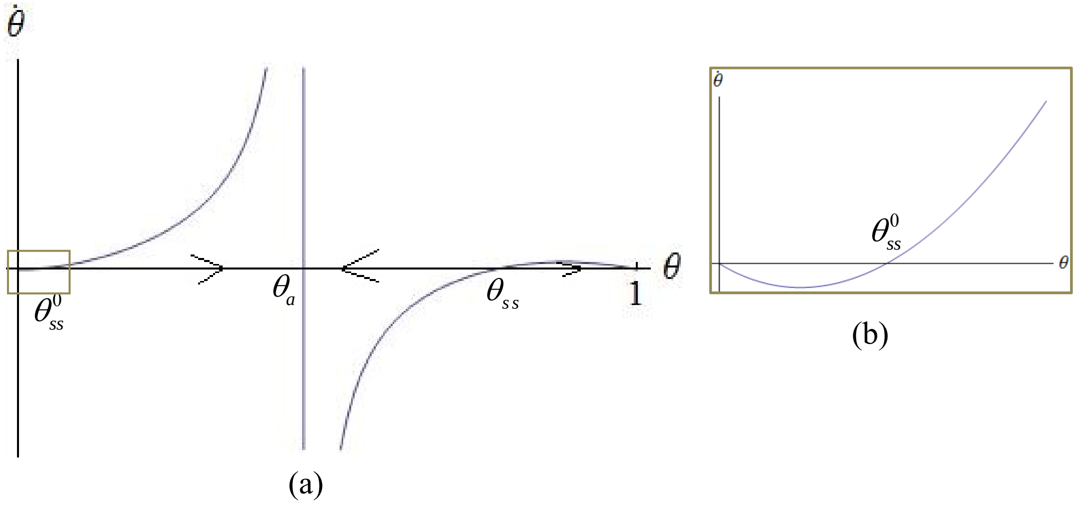

Figure 7. Vector field of θ. Panel (a) shows the whole vector field and panel (b) highlights the portion where θ is low.

<u>Proposition 2</u>: Given initial K_0 and the transversality condition (22), on the equilibrium path $\theta(t) = \theta_{ss}$ or $\theta(t) = \theta_{ss}^{0}$ $\forall t$.

The formal proof is given in Appendix F. As a sketch, observe that as $\theta \to 1$, (22) cannot hold since the limit explodes as the power of the exponential term becomes positive. So, any path starting from some $\theta > \theta_{ss}$ cannot be consistent with household's optimization. Any path starting from some $\theta_a < \theta < \theta_{ss}$ is not feasible since $\dot{\theta}$ tends to negative infinity and any path starting from some $0 < \theta < \theta_a$ is not feasible since $\dot{\theta}$ tends to positive infinity. Finally, any path where $\theta \to 0$ does not satisfy the transversality condition. Thus, $\theta(t) = \theta_{ss}$ and $\theta(t) = \theta_{ss}^0$ are the only feasible paths that are consistent with the market equilibrium.

There are two problems with this model. First and foremost, the model fails to predict a single solution for the differential equation system at hand. This problem haunts any modeler who finds more than a single equilibrium as a result of her model. It is a recurrent problem in the field of game theory and lots of methods have been devised to "refine" the solution set further. In our case, $\theta(t) = \theta_{ss}^0$ can be seen as an abnormality created by some small effect in the model and may as well be ignored. But that would be an afterthought rather than a prediction of the model.

The second problem is that although the model is dynamic in its components the solution turns out to be static in terms of θ, which fails to explain how the adjustment to the steady state occur. Instead, we settle on a level of urbanization and stay on that while the economy grows. So, the model actually fails to explain the simultaneous growth with the change in urbanization. Hence, it is a model that might be relevant in understanding the level of urbanization achieved by most developed countries through comparative statics but fails to achieve how the economy reaches there.

However, Murphy, Shleifer and Vishny (1989) offer a different interpretation for the existence of multiple equilibria in their model based on aggregate demand externalities. Their model features a game theoretic approach to investment decision among firms in the process of development and using it they find two equilibria. In one of them, there is no investment and in the other there is positive investment and they call the process associated with the second equilibrium “the big push.” As they show, the equilibrium with positive investment Pareto dominates the other. However, they do not see this fact as a shortcoming of their model in prediction but as a model which can explain the existence of the development trap. However, Acemoglu (2009) criticizes their take on their results. He contends that a model explaining a development trap should do so based on the parameters and conditions of the model rather than explanations based on factors which are not included in the model.

Behavior of All Endogenous Variables in Equilibrium

Behavior of the other variables can be derived given that $\theta(t)=\theta_s \quad \forall t$ where $\theta_s \in (\theta_{ss}^0, \theta_{ss})$ obtain the behavior of K, solve (19) by plugging in the value of θ:

$$K(t)=K(0)\cdot \exp\left\{ \left(\theta_s L\right)^{\frac{1-\alpha_U}{\alpha_U}} - \frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\left(\theta_s L\right)^{\frac{1-\alpha_U}{\alpha_U}}\frac{1-\theta_s}{\theta_s} - \delta \right\}$$

$$\Rightarrow K(t)=K(0)\cdot \exp\left\{ \alpha_U \left(\theta_s L\right)^{\frac{1-\alpha_U}{\alpha_U}} - \rho - \delta \right\} \tag{24}$$

To get the behavior of C_U, we can plug in (24) into (18) to find

$$C_U(t)=K(0)\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\left(\theta_s L\right)^{\frac{1-\alpha_U}{\alpha_U}}\frac{1-\theta_s}{\theta_s}\cdot \exp\left\{ \alpha_U \left(\theta_s L\right)^{\frac{1-\alpha_U}{\alpha_U}} - \rho - \delta \right\} \tag{25}$$

Observe from (24) and (25) that both variables grow at the same rate on the equilibrium path. So, we are on the balanced growth path from the start of time. This is to be predicted since the model resembles a standard endogenous growth model with Romer-type externality in the production function. Another observation is that the growth rate is positively related to the urbanization level. So, higher the urbanization level, higher the rate of investment and higher the growth rate of urban consumption.

Behavior of the production levels can be found using (24), θ_{ss} and the relevant production function:

$$Y_U(t) = K(0)\left(\theta_s L\right)^{\frac{1-\alpha_U}{\alpha_U}} \cdot \exp\left\{\alpha_U \left(\theta_s L\right)^{\frac{1-\alpha_U}{\alpha_U}} - \rho - \delta\right\}$$

$$Y_R(t) = C_R(t) = D^{\alpha_R}\left(K(0)\left(\theta_s L\right)^{\frac{1-\alpha_U}{\alpha_U}}\right)^{\psi(1-\alpha_R)}(1-\theta_s)L^{(1-\alpha_R)} \times\left[\exp\left\{\alpha_U \left(\theta_s L\right)^{\frac{1-\alpha_U}{\alpha_U}} - \rho - \delta\right\}\right]^{\psi(1-\alpha_R)}$$

Observe that growth rate of rural production is proportionate to the growth rate of urban production and the other variables. It is higher than the growth rate of urban production if $\psi(1-\alpha_R) > 1$ holds and vice versa. Hence, a higher urbanization level leads to higher consumption of both goods, which means that a higher level of urbanization Pareto dominates any path with lower level of urbanization. So, path $\theta(t) = \theta_{ss}$ Pareto dominates $\theta(t) = \theta_{ss}^0$.

Using the intratemporal condition, (8), we can find the behavior of prices on the equilibrium path:

$$p(t) = \frac{\left(K(0)\left(\theta_s L\right)^{\frac{1-\alpha_U}{\alpha_U}}\right)^{1-\psi(1-\alpha_R)} \dfrac{(1-\alpha_U)}{(1-\alpha_R)}\dfrac{1-\theta_s}{\theta_s}}{D^{\alpha_R}\left((1-\theta_s)L\right)^{(1-\alpha_R)}} \cdot \left[\exp\left\{\alpha_U \left(\theta_s L\right)^{\frac{1-\alpha_U}{\alpha_U}} - \rho - \delta\right\}\right]^{1-\psi(1-\alpha_R)} \qquad (26)$$

From (26), we can see that prices increase if $\psi(1-\alpha_R) > 1$ holds, which means that the rate of increase in productivity is higher in the urban sector than the rural sector. This is the case which the urbanization literature usually assumes. However, the structural change literature usually assumes that the exogenous growth rate in agriculture is higher than the growth rate in urban sectors so that as discussed in Chapter 2, when substitutability between goods is low, labor moves towards the sector that is growing slower (Ngai & Pissarides, 2007; Kongsamut, Rebelo & Xie, 1997).

Bairoch (1985/1988) actually states that at times of great leaps in industrialization, agriculture grows at a greater rate than manufacturing or any other modern sector. While our model can accommodate both assumptions without changing the other implications of the model, it is also consistent with Bairoch's observation since the model involves a spillover effect from the urban, industrialized sector.

Comparative Statics

As we have seen, the solution consists of a constant urbanization level and the behavior of the other variables on the equilibrium path directly depends on that level. So, it is worthwhile to investigate the effect of exogenous variables on the equilibrium level of urbanization and compare the relationship to what the literature suggests. These relationships are summarized in Proposition 3.

Proposition 3: Change in θ_{ss} with respect to exogenous variables in the model is

$$\frac{d\theta_{ss}}{d\left(\beta_U/\beta_R\right)}>0;\ \frac{d\theta_{ss}}{d\alpha_R}>0;\ \frac{d\theta_{ss}}{d\alpha_U}<0;\ \frac{d\theta_{ss}}{d\rho}<0$$

Change in θ_{ss}^0 with respect to exogenous variables in the model is

$$\frac{d\theta_{ss}^0}{d\left(\beta_U/\beta_R\right)}<0;\ \frac{d\theta_{ss}^0}{d\alpha_R}<0;\ \frac{d\theta_{ss}^0}{d\alpha_U}>0;\ \frac{d\theta_{ss}^0}{d\rho}>0$$

Proof involves the use of Implicit Function Theorem and it is given in Appendix F. We provide intuition for comparative statics around θ_{ss}. The first relationship states that more labor is allocated to the more desirable good's production. Thus if the taste for the urban good relative to the rural good is higher, then more labor is allocated to the urban sector. Hence, a higher level of urbanization is attained.

An increase in α_R means a decrease in the share of labor in the rural sector. So, each unit of labor is more productive in the urban sector than the rural sector relative to the case where α_R does not increase. By homotheticity of preferences, the intratemporal condition implies that c_U / pc_R is constant for any value of α_R at any point in time. Thus, allocating more labor to the urban sector brings more utility to the consumer statically. Observe that α_R does not enter the intertemporal condition. Hence, it does not affect the dynamics of optimization problem.

Similarly, an increase in α_U means a decrease in the share of labor in the urban sector and the static optimization at any point in time affects the equilibrium urbanization level in the opposite direction. However, change in α_U also means that interest rates increase keeping everything else constant, which makes investment more profitable and so it encourages more urban production. Given the functional forms that we specified, static gains turn out to be higher than the dynamic gains at θ_{ss} so that some of the urban labor shifts towards the rural sector.

If the households are less patient, then investment demand decreases. Thus, by the intratemporal margin, some of the labor that would otherwise be used in the urban sector to cater to the investment demand is shifted to the rural sector.

CHAPTER 4
THE MODEL UNDER IMPERFECT LABOR MOBILITY

In the previous chapter, we assumed that labor can move instantly between the two sectors. The immediate implication is that the wage rate in both sectors equalize at any moment in time. As a result, the dynamics turned out to be nonexistent in a model where the aggregate production function is linear in capital in equilibrium. The steady state level of urbanization remained constant at its initial level dictated by the parameters of the model.

The model that we propose in this chapter features some imperfection in labor's mobility. Instead of perfect mobility of labor, the model in this chapter incorporates a behavioral assumption in the process of migration, where the households partially adjusts her labor allocation proportionately to the wage difference between the sectors. For example, if the wage rate in the urban sector is higher, more labor will be allocated to the urban sector but the adjustment will not be to the degree where the wage rates in the urban and rural sectors equalize.

Definition of the Environment

Characteristics of the technology and preferences are the same as the model in the previous section. The only difference is that there is a migration function that governs how much labor will move at a given time instant based on the wage difference between the two sectors. Following Mas-Colell and Razin (1973), the "migration" function we use is

$$\frac{\dot{\theta}}{\theta} = f\left(\frac{\omega_U - \omega_R}{\omega_R}\right), \tag{27}$$

where f is continuously differentiable. Also, we assume for function f that $f(0)=0$ and $f'(\cdot)>0$. In other words, when there is a difference between the wages in urban and rural sectors, there is migration towards the sector with higher wages and when the wages are equal, urbanization halts. In its essence, this is a behavioral assumption rather than a technological one.

Characterization of the Equilibrium

Problems of urban and rural firms stay the same as the model in the previous section, so equilibrium wages and interest rates are given by the same expressions. Household's solution is also the same except for the condition given by equation (9). Instead, we use (27) to derive the law of motion for urbanization. The market clearing conditions are also the same.

The main difference between the characterizations is that we can no longer derive C_U in terms of the other endogenous variables of the model. Hence, the law of motion for capital becomes

$$\frac{\dot{K}}{K}=\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}}-\frac{C_U}{K}-\delta\,. \tag{28}$$

In equilibrium, plugging in the wage equations, the migration equation becomes

$$f\left(\frac{\frac{(1-\alpha_U)}{\theta L}K\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}}-p\frac{(1-\alpha_R)}{(1-\theta)L}Y_R}{p\frac{(1-\alpha_R)}{(1-\theta)L}Y_R}\right)=f\left(\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{K}{pY_R}\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}}\frac{(1-\theta)}{\theta}-1\right) \tag{29}$$

Since the intratemporal condition given by (8) still holds and the production of rural good is equal to its consumption of it, we can write (29) as

$$\frac{\dot{\theta}}{\theta}=f\left(\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{K}{C_U}\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}}\frac{(1-\theta)}{\theta}-1\right) \tag{30}$$

C_U 's growth rate is given by (20). So, (20), (28) and (30) define the dynamics of this system given K_0, θ_0 and the transversality condition, which equals in this case to

$$\lim_{t\to\infty}K(t)\cdot\exp\left\{-\int_0^t(\alpha_U\left(\theta(s)L\right)^{\frac{1-\alpha_U}{\alpha_U}}-\delta)ds\right\}=0 \tag{31}$$

The analysis of this third-order differential equation system requires three dimensional methods unless we apply some transformation to reduce the complexity of the system. So, let $\chi=C_U/K$, and express the system in terms of χ, K and θ:

$$\frac{\dot{K}}{K}=\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}}-\chi-\delta \tag{32}$$

Using (28) and (30), we can obtain the growth rate of χ:

$$\frac{\dot{\chi}}{\chi}=\frac{\dot{C}_U}{C_U}-\frac{\dot{K}}{K}=\alpha_U\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}}-\delta-\rho-\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}}+\frac{C_U}{K}+\delta$$

$$\Rightarrow\frac{\dot{\chi}}{\chi}=(\alpha_U-1)\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}}+\chi-\rho \tag{33}$$

Applying the transformation to (30) , we obtain

$$\frac{\dot{\theta}}{\theta}=f\left(\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}}}{\chi}\frac{(1-\theta)}{\theta}-1\right) \tag{34}$$

Observe that (33) and (34) does not include any K. Thus, we can analyze the dynamics simply by analyzing the system implied by these two equations to determine the paths of θ and χ given the boundary conditions θ_0 and the transversality condition, which becomes using (31) and (32)

$$\lim_{t\to\infty} K(0)\cdot\exp\left\{\int_0^t \left((1-\alpha_U)\left(\theta(s)L\right)^{\frac{1-\alpha_U}{\alpha_U}} - \chi(s)\right)ds\right\} = 0\,. \tag{35}$$

Thus, we can determine the path of K given K_0 using (32) after solving the system for the paths of χ and θ using (33) and (34).

Behavior of the Transformed System

To characterize the behavior of the system, we draw the phase diagram in (θ,χ) plane. At the steady state $\dot{\chi}=0$ and $\dot{\theta}=0$ must hold. We can conjecture that such a steady state exists by what we know about the behavior of the aggregate variables in the steady state of a regular endogenous growth model. As shown in the previous chapter, at the steady state K and C_U grow at the same rate. Therefore, if there is a steady state where θ is constant, χ will be a constant, too.

Inspecting (33), $\dot{\chi}=0$ implies $\chi=0$ or

$$\chi = (1-\alpha_U)\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}} + \rho, \tag{36}$$

which is a convex and increasing function in θ when α_U is less than $1/2$ (see Appendix D for detailed characterization). To determine the arrows of motion, we need to determine the regions where rate of change is positive or negative. Observe from (33) that when $\chi>0$, $\dot{\chi}>0$ if and only if $\chi > (1-\alpha_U)\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}} + \rho$ which means that χ is increasing above the $\dot{\chi}=0$ curve. Hence, it decreases below the curve.

From (34), $\dot{\theta}=0$ when $\theta=0$ or when

$$\chi = \frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}}\frac{(1-\theta)}{\theta}, \tag{37}$$

by the property of the migration function that migration flow is zero when wages are equal to each other. In Appendix D, we show that (37) is a hump-shaped function in θ when α_U is less than $1/2$.

The region where $\dot{\theta}>0$ holds can be determined from (34), which is implies that $\dot{\theta}>0$ when $\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}(\theta L)^{\frac{1-\alpha_U}{\alpha_U}}\frac{(1-\theta)}{\theta}>\chi$ holds given $\theta>0$. In other words, urbanization level is increasing below the $\dot{\theta}=0$ locus and decreasing above it.

Figure 8 shows the phase diagram with the arrows of motion indicated on it based on our findings above.

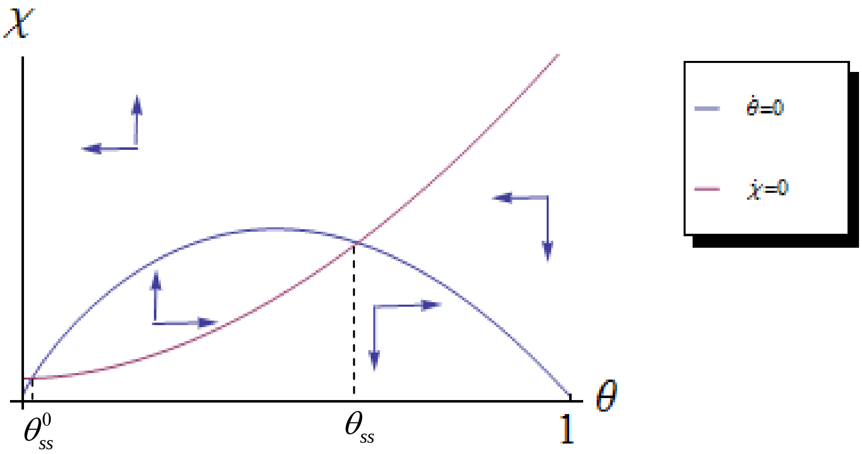

Figure 8. Phase plane in (θ,χ). Arrows of motion are indicated.

As can be seen from Figure 8, one set of steady states consists of the solution to the equations (36) and (37), which implies that the steady state urbanization level is the solution to the following equation:

$$(1-\alpha_U)(\theta_{ss}L)^{\frac{1-\alpha_U}{\alpha_U}}+\rho=\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}(\theta_{ss}L)^{\frac{1-\alpha_U}{\alpha_U}}\frac{(1-\theta_{ss})}{\theta_{ss}}$$

Rearranging it, the steady state θ is given by

$$\left(\theta_{ss}L\right)^{\frac{1-\alpha_U}{\alpha_U}}\left(1-\alpha_U\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{1-\theta_{ss}}{\theta_{ss}}\right)+\rho=\Gamma(\theta)=0\,,$$

which is the same as (23) in the previous chapter. Therefore, any steady state implied by this equation in the previous model is also a steady state in this model. We will continue denoting these steady states as θ_{ss} and .

There are actually two more steady states in this system. These are when $\chi=0$ and $\theta=0$ both hold and when both $\chi=0$ and $\theta=1$ hold. Observe that these steady states are also the steady states in the previous model.

Stability properties of various steady states can be inferred from the phase diagram in Figure 8. θ_{ss} turns out to be a saddle node, with a stable manifold coming in and a non-stable manifold and going out of it. θ_{ss}^0 is an unstable node with all paths going out of it. (1,0) is a stable node in the relevant region since any path below the stable manifold converges to it (except for any path that involves $\theta<\theta_{ss}^0$ and passes through the small region that is below $\dot{\chi}=0$ and above $\dot{\theta}=0$). Observing further, (0,0) is a saddle node, where the unstable manifold is the χ-axis and the stable manifold is the θ-axis.

<u>Proposition 4</u>: If $\theta_{ss}^0<\theta_0<\theta_{ss}$, then on the equilibrium path both θ and χ are increasing and the system converges to θ_{ss}.

We give a graphical proof. Knowing the arrows of motion and the nullclines we can characterize five different types of paths starting from an arbitrary $\theta\in(\theta_{ss}^0,\theta_{ss})$. Figure 9 demonstrates these paths.

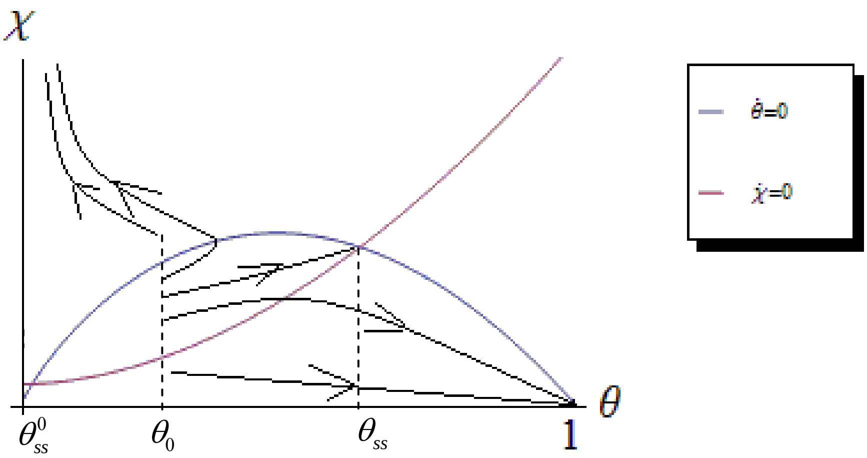

Figure 9. Possible paths of the system starting from an initial θ_0.

The paths which start from a point above the stable manifold cannot be equilibrium paths since χ goes to infinity, which means from (32) that K's growth rate goes to negative infinity, which in turn implies that K goes to zero in finite time. This implies that Y_U becomes zero, so makes a discrete jump to zero. This contradicts the assumption that we are on a market clearing equilibrium path.

The paths which start from a point below the stable manifold cannot be solutions to this system of differential equations either since they converge towards the node where $\theta = 1$ and $\chi = 0$, which violates the transversality condition:

$$\lim_{t\to\infty} K(0)\cdot\exp\left\{\int_0^t (1-\alpha_U)L^{\frac{1-\alpha_U}{\alpha_U}}\,ds\right\} = \lim_{t\to\infty} K(0)\cdot\exp\left\{(1-\alpha_U)L^{\frac{1-\alpha_U}{\alpha_U}}\,t\right\} = \infty \neq 0$$

So, given the level of urbanization level the system settles on the stable manifold at time zero by the adjustment of initial χ accordingly and converges to θ_{ss}. Let us check if the transversality condition is satisfied around the steady state:

$$\lim_{t\to\infty} K(0)\cdot\exp\left\{\int_0^t \left((1-\alpha_U)\left(\theta(s)L\right)^{\frac{1-\alpha_U}{\alpha_U}} - \chi(s)\right)ds\right\} = \lim_{t\to\infty} K(0)\cdot\exp\{-\rho t\} = 0\,,$$

where the first equality comes from (36), which is satisfied at the steady state.

Behavior of Capital Accumulation in Transition

The characterization in the previous section only applies to the transformed system, which enables us to specify the qualitative behavior of the urbanization level. However, from the analysis of the transformed system, we cannot directly infer the qualitative behavior of capital on the stable path. Still, we can investigate the behavior of capital, K, around the steady state.

Given the optimal path of θ and χ, behavior of the growth rate of capital is given by (32). Linearizing around the steady state:

$$\frac{\dot{K}}{K} = \left(\theta_{ss}L\right)^{\frac{1-\alpha_U}{\alpha_U}} - \chi_{ss} - \delta + \frac{1-\alpha_U}{\alpha_U} L^{\frac{1-\alpha_U}{\alpha_U}} \theta_{ss}^{\frac{1-2\alpha_U}{\alpha_U}} (\theta - \theta_{ss}) - (\chi - \chi_{ss}) \qquad (38)$$

Observe that $\left(\theta_{ss}L\right)^{\frac{1-\alpha_U}{\alpha_U}} - \chi_{ss} - \delta$ is the steady state growth rate. Let us denote it by γ. Then, we can rewrite (38) as

$$\frac{\dot{K}}{K} - \gamma = \frac{1-\alpha_U}{\alpha_U} L^{\frac{1-\alpha_U}{\alpha_U}} \theta_{ss}^{\frac{1-2\alpha_U}{\alpha_U}} (\theta - \theta_{ss}) - (\chi - \chi_{ss}), \qquad (39)$$

whose sign is ambiguous assuming that the system is on the stable path starting from some $\theta_0 < \theta_{ss}$ since for any θ and χ on the stable path $(\theta - \theta_{ss})$ and $(\chi - \chi_{ss})$ are negative as can be seen from Figure 8. Then, to be able to characterize the behavior of K around the steady state further, we need to characterize the transformed system around the steady state. The behavior of the system around the steady state can be approximated by the following linear system (see Appendix E for the derivation):

$$\begin{bmatrix} \dot{\chi} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \chi_{ss} & \chi_{ss} \dfrac{(1-\alpha_U)^2}{\alpha_U} \theta_{ss}^{\frac{1-2\alpha_U}{\alpha_U}} L^{\frac{1-\alpha_U}{\alpha_U}} \\ -f'(\bullet)\dfrac{\theta_{ss}}{\chi_{ss}} & \dfrac{f'(\bullet)}{\alpha_U}\left(\dfrac{1-2\alpha_U-(1-\alpha_U)\theta_{ss}}{1-\theta_{ss}}\right) \end{bmatrix} \begin{bmatrix} \chi - \chi_{ss} \\ \theta - \theta_{ss} \end{bmatrix}$$

Since we know that (θ_{ss}, χ_{ss}) is a saddle node, the eigenvalue associated with the stable path, let us denote it by μ, is negative. Then, given θ_0 and the transversality condition, the linearized system has the following solution:

$$\theta(t) - \theta_{ss} = (\theta_0 - \theta_{ss})e^{\mu t}$$

Then, after considerable algebra (see Appendix E for details), growth rate of capital around the steady state can be derived as

$$\frac{\dot{K}}{K} = \gamma + \frac{1-\alpha_U}{\alpha_U}\left(\theta_{ss}L\right)^{\frac{1-\alpha_U}{\alpha_U}}\left(1-(1-\alpha_U)\frac{\chi_{ss}}{\chi_{ss}-\mu}\right)(\theta-\theta_{ss}) \tag{40}$$

Observe that the second term of (40) is negative since $\mu < 0$, $0 < \alpha_U < 1$ and $\theta - \theta_{ss} < 0$. Then, growth rate of capital in transition is positive only when the system is arbitrarily close to the steady state, assuming that growth rate is positive at the steady state. However, if the system is somewhat further away from the steady state, such that the second term in (40) dominates the steady state growth rate, then growth rate of capital might be negative.

CHAPTER 5
CONCLUSION

This thesis investigated the implications of the existence of a specific type of externality stemming from the urban sector on urbanization and structural change using two models, where the second model is a variation of the first one. Theoretically, the novelty of the models is that they include an externality that is based on the level of output from the urban sector on both sectors while incorporating none of the assumptions which the standard models in structural change literature adopt, and which derive structural change exogenously. Also, the model views urbanization as a transitory phenomenon unlike the structural change models.

In the case of perfect labor mobility, we find that there are no transitional dynamics and that the urbanization level is set at some constant level at any time over the whole path. However, other aggregates, such as capital and production, all grow at a constant rate at the steady state and the model is consistent with balanced growth. Thus, this model can be interpreted as a model that predicts the value of steady-state urbanization level for a developed economy. Theoretically, it can be viewed as a generalization of Matsuyama (1992), which does not include capital in the model and makes a point prediction about the urbanization level.

The caveat of this model is that, the prediction of the model is not clear since there are two equilibrium paths, one of which corresponds to a failure in urbanization and the other is the case where the economy becomes urbanized. However, the comparative statics around the urbanized case lead to more plausible outcomes. Hence, if we are to empirically motivate the selection among equilibria, the urbanized equilibrium is the more likely one in real life for reasons not incorporated into the model with perfect labor mobility.

To get transitional dynamics, we further assumed that labor does not move instantaneously in Chapter 4. Instead, we assume that labor shares partially adjusts to the desired levels. This adjustment in the model actually acts as a refinement of the first model in terms of determination of the long run behavior of the model. Both steady states of the first model are still the steady states under imperfect labor markets but only one of them can be achieved without measure zero starting from an initial urbanization level. Thus, incorporating a behavioral migration function not only enables transitional dynamics but also acts as a refinement tool.

In transition, urbanization level increases when the economy starts from a low level of urbanization level, which is the empirically relevant case. However, the cost of the complexity added to the model is that the transitory behavior of capital accumulation cannot be inferred globally over the whole stable path. When we approximate the growth rate of capital around the steady state, we see that it can be negative if the approximation is still close enough to the actual behavior at the earlier stages of transition. This is a shortcoming associated with the model's predictions.

This study assumes a behavioral assumption on migration to obtain transitional dynamics. However, a different, more neoclassical, approach would be to assume a cost to migration as a friction in the labor market. This could potentially lead to different dynamics and make the decision-making process of the migrant more explicit. Hence, we could potentially draw more conclusions from our model.

Another way to enrich the model would be to disaggregate the urban sector into manufacturing and service sectors to better understand the relationship between urbanization and structural change.

APPENDICES

APPENDIX A: DERIVATION OF THE SOLUTION TO THE REPRESENTATIVE HOUSEHOLD'S PROBLEM

Hamiltonian associated with the optimal control problem is

$$H = e^{-\rho t}\left(\beta_R \ln c_R + \beta_U \ln c_U\right) + \lambda e^{-\rho t}(\omega_U \theta + \omega_R(1-\theta) + r_D d + rk - pc_R - c_U - \delta k - \dot{k})$$

The first order conditions are

$$\frac{\partial H}{\partial c_U} = \frac{\beta_U}{c_U} - \lambda = 0 \tag{41}$$

$$\frac{\partial H}{\partial c_R} = \frac{\beta_R}{c_R} - \lambda p = 0 \tag{42}$$

$$\frac{\partial H}{\partial \theta} = \lambda(\omega_U - \omega_R) = 0 \tag{43}$$

Dividing (42) by (41) implies (8), and (43) immediately gives (9) in the main text since at an interior solution λ>0.

From (41), take λ to the right hand side; take logarithms of both sides and then, take the derivative of both sides with respect to time, which yields

$$-\frac{\dot{c}_U}{c_U} = \frac{\dot{\lambda}}{\lambda} \tag{44}$$

The Euler condition is

$$\frac{\partial H}{\partial k} = \frac{d}{dt}\left(\frac{\partial H}{\partial \dot{k}}\right) \Rightarrow e^{-\rho t}(r-\delta) = \frac{d}{dt}\lambda e^{-\rho t}$$

$$\begin{aligned} &\Rightarrow \lambda(r-\delta) = \rho\lambda - \dot{\lambda} \\ &\Rightarrow r - \delta - \rho = -\frac{\dot{\lambda}}{\lambda} \end{aligned} \tag{45}$$

Inserting (44) into (45) yields (10) in the main text.

APPENDIX B: DERIVATIONS OF THE CHANGE IN URBANIZATION LEVEL AND THE TRANSVERSALITY CONDITION

Take natural logarithms of both sides of (18) and then differentiate the whole expression with respect to time:

$$\frac{\dot{C}_U}{C_U} = \frac{\dot{K}}{K} + \frac{1-\alpha_U}{\alpha_U}\frac{\dot{\theta}}{\theta} - \frac{\theta}{1-\theta}\frac{\dot{\theta}}{\theta} - \frac{\dot{\theta}}{\theta} \tag{46}$$

Inserting (16) into (15) and then, dividing by K, we obtain the growth rate of capital:

$$\frac{\dot{K}}{K} = \left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}} - \frac{C_U}{K} - \delta \tag{47}$$

From (18):

$$\frac{C_U}{K} = \frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}}\frac{1-\theta}{\theta} \tag{48}$$

Combining (47) and (48) gives (19). Then, substituting (19) and (20) into equation (46), we get (21).

To get the transversality condition, first, integrate (19) over time and plug in the initial condition:

$$K(t) = K(0)\cdot\exp\left\{\int_0^t\left(\left(\theta(s)L\right)^{\frac{1-\alpha_U}{\alpha_U}} - \frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\left(\theta(s)L\right)^{\frac{1-\alpha_U}{\alpha_U}}\frac{1-\theta(s)}{\theta(s)} - \delta\right)ds\right\} \tag{49}$$

Plug (49) into (11) while replacing $r(s) = \alpha_U\left(\theta(s)L\right)^{\frac{1-\alpha_U}{\alpha_U}}$:

$$\lim_{t\to\infty} K(0)\cdot\exp\left\{\int_0^t\left(\left(\theta(s)L\right)^{\frac{1-\alpha_U}{\alpha_U}} - \frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\left(\theta(s)L\right)^{\frac{1-\alpha_U}{\alpha_U}}\frac{1-\theta(s)}{\theta(s)} - \delta\right)ds\right\}$$
$$\times\exp\left\{-\int_0^t(\alpha_U\left(\theta(s)L\right)^{\frac{1-\alpha_U}{\alpha_U}} - \delta)ds\right\} = 0,$$

which yields (22) after simplification.

APPENDIX C: CHARACTERIZATION OF THE VECTOR FIELD OF URBANIZATION LEVEL

From equation (21), we can write urbanization rate as

$$\dot{\theta} = \frac{\alpha_U \theta(1-\theta)}{\alpha_U - (1-\alpha_U)(1-\theta)} \Gamma(\theta)$$

Observe that the denominator, $\alpha_U - (1-\alpha_U)(1-\theta)$, is equal to zero say at some θ_a, which implies that $\theta_a = 1 - \frac{\alpha_U}{1-\alpha_U} \in (0,1)$. For $\theta < \theta_a$ the denominator is negative and when $\theta > \theta_a$ the denominator is positive. For reasonable parameter values $\theta_{ss}^0 < \theta_a < \theta_{ss}$ holds, and we assume as such in the following analysis.

When $\theta \in (0, \theta_{ss}^0)$, since the denominator is negative and all other terms are positive $\dot{\theta}$ is less than zero. When $\theta \in (\theta_{ss}^0, \theta_a)$, $\dot{\theta}$ is more than zero and explodes to infinity as $\Gamma(\theta)$ and the denominator is negative and the denominator is approaching towards zero. When $\theta \in (\theta_a, \theta_{ss})$, $\dot{\theta}$ is less than zero again as denominator changes signs and takes off from negative infinity. When $\theta \in (\theta_{ss}, 1)$, $\Gamma(\theta)$ is positive but decays towards 1 as $(1-\theta)$ gets smaller and smaller. So, we get the curve in Figure 7 in the main text.

APPENDIX D: CHARACTERIZATION OF THE NULLCLINES OF THE SYSTEM IN THE CASE OF IMPERFECT LABOR MOBILITY

$\dot{\chi} = 0$ implies the nullcline given in (36),

$$\chi = (1-\alpha_U)\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}} + \rho$$

On this curve, when $\theta = 0$, $\chi = \rho$ and when $\theta = 0$, $\chi = (1-\alpha_U)L^{\frac{1-\alpha_U}{\alpha_U}} + \rho$. Taking derivatives:

$$\frac{d\chi}{d\theta} = (1-\alpha_U)\frac{1-\alpha_U}{\alpha_U}L^{\frac{1-\alpha_U}{\alpha_U}}\theta^{\frac{1-2\alpha_U}{\alpha_U}} > 0$$

$$\frac{d^2\chi}{d\theta^2} = (1-\alpha_U)\frac{1-\alpha_U}{\alpha_U}\frac{1-2\alpha_U}{\alpha_U}L^{\frac{1-\alpha_U}{\alpha_U}}\theta^{\frac{1-3\alpha_U}{\alpha_U}} > 0$$

Both inequalities follow from the assumption that $\alpha_U < 1/2$ holds. Hence $\dot{\chi} = 0$ nullcline starts at a positive level ρ and is an increasing convex function.

$\dot{\theta} = 0$ implies the nullcline given in (37):

$$\chi = \frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}}\frac{1-\theta}{\theta}$$

As proved in Appendix F in Proof of Proposition 1, as $\theta \to 0$, $\chi \to 0$. It is easy to see that $\chi = 0$ when $\theta = 1$. Let us differentiate the nullcline to characterize it further.

$$\frac{d\chi}{d\theta} = \frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}L^{\frac{1-\alpha_U}{\alpha_U}}\left(\frac{1-2\alpha_U}{\alpha_U}\theta^{\frac{1-3\alpha_U}{\alpha_U}} - \frac{1-\alpha_U}{\alpha_U}\theta^{\frac{1-2\alpha_U}{\alpha_U}}\right),$$

which is of ambiguous sign but it vanishes at a unique θ, say at θ_c, where

$$\theta_c = \frac{1-2\alpha_U}{1-\alpha_U} \in (0,1),$$

given that $\alpha_U < 1/2$ holds. Further differentiation determines what type of critical point θ_c is:

$$\frac{d^2\chi}{d\theta^2} = \frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)} L^{\frac{1-\alpha_U}{\alpha_U}} \left(\frac{1-2\alpha_U}{\alpha_U}\frac{1-3\alpha_U}{\alpha_U}\theta^{\frac{1-4\alpha_U}{\alpha_U}} - \frac{1-\alpha_U}{\alpha_U}\frac{1-2\alpha_U}{\alpha_U}\theta^{\frac{1-3\alpha_U}{\alpha_U}} \right)$$

Let θ_{curve} be the point where the second derivative vanishes. Then,

$$\theta_{curve} = \frac{1-3\alpha_U}{1-\alpha_U},$$

which is negative if $1/3 < \alpha_U < 1/2$ and positive if $0 < \alpha_U < 1/3$ holds. Observe that for any $\theta > \theta_{curve}$ the curve is concave. Thus θ_c is a local maximum.

Hence we can conclude that if $1/3 < \alpha_U < 1/2$ holds, then the locus is a hump-shaped concave function in the region $\theta \in [0,1]$ and if $0 < \alpha_U < 1/3$ holds, then the nullcline is convex in a small portion of the relevant region, which does not affect the dynamics qualitatively.

APPENDIX E: DERIVATION OF THE BEHAVIOR OF CAPITAL ACCUMULATION AROUND STEADY STATE

The system is characterized by the following differential equations given the boundary conditions:

$$\dot{\chi} = \chi\left[(\alpha_U - 1)\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}} + \chi - \rho\right]$$

$$\dot{\theta} = \theta f\left(\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}}}{\chi}\frac{1-\theta}{\theta} - 1\right)$$

Differentiating the equations:

$$\frac{\partial\dot{\chi}}{\partial\chi} = (\alpha_U - 1)\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}} + 2\chi - \rho$$

$$\frac{\partial\dot{\chi}}{\partial\theta} = \chi\frac{1-\alpha_U{}^2}{\alpha_U}\theta^{\frac{1-2\alpha_U}{\alpha_U}}L^{\frac{1-\alpha_U}{\alpha_U}}$$

$$\frac{\partial\dot{\theta}}{\partial\chi} = -f'(\bullet)\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}}}{\chi^2}(1-\theta)$$

$$\frac{\partial\dot{\theta}}{\partial\theta} = f(\bullet) + \theta f'(\bullet)\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{L^{\frac{1-\alpha_U}{\alpha_U}}}{\chi}\left(\frac{1-2\alpha_U}{\alpha_U}\theta^{\frac{1-3\alpha_U}{\alpha_U}} - \frac{1-\alpha_U}{\alpha_U}\theta^{\frac{1-2\alpha_U}{\alpha_U}}\right)$$

Evaluate at the steady state:

$$\frac{\partial\dot{\chi}}{\partial\chi} = (\alpha_U - 1)\left(\theta_{ss} L\right)^{\frac{1-\alpha_U}{\alpha_U}} + 2\chi_{ss} - \rho = \chi_{ss}$$

$$\frac{\partial\dot{\chi}}{\partial\theta} = \chi_{ss}\frac{1-\alpha_U{}^2}{\alpha_U}\theta_{ss}^{\frac{1-2\alpha_U}{\alpha_U}}L^{\frac{1-\alpha_U}{\alpha_U}}$$

$$\frac{\partial \dot{\theta}}{\partial \chi} = -f'(\bullet)\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{(\theta_{ss}L)^{\frac{1-\alpha_U}{\alpha_U}}}{\chi_{ss}^{2}}(1-\theta_{ss})$$

$$= -f'(\bullet)\frac{\theta_{ss}}{\chi_{ss}}$$

$$\frac{\partial \dot{\theta}}{\partial \theta} = \theta_{ss} f'(\bullet)\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{L^{\frac{1-\alpha_U}{\alpha_U}}}{\chi_{ss}}\left(\frac{1-2\alpha_U}{\alpha_U}\theta_{ss}^{\frac{1-3\alpha_U}{\alpha_U}} - \frac{1-\alpha_U}{\alpha_U}\theta_{ss}^{\frac{1-2\alpha_U}{\alpha_U}}\right)$$

$$= f'(\bullet)\frac{\theta_{ss}}{1-\theta_{ss}}\left(\frac{1-2\alpha_U}{\alpha_U}\frac{1}{\theta_{ss}} - \frac{1-\alpha_U}{\alpha_U}\right)$$

$$= \frac{f'(\bullet)}{\alpha_U}\left(\frac{1-2\alpha_U-(1-\alpha_U)\theta_{ss}}{1-\theta_{ss}}\right),$$

where the derivations follow from evaluating (34) at the steady state:

$$\chi_{ss} = \frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}(\theta_{ss}L)^{\frac{1-\alpha_U}{\alpha_U}}\frac{1-\theta_{ss}}{\theta_{ss}},$$

Then, the linearized system becomes

$$\begin{bmatrix}\dot{\chi}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\chi_{ss} & \chi_{ss}\frac{(1-\alpha_U)^2}{\alpha_U}\theta_{ss}^{\frac{1-2\alpha_U}{\alpha_U}}L^{\frac{1-\alpha_U}{\alpha_U}}\\ -f'(\bullet)\frac{\theta_{ss}}{\chi_{ss}} & \frac{f'(\bullet)}{\alpha_U}\left(\frac{1-2\alpha_U-(1-\alpha_U)\theta_{ss}}{1-\theta_{ss}}\right)\end{bmatrix}\begin{bmatrix}\chi-\chi_{ss}\\ \theta-\theta_{ss}\end{bmatrix} \tag{50}$$

Given θ_0 and the transversality condition as given in the main text, the solution to the linearized system is

$$\theta(t)-\theta_{ss} = (\theta_0-\theta_{ss})e^{\mu t} \tag{51}$$

The evolution θ is

$$\dot{\theta} = -f'(\bullet)\frac{\theta_{ss}}{\chi_{ss}}(\chi-\chi_{ss}) + \frac{f'(\bullet)}{\alpha_U}\left(\frac{1-2\alpha_U-(1-\alpha_U)\theta_{ss}}{1-\theta_{ss}}\right)(\theta-\theta_{ss})$$

Then, we can solve for $\chi-\chi_{ss}$:

$$
\begin{aligned}
\chi-\chi_{ss} &= \frac{\chi_{ss}}{\alpha_U\theta_{ss}}\left(\frac{1-2\alpha_U-(1-\alpha_U)\theta_{ss}}{1-\theta_{ss}}\right)(\theta-\theta_{ss})-\frac{\dot{\theta}}{\theta_{ss}}\frac{\chi_{ss}}{f'(\bullet)} \\
&= \frac{\chi_{ss}}{\theta_{ss}}\left(\frac{1-2\alpha_U-(1-\alpha_U)\theta_{ss}}{\alpha_U(1-\theta_{ss})}-\frac{\mu}{f'(\bullet)}\right)(\theta-\theta_{ss})
\end{aligned}
\tag{52}
$$

The second equation above comes from taking logarithms and differentiating (51):

$$
\ln(\theta(t)-\theta_{ss})=\ln(\theta_0-\theta_{ss})+\mu t \Rightarrow \frac{\dot{\theta}}{\theta-\theta_{ss}}=\mu
$$

Plug (52) into (39):

$$
\frac{\dot{K}}{K}-\gamma=\left(\frac{1-\alpha_U}{\alpha_U}L^{\frac{1-\alpha_U}{\alpha_U}}\theta_{ss}^{\frac{1-2\alpha_U}{\alpha_U}}-\frac{\chi_{ss}}{\theta_{ss}}\left(\frac{1-2\alpha_U-(1-\alpha_U)\theta_{ss}}{\alpha_U(1-\theta_{ss})}-\frac{\mu}{f'(\bullet)}\right)\right)(\theta-\theta_{ss})
$$

(53)

The characteristic equation of (50) implies

$$
(\chi_{ss}-\mu)\left(\frac{f'(\bullet)}{\alpha_U}\left(\frac{1-2\alpha_U-(1-\alpha_U)\theta_{ss}}{1-\theta_{ss}}\right)-\mu\right)-\chi_{ss}\frac{(1-\alpha_U)^2}{\alpha_U}\theta_{ss}^{\frac{1-2\alpha_U}{\alpha_U}}L^{\frac{1-\alpha_U}{\alpha_U}}f'(\bullet)\frac{\theta_{ss}}{\chi_{ss}}=0
$$

$$
\Rightarrow (\chi_{ss}-\mu)f'(\bullet)\left(\frac{1-2\alpha_U-(1-\alpha_U)\theta_{ss}}{\alpha_U(1-\theta_{ss})}-\frac{\mu}{f'(\bullet)}\right)=\frac{(1-\alpha_U)^2}{\alpha_U}\left(\theta_{ss}L\right)^{\frac{1-\alpha_U}{\alpha_U}}f'(\bullet)
$$

$$
\Rightarrow \frac{1-2\alpha_U-(1-\alpha_U)\theta_{ss}}{\alpha_U(1-\theta_{ss})}-\frac{\mu}{f'(\bullet)}=\frac{(1-\alpha_U)^2}{\alpha_U(\chi_{ss}-\mu)}\left(\theta_{ss}L\right)^{\frac{1-\alpha_U}{\alpha_U}}
$$

Plugging into (53), we get

$$
\begin{aligned}
\frac{\dot{K}}{K}-\gamma &= \left(\frac{1-\alpha_U}{\alpha_U}L^{\frac{1-\alpha_U}{\alpha_U}}\theta_{ss}^{\frac{1-2\alpha_U}{\alpha_U}}-\frac{\chi_{ss}}{\theta_{ss}}\frac{(1-\alpha_U)^2}{\alpha_U}\left(\theta_{ss}L\right)^{\frac{1-\alpha_U}{\alpha_U}}\right)(\theta-\theta_{ss}) \\
&= \left(\frac{1-\alpha_U}{\alpha_U}\frac{\left(\theta_{ss}L\right)^{\frac{1-\alpha_U}{\alpha_U}}}{\theta_{ss}}-\frac{1-\alpha_U}{\alpha_U}(1-\alpha_U)\frac{\chi_{ss}}{\chi_{ss}-\mu}\frac{\left(\theta_{ss}L\right)^{\frac{1-\alpha_U}{\alpha_U}}}{\theta_{ss}}\right)(\theta-\theta_{ss}),
\end{aligned}
$$

which yields (40) in the main text.

APPENDIX F: PROOFS OF THE PROPOSITIONS IN THE TEXT

Proof of Proposition 1

Taking the limit as $\theta \to 0$

$$\lim_{\theta\to 0}\Gamma(\theta) = (\theta L)^{\frac{1-\alpha_U}{\alpha_U}}\left(1-\alpha_U - \frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{1-\theta}{\theta}\right) + \rho = 0.\infty + \rho$$

Observe that the first term is indeterminate, so it can be transformed as

$$\frac{(\theta L)^{\frac{1-\alpha_U}{\alpha_U}}}{\dfrac{1}{1-\alpha_U - \dfrac{\beta_U}{\beta_R}\dfrac{(1-\alpha_U)}{(1-\alpha_R)}\dfrac{1-\theta}{\theta}}} = \frac{(\theta L)^{\frac{1-\alpha_U}{\alpha_U}}}{\dfrac{\beta(1-\alpha_R)\theta}{\beta(1-\alpha_R)\theta - \beta_U(1-\alpha_U)(1-\theta)}}$$, which

goes to the $\left[\frac{0}{0}\right]$ indeterminate form so that we can apply L'Hôpital's rule:

$$\lim_{\theta\to 0}\frac{(\theta L)^{\frac{1-\alpha_U}{\alpha_U}}}{\dfrac{\beta_R(1-\alpha_R)\theta}{\beta_R(1-\alpha_R)\theta - \beta_U(1-\alpha_U)(1-\theta)}} = \lim_{\theta\to 0}\frac{\dfrac{1-\alpha_U}{\alpha_U}\theta^{\frac{1-2\alpha_U}{\alpha_U}}L^{\frac{1-\alpha_U}{\alpha_U}}}{\dfrac{-\beta_R(1-\alpha_R)\beta_U(1-\alpha_U)}{\left(\beta_U(1-\alpha_U)\right)^2}} = 0$$

if we make the empirically relevant assumption that $\alpha_U < 0.5$. So,

$$\lim_{\theta\to 0}\Gamma(\theta) = \rho$$

When we evaluate the function at $\theta = 1$ we get

$$\Gamma(1) = L^{\frac{1-\alpha_U}{\alpha_U}}(1-\alpha_U) + \rho$$

To characterize the function further let us differentiate it:

$$\begin{aligned}\frac{\partial\Gamma(\theta)}{\partial\theta} &= \frac{1-\alpha_U}{\alpha_U}L^{\frac{1-\alpha_U}{\alpha_U}}\theta^{\frac{1-2\alpha_U}{\alpha_U}}\left(1-\alpha_U - \frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\left(\frac{1}{\theta}-1\right)\right) + \theta^{\frac{1-\alpha_U}{\alpha_U}}\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{1}{\theta^2} \\ &= L^{\frac{1-\alpha_U}{\alpha_U}}\theta^{\frac{1-2\alpha_U}{\alpha_U}}\left(\frac{1-\alpha_U}{\alpha_U}\left(1-\alpha_U - \frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\right) + \frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{1}{\theta}\left(1-\frac{1-\alpha_U}{\alpha_U}\right)\right)\end{aligned} \quad (54)$$

Let θ_c be the critical point where derivative vanishes. Then,

$$\theta_c = \frac{\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\left(\frac{1-\alpha_U}{\alpha_U}-1\right)}{\frac{1-\alpha_U}{\alpha_U}\left(1-\alpha_U+\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\right)} = \frac{\frac{1-\alpha_U}{\alpha_U}\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}-\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}}{\frac{1-\alpha_U}{\alpha_U}\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}+\frac{1-\alpha_U}{\alpha_U}(1-\alpha_U)}$$

Then $\theta_c \in (0,1)$ given that $\alpha_U < 0.5$ and it is unique. Observe from (54) that when $\theta < \theta_c$, the derivative is negative and when $\theta > \theta_c$ the derivative is positive. So $\Gamma(\theta)$ is decreasing when $\theta < \theta_c$ and increasing when $\theta > \theta_c$. To be able to say there is a θ_{ss} such that $\Gamma(\theta_{ss}) = 0$, we need to see when $\Gamma(\theta_c) < 0$ would hold. Then, by the intermediate value theorem, there are two roots to the equation. At θ_c

$$\Gamma(\theta_c) = \left(\theta_c L\right)^{\frac{1-\alpha_U}{\alpha_U}}\left(-\left(1-\alpha_U+\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\right)\right)+\rho\,,$$

which is less than zero when

$$\left(\theta_c L\right)^{\frac{1-\alpha_U}{\alpha_U}}\left(1-\alpha_U+\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\right) > \rho\,.$$

This is the plausible case to consider since the value of ρ is usually considered to be around 0.04, and α_U is taken around 0.35. So, as argued above, we can conclude by the intermediate value theorem that there are two roots to $\Gamma(\theta) = 0$. Let us denote the smaller root by θ_{ss}^0 and the larger root by θ_{ss}.

The shape of $\Gamma(\theta)$ can be qualitatively drawn as in Figure 10.

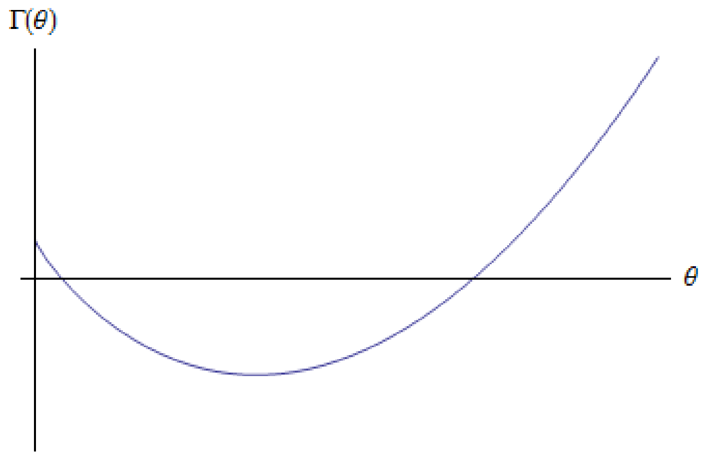

Figure 10. Qualitative characterization of $\Gamma(\theta)$.

Proof of Proposition 2

Qualitatively there are six possible types of paths that can be an equilibrium path.

Case 1: $\theta_0 > \theta_{ss}$

By Figure 7, we know that θ converges to 1 on such a path. However, at the steady state transversality condition is violated:

$$\lim_{t\to\infty} K(0)\cdot\exp\left\{\int_0^t L^{\frac{1-\alpha_U}{\alpha_U}}\left(1-\alpha_U\right)ds\right\} = \lim_{t\to\infty} K(0)\cdot\exp\left\{L^{\frac{1-\alpha_U}{\alpha_U}}\left(1-\alpha_U\right)t\right\} = \infty$$

So, such a path cannot be an equilibrium path.

Case 2: $\theta_a < \theta_0 < \theta_{ss}$

From Figure 7, we know that $\dot{\theta} \to -\infty$, which means that θ reaches zero in finite time, which would cause a discrete jump in C_U to 0, which cannot be an equilibrium path assuming interior solution.

Case 3: $\theta_{ss}^0 < \theta_0 < \theta_a$

Similar to Case 2. From Figure 7, we know that $\dot{\theta} \to \infty$, which means that θ reaches one in finite time, which would cause a discrete jump in C_R to 0, which cannot be an equilibrium path assuming interior solution.

Case 4: $0<\theta_0<\theta_{ss}^0$

From Figure 7, we know θ converges to 0 on such a path. However, at the steady state transversality condition is violated. As shown in the proof of Proposition 1,

$$\lim_{\theta\to 0}\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}}\left(1-\alpha_U-\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{1-\theta}{\theta}\right)=0$$

$$\Rightarrow \lim_{t\to\infty}K(0)\cdot\exp\left\{\int_0^t\left(\theta(s)L\right)^{\frac{1-\alpha_U}{\alpha_U}}\left(1-\alpha_U-\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{1-\theta(s)}{\theta(s)}\right)ds\right\}\neq 0$$

Cases 5 & 6: $\theta(t)=\theta_{ss}^0$ & $\theta(t)=\theta_{ss}$

When the system starts on any of these steady states, it stays there since $\dot{\theta}=0$ on these paths and the transversality condition holds, $\lim_{t\to\infty}K(0)\cdot\exp\{-\rho t\}=0.$ The term in the exponential comes from the fact that at these steady states

$$\Gamma(\theta)=\left(\theta L\right)^{\frac{1-\alpha_U}{\alpha_U}}\left(1-\alpha_U-\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{1-\theta}{\theta}\right)+\rho=0$$

as given by Proposition 1. Hence these paths are both solutions to the given dynamic system.

Proof of Proposition 3

Applying implicit function theorem to (23)

$$\frac{d\theta_{ss}}{dx}=-\frac{\partial\Gamma(\theta_{ss})/\partial x}{\partial\Gamma(\theta_{ss})/\partial\theta_{ss}}$$

$$\frac{d\theta_{ss}}{d\left(\beta_U/\beta_R\right)}=-\frac{-\left(\theta_{ss}L\right)^{\frac{1-\alpha_U}{\alpha_U}}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{1-\theta_{ss}}{\theta_{ss}}}{L^{\frac{1-\alpha_U}{\alpha_U}}\theta_{ss}^{\frac{1-2\alpha_U}{\alpha_U}}\left(\frac{1-\alpha_U}{\alpha_U}\left(1-\alpha_U-\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\right)+\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{1}{\theta_{ss}}\left(1-\frac{1-\alpha_U}{\alpha_U}\right)\right)}$$

Observe that around θ_{ss} the $\partial\Gamma(\theta_{ss})/\partial\theta_{ss} > 0$ (see Proof of Proposition 1). Then,

$$\frac{d\theta_{ss}}{d\left(\beta_U/\beta_R\right)} > 0$$

$$\frac{d\theta_{ss}}{d\alpha_R} = -\frac{-\left(\theta_{ss}L\right)^{\frac{1-\alpha_U}{\alpha_U}}\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)^2}\frac{1-\theta_{ss}}{\theta_{ss}}}{L^{\frac{1-\alpha_U}{\alpha_U}}\theta_{ss}^{\frac{1-2\alpha_U}{\alpha_U}}\left(\frac{1-\alpha_U}{\alpha_U}\left(1-\alpha_U-\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\right)+\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{1}{\theta_{ss}}\left(1-\frac{1-\alpha_U}{\alpha_U}\right)\right)} > 0$$

$$\frac{d\theta_{ss}}{d\alpha_U} = -\frac{-\left(\theta_{ss}L\right)^{\frac{1-\alpha_U}{\alpha_U}}\left(1-\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{1-\theta_{ss}}{\theta_{ss}}\right)}{L^{\frac{1-\alpha_U}{\alpha_U}}\theta_{ss}^{\frac{1-2\alpha_U}{\alpha_U}}\left(\frac{1-\alpha_U}{\alpha_U}\left(1-\alpha_U-\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\right)+\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{1}{\theta_{ss}}\left(1-\frac{1-\alpha_U}{\alpha_U}\right)\right)}$$

When (23) holds, the term in the parentheses in the numerator is negative, so

$$\frac{d\theta_{ss}}{d\alpha_U} < 0$$

$$\frac{d\theta_{ss}}{d\rho} = -\frac{1}{L^{\frac{1-\alpha_U}{\alpha_U}}\theta_{ss}^{\frac{1-2\alpha_U}{\alpha_U}}\left(\frac{1-\alpha_U}{\alpha_U}\left(1-\alpha_U-\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\right)+\frac{\beta_U}{\beta_R}\frac{(1-\alpha_U)}{(1-\alpha_R)}\frac{1}{\theta_{ss}}\left(1-\frac{1-\alpha_U}{\alpha_U}\right)\right)} < 0$$

The relationships are reversed for θ_{ss}^0 since $\partial\Gamma(\theta_{ss}^0)/\partial\theta_{ss}^0 < 0$ as shown in Proof of Proposition 1.

BIBLIOGRAPHY

Acemoglu, D. (2009). *Introduction to Modern Economic Growth.* Princeton: Princeton University Press.

Acemoglu, D. & V. Guerrieri (2008). "Capital Deepening and Nonbalanced Economic Growth." *The Journal of Political Economy*, 116, 467-498.

Annez, P. C. & R. M. Buckley (2009). "Urbanization and Growth: Setting the Context." In M. Spence, Annez, P. C. & R. M. Buckley (Eds.), *Urbanization and growth* (pp. 1-45). Washington, DC: Commission on Growth and Development, World Bank.

Bairoch, P. (1988). *Cities and Economic Development: From the Dawn of History to the Present* (C. Braider, Trans.). Chicago: University of Chicago Press. (Original work published in 1985).

Barro, R. J. & X. Sala-i Martin (2004). *Economic Growth* (2nd ed.). Cambridge, Mass: MIT Press.

Baumol, W. J. (1967). "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis." *The American Economic Review*, 57, 415-426.

Capasso, S. & M. R. Carillo (2009). "The Legacy of Dualism in New Growth Theory." In N. Salvadori, P. Commendatore & M. Tamberi (Eds.), *Geography, Structural Change and Economic Development: Theory and Empirics* (pp. 203-226). Cheltenham, UK; Northampton, MA: Edward Elgar.

Echevarria, C. (1997) "Changes in Sectoral Composition Associated with Economic Growth." *International Economic Review*, 38, 431-452.

Gollin, D., S. Parente & R. Rogerson (2002). "The Role of Agriculture in Development." *The American Economic Review (Papers and Proceedings)*, 92, 160-164.

Harris J. R. & M. P. Todaro (1970). "Migration, Unemployment and Development: A Two-Sector Analysis." *The American Economic Review*, 60, 126-142.

Henderson, J. V. (2005) "Urbanization and Growth." In P. Aghion and S. N. Durlauf (Eds.), *Handbook of Economic Growth* (pp. 1543-1591). Amsterdam: Elsevier.

Hohenberg, P. M. & L. H. Lees (1995). *The Making of Urban Europe: 1000-1950*. Cambridge, Mass.: Harvard University Press.

Jacobs, J. (1969). *The Economy of Cities*. New York: Random House.

Kongsamut, P., S. Rebelo & D. Xie (1997). "Beyond Balanced Growth." NBER Working Paper No. 6159. National Bureau of Economic Research.

Kongsamut, P., S. Rebelo & D. Xie (2001) "Beyond Balanced Growth." *The Review of Economic Studies*, 68, 869-882.

Kuznets, S. (1966). *Modern Economic Growth: Rate, Structure, and Spread*. New Haven: Yale University Press.

Laitner, J. (2000). "Structural Change and Economic Growth." *The Review of Economic Studies*, 67, 545-561.

Lewis, W. A. (1954). "Economic Development with Unlimited Supplies of Labour." *The Manchester School*, 22, 139–191.

Lucas, R. E. (1988). "On the Mechanics of Economic Development." *Journal of Monetary Economics*, 22, 3-42.

Lucas, R. E. (2004). "Life Eamings and Rural-Urban Migration." *The Journal of Political Economy (Papers in Honor of Sherwin Rosen)*, 112, S29-S59.

Mas-Colell, A. & A. Razin (1973). "A Model of Intersectoral Migration and Growth." *Oxford Economic Papers*, 25, 72-79.

Matsuyama, K. (1992). "Agricultural Productivity, Comparative Advantage, and Economic growth."*Journal of Economic Theory*, 58, 317-334.

Mokyr, J.(1995). "Urbanization, Technological Progress, and Economic History." In H. Giersch (Ed.), *Urban Agglomeration and Economic Growth* (pp. 3-37). New York: Springer-Verlag.

Murphy, K. M., A. Shleifer & R. W. Vishny (1989)" Industrialization and the Big Push." *Journal of Political Economy*, 97, 1003-1026.

Ngai L. R. & C. A. Pissarides (2007). "Structural Change in a Multisector Model of Growth." *The American Economic Review*, 97, 429-443.

Quigley, J. M. (2009). "Urbanization, Agglomeration, and Economic Development." In M. Spence, Annez, P. C. & R. M. Buckley (Eds.), *Urbanization and growth* (pp. 115-132). Washington, DC: Commission on Growth and Development, World Bank.

Ranis, G. & J. C. Fei (1961). “A Theory of Economic Development.” *The American Economic Review*, 51, 533-565.

Romer, P. M. (1986). “Increasing Returns and Long-Run Growth.” *Journal of Political Economy*, 94, 1002-1037.

Simmie J. (2001). “Innovation and Agglomeration Theory.” In J. Simmie (Ed.), *Innovative* Cities (pp. 9-52). London; New York: Spon Press.

Turkish Statistical Institute (2010). *Statistical Indicators: 1923-2009.* Ankara: Turkish Statistical Institute, Printing Division.

United Nations (2006). *Diverging Growth and Development, World Economic and Social Survey.* New York: United Nations.