GREY PREDICTION BASED CONTROL OF A NON-LINEAR LIQUID LEVEL SYSTEM USING PID TYPE FUZZY CONTROLLER

by

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ABSTRACT

GREY PREDICTION BASED CONTROL OF A NON-LINEAR LIQUID LEVEL SYSTEM USING PID TYPE FUZZY CONTROLLER

This thesis proposes a grey system theory based fuzzy PID controller that has a prediction capability. Although fuzzy control theory and grey system theory have completely different mathematical basics, both deal with uncertain information. In the thesis, a short description of both are given and their performance are compared on a non-linear liquid level control system. The grey model developed is examined under several different conditions and it is shown that the proposed grey fuzzy PID controller has better self-adapting characteristics. The simulation results indicate that the proposed controller has the ability to control the non-linear system accurately with a little amount of overshoot and with no steady-state error. It has, in these respects, better performance than the conventional controllers. The thesis is also intended to serve as a first reading on grey system theory and grey prediction based controllers. The fundamental concepts and mathematical basics of grey system theory are therefore explained in simple terms.

ÖZET

NON-LİNEER SIVI SEVİYE SİSTEMİNİN GRİ ÖNGÖRÜSEL TEMELLİ PID TİPİ BULANIK DENETLEYİCİ İLE DENETİMİ

Bu tez, öngörüsel yeteneğe sahip, gri sistem teorisi tabanlı bulanık PID denetleyici önermektedir. Bulanık denetim teorisi ve gri sistem teorisi bütünüyle farklı matematiksel temellere sahip olmalarına rağmen, her iki teori de kesin olmayan bilgiyle ilgilenmektedir. Bu tezde, her iki teorinin de kısa açıklamaları verilmiş ve başarımları, nonlineer bir sıvı seviye kontrol düzeneği üzerinde karşılaştırılmıştır. Geliştirilen gri model birçok farklı durumda sınanmış ve önerilen gri bulanık PID denetleyicinin daha iyi kendini uyarlama yeteneğine sahip olduğu gösterilmiştir. Benzetim sonuçları önerilen denetleyicinin, nonlineer bir sistemi düşük bir aşımla ve kalıcı hal hatası olmaksızın hassas bir şekilde kontrol edebildiğini göstermiştir. Önerilen denetleyici, söz konusu yönlerden, geleneksel denetleyicilerden daha iyi başarım elde etmiştir. Bu tezin bir diğer amacı da gri sistem teorisi ve gri öngörüsel temelli kontrolörler hakkında hakkında genel bir bilgi birikimi sağlamaktır. Gri sistem teorisinin genel kavramları ve matematiksel temelleri yalın bir şekilde açıklanmıştır.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS/ABBREVIATIONS

A_i	The linguistic values of the error
B_j	The linguistic values of the change rate of error
D	First order accumulating generator
$\bar{D}$	First order inverse accumulating generator
e	The system error
$\dot{e}$	The change rate of system error
$e(k)$	The k^{th} error
E	The universe of discourse of e
$\dot{E}$	The universe of discourse of $\dot{e}$
H	Prediction horizon
K	Scaling factor for e
K_d	Scaling factor for $\dot{e}$
K_p	Proportional parameter
K_I	Integral parameter
K_D	Derivative parameter
T_s	Sampling time of the simulations
u	Input to the controlled process
U	The universe of discourse of u
$x^{(0)}$	The original sequence
$x^{(i)}$	The i^{th} generated sequence
$x_p^{(0)}(k)$	The k^{th} predicted value
$z^{(1)}(k)$	The k^{th} background value
α	The weight of PD controller in PIDFC
α_g	Adaptive value in the definition of $z^1(k)$
β	The weight of PI controller in PIDFC
μ	Degree of membership
$\sigma(k)$	Class ratio
$\otimes$	A grey number

AGO	Accumulating Generation Operation
ARPE	Average Relative Percentage Error
CGSA	Chinese Grey System Association
C-o-A	Center-of-Area
C-o-M	Center-of-Maximum
GM	Grey Model
IAGO	Inverse Accumulating Generation Operation
MIMO	Multiple Input Multiple Output
PID	Proportional Integral Derivative
PDFC	PD type fuzzy controller
PIFC	PI type fuzzy controller
PIDFC	PID type fuzzy controller
RPE	Relative Percentage Error
RPM	Rotation Per Minute
SISO	Single Input Single Output

1. INTRODUCTION

Conventional control theory uses a mathematical model to define the relationships between the inputs and the outputs of a system. If an accurate mathematical model of the system can be derived, a conventional PID controller can generally result in a satisfactory performance. Conventional control theory is a long studied subject and as a result, the design of a PID type controller is very well known. Moreover, the implementation of PID controllers is simple and inexpensive.

In real life, the mathematical model of a physical system cannot be defined exactly; there are always some uncertainties. The real world is nonlinear, uncertain and always contains incomplete data. Time-varying nature of parameters, noise and/or disturbances, saturation and time-delay characteristics of an industrial process are the main uncertainties and therefore make the design of conventional controllers very complex. A control method that has the ability to handle these difficulties would very much be welcomed. Traditional model-free control approaches, such as neural networks and fuzzy models threat all training data equally without preference in developing their models. Alternately, grey predictors make predictions using the most recent data, because the most recent data carry more information than the data far away from the present. Grey predictors have the ability to manage the uncertain information and use the data effectively. As a result, grey predictors give more accurate prediction results for time series predictions.

Grey system theory, which has a certain prediction capability, provides an alternative approach to various kinds of conventional control methods. In most control applications, the control signal is a function of the error present in the system at the present time. In other words, conventional control algorithms are based on the errors occurred. However, in grey system theory, prediction error is used instead of current measured error. Similarly, during the development of a grey PID type fuzzy controller, the prediction error is considered as the error of the system.

2. INTRODUCTION TO FUZZY LOGIC

While Boolean logic results are restricted to 0 and 1, fuzzy logic results are between 0 and 1. In other words, fuzzy logic defines some intermediate values between sharp evaluations like absolute true and absolute false. That means fuzzy sets can handle some concepts that we commonly meet in daily life, like "very old", "old", "young", "very young". Fuzzy logic is more like human thinking because it is based on degrees of truth and uses linguistic variables.

Although the concept of fuzzy logic and the concept of probability seem similar, they are quite different. While probability makes guesses about a certain reality, fuzzy logic does not make probability statements but represents membership in vaguely defined sets. For instance, if 0.5 is defined as a probability value for the oldness of a person, it can be said that there is a chance that he/she can be old. It is not known whether he/she is old or young. However in fuzzy logic, if 0.5 is defined as the degree of membership in the set of young and old people, we have some knowledge about his/him and he/she is positioned in the middle of young and old people.

2.1. The History of The Fuzzy Logic

The fuzzy theory was first introduced into the scientific literature in 1965 by Professor Lotfi A. Zadeh at the University of California at Berkeley who proposed a set theory that operated over the range [0;1]. He published a paper titled "Fuzzy Sets" in the journal Information and Control [1].

Fuzzy logic was not an acceptable theory for the scientists at that time because it contained vagueness in the engineering field. However, since 1970s, this approach to set theory has been widely applied to control systems. The principles of fuzzy logic were used to control a steam engine by Ebraham Mamdani of University of London in 1974 [2]. It was a milestone for fuzzy logic. The first industrial application was a cement kiln built in Denmark in 1975. In the 1980s, Fuji Electric applied fuzzy logic theory to the control of a water purification process. As a challenging engineering project, in 1987, Sendai Railway system that had automatic train operation control was built with fuzzy logic principles in Japan. Fuzzy control techniques were used in all the critical operations in the control of the train, such as accelerating, breaking, and stopping operations. In 1987, Takeshi Yamakawa used fuzzy control in an inverted pendulum experiment which is a classical control problem. After these successful applications, not only the engineers but also the social scientists applied fuzzy logic into different areas. In todays technology, many companies use fuzzy logic in their engineering projects like air conditioners, video cameras, televisions, and washing machines. However, it is a well-known fact that fuzzy logic has always been more popular in Eastern countries because of the philosophical and religious view of the Eastern culture.

2.2. Basic Concepts of Fuzzy Logic

2.2.1. Membership Functions

Fuzzy logic is claimed to be much closer in spirit to human thinking and natural language [3]. The way of human thinking is realized with membership functions which define how every point in the input space is mapped to a membership values space. The membership values in fuzzy sets are in the range of [0;1]. If an axiom is absolutely true, the membership value in fuzzy sets is 1. Similarly, if it is absolutely false, membership value in fuzzy sets is 0. The output of the membership function is called antecedent(μ). While the input values for a membership function are crisp inputs, they are changed into fuzzy variables by the membership functions.

Figure 2.1 and figure 2.2 show the difference between classical binary logic and fuzzy logic. In, figure 2.1, the membership function is sharp-edged that means a small change of input values might cause a big changes in output values. In figure 2.2, the membership function is continuous and smooth that is more like human thinking [4].

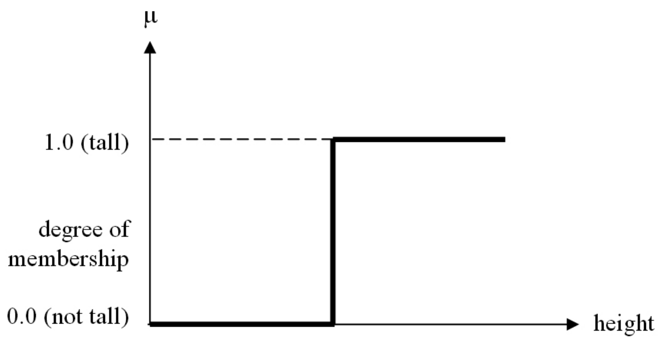

Figure 2.1. A possible description of the vague concept “tall” by a crisp set

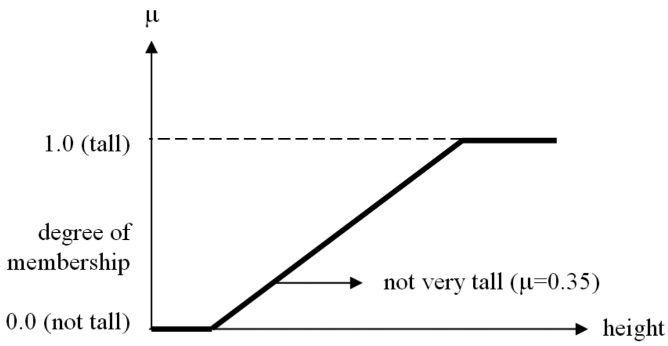

Figure 2.2. A possible description of the vague concept “tall” by a fuzzy set

2.2.2. If-Then Rules

Fuzzy controllers have always some conditional “if-then” rules. These rules are written by an expert who know and understand that system very accurately. Although many rules can be written to describe a system in more detail, in general, a low number of rules are sufficient for a fuzzy controller to control that system.

2.3. Advantages and Disadvantages of Fuzzy Control

Classical control theory uses a mathematical model to define the relationships between the inputs and the outputs of a system. The most common type of these controllers are PID controllers. After they take the output of the system and compare the desired input, they generate a proper control signal based on the error value. The most serious disadvantage of these controllers is that PID controllers usually assume the system to be linear or at least it behaves as a linear system in some range.

If an accurate mathematical model of a control system is available, a classical PID controller can make the performance of the system quite acceptable. Scientists have been working on the classical control theory for a long time. Nowadays, not only the design of a PID type controller is a very well-known subject but also the implementation is simple and cheap.

There are some reasonable causes why fuzzy logic has been famous for the last decades. In real life, an accurate mathematical model of a control process will not generally be available, even it may not exist. The real world is nonlinear, uncertain and contains always incomplete data. If the mathematical model is not known by the designer, there is no way to come up with a good PID controller design. Even in those cases, when the mathematical model is known relatively accurate, the parameters of the system are likely to change by some outside factors, like heat or pressure, etc... In such cases, a good way of controlling the system is to design a controller that does not need the exact mathematical model of that system. Fortunately, fuzzy controllers have ability to control a system with just some limited expert knowledge. Another advantage of fuzzy logic is its flexibility. Besides, fuzzy controllers are low-cost implementations based on cheap sensors.

Although fuzzy control fills an important gap in controller design methodologies that require a full mathematical clarity about a system, it has also some serious drawbacks. First of all, because fuzzy control is a method of nonlinear variable structure control, deriving their analytical structures is the first step for analytical study. However, this step is very difficult and sometimes impossible [5].

The second disadvantage is the number of the design parameters. Although a classical PID controller has only three design parameters, the number of parameters for a fuzzy controller can be very large. The number and the shape of input and output fuzzy sets, scaling factors and fuzzy AND and OR operators characteristics must be determined by the designer. Moreover, there are no clear relationships between these parameters and the controller's performance [5].

3. INTRODUCTION TO FUZZY CONTROL

The main idea relying at the back of fuzzy logic control is very well explained by Kickert and Mamdani as:

- *"The basic idea behind this approach was to incorporate the experience of a human process operator in the design of controller. From the set of linguistic rules which describe the operator's control strategy a control algorithm is constructed where the words are defined as fuzzy sets. The main advantage of this approach seem to be the possibility of implementing rule of thumb experience, intuition, heuristics and the fact that it does not need a model of the process."*

Kickert and Mamdani

A fuzzy controller can be divided into four main sub-groups that are fuzzification, inference, rule base and defuzzification as shown in the Figure 3.1:

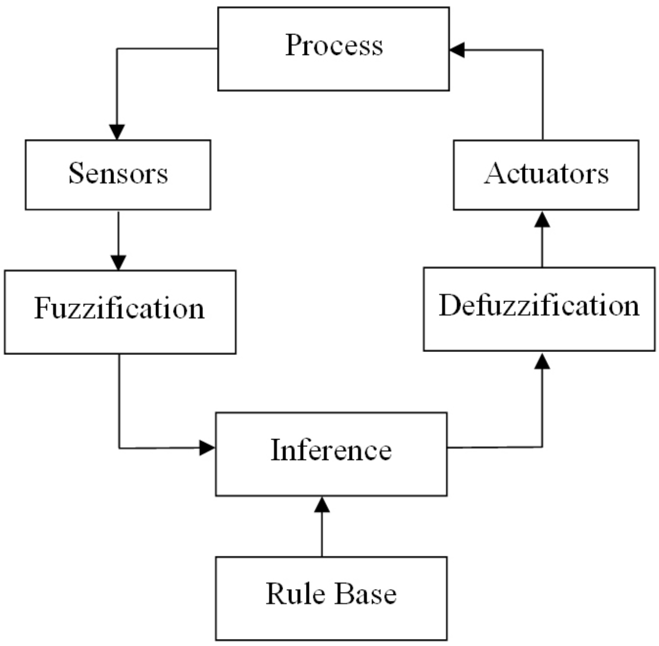

Figure 3.1. Fuzzy controller block diagram

3.1. The Basics of Fuzzy Logic Control

The inputs of a fuzzy controller are always crisp inputs that are fuzzified in *fuzzification process* based on the rules in the *rule base*. After the fuzzy decisions are made by the *inference*, the output of the fuzzy controller is converted into a crisp value. This is called as *defuzzificaon process*.

3.1.1. Fuzzification

Fuzzification is the process of converting a crisp input value to a fuzzy value that is performed by the use of the information in the knowledge base.

Although various types of curves can be seen in the literature, triangular and trapezoidal membership functions are the most common ones used in fuzzification process. These types of membership functions can easily be implemented by embedded controllers.

The membership functions are defined mathematically with some constants. In order to fine-tune the performance of a fuzzy controller, the constants and the shape of the membership functions can be adapted.

3.1.2. Rule Base

In this step, the expert knowledge is formulated as a finite number of rules. The rule base contains the rules that are to be used in making decisions. These rules are generally based on personal experiences and intuition. However, in some cases, the rules can be obtained by using neural networks, genetic algorithms or some empirical approaches [6].

A rule is composed of two main parts: an antecedent block (between the IF and THEN) and a consequent block(following THEN).

If (antecedent) then (consequent)

Although the antecedent and the consequent parts have single arguments in above, a rule can be written with multiple arguments. While single arguments are used in SISO systems, multiple arguments are used in dealing with MIMO systems.

3.1.3. Inference

Fuzzy decisions are produced in this process using the rules in the rule base. During this process each rule is evaluated separately and then a decision is made for each individual rule. The result is a set of fuzzy decisions. Logical operators, such as "AND","OR", and "NOT" define how the fuzzy variables are combined.

3.1.4. Defuzzification

The final step is defuzzification process where the fuzzy outputs are translated into a single crisp value, like the fuzzification process, by the degree of membership values. Defuzzification is an inverse transformation compared with the fuzzification process, because in this process, the fuzzy outputs are converted into crisp values to be applied into the system.

There are several heuristic defuzzification methods. For instance, some methods produce an integral output considering all the elements of the resulting fuzzy set with the corresponding weights. One of the widely used methods is the Center-of-Area (C-o-A) method that takes the center of gravity of the fuzzy set. Some other methods can be mentioned such as Center-of-Maximum (C-o-M) method that uses only the peaks of the membership functions.

4. COMBINING FUZZY AND PID TYPE CONTROL

4.1. Analysis of a Fuzzy Controller

In this section, a zero order Takagi-Sugeno type fuzzy controller with product-sum inference method is briefly explained [7, 8, 9]. Let us suppose that the fuzzy controller is a two-input and one-output one. The two inputs to the fuzzy controller are the error e and the change of the rate of error $\dot{e}$, and the output of the fuzzy controller (that is the input to the controlled process) is u. The universes of discourses of e, $\dot{e}$ and u are $E \subset R$, $\dot{E} \subset R$ and $U \subset R$, respectively. The linguistic values of e and $\dot{e}$ are denoted as A_i and B_j, respectively. These membership functions are referred to as $A_i(\mathrm{e})$ and $B_j(\dot{\mathrm{e}})$. The center of gravity method is applied in the defuzzification process to obtain the controller output, u. An example fuzzy controller rule is expressed below:

$$\text{if } e \text{ is } A_i \text{ and } \dot{e} \text{ is } B_j \text{ then } u \text{ is } u_{ij}.$$

where $u_{ij} \in U$ is a crisp value instead of a fuzzy subset. The u_{ij}s are not necessarily different from each other . The fuzzy controller with such kind of control rules is called crisp type fuzzy controller [8, 9].

In the following discussion, fuzzy control rule base, assumed to be complete, this means the number of control rules are equal to IxJ [10].

In this thesis, the triangular membership functions will be employed for each fuzzy linguistic value of the error e and the change rate of error $\dot{e}$ as shown in Figure 4.1.

The cores of fuzzy sets A_i and B_j are denoted as e_i and $\dot{e}_j$, respectively. The supporting set of A_i is shown as $[e_{i-1}, e_{i+1}]$ and the set of B_j is $[\dot{e}_{j-1}, \dot{e}_{j+1}]$.

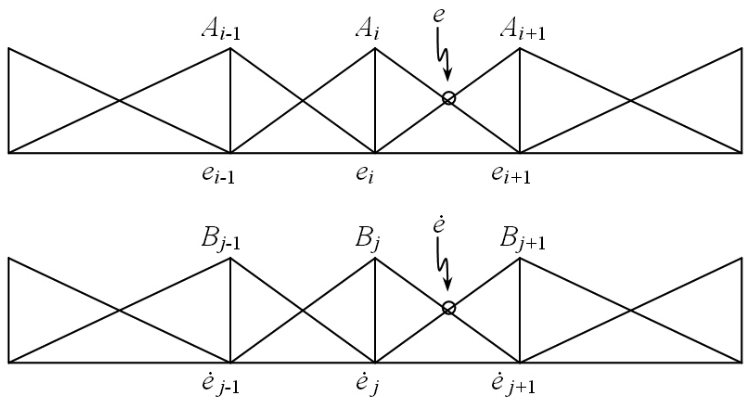

Figure 4.1. Membership functions of A_i and B_j

As indicated in Figure 4.2, every point on the e-$\dot{e}$ plane represents an output of the controller, which is a nonlinear function. An analytical solution is not available for such kind of non-linearity. However, using some linearization methods as applied in classical and modern control theory, an approximation of u can be obtained for some small deviations about the nominal values of e and $\dot{e}$.

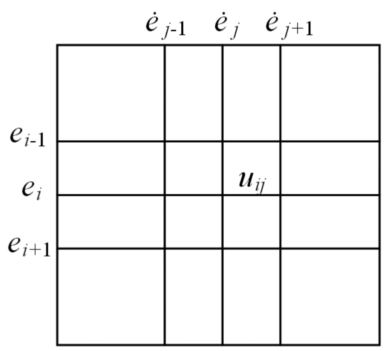

Figure 4.2. The net on the (e-$\dot{e}$) plane

The non-linear controller is of the general form:

$$u = f(e, \dot{e}, t) \tag{4.1}$$

When e=e_i and $\dot{e}$=$\dot{e}_j$, in other words, if the node is (e_i,$\dot{e}_j$), then the output of the controller will be,

$$u = u_{ij} \tag{4.2}$$

Therefore, the result will become,

$$u = f(e, \dot{e}, t) = u_{ij} \tag{4.3}$$

A linearization can be done around a node u_{ij} of the e-$\dot{e}$ plane for small excursions from e, $\dot{e}$ and u as,

$$\delta e = e - e_i \tag{4.4}$$

$$\delta \dot{e} = \dot{e} - \dot{e}_j \tag{4.5}$$

$$\delta u = u - u_{ij} \tag{4.6}$$

If the values of δe, $\delta \dot{e}$ and δu are small enough, Equation 4.1 can be linearized as:

$$\delta u = \left[\frac{\delta f}{\delta e}\right]_n \delta e + \left[\frac{\delta f}{\delta \dot{e}}\right]_n \delta \dot{e} \tag{4.7}$$

Every neighborhood of each node (or nominal point) will be divided into four different quadrants by the two net lines that across at the node.

For simplicity, only the case of one quadrant, i.e. where $\delta e \geq 0$ and $\delta \dot{e} \geq 0$ shall be considered:

$$\delta u = \left[\frac{\delta f}{\delta e}\right]_n \delta e + \left[\frac{\delta f}{\delta \dot{e}}\right]_n \delta \dot{e} = \frac{u_{(i+1)j} - u_{ij}}{e_{i+1} - e_i} \delta e + \frac{u_{(j+1)i} - u_{ij}}{\dot{e}_{j+1} - \dot{e}_j} \delta \dot{e} \tag{4.8}$$

$$u - u_{(ij)} = \frac{u_{(i+1)j} - u_{ij}}{e_{i+1} - e_i}(e - e_i) + \frac{u_{(j+1)i} - u_{ij}}{\dot{e}_{j+1} - \dot{e}_j}(\dot{e} - \dot{e}_j) \tag{4.9}$$

Therefore

$$u = \left[u_{ij} - \frac{u_{(i+1)j} - u_{ij}}{e_{(i+1)} - e_i}e_i - \frac{u_{(j+1)i} - u_{ij}}{\dot{e}_{j+1} - \dot{e}_j}\dot{e}_j\right] + \frac{u_{(i+1)j} - u_{ij}}{e_{(i+1)} - e_i}e + \frac{u_{(j+1)i} - u_{ij}}{\dot{e}_{j+1} - \dot{e}_j}\dot{e} \tag{4.10}$$

$$u = A + Pe + D\dot{e} \tag{4.11}$$

where

$$A = \left[u_{ij} - \frac{u_{(i+1)j} - u_{ij}}{e_{(i+1)} - e_i}e_i - \frac{u_{(j+1)i} - u_{ij}}{\dot{e}_{j+1} - \dot{e}_j}\dot{e}_j\right] = u_{ij} - Pe_i - D\dot{e}_j$$

$$P = \frac{u_{(i+1)j} - u_{ij}}{e_{(i+1)} - e_i}$$

$$D = \frac{u_{(j+1)i} - u_{ij}}{\dot{e}_{j+1} - \dot{e}_j}$$

As can be seen, the product-sum crisp type fuzzy controller behaves approximately like a PD controller in the neighborhood of the node point of the net of e-$\dot{e}$ plane. The equivalent proportional and derivative control coefficients are P and D respectively. When the error and the change rate of error moves on the e-$\dot{e}$ plane, the PD parameters switch from one set to another. So, this kind of fuzzy controller can be regarded as a parameter time-varying PD controller and it can be named as PD type fuzzy controller(PDFC) [11].

It is a well-known fact in conventional PID control theory that if the controlled system is type “0”, P or PD type controller will yield a steady-state error for step response. Although PI controller improves the steady-state error, it can deteriorate the transient characteristics, i.e. it slows the response down. Since the mathematical models of most industrial process systems are of type 0, obviously there would exist a steady state error if they are controlled by PD type controller [11].

4.2. PID Type Fuzzy Controller Structure

In a PID controlled system, the performance of the system is determined by its proportional parameter K_P, integral parameter K_I, and the derivative parameter K_D. The proportional control law can guarantee the fast response of the system, the integral control law can eliminate the steady state error and the derivative control law can increase the damping of the system thus reduce the overshoot and oscillating times of the system response. Thus a PID controller can yield a system with fast rise time and small overshoot and non steady state error [11].

In order to eliminate the steady-state error of the control system, one can substitute the input $\dot{e}$ (the change rate of error) of the fuzzy controller with the integration of the error. This means a fuzzy controller behaving like a parameter time-varying PI controller.

In order to design a PID type fuzzy controller (PIDFC), one can design a fuzzy controller with with three inputs, error, the change rate of error and the integration of the error. Handling the three variables is however, in practice, quite difficult. Besides, adding another input to the controller will increase the number of rules exponentially. This requires more computational effort, which leading to larger execution time.

Because of the drawbacks mentioned above, a PID type fuzzy controller consisting of only the error and the change rate of error is used in the proposed method. This system allows a PD and PI type fuzzy controllers to work in parallel [11, 12]. An equivalent structure is shown in Figure 4.3, where β and α are the weights of PI and PD type controllers, respectively. Similarly, K and K_d are the scaling factors for e and $\dot{e}$, respectively.

As the α/β ratio becomes larger, the effect of the derivative control increases with respect to integral control [13].

The output of the controller can be expressed as:

$$u_c = \alpha u + \beta \int udt = \alpha(A + PKe + DK_d\dot{e} + \beta \int (A + PKe + DK_d\dot{e})dt$$

$$= \alpha A + \beta At + (\alpha KP + \beta K_d D)e + \beta KP \int edt + \alpha K_d D\dot{e} \tag{4.12}$$

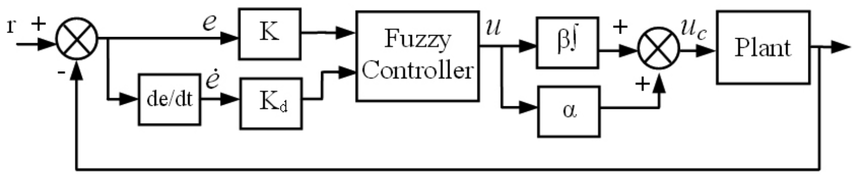

Figure 4.3. PID type fuzzy controller structure

This controller is called as PID type fuzzy controller (PIDFC). Equivalent proportional, integral and derivative components are $\alpha KP + \beta K_d D$, βKP and $\alpha K_d D$ [11].

5. MODELING

5.1. Description of Controlled Object

A model for a nonlinear liquid-level system will be obtained in this part of the thesis [13]. Figure 5.1 shows a simple system, the objective of which is to control the level of the liquid in a tank by adjusting the input flow rate. Such a simple system is considered in order to be able to compare the results to be obtained with those in the literature.

In this system, Q_{in} and Q_{out} are the maximum liquid flow rates in m^3/s for input and outlet, respectively.

The controlled input liquid flow rate q_{in} is given by:

$$q_{in} = Q_{in} \sin(\phi(t)) \qquad \phi(t) \in [0, \pi/2] \tag{5.1}$$

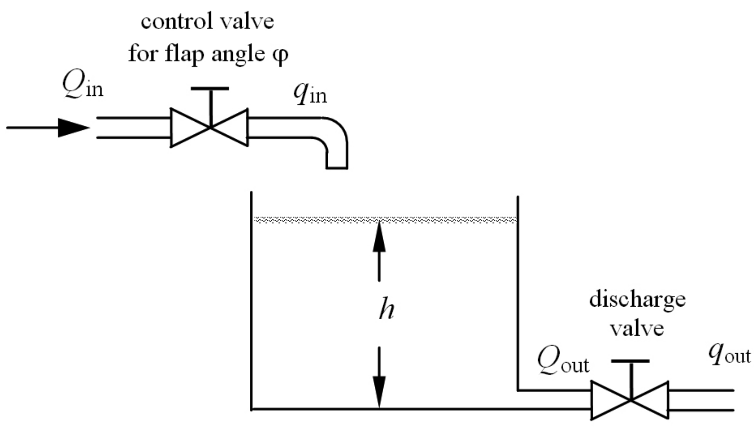

Figure 5.1. A nonlinear liquid-level system

The output liquid flow rate q_{out} (that equals Q_{out} since no control is applied) is defined as:

$$q_{out} = a_{out}\sqrt{2gh(t)} \tag{5.2}$$

where a_{out} is surface area of the outlet and g is the gravitational constant.

The output variable h, which is the level of the liquid, is calculated as:

$$h(t) = h(0) + \frac{1}{A}\int_0^t (q_{in}(\tau) - q_{out}(\tau))d\tau \tag{5.3}$$

where, A is the surface area of the tank.

The numerical values used in this thesis are:

$A = 1m^2$,

$a_{out} = 0.01m^2$,

$Q_{in} = 0.12m^3/s$, and

$h(0) = 0$.

6. SIMULATION RESULTS FOR PID TYPE FUZZY CONTROLLER

6.1. Rule Base and Membership Functions

In a conventional fuzzy inference system, an expert, who is familiar with the system to be modeled, decides on the number of rules. The fuzzy PID type control rule base employed in this thesis is shown in Table 6.1. The membership functions of error, change rate of error and control signal, shown in Figure 6.1., are chosen as triangular membership functions.

Table 6.1. A general fuzzy PID type rule base

$e/\dot{e}$	**NL**	**NM**	**NS**	**ZR**	**PS**	**PM**	**PL**
PL	ZR	PS	PM	PL	PL	PL	PL
PM	NS	ZR	PS	PM	PL	PL	PL
PS	NM	NS	ZR	PS	PM	PL	PL
ZR	NL	NM	NS	ZR	PS	PM	PL
NS	NL	NL	NM	NS	ZR	PS	PM
NM	NL	NL	NL	NM	NS	ZR	PS
NL	NL	NL	NL	NL	NM	NS	ZR

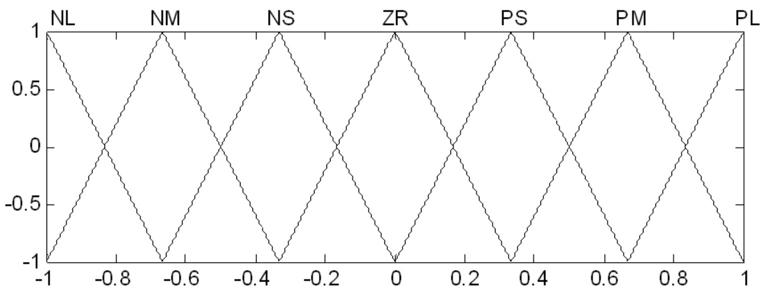

Figure 6.1. The membership functions of e, $\dot{e}$ and u.

6.2. PD Type Fuzzy Controller(PDFC)

Figure 6.2 shows the response of the model to PD type fuzzy controller. In Figure 6.2, $\alpha = 30$ and $\beta = 0$, which are derivative and integral controller constants. It is obvious that the system has a steady-state error. Sampling time of the simulation T_s=1 sec. The numerical values are selected as $K = 1$ and $K_d = 0.1$ in the simulation.

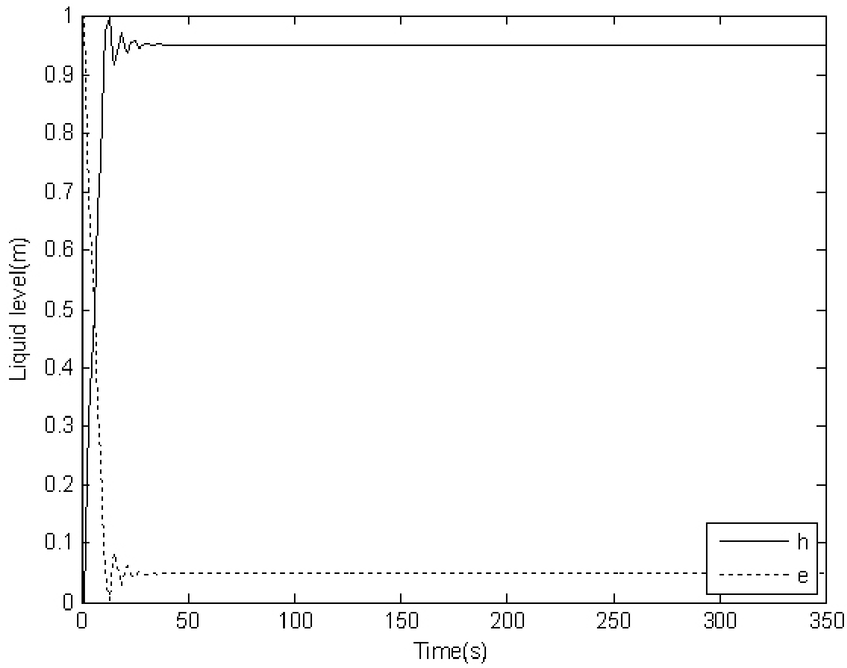

Figure 6.2. The response of the model for a unit step reference input and the error for $\alpha = 30$, $\beta = 0$

6.3. PI Type Fuzzy Controller(PIFC)

Figure 6.3 shows the response of the model to PI type fuzzy controller for $\alpha = 0$ and $\beta = 0.2$. It is seen that although the system does not have a steady-state error, it is too slow and has a big overshoot. Sampling time of the simulation T_s=1 sec. The numerical values are selected as $K = 1$ and $K_d = 0.1$ in the simulation.

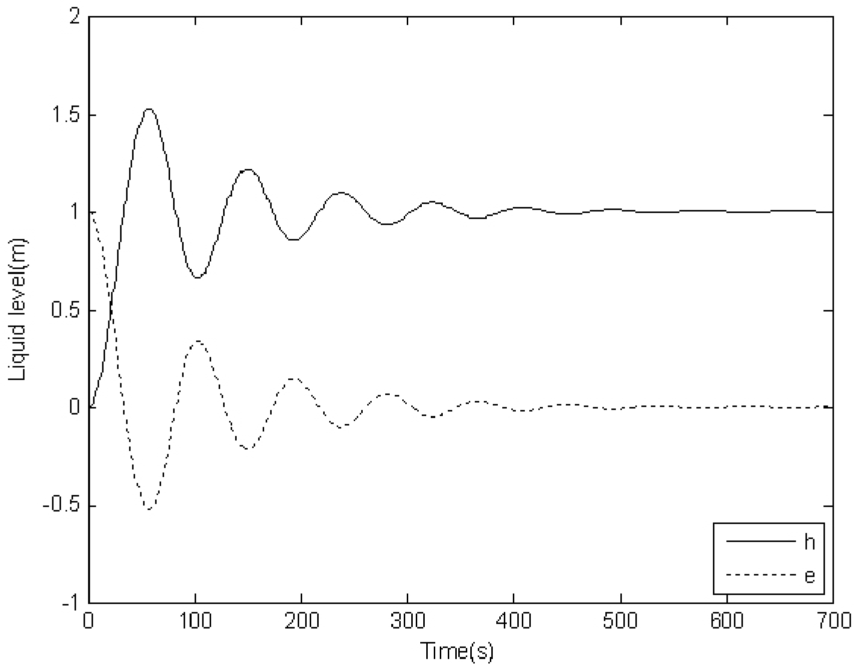

Figure 6.3. The response of the model for a unit step reference input and the error for $\alpha = 0$, $\beta = 0.2$

6.4. PID Type Fuzzy Controller(PIDFC)

Figure 6.4-6.6 show the response of the model to PID type fuzzy controller with different coefficients. In Figure 6.4, $\alpha = 0.5$ and $\beta = 0.2$, which are derivative and integral controller constants, and it is seen that the system is too slow and has a big overshoot. In Figure 6.5, $\alpha = 5$ and $\beta = 0.5$, speed of the system increases but the overshoot is still high. In Figure 6.6, $\alpha = 8$ and $\beta = 0.5$, as the speed of the system is quite good, the overshoot is reasonable. If α is selected bigger, Figure 6.7 is obtained. In Figure 6.7, $\alpha = 30$ and $\beta = 0.5$, although the speed of the system is quite good and the overshoot is reasonable, the system has an oscillatory characteristic. Sampling time of the simulations T_s=1 sec. The numerical values are selected as $K = 1$ and $K_d = 0.1$ in the simulations.

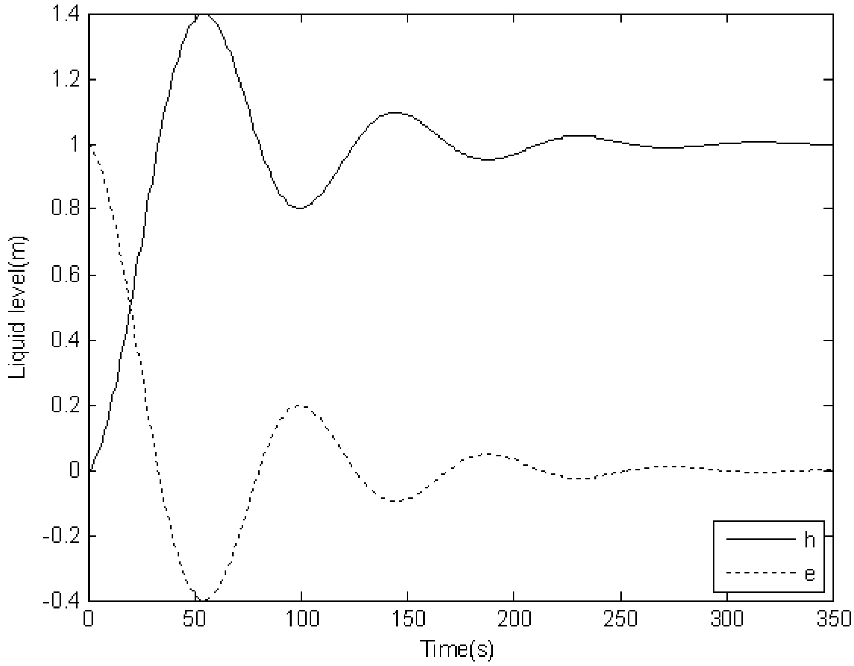

Figure 6.4. The response of the model for a unit step reference input and the error for $\alpha = 0.5$, $\beta = 0.2$

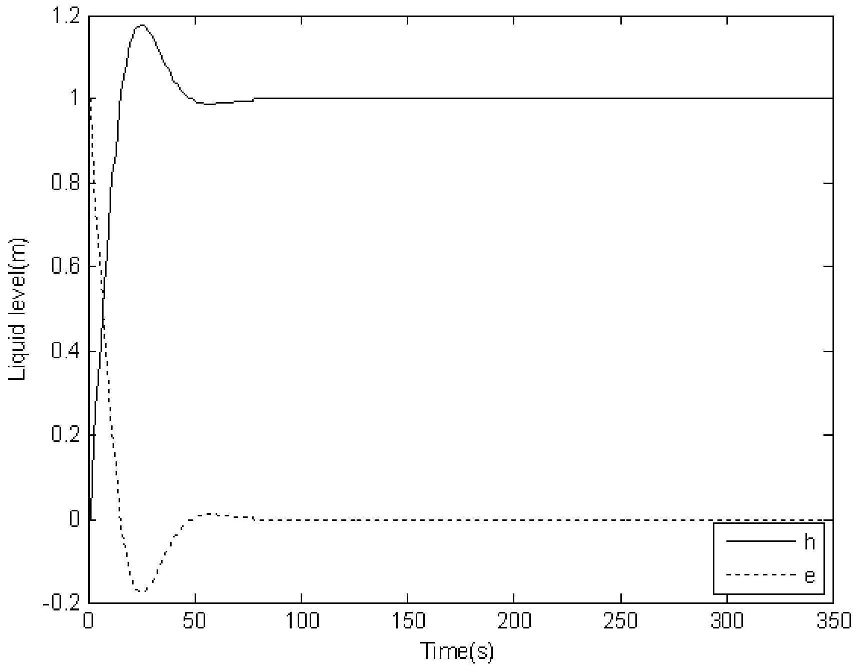

Figure 6.5. The response of the model for a unit step reference input and the error for $\alpha = 5$, $\beta = 0.5$

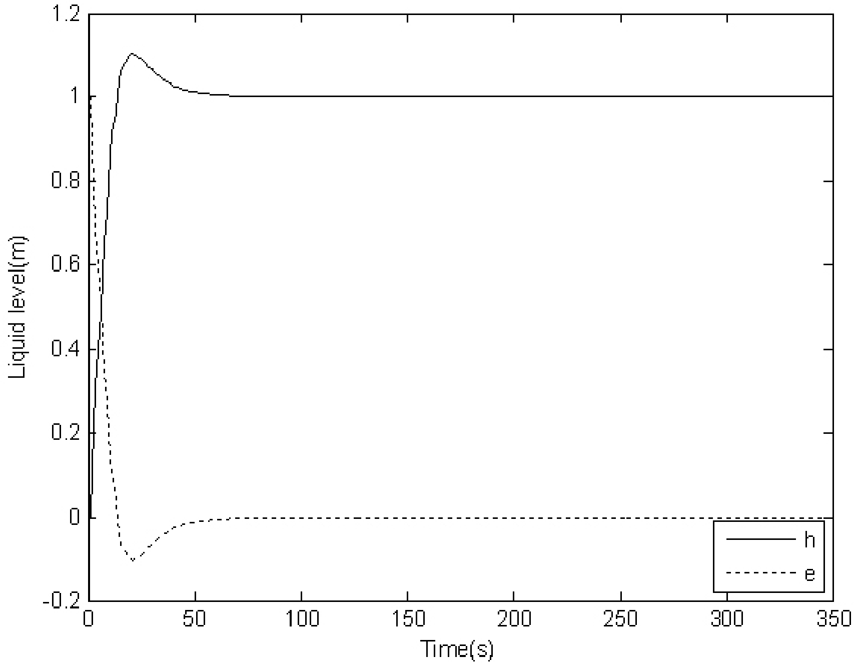

Figure 6.6. The response of the model for a unit step reference input and the error for $\alpha = 8$, $\beta = 0.5$

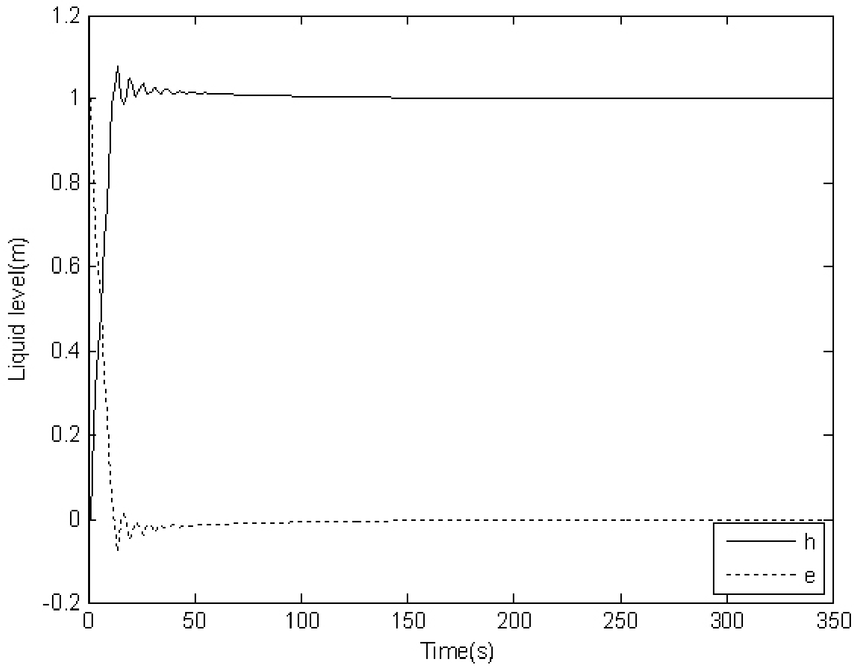

Figure 6.7. The response of the model for a unit step reference input and the error for $\alpha = 30$, $\beta = 0.5$

7. INTRODUCTION TO GREY SYSTEM THEORY

7.1. The Beginning of the Grey System Theory

Grey system theory was first introduced by Professor Deng Ju-long from China in the international journal "Systems and Control Letters" in 1982 [14]. Professor Roger W. Brockett of Harvard University, the editor of the journal, commented on Professor Deng's first article about grey system theory as follows: "Grey system is an initiative work and all the results are new". The theory is distinguished with its ability to deal with the systems that have partially unknown parameters. Thus, it is easily applicable to real-time control systems.

During the last two decades, the grey system theory has been developed rapidly and caught the attention of researchers with successful real-time practical applications. It has been commonly applied to the analysis, modeling, prediction, decision-making and control of various systems such as social, economic, financial, scientific and technological, agricultural, industrial, transportation, mechanical, meteorological, ecological, hydrological, geological, medical, military, etc., systems. Moreover, some universities located in Australia, China, Japan, Taiwan, USA, have offered courses or workshops on grey system theory [15]. Chinese Grey System Association (CGSA) was established in 1996. A conference on grey system theory and applications is held by CGSA every year. As an academic magazine, "The Journal of Grey System" is an international academic periodical which was published in England in 1989 for the first time. The Chief-Editor of the journal was Professor Julong Deng. More than 300 kind of academic periodicals accept and publish the grey system related articles in the world [17].

A grey prediction controller for an unknown system model was proposed by Cheng in 1986 [18]. In 1994, Huang proposed the basic structure of grey prediction fuzzy model to control robotic motion and inverted pendulum which is a classical control problem [19, 20]. Then, Huang offered a genetic-based fuzzy grey prediction model to compensate the output of grey system [21].

In 1998, Wong proposed a switch grey prediction fuzzy controller to find the appropriate forecasting prediction horizon of the grey predictor [22].

In 2002, Lin offered a designing technique about how to search the optimized inner parameters of grey model [23]. The inner parameters of grey predictors, which will be explained in detail in the following chapters, play an important role on improving the accuracy of grey controllers.

7.2. Fundamental Concepts of Grey System Theory

In control theory, a system can be defined with a color that represents the amount of clear information about that system. For instance, a system can be called as "black box" if its internal characteristics or mathematical equations that describe its dynamics are completely unknown. On the other hand if the description of the system is, completely known, it can be named as white system. Similarly, a system that has both known and unknown information is defined as a grey system. In real life, every system can be considered as a grey system because there are always some uncertainties [16].

In real life, due to noise from both inside and outside of the system of our concern (and the limitations of our cognitive abilities!), the information we can reach about that system, is always uncertain and limited in scope [17].

There are many situations with incomplete information in industrial control systems. This is due to the lack of modeling information or the fact that the correct observation and control variables are or cannot be employed. For instance, the data collected from a motor control system always contains some grey characteristics due to the time-varying parameters of the system and measurement difficulties. Similarly, it is difficult to forecast the electricity consumption of a local area accurately because of various kinds of social and economic factors. These factors are generally random and make it difficult to obtain a sensitive model.

"Incomplete information" or "lack of data" are the most important characteristics of grey systems. In different circumstances, the meaning of being "grey", "black" and "white" can be summarized as in Table 7.1 [15].

Table 7.1. The meaning of black, grey and white

	Black	**Grey**	**White**
Information	unknown	incomplete	known
Appearance	dark	grey	bright
Process	new	replace old with new	old
Property	chaos	complexity	order
Methodology	negative	transition	positive
Attitude	indulgence	tolerance	surety
Conclusion	no result	multiple solution	unique solution

7.3. The Differences Among Probability and Statistics, Fuzzy Theory and Grey Theory

Although probability and statistics, fuzzy theory and grey system theory deal with uncertain information, different methods and mathematical tools are used to analyze the data.

While fuzzy mathematics mainly deals with problems associated with cognitive uncertainty by experience with the help of affiliation functions, probability and statistics need special distributions and samples of reasonable size to draw inferences. These very different approaches have a serious difficulty in such situations either without any prior experience or without satisfying any special distributions and with small sample size [15]. Grey system theory and grey controllers have great advantages in such kinds of systems, because grey controllers have the ability to handle the uncertain information and use the data effectively. Grey controllers investigate the behavioral characteristics of a system using a sequence of definite white numbers. The characteristic data obtained from the system is supposed to contain, if there is, the laws of development of the system.

The methods of probability and statistics study the uncertain data from a stochastic point of view. They focus on the statistical laws existing in the history of the uncertain data and the probability of each data within possible outcomes [17].

The complexity and chaotic characteristics of data generally mislead the controller. In order to eliminate this problem, some sequence operators are used in grey controllers. It is argued that if the randomness of the data obtained from a grey system is somehow smoothed, it is easier to derive the any special characteristics of that system [16].

The differences among probability and statistics, fuzzy theory and grey theory are described in Table 7.2 [15]:

Table 7.2. Probability and statistics, fuzzy theory and grey theory

	Grey Systems Theory	**Probability and Statistics**	**Fuzzy Mathematics**
Intention	small sample uncertainty	large sample uncertainty	cognitive uncertainty
Basis	hazy integration	cantor set	fuzzy integration
Foundation	information coverage	probability distribution	function of affiliation
Means	generation	statistics	marginal sampling
Characteristic	few data points	lots of data points	experience
Requirement	allowing any distribution	special distribution	function
Objective	laws of reality	laws of statistics	historic cognitive expression

8. GREY SYSTEM MODELING

8.1. Grey Numbers

Grey numbers, grey algebraic and differential equations, grey matrices and their operations are used to deal with grey systems. The symbol of "$\otimes$" is used to represent a grey number. Grey number is such a number whose value is not known exactly but it is taking values in a certain range. Grey numbers might have only upper limits, only lower limits or both. Grey algebraic and differential equations, also grey matrices, have grey coefficients.

8.2. Generations of Grey Sequences

The main task of grey system theory is to extract realistic governing laws of a system using available data. This process is known as the generation of the grey sequence [15].

It is argued that even though the available data of the system, which are generally white numbers, is too complex or chaotic, they always contain some governing laws. If the randomness of the data obtained from a grey system is somehow smoothed, it is easier to derive the any special characteristics of that system.

For instance, the following sequence that represents the speed values of a motor might be given:

$$X^{(0)} = (820, 840, 835, 850, 890)$$

It is obvious that the sequence does not have a clear regularity.

If accumulating generation is applied to original sequence, $X^{(1)}$ is obtained which has a clear growing tendency.

$$X^{(1)} = (820, 1660, 2495, 3345, 4235)$$

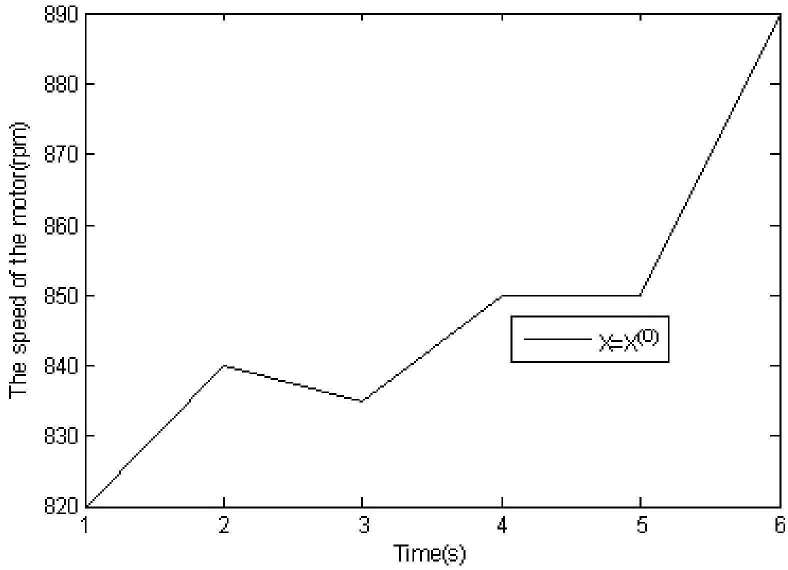

Figure 8.1. The original data set

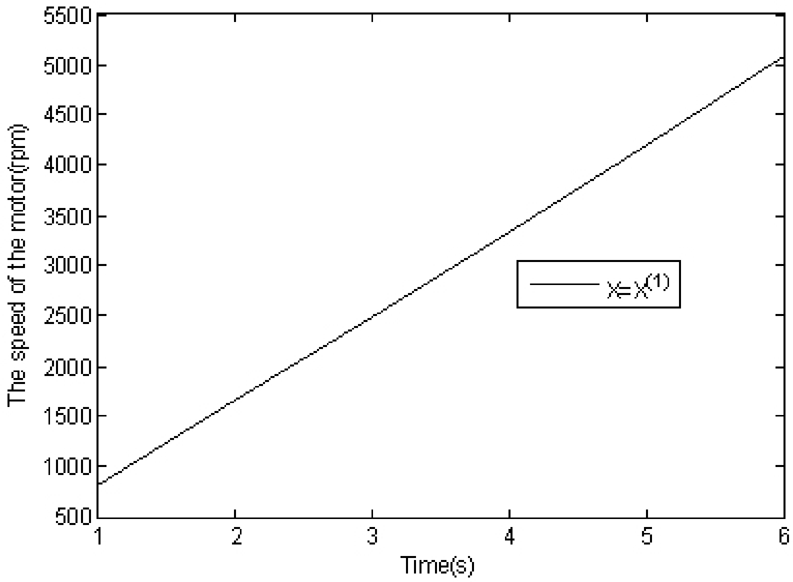

Figure 8.2. The accumulated data set

8.2.1. Accumulating Generating Operation (AGO)

In order to see the special characteristics or laws hidden in the chaotic data, Accumulating Generating Operation (AGO) is used. Accumulating generation process is a method of whitening a grey process [15].

Consider a non-negative sequence of data $X^{(0)}$ and D is a sequence operator:

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n)\right) \tag{8.1}$$

and

$$X^{(0)}D = \left(x^{(0)}(1)d, x^{(0)}(2)d, ..., x^{(0)}(n)d\right) \tag{8.2}$$

where

$$x^{(0)}(k)d = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, 3....., n \tag{8.3}$$

then D is called a first order accumulating generator of $X^{(0)}$, denoted as 1-AGO. The rth-order operator D^r of $X^{(0)}$ is obtained by applying D operation r times, denoted as r-AGO.

$$X^{(0)}D = X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(n)\right) \tag{8.4}$$

and

$$X^{(0)}D^r = X^{(r)} = \left(x^{(r)}(1), x^{(r)}(2), ..., x^{(r)}(n)\right) \tag{8.5}$$

where

$$x^{(r)}(k) = \sum_{i=1}^{k} x^{(r-1)}(i), k = 1, 2, 3....., n \tag{8.6}$$

8.2.2. Inverse Accumulating Generating Operation (IAGO)

Inverse Accumulating Generating Operation (IAGO) is the process of returning the original data after an accumulating generation process [15].

Consider a non-negative sequence of data $X^{(1)}$ and $\bar{D}$ is a sequence operator:

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(n)\right) \tag{8.7}$$

and

$$X^{(1)}\bar{D} = X^{(0)} = \left(x^{(1)}(1)\bar{d}, x^{(1)}(2)\bar{d}, ..., x^{(1)}(n)\bar{d}\right) \tag{8.8}$$

where

$$x^{(1)}(k)\bar{d} = x^{(1)}(k) - x^{(1)}(k-1), k = 1, 2, 3....., n \tag{8.9}$$

then $\bar{D}$ is called a first order inverse accumulating generating operation of $X^{(1)}$, denoted as 1-IAGO. The rth-order operator $\bar{D}^r$ of $X^{(r)}$ is obtained by applying $\bar{D}$ operation r times, denoted as r-IAGO.

$$X^{(1)}\bar{D} = X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n)\right) \tag{8.10}$$

and

$$X^{(r)}\bar{D} = X^{(r-1)}\bar{D}^{r-1} = \left(x^{(r-1)}(1), x^{(r-1)}(2), ..., x^{(r-1)}(n)\right) \tag{8.11}$$

where

$$x^{(r-1)}(k) = x^{(r)}(k) - x^{(r)}(k-1), k = 1, 2, 3....., n \tag{8.12}$$

8.3. Grey Differential Equations

Consider the following differential equation:

$$\frac{dx}{dt} + ax = b \tag{8.13}$$

$\frac{dx}{dt}$ is called the derivative of the function, x is the background value of $\frac{dx}{dt}$, a and b are the parameters of the differential equation.

The following equation

$$x^{(0)}(k) + az^{(1)} = b \tag{8.14}$$

where

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1) \tag{8.15}$$

is a grey differential equation.

8.4. GM(n,m) Model

Grey models can predict the future outputs of systems with high accuracy without knowing the mathematical model of the actual system.

In grey systems theory, GM(n,m) denotes a grey model, where n is the order of the difference equation and m is the number of variables. Although various types of grey models can be mentioned, the research reports available in the literature focus on GM(1,1) model in their predictions because of its computational efficiency. Because the grey controllers are more successful on real-time systems, the execution time of the control algorithm is always the most important parameter for the researchers after the accuracy.

8.5. GM(1,1) Model

GM(1,1) type of grey model is most widely used in the literature, pronounced as “Grey Model First Order One Variable”. This model is a time series forecasting model. The differential equations of the GM(1,1) model have time-varying coefficients. In other words, the model is renewed as the new data become available.

The GM(1,1) model can only be used in positive data sequences [24]. In this paper, a non-linear liquid level tank is considered. It is obvious that the liquid level in a tank is always positive, so that GM(1,1) model can be used to forecast the liquid level.

In order to smooth the randomness, the primitive data obtained from the system to form the GM(1,1) is subjected to an operator, named Accumulating Generation Operation (AGO)[24]. The differential equation (i.e. GM(1,1)) thus evolved is solved to obtain the n-step ahead predicted value of the system. Finally, using the predicted value, the inverse accumulating operation (IAGO) is applied to find the predicted values of original data.

Consider a single input and single output system. Assume that the time sequence $X^{(0)}$ represents the outputs of the system:

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n)\right), n \geq 4 \tag{8.16}$$

where $X^{(0)}$ is a non-negative sequence and n is the sample size of the data. When this sequence is subjected to the Accumulating Generation Operation (AGO), the following sequence $X^{(1)}$ is obtained. It is obvious that $X^{(1)}$ is monotone increasing.

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(n)\right), n \geq 4 \tag{8.17}$$

where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, 3....., n \tag{8.18}$$

The generated mean sequence $Z^{(1)}$ of $X^{(1)}$ is defined as:

$$Z^{(1)} = \left(z^{(1)}(1), z^{(1)}(2), ..., z^{(1)}(n)\right) \tag{8.19}$$

where $z^{(1)}(k)$ is the mean value of adjacent data, i.e.

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1), k = 2, 3, ..., n \tag{8.20}$$

The least square estimate sequence of the grey difference equation of GM(1,1) is defined as follows [24]:

$$x^{(0)}(k) + az^{(1)}(k) = b \tag{8.21}$$

The whitening equation is therefore as follows:

$$\frac{dx^{1}(t)}{dt} + ax^{(1)}(t) = b \tag{8.22}$$

In above, $[a, b]^T$ is a sequence of parameters that can be found as follows:

$$[a, b]^T = (B^T B)^{-1} B^T Y \tag{8.23}$$

where

$$Y = \left[x^{(0)}(2), x^{(0)}(3), ..., x^{(0)}(n)\right]^T \tag{8.24}$$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ . & . \\ . & . \\ . & . \\ -z^{(1)}(n) & 1 \end{bmatrix} \tag{8.25}$$

According to equation (8.22), the solution of $x^{(1)}(t)$ at time k:

$$x_p^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} + \frac{b}{a} \tag{8.26}$$

To obtain the predicted value of the primitive data at time (k+1), the IAGO is used to establish the following grey model.

$$x_p^{(0)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a} \right] e^{-ak}(1 - e^{a}) \tag{8.27}$$

and the predicted value of the primitive data at time (k+H):

$$x_p^{(0)}(k+H) = \left[x^{(0)}(1) - \frac{b}{a} \right] e^{-a(k+H-1)}(1 - e^{a}) \tag{8.28}$$

The parameter $(-a)$ in the GM(1,1) model is called “development coefficient” which reflects the development states of $X_p^{(1)}$ and $X_p^{(0)}$. The parameter b is called “grey action quantity” which reflects changes contained in the data because of being derived from the background values [15].

8.6. GM(1,1) Rolling Model

GM(1,1) rolling model is based on the forward data of sequence to build the GM(1,1). For instance, using $x^{(0)}(k)$, $x^{(0)}(k+1)$, $x^{(0)}(k+2)$ and $x^{(0)}$(k+3), the model predicts the value of the next point $x^{(0)}(k+4)$. In the next steps, the first point is always shifted to the second. It means that $x^{(0)}(k+1)$, $x^{(0)}(k+2)$, $x^{(0)}$(k+3) and $x^{(0)}$(k+4) are used to predict the value of $x^{(0)}$(k+5). This procedure is repeated till the end of the sequence and this method is called rolling check [26].

GM(1,1) rolling model is used to predict the long continuous data sequences such as the outputs of a system, price of a specific product, trend analysis for finance statements or social parameters, etc...

8.7. Error Remedy of GM(1,1) Model

In order to improve the accuracy of GM(1,1), Remnant GM(1,1) model can be established that uses the error sequence to remedy the original model [15].

Consider $X^{(1)}$ is the 1-AGO sequence of $X^{(0)}$ and the response of the GM(1,1) is:

$$x_p^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} + \frac{b}{a} \tag{8.29}$$

Then, equation (8.30) can be called as “restored value through derivatives”.

$$dx_p^{(1)}(k+1) = (-a)\left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} \tag{8.30}$$

$$\epsilon^{(0)} = \left(\epsilon^{(0)}(1), \epsilon^{(0)}(2), ..., \epsilon^{(0)}(n)\right) \tag{8.31}$$

where $\epsilon^{(0)}(k)$ is the error sequence of $X^{(1)}$:

$$\epsilon^{(0)}(k) = x^{(1)}(k) - x_p^{(1)}(k) \tag{8.32}$$

If there exists k_0 satisfying:

1. For any $k \geq k_0$, $\epsilon^{(0)}(k)$ has the same sign,

2. $n - k_0 \geq 4$

$$\left(\left|\epsilon^{(0)}(k_0)\right|, \left|\epsilon^{(0)}(k_0+1)\right|, \ldots\ldots, \left|\epsilon^{(0)}(n)\right|\right) \tag{8.33}$$

$$\left(\epsilon^{(0)}(k_0), \epsilon^{(0)}(k_0+1), \ldots\ldots, \epsilon^{(0)}(n)\right) \tag{8.34}$$

Consider $\epsilon^{(1)}$ is the 1-AGO sequence of $\epsilon^{(0)}$

The time response sequence is given as:

$$\epsilon_p^{(1)}(k+1) = \left[\epsilon^{(0)}(k_0) - \frac{b\epsilon}{a\epsilon}\right] e^{-a\epsilon(k-k_0)} + \frac{b\epsilon}{a\epsilon}, k \geq k_0 \tag{8.35}$$

$$\epsilon_p^{(0)}(k+1) = (-a\epsilon)\left[\epsilon^{(0)}(k_0) - \frac{b\epsilon}{a\epsilon}\right] e^{-a\epsilon(k-k_0)}, k \geq k_0 \tag{8.36}$$

The sign of the error modification value $\epsilon_p^{(0)}(k+1)$ must be the same as the error $\epsilon^{(0)}$. If $\epsilon^{(0)}$ is used to modify $X_p^{(1)}$, the time response sequence after the modification is below. The model below is called the *"GM(1,1) model with error remedy"* or *"Remnant GM(1,1) Model"* :

$$x_p^{(1)}(k+1) = \begin{cases} \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} + \frac{b}{a}, & k < k_0 \\ \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} + \frac{b}{a} \pm a\epsilon \left[\epsilon^{(0)}(k_0) - \frac{b\epsilon}{a\epsilon}\right] e^{-a\epsilon(k-k_0)}, & k \geq k_0 \end{cases} \quad (8.37)$$

If

$$x_p^{(0)}(k) = x_p^{(1)}(k) - x_p^{(1)}(k-1) = (1 - e^a)\left[x^{(0)}(1) - \frac{b}{a}\right] e^{-a(k-1)} \quad (8.38)$$

then the sequence of data after the error remedy is as below. This model is called *"the error remedy model of inverse accumulating restoration"*.

$$x_p^{(0)}(k+1) = \begin{cases} (1 - e^a)\left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak}, & k < k_0 \\ (1 - e^a)\left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} \pm a\epsilon \left[\epsilon^{(0)}(k_0) - \frac{b\epsilon}{a\epsilon}\right] e^{-a\epsilon(k-k_0)}, & k \geq k_0 \end{cases} \quad (8.39)$$

If

$$x_p^{(0)}(k+1) = (-a)\left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} \quad (8.40)$$

then the sequence of data after the error remedy is as below. This model is called *"the error remedy model of derivative restoration"*.

$$x_p^{(0)}(k+1) = \begin{cases} (-a)\left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak}, & k < k_0 \\ (-a)\left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} \pm a\epsilon \left[\epsilon^{(0)}(k_0) - \frac{b\epsilon}{a\epsilon}\right] e^{-a\epsilon(k-k_0)}, & k \geq k_0 \end{cases} \quad (8.41)$$

The error values of the model can be obtained from the following equation:

$$\epsilon_p^{(0)}(k+1) = (1 - e^{a\epsilon})\left[\epsilon^{(0)}(k_0) - \frac{b\epsilon}{a\epsilon}\right] e^{-a\epsilon(k-k_0)}, k \geq k_0 \quad (8.42)$$

8.8. The Grey Verhulst Model

The Verhulst model was first introduced by a German biologist Verhulst. The main purpose of Velhulst model is to limit the whole development for a real system and it is effective in describing some increasing processes, such as an S curve which has a saturation region.

The grey Verhulst model can be defined as [27]:

$$\frac{dx^{(1)}}{dx} + ax^{(1)} = b\left(x^{(1)}\right)^2 \tag{8.43}$$

Grey difference equation of equation (8.43) is

$$x^{(0)}(k) + az^{(1)}(k) = b\left(z^{(1)}(k)\right)^2 \tag{8.44}$$

$$x^{(0)}(k) = -az^{(1)}(k) + b\left(z^{(1)}(k)\right)^2 \tag{8.45}$$

Similar to the GM(1,1) model

$$[a, b]^T = (B^T B)^{-1} B^T Y \tag{8.46}$$

where

$$Y = \left[x^{(0)}(2), x^{(0)}(3), ..., x^{(0)}(n)\right]^T \tag{8.47}$$

$$B = \begin{bmatrix} -z^{(1)}(2) & \left(z^{(1)}(2)\right)^2 \\ -z^{(1)}(3) & \left(z^{(1)}(3)\right)^2 \\ . & . \\ . & . \\ . & . \\ -z^{(1)}(n) & \left(z^{(1)}(n)\right)^2 \end{bmatrix} \tag{8.48}$$

The solution of $x^{(1)}(t)$ at time k:

$$x_p^{(1)}(k+1) = \frac{ax^{(0)}(1)}{bx^{(0)}(1) + \left(a - bx^{(0)}(1)\right) e^{ak}} \tag{8.49}$$

Applying the I-AGO, the solution of $x^{(0)}(t)$ at time k:

$$x_p^{(0)}(k) = \frac{ax^{(0)}(1)\left(a - bx^{(0)}(1)\right)(1 - e^{a})e^{a(k-2)}}{\left(bx^{(0)}(1) + \left(a - bx^{(0)}(1)\right) e^{a(k-1)}\right)\left(bx^{(0)}(1) + \left(a - bx^{(0)}(1)\right) e^{a(k-2)}\right)} \tag{8.50}$$

As can be seen, in equation (8.49), if $a < 0$, then

$$\lim_{k\to\infty} x_p^{(1)}(k+1) \to \frac{a}{b}$$

It means that the saturation point in equation (8.49) is $\frac{a}{b}$ which limits the prediction value. It is also the saturation point of $x_p^{(0)}(k)$ [27].

It also means that when k is sufficiently large, $x_p^{(1)}(k+1)$ and $x_p^{(1)}(k)$ will be very close. Because of this feature of grey Verhulst model, it is commonly used to describe and to predict processes with a saturation region.

9. MODEL ACCURACY EXAMINATION

9.1. Error Analysis Standards

Grey prediction is an action based on discussions of the past and tell about the future of the system. The original data obtained from the system is studied and discovered some development laws of the system.

The accuracy and the feasibility of a model need to be checked using various criteria. Only the models passing all the checks of different criteria can be used as prediction models [25].

To demonstrate the accuracy of the proposed forecasting models, the actual value and the forecasted value can be compared.

Equation (9.1), (9.2) and (9.3) are the three accuracy evaluation standards that are used to examine the accuracy of the models in this thesis.

$$\epsilon(Error) = x^{(0)}(k) - x_p^{(0)}(k) \tag{9.1}$$

$$RPE(Relative\ Percentage\ Error) = \frac{|\epsilon(k)|}{x^{(0)}(k)} 100\% \tag{9.2}$$

$$ARPE(Average\ Relative\ Percentage\ Error) = \frac{1}{n-1} \sum_{k=2}^{n} \frac{|\epsilon(k)|}{x^{(0)}(k)} \tag{9.3}$$

9.2. Test of GM(1,1) Model

Consider the following sequence that represents the speed values (rpm) of a motor are given:

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), x^{(0)}(4), x^{(0)}(5)\right) \tag{9.4}$$

$$X^{(0)} = (820, 840, 835, 850, 890) \tag{9.5}$$

GM(1,1) model can be used to simulate $X^{(0)}$ and seen its simulations accuracy:

Step 1: Apply 1-AGO on $X^{(0)}$

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), x^{(1)}(4), x^{(1)}(5)\right) \tag{9.6}$$

$$X^{(1)} = (820, 1660, 2495, 3345, 4235) \tag{9.7}$$

Step 2: Find $z^{(1)}(k)$ using consecutive neighbor generation to $X^{(1)}$

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1) \tag{9.8}$$

$$Z^{(1)} = \left(z^{(1)}(1), z^{(1)}(2), z^{(1)}(3), z^{(1)}(4), z^{(1)}(5)\right) \tag{9.9}$$

$$Z^{(1)} = (820, 1240, 2077.5, 2920, 3790) \tag{9.10}$$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ -z^{(1)}(4) & 1 \\ -z^{(1)}(5) & 1 \end{bmatrix} = \begin{bmatrix} -1240 & 1 \\ -2077.5 & 1 \\ -2920 & 1 \\ -3790 & 1 \end{bmatrix} \tag{9.11}$$

$$Y = \left[x^{(0)}(2), x^{(0)}(3), x^{(0)}(4), x^{(0)}(5)\right]^T = [840, 835, 850, 890]^T \tag{9.12}$$

$$[a, b]^T = (B^T B)^{-1} B^T Y = [-0.0195, 804.7900]^T \tag{9.13}$$

Step 3: Construct the model

$$\frac{dx^{(1)}}{dt} - 0.0195x^{(1)} = 804.79 \tag{9.14}$$

The time response sequence:

$$x_p^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} + \frac{b}{a} = 42091e^{0.0195k} - 41271 \tag{9.15}$$

Step 4: Solve for the simulation value of $X^{(1)}$

$$X_p^{(1)} = \left(x_p^{(1)}(1), x_p^{(1)}(2), x_p^{(1)}(3), x_p^{(1)}(4), x_p^{(1)}(5)\right) \tag{9.16}$$

$$X_p^{(1)} = (820, 1648.8, 2494, 3355.8, 4234.5) \tag{9.17}$$

Step 5: Restore to find the simulation value of $X^{(0)}$

$$x_p^{(0)}(k) = x_p^{(1)}(k) - x_p^{(1)}(k-1) \tag{9.18}$$

and

$$X_p^{(0)} = \left(x_p^{(0)}(1), x_p^{(0)}(2), x_p^{(0)}(3), x_p^{(0)}(4), x_p^{(0)}(5)\right) \tag{9.19}$$

$$X_p^{(0)} = (820, 828.8, 845.2, 861.8, 878.7) \tag{9.20}$$

Step 6: Calculate the error

Table 9.1. The error of the GM(1,1) Model

	Actual value	**Predicted value**	**Error** $\epsilon(k)$	**RPE**
No	$x^{(0)}(k)$	$x_p^{(0)}(k)$	$x^{(0)}(k) - x_p^{(0)}(k)$	$\frac{\lvert\epsilon(k)\rvert}{x^{(0)}(k)}100\%$
2	840	828.8	11.2	1.33 %
3	835	845.2	-10.2	1.22 %
4	850	861.8	-11.8	1.39 %
5	890	878.7	11.3	1.27 %

$$ARPE = \frac{1}{n-1}\sum_{k=2}^{n}\frac{|\epsilon(k)|}{x^{(0)}(k)}$$

$$ARPE = \frac{1}{4}\sum_{k=2}^{5}\frac{|\epsilon(k)|}{x^{(0)}(k)} = 1.302\%$$

9.2.1. The Influence Factor of Error

The general definition of $z^{(1)}(k)$ in GM(1,1) model is:

$$z^{(1)}(k) = \alpha_g x^{(1)}(k) + (1 - \alpha_g) x^{(1)}(k-1) \tag{9.21}$$

In the literature of grey system theory, α_g is equal to 0.5 in most cases. However, it is obvious that the selection of the value of α_g has a role on the construction of the GM(1,1) model. In order to see the effect of different α_g values on the accuracy of predictions, three different numerical values of α_g that are 0.1, 0.5 and 0.9 will be tested below:

Consider the same sequence in the previous section:

$$X^{(0)} = (820, 840, 835, 850, 890) \tag{9.22}$$

Step 1: Apply 1-AGO on $X^{(0)}$

$$X^{(1)} = (820, 1660, 2495, 3345, 4235) \tag{9.23}$$

Step 2: Find $z^{(1)}(k)$ using consecutive neighbor generation to $X^{(1)}$

$$z^{(1)}(k) = \alpha_g x^{(1)}(k) + (1 - \alpha_g) x^{(1)}(k-1) \tag{9.24}$$

Step 3: Find the sequence of $z^{(1)}(k)$ and the B,Y matrices:

$$B = \begin{bmatrix} -(1660\alpha_g + 820(1-\alpha_g)) & 1 \\ -(2495\alpha_g + 1660(1-\alpha_g)) & 1 \\ -(3345\alpha_g + 2495(1-\alpha_g)) & 1 \\ -(4235\alpha_g + 3345(1-\alpha_g)) & 1 \end{bmatrix} \tag{9.25}$$

$$Y = [840, 835, 850, 890]^T \tag{9.26}$$

Step 4: Construct different GM(1,1) models for different α_g values:

- When $\alpha_g = 0.5$, the values of the parameters $a = -0.0195$ and $b = 804.7900$

$$X_p^{(0)} = (820, 828.8, 845.2, 861.8, 878.7) \tag{9.27}$$

- When $\alpha_g = 0.1$, the values of the parameters $a = -0.0196$ and $b = 811.2479$

$$X_p^{(0)} = (820, 835.5, 852.1, 868.9, 886.2) \tag{9.28}$$

- When $\alpha_g = 0.9$, the values of the parameters $a = -0.0194$ and $b = 798.3994$

$$X_p^{(0)} = (820, 822.3, 838.4, 854.9, 871.6) \tag{9.29}$$

Step 5: Calculate the error for different α_g values:

Table 9.2. The error of the GM(1,1) Model for $\alpha_g = 0.1$

	Actual value	**Predicted value**	**Error** $\epsilon(k)$	**RPE**
No	$x^{(0)}(k)$	$x_p^{(0)}(k)$	$x^{(0)}(k) - x_p^{(0)}(k)$	$\frac{\lvert\epsilon(k)\rvert}{x^{(0)}(k)}100\%$
2	840	835.5	4.5	0.54 %
3	835	852.1	-17.1	2.00 %
4	850	868.9	-18.9	2.18 %
5	890	886.2	3.8	0.43 %

$$ARPE = \frac{1}{n-1}\sum_{k=2}^{n}\frac{|\epsilon(k)|}{x^{(0)}(k)} = \frac{1}{4}\sum_{k=2}^{5}\frac{|\epsilon(k)|}{x^{(0)}(k)} = 1.288\%$$

Table 9.3. The error of the GM(1,1) Model for $\alpha_g = 0.9$

	Actual value	**Predicted value**	**Error** $\epsilon(k)$	**RPE**
No	$x^{(0)}(k)$	$x_p^{(0)}(k)$	$x^{(0)}(k) - x_p^{(0)}(k)$	$\frac{\lvert\epsilon(k)\rvert}{x^{(0)}(k)}100\%$
2	840	822.3	17.7	2.15 %
3	835	838.4	-3.4	0.41 %
4	850	854.9	-4.9	0.57 %
5	890	871.6	18.4	2.10 %

$$ARPE = \frac{1}{n-1}\sum_{k=2}^{n}\frac{|\epsilon(k)|}{x^{(0)}(k)} = \frac{1}{4}\sum_{k=2}^{5}\frac{|\epsilon(k)|}{x^{(0)}(k)} = 1.309\%$$

The results, for $\alpha_g = 0.1$ ARPE=1.288, for $\alpha_g = 0.5$ ARPE=1.302 %, for $\alpha_g = 0.9$ ARPE=1.309 %, show that although there is an small effect of different α_g values on the predictions, it does not affect the accuracy of the predictions very seriously.

9.3. Test of GM(1,1) Model After Error Remedy

Consider the following sequence that represents the speed values (rpm) of a motor:

$$\begin{aligned} X^{(0)} &= (x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), x^{(0)}(4), x^{(0)}(5), x^{(0)}(6), x^{(0)}(7), x^{(0)}(6), \\ &\quad x^{(0)}(7), x^{(0)}(8), x^{(0)}(9), x^{(0)}(10), x^{(0)}(11), x^{(0)}(12), x^{(0)}(13)) \end{aligned} \tag{9.30}$$

$$X^{(0)} = (550, 650, 880, 750, 880, 940, 750, 620, 530, 560, 490, 510, 470) \tag{9.31}$$

Step 1: Apply 1-AGO on $X^{(0)}$

$$X^{(1)} = (550, 1200, 2080, 2830, 3710, 4650, 5400, 6020, 6550, 7110, 7600, 8110, 8580) \tag{9.32}$$

Step 2: Find $z^{(1)}(k)$ using consecutive neighbor generation to $X^{(1)}$

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1) \tag{9.33}$$

$$Z^{(1)} = (875, 1640, 2455, 3270, 4180, 5025, 5710, 6285, 6830, 7355, 7855, 8345) \tag{9.34}$$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ . & . \\ . & . \\ . & . \\ -z^{(1)}(13) & 1 \end{bmatrix} = \begin{bmatrix} -875 & 1 \\ -1640 & 1 \\ . & . \\ . & . \\ . & . \\ -8345 & 1 \end{bmatrix} \tag{9.35}$$

$$Y = [650, 880, 750, 880, 940, 750, 620, 530, 560, 490, 510, 470]^T \tag{9.36}$$

$$[a, b]^T = (B^T B)^{-1} B^T Y = [0.0490, 913.6683]^T \tag{9.37}$$

Step 3: Construct the model

$$\frac{dx^{(1)}}{dt} + 0.0490x^{(1)} = 913.6683 \tag{9.38}$$

The time response sequence:

$$x_p^{(1)}(k+1) = -18080e^{-0.0195k} + 18630 \tag{9.39}$$

Step 4: Calculate the $X_p^{(1)}$ using Equation (9.39):

$$\begin{aligned} X_p^{(1)} &= (550, 1415.3, 2239.2, 3023.6, 3770.6, 4481.7, 5158.9, \\ &\quad 5803.6, 6417.4, 7001.9, 7558.4, 8088.3, 8592.8) \end{aligned} \tag{9.40}$$

Step 5: Restore to find the simulation value of $X^{(0)}$

$$\begin{aligned} X_p^{(0)} &= (550, 865.30, 823.89, 784.46, 746.91, 711.17, 677.13, \\ &\quad 644.72, 613.86, 584.48, 556.51, 529.88, 504.52) \end{aligned} \tag{9.41}$$

Step 6: Calculate the error

Table 9.4. The error of the GM(1,1) Model

	Actual value	**Predicted value**	**Error** $\epsilon(k)$	**RPE**
No	$x^{(0)}(k)$	$x_p^{(0)}(k)$	$x^{(0)}(k) - x_p^{(0)}(k)$	$\frac{\lvert\epsilon(k)\rvert}{x^{(0)}(k)}100\%$
2	650	865.30	-215.30	24.88 %
3	880	823.89	56.11	6.81 %
4	750	784.46	-34.46	4.39 %
5	880	746.91	133.09	17.81 %
6	940	711.17	228.83	32.18 %
7	750	677.13	72.87	10.76 %
8	620	644.72	-24.72	3.83 %
9	530	613.86	-83.86	13.66 %
10	560	584.48	-24.48	4.19 %
11	490	556.51	-66.51	11.95 %
12	510	529.88	-19.88	3.75 %
13	470	504.52	-34.52	6.84 %

$$ARPE = \frac{1}{n-1}\sum_{k=2}^{n}\frac{|\epsilon(k)|}{x^{(0)}(k)} = \frac{1}{12}\sum_{k=2}^{13}\frac{|\epsilon(k)|}{x^{(0)}(k)} = 11.76\%$$

As can be seen, the average relative percentage error is very large. It is necessary to change the model GM(1,1) and apply a GM(1,1) model with error remedy to obtain more accuracy on the predictions:

A value for k_0 must be selected satisfying:

1. For any $k \geq k_0$, $\epsilon^{(0)}(k)$ has the same sign,

2. $n - k_0 \geq 4$

If $k_0 = 9$ is selected:

$$\begin{aligned}\epsilon^{(0)} &= \left(\epsilon^{(0)}(9), \epsilon^{(0)}(10), \epsilon^{(0)}(11), \epsilon^{(0)}(12), \epsilon^{(0)}(13)\right) \\ &= (-83.86, -24.48, -66.51, -19.88, -34.52) \end{aligned} \tag{9.42}$$

The absolute value of $\epsilon^{(0)}$:

$$\epsilon^{(0)} = (83.86, 24.48, 66.51, 19.88, 34.52) \tag{9.43}$$

To obtain the time response sequence of the predicted values of the error $\epsilon_p^{(1)}$ (the 1-AGO sequence of $\epsilon_p^{(0)}$), the model GM(1,1) is established:

$$\epsilon_p^{(1)}(k+1) = -1009.7e^{-0.0389(k-9)} + 1093.6 \tag{9.44}$$

$$\epsilon_p^{(0)}(k+1) = 39.2545e^{-0.0389(k-9)} \tag{9.45}$$

$$x_p^{(0)}(k+1) = x_p^{(1)}(k+1) - x_p^{(1)}(k) = (1-e^a)\left[x^{(0)}(1) - \frac{b}{a}\right]e^{-ak} \tag{9.46}$$

$$x_p^{(0)}(k+1) = 908.7976e^{-0.049k} \tag{9.47}$$

$$x_p^{(0)}(k+1) = \begin{cases} 908.7976e^{-0.049k}, & k < 9 \\ 908.7976e^{-0.049k} - 39.2545e^{-0.0389(k-9)}, & k \geq 9 \end{cases} \tag{9.48}$$

After the model modification, the prediction results can be seen in Table (9.5) and the accuracy of the models can be compared for $k > 9$:

Table 9.5. The error of the GM(1,1) Model with error remedy

	Actual value	**Predicted value**	**Error** $\epsilon(k)$	**RPE**
No	$x^{(0)}(k)$	$x_p^{(0)}(k)$	$x^{(0)}(k) - x_p^{(0)}(k)$	$\frac{\lvert\epsilon(k)\rvert}{x^{(0)}(k)}100\%$
10	560	545.23	14.77	2.64 %
11	490	518.75	-28.75	5.87 %
12	510	493.56	16.44	3.22 %
13	470	469.58	0.42	0.09 %

The value of ARPE for the GM(1,1) model with error remedy is:

$$ARPE = \frac{1}{4}\sum_{k=10}^{13}\frac{|\epsilon(k)|}{x^{(0)}(k)} = 2.95\%$$

The value of ARPE for GM(1,1) model that can be calculated from Table (9.4) was:

$$ARPE = \frac{1}{4}\sum_{k=10}^{13}\frac{|\epsilon(k)|}{x^{(0)}(k)} = 7.30\%$$

As can be seen, the accuracy of the prediction is better for remnant GM(1,1) model when compared to GM(1,1) model. Although the predictions of GM(1,1) model is satisfactory for the monotonic processes of change, remnant GM(1,1) model is needed when the sequence of data has an oscillatory characteristic.

9.4. Test of Grey Verhulst Model

Consider the R-L circuit below.

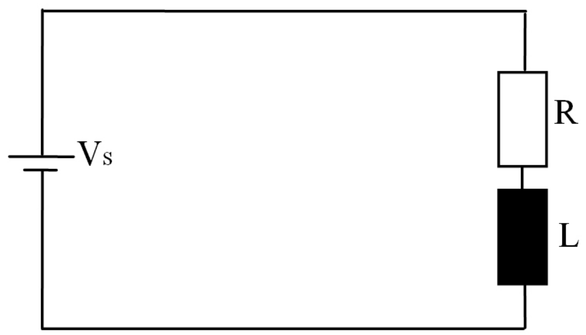

Figure 9.1. The R-L circuit

Source voltage $V_s = 12Vdc$, $R = 1\Omega$ and $L = 10mH$. The data sequence $X^{(0)}$ represents the current measurement values obtained from the circuit at every 0.005s :

$$\begin{aligned} X^{(0)} &= (x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), x^{(0)}(4), x^{(0)}(5), x^{(0)}(6), x^{(0)}(7), x^{(0)}(6), \\ &\quad x^{(0)}(7), x^{(0)}(8), x^{(0)}(9), x^{(0)}(10), x^{(0)}(11), x^{(0)}(12), x^{(0)}(13)) \end{aligned} \tag{9.49}$$

$$\begin{aligned} X^{(0)} &= (0, 4.72163, 7.58545, 9.32244, 10.37598, 11.01498, 11.40255, \\ &\quad 11.63763, 11.78021, 11.86669, 11.91914, 11.95096, 11.97025) \end{aligned} \tag{9.50}$$

The last five data of the sequence can be used for the analysis. In this case, the sequences of the original data can be regarded as $X^{(1)}$. The construction of the model is below:

Step 1: Build the original sequence

$$X^{(1)} = (11.78021, 11.86669, 11.91914, 11.95096, 11.97025) \tag{9.51}$$

Step 2: Find I-AGO sequence $X^{(0)}$ of $X^{(1)}$ is

$$X^{(0)} = (11.78021, 0.08648, 0.05245, 0.03181, 0.01929) \tag{9.52}$$

Step 3: Find $Z^{(1)}$ using consecutive neighbor generation on $X^{(1)}$

$$Z^{(1)} = (11.78021, 11.82345, 11.89292, 11.93505, 11.96061) \tag{9.53}$$

Step 4: Construct B and Y matrices

$$B = \begin{bmatrix} -z^{(1)}(2) & \left(z^{(1)}(2)\right)^2 \\ -z^{(1)}(3) & \left(z^{(1)}(3)\right)^2 \\ -z^{(1)}(4) & \left(z^{(1)}(4)\right)^2 \\ -z^{(1)}(5) & \left(z^{(1)}(5)\right)^2 \end{bmatrix} = \begin{bmatrix} -11.8235 & 139.7940 \\ -11.8929 & 141.4415 \\ -11.9351 & 142.4455 \\ -11.9606 & 143.0561 \end{bmatrix} \tag{9.54}$$

$$Y = [0.0865, 0.0525, 0.0318, 0.0193]^T \tag{9.55}$$

Step 5: Find the coefficients a and b

$$[a, b]^T = (B^T B)^{-1} B^T Y = [-0.4989, -0.0416]^T \tag{9.56}$$

Step 6: The solution of $x^{(1)}(t)$ at time k:

$$x_p^{(1)}(k+1) = \frac{-0.4989x^{(0)}(1)}{-0.0416x^{(0)}(1) + \left(-0.4989 + 0.0416x^{(0)}(1)\right) e^{-0.4989k}} \tag{9.57}$$

Table 9.6. The error of the grey Verhulst model

	Actual value	**Predicted value**	**Error** $\epsilon(k)$	**RPE**
No	$x^{(1)}(k)$	$x_p^{(1)}(k)$	$x^{(1)}(k) - x_p^{(1)}(k)$	$\frac{\lvert\epsilon(k)\rvert}{x^{(1)}(k)} 100\%$
2	11.86669	11.86528	0.00141	0.01185 %
3	11.91914	11.91754	0.00160	0.01344 %
4	11.95096	11.94949	0.00146	0.01222 %
5	11.97025	11.96898	0.00127	0.01061 %

The value of ARPE for grey Verhulst model is:

$$ARPE = \frac{1}{4} \sum_{k=2}^{5} \frac{|\epsilon(k)|}{x^{(1)}(k)} = 0.012\%$$

If the circuit is analyzed and obtained the current waveform, it can be seen that $x^{(1)}(6) = 11.98196A$ at $t = 0.065s$. The grey Verhulst model predicts the value as $x_p^{(1)}(6) = 11.98085A$. So, the error of prediction is:

$$\epsilon_p^{(1)}(6) = x^{(1)}(6) - x_p^{(1)}(6) = 11.98196 - 11.98085 = 0.00111 \tag{9.58}$$

$$RPE = \frac{\epsilon_p^{(1)}(6)}{x^{(1)}(6} = 0.00926\% \tag{9.59}$$

As it was mentioned in the chapter (8.8), the saturation point of $x_p^{(0)}(k+1)$ is:

$$\lim_{k \to \infty} x_p^{(1)}(k+1) \to \frac{a}{b} = \frac{-0.4989}{-0.0416} = 11.99279 \tag{9.60}$$

While the steady-state current of the circuit can be found using some electrical analysis methods as 12A, the prediction of the model is 11.99279A which means a highly accurate prediction.

9.5. Review of the Grey Models

The general form of a grey model is GM(n,m), where n is the order of the difference equation and m is the number of variables. The computing time period increases exponentially as n and m increases. However, in most cases, the prediction accuracy may not increase with large values of n and m. So that, GM(1,1) model is the most common grey model studied in the literature of grey system theory.

The model accuracy examination results show that GM(1,1) model is able to make very accurate predictions for long-term forecasting of the monotonous variety processes.

In the literature, as the ratio $\sigma(k)$ defined below stays in the interval of $\sigma(k) \in (0, 1]$, a grey model can be built.

$$\sigma(k) = \frac{x(k-1)}{x(k)} \qquad k \geq 2 \tag{9.61}$$

However, the model GM(1,1) is imperfect when the primitive data sequence increases in the shape of S or when it has a saturation region. In these cases, the forecasting error of GM(1,1) will increase and the accuracy of the predictions will be unacceptable. In order to solve this problem, remnant GM(1,1) model or grey Verhulst model can be used. Grey Verhulst model is especially applicable for the systems with S-shape which has a saturation region. Because the output data sequence of non-linear liquid level control system used in this thesis does not have these kind of characteristics, GM(1,1) model gives very accurate prediction results.

GM(1,1) model predicts the future values of a time sequence with less computation time when compared the other prediction methods, because it always uses small data set for its predictions. As a result, GM(1,1) model is successful on real time modeling and real time control of the industrial systems.

10. SIMULATION RESULTS FOR GREY CONTROLLERS

A number of simulation studies have been carried out on the plant described in the modeling section. Various kinds of grey controllers based on GM(1,1) model are used to control the liquid level.

The sampling time of the simulations is set as T_s=1 sec. The numerical values are selected as $K = 1$ and $K_d = 0.1$. At each time that the grey controller produces an output, 5 most recent sample data are used to construct the GM(1,1) model.

10.1. Grey PID Control

In most control applications, the control signal is a function of the error present in the system at the given time. In grey system theory, prediction error is used instead of current measured error [28]. Similarly, during the development of a grey PID controller, the prediction error is considered as the error of the system. Figure 10.1 shows the general structure of a grey PID controller.

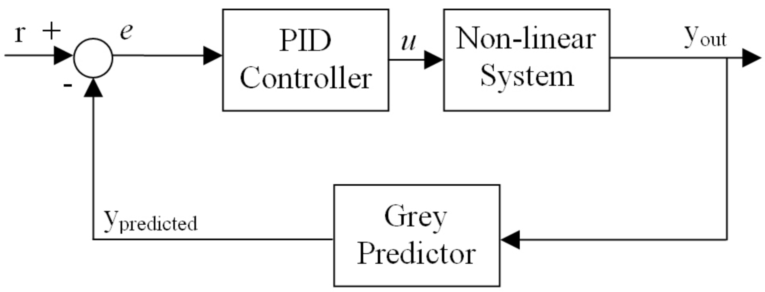

Figure 10.1. Grey PID controller structure

Figure 10.2-10.4 show the step responses of the model to conventional PID controller and grey PID controller. In figure 10.2, $K_P = 1.275$, $K_D = 0.175$ and $K_I = 0.25$, which are proportional, derivative and integral controller constants, it is obvious that although the system has a big overshoot in conventional PID controller, grey PID controller can decrease the overshoot with its prediction ability. In Figure 10.3, $K_P = 1$,

$K_D = 0.1$ and $K_I = 0.1$, the speed and the overshoot of the conventional PID controller is not as successful as grey PID controller.

Figure 10.4 compares the unit step response of the model with a grey PID controller with different prediction horizons. As can be seen, a bigger step size of the grey controller will cause over compensation, resulting in a slow system response. Figure 10.4 also shows that when the prediction horizon is increased dramatically, the system will have an oscillatory characteristic.

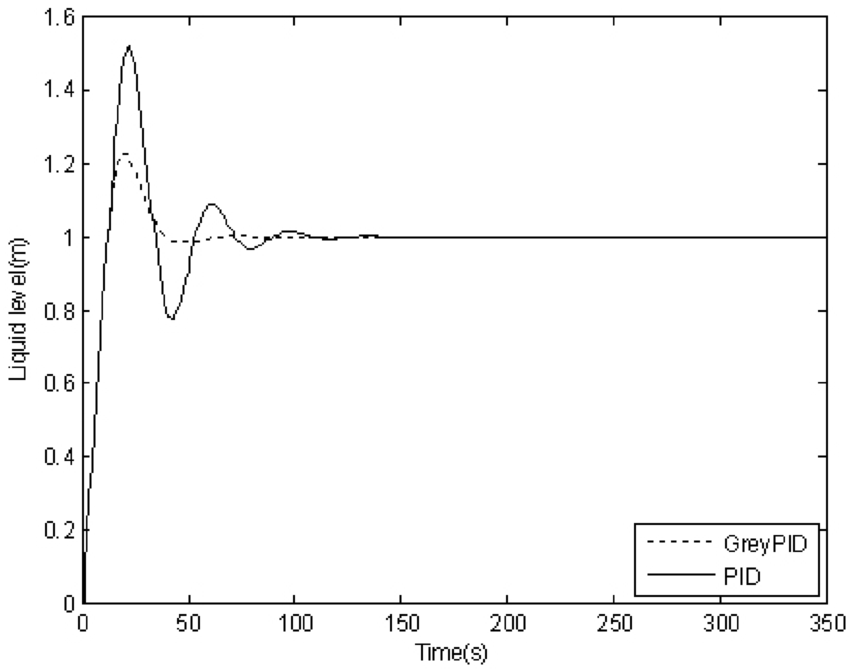

Figure 10.2. Step responses of the model to conventional PID and GreyPID controllers for $K_P = 1.275$, $K_D = 0.175$ and $K_I = 0.25$

Figure 10.2-10.4 show that when the prediction horizon is selected properly, control quality of a grey PID controller is always better than a conventional PID controller.

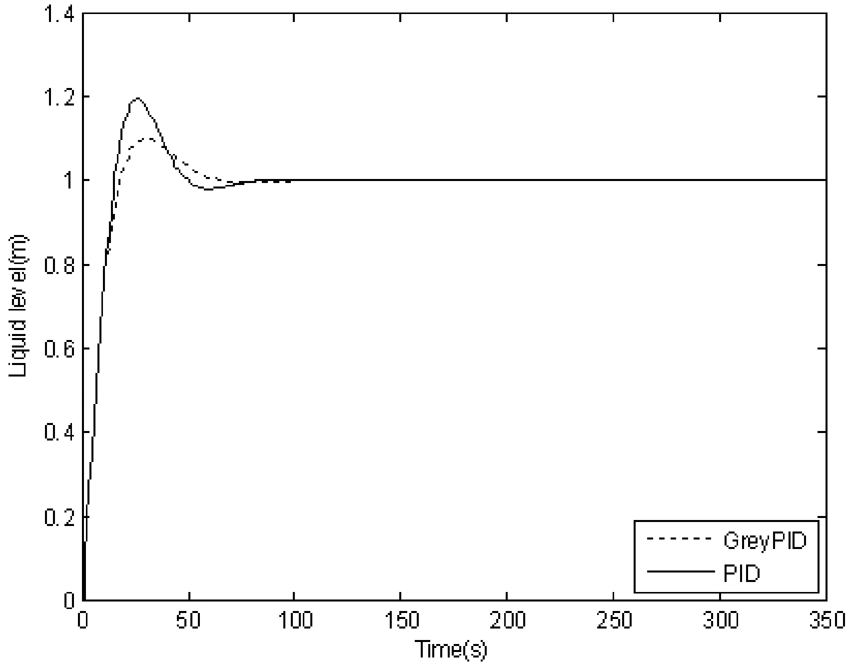

Figure 10.3. Step responses of the model to conventional PID and GreyPID controllers for $K_P = 1$, $K_D = 0.1$ and $K_I = 0.1$

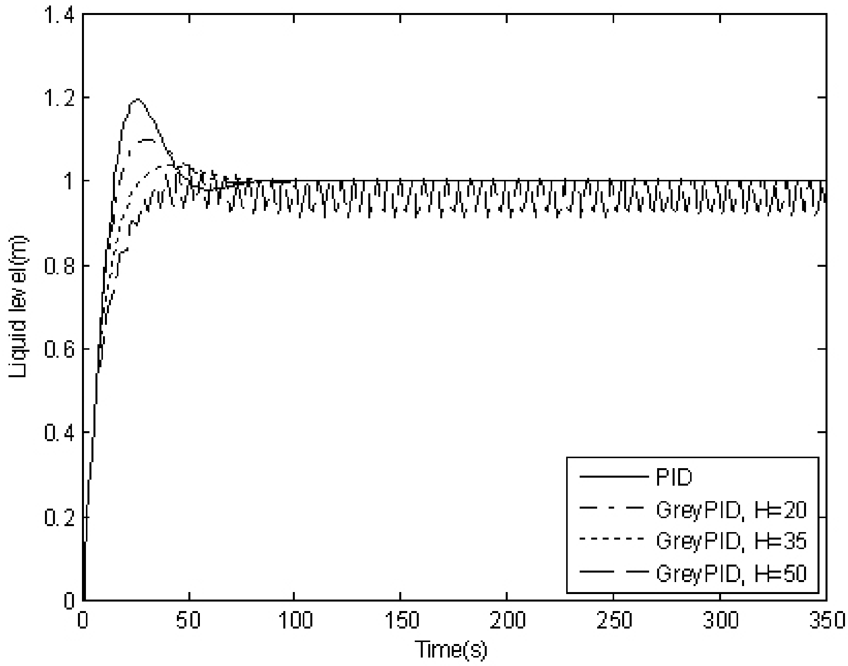

Figure 10.4. Step responses of the model to conventional PID and Grey PID controllers for different prediction horizons for $K_P = 1$, $K_D = 0.1$ and $K_I = 0.1$

10.2. Grey Fuzzy Control

Figure 10.5 shows the general structure of a grey PIDFC controller. Similar to grey PID control, during the development of grey PIFC, PDFC and PIDFC, the prediction error is considered as the error of the system. The numerical values are selected as $K = 1$ and $K_d = 0.1$ in the simulations.

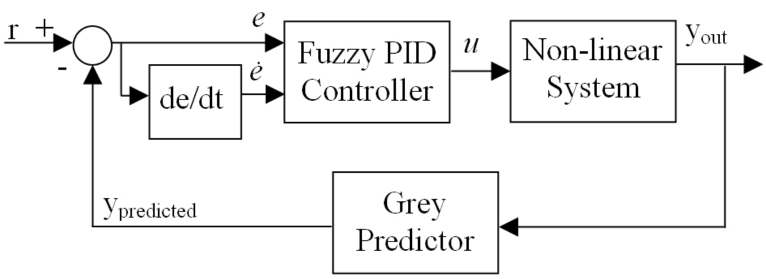

Figure 10.5. Grey PID type fuzzy controller structure

10.2.1. Grey Fuzzy PI Control

In Figure 10.6, although the performance of grey PIFC is better than PIFC, the system has a big overshoot and a slow response with both of the controllers.

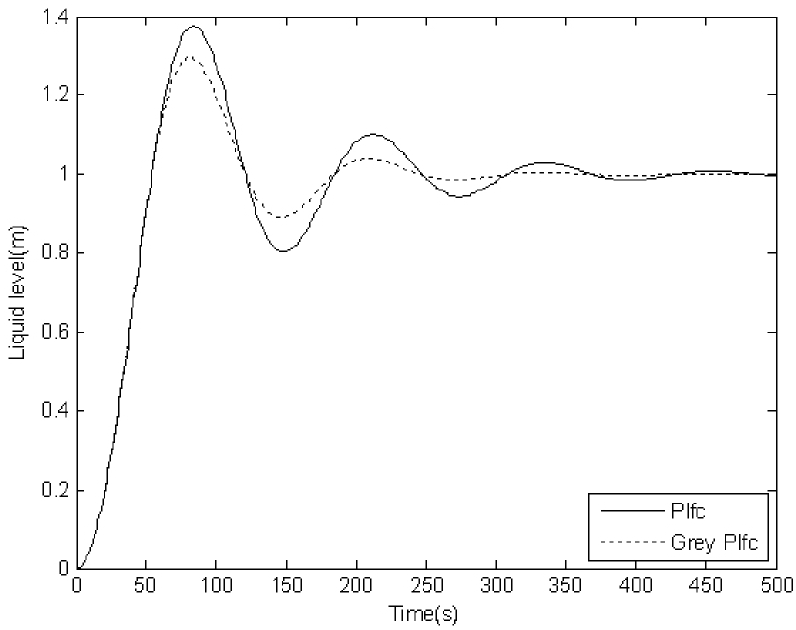

Figure 10.6. Step responses of the model to PIFC and Grey PIFC for $\alpha = 0$, $\beta = 0.1$

10.2.2. Grey Fuzzy PID Control

Figure 10.7 compares the unit step response of the system with a PIDFC and a grey controller with different prediction horizons. As can be seen, when the step size of the grey controller is large, it will cause over compensation, resulting in a slow system response. Conversely, a smaller step size will make the system respond faster but cause larger overshoots [29]. The response with H=20 is better than the one obtained with the fuzzy PID type controller. Further simulations, shown in Figure 10.8 and Figure 10.9 are carried out with this value of H to determine the best parameters of the Grey Controller. The response shown in Figure 10.9 has a fast rise time and reasonable overshoot.

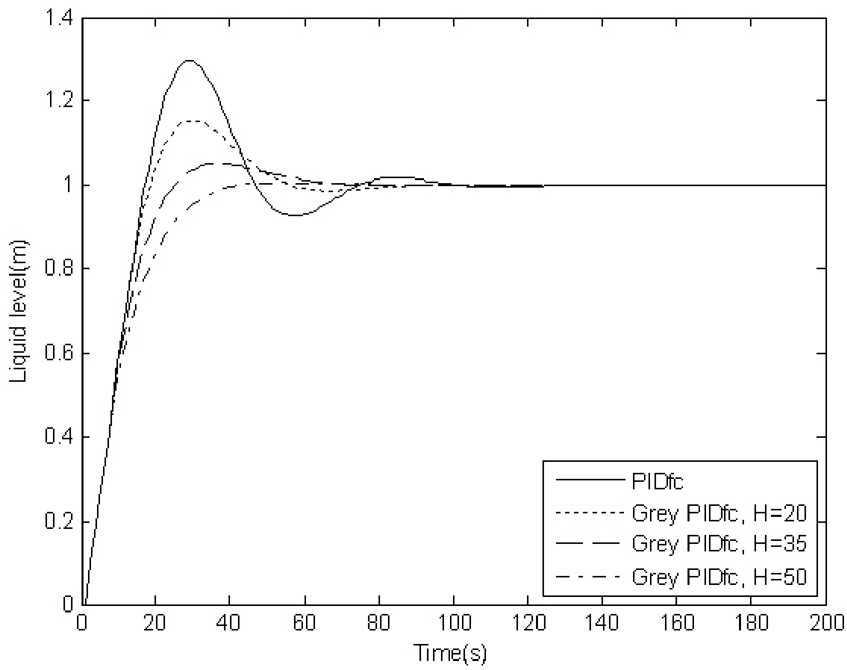

Figure 10.7. Step responses of the model to Grey PIDFC with different prediction horizons for $\alpha = 3$, $\beta = 0.5$

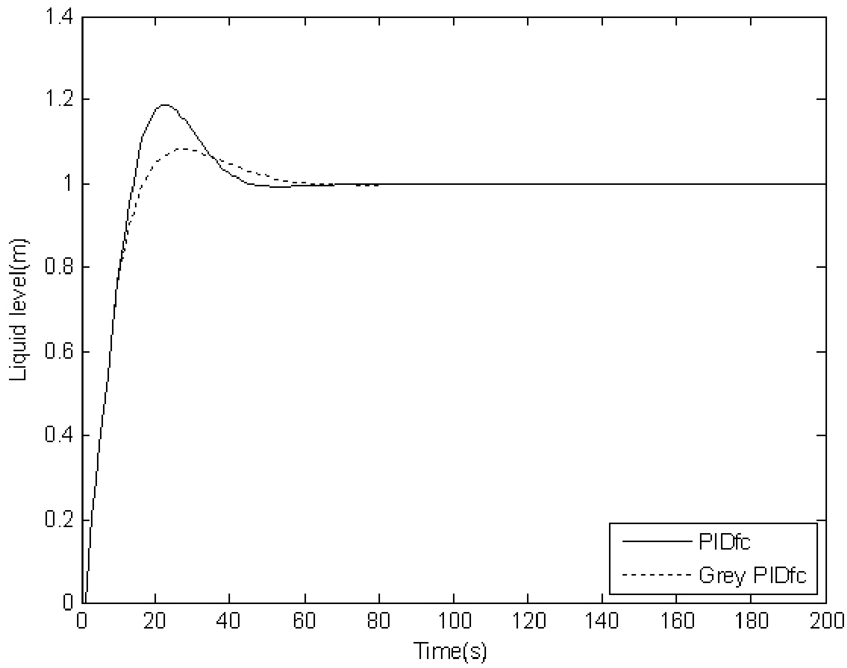

Figure 10.8. Step responses of the model to PIDFC for $\alpha = 6$, $\beta = 0.6$, $H = 20$

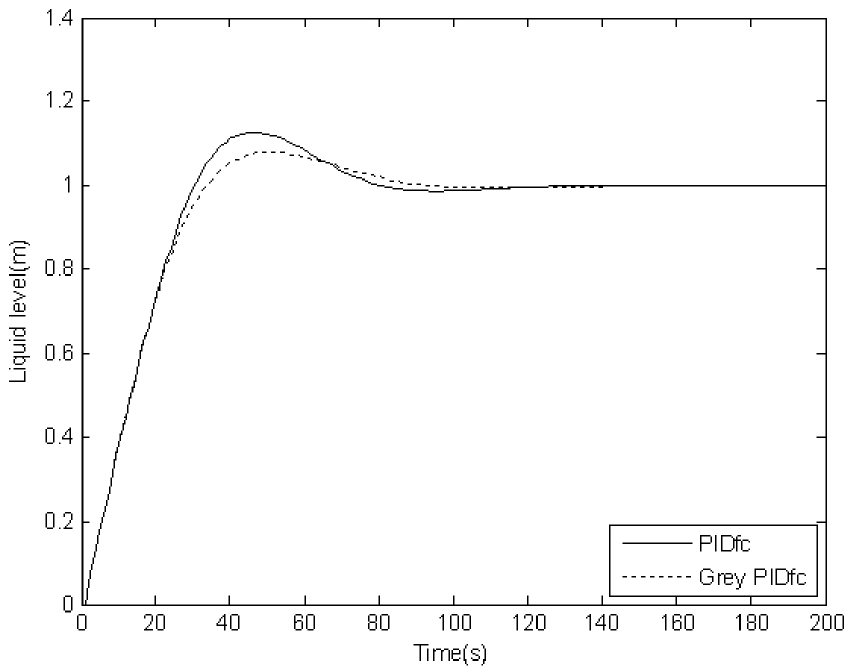

Figure 10.9. Step responses of the model to PIDFC for $\alpha = 2.4$, $\beta = 0.2$, $H = 20$

10.3. Noise Response of the System

Figure 10.10-10.12 show the unit step responses of the system to PID, grey PID, PIDFC and grey PIDFC with the band-limited white noise at the output measurement. The noise power, which is the height of the power spectral density of the white noise, is equal to 0.003. The correlation time of the noise is equal to 1 sec.

The three figures show that although noise response of PID control and fuzzy PID control is acceptable, the performance of grey controllers is always better than conventional controllers.

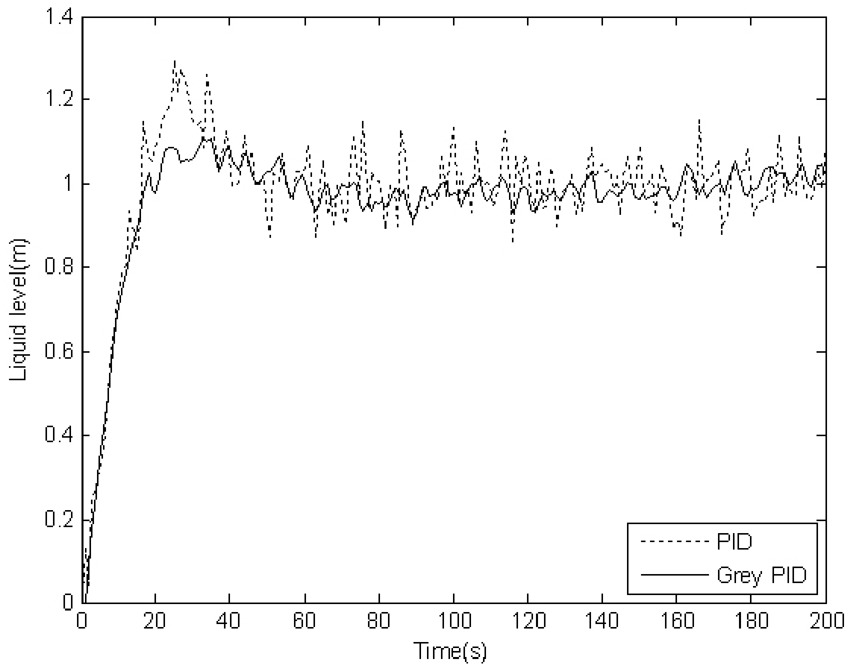

Figure 10.10. Grey PID controller with the band-limited white noise at the output measurement for $K_P = 1$, $K_D = 0.1$ and $K_I = 0.1$, $H = 20$

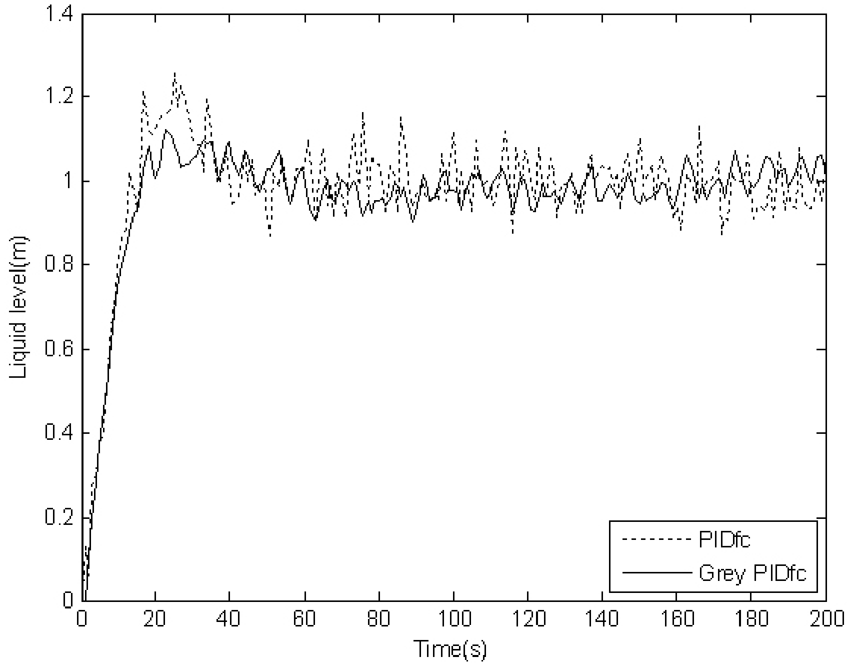

Figure 10.11. Grey PIDFC with the band-limited white noise at the output measurement for $\alpha = 6$, $\beta = 0.6$, $H = 20$

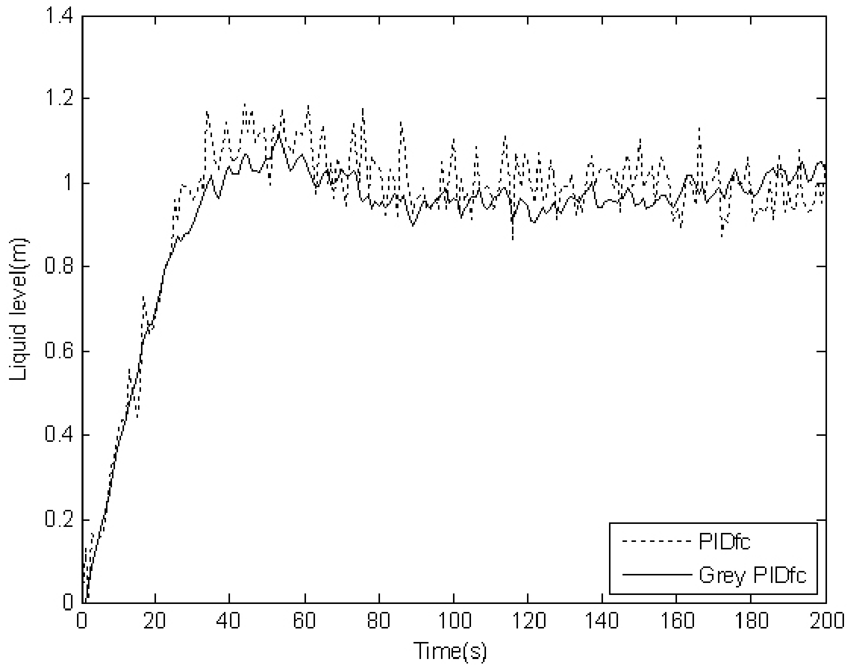

Figure 10.12. Grey PIDFC with the band-limited white noise at the output measurement for $\alpha = 2.4$, $\beta = 0.2$, $H = 20$

11. CONCLUSIONS

In this thesis, it is shown that grey prediction approach is an efficient way of controlling highly non-linear, uncertain systems. The controller described is a combination of grey prediction approach with a PID type fuzzy controller. The proposed grey predictor is based on the online prediction of outputs of a non-linear system using a first order grey model without the dynamic model of the system. The simulation results presented indicate that grey prediction model can forecast the future outputs of a grey system to be used to overcome the drawbacks met with conventional controllers. Additionally, the system controlled with a grey parameter estimator has a better noise response capability.

The model accuracy examination results show that GM(1,1) model is able to make accurate predictions for long-term forecasting of the monotonous type of processes. However, the model GM(1,1) cannot give the same performance when the primitive data sequence increases like as in an S curve or it has a saturation region. As a solution to these kinds of problems, the GM(1,1) model with error remedy or grey Verhulst model can be used. Because the output data sequence of non-linear liquid level control system used in this thesis does not have any saturation region, the use of GM(1,1) model results in good performance.

GM(1,1) model predicts the future values of a time sequence with less computation time when compared the other prediction methods, because it always uses small data set for its predictions. As a result, GM(1,1) model is successful on real time modeling and real time control of the industrial systems.

REFERENCES

1. Zadeh, L., "Fuzzy Sets", *Information and Control*, Vol. 8, 1965, pp. 338-353.

2. Mamdani, E. H., "Applications of fuzzy algorithms for control of a simple dynamic plant", *Proceedings of IEE*, Vol. 121,12,1974, pp. 1585-1588.

3. Lee, C. C., "Fuzzy Logic in Control Systems: Fuzzy Logic Controller Part I", *IEEE Transactions on Systems, Man and Cybernetics,* Vol. 20, No. 2, March/April 1990.

4. Park, S., *"Fuzzy Inference System in Hexor robot based on sonar radar"*, http://www.ee.pdx.edu/mperkows/class478-2005/robot-2005/seho-final-robotics.doc, 2006.

5. Ying, H., *Fuzzy Control and Modeling*, IEEE Press, 2000.

6. Ross, T., *Fuzzy Logic with Engineering Applications*, McGraw-Hill, 1995

7. Muzimoto, M., "Min-max-gravity method versus product-sum-gravity method for fuzzy controls", *Proceedings of IV IFSA Congress*, Brussels, Part E, 1991, pp. 127-130.

8. Takagi, T. and M. Sugeno, "Derivation of fuzzy control rules from human operator's control actions", *Proceedings of IFAC Conf. on Fuzzy Information*, Vol. 1, Marseille, 1983, pp. 55-60.

9. Takagi, T. and M. Sugeno, "Fuzzy identification of fuzzy systems and its application to modeling and control", *IEEE Trans. Systems Man Cybernet 15*, 1985, pp. 116-132.

10. Zhuang, P., H. Min and X. Wei, "Pad-analysis of stability of fuzzy control systems", *Fuzzy Sets and Systems*, Vol. 38, 1990, pp. 27-42.

11. Qiau, W. and M. Muzimoto, “PID type fuzzy controller and parameter adaptive method”, *Fuzzy Sets and Systems*, Vol. 78, 1995, pp. 23-35.

12. Woo, Z., H. Chung and J. Lin, “A PID type fuzzy controller with self tuning scaling factors”, *Fuzzy Sets and Systems*, Vol. 115, 2000, pp. 321-326.

13. Engin, S., J. Kuvulmaz and V. Omurlu, “Fuzzy control of an ANFIS model representing a nonlinear liquid-level system”, *Neural Computing and Applications* , Vol. 13, 2004, pp. 202-210.

14. Deng, J. L., “Control problems of grey system”, *Syst. and Contr. Lett.*, Vol. 1, 1982, pp. 288-294.

15. Liu, S. F. and Y. Lin, *An Introduction to grey systems*, PA, USA, IIGSS Academic Publisher, 1998.

16. Kayacan, E. and O. Kaynak, “Grey Prediction Based Control of a Non-Linear Liquid Level System Using PID Type Fuzzy Controller”, *IEEE International Conference on Mechatronics*, Budapest, 2006. (accepted for publication)

17. Lin, Y. and S. Liu, “A historical introduction to grey systems theory”, *Proceedings of IEEE Int. Conf. on Systems, Man and Cybernetics*, Vol. 1, The Netherlands, 2004, pp. 2403-2408.

18. Cheng, B. “The grey control on industrial process”, *Huangshi College J.*, Vol. 1, 1986, pp. 11-23.

19. Huang, S. J. and C. L. Huang, “Control of an inverted pendulum using grey prediction model”, *Proceedings of IEEE Conf. Ind. Applications*, Vol. 3, Denver, CO, 1994, pp. 1936-1941.

20. Huang, S. J. and C. F. Hu, “Predictive fuzzy controller for robotic motion control”, *Proceedings of IEEE Conf. Ind. Applications*, Vol. 2, Orlando, FL, 1995, pp. 1668-1672.

21. Huang, Y. P. and S. F. Wang, "Identifying the fuzzy grey prediction model by genetic algorithms", *Proceedings of IEEE Int. Conf. Evolutionary Comput.*, Nagoya, Japan, 1996, pp. 720-725.

22. Wong, C. C. and C. C. Chen, "Design of fuzzy control system with a switching grey prediction", *Proceedings of IEEE Int. Conf. Fuzzy Systems*, Vol. 1, Anchorage, AK, 1998, pp. 567-571.

23. Lin, K. H. and B. D. Liu, "Design of predictive fuzzy control system using GESA-based grey predictor", *J. Chinese Institute of Electrical Engineering*, Vol. 10, no 2, 2002, pp. 145-154.

24. Deng, J. L., "Introduction to grey system theory", *The Journal of Grey System*, Vol. 1, 1989, pp. 1-24.

25. Deng, J. L., "Five kinds of grey forecasting", *Fuzzy Math.*, Vol. 2, 1985, pp. 33-42.

26. Wen, K. L., *Grey Systems*, Tucson, USA, Yang's Scientific Press, Vol. 4, 2004.

27. Wen, K. L., T. C. Chang, F. H. Chen, and Y. W. Chyu, "The grey Verhulst model for population prediction in Taiwan-Fukien", *The Journal of Grey System*, Vol. 12, 2000, pp. 61-64.

28. D. F. Wang, P. Han, W. Han, and H. J. Liu, "Typical Grey Prediction Control Methods and Simulation Studies", *Second International Conference on Machine Learning and Cybernetics*, Vol. 12, 2000, pp. 513-518.

29. S. Y. Wang, "Design of a Linear-Like Fuzzy Grey Prediction Controller for a Three-Phase Induction Motor Speed Control System", *Proceeding of the 1998 Conference on Grey Theory and its Applications*, pp. 457-463, 1998.