A CRITICAL EXAMINATION OF THE PHILOSOPHICAL DEFENSE OF FREE LOGICS

EMRE ALTAN

BOĞAZİÇİ UNIVERSITY

2014

A CRITICAL EXAMINATION OF THE PHILOSOPHICAL DEFENSE OF

FREE LOGICS

Thesis submitted to the

Institute for Graduate Studies in the Social Sciences

in partial fulfillment of the requirements for the degree of

Master of Arts

in

Philosophy

by

Emre Altan

Bogaziçi University

2014

Thesis Abstract

Emre Altan, “A Critical Examination of the Philosophical Defense of Free Logics”

In classical quantification theory, each *singular term* available in the formal language must denote some member of the quantificational domain. In this sense, standard systems of predicate logic do not allow for non-denoting singular terms of our ordinary discourse. In a non-standard family of logics called *free logics*, this classical requirement is dispensed with. This study is about these non-standard logics. It has a two-fold aim. Firstly, to provide a survey of free logics. To this end, I introduce definitions and characteristics of these systems, present axiomatic formulations of certain types of free logics, give a summary of different semantic approaches in free logics, and finally provide a brief historical information about their origins. Secondly, I discuss whether the adoption of free logics instead of classical logic is justified or not. To this end, I present and discuss six different kind of motivations behind the adoption of free systems. I ultimately conclude that none of the motivations provides us with enough reason to replace classical quantification theory with free logics. The study ends with a suggestion of a more successful argument in favor of free systems.

Tez Özeti

Emre Altan, “Serbest Mantığın Felsefi Savunusunun

Eleştirel bir İncelemesi”

Klasik niceleme mantığında, formel dildeki her *tekil terimin* evrenin bir değerine gönderge yapması gerekir. Bu anlamda, standart yüklemler mantığı günlük dildeki göndergesiz tekil terimlere izin vermez. *Serbest mantık* adı verilen standart olmayan bir mantık ailesinde, söz konusu klasik gereklilik geçersiz kılınır. Bu çalışma söz konusu standart olmayan mantık sistemleri hakkında. Çalışmanın ikili bir amacı var. Birincisi, serbest mantık hakkında genel bir bakış sunmak. Bu amaçla; serbest mantık tanımlanıyor ve karakteristikleri veriliyor, belirli serbest mantık türlerinin aksiyomatik formülasyonları sunuluyor, serbest mantıktaki farklı semantik yaklaşımlar özetleniyor ve son olarak bu sistemlerin kökenlerine dair tarihsel bilgi veriliyor. Çalışmanın ikinci amacı ise, klasik mantık yerine serbest mantığı geçirmenin haklı olup olmadığını tartışmak. Bu amaçla, serbest sistemleri benimsemenin altında yatan altı farklı türden motivasyon inceleniyor. Nihai olarak, söz konusu motivasyonların hiçbirinin klasik niceleme mantığının yerini serbest mantığın alması için yeterli neden sunmadığı sonucuna varılıyor. Çalışma, serbest mantık lehine verilebilecek daha başarılı bir argüman önerisiyle bitiyor.

ACKNOWLEDGEMENTS

I would like to thank all the jury members, Prof. Dr. Berna Kılınç, Assist. Prof. Ali Karatay and Assist. Prof. T. Necati Ilgıcıoğlu, for all their guidance and assistance. They provided everything I needed for this thesis.

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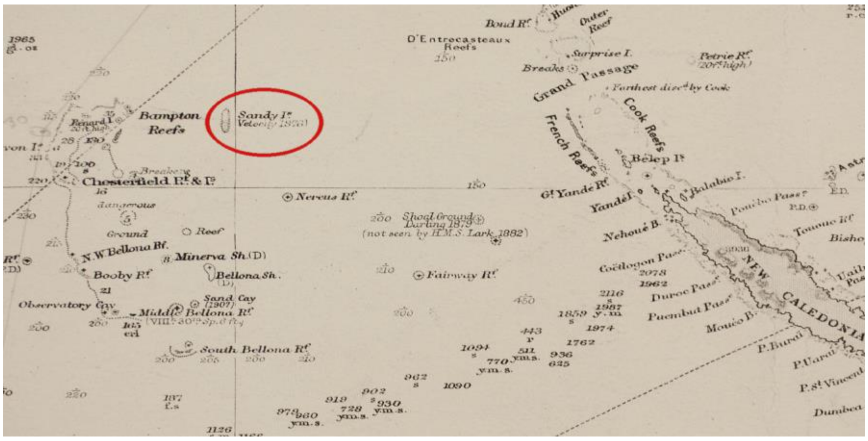
PREFACE

Sandy Island: The Island That Never Was



In the year of 1876, the British vessel *Velocity* reported a Sandy Island between Australia and New Caledonia in the Coral Sea at approximately 19°S and 160°E. Starting from the late nineteenth century, the island thus started to appear on charts.[1] It was on world maps ever since. It even appeared on the satellite views that were provided by Google Earth.

As far as is known, there are two earlier references to Sandy Island. According to one source, Sandy Island was first noted and charted by the renowned captain James Cook in 1774. Although the coordinates Cook had provided differed a few degrees[2] with those *Velocity* reported after more than a century later, this may be the earliest appearance of Sandy Island on a chart. Another source dates back the

[1] The above map is a 1908 edition of a chart created by Hydrographic Office of the British Admiralty in 1876. This seems to be the first official appearance of Sandy Island on a map.

[2] About 420 km, to be more precise.

discovery of Sandy Island to 1792 and reports the discoverer as the French navigator Joseph Bruny d'Entrecasteaux.

According to *Wikipedia*, Sandy Island is about 24 km long and its width is 5 km. Its precise coordinates are 19.22°S and 159.93°E.[3] The island is unpopulated, and it belongs to French governed New Caledonia. Last, but certainly not the least, it has been recently discovered that Sandy Island does not exist and have never existed. In other words, it turned out to be a *phantom island* as they are usually called. The related *Wikipedia* entry describes Sandy Island as follows:

> Sandy Island (sometimes also called in French: Île de Sable) is a non-existent island that was charted for over a century as being located near the French territory of New Caledonia between the Chesterfield Islands and Nereus Reef in the eastern Coral Sea.[4]

How a non-existent island managed to appear on world maps for more than a century and even appeared on Google Earth remains unclear.

Sandy Island is one of the newest members of the set of phantom (or non-existent) islands. It is interesting to note that the number of the individuals in this set is not as low as one might expect. The related *Wikipedia* entry lists 46 phantom islands and these are only those that have their own entry in this website. Although probably only one of them managed to appear on Google Earth, all of the phantom islands are assumed to be existent for some period, and most of them found their ways into various sorts of maps.

Now that we know that Sandy Island does not exist, we can safely say the expression 'Sandy Island' does not refer to an existing thing; in other words, it is an *empty singular term* as we will usually call such expressions in this study. Classical

[3] "Sandy Island, New Caledonia" *Wikipedia,* last modified May 19, 2014, http://en.wikipedia.org/wiki/Sandy_Island,_New_Caledonia.

[4] Ibid.

quantification theory, as we shall see in the next chapter, is subject to a number of difficulties and problems in its treatment of these sorts of expressions. As we shall see as well, there are classical and quasi-classical solutions to these problems, but none of them seem to be satisfactory.

It seems weird enough to me that a well-established system such as classical quantification theory does not provide a straightforward way to deal with expressions such as ‘Sandy Island’, which at least for a while stood for something that appeared on maps and even on Google Earth. Needless to say, the expression itself still plays an important role in a wide and varied range of discourses.

For worse, phantom islands are not the only things that have been mistakenly assumed to be existent for some period. To begin with, just like phantom islands, there are also *phantom settlement*s as well. To name an example, *Argleton* is one of the well-known non-existent settlements, which by the way also found its way to Google Maps and Google Earth. Besides phantom islands and phantom settlements, there are many other things that are mistakenly believed for some time to be existent. Let us call them error objects. *Phlogiston*, the hypothetical chemical contained in combustible bodies, and *Vulcan*, the planet that never was, are some of the well-known examples of error objects from the history of science. Having said this, error objects certainly do not exhaust the things that are non-existent. Fictional (or mythological) creatures, characters and places, past and future objects, possible objects or even impossible objects can be named among the kind of things that are non-existent.

It is not that seldom that we talk about things of the kind I listed or exampled above. Empty singular terms have a significant place in discourse. Even so, classical quantification theory does not provide a straightforward way to deal with such expressions. This was in the first place what has driven me to this topic.

Needless to say, this study is neither about Sandy Island nor any other phantom island. Rather it is a study about the logic of expressions involving terms such as ‘Sandy Island’. Several logical systems, called *free logics*, have been proposed, which in a comparison to standard systems of logic, provide a more straightforward treatment of empty singular terms.

This study aims to provide answers to two simple questions about free logics. The first of them is what free logics are, and the second is why free logics should be preferred over classical quantification theory. The general outline of the thesis will be as follows.

The first chapter is intended to serve as an introduction to free logics. In this chapter, I present the notion of an empty singular term and some of the problems that arise in connection with such expressions in classical quantification theory. Afterwards, I discuss whether or not these problems can be solved in classical logic. The purpose of this chapter is to introduce a framework that will help to understand the background behind free logics.

In the second chapter, I address the first question I mentioned above. In this sense, the chapter constitutes the presentation part of this study. It is primarily comprised of four subsections. In the first section, I present some of the definitions proposed for free logics and how these systems can be characterized. Next, I introduce some of the formal systems of free logics. The section afterwards is devoted to the presentation of various kinds of free semantics. Finally, in the fourth section I give a brief summary about the origins of free logics.

The third chapter is the argumentative part of this study and constitutes the main body of the work. It seeks an answer to the second question I presented above, i.e., the question why one should replace classical quantification theory with free logics. To this end, it surveys the answers that have already been given (or can be given) to this question and discusses whether they are successful or not in justifying free logics.

The study ends with the fourth chapter, which is the conclusion part where I simply provide a summary of the entire work.

CHAPTER I

CLASSICAL LOGIC AND EMPTY SINGULAR TERMS

The aim of this chapter is to introduce the general framework behind free logics. In order to do this, I first introduce the notion of an empty singular term. Second, I discuss some of the problems that arise in connection with empty singular terms in classical logic. Finally, I present and discuss some of the classical (or quasi-classical) solutions to these problems.

Empty Singular Terms

In classical quantification theory, hereafter CQT, there is not an easy and smooth way to analyze sentences of the form 'there is no such thing as *t*' or '*s* does not exist' where *t* and *s* stand for some proper name or definite description. These kinds of statements, usually known as singular negative existential statements, are typically used to deny the existence of something, and hence, contain non-referring expressions, or expressions referring to non-existent objects.[5] To name a few,

[5] This may be a good place to underline a few terminological matters. To begin with, unless there is a risk of ambiguity, the difference between a non-referring expression and an expression referring to a non-existent is ignored. The difference can be best understood with an example. Take the term 'Pegasus', for instance. For some people, it is simply a non-referring expression because it denotes no entity. For others, however, it is an expression referring to a non-existent thing because even tough Pegasus does not exist as a real thing, the term itself still refers to a mythological creature. Hence, for any position that maintains that there are non-existent things, the difference is important. However, for those who reject the idea of non-existent things, the distinction loses its importance. Regardless, I mainly use 'empty singular term' throughout this study and it can be understood in a way to stand for both. In other words, an empty singular term may be a non-referring expression as well as an expression referring to a non-existent. However, also note that I occasionally use 'non-denoting expression' (or 'non-referring expression') and most of the time it simply means the same as 'empty singular term'. Second, the expressions 'denote' and 'refer to' are used interchangeably. Similarly, no distinction is made between 'object' and 'thing'. Finally, expressions such as 'existing things' and 'non-existing thing' or non-existents are used occasionally. Note that this is not because I believe

‘Pegasus’, ‘Zeus’, ‘Sherlock Holmes’, ‘Vulcan’, ‘the present king of France’, ‘the golden mountain’, ‘the round square’ are among some of the well known examples of such expressions from the literature of philosophy of language.

Negative existential statements are not the only kind of sentences where a non-referring expression finds a use in ordinary discourse. Natural languages allow for such expressions, and they are often adopted in different kinds of statements, especially in fictional, modal and scientific discourses. ‘Pegasus is white’, ‘Sherlock Holmes might have existed’, ‘Vulcan is the planet causing perturbations in the orbit of Mercury’ are examples for such statements.

In what follows, any such expression will be called as an empty singular term and shortened as EST. Note that this choice of words already embodies three different aspects of the kind of expressions we are considering; namely, they are (I) terms, (ii) empty, and (iii) singular. Let us first make clear what each of these notions means. For providing answers for (i) what a term is, (ii) when a term is empty, and (iii) which terms are singular will give us a definition of what an EST is.[6]

To begin with, a *term* of natural language is a possibly referring expression. In other words, a linguistic expression that has the function of referring to an individual or to several things is a term of natural language. Accordingly, proper names such as ‘Sherlock Holmes’ and ‘Arthur Canon Doyle’ as well as ‘the unicorn’ and ‘the horse’, definite descriptions such as ‘the present king of France’ and ‘the present president of France’, demonstratives such as ‘this table’ and ‘that woman,’ and

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there are objects that are non-existent, but because I want to adopt a language that is as much neutral as possible to different ontological views.

[6] Note that the suggested definitions are not meant to be admissible by everyone. There may be different views on how to define each of these notions. Our purpose here is only to clarify what we understand by the expression ‘empty singular term’.

pronouns such as ‘he’ and ‘it’ are all terms of natural language since they all can be used to refer to something.

Second, a term is *empty* just in case it fails to refer. A simple example for this would be ‘Pegasus’. It is usually considered as an empty term because even though it purports to refer to an individual, there is no existing thing such that this term denotes. To put it simply, ‘Pegasus’ is an empty term because there is no such thing as Pegasus.[7]

Third, a term of natural language can be either *general* or *singular*. If a term has the function of referring to an individual, then it is a singular term. A general term, on the other hand, “is one true of each, severally, of any number of objects.”[8] Hence, the extension of a general term is a (possibly empty) set of things to which the general term applies. According to this criterion, ‘Sherlock Holmes’ and ‘Arthur Canon Doyle’ are both singular terms while ‘unicorn’ and ‘horse’ are general.[9]

Based on the definitions above, an EST of natural language is a linguistic expression that purports to refer to an individual yet fails to achieve this. Such expressions usually appear in natural languages in two different ways. First, as proper names that have no (real) denotations; and second, as *improper* (or *incomplete*) definite descriptions. While ‘Pegasus’, ‘Sherlock Holmes’ and ‘Vulcan’

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[7] As noted earlier, there are views according to which ‘Pegasus’ is *not* non-denoting but it refers to a mythological creature. Based on this account, the view that ‘Pegasus’ is an empty term/name may be challenged. Nevertheless, we will ignore any such position for now and simply consider all such expressions as ESTs.

[8] Willard Van Orman Quine, *Word and Object* (Cambridge, MA.: MIT Press, 1960), 90--91.

[9] Thus, an empty term of natural language can be either singular or general. In order to illustrate the difference between empty singular terms and empty general terms, let us contrast ‘Pegasus’ with ‘unicorn’. Given that there are no unicorns in our world, the general term ‘unicorn’ is true of no thing/entity, therefore it is an empty term just like ‘Pegasus’ is. That being said, there is an important difference between them. While both terms fail to refer, the former purports to refer to an individual whereas the latter purports to refer to a kind. Note that only empty terms of the singular kind will be of our concern. The reason for this is that the treatment of empty general terms in classical logic is not as problematic as the treatment of ESTs. We will see this later.

are examples for the former kind, ‘the present king of France’, ‘the golden mountain’ and ‘the round square’ are examples for the latter.[10]

The Problems with Empty Singular Terms

As we mentioned at the beginning of the last section, ESTs are an important part of our ordinary discourse. Even so, philosophical problems that can be associated with them are wide-ranging.

One of the oldest and perhaps most acknowledged problem is known as the problem of empty names. Proper names that have no referents give rise to serious difficulties especially for *direct reference* theories according to which the only semantic value of a proper name is the individual it denotes. The problem in its simplest can be put as follows: If direct reference theories are true, empty names should have no meaning; there are, however, many names that do not refer, yet are meaningful. As should be clear already, it is essentially a semantic problem.

The problem of empty names is not the only philosophical problem that arises in connection with the presence of ESTs in natural languages. What is known as the problem of negative existentials is another important problem that has attracted interest of many philosophers. This problem can be simply put as follows: In order to deny the existence of something, one must assume the existence of that very thing; for to be able to make a true claim about something, it has to exist in some sense. In this sense, negative existential statements pose an ontological problem.

[10] We could also add some demonstratives to this list. For instance, a complex demonstrative such as ‘this table’ while there is no table or ‘this unicorn’ may be examples for empty demonstratives. For the sake of simplicity, however, we will usually ignore them.

A wide array of solutions to these problems has been suggested by different philosophers and significant philosophical theories emerged from these attempts, yet the problems themselves remained under philosophical dispute. There are other problems that can be associated with ESTs, perhaps too many to be mentioned here, yet none of these will be of our concern directly. Our focus will be instead on logic, and particularly, on the problems that arise in connection with ESTs in CQT.

Before introducing these problems, however, let us first settle the question of what the formal counterparts of ESTs are. To this end, let us first try to answer this question: How are singular terms represented in CQT?[11] In order to answer it, we will need a formal language with some semantics.

Let *L* be a classical first-order language such that its *vocabulary* consists of *variables*, *connectives*, *parentheses* and some *individual constants* and *n-place predicates*. Individual constants together with variables constitute the *terms* of *L*. A *model* (or an *interpretation*) for *L* is a pair $<D, I>$ such that *D* is a non-empty set, and *I* is a function that assigns to each individual constant some member of *D* and to each *n*-place predicate a set of *n*-tuples of members of *D*. A *variable assignment* α is a function that assigns to variables individuals in *D*.

According to the usual interpretation of classical semantics, individual constants of *L* are thought to be naming individuals in *D*, and hence, correspond to what we call proper names of natural languages. In classical semantics, *free variables* are also treated like individual constants in that they are assigned members of *D* by the variable assignment α.[12] Intuitively, they stand for pronouns of natural

[11] Note that there are different answers to this question as well, as was the case with the definition of ESTs.

[12] Consider, the schema $\exists xA$, for instance. We hold any instance of it true if *A* is true for some assignment of a value $\alpha(x)$ in *D* to the variable *x*.

languages. Thus, for a given language *L*, its individual constants together with its variables (i.e., terms of *L*) can be thought of as representing singular terms of natural languages. If the *description operator* were added to the vocabulary of *L*, then definite descriptions formed by the descriptor operator (e.g., ιxA where *A* is a formula and *x* is a variable of *L*) would be among the terms of this augmented language, say L^{dd}. In this case, definite descriptions are also assigned members of *D* by the *interpretation function I*, and for any view that does not subscribe to Russell's theory of definite definitions,[13] they should be regarded among the formal expressions that correspond to singular terms of natural languages, as well.[14]

To sum up, for a given language L^{dd}, we hold that its individual constants, variables and definite descriptions (i.e., terms of L^{dd}) correspond to what we call singular terms of natural languages. That being said, in what follows, we will usually ignore variables as singular terms.[15] Definite descriptions will also be ignored for the most part.[16] Note that these simplifications mean in effect that singular terms of natural languages are being restricted to proper names.

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[13] This theory entails that definite descriptions are not singular terms.

[14] Note that if the vocabulary also included *function symbols*, then *n*-place function symbols followed by *n* terms (e.g., $ft_1...t_n$) would be terms as well. For the sake of simplicity, however, we will exclude function symbols from formal language.

[15] The reason for this choice is that our real concern lies with ESTs. Since variables are assigned arbitrary members of *D* and there is always something in *D* to be assigned to a free variable when *D* is non-empty, the only way for these terms to be non-denoting is when the domain *D* is empty as in *inclusive logics*. However, not all systems of free logics are inclusive.

[16] There are two different reasons for ignoring definitive descriptions. For one thing, not everyone regards definite descriptions as genuine singular terms. Russell's theory of definite descriptions is the obvious example for such a position. Moreover, in classical semantics definite descriptions are usually treated just like individual constants. Therefore, ignoring them will simplify our discussion without any significant loss. That being said, if one regards definite descriptions as genuine singular terms, almost everything we will say about individual constants of *L* can be easily generalized in a way that would also apply to definite descriptions of L^{dd}.

Now that we have settled how singular terms are represented in formal languages, let us see what the formal counterparts of ESTs are. To this end, let us take a look at how singular terms are interpreted in CQT.

As we have already seen, CQT requires that, in each model, the interpretation function assign to each individual constant some member of the domain over which the *bound variables* range. Formally put, for every model $M = \langle D, I \rangle$ and for each individual constant a, there is a member d in D such that $I(a) = d$. That means that, in CQT, ESTs do not have direct formal counterparts. To put it differently, CQT does not allow for ESTs.[17] Now let us look at some of the results of this.

If we generalize the above requirement so that it would apply not just to constants but to all terms, then for a given classical first-order language L with *identity*, say $L^{=}$, every instance of

(1) $\exists x(x = t)$

where t can be substituted by any term of $L^{=}$, is a *theorem* and a *valid sentence*. In classical first-order logic without identity, on the other hand, for every formula A and for every term t

(2) $A(t/x) \supset \exists xA$

and

(3) $\forall xA \supset A(t/x)$

are likewise theorems and valid sentences.

[17] Consider, 'Sandy Island', for instance. It surely is an empty name; that is, it denotes no existing thing. As noted earlier, the formal counterparts of names are individual constants. However, no individual constant can stand for 'Sandy Island' in CQT because the interpretation function assigns to each constant some member of the quantificational domain.

Let us make an important remark before we proceed. Although the schemas given above may seem to be problematic at first glance, they are, in fact, not more than just strings of symbols in these forms. Consider (1) for instance. It simply follows from the requirement that each term of $L^{=}$ must receive an interpretation in D. All that can be said based on it is that CQT does not allow for terms that do not have an interpretation in D. However, in addition to this requirement, if we also adopt the usual understanding of the relation between terms and members of D as terms denoting their interpretations and following Quine interpret the *quantifiers* as having *existential import*[18], it follows that each term of $L^{=}$ must denote some existent object. It is only now that we have a conclusion that may be problematic. Note that, for this conclusion to follow, three different assumptions are made. Let us take a more detailed look at each of them.[19]

The first premise is the classical requirement that the interpretation function assigns to each individual constant of $L^{=}$ some member of the domain D. Let us put this premise into the following form:

(4) Each individual constant of $L^{=}$ receives an interpretation in D.

Clearly, this condition does not imply anything about denotation or existence. All (4) says is that to every constant of $L^{=}$, an interpretation in D is assigned. It does not imply anything either about the nature of the relation between individual constants and their interpretations in D or anything regarding the nature of the members of D.

As the second premise, we have a particular understanding of classical semantics. According to it, the interpretation function for constants is understood as

[18] A term *t* has existential import just in case *t* exist.

[19] For the sake of simplicity, in the following discussion, terms of the formal language will be restricted to individual constants.

the denotation relation between the constants of L and the members of D. Thus, $I(t_0) = d_{0,}$ for instance, is understood as the constant t_0 denoting the individual d_0. This premise can be put into the following form:

(5) Individual constants of $L^=$ denote their interpretations in D.

Combined with the previous premise, it follows that in classical first-order logic,

(6) each individual constant of $L^=$ denotes some member of the domain D,

or simply, that each constant denotes. That being said, since it is not clear yet what the members of D represent, we still do not have an implication regarding existence.

Finally, as the third premise, a particular reading of quantifiers is adopted which is most fully expressed by Quine. According to this reading of quantifiers, they are interpreted as having existential import. Consider Quine's famous dictum that "to be is to be the value of a bound variable."[20] Given that bound variables range over D and the values of bound variables are the individuals of D, that means that members of *quantificational domain* are exactly what there is (or exists). Hence, with this particular interpretation of quantifiers, the domain of quantification becomes a set of existent things and the individuals in D are thus existing things. There is more than one way to formulate this premise; we could simply say, for instance, that quantifiers have existential import or that members of D are existent things, but our choice is the following:

(7) The quantificational domain D is a set of existent things.

[20] Willard Van Orman Quine, "On What There Is," *Review of Metaphysics* 2, no. 5 (1948/1949): 21--38

Now if we combine this result with the previous conclusion that each constant must denote, it follows that

(8) each individual constant of $L^=$ denotes an existent.

To conclude, the classical requirement that to each individual constant some member of the quantificational domain must be assigned together with the usual understanding of classical semantics and the existential reading of quantifiers imply that each constant of a formal language must denote some existent thing. If this result is generalized in a way that it would apply to all terms of the formal language, then it follows that CQT presupposes that its terms have existential import.[21] In other words, CQT contains an implicit existence assumption.

This, however, does not seem to be the case with the singular terms of natural languages. In ordinary discourse, we seem to use singular terms without requiring them to denote existents. To put it differently, natural languages contain ESTs, which are not permitted in CQT. Some of the problems arising from this divergence should be obvious. For one thing, classical reasoning becomes unreliable in its application to ESTs. For instance, from (1) above,

(9) There is (or exists) something identical with Pegasus

or simply,

(10) Pegasus exists

is obtained if we allow *t* in (1) to be the singular term ‘Pegasus’. This conclusion, however, seems to be against our intuitions since we usually maintain that Pegasus

[21] For a similar discussion of these points, see Ermanno Bencivenga, “Free Logics,” in *Handbook of Philosophical Logic*, ed. Dov M. Gabbay and Franz Guenthner, 2nd ed., vol. 5 (Dordrecht: Kluwer, 2002), 147.

does not exist. Similarly, if we allow *t* in (2) to be ‘Santa Claus’, and *A* to be the general term ‘lives at the North Pole’, this schema yields

(11) If Santa Claus lives at the North Pole, there is something living at the North Pole,

which would be regarded by most people as false with a true antecedent and a false conclusion. One can also give false instances of (3). For example, from the seemingly true premise ‘everything exists’ we can infer the false conclusion ‘Vulcan exists’ if we allow *t* to be ‘Vulcan’ and *A* to be the general term ‘exist’ in the rule version of (3).

A somewhat related problem is that the existence assumption tacit in CQT blurs the distinction between the inferences for which this assumption is relevant and for which it is not. Consider, for instance, an inference of the following form:

(12) $A(t/x) \supset \neg\neg A(t/x)$

In contrast to (1), (2) and (3), this schema does not seem to require the term *t* to be denoting to yield a true sentence. In other words, one can substitute an EST for *t* here but still have a valid inference, which was not the case with schemas (1), (2) or (3) as we saw above.[22]

From an ontological point of view, the assumption that each term of the formal language denotes some existing object seems to be introducing into logic some sort of existential commitment and makes way for some undesirable results. The fact that

[22] For an overview of this and the previous problem, see Karel Lambert, “Free Logics,” in *Blackwell Guide to Philosophical Logic*, ed. Lou Goble (Oxford: Blackwell Publishing, 2001), 263.

it is quite simple by classical reasoning to prove the existence of God may be enough to show this point.[23]

More problems can be mentioned here, but their general character should be already clear enough.[24] To put it simply, natural languages contain ESTs but CQT does not allow for such expressions and this incompatibility between natural languages and classical logic gives rise to various problems or difficulties.

Given that non-referring expressions and sentences containing them play an important part of ordinary discourse, it should be quite reasonable to want to reform CQT in such a way that it can accommodate ESTs without any ontological commitment or undesirable consequences following from them. This is indeed what free logics are proposed for. However, before going into details of these systems, let us briefly look at whether or not the problems mentioned above can be solved in classical (or quasi-classical) framework.[25]

Classical Solutions

Let us begin this section with a remark. In the preceding discussion, we mentioned some problems that seem to stem from an incompatibility between the presence of ESTs in natural languages and the existence assumption tacit in CQT, i.e., the

[23] Scott Lehmann, "More Free Logics," in *Handbook of Philosophical Logic*, ed. Dov M. Gabbay and Franz Guenthner, 2nd ed., vol. 5 (Dordrecht: Kluwer, 2002), 204 offers a proof for this:

1. $x=x$	(Axiom)
2. $\forall x(x=x)$	UG(1)
3. $\forall x(x=x) \supset g=g$	(Axiom)
4. $\forall x(x\neq g) \supset g\neq g$	(Axiom)
5. $\exists x(x=g)$	T(2,3,4)

[24] Rolf Schock, *Logics Without Existence Assumptions* (Stockholm: Almqvist & Wiksell, 1968), 11--12 may be a useful source for a brief summary of these problems.

[25] By 'classical' or 'quassi-classical framework' here, I simply mean one that includes the classical requirement that each term must receive an interpretation.

assumption that each term of the formal language denotes an existent.[26] Let us briefly elaborate on this point.

Recall the first problem we mentioned above. There we were able to infer (9), for instance, from the schema (1), and this conclusion seemed to be against our intuitions. Now if there were no ESTs in the language, then there would be no expressions to substitute for *t* in (1) so that the schema would not lead to such problematic results. On the other hand, if CQT did not require its terms to denote existents, then (1) would not be a theorem schema and conclusions such as (9) or (10) would not follow from it. A similar line of reasoning can also be applied to other examples mentioned above, but the general strategy to overcome the difficulty should be already clear. Very simply put, since the problems we considered above stem from an incompatibility between the presence of ESTs in natural language and the classical assumption that each term denotes an existent, any solution to these problems must entail eliminating one or the other.[27]

Let us begin with the first and perhaps the most obvious strategy. Accordingly, we want to get rid of the existence assumption, which was formulated as (8) above. Recall that (8) is a result of three premises. Given that (4) is a classical requirement, and we do not want to abandon classical framework for now, and that there are no real alternatives for (5), this strategy in effect boils down to abandoning (7) which expresses the standard interpretation given to quantifiers. Once Quine's dictum is dropped, quantifiers would be ranging over non-existent objects as well, and the quantificational domain would consist of both existent objects and non-existent ones.

[26] Note that this is just another way to express what we have said in the end of the previous section. For the assumption in question already entails the conclusion that CQT does not allow for ESTs.

[27] For a discussion of this point, see Bencivenga, "Free Logics," 151--52.

As a result, (8) would not be valid anymore. Let us call this quasi-classical solution as the Meinongian strategy.

Apart from the well-known problems of Meinong’s doctrine, it can be argued that the Meinongian strategy fails when it comes to eliminating ontological commitments from logic. For while saving us from one kind of existential commitment, it brings another, namely that there are non-existent objects. Besides, this strategy does not actually allow for non-referring expressions. For according to it, all terms refer, either to existing things or to non-existent ones. Obviously, this might be a problem for those who make a distinction between non-referring expressions and expressions referring to non-existents.

There are of course many variations of this strategy and some of them can be argued to be immune to these shortcomings. One such account is known as *noneism*. Graham Priest, in his “How the Particular Quantifier Became Existentially Loaded Behind Our Backs,” [28] shows how the *existential quantifier* has become *existentially loaded* in modern logic through Frege, Russell and Quine. He rejects Quine’s dictum and adopts instead a different reading of quantifiers according to which existential quantifier should be simply read as ‘some’, “leaving it open whether the some in question exist or not.”[29] At first glance, his account seems to be immune to the difficulties Meinongian theories are usually subject to; however, it has its own problems.[30]

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[28] Graham Priest, “The closing of the mind: How the Particular Quantifier Became Existentially Loaded Behind Our Backs,” *The Review of Symbolic Logic* 1, no. 1 (June 2008): 42--55.

[29] Ibid., 42.

[30] For a brief summary of these problems, see Maria Reicher, “Nonexistent Objects”, The Stanford Encyclopedia of Philosophy (Winter 2012 Edition), ed. Edward N. Zalta, Available [online]: http://plato.stanford.edu/archives/win2012/entries/nonexistent-objects/ [20 April 2014].

The second strategy to overcome the problems mentioned above is keeping the existential reading of quantifiers but eliminating instead non-referring expressions from the language. Two different ways to achieve this have been suggested, and both are very well known. Let us briefly consider them.

The first alternative, originally due to Frege and later developed by Carnap, is usually known as the chosen object theory (or method). To put it simply, the idea is to choose an arbitrary denotation for all non-denoting expressions of the language. In this way, ESTs are made to be referring, and thus, eliminated from the language.[31]

The other alternative is due to Russell and it is his renowned theory of definite descriptions. Very simply put, Russell's theory denies the status of singular term to almost all expressions of natural languages one usually considers as referring except maybe some demonstratives such as 'this' and 'that' and what Russell calls egocentric particulars such as 'here' and 'now'. In this way, ESTs are eliminated from the language. To give an example, according to Russell, 'Pegasus' is a *disguised* definite description. As a result of this, the *logical* (or *real*) *form* of a statement containing 'Pegasus' does not actually contain a reference to Pegasus or whatsoever.

The shortcomings of both alternatives are well known and do not need a detailed treatment here.[32] The chosen object theory, besides being too artificial, forces us to identify all ESTs with an arbitrary object and thereby make sentences such as 'Pegasus is the present king of France' true for it takes 'Pegasus' and 'the

[31] For details of Frege's account, see Gottlob Frege, "On Sense and Reference" in *Translations from the Philosophical Writings of Gottlob Frege*, 2nd ed., ed. Peter Geach and Max Black (Blackwell: Oxford, 1960), 69--71. For Carnap's account, see Rudolph Carnap, *Meaning and Necessity: A Study in Semantics and Modal Logic* (Chicago: University of Chicago Press, 1956), 32--38.

[32] Note that the solutions mentioned in this section are addressed in the third chapter in a more detailed way.

present king of France' as both denoting the same thing. Russell's theory of definite descriptions, on the other hand, is an inaccurate representation of natural languages since it denies the status of singular terms to almost all expressions we normally consider as referring.

There have been of course many suggestions to defend both alternatives, but we do not need to pursue them here. Overall, we can conclude that there has not been yet a conclusive solution within the classical or quasi-classical framework to the problems posed by ESTs. There is, however, another option. Instead of giving quantifiers some other interpretation or trying to eliminate ESTs altogether from the language, we can admit such expressions into the formal language while rejecting inferences whose validity depends on the assumption that all singular terms denote. This, obviously, requires abandoning the classical framework and is exactly what free logicians propose.

CHAPTER II

WHAT ARE FREE LOGICS?

A Brief Introduction

In the previous chapter, I explained what an EST is and presented some of the problems CQT faces in connection with them. One possible way out of this difficulty was trying to solve these problems without abandoning the classical framework. I looked at some of the previous attempts in this direction; nevertheless, concluded that none of them is entirely successful. Another possible strategy is modifying certain classical principles and rules in a way that leads to free logics. In what follows, I present this solution. To put it very simply, the chapter is centered on the following question:

Q1 What are free logics?

The presentation starts with definitions. I look at some of the definitions put forth for free logics and determine commonalities among them, and thereby, specify the characteristic features of these systems. Next, I present proof theoretical developments of free logics. The presentation is restricted to axiomatic systems, which are only available for certain kinds of free logics. A special emphasis is given to the question of how the differences between free logics and CQT find a syntactical expression. After presenting formal systems, I look at their interpretations. As will be seen, semantic approaches in free logics greatly vary depending on a number of questions. After presenting each kind of semantics, I briefly mention its advantages and disadvantages over others. Presenting proof and model theory of free logics will not only provide us with a more formal presentation of these systems, but it will also give us a different perspective on how they differ from CQT. The chapter ends with a brief section about the origins of free logics.

Definition, Characteristics, and Types

The term 'free logic' is coined by Karel Lambert as an abbreviation for "logic free of existence assumptions with respect to its terms, singular and general, but whose quantifiers are treated exactly as in standard quantifier logic."[33] Edgar Morscher and Peter Simons formalize Lambert's definition as follows:

> A logical system *L* is a free logic iff (i) *L* is free of existential presuppositions with respect to the singular terms of *L*, (ii) *L* is free of existential presuppositions with respect to the general terms of *L*, and (iii) the quantifiers of *L* have existential import.[34]

Let us look at this definition closer. The first condition practically means that, in free logics, singular terms do not have existential import. That is to say that free logics allow for terms that have no denotations in the domain of quantification. The second condition may be somewhat ambiguous, so a clarification is in order. In non-empty domains, a statement such as '$\exists xPx \vee \exists x\neg Px$' is always true no matter what the predicate P is, and this can be regarded as an existential presupposition with respect to general terms. However, the idea underlying (ii) is different. This condition must be understood in parallel to (i). To be more precise, free logics allow for empty

[33] Lambert, "Free Logics," 258.

[34] Edgar Morscher and Peter Simons, "Free Logic: A Fifty-Year Past and an Open Future," in *New Essays in Free Logic: In Honour of Karel Lambert,* ed. Edgar Morscher and Alexander Hieke (Dordrecht: Kluwer, 2001), 2.

predicates, i.e., predicates that are not satisfied by anything.[35] Having said this, given that CQT is already free of existence assumptions with respect to its general terms (in the sense that it allows for empty predicates) the condition (ii) can be ignored. Finally, the third condition means that the quantificational domain is exactly the set of existing things. The main features of free logics given in Lambert's definition can be summarized as follows: In free logics, singular terms need not denote and the domain of quantification equals to the set of existing things.

A somewhat clearer definition of free logics is due to Bencivenga. The definition he provides is as follows:

> A free logic is a formal system of quantification theory, with or without identity, which allows for some singular terms in some circumstances to be thought of as denoting no existing object, and in which quantifiers are invariably thought of as having existential import.[36]

Bencivenga's definition emphasizes the importance of the specific interpretation given to semantics. As we have seen in the previous chapter, the existence assumption usually associated with CQT not only rests on formal interpretations given to these systems, but also on the intuitive readings of these interpretations. Bencivenga's use of 'thought of' is intended to serve this purpose. Other than that, his definition embraces the same features of free logics that have been already given in Lambert's definition.

Finally, Lehmann provides the following:

> By a free logic is generally meant a variant of classical first-order logic in which constant terms may, under interpretation, fail to refer to individuals in the domain D over which the bound variables range, whether because they do not refer at all or because they refer to individuals outside D. If D is identified with what is assumed by the given interpretation to exist, in accord with Quine's dictum that 'to be is

[35] In this sense, there is no difference between free logics and CQT. Both allow for empty predicates.

[36] Bencivenga, "Free Logics," 158.

> to be the value of a [bound] variable,' then a free variation of classical semantics does not require that all constant terms refer to existent, and in this sense such terms lack existential import.[37]

A closer look at Lehmann's definition reveals that it embraces the same characteristic features of free logics given in the previous definitions.[38] These features can be put as follows:

(i) Quantifiers have existential import; in other words, the domain of quantification over which the bound variables range is the same as the set of existing objects.

(ii) Some terms of the formal language may fail to denote members of the quantificational domain. (They may denote individuals outside the domain or fail to denote at all.)

As we have already seen, certain principles or rules of CQT are unreliable in their application to ESTs. These classical principles and rules whose validity depends on the assumption that all terms are denoting (existents) are modified in free logics. An important characteristic of these systems is thus the rejection of the principle called universal specification. In free logics, universal specification

US $\forall xA \supset A(t/x)$

or its inferential counterpart, the rule of universal instantiation

UI $\forall xA \vdash A(t/x)$

of CQT are replaced by their restricted versions

[37] Lehmann, "More Free Logic," 197.

[38] There are of course differences between the three definitions; nevertheless, these differences are negligible for our purposes.

RUS $\forall xA \supset (E!t \supset A(t/x))$

and

RUI $\forall xA, E!t \vdash A(t/x)$.[39]

The duals of US and UI are weakened as well. The rule of existential generalization,

EG $A(t/x) \vdash \exists xA$

for instance, is invalid in free logics and replaced by the weaker rule

REG $A(t/x), E!t \vdash \exists xA$

The necessity to modify these principles and rules in a logic that allows for some singular terms to be empty should be straightforward. Consider US for instance. The fact that every member of the quantificational domain satisfies A obviously does not guarantee the truth of $A(t/x)$ because the term t may not denote a member of the domain. The same line of reasoning also applies to EG. By modifying these principles and rules, the existence assumption tacit in CQT is said to become explicit in free logics so that it is independently true or false for each singular term to be substituted in.

Admission of ESTs into the formal language obviously requires an account on how to assign truth-values to *atomic formulas* containing them. Free logics are thus divided into three major classes depending on how these formulas are evaluated for truth-values. These classes are called *negative*, *positive* and *neutral* (also called *non-*

[39] In some formal systems of free logics, the one-place predicate '$E!$' is introduced in order to distinguish singular terms that denote existing things from those that do not. Intuitively, '$E!t$' means that t exist. If the identity predicate is present in the language, it is possible to define '$E!$' in terms of identity as follows: $E!t =_{df} \exists x(x = t)$. In systems without identity, '$E!$' is undefinable and must be taken as primitive.

valent) free logics. Briefly put, in negative free logic, all atomic statements containing an EST are evaluated as false; in positive free logic, some of these statements are evaluated as true, and in neutral free logic all such statements (with the exception of those of the form '*t* exists') are evaluated as truth-valueless.[40]

Proof Theory

Now we are ready to present proof theoretical developments of free logics. As usual, I start with describing a formal language by listing its vocabulary and formation rules. Afterwards, I present axiomatic formulations of some free logics. Let us begin with making some preliminary remarks that should help to clarify what will and will not be covered in what follows.

First of all, recall that free logics are divided into three major classes depending on how atomic statements containing ESTs are evaluated for truth-value. Correspondingly, proof theoretical developments of free logics are essentially of three sorts. That being said, not every class of free logics will be covered here. Although there are natural deduction and tableau versions of every class of free logics, we will confine ourselves to axiomatic systems that are only available for positive and negative free logics. Nevertheless, that should be more than enough for our purposes.

[40] Now is a good time to underline a terminological matter. In the last chapter, we have restricted the use of the expression 'EST' to natural languages. Here, however, we seem to be talking about certain expressions of some formal language, yet use the same expression. This, in fact, is simply a matter of convenience. To be precise, an EST is an expression of natural language. Its formal counterpart, on the other hand, can be defined as follows: Given a formal language L and an interpretation $M = \langle D, I\rangle$ for L, let t be a term of L which has not been assigned a member of the quantificational domain D by the interpretation function I. From the definition, it must be already clear why we want to use 'EST' for both types of languages. We could of course introduce a shorter expression to use while talking about formal languages, however, given that the context almost always prevents any sort of confusion, that should be hardly necessary. In case there is a risk of ambiguity, we shall explicitly state what is meant.

Second, axiomatic formulations of a certain kind of free logics also differ depending on whether the identity predicate '=' and the existence predicate '*E*!' is in the primitive vocabulary of the formal language or not. For instance, there are formal systems of positive free logic that have neither of the predicates as well as positive systems that include both. Obviously, these systems have different axioms. Instead of covering every axiomatic formulation of each class of free logics, our presentation will be mainly based on a formal language that has both predicates in its primitive vocabulary. Nonetheless, we will specify the necessary changes if the language had only one of them or neither.

Finally, let us also note that the formal systems that will be presented below have a lot in common with standard systems of logic, so we will occasionally skip some formalities that are otherwise necessary.

The Formal Language *FL* of Free Logics

The vocabulary of *FL* is the same as that of the language of classical first-order logic with identity ($\text{FOL}^{=}$) except that it is augmented with a one-place predicate '*E*!' for singular existence.[41] To be specific, the primitive vocabulary of *FL* consists of the following signs:

(i) connectives: $\supset$, $\neg$

(ii) the universal quantifier $\forall$

(iii) logical predicates: =, *E*!

(iv) variables x, y, z, ..., with or without subscripts

[41] The formula '$E!t$' is well-formed if t is a term of the formal language. It should be simply read as 't exists', whatever t may be.

(v) n-place predicates P, Q, R, ..., with or without subscripts

(vi) individual constants a, b, c, ..., with or without subscripts

(vii) parentheses

Any variable or individual constant is a term of *FL*. In the metalanguage of our language, we will use the metavariables 's', 't', 't_1', 't_2', etc. for terms and 'A', 'B', 'C', etc. for formulas. The remaining symbols of *FL* will be used autonymously.

Now that we have described the primitive vocabulary of *FL*, it is time to specify the *formation rules* for *well-formed formulas* (wffs). A wff of *FL* can be defined recursively as follows:

(i) If P is an n-place predicate and $t_1, t_2, \ldots, t_n$ are terms of *FL*, then $Pt_1t_2\ldots t_n$ is a wff of FL.

(ii) If t_1 and t_2 are terms of *FL*, then $t_1 = t_2$ is a wff of *FL*.

(iii) If t is a term of *FL*, then $E!t$ is a wff of *FL*.

(iv) If A and B are formulas of *FL* and x is a variable of *FL*, then $\neg A$, $A \supset B$, and $\forall xA$ are wffs of *FL*.

(v) Nothing else is a wff of *FL*.

A formula A of *FL* is an *atomic formula* if it has one of the forms (i), (ii) or (iii). An *occurrence* of a variable x in a well-formed formula A of *FL* is *bound* if that occurrence is in a part $\forall xB$ of A, otherwise it is free. A formula A of *FL* is a *closed formula* or *sentence* of *FL* if A has no free occurrences of variables; otherwise it is an *open formula*.

In our metalanguage, the substitution operation $A(s/t)$ refers to the result of substituting s for t as in follows:

(i) $A(a/x)$ is the result of replacing every free occurrence of x in A with a,

(ii) $A(x/a)$ is the result of replacing every occurrence of a in A with a variable x that is free for a in A,

(iii) $A(x/y)$ is the result of replacing every free occurrence of y in A with a variable x that is free for y in A.

Finally, the substitution operation $A(s//t)$ refers to the result of replacing one or more occurrences of t in A with s, if at all.

Connectives other than $\forall$, $\supset$, and $\neg$ ($\leftrightarrow$, $\vee$, $\wedge$, $\exists$) can be introduced into *FL* with the usual definitions.

Finally, note that by dropping the existence predicate '$E!$' from the primitive vocabulary of *FL*[42], we get a restricted language, say $FL^{=}$, and by also dropping the identity predicate, we get an even restricted language, say FL^{-}.[43]

Now we are ready to present axiomatic systems of positive and negative free logics, PFL and NFL respectively, formulated in the language *FL*.

The Axiomatic System PFL

An axiom of PFL is either a tautology of $\mathrm{FOL}^{=}$ or any closed formula of the following form:

(A1) $A \supset \forall xA$

(A2) $\forall x(A \supset B) \supset (\forall xA \supset \forall xB)$

(A3) $\forall xA \supset (E!t \supset A(t/x))$

[42] That does not mean of course that the vocabulary would not include '$E!$' at all; rather than being a primitive symbol, it may be taken as defined as in follows: $E!t =_{\mathrm{df}} \exists x(x = t)$.

[43] The necessary adaptations of the formation rules for the languages $FL^{=}$ and FL^{-} should be obvious, hence they are omitted here.

(A4) $\forall x E!x$

(A5) $\forall x A(x/t)$ if A is an axiom

(A6) $t = s \supset (A \supset A(s//t))$

(A7) $t = t$

The only rule of inference for PFL is

MP From A, $A \supset B$, infer B.

The given set of axiom schemata together with MP establishes the axiomatic basis of PFL. The metalogical concepts (*derivation*, *proof*, *theorem*, etc.) are defined as usual, and hence will be skipped here.[44]

Let us take a closer look at the axioms of PFL to see how they reflect the difference between free logics and CQT. Note that (A1), (A2) and (A5) are classical axiom schemas. (A3) and (A4), on the other hand, are peculiar to free logics, to be more precise, to PFL.

The schema (A3) is the restricted version of the classical principle called universal specification, which we have shortened as US above. Recall that weakening this principle was an important characteristic of free logics that distinguishes these systems from standard systems. (A3) exactly serves this purpose. It restricts the use of US to existing objects. The schema (A4), on the other hand, establishes a connection between quantification and existence; it assures that the universal quantifier has existential import; in other words, it assures that we quantify only over existing objects. As we saw before, this is another important feature of free

[44] For the first examples of formal systems similar to PFL, see Karel Lambert, "Free Logic and the Concept of Existence," *Notre Dame Journal of Formal Logic* 8, no. 1 and 2 (April 1967): 139; and, Robert K. Meyer and Karel Lambert, "Universally Free Logic and Standard Quantification Theory," *The Journal of Symbolic Logic* 33, no. 1 (March 1968): 9.

logics as well. To put in a nutshell, we can say that (A3) together with (A4) restricts US to members of the quantificational domain which equals to the set of existing things.

Since (A3) and (A4) are the only axiom schemas that are peculiar to free logics, by dropping them from the given set of axiom schemata and adding US as an axiom schema to the set (or its inferential counterpart UI as a rule of inference), an axiomatic basis of FOL$^{=}$ can be easily obtained.

Axiomatic systems of positive free logic can also be formulated in languages weaker than *FL*. For instance, note that the biconditional

$E!t \leftrightarrow \exists x(x = t)$

is derivable in PFL. It could be instead taken as a definition, however. That is, '*E*!', rather than being a primitive predicate, could be taken as a defined predicate with the following definition:

DEF $\quad E!t =_{\mathrm{df}} \exists x(x = t)$

Note that this would give us a weaker language, which is similar to what we have called *FL*$^{=}$ above. A formal system PFL$^{=}$, formulated in the language *FL*$^{=}$, can be axiomatized with the same set of axiom schemata given for PFL except (A3). This axiom schema becomes redundant in the resulting system because it would then follow from DEF and the other axioms.

It is also possible to formulate an axiomatic system of positive free logic in a language that is even weaker than *FL*$^{=}$. Consider, for instance, a formal language *FL*$^{-}$ that has neither the identity nor the existence predicate in its primitive vocabulary. A formal system PFL^{-} formulated in the language *FL*$^{-}$ can be obtained by changing the schemas (A3) and (A4) of PFL with

(A3*) $\forall y(\forall x A \supset A(y/x))$

and

(A4*) $\forall x \forall y A \supset \forall y \forall x A$,

respectively, and dropping the identity axioms (A6) and (A7) from the given set of schemata. Note that the modified version of (A3), namely (A3*), would serve the same purpose as before.[45]

<u>The Axiomatic System NFL</u>

Now we are ready to present the axiomatic system NFL of negative free logics. A few changes in the system PFL would be enough to obtain it. To this purpose, recall that negative semantics requires that all atomic formulas containing an EST be false. NFL should reflect this requirement axiomatically. Accordingly, we need an additional axiom schema that assures that an atomic formula of the form *Pt* is true only if *t* is denoting. Moreover, since self-identity statements are no exception to this requirement, any statement of the form $t = t$ should be false if *t* is non-denoting. Hence, the schema (A7) needs to be modified for NFL. These are the only necessary changes that need to be made in PFL to obtain the axiomatic basis of NFL. To be more precise, replacing (A7) with

(A7*) $\forall x(x = x)$

and adding the schema

[45] For systems similar to PFL$^=$ and PFL$^-$, see respectively Hugues Leblanc and R. H. Thomason, "Completeness Theorems for some Presupposition-free Logics," *Fundamenta Mathematicae 62*, no. 2 (1968): 130--131; and, Karel Lambert, "Existential Import Revisited," *Notre Dame Journal of Formal Logic* 4, no. 4 (Oct. 1963): 290--91.

(A8) $A(t/x) \supset E!t$, where the variable x is free and $A(t/x)$ is atomic,

to the set of axiom schemata given for PFL, the formal system NFL can be obtained.[46]

It is also possible to formulate an axiomatic system of negative free logic in the weaker language $FL^{=}$. The corresponding system NFL$^{=}$, formulated in this language, can be obtained from NFL by changing (A4) with

(A4**) $\forall x \exists y(x = y)$.

If the predicate '$E!$' is taken as defined rather than primitive as in PFL$^{=}$, the resulting system NFL$^{=}$ also does not require (A3) as an axiom schema since it would then become derivable.[47] Finally, as for the language FL^{-}, there is no axiomatization of negative free logic in it.

Note that the schemas (A3) and (A4) in NFL as well as (A3) and (A4**) in NFL$^{=}$ serve the same purposes before. The additional axiom schema (A8), on the other hand, is peculiar to negative free logic, and it reflects its special semantical convention.

Let us close this section with a remark. If the differences in formulations reflecting special semantic conventions are left aside, all systems considered above axiomatically reflect two characteristic features of free logics: First, they all involve a restricted version of US either as an axiom schema or as a derived principle, and second, depending on the formal system, either via (A4), (A4*) or (A4**), quantifiers are granted existential import.

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[46] For an example of a formal system similar to NFL, see Rolf Schock, *Logics Without Existence Assumptions*, 94.

[47] For a system similar to NFL$^{=}$, see Tyler Burge, "Truth and Singular Terms," *Noûs* 8, no. 4 (November 1974): 311--12.

Model Theory

In the previous section, we saw some of the formal systems of free logics and also noted a few differences between them. When it comes to the interpretations given to these systems, differences become more diversified. Hence, it is meaningful to start a discussion on model theory with explaining how semantic approaches for free logics differ in the first place.

As noted earlier, free logics, in contrast to CQT, allow for some singular terms to be empty. Admitting such expressions into the language naturally gives rise to two major semantic questions that need to be answered:

SQ1 Are ESTs such that they denote individuals outside the quantificational domain or they do not denote at all?

SQ2 How should the atomic formulas containing ESTs be evaluated for truth-value? (i) Are some of these statements true, (ii) all of them false, or (iii) all of them truth-valueless?

As is already clear from SQ1 itself, there are two ways in which singular terms may fail to denote members of the quantificational domain: (i) ESTs may denote individuals outside the range of quantifiers, or (ii) they may not denote at all. The former account leads to *dual-domain semantics* in which the domain of discourse is divided into two domains, typically an *inner domain* over which the bound variables range, and an *outer domain* whose principal function is to provide denotations to ESTs. The latter account, on the other hand, leads to *single-domain semantics* in which there is only one domain but the interpretation function on individual constants is partial.[48] Finally, as we have already seen, the answer given to SQ2 leads to (i) positive, (ii) negative, and (iii) neutral semantics.

Moreover, depending on the answer given to SQ2, more questions may follow which in turn will lead to further variations in free semantics. To give an example, suppose that we adopt a positive semantics with single domain models. Accordingly, some of the atomic formulas containing ESTs would be true. Some others, however, would be probably evaluated as truth-valueless.[49] Then, however, the following question arises: How shall we assign truth-values to complex formulas that have truth-value gaps at the atomic level?[50] Clearly, the adopted strategy to deal with such complex formulas leads to further variations in positive semantics.

To sum up, free semantics come in various forms depending on a number of questions. In what follows, I shall look at some of the different semantic approaches in free logics. These are, respectively, (i) dual-domain semantics, (ii) negative semantics, (iii) *story semantics*, (iv) *supervaluational semantics*, (v) *proto-semantics*, and (vi) neutral (or non-valent) semantics.[51]

Dual-Domain Semantics

Dual-domain semantics differs from classical semantics primarily in its model structures. Unlike classical models, a dual-domain model includes an additional

[48] Consider a language L and a model $M = \langle D, I \rangle$ for it where D is the domain of objects and I is a partial function interpreting individual constants and predicates of FL. In M, $I(a)$ may be undefined for some individual constant a of FL, i.e. there may be no $d \in D$ such that $I(a) = d$.

[49] e.g., 'Pegasus is six-feet long'.

[50] Note that this issue is not restricted to positive semantics; neutral semantics, which also allow truth-value gaps at the atomic level, faces the same question.

[51] As we will see, (ii), (iii), (iv), (v), and (vi) are semantics with single-domain models. Also (iii), (iv), and (v) are positive semantics with single-domain models.

domain besides the quantificational domain whose principal function is to provide denotations to ESTs. The interpretation function, on the other hand, is total as in classical semantics. Accordingly, in a dual-domain semantics, (i) all individual constants[52] of the formal language have a denotation (although these need not to be in quantificational domain), and (ii) predicates are assigned extensions from the union of the two domains.[53]

Dual-domain models are most widely used in semantical developments of positive free logic; hence, our discussion of dual-domain semantics will be restricted to positive kind.[54] There are three slightly different kinds of model structures that are used in dual-domain semantics. Of these, we shall only look at the first two.

In the first kind, a dual-domain model structure is an ordered triple $\langle D_o, D_i, I_1\rangle$ where D_o (the outer domain) and D_i (the inner domain) are disjoint and possibly empty sets whose union is non-empty, and I_1 is a total interpretation function such that it assigns to each individual constant *a* of the formal language, a member of $D_o \cup D_i$, and to each *n*-place predicate *P*, a set of *n*-tuples of members of $D_o \cup D_i$. Let

[52] Here and after, we will assume that our formal language does not include the descriptor operator and function symbols. Hence, in our language, singular terms are represented only with individual constants.

[53] Dual-domain model structures were developed independently by Lambert and Nuel Belnap in the fifties. Their first appearance in the literature is in a critical review by Alanzo Church of a paper written by Lambert. For the review, see Alanzo Church, review of “Existential Import Revisited,” by Karel Lambert, *The Journal of Symbolic Logic* 30, no. 1 (March 1965): 103--04. For the first examples of dual-domain models, see (i) Leblanc and Thomason, “Completeness Theorems for some Presupposition-free Logics,” 125--64, (ii) Nino Cocchiarella, “A Logic of Possible and Actual Objects,” *Journal of Symbolic Logic* 31 (1966): 688--89 (Abstract), and (iii) Dana Scott, “Existence and Description in Formal Logic” in *The Philosophical Applications of Free Logic*, ed. Karel Lambert (New York: Oxford University Press, 1991), 28--48. Note that although all of the model structures adopted in these three papers have dual-domains, they differ from each other in important respects.

[54] Since all atomic formulas containing ESTs are uniformly false in negative semantics, and truth-valueless in neutral semantics, there is hardly a reason to introduce an additional domain of objects to assign its members to ESTs.

us call this kind of model structure as FM_1 model structure, and a model based on it as an M_1 model.[55]

Given the language FL^{56} and a model $M_1 = \langle D_o, D_i, I_1\rangle$ for it, closed formulas of FL are assigned truth-values T or F by the valuation function V_1 as follows:

(1a) $V_1(Pa_1\ldots a_n) = T$ iff $\langle I_1(a_1),\ldots, I_1(a_n)\rangle \in I_1(P)$; $=F$ otherwise.

(1b) $V_1(a = b) = T$ iff $I_1(a) = I_1(b)$; $=F$ otherwise

(1c) $V_1(E!a) = T$ iff $I_1(a) \in D_i$; $=F$ otherwise

(1d) $V_1(\neg A) = T$ iff $V_1(A) = T$; $=F$ otherwise

(1e) $V_1(A \supset B) = T$ iff $V_1(A) = F$ or $V_1(B) = T$; =F otherwise

(1f) $V_1(\forall xA) = T$ iff for each $d \in D_i$, $V_{1(a,d)}$ $(A(a/x)) = T$ (where the individual constant a does not occur in A and $V_{1(a,d)}$ is the valuation function on the model $\langle D_o, D_i, I_{1*}\rangle$ such that I_{1*} is just like I_1 except that $I_{1*}(a) = d$; $=F$ otherwise.

The inner domain D_i of an M_1 model is usually interpreted as the set of existing things whereas the outer domain D_o is interpreted as the set of non-existing things.[57] Given the fact that for each individual constant a of the formal language,

[55] FM_1 model structures were developed by Hugues Leblanc and R. H. Thomason. For an example of an M_1 model, see Leblanc and Thomason, "Completeness Theorems for some Presupposition-free Logics," 143. In this paper, the authors present a formal system similar to $PFL^=$ and prove that it is sound and complete with respect to a semantics outlined above.

[56] Note that the language *FL* here is the same *FL* of the previous section.

[57] Depending on the interpretation, members of the outer domain can be past/future objects, merely possible objects, impossible objects, error objects and so on. They can be even linguistic expressions so that they are not necessarily non-existent. For this kind of an approach, see Meyer and Lambert, "Universally Free Logic and Standard Quantification Theory," 8--26. The paper also offers a proof that their semantics yields soundness and completeness for a system similar to PFL but without the identity.

$I_1(a) \in D_o \cup D_i$; and for every *n*-place predicate P, $I_1(P) \subseteq (D_o \cup D_i)^n$, 'non-existing' objects in the outer domain can have *positive properties* in an M_1 model.[58]

The second kind of model structure adopted in dual-domain semantics is an ordered triple $\langle D_p, D_a, I_2 \rangle$ where the domain D_p is a non-empty set, the domain D_a is a possibly empty subset of D_p, and I_2 is a total function interpreting individual constants and predicates on D_p. Let us call this kind of model structure as FM_2 model structure, and models based on it, as M_2 models.[59]

Given the formal language *FL* and an interpretation M_2 for it, truth-values of closed formulas of *FL* are computed by the total valuation function V_2 that is defined in a similar way to V_1 of course with the necessary modifications.

The domain D_p of an M_2 model is usually interpreted as the set of possible objects. The other domain D_a, on the other hand, is interpreted as the set of actual objects. Since predicates are assigned extensions from the domain D_p, possible objects of D_p, just like the non-existent objects of D_o, can have positive properties even though they are non-actual.

A final point to note about M_2 models is that the domain D_p is usually the range of another pair of quantifiers which, unlike the quantifiers ranging over D_a, have no existential import. Since D_p is interpreted as the set of possible objects, we can say that the additional pair of quantifiers allows for quantification over possible objects.[60]

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[58] Consider the sentence 'Pegasus has wings', for instance. Suppose that Pegasus does not exist. Then, according to the semantics that is outlined above, the denotation of 'Pegasus' should be in the outer domain. Having said this, since predicates are interpreted over the union of two domains, the denotation of 'Pegasus' can still be in the extension of the predicate 'has wings', and hence, the sentence can be true. Note that this usually accords with our intuitions.

[59] For an outline of a semantics based on M_2 models, see Nino Cocchiarella, "A logic of possible and actual objects," 688.

[60] Note that US is valid with this additional pair of quantifiers. Nevertheless, given the fact that these quantifiers are not existentially loaded, no problematic consequences arise. This also hints at

One of the main advantages of dual-domain semantics is that it requires minimal change in classical semantics.[61] From a formal point of view, on the other hand, dual-domain semantics is more convenient when compared to some of its alternatives.[62] As one of its drawbacks, we can mention that, in introducing a second domain of non-existents, we face the charge of ontological extravagance. Given that one of the major sources of motivation for free logics is the desire to 'save' logic from existential assumptions, this can be a serious problem. There are ways to avoid this criticism;[63] however, any such approach often results in an increased artificiality and complexity.[64]

Single-Domain Semantics

In single-domain semantics, the domain is single as in classical semantics and typically represents the set of existing things. The interpretation function, on the other hand, is only partial. That is to say that ESTs are not assigned denotations in this kind of semantics.

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how a system of free logic can be obtained from FOL: Pairing the quantifiers ranging over D_p with the usual quantifiers of FOL and making the quantifiers ranging over D_a defined in terms of $E!$ and the first pair of quantifiers.

[61] Consider an M_1 model for instance. As we have seen above, we assign to each individual constant of the formal language some member of $D_o \cup D_i$, interpret the predicates over the union $D_o \cup D_i$, and compute the truth-values of atomic formulas in the usual Tarskian way. To give an example, as noted earlier, the sentence 'Pegasus has wings' is true with respect to an M_1 model, simply because Pegasus is in the extension of the predicate 'has wings'.

[62] For instance, unlike the supervaluational semantics, which is the other popular alternative of positive semantics, dual-domain semantics yields strong completeness.

[63] e.g., Meyer and Lambert's approach that is mentioned in the footnote 57.

[64] For a good discussion of the drawbacks of dual domain semantics, see Bencivenga, "Free Logics," 167--69.

Single-domain models are used in semantical developments of all three kinds of free logics. The simplest version is negative semantics, so let us start with it.

Single-Domain Negative Semantics

The kind of model structure used in a single-domain negative semantics is an ordered pair $\langle D, I_3 \rangle$ where the domain D is a possibly empty set and I_3 is a partial interpretation function such that for every individual constant a, it either assigns to a some member of D or it is undefined[65], and for every n-place predicate P, it assigns to P a set of n-tuples of members of D. Let us call this kind of model structure as FM_3 model structure, and a model based on it as an M_3 model.

Recall that negative semantics requires that all atomic formulas containing at least an EST be false. That is to say that, given an M_3 model, the truth of an atomic formula of the form Pa guarantees that $I_3(a)$ is defined. Atomic formulas that do not contain ESTs, on the other hand, are assigned truth-values in the usual way. Given the language FL and a model M_3 for it, the valuation function V_3 is defined in a way that reflects these requirements. To be more specific, the condition (1a) of V_1 is changed with

(3a) $V_3(Pa_1 \ldots a_n) = \mathrm{T}$ iff $I_3(a_1), \ldots, I_3(a_n)$ are all defined and $\langle I_3(a_1), \ldots, I_3(a_n) \rangle \in I_3(P)$; $=\mathrm{F}$ otherwise.

This ensures the falsehood of atomic formulas containing ESTs. (1b) and (1c) are also modified so that they include an additional condition that ensures that the

[65] $I_3(a)$ is undefined iff there is no $d \in D$ such that $I_3(a) = d$.

relevant individual constants are not uninterpreted. Finally, the inner domain D_i in the definition of V_1 is replaced with D where necessary.[66]

One of the main merits of single-domain negative semantics is that it offers a simple and straightforward procedure for assigning truth-values to statements: At the atomic level, all formulas containing ESTs are accepted as false. This decision, however, seems to be without a foundation, and any appeal to ordinary language for this would not be enough to provide the necessary justification for it. This is one of the most important drawbacks of negative semantics. Another difficulty it faces concerns the logical form of statements, which can be best understood with an example. Consider the formula $Pa \supset E!a$ for instance. Although it is logically true in negative semantics, one of its substitution instances $\neg Pa \supset E!a$ (where P is replaced with the complex predicate $\neg P$) is only contingently true.[67] As a final drawback, we can mention that the distinction that is made between *primitive* and *non-primitive* predicates in negative semantics is not easy to draw in natural languages as in formal languages.[68]

[66] For a semantics of the kind outlined above, see Burge, "Truth and Singular Terms," 314. In this paper, Burge presents an axiomatic system similar to $\text{NFL}^{=}$ (but without '$E!$'), and proves that it is sound and complete with respect to his semantics. For another semantics that is based on M_3 models, see Schock, *Logics Without Existence Assumptions*, 38--39. Schock presents a natural deduction version of negative semantics which is formulated in a language similar to *FL* and proves that it is sound and complete with respect to his semantics.

[67] The example, as well as the criticism mentioned here, is due to John Nolt. See John Nolt, "Free Logics," in *Philosophy of Logic*, ed. Dale Jacquette, vol. 5, *Handbook of the Philosophy of Science*, ed. Dov M. Gabbay, John Woods and Paul Thagard (Amsterdam: Elsevier, 2006), 1033.

[68] Suppose a is an individual constant such that it is not assigned a member of the quantificational domain and P is a one-place primitive predicate. Then Pa is false, but $\neg Pa$ is true with respect to an M_3 model. Hence, in a single-domain negative semantics, truth-values of formulas containing ESTs seem to depend on the choice of primitive predicates. However, the distinction between primitive and complex predicates is hard to hold in natural languages for it is not always determined which predicate is primitive and which is complex. For a brief discussion of this point and others mentioned above as well as for other drawbacks of negative semantics, see Nolt, "Free Logics," 1033--35.

Single-Domain Positive Semantics

We now turn to positive semantics with single-domain models. Recall that positive semantics require that some of the atomic formulas containing ESTs be true. Since ESTs are not assigned denotations in single-domain semantics, a positive semantics with a single-domain model requires a non-standard procedure for assigning truth-values to such formulas.[69] The simplest kind of single-domain positive semantics is story semantics.

Story Semantics

The kind of model structure adopted in story semantics is an ordered triple $<D, I_4, S>$ where the domain D is a possibly empty set, I_4 is a partial interpretation function just like I_3, and the *story S* is a possibly empty set of atomic formulas containing ESTs. Let us call this kind of model structure as FM_4 model structure, and a model based on it as an M_4 model.[70]

In story semantics, the truth-values of the atomic formulas that do not contain ESTs are computed in the usual Tarskian way. Atomic formulas containing ESTs, on the other hand, are assigned truth-values by the story S. Very simply put, if the story S includes the atomic formula Pa, where P is a one-place predicate, and a is an individual constant such that $I_4(a)$ is undefined, then it is accepted as true. The

[69] For instance, suppose $I(a)$ is undefined for some individual constant a, yet we require the atomic formula Pa to be true for some n-place predicate P. Obviously, the standard definition of a valuation function, i.e., Pa is true if $I(a) \in I(P)$, would not be helpful in this case.

[70] Story semantics has been developed by Lambert and Van Fraassen. For details, see Karel Lambert and Bas C. van Fraassen, *Derivation and Counterexample: An Introduction to Philosophical Logic* (Encino, CA - Belmont, CA: Dickinson, 1972), 179 ff.

valuation function V_4 is defined in a way to ensure this. Accordingly, truth conditions of an atomic formula with respect to a M_4 model can be given as follows:

Given the language *FL* and a model M_4 for it, the atomic formula $Pa_1...a_n$ is true either if $I_4(a_i)$ is defined for all i such that $1 \leq i \leq n$, and $<I_4(a_1), ..., I_4(a_n)> \in I_4(P)$, or $I_4(a_i)$ is undefined for some i such that $1 \leq i \leq n$ but $Pa_1...a_n \in S$ where $a_1, ..., a_n$ are all individual constants and P is a n-place predicate; otherwise it is false.

In a story semantics, the story S is usually supplemented with some logical laws. For instance, for every individual constant a which is not assigned a denotation by the interpretation function, the story S includes the formula $a = a$. That means that even if a does not denote, story semantics makes $a = a$ true nevertheless. There are other similar laws, but we do not need to present them all here; the idea should be already clear.

Story semantics can be seen as a non-referential version of dual-domain positive semantics. Most importantly, it also allows for some non-existents to have positive properties but without appeal to an additional domain as in dual-domain semantics. For instance, even though ‘Pegasus’ does not denote an existent, the statement ‘Pegasus has wings’ can still be true in story semantics simply because the statement itself is part of the story. For a major drawback of story semantics, recall that the story S is supplemented with some logical laws. These laws make it open to the charge of being arbitrary or without foundation.[71] Another criticism story semantics faces is concerned with its *bivalence*. Given that stories are *incomplete* in the sense that they do not always provide us with all the details, it is natural to expect

[71] For this kind of a criticism, see Bencivenga, “Free Logics,” 176.

that the truth-values of certain statements be indeterminate. However, bivalent semantics does not allow for that.[72]

Supervaluational Semantics

Another kind of positive semantics with single-domain models is *supervaluational* semantics. In this kind of semantics, atomic formulas containing ESTs (except identity statements) are evaluated as truth-valueless.[73] The truth-values of complex-formulas composed of such formulas, on the other hand, are computed with a non-standard procedure. In its more popular version, the idea is simply to consider what the truth-value of an atomic formula would be if all of its singular terms were denoting.

The model structure adopted in supervaluational semantics is the same as the FM_3 model structure of negative semantics. Thus, an M_5 model is an ordered pair $\langle D, I_5\rangle$ where D is a possibly empty set, and I_5 is a partial interpretation function. However, the valuation function V_5 of an M_5 model is different from V_3 of an M_3 model in that it is only a partial function. That means, in an M_5 model, atomic formulas containing ESTs are evaluated not as false as in negative semantics, but as truth-valueless. As noted earlier, truth-values of the statements that are composed of atomic formulas containing ESTs are computed in a non-standard way. Let us see this two-fold procedure.

[72] For example, none of the stories about Pegasus provide us with any information about its length. Nevertheless, in story semantics, the sentence 'Pegasus is six feet long' must be either true or false. However, there does not seem to be a way to decide which. To be sure, the same criticism can be directed to dual-domain semantics too. Note that the example and the criticism are due to Bencivenga. For details, see Bencivenga, "Free Logics," 168.

[73] Thus, supervaluational semantics is non-bivalent.

First, a *completion* (or a *complete supermodel*) M^c of a model M_5 is defined. Accordingly, M^c is an ordered pair $\langle D^c, I^c \rangle$ where D^c is a non-empty superset of D, and I^c is a total interpretation function such that for every individual constant a, $I^c(a) \in D^c$ and $I_5(a) = I^c(a)$ when $I_5(a)$ is defined, and for every n-place predicate P, $I(P) \subseteq I^c(P)$. In other words, a completion of an M_5 model is a bivalent single-domain model whose domain D^c is a superset of the domain D, on which each term is assigned a member of D^c, and each n-place predicate is assigned a set of n-tuples of members of D^c. Truth conditions of the formulas with respect to a completion M^c of a model M_5 are defined in a standard way.

Next, a *supervaluation S* over a model M_5 is constructed from the set of all completions of M_5. Accordingly, a statement A is true on a supervaluation S of a model M_5 (also called supertrue) if all completions of M_5 make it true, false (also called superfalse) if they all make it false, and truth-valueless if they disagree.[74]

Supervaluational semantics has an important advantage over its alternatives that we have considered so far. On such a semantics, atomic formulas containing ESTs are not assigned truth-values[75] while, at the same time, the lack of truth-value of an atomic formula is not automatically carried to complex formulas constituted of

[74] Supervaluations were originally developed by Bas C. van Fraassen. For his version of supervaluational semantics, see Bas C. van Fraassen, "Singular Terms, Truth-Value Gaps, and Free Logic," in *The Philosophical Applications of Free Logic*, ed. Karel Lambert (New York: Oxford University Press, 1991), 82--97. In his original version, in order to compute the truth-value of a statement constituted of empty-termed atomic formulas, we consider the possible truth-values the statement would get if the empty-termed atomic formulas were assigned a truth-value. The version that was presented above is due to Bencivenga. For details of his approach, see Ermanno Bencivenga, "Free semantics," in *The Philosophical Applications of Free Logic*, ed. Karel Lambert, (New York: Oxford University Press, 1991): 98--110. Finally, for a suggestion about how supervaluations can be used in neutral free logic, see Brian Skyrms, "Supervaluations: Identity, Existence, and Individual Concepts," *The Journal of Philosophy* 65, no. 16 (August 1968): 477--82.

[75] For instance, the statement 'Pegasus is six feet long' is neither supertrue nor superfalse since the possible truth-values of it disagree if 'Pegasus' were denoting.

it. In this way, some of the classical principles are saved from being truth-valueless.[76] That being said, this kind of semantics has a major drawback. Positive free logic is only weakly complete on supervaluational semantics so that the connection between *logical truth* and *logical consequence* is lost.[77] Another important drawback of supervaluational semantics is that the well-established correspondence theory of truth is not anymore enough to justify the truth of certain statements.

Proto-semantics

Another kind of positive semantics with single-domain models is due to G. Aldo Antonelli and called proto-semantics.[78] The idea behind his semantic approach is to build a positive model theory that will overcome the shortcomings of both supervaluational semantics and dual-domain semantics. Accordingly, as one might expect, proto-semantics is bivalent while the model structure on it has only a single domain. As in supervaluational semantics, atomic formulas that do not contain ESTs are assigned truth-values in the standard way whereas for atomic formulas containing ESTs, a non-standard treatment is adopted. Let us briefly see the details of this semantic approach.

[76] Let me explain this with an example. Consider the formal language *FL* and an M_5 model for it. Let $I(a)$ be undefined for some individual constant a. Then the statement $Pa \vee \neg Pa$ (where P is an arbitrary one-place predicate) is supertrue since all completions of M_5 make this formula true. For if a were denoting, either Pa or $\neg Pa$ would be true, so that their conjunction $Pa \vee \neg Pa$ is true in either case.

[77] For instance, although on every supervaluation on which Pa is true, $E!a$ is also true, the formula $Pa \supset E!a$ is not logically valid, i.e., although we have $Pa \vDash E!a$, we do not have $Pa \vdash E!a$.

[78] For the original article where Antonelli introduces proto-semantics, see G. Aldo Antonelli, "Proto-Semantics for Positive Free Logic," *Journal of Philosophical Logic* 29, no. 3 (June 2000): 277--94. For a brief summary of Antonelli's approach, see Nolt, "Free Logics," 1031--32.

A model M_6 in proto-semantics is a triple $<D, \rho, I_6>$ where D is a possibly empty set, ρ is a partial *reference function* such that for every individual constant a, $\rho(a)$ is either a member of D or it is undefined; and I_6 is an interpretation function that assigns to each n-tuple of individual constants a *proto-interpretation*. A proto-interpretation π is a function that assigns to each n-place predicate P a *signed extension* $\pi(P)$[79] such that $\pi(P)$ is a pair either of the form $<S,+>$ or $<S,->$ where S is the ordinary extension of the predicate P (i.e., a subset of D^n) and + and - are markers whose function is to assign truth-values to atomic formulas containing ESTs. Truth conditions of atomic formulas of the form Pa can be given as follows:[80]

Given the language *FL* and a model M_6 for it, let a be an individual constant such that I_6 assigns to a the proto-interpretation π, and P be a one-place predicate such that $\pi(P) = <S, \pm>$.[81] Then the truth-value of the atomic formula Pa is computed as follows:

(i) If $\rho(a)$ is defined (i.e. if there is a $d_a \in D$ such that $\rho(a) = d_a$), then Pa is true if $I_6(a) \in S$, false otherwise.

(ii) If $\rho(a)$ is undefined, then Pa is true if $\pi(P) = <S,+>$, and false if $\pi(P) = <S,->$.

In other words, the atomic formula Pa is true if and only if either a denotes some individual d and $d \in S$; or a does not denote, and S has a positive sign +.[82]

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[79] A signed extension is simply an extension which is associated with a + or - sign.

[80] Note that the restriction of the valuation function to atomic formulas of the form Pa is for the sake of simplicity.

[81] If the sign of a signed extension is not specified, we write $\pi(P) = <S, \pm>$.

[82] For the sake of simplicity, we assumed that P is a one-place predicate. If it were n-place instead, the proto-interpretation π would be assigned to the n-tuple of individual constants i.e., to $<a_1, ..., a_n>$. In that case, truth conditions of the atomic formula $Pa_1...a_n$ would be as follows: (i) If $\rho(a_i)$ is defined

One of the merits of proto-semantics is that it allows for some statements containing ESTs to be true. In this sense, it accords with our intuitions as regards statements such as ‘Pegasus has wings’ or ‘Sandy Island is a phantom island’. Furthermore, it is also immune to some of the major criticisms dual-domain semantics and supervaluational semantics face. Very simply put, it does not appeal to non-existent objects as in dual-domain semantics, and unlike supervaluational semantics, it is bivalent. However, it suffers from the arbitrariness of assigning truth-values to atomic formulas containing ESTs. For instance, in an M_6 model, the statement ‘Pegasus has wings’ is true since although ‘Pegasus’ is not in the extension of the predicate ‘has wings’ it is still assigned a proto-interpretation with a + sign. However, not every case is that simple.[83]

Single-Domain Neutral Semantics

The last kind of single-domain semantics we will look at is neutral semantics. Like supervaluational semantics, it is also non-bivalent and allows for truth-value gaps both at atomic and complex levels.

The model structure adopted in neutral semantics is of FM_2 kind. That is, it has a single domain and the interpretation function is partial. Since neutral free logic requires all atomic formulas containing ESTs to be truth-valueless, the valuation function is defined in a way to ensure this. Atomic formulas that do not contain an EST, on the other hand, are assigned truth-values in the standard way.

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for all i, $1 \leq i \leq n$, then $Pa_1...a_n$ is true if $<a_1, ..., a_n> \in S$, and false otherwise. (ii) If $\rho(a_i)$ is undefined for some i, $1 \leq i \leq n$, then $Pa_1...a_n$ is true if $\pi(P) = <S,+>$, and false if $\pi(P) = <S,->$.

[83] For instance, what about the truth-value of the sentence ‘Pegasus is six-feet long’? Is it true or false? But maybe this is a price worth paying for bivalence.

Neutral free semantics have variants depending on how complex formulas composed of empty-termed atomic formulas are evaluated for truth-value. One of the better-known and simpler variant is ‘No input No Output (NINO)’ semantics by Lehmann, which simply makes all such complex formulas (except quantified statements) false.[84]

Apart from its obvious merits, NINO semantics has a major drawback. Certain logical laws are only weakly valid, i.e., not false on any model; and some others are not even weakly valid on it. As a matter of fact, each kind of neutral semantics invalidates some classical principles. The reason for this should be evident; the lack of truth-value at the atomic level is carried to formulas that are more complex.

Origins

The answer to the question of what free logics are would not be complete without at least mentioning certain facts about their history. Thus, before concluding this chapter, I shall briefly look at the history of free logics. Note that, in sections about model and proof theory, I have already provided some references to various sources. This should have already given an idea about how free logics have been developed. For this reason, I will rather concentrate on their origins in what follows.

The suggestion to change the classical principles or rules whose validity depends on the classical assumption that all singular terms are denoting was first put forward by Henry Leonard in his “The Logic of Existence”[85] Today, this article is

[84] For details of NINO semantics, see Scott Lehmann, “Strict Fregean Free Logic,” *Journal of Philosophical Logic* 23, no. 3 (June 1994): 307--36; and, Lehmann, “More Free Logic,” 233--37.

[85] Henry S. Leonard, “The Logic of Existence,” *Philosophical Studies* 7, no. 4 (June 1956): 49--64.

commonly regarded as the pioneering paper of free logics. Hence, it deserves a separate mention.

In this landmark paper, although using a defective definition of ‘*E*!’, Leonard proposes a weakened version of EG. He introduces into the formal system a one-place predicate ‘*E*!’ in order to express singular existence. Accordingly, he suggests adding ‘*E*!*t*’ for every singular term *t* as an antecedent to EG. Nevertheless, he does not provide a formal system for his new logic. His motivation is to develop a system of logic that has a wider applicability than CQT and is capable of discriminating between inference patterns for whose validity it is relevant that its terms are non-empty and for those it is not.

Lambert, in an article published two years later, suggests ‘harmonizing’ Leonard’s logic with the suggestions of Nicholas Reschers who in the footsteps of Leonard proposed to modify his definition.[86]

In a few years later, Jaako Hintikka publishes a paper where he discusses the relation between the problems that arise in connection to the application of certain classical rules to ESTs and Quine’s thesis ‘to be is to be a value of a bound variable’.[87] He argues against Russell’s theory of definite descriptions as a way out the difficulties we have mentioned in the previous chapter. Hintikka’s suggestion is rather an amendment in CQT. He provides rules for his new system of logic and labels it as “a logic without existential presuppositions.”

[86] Karel Lambert, “Notes on ‘E!’,” *Philosophical Studies: An International Journal for Philosophy in the AnalyticTradition* 9, no. 4 (June 1958): 60--63.

[87] Jaako Hintikka, “Existential Presuppositions and Existential Commitments,” *The Journal of Philosophy* 56, no. 3 (January 1959): 125--137.

In almost the same time with Hintikka's paper, Hugues Leblanc and Theodore Hailperin published another paper following Leonard's proposal.[88] They presented a natural deduction system of free logics without the predicate '*E*!'. The amended rules they provide amount to restricted versions of EG and UI.

These four papers, all published in the same period, can be seen as the first publications in free logics. Starting with Lambert's "Existential Import Revisited", a number of papers were published where various formalizations of free logics have been put forward. After a decade, the development of free logics took a turn into semantics and the applications of these systems.

Conclusion

The aim of this chapter was to provide a survey of free logics. I started my presentation with an overview of the definitions. I identified commonalities between these definitions and formulated the characteristics of these systems. The second section was devoted to proof theory. I presented axiomatic systems of positive and negative free logics. I also concentrated on the issue of how systems of free logics syntactically differ from CQT. In the third section, I gave an overview of the different semantic approaches in free logics. For each kind of free semantics I have discussed, I mentioned some of its major strengths and weaknesses. I concluded the chapter with a brief section about the origins of free logics.

[88] Hugues Leblanc and Theodore Hailperin, "Nondesignating Singular Terms," *The Philosophical Review* 68, no. 2 (April 1959): 239--243.

CHAPTER III

WHY FREE LOGICS?

A Brief Introduction

In the preceding chapter, we have seen how free logics are defined as well as some of the formal systems and semantics of certain kinds of free logics. By now, we must have a good idea what free logics are and in what respects they differ from standard systems of logic. Free logicians call for a modification of CQT; nevertheless, neither the 'undiscovery' of Sandy Island nor any other phantom island or some other non-existent object is enough to motivate such a substantial revision. It must be very natural then for the following question to arise:

Q2 Why should one adopt free logics instead of CQT?

This chapter is centered around this question.

Note that Q2 can be addressed as a two-fold question. On the first level, it is concerned with the motivations behind the development of free systems since each of them comprises an answer to Q2. As to that matter, the previous presentation must have already hinted at some of the major sources of motivations to develop free logics. However, the sole practice of describing the motivations should not be enough for our purposes. After all, ours is not just a historical concern for why and how free logics have emerged; we also want to decide whether motivations succeed in providing good reasons for a revision of CQT in the way free logicians suggest. In other words, we want to find out whether there is a good justification for the adoption of free logics. Accordingly, the second part of this study is about the motivations behind free systems and their critical evaluations.

To this end, I address six different kinds of motivations that one usually finds in the literature, either explicitly or implicitly, and a further one that I propose. First, let me briefly introduce these six motivations.

The first motivation is based on the idea that logic should be independent of ontological assumptions but that CQT contains an existence assumption and that this assumption is an ontological one. The second motivation proceeds from certain instances of classically valid inference patterns that seem to have true premises but false or controversial conclusions. The idea is that since these inference patterns are invalid in free logics, undesirable results following from them are blocked in these systems. The third motivation rests on the claim that free logics have a wider range of applicability in comparison to CQT. To be more specific, the assertion is that, free logics, in contrast to CQT, can be directly applied to statements that include ESTs. The fourth motivation is concerned with the incongruent treatments of singular and general terms in CQT. The underlying claim is that free logics offer a more 'consistent' treatment of terms. The fifth motivation is based on the claim that free logics reflect the logic of ordinary language better than CQT. The sixth motivation is based on the relation between logical form and empirical facts. The idea is that logical form should be independent of empirical matters, which does not seem to be the case in CQT.

For each kind of these motivations, I offer a thorough discussion whose general outline will be usually as follows: First, I introduce the motivation that will be addressed in that section. I provide quotations from free logicians whenever it is possible. After having clarified the motivation in question, I put it into an argument form. Finally, I offer a critical assessment of the argument I have provided to decide whether or not the relevant motivation is successful in justifying free logics. Note that, I will occasionally go back after a negative evaluation of an argument and consider other possible formulations for it. Roughly speaking, this is how I proceed in the discussions about the motivations that I have listed above.

The conclusion I will arrive at the end of this discussion is that neither of these motivations is conclusive enough to justify a modification of CQT in a way free logicians suggest. I close the chapter with a further kind of motivation that I believe hints at the right justification of free logics. This motivation rests on the claim that free logics provide a more suitable setting for a number of contexts compared to their classical alternative. For reasons that will become clear later, unlike the previous motivations, I do not offer a critical evaluation of it. Instead, I simply mention some of the contexts for which free logics may be more 'suitable' than CQT, and address one of them particularly. In this sense, the chapter ends with a suggestion of how a successful argument for free logics might be like.

Before starting, let me digress for a moment to mention a few preliminaries and remarks that will be helpful for the discussion that follows.

Some Preliminaries and Remarks

As the presentation so far might already suggest, the assertion that CQT contains an assumption that all singular terms are denoting (existents) bears a great significance in the adoption of free systems. Most free logicians, including those mentioned in the last section, in some way or the other, felt an uneasiness about this classical assumption and wanted to get rid of it. From a historical point of view, this was for the most part what motivated these logicians to develop free systems of quantification theory. Thus, the claim that CQT inherits an existence assumption played a vital role in the development of free logics.[89]

However, the uneasiness free logicians felt about this existence assumption was not of one kind. Some felt that it ‘contaminated’ logic with ontological facts; others thought that the quantification theory without this assumption would be more powerful in a certain sense, while some others believed that CQT fails to reflect ordinary language because of this assumption. Despite the differences, however, all of these motivations have a common point; they are all grounded on the claim that CQT contains a certain implicit assumption that needs to be abandoned.

Throughout the chapter, I will suggest a number of arguments that rest on the assertion that CQT contains an implicit assumption of singular existence. In other words, all of the arguments that will be put forward in this chapter include either

(P1) CQT contains an implicit assumption of singular existence, i.e., the assumption that all singular terms available in the formal language are denoting (existents),[90]

or some equivalent statement as a premise. Thus, it is convenient to settle beforehand whether this premise is acceptable or not.

Recall from the first chapter, that (P1) can be seen as a consequence of three different premises. First, there is the classical requirement that each singular term of

[89] This is also obvious from the label given to these systems. ‘Free logics’ is short for ‘presupposition-free logics’.

[90] Note that, from now on, whenever I say ‘denoting’ it means ‘denoting an existent’. I will occasionally omit the later part.

a formal language[91] must receive an interpretation in the quantificational domain. Second, a special understanding of the interpretation function is adopted so that it is understood as the denotation relation between singular terms and members of the domain. Finally, quantifiers are given an existential reading so that the domain of quantification becomes the set of all and the only existing things at a given situation. Accordingly, if one wants to reject (P1), one has three options. Let us consider each of these options in turn.

To begin with, the first condition is a standard requirement of all classical systems. As for the second condition, although there may be alternative approaches, it is nevertheless very natural to see the interpretation function as assigning denotations to singular terms. Finally, as far as CQT is considered, the third condition is also sound. To be sure, one can reject this specific interpretation of quantifiers and adopt some other; nevertheless, it is far from controversial that this existential reading of quantifiers is an established part of CQT. To conclude then, (P1) is acceptable.

Note that this was mainly a semantic reasoning for the truth of (P1). After all, it is classical semantics in which a model is defined in a way that each singular term available in the language is assigned an interpretation in quantification domain. Starting off from a syntactical point of view, it might be even easier to support (P1). To this end, it should be enough to recall that

(13) $\exists x(x = t)$

[91] Let me underline a terminological matter. As already noted in the first chapter, I use the expression '(empty) singular term' autonomously with respect to ordinary and formal languages. In a formal language, a singular term is simply a term (an individual constant, a definite descriptions, a n-place function symbol followed by n terms, etc.) that stands for a singular term of ordinary language. That being said, there will be times when both kinds of expressions are required to be mentioned in the same context. For those situations, I shall adopt the following convention in order to prevent a possible misunderstanding: I shall reserve 'EST' to formal language and 'non-referring (or non-denoting) expression' to ordinary language.

is a theorem schema in CQT. It intuitively says that there is something that is the denotation of the singular term *t*, whatever that may be. To conclude, I can see no reason for why one doubt the truth of (P1).

As already stated above, in what follows, I will offer a number of arguments in favor of free logics[92] that include (P1), or some equivalent of it, as a premise. I can think of two different forms of arguments. I shall call them as the strong and the weak forms. The strong argument form, shortened as SA-form hereafter, is as follows:

(P1) CQT contains an implicit assumption of singular existence, i.e., the assumption that all singular terms available in the formal language are denoting.[93]

(P2) The presence of this assumption in CQT is a defect.

(P3) Therefore, the assumption of singular existence should be abandoned.

To obtain the weak argument form, WA-form hereafter, it is enough to replace (P2) in SA-form with the following premise:

(P2*) CQT is improved if the assumption of singular existence is abandoned.

Without a doubt, both of the arguments are invalid in the forms given above. Nevertheless, they both contain implicit premises that have been omitted here but can

[92] To be more precise, in favor of the abandonment of the classical assumption that each singular I s denoting.

[93] Let me clear a terminological matter once and for all. I will occasionally use the shortened expression 'assumption of singular existence' sometimes also prefixed by 'classical' or 'implicit'. As should be clear from the premise, the full version is as follows: 'The (classical/implicit) assumption that all singular terms available in the formal language are-denoting (existents).'

be easily added to make the arguments valid.[94] Thus, I will occasionally take it granted that (P3) follows from the remaining premises in each case. In addition to this, I will also assume, at least for the most part, that (P3) leads to free logics.[95]

Given these assumptions, since we have already cleared (P1), the success of any argument based on one of these forms rests on the decision whether (P2) or (P2*) is accepted or not. Accordingly, in the discussion to follow, I may sometimes refrain myself from giving the argument in its complete form and concentrate on the question whether these key premises are justified or not. Obviously, there are more than one way to argue in favor of (P2) or (P2*), so let us begin with what is perhaps the most natural but also the most naive one.

The Motivation from the Ideal of Logic

As we have seen, in classical logic one seems to be committed to the assumption that all singular terms have existential import. A very natural response to this would be to regard this assumption as introducing into logic an ontological commitment.[96] Accordingly, if one believes that logic should be 'pure' or 'neutral' in the sense that it should be independent from all sorts of empirical, epistemological or ontological facts, then it might be very natural for her to call for the abandonment of the

[94] In order to make the arguments valid, first of all, we need an additional premise that will logically connect (P2) and (P2*) with (P3). As regards the SA-form, for instance, one must add something like the following: 'If the presence of an assumption in a quantification theory is a defect, than one should get rid of it'. Also note that neither of the arguments has the conclusion that one should adopt free logics instead of CQT. If that is what one intends to argue for, then both arguments are in need of an additional premise that will make the intended conclusion to follow from (P3).

[95] As already noted, the assumption of singular existence is a consequence of three premises. Note that only one of them, i.e., dispensing with the requirement that each singular term must receive an interpretation in the domain, leads to free logics.

[96] The fact that the existence of God can be easily proven in CQT may be considered as a demonstration of this general fact.

assumption in question.[97] This is roughly the claim underlying the motivation in question. Bencivenga, in the following passage, describes such a position:

> Whether they regard metaphysics as sheer 'nonsense' or as a set of 'synthetic' statements to be neatly distinguished from the 'analytic' ones constituting their discipline, many logicians like logic to be metaphysically 'pure', or not to carry any metaphysical 'baggage' (…)[98]

As a matter of fact, this kind of motivation is usually associated with another existence assumption of CQT and the call for its abandonment. Nevertheless, it may be easily adapted to serve as an argument in favor of free logics. This is indeed what we will do in what follows, but first, let us consider the original motivation.

The assumption that all singular terms of the formal language are denoting is certainly not the only assumption of CQT. Above all, classical systems also require the quantificational domain to be non-empty. Provided that the domain of quantification is interpreted as the set of existing things, it follows that CQT is committed to the existence of at least one thing. Hence, the requirement of a non-empty domain operates as another existence assumption in CQT. Syntactically, this can be best seen if we consider that formulas such as

(14) $\exists x(x = x)$,

(15) $\exists xPx \lor \exists x \neg Px$,

as well as any instance of the following schema

(16) $\forall xA \supset \exists xA$

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[97] The reason I have called this sort of a motivation naive above should be more or less evident from this claim. I believe that in logics, as in any place else, it is not possible to find such a 'pureness'. I will come to this later.

[98] Bencivenga, "Free Logics," 150.

are theorems of $FOL^=$.

Some philosophers/logicians felt an uneasiness about this classical requirement of CQT for reasons that will be clear in a moment, and some of them argued in favor of its abandonment. Let us look at a few passages that would exemplify this sort of motivation, which are mentioned by A. P. Rao in a similar discussion.

Russell, in his *Introduction to Mathematical Philosophy*, calls this requirement a "logical impurity." In a footnote, he writes the following: "The primitive propositions in *Principia Mathematics* are such as to allow inference that at least one individual exists. But I now view this as a defect in logical purity."[99] Carnap, in the following passage from his *The Logical Syntax of Language* shows that he shares a similar position:

> If logic is to be independent of empirical knowledge, then it must assume nothing concerning the existence of objects (...) But if in order to separate logic as sharply as possible from empirical science, we intend to exclude from the logical system any assumption concerning the existence of objects (…)[100]

Finally, A. P. Rao quotes Ludwig Wittgenstein as saying the following in a letter written to Russell: "A proposition like $\exists x_i\,(x_i = x_i)$ is, for example, really a proposition of physics. The proposition $(x_i)\,(x_i = x_i) \rightarrow \exists x_j\,(x_j = x_j)$ is not a proposition of logic. It is for physics to say whether anything exists."[101]

To sum up, these philosophers, among many others, believed that the assumption that at least one thing exists must be grounded on empirical knowledge and that logic should be ideally independent of empirical facts. As regards the

[99] Bertrand Russell, *Introduction to Mathematical Philosophy*, 2nd ed. (London: G. Allen & Unwin, ltd., 1920), 203.

[100] Rudolph Carnap, *The logical syntax of language*, trans. Amethe Smeaton (Chicago & La Salle, IL: Open Court, 2002), 140.

[101] A. P. Rao, "A Survey of Free Logics," *Modern Logic* 6, no. 2 (April 1996): 126.

philosophers quoted above, their attitude in this respect did not lead them to revise their quantification theory. Some others, however, did this and developed systems of logic that allow for empty domains. As noted in a previous footnote, such systems are usually known as inclusive logics after Quine.

The moral to be drawn from the above discussion is that the attitude of these philosophers towards the assumption in question has been a major source of motivation for the development of inclusive logics. To be sure, that kind of motivation can be easily adapted to free logics. After all, free logics also disallow a classical requirement even if it is a different one. The only difference may be that the assumption of singular existence is usually regarded as an ontological assumption instead of an empirical one. Thus, a simple change of ‘empirical’ with ‘ontological’ above should give us pretty much what we need.

In view of this, one can argue in favor of free logics in the following way. Since the classical assumption of singular existence is a ontological assumption in essence, and since logic should be free of ontological knowledge, the presence of this assumption is a defect of CQT, and therefore, ought to be abandoned. The following passage from Lambert provides us with a description of this sort of a position:

> For example, there is a primordial intuition that logic is a tool that the philosopher uses (or should use, given Langford’s famous complaint) and ideally should be neutral with respect to the ontological, epistemological, ethical, etc., truth just as the various mathematical tools available to the empirical scientist -calculus, statistics, algebra, etc.- are presumed neither to create nor to pre-determine the empirical facts. The tool of logic is used to help decide among the various opinions what the philosophical truth really is -or at least it should according to primordial intuition (and Langford.) So if there are preconditions to logic that have the effect of settling what there is and what there is not, they ought to be eliminated because they corrupt the ideal of logic as a philosophical tool.[102]

[102] Karel Lambert, “The Philosophical Foundations of Free Logic,” in *Free Logic: Selected Essays*, ed. Karel Lambert (Cambridge: Cambridge University Press, 2003), 141.

In order to make a systematic evaluation of this motivation, let us put it into an argument form. I shall begin with the strong version. The argument, which I will label as SA1, is as follows:

($_s$1.1) CQT contains an implicit assumption of singular existence, i.e., the assumption that each singular term available in the formal language is denoting.

($_s$1.2) Logic should be free of existence assumptions.

($_s$1.3) Therefore, the assumption of singular existence should be abandoned.

Certain premises of SA1 are missing but I assume that they can be easily added in a way that was already hinted in a previous footnote.[103] For the time being, I assume that the argument is valid. Accordingly, since we have already cleared ($_s$1.1) and ($_s$1.3) is merely a consequence, the decision whether the argument is sound or not ultimately depends on how ($_s$1.2) is evaluated. Therefore, let us see if this premise is justified or not.

Keeping the passage from Lambert in mind, one possible justification for ($_s$1.2) may be the claim that logic should be independent of ontology all together with the assertion that existential assumptions are ontological in essence.[104] Let us label any justification developed along this line as J($_s$1.2). To put it in a form we can work with, it comprises of the following assertions:

J($_s$1.2)$_i$ Existence assumptions are ontological in nature.

J($_s$1.2)$_{ii}$ Logic should be independent of ontological facts.

[103] See the footnote 94.

[104] Leonard, for instance, in a talk he gave, says, "Any assumption that there is a language with such and such variables and such and such or so and so many substituends for those variables is an ontological assumption." Henry S. Leonard, "Essences, Attributes, and Predicates," *Proceedings and Addresses of the American Philosophical Association* 37 (1963-1964): 37.

I do not find J($_s$1.2) as a tenable justification for ($_s$1.2). Some of these reasons are in order.

To be sure, J($_s$1.2)$_i$ is a claim that is not easy to evaluate. Not only it may have has been ill formulated, its content also seems to be somewhat vague. Hence, we better suspend judgment and put it to one side for the time being. Let us instead concentrate on the other claim.

First of all, a closer look at J($_s$1.2)$_{ii}$ would easily reveal that J($_s$1.2) does not actually go very far towards justifying ($_s$1.2). After all, what it basically says is that logics should be free from assumptions that are grounded on ontological knowledge. That assertion, however, may not be that much different from the original premise ($_s$1.2) in content. If so, J($_s$1.2) can hardly be considered as a justification for ($_s$1.2), if it is tantamount it.

Nevertheless, let us assume otherwise, that is, that J($_s$1.2) is a proper justification for ($_s$1.2). In that case, just like ($_s$1.2), J($_s$1.2)$_{ii}$ would also be in need of a justification. Needless to say, one cannot appeal for this to the idea that 'what there is and what there is not' is a matter better be resolved somewhere else than in logic because of the obvious threat of circularity.

But perhaps one does not need a justification for J($_s$1.2)$_{ii}$. Recall the passage quoted from Lambert above. The assertion that logics should be free of all sorts of non-logical truth is called a "primordial intuition" there and this choice of words may be for a good reason. Intuitions, after all, usually do not need justifications. They are simply as they are, one either agrees with an intuition or not. If so, the question that needs to be answered is whether J($_s$1.2)$_{ii}$ is an acceptable intuition or not. And as to that matter, I would go with the second option. That is, I do not hold this 'intuition'. In what follows, I will suggest two different reasons for this.

First of all, $J(_s1.2)_{ii}$ rests on a mere idealization. I do not think that logic can be as pure (or neutral) as one might desire. After all, the assumption in question is not the only assumption that is present in classical logic, there are others as well, and some of them, if not all, seem to be founded on empirical, ontological or epistemological truth.

To give a simple example, in classical semantics, each singular term is associated with only one member of the quantificational domain. Given the standard interpretations, one might ask why singular terms is such that they denote only one thing?[105] This assumption does not seem to be founded on logical truth either. On the other hand, it is quite a standard convention, and to my knowledge at least, no one suggests dropping it. It is possible to come up with other examples; nevertheless, my intention should be already clear. CQT contains a number of assumptions that reside outside logical truth but not all of them are abandoned. To conclude, any desire to 'free' logics from all sorts of truth other than the 'logical' seems unrealistic.

Second, if we are to accept $J(_s1.2)_{ii}$, then in the name of being consistent, we should also abandon all sorts of assumptions of similar character. To put it differently, with this kind of a motivation in mind, dispensing with one existential assumption while retaining others would be inconsistent; if you are to abandon one, then you should abandon all. For the least, a free quantification theory developed through this line of thought should allow for empty domains.[106] This reasoning certainly applies to any other classical assumption as well. Hence, it turns out that SA1 is not much of an argument for free logics, but for a logic that is independent of all sorts of facts that reside outside logical truth. For instance, with this

[105] Or, one and the same thing for that matter.

[106] Such systems of logic, i.e., systems that are both free and inclusive, are usually called universally free logics.

understanding, SA1 can be equally adapted for universally free logics. However, just like there are ‘un-free’ systems of inclusive logics, there are free systems that are not inclusive as well. In conclusion, the claim J($_s$1.2)$_{ii}$ seems to be too strong in the sense that it leads to more than we need, if not to more than we can afford.

In our discussion regarding J($_s$1.2), we have so far concentrated on J($_s$1.2)$_{ii}$. As for J($_s$1.2)$_i$ let us at least say that it is a claim that is far from uncontroversial. The assertion that existence assumptions are ontological in nature is an assertion that is at least in need of a justification. This point, together with the reasons for rejecting J($_s$1.2)$_{ii}$, should be enough to conclude that J($_s$1.2) is not tenable. To sum up the whole discussion about ($_s$1.2), our conclusion is that it is not justified at least in the way suggested in the passages quoted above.

Another difficulty SA1 seems to be facing is concerned with its form. Recall that we have assumed above that the argument is valid, or can be made so provided that the omitted premises were added. That means that we have assumed that ($_s$1.3) follows from the remaining premises. This, however, might not be the case. Let us consider this possibility now.

As a matter of fact, SA1 bears an important weakness just like any other argument to establish a *desiradum*. Even if we accept the premise ($_s$1.2), this does not necessarily lead to the conclusion that CQT is in need of a revision. For although not stated explicitly, ($_s$1.2), and its alleged justification J($_s$1.2), is about an ideal. For this reason, the best one can achieve along this line of reasoning is that the assumption of singular existence should be ideally abandoned. Accordingly, in order to make the argument valid, ($_s$1.3) needs to be replaced with,

($_s$1.3*) Therefore, the assumption of singular existence should *ideally* be abandoned.

In this way, the validity is saved. At the same time, however, things seem a bit different now. Above all, it is not that easy anymore to reach from ($_{s}1.3^{*}$) to the desired conclusion that free logics should replace CQT. After all, all that ($_{s}1.3^{*}$) says is that ideally we should dispense with the assumption of singular existence. In practice, however, things may be different.

Carnap, for instance, with regard to the classical requirement that the quantificational domain be non-empty, says in the very same passage quoted above, that "in practice, this distinction is immaterial since we are usually concerned with non-empty domains."[107] Accordingly, although he thinks that logics should ideally not assume the existence of things, this belief does not lead him to revise the theory of quantification he adopts, for, in practice, systems with non-empty domains are as good as systems allowing for empty domains.[108]

It seems that the same line of reasoning can be applied to our argument as well. After all, logic is a tool that has a wide range of applicability and some of the decisions about it can be made with respect to the needs of the particular context to which the logic is applied to. To put it differently, even if we admit that abandoning the assumption of singular existence is more 'ideal', that does not mean that we should actually do this. To conclude, it seems that ($_{s}1.3^{*}$) does not lead to the conclusion we want to arrive at ultimately, i.e., that the assumption of singular existence should be dispensed with.

To be sure, there are other ways for saving the validity of the argument. For instance, one might try to change ($_{s}1.2$) instead of ($_{s}1.3$). One of the obvious candidates is as follows:

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[107] Carnap, *The Logical Syntax of Language*, 140.

[108] As a matter of fact, for matters of convenience, systems requiring non-empty domains can even be for the better.

($_s$1.2*) Logics should be free of existence assumptions, otherwise they are defective.

In this way, 'the ideal' in a sense becomes connected with 'the practical'. Consequently, the previous objection from the ideal argument would not apply to it anymore. For it can be easily shown that ($_s$1.3), not ($_s$1.3*), follows from the remaining premises. Let us consider this alternative now.

Although this strategy seems to be more promising at first sight, ($_s$1.2*) has an important drawback: It seems too strong. To put it simply, not every divergence from an ideal can be seen as a defect that is in need of a fix. In logic, particularly, there are many requirements that may not be ideal; nevertheless, they seem to serve their purposes. The classical requirement that the quantificational domain be non-empty may be a good point to illustrate this point. As we have already noted with respect to Carnap's passage above, this requirement may not be ideal; however, it also seems far from being a defect that needs to be fixed. On this ground, I think that ($_s$1.2*) must be rejected.

Now that we have seen that SA1 is too strong, perhaps we can try to weaken the argument. One possible formulation, which we will label as WA1, is as follows:

($_w$1.1) CQT contains an implicit assumption of singular existence, i.e., the assumption that each singular term available in the formal language is denoting.

($_w$1.2) CQT is improved if one of its existence assumptions is abandoned.[109]

($_w$1.3) Therefore, the assumption of singular existence should be abandoned.

[109] One of the many other possible formulations would be as follows: A logical theory with fewer existence assumptions is better.

As before, I shall assume that the argument is valid. Hence, its success depends again on how ($_w$1.2) is to be evaluated. In this regard, the first thing to notice is that the same objections mentioned above are also valid for ($_w$1.2) or can be easily adapted for it. Let us briefly consider them in turn.

To begin with, ($_w$1.2), just like ($_s$1.2), also appears to be in need of a justification. As before, however, any appeal to an alleged relation between ‘the logical’ and ‘the ontological’ may not be enough for this purpose. It also does not seem to be less depending on an ideal. Furthermore, once ($_w$1.2) is accepted, it becomes applicable to all sorts of existential/ontological assumptions as well. One interesting fact to note is that the reasoning underlying ($_w$1.2) can be repeatedly applied to a theory until it is exhausted of all existence assumptions. In this respect, ($_w$1.2) seems to be boiling down to the previous claim that logics should be free of existence assumptions, namely ($_s$1.2). Needless to say, it would eventually face the same difficulties with ($_s$1.2). Finally, one can easily note that the objection from the relation between the ‘ideal’ and the ‘practice’ would also extend to WA1. To sum up this discussion then, I shall say that the weak version of the argument faces more or less the same difficulties with the strong one.

One possible way out of these difficulties might be to leave the justification based on the ideal relation between logic and existential assumptions aside, and modify the argument so that it becomes about any assumption of any sort. More precisely, ($_w$1.2) can be reformulated in the following way:

($_w$1.2*) CQT is improved if one of its assumptions is abandoned.[110]

[110] In a parallel to footnote 109, the premise could be formulated as follows: A logical theory with fewer assumptions is better than one that has more assumptions.

As an example along a similar line of thought, consider the following passage from Hintikka:

> Since this system is obtained simply by omitting one of the old rules, it is more elegant than the traditional quantification theory in the philosophically relevant sense in which the merits of a system are measured by the paucity of its basic assumptions.[111]

It appears that one might even drop 'CQT' in ($_w$1.2*) and assert the same for all theories of any sort. Now let us discuss whether this claim is tenable.

At first glance, ($_w$1.2*), at least as a general principle, looks promising. After all, there is no reason for why economy or simplicity, as elsewhere in science, should not be preferable in logics. That being said, the premise has a major drawback. It is too general to allow for a critical evaluation. Apparently, in order to decide on its faith, one needs to answer the following question: Are logical theories always improved if some of their assumptions are dropped? Although 'economy' may be preferable in general, and for a good reason I think, I shall still answer this question in the negative. Yet it does not seem too easy to challenge ($_w$1.2*) in this general way, so let us instead focus on particular cases where this claim may fail.

To this end, recall that quantifiers are existentially loaded in CQT. That is to say that quantifiers are interpreted as having existential import in standard systems. Now having ($_w$1.2*) in mind, it is very natural for the following questions to arise: (i) Should this assumption, i.e., the assumption that quantifiers are existentially loaded, be also dropped? (ii) If so, why? (iii) And in what sense will the theory improve?

More questions might follow but our intention should be already clear by now. If one answers the first question above in the negative, that constitutes a clear

[111] Hintikka, "Existential Presuppositions and Existential Commitments," 131--32.

counterexample to ($_{w}1.2^{*}$). On the other hand, if she answers in the positive but cannot give precise answers to follow-up questions, then it might be the case that she does not have enough reason to accept ($_{w}1.2^{*}$). Of course if she answers the first question in the positive and also provides precise answers to follow-up questions, this challenge does not pose a problem for ($_{w}1.2^{*}$). In this case, however, I think one can come up with other assumptions of classical logic that will eventually face the same questions above. All in all, I do not find ($_{w}1.2^{*}$) as a general principle acceptable. Therefore, I conclude that WA1 is inconclusive.

Perhaps, the moral to be drawn from the example given above is that a general principle like ($_{w}1.2^{*}$) must be evaluated with respect to particular cases. To put it in a different way, whether or not a theory is improved by abandoning an assumption may actually depend on the context it is applied to. That is to say, one has to show that CQT is improved in a certain way if the assumption of singular existence were abandoned. Roughly speaking, this amounts to what we will try to do in the remaining parts of this study.

To sum up the discussion, for the reasons I have explicitly stated above, I find neither SA1 nor WA1 tenable. I therefore conclude that the motivation based on the ideal of logic, at least in the way formulated above, does not go very far towards justifying free logics.

The Motivation from Undesirable Results

Our previous discussion revealed that the argument from the ideal of logic does not provide us with enough reason for the abandonment of the classical assumption of singular existence. Perhaps it would be a better strategy to concentrate on the consequences of this assumption, rather than itself. To this end, recall from the first chapter that certain rules of CQT make it possible to infer false conclusions from true statements.[112] The following passage from Quine should be enough to refresh our memory:

> But this rule of inference leads from the truth '(*x*) (*x* exists)' not only to the true conclusion 'Europe exists' but also to the controversial conclusion 'God exists' and the false conclusion 'Pegasus exists' if we accept 'Europe', 'God', and 'Pegasus' as primitive names in our language.[113]

In free logics, on the other hand, as we have already seen, the rule in question is invalidated, and thereby, inferences as such are blocked. Accordingly, the desire to avoid *unwanted results*[114] can be another source of motivation for the adoption of free systems. More precisely, some free logicians see the presence of the assumption of singular existence as paving way for some unwanted results, and either because they see this as a defect or as a disadvantage for the quantification theory, they suggest disallowing it. Roughly speaking, this is the kind of motivation that we will address in what follows.[115]

As before, the first thing to do is to make the relevant motivation as clear as possible. To this end, I will present a few examples of what we have called unwanted results above that supposedly follow from certain classical rules or laws. Although I will offer a few comments and remarks about the general character of these results,

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[112] or controversial conclusions as in the case of the existence of God, for instance.

[113] Willard Van Orman Quine, *Mathematical Logic*, rev. ed. (Cambridge, MA: Harvard University Press, 1981), 150.

[114] I use the expressions 'unwanted' and 'undesired' as to include both false and controversial situations. Accordingly, in the discussion to follow, an 'undesired result' means either of the following: (i) An inference with a true premise and a false or controversial conclusion, or (ii) an instance of a law that is false or controversial.

[115] Note that in the first chapter where we have introduced free logics, we made use of some examples of undesired results. Hence, the present discussion shows some parallelism with the previous presentation. Having said this, I will try to avoid repeating myself as much as possible.

the examples themselves should be for the most part enough for our purposes. After clarifying the motivation, I will formulate it as an argument for free logics and offer a critical evaluation of it. So let us start with examples. Leonard, in his "The Logic of Existence" gives the following inference as an example:

(17) Santa Claus lives at the North Pole;

so,

(18) There is something living at the North Pole.

The inference from (17) to (18) seems to be authorized by the rule of existential generalization (EG), i.e., the rule that permits one to infer from any statement of the form 'such-and-such is so-and-so', the conclusion that 'there is something that is so-and-so'.[116] As a valid rule of inference, it must be truth preserving; that is, for every inference having the form of EG, its conclusion must be true if its premise is true. However, that does not seem to be the case with the inference from (17) to (18). More precisely, it seems to be an instance of EG and (17) seems to be true; nevertheless, assigning the same truth-value to (18) would probably not be in agreement with our intuitions. If so, something must have gone amiss. For those who may have doubts about the falsity of (18) and have no objection to accept 'existence' as a first order predicate, a better example may be the inference from the statement

(19) Pegasus does not exist,

to the conclusion that

[116] In symbols, infer $\exists xA$ from $A(t/x)$, i.e., $A(t/x) \vdash \exists xA$.

(20) There is something that does not exist.[117]

This time, the truth-value to be assigned to the conclusion seems less controversial; for most people at least, (20) is simply false.

A natural anticipation at this point is that the dual of EG, i.e., the rule of universal instantiation (UI), which permits one to infer from any statement of the form ‘everything is such-and-such’, the conclusion that ‘*t* is such and such’, has similar undesirable instances, as well. Compared to its dual, it may be harder to come up with such examples for there are not many predicates that apply to everything. Even so, there is at least one such predicate that most people would hopefully agree. Consider the inference from

(21) For all objects *x*, there is an object *y* such that *x* is the same as *y*[118]

to the conclusion that

(22) There is an object *y* such that Vulcan is the same as *y*.[119]

The inference seems to be an instance of UI, hence it must be truth preserving as a valid inference. However, although (21) seems to be a true statement, the intuition tells us that (22) is false.

Our final example of an unwanted result is a line of reasoning that will utilize both UI and universal generalization (UG). For this, we will also require identity theory. In $FOL^{=}$,

[117] Note that ‘something does not exist’ and ‘there exist something that does not exist’ express the same proposition contained in (20). It may seem clearer from these formulations that (20) is false.

[118] Itself of course.

[119] The example is due to Lambert, which can be found in Lambert, “Free Logics,” 263.

(23) $\forall x(x = x)$

is a theorem that follows from *the axiom of self-identity* and UG. From (23), it follows by UI

(24) Sandy Island = Sandy Island

if we substitute 'Sandy Island' for x in (23); and by applying EG to this statement, we obtain

(25) There is something the same as Sandy Island

or simply, that

(26) Sandy Island exist.[120]

In light of the discovery that there is no Sandy Island, (26) is certainly false. Thus by supposedly valid inferences, we seem to have arrived at a false conclusion from a true premise again.[121]

Needless to say, any singular term can be substituted for 'Sandy Island' in the above reasoning. That is to say that it not only yields a false conclusion such as (26) but also controversial ones such as

(27) God exists

or,

(28) Homer exists,

[120] I take it granted that $\exists x(x = t)$ expresses singular existence.

[121] One can of course argue that the identity theory is to blame here; however, since this objection clearly does not apply to the previous inferences, I will simply omit it.

of which we do not know whether they are true or not. Nevertheless, according to the above reasoning, they must be true since they follow from an axiom by valid inferences. Obviously, something must have gone wrong here.

Another way to look at the same problem is to consider that

(29) $\exists x(x = t)$

is a theorem of $FOL^{=}$.[122] It intuitively says that for every singular term *t*, whatever it is, there is something in the quantificational domain such that it is the denotation of *t*. Obviously, in case an EST (or a singular term that may be empty) is substituted for *t* in (29), it would yield a false (or controversial) conclusion.

The examples given so far should be enough for our purposes. The moral to be drawn is that certain classical rules are unreliable in their applications with respect to empty (or may-be-empty) singular terms. To be precise, if such terms were substituted for individual constants, these rules may lead to false (or controversial) conclusions from true premises.[123]

Before going further, we will mention one last example, which is a bit different from the ones we have seen and is also from a more recent discussion. Timothy Williamson, in his paper titled “Necessary Existents”[124] presents an argument for the claim that, necessarily, he exists; in other words, that he is a ‘necessary existent’. Andrew Bacon, in return, argues in his “Quantificational Logic and Empty

[122] The reasoning given above already entails that. (1) $\forall x(x = x)$ [Ax.], (2) $t = t$ [1, UI], (3) $\exists x(x = t)$ [2, EG]

[123] This is obviously a serious problem, for it shows that the system is unsound.

[124] Timothy Williamson, “Necessary Existents” in *Logic, Thought and Language,* ed. Anthony O'Hear (Cambridge: Cambridge University Press, 2002), 233--51.

Names"[125] that Williamson's conclusion is a result of combining CQT with modal logic. His reasoning is as follows: Since (29) is a theorem schema in CQT, by the rule of necessitation[126] we obtain

(30) $\Box\exists x(x = t)$,

and by applying UG to (30), we get

(31) $\forall y\Box\exists x(x = y)$,

which intuitively says that 'necessarily, everything exists'. This conclusion certainly does not seem to be very intuitive. According to Bacon, there are two possible ways out of this difficulty, one can either drop the necessitation rule or modify UI and EG.[127] Apparently, his choice is the latter.

Accordingly, it seems that CQT not only requires Sandy Island to exist, but also to exist necessarily if combined with standard Kripke-style modal logic. About the truth-value of this conclusion, there would be little, if any, disagreement. Obviously, we cannot dwell on all the details of Bacon's argumentation; however, it may be helpful to note that if the above reasoning is correct it shows that the problem with EG and UI may not be such that it only arises in connection with ESTs as our

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[125] Andrew Bacon, "Quantificational Logic and Empty Names," *Philosophers' Imprint* 13, no. 24 (December 2004): 1--21.

[126] According to this rule, if A is a theorem, then $\Box A$ is likewise a theorem, i.e., if $\vdash A$, then $\vdash\Box A$

[127] Let us note that not every modification of UI and EG leads to free logics. For instance, as has often been noted, the axiom schema $\forall x\forall yA \supset \forall y\forall xA$ appeared in the same year both in Lambert's formalization of positive free logic and Kripke's formal system of quantified logic. The schema in effect restricts the application of UI to members of the quantificational domain. Another example is due to Lambert. In his "The Philosophical Foundations of Free Logic," he presents the quantificational part of the formal system of modal logic that is developed by Kit Fine. Although Fine's system is not free, it is syntactically equal to the formal system of free logic that is introduced by Lambert and Meyer in their "Universally Free Logic and Standard Quantification Theory." Needless to say, both systems included the axiom schema $\forall xA \supset (E!t \supset A(t/x))$. For the relevant discussion, see Lambert, "The Philosophical Foundations of Free Logic," 133.

previous examples suggest. After all, the conclusion that Williamson (or any other person or object for that matter) is a necessary existent seems problematic enough, and if it is EG and UI that is to be blamed here, that would probably give us enough reason to modify CQT in a way free logicians suggest.

The examples given so far must have made the present motivation clear. In short, we can say that CQT leads to some undesirable results, and the underlying problem seems to be the classical assumption that all singular terms are denoting. Now let us formulate this as an argument. We will begin with the strong version.[128] One of the many possible formulations is as follows:

($_s$2.1) CQT contains an implicit assumption of singular existence, i.e., the assumption that all singular terms available in the formal language are denoting.

($_s$2.2) Certain rules of CQT rest on the assumption of singular existence.

($_s$2.3) In a particular application of one of these rules, if the assumption of singular existence is not satisfied, it leads to a false or controversial conclusion from a true premise.

($_s$2.4) The presence of the assumption of singular existence in CQT is a defect.

($_s$2.5) Therefore, the assumption of singular existence should be abandoned.

Let us call this argument as SA2. As before, we assume that the argument is valid, or more precisely, can be easily made so.[129] Now let us consider whether it is conclusive or not. To this end, I shall consider the premises in turn.

[128] Roughly put, the idea is as follows: As a result of the assumption of singular existence, some instances of classical rules or principles yield falsehoods in their applications to ordinary discourse. Therefore, this assumption makes CQT unsound. Also note that in the formulations to follow, I will omit the law versions of EG and UI and mention only rules. This is just a matter of convenience.

[129] To make the argument valid, first of all, we need premise(s) that would logically connect ($_s$2.3) with ($_s$2.4). Although a bit informal, one such possible addition is as follows: (i) There are particular

To begin with, the premise ($_s$2.2) seems justified enough, at least with the standard interpretation given to quantifiers. Intuitive readings of the relevant rules or laws can make this point clearer. For this purpose, consider the law version of UI, i.e., the principle of specification (US), for instance. Given the standard interpretation of quantifiers, US is read as follows:

(32) If every *existent* thing is such and such, then *t* is such and such.

If this is the correct reading of US, it is more obvious in this form that US rests on the assumption that the singular term *t*, whatever it may be, has existential import. After all, the inference from the antecedent that says that 'being such-and-such is applicable to every existing thing' to the consequence that '*t* is such-and-such' is acceptable only if *t* denotes an existent.

Another support for ($_s$2.2) can be found in a response of Frege to Bernhard Punjer about the validity of EG.[130] Apparently, Punjer had challenged the validity of EG, on the ground that some singular terms can be empty. In particular, he claimed that the inference from 'Sachse is a man' to 'There exist a man' is valid only in presence of the additional premise 'Sachse exists'. In Lambert's translation, Frege's response was as follows:

> If 'Sachse exist' means 'The word 'Sachse' is not an empty sound, but designates something' then it is correct that the condition be fulfilled. However, this condition is not a new premise, but the obvious precondition of all our words. *The rules of logic always assume that the*

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applications of these rules in which the assumption of singular existence is not satisfied. (ii) These rules may sometimes lead to false or problematic results. (iii) If certain rules may sometimes lead to false or problematic results as a result of an assumption being unsatisfied, the presence of this assumption in that system is a defect. Note that we must also add a premise to connect ($_s$2.4) with ($_s$2.5) such as the following: "If the presence of an assumption in a theory is a defect, it must be abandoned."

[130] Frege's response has been translated and cited by Lambert. See Lambert, "The Philosophical Foundations of Free Logic," 143.

> *words used are not empty*, that sentences are expressions of judgments, that one is not merely playing with words.[131]

I take Frege's words, especially the italicized part, as another support for ($_s$2.2). To be sure, his response is inclusive of all singular terms available in formal language but we will come to this later. For the time being, I conclude that ($_s$2.2) is true.

Let us look now whether ($_s$2.3) is justified or not. It roughly says that if, for instance, EG or UI is to be applied to an EST[132], it may lead to a false result. A simple explanation for this would be as follows. Given ($_s$2.2), the rules of EG and UI are valid only if the assumption of singular existence is satisfied. If a non-denoting expression is substituted for a singular term in one of these rules, the assumption remains unsatisfied, and as a result of this, the rule may lead to a false conclusion. For a more general explanation in support of ($_s$2.3), let us look at the following passage from Leonard:

> The presuppositions are of tremendous importance when one undertakes to apply the abstract system to a concrete subject matter. Unless the field of application satisfies the presuppositions, both the explicit and the tacit ones, the application is invalid and the result of the application can be serious errors of belief about the subject matter in question.[133]

The idea seems intuitive enough. In addition to this, I can see no reason for why this general explanation should not apply to ($_s$2.3).

In light of the explanations given above, ($_s$2.3) seems to be a reasonable claim. The obvious support for this premise, however, comes from particular examples. After all, if one shows that there are indeed such cases as envisaged in ($_s$2.3), this

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[131] Lambert, "The Philosophical Foundations of Free Logic," 143 (my emphasis).

[132] Strictly speaking, it is not the terms that EG or UI are applied to, but rather the statements. For matters of convenience, however, I will occasionally use this expression.

[133] Leonard, "The Logic of Existence," 50.

would be enough to accept the premise as true. In this regard, I take the examples given earlier as providing enough reason to accept $(_{s}2.3)$.[134]

Since we have assumed that the argument is valid, its conclusion, i.e., $(_{s}2.5)$, follows from the premises. Now that we have already cleared the first three premises, it remains to evaluate $(_{s}2.4)$ in order to come to a decision regarding SA2. To be sure, the premise is not acceptable in this form because it lacks a justification. In order to make a proper evaluation of it, we need to consider the omitted premises that serve as a logical connection between $(_{s}2.3)$ and $(_{s}2.4)$ and thereby provide a justification for $(_{s}2.4)$.[135]

A closer look to the omitted premises would easily reveal that $(_{s}2.3)_{ii}$ follows from $(_{s}2.3)$ and $(_{s}2.3)_{i}$. The other premises, i.e., $(_{s}2.3)_{i}$ and $(_{s}2.3)_{iii}$, on the other hand, seem to be in need of evaluation. If this assessment would reveal that either one of the premises is false, we can safely conclude that SA2 fails. As a matter of fact, standard objections to SA2 target the premise $(_{s}2.3)_{i}$, so let us begin our evaluation with it. To remind, the premise was as follows:

$(_{s}2.3)_{i}$ There are particular applications of these rules in which the assumption of singular existence is not satisfied.[136]

To be sure, if the premise is false, i.e., if there are no applications of these rules in which the assumption is not satisfied, there will be no inferences from true premises to false or controversial conclusions by $(_{s}2.2)$, and hence, no reason to see the presence of this assumption as a defect and to dispense with it. Accordingly, standard

[134] In particular, I mean the inference from (19) to (20) and the derivation from (23) to (26) or (27).

[135] For the omitted premises, see the footnote 129. To refer to these premises, I will add the indexicals I used there as suffixes to $(_{s}2.3)$. Accordingly, $(_{s}2.3)_{i}$ refers to (i) of the footnote 129, for instance.

[136] The field of application that is meant here is of course ordinary discourse.

objections that we will consider in what follows, if successful in their challenge to $(_s2.3)_i$, would block SA2.

Let us first note that there are two obvious ways for $(_s2.3)_i$ to fail. First, if it turns out that non-denoting expressions of ordinary language are not genuine singular terms, then there would be no ESTs in the language at all, and hence, no particular application of any rule in which the assumption of singular existence remains unsatisfied. Second, if all ESTs are made referring by assigning an arbitrary denotation to them, then the language would again not include any EST at all, and this would eventually lead to the same result. Standard objections to SA2 are roughly based on these two strategies. Let us see now what they are and how to evaluate them.

Objection 1: Russell's Theory of Definite Descriptions (and Names)

The first objection is based on Russell's theory of definite descriptions as well as on his account of proper names. I will call them Russell's Theory and shorten it as RT. In the second chapter, while introducing free logics, we have briefly mentioned how RT operates as a classical fix for the problems CQT is subject to. Now we will formulate it as an objection to $(_s2.3)_i$ and to SA2 eventually. Russell's treatment of definite descriptions and empty names are well known, so we do not need to dwell on in detail; a brief presentation should be more than enough for our purposes.

According to Russell, grammatical forms (also called *surface forms*) of ordinary language sentences do not always reflect their *logical forms*. This idea can be best explained with an example. Consider the following statement:

(33) The King of France is bald.

According to Russell, (33) does not have the subject-predicate form as one might expect. The reason underlying Russell's belief is that the grammatical subject of (33), i.e., the definite description 'the King of France', is not the logical subject of this statement, for it does not stand for anything existing. Consequently, Russell analyzes (33) as to mean the same as that there is one and only one thing that is King of France, and that everything that is King of France is bald. To put it differently, for Russell, the statement 'The *F* is *G*' involving the definite description 'The *F*' is a complex formula involving variables and quantifiers which consists of three distinct claims; namely, (i) an existence claim ('there is an *F*'), (ii) a uniqueness claim ('there is at most one *F*'), and (iii) a predication ('whatever that is *F* is *G*').

On Russell's account then, definite descriptions turn out to be quantificational expressions rather than singular terms. This, in fact, is what really matters for us. For if this treatment of definite descriptions is accepted, that means that one kind of EST is eliminated from the language; in other words, improper definite descriptions cannot take the position of singular term placeholders anymore for they are not genuine singular terms now. As for the second kind of EST, i.e., empty names, RT provides a solution as well. To put it simply, the treatment of definite descriptions is extended to empty names. Let us look at this account now.

According to Russell, ordinary names (except those that are guaranteed to denote such as 'I', 'this' and 'that'[137]) are disguised definite descriptions. On his account, 'Pegasus', for instance, is a definite description just like 'the present King of France' is.[138] Accordingly, a statement containing an empty name is paraphrased

[137] i.e., logical names as Russell calls them.

[138] To be sure, not only empty names like 'Pegasus', but also proper names like 'Socrates' are definite descriptions on Russell's account. However, it is empty names that really matters for us for the time being. The examples are chosen to reflect this.

in terms of the definite description the name abbreviates and then analyzed in the way explained earlier. The consequence to be drawn from this is that, in this way, empty names are also eliminated from the language; in other words, they cannot take the position of singular term placeholders anymore.

In what follows, we will look at how an objection based on RT can be developed against SA2. Before starting with this task, however, there is another account of empty names that deserves at least a mention here. It is due to Quine and as we will see in a moment, it can be seen as a very natural adaptation of RT. Thus, I include it under this heading.

Quine, in contrast to Russell, holds that empty names are genuine singular terms. Nevertheless, he believes that their presence in the canonical language is a defect that needs to be eliminated. The reason underlying this belief seems to be his conviction that ESTs introduce into *canonical language* truth-value gaps. Since Quine does not think that empty names are meaningless and accepts them as genuine singular terms, he needs a method to 'absorb' them into the canonical language in a way that would not give rise to truth-value gaps. To this end, RT seems to provide all the means Quine needs. He thus adapts Russell's treatment of empty names.[139] Accordingly, statements containing empty names are paraphrased in terms of predicates, variables and connectives and analyzed as in RT. To give an example, the statement

(34) Pegasus has wings.

gets paraphrased as

[139] To be sure, Quine's reasons to adopt Russell's treatment of empty names is very different from Russell's. As has been often underlined in the literature, his approach is mainly grounded on matters of theoretical convenience. For our purposes, however, this has little significance if any. What matters for us is that Quine's treatment of empty names boils down to Russell's, at least to a great extent.

(35) There is a unique thing that Pegasizes and it has wings.[140]

The similarities and differences between Russell and Quine on their accounts on ESTs should be obvious from above. Given our purposes, the similarities seem to outweigh the differences. As a matter of fact, as far as it is the issue of providing a solution to the problem of undesirable results, I see no reason for not reducing Quine's account to Russell's. For this reason, the following discussion will be essentially based on RT. However, not that most of what we will say in what follows also applies to Quine's approach.

It should be obvious by now how a typical objection to $(_s2.3)_i$ along RT can be developed. I shall call any such objection as OBJ_1. To put it very simply, it will be based on the assertion that ordinary language does not actually contain ESTs.[141] How this succeeds in blocking SA2 should also be quite obvious. If the claim is true, i.e., if there were no ESTs, the classical assumption of singular existence would be satisfied for any singular term of ordinary language, and hence, no problems of the sort we have seen above would arise.[142] In order to illustrate how it succeeds in invalidating $(_s2.3)_i$, let us consider the same examples we have given above.

Let us begin with the inference from (17) to (18). According to RT, the statement (17) does not have the subject-predicate form; hence it is not an instance of $A(t/x)$. For t in $A(t/x)$ is a placeholder for singular terms and since 'Santa Claus' is a

[140] Let us note that this method of paraphrase easily extends to all proper names. That is, any proper name, be it empty or not, can be paraphrased in terms of a predicate and analyzed accordingly. Thus, through the method of paraphrase, not empty names but all proper names become eliminable. For the time being, however, we leave this issue to one side.

[141] To be sure, this claim needs to be adapted for Quine's account. After all, he admits that natural languages contain ESTs. A possible formulation for this would be as follows: For matters of convenience, it is better to suppose that the language does not contain ESTs.

[142] As noted in the first chapter, the problem underlying undesired results seems to be related to a disagreement between natural languages, which contain non-denoting expressions, and CQT, which does not allow for such expressions. In view of this, one can say that Russell-Quine strategy solves this conflict in favor of logic by eliminating all singular terms from the language.

definite description (an improper one indeed), and hence, not a singular term, it cannot be substituted for t here. As a consequence, the rule of EG does not apply to (17), and the inference from (17) to (18) is not valid anymore. In conclusion, even if (17) is true and (18) is false, since the latter does not follow from the former, the example does not pose a problem for EG and CQT.

The same line of reasoning can also be applied to the inference from (21) to (22), which is supposedly authorized by the rule of UI. Since according to RT, (22) is not an instance of $A(t/x)$, it does not follow from (21) by UI. Hence, the inference is invalid. In a similar manner, the derivation from (23) to (26) fails because the inference from (24) to (25) is invalid, for the latter is not an instance of $A(t/x)$ according to RT, and hence, the inference in question is not a proper application of EG. Finally, note that the expressions not known to denote such as 'God' and 'Homer' are treated in the same way as empty names according to RT, so no problems arise since they cannot be substituted for t in $\exists x(x = t)$ as well.

As explained and illustrated, OBJ_1 easily blocks the inferences of the sort we have considered above. Thus, if Russell-Quine's treatment of names and definite descriptions is accurate, the premise $(_s2.3)_i$ is false, and the argument SA2 is unsound. Is this so indeed?

Let me say in advance that I do not find RT acceptable. In particular, I do not think that names are definitional abbreviations for definite descriptions. To be sure, any decent assessment of Russell's theory deserves much more space than we can spare here. Hence, I will confine myself to mentioning two distinct reasons for rejecting the Russellian objection to SA2.

The foremost of these reasons is that even if we agree that definite descriptions are paraphrasable in terms of quantificational expressions in the way Russell suggests, this does not necessarily mean that definite descriptions are not genuine singular terms. The reason Russell denies the status of singular term to definite descriptions seems to be related to the fact that some of them may fail to pick up a unique object. If they were genuine singular terms, he thinks, a failure of reference would never arise.[143] I do not find this idea convincing. That is, I see no reason to preclude definite descriptions altogether from singular term placeholders. I even think that there is a good sense that definite descriptions should be considered as referring expressions. After all, it seems quite intuitive that if a certain definite description is satisfied by an object, then it is a referring expression and the referent of it is the unique object that satisfies the description. However, the idea is far from being uncontroversial, so let us concentrate on the other objection.

Russell's claim that ordinary names are disguised definite descriptions is open to a serious objection. After all, there are many proper names for which it is not clear for which definite description they stand. I also see no reason for why some names cannot be used to refer to objects about which we know almost nothing. Challenges to Russell's account of names are abound in the literature, and there is an alternative account of names known as the causal theory of names. We can conclude that Russell's account of proper names is at least highly controversial.[144]

The reasons given above should be sufficient to reject OBJ_1. Nevertheless, I shall conclude this discussion with another reason that will rest on a purely pragmatic ground. For this, let us suppose that Russell's treatment of definite descriptions and

[143] This thought seems to be related to his view of meaning. According to Russell, the meaning of a referring expression is the object it stands for.

[144] Quine's account, at least to some extent, seems to be immune to the sort of criticism we have mentioned above. After all, he does not subscribe to the idea that proper names are not referring expressions. All he claims seems to be that adopting a canonical language that does not contain ESTs has certain advantages. There are other ways, however, to challenge Quine's account. For an example from the perspective of free logics, see Lambert, "The Philosophical Foundations of Free Logic," 152.

names is accurate. Thus we agree that 'The *F*' is not a genuine singular term, that the statement 'The *F* is *G*' is not a subject-predicate sentence, and that this statement is analyzed in some special way. We also accept that empty names are disguised definite descriptions. In this case, it may be very natural to ask whether it would not be more convenient to treat improper definite descriptions and empty names as singular terms even if they are not so indeed. Roughly speaking, the idea rests on a strategy similar to Quine's but in a somewhat reverse direction. If it can be shown that regarding ESTs as singular terms is more 'convenient', then we can reject OBJ_1 on this pragmatic ground.[145]

I conclude that any objection that is developed along RT would be inadequate in its challenge to SA2.

Objection 2: Frege's Chosen Object Theory

Let us consider now the other standard objection to SA2, which will be essentially based on what is known as chosen object theory (COT). The theory is originally due to Frege, but I will also mention Carnap's version of it. In what follows, I start with a brief presentation of COT, then show how this theory provides a challenge to SA2, and finally discuss whether an objection based on COT succeeds or not. I shall call the objection based on COT as OBJ_2.

First of all, let me say a few things regarding the general thought behind OBJ_2. To begin with, as in OBJ_1, the problem underlying the examples such as those we have seen above is analyzed in a similar manner; to be precise, there is a certain conflict between ordinary language and CQT that gives rise to some undesirable

[145] For a similar discussion, see Lehmann, "More Free Logic," 216--17.

results, and it is the language, not the logic, that seems to be at fault for this. Although originally intended for mathematical discourse, the following passage from Frege's "On sense and Reference" provides an example for this line of thought:

> The logic books contain warnings against logical mistakes arising from the ambiguity of expressions. I regard as no less pertinent a warning against apparent proper names having no reference. The history of mathematics supplies errors which have arisen in this way. [146]

In the same paragraph, he writes, "It is therefore by no means unimportant to eliminate the source of these mistakes, at least in science, once and for all."[147] Hence, the general strategy to solve the problem is essentially the same as in OBJ_1. If, somehow, ESTs were eliminated from the language, undesirable results such as those we have seen would not arise. Although the analysis of the problem and the strategy to overcome it are essentially the same as OBJ_1, since COT is different from RT in substantial ways, OBJ_2 may also be different from OBJ_1. So let us see the details of Frege's theory.

First of all, Frege, unlike Russell, regards definite descriptions and empty names as singular terms. Hence, he admits that natural languages contain ESTs. Yet Frege considers the presence of such expressions in ordinary discourse as a defect of natural languages. As regards their presence, he writes, "Now languages have the fault of containing expressions which fail to designate an object (although their grammatical form seems to qualify them for that purpose) because the truth of some sentence is a prerequisite."[148] On this basis, he argues for the elimination of ESTs in

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[146] Frege, "On Sense and Reference," 70. In another translation, the passage is as follows: "I deem it at least as appropriate to issue a warning against proper names that have no nominate. The history of mathematics has many a tale of errors which originated from this source." Gottlob Frege, "On Sense and Nominatum," in *Readings in Philosophical Analysis,* ed. Herbert Feigl and Wilfrid Sellars (New York: Appleton-Century-Crofts, 1949), 96.

[147] Frege, "On Sense and Reference," 70.

[148] Ibid, 69.

the construction of a ‘logically perfect language’. The following passage makes this point clear:

> A logically perfect language should satisfy the conditions, that every expression grammatically well constructed as a proper name out of signs already introduced shall in fact designate an object, and that no new sign shall be introduced as a proper name without being secured a reference.[149]

As for the ordinary language, his suggested method for this ‘elimination’ is to assign to all ESTs an arbitrary denotation in their formal translations.[150] For instance, regarding ‘the negative square root of 4’, he says:

> We have here the case of a compound proper name constructed from the expression for a concept with the help of the singular definite article. This is at any rate permissible if one and only one single object falls under the concept. [footnote] In accordance with what was said above, an expression of the kind in question must actually always be assured of a *Bedeutung*, by means of a special stipulation, e.g., by the convention that its *Bedeutung* shall count as 0 when the concept applies to no object or to more than one.[151]

In this way, all non-denoting expressions of ordinary language are made referring. Roughly speaking, this is the gist of Frege’s chosen object theory.

Carnap’s version of COT deserves at least a mention here. He introduces a *null-thing* as the common denotation of all ESTs. An important difference from Frege’s version of COT is that Carnap’s null-entity does not seem to be a member of the quantificational domain.[152] This makes Carnap’s version of COT, at least to some

[149] Ibid, 70.

[150] In one version of his theory, this chosen object is an arbitrary denotation, say *, which is a member of the quantificational domain, and in another version, it is a set. As we will see shortly, Carnap’s suggestion for the chosen object is the null-thing, which seems to reside outside the quantificational domain.

[151] Frege, “On Sense and Reference,” 71.

[152] In this sense, it appears to be a non-existent object.

extent, immune to some of the standard objections Frege's version faces that we will see later.

There is certainly more to COT; nevertheless, the points made above should be enough for us to formulate OBJ_2: Since all non-referring expressions of ordinary language are assigned an arbitrary denotation in their formal treatments, there will be no particular applications of EG and UI in which the assumption of singular existence remains unsatisfied. Thus, if COT is accepted, the premise $(_s2.3)_i$ becomes false, and hence, the argument SA2 fails. This, in general, is how OBJ_2 provides a challenge to SA2. Before deciding whether it is successful or not in this challenge, let us consider once again the examples given before as we have done with respect to OBJ_1, and see if they still support the premise $(_s2.3)_i$ or not.

Let us begin with the inference from (17) to (18). On Frege's account, it is a proper instance of EG; hence, the inference is valid. Nevertheless, since 'Santa Claus' refers to an arbitrary object now, say the number zero, (17) is not true anymore, for obviously it is not one of the properties of the number zero to live in the North Pole. In consequence, the inference in question is not anymore an example of a valid inference from a true statement to a false conclusion.

As for the inference from (19) to (20), given that existence is not a predicate to be applicable to singular terms on Frege's account, it is not a valid instance of EG, and thus, no problem arises. If, on the other hand, we were to paraphrase (19) as

(19^*) There is no object *x* such that Pegasus is the same as *x*, and paraphrase (20) also in a similar fashion, the premise would be no longer true, for there is indeed such an object, i.e., an object that is the same as Pegasus.[153] To sum up, in either case the problem disappears.

Let us look at the inference from (21) to (22) now. On Frege's account it is still a valid inference in virtue of UI; however, since the conclusion that there is an object *y* such that Vulcan is the same as *y* is not false anymore, no difficulties arise.

Finally, the theorem schema (29), which seems to encapsulate the problem underlying the reasoning from (23) to (26) as well as to (27) or (28), is not problematic anymore.[154] It intuitively says that for every singular term *t*, whatever it may be, there is something in the quantificational domain such that it is the denotation of *t*, and this is obviously a valid principle on COT, for to every singular term, including an empty one, is assigned a denotation according to this method. To conclude, neither of the examples supports $(_{s}2.3)_{i}$ anymore.

Based on the above considerations we can safely conclude that, if COT is acceptable, OBJ_2 succeeds in refuting SA2. That is to say that the success of OBJ_2 ultimately rests on whether Frege's method is acceptable or not. However, I do not find COT to be a tenable theory. As it was the case with RT, we cannot do full justice to COT here. I rest content with confining confine myself to offering two important reasons for rejecting COT.

First of all, COT gives rise to a number of strange results. I shall briefly mention three of them. To begin with, a statement like

(36) Sherlock Holmes is Pegasus

[153] To be explicit, this is the chosen object whatever it may be.

[154] Note that this theorem schema does not anymore say that *t* exists, for existence is a second level concept according to Frege that only applies to concepts, i.e., to predicates.

turns out to be true if COT and the axiom of identity is accepted. For both terms in (36) denote the same object or set. I find this result unacceptable especially if we compare it with

(37) Sherlock Holmes lives at Baker Street 221B,

which, unlike (36), seems to be true, at least intuitively.

Second, there is another complication that arises in connection with statements containing ESTs on COT. Let the arbitrary object be ‘number zero’ as Frege suggests in one of the passages quoted above. Now consider, the following statement:

(38) Sherlock Holmes is the same as the number zero.

Let us also note that Frege considers numbers as objects. Hence, they exist. Accordingly, since Sherlock Holmes turns out to be the same as the number zero on this account, (38) implies that Holmes must also be an existent. This, however, does not agree with our intuitions.[155]

Third and finally, nearly all atomic sentences containing ESTs turn out to be ‘untrue’ on a semantics based on COT. Consider, for instance, the following example:

(39) Sandy Island is a phantom island.

Since neither the number zero nor the null set or any other chosen object is among the phantom islands, (39) is false on such a semantics, at least as far as Frege’s version of COT is considered. However, there might be a good sense in accepting the statement as true, especially if we compare it with

[155] It should be obvious that I do not agree with Frege on his claim that existence is a second-order concept. Otherwise, I would not talk about singular existence in the first place.

(40) Sandy Island is a Greek island.

The latter is beyond doubt false.[156] The moral to be drawn from this comparison is that we usually accept some ordinary language sentences containing ESTs as true,[157] and it seems for a good reason, but a semantics based on COT does not allow for such statements to be true.[158]

Let us see now the second reason to reject COT. It is obvious that a logic based on COT does not reflect the logic of ordinary language. Most certainly, that was never Frege's intention; after all, what he wanted was to construct an ideal language for scientific discourse. Hence, such a criticism may be unfair.[159] In any event, since COT is originally intended for scientific discourse, it would be fairer to evaluate it in this respect.

As Frege also acknowledges in one of the passages quoted above, it is obvious from the history of science that ESTs have played a role in scientific discourse. Nevertheless, this may not be a defect as Frege thought. Why should logic be not applicable to a statement containing 'Sandy Island' or 'Vulcan', especially if we can make valid inferences from it? To put it differently, there may not be strong reason for depriving scientific discourse of ESTs. After all, an ideal language for scientific discourse may not be one in which ESTs are not allowed for, but on the contrary, are permitted.

[156] Or it is untrue, if truth-value gaps are allowed or a third truth-value is introduced.

[157] For instance, although we cannot know for sure, it is very probable that no one has raised an objection to *Wikipedia* about the truth of the entry about Sandy Island. Propositional attitude reports, statements about fiction, counterfactual statements or sentences with intensional content are examples of such statements.

[158] Hence, if one attributes truth to some statements containing ESTs, then COT does not seem to be the right course to take.

[159] Nevertheless, if one believes that the more the logic approximates to ordinary language, the better it is, then COT may not be the right choice. Later, I will consider an argument along this line of thought, but for now let us put this aside.

To conclude, for the reasons I have stated above, I do not find COT as an acceptable theory. Consequently, I reject OBJ_2 as a challenge against SA2. However, there are other ways to counter the argument. In what follows, we shall consider two of them.

Objection 3: A Non-Standard Interpretation of Quantifiers

The next objection we will consider is in the footsteps of Meinong and is roughly based on a denial of existential import to quantifiers. Let us call it OBJ_3. Unlike the previous objections, OBJ_3 does not pose a direct challenge to SA2; on the contrary, it is grounded on the truth of its conclusion. Nevertheless, it targets the connection between the conclusion of SA2 and the need for a revision of CQT in the way free logicians suggest. Strictly speaking then, OBJ_3 is not actually an objection to SA2, but a claim to the effect that the relevant argument does not justify the adoption of free logics.

To see how this challenge works, let us suppose that SA2 is sound. Hence, we agree with its conclusion that the assumption of singular existence needs to be dispensed with. Does that necessarily mean that one should adopt free logics instead of CQT? If we consider the fact that this assumption is a result of three premises and only the abandonment of one of them results in free logics, i.e., the abandonment of the classical requirement that each singular term of the formal language must receive an interpretation in the quantificational domain, the answer is obviously no.[160]

As should be obvious from above, there are, in fact, other options to block undesirable results. OBJ_3 is based on one of these other options. Rather than

[160] To be sure, this line of thought applies to all arguments with the same conclusion, namely (P3). Thus, what we will say in this section applies to them as well.

admitting ESTs into formal language as advocated by free logicians, it rests on the strategy of abandoning the existential reading of quantifiers to overcome the difficulties that arise in connection with ESTs.

Meinongian roots of OBJ_3 should be already obvious. Once the existential reading of quantifiers is dropped, singular terms of the formal language are relieved of the requirement to denote existent things. Under this non-standard interpretation, each singular term of the formal language must denote, but not every one of them denotes an existent, some may denote non-existents. The connection that is usually thought to be necessary between quantification and existence is thus broken.

I shall refrain myself from giving a detailed presentation of this position, since how it provides a challenge to free logics should be clear already. I shall rather illustrate how this non-standard interpretation of quantifiers manages to block undesirable results such as those we have seen above. To be sure, there are many variations in this line of thought; in what follows, I will consider one that seems to me as the most promising.

Czesław Lejewski, in his "Logic and Existence,"[161] discusses problematic instances of EG and UI just like we have done at the beginning of this section. The examples he gives are similar to those we have considered above. Nevertheless, his suggestion to block such inferences is different from the one of free logicians. To this end, he makes a distinction between two different interpretations of quantifiers. As will be clear in a moment, the difficulties of the relevant sort arise only with one of these interpretations. On this basis, he suggests adopting the other interpretation as a solution to the problem of undesirable results. To have a better idea about his account, let us see what these interpretations are.

[161] Czesław Lejewski, "Logic and Existence," *The British Journal for the Philosophy of Science* 5, no. 18 (August 1954): 104--19.

According to what he calls as “restricted interpretation,” quantifiers have existential import. That is to say that quantifiers range over existent objects. Accordingly, an expression such as ‘$\exists xFx$’ is read as ‘there exists an x such that x is F’. Needless to say, this is the standard interpretation given to quantifiers in accordance with the line from Frege and Russell to Quine.

According to what he calls as “unrestricted interpretation,” on the other hand, quantifiers do not have existential import. That means that quantifiers, under this interpretation, not only range over existent things as in CQT, but they range over non-existents, as well. The set of existent objects thus becomes only a portion of the domain of quantification, which can be thought of as the set of everything, be it existing or non-existing. Under the unrestricted interpretation of quantifiers, the correct reading of the same expression above is ‘for some x, x is F’.[162]

In light of this distinction, it can be easily observed that the problematic instances of EG and UI can be associated with the restricted interpretation of quantifiers. For instance, the inference from (17) to (18) yields falsehood only if we interpret the quantifier in (18) as existentially loaded in accordance with the restricted interpretation. After all, in real world, there is no such person. However, if we instead adopt the unrestricted interpretation, the correct reading of (18) becomes,

(18*) For some x, x lives at the North Pole.

[162] Note that this reading is different from ‘there is (but not exists necessarily) an x such that x is F’ which is imposed by a distinction between ‘being’ and ‘existence’ as usually made in Meinongian theories. According to these theories, the set of existent objects is only a part of the set of all objects that have being. Thus, two sets of quantifiers are introduced; a *restricted* one that ranges over existent objects and an *unrestricted* one that ranges over the objects that *are*. In the end, however, both quantifiers seem to be existentially loaded. Under Lejewski’s interpretation, however, unrestricted quantifiers seem to be neutral to the issue of existence. The reading ‘for some x, x is F’’ neither implies that x exists or that it has some sort of being. To make this point precise, instead of the usual label ‘existential quantifier’, Lejewski employs ‘particular quantifier’ for the dual of the universal quantifier.

Read in this way, there seems to be nothing wrong with the statement. For in accordance with (17), Santa Claus indeed lives at the North Pole.[163] Similarly, the conclusion (20) is not problematic anymore. The apparent contradiction disappears if the statement is read as follows:

(20*) For some *x*, *x* does not exists.

In the same manner, the remaining problems can be shown to disappear with this new interpretation of quantifiers. To conclude, if we adopt the unrestricted interpretation for quantifiers, no problems such as those we have sampled above would arise.[164] Now that we have made it clear what OBJ_3 is and how it provides a challenge to SA2, it is time to evaluate it.

Let me say beforehand that I am more sympathetic to OBJ_3 in a comparison to OBJ_1 or OBJ_2. In particular, I think that there is a certain appeal in the idea that there is nothing necessary about the connection between quantification and existence. On the other hand, starting from Russell, there has traditionally been a suspicion towards any sort of Meinongian thought. Introducing non-existent objects into logic has been typically received with a discontent among philosophers, usually with the conviction that logic should be neutral with respect to ontological matters; in other words, it is usually argued that it is not for logic to decide whether certain things exist or not.

Let it suffice to mention here one or two merits of adopting unrestricted interpretation of quantifiers instead of the standard one. To begin with, albeit its Meinongian roots, at least the version we have considered above seems to be free of

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163 If for some reason one maintains that (18*) is false, then certainly (17) is also false. Thus, no problem arises.

164 A recent version of this approach can be found in the writings of Priest. In his "How the Particular Quantifier Became Existentially Loaded Behind Our Backs," he attempts to show how quantifiers have become existentially loaded through Frege, Russell and Quine.

any ontological commitment. After all, the key idea is that the particular quantifier is not existentially loaded. In addition to this, a semantics with unrestricted interpretation of quantifiers seems to be a better approximation of natural languages. As some of the examples above suggest, it seems that, in ordinary discourse, we seem to quantify over things of whose ‘non-existence’ we do not hesitate. These are the two main reasons why one might want to follow OBJ_3.

Nonetheless, the suspicion towards any theory with Meinongian roots is still very strong in philosophical discourse. This, of course, is not a reason by itself to reject OBJ_3; however, it is for sure that the merits we have stated above would not be convincing for most people. Another reason for why one might want to reject OBJ_3 is that the standard interpretation of quantifiers is very well established. On this basis, it may be argued that it is not the best choice to change the interpretation given to quantifiers for matters of convenience. Given these objections, it seems to be more reasonable to conclude that OBJ_3 is unsuccessful as a challenge against free logics.

Clearly, for those who reject OBJ_3, the first two objections, i.e., OBJ_1 and OBJ_2 might work. If neither of them works, however, there remains another solution to block inferences such as those sampled above. I shall call it the restriction method, in short, RM.

Objection 4: Restriction Method

Let us begin the discussion about RM with a passage from Hintikka:

> It was pointed out by Quine a long time ago that the existence of an entity to which a term *t* refers is a necessary and sufficient condition for the success of existential generalization as applied to *t*. (This is the

> starting-point and, as we shall see, the gist of QT.) It follows that empty terms cannot be substituted for *a* in (2) (b).[165]

In some other place, he writes:

> One might perhaps also hope to limit the substitution-values of free individual symbols to some syntactical category which is restricted narrowly enough to guarantee that existential presuppositions are satisfied by all its members. For instance, it might seem that the category of proper names fills the bill satisfactorily enough. Proper names are then thought of as mere identifying labels attached to individuals we know to exist, without any descriptive content. Hence there does not seem to be any use for them in connection with nonexisting individuals, on which no labels can be pasted.[166]

These quotations should be enough to give us a general idea about RM. In fact, the idea is very simple and straightforward. It seems that certain rules of CQT may lead to undesirable results if applied to ESTs. Accordingly, if we somehow restrict the application of these rules to denoting expressions, inferences that seem unacceptable would have been blocked. The question, of course, is how to restrict their application to denoting expressions.

According to the former passage, Hintikka seems to be thinking that this restriction is a prerequisite of EG and UI. However, the reason he gives for this claim is not very convincing. To be sure, *t* having existential import is a necessary and sufficient condition for the success of EG applied to *t*. However, from this fact it does not follow that one is not allowed to replace *t* in EG with an EST. Thus, we need another way for this restriction.

One radical way to do this would be to dictate a rule that authorizes only denoting expressions of natural languages to be substituted for singular terms of the

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[165] Hintikka, "Existential Presuppositions and Existential Commitments," 130. Note that '(2) (b)' refers to Hintikka's version of EG that is applied to individual constants. 'QT' is short for Quine's thesis 'to be is to be a value of a bound variable'. Also note that, the symbol *a* refers to an individual constant and *t* to a term.

[166] Jaakko Hintikka, "Studies in the Logic of Existence and Necessity," *The Monist* 50, no. 1 (January 1966): 55--76.

formal language. In other words, for any formal expression that includes a singular term *t*, we establish a rule to the effect that *t* is such that only denoting terms of natural languages are eligible to be substituted for it. This, in fact, is what we call RM. Let us see first how it succeeds in blocking SA2.

If we adopt RM, the rules of EG and UI become applicable only to statements whose constituent terms are all denoting, and consequently, there will be no particular application of these rules in which the assumption of singular existence is not satisfied. That is to say that the premise $(_s2.3)_i$ would be false and SA2 unsound. I shall call any such objection based on RM as OBJ_4.

To be sure, RM renders the previous examples invalid. To illustrate, it is no longer possible to assign to the statement 'Santa Claus lives at the North Pole' the form $A(t/x)$ as required for EG, for 'Santa Claus' cannot be substituted for *t* anymore. Hence, (18) would not follow from (17) anymore. The same line of reasoning certainly applies to other examples, as well. Thus, we can safely say that, OBJ_4, if accepted, makes SA2 unsound.

Unlike the previous objections, I will not offer an assessment of OBJ_4, at least not immediately, for reasons that will be clear in a moment. To explain this, let me underline an important matter regarding RM. Although it is very clear what RM requires us to do and not to do, it is not that clear how this method should be seen. RM, in fact, can be interpreted in different ways. After all, all it says is that one is not allowed to substitute a non-denoting expression for a singular term of formal language. It can be seen as a placeholder for certain requirements of various theories or methods as well as a standalone rule or a necessity of CQT. Let us look at these options in turn.

First of all, RM can be understood as a practical rule that encapsulates and expresses the requirements of certain theories or methods. Let me explain this with an example. Recall Quine's claim that ESTs must not be introduced into logic for reasons of theoretical convenience. If one agrees with Quine on that, substituting a non-denoting expression for a singular term of formal language should not be allowable. RM establishes just that. In this sense, it behaves as a placeholder for a specific requirement that comes with Quine's account. Note that RM is also in accordance with RT. For instance, recall that ESTs such as 'Pegasus' or 'the present king of France' are not singular terms for Russell; hence they are not substitutable for singular terms of formal language, for they are nothing but placeholders for referring expressions. RM ensures that ESTs are not substituted for singular terms.

Note that, if RM is understood as a requirement that follows from Russell-Quine line rather than a different theory or method, any challenge to OBJ_4 would be at the same time a challenge to OBJ_2. Although we have already considered some of the reasons for rejecting OBJ_2; nevertheless, let us note that all we will say against OBJ_4 can be equally applied to OBJ_2 according to this understanding of RM. In any event, it is evident that any argument against OBJ_4 provides a challenge to SA2, either directly or indirectly.

RM can also be seen as a requirement that follows from the assumption of singular existence tacit in CQT. The idea is roughly as follows. As we have seen, CQT assumes that singular terms available in the formal language are denoting. Accordingly, expressions one can substitute for singular terms must be in accordance with this assumption. To put it differently, singular terms of formal language are only placeholders for denoting expressions of natural languages; hence, one is not allowed to substitute a non-denoting expression for a singular term of formal language. Accordingly, RM is a classical requirement of CQT.[167]

Understood in this way, RM actually does not bring about an additional claim on behalf of CQT. It rather dictates how the classical requirement that all singular terms must denote should be read. A simpler way to express the same idea would be to say that CQT does not allow for ESTs. Note that the difference between this formulation and (P1) would be more like a matter of emphasis according to this understanding of RM. They, however, express the same proposition.

Finally, RM can also be seen as an ad-hoc rule. In other words, it can be understood as an expedient to prevent certain peculiarities caused by the application of CQT to ordinary discourse. As a stand-alone rule, it serves as an artificial solution to the problem that underlies the examples we have witnessed.

If understood as a classical requirement or as an ad-hoc rule, RM can be seen as a standard practice that comes with CQT rather than an objection to SA2. In this sense, any challenge to RM is also a challenge to CQT, and hence, provides us with an argument in favor of free logics. On the other hand, if interpreted as an objection to SA2, it again provides us with a reason to adopt free logics. After all, if RM is false and OBJ_4 fails, then SA2 may succeed in motivating free logics.

In what follows, I will consider four different kinds of motivations for free logics all of which are such that they either directly target RM or challenge CQT on a basis that rests on the assumption that CQT entails RM. In this sense, they all provide a challenge to OBJ_4. Nevertheless, in light of the remarks made above, they can also be regarded as standalone arguments for free logics. For matters of convenience, I will pursue the second option. That is, I will consider the next four motivations as

[167] Note the similarity between this understanding of RM and Hintikka's claim in the passage noted in the footnote 165.

standalone arguments for free logics. This was indeed why I have not offered an evaluation of OBJ_4 above, for the next four motivations will serve this purpose as well. Accordingly, if the forthcoming discussion were to reveal that one of the arguments is justified, that would show that OBJ_4 fails, and since it is the last objection in hand, it would give us with enough reason to conclude that SA2 succeeds. Until then, however, we assume that OBJ_4 is successful and that SA2 fails.

As a final remark, let us note that we have not attempted to formulate a weak argument for the present motivation. The reason for this should be quite obvious. To be precise, the objections considered above are equally applicable to an argument in WA-form. Thus, if OBJ_4 is successful as we have assumed it to be, that means that the weak argument would also fail, for it would also contain $(_s2.3)_i$ or en equivalent of it as a premise. Now, it is time to consider further motivations.

The Motivation from a Wider Range of Applicability

The next motivation we will consider is concerned with certain disadvantages of RM. As before, I shall begin with a few examples from the literature and clarify the relevant motivation step by step. Let us start with a passage from Leonard. In his "The Logic of Existence," he argues that an 'abstract system' would have a wider potential for application if its assumptions were made explicit. Consider the following passage:

> The remedy is to make the presuppositions explicit. Upon doing this, one can also widen the potentials for application of the abstract system by formally discriminating those cases in which the presupposition is relevant and those in which it is not.[168]

[168] Leonard, "The Logic of Existence," 50.

What he has in mind seems to be that if the assumption of singular existence were to be made explicit, the resulting logic would allow for discrimination between inferences for those this assumption is relevant and for those it is not, and thereby has a wider range of applicability. As he observes shortly after, finding a symbolic way to express singular existence is not enough for this task. He writes:

> Can revisions of the modern logic make the handling of singular existence explicit? Were an effort in this direction to be made, we should want to do more than symbolize explicitly the tacit presuppositions of the modern logic, just as the modern logic did more than render explicit the tacit presuppositions of traditional logic. (...) What we want is to be able to discriminate between those modes of inference for whose validity this assumption is irrelevant and those modes for which it is relevant.[169]

Leonard's solution is to make the assumption of singular existence an explicit claim such that it can be true or false depending on the relevant singular term. To put it differently, he suggests adding this assumption as an additional premise to inference patterns that rely on it.[170]

To be sure, Leonard's idea can be easily turned into a claim to the effect that, CQT, in comparison to free logics, has a limited range of applicability, and unlike its alternative, it does not allow for a discrimination between different inference patterns as described above. We will formulate this claim in a while, but first, let us see in what sense the scope of applicability of CQT is limited. To this end, consider the following passage from Hintikka:

> (…) one cannot apply the usual logic of quantification to empty names and other empty singular terms without some special explanation,

[169] Ibid, 56.

[170] For an illustration of this idea, compare US [in symbols, $\forall xA \supset A(t/x)$] with double negation (DN) [in symbols, $A(t/x) \supset \neg\neg A(t/x)$]. Unlike the latter principle, the former requires that t is denoting to be a valid law. That is, US is only valid if t has existential import. The first thing we need is to find a formal way to express this. According to Leonard, E!t is suitable for this job. (Note that his definition of 'E!' is defective though.) Then, we must add this as an antecedent to US. The principle thus becomes: $\forall xA \supset (\text{E!}t \supset A(t/x))$. DN, on the other hand, does not require any such modification, for it does not rely on this assumption. In this way, it becomes possible in Leonard's system to discriminate between these different patterns of inferences.

> notwithstanding the fact that they were seen to be legitimate and useful constituents of our discourse.[171]

The idea is simple and straightforward. Since RM does not allow non-denoting expressions to take the place of singular terms of formal language, rules and laws of CQT cannot be applied to such expressions. To be sure, as Hintikka also draws attention above, there are certain theories or methods that offer alternative ways to handle them. The method of paraphrase due to Russell and Quine, for instance, provides the necessary means for the analysis of statements that contain ESTs. If, however, one is discontent with these theories and rejects them, CQT then becomes inapplicable to ESTs of natural languages. It is in this sense that the range of the applicability of CQT is restricted. For an illustration of this idea, let us look at the following example given by Lambert:

> For example, the true statement 'Everything identical with the perfect frictionless plane allows for unrestricted movement over it' and the evaluation of its alleged consequences now falls outside the purview of logic.[172]

Apparently, he believes that the rules and laws of CQT cannot be applied to a statement containing the definite description 'everything identical with the perfect frictionless plane' since no existing object satisfies this description. In any case, it must be clear by now in what sense CQT is restricted. Now let us see why free logicians usually think that this should be considered as a limitation. Hintikka says,

> (…) there is nothing so sacred about the idea of a name that one could not conceivably drop the assumption that it refers to something. I would go as far as to hold that a formal reconstruction of the logic of our

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[171] Hintikka, "Existential Presuppositions and Existential Commitments," 130.

[172] Karel Lambert, "Existential Import, 'E!' and 'The'," in *Free Logic: Selected Essays*, ed. Karel Lambert (Cambridge: Cambridge University Press, 2003), 19 (my emphasis). Note that this article is an exposition of ideas contained in several papers. For the original papers including this idea, see his "Free Logic and The Concept of Existence" and "Existential Import Revisited."

> language in which this cannot be done without breaking the rules of the game just is *not comprehensive and flexible enough*.[173]

Hintikka thinks that CQT is not ‘comprehensive’ and ‘flexible’ enough since it does not allow for ESTs. Although the idea is straightforward enough, let us look at the same issue in reverse order. In other words, let us consider what we would gain if the quantification theory could be applied to ESTs to see why CQT has a limited range of applicability. For this, let us look at Hintikka again:

> This enables us to reconstruct formally in our new system a number of arguments which cannot be accommodated in the traditional quantification theory without resorting to some artificial methods such as the replacement of proper names by descriptions.[174]

Simply put, Hintikka believes that a logic with a wider scope of applicability in the sense we are considering would allow us to express and formulate a number of arguments that cannot be expressed and evaluated in CQT unless we resort to RT or another similar method. In another place, he expresses the same idea by saying “it gives us a somewhat richer (more flexible) system in which we can express certain things we could not express before.”[175] To conclude, if Hintikka is right, unless one does not abandon CQT, one seems to be facing two options; she either accepts the limited scope of CQT, or enters Russell-Quine or any other method/theory that serves the same purpose.

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[173] Hintikka, “Existential Presuppositions and Existential Commitments,” 127.

[174] Ibid, 134.

[175] Hintikka, “Studies in the Logic of Existence and Necessity,” 67. The whole of the passage that contains this sentence is as follows: “For the modification of the conditions (C.E) and (C.U) which turns them into ($C.E_o$) and ($C.U_o$) really gives us something new. It gives us a somewhat richer (more flexible) system in which we can express certain things we could not express before. For instance, we can now meaningfully deny the existence of individuals; formulas of the form $\sim(Ex)(x = a)$ are not all disprovable any more. Hence such sentences as ‘Homer does not exist’ can be translated into our symbolism without any questionable interpretation of the proper name ‘Homer’ as a hidden description. if anybody should set up a chain of arguments in order to show the nonexistence of Homer, we could hope to translate into our symbolism without too many clumsy circumlocutions. In this sense, the use of an expression for existence is not only possible but serves a purpose.” (Ibid.)

Obviously, Hintikka is not comfortable with either of these choices. He says, "what we have found shows that one can omit the critical rule and obtain a system whose applicability is not limited to singular terms that do refer."[176] The idea is quite straightforward; by omitting (or modifying in our case) the classical rules of EG and UI, one can end up in a logic that can be applied to ESTs without requiring to rely on controversial theories. Obviously, the resulting system would have a wider range of applicability in the sense explained above.

The passages quoted so far should be more than enough to give a decent idea about the sort of motivation we are considering. Some free logicians think that CQT cannot be applied to ESTs at least not without resorting to theories such as Russell's. Nonetheless, given that these expressions are a useful part of our ordinary discourse, this limitation can be considered as a defect or disadvantage. I shall formulate this motivation as an argument below, but first, let us see how it presents a challenge to RM and OBJ_4 to make the connection made earlier more evident.

Recall that RM restricts singular terms of formal language to denoting expressions of ordinary language. Accordingly, this method also restricts the range of applicability of quantification theory since it is no more possible to reason with ESTs. For example, although 'Sandy Island does not exist' seems to be a true sentence, or even, 'Pegasus is white' as many believes so, there is no way to express these statements in CQT unless one takes the Russell-Quine route or adopt a similar method of paraphrase for names. Moreover, RM also blurs the distinction between inferences whose validity depends on the assumption of singular existence and those which do not. After all, not every inference requires that its constituent terms to be denoting in order to be valid. For instance, by adopting RM, not only EG and UI, but

[176] Hintikka, "Existential Presuppositions and Existential Commitments," 135.

double negation[177] also becomes inapplicable to statements including ESTs. However, there seems to be nothing wrong with the statement 'if Pegasus is white, then Pegasus is *not* non-white'.

Accordingly, if the argument from a wider applicability succeeds, one can safely reject OBJ_4 on this basis. As a result, SA2 becomes a plausible argument, at least in lack of any other objection to $({}_{s}2.3)_{i}$. That is to say, the motivation at hand, can be formulated as an argument against OBJ_4. However, as I have already said before, I will address it as a standalone argument given in favor of free logics. Before starting, however, let me make a remark.

The present motivation is actually about a certain ideal relation between CQT and ordinary discourse. After all, the issue at hand is the applicability of logic to ordinary language. This relation may find a better expression in RM as we have noticed previously. In light of this, it might be more convenient to modify (P1) for the present motivation so that this relation would be emphasized. One of the possible formulations would be as follows:

($P1^*$) CQT does not allow for ESTs.

However, I shall not do such a modification in (P1), which was common to all arguments given so far. The reason for this is to implement a continuity throughout the arguments. That being said, the obvious reference to ordinary language in the arguments to follow should be kept in mind.

I shall first put the present motivation into an SA-form. One of the possible formulations, which I think approximates towards Leonard's idea the most is as follows:

177 In symbols, $A(t/x) \supset \neg\neg A(t/x)$.

($_s$3.1) CQT contains an implicit assumption of singular existence, i.e., the assumption that all singular terms available in the formal language are denoting.

($_s$3.2) The presence of the assumption of singular existence in CQT is a defect because it restricts the range of applicability of CQT and blurs the distinction between the inferences for which this assumption is relevant and those for which it is not.

($_s$3.3) Therefore, the assumption of singular existence should be abandoned.[178]

I shall label this argument as SA3. As before, we assume that SA3 is valid.[179] Accordingly, its success depends on ($_s$3.2), so let us see if this is an acceptable claim or not. Note that, in a parallel to the remark made above, the implicit reference to ordinary language in SA3 should be kept in mind. That is to say, the claim that the assumption of singular existence restricts the range of applicability of CQT must be understood together with RM.

The premise ($_s$3.2) comprises of two distinct claims. First, there is the compound claim that CQT has a limited range of applicability and the distinction between certain patterns of inference are blurred in it; and second, that these aspects of CQT should be considered as a defect. As for the first of these claims, both assertions seem to be acceptable in light of the quotations above. To be precise, the

[178] In fact, Leonard offers two different arguments. The conclusion of the first one is that the assumption of singular existence needs to be made explicit. However, he later observes that the only advantage of this new system would be to allow for certain statements previously not expressible to be expressed. Therefore, he offers another argument according to which the assumption of singular existence not only needs to be made explicit but it must also become deniable. Our formulation above, in fact, follows this argument. The following passage from Hintikka makes this point clear: “How can we rid our logic of existential presuppositions? (...) In order to be able to eliminate the presuppositions we want to be able to express the existence of the reference of a singular term (say the term a) in such a way that its existence can also be meaningfully denied.” Hintikka, “Studies in the Logic of Existence and Necessity, 62.

[179] The ways of doing so must be more or less clear from the previous discussions.

assumption of singular existence indeed seems to be restricting the range of applicability of quantification theory, for it does not allow for non-denoting expressions to be substituted for singular terms of formal language. The claim about the blurring of the distinction between different types of inferences also seems to be sound as we have seen earlier. That being said, we are nowhere near to judge these aspects of CQT as defect. For the broadest possible range of applicability does not have to be a must for a theory of quantification. When viewed from this aspect, the second claim of ($_s$3.2) does not seem very plausible.

Let me explain this idea with some examples. For instance, CQT also cannot be applied to expressions of natural languages that do not even purport to refer, but to my knowledge, no one regards this as a defect. Similarly, there are many ordinary language sentences that cannot be translated into CQT. After all, CQT handle only sentences with propositional content. These two points should be enough to show that the mere fact that a logical system cannot be applied to some part of natural language cannot be considered as a defect.

This, of course, changes if one adopts a certain ideal relation between logic and language. For instance, if ($_s$3.2) is backed up with the claim that logic should be fully applicable to ordinary language, any divergence from this ideal would then be a defect and the original claim becomes acceptable. Nevertheless, it is obvious that this assertion is not tenable. Thus, we can safely conclude that ($_s$3.2) must be rejected. Consequently, SA3 fails. However, perhaps there is a more sympathetic reading of the passages quoted above.

Indeed, a closer look reveals that ($_s$3.2) does not fairly reflect the idea expressed in the quoted passages. The idea is as follows: CQT does not allow for ESTs, and in this sense, it has a more limited range of applicability in comparison to free logics. Moreover, in contrast to free logics, CAQ is incapable of discriminating between inferences for which the existence of the objects their constituent terms purport to refer is relevant and those for which it is not. Now if this idea is backed up with the general claim that a logical system with a wider applicability and one that is capable of distinguishing between certain different modes of inferences is preferable, then we seem to have an argument in favor of free logics. Expressed in this way, it is obvious that the argument rests on a comparison between CQT and free logics. Thus, it will be more convenient to formulate the relevant motivation as a weak argument. For this, it should be enough to replace ($_s$3.2) of SA3 with the following:

($_w$3.2) CQT is improved if the assumption of singular existence is abandoned because (i) it restricts the range of applicability of CQT and (ii) blurs the distinction between the inferences for which this assumption is relevant and those for which it is not.

I shall label the resulting argument as WA3. Certainly, the argument seems to be more promising in this form. Let us see if it is so indeed. To this end, we must decide whether ($_w$3.2) is acceptable or not.

To begin with, let us note that ($_w$3.2) is also a two-fold premise. On the first level, there is a compound claim consisting of two distinct assertions, which I have labeled as (i) and (ii) in the above formulation of ($_w$3.2). As regards this level, both assertions seem reasonable as we have already noticed above.[180] Thus, it is clear in what sense the system would be improved if the relevant assumption were

[180] There may be of course objections to this. Consider Russell's position, for instance. Since he believes that ESTs are not singular terms, for him, there would be nothing to be gained in terms of applicability if the assumption of singular existence were abandoned. To put it differently, Russell would reject the claim that CQT has a narrower range of applicability in a comparison to free logics. As I have already made myself clear about this, however, I do not find RT tenable.

abandoned. To be precise, the resulting system, i.e., the one without the assumption, would have a wider range of applicability and would allow for the discrimination between certain types of inferences. That being said, why this should be considered as an improvement over CQT is not that clear from the premise. In other words, the second level of ($_{w}$3.2), i.e., the claim that a system without the aspects expressed in (i) and (ii) is an improvement over CQT, seems unfounded. Thus, ($_{w}$3.2) seems to be in need of a justification with respect to its claim of improvement. Accordingly, there are two questions that are in need of an answer on behalf of ($_{w}$3.2) that can be informally expressed as follows:

(i) Why is a wider range of applicability a virtue/merit for a logical system?

(ii) What exactly are to be gained from being able to distinguish inferences whose validity depend on the assumption of singular existence and those whose validity do not?

Obviously, the success of ($_{w}$3.2) depends on the answers given to these questions. Unless one suggests plausible answers, the premise ($_{w}$3.2) will remain unjustified, and consequently, the argument WA3 would have no appeal. Let us see now if we can achieve this.

I shall begin with (ii). As we have seen in the last section, there are certain inference patterns of CQT that lead to undesirable results if applied to ESTs of ordinary language. That is to say, their validity require that the assumption of singular existence to be fulfilled for each and every singular term contained. RM obviously ensures that. However, there are other inference patterns of CQT whose validity do not require this assumption to be fulfilled. Nonetheless, by restricting singular terms of formal language to denoting expressions of natural language, these inferences also become inapplicable to statements containing non-referring expressions.[181] The standard policy thus seems to result in a narrower range of application. However, if we were able to distinguish these different inference patterns, we could apply the inference patterns that do not depend on the assumption of singular existence to statements containing non-denoting expressions. Accordingly, (ii) can be answered in terms of a wider range of applicability of logic. The answer of the question why this should be desirable lies in the first question though, so let us consider (i) now.

A possible answer to (i) is founded on a certain view about the ideal relation between logic and language. According to this view, logic should reflect natural language, which, among other expressions, also contains ESTs. If this view is supplemented with the claim that a logical system with a broader potential in terms of its applicability to ordinary language in effect means that it reflects the logic of the natural language better, that would give us a justification for the claim that the wider the range of applicability of a logical system, the better it is.

Another possible answer to (i) is more pragmatic one: A wider range of applicability is desirable for a formal system because it simply means that it has more uses. For instance, even if Russell is right, there are still many people who regard non-referring expressions of ordinary language as singular terms. A formal system with a wider potential in terms of applicability to ESTs means a system that can be used to analyze their reasoning as well.

I find neither of these extremes tenable; nevertheless, I cannot deny that there is a certain appeal in the claim that a logical system with a wider range of applicability is preferable. If it is indeed so, the answer to (i) must lie somewhere in

[181] In a previous footnote we have already illustrated this idea. See the footnote 170.

the middle of the extremes. In any case, although we do not have a proper explanation for it, we assume that the following principle is true:

J($_w$3.2) Other things being equal, a logical system with a wider potential in terms of its applicability to ordinary language is an improvement over its alternative.

This principle seems to provide a justification for ($_w$3.2). Let us see now if it actually applies to our case or not.

To begin with, in logic, as elsewhere in science, if the gains with a certain amendment is not worth the change, the traditional is preferable. That means, even though free quantification theory has a wider range of applicability, it does not mean that it is an improvement over CQT, for the decision ultimately depends on a comparison between these systems.

An obvious support for this conclusion comes from the view that there is little to gain with a formal system that is applicable to ESTs. In other words, although free logics have a wider range of applicability compared to CQT, this advantage may be negligible. Consider the following:

> Nevertheless existential presuppositions do not seem to matter greatly as long as we consider only descriptive uses of language in the narrow sense of the term in which descriptive uses of language are contrasted to the use of language e.g., for the purpose of formulating hypotheses, verifying and falsifying them, making counterfactual statements etc. The innocence of these presuppositions in descriptive contexts is not very surprising, however, for there is obviously little that can be said by way of pure description of nonexistent individuals.[182]

According to Hintikka, as long as the sole function of logic is to 'describe', there would be little, if any, to be gained with an extension of CQT in the way suggested.

[182] Hintikka, "Studies in the Logic of Existence and Necessity," 60.

After all, as he also observes, one can only ‘describe’ objects that have existence, at least in the ‘narrower sense’ of the term.

At first glance, this idea has a certain intuitiveness. After all, it is not that often that we need to reason about Sandy Island or any other non-existent island. A little reflection, however, would show that it is not a very strong claim. There are many ordinary language sentences that have functions other than ‘describing’ yet logical rules or laws are applicable to them. For instance, it is perfectly possible for someone to believe that Sandy Island exists just as people did for more than a decade. However, there should be no reason for her belief to lie outside the scope of the quantification theory.

A possibly better claim to the same effect would be to accept that there are things to be gained with the amendment of CQT, however, point out the fact that CQT is a well-established system and that the gains would not be worth it. Clearly, the success of a claim as such ultimately depends on a comparison of the gains and the losses. As for our case, free logics seem to provide us with a wider range of applicability when it comes to ESTs in exchange of certain well-established classical rules and laws. Hence, it appears that the decision whether or not to accept the modification depends on how much one values a logical system capable of handling ESTs directly. To put it simply, if one values this more than the losses, the amendment is worth it; if not, it is not. However, any such ‘valuation’ should probably depend on the context the logic is applied to.

The conclusion to be drawn from the above discussion is that whether or not J($_w$3.2) provides enough justification for ($_w$3.2) actually depends on a comparison of the gains and the losses of the modification required for free logics. I, myself, believe that the gains is not enough in general by itself to motivate such a substantial amendment in CQT.[183] On this basis, I reject the premise ($_w$3.2), and consequently, the argument WA3.[184] To conclude, the discussion about the motivation from a wider range of applicability, I believe that it fails in justifying free logics.

Let me close this section with a remark. Although the weak argument does not seem to be enough to provide the justification we are looking for free logics, it may still give us an insight about why one might adopt free logics instead of classical logic. With respect to a specific context, the gains with a free quantification theory may be worthwhile. If the motivation from a wider range of applicability is formulated as an argument in this restricted sense, it may succeed in justifying free logics for that particular context. To this, we will come later.

The Motivation from Inconsistency

In CQT, there is a difference in treatment of singular and general terms. The next motivation we will consider proceeds from this dissimilarity. To see this, let us first consider the traditional square of opposition that dates back to Aristotle:

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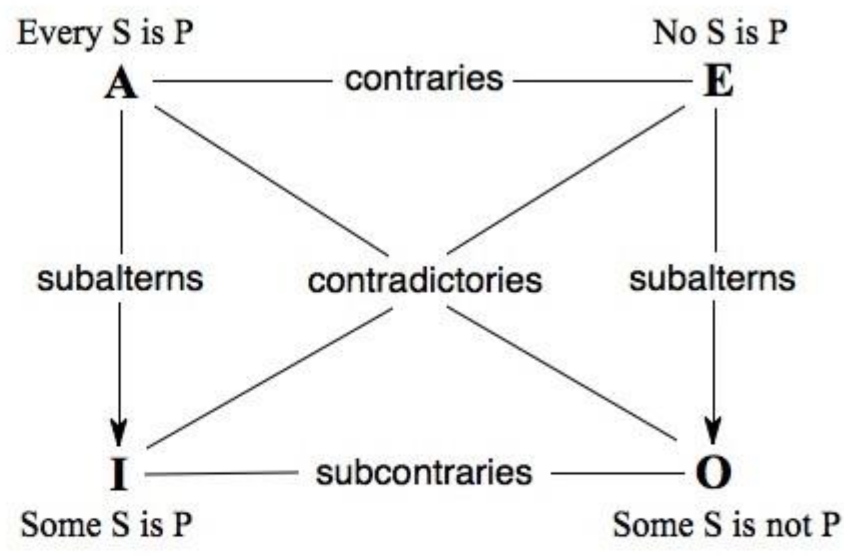
[183] For a judgment in the opposite direction, consider the following passage from Hintikka: "Since this system is obtained simply by omitting one of the old rules, it is more elegant than the traditional quantification theory in the philosophically relevant sense in which the merits of a system are measured by the paucity of its basic assumptions. (...) In spite of its economy, our 'quantification theory without existential presuppositions' may serve all the same purposes as the traditional quantification theory." Hintikka, "Existential Presuppositions and Existential Commitments," 131--32. He also writes, "The probable reason why this has not been attempted before is that such an enterprise may be feared to give rise to a system in which the virtues of the ordinary formulations of quantification theory are lost, e.g., in which the transformation rules are more complicated than usual. However, these fears can be shown to be idle." Ibid., 131.

[184] Note that even if we accept the claim that free quantification theory is an improvement over CQT, this does not necessarily mean that we should abandon CQT. Let us call this claim CL. The idea underlying CL is that ($_w$3.3) does not follow from ($_w$3.2). However, recall that we have assumed that the argument is valid. Additionally, the general thought behind CL is in a sense already contained in the discussion about J($_w$3.2). To put it simply, there is a certain parallelism between CL and the claim 'even though free quantification theory has a wider range of applicability, it still cannot be considered as an improvement over CQT'.



In traditional logic, it was assumed that general terms have existential import, i.e., that all predicates are true of something. As a consequence of this, the inferences from A to I statements as well as those from E to O statements were counted valid.[185] In symbols, the following inference patterns were traditionally valid:

(41) $\forall x(Sx \supset Px)$

$\therefore \exists x(Sx \wedge Px)$

(42) $\forall x(Sx \rightarrow \neg Px)$

$\therefore \exists x(Sx \ \& \ \neg Px)$

In presence of the assumption that general terms have existential import,[186] inferences of the form (41) and (42) were valid. To illustrate this idea, let us consider the former. In accordance with the assumption of general existence, the general term

[185] A statements are those having the form 'All *S* are *P*', i.e., $\forall x(Sx \rightarrow Px)$; I statements are those having the form 'Some *S* are *P*', i.e., $\exists x(Sx \wedge Px)$; E statements are those having the form 'No *S* is *P*', i.e., $\forall x(Sx \rightarrow \neg Px)$; and O statements are those having the form 'Some *S* is not *P*', i.e., $\exists x(Sx \wedge \neg Px)$.

[186] Let us call this assumption as the assumption of general existence.

S must have existential import. In this case, the validity of (41) can be simply shown as follows:

There are things that are *S*. Let *t* be one of them. Suppose further that all *S*'s are *P*, that is, that the things that are *S* are also *P*. Then *t* is *P* since it is *S*. That means, however, that there are *S's* that are *P's*; in other words, that some *S* is *P*. Thus, we have the desired conclusion.

Now if the assumption of general existence is dropped, (41) becomes invalid since then we would no longer be entitled to claim that there are *S*'s in the first place. For if *S* were replaced by an empty general term, i.e., a predicate that is true of nothing, this claim would be obviously false. Accordingly, it would no longer be possible to infer that there are things that are *S* and *P*. Certainly, we could add this claim as an assumption, but then, the best we could infer from 'All *S* are *P*' would be the conclusion that 'if there are *S*'s, some *S* are *P*'.

The traditional way out of this difficulty was to restrict the general term placeholders *S* and *P* in the above inferences to general terms that have existential import, i.e., to predicates that are at least true of one (existing) thing. In this way, any problematic result that may have been obtained as a result of applying the inference pattern to an empty general term was permanently blocked. At the same time, however, it was no longer possible to discriminate between an inference pattern whose validity requires that its constituent general terms to have existential import and one whose validity do not require this. Moreover, the restricted system had a narrower range in terms of its applicability to ordinary language.[187]

[187] Similarities between the restriction of general terms to those having existential import in traditional logic and the restriction of singular terms to those having existential import in modern logic should be obvious. As a matter of fact, the motivation from a wider range of applicability is usually supported with a reference to this contrast between traditional and modern logic in their attitudes towards general existence.

Modern logic starting from Frege had a different solution to this problem. It acknowledged the fact that general terms can be empty, and accordingly, revised inferences of the form (41) and (42) so that the assumption of general existence appeared as an additional premise.[188] For instance, the inference pattern from A to I statements were modified so that it only holds if the premise 'there is an *x* such that *x* is *S*' is added. To put it in symbols, the inference pattern (41) of traditional logic is replaced with

(43) $\forall x\, (Sx \supset Px)$

$\exists x Sx$

$\therefore \exists x\, (Sx \wedge Px)$

in modern logic.[189] In his way, the implicit assumption of general existence had been abandoned, and *general inferences* become free of existence assumptions. The same, however, cannot be said for *singular inferences* as we have already seen. Accordingly, there remained a certain inconsistency in modern logic's attitude towards singular and general inferences, and some free logicians think that this dissimilarity is not acceptable. For an example of such a position, consider the following passage from Lambert:

> (...) the fact of the matter is that there is a certain inconsistency of attitude in standard text-book logic toward the question of logical form. Does the mere fact that we are dealing with singular terms rather than with general terms alone make that much of a difference? If this is so, why shouldn't

[188] Note that this solution does not suffer from the disadvantages of the traditional solution we have seen above. For one thing, it permits unrestricted application of these inference patterns to general terms, be it non-empty or not. Second, the distinction between inferences whose validity require that its constituent general terms to be non-empty from those whose validity do not require this is not blurred. To give an example, the inference from an A statement to the negation of an O statement holds even if the assumption of general existence is not fulfilled. In symbols, one can safely infer from $\forall x(Sx \supset Px)$, the conclusion $\neg\exists x(Sx \wedge \neg Px)$ without the additional premise $\exists x Sx$. Compare this to the revised version of (41), i.e., (43) below.

[189] Notice the similarity between this version of (41) and the restricted version of UI.

> we expect formal differences to be occasioned by the difference between general denotative terms like ‘man’ and general attributive terms like ‘pretty’? Yet standard text-book logic tells us they are to be treated formally in the same way. When viewed from this angle, standard text-book logic’s treatment of singular and general inference has a faint odor of adhocness about it.[190]

In another place, he describes this alleged inconsistency as a “theoretical schizophrenia in the conventional logic of terms.”[191] Lambert’s idea is straightforward. In modern logic, the traditional method of restriction for general terms is rejected. Instead of this, CQT allows for empty general terms. Nevertheless, when it comes to singular terms, the same logic does not recognize the possibility of some singular terms to be empty but instead adopts the restriction method for them. On this basis, some free logicians such as Lambert think that CQT is inconsistent in its treatment of singular and general terms. To be sure, the same logicians also maintain that free logics handle both types of terms in a more consistent way. For, in these systems, restriction method is rejected for both types of terms and both are allowed to be empty.[192]

The motivation in question should be more or less clear from the quotations presented above. So let us try to formulate it as an argument. This time, however, I will refrain myself from giving the arguments in their complete forms. I will rather present the key premises of the arguments and concentrate on them. I begin with the strong argument, which I will label as SA4. The key premise on which the success of SA4 depends is as follows:

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[190] Lambert, “Free Logic and the Concept of Existence,” 137.

[191] Lambert, “The Philosophical Foundations of Free Logic,” 141.

[192] Note that this is only one of the possible interpretations of the claim of inconsistency. Since it seems to me as the most promising one, I have mentioned this version here.

($_s$4.2) The presence of the assumption of singular existence in CQT is a defect because it makes the system inconsistent in its treatment of singular and general terms.

The premise comprises of more than one claim. First, (i) there is the claim of inconsistency. Second, (ii) there is another claim that this consistency is caused by the assumption of singular existence. Finally, (iii) the relevant assumption is asserted to be a defect since it is the reason of the alleged inconsistency. Accordingly, any line of reasoning that will refute one of these claims would be enough to reject ($_s$4.2) and conclude that SA4 is unsound.

At first glance, the obvious way to challenge ($_s$4.2) seems to argue against the claim of inconsistency, which can be expressed as follows:

($_s$4.2)$_i$ CQT is inconsistent in its treatment of singular and general terms.

For if CQT were not inconsistent in the way suggested, there would be no reason to accept ($_s$4.2). The claim of inconsistency itself, however, is not very clear from the above formulation. To be more precise, ($_s$4.2)$_i$ does not offer an explanation for why and in what sense CQT is inconsistent. In order to argue against it, we obviously need a better-expressed claim. Hence, let us first try to clarify in what sense CQT is inconsistent. To this end, the quotations presented earlier would be helpful.

One of the possible interpretations of ($_s$4.2)$_i$ rests on the fact that CQT allows for some general terms to be empty yet does not allow the same for singular terms. The claim of inconsistency can be understood in this respect. Let us formulate this interpretation as follows:

(i) CQT contains an assumption of singular existence but not an assumption of general existence; hence, it is inconsistent in its treatment of singular and general terms.

Let us label (i) as $I^1({}_s4.2)_i$ and see if it is acceptable or not.

I shall begin with what is probably the most obvious way to argue against $I^1({}_s4.2)_i$. The assertion of inconsistency seems to be resting on an alleged parallelism between singular and general terms. However, this presumption can be rejected. On this basis, one can argue that the dissimilarity in CQT's attitude towards singular and general existence cannot be considered as an inconsistency. An objection in this line of thought would be as follows: It is true that singular and general terms are treated in CQT differently, to be precise; CQT admits the possibility of the latter being empty, but not of the former. Nevertheless, these two types of terms are essentially different from each other, and hence, it should be very natural for them to be handled in different ways. Hence, the relevant difference cannot be considered as an inconsistency. After all, not every difference can be regarded as an inconsistency.

Recall Lambert's question that we have quoted above: "Does the mere fact that we are dealing with singular terms rather than with general terms alone make that much of a difference?"[193] Now, if there are indeed some essential differences between singular and general terms, I see no reason for why this question cannot be answered in the negative. For if they are essentially different, then there should be nothing wrong with the difference in their treatments. Thus, one can simply response to Lambert as follows: "Yes, it makes that much difference." If one thinks otherwise, the burden of proof is on her side. In other words, in order to be entitled to raise the claim of inconsistency, one needs to show that there is a certain agreement between

[193] Lambert, "Free Logic and the Concept of Existence," 137.

singular and general terms such that it makes the difference in their treatments an inconsistency.

A possible attempt in this line would be to claim that singular and general terms differ only on the number of the objects they refer to. After all, it seems that singular terms are such that they refer to exactly one thing whereas general terms refer to multiple of things. If this is an accurate characterization of their distinction, then the difference between singular and general terms should not be enough by itself to justify such a substantial dissimilarity in their formal treatments.

In response to this, I do not think that the difference between singular and general terms can be reduced to the issue of the number of objects they refer to. After all, there are some essential differences between them that are not quantitative at all. A better characterization of general terms is one that takes into account that they are predicated of singular terms. As a matter of fact, general terms are usually characterized as being true of certain things while singular terms are characterized as referring expressions that purport to single out a specific object. This way of describing them seems to me enough to conclude that the above attempt is unsuccessful.

To be sure, there may be other attempts to support the alleged parallelism between singular and general terms; however, the ultimate fact is that the burden of proof lies on the side of those who raise the claim of inconsistency. I do not think that any of them would succeed in giving enough support for $I^1({}_s4.2)_i$. Nevertheless, I shall mention another kind of objection to this interpretation.

A more specific objection to $I^1({}_s4.2)_i$ would be based on the certain view one adopts in philosophy of language. For a Russellian, for instance, there is very little reason, if any at all, to accept $({}_s4.2)_i$ if the claim of inconsistency is interpreted in the way suggested above. According to RT, singular terms are always denoting whereas general terms can be empty. For a Russellian, therefore, the difference in CQT's attitude toward singular and general existence is perfectly reasonable.[194] Independent of the question whether RT is accurate or not, this shows that $I^1({}_s4.2)_i$ needs to be supported with a certain view of singular and general terms. In lack of a view as such, the claim remains unjustified.

The conclusion to be drawn from the discussion above is that $I^1({}_s4.2)_i$ is in need of a justification. To put it differently, one has to show that the difference in CQT's attitude towards singular and general existence is indeed an inconsistency. In lack of a justification, there would be no reason to accept $I^1({}_s4.2)_i$. That of course does not mean that we should reject $({}_s4.2)_i$. It may be the suggested interpretation that is at fault here, so let us look at some of the other interpretations of the claim of inconsistency.

Another possible interpretation for $({}_s4.2)_i$ is one that is based on modern logic's different policies towards the restriction method as applied to singular and general terms. Let RM_g stand for the traditional version of the restriction method that restricts general terms to those having existential import. As we have seen above, RM_g is explicitly rejected in CQT. Instead, the traditional inference patterns that rely on the assumption of general existence are invalidated. The reason for the refusal of RM_g is usually explained in terms of certain adverse results of this method, i.e., a narrower scope of applicability of the logic and the blurring of certain formal distinctions. The same results, however, as we have already seen in our previous discussion, also show up as a result of the adoption of RM in CQT. This time,

[194] On RT, the assumption of singular existence is fulfilled for each and every singular term whereas the assumption of general existence is unfulfilled for some general terms.

however, these results are not seen enough for a change of the inference rules and principles that rely on the assumption of singular existence.

When considered from this point of view, the difference in CQT's attitude towards the restriction method as applied to different kinds of terms can be regarded as an inconsistency. After all, if the reasons for the refusal of RM_g were enough to motivate an amendment in traditional logic's treatment of general inferences, they should be also enough for the refusal of RM and for a similar amendment in modern logic's treatment of singular inferences. This interpretation of the claim of inconsistency can be expressed as follows:

(ii) In CQT, the restriction method for general terms is rejected for certain reasons.[195] However, although the same reasons are also valid for the restriction method for singular terms, it is retained anyway. In this sense, CQT is inconsistent in its treatment of singular and general terms.

Let us label this interpretation as $I^2({}_s4.2)_i$ and discuss whether it is tenable or not.

To begin with, how one evaluates $I^2({}_s4.2)_i$ is closely related to how one evaluates the argument from a wider applicability. Recall that, in the relevant discussion, we concluded that the success of the argument depends on a comparison of the gains and the losses of a particular amendment. If this conclusion is accepted, then it should be perfectly possible for the argument from a wider range of applicability to succeed in justifying the amendment in traditional logic with respect to general terms while it fails to justify a similar amendment in CQT with respect to singular terms. To put it differently, although the same reasons are valid in both cases, the gains of 'freeing' general terms from existential presuppositions may have

[195] i.e., a narrower range of applicability and the blurring of certain formal distinctions. For the sake of simplicity, I will mainly mention the former in the following discussion.

been worthwhile whereas in case of singular terms it is not. Proceeding from this, one can conclude that there is no inconsistency in CQT's refusal of the restriction method for general terms while accepting it for singular terms.

Certainly, this argumentation needs to be supported by a comparison of the specific gains and losses with respect to the adoption of the restriction method as applied to each kind of terms. To give a simple example, one can, for instance, try to show that empty predicates hold a more central position in ordinary discourse as compared to ESTs. If so, that means that RM_g would result in a greater loss in terms of applicability compared to RM. All in all, for a complete assessment of $I^2({}_s4.2)_i$, one needs a thorough discussion. For obvious reasons, however, we cannot do this here. Nevertheless, that does not mean that we should withhold judgment about $I^2({}_s4.2)_i$. In general, I see no reason for why a wider range of applicability cannot be preferred only in case of general terms.

Another way to argue against $I^2({}_s4.2)_i$ is to indicate that there are alternative ways to analyze statements containing ESTs while there is none for empty predicates. Following Quine, for instance, any singular term can be paraphrased in terms of variables, predicates and connectives, and thus, incorporated into the formal system.[196] That is to say, for someone who follows Quine's method of paraphrase, RM turns out to be not that restricting after all. To be sure, on this account, ESTs are not treated as singular terms, however, what really matters is that the method of paraphrase offers an alternative way to reason with such expressions. On the other hand, the same method obviously cannot be applied to empty predicates; and in lack of any other alternative method for this, one can safely conclude that RM_g brings

[196] According to Quine, for instance, the statement 'Pegasus does not exists' means the same as the statement 'nothing pegasizes', which does not contain any EST. That is to say, Quine's canonical language offers an alternative way for the formal treatment of statements containing ESTs.

about a greater loss of applicability in comparison to RM. On this basis, the claim of inconsistency can be rejected.

To conclude the discussion regarding $I^2({}_s4.2)_i$, I believe that the two objections expressed above should be enough to reject it. Let us consider one last interpretation for $({}_s4.2)_i$. For this, let us look at the following passage from Antonelli:

> Since Aristotle, and certainly until the school of Port Royal, logic has regarded general terms as having existential import: as a consequence, the inference from "All P's are Q's" to "Some P's are Q's" was regarded as valid. On the other hand, modern symbolic logic, beginning with Frege, has done away with this assumption. But a certain asymmetry remained, in that the inference from "Everything the same as t is P" to "Something the same as t is P" (where t is a singular term) was still regarded as valid: free logic can be viewed as an effort to remedy this situation, by allowing for the case in which t has no existential import.[197]

Antonelli hints at another way the claim of inconsistency can be interpreted. Recall that CQT invalidates the traditionally valid inference pattern from statements of the form 'every *S* is *P*' to those of the form 'some *S* is *P*'. Now, if the predicate *S* is replaced with

(44) $= t$

in the above pattern, the inference becomes valid again. That is to say, it seems that we do not need the additional assumption that the general term 'is the same as *t*' is non-empty for the validity of the inference as normally required in modern logic. In other words, as far as the general term 'the same as *t*' is considered, the traditional inference pattern (41) is valid. The reason for this should be obvious. Since the singular term *t* is always non-empty in modern logic, the general term 'is the same as *t*' is also non-empty, i.e., it has existential import.

[197] Antonelli, "Proto-Semantics for Positive Free Logic," 277.

Although modern logic disallowed the traditional assumption of general existence, it seems that a specific predicate remained as having existential import and the reason underlying this seems to be the assumption of singular existence. To put it differently, because of the assumption of singular existence present in CQT, existential import are denied to all general terms except one. The claim of inconsistency can be understood in this sense. I will refrain myself from giving a separate formulation for this interpretation, let us just suppose that we have done this and simply label it as $I^3({}_s4.2)_i$.

I shall only offer a brief evaluation of $I^3({}_s4.2)_i$, for it seems to be a very weak claim. As a matter of fact, Antonelli, in the above passage, defines this aspect of CQT as an “asymmetry”. This choice of words, I believe, is accurate. If one has no difficulty at accepting that there is an essential difference between singular and general terms, then there should be no reason to regard this asymmetry as a problem. After all, modern logic does not require general terms to have existential import, however, that does not mean that certain predicates cannot have existential import. There should be nothing inconsistent about this. On this basis, I conclude that $I^3({}_s4.2)_i$ fails, as well.

Now it is time to end the evaluation of $({}_s4.2)$. Recall that $({}_s4.2)$ included three separate claims. In the above evaluation, we have concentrated only on the claim of inconsistency, which we have labeled as $({}_s4.2)_i$, since it seems to be the weakest claim. We have considered three different interpretations of this claim. On neither of these interpretations, $({}_s4.2)_i$ appeared as an acceptable claim. On this basis, at least in lack of any other interpretation for $({}_s4.2)_i$, we can safely conclude that one does not have enough reason to accept $({}_s4.2)$. As a consequence, SA4 is inconclusive.

As a final remark, let us note that we have not attempted to formulate a weak argument for the present motivation at all. In such a formulation, the key premise would entail a claim that the more a formal system is consistent in terms of its treatment of singular and general terms the better it is. This claim, however, would eventually suffer from the same weaknesses with ($_s$4.2) since it would be also founded on a claim of inconsistency. Accordingly, most of the reasoning we have suggested above would apply to it.

Ultimately, we conclude that the motivation from inconsistency is not enough by itself to justify the modification of CQT in the way suggested.

The Motivation from a Better Reflectivity

The motivation to be addressed in this section is based on the idea that free logics reflect the logic of ordinary language better than CQT. If this claim were to be supplemented with an argument showing that this is indeed a desirable quality for a formal quantification theory to have, it would easily turn into another argument for free logics. Let us first make a remark.

In the previous discussion, we have addressed the claim that free quantification theory, in a comparison to CQT, has a wider range of applicability. This claim rests on a relation between logic and ordinary language. Rather than this relation, however, we mostly concentrated on the question whether this can justify free logics or not. Let us suppose for a moment that this claim is true, i.e., that free quantification theory has a wider potential in terms of its applicability to ordinary language. Now another claim that is often expressed as regards the relation between free logics and ordinary language is that free logics, in a comparison to CQT, reflect the logic of ordinary language better. Let us call this the claim of a better reflectivity. Given both claims, it is very natural for the following question to arise: Is here is a parallelism between the claim of a wider range of applicability and the claim of a better reflectivity?

At first glance, the idea seems appealing. After all, if a logical system has a wider scope in terms of its applicability to ordinary discourse, it may be said that it reflects the logic of it better. In addition to this, if this relation also holds for the other way around, then the two claims might be the same claim. If there is indeed such a parallelism between these claims, one way to argue for the assertion that free logics reflect the logic of ordinary language better would be to show that free systems have a wider range of applicability in comparison to CQT. Especially given the vagueness of the claim of better reflectivity, this might be a plausible option. One could then evaluate this claim with reference to the previous discussion about the motivation from a wider range of applicability.

As to the matter whether or not the claims mentioned above are the same or whether there is a parallelism between them, I shall withhold judgment about it. In what follows, I will consider the motivation from a better reflectivity as a standalone argument for free logics. In general, however, I would not reject the idea that there is a certain parallelism between these claims.

As should be evident from above, free logics are usually believed by its proponents to be reflecting the logic of the ordinary language better than CQT. For an example, consider Hintikka’s following words: “Our modified quantification theory is a better approximation toward the logic of ordinary language than the traditional theory."[198] Another free logician, Lambert, in one of his early papers, supports Hintikka on this:

> (...) this paper (...) is not in agreement with the autistic doctrine that formal logic is in principle incapable of correctly representing the logic of ordinary discourse. Along with Hintikka, 'I do not see why the possible aberrations [in logic] from [ordinary discourse] could not also be dealt with by methods [like] those used here.'[199]

It seems evident from these passages that for reasons we will try to clarify below, these free logicians think that free logics, in a comparison to CQT, reflect the logic of ordinary language better.[200] The present motivation is centered on this claim. To put it simply, the idea is as follows: Since free logics reflect the logic of ordinary language better than CQT, they should be preferred over it. Let us put this motivation into an argument form.

To begin with, the idea have almost no appeal in an SA-form of argument. In any event, let me offer a brief evaluation of it. A possible formulation of the key premise of an SA-argument, call it SA5, would be as follows:

($_s$5.2) (i) The presence of this assumption is a defect because (ii) a quantification theory should reflect the logic of ordinary language; however, (iii) CQT fails to do that.

Obviously, (iii) is in need of a justification. However, there is an obvious connection between (iii) and the claim of better reflectivity. In what follows, at least with respect to the weak-argument, we will offer possible justifications for this claim, so for now,

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198 Hintikka, "Existential Presuppositions and Existential Commitments," 136.

199 Karel Lambert, "Notes on 'E!':II," *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition* 12, no. 1 and 2 (January - February 1961): 4. The methods meant here is of course the methods of free logics.

200 I hold that the following propositions express the same in content: (i) *x* reflects the logic of the ordinary language better, (ii) *x* represents the logic of ordinary language more accurately, and (iii) *x* is a better approximation toward the logic of ordinary language.

let us accept (iii) as true and concentrate on (ii). Let $({}_{s}5.2)_{ii}$ stand for this claim. Here is our evaluation of it.

First of all, $({}_{s}5.2)_{ii}$ seems to be a mere idealization. The issue whether or not a quantification theory can completely reflect the logic of the ordinary language should be at least regarded as controversial.[201] Let me express this idea in a different way. $({}_{s}5.2)_{ii}$ rests on the assumption that a quantification theory can in principle reflect the logic of ordinary language.[202] That means, if logical systems are incapable of reflecting the logic of ordinary language, then there would be no reason to accept $({}_{s}5.2)_{ii}$. Hence, in order to come to a decision about $({}_{s}5.2)_{ii}$, one needs to evaluate this assumption first. Having said this, we cannot do justice to this issue here. It is a complicated claim and deserves a thorough evaluation. However, let me say that even if the assumption is sound and $({}_{s}5.2)_{ii}$ is true, it would certainly not plausible to consider it as a defect for a logical system not to approximate towards its ideal. In other words, (i) of $({}_{s}5.2)$ is highly controversial.

Second, although the idea expressed in $({}_{s}5.2)_{ii}$ may have a certain attractiveness at first glance, I do not think that it can be justified easily. To put it differently, even if logic is capable of reflecting the logic of ordinary language, whether this is a desirable thing or not is controversial. As a matter of fact, $({}_{s}5.2)_{ii}$ is a claim that has been often targeted. For instance, let us recall Frege's account on the relation between logic and ordinary language. He regards ordinary language defective and argues for the construction of a formal language. in which these 'defect's are

[201] One could of course try to weaken this claim so that it would not rest on a relation of complete reflection. For example, the claim can be revised as follows: A quantification theory must reflect the logic of the ordinary language as much as possible. This idea, however, seems to me as much an idealization as the previous one.

[202] Lambert, for instance, in the passage quoted just above, explicitly says that he is in agreement with this assumption. To repeat, his exact words are follows: "(...) this paper (...) is not in agreement with the autistic doctrine that formal logic is in principle incapable of correctly representing the logic of ordinary discourse." Lambert, "Notes on 'E!':II," 4.

eliminated. That means, at least for Frege, quantification theory need not reflect the logic of ordinary language. As a matter of fact, almost every pragmatic stance on the relation between logic and language, would entail a refusal of $(_s5.2)_{ii}$.

The two reasons offered above should be enough to conclude that $(_s5.2)_{ii}$ is not acceptable. Consequently, SA5 fails. Let us try the weaker form of argument. I shall again refrain myself from presenting the argument in its full form. The key premise should be enough for an evaluation of the weak argument, which we shall call WA5. The premise is as follows:

$(_w5.2)$ (i) CQT is improved if the assumption of singular existence is abandoned because (ii) the resulting system would reflect the logic of the ordinary language better.

In what follows, I will offer a critical evaluation of $(_w5.2)$. Note that, however, it is not very clear from the premise itself as well as from the passages above, in what sense free logics reflect the logic of ordinary language better. Thus, the claim of better reflectivity is in need of a clarification. We could of course do that, but another way to advance would be to suppose that it is clear in what sense the claim of a better reflectivity should be understood, but regard it as an unjustified claim and offer possible justifications for it. This is how we will proceed in what follows. Let us start the discussion with a remark again.

It can be easily noted that $(_w5.2)$ rests on an assumption, which we can express as follows: If a certain revision of a quantification theory reflects ordinary language better than the original, then the revision should be considered as an improvement and the revised system should be preferred over the original. Let me give an example of such a position. Burge, in an attempt to counter RT, says that "Russell's elimination of singular terms pays well-known dividends but fails to account for natural language as it is actually used."[203] It is clear from the context that he believes that 'accounting for natural language' is for the better.

There are of course some well-known views against this assumption. After all, reflecting the ordinary language might not be an accurate criterion for logic as Frege for instance believed. We have seen that Carnap or Quine, although in different contexts, had similar ideas on this. Thus, despite all its appeal, there is nothing so indispensable about the idea that a quantification theory that represents the ordinary language better should be preferred. Nevertheless, for the sake of the argument, I shall accept this assumption true for the following discussion.

After this remark, now it is time to evaluate ($_w$5.2). For our purposes, it should be enough to concentrate on the second part of the premise that I have labeled as (ii) in the above formulation of ($_w$5.2). Since the 'resulting system' in question here is nothing but free logics, this claim can be simplified as follows:

($_w$5.2)$_{ii}$ Free logics reflect the logic of ordinary language better.

As made it clear above, I will introduce and discuss possible justifications for this claim. To this end, I shall look at some passages again.

Leonard, in a talk he gave, says, "the wide language (...) that I envisage, would be like English in counting all of these sentences as instances of the basic subject-predicate pattern of sentence."[204] Note that by 'the wide language', he means a free language; and by 'all of these sentences', he means ordinary language sentences of the form 'such-and-such is so-and-so' where 'such-and-such' is an EST.

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[203] Burge, "Truth and Singular Terms," 310.

[204] Leonard, "Essences, Attributes, and Predicates," 29.

Accordingly, Leonard's idea seems to be that ordinary language sentences are taken to be more at their surface value in free logics. This idea may serve as a possible justification for $({}_{w}5.2)_{ii}$. Let $J^1({}_{w}5.2)_{ii}$ stand for a justification as such.

At first glance, $J^1({}_{w}5.2)_{ii}$ has a certain appeal. After all, in free logics, subject-predicate sentences of natural languages are not assigned a separate logical form besides their grammatical forms. To illustrate, in free logics, an ordinary language sentence like 'such-and-such is so-and-so' is analyzed in the same uniform way independent of the fact whether the singular term 'such-and-such' denotes an existent or not. In either case, the sentence is translated into the formal language in the same way. In this sense, it is closer to the grammatical/surface form of the sentence. If we think about CQT, however, the same sentence must be paraphrased if 'such-and-such' is non-denoting in order to be translated into the formal system.[205] In this sense, free logics seem to reflect the logic of ordinary language better than CQT.

That being said, the view loses some of its appeal once we consider another alternative of CQT. A quantification theory with an unrestricted interpretation of quantifiers seems to be taking ordinary language sentences also at their surface values.[206] In Lejewski's system, for instance, which we have briefly mentioned earlier, the sentence 'such-and-such is so-and-so' where 'such-and-such' is an EST is translated into the formal system in the same way as in free logics. On this basis, it seems that $J^1({}_{w}5.2)_{ii}$ can be adapted so that it would equally serve as a justification for the claim that Meinongian logic is an improvement over CQT. Most free

[205] Let me illustrate this. In free logics, the sentence 'Pegasus is a winged horse' is translated into the formal system as Wp where the predicate W and the individual constant p respectively stand for the general term 'is a winged horse' and the (empty) singular term 'Pegasus'. In CQT, however, the same sentence is symbolized as follows: $\exists x(\forall y(Py \leftrightarrow x = y) \wedge Wx$.

[206] I shall call such logics, although perhaps not entirely accurate, as Meinongian logics in the following discussion.

logicians, however, would not accept this claim.[207] In any case, WA5 can be considered as equally valid for Meinongian logics. This fact should be enough by itself to conclude that WA5 is inconclusive in justifying an amendment in CQT in the way free logicians suggest.

Another possible justification for $({}_{w}5.2)_{ii}$ is grounded on a comparison between free logics and CQT in terms of their allowance for ESTs. Bencivenga, while talking about abandoning the assumption of singular existence, hints at how this idea can be turned into an argument for free logics:

> The need for such a "liberation" is usually articulated as follows. Natural language contains non-denoting singular terms: empty names like 'Pegasus' and improper descriptions like 'the round square'. Classical logic, on the other hand, allows for no such terms.[208]

The following passage from another philosopher, namely Richard Grandy, also shows an example of a similar position:

> I take the serious motivation for extending logic (and truth theory) to include non-denoting singular terms to be given by sentences such as the original Russell example, i.e., 'The present King of France is bald'. In everyday talk we use singular terms which we believe, but do not know, denote; it seems desirable to have a logic which acknowledge this fact.[209]

The idea should be clear enough. Free logics, in contrast to CQT, allow for ESTs, which are already contained in the ordinary language. In this sense, they reflect the logic of ordinary language better than CQT. Let $J^2({}_{w}5.2)_{ii}$ stand for a justification in this line of thought.

Note that theories such as Russell's provide us with a reason to reject $J^2({}_{w}5.2)_{ii}$. After all, the expressions we normally consider as ESTs turn out to be as no genuine

[207] After all, as we have seen earlier, one of the two important features of free logics was that quantifiers have existential import.

[208] Ermanno Bencivenga, "Free From What?," *Erkenntnis* 33, no. 1 (July 1990): 9.

[209] Richard E. Grandy, "A Definition of Truth for Theories with Intensional Definite Description Operators," *Journal of Philosophical Logic* 1, no. 2 (May 1972): 140.

singular terms on RT. That means that RT entails the view that ordinary language does not contain ESTs. Accordingly, for a Russellian on this, quite on the contrary to $J^2({}_w5.2)_{ii}$, it is CQT, not free logics, that reflects the logic of ordinary language better. Nevertheless, I shall ignore any such position for the following discussion.

The justification $J^2({}_w5.2)_{ii}$, just like the previous one, seems reasonable for the claim of a better reflectivity. That being said, it has a similar drawback. If one accepts that $J^2({}_w5.2)_{ii}$ provides enough reason to justify $({}_w5.2)_{ii}$, then she must also accept that it provides enough reason to justify the claim that Meinongian logics reflect the logic of ordinary language better, for they allow ESTs as well. If that is not something one can afford to accept, than one should reject $J^2({}_w5.2)_{ii}$.

Another possible justification for the alleged relation between free logics and ordinary language in terms of reflectivity is based on certain semantic intuitions. Grandy's following words show what these intuitions are: "Motivation is usually cited as the obvious truth of some such sentences as 'Pegasus is a horse' or 'Hamlet loved Ophelia' (...)"[210] How this serves as a justification for $({}_w5.2)_{ii}$ should be more or less clear: It seems that we sometimes attribute truth to some subject-predicate sentences whose subject terms are non-denoting. Since free logics, in particular, the positive variety, are in accordance with these semantic intuitions, it reflects ordinary language better. Let us call any justification in this line of thought as $J^3({}_w5.2)_{ii}$ and see if it is acceptable or not.

There is more than one obvious problem with $J^3({}_w5.2)_{ii}$. First of all, not everyone would accept that the kind of statements Grandy mentions as true. A Fregean, for instance, would evaluate these statements as truth-valueless, for neither 'Pegasus' nor 'Hamlet' is denoting. In a similar manner, a Russellian would evaluate

[210] Ibid.

these statements as false. In short, there are at least two different ways to evaluate this kind of sentences besides accepting them as true.

Another problem with $J^3({}_w5.2)_{ii}$ is that it essentially suffers from the same shortcomings as the previous justifications. To be precise, if we accept that kind of statements as true, they would also be true in models of Meinongian logics. That is to say, with this kind of a justification, the premise $({}_w5.2)$ can be equally adapted as a premise for an argument in favor of Meinongian logics. Thus, as before, we conclude that $J^3({}_w5.2)_{ii}$ is not a proper justification for $({}_w5.2)_{ii}$.

The discussion about $({}_w5.2)_{ii}$ so far suggests that this claim is unjustified, at least as far as those justifications we have mentioned above are considered. To be sure, there may be other justifications for $({}_w5.2)_{ii}$. Regardless, however, there is another way to challenge WA5, which we have already hinted at in the beginning of this section. The real weakness of $({}_w5.2)$ seems to reside in the other claim it includes, i.e., the claim that a quantification theory that reflects the logic of ordinary language better is an improvement. Let us concentrate on this claim now.

Although this idea may have a certain appeal at first glance, it also requires a justification. However, this does not seem to be an easy task either, so I will refrain myself from engaging in it. Instead, considering its intuitiveness, I shall simply accept it as true. The claim, in a more general form, can be expressed as follows:

(i) Let A be a quantification theory and A^* a revision of it. Suppose A^* reflects the logic of ordinary language better than A. In this case, A^* should be regarded as an improvement over A.

The formulation above shows what is actually missing in an argumentation based on (i). Obviously, (i) would justify replacing A with A^* only if A and A^* are equal in all other respects. After all, if A^*, for instance, is not sound with respect to a usual semantics, but A is, then, this may be certainly a reason to choose A over A^*. To conclude, even if we accept the claim that free logics reflect the logic of ordinary language better than CQT, and furthermore, that this is indeed an improvement, that still not does give us enough reason to choose free logics over CQT. There are other factors to consider about free logics and CQT before we can come to such a decision.

In conclusion, I believe that the premise ($_w$5.2), no matter how much it may seem appealing, should be rejected. That being said, as hinted above, there may be some contexts where, for some reason, reflecting the logic of ordinary language is better can be an important factor. In this case, if the argument of a better reflectivity were restricted to this specific context, it might succeed. Nevertheless, if considered as a general argument in favor of free logics, we conclude that WA5 fails. Considered together with the conclusion we arrived at about SA5, we can safely conclude that the motivation from a better reflectivity is not enough to justify the adoption of free logics instead of CQT.

The Motivation from Logical Form

The next motivation to be addressed is based on a certain view of logical form. The idea is roughly that logical form should be independent of empirical facts. To give a better description of it, let us look at some passages again. We shall begin with the following passage from Leonard:

> The question as to whether or not an expression designates is not always one about *form* or purport; rather it is frequently a question of *fact*, and is always so in the case of expressions which purport to designate. Hence, applications of modern logic wait on the systematic exploration of

specific matters of fact: do such and such expressions designate; that is, do the things they purport to designate have singular existence?[211]

It seems that it is the troublesome rule of EG what Leonard has in mind when he says 'applications of modern logic' here. As we have seen, EG with respect to a singular term *t* is a valid inference only if *t* has existential import. This, however, Leonard says, is a question of fact but not one of logical form. Although not stated in this passage explicitly, he apparently believes that the application of logical rules or laws to an ordinary language statement should not be dependent on the question whether a term denotes or not.

Note that this idea provides a direct challenge against RM. The suggestion to restrict singular terms of formal language to referring expressions of ordinary language implies that one needs to make sure that an expression is denoting ann existent before substituting it for a singular term of formal language. Alternatively, the same idea can also be formulated as a direct challenge against RT or any other theory in its footsteps. For, according to RT, the decision whether a statement such as 'such-and-such is so-and-so' is a subject-predicate sentence or not is grounded on the question whether 'such-and-such' denotes or not. In what follows, I shall look at both of these alternatives.

For the former alternative, i.e., for how it provides a challenge to RM, consider the following passage from Lejewski:

> It follows from his remarks that before we can safely use certain laws established by logic, we have to find out whether the noun-expressions we may like to employ, are empty or not. This, however, seems to be a purely empirical question.[212]

[211] Leonard, "The Logic of Existence," 53--54 (my emphasis).

[212] Lejewski, "Logic and Existence," 108. Note that Lejewski targets Quine in this passage. He attributes the method what we labeled as RM to him.

The connection to RM is obvious in Lejewski's words. Say, we want to apply EG to an ordinary language statement such as 'such-and-such is so-and-so'. Before we can safely do that, we have to be sure that the expression 'such-and-such' is denoting an existent so that it can take the place of the singular term t in $A(t/x)$ of EG. This restriction, of course, is applicable to every logical rule and law. After reminding that this restriction not only applies to ESTs, but also to "noun-expressions of which we do not know whether they are empty or not,"[213] Lejewski continues as follows:

> This state of affairs does not seem to be very satisfactory. The idea that some of our rules of inference should depend on empirical information, which may or may not be forthcoming, is so foreign to the character of logical enquiry (…)[214]

It is clear from Lejewski's words that he believes that whether a logical rule is applicable to a statement or not should not in any way depend on empirical information. The reason for this, he believes, is that this is against "the character of logical enquiry."[215] Hintikka, in the following passage, shows a similar position:

> Existential presuppositions in effect prejudge all questions concerning the existence of individuals referred to by singular terms which occur in our model sets or which can be substituted for our free individual symbols. They thus imply the unsatisfactory conclusion that a decision concerning the syntactical status of a term may depend on the decision of the factual question concerning the existence of the individual to which it purportedly refers.[216]

Evidently, Hintikka is not comfortable with the idea that the decision whether or not an expression of natural language is eligible take the place of a singular term of

[213] Ibid.

[214] Ibid.

[215] For another passage in this direction, consider the following: "There is, however, a serious downside to the current way of fixing Quine's theory of predication. Anyone thinking of predication as kind of a logical form (...) is bound to be upset by the realization that in many cases what qualifies as a predication will depend on empirical facts." Lambert, "Predication and Existentiality," in *Free Logic: Selected Essays*, ed. Karel Lambert (Cambridge: Cambridge University Press, 2003), 103.

[216] Hintikka, "Studies in the Logic of Existence and Necessity," 60. Note that it is individual constants of formal language what Hintikka means by 'free individual symbols'.

formal language depends on the question whether the expression has existential import or not. Proceeding from this, he argues in favor of the elimination of 'existential presuppositions'. Hintikka's words already show how this idea can be turned into an argument for free logics.

Now let us consider the other alternative, i.e., how the consideration as regards the relation between logical form and empirical facts provides a challenge for RT. To this end, consider another passage again from Hintikka:

> The reason why (C) is objectionable is that we do not know for sure whether the name ' Homer' had a bearer or not, for existential generalization with respect to a term is a valid inference only if the term in question really refers to something or somebody. But from the point of view of logic there is little to choose between different names. The question whether a putative name has a bearer is always a contingent one; such questions as 'Did N.N. really exist?' cannot be answered by purely logical means. Hence it does not suffice to discard names which are admittedly empty; one is involved in the quixotic attempt to explain away all or nearly all proper names.[217]

Although it may not be as clear as one might want, Hintikka's target seems to be RT this time. As we have seen, Russellian solution to the problem of empty names simply consists in their elimination from the language. However, since the decision whether a term is denoting or not depends on empirical information, which we may or may not know, Russellian solution must be extended to all proper names as Hintikka draws attention above. It appears that he is not happy with this consequence. Needless to say, the previous consideration with respect to RM applies to the Russellian way out of the difficulty as well. Simply put, the idea that the decision whether a sentence has a subject-predicate form or not depends on factual matters is highly controversial.

[217] Jaakko Hintikka, "Towards a Theory of Definite Descriptions," *Analysis* 19, no. 4 (March 1959): 80. Note that '(C)' stands for the following inference: "Homer was a Greek poet who composed the Iliad; therefore there was a Greek poet who composed the Iliad."

The passages quoted above should be enough to make the present motivation clear. Roughly put, the question whether a term is empty or not depends on empirical knowledge, however, logical forms of statements should be independent of empirical matters. As before, we will formulate this motivation as an argument in order to make a proper evaluation of it. But first, let us make some remarks.

As we have seen above, the motivation in question presents a challenge to both RM and the Russellian solution to the problems that arise in connection to ESTs. On the other hand, it is clear that it does not provide a challenge to Fregean solution. The reason for this is that Frege admits non-denoting expressions as genuine singular terms and allows for their unrestricted substitution for singular terms of a formal language.[218]

Second, although the consideration regarding logical form concerns both RT and RM, I shall only concentrate on RM in what follows for reasons that should be more or less obvious. For one thing, we have already taken a stance against any solution along RT. Furthermore, as we have noted earlier, any objection to RM can be easily turned into an objection to RT. For these reasons, I shall be contended with a brief mention of how the motivation in question can be formulated as an objection against any solution based on the method of paraphrase.

As before, I shall refrain myself from giving a full argument for the relevant motivation. The argument will be in an SA-form first. This time I will also not offer a precise formulation of the key premise. Given the quotations above, it should be more or less clear anyway. An informal description of the premise should be enough for the discussion to follow.

[218] The same reasoning certainly applies to the Meinongian solution as well.

Because of the assumption of singular existence, RM requires singular terms of a formal language to be restricted to denoting expressions of natural languages language. As a result of this, logical rules or principles become applicable only to statements that do not include non-denoting expressions. For example, EG can be applied only to subject-predicate statements where the subject denotes an existent. Accordingly, the inference from 'Santa Claus lives at the North Pole' to 'there exists something living at the North Pole' is invalid since the former statement does not have the required logical form $A(t/x)$. This idea, however, raise several difficulties. For one thing, it entails the view that the logical form to be assigned to a statement depends on the question whether the terms it includes denote existents or not. Let me illustrate this with a few examples. Compare the following two statements:

(45) Sandy Island was charted by James Cook.

(46) New Caledonia was charted by James Cook.

Since Sandy Island turned out to be a phantom island, (45), unlike (46), does not have the logical form $A(t/x)$. However, the fact that they have different logical forms have revealed itself only after the recent 'undiscovery' of Sandy Island. Although this example is probably enough to show how unintuitive the idea is, let me offer two more examples.

In the very beginning of this study, we have mentioned that Sandy Island is a French territory, at the least for the period it was assumed to be existing. Therefore, it is also known as *Île de Sable*, which is the French equivalent of 'Sandy Island'. As a matter of fact, there are more than one 'Sable' island. Another island that is known by the same name is a small island situated 300 km southeast of Halifax, Nova Scotia. Now consider the following sentence:

(47) *Île de Sable* is an island.

In CQT, the logical form to be assigned to this sentence changes depending on the question which of the Sable islands it is meant. If it is the ‘phantom’ one, than the sentence has the form:

(48) $\exists x(\forall y(Sy \leftrightarrow x = y) \wedge Ix$[219]

If, on the other hand, it is the existent Sabre island that is meant, the same statement has the following form:

(49) Is[220]

As if it is not weird enough to assign different logical forms to the ‘same’ statement, if it was a few years ago, i.e., before a group of scientist ‘undiscovered’ Sandy Island, we would assign the exact same form, namely (47), to the very same statement, no matter which Sabre island it is meant.

For another example to illustrate the problem, let us mention another phantom settlement. According to *Wikipedia*, “Agloe, New York was invented on a 1930s map as a phantom settlement. In 1950, a general store was built there and named Agloe General Store, as that was the name seen on the map. Therefore, the phantom settlement actually became a real one.”[221] Accordingly, the statement ‘Agloe, New York is such-and-such’ had the subject-predicate form from 1930s until it has been discovered to be a phantom settlement. Starting from this ‘undiscovery’ until 1950s,

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[219] Note that the predicate I stands for the predicate ‘is an island’ and S stands for some predicate abstracted from ‘Sable Island’, say, ‘Sable islandizes’.

[220] The individual constant s refers to (existent) *Île de Sable*.

[221] “Agloe, New York” *Wikipedia*, last modified June 4, 2014, http://en.wikipedia.org/wiki/Agloe,_New_York

the very same statement had another logical form, one that is similar to (46). Then, in the 1950s, the statement has again changed its logical form after the foundation of a real settlement with this name.

The conclusion to be drawn from these examples should be clear. We usually believe that logical form should be independent of empirical facts; however, the above examples clearly show that this is not the case as far as CQT is considered. In other words, RM entails the conclusion that the logical form of an ordinary language statement depends on factual matters. This conclusion, however, does not seem to be very intuitive. Accordingly, developing a logical system that is invariant to empirical matters can be another source of motivation for free logics.

Let the premise ($_s$6.2) stand for such a claim. I shall call the resulting argument as SA6. A closer look at it would easily reveal that the premise includes more than one claim; the key claim, however, is the following one:

($_s$6.2)$_i$ Logical form should be independent of factual matters.

If ($_s$6.2)$_i$ is accepted, I see no reason for why ($_s$6.2) should fail. Consequently, SA6 would succeed in justifying free logics. So let us concentrate on this claim.

I admit that ($_s$6.2)$_i$ has a certain appeal at first glance. After all, the form of a statement supposed to be what is left when the statement is abstracted from its contents. Hence, there is a sense in the claim that the logical form of a statement should not contain anything that can be associated with empirical facts. To put it differently, logical form should be 'fact-free'. If so, assigning different forms to (45) and (46), for instance, seems unjustified.

That being said, albeit all its appeal, the idea that logic should be *completely* independent of empirical matters has been often challenged, most famously by

Quine. His position on this issue well beyond the scope of this thesis. Nevertheless, let us look at one of the possible responses to him. For this, consider the following passage from Meyer and Lambert:

> Yet it is ridiculous that from x = x the logician may assert "Caesar = Caesar", withhold comment on "Pegasus = Pegasus" (but see previous footnote), and ring up his archeological colleague with respect to "Romulus = Romulus". To be sure, Quine's convincing refutation of the myth of analyticity disposes of the corresponding myth that the logician may be wholly unmoved by the facts of life, but we do not expect to hear him mutter, while reading the paper over his morning coffee, "By God, Romulus is self-identical after all!"[222]

The example seems convincing enough. It may not be that unreasonable to expect that logic, at least to some extent, be independent of empirical facts. Thus, the argument against the view that logic should be completely independent of empirical matters may not actually invalidate $({}_{s}6.2)_{i}$ if understood in this restricted sense.

That being said, it seems not easy to justify this claim. In the first place, one needs a certain view of logical form for this. To put it differently, first of all one has to make clear what logical form is before giving a justification for the claim that it must be independent of empirical facts. There seems to be no such standard view, however. Second, it is hard to analyze the assertion into its components.

To be sure, we could try to find a more general principle from which $({}_{s}6.2)_{i}$ would follow. There is already an obvious candidate for this. Recall the general idea underlying the motivation from the ideal of logic is based on. With its obvious roots in logical positivism, it indeed seems to provide a justification for $({}_{s}6.2)_{i}$. Nevertheless, since I have already dismissed this general claim in the relevant discussion, I shall not repeat myself here.

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[222] Robert K. Meyer and Karel Lambert, "Universally Free Logic and Standard Quantification Theory," 10.

As a matter of fact, it seems that one might even argue for the contrary claim, i.e., that logical form is not independent from empirical facts. Lehmann, in a similar discussion regarding logical form, gives the following example:

> When Alice says, 'I'm hungry', Bill adds, 'But I'm not', there is no contradiction: the form of what Alice says is represented by Ha and the form of what Bill says by ¬Hb. If logical forms reflect the different uses of indexicals like 'I', as they must in one way or another, then why may they not reflect empirical facts as whether "terms" refer?[223]

His objection seems reasonable enough. Accordingly, we can conclude that $(_s6.2)_i$, how intuitive it may sound, is not justified. So let us try to weaken the argument.

As before, the weaker form of the argument is grounded on a comparison between CQT and free logics. Independent of the fact whether $(_s6.2)_i$ is true or not, one thing is for sure. In free logics, logical forms of statements seem to be less dependent on factual matters. To see this, consider again (45) and (46). In free logics, these statements do not differ with respect to their logical forms; both are treated as subject-predicate sentences. Accordingly, if it is for the better for a logical system to be less dependent on factual matters with respect to logical form, that would give us an argument in an WA-form. I shall call any argument developed in this line as WA6. The key premise of the argument is simply that a logical system that is less dependent on empirical facts is better,[224] other things being equal of course. Let us call this premise $(_w6.2)$ and see if it is tenable or not.

The first thing to notice about $(_w6.2)$ is that it appears very intuitive at first glance. After all, as far as free logics are considered, there is no need for an exploration whether the places 'Sandy Island' and 'New Caledonia' purport to denote really exist or not before we can assign logical forms to statements including

[223] Lehmann, "More Free Logic," 213--14.

[224] To be more precise, a logical system in which logical form of statements is less dependent on empirical facts is better.

them such as (45) and (46). That means that free logics allow us to reason with less empirical information. Considered from this point of view, this seems to be an advantage over CQT. This may provide a justification for ($_w$6.2), so let us see if this idea is plausible or not.

To begin with, I can see no reason for why anyone might not want that. After all, ESTs are a useful part of our ordinary discourse. Certainly, it should be desirable to have a logical system that depends less on empirical facts, which we may or may not know. To be sure, there are alternative ways for reasoning with ESTs without abandoning the general framework of CQT. However, in free logics, this is done in a more direct and less controversial way. Thus, the justification seems plausible.

Nevertheless, there is a drawback of this idea, and it lies in the second part of it. The same consideration we have offered about ($_w$5.2) in the end of the previous section would apply to ($_w$6.2), as well. To put it simply, the premise ($_w$6.2) actually rests on a comparison. However, it is hardly that we have a fair comparison in a specific respect unless we other things are equal. Considered from this aspect, ($_w$6.2) may not be acceptable. After all, not only CQT is a well-established system; in practice, it also seems to provide all the means one usually needs. Considered from this point of view, I do not find WA6 tenable.

Having said this, the same argument may succeed if restricted to a specific context where there is not such a big difference between both alternatives. After all, I see no reason for why being less dependent on empirical facts cannot be a major factor in a particular comparison. Nevertheless, especially given the great extent of the claim, I conclude that WA6, just like SA6, fails to justify an amendment in CQT in the way free logicians suggest.

Thus ends our discussion regarding the motivation from logical form. To sum up, we have arrived at the conclusion that the relevant motivation, at least in the general form considered above, fails to justify free logics. Moreover, we have also briefly introduced the idea that the argument may succeed if restricted to a specific context. In fact, this is more or less the general thought behind the next and the final motivation we will consider.

A Further Motivation to Consider

Up to now, we have considered six different kinds of motivations for free logics that one usually finds in the literature. The discussion so far have hopefully showed that none of them succeeds in justifying the adoption of free systems instead of CQT. I shall close the discussion with a suggestion. To this end, I will address a further kind of motivation, which I hope demonstrates how a more successful argument for free logics can be developed.

As I have already stated at the very beginning of the chapter, the motivation to be addressed now, hereafter shortened as M7, will be substantially different in certain respects from those we have considered so far. In what follows, I shall mention two of these differences, which would hopefully help to clarify some important points about M7.

The first difference is related to the scope of the claim which M7 is based on. All of the previous motivations were such that they can be associated with the general claim that free logics should replace CQT no matter what the context the logic is applied to.[225] The conclusion that all arguments fail may be suggesting that we should perhaps restrict the scope of this claim. This is the main idea behind M7. Simply put, M7 will not include a general claim as before but rather a restricted one. More precisely, the claim that free logics should replace CQT will be restricted to certain contexts this time. In this sense, it is possible to say that M7 operates more like a placeholder for arguments given in favor of the adoption of free logics in a particular context.

As a second difference, no through consideration will be offered for M7 as has been done for the previous motivations. I will not formulate M7 as an argument and offer a critical assessment of it. The reason for this choice is partly related to the first point made above. Since M7 is *indexed* to certain contexts, it would not be fair to treat it as a stand-alone argument. On the other hand, addressing it in this restricted sense obviously deserves a more intensive treatment than we can offer here. However, given that our intention is merely to conclude the discussion with a suggestion of how a successful argument for free logics might be like, we do not need a thorough consideration of M7 anyway. It would be enough for our purposes to treat M7 as a suggestion and show how it may succeed in justifying free logics.[226]

The points made above will be more evident after we clarify M7, so let us start with this task now. Our starting point is a similar discussion about the motivations behind free logics that can be found in Lehmann's "More Free Logic." In this paper, Lehmann groups the motivations for free logics under four headings of which one he

[225] For instance, consider the last motivation above. The idea was basically that in free logics, in contrast to CQT, logical form is not dependent on factual matters so that the former should replace the latter. Thus, the claim is valid no matter what the context is the logic would be applied to.

[226] Note that this also means that I will be less rigorous in the discussion to come. For the most part, I will not be as formal and explanatory as I was before and often omit some points.

calls “Logical Habitats for Philosophical Doctrines”[227]. He describes the general idea underlying this type of motivation as follows: “Certain philosophical doctrines can be expressed in first-order languages, classically conceived, only with difficulty and in ways that will seem artificial.”[228] The claim seems quite straightforward. In his own words,

> Free semantics provides a more neutral logical setting than classical semantics for certain philosophical views. It permits distinctions upon which they depend to be made in a straightforward way and does not prejudice the case against them by rendering important claims logically false. [229]

His choice of words here may seem a bit strange, so let us first concentrate on this issue. To begin with, there seems to be no reason for why the claim of ‘neutrality’ should be restricted to philosophical views. It appears, however, that what Lehmann has in mind with this expression is more comprehensive than one usually attributes to it. In what follows, I will rather use ‘contexts’ instead of ‘philosophical views’. Second, the use of ‘free/classical semantics’ reflects the semantical approach he adopts in his paper, however, ‘free/classical logic’ seems more accurate for us. Finally, the expression ‘a more neutral setting’ may be somewhat ambiguous. Perhaps, one could simply say ‘more suitable’ instead. However, since the second sentence in the quotation above, as well as Lehmann’s examples following, make the meaning of the expression clear, I will stick to it. In light of these considerations, the claim can be simply expressed as follows:

CL1 Free logics provide a more neutral setting than CQT for certain contexts.

[227] Lehmann, “More Free Logic,” 206--11.

[228] Ibid, 202.

[229] Ibid, 206.

The obvious way to support CL1 is to show some of these contexts, which is exactly what Lehmann does in his article. According to him, Meinong's doctrine is one of these contexts. He argues that Meinong's claim that there are non-existent objects finds a more neutral and simpler expression in free logics. Consider, the sentence '*a* does not exist', for instance.[230] In CQT, this sentence cannot be expressed in the simple form $\neg\exists x(a = x)$ because this is a logical falsehood, but the statement itself may be true. To be sure, the same sentence can be expressed in the complex form $\neg\exists x\exists y(\forall z(Az \leftrightarrow z = y) \wedge y = x$ (where A is a predicate abstracted from a) following RT without ending up in a logical falsehood. However, this form is much more complicated compared to $\neg\exists x(a = x)$ which is the usual expression the sentence in question finds in free logics.[231] The conclusion to be drawn from this example is that free logics provide a more neutral logical setting for Meinong's views.

There are other contexts that are addressed in Lehmann's paper. I will not give the details but be contended with a list of them. The contexts which allegedly finds a more neutral expression in free logics are (i) "standard Kripke-style modal semantics," (ii) intuitionism and constructivism, (iii) "standard examples of contingent a priori truths," (iv) Aristotle's idea that "true predication requires a subject," (v) Russell and Meinong's claim that "ascribing to a sentence a subject predicate form requires that the subject-term refer," (vi) Strawson's presupposition theory and (vii) mereology.[232]

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[230] Note that I am following Lehmann's own example here.

[231] There are of course other systems besides free logics for handling these kinds of statements. For instance, one can introduce another set of quantifiers as has been often done. Lehmann briefly considers these approaches, however, I will simply omit them here. Regardless, the idea is that free logics provide a more suitable setting for expressing Meinong's views compared to classical or quasi-classical alternatives.

[232] The details can be found in Lehmann, "More Free Logic," 207--11. The expressions enclosed in quotation marks above are due to Lehmann.

As noted earlier, I will not offer a thorough discussion of CL1 and the reason underlying this choice should be clearer now. Such a discussion would require to address all of the contexts listed above as well as others that may have been omitted here.[233] In fact, even if we were to consider all such contexts and conclude that free logics do indeed provide a more neutral setting for them, that would probably still not be enough to conclude that CL1 is true. For simply there may be other contexts where CQT has the same advantage over free logics. For these reasons, it is best to leave CL1 aside and try to come up with a claim that would be more suitable for our discussion.

Recall that my aim is to show how a successful argument in favor of free logics can be developed. The key idea was to restrict the argument as regards the adoption of free systems to specific contexts. Consider the following claim for instance:

CL2 Free logics provide a more neutral logical setting than CQT for modal discourse.

The same idea can be expressed more simply as follows:

CL2* Modal logic is/should be free.[234]

Note that coming to a conclusion about CL2* does not seem to be as hard as it was for CL1. For the decision ultimately depends on a consideration of CL2* itself. To be sure, the same is also valid for other contexts that have been listed above as well as

[233] To name a few of them, one can mention theory of definite descriptions, theory of partial functions, set theory, and programming. Needless to say, free logics have a wide and varied range of applicability.

[234] Strictly speaking, these formulations may not be expressing the same idea; however, for the discussion to follow, I shall leave the differences aside.

others Lehmann does not mention in his paper. Since our aim is merely to sample a more successful argument for free logics, let us concentrate on $CL2^*$.

The assertion that modal logic should be free is usually justified with a reference to counterfactual situations where some existing thing is pretended to be non-existing. Consider the following passage for instance:

> When we cease merely to report or to register what is true of the actual world and start to discuss what might not have happened or what could have happened, existential presuppositions soon become awkward. Surely it ought not to be logically inadmissible to try to say something of what might have happened if some particular individual had not existed, e.g., if there had been no Napoleon.[235]

The idea seems straightforward. To put it very roughly, one of the points of modal logic is to analyze propositions about counterfactual state-of-affairs, and there seems to be no reason for why some of them should not include non-existing objects. However, the assumption of singular existence and the rules following from it restrains the expression of state-of-affairs where some existing thing is pretended to be non-existing, for it simply does not allow for ESTs. In this sense, free logics seem to be providing a more suitable setting for modal discourse.

One way to argue in favor of $C2^*$ is to point out that, in modal logic, the assumption of singular existence is not valid. Let me illustrate this idea with an example. Consider the following sentence:

(50) This thesis might not have been written.

Intuitively speaking, the sentence seems to be true. After all, there was a good chance that I could choose a different topic or that I might not study philosophy in the first place so that this thesis would not have been written, save for the possibility that my

[235] Hintikka, “Studies in the Logic and Existence,” 61.

parents may have never met and I may not have been born at all. Thus, (50) seems to be true. This sentence is expressed in modal logic in the following form:

(51) $\Diamond\neg Wt$

where the predicate *W* stands for the property of having being written and the individual constant *t* refers to this thesis. Intuitively, (51) says that there is a world where for whatever reason it is not the case that this thesis has been written. Thus, the truth of (50) is demonstrated with a reference to a *possible world* where this thesis has not been written. In that world, this thesis obviously does not exist. That is to say that the constant *t* which refers to this thesis in this world, does not have a denotation in that hypothetical world. Thus, it seems that standard Kripke-style modal logic is free.[236]

Another way to argue in favor of $CL2^*$ is to address the problems that arise as a result of combining modal logic with CQT. Recall, for instance, Bacon's paper that we have mentioned earlier. His idea was simply that classical modal logic leads to *necessitism*, i.e., the view that entails that everything necessarily exists.[237] If one shows that free modal logic is immune to these problems, this might give us a good reason to adopt free logics instead of CQT in modal discourse.

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[236] The suggested reasoning certainly rests on what is known as the *world-relative domain* approach. In modal logic, there are two different semantic approaches to the issue of the domain of quantification. According to *fixed* or *constant domain* approach, there is a single domain of quantification for all worlds, which include all individuals of all worlds, i.e., every individual be it actual or possible. According to world-relative domain approach, on the other hand, each world has its own domain of quantification, which includes only those individuals that exist in that world. Accordingly, for the suggestion above to succeed, one obviously needs to argue against the fixed domain approach first.

[237] A similar claim can also be found in James W. Garson, "Application of Free Logic to Quantified Intensional Logic" in *The Philosophical Applications of Free Logic*, ed. Karel Lambert (New York: Oxford University Press, 1991), 111--42. See especially page 112.

A further support for basing modal logic on a free quantification theory rather than CQT can be found in a particular similarity between certain axiomatizations of these systems. As has been often drawn attention, Kripke's axiomatization of modal logic and Lambert's axiomatization of positive free logic (without identity and existence) share an important similarity. As pointed out in a previous footnote[238], the axiom schemata

(A4*) $\forall x \forall y A \supset \forall y \forall x A$

is common to both systems.[239] Although Kripke's system is not free, this point may still be suggesting that free logics provide a more natural alternative for modal logic than CQT.

Certainly, any decent discussion about the question whether CL2* is true or not requires much more space than we can spare here. As a matter of fact, this is an issue that has been received a considerable attention in the literature and the best we could offer would be probably not more than presenting the available views.[240] Let us be contended with the brief and somewhat shallowed support given for C2* above. However, even if the above support is found to be not enough, I think it still demonstrates how a more successful argument in favor of the adoption of free logics can be developed.

As a final point, recall that, for some of the motivations we have considered before, we arrived at the conclusion that if they were to be restricted, they might

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[238] See the footnote 127.

[239] In the previous chapter we gave the details of Lambert's system that we have labeled as PFL$^-$. For Kripke's system, see Saul Kripke, "Semantical Considerations on Modal Logic" in *Reference and Modality*, ed. Leonard Kinsky (Oxford: Oxford University Press, 1971), 69.

[240] The first source to look at for this issue is Garson's "Application of Free Logic to Quantified Intensional Logic." For an overview of the issue, see Nolt, "Free Logics," 147--51.

succeed. Accordingly, an argument that is developed along the line suggested above, can be further supported by some of our previous arguments.

To sum up this discussion, I think the right way to argue for free logics is to restrict the claim that free logics should replace CQT to specific contexts. I gave an example in this line of thought. As the relevant discussion might have already suggested, I am especially sympathetic to the view that modal logic should be free. This, however, is in no way the only context free logics provide a more neutral setting than CQT. In the literature, one can find a wide range of arguments in favor of the adoption of free logics in various contexts, and in general, I think that they all provide better attempts for the justification of free logics.

Conclusion

Recall the question that was posed at the beginning of this chapter: Why should one adopt free logics instead of CQT? Simply put, this chapter was centered on this very question which I have labeled as Q2 above. My aim was to survey different answers that has been or can be given to this question.

To this end, I examined six different motivations for free logics which are expressed the most in the literature. Each of these motivations provided a different answer to Q2. I presented and critically evaluated them. The discussion ultimately revealed that none of them succeeds in justifying the adoption of free logics instead of classical logic.

At the end of the chapter, I considered the option to restrict the claim as regards the adoption of free logics to specific contexts. To this end, I introduced another kind of motivation that seemed more promising to me. In order to demonstrate the idea underlying the motivation, I addressed modal discourse and briefly discussed the question whether modal logic is/should be free or not. My aim was to conclude the chapter with an example of a more successful answer to Q2 and I think the discussion regarding modal logic offers such an answer.

The conclusions I ultimately arrived at in the end of the chapter can be summarized as follows:

(i) The arguments given in favor of the adoption of free logics instead of CQT are not justified.

(ii) If some of these arguments were restricted to specific contexts, they might offer better challenges to CQT.

(iii) A more successful way to argue in favor of free logics is to restrict the claim regarding the adoption of free logics to specific contexts.

I, therefore, think that Q2 should be discussed with respect to specific contexts.

CHAPTER IV

CONCLUSION

This study was about logics that allow for expressions such as ‘Sandy Island.’ My aim was two-fold. Firstly, to give a presentation of these systems, and secondly, to discuss the question whether free logics should replace classical logic or not. To be sure, these aims were interconnected in that the latter question cannot be discussed without the former. In accordance with these ends, the study comprised of three chapters; one intended as an introduction to free logics, and two intended as to address the different questions mentioned above.

In the first chapter, I presented the general framework behind free logics. I first introduced the notion of an empty singular term. I then mentioned some of the problems that arise in connection with such expressions in CQT. I finally discussed some of the ‘un-free’ solutions to these problems and concluded that none of them is conclusive. The idea behind this chapter was to familiarize the reader with a certain kind of motivation behind the development of free systems of logic.

In the second chapter, I aimed to give a detailed answer to the question of what free logics are. This constituted the presentation part of this study. I started the chapter with some of the proposed definitions for free logics. I then put forward the characteristics of these systems. Finally, I introduced different kinds of free logics and thereby concluded the first section.

In the second section, I presented axiomatic systems of positive and negative free logics. As usual, for each system, I first fixed the formal language and then give the axiom schemata. I mentioned the necessary changes if the language was different in a certain respect. In general, I had two emphases in this section. First, I aimed to show how systems of free logics syntactically differ from CQT. Second, I intended to show how the characteristics of free systems find a syntactical expression.

The third section was devoted to free semantics. I started the section with an attempt to show why free semantics vary in the first place. To remind, free semantics differed on two major semantics questions. The first of these questions was whether ESTs denote individuals outside quantificational domain or they do not denote at all. The second question was how to evaluate the atomic formulas containing ESTs for truth-value. I introduced different semantic approaches which mainly differed on the answers they provide to these questions. For each kind of free semantics I have addressed, I gave its essential features, introduced the adopted model structure, presented the valuation function, and mentioned some of its major strengths and weaknesses over others.

I concluded the second chapter with a fourth section which provided a brief summary about the origins of free logics.

The third chapter was the argumentative part of this study. Simply put, in this chapter, I aimed to give an answer to the question of why one should adopt free logics instead of CQT, which I have labeled as Q2. To this end, I intended to survey the answers that have been already given to this question.

This survey was more or less equivalent to a study of the motivations behind free logics. Thus, I examined six different kinds of motivations for free logics that have been usually put forward by free logicians. I discussed whether the answers they provide to Q2 are justified or not. To this end, I formulated each motivation as an argument and offered a critical evaluation of it. I ultimately concluded that none of them succeeds in justifying the adoption of free logics instead of CQT.

After the discussion has revealed that none of the usual answers given to Q2 is justified, I considered the option to restrict the claim as regards the adoption of free logics to specific contexts. With this thought in mind, at the end of the chapter I addressed another kind of motivation that was different from the preceding ones in certain respects. This time, the idea was that free logics are more suitable than CQT for certain contexts. After briefly introducing some of these contexts, I concentrated on modal discourse, and in particular, on the question whether modal logic should be free or not. After a brief discussion, I sided with the assertion that free quantification theory provides a more successful alternative for modal logic in comparison to CQT.

The discussion with respect to the modal discourse was intended to demonstrate how one might successfully argue in favor of free logics. The conclusion I arrived at in the end is that if the claim regarding the adoption of free logics were restricted to a specific context, it might offer a more successful way for the justification of the adoption of these systems. The ultimate consequence to be drawn from this is that the question posed at the beginning of this chapter should be discussed with respect to specific contexts if one wants to give a more successful answer to it. For instance, the question

$Q2_1$ Why should one adopt free logics instead of CQT in modal logic?

which could be definitely another subject of a study such as this one, seems to be having a greater chance to have a more successful answer compared to Q2.

This study started with Sandy Island and so thus shall it end. Recall that we have defined free logics as the logics of ‘Sandy Island’ and similar expressions. Thus, the real question we should ask is, perhaps, where and in what context one may need a logic for ‘Sandy Island’.

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