IDENTITY, QUANTIFICATION AND SORTALS

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DECLARATION OF ORIGINALITY

I, Adil Alibaş, certify that

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- this thesis contains no material that has been submitted or accepted for a degree or diploma in any other educational institution;
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ABSTRACT

Identity, Quantification and Sortals

The topic of this thesis is the exploration of the relations between first-order quantification and identity and between identity conditions and sortals. In the first chapter, I will formulate a version of the thesis that quantification involves identity. I will formulate two principles that together entail this thesis and defend those principles from the arguments against them in the literature. In the second chapter, I will turn to the sortalist idea that the answers to identity questions depend on the identity conditions provided by sortal terms and point out the tension between two sortalist theses and their application in the standard semantics. Then, I will develop a version of situation semantics for first-order logic by adapting Fine's truth-maker semantics for propositional logic. Finally, I will argue that when the sortalist theses are applied in this semantics, the tension evaporates. In the third chapter, I will apply the semantics that I developed to the thesis that identity conditions can be taken as abstraction principles and to the thesis of sortal essentialism. I will show that the semantics that I developed is a versatile tool that can shed light on the problems involving identity that can arise in other branches of philosophy, for example, in philosophy of mathematics and modal metaphysics.

ÖZET

Özdeşlik, Niceleme ve Türeller

Bu tezin konusu birinci-derece niceleme ile özdeşlik arasında ve özdeşlik koşulları ile türeller arasındaki ilişkinin incelenmesidir. İlk bölümde nicelemenin özdeşliği içerdiği tezinin bir versiyonunu formule edeceğim. Birlikle alındığında bu tezi gerektiren iki prensip formule edeceğim ve bu prensipleri literatürdeki eleştirilere karşı savunacağım. İkinci bölümde türelcilerin fikri olan özdeşlik sorularına cevapların türeller tarafından sağlanan özdeşlik koşullarına bağlı olduğuna yönelecek ve türelcilerin savunduğu tezlerin standard semantik kullanan uygulamalarının gerilimini göstereceğim. Ardından, Fine'ın önermeler mantığı için kurduğu doğru-yapıcı semantiğine dayanarak birinci-derece niceleme mantığı için bir çeşit durum semantiği geliştireceğim. Son olarak türelcilerin tezlerinin bu geliştirilen semantik içindeki uygulamalarında gerilimin yok olduğunu iddia edeceğim. Üçüncü bölümde bu geliştirdiğim semantiği özdeşlik kouşullarının soyutlama prensipleri olarak alınabilceği tezine ve türel özcülük tezine uygulayacağım. Geliştirdiğim semantiğin felsefenin diğer alanlarında ortaya çıkan problemere ışık tutabilecek çok yönlü bir araç olduğunu matematik felsefesi ve kipsel metafizikten örneklerle göstereceğim.

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CHAPTER 1

INTRODUCTION

The topic of this thesis is the exploration of the relations between first-order quantification, identity, sortals and identity conditions. That there is a need for this kind of an exploration is best explained with an episode in the history of philosophy. Model theory of first-order logic has always treated the identity predicate differently from other predicates. Given a domain of quantification, we need to specify the extensions of all the other predicates, but there is no such need for the identity predicate. The reasoning was that we were treating the identity predicate as a logical constant. Just as the logical constants like truth functions, for example, conjunction and disjunction, do not change their interpretations from model to model, the identity predicate is also interpreted similarly. The meaning of identity is standardly conceived as the smallest reflexive relation. Whenever we are given a domain, we find the smallest reflexive relation in that domain and that is the extension of the identity predicate. To make the long story short, there was a single predicate that is identity which is treated as logical constant, i.e. having a fixed meaning.

Then came the challenge of such sortalists as Geach (1980) and Wiggins (1967). They argued that there was not only one identity predicate, but rather several distinct identity predicates indexed with some sortal. Besides, the extensions of those identity predicates were not absolute, but depended on the identity conditions provided by those sortals. On the other hand, Quine (1964) has defended the status quo with a novel idea that quantification involves identity. However, Quine's defense has been less than satisfactory.

That brings us to the topic of the first chapter. In the first chapter, I will formulate a version of the thesis that quantification involves identity. The version that I will defend will be different from Quine's and will be based on different principles. I will show that this formulation is not vulnerable to the attacks against Quine's version. Moreover, I will defend the principles that I used to justify the thesis and defend those principles from the attacks in the relevant literature.

However, we should not be dismissive of sortalists. Their thesis that the answers to identity questions depend on identity conditions provided by sortals demand a fair hearing. Moreover, even Quine, the champion of the status quo, appears to agree with sortalists. Ironically, one of his famous slogans is that no entity without identity (Quine, 1969), i.e. we should not add objects to our ontology for which we cannot provide clear identity conditions,

That brings us to the topic of the second chapter. In the second chapter, I will formulate two theses about sortal identity conditions which create tensions for the sortalists when applied by using the tools of the standard semantics. However, I will argue that the problem is not with the theses but their applications. I will show that when these theses are applied to a version of situation semantics, the tensions evaporate. Then, I will develop a version of Fine's truth-maker semantics (Fine, 2017) for propositional logic, into a semantics for first-order logic. This is the kind of semantics in which sortalist principles flourish. Finally, I will briefly comment on the relationships between the semantics that I developed and the standard semantics.

The last chapter will be a very brief exploration of some other possible uses of the semantics that I have developed. I will give two example cases to which the semantics that we developed can be applied. Although these examples are from different philosophical branches, they are closely related to the issues we discuss all through this thesis. The first case is from philosophy of mathematics. The idea is that we can use identity conditions as abstraction principles to introduce new mathematical entities. I will give the example of Julius Caesar problem by Frege (1884) as an instance of a problem that abstractionists face, but which can be surmounted if we exploit the resources of the new semantics. The second case is from modal metaphysics. It is the idea that objects instantiate sortals essentially. I will show that the semantics that I have developed embodies sortal essentialism. However, we can make modifications to avoid that result. Moreover, I will argue that showing clearly the kind of the semantic modifications it takes in order to move from essentialism to a non-essentialism about sortals, offers us new opportunities to evaluate the essentialist and non-essentialist positions.

Although I have introduced the topic of this thesis by mentioning a historical episode of disagreement with great pedigree, the purpose of the thesis is not historical or exegetical at all. The thesis is argumentative. The purpose of the thesis is to defend, first, the claim that quantification involves identity, second, to argue that this is not a fact that blocks the sortalist theses about identity conditions, and third to claim that if we develop a version of situation semantics then the investigation of the identity conditions and application condition of our terms is of paramount importance even in quantification, and finally, to show the semantics that I have developed is not limited in use but can be applied in other areas of philosophy.

CHAPTER 2

QUANTIFICATION AND IDENTITY

In this chapter, I will argue that in standard semantics of first-order logic, the interpretation of the identity predicate is determined by the resources that we use to evaluate the quantificational structure of the sentences. Similar theses have been proposed under the slogan that quantification involves identity (Dummett, 1991), (Quine, 1964). However, that slogan means different things in the writings of different philosophers. Even though one can interpret my thesis as a version of that slogan, it is a more sharpened proposition than theirs.

My defense of the thesis will have two components. One is a factual component that claims that interpretation of quantificational sentences requires recourse to the identity predicate of the meta-language. The second is a methodological principle that says the identity predicate of the language must be faithful to the identity predicate of the meta-language that is used to interpret it. After I explain what I mean by these components, I will show how these two components lead to the desired result. Then I will argue for these components.

Let us review some preliminaries and fix some terminology. First, the topic is about first-order languages that contain quantification. Second, what we are investigating is the standard account of identity. So, I will assume that each of the identity predicates, whether it is an object-level predicate or a meta-level one, is reflexive and satisfies Leibniz's law, i.e. it satisfies '$\forall x(x = x)$' and '$\forall xy(x = y \rightarrow Fx \rightarrow Fy)$'. Third, by standard semantics, I mean an assignment of a tuple <D,I> where 'D' is the non-empty domain of quantification and 'I' is an interpretation function that assigns extensions to predicates and denotations to terms. Fourth, I assume that this semantics is conducted in a meta-language which is also first-order and involves its own identity predicate.

Now I can explain the factual component of the thesis. In order to interpret quantificational sentences of the object language, one must use the identity predicate of the meta-language. Consider two sentences that are targeted by Hawthorne (2003), '$\exists x(Fx \land Gx)$' and '$\exists x \exists y(Fx \land Gy)$'. The difference between these two sentences stems from the fact that according to the first sentence the value that satisfies the predicate 'F' is the same value that satisfies the predicate 'G'. However, according to the second sentence the value that satisfies 'F' need not be identical to the value that satisfies 'G'. Therefore, the difference between the interpretation of these sentences depends on whether the values that satisfy these predicates are identical or not (Hawthorne, 2003). So, one needs recourse to the extension of the meta-level identity predicate to distinguish these two sentences.

The previous example showed that one makes a recourse to the meta-level identity predicate in order to distinguish the two formulas. A similar example will show that in order to interpret a formula one must defer to the meta-level identity predicate. Moreover, the same example will show that even for a first-order language that does not contain an identity predicate, the meta-level identity predicate must be determined. Consider the following, let <D,I> be an interpretation such that for some and only one value of 'x' it satisfies 'Fx' and for some and only one value of 'y' it satisfies 'Gy'. The question is whether this interpretation satisfies '$\exists x(Fx \land Gx)$'. Observe the following facts.

A) If the interpretation satisfies the sentence, then val(x) = val(y)

B) If the interpretation does not satisfy the sentence, then val(x) $\neq$ val(y)

Also observe that an interpretation must decide every sentence, i.e. it either satisfies or does not satisfy each sentence. Therefore, given that the interpretation depends on the identity of the values, any determinate interpretation must have a determinate meta-level identity predicate in order to evaluate quantificational sentences, even the ones that do not contain object-level identity predicate.

Third, observe that changing the identity predicate of meta-language is in effect changing the domain. Therefore, given a domain, the associated meta-level identity predicate is uniquely determined. An example will show what I mean by that. Let us start with a two membered domain of quantification, for example, D={a,b} and val(x) = a and val(y) = b and $ext(F) = \{a\}$ and $ext(G) = \{b\}$. Then this model does not satisfy '$\exists x(Fx \wedge Gx)$'. That is because, given we are working with a two membered domain, $a \neq b$ according to the meta-language identity. However, let us try to change that fact and assume $a = b$. If everything remains the same this new interpretation satisfies '$\exists x(Fx \wedge Gx)$'. That is because there is a value that satisfies both of those predicates. So, by changing the meta-level identity predicate we changed its satisfaction relation to some sentences. Yet, observe that that change of identity predicate did not leave the domain unchanged. Previously, we had a two membered set as a domain, now the domain of quantification is a singleton. What this shows is that changing the meta-level identity is in effect changing the domain of quantification.

Let us take stock. So far, I have shown three facts through examples. First, in order to evaluate first-order quantificational sentences, even the ones that do not contain identity predicate, one must refer to the meta-language identity predicate. Second, in order to evaluate first-order quantificational sentences, even the ones that do not contain identity predicate, the meta-language identity predicate must be determined. Third, in standard semantics for first order logic, the meta-level identity predicate comes with the domain, so changing the extension of meta-language identity is in effect changing the domain. Hence, it is the domain that determines the meta-level identity predicate, in the sense that no change in the identity predicate without a change in the domain. These three taken together means that a determinate meta-level identity predicate comes with the domain of quantification and furthermore, interpretation of first-order sentences must refer to that identity predicate.

So far, I have argued that in order to interpret quantificational sentences, one must make use of meta-language's identity predicate which is externally determined. However, I haven't said anything about the object-level identity predicate. The following methodological principle will bridge the two.

- Faithfulness: The object level identity predicate must be faithful to the meta-level identity predicate.

Now I need to unpack what I mean by faithfulness.

- An identity predicate $=_1$ is faithful to its meta language identity predicate $=_2$, if and only if, if $(d_i, d_j) \in ext(=_1)$ then $d_i =_2 d_j$.
- An identity predicate $=_1$ is minimal if and only if, for all meta-languages M with the domain D, $(d_i, d_j) \in ext(=_1)$ then $d_i =_M d_j$.

Observe that any minimal identity predicate is faithful to its meta-language identity predicate. That is because a minimal identity predicate is faithful to all meta-language interpretation with the same domain. Therefore, minimality is a much stronger condition than faithfulness.

This fact explains why the faithfulness requirement is already a part of standard semantics. In standard semantics we assume that all our identity predicates are minimal. Therefore, we need not also mention that they are faithful, given that minimality entails faithfulness. However, in this paper we will be more general and include non-minimal interpretations. In addition, given that faithfulness is a weaker assumption we will avoid criticism made against minimalism by basing our argument on the weaker assumption.

Now I will show that faithfulness requirement together with a given meta-language identity predicate fully determines the extension of the object-level identity predicate. Let us assume we have a formal language that we interpret which does not contain identity. We have argued that in order to interpret that language we need to be able to refer to the meta-language identity predicate, and argued that the meta-language identity predicate is fixed once a domain is fixed. Let us assume that we determined a domain, and acquired the corresponding meta-level identity predicate. Now we want to add the identity predicate to our formal language. What should its extension be? Observe that we want the extension to be reflexive so, if $d_i =_{Meta} d_j$ then $(d_i, d_j) \in ext(=_{object})$. If we furthermore assume that the object language identity predicate is faithful, then if $(d_i, d_j) \in ext(=_{object})$ then $d_i =_{Meta} d_j$. These two taken together mean that $(d_i, d_j) \in ext(=_{object})$ if and only if $d_i =_{Meta} d_j$. Therefore, once the meta-language identity predicate is determined so is the extension of object language identity predicate.

So far I have explained two principles. The first one was that interpretation of quantification uses meta-language identity predicate. The second one was the idea that object-level identity predicate must be faithful to the meta-level identity predicate. Furthermore, I have shown that these two principles together entail that the extension of the identity predicate is fixed once a domain is given. Now I will defend these two principles.

2.1 Wehmeier's objections

Recall, if '$\exists x(Fx \wedge Gx)$' follows from 'Fx' and 'Gy' then the value of 'x' under the interpretation is the same with the value of y under the interpretation. We assumed that fact involves identity. Therefore, we have taken all the following as equivalent.

A) 'x' and 'y' have the same value under the interpretation.

B) The value of 'x' and the value of 'y' is the same.

C) val(x) = val(y)

However, Wehmeier (2017) thinks that one need not interpret a sentence in the form of (A) with a sentence of the form of (B). He invites us to consider the following sentences.

D) John and James love the same woman.

E) The woman who is loved by John and the woman loved by James is the same

F) Lover-of(John) = Lover-of(James)

Furthermore, he points out that (D) and (E) do not have the same meaning. Assume that James loves Marry and Jane, and James loves Marry and Cathy. Wehmeier argues that in this situation (D) is true but (E) is false. That is because the definite description in (E) presupposes uniqueness, which is not presupposed by (D). Therefore, the step from (D) to (F) is blocked.

He argues that the chain A-B-C has the same structure as the chain D-E-F and the transition from D to F is blocked, so the same thing must be said about A to C. Therefore, he thinks that having the same value does not involve identity.

Let us assume that he is right that transition from D to F is illegitimate. However, this does not mean that his argument is successful. He needs to show that the same illegitimacy occurs in A to C. The reason for illegitimacy was that the function 'lover-of(x)' may not be single valued. If the same illegitimacy infects first-order semantics, we should expect also the possibility that the function 'value-of(x)' to be non-single valued. However, that the valuation function is single valued is essential for first-order semantics. Therefore, his argument fails. Now I will show that being single valued is necessary for the valuation function.

Let us assume, for reductio, that a term, be it a variable or a singular term has more than one value associated with it. Let 'x' be associated with $d_1, \ldots, d_n$. There are two cases to be considered. The first case is when the values of 'x' to be coordinated. Call the values of a term coordinated if and only if for all predicates 'F', if $\exists d_i d_i \in ext(F)$, then $\forall d_j d_j \in ext(F)$. Observe that if the values of the term are coordinated then for some value d_i of 'x', $d_i \in ext(F)$ if and only if for all values d_j of 'x' $d_j \in ext(F)$. Therefore, without loss of generality, we can take 'Fx' to be true in our model when all the values of 'x' are in the extension of the predicate 'F'.
Let I_1, be an interpretation that assigns multiple coordinated values to terms. Now I will give a recipe for a construction of a new interpretation which is single valued. If $val_{I_1}(x)$ is associated $d_1, \ldots, d_n$, then $val(x)_{I_2} = \{d_1, \ldots, d_n\}$, and the extensions $\{d_1, \ldots, d_n\} \in Ext(F)_{I_2} \leftrightarrow \forall d_j, d_j \in Ext(F)_{I_1}$. The fact is that for all sentences of first-order logic these two interpretations agree, i.e. if the first interpretation assigns truth to a sentence so does the second interpretation and vice versa. Therefore, coordinated multiple values is just a notational variant of single valued interpretations.

The second case is when the values are not coordinated. In this case we have two sub-cases. When will 'Fx' be true? Is it when all values are in the extension of the predicate or is it when some of the values are in the extension of the predicate? We have seen these two cases are the same when the values are coordinated but they come apart when the values are not coordinated. As the first sub-case, let us investigate taking ‘Fx’ to be true when some of the values of ‘x’ is in the extension of ‘F’. Recall these values are not coordinated, therefore it is possible for some value d_i, d_j of ‘x’ $d_i \in ext(F)$ but $d_j \notin ext(F)$. Then, according to that interpretation ‘$(Fx \wedge \neg Fx)$’ is true. Since some value of ‘x’ is in the extension of ‘F’, so ‘Fx’ is true. Moreover, some of the values are not in the extension, therefore, the negation is also true. Consequently, this sub-case leads to a change of logic. As a result, this interpretation is not suitable for providing a semantics for classical first-order logic.

The case for the second sub-case is similar. In this sub-case, ‘Fx’ is true when all the values are in the extension of ‘F’. Again we assume the values are not coordinated, so it is possible to find d_i, d_j of x $d_i \in ext(F)$ but $d_j \notin ext(F)$. This time this interpretation makes ‘$(Fx \vee \neg Fx)$’ false. ‘Fx’ is false because not all values of ‘x’ is in the extension; the negation is false because not all values are excluded by the extension. Consequently, this sub-case also leads to a change of logic. As a result, this interpretation is not suitable for providing a semantics for classical first-order logic.

So far, we have assumed that the only change in interpretation was that terms, which are variables and constants, received multiple valuations and we have seen that it leads to problems. One might say that predicates must also have a different interpretation. So far, we assumed they receive a subset of the domain as an interpretation. One might say one should assign sets of subsets to the extensions of predicates in order to match the multiple values of the terms. I do not have any problem with this suggestion other than that it describes the set theoretical semantics of plural logic. Therefore, there is nothing wrong to devise such a semantics but it is not a semantic for classical first order logic, but a semantics for a different kind of logic.

In conclusion, we have seen that multiple valued interpretations are equivalent to single valued ones if the values are coordinated. They change the laws of logic if they are not coordinated. They give semantics for logics other than the first-order logic when extended in a way that they assign multiple values to predicates. Therefore, having a single valued function essential for the semantics of first-order classical logic.

Turning back to the Wehmeier's argument; he tried to show that interpretation of first-order languages does not involve identity by arguing that having the same value does not imply identity if single values are not assumed. I have shown that this assumption is not optional. Therefore, the argument from identity of values to the involvement of identity in interpretation still stands.

2.2 Humberstone and Townsend's objections

Another criticism by Humberstone and Townsend (1993) involves the criticism of both our components. In order to explain that criticism we need to have some stage setting. As I explained before several philosophers subscribed to the slogan that quantification involves identity. Quine was one of them. However, we shall see his interpretation of the slogan was different from mine. Therefore, criticisms made against him do not necessarily apply to our case. Now I will explain the Quinean interpretation of the slogan and expound Humberstone and Townsend's criticism of it. Then I will show why our interpretation is immune to that criticism.

First observation of Quine (1970) was that a first order language can have at most one identity predicate. What I mean by that is that assuming we have two predicates that are reflexive and support Leibniz's Law they must be co-extensional. Assume we have '$=_1$' and '$=_2$' both reflexive and support Leibniz's law. Let for some a and b, $a =_1 b$. By reflexivity of '$=_2$', $a =_2 a$. Given that 'a' satisfies the predicate $a =_2 x$, and '$=_1$', supports Leibniz's Law 'b' must also satisfy that predicate. Therefore, $a =_2 b$. Which means, if $a =_1 b$, then $a =_2 b$. The reverse argument is similar. Therefore, if there are two predicates that are reflexive and satisfy Leibniz's Law, then for all a and b, $a =_1 b \; iff \; a =_2 b$. As a result, these two predicates are co-extensional. Assuming that only extensions matter in first-order logic, these two predicates are the same.

However, this observation is not enough for the thesis that quantification involves identity. It says that once you have an identity predicate in your language, you cannot add a different identity predicate. Nevertheless, it does not tell you anything about the extension of the first identity predicate. In other words, this observation establishes that extensions of two predicates co-vary but it does not pick a unique extension for either of them.

Quine (1950) extends his thesis with a second observation. He observes that if we add identity of indiscernibles, '$\forall x \forall y((Fx \leftrightarrow Fy) \leftrightarrow x = y)$' as an axiom schema, then identity predicate receives a unique extension. If we combine these two observations, we get a unique extension for all possible identity predicates. The reason for this result is that if we add the identity of indiscernibles, we get a unique extension. Moreover, by the first observation, any other predicate that is reflexive and support Leibniz's law must be co-extensional with it.

However, the problem, as pointed out by Geach (1972) is that the extension that we get by adding identity of indiscernibles as an axiom need not be faithful to the meta-language. Let $D = \{a, b, c\}$ and ext(F) = {a,b} and ext(G) = {c}. Which means that 'a' and 'b' are indiscernible. Then the extension we get by identity of indiscernibles includes (a,b), i.e. $(a, b) \in ext(=)$ but $a \neq b$. Therefore, the identity predicate that we defined is not faithful to the meta language identity predicate.

At this point Quine (1950) made yet another observation. We can transform any interpretation to which the identity predicate defined by identity of indiscernibles is not faithful, into one to which object language predicate is faithful. This transformation is simple. Let D_1 be a domain consisting of objects $d_1, \dots, d_n$ we will get another domain D_2 which consist of sets $s_1 \dots s_n$. First thing that we follow is to satisfy the following if $(a, b) \in ext_{D_1}(=)$ then if $a \in s_i$ then $b \in s_i$. So, if the defined identity predicate constructed in a way that they are identical, they get into the same set. Then we declare the extensions of predicates, following if $d \in ext_{D_1}(F)$, then $\forall s_j d \in s_j, s_j \in ext_{D_2}(F)$. Finally, declare the new valuation of the terms as the set that contains the old valuation, i.e. $val_{D_2}(x)$ is the set $s_j, val_{D_1}(x) \in s_j$. The definition is proper, because for all d in D_1 there is only one s in D_2 that contains it. This is because the defined identity predicate divides the domain into non-overlapping equivalence classes, so that for each d in D we will only have one set s that contains d.

This observation by itself is not sufficient for the purpose at hand. However, Quine (1950) adds as a maxim that in first-order quantificational logic either the extension of the defined identity predicate is minimal or if it is not minimal only appropriate domain to interpret that language is one that can be got by the transformation that we explained. Now we can observe that any extension that identity predicate gets from this transformed domain is also minimal. Therefore, Quine's maxim is in effect a declaration that the defined identity predicate is assumed to be minimal. We have seen previously that every minimal identity relation is faithful to its meta-language identity predicate. Therefore, Quine secures faithfulness by imposing minimality.

Taking all these together, we have only one identity predicate that is reflexive and supports Leibniz's law. We can fix its unique extension by defining it by identity of indiscernibles. That extension is guaranteed to be faithful to its meta-language if we assume that extension to be minimal by accepting as appropriate only meta-languages that are got by the transformation that we observed before. Therefore, in first order logic we can *define* identity predicate using resources of quantification. Therefore, quantification involves identity.

Now I will explain Humberstone and Townsend's criticism of the thesis that quantification involves identity. We will see that their criticism is applicable only when the minimality is assumed. We have seen that the assumption of minimality is central in Quine's case for the thesis, so their criticism applies to Quine's defense. However, I will also show that since we did not assume minimality but only faithfulness, our defense of the thesis is immune from their attacks.

First of Humberston and Townsend 's (1994) observations is that ability to define its own identity predicate is a property of second-order logic. Therefore, if we say we can define identity predicate in some quantificational language, we must also say that this quantificational language must have the resources of second-order logic. However, we have seen that Quine made such a claim. Therefore, some of the assumptions we use must attribute second order power to quantification.

Which assumption of Quine does lead to the result? It is the minimality assumption. He started by defining the identity predicate with the indiscernibility of identicals. Therefore, when we have 'a' and 'b' having all the same properties we declared them to be identical. Then, by claiming that this identity predicate is minimal we also blocked that there can be other predicates corresponding to other properties.

Assume for contradiction with minimality that there are other properties that are not yet represented in our language. Even though 'a' satisfies every property that 'b' satisfies among the properties that we represented, we could have added a new predicate 'N' that represents some other property such that it allows us to discern 'a' from 'b'. Therefore, by using this new predicate, identity of indiscernibles definition would not have declared them to be identical. By assuming that the identity predicate is minimal, we thereby assumed that its extension is the subset of all reflexive relations. Now assume that every identity relation is reflexive. Therefore, if the relation containing (a,b) is minimal then (a,b) must be an element of the new identity relation that is defined by identity of indiscernibles using 'N'. It means that 'N' does not discern between 'a' and 'b'. Contradiction. Therefore, there are no properties that are not represented in our language.

So far, we have seen that Humberstone and Townsend's observation has a bite against Quine because it is based on Quine's assumption of minimality. Since my thesis does not use minimality but faithfulness, the thesis is immune from the threat of assuming second-order powers for first-order logic. However, they have a second observation that is more related to the way we argued for our thesis.

Remember we said that the interpretation of '$\exists x(Fx \wedge Gx)$' makes recourse to identity because of the fact that the same variable occurs twice in the sentence. Humberstone and Townsend (1994) observe that variable co-occurrence can happen in the definition of pairs. The set $\{(d,d): d \in D\}$ uses the same device of variable co-occurrence. Their observation is that if the first-order sentence makes recourse to identity then the set definition also does. We do not disagree with this interpretation.

However, they further argue that if the definition of the set makes recourse to identity, we can use it in our meta-language to define its identity predicate. And if we assume our first-order language is faithful to meta-language one, it corresponds to this defined one. Therefore, we would be able to define identity predicate, which we have seen requires second-order power. They argue that in order to avoid this conclusion we should deny the definition of the set uses identity (1994). However, we already agreed that the definition of the set uses identity if and only if first-order variable co-occurrence uses identity. Therefore, they argue, we should conclude that first-order variable co-occurrence does not require the use of identity.

This argument seems to be directly related to our two theses. Therefore, it appears that even though we do not require minimality, our two theses together entail definability of identity predicate and consequently has the same problems with Quine's proposal. However, I will show that Humberstone and Townsend's argument itself assumes minimality. Because we do not subscribe to that thesis, we need not accept its conclusion.

Let us start with the observation that if '$\exists x(Fx \wedge Gx)$' requires identity, then so does '$\{(d,d): d \in D\}$'. We accept that '$\exists x(Fx \wedge Gx)$' requires identity, therefore we should also accept '$\{(d,d): d \in D\}$' also requires identity. Now observe that '$\{(d,d): d \in D\}$' is a sentence of set theory. However, we assumed that meta-language of our interpretation is conducted in first-order set theory. Therefore, this is a sentence of our meta-language. The question is now whether we can use this sentence of meta-language to define the identity predicate of the same language. Previously we said that '$\{(d,d): d \in D\}$' makes recourse to identity. Now we have two options, either it makes recourse to its own identity predicate, or it makes recourse to the identity predicate of its meta-language. I will show in both cases it cannot define its own identity predicate without the assumption of minimality.

Let us take the first case that in order to interpret '$\{(d,d): d \in D\}$' we must use the identity predicate of the same meta-language. First, observe that I am not committed to this claim. I do not say that in order to interpret quantified formulas we have to use the identity predicate of the same language. I say we have to use the identity predicate of its meta-language. Therefore, I can easily dismiss this case as irrelevant to my thesis. However, its failure even in its own terms makes my further exposition easy. So let us examine this case.

So, by declaring '$\{(d,d): d \in D\}$' to be the extension of the identity predicate we try to define an extension. So the meta-language tries to form this extension by going through all the pairs (a,b) and if a=b, according to its own identity predicate, then it declares (a,b) is a member of this set. However, observe that the meta-language is a first-order language. So, its identity predicate need not be minimal. Therefore, it can declare 'a' and 'b' to be identical even when they are not according to its own meta-language. So, the extension it comes up with need not be minimal. Therefore, the first-order language which gets this extension by faithfulness to its meta-language does not necessarily get a minimal identity predicate. Consequently, it does not have the power of the second-order language.

Now we turn to the case in order to interpret '$\{(d,d): d \in D\}$' we use the meta-language identity predicate. This is a thesis we subscribe to. Therefore, if this case also does not lead to minimal extension for the identity predicate, we are immune to the criticism of Humberston. Now, our meta-language tries to form this set by going over the pairs (a,b) and if a = b according to its meta-language it adds (a,b) to the extension. Assume we fixed that extension as the extension of the first-order language that we study. Then the identity predicate of the first-order language is faithful to its own meta-language. This is because if $(a, b) \in ext(=_1)$ then $(a, b) \in \{(d, d) : d \in D\}$, which implies according to the construction above, $a =_{meta2} b$.

By the definition of '$\{(d, d) : d \in D\}$' we guaranteed that the identity predicate of the first language is faithful but we have seen that faithfulness is weaker than minimality. Is it also minimal? The answer is that it is not unless the identity predicate of the meta-language of the meta-language is also minimal. The reason is that if identity predicate of the meta-language is not minimal it declares a=b even though they are not identical according to its own meta-language. Therefore, the first meta-language declares (a,b) an element of the set $\{(d, d) : d \in D\}$. In that case the set we define is not guaranteed to be minimal. Therefore, if we assign that set as the extension of the first-order identity predicate, we did not assign it a minimal extension. Therefore, even with definition of the extension of the predicate by '$\{(d, d) : d \in D\}$' we are not guaranteed minimality. Therefore, we did not assume second-order powers for neither our first-order language nor for any meta-language that we use to interpret that language.

In conclusion, Humberstone and Townsend's argument assumes that the meta-languages that we use to interpret first-order quantification have minimal identity predicates. If we drop that assumption, then we can avoid the unwelcome conclusion of his argument.

2.3 Conclusion

In this chapter I have argued that once a domain is given in a meta-language of a first-order theory, even when it only contains quantificational apparatus, the extension of the identity predicate of that theory is uniquely determined. I have identified two theses that together entail that conclusion. I have explained the theses, and defended them against objections.

If my defense is correct and we have to accept these theses, then we have to accept its consequences. One consequence of this is that we cannot treat identity as any other predicate. Once a domain is given, we are free to choose which subsets of that domain to assign as the extension of predicates other than identity. We can devise different schemes for different assignments. However, when it comes to the assignment of the extension of identity predicate, neither is there a need, nor a place for different schemes.

On the other hand, some might think that there is a place for different schemes and conditions for determining the extension of the identity predicate. Furthermore, they might think that there is a need for devising schemes and conditions for assigning extension to the identity predicate. This chapter shows on a general and abstract level that they need to employ a non-standard semantics.

In the next chapter, I will show the tension between standard semantics and conditions of identity more concretely. In that chapter I will also provide a new semantics that makes room for different schemes and conditions for the identity predicate.

CHAPTER 3

SORTALS AND SITUATION SEMANTICS

In the previous chapter, I argued on general grounds that in standard semantics the extension of identity predicate is determined once you fix a domain in order to interpret quantificational structure of the first-order sentences. In this chapter, I will introduce the sortalist way of thinking that claims there is a special class of predicates; sortals. In this way of thinking, sortals play an ineliminable role in answering identity questions. Therefore, sortalists think that interpretation of sentences that involve identity must make a recourse to the sortals and the conditions associated with them. I will show that this sortalist way of thinking clashes with standard semantics, not only in the general way that is described in the previous chapter, but in a specific way involving a clash of the principles they espouse and the way these principles put into use in the semantics.

The plan of this chapter is as follows. First, I will introduce two sortalist principles and their use in standard semantics. Then, I will show that the principles and their use do not cohere well. Second, I will introduce the basic idea of a situation semantics and outline the use of sortalist principles in this semantics. Moreover, I will show that in this setting, the principles and their uses cohere well. Third, I will introduce the complete situation semantics for first-order logic and show that this semantics is not merely ad-hoc solution to accommodate sortalist principles but can be extended to give a full semantics for first order logic.

3.1 Sortals and identity and application conditions

It is commonly argued that sortal terms play an important role in answering identity and existence questions. The difference between a sortal term and other terms is that the former is equipped with application conditions and identity conditions but, the latter has only application conditions. The idea is that in order to introduce a term you must semantically specify an application condition, but to introduce a sortal term you have to introduce an identity condition separately (see Dummett, 1973 and Thomasson, 2007). I will cash the independent introduction of an identity condition in terms of the following thesis.

Independence Thesis: Providing an application condition for any sortal does not determine a single identity condition appropriate to it. The identity condition must be specified separately.

Difference Thesis: Two sortal terms can have the same application condition yet can differ on their identity conditions.

Observe that the Difference Thesis is a corollary to the Independence Thesis. Assume for contraposition that the Difference Thesis does not hold. Then, we have two sortals S_1, S_2 that have the same identity conditions whenever they have the same application conditions. Assume that we have specified the application conditions and the identity conditions of S_1. Now, if we specify the application conditions of S_2 as the same with S_1, it automatically follows that S_2 has to have the identity conditions of S_1. Therefore, we do not need to specify the identity conditions of S_2. Consequently, we must deny the Independence thesis. Contra-positively, the Independence thesis entails the Difference Thesis as a corollary.

However, these two theses do not cohere well with the standard semantics. By standard semantics I mean semantics that we dealt on the previous chapter which consist of a domain of quantification and extensions of predicates (or relations) as subsets of D^n. In this standard way of thinking, the role of application conditions and identity conditions are cashed as follows:

Objectual Application: Application conditions of a term determine a subset of the domain as the extension of the term.

Objectual Identity: Identity conditions of the sortals determine a subset of D^2 as the extension of the identity predicate.

Now I will show one of the tensions between Difference Thesis and standard semantics. Assume $S_1$and S_2 are sortals that have the same application conditions yet they have different identity conditions. By Objectual Application, the extension of $S_1$and S_2 are the same. Let a,b be arbitrary members of the common extension. Assume by the perspective of the identity conditions of S_1, (a,b) is an element of the extension of the identity predicate. Since S_1 and S_2have different identity conditions, it is possible that from the perspective of the identity condition of S_2, (a,b) is not in the extension of the identity predicate.

We obviously have a contradiction. (a,b) cannot be both a member and not a member of the extension of the identity predicate. We must choose one or the other. Let us assume that we have chosen to say that it is a member of the extension. In that case we say a is identical to b. However, if we follow that decision we are committed to idea that identity conditions of S_2 is not operative on the objects that satisfy the application condition ofS_2. We disregard the effect of S_2's identity conditions on the extension of the identity predicate. So, S_2's having an identity condition is superfluous in this case. Therefore, there is no distinction between $S_2$and any other non-sortal predicate. If we say that a is not identical to b, this time we have to say the same thing about S_1's identity conditions. In either case, we end up denying the effect of a sortal's identity condition.

The contradiction that we see arises because two sortals make different claims about a single identity predicate. If there were different identity predicates answering the demands of these two sortals, we would not have a contradiction. So, another option to solve the apparent contradiction is to introduce an identity predicate for each distinct sortal. This is called the relativization of the identity predicate. We say a is the same S_1 with b, but a differentS_2. This is the move taken by Geach (1980).

However, the problem with this move is that not all sortalists want to subscribe to relative identity thesis. The problem with the relative identity is that it revises the logic. The reason for this is that, as we have seen in the previous chapter, if there are two identity predicates that obey the Leibniz' Law they must be co-extensional. Therefore, in order to preserve the different extensions of different identity predicates, relative identity thesis denies that the identity predicates obey the Leibniz's Law. Therefore, relative identity does not satisfy the principles of classical logic. Consequently, this move does not solve the tension between standard semantics and sortalist theses, but it is a call for the abandonment of the classical logic.

Given the tension between the principles and the standard semantic, if he does not want to be revisionary one final option for sortalist is to deny the principles. Therefore, this option forces us to give up the Difference Thesis. One way to do that is to say that if extensions of two sortals intersect, then there is a sortal that includes the extension of the both, called the ultimate sortal. It is this ultimate sortal's identity conditions that governs those sortals. Therefore, two sortals that have same application conditions must have the same identity conditions. This is the move taken by Wiggins (1980).

Each maneuver is unattractive from the perspective of a sortalist. The first maneuver that we considered forces us to downplay the role of identity conditions of some sortals. The second one forces us to adopt the relative identity thesis which not all sortalists are willing to adopt and which is incompatible with classical logic. The last one, forces us to give up the Difference Thesis, which is not a solution but an admission of defeat.

Next, I will show the tension between the Independence Thesis and the standard semantics. My argument for this is a modification of Noonan and Curtis's (2018) argument. First, I will paraphrase their argument in full. Then, I will adopt their argument in a different way so that this argument's relation to the Objectual Application and the Objectual Identity is more perspicuous.

Here is Noonan and Curtis's argument in full. Assume you are given the application conditions of a sortal term, $\forall x(Sx \leftrightarrow \phi x)$, i.e. necessary and sufficient conditions for being an S. Let F be a candidate identity condition in the form of $\forall x \forall y(Sx \wedge Sy \ \rightarrow (x = y \rightarrow Rxy) \wedge \forall x \forall y(Sx \wedge Sy \ \rightarrow (Rxy \rightarrow x = y)$

Noonan and Curtis invite us to observe that the first conjunct is equivalent to $\forall x(Sx \rightarrow Rxx)$, which tells us that Rxx is a sufficient condition for being an S. However, the application condition of the sortal also gives the same information. Therefore, once the application condition is given there is need to introduce the first conjunct of the identity condition.

For the second conjunct, Noonan and Curtis invite us to observe that it is equivalent to $\forall x \forall y(Rxy \wedge x \neq y \rightarrow \neg Sx \vee \neg Sy)$, which says that there are no two distinct R-related S's. They think that if you are given the application conditions of a sortal and all the facts that hold before the introduction of the sortal term, then truth value of the second conjunct is determined. Therefore, there is no need for identity conditions. Application conditions plus facts determine the identity conditions.

The problem with this argument is that it is not clear whether the argument is intended to be syntactical or semantical. The first part appears to be a straightforward syntactical observation that if those identity conditions and application conditions are valid in our language, then '$\phi x \rightarrow Rxx$' is also valid. However, observe the second part where he assumes you are given all the facts. Are they given as syntactic sentences of the language? Nonetheless, this assumes that the language under consideration is capable of expressing all the facts, i.e. it does not lack any predicate that might be used to express a property that is a part of all the facts. Yet, we have seen that if that is the case, the language under consideration is capable of defining its own identity predicate. Therefore, it is not a first-order language.

However, if we can interpret the idea of having all the facts as knowing the extensions of all the predicates, therefore as a semantic thesis, then I think the argument goes through. So, I adopt his argument in a full semantic guise.

My adoption of his argument is as follows. Assume you are given a model for a language that does not contain S. This model determines all the facts including identity facts except for facts about S's and their identities. Now, assume we introduce a sortal term to this language by specifying a non-circular application condition for S's. $\forall x(Sx \leftrightarrow \phi x)$, where ϕ is a sentence of the previous language.

Given the model provided an extension for each predicate for the previous language it provides the extension of ϕ and it is the same with the extension of S in the new model. Now take a and b in the extension of S. Given we work with the same domain of quantification, if (a,b) is in the extension of identity predicate of the previous model, it is also in the extension of the identity predicate in the new model. If it was not in the extension of the identity predicate of the previous model, then neither is it in the extension of the new model. Therefore, once application conditions of a term are given, the job of identity condition, which is determining the extension of the identity predicate by the Objectual Identity, is done by the application conditions plus the facts that do not contain any S term.

Now I will present two objections to the previous argument. My main response to both of these objections is that they are not available to sortalists.

One might contend that the argument assumes that after the introduction of the new term, extension of the identity predicate remains the same. Nevertheless, that is not always true. Define the object language of identity predicate using identity of indiscernibles, i.e. add every instance of $\forall x \forall y((\phi x \leftrightarrow \phi y) \rightarrow x = y)$, as an axiom. Consider the following model D={1,2,3,4} ext(F)={1,2}, ext(G)={3,4}. The extension of the defined identity predicate is ext(=)={(1,1),(2,2),(3,3),(4,4),(1,2),(3,4),(2,1),(4,3)}. Now add ext(H)={1,3} to this model. The new extension of the defined identity predicate is ext(=)={(1,1),(2,2),(3,3),(4,4)}. For example, (2,3) was an element of the previous extension of the identity predicate but it is not an element of the new extension.

My main reply to this argument is that this move is not available to sortalists. Remember that sortalists uphold the Independence Thesis and the Difference Thesis.

Each sortal must be introduced to the language with its identity conditions. Yet, identity of indiscernibles is an identity condition that is independent of sortals. It functions as a common identity condition for each sortal. So, it contradicts the Difference Thesis. Second, given the application conditions of sortals, identity of indiscernibles determine all identity facts, contradicting the Independence Thesis.

My other reply is that even if a sortalist makes this move, it does not disrupt the argument. First, in order for the objection to work, the extension of the added predicate must be not definable in the previous language. Otherwise, the extension of the identity predicate remains the same, thus blocking the objection. However, we assumed that the application condition of S is in the form $\forall x(Sx \leftrightarrow \phi x)$ and ϕ is a sentence of the previous language. This means that the extension of the new introduced term can be definable using the previous language. Consequently, the objection does not work.

Let us assume that we are given an application condition that cannot be definable in the previous language so that the extension of the new term is undefinable. Next, let us waive the worries of circularity, unintelligibleness, or introduction failure. But observe two facts. First, if we use this method, each introduction of a new term only makes the extension of identity predicate get smaller, i.e. it eliminates some pairs from the extension. Therefore, the objection does not disrupt the fact that if (a,b) is not in the previous extension of the identity predicate, then it is not in the latter extension. Second, observe that none of the eliminated pairs is in S^2. What this means is that if (a,b) is in in the extension of the previous identity predicate and if both a and b in the extension of the S, then (a,b) is not one of the eliminated pairs. So, (a,b) is still in the extension of the new identity predicate. Hence, the objection does not disrupt the fact that for a,b that are in ext(S) if (a,b) is in the previous extension of the identity predicate, then it is in the extension of the new identity predicate. As a result, the extension of the identity predicate is determined by the application conditions of the sortal alone, without any need of the separate use of the identity conditions.

Someone else might object to the argument by claiming that the argument assumed that the model before the introduction of the sortal term decides all the sentences, including identity sentences, except for the sentences that contain S. If you gave up those identity facts and added the sortal, the extension of identity predicate would be indeterminate, unless some identity condition was introduced.

Again my main reply to this argument is that I think this objection is not available to sortalists. Most sortalists believe that the application conditions and the identity conditions of the new introduced sortal can make use of the identity facts that are not related to the sortal introduced. For example, Frege (1884) thought the possibility that you can introduce sortal "direction" using "direction-of(x) is the same as the direction-of-y if and only if x is a line and y is a line, and x and y are parallel". Or one might introduce the sortal 'Set' using the identity condition in effect that two sets are identical if and only if they have the same members. If we are working in a set theory with ur-elements, given that they are not sets, the identity condition for sets makes appeal to the identity and the distinctness of the ur-elements, which must be independently given in the original language.

So far, I have shown that there is a tension between the Independence Thesis and the Difference Thesis and the standard semantics. The reason for this tension is that the only way to apply those theses to the standard semantics is through the Objectual Application and the Objectual Identity. Given that our model consists of a domain and an assignment of extensions to predicates, the only thing that sortal application conditions determine is the extension of the sortal predicate. That is the main gist of the Objectual Application. Moreover, the only predicate that identity conditions supposed to control is the identity predicate, and the only job of the identity conditions is to determine the extension of the identity predicate, which is the gist of the Objectual Identity.

Given this tension one might respond by saying that we have to reject sortalist theses of the Independence and Difference. However, this response is too quick to attribute the blame to the theses. After all, the tension arose because of the fact that the application of those theses are restricted to Objectual Identity and Objectual Application. The reason for this restrictions is the meager resources of the standard semantics. In the next section, I will introduce the idea of situation semantics. There we will be able to find different ways to apply the theses, thereby overcoming those restrictions. I will call these applications of the sortalist principles Situational Application and Situational Identity. Then, I will show that the Independence Thesis and the Difference Thesis can be upheld in a consistent way in this semantics.

3.2 Sortals and situation semantics

The idea that I will pursue derives its inspiration from the accounts of two different lines of thought. First, Thomasson (2007) makes a distinction between object-level vs. frame-level application conditions and identity conditions. Second, Fine (2017) gives a truth-maker semantics for propositional logic. I will extend Fine's truth-maker semantics to the first-order logic. Then, I will combine these two lines of thought by making application conditions and identity conditions operating on these truth-makers, which I call situations, rather than directly on the objects. Thus, the application and identity conditions mentioned by sortalists will not be object-level but frame-level.

First of all, Thomasson (2007) following Dummett (1973), makes the following observation. The term ‘book’ in the sense of a physical object has the same application condition with the term ‘book’ in the sense of a literary work. However, these two terms do not have the same identity condition. That is because in order to re-identify a ‘book’ in the physical object sense, you have to track spatio-temporal continuity, but you don’t need to do the same in order to re-identify a ‘book’ in the sense of literary term.

Then, Thomasson (2007) points out, in the terminology of this paper, that the term ‘book’ in the sense of a physical object and the term ‘book’ in the sense of a literary term cannot have the same application condition in the sense of the Objectual Application. Recall that the Objectual Application says that the application conditions of a term determine which objects in the domain satisfy that term. If two terms have the same application condition, they must have the same extension, which implies every object that satisfies one also satisfies the other. However, no book in the sense of a physical object is also a literary work.

Consequently, we need to make a distinction between object-level identity and application conditions and frame-level identity and application conditions. Those frame-level conditions are about hypothetical and actual situations. A frame-level application conditions determine under which possible or actual situations the term applies, and a frame-level identity conditions determine under which possible or actual situations the term co-applies (Thomasson, 2007).

If we are tempted by the term situation, we would use the situation semantics in order to model what Thomasson means by frame-level conditions. However, in the literature, situation semantics refers to a specific semantics developed by Barwise and Perry (1983). The problem with this formulation about the task at hand is that they identify situations with structured entities that contain individuals and relations. Hence, whether two situations support the same term is determined by the individuals and relations these two situations contain. Therefore, if we adopted this version, our frame-level application and identity conditions would reduce to the Objectual Application and the Objectual Identitiy. Hence they would not have a difference from object-level conditions.

Hence, we need a generalization of situation semantics in which we are neutral to the internal constitutions of the situations. Fine (2017) does exactly that in his truth-maker semantics. He starts with a set of situations and he builds his semantics using operations on this set, without postulating anything about the internal structure of the situations. He gives a semantics for propositional logic, classic or otherwise. And he shows how this semantics can be extended to conditionals and counterfactuals. The only thing that is missing for our purposes is the extension of this semantics to first-order logic.

In the following I will present a version of Fine's truth-makers semantics with some major divergences. The reason for this divergence is that Fine develops his theory specifically by bearing propositional logic in mind. Given that we will try to apply it to first-order logic, sometimes changes will be necessary, and even when they are not necessary some changes will be useful for the purpose of presentation. The following will be my adoption of Fine's semantics. As Fine himself admits that this truth-maker semantics is a generalization of situation semantics (Fine, 2017), I will keep using the term ‘situation’ to refer to the proposed truth-makers of this semantics.

The basic idea of situation semantics is that a possible world is not a totality of propertied objects, but rather a collection of situations. Let S be the set of situations. Given that we want to model worlds as collection of situations, S must be equipped with a collection or fusion operator.

Situation Frame: Let $< S, \sqcup >$ be a situation frame such that $S \neq \emptyset$ and if $X \subseteq S, and\ X \neq \emptyset, then \sqcup X$ is defined and $\sqcup X \in S$.

What this condition ensures is that if s and s' are two situations, their fusion $s \sqcup s'$ exists and is also a situation.

Let W be a relation $W \subseteq S \times \mathbb{N}$ such that $sWn\ and\ s'Wn\ \leftrightarrow (s \sqcup s')Wn$.

And identify each possible world with $w_n = \sqcup \{s : sWn\}$

The relation W tells us which situations are in the same possible world. The condition imposes that if s and s’ are in the same possible world, so is their fusion, and if a situation is in a world, so are its parts. Then we identify possible worlds with the sum of the situations that are in the same world.

Call $MF =< S, W, \sqcup >$ a modalized frame.

In this setting, by application conditions we do not directly determine the set of objects that a predicate applies to, but a set of situations that supports the predicate. Also, identity conditions determine the set of situations that supports the co-application of the predicate in question. Therefore, we replace the Objectual Application and the Objectual Identity with the followings.

Situational Application: Application conditions determine a set of situations as the situations in which the predicate is satisfied

Situational Identity: Identity conditions determine sets of situations such that situations that are the member of the same set support the existence of the same object.

We impose two conditions on the sets determined by Situational Application and Situational Identity.

(1) $App[F] \subseteq S$.

(2) $Id[F] \subseteq \wp(S)$ such that if $s \in App[F]$, then $\exists X \in Id[F]$ s.t $s \in X$, and if $X, Y \in Id[F]$, then $X \cap Y = \emptyset$.

What these conditions ensure is that first, application conditions of a term determine a subset of the situations as the one in which it applies. Second, identity conditions of a term determine a set of subsets of S, such that each subset consists of situations that support the existence of the same object and no other object.

So far, I have talked about the general form of the situation semantics. I haven't said anything about how it can provide a semantics for first-order logic. I have not said what I mean when I say a situation supports the existence of an object. These omissions are on purpose. Now I will show that only with these modifications we can solve the tension that arises from the Independence Thesis and the Difference Thesis. For the rest of this section, I invite you to rely on your intuitive understanding that is provided by the examples given by Thomasson above. In the next section I will give a full semantics, then you can turn back to the following argument and see that it works with the meaning provided to the phrase 'the situation supports the existence of the object.

Remember that on the previous section we have seen that the Independence Thesis and the Difference Thesis lead to problems when implemented with the Objectual Application and the Objectual Identity. We rejected the denial of Independence Thesis and the Difference Thesis based on the tension as too rash, saying that we can find different implementations of the theses. Previously, I have provided different implementations, namely the Situational Application and the Situational Identity. Now I will show that the theses do not cause any problems with these implementations.

Consider the following set of situations, $S = \{s_1, s_2, s_3, s_1 \sqcup s_2, s_1 \sqcup s_3, s_2 \sqcup s_3\}$, and assume that we are given a relation on the this set as follows, $W = \{(s_1, 1), (s_2, 1), ((s_1 \sqcup s_2), 1), (s_2, 2), (s_3, 2), (s_2 \sqcup s_3, 2)\}$. From these two pieces of information we get $w_1 = s_1 \sqcup s_2$ and $w_2 = s_2 \sqcup s_3$, according to definition of worlds that we have given.

Now assume $App[F] = \{s_1, s_2\}$, i.e. we are given a sortal and by Situational Application the application conditions of that sortal determined these situations to be the ones that satisfy that sortal. Then, there are two possible sets which can be designated by the Situational Identity as the sets that the identity conditions of the sortal determine as supporting the existence of the same object. The possible sets are (1) $Id[F] = \{\{s_1\}, \{s_2\}\}$ or (2) $Id[F] = \{\{s_1, s_2\}\}$. However, observe that neither choice is forced upon us. Therefore, we can have two different identity conditions that correspond to these two choices. Since we are free to choose one of the sets as the set we get by the Situational Identity, we are free to choose which identity condition that correspond to those choices. Hence, when you introduce a sortal term, fixing the application conditions of the sortal does not fix the identity conditions. If you choose (1), that it means w_1 contains two distinct F's. If you choose (2), it means that w_1 contains only one F. Therefore, a given application condition does not determine an identity condition associated with it, so that the identity condition must be specified independently. That indicates we can satisfy the Independence Thesis.

Let us now turn to the case of the Difference Thesis. The denial of the Difference thesis says that if there are two sortals that have the same application condition, then they must have the same identity condition. Let us assume that you have chosen (1) as the set that is determined by the identity condition of ‘F’. And assume that you said that ‘F’ and ‘G’ have the same application condition, which means $App[G] = \{s_1, s_2\}$. If the denial of the Difference thesis is correct, then the only set as the set determined by the identity condition of ‘G’ is (1). However, we have seen in the previous paragraph that (1) is not the only set that we can choose. We can also choose (2). So, we can choose another identity condition that would correspond to this choice. Therefore, in this implementation we can uphold the Difference Thesis.

In conclusion, there is nothing wrong with the Independence Thesis and the Difference Thesis. What is problematic is rather their implementation in standard semantics. On the contrary, if we switch to situation semantics, they receive reasonable implementations. The solution is not dependent on the intricacies of the semantics, which I will explain in the next section, but rather follows immediately from the change of basic entities of semantics, that is, from objects to situations. Therefore, now sortalists have a choice. Either they stick to the standard semantics, in which case they need to deny those theses or they should switch to a version of the situation semantics.

In the next section I will describe how this situation frame provides semantics for first-order logic. However, keep in mind that the solution I have presented does not depend on the other finer details of this semantics. I present the full semantics in order to prove that the solution of these problems do not prohibit us from having a full first-order logic with its associated semantics.

3.3 First-order situation semantics for sortalists

3.3.1 The language

Before describing the semantics we have to describe the language. Standard first-order language treats each predicate the same. On the other hand, sortalists believe that there is a distinction between predicates that correspond to sortal terms, and those which do not. We will syntactically mark this difference. In our language, first, there are sortal predicates. Also, each of our predicates is assigned to a type for each of their argument places. These types will include the sortals, and a dummy type S, which signifies that the term is used adjectivally.

Examples of sortals: (1) S_i examples: "… is an animal", "… is an artifact".

Examples of typed n-place relations: $R_k^n$with associated sortals for each place:

(1) 1-place: "…is a boy" its type is $< S_i >$ where S_i is "…is human".

(2) 1-place: "…is yellow" it is type is <S>, which means it means it is used adjectivally.

(3) 2-place: "… builds,,," its type is <human,artifact>.

(4) 2-place: "… sees,,," its type is <human, S> which is adjectival in its second place.

(5) 2-place: "… is bigger than,,," its type is <S,S> is fully adjectival.

The rest of the language is as usual.

3.3.2 Situation semantics

In situation semantics, we start with a modalized situation frame as defined above. Then, in order to interpret the sentences of first-order logic, we define two functions App[…] and Id[…] as representing the interpretations of predicates according to their application and identity conditions. Call these functions together with the modalized frame a model. We specify the model in a way that the following conditions are met. I will provide commentary on the intended effect of those conditions on our semantics.

(1) $App[S_i] \subseteq S$, application conditions for sortals determine a subset of S as the situations that support the sortal.

(2) $App[R_i^n] \subseteq S^n$, application condition for any relation determine a subset of S^n for all n-ary relations.

(3) $Id[S_i] \subseteq \wp(S)$, identitiy conditions for sortals determine a set of sets members of which support the same co-application conditions.

(4) $ID = \bigcup_i Id[S_i]$, ID is the union of all sets determined by identity conditions of different sortals, it is the aggregation of the contributions of the distinct sortals.

Furthermore, we assume we are given a function, $d: D \mapsto ID$, where D is any set of objects which can be put to one-to-one correspondence to ID. D is the domain of objects that is generated by sortal identity conditions.

Moreover, we assume we are given a function $val: Term \mapsto D$ as the interpretation of the singular terms and variables of the language.

(5) $d_n \in_0 s\ iff\ s \in\ d(d_n)$ and $d_n \in s\ iff\ s = s_1 \sqcup \ldots \sqcup s_n\ and\ d \in_0 s_i$. We say an object d is in s, iff the object is mapped to a set in ID and and that set contains s. Then we generalize to all situations by saying, d is in S iff d is s for some part of S. This is the sense in which we say that a situation supports the existence of an object.

(6) $Id[S_i] \subseteq \wp(App[S_i])$, the identity conditions of a sortal classify only the situations that the sortal applies.

(7) $if\ s \in App[S_i] then\ \exists X \in Id[S_i], s \in X$, each situation that the sortal applies is classified.

(8) $if\ X, Y \in Id[S_i], then\ X \cap Y \neq \emptyset$, that identity condition puts each situation to a single co-application set.

(9) $if\ i \neq j, then\ Id[S_i] \cap Id[S_j] = \emptyset$, distinct sortals determine distinct co-application conditions, which means situations that support the existence of the same object must also support the same sortal.

(10) $if\ Typ(R_k^n) =< S_{i1}, .. S_{jn} >, then\ App[R_k^n] \subseteq App[S_{i1}] \times \ldots \times App[S_{jn}]$, the application conditions of predicates are dependent on the sortals that the type of the predicate contains.

3.3.3 Situations to truth

If we are given a situation model described as above, we say that a situation satisfies a sentence if and only if:

A) $s \vDash St$ iff $s = s_a \sqcup s_b$ $val(t) \in s_a, s_a \in App[S]$ and $d\big(val(t)\big) \in Id[S]$.

B) $s \vDash R^n t_1, \dots, t_n$ iff $s = s_1 \sqcup \dots \sqcup s_n, val(t_i) \in s_i$ and $< s_1, \dots, s_n >\in$ $App[R^n]$ and $if\ Typ(R^n)i = S_k, then\ d(val(t_i) \in Id[S_k]$.

C) $s \vDash t_1 = t_2$ iff $s = s_1 \sqcup s_2, val(t_i) \in s_i$ and $val(t_1) = val(t_2)$.

D) $s \vDash \neg\phi$ iff $s \nvDash \phi$.

E) $s \vDash \phi \wedge \psi$ iff $s \vDash \phi\ and\ s \vDash \psi$.

F) $s \vDash \forall x \phi$ iff $s \vDash \phi[t \backslash x], \forall t\ val(t) \in s$.

(A) means that a situation supports an atomic sentence containing a sortal if three conditions are satisfied.(i) the object assigned to t must exist at some part of s,(ii) the part that contains the object is in the application condition of the sortal in question, (iii) the object in question has the appropriate identity condition.

(B) For other n-place relations (i) and (ii) are in place. (iii) only holds for places associated with sortals. If the place is adjectival, it does not care about the identity conditions.

(C) For identity, (i) is also in place, only other requirement is that two terms are assigned to the same object in the domain.

(D) And (E) are standard. (F) means that if every object that is in s satisfies the sentence, the quantified sentence is satisfied. So, a situation only quantifies over the objects in it.

This completes the description of the situation semantics for first-order logic, with the sortalist language.

3.3.4 Observations

First of all, observe that we also have a domain in situation semantics. Yet, this domain is determined only after the application conditions and identity conditions of the sortals are given. The reason for this is that a domain is a set of objects that can be mapped to the set ID, which is the union of the sets that are generated by the identity conditions of the sortals. This is the fact that solves the tension between thesis of the first chapter and the independence of sortal identity conditions.

Remember that in the first chapter I have argued that once a domain is given, the extension of the identity predicate is determined. However, in this chapter we have argued that we can subscribe to the intuition of sortalist that identity questions are settled only after the identity conditions of the sortals are given. These two might seem to be in tension but they are not. In the situation semantics, the domain of quantification arises only after the identity conditions of the sortals are laid down. Therefore, the identity conditions determine the domain. Only then, the domain fixes the extension of the identity predicate.

This observation that the situation semantics also generates a domain leads to the following. Given a situation semantics we can define a standard semantics that corresponds to it. Let us assume that we are given a situation semantics and a world w in this semantics. Let the domain of the standard semantics be $D_2 = \{d \in D: d \in w\}$. Define the extensions of the predicates as $ext(F^n) = \{(d_1, \dots, d_n) \in D_2: w \vDash F(d_1, \dots, d_n)\}$. It is a routine verification to check that the world w and the standard model we defined satisfy the same sentences. Therefore, we can take every standard semantics as generated by a situation semantics. Consequently, we can say that the reason why the identity facts are determined in a standard semantics is that those facts are settled by the situation semantics that generates the standard semantics.

The second observation is that every standard semantics can be turned into a situation semantics if we assume there exist a most general sortal that applies to everything, for example ‘thing’ and we treat every predicate as adjectival. Let $M =< D, R_i^n >$ be a standard model. Define S=D and $W = \{< d, 1 >: \forall d \in D\}$ and $App[F] = ext(F)$ and $Id['thing'] = \{\{d\}: d \in D\}$. This constitutes a situation semantics. Again it is easy to see that these two models satisfy the same sentences. This means the situation semantics is a generalization of a standard semantics. Moreover, this shows that ones who stick to the standard semantics are committed to the idea that there is a sortal that applies to everything and every other predicate is used adjectivally.

3.4 Conclusion

In this chapter, I have argued that some theses upheld by sortalists, namely the Independence thesis and the Difference thesis, create problems when applied to standard semantics. I have explained those problems. Then, I have provided the bare bones of a different semantic framework in which the sortalist theses acquire non-problematic applications. Finally, I have provided the full exposition of the semantic framework. I have shown how it can provide an interpretation of first-order quantificational logic. Then, I briefly explored the relation between the new semantics and the old standard semantics.

If the only work that could be done with this semantics were solving the specific problems explained in this chapter, then its introduction would be a futile endeavor. However, this is not the case. In the next chapter, I will show that this semantics can be applied to shed light on different philosophical questions.

CHAPTER 4

APPLICATIONS

In the chapter two I introduced the situation semantics for first-order logic, and argued that it solves the problems that I introduced in chapter one and chapter two. In this chapter, I will show that the advantages of this tool is not limited to those specific problems, but it can also be applied to other problems and it can shed light on the other areas of philosophy.

In this chapter, I will apply the situation semantics on two different philosophical areas. The first one is a part of philosophy of mathematics. I will introduce Frege's Julius Caesar problem, analyze it using situation semantics and argue for a possible solution. The second one is a part of modal metaphysics. I will introduce the idea of sortal essentialism, and show that the situation semantics formulated above is committed to sortal essentialism. Then I will describe the changes necessary to avoid that commitment. I will next argue that we can use the perspective that we gain from seeing the semantical implication of the change, to explore new grounds for evaluating these two positions.

4.1 Julius Caesar problem for abstractionists

Abstractionists in philosophy of mathematics think that we can introduce new mathematical objects in our theory using abstraction principles. Frege (1884) thought this method of introducing objects into mathematics and rejected it on the basis of his Julius Caesar problem.

One form an abstraction principle can take is a form of identity statement. Abstractionists think that we can introduce the numbers in our theory by adding the following as an axiom: The number of F's is identical to the number of G's if and only if F's and G's can be put into one to one correspondence (see Ebert et al, 2016). Frege (1884) pointed out that this statement can be true in a world where Julius Caesar is identical to number one. Or it can also be true in a world in which they are not identical. In other words, the abstraction principle is not strong enough to fix the truth value of every sentence that contains a number term. Frege took this to be a fatal flaw and dismissed the introduction of numbers by abstraction principles.

Now I will introduce how similar problems can arise in situation semantics and describe how we can fix them. Consequently, I will argue that one possibility for an abstractionist is to subscribe to the situation semantics and avoid the Julius Caesar problem.

Instead of dealing with numbers I will deal with lines and their directions. Their case is identical to the case of Julius Caesar problem. Let us assume that we have the following situation model:

Let S be the sets of equations of the form $by = a_0 + a_1x^1 + \cdots + a_nx^n$, let $\sqcup$ be the union of sets containing those equations. Let $W = \{< s, 1 >: \forall s \in S\}$, $App[Line] = \{s \in S: 2 < i, a_i = 0\}$, and $Id[Line] = \{X_1, X_2 \ldots\}$ such that $s_i, s_j \in X_t \; iff \, \frac{b_i}{b_j} = \frac{a_{0i}}{a_{0j}} = \frac{a_{1i}}{a_{1j}}$, and $App[Parallel^2] = \{(s_1, s_2)\}$ such that $\frac{a_{0i}}{a_{0j}} = \frac{a_{1i}}{a_{1j}}$. Then it follows that $ID = Id[Line]$ because it is the only sortal in our language.

What I have described above is just analytic geometry in the guise of situation semantics. We identified our situations as the polynomials. We said that predicate 'line' applies in linear equations and two linear equations represent the same lines when these equations are multiples of each other. Then we added that the predicate 'parallel' applies on lines if the equations that represent those lines have the same slope. The only objects that our semantics yet recognizes are lines, since ‘line’ is the only sortal there is our language.

Now we can introduce the term ‘direction’ in our language by an abstraction principle. The direction of a line x is identical to direction of a line y if and only if x and y are parallel. Next I will describe two extensions of the situation model, which will show that Julius Caesar problem arises also in situation semantics.

Extension one: Let $App[direction] = App[line]$ and let $Id[direction] = \{X_1, \dots X_n\}$ such that $s_i, s_j \in X_k \leftrightarrow \frac{a_{0i}}{a_{0j}} = \frac{a_{1i}}{a_{1j}}$. Let $App[xdirectionofy] = \{(s, s) \in App[direction] \times App[line]$. In this extension, we have arranged it so that in each situation that contains a line there is also a direction which is the direction of that line. Two situations contain the same direction when these situations as equations have the same slope. In this extension $ID = Id[line] \cup Id[direction]$, which means that we have added new objects into our domain.

Extension two: Let $App[direction] = \{s\text{: } 2 < i, a_i = 0 = b\}$ and let $App[xdirectionofy] = \{(s_1, s_2)\text{: } \frac{a_{0i}}{a_{0j}} = \frac{a_{1i}}{a_{1j}}$. In this extension, we have arranged it so that directions are lines that pass from the origin, and a line that passes from the origin is the direction of all the lines that have the same slope with it. In this extension $ID = Id[line]$, so we have not added any new objects into our domain. That is because the new things, i.e. directions, turned out to be some of the lines that our previous model talks about.

However observe that each extension satisfies the abstraction principle, i.e.

(1) $W \vDash_1 \forall x(direction(x) \rightarrow \exists z\big(x = dir(z) \wedge line(z)\big) \wedge$

$\forall x \forall y directionof(x) = direction\ of(z) \leftrightarrow Parallel(x, y)).$

(2) $W \vDash_2 \forall x(direction(x) \rightarrow \exists z(x = dir(z) \wedge line(z)) \wedge$

$\forall x \forall y directionof(x) = direction\ of(z) \leftrightarrow Parallel(x, y))$.

Moreover, observe that

(3) $W \vDash_2 \forall x(direction(x) \rightarrow line(x))$.

(4) $W \nvDash_1 \forall x(direction(x) \rightarrow line(x))$.

This is exactly the Julius Caesar problem for directions. There are models that satisfy the abstraction principle that extend the original model which do not agree on the truth value of the sentences about directions. Consequently, this means that the abstraction principle is not sufficient to fix the meaning of the new term 'direction'.

Observe the reason for the availability of two different extensions. The original model fixed the situations that contain a line and when these lines are parallel. However, it did not fix the situations that contain a direction. We abused that fact and assigned two different application sets for the term direction. The abstraction principle affected these two sets similarly. To each member of these sets it assigned only the lines that have the same slope. In this way, we created divergent model extensions. Therefore, if the abstraction principle determined a unique application set for directions, we would not face the problem.

Now that we have seen that the same problem arises for the situation semantics, we can describe the solution of it. The reason that we can solve this problem is that situation semantics has more resources. We have seen that the worlds of each model satisfy the abstraction principle. However, in situation semantics worlds are just sums of situations. Then, demanding that the abstraction principle to be valid in a world is not the only option. We can demand that the abstraction principle to be valid in all situations. Call this notion frame-level validity.

Next, I will describe how the frame-level validity of the abstraction principle determines a unique extension of the original model. Let M be any model, not just the analytic geometrical one that we described. It fixes the situations in which the term ‘line’ and ‘parallel’ apply. Let s be one of them. So, there is a line in this situation, call it l. Consider the instantiation of the abstraction principle on l, if l is parallel to l then direction of l is identical to direction of l. Given that all lines are parallel to themselves, ‘direction of l=direction of l’ must be true in s. Furthermore, this sentence is true in situation semantics if the direction of l is in s. That means first, there is a direction in s and that is the direction of the line in s. Since s was arbitrary, for all s in App[line] first, there is a direction in s; second, that direction is the direction of the line in s. The first means that $App[line] \subseteq App[direction]$. The second means that $App[directionof] \subseteq \{(s,s): App[direction] \times App[line]$.

Next consider any situation s containing a direction, call it d. Our abstraction principle says that for any direction, there is a line that it is the direction of. In order for that to be true in s, first there must be a line in s, second that line must have l as its direction. Since s was arbitrary; the first means that $App[direction] \subseteq App[line]$ and the second means that $\{(s,s) \in App[direction] \times App[line] \subseteq App[directionof]$

If we take all these together, that means $App[line] = App[direction]$ and $App[directionof] = \{(s,s) \in App[line] \times App[line]$. Therefore, if the abstraction principle is frame-level valid, it determines a unique set of situations that the new term applies. That application set must be the set that is the application set of the term that we used to define it in our abstraction principle.

Moreover, we did not use any principle that is specific to lines and parallelism. This argument can be generalized to any abstraction principle, excluding contradictory or problematic ones. Therefore, situation semantics provides the promise of safety for abstractionist. They can make progress with their favorite abstractionist principles avoiding Julius Caesar problem using situation semantics.

This is not to say that this is the only problem for abstractionist, nor is it to say that switch to situation semantics is the sure way to guarantee success for the abstractionist. However, for the purposes of this chapter what is important is that the situation semantics can provide new tools for solving age old problems.

4.2 Sortal essentialism

Sortal essentialism is the thesis that if an object is a member of a sortal kind, then in all possible worlds it exists, it is a member of that sortal. I will show that the semantic that I described in the previous chapter upholds sortal essentialism.

Assume an object x is a member of the sortal kind S in a possible world W. Is it possible that there is a possible world in which that object is not a member of S? The question is, in our symbolism, whether $W \vDash Sx$ and $W \sqcup W' \vDash x = x$ entail $W' \vDash Sx$. I will argue that this entailment holds, therefore the answer to the question is no.

According to our semantics $s \vDash x = x$ when $s = s_1 \sqcup s_2, (val(t_i)) \in s_i$ and $val(t_1) = val(t_2)$. $val(x) = val(x)$ is already satisfied and we can take without loss of generality that $s_1 = W, s_2 = W'$ which means $val(x) \in W\ and\ (val(x) \in W'$. Furthermore, $W \vDash Sx$ means that $W \in App[S]$ and $d(val(x)) \in Id[S]$. Recall we imposed that $Id[S_i] \subseteq \wp(App[S_i])$, which together with $d(val(x)) \in Id[S]$

entails that $d(val(x)) \subseteq (App[S])$ Now observe that $val(x) \in W'$ means that $W' \in d(val(x))$ which together with $d(val(x)) \subseteq (App[S])$ amounts to $W' \in App[S]$.

Now putting all these together, we get $val(x) \in W', W' \in App[S], d(val(x) \in Id[S]$, which is exactly the condition for $W' \vDash Sx$. Therefore, our semantics upholds sortal essentialism. The reason for it is as follows. Recall our domain of quantification is just a set that can be mapped to ID. Therefore, ID determines the objects of quantification. Moreover, for each member of ID there is only one sortal identity set that contains it. Therefore, for each object only one sortal identity set determines all the situations that contain that object. Furthermore, we assume that an identity set of a sortal is a subset of the application set of that sortal. As a result, all the situations that contain that object falls under the application set of this sortal. These together entail sortal essentialism.

However, Mackie (2006) argues that sortal identity conditions do not entail sortal essentialism. Her interpretation of sortal identity conditions is that these conditions govern objects and situations that are in the same possible world. According to her, when objects are members of different possible worlds, sortal identity conditions are silent about the identity or distinctness of those objects.

In order to change our semantics to a Mackiean semantics we have to make three changes. First, we should add that sortal identity sets contain only sets the members of which are members of the same possible world.

1) $\forall X \in Id[S], s_i, s_j \in X \leftrightarrow \exists k\ (s_i, k), (s_j, k) \in W$

Second, recall that ID is the set that determines the domain of quantification. Previously, we took ID to be the union of the Id[S]'s, since for each object only one sortal determined its existence in all situations and possible worlds. So, our job of specifying a domain was done when we specified the application and identity sets of our sortals. Now that we have restricted the power of sortals, we also need a method to determine ID. Restricted identity sets of sortals determine world-mate situations that contain the same object. If we combine several of those we will get a full set of situations that contain the same object. We will assume that we are given a method such that each element of ID is a union of some world-mate situations given by Id[S].

2) $\forall X \in ID, \exists i, j, \dots, k: X = Id[S_i] \cup Id[S_j] \cup \dots \cup Id[S_k]$

Third, since previously for each object there was only one sortal that determines its identity conditions, we said that $s \vDash Sx$ only if $d(val(x)) \in Id[S]$. However, now we will only demand that only the part of $d(val(x))$ that contains s to be in Id[S].

3) $s \vDash St$ iff $s = s_a \sqcup s_b$ $val(t) \in s_a, s_a \in App[S]$ and $\{s': \exists w, s, s' \in w \wedge s' \in d(val(x)\} \in Id[S]$

The reason why I go over these details is to show that the situation semantics for first-order logic is a powerful tool that we can use to evaluate our philosophical positions. Now that we have seen the kinds of changes that must be made in order to switch from an essentialist to a Mackiean semantics, we can evaluate the merits or costs of those adjustments. Therefore, situation semantics provides us with the tools to exactly formulate the commitments of these theories, and opens up possibility of progress in these topics.

Let me give you two examples of possible cost analysis that we can make after we see the changes in Mackiean semantics. First, consider the reason why one would subscribe to sortalism. One is committed to the idea that in order to answer identity and existence questions we must use the application and identity conditions of the sortals. However, in Mackiean semantics we see that these are not enough, we also need to be given, ab extra, some ways of deciding trans-world identities. However, this is not a stable position. Remember that world are just some big situations. If we believe that some identity questions between two situations are not answerable by sortal identity conditions why should we believe that some others are? Therefore, the move from full sortalist picture to Mackiean semantics undermines the sortalist motivation.

Second, Mackiean semantics creates a big distinction between application and identity conditions. In full semantics we started with, the idea that both application conditions and identity conditions govern all the actual and merely possible situations. Application conditions would give, among all the actual and merely possible situations, the situations that a term applies. Identity conditions would give, among all the actual and merely possible situations, the situations that contains the same object. However, in Mackiean semantics application conditions of sortals keep doing their work of accessing and deciding among the actual and merely possible situations. Yet, identity conditions of sortals can only decide among the situations that are possible with respect to each other, i.e. which are in the same possible world.

Parity of reasoning would require that we make the same restrictions for the application conditions. However, that would be disastrous. Our presumed ability to think and decide about merely possible situations, using the application conditions of our concepts is fundamental to our philosophical endeavors. How many philosophical theories are rejected because they cannot accommodate a merely possible situation that did not or will not arise in our world? If the application conditions of our concepts only decided about situations that arises in our world counter-factual thinking would be impossible.

If we are not willing to give up merely possible reach of the application conditions of our concepts, we must be provided with reasons why the parity with the application conditions is severed.

These are just some of the possible ways that we can criticize the change from essentialist semantics to Mackiean semantics. For the purpose of this chapter what is important is not whether they are persuasive or not. What is important is that the situation semantics can formulate both of the essentialist and Mackiean options and can give us new ways to attack the problem of deciding between them.

4.3 Conclusion

In this chapter, I have given examples of two philosophical positions, namely abstractionism and sortal essentialism, and shown how the situation semantics that I have developed can be applied to those positions. The main purpose of this chapter was that the semantics that we provided is not of limited use, but has a broad application. Moreover, it has more resources than the standard semantics, which makes it capable of offering different analyses of philosophical positions, and capable of offering more ways to develop those positions.

I have shown that in our semantics we can define different notions of validity, namely world validity and frame-level validity. I have argued that if we apply different notions of validity to be applicable to abstraction principles, the result of the abstraction principle on the model would change. I have argued that if we impose frame-level validity on the abstraction principle, it can have the power of determining a unique extension, thereby avoiding Frege's Julius Caesar objection.

Lastly, I have shown that the semantics that I have introduced is committed to sortal essentialism. Then, I have described the changes that can be done in order to avoid sortal essentialism. I have emphasized two facts. The first is that situation semantics is a versatile tool than can be used to formulate more perspicuously the differences between opposite positions in a topic. The second is that by analyzing the changes needed in our semantics to apply different positions, we can make progress in evaluating those different positions.

CHAPTER 5

CONCLUSION

This thesis has been an exploration of the relations between first-order quantification, its semantics and identity. In the first chapter, I argued, in the slogan form, that the first-order quantification involves identity. We put flesh on this slogan by interpreting it as saying that the extension of a possible identity predicate of a first-order language is uniquely determined by the resources we use to interpret the quantificational structure of the sentences of that language. We identified two theses that together entail that conclusion and defended those theses from the arguments against them in the literature.

In the second chapter, we have seen how the results of the first chapter might be seen as a restriction from a sortalist perspective. We have seen that the demands of sortalists, i.e. that the identity conditions of sortals must be independent and flexible, lead to problems when applied to standard semantics. Then we have explored a different semantical framework, and seen that the sortalist theses would not lead to the problems we described earlier, when applied to this semantics. Then we fully developed the semantic framework of situation semantics for first-order logic and explored its relations to the standard semantics. Lastly, we observed that the result of the first chapter is not a restriction, not even from a sortalist perspective. The sortalist can also believe that once a domain of quantification of is given the extension of the identity predicate is determined. The caveat that they would add is that the domain of quantification is determined by the application and identity conditions of the sortals.

In the last chapter, we have seen that the use of the semantical framework that I developed in the second chapter is not limited to dissolving the tension between sortalism and the fact that quantification involves identity. We have seen that it can shed light on other philosophical positions involving identity. We have explored two examples. These examples were the use of identity conditions as abstraction principles, and essentialism as a thesis about trans-world identity. In each case, the new resources of the situation semantics which the standard semantics lack, proved useful in formulating more sharpened versions of philosophical positions that would facilitate their evaluation. Moreover it proved to be useful in offering solutions to the problems that might not be solved with the resources of standard semantics.

Philosophical problems that we investigated in the last chapter were given as an example of the uses of the semantics that we developed. However, the tentative results that we obtained by applying the semantic framework suggest that we can make progress in those areas using that framework. Therefore, application of the semantics that we developed to those problems is one of the ways that can be taken as further research.

In short, in this study it is argued that quantification involves identity but this is not a fact that blocks the sortalist theses about identity conditions and if we develop a version of situation semantics then the investigation of the identity conditions and application condition of our terms is of paramount importance even in quantification. Moreover, the semantics that is developed is not limited in use but can be applied in other areas of philosophy.

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