REAL INTEREST RATES AND FRACTIONAL INTEGRATION : THE CASE OF EMERGING MARKETS

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REAL INTEREST RATES AND FRACTIONAL INTEGRATION :
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Thesis Abstract

Hale Koç, "Real Interest Rates and Fractional Integration : The Case of Emerging Markets”

This thesis is composed of two essays applying fractional integration analysis to (i) real interest rate series of Turkey using various definitions (ii) real interest rate series of 19 emerging economies. Despite the existence of papers discussing fractional dynamics in real rates of U.S. and OECD countries, very little is known about this kind of dynamics in the real interest series of emerging markets.

Augmented efficient fractional unit root test is implemented to conduct inference on the order of integration real rate series of Turkey and emerging markets. Possible existence of deterministics in the data is taken into account by considering two cases of the test- no trend and linear trend. As far as Turkish real rate series are concerned empirical results suggest that 2 out of the 6 real interest rate series studied are characterized by fractional unit roots. Results are observed to be responsive to maturity. On the contrary, inclusion of a linear trend doesn't alter the results. When economic causes of persistence are concerned, macroeconomic fundamentals like inflation, fiscal deficit, uncertainty as well as credibility and risk premium seem to play role.

As far as emerging economies are concerned results suggest that for the majority of the countries studied, estimated order of integration is above 0.5, pointing out the existence of non?stationary but mean reverting dynamics in the data. Upon implementation of the test, null hypothesis of a unit root is rejected in favor of a general fractional alternative for 9 emerging countries. It is also observed that test results are responsive to inclusion/exclusion of deterministics. Presence of fractional dynamics in more than half of the countries is in line with what is previously found for developed economies.

Tez Özeti

Hale Koç, Reel Faiz Serileri ve Parçalı Bütünleşme : Gelişmekte Olan Ülkeler Vakası

Bu tez parçalı bütünleşme analizini (i) çeşitli tanımlar kullanılarak oluşturulan Türkiye reel faiz serilerine (ii) gelişmekte olan ülkelerden 19unun reel faiz serilerine uygulayan 2 denemeden oluşmaktadır. Literatürde parçalı dinamikleri Amerika ve Ekonomik Kalkınma ve İşbirliği Örgütü üyesi ülkeler için araştıran çalışmaların varlığına karşın, gelişmekte olan ülkeler özelinde bu tip dinamikler hakkında sınırlı bilgi mevcuttur.

Arttırılmış, verimli parçalı birim kök testi, reel faiz serilerinde durağanlık derecesi hakkında çıkarsama yapmak için kullanılmıştır. Olası trend davranışını göz önüne almak için, iki vaka trendsiz vaka ve lineer trend vakası incelenmiştir. Türkiye reel faiz serileri için, incelenen 6 seriden 2 tanesinin parçalı birim kök yapısına sahip olduğu görülmüştür. Sonuçların vadeye duyarlı olduğu, trend varlığına ise duyarlı olmadığı gözlemlenmiştir. Bulguların arkasındaki olası ekonomik sebepler ise, enflasyon gibi temel makroekonomik değişkenler, bütçe açığı, ekonomik ve politik belirsizlik, risk primi olarak sıralanabilir.

Gelişmekte olan ülkelere bakarsak, incelenen ülkelerin yarısından fazlası için durağanlık derecesi 0.5in üzerinde tahmin edilmiştir. Bu tahmin serilerin uzun hafızaya sahip olduğunu, durağan olmadıklarını, ancak uzun vadede ortalamalarına geri döndüklerini işaret etmektedir. Parçalı birim kök testinin uygulanması ile, birim kök boş hipotezi 9 gelişmekte olan ülke serisi için reddedilmiş, parçalı birim kök alternatifi kabul edilmiştir. Türkiye serilerinin aksine, sonuçların trend varlığına duyarlı olduğu gözlemlenmiştir. Reel faiz serilerinde parçalı dinamiklerin varlığı, daha önce gelişmiş ülkeler için ortaya konulan bulgularla uyum içerisindedir.

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CONTENTS

CHAPTER 1

REAL INTEREST RATES AND FRACTIONAL INTEGRATION : THE CASE OF TURKEY

Introduction

Real interest rate occupies a pivotal position in financial and macroeconomic modeling. Besides several others, it lies in the heart of the Fisher equation, the neoclassical growth model, and the consumption based capital asset pricing model (CCAPM). Without formal testing, many of these models either implicitly assume or explicitly predict real interest rates to be stationary. For example, Fisher hypothesis suggests that expected inflation and nominal interest rate move one-for-one in the long run. Then nominal interest rate and expected inflation series share the same integration properties. In order for Fisher hypothesis to hold expected real interest rate must be stationary. Another example is CCAPM. According to the CCAPM model consumption growth rate and real interest rate must have similar integration properties. It is an empirically well established fact that consumption growth rate series is $I(0)$, implying the stationarity of the real interest rate.

Given the theoretical significance of the topic, considerable amount of work is devoted to understanding the long-run behavior of real interest rates. Rose (1988) is among the first who suggest evidence in this line of research. Using data from 18 OECD countries on ex post real interest rates of different maturities, he shows that the real interest rate contains a unit root in various industrialized countries. Based on findings, Rose argues that evidence of real interest rate series following a random walk, together with the finding of stationary consumption growth rate undermine the validity of the CCAPM. Following Rose (1988), many studies have failed to reject the null hypothesis of a unit root, substantiating the inconsistency between theoretical and empirical work (see King et al., 1991, Rapach, 2003 among others).

In a recent survey Neely and Rapach (2008) report a number of studies that found evidence in the opposite direction, i.e., that real interest rate series are stationary. Mishkin (1992) examines nominal Treasury bill rate and the CPI-based inflation rate for the U.S. over the full period of 1953-1990 and sub periods; and fails to reject the null hypothesis that the series have a unit root component. However when he tests for a cointegrating relationship between the two series, he rejects the null hypothesis of no cointegration. So he concludes that U.S. real interest rate series follows an $I(0)$ process. Therefore it can be concluded that empirical evidence on the long-run properties of real interest rate series is rather mixed.

Fractional integration analysis is an alternative method which may offer an explanation for the contradicting results obtained within the empirical literature and to the inconsistency between theory and the data. Standard unit root analysis concerns itself with testing the null hypothesis of an $I(1)$ against the alternative of an $I(0)$ process. However in the presence fractional unit roots, choosing between only two alternatives may be too restrictive. As a consequence hypotheses of $I(1)$ and $I(0)$ may be rejected simultaneously. By abandoning the binary distinction between $I(1)$ and $I(0)$ processes; fractional integration analysis is capable of providing a more comprehensive description of the data generating process. Furthermore taking fractional alternatives into account may explain the divergence between the model and empirical findings. Within the context of fractional integration, requirement of stationarity is replaced by mean reversion which is a broader concept than stationarity. For instance in the case of Fisher hypothesis it is enough to show that real rate series revert back to their means as implied by an fractionally integrated $I(d)$ process with $0 \leq d < 1$.

What lies behind the ability of fractional integration in describing long run behavior better than binary framework is simple: The former is the generalization of the latter and allows for a wider range of dynamics. In the binary framework, a series is stationary (non-stationary) only when it is an $I(0)$ ($I(1)$) process. Series revert back to their means only when they are $I(0)$. However within the context of fractional integration, all $I(d)$ series with $0 < d \leq 1/2$ is concluded to be stationary, although they return to their means at a lower rate compared to the $I(0)$ case. Mean reversion property is also verified for the $I(d)$ case with $1/2 < d < 1$. These series are non-stationary but mean reverting processes. They are characterized by persistency, i.e., the effects of a shock live long (longer than a stationary series) but die eventually, allowing the series to restore their means.

A few papers apply fractional integration analysis to real interest rate series and document existence of fractional unit roots in the data. Lai (1997) provides a reappraisal of the existing evidence on the integration properties of 3 different monthly series of U.S. real interest rates, and reports that the series examined exhibit mean reversion, but in a special manner not captured by the usual stationary process. Karanasos et al., (2006) analyze monthly long-term government bond yield data for the U.S. spanning the period from 1876 to 2000 and assert that U.S. real rate series displays near integrated behavior, which is a type of stationary behavior that is difficult for standard unit root tests to detect. Kasman et al., (2006) focusing on 33 developed and developing countries, first test for the existence of a long run relation between nominal interest rates and inflation via conventional cointegration tests, and fail to reject the null hypothesis of no cointegrating relationship, for most countries. However, as a further analysis, they next apply fractional cointegration tests. For the majority of the countries including Turkey, they find the two series to be fractionally cointegrated, implying the validity of Fisher hypothesis.

This study contributes to the literature by providing evidence on whether

Turkish real interest rate series are characterized by fractional integration. To our knowledge, this is the first study which investigates fractional dynamics in Turkish real interest rate series. 6 different definitions of real interest rate are employed. The reason behind this is twofold: (i) Calculation of ex ante real interest rate series requires the use of expected inflation series which is unobserved. Different assumptions on this unobserved variable lead to different results. (ii) Degree of integration depends on whether short-term or long-term real rates are analyzed, whether inflation expectations or actual inflation is used, etc.

Knowledge on the order of integration of real interest rate series in Turkey is important for several reasons. First of all Turkey is a country whose economic history is depicted by prolonged periods of high inflation and failed stabilization attempts. Degree of fractional integration is a crucial input in developing effective stabilization policy. Moreover Turkey is a small, open economy with integrated capital markets. Hence it is frequently exposed to external shocks, as well as internal shocks. Determining the order of integration helps to measure how resilient the economy is to these kinds of shocks. Finally Turkish economy is characterized by a low level of savings both in public and private sectors, and consequently by a high dependence on borrowing. Therefore the effects of shocks that cause the real interest rates persistently deviate from their means are not confined to the series themselves. By determining the amount of funds available in a given period, these shocks limit or relax the country's economic capabilities.

In this study, we draw inference on the order of integration in Turkish real interest rate series by applying augmented efficient fractional Dickey Fuller test, proposed by Lobato and Velasco (2007). Possible existence of deterministics in the data is taken into account by considering two cases of the test- no trend and linear trend. Empirical results suggest that 2 out of the

6 real interest rate series studied are characterized by fractional unit roots. Results are observed to be responsive to maturity as 360-day yield is concluded to contain a unit root but its 30-day counterpart doesn't. On the contrary, inclusion of a linear trend doesn't alter the results. As far as the economic causes of persistence are concerned, macroeconomic fundamentals like inflation, fiscal deficit, uncertainty as well as credibility and risk premium seem to play role.

The plan of the study is as follows: Section 2 discusses the data used in this study and describes how the real interest rate series are constructed. While section 3 establishes the theoretical basis of empirical methodology, section 4 summarizes the empirical results and section 5 concludes.

The Data

Our aim is to establish the fractional dynamics of ex ante real interest rate which is defined as the difference between nominal interest rate and expected inflation. However calculation of ex ante real interest rate is not a straightforward task, since expected inflation is not directly observable. Empirical literature relies on different methods to solve the problem of unobserved inflation expectations. In this study, we apply 4 of these methods.

The first method uses inflation forecasts or surveys of inflation expectations.

$r^{sr,360}$ is based on the survey of inflation expectations data collected by the Central Bank of Turkey where the term "sr" in the superscript stands for the use of survey data. The series is obtained by subtracting the 12-month ahead expected rate of inflation from 360-day Turkish government bond yield. The resulting real rate series extends from 2001:M8-2008:M12.

Other methods use actual inflation as a proxy for expected inflation. Under method two, inflation series is assumed to follow a random walk, i.e.,

$\pi_{t+1} = \pi_t + \varepsilon_{t+1}$ where π_t is actual inflation rate at time t, and ε_{t+1} is the random disturbance term. In this case, current period's inflation is the best predictor of the next period's inflation. Then the second method obtains the real interest rate as follows:

$$E_t\pi_{t+1} = \pi_t \Rightarrow r_t^{rw} = i_t - \pi_t$$

where i_t is the actual nominal interest rate at time t, and r_t^{rw} is the real interest rate. The term "rw" in the superscript refers to the assumption that inflation series are characterized by a random walk.

$r^{rw,30}$ and $r^{rw,360}$ series are obtained with reference to method 2. $r^{rw,30}$ is given by nominal 30-day Turkish government bond yield minus current month's CPI based inflation rate. Similarly $r^{rw,360}$ is given by nominal 360-day Turkish government bond yield minus current year's CPI based inflation rate. Both series cover the period 1995:01-2008:12

Alternatively one may assume economic agents have perfect foresight in forming inflation expectations, i.e., $E_t\pi_{t+1} = \pi_{t+1}$. Hence the third method calculates

$$r_t^{pf} = i_t - \pi_{t+1}$$

where r_t^{pf} is the real interest rate. The term "pf" in the superscript represents the assumption of perfect foresight.

Calculation of $r^{pf,30}$ and $r^{pf,360}$ series are based on method 3. While $r^{pf,30}$ is the difference between nominal 30-day Turkish government bond yield and the following month's CPI inflation rate, $r^{pf,360}$ equals the the difference between nominal 360-day Turkish government bond yield and the following year's CPI inflation rate. 1995:01-2008:11 and 1995:01-2007:12 are the respective sample periods for the real rate series.

TURKSTAT's online database is the source for CPI-based inflation series, and Turkish government bond yield curve is made available by *Risk-turk*.[1]

Finally in method 4 real interest rates are constructed by adding the country-specific sovereign spreads to the international risk-free real rate (see Fernandez-Villaverde et al., 2009 for a similar treatment). As is the standard view in the literature, the U.S. rate is taken to be the international risk-free nominal interest rate. Subtracting the expected CPI-based inflation for the U.S. from the nominal interest rate leads to international risk free real rate. 10-year treasury constant maturity rate is used as U.S. nominal interest rate, since bonds with 10 year maturities are included in the calculation of EMBI Global index. 10 year ahead expected inflation is based on the Survey of Professional Forecasters conducted quarterly by the Federal Reserve Bank of Philadelphia, in which participants are asked for their expectation of the average CPI inflation rate over the next 10 years.

For country-specific sovereign spreads J.P. Morgan's Emerging Market Bonds Index (EMBI) Global Spread is used. Hence applying method 4, $r^{rw,embi}$ is given by adding EMBI Turkey spread to the international risk free real rate. The term *embi* in the superscript represents the use of EMBI data and the series extends from 1996:M7 from 2008:M12.

J.P. Morgan's Emerging Market Bonds Index (EMBI) Global Spread is the source for the daily country-specific spread. Spread series are converted to monthly frequency by taking average of each month. Data on U.S. 10-year treasury constant maturity rate is obtained from St. Louis Fed's FRED database at monthly frequency. Expected inflation series are retrieved from Philadelphia Fed at quarterly frequency and interpolated to monthly frequency via cubic spline method.

[1]The original Turkish bonds and bills data have been collected from Reuters. Then Nelson-Siegel method has been used for estimating the yield curve.

Empirical Methodology

Lack of power of standard unit root tests in the presence of fractional alternatives is a widely known empirical finding. These tests may mistakenly claim a series to contain unit roots when it actually contains near unit roots, leading erroneous theoretical or empirical results.

Fractional Dickey Fuller test and its variants offer a way out of these potential mistakes motivated by power and efficiency gains achieved in the case of fractional unit roots.

Fractional Dickey Fuller Test

Dolado *et al.*, (2002) have designed a simple Wald test (FDF test thereafter) which extends the widely applied Dickey Fuller test to fractional alternatives. Indeed, while the latter limits itself to the classical distinction between cases of $I(1)$ and $I(0)$, FDF test concerns itself with the more general distinction between $I(1)$ and $FI(d_1)$ processes with $d_1 \in \mathbb{R}$, and $0 \leq d_1 < 1$. Thus standard Dickey Fuller test can be restored setting d_1 equal to 0.

In the basic setup y_t is a fractionally integrated series whose true order of integration is d. Data generating process is assumed to be given by

$$\Delta^d y_t 1\{t > 0\} = \varepsilon_t \tag{1}$$

where $1\{\cdot\}$ denotes the indicator function, $\{\varepsilon_t\}_{t=1}^{T}$ is a sequence of zero mean, finite variance *iid* random variables, and T is the sample size.

In order to be able to test the null hypothesis $H_0 : d = 1$; against the alternative $H_1 : d < 1$, FDF test makes use of the following regression model:

$$\Delta y_t = \phi_1 \Delta^{d_1} y_{t-1} + u_t \tag{2}$$

It is straightforward to show that the null and alternative hypotheses can be restated in terms of ϕ_1. Assuming that $u_t = \varepsilon_t$, $\phi_1 = 0$ corresponds to the case $\{y_t\}$ contains a unit root. On the other hand, $\phi_1 < 0$ implies that the y_t series is fractionally integrated of order d_1. Consequently, it is suggested that the null hypothesis versus the alternative can be tested by checking for the statistical significance of parameter ϕ_1 via construction of a simple t statistic.

Efficient Fractional Dickey Fuller Test

Lobato and Velasco (2007) argue that the use of $\Delta^{d_1} y_{t-1}$ is not the optimal choice among the class of regressors, which calculate a t ratio whose asymptotic null distribution is standard normal. They argue that the model given in equation (2) is misspecified because under the alternative hypothesis, no pair (ϕ_1, d_1) can be found that will make the error series, u_t, serially uncorrelated and independent of the regressor, $\Delta^{d_1} y_{t-1}$. Consequently, the estimated OLS coefficient and the resulting t statistic are inefficient in the sense that FDF test does not maximize power. Lobato and Velasco (2007) propose an alternative Wald type test (EFDF test thereafter) which uses the regression model

$$\Delta y_t = \phi_2 z_{t-1}(d_2) + u_t \tag{3}$$

where $z_{t-1}(d_2)$ is defined as

$$z_{t-1}(d_2) = \frac{(\Delta^{d_2-1} - 1)}{1 - d_2} \Delta y_t$$

By testing for the significance of the coefficient ϕ_2 with $d_2 > 0.5$ via a left-sided t test , Lobato and Velasco (2007) propose to test for the null hypothesis where y_t is believed to be random walk against the alternative y_t is claimed to contain a fractional unit root, with differencing parameter $d_2 < 1$.

Notice that previous discussion is built on the assumption that data generating process is given by (1) where ε_t is iid. Lobato and Velasco (2007) construct a generalized version of the EFDF test, augmented EFDF (AEFDF) test, in the presence of autocorrelation among $\Delta^d y_t$, that is data generating process is given by

$$\alpha(L)\Delta^d y_t 1\{t > 0\} = \varepsilon_t \tag{4}$$

where $\alpha(L) = 1 - \alpha_1 L - ... - \alpha_p L^p$ is a polynomial in the lag operator with all roots outside the unit circle.

In this case they propose to use a two-step procedure that will result in an efficient test under the assumption that DGP is characterized by (4).[2] The first step is

$$\Delta^{\widehat{d_2}} y_t = \sum_{j=1}^{p} \Delta^{\widehat{d_2}} y_{t-j} + u_t \tag{5}$$

where an autoregression in the fractional difference of the series is estimated by OLS to obtain coefficients of $\alpha(L)$. The second step is

$$\Delta y_t = \phi_2[\widehat{\alpha}(L) z_{t-1}(\widehat{d_2})] + \sum_{j=1}^{p} \alpha_j \Delta y_{t-j} + \nu_t \tag{6}$$

where $\widehat{\alpha}(L)$ is the estimate coming from (5).

In the derivation of the AEFDF test, Lobato and Velasco (2007) didn't allow for deterministics in the data generating process. As a further generalization, Dolado *et al.*, (2006) investigate how to implement the test in the case of deterministics which may take the form of drifts, linear or quadratic trends etc. With a focus on the role of a linear trend (since many economic

[2] Power of the test falls substantially (up to 50%) when the augmented version of the test is applied to series that don't contain serial correlation

time series are characterized by this type of trend in levels), they show analysis proceeds in the same way as in AEFDF test once the series is demeaned (in the case of a linear trend) or detrended (in the case of a quadratic trend).

It should be noted that to make any of the tests discussed (FDF, EFDF or AEFDF) feasible, value of the fractional differencing parameter needs to be estimated. Lobato and Velasco (2007) demonstrate that both EFDF and AEFDF tests are efficient not only when parametric $T^{1/2}$ consistent estimators are used but also when the estimation is based on a semi-parametric estimator with an appropriate choice of bandwidth parameter.[3]

Motivated by the power and efficiency gains achieved over its predecessor, AEFDF test (with deterministics) will be implemented in this study to conduct inference on the degree of integration of real interest rate series, r_t, of Turkey. To perform the test, fractional differencing parameter of the real interest rate series is estimated by means of the feasible exact local Whittle estimator (FELW thereafter) developed by Shimotsu and Phillips (2005). The reason why FELW is chosen over other alternative estimators, all of which ensure AEFDF test to be efficient, is because it is a good-general purpose estimator that preserves its desirable properties for a wide range of stationary and non-stationary values of the differencing parameter.

Empirical results

In order to establish the long-run properties of real interest rate series, we first carry out the analysis in traditional $I(0)/I(1)$ domain, which is the standard practice in the literature. We utilize the Augmented Dickey Fuller (ADF) test and the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test. These two tests complement each other as the former takes non-stationarity

[3] The test is inefficient but standard normal when an inconsistent estimator is employed.

as its null hypothesis, while the latter assumes stationarity under the null. Table 1 shows the results for each Turkish real interest rate series.

Inspection of Table 1 reveals that ADF test rejects the $I(1)$ hypothesis for 30-day and 360-day Turkish government bond yields. This conclusion is true whether the real interest rate series are calculated via method 2 or method 3, and corroborated by what is found by KPSS test. Hence both ADF and KPSS tests agree on the fact that Turkish government bond yield series are stationary, $I(0)$ processes. On the contrary, when 360-day Turkish real government bond series is calculated under method 1, the resulting $r^{sr,360}$ series is reported to be a non-stationary, $I(1)$ process by both tests. Finally ADF and KPSS tests both claim $r^{rw,embi}$ series to contain unit roots.

No contradiction is observed in the results of the ADF and KPSS tests. The tests unanimously infer $r^{pf,30}$, $r^{rw,30}$, $r^{pf,360}$ and $r^{rw,360}$ $I(0)$ and the remaining $r^{rw,embi}$ and $r^{sr,360}$ as $I(1)$ processes. However, as mentioned previously, generic unit root tests have little power to identify between unit roots and near unit roots. This deficiency justifies implementation of a fractional unit root test, which explicitly take fractional alternatives into account.

Recall that to make the AEFDF test feasible, value of fractional differencing parameter needs to be estimated. Table 2 presents the estimates for this parameter, $\hat{d}$, calculated by a variety of estimators. These estimates will not only enable us to implement the AEFDF test, but also give us the first idea on the order of integration in Turkish real interest rate series.

Table 2 shows that, all of the estimates for $r^{rw,embi}$, $r^{pf,360}$, $r^{rw,360}$ and $r^{sr,360}$ are above 0.5, indicating the possible existence of non-stationary dynamics in these series.

To get the first formal inference on the degree of integration, asymptotic 95% confidence intervals are constructed around the FELW estimates. The standard error of the FELW estimator is given by $\sqrt{1/4m}$ where $m = T^{0.65}$, and T. Then calculation of 95% interval follows $\widehat{d} \pm 1.96 \times 1/\sqrt{4m}$ where $\widehat{d}$ is the estimated order of integration. For $r^{rw,embi}$ and $r^{sr,360}$ series confidence intervals include $d = 1$ value, implying that the null hypothesis of a unit root won't be rejected. On the other hand for $r^{pf,30}$, $r^{rw,30}$, $r^{pf,360}$ and $r^{rw,360}$ the interval range is (0.4,1). For these series random walk hypotheses are rejected. But in the face of this rejection, it is ambiguous whether they are stationary ($d < 0.5$) or non-stationary but mean reverting ($d > 0.5$) processes.

Given the estimates provided by various estimators of the differencing parameter, as well as some inconclusive results by the confidence intervals; it will be illuminating to examine degree of fractional dynamics via a formal test. This is where the AEFDF test, as discussed in the previous section, will be utilized. The test will tell whether $d = 1$ or $d < 1$, either supporting the unit root hypothesis or rejecting it in favor of a fractional alternative.

Table 3 contains the results of the AEFDF test calculated by selecting the optimal lag length on the basis of AIC. To allow for deterministics, cases of no trend and linear trend are considered.

AEFDF test rejects the null hypothesis of a unit root in favor of a fractional alternative only for $r^{pf,360}$ and $r^{rw,360}$ series. The outcome of the test doesn't change neither when the series are calculated under method 2 (by assuming inflation follows a random walk) nor under method 3 (by assuming agents have perfect foresight). Additionally, test results are not responsive to inclusion of deterministics. Therefore, it can be concluded that 360-day real government bond yield series contain fractional unit roots. Going back to table 2 gives rise to the further conclusion that 360-day real return series are non-stationary but mean reverting processes as estimates of the fractional differencing parameter are above 0.5 for both $r^{pf,360}$ and $r^{rw,360}$.

Lower left panel of figure 1 shows evolution of $r^{pf,360}$ and $r^{rw,360}$ series.

Inspection of the figure reveals that through 1995:01-2008:12, 360-day real government bond yield series are characterized by two distinct periods. The first period from 1995:01 to 2001:02 is when we observe a prolonged period of high real interest rates, high inflation and high volatility and high risk in the aftermath of April, 1994 crisis. Although Turkey made a serious stabilization attempt in June, 1998 by starting to follow a disinflation programme under the guidance of IMF; the programme resulted in significant deterioration of fiscal account. Together with the political uncertainty caused by the elections held in April, 1999 and the weak coalitional governments established afterwards, fiscal deficit led to higher real interest rates. As another attempt, in December, 1999 the government decided to adopt a 3-year exchange rate based stabilization programme, again under the guidance of IMF. Within the first year after the implementation of the programme, interest rates and inflation began to fall. However worse than the previous one, this attempt ended with eruption of crises of November, 2001 and February, 2002. The idea of breaking the high inflation inertia via a crawling peg exchange rate regime turned out to be a failure where the government was forced to let the currency float. Furthermore, high inflation returned and interest rates skyrocketed. Hence characterized by two economic crises, two failed stabilization attempts and political distrust; persistently high levels of real interest rates in 1995:01-2001:02 can be explained by high risk premium, high government deficit, high and persistent inflation and lack of credibility.

In contrast, the second period from 2001:02 to 2008:12 is when we observe a prolonged period of relatively lower real interest rates, lower inflation and lower volatility. As mentioned in the previous paragraph currency crisis of February, 2001 resulted in the adoption of free-float exchange rates regime. Moreover, immediately following the crisis, Central Bank of Turkey gained its independence from political authority in October, 2001 and announced its decision to follow an implicit inflation targeting policy until the conditions are ready for a full targeting regime. In its announcement, Central Bank of Turkey clearly specified that its sole objective is to restore and maintain price stability. This objective is also one of the two basics of the post-crisis programme of IMF for Turkey. As the second basic, IMF required the government to run a public budget surplus. By achieving fiscal and monetary targets, the aim of the programme was to enhance credibility of the country and reduce risk premium as a consequence.

When we look at the inflation and real interest rate levels of the 2001:02-2008:12 period, inflation target and lower level of real interest rates seem to be achieved. Indeed, inflation is lowered from its average 80% level in the first period to its average 20% level in this period. Real interest rate moved down to 12% from 30% on average. The story behind this reduction is as follows: Turkish lira appreciated significantly as a result of the massive capital flows following the unexpected switch to free float regime right after the crisis. This appreciation caused the value of the foreign currency denominated government debt to decline, inducing a consequent decline in risk perception and risk premium. Lower risk premium causes a significant decline in inflation expectations in turn giving acceleration to inflation targeting policy and fight with high interest rates. However as Yeldan (2006) puts, real interest rates are still quite slow to adjust in the face of achievements in disinflation policy. As also stated in Kannan (2008), despite considerable improvements in macroeconomic fundamentals (reduced fiscal deficit, lower inflation, restructuring of banking sector etc.) Turkey continues to pay persistently higher real interest rates, compared to other emerging economies with similar characteristics. Kannan attributes this puzzling phenomenon by lack of credibility and existence of a risk premium. He argues that credibility is an important concern for countries with a history of macroeconomic volatility and failed stabilization attempts, like Turkey. Doubts over the sustainability of the disinflation programme positively distorts inflation expectations, leading higher nominal and real interest rates. Therefore it can be said that despite improved macroeconomic fundamentals, real interest rates of Turkey are persistently high -although lower in comparison to 1995:01-2001:02 period- due to lack of credibility, biased inflation expectations and positive risk premium in 2001:02-2008:12 period.

In contrast with 360-day real government bond yield series, AEFDF test fails to reject the null hypothesis of a unit root for their 30-day counterparts, implying that different maturities are characterized by different long run behavior. In particular, while $r^{pf,360}$ and $r^{rw,360}$ are described by long memory and mean reverting behavior, infinite memory and no mean reversion describe $r^{pf,30}$ and $r^{rw,30}$ series. A tentative explanation for this finding is that short maturities may be more vulnerable to shocks than long maturities. Indeed, more fundamental shocks may be needed to permanently shift a long term real interest rate series away from its equilibrium, whereas smaller shocks can be enough to create permanent disturbances in short term real rate series. Alternatively, as upper right panel of figure 1 shows, $r^{pf,30}$ and $r^{rw,30}$ contains more noise than $r^{pf,360}$ and $r^{rw,360}$ series, and test results may be reflecting this.

As far as the $r^{rw,embi}$ series are concerned there is unified support for the presence of unit roots as AEFDF besides ADF and KPSS tests conclude the series to be a random walk. Upper left panel of figure 1 shows $r^{rw,embi}$ series, as well as U.S. real rate and Turkey spread series. Inspecting the figure reveals that movements in $r^{rw,embi}$ series are driven mainly by the country specific spread over the U.S. real rate. In the face of this observation, detecting unit roots in spread series is a relevant issue. If Turkish spread is also concluded be a random walk, then we can infer that there exist such shocks that they permanently affect $r^{rw,embi}$ series via their effect on spread.

AEFDF test carried out for Turkish spreads detects unit roots in the series. This finding can be explained with reference to both country-specific and global dynamics. During 1996:07-2001:02 period, $r^{rw,embi}$ and Turkey spread series are characterized by an upward trend and large volatility. Turkey's pronounced large public deficit, political instability and high inflation contributed to the market's perception of Turkey as a risky country. These negative shocks are reflected in the country's high sovereign spread. Following the February, 2001 crisis, Turkey switched to a free float exchange rate regime, implemented implicit/explicit inflation targeting, started to restructure its banking sector; contributing to a better macro-environment in the country. Together with a worldwide scale wave of optimism and increased risk appetite, market's perception of risk has been pulled down to a lower level. However, given the stickiness in risk perceptions all these positive shocks are not enough to offset the effect on spread series of negative shocks, explaining the unit root component.

$r^{sr,360}$ is the final series for which AEFDF test fails to reject the null hypothesis of a unit root. This result is supported by ADF and KPSS tests which also conclude the series to contain a unit root. Therefore, it can be said that $r^{sr,360}$ series follows a random walk.

Lower right panel of figure 1 shows $r^{sr,360}$ as well as Turkish expected inflation series. First of all it should be noticed that sample period for the series extends from 2001:08 to 2009:01. The implication of this observation is that it will deprive us of the chance to present the results in comparison with other series. Due to unavailability of data, $r^{sr,360}$ is shorter than the remaining 5 real interest rate series. Hence, while interpreting the results one should bear in mind that we don't know what would happen if the series were extended back till 1995:01. Another thing to notice in the figure is the smooth downward trend in inflation expectations, embodying the success of implicit/explicit inflation targeting regime in controlling inflation expectations. With central bank starting to use nominal interest rate as a stabilization tool, lower inflation expectations translate into a new equilibrium characterized by lower level of nominal and real interest rates.

Conclusion

This study provides evidence on the fractional order of integration in six different definitions of the Turkish real interest rate series, through application of fractional integration analysis. AEFDF test by Lobato and Velasco (2007) is implemented to conduct inference on the order of integration. Test results with and without a linear trend are reported to allow for deterministics in the data. Following standard practice in the literature, analysis is first carried out in the traditional $I(0)/I(1)$ domain. Afterwards, as a first step in detecting fractional unit roots, estimates of the fractional differencing parameter, d, by various estimators are reported. What merits attention is that literally all estimates for all series are above 0.5 suggesting that Turkish real interest rate series are characterized by non-stationary but mean reverting behavior. With implementation of AEFDF test, only 360-day real government bond yield series are concluded to contain fractional unit roots. For the remaining 4 series, null hypothesis of a unit root fails to be rejected implying that Turkish real interest rate series are mostly defined by random walk.

As far as the economic causes of persistence are concerned, macroeconomic fundamentals like inflation, fiscal deficit seem to be important. Indeed high and persistent level of 360-day real return in the pre 2001 period can be attributed to these characteristics, as well as risk premium and credibility. Yet for the post 2001 period although macroeconomic fundamentals improve and pull the level of the series downwards; issues like credibility and risk premium continue to play role.

CHAPTER 2

REAL INTEREST RATES AND FRACTIONAL INTEGRATION : THE CASE OF EMERGING ECONOMIES

Introduction

Real interest rate occupies a key position in financial and macroeconomic models. Stationarity of real interest rate series is either the assumption or the implication of many of these models. One example is the Fisher hypothesis. What Fisher hypothesis suggests is that expected inflation and nominal interest rate move one-for-one in the long run. If changes in former is fully reflected in the changes in the latter, then ex ante real interest rate -difference between the nominal interest rate and the expected inflation- should stay unchanged.[4] So Fisher hypothesis implies nominal interest rate and expected inflation series to share the same integration properties; and real interest rate series to be stationary. Another example is consumption based capital asset pricing model (CCAPM). Euler condition of the model predicts that the real interest rate and the consumption growth have similar integration properties. In applied work, consumption growth series is documented to be integrated of order 0. Hence CCAPM model requires real interest rates series to be also stationary, $I(0)$ series.

Empirical validation of these models depends on establishing stationarity property of real interest rate series in data. To this end, considerable amount of empirical work is devoted to understand the behavior of the series. Rose (1988) is among the first who suggest evidence in this line of research. Using data from 18 OECD countries on ex post real interest rates of different maturities, he shows that the real interest rate contains a unit root in various industrialized countries. Based on his findings, Rose argues that evidence of

[4] Long run neutrality of money, claiming movements in real variables are independent of the nominal movements is the basis of Fisher hypothesis.

real interest rate series following a random walk, together with stationary consumption growth rate undermine the validity of the CCAPM. Following Rose (1988), many studies have failed to reject the null hypothesis of a unit root in real interest rate series. Koustas and Serletis (1999) studying various short-term nominal interest rate series and CPI based inflation for 11 OECD countries, document the non stationarity of real interest rates upon implementation of unit root and cointegration tests. Rapach and Weber (2004) reach the same conclusion for an extended set of long-term real interest rates of 16 OECD countries. King *et al.* (1991) and Rapach (2003) establish the presence of $I(1)$ dynamics in real rate series of U.S. and industrialized countries.

On the other hand, the number of studies that found evidence in the opposite direction, i.e., that real interest rate series are stationary, is rather limited. Mishkin (1992) examines nominal Treasury bill rate and the CPI-based inflation rate for the U.S. over the full period of 1953-1990 and sub periods. Although he fails to reject the null hypothesis that the series have a unit root component, he proves the existence of a cointegrating relationship between nominal interest and inflation rate. Thus he concludes that U.S. real interest rate series are stationary and Fisher parity is valid.

Broadly, two results emerge: Empirical evidence favors the existence of unit roots in the real interest rate series. The implication of this result for the above mentioned models is the rejection of these models by the data. Moreover, non-absence of studies claiming stationarity of the real rate series implies that there is no unified support for unit roots and the results are mixed.

Fractional integration analysis is an alternative method which may offer an explanation for the contradicting results obtained within the empirical literature and for the rejection of theoretical models by the data. Traditional stationarity analysis limits itself with the $I(0)/I(1)$ dichotomy. Presence of fractional integration in real interest rate series, which is not consistent with either an $I(0)$ or $I(1)$ process, may be the cause of diverging results. By abandoning the binary distinction between $I(1)$ and $I(0)$ processes, fractional analysis may describe the long run behavior of real interest rate series better. Furthermore, taking fractional dynamics into account leads us to reconsider the stationarity assumption which is only implied by an $I(0)$ process in the $I(0)/I(1)$ framework. By relaxing the imposition of integer orders of integration only, it may no longer be required to prove the stationarity of the real interest rate series in order to verify the models empirically. For instance, in the case of Fisher hypothesis it is enough to show that real rate series revert back to their means as implied by an fractionally integrated $I(d)$ process with $0 \leq d < 1$.

What lies behind the ability of fractional integration in describing long run behavior better than the binary framework is simple: The former is the generalization of the latter and allows for a wider range of dynamics. In the binary framework, a series is stationary-non-stationary- only when it is an $I(0)$-$I(1)$- process. Series revert back to their means only when they are $I(0)$. However, within the context of fractional integration, all $I(d)$ series with $0 < d \leq 1/2$ is concluded to be stationary, although they return to their means at a lower rate compared to the $I(0)$ case. Mean reversion property is also verified for the $I(d)$ case with $1/2 < d < 1$. These series are non-stationary but mean reverting processes. They are characterized by persistency, i.e., the effects of a shock live long (longer than a stationary series) but die eventually, allowing the series to restore their means.

There has been a number of studies which provide application of fractional integration analysis to the real interest rate series of U.S. and OECD countries and report existence of fractional unit roots. Lai (1997) presents a reappraisal of the existing evidence on the integration properties of 3 different monthly series of U.S. real interest rates, and reports that the series examined exhibit mean reversion, but in a special manner which is not captured by the usual stationary process. Karanasos *et al.*, (2006) analyze monthly long-term government bond yield data for the U.S. spanning the period from 1876 to 2000 and assert that U.S. real rate series displays near integrated behavior, which is a type of stationary behavior that is difficult for standard unit root tests to detect. Tabak (2007) presents evidence that different definitions of Brazilian real interest rates, as well as interest rate spreads are fractionally integrated. The degree of integration is higher after the implementation of an inflation targeting regime, implying that monetary policy may determine the order of integration of the real interest rate series. Yoon (2009) focusing on tax adjusted ex-post real interest rate series of 13 industrialized countries also documents that post-war real interest rates are characterized by a high order of integration.

Despite the existence of papers discussing fractional dynamics in real rates of U.S. and OECD countries, very little is known about this kind of dynamics in the real interest series of emerging markets. This study contributes to the literature by reporting evidence on the fractional orders of integration in the real interest rate series for 19 emerging economies, through application of a fractional unit root test. Having knowledge on the order of integration of real interest rates is important for emerging economies for several reasons: First of all, degree of fractional differencing is a crucial variable for development of effective stabilization policy. This is a relevant concern especially for emerging economies whose history witnessed failed stabilization attempts. In particular, if real interest rate series are concluded to contain unit roots, then in the case of shock hitting the economy, specific policy action is needed to stabilize the series back to their equilibrium. However, if it is known that the real interest rate series are characterized by fractional unit roots where the degree of integration is smaller than 1, then there is no (or less) need for such an action, since the series revert back to their means sometime in the future. Furthermore, degree of integration is a crucial variable also in assessing macroeconomic risk. As mentioned above, in the case of fractional differencing parameter lying in (1/2,1) interval, real interest rate series are characterized by persistency. In the case of a shock, a persistent series will deviate away from its mean for longer periods of time compared to a stationary series. This feature of persistent series affects the forecasting precision in a negative way, contributing to macroeconomic risk. Investors perceive the countries, whose real interest rate series have proven to be persistent as risky, and in consequence capital flows will be limited. Finally, it is usually stated that the effect of shocks to real interest rate series is not confined to the series itself. For instance, persistent shocks can severely affect the accumulation of physical and human capital imposing binding restrictions on growth, which is one of the most crucial concerns of emerging economies.

Motivated by the aforementioned reasons, augmented efficient fractional unit root test is implemented to conduct inference on the order of integration in real interest rate series of 19 emerging countries. To allow for possible deterministics in the data, two cases of the test-no trend and linear trend are considered. As the first step of implementing the test, degree of integration is estimated by various estimators. Results suggest that for the majority of the countries studied, estimated order of integration is above 0.5, pointing out the existence of non-stationary but mean reverting dynamics in the data. Upon implementation of the test, null hypothesis of a unit root is rejected in favor of a general fractional alternative for 9 emerging countries. It is also observed that test results are responsive to inclusion/exclusion of deterministics. For 4 out of the 9 fractionally integrated series, inference changes between cases of no trend and linear trend. Presence of fractional dynamics in more than half of the countries is in line with what is previously found for developed economies.

The outline of the study is as follows: Section 2 describes the data and explains how the real interest rate series are constructed. Section 3 establishes the theoretical basis of empirical methodology, and section 4 summarizes the empirical results. Section 5 presents the conclusions. Appendix contains the figures and tables used in this study.

The Data

The data set includes real rates for 19 emerging market countries. Table 4 shows the countries studied and respective sample periods covered.

The real interest rates are constructed by adding the country-specific sovereign spreads to the international risk-free real rate (see Fernandez-Villaverde et al., 2009 for a similar treatment).For country-specific sovereign spreads J.P. Morgan's Emerging Market Bonds Index (EMBI) Global Spread is used. EMBI Global is a weighted index which tracks the U.S. Dollar denominated emerging market debt, and currently covers 27 emerging countries. Using EMBI data is advantageous as it provides a common ground for calculating real interests, making comparisons between different countries easier.

According to the standard view in the literature, the U.S. rate is taken to be the international risk-free nominal interest rate. Subtracting the expected CPI-based inflation for the U.S. from the nominal rate leads to international risk free real rate. 10-year treasury constant maturity rate is used as U.S. nominal interest rate, because bonds with 10 year maturities are included in the calculation of EMBI Global index. 10 year ahead expected inflation is based on the Survey of Professional Forecasters conducted quarterly by the Federal Reserve Bank of Philadelphia, in which participants are asked for their expectations of the average CPI inflation rate over the next 10 years.

Datastream is the source for EMBI Global spread. Daily spread series are converted to monthly frequency via taking averages. Data on U.S. 10-year treasury constant maturity rate is obtained from St. Louis Fed's FRED database at monthly frequency. Expected inflation series are retrieved from Philadelphia Fed at quarterly frequency and interpolated to monthly frequency via cubic spline method.

Figure 2 plots the data series constructed. Table 5 and table 6 report the summary statistics for country spreads and real interest rates. Several features emerge: First of all, movements in the real interest rate series of the emerging market countries are driven mainly by their respective spreads. However, the 2002-2007 period is an exception for many countries when the series closely follow U.S. real interest rates. In this period, spreads are quite low indicating a positive mood in global scale rather than country-specific developments. However as the minimum statistic in table 5 shows, country spreads are never negative -even at their lowest levels, implying that real interest rates of emerging economies are always above the U.S. rate. This is not surprising since the majority of the emerging economies have undergone severe economic and political crises throughout the period studied. Latin American countries are victims of Tequila crisis, Asian countries suffered from Asian flu, Russia experienced Russian financial crisis, Pakistan went through political instability for prolonged periods of time, etc. In figure 2 we also see the effect of subprime mortgage crisis which is a global scaled crisis originated in the U.S. It reveals itself as the upward spikes in the real interest rate series of emerging economies and as the downward spike in the real interest rate series of U.S. between 2007-2010. Table 5 also reveals that high volatility translates into high means in the spreads. Argentina, Russia and Brazil are the top three countries with highest spread volatility, and they are also the top three countries paying highest spread on average. This observation demonstrates the connection between risk and spreads. Inspection of table 6, in comparison with table 2 supports the claim that movements of real interest rate series are mainly driven by the movements in spread series. According to table 6, countries with the highest volatility in their real interest rates are paying the highest interest in real terms on average. Argentina, Russia and Brazil are again the top three countries with highest real rate volatility and highest real rate mean.

Empirical Methodology

Lack of power of standard unit root tests in the presence of fractional alternatives is a widely known empirical finding. These tests may mistakenly claim a series to contain unit roots when it actually contains near unit roots, leading to erroneous theoretical or empirical results. Alternatively, null hypotheses of stationarity and unit roots can be rejected for the same series by different tests, leading to inconclusive results.

Fractional Dickey Fuller test and its variants offer a way out of these potential errors motivated by power and efficiency gains achieved in the case of fractional unit roots.

Fractional Dickey Fuller Test

Dolado *et al.*, (2002) have designed a simple Wald test (FDF test thereafter) which extends the widely applied Dickey Fuller test to fractional alternatives. Indeed, while the latter limits itself to the classical distinction between cases of $I(1)$ and $I(0)$, FDF test concerns itself with the more general distinction between $I(1)$ and $FI(d_1)$ processes with $d_1 \in \mathbb{R}$, and $0 \leq d_1 < 1$. Thus standard Dickey Fuller test can be restored setting d_1 equal to 0.

In the basic setup y_t is a fractionally integrated series whose true order of integration is d. Data generating process is assumed to be given by

$$\Delta^d y_t 1\{t > 0\} = \varepsilon_t \tag{7}$$

where $1\{\cdot\}$ denotes the indicator function, $\{\varepsilon_t\}_{t=1}^{T}$ is a sequence of zero mean, finite variance *iid* random variables, and T is the sample size.

In order to be able to test the null hypothesis $H_0 : d = 1$; against the alternative $H_1 : d =< 1$, FDF test makes use of the following regression model:

$$\Delta y_t = \phi_1 \Delta^{d_1} y_{t-1} + u_t \tag{8}$$

It is straightforward to show that the null and alternative hypotheses can be restated in terms of ϕ_1. Assuming that $u_t = \varepsilon_t$, $\phi_1 = 0$ corresponds to the case $\{y_t\}$ contains a unit root. On the other hand, $\phi_1 < 0$ implies that the y_t

series is fractionally integrated of order d_1. Consequently, it is suggested that the null hypothesis versus the alternative can be tested by checking for the statistical significance of parameter ϕ_1 via construction of a simple t statistic.

Efficient Fractional Dickey Fuller Test

Lobato and Velasco (2007) argue that the use of $\Delta^{d_1} y_{t-1}$ is not the optimal choice among the class of regressors, which calculate a t ratio whose asymptotic null distribution is standard normal. They argue that the model given in equation (2) is misspecified because under the alternative hypothesis, no pair (ϕ_1, d_1) can be found that will make the error series, u_t, serially uncorrelated and independent of the regressor, $\Delta^{d_1} y_{t-1}$. Consequently, the estimated OLS coefficient and the resulting t statistic are inefficient in the sense that FDF test does not maximize power. Lobato and Velasco (2007) propose an alternative Wald type test (EFDF test thereafter) which uses the regression model

$$\Delta y_t = \phi_2 z_{t-1}(d_2) + u_t \tag{9}$$

where $z_{t-1}(d_2)$ is defined as

$$z_{t-1}(d_2) = \frac{(\Delta^{d_2-1} - 1)}{1 - d_2} \Delta y_t$$

By testing for the significance of the coefficient ϕ_2 with $d_2 > 0.5$ via a left-sided t test , Lobato and Velasco (2007) propose to test for the null hypothesis where y_t is believed to be random walk against the alternative y_t is claimed to contain a fractional unit root, with differencing parameter $d_2 < 1$.

Notice that previous discussion is built on the assumption that data generating process is given by (1) where ε_t is iid. Lobato and Velasco (2007) construct a generalized version of the EFDF test, augmented EFDF (AEFDF) test, in the presence of autocorrelation among $\Delta^d y_t$, that is data generating process is given by

$$\alpha(L)\Delta^d y_t 1\{t > 0\} = \varepsilon_t \tag{10}$$

where $\alpha(L) = 1 - \alpha_1 L - ... - \alpha_p L^p$ is a polynomial in the lag operator with all roots outside the unit circle.

In this case they propose to use a two-step procedure that will result in an efficient test under the assumption that DGP is characterized by (4).[5] The first step is

$$\Delta^{\widehat{d_2}} y_t = \sum_{j=1}^{p} \alpha_j \Delta^{\widehat{d_2}} y_{t-j} + u_t \tag{11}$$

where an autoregression in the fractional difference of the series is estimated by OLS to obtain coefficients of $\alpha(L)$. The second step is

$$\Delta y_t = \phi_2[\widehat{\alpha}(L) z_{t-1}(\widehat{d_2})] + \sum_{j=1}^{p} \alpha_j \Delta y_{t-j} + \nu_t \tag{12}$$

where $\widehat{\alpha}(L)$ is the estimate coming from (5).

In the derivation of the AEFDF test, Lobato and Velasco (2007) didn't allow for deterministics in the data generating process. As a further generalization, Dolado *et al.*, (2006) investigate how the test may be implemented in the case of deterministics which may take the form of drifts, linear or quadratic trends etc. With a focus on the role of a linear trend (since

[5]Power of the test falls substantially (up to 50%) when the augmented version of the test is applied to series that don't contain serial correlation

many economic time series are characterized by this type of trend in levels), they show analysis proceeds in the same way as in AEFDF test once the series is demeaned (in the case of a linear trend) or detrended (in the case of a quadratic trend).

It should be noted that to make any of the tests we have discussed (FDF, EFDF or AEFDF) feasible, value of the fractional differencing parameter needs to be estimated. Lobato and Velasco (2007) demonstrate that both EFDF and AEFDF tests are efficient not only when parametric $T^{1/2}$ consistent estimators are used but also when the estimation is based on a semiparametric estimator with an appropriate choice of bandwidth parameter.[6]

Motivated by the power and efficiency gains achieved over its predecessor, AEFDF test (with deterministics) will be implemented in this study to conduct inference on the degree of integration of real interest rate series, r_t, of emerging market economies. To perform the test, fractional differencing parameter of the real interest rate series is estimated by means of the feasible exact local Whittle estimator (FELW thereafter) developed by Shimotsu and Phillips (2005). The reason why FELW is chosen over other alternative estimators, all of which ensure AEFDF test to be efficient, is because it is a good-general purpose estimator that preserves its desirable properties for a wide range of stationary and non-stationary values of the differencing parameter.

Empirical Results

In order to establish the long-run properties of real interest rate series, analysis is first carried out in traditional $I(0)/I(1)$ domain, which is the standard practice in the literature. Augmented Dickey Fuller (ADF) and Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) tests are utilized to that end. Results

[6]The test is inefficient but standard normal when an inconsistent estimator is employed.

are presented in table 7.

Close inspection of Table 7 reveals that ADF test fails to reject the $I(1)$ hypothesis for more than half of the countries studied. However this failure of ADF test doesn't necessarily imply that the real interest rate series are characterized by the presence of unit roots. It is a widely established empirical fact that conventional augmented DF test usually has low power against local alternatives: While testing for a highly persistent variable, it is very seldom that ADF rejects the unit root null in favor of the alternative. The basic intuition behind this power reduction is that null hypothesis will not be rejected as long as there is no strong evidence against it.

As a possible remedy KPSS test, which takes stationarity as the null hypothesis, is implemented. In the presence of near unit roots it is possible that KPSS test has better power properties than those of ADF since it will not reject the null hypothesis of an $I(0)$ process until it is convinced that the series at hand is non-stationary. According to Table 7, KPSS test rejects the $I(0)$ hypothesis for more than half of the countries.

The results of the ADF and KPSS tests together imply that 7 countries in the sample are characterized by unit roots. On the other hand, for 8 out of the 12 remaining countries ADF and KPSS tests produce contradicting results, signalling the possible presence of fractional unit roots.

However, as mentioned previously, generic unit root tests have little power to identify between unit roots and near unit roots. Together with the conflicting results produced, this deficiency makes implementation of a fractional unit root test, which explicitly take fractional alternatives into account, more than necessary.

Recall that to make the AEFDF test feasible, value of fractional differencing parameter needs to be estimated. Table 8 presents the estimates for this parameter, $\hat{d}$, calculated by a variety of estimators. These estimates will not only enable us to implement the AEFDF test, but also give us the first idea on the order of integration in Turkish real interest rate series.

Table 8 reveals that the estimated values of d are always above 0.5 for all countries, indicating that non-stationary dynamics are embedded in the data. Asymptotic 95% confidence intervals[7] are constructed to get the first formal inference on the order of integration of emerging real interest rate series. Since FELW estimates are what we will use while implementing the AEFDF test, we will focus on the intervals calculated with this estimator. For 12 of the 19 countries, confidence intervals include the value $d = 1$, suggesting that null hypothesis of a unit root won't be rejected. On the other hand, for Mexico, Morocco,Peru, Philippines, Poland and Thailand interval range is (0.5,1). This implies that the series have long memory and exhibit long-term persistence. It is probable that they are generated by non stationary but mean reverting processes, meaning that unlike the unit root case, the effects of a random shock will decrease gradually in time, but at a much more slower rate compared to the stationary case. The confidence interval of Pakistan covers values above 1 only indicating that the order of integration for real interest rate series of Pakistan is larger than 1.

Given the estimates provided by various estimators of the long memory parameter, the next step is to implement AEFDF test which takes estimated d values as an input. The test will tell whether $d = 1$ or $d < 1$, either supporting the unit root hypothesis or rejecting it in favor of a general fractional alternative. Table 9 contains the results of the AEFDF test calculated by choosing the optimal lag length on the basis of AIC. Cases of no trend and linear trend are considered.

AEFDF test rejects the null hypothesis of $I(1)$ against the alternative of $I(d)$ for 9 out of 19 emerging countries. Brazil, Indonesia, Malaysia, Peru,

[7] The formula is $\widehat{d} \pm 1.96 \times 1/\sqrt{4m}$ where $\widehat{d}$ is the estimated d value, $m = T^{0.65}$, and T is the sample size

and Thailand are inferred to be characterized by fractional orders of integration, no matter whether a trend is included or not. While Hungary and Pakistan are concluded to be fractionally integrated in the absence of a trend, inclusion of trend is needed in order to be able to reach the same conclusion for Philippines and South Africa. These results show that inference depends on the inclusion/exclusion of trends, justifying our choice of the version of the AEFDF test which explicitly take deterministics into account.

Returning back to Table 7 reminds that real rates of Brazil, Malaysia and South Africa are concluded to contain a unit root by both ADF and KPSS tests. According to AEFDF test however, these series contain unit roots but fractional ones, that is they are non-stationary as ADF and KPSS tests claim but they turn back to their means at a slower rate that cannot be captured by the standard tests. Although they have long memory like a unit root process, they revert back to their means unlike a unit root process.

Real interest rate series of Philippines and Thailand are also concluded to be fractionally integrated by the AEFDF test. For these series, ADF and KPSS tests favor the result of stationarity. On the other hand AEFDF test reject the null hypothesis of non-stationarity. Hence although it is true that the series show mean reversion as a stationary process, this happens at a slower rate than ADF and KPSS tests imply.

For Hungary, Indonesia, Pakistan and Peru null hypotheses of unit roots and stationarity are both rejected. It was conjectured before that this contradiction may be a signal of the presence of fractional unit roots. Our conjecture is verified by AEFDF test rejecting the null of a unit root over the fractional alternative for these countries. It is concluded the real interest series of Hungary, Indonesia, Pakistan and Peru are fractionally integrated.

In the end, it can be said that long memory properties of the 9 emerging countries mentioned above, which are found to be fractionally integrated, is distinct from both stationary and non-stationary processes. To be more specific, real interest series of these economies are characterized by two features: persistence and mean reversion. For instance when a shock is given to real rates of Brazil, its effect on the series will last for longer periods compared to real rates of Mexico, which is concluded to be stationary. This is persistence. However the effects of the shock will dissipate eventually, and the series will revert back to their means. This is mean reversion. Memory of the series is long but not infinite.

Finding of a fractional unit root for 9 out of 19 emerging economies is consistent with what is previously found for developed countries. Lai (1997), Pipatchaipoom et al. (2005), Neely and Rapach (2008) all report estimates of d for U.S. real rate series within the range 0.7-0.8. By focusing on real rates of 13 developed countries, Rapach and Wohar (2004) claim that a very high degree of persistence in international real interest rate series is a stylized fact. For a sample of 13 industrialized countries, Yoon (2008) argues the estimated value of d lies within the interval 0.8-1. Couchman *et al.,* (2006) examining a mix of 16 developed and developing countries conclude that for the majority of the countries fractional integration parameter lies between 0 and 1, implying mean reversion. Overall, fractional integration tests indicate that real interest series for many developed and emerging economies are mean reverting and quite persistent. This similarity in findings can imply two things for emerging markets: (i) Empirical evidence suggests that differences in economic policy lead different persistence properties in real rates. As we observe convergence in exchange rate regimes, in inflation stabilization programmes etc.; we may expect to observe degree of persistence to converge among emerging and industrialized countries. (ii) It is also empirically documented that differences in persistence properties are led by different shocks. If the notion of a single world market is valid, if developing economies are well integrated into the developed economies; then it will not be wrong to argue that real interest rate series of these countries are exposed to similar shocks.

Conclusion

This study contributes to the literature by reporting evidence on the fractional orders of integration in the real interest rate series for 19 emerging economies, through application of fractional integration analysis. Augmented efficient fractional Dickey Fuller (AEFDF) test is implemented to conduct inference on the order of integration. To allow for possible deterministics in the data, two cases- no trend and linear trend are considered. Following standard practice in the literature, two traditional unit root tests, ADF and KPSS tests are performed, preceding the implementation of the test. Results in the $I(0)/I(1)$ domain are reported. As a first step in detecting fractional unit roots, estimates of the fractional differencing parameter, d, by various estimators are reported. What merits attention is that for the majority of countries estimates are above 0.5, suggesting the presence of non-stationary but mean reverting dynamics in the data. Upon implementation of the AEFDF test, null hypothesis of a unit root is rejected in favor of a general fractional alternative for 9 emerging countries. It is also observed that test results are responsive to inclusion/exclusion of deterministics. For 4 out of the 9 fractionally integrated series, inference changes between cases of no trend and linear trend. In the light of the above mentioned, it can be said that half of the emerging market real interest rate series are characterized by fractional integration. However, many extensions exist which hold the possibility of altering what we find in this study. Incorporating deterministic and/or stochastic structural breaks which are evident characteristics of many real rate series of emerging markets may be one improvement (see Gil-Alana, 2004; Gil-Alana, 2008). Conducting the analysis in the panel data framework comes out as another extension. Exploiting common dynamics among countries may give way to different results. Finally, an analysis of fractional cointegration both in time series and panel data framework seems to be a promising pursuit (see Chen and Hurvich, 2003, Robinson and Hualde, 2003).

APPENDIX

Table 1. Results of the Standard Unit Root Tests

	$r^{rw,embi}$	$r^{pf,30}$	$r^{rw,30}$	$r^{pf,360}$	$r^{rw,360}$	$r^{sr,360}$
ADF	-0.53	-6.87***	-7.91***	-3.66**	-4.41***	-1.43
KPSS	0.25***	0.08	0.08	0.05	0.09	0.24***

Notes:

(i) In both tests, a constant and/or trend term is included when significant.

(ii) Optimal lag length in ADF test is chosen based on Schwarz information criterion.

(iii) (***) and (**) indicate statistical significance at 1% and 5% levels respectively.

Table 2. Estimates of the Degree of Fractional Integration

	$r^{rw,embi}$	$r^{pf,30}$	$r^{rw,30}$	$r^{pf,360}$	$r^{rw,360}$	$r^{sr,360}$
LW	0.914	0.551	0.585	0.816	0.827	0.739
ELW	1.029	0.573	0.619	0.861	0.880	0.853
FELW	0.929	0.654	0.651	0.649	0.645	0.840
FELWT	0.925	0.330	0.363	0.586	0.572	0.951

Notes:

(i) LW, ELW, FELW, FELWT stand for the local Whittle, exact local Whittle, feasible exact local Whittle, feasible exact local Whittle with detrending estimators of the fractional differencing parameter, respectively.

(ii) $T^{0.65}$ is the number of frequencies employed while evaluating the Whittle likelihood function.

(iii) MATLAB codes for the estimators are made available on http://www.econ.queensu.ca/faculty/shimotsu/ by Katsumi Shimotsu.

Table 3. AEFDF Test

Series	No trend AEFDF test	Lag	Linear trend AEFDF test	Lag
$r^{rw,embi}$	-1.14	2	-1.16	2
$r^{pf,30}$	-0.55	5	-0.89	2
$r^{rw,30}$	2.49	4	0.99	4
$r^{pf,360}$	-2.35**	1	-2.96***	1
$r^{rw,360}$	-2.63***	1	-3.42***	1
$r^{sr,360}$	0.55	3	-0.90	3

Notes:

(i) (***) and (**) denote statistical significance at 1% and 5% levels, respectively.

(ii) Maximum lag length is chosen to be 5 in the test.

(iii) MATLAB procedures to construct the test statistic and calculate finite sample critical values are borrowed from Peter Sephton on http://web.business.queensu.ca/faculty/PSephton/EFDF.

Table 4. List of Countries Studied

Country	Sample Period
Argentina	1994:01 2010:04
Brazil	1994:05 2010:04
Chile	1999:06 2010:04
China	1999:04 2010:04
Colombia	1997:03 2010:04
Egypt	2001:08 2010:04
Hungary	1999:02 2010:04
Indonesia	2004:06 2010:04
Malaysia	1996:11 2010:04
Mexico	1994:01 2010:04
Morocco	1998:01 2006:11
Pakistan	2001:07 2010:04
Peru	1997:04 2010:04
Philippines	1998:02 2010:04
Poland	1994:11 2010:04
Russia	1998:01 2010:04
South Africa	1995:01 2010:04
Thailand	1997:06 2006:03
Turkey	1996:07 2010:04

Table 5. Descriptive Statistics for Country Spreads

Country	Sample size	Mean	Std.Dev.	Minimum	Maximum
Argentina	196	17.40	19.73	2.03	68.56
Brazil	192	6.61	3.94	1.47	20.67
Chile	131	1.46	0.69	0.55	3.82
China	193	1.06	0.47	0.39	2.92
Colombia	158	4.16	2.06	1.08	9.81
Egypt	105	1.76	1.27	0.28	5.10
Hungary	135	1.11	1.09	0.13	5.81
Indonesia	71	3.19	1.73	1.44	8.86
Malaysia	162	1.89	1.45	0.53	10.32
Mexico	196	3.91	2.76	0.97	18.67
Morocco	107	3.77	2.25	0.54	11.48
Pakistan	106	5.78	5.11	1.37	21.35
Peru	157	4.04	1.97	1.04	9.36
Philippines	148	3.99	1.43	1.38	9.40
Poland	186	1.99	1.52	0.41	9.22
Russia	148	8.83	12.76	0.95	59.61
South Africa	184	2.31	1.33	0.58	6.65
Thailand	106	1.57	1.20	0.41	7.27
Turkey	166	4.46	2.34	1.58	10.47

Table 6. Descriptive Statistics for Real Interest Rates

Country	Sample size	Mean	Std.Dev.	Minimum	Maximum
Argentina	196	19.85	19.38	4.34	70.70
Brazil	192	9.05	4.30	3.21	22.12
Chile	131	3.50	1.02	2.12	5.80
China	193	3.50	1.09	1.34	5.69
Colombia	158	6.37	2.34	3.15	11.89
Egypt	105	3.49	1.35	1.45	7.15
Hungary	135	3.18	1.07	1.05	6.33
Indonesia	71	4.86	1.33	3.37	9.87
Malaysia	162	4.13	1.70	2.24	12.63
Mexico	196	6.36	3.31	2.87	22.57
Morocco	107	6.15	2.65	2.68	13.79
Pakistan	106	7.52	4.88	3.40	21.60
Peru	157	6.23	2.39	2.87	11.67
Philippines	148	6.11	1.73	3.33	11.71
Poland	186	4.38	2.11	1.60	13.12
Russia	148	10.95	13.14	2.78	61.62
South Africa	184	4.68	1.60	2.45	8.65
Thailand	106	4.01	1.65	1.66	9.58
Turkey	166	6.73	2.48	3.49	13.21

Table 7. Results of the Standard Unit Root Tests

Country	ADF test	KPSS test
Argentina	-1.09	0.26
Brazil	-3.31*	0.18**
Chile	-1.66*	0.29***
China	-2.41	0.14*
Colombia	-2.96	0.21**
Egypt	-1.78*	0.21**
Hungary	-0.81	0.31
Indonesia	-0.80	0.16
Malaysia	-2.99	0.15**
Mexico	-3.75**	0.14*
Morocco	-3.67**	0.12*
Pakistan	-1.61	0.23
Peru	-3.74**	0.17**
Philippines	-3.53**	0.10
Poland	-2.61***	0.20**
Russia	-6.09***	0.20**
South Africa	-1.35	0.16**
Thailand	-4.98***	0.14*
Turkey	-0.77	0.23***

Table 8. Estimates of the Degree of Fractional Integration

Country	LW	ELW	FELW	FELWT
Argentina	0.974	1.006	1.000	1.001
Brazil	0.853	0.909	0.855	0.859
Chile	0.988	1.037	0.996	1.002
China	0.916	1.040	0.824	0.811
Colombia	0.837	0.932	0.856	0.819
Egypt	0.975	1.009	0.981	0.986
Hungary	0.989	1.136	1.009	1.009
Indonesia	0.744	0.803	0.960	0.957
Malaysia	0.857	0.964	0.862	0.837
Mexico	0.809	0.903	0.797	0.631
Morocco	0.826	0.928	0.750	0.653
Pakistan	1.253	1.274	1.290	1.291
Peru	0.836	0.935	0.771	0.741
Philippines	0.811	0.948	0.766	0.705
Poland	0.920	0.997	0.811	0.867
Russia	0.991	1.029	1.025	1.015
South Africa	0.779	0.867	0.866	0.873
Thailand	0.903	1.041	0.776	0.675
Turkey	0.863	0.917	0.913	0.901

Table 9. Results of the AEFDF Test

Country	No trend AEFDF test	Lag	Linear trend AEFDF test	Lag
Argentina	-1.17	2	-1.15	2
Brazil	-2.23**	1	-2.60**	1
Chile	-0.80	3	-1.31	3
China	-0.03	3	-1.71	2
Colombia	-0.98	2	-1.10	2
Egypt	-1.12	1	-1.71	1
Hungary	-1.88**	2	-1.65	2
Indonesia	-2.10**	2	-2.02*	2
Malaysia	-2.04**	2	-2.14*	2
Mexico	0.44	5	0.26	5
Morocco	0.18	2	-1.16	2
Pakistan	-2.10**	1	-1.26	1
Peru	-2.82***	1	-3.76***	1
Philippines	-1.22	2	-2.45**	2
Poland	-0.10	1	-1.00	1
Russia	-0.08	5	-0.25	5
South Africa	-0.99	2	-3.13***	5
Thailand	-3.20***	1	-3.75-***	1
Turkey	-1.16	2	-1.29	2

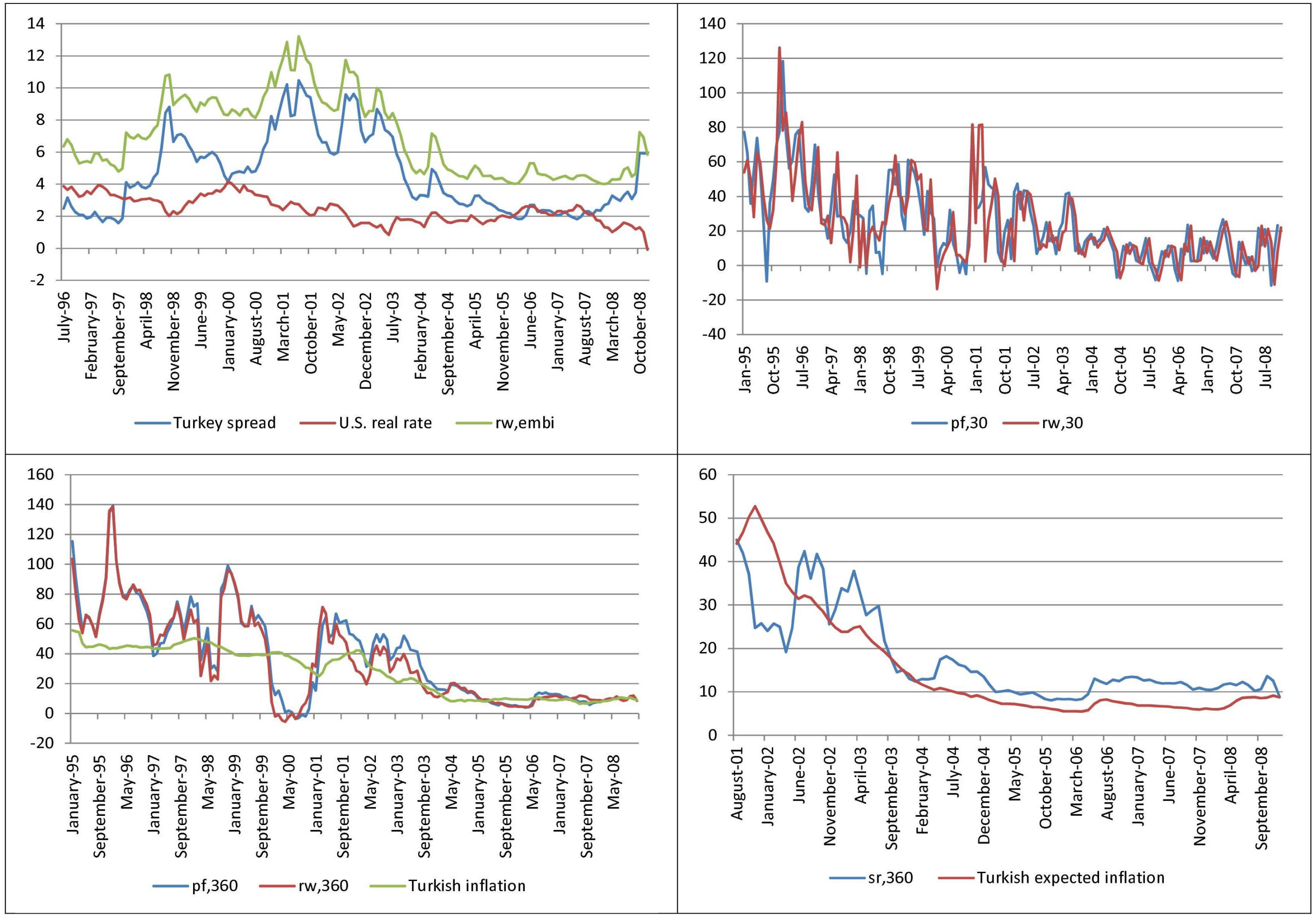


Figure 1: Evolution of Real Interest Rates in Turkey

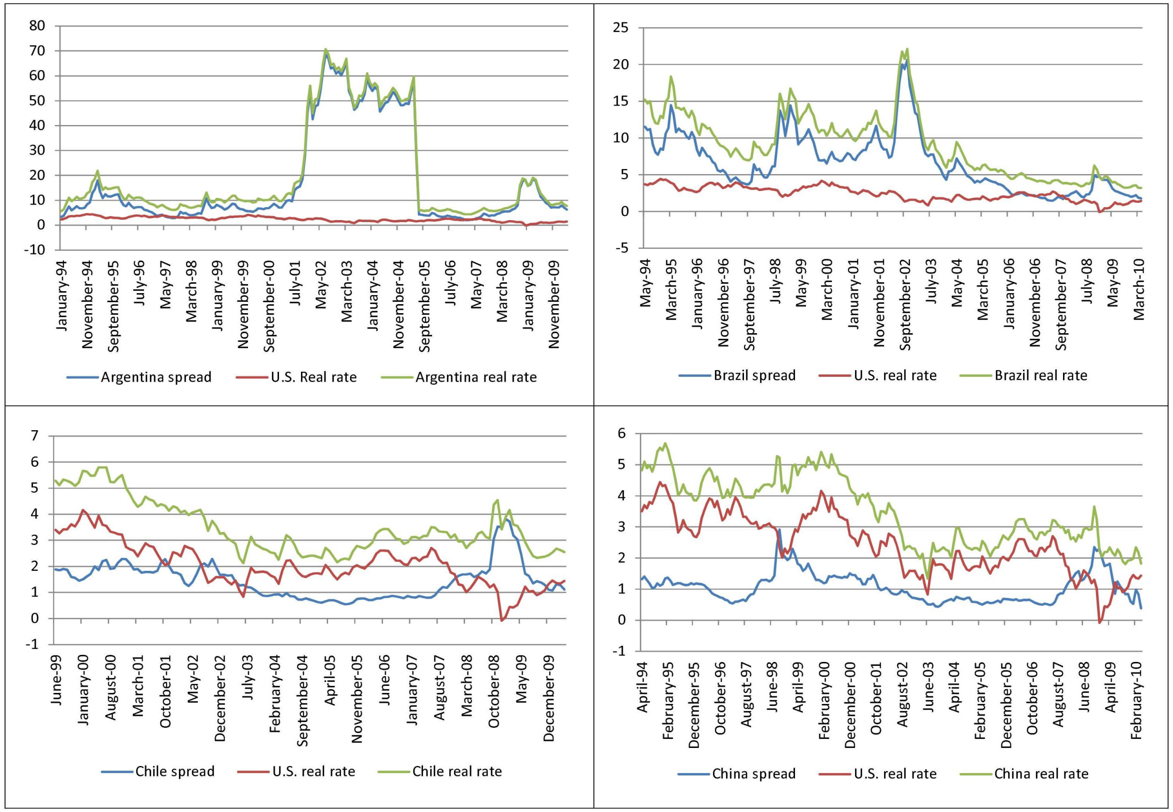
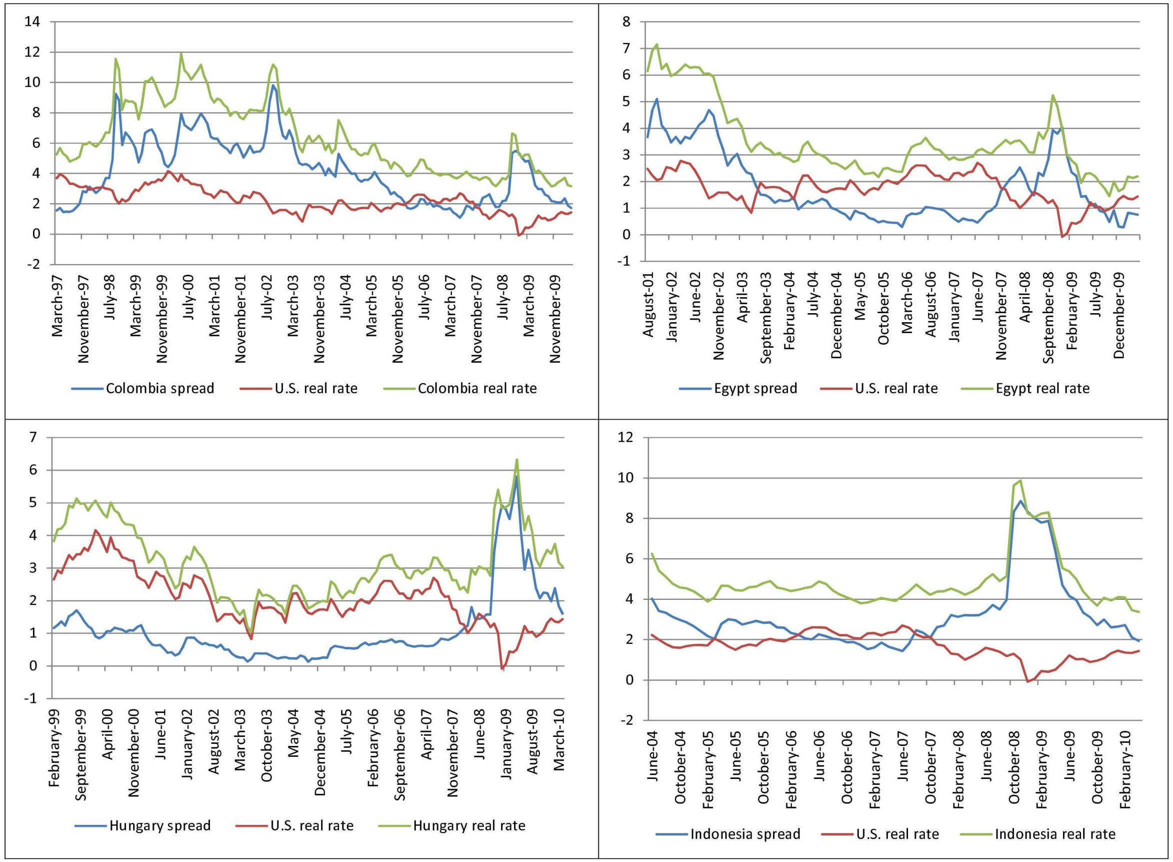


Figure 2: Country Spreads, U.S. Real Interest Rate and Country Real Interest Rates



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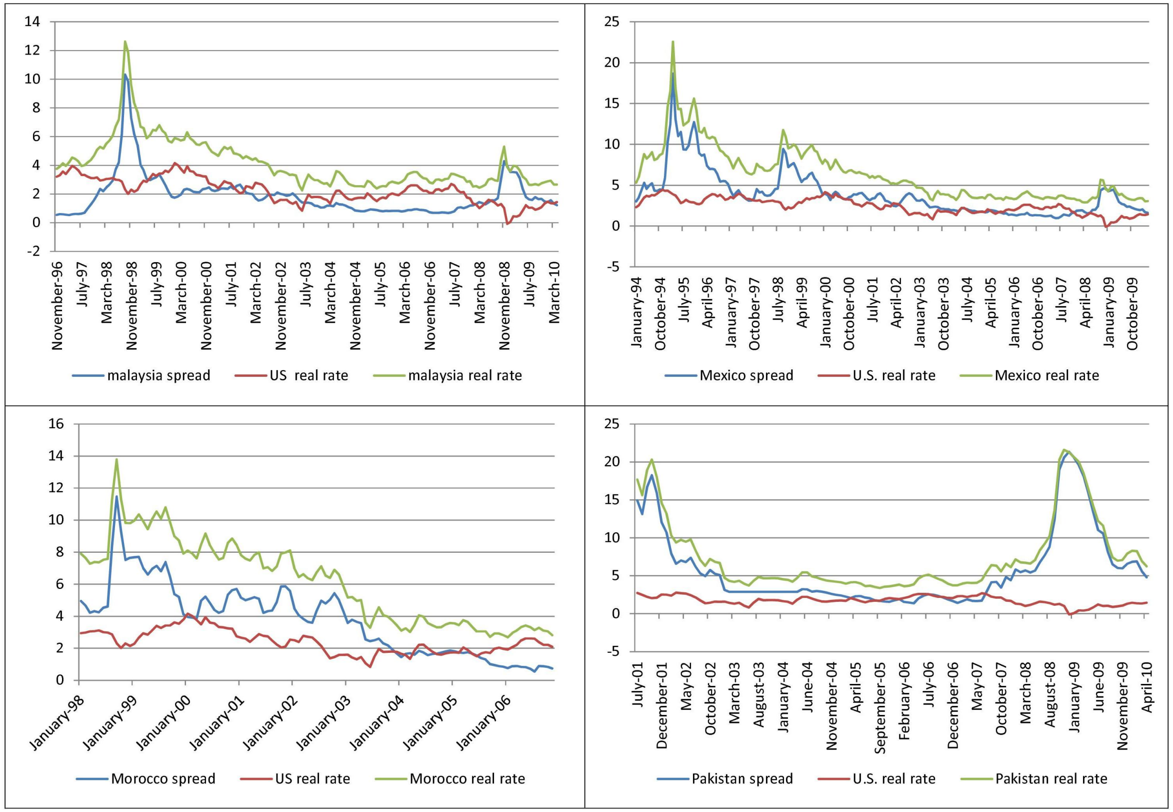

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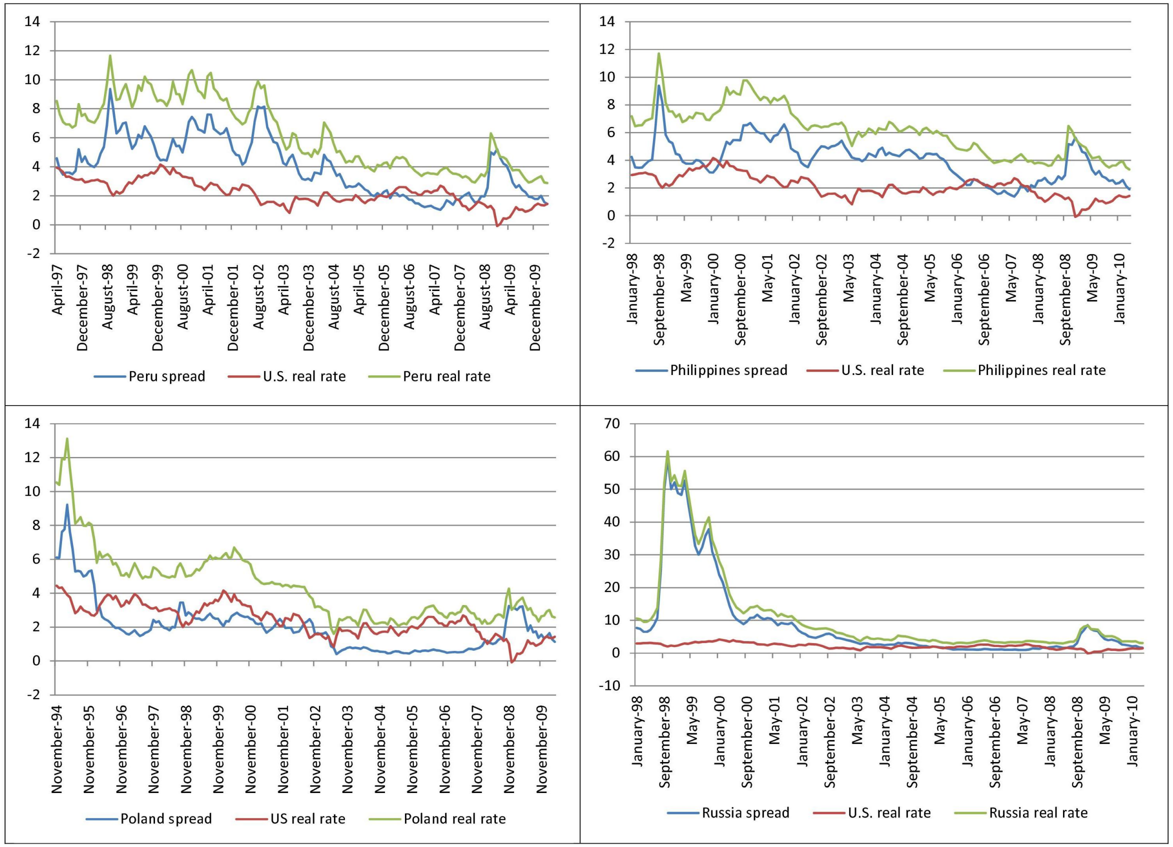

Figure 2: Country Spreads, U.S. Real Interest Rate and Country Real Interest Rates

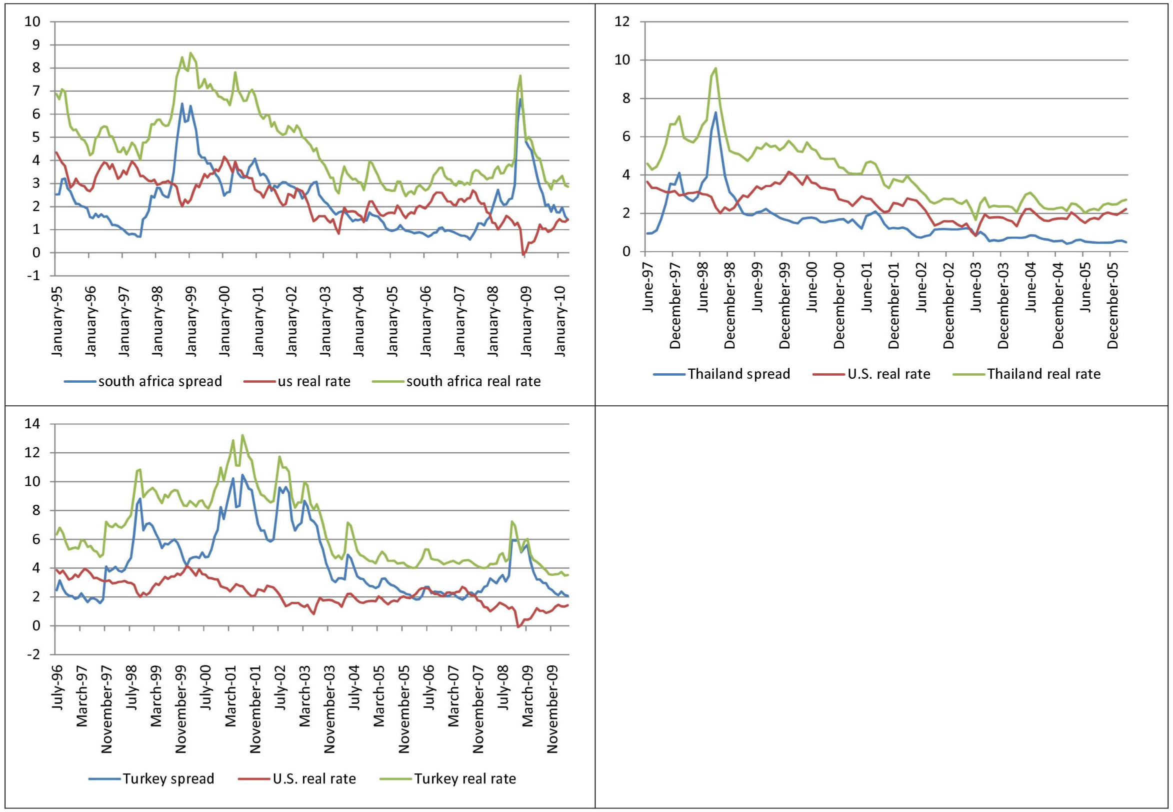

Figure 2: Country Spreads, U.S. Real Interest Rate and Country Real Interest Rates

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