THE IMPORTANCE OF EQUAL SIGN UNDERSTANDING IN 4TH AND 5TH GRADES: A MIXED METHOD STUDY

by

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Dedicated to the strongest woman I have ever known, my lovely mother, Aynur Şenkule
and to my awesome sister, Sıla Gözüm
for all their love and support

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ABSTRACT

THE IMPORTANCE OF EQUAL SIGN UNDERSTANDING IN 4TH AND 5TH GRADES: A MIXED METHOD STUDY

The aim of the current study was to investigate fourth and fifth grade students' equal sign conceptions in the light of the revised Turkish mathematics curriculum (MoNE, 2018) that includes newly added objectives related to the equal sign concept in fourth grade. The study was conducted in a private school in Istanbul, Turkey, during the spring semester of the 2018-2019 academic year. A total of 53 students-30 fourth grade and 23 fifth grade-participated in the study. Both the arithmetic achievement test and the equal sign test (EST) taken from Capraro *et al.* (2011) were used as instruments. After applying the tests, the semi-structured interviews were conducted to gain a deeper insight into the students' equal sign conceptions. Moreover, the fourth and fifth grade students' textbooks and other course materials were analyzed in terms of 13 standard and non-standard equal sign contextual presentations. According to the quantitative analysis results, the average score of fifth-grade students was 11.21 whereas the fourth-grade students' average score was 8.13 out of 13 in the EST. The analysis of the course materials displayed that standard contextual presentations of the equal sign, that are only helpful for teaching the operational view, have higher frequencies in fourth grade, whereas non-standard presentations are dramatically high in fifth grade. The results of the semi-structured interviews supported that both the fourth and fifth grade students have some misunderstandings of the relational meaning according to the error patterns found in the EST results. The educational implications, limitations, and recommendations for future studies were also discussed.

ÖZET

4. VE 5. SINIFLARDA EŞİTTİR İŞARETİNİ ANLAMANIN ÖNEMİ: KARMA YÖNTEM ÇALIŞMASI

Bu çalışmanın amacı, eşittir işareti kavramına ilişkin müfredata yeni eklenen, eşittir işaretini tüm açılarıyla anlamayı hedefleyen kazanımlar ışığında (MEB, 2018), dördüncü ve beşinci sınıf öğrencilerinin eşittir işareti kavramını anlayışlarını incelemektir. Araştırma, 2018-2019 eğitim-öğretim yılının bahar döneminde İstanbul, Türkiye'de özel bir okulda gerçekleştirilmiştir. Çalışmaya 30 dördüncü sınıf ve 23 beşinci sınıf olmak üzere toplam 53 öğrenci katılmıştır. Aritmetik başarı testi ve eşittir işareti testi (EST) (Capraro vd., 2011) bu çalışmada ölçme aracı olarak kullanılmıştır. Testler uygulandıktan sonra, yarı yapılandırılmış görüşmeler, öğrencilerin eşittir işareti kavramları hakkında daha derin bir fikir edinmek için yapılmıştır. Ayrıca, dördüncü ve beşinci sınıf öğrencilerinin ders kitapları ve diğer ders materyalleri, eşittir işaretinin 13 farklı standart ve standart olmayan farklı sunumları açısından analiz edilmiştir. Nicel analiz sonuçlarına göre, eşittir işareti testinde öğrencilerin ortalama puanı 13 üzerinden dördüncü sınıflar için 8.13, beşinci sınıflar için 11.21 olarak hesaplanmıştır. Ders materyallerinin analizi, ilişkisel görüşü öğretmeye yardımcı olmayan eşittir işaretinin farklı standart sunumlarının dördüncü sınıfta daha yüksek orana sahip olduğunu, standart olmayan sunumların ise beşinci sınıfta çarpıcı biçimde yüksek olduğunu göstermiştir. Yarı yapılandırılmış görüşmelerin sonuçları, eşittir işareti testi sonuçlarında bulunan hata kalıplarına göre hem dördüncü hem de beşinci sınıf öğrencilerinin ilişkisel anlamı yanlış anladıklarını desteklemektedir. Çalışmada gelecekteki çalışmalar için eğitimsel çıkarımlar, kısıtlamalar ve öneriler de tartışılmıştır.

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LIST OF ACRONYMS/ABBREVIATIONS

EST	Equal Sign Test
NS	Non-Standard Presentation of the Equal Sign
S	Standard Presentation of the Equal Sign
TA	Thematic Analysis

1. INTRODUCTION

Every student has encountered with the equal sign in almost all mathematics classes in various topics as early as kindergarten. The equal sign concept and how students evaluate and understand the equal sign have been studied for years by many researchers and educators for the sake of revealing students' understanding of the equal sign (Alibali *et al.* 2007; Baroody & Ginsburg, 1983; Behr *et al.*, 1976; Capraro *et al.*, 2011; McNeil, 2007; Li *et al.*, 2008; Renwick, 1932) and its effects on students' pre-algebra learning (Byrd *et al.*, 2015; Carpenter *et al.*, 2003; Knuth *et al.*, 2006; McNeil *et al.*, 2006).

Most of the studies have shown that students' understanding of the equal sign concept is quite shallow as they mostly focus on the operational meaning of the equal sign and they usually recognize it only as a mathematical symbol or a punctuation mark rather than a representation of the relation between two expressions that are equivalent (Baroody & Ginsburg, 1983; Renwick, 1932). However, the equal sign is more than an operator in mathematical equations as it has relational and balance meanings. Without the relational meaning of the equal sign (i.e., balanced on both sides of the equal sign, or both sides are equivalent), students would be struggling to memorize some mathematical equation rules rather than understanding the real meaning behind algebra (Jacobs *et al.*, 2007; Seo & Ginsburg, 2003).

In the literature, there are some studies mentioning the effects of understanding the equal sign concept on students' pre-algebra learning. For instance, Alibali *et al.* (2007) argued that it was very important to emphasize both the operational and relational meanings of the equal sign as it affected students' success in pre-algebra learning. Moreover, Knuth *et al.* (2006) found a positive correlation between students' understanding of the equal sign concept and their success in solving algebraic expressions. When students were exposed to deal with arithmetic-specific interpretations (i.e., add-up the numbers) rather than non-relational interpretations (i.e., the answer, the result) of the equal sign, it was observed that it hindered students' pre-algebra learning (McNeil & Alibali, 2005; Byrd *et al.*, 2015). Byrd *et al.* (2015) concluded that arithmetic-specific views of the equal sign might cause students to focus only on arithmetic operations in an equation without thinking about the position of the equal sign. Also, when students were exposed to arithmetic-specific views of the equal sign constantly, it might cause resistance for students to understand the relational view.

On the other hand, how to teach the equal sign concept has been a controversial topic for many researchers. Over the years, several curriculum modifications have been studied and the results have shown that the curricula with the objectives for teaching the equal sign concept within the relational view could be more helpful for students' understanding of the equal sign concept (Baroody & Ginsburg, 1983; Carpenter *et al.*, 2003; McNeil, 2008; Hattikudur & Alibali, 2010; McNeil *et al.*, 2011; Molina, *et al.*, 2009). For instance, Baroody and Ginsburg (1983) studied a mathematics curriculum developed by Wynroth (1975) to see the effects of curriculum change related to the equal sign concept. They argued that the Wynroth mathematics curriculum design was fairly successful to improve students' relational understanding of the equal sign since it focused on the relational meaning of the equal sign rather than emphasizing the operational meaning.

The equal sign literature that supports curriculum modifications to teach the equal sign concept looks like affecting some changes in the Turkish mathematics curriculum as well. In the previous Turkish mathematics curriculum (2013), there was only one objective to emphasize the equal sign concept in 7th grade (M.7.2.2.1) (MoNE, 2013). With the revision in the Turkish mathematics curriculum in 2018, three new objectives-one in second grade and two in fourth grade-were added to the curriculum (see Table 1.1). The objective in seventh grade was reworded keeping the equal sign concept.

Table 1.1. Curriculum objectives related to the equal sign concept.

	The Previous Turkish Mathematics Curriculum (MoNE, 2013)	The Revised Turkish Mathematics Curriculum (MoNE, 2018)
Second Grade	NA	M.2.1.3.5. Students will be able to realize the balance meaning of the equal sign between two mathematical equations[1]
Fourth Grade	NA	M.4.1.5.7. Students will be able to find the missing value in one of two mathematical equations and explain how the equality is established between them. (e.g., 8 + _ = 15 - 3, 12 : 4 = _ + 1, and 6 x _ = 48 - 12)[2] M.4.1.5.8. Students will be able to describe the operations that must be done to establish equality between the two mathematical expressions that are not equal. (e.g., Explain how to make two sides of the equation 8 + 5
Seventh Grade	M.7.2.2.1. Students will be able to understand the principle of conservation of equality in mathematical equations.[4]	12 - 3 equal to each other.)[3] M.7.2.2.1. Students will be able to understand the principle of conservation of equality.[5]

[1]M.2.1.3.5. Eşit işaretinin matematiksel ifadeler arasındaki ”eşitlik” anlamını fark eder. Eşit işaretinin her zaman işlem sonucu anlamı taşımadığı, eşitliğin iki tarafındaki matematiksel ifadelerin denge durumunu da (eşitliğini) gösterdiği vurgulanır. Örneğin 5+6=10+1; 15-3= 18-6; 8+7 = 20-5; 18= 16+2 (MoNE, 2018).

[2]M. 4.1.5.7. Aralarında eşitlik durumu olan iki matematiksel ifadeden birinde verilmeyen değeri belirler ve eşitliğin sağlandığını açıklar. Örneğin; 8 + _ = 15 - 3, 12 : 4 = _ + 1, 6 x _ = 48 - 12 (MoNE, 2018).

[3]M.4.1.5.8. Aralarında eşitlik durumu olmayan iki matematiksel ifadenin eşit olması için yapılması gereken işlemleri açıklar. Örneğin 8+5 $\neq$ 12-3 ifadesinde eşitlik durumunun sağlanabilmesi için yapılabilecek işlemler üzerinde durulur (MoNE, 2018)

[4]M.7.2.1.2. Denklemlerde eşitliğin korunumu ilkesini anlar (MoNE, 2013).

[5]M.7.2.2.1. Eşitliğin korunumu ilkesini anlar (MoNE, 2018).

Not only the curriculum but also the course materials and textbooks should be taken into consideration while teaching the equal sign concept. Some studies in the literature indicated that teachers' choices of textbooks and other course materials affected both the ways of teaching the topics and students' learning processes (Malzahn, 2002; Reys *et al.*, 2004; Li *et al.*, 2008). Tornroos (2005) also claimed that even a partial analysis of textbooks could provide valuable data to evaluate and explain students' mathematics achievement. Capraro *et al.* (2011); however, took it to the next level and analyzed four countries textbooks within a broader perspective by coding second and sixth grades textbooks page by page in terms of various equal sign presentations. They focused on the relation between students' achievements on the equal sign test (EST) and different equal sign usage in the textbooks that students have mostly been exposed to. They claimed that the analysis of textbooks could be helpful in determining how textbooks have affected students' achievements. To that extent, in the current study, the textbooks and other lecture materials that fourth and fifth grade students have used were analyzed page by page in order to reveal whether there was a connection between the course materials and students' understanding of the equal sign as reflected by their performances on the EST.

1.1. Purpose of the Study

The current study has a twofold purpose: (1) to investigate fourth and fifth grade students' understanding of the equal sign concept within the various standard (S) and non-standard (NS) contexts, and (2) to analyze the textbooks and other lecture materials that students were exposed to. In order to accomplish those purposes, the EST was conducted with fourth and fifth grade students, the course materials were analyzed in terms of various equal sign usage, and semi-structured interviews were conducted with some of the students to reveal students' understandings of the equal sign.

1.2. Significance of the Study

In the literature, there are several studies that provide the framework for the current study design as they mostly focused on students' conceptions of the equal sign in different grade levels (Baroody & Ginsburg, 1983; McNeil, 2007; Alibali *et al.*, 2007; Li *et al.*, 2008; Capraro *et al.*, 2011). Baroody and Ginsburg (1983) worked with students from grades 1 to 3, whereas McNeil (2007) from grades 1 to 4 in order to understand elementary school students' conceptions of the equal sign in terms of the effects of instruction and equivalence problems, respectively. Alibali *et al.* (2007), on the other hand, studied middle school students? (from grade 6 to 8) understandings of the equal sign. Li *et al.* (2008) preferred to focus only on sixth graders' equal sign perceptions with the help of comparing Chinese and American students. Likewise, Capraro *et al.* (2011) conducted a study to compare both grade 2 and grade 6 students' equal sign understandings from four different countries, Turkey, China, Korea, and the USA. They developed instruments which are the arithmetic achievement test and the equal sign test (EST) to reveal students' error patterns of equal sign conceptions.

It is also important to mention the difference between the terms 'error pattern' and 'misconception'. Ashlock (2010) explains error patterns as systematic procedures that students apply in arithmetic operations, but mostly not providing the correct answer. Even if error patterns sometimes produce the correct answer by chance, students assume that they have learned the correct procedure. In the long term, those error patterns turn to misconceptions for students. Although the studies in the literature have used the term 'misconception', using the term 'error patterns' would be more appropriate for the current study because the instruments used in this study were not sufficient to diagnose students' misconceptions, nor were they intended to measure it.

While looking at the aforementioned studies, it can be seen that fourth and fifth grade students have never been studied within a single study in terms of their equal sign conceptions within standard and non-standard contextual presentations. Also, in the last national mathematics curriculum (MoNE, 2018), there are two new objectives directly related to the understanding of the equal sign concept added to the fourth-grade mathematics content (see Table 1.1). When some objectives are added to the curriculum, they should also be involved in the textbooks because the content of textbooks should meet all the objectives of the relevant curriculum (The Board of Education [Talim ve Terbiye Kurulu, TTKB], 2019). When they take place in the textbooks, it is expected that they will be discussed and taught in class as well. Thus, according to these newly added objectives to the fourth grade, it can be said that fourth graders have become familiar with the equal sign concept. Therefore, insights from fourth grade students could give meaningful inferences about students' understanding of the equal sign concept.

It is also important to mention that this revised curriculum (MoNE, 2018) has been applied in all grade levels simultaneously, not gradually. Thus, according to the previous mathematics curriculum (MoNE, 2013), the fifth graders have never encountered any objectives related to the equal sign concept when they were in fourth grade, and so their textbooks did not involve any objectives about the equal sign directly. Therefore, insights from fifth grade students could also give meaningful inferences about the equal sign. In short, since 4th and 5th grades follow each other, using fourth and fifth grade samples in the current study may allow further research to build a significant theoretical framework about students' conceptions of the equal sign.

On the other hand, it is important to examine how the equal sign is presented to students in the textbooks and other lecture materials in order to understand how they influence students' understanding of the equal sign concept (Capraro *et al.*, 2011; McNeil *et al.*, 2006; Seo & Ginsburg, 2003; Tornroos, 2005). Thus, analyses of textbooks and other lecture materials were conducted in the current study. Furthermore, semi-structured interviews with some students were conducted after applying the equal sign test (EST) to make deeper inferences about students' way of thinking about the equal sign concept while answering the EST items.

In the current study, students' conceptions of the equal sign were examined on a basis of the fourth and fifth grade students' achievements in the equal sign test (EST), and the textbooks and other lecture materials that they use in mathematics lessons.

The research questions are as follows:

- What are the fourth and fifth grade students' error patterns of the equal sign concept in the light of their equal sign test (EST) performances and semi-structured interviews?
- What different meanings and contextual presentations of the equal sign do the revised fourth and fifth grade mathematics curricula (2018), textbooks and other lecture materials include?

2. LITERATURE REVIEW

In this chapter, the literature related to the equal sign concept and students' error patterns and misconceptions about the equal sign will be reviewed. It aims to provide a substantial basis for the instrument by focusing on essential elements of the equal sign concept and students' misconceptions about the equal sign. Previously conducted studies on the equal sign concept and instruments that were constructed to measure students' conceptions of the equal sign will be discussed. It consists of three main sections: i) the equal sign concept, ii) the equal sign error patterns and misconceptions, and iii) textbooks and other lecture materials.

2.1. The Equal Sign Concept

Over many years, the equal sign concept and students' conceptions of the equal sign have been studied. Some studies have already displayed that comprehending the meaning of the equal sign has been very challenging for students, and most of the time students failed to understand the equal sign concept (Alibali *et al.*, 2007; Baroody & Ginsburg, 1983; Behr *et al.*, 1976; Capraro *et al.*, 2011; Li *et al.*, 2008; McNeil, 2007; Renwick, 1932).

Kieran (1981) explained the meaning of the equal sign as one of the relational symbols meaning that two sides are not only equal but also exchangeable. However, most students are prone to comprehend the equal sign as an operator, that means giving the results of arithmetic-specific operations (Alibali, 2005; Baroody & Ginsburg, 1983; Behr *et al.*, 1976; Capraro *et al.*, 2011; Kieran, 1981; McNeil & Byrd *et al.*, 2015). As long as students think that they use the equal sign only to run arithmetic operations, they fail to understand relational and balance purposes of the equal sign. Therefore, they misinterpret some non-standard (NS) contexts (i.e., $\_ + 8 = 7 + 5$; $8 + 4 = \_ + 5$) and fail to understand that one side of an equation is related to the other side, in fact, both sides give the same result to preserve the balance (Capraro *et al.*, 2011).

On the other hand, Behr *et al.* (1976) mentioned three different properties of an equation-reflexive, symmetric, and transitive properties-that students are hoped to exhibit in order to comprehend non-standard equality sentences properly. Baroody and Ginsburg (1983) also suggested that emphasizing three fundamental properties of the equal sign was important to establish a relational understanding because students mostly expected that an arithmetic equation consists of two numbers on the left side, the result on the right side, and in between, the equal sign as an operational symbol like in standard forms. Firstly, students should accept that $12 = 5 + 7$ and $5 + 7 = 12$ are the same-the property of symmetry. Secondly, they should accept identity statements-the reflexive property-such as $18 = 18$. Lastly, if students accept that $3 + 8 = 11$ as well as $11 = \mathrm{XI}$, then they could deduce that $3 + 8 = \mathrm{XI}$, which is the transitive property. In order to support students' relational understanding of the equal sign concept, being exposed to arithmetic problems in non-standard forms such as $5 = 5$, $9 = 4 + 5$, and $3 + 6 = 7 + 2$, instead of in standard forms like $8 + 4 = 12$, is crucial for supporting students' understanding of the equal sign concept (Baroody & Ginsburg, 1983).

Several studies suggested that students might benefit from encountering non-standard equal sign presentations for improving their equal sign understanding (Baroody & Ginsburg, 1983; Denmark *et al.*, 1976; McNeil *et al.*, 2011; Seo & Ginsburg, 2003). Since most students have a cognitive barrier for comprehending the relational meaning of the equal sign, it is important to emphasize non-standard presentations which seem harder for students such as $6 + 3 = 4 + \_$ and $\_ = 6 + 6$ (Baroody & Ginsburg, 1983). Denmark *et al.* (1976) assumed that providing the forms of equality as $a = b$, $a = b \cdot c$, and $a \cdot b = c \cdot d$ (i.e. non-standard forms) helps students construct the relational view of the equal sign rather than operational. Also, McNeil *et al.* (2011) supported this idea and concluded that using non-standard contextual presentations of the equal sign had very useful impacts on students' mathematics achievements.

2.2. The Equal Sign Error Patterns and Misconceptions

The equal sign is used in several different ways in arithmetic equations such as an answer sign, as a sign emphasizing the equivalence of two mathematical expressions, and as a symbol used in solving algebraic equations. In mathematics, the equal sign is presented at almost all levels; however, many students at different grade levels do not have enough understanding of the equal sign concept (Behr *et al.*, 1976; Baroody & Ginsburg, 1983; Byrd *et al.*, 2015; Jacobs *et al.*, 2007; Renwick, 1932; Seo & Ginsburg, 2003).

Knuth *et al.* (2006) indicated that standard contexts made students see the equal sign as an operator, whereas non-standard contexts supported the relational and balance meanings of the equal sign. In the same manner, McNeil *et al.* (2006) claimed that non-standard presentations of the equal sign were more effective than standard presentations in terms of developing middle school students' relational understandings of the equal sign concept. Behr *et al.* (1976) observed that most of the first to sixth grade students could solve correctly the equations given with a particular form such as $3 + 5 = 8$ (i.e., standard presentations) whereas they struggled with equations like $15 = 8 + 7$, $3 + 5 = 7 + 1$, and $15 = 15$ (i.e., non-standard presentations). Based on their interviews with students, Behr *et al.* (1976) concluded that students did not view the equal sign as relational but as a sign to carry out mathematical operations from left to right only to find the answers. This limited conception of the equal sign might affect students' learning of further mathematical concepts, especially in pre-algebra learning (Behr *et al.*, 1976).

Some studies in the literature have revealed that misconceptions about the equal sign have an impact on students' learning of algebra in further grades (Byrd *et al.*, 2015; Carpenter *et al.*, 2003; Knuth *et al.*, 2006, 2008; McNeil *et al.*, 2006; Renwick, 1932; Rittle-Johnson & Alibali, 1999). According to Renwick (1932), students' early arithmetic training and misinterpretations in ages 10 to 12 prevented understanding a mathematical equation as a unity and led to comprehending the equal sign as only a sign that gives the answer. Therefore, it caused a misconception about the meaning of the equal sign and affected students' understanding of pre-algebra. Without a proper understanding of the equal sign concept, it is difficult to make sense of algebraic expressions in further grades (Knuth *et al.*, 2008).

On the other hand, Kieran (1981) described the intuitive behavior of students' equality concept. She claimed that before entering school, students have already developed an intuitive understanding of the equal sign as an operator. When students entered the school, they brought those preconceptions about equality that influence their perceptions of arithmetic operations with them, and later it affected their understanding of algebra by turning to entrenched misconceptions. Likewise, Baroody and Ginsburg (1983) supported this idea and argued that external factors-family and media at home, teachers and textbooks in school-had an important impact on students' perspectives in terms of emphasizing the operational view of the equal sign rather than relational. Byrd *et al.* (2015) suggested that it might be important to assess the equal sign concept in earlier grades to prevent those preconceptions from turning to misconceptions in further grades.

Another misinterpretation of the equal sign concept emerged from using the equal sign in an equality string (i.e., $3 + 6 = 9 + 2 = 11 + 5 = 16$) (Knuth *et al.*, 2006). Renwick (1932) claimed that as long as children comprehend the equal sign as an operator rather than a bridging symbol, they keep losing the real meaning of the equal sign and seeing it as a distinctive symbol, that is a widespread misconception even among educated adults (e.g., $(44\text{-}30)^2 = 14^2 = 196 \text{ x } 2 = 392$). Capraro *et al.* (2011) explained this kind of usage of the equal sign with the term running equal sign which creates a misconception in students' minds that shows the equal sign only as an operator and overshadows the balance meaning. Therefore, this kind of interpretation of the equal sign should be avoided to use in order to support the relational meaning of the equal sign (Capraro *et al.*, 2011).

In general, students' understanding of the equal sign takes place in two basic categories which are operational and relational purposes (McNeil *et al.*, 2006). Mostly, students fail to recognize the relational use of the equal sign because they do not see it as a sign expressing both sides having the same value. Thus, it affects students' future achievement in algebra (Byrd *et al.*, 2015; Carpenter *et al.*, 2003; Kieran, 1981; Knuth *et al.*, 2006, 2008; McNeil *et al.*, 2006; Renwick, 1932; Rittle-Johnson & Alibali, 1999).

2.3. Textbooks and Other Lecture Materials

Several curriculum modifications have been studied over the years and the results have shown that curriculum revisions with the objectives about the equal sign concept could be beneficial for improving students' understandings of the equal sign (Baroody & Ginsburg, 1983; Carpenter *et al.*, 2003; Hattikudur & Alibali, 2010; McNeil, 2008; McNeil *et al.*, 2011; Molina *et al.*, 2009). For instance, for instance, Baroody and Ginsburg (1983) investigated a mathematics curriculum developed by Wynroth (1975) to see the effects of curriculum change related to the equal sign concept. They argued that the Wynroth mathematics curriculum design was fairly successful to improve students' relational understanding of the equal sign since it focused on the relational meaning of the equal sign rather than emphasizing the operational meaning.

While considering the curriculum revisions, course materials and textbooks should also be taken into consideration while teaching the equal sign concept because some studies have shown that most of the teachers rely heavily on textbooks and other supporting materials in their lectures (Malzahn, 2002; Reys *et al.*, 2004). Teachers' choices of textbooks mostly affect what to teach, how to teach, and how students learn the content (Li *et al.*, 2008). Therefore, even a partial analysis of textbooks could provide valuable data to evaluate and explain students' mathematics achievement (Tornroos, 2005).

Various presentations of the equal sign concept in textbooks have an impact on constructing students' equal sign understandings as well as misinterpretations (McNeil *et al.*, 2006; Seo & Ginsburg, 2003). Seo and Ginsburg (2003) examined how the equal sign was presented in elementary textbooks and other lecture materials and found that standard presentations of the equal sign (i.e., a + b = c, a - b = c etc.) were highly used in textbooks, unlike the non-standard presentations (i.e., a = a, b + _ = _ + b, _ =

c + d etc.). They concluded that this limited presentation of the equal sign concept in textbooks did not support students' relational understanding of the equal sign; on the contrary, it led students to create misunderstanding of the equal sign as an operator only. Additionally, McNeil *et al.* (2006) examined four middle school textbooks in terms of equal sign usage. They found that there were also limited presentations of the non-standard usage of the equal sign, especially equations with operations on both sides. Thus, they concluded that middle school students still had misunderstandings of the equal sign due to the limited equal sign presentations displayed in textbooks.

Capraro *et al.* (2011) took the aforementioned studies to the next level and focused on eleven different standard and non-standard presentations of the equal sign in four different countries' textbooks in order to reveal how textbooks have an impact on the improvement of students' conceptual understandings within a broader perspective. They claimed that the analysis of textbooks could be helpful for determining the impact of different equal sign presentations on students' achievements, and they found that using more non-standard presentations in textbooks have supported students' performances according to the equal sign test results. Thus, in the current study, on a similar vein, the textbooks and other lecture materials that the fourth and fifth grade students have used were analyzed page by page in terms of different equal sign presentations in order to gain deeper insight into students' achievements and failures on the equal sign concept.

To sum up, according to the literature, the studies investigating the equal sign concept have revealed that students failed to understand the relational meaning of the equal sign due to being exposed mostly to the operational view (Alibali *et al.*, 2007; Baroody & Ginsburg, 1983; Behr *et al.*, 1976; Byrd *et al.*, 2015; Capraro *et al.*, 2011; Kieran, 1981; Li *et al.*, 2008; McNeil & Alibali, 2005; McNeil, 2007; Renwick, 1932). In the long term, this situation has created misconceptions in students' minds and made students fail to learn algebra in further grades (Behr *et al.*, 1976; Byrd *et al.*, 2015; Carpenter *et al.*, 2003; Knuth *et al.*, 2006, 2008; McNeil *et al.*, 2006; Renwick, 1932; Rittle-Johnson & Alibali, 1999). In order to prevent students to fail in making sense of the equal sign as well as algebra, it is suggested that textbooks and other lecture materials should include more non-standard presentations of the equal sign and less standard ones that imply the operational meaning (Capraro *et al.*, 2011; McNeil *et al.*, 2006; Seo & Ginsburg, 2003; Tornroos, 2005).

3. METHOD

The current study is a mixed-method study, the QUAN-Qual model, which is also known as explanatory sequential mixed methods design. In this model, the quantitative part of the study leads to the qualitative part as the researcher collects quantitative data first, then according to the results s/he plans the qualitative part (Creswell, 2014; Gay, Mills, & Airasian, 2009). That is why this design is well-suited for the current study as this study focus on understanding students' equal sign conceptions by collecting further data through the qualitative part.

The current study has been carried out within two phases in the spring semester of the 2018-2019 academic year. As the quantitative part of the current study, students took the arithmetic achievement test and the equal sign test (EST). The arithmetic achievement test included only two questions given at the beginning of the EST in order to check students' performances on arithmetic operations. If the students failed in those questions, their EST results were taken out of the data. As the qualitative part of the study, after applying the test, the semi-structured interviews were conducted with some selected students to gain a deeper understanding of students' equal sign conceptions. Textbooks and other lecture materials that were used in their mathematics classes (i.e. the booklets prepared by their teachers at school, Sadik Uygun Workbook, Pearson International Mathematics 1 Book) were examined page by page in accordance with various equal sign contextual presentations.

3.1. Participants

Participants of this study are chosen according to convenience sampling. The main reason behind using this sampling approach is that it allows easy access to the participants (Gay *et al.*, 2009). Also, the participants are chosen from the fourth and fifth grades in order to examine their different experiences about the equal sign concept related to the revised Turkish mathematics curriculum. The Turkish mathematics curriculum was revised in 2018 and began to be implemented in the 2018-2019 academic year. Based on the revision, the fourth graders were exposed to newly added equal sign objectives (see Table 1.1) while the fifth graders were not exposed to any objectives about the equal sign concept in either fourth or fifth grade.

There were two fourth grade and one fifth grade classrooms in the school where the study was conducted. Two fourth grade classrooms (n = 30) and one fifth grade classroom (n = 23) were selected. The researcher was the mathematics teacher of both grades in that year. The school was a private school in Göktürk district of Istanbul. All of the students' mother tongue was Turkish except three students: one Russian (4th grade), Arabic (5th grade), and English (5th grade). The socioeconomic status (SES) of the participants was above average. Besides the national mathematics curriculum, the school applies the International Baccalaureate Program (IB): Primary Years Program (PYP) (K-4) and Middle Years Program (MYP) (Grade 5-8). Also, in grade 5, mathematics lessons are taught in English; thus, their course materials and textbooks are in English. It is expected that there will be no significant difference between the fourth and fifth graders' perceptions about the test items, since the EST items were language-free, and the arithmetic achievement test and the EST explanations were given in Turkish. In the fifth-grade classroom, the explanations of the EST were made in English verbally as well.

As the second phase of the study, semi-structured interviews have been conducted with seven fourth grade students and five fifth grade students who were chosen according to their EST scores and their responses to the EST items. Some of their responses were unexpected such as adding all numbers up that they saw in an equation and writing a number's divisor as its equal. Also, as they were the students of the researcher, some interviewees who were performing below the average at math classes but had higher scores in the EST and some of them who were performing above the average at math classes but had lower scores in the EST were specifically chosen for the interviews. There were also some students who were performing below or above the average on both math classes and the EST chosen for the interviews. Therefore, it can be said that they were purposefully selected. Information about the interview participants is presented in Table F. Also, instead of using their original names, pseudonyms are given to the selected participants in order to provide confidentiality.

Table 3.1. The teacher's descriptions of interviewees and the EST questions asked to them

Interviewees	**The teacher's descriptions of interviewees**	**EST questions asked to them**
1- Engin (4th Grade) EST score*: 12/13 Below average**	He is below average in math. Since his mother tongue is Russian, he had problems reading and understanding word problems in Turkish. Therefore, his exam results were mostly below the average (around 60). However, he earned a very good score in the EST. He showed how he solved questions step by step and clearly during the interview. Also, since the beginning of the fall semester, he said that he was attending a mental arithmetic course. Probably the mental arithmetic course contributed to his arithmetic operation skills. One of the students I was surprised at his success in the test.	Q1, Q2, Q3, Q8, Q9, Q12
2- Remzi (4th Grade) EST score*: 12/13 Below average**	He is below average in math. He came from England in the 4th grade. Thus, he had serious problems with reading and understanding word problems in Turkish. Therefore, his exam results were mostly below the average. But he displayed very good success in the EST. He explained the answers very well during the interview. One of the students I was surprised at his success in the test	Q3, Q5, Q8, Q9, **Q12**
3- Deniz (4th Grade) EST score*: 9/13 Above-average**	She always performs above average in math exams (around 95). But in the test, she did only 9 questions correctly. In the interview, she explained all the questions very well without any help and gave all the correct answers. One of the students I was surprised with the performance of her (only 9 correct answers) in the test.	**Q1, Q2, Q5, Q8, Q9, Q12**
4- Nazlı (4th Grade) EST score*: 7/13 Above-average**	She is a successful student in mathematics and has always above average scores (around 90). But in the EST, she received a low score (7 out of 13). She showed that she had misunderstandings about the equal sign concept with her explanations during the interview. One of the students I was surprised with the performance of her in the test.	Q3, Q5, Q8, Q9, **Q12**

Table 3.1. The teacher's descriptions of interviewees and the EST questions asked to them (cont.).

Interviewees	**The teacher's descriptions of interviewees**	**EST questions asked to them**
5- Ebru (4th Grade) EST score*: 3/13 Above-average**	She is good at mathematics. Her exam averages were also good (above 90). My expectation was high, but I have seen during the interview that she had many misunderstandings about the equal sign concept. As a result of the test, she gave only 3 correct answers, and it surprised me in a bad way.	Q1, Q2, Q5, Q6, **Q8, Q9, Q10, Q11, Q12**
6- Ali (4th Grade) EST score*: 3/13 Average**	He has average scores in math (around 80). He is always concerned about math classes, and he has confusion about four operations. He had only 3 correct answers in the EST. During the interview, he also showed a very biased attitude towards to the test and mentioned that he did not know the topic, so could not do the questions. But as he gave the answers, he discovered that he actually knew many things and he could explain some of the questions properly. He just needed to pay attention to the questions.	Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q10, **Q11, Q12**
7- Melek (4th Grade) EST score*: 2/13 Average**	She is an average student in mathematics. She is diligent, she has average exam scores (around 80). In the EST, she had only 2 correct answers. Her score was below my expectations from her. But as far as I can see during the interview, she had some misconceptions about the equal sign concept.	Q1, Q2, Q3, Q4, **Q5, Q8, Q9, Q10, Q11, Q12**
8- Ahmet (5th Grade) EST score*: 3/13 Below average**	He is quite indifferent to the math classes. His exam scores are also below average (around 60). I also think that this situation constitutes prejudice against mathematics because he displayed such an attitude like 'I don't know, I can't do it' during the interview. He had only 3 correct answers in the test. But in the interview, he gave explanatory and correct answers to some of the questions which I did not expect from him. Maybe he is not good at the quantitative part, but he is definitely good at explaining questions.	Q1, Q2, Q3, Q4, **Q5, Q8, Q9, Q10, Q11, Q12**

Table 3.1. The teacher's descriptions of interviewees and the EST questions asked to them (cont.).

Interviewees	**The teacher's descriptions of interviewees**	**EST questions asked to them**
9- Erdem (5th Grade) EST score*: 11/13 Above-average**	He has good scores in math exams (around 90). He established cause-effect relations well and explained the answers to the questions in detail during the interview. He had 11 correct answers in the EST. In the interview, he explained the questions very accurately as I expected from him.	**Q5, Q8, Q9, Q10, Q12**
10- Emre (5th Grade) EST score*: 12/13 Above-average**	He is pretty good at math, always gets above-average scores in exams. His test score did not surprise me, and he achieved a high score (12 out of 13). In the interview, he explained his answers to the problems very well by establishing a cause-effect relationship.	**Q5, Q8, Q9, Q12**
11- Gamze (5th Grade) EST score*: 10/13 Above-average**	She is a diligent student who has average scores (around 80) in math exams. She displayed that she had many misunderstandings about the equal sign concept during the interview. In the EST, she answered 10 questions correctly. After all, she did not surprise me.	Q5, Q6, Q9, Q10, **Q12**
12- Akın (5th Grade) EST score*: 12/13 Above-average**	He is a very good student in math classes and always takes over 90 in the exams even without studying. He also explained each question in detail with a cause-effect relationship. He already clearly demonstrated his knowledge in the EST and during the interview.	Q3, Q6, Q10, Q12
Note. As the researcher, I had access to detailed and rich information about each student because I was their teacher during the 2018-2019 academic year. Questions in bold are the common ones asked to the students.		
*students' EST scores		
**students' mathematics achievements		

3.2. Instrument

3.2.1. The Equal Sign Test (EST)

At the beginning of the EST, students' knowledge about arithmetic operations were tested with an arithmetic achievement test included only two questions-taken from Capraro *et al.*'s study (2011)-given in Table 3.1. The purpose of the arithmetic achievement test was to prevent any arithmetic deficiency related to students' misunderstandings of the equal sign concept influence their performances on arithmetic operations in the EST. The arithmetic achievement test included only two questions (i.e. questions a and b) given at the beginning of the EST, and if the students failed in those questions, then their EST results were removed from the data.

Table 3.2. Arithmetic achievement test.

Arithmetic Achievement Test Items			
a) Which of the following is the result of 27 + 46?			
63	613	73	713
b) Match the equations with the correct results given below.			
		72	
	5 x 8	9	
	14 - 6	16	
	9 + 7	**40**	
	8 x 9	8	
		63	

Just after completing two questions of the arithmetic achievement test, students took the equal sign test (EST) given in Table 3.2 that is developed by Capraro *et al.* (2011) for sixth grade students. Since the fourth and fifth graders have learned the basic arithmetic operations, it is expected that they can answer the test items correctly. Also, the test was shown to four mathematics teachers who taught fourth and fifth grades whether the items were appropriate for the students in order to provide evidence for the validity of the EST items. Consequently, it was concluded that expecting the participants of this study to able to complete the EST successfully.

The EST format was changed by putting empty boxes at the end of each question to observe students' ways of thinking and solving the questions step by step in order to be alert for error patterns. The aim was diagnosing students' error patterns that they learn while learning concepts and computation procedures in order to look for evidence that indicates how students think while solving problems (see Appendix A [for the English version of the instrument] and Appendix B [for the Turkish version of the instrument]).

The equal sign test items are all in non-standard contexts except the second part of question 9 (Q9-b*: _ + 3 = 5 + 7 = _*_) that is in standard context. The reasons for using non-standard contextual presentations of the equal sign as items of the EST may be to reveal whether students are familiar with the non-standard presentations and to determine their error patterns related to their misunderstandings of the equal sign concept. Also, Capraro *et al.* (2011) calculated Cronbach's alpha internal consistency reliability of the test as .90 in their study. Cronbach's alpha for the current study was calculated as 93.

The EST was developed as language-free in order to remove possible language-based assumptions of students, thus there are no words used in the test items. Instead, there are some equations with missing values to complete each by putting the appropriate number in the blanks. The items are formed according to the equal sign contextual presentations such as operation on both sides (i.e., 6 + 3 + 7 = 5 + _), operation on the left or right side only (i.e., 47 + _ = 63 or 18 = _ - 8), or no explicit operations on either side, reflexive (i.e., 160 = _). These three different presentations can be commonly seen in other studies (e.g., Knuth *et al.*, 2008; Byrd *et al.*, 2015) that have examined students' equal sign conceptions and on the standardized achievement tests (Capraro *et al.*, 2011).

On the other hand, some items of the EST can be solved qualitatively which means without performing an operation to find the correct answer (i.e., 13 + 51 = 51

+ _, 8 + _ = _ + 7 or 160 = _). Therefore, it is hard to gain insights into students by just looking at the answers of students. For instance, in question 1 (13 + 51 = 51 + _), students can easily obtain the correct answer without doing any operation because of the reflexive property by simply putting 13 in the blank. Likewise, in question 3 (8 + _ = _ + 7), students can solve the problem by putting 7 and 8 in the blanks respectively to make both sides of the equal sign look alike without any quantitative process, which is called a carry strategy in previous studies (Gather *et al.*, 1998). Therefore, in order to gain a deeper understanding of students' responses to those qualitative test items, the semi-structured interviews were conducted in the current study.

Table 3.3. The equal sign test (EST).

Equal Sign Test Items	
Question 1 (Q1)	13 + 51 = 51 + _
Question 2 (Q2)	6 + 3 + 7 = 5 + _
Question 3 (Q3)	8 + _ = _ + 7
Question 4 (Q4)	160 = _
Question 5 (Q5)	15 - 7 = _ + 5
Question 6 (Q6)	6 x _ = 40 - _
Question 7 (Q7)	47 + _= 63
Question 8 (Q8)	15 - 7 = 5 + _
Question 9 (Q9)	_ + 3 = 5 + 7 = _
Question 9-a (Q9-a)	_*_ + 3 = 5 + 7 = _
Question 9-b (Q9-b)	_ + 3 = 5 + 7 = _*_
Question 10 (Q10)	_ + 5 = 2 x 8
Question 11 (Q11)	13 + 51 = 24 + _
Question 12 (Q12)	18 = _ - 8
Note. Q9-a considers only the first portion of item 9, and Q9-b considers only the second portion of item 9. All questions represent the non-standard presentation of the equal sign except Q9-b that is standard presentation.	

3.2.2. Semi-Structured Interviews

Interviews are valuable data for studies because they enable researchers to gain deeper insights into how students make reasoning for their understanding of mathematical ideas, concepts, procedures, and misconceptions (Ashlock, 2010). For the qualitative part of the study, semi-structured interviews were conducted with some of the students in order to gain a deeper understanding of their equal sign concepts. It is called semi-structured because not all students gave answers to the same questions and the interview protocol had to change for one student to another (Ashlock, 2010). The students were chosen for interviews according to their common errors in the EST results. To that extent, 12 students were chosen for the interviews. In general, questions 5, 8, 9, and 12 in the EST were asked to all of the interviewees because these items had the most common incorrect answers. Some of the interviewees were asked to explain other questions related to their incorrect answers to the EST items. In Table 3.1, the general information about each interviewee, students' profiles, and which EST questions were asked to them during the interviews are given.

I followed Ashlock's (2010) suggestions for a diagnostic interview to form the interview questions. Then, those questions were modified, and new questions were added due to the semi-structured interviews' flow since some interview questions emerged according to students' individual answers during the interviews. The interview protocol is given in Appendix C (in English) and Appendix D (in Turkish).

3.3. Procedure

3.3.1. Examining the Equal Sign Test (EST)

In the fall semester of 2018-2019 academic year, with the implementation of the revised curriculum, fourth grade students were instructed on focusing on the equal sign concept. The new objectives related to the equal sign concept are as follows (MoNE, 2018):

- M.4.1.5.7. Students will be able to find the missing value in a given mathematical equation and explain how the equality is established with proper four operations. (i.e., $8 + \_ = 15 - 3$, $12 : 4 = \_ + 1$, and $6 \times \_ = 48 - 12$)
- M.4.1.5.8. Students will be able to describe the operations that must be done to establish equality between the two mathematical expressions that are not equal. (i.e., Explain how to make two sides of the equation $8 + 5 \neq 12 - 3$ equal to each other.)

On the other hand, fifth grade students have never been instructed the equal sign concept when they were in fourth grade because there was no specific objective about the equal sign in the previous curriculum (MoNE, 2013).

In the spring semester of 2018-2019 academic year, after the equal sign objectives were taught to the fourth graders in the fall semester, the equal sign test (EST) developed by Capraro *et al.* (2011) was applied to the fourth and fifth grade students in order to assess their understanding of the equal sign concept.

Before the EST, there was an arithmetic achievement test (see Table 3.2) that has only two questions to decide whether the students know how to perform arithmetic operations by selecting an option and matching strategies without using the equal sign. The purpose of applying the arithmetic achievement test was to prevent any arithmetic deficiency related to students' misunderstandings of the equal sign concept. The results of this test were not included to the analysis part of the study.

At the beginning of the EST, the students were informed that they could use their time as long as they wanted, but no questions would be accepted by the researcher in order to prevent giving any clue or misleading answer to the students' questions. Also, they were informed that they should take the EST seriously even if the test would not be graded, and they should perform their best in the test without leaving any question empty. In the empty boxes in front of each question, they should also explain their way of thinking during the solution process by using mathematical equations, illustrations, or at least explanatory short sentences, in order to understand how the students have thought while solving each question in the test.

The test was not given at the same time for all students because there was no big enough exam room in the school to hold 53 students. Therefore, the test applied in three sections in three different classrooms in a day. The duration of the test depended on students' different paces, thus there was no limitation for timing. The duration ranged from 15 minutes to 45 minutes.

3.3.2. Conducting Semi-Structured Interviews

During the interviews, without showing students their previous test answers, the questions chosen before the interviews according to the students' all potentially misconception related answers were shown to them (see Table 3.4). How they responded to the questions, and how they made reasoning for their answers were asked. It was said that they could take their time as long as they wanted, and feel free to ask any part that they could not understand. They could also use base ten blocks as manipulatives to show their answers. They were reminded that they should think out loud in order to make clear deductions and let the interviewer comprehend their way of thinking as well.

Interviews were recorded by using an audio recorder and then the audio recordings were transcribed word by word. The duration of the interviews ranged from 6 minutes to 22 minutes, with a total of 125 minutes for the 12 interviews.

3.3.3. Coding Course Materials

After collecting data from the EST results and the semi-structured interviews, course materials used in fourth and fifth grades were coded page by page with regards to the various contextual presentations of the equal sign. It is important to mention that only some parts related with the fifth-grade Turkish curriculum contents of Pearson International Mathematics 1 Book were coded, not the whole book, since it includes sixth and seventh grade Turkish curriculum contents as well.

In order to check inter-rater reliability, before coding all pages, 10 pages from the course materials were chosen randomly. After I coded the pages in terms of various equal sign presentations, my advisor also checked the same pages whether the codes were consistent. Then we decided to add two more non-standard contextual presentations of the equal sign that are (i) without equal sign type-b (e.g., $9.08 + 12.16$; given without any instruction) and (ii) running equal sign (e.g., $\_ + 3 = 5 + 7 = \_$) since they had dramatically high percentages in the course materials.

The reasons to check all course materials according to the standard and non-standard contexts of the equal sign presentations were to gain a deeper comprehension about what kind of contextual presentations students have faced with during the mathematics courses, and whether the course materials provide the needs of the students and support their understanding about the equal sign concept according to the revised national mathematics curriculum (2018). Course materials used in fourth and fifth grades are given in Table 3.4.

Table 3.4. Course materials.

	Fourth Grade	**Fifth Grade**
Course materials	• Sadik Uygun Mathematics Workbook (Aydin, 2018) • Course Booklets (Prepared by the fourth-grade mathematics teachers at the school)	• Pearson International Mathematics for the Middle Years-1 (McSeveny, 2007) • Course Booklets (Prepared by the fifth-grade mathematics teachers at the school)

3.4. Analysis

There are three main parts of analyzing the findings of the study: The EST analysis, course materials analysis, and interview analysis.

3.4.1. The EST Analysis

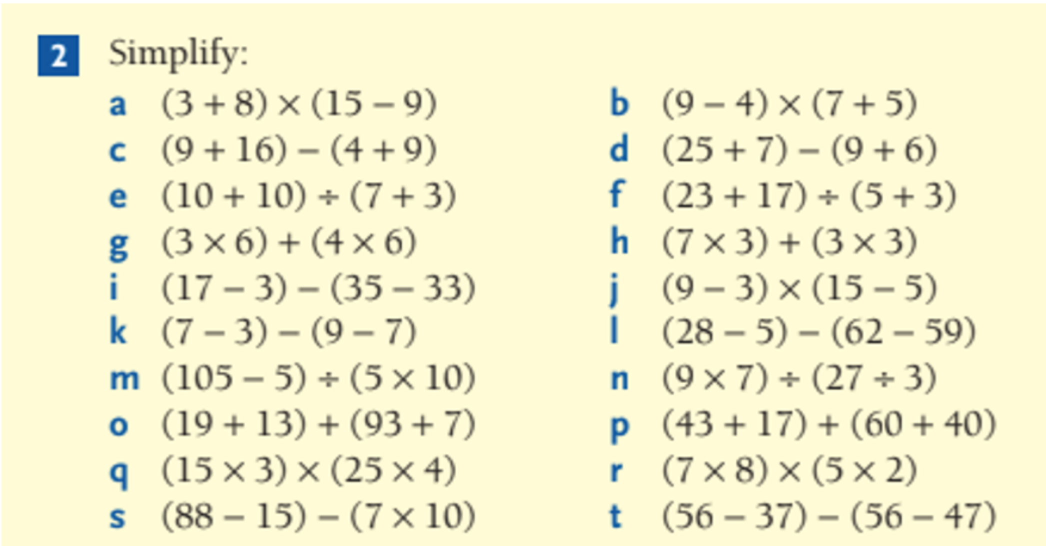
At the beginning of the EST, there were two questions given as an arithmetic achievement test to decide whether students know how to do four operations. According to the arithmetic achievement test results, there was no wrong answer which meant none of the students' incorrect answers in the EST were related with any arithmetic deficiency, and so all students' test results were included into the data set.

After applying the equal sign test (EST), the students' test results were coded, as 1 for correct answers and 0 for incorrect or incomplete answers. Also, the students' answers for each question in the test were written next to the correct/incorrect columns in order to see whether there was any error-pattern between students' responses. Then, the sum of their score was calculated in another column at the end of the table to analyze fourth and fifth grade students' overall achievements in the test.

3.4.2. Course Materials Analysis

The course materials mentioned in Table 5 that the fourth and fifth grade students use in their mathematics classes were coded page by page to examine the various contextual presentations of the equal sign. There are thirteen different presentations determined in order to code the course materials. Eleven of them are taken from Capraro *et al.*'s study (2011). Two of them are standard presentations that are (i) operation on the left side only (e.g., $3 + 15 = \_$), and (ii) the equivalency bar (e.g., $\begin{array}{r}11\\ +3\\ \hline 14\end{array}$). Nine of them are non-standard presentations: (i) name part of the operation (e.g., $4 \_ 4 = 8$; place a + sign on the line), (ii) using arrow to connect (e.g., 7 -> 3 + 4), (iii) filling missing numbers (e.g., $5 + \_ = 9$), (iv) no explicit operations on either side (reflexive) (e.g., 12 inches = 1 foot; $150 = \_$), (v) operation on the right side only (e.g., $\_ = 7 + 9$), (vi) operations on both sides (e.g., $6 + \_ = 7 + \_$), (vii) use/insert relational symbols (e.g., $6 \_ 9$; insert $<$, $>$, or $=$), (viii) verbal presentation (e.g., 7 + 3 is the same as _; 2 x 5 - possible solution), and (ix) without equal sign (e.g., 7 + 3; match to an equivalent quantity).

The last two of them were newly added as non-standard presentations: (a) without equal sign type-b and (b) running equal sign. Without equal sign type-b presentation represents the equations given without any instruction (see Figure 1) and running equal sign presentation represents equations given sequentially and connected (Knuth *et al.*, 2006) (e.g., $\frac{3}{5} = \frac{3x5}{5x5} = \frac{15}{25}$, $2 + 3 = 5 + 7 = 12$) which creates a misconception in students' minds that shows the equal sign only as an operator and overshadows the balance meaning (Capraro *et al.*, 2011). The reason for including those newly occurred non-standard contextual presentations while coding course materials is that they occupy remarkable places in the textbooks.



2 Simplify:

a	$(3 + 8) \times (15 - 9)$	**b**	$(9 - 4) \times (7 + 5)$
c	$(9 + 16) - (4 + 9)$	**d**	$(25 + 7) - (9 + 6)$
e	$(10 + 10) \div (7 + 3)$	**f**	$(23 + 17) \div (5 + 3)$
g	$(3 \times 6) + (4 \times 6)$	**h**	$(7 \times 3) + (3 \times 3)$
i	$(17 - 3) - (35 - 33)$	**j**	$(9 - 3) \times (15 - 5)$
k	$(7 - 3) - (9 - 7)$	**l**	$(28 - 5) - (62 - 59)$
m	$(105 - 5) \div (5 \times 10)$	**n**	$(9 \times 7) \div (27 \div 3)$
o	$(19 + 13) + (93 + 7)$	**p**	$(43 + 17) + (60 + 40)$
q	$(15 \times 3) \times (25 \times 4)$	**r**	$(7 \times 8) \times (5 \times 2)$
s	$(88 - 15) - (7 \times 10)$	**t**	$(56 - 37) - (56 - 47)$

Figure 3.1. Without equal sign type-b example. Source: 'Pearson International Mathematics for the Middle Years-11, McSeveny, 2007.

Frequencies of the standard and non-standard contextual presentations of the equal sign were calculated to display their weight in the course materials and to determine whether they have an impact on students' perceptions about the equal sign concept. Moreover, Capraro *et al.*'s study (2011) findings of second and sixth grade Turkey and USA will be added to the result part in order to assist interpretations of students' equal sign conceptions in a continuum from grade 2 to 6.

3.4.3. Interview Analysis

After examining the EST, diagnostic interviews were conducted with some of the students to understand their equal sign conceptions more deeply. Interview records were transcribed word by word to analyze students' answers and to see possible error patterns. It was expected that the transcriptions would enable the researcher to gain a deeper insight into students' misunderstandings of the equal sign. In order to determine the students' common thoughts and misunderstandings, the transcribed interviews were coded by using the thematic analysis (TA) method was used (Braun & Clarke, 2013).

There are specific reasons for using the thematic analysis (TA) method to analyze the qualitative data of this study. Firstly, Boyatzis (1998) claims that thematic analysis (TA) is a useful approach that builds a bridge between qualitative and quantitative research which depends on a particular definition of qualitative research as providing some tools for collecting and analyzing data. However, seeing TA as only a tool for quantitative data is not accepted by many qualitative researchers (Terry, Hayfield, Clarke, & Braun, 2017). Instead, Terry *et al.* (2017) claim that their new approach to TA is expanded and it offers theoretical flexibility and potential for using TA for empirical purposes as well. Secondly, the thematic analysis method is flexible with regards to framing research design which means that it is applicable to explore questions about participants' experiences, perspectives, factors that influence their lives in both implicit and explicit norms and social constructions in particular contexts (Braun & Clarke, 2006). Thus, this method serves the purposes of this research is understanding students' equal sign conceptions related to their experiences, as well as external factors-their teachers, course materials, and the related objectives in the national mathematics curriculum-possibly influential in the occurrence of those error patterns.

The thematic analysis method highlights identifying, analyzing, and interpreting patterns of themes that seem very useful for analyzing the qualitative data of the current study because the point of the study is finding students' common misunderstandings of the equal sign concept. Also, according to TA method, the recommended project sample size for master's projects is between 6 and 15 people which suits this study with 12 students in the interviews (Braun & Clarke, 2013).

According to TA, there are six steps to follow to analyze the data: Familiarization, coding, generating themes, reviewing themes, defining and naming themes, and writing the report (Braun & Clarke, 2013). In the familiarization phase, familiarizing with the qualitative data can even begin during the data collection part, while conducting interviews in that case. It is a process that allows the researcher to generate their own ideas related to the research questions, to be curious to ask questions about the data and to make early assumptions. By doing that, the researcher should consider the entire data set as a whole and become familiar with the qualitative data in general. After developing a general idea upon the overall data, the researcher can start to generate codes. The coding phase involves creating meaningful labels systematically while highlighting important ideas in the interview transcriptions related to the research topic. In this phase, the patterns between similar codes can be seen and it leads the researcher to construct themes, which is the next step. In the constructing themes phase of the TA, the researcher starts to identify the similar codes under a more general theme related to the topic. However, in this third step, the developed themes can be seen as a first draft and changeable after reviewing all themes, which is the fourth phase. In the fourth phase of the TA, the constructed themes should be reviewed to ensure that they serve well to the whole dataset as well as the research purposes. After being sure of the themes, they should be defined and named which is the fifth phase. In this phase, the definitions of each determined theme should be written and then according to those summaries, each theme should be named with a couple of words. In the last phase of the TA, writing the report, the researcher should gather all the codes, themes, data analysis, and connections in order to reach an output to give answers to the research questions. Producing the report offers researchers an opportunity to make the last changes in analyzing data (Terry *et al.*, 2017).

Although those steps look like in sequential order, as in every research study, the analysis is recursive that moves back and forth between different steps. For instance, by coding each interview, there can be some other codes raised from other interviews and this situation can change generating and reviewing themes at the end (Braun &

Clarke, 2013).

Within the light of the thematic analysis approach (TA) (Braun & Clarke, 2013), all transcribed interviews were reviewed in order to catch any pattern between students' responses. The similar responses of the students were highlighted with the same colored pen, and each pen took a different meaning. For instance, the red-colored pen was used for expressing the equality meaning of the equal sign while the blue-colored pen for the misunderstanding of the balance meaning of the equal sign. Then each interview was coded, and the determined codes are given in Table 3.5. After determining the codes of transcribed interviews, according to the similar codes, themes were generated, and the codes were collected under each theme. Afterward, the themes were reviewed once again whether they served the purpose of the qualitative part of this study in order to give a reliable answer to the research questions. Then, the themes were defined and named with respect to the research purposes are given in Table 3.5.

Table 3.5. The determined codes and the themes of the interview transcriptions.

The Determined Codes	**The Themes**
• Understanding of the operational meaning of sign • Equality meaning of the equal sign (reflexive i.e., $160 = 160$ and $13 + 51 = 51 + 13$) • Balance meaning of the equal sign (i.e., $8 + \_ = \_ + 7$) • Relational meaning of the equal sign (i.e., using $>$, $<$ or $=$ symbols while comparing)	• Understanding the equal the equal sign concept
• Doing operations without considering the position of the equal sign. • Running equal sign misconception (i.e., $\_ + 3 = 5 + 7 = \_$) • Misunderstanding the balance meaning of the equal sign • Misunderstanding the reflexive meaning of the equal sign • Limited understanding of the equal sign concept • Not knowing the meaning of the equal sign (more like having a bias against it)	• Having misconceptions/ misunderstandings about the equal sign concept

The last phase of the TA approach was writing the report. After naming themes, the report of analyzing interview data was written like above. In the writing-up process, the transcribed data were reviewed once again and rearranged according to the 15-point checklist for a good TA (see Appendix E) (Braun & Clarke, 2006).

4. RESULTS

In this section, the findings of the current study will be presented in two parts related with the research questions: (i) The students' error-patterns of the equal sign concept in the light of the EST performances and interviews, and (ii) different meanings and contextual presentations of the equal sign in the light of the EST performances and the equal sign presentations in textbooks.

4.1. The Students' Error-Patterns of the Equal Sign Concept In The Light of the EST Performances and Interviews

The students' test scores ranged from 2 to 13. Four students from the fourth grade (13%) and 14 students from the fifth grade (58%) answered all the questions correctly in the test. The average score of the fourth-grade students' EST results is 8.13 out of 13, whereas the fifth graders' average score is 11.21.

When the EST items were analyzed one by one according to the fourth and fifth grade students' answers, some potential error patterns in the equal sign were found. In Table 4.1, the EST questions responded with common wrong answers (i.e., error patterns) are given. Questions 1, 2, 5, 8, 9-a, 10, 11, and 12 have the same type of wrong answers given by the students. According to the common incorrect answers of students, those answers were categorized as error type-1 and error type-2 in the current study, since incorrect answers given for the other questions can be found by 'performing the operations given before the equal sign only' (i.e., error type-1) or 'doing all calculations without thinking about the place of the equal sign in the equations' (i.e., error type-2), except for the question 9-a. For instance, in question 1, the answer 115 emerged from adding all numbers in the equation (i.e., $13 + 51 + 13 = 115$). The same issue can be seen in the given answers for question 2 (i.e., $6 + 3 + 7 + 5 = 21$) and question 11 (i.e., $13 + 51 + 24 = 88$).

On the other hand, in questions 5 and 8, basically the same equations with different places of the blank, the common incorrect answer 13 was attained by doing all calculations without considering the equal sign in the middle (i.e., 15 - 7 + 5 = 13). The same issue can be seen in the given answers for question 10 (i.e., 2 x 8 + 5 = 21) and question 12 (i.e., 18 - 8 = 10).

As an exception, the incorrect answer given for question 9 is for only the first part (the left-hand side) of the equation (i.e., question 9-a: _ + 3 = 5). For the rest of question 9, that is question 9-b, all students gave the correct answer. Also, it is important to note that question 9-b is the only standard presentation of the equal sign in the EST. Similar situations stand for questions 5 and 8. The other common incorrect answer for those questions is 8, that is the answer given for only the first part (the left-hand side) (i.e., 15 - 7 = _) of the equation.

Table 4.1. Error Types Presented in the EST.

Questions in the EST	Correct Answer Given to The Question	Incorrect Answers Given to The Question	Error Type-1	Error Type-2	The Number of Students Who Answered Incorrectly	
					4th Grade (n = 30)	5th Grade (n = 24)
Q1 (13 + 51 = 51 + _)	13	115		13 + 51 + 13	4 (13%)	1 (4%)
		64	13 + 51		7 (23%)	1 (4%)
Q2 (6 + 3 + 7 = 5 + _)	11	21		6 + 3 + 7 + 5	8 (26%)	1 (4%)
		16	6 + 3 + 7		5 (16%)	1 (4%)
Q5 (15 - 7 = _ + 5)	3	13		15 - 7 + 5	3 (10%)	0 (0%)
		8	15 - 7		12 (40%)	1 (4%)
Q8 (15 - 7 = 5 + _)	3	13		15 - 7 + 5	5 (16%)	1 (4%)
		8	15 - 7		10 (33%)	2 (8%)
Q9-a (_*_ + 3 = 5 + 7 = _)	9	2	5 - 3*		11 (37%)	1 (4%)
Q10 (_ + 5 = 2 x 8)	11	21		2 x 8 + 5	4 (13%)	0 (0%)
Q11 (13 + 51 = 24 + _)	40	88		13 + 51 + 24	5 (16%)	1 (4%)
Q12 (18 = _ - 8)	26	10		18 - 8	12 (40%)	3 (13%)

Note. *It is different from error type-2 since it is due to running equal sign misconception.

One of the common error types (error type-1) seen in questions 1, 2, 5, and 8 is that students do the operations only on the left-hand side of the equal sign to find the answers. For example, in question 1 (13 + 51 = 51 + _), for answer 64, they calculated only the left-hand side of the question without taking the right side of the equal sign into consideration.

Ahmet (5th grade student): Teacher, I guess it will be definitely wrong, but whatever... I add 51 and 13, the answer is 64... Then, I may add 64 and 51... I do not know...

In question 5 (15 - 7 = _ + 5), although the correct answer is 3, 13 students responded to the question as 8, and three students as 13. Also, in question 8 (15 - 7 = 5 + _), that is almost the same as question 5, 12 students answered as 8 and six students as 13, which are the same wrong answers as some students gave for question 5. According to the interview transcripts, the reason for the answer 8 occurred due to calculating only the left-hand side of the equal sign.

Ahmet (5th grade student): Hmm... 8... 21... 13 can be written. I subtracted 7 from 15.

Then, I added 5 and it made 13. In question 8, it can be the same thing. 15, 7... Then it makes... 8.

Surprisingly, although some students answered question 8 correctly, they answered question 5 incorrectly. They explained this situation during the interviews. Because of the place of the blank in question 5, they wanted to proceed the equation and put one more equal sign at the end, hence their final result was 13, that is called the equality string, or the running equal sign misconception in the literature (Knuth *et al.*, 2006; Capraro *et al.*, 2011).

Gamze: First, we subtract 7 from 15. It is 8. Then, I should add 8 and 5... I don't think that we should write something else here...

Researcher: Then, what is your answer when you add 8 and 5

Gamze: 13.

Researcher: Where will you write 13 then? In the blank? There is no other blank in the question.

Gamze:... I can write it myself in the end.

Researcher: Okay. Let's look at question 8. What does it say?

Gamze: Hmm... First, we will add them... No, subtract them. When we add 5 with something, it is 8. So, the answer is 3.

Also, when the places of the blanks are different in these questions, even though the result does not change, students think that they are very different questions because of their different non-standard presentations.

On the other hand, error type-2 was found in all the questions in Table 4.1 in the question without considering the equal sign to get answers. For instance, in question 1 ($13 + 51 = 51 + \_$), for answer 115, some students said that they just added the all numbers up without thinking about the place of the equal sign.

Ebru (4th grade student): At first, I add 51 and 13. I found 64. Then, I add 51 to this number I found. The answer is 115.

In the second question ($6 + 3 + 7 = 5 + \_$) of the EST, for the answer 21, students added all numbers as in the first question that supports error type-2.

Ebru (4th grade student): At first, in the same way, 7, 6, 3, equal. I added 7 and 6... makes 13. When we add 3 more, it becomes 16. Then, I add this 5 to 16. So, this time it makes 21.

Ahmet (5th grade student): I add them like this: The answer is 16. Then, again the same thing. Then, 21. The question says that add 6 and 3, then, add 7 more, then, add 5 more.

In question 5 (15 - 7 = _ + 5) for answer 13, after calculating the left-hand side of the equal sign, because of the plus sign in front of 5, the students added 5 to what was initially calculated and found a wrong answer at the end. Likewise, in question 8 (15 - 7 = 5 + _), that is almost the same as question 5, six students answered as 13, which are the same wrong answers as some students gave for question 5. This is due to the lack of understanding of the equal sign concept.

Ebru (4th grade student): In this question (while pointing the question 5), at first, we subtract 7 from 15... 14,13,12,11,10,9,8. 8 left. Then, 8 + 5 = 13. In this question (while pointing question 8), again the same thing, at first, we subtract 7 from 15 because there is a minus sign. 10 left. 14,15... 10,9,8... Wait a minute. Did not I solve the same question on the last page? They are exactly the same questions. Then again, 13.

Remzi (4th grade student): ... We will subtract 7 from 15 and add the result and 5... It makes 13 when we add 8 and 5.

Although the correct answer to question 10 (_ + 5 = 2 x 8) is 11, four students responded as 21. As in questions 1, 2, 5, and 8, the misunderstandings emerged due to the fact that those students tended to calculate the given questions without thinking of the place of the equal sign and putting the equal sign always at the end just as an operational symbol. Thus, it can be said that there is a misunderstanding about the balance meaning of the equal sign. Also, the same situation in question 10 can be seen in question 11 (13 + 51 = 24 + _) that six students responded as 88 even though the correct answer is 40 with the same misleading logic behind.

Ebru (4th grade student): Well... Here at first, we multiply 8 and 2 because there is a multiplication sign. 8 times 2 equals 16. Then, since there is 6 and 5, I should add 16 and 5... It makes 21. I think the answer is 21.

In question 12 (18 = _ - 8), 15 students gave the wrong answer as 10, although 26 is the correct answer. The issue here is that those students tend to ignore seeing the real place of the equal sign and they just focus on the minus sign and the numbers given to them. Then, they just want to subtract the smallest number from the biggest number given in the question.

Researcher: Let's look at the last question together. Can you read it out loud?
Nazlı (4th grade student): 18 minus a number equals 8.
Researcher: The first sign is the equal sign. Can you reread it?
Nazlı: 18 equals a blank minus 8.
Researcher: So, what becomes in the blank here?
Nazlı: 10.
Researcher: Okay, let's write 10 here. Now, again, read the equation out loud.
Nazlı: 18 equals 10 minus 8. It can't be!
Researcher: So, what are we going to do now?
Nazlı: ...
Researcher: You can change, delete your answer if you want.
Nazlı: It is 18 then... To make it equal.
Researcher: Okay. Let's write 18 and reread it out loud.
Nazlı: 18 equals 18 minus 8... Okay, it is 10.

On the other hand, in question 9-a ($\_*\_ + 3 = 5 + 7 = \_$), students saw it as an equality string (Knuth *et al.*, 2006) which causes running equal sign misconception (Capraro *et al.*, 2011) as it can be seen in the given incorrect answer as 2. As a result, some students thought about the question in such a way that equal signs followed each other and the left-hand side needed to be equal to the first number on the right side which was 5.

Ali (4th grade student): It is 2 because there is 5 over there, so I found 2.
Nazlı (4th grade student): 2 plus 3 makes 5. 5 plus 7 makes 12.

Some students, however, explained the relational meaning of the equal sign in question 9 in detail. Since the question includes two different equal sign presentations-one standard and one non-standard-and it gives three different sides equal to each other, it was one of the determinant questions to understand students' comprehension of the equal sign concept in the EST. There were such explanations from various students as follows:

Deniz (4th grade student): This time, we will add 5 and 7 at first. It makes 12, then we will subtract 3 from 12, 9.
Researcher: Why did you subtract 3 from 12?
Deniz: To find the number here (while pointing the first blank, that is question 9-a). Because the sum of 3 and 9 equals the sum of 5 and 7.
Erdem (5th grade student): 5 + 7 is 12. Since it is 12, ... 12 - 3 is 9. This is 9, and this is 12.
Emre (5th grade student): 5 + 7 is 12. When we add 9 and 3, it makes 12. Then, it should be 9 as well.

Besides the students' common error types, according to the interview transcriptions, some students displayed that they had an adequate understanding of the equal sign concept by explaining the right answers of the EST questions step by step. They showed that they comprehended the balance and reflexive meanings of the equal sign. For instance, Melek, a 4th-grade student, explained her answer for both question 5 and question 8 by using the relational meaning of the equal sign as follows:

Melek (4th grade student): Here, it says that 7 subtracted from 15, so it should be 8. I mean, this should be 3 (while pointing the blank) because when we subtract them it is 8 and when we add them (while showing the right-hand side of the equation) it should be 8 as well. That's why they are equal.
Researcher: Okay, then. What about question 10?
Melek: (Surprised) Aa! The 10th question is the same. I mean, when we do subtraction, it is 8. When we add them (while showing the right-hand side of the equation), I will add with 3, it is 8 again.

In another example to show students' understanding of the equal sign, Engin, a 4th-grade student, explained his answers to question 1 and 2 within a proper understanding of the relational meaning as follows:

Engin (4th grade student): Teacher, they are the same numbers, but in reversed places (while pointing question 1). So, the answers will be the same. For example, then we add them (while showing 13 + 51), we find a number and equal... When we change the places of these numbers, the answer will be the same... I answered this question as 13.
Engin (4th grade student): Teacher, in this question (Q2), the addition of these... We add this 5 and which number would be equal, I mean their results... There should be 11. When I add all these numbers up, it will be 16. Then, in order to equalize, I subtract 5 and find 11.

In general, the students' error-patterns in Table 4.1 showed that the fourth-grade students gave mostly wrong answers to the questions in the EST. This situation is quite surprising because the objectives related to the equal sign concept are taught to the fourth graders in the fall semester of the 2018-2019 academic year, and it would be expected that they could give more correct answers to the questions. With the help of the semi-structured interviews, students' misinterpretations of the equal sign concept and their way of thinking while solving the mathematical equations became more understandable according to their error-patterns displayed in the EST results.

4.2. Different Meanings and Contextual Presentations of the Equal Sign in the Light of the EST Performances and the Equal Sign Presentations in Textbooks

Different contextual presentations of the equal sign in textbooks are discussed an impact on constructing students' equal sign understandings (McNeil *et al.*, 2006; Seo & Ginsburg, 2003). Hence, it is important to take a look at the frequencies of various equal sign contextual presentations displayed in the fourth and fifth grade textbooks in order to reveal what different meanings and contextual presentations of the equal sign include the revised curriculum (2018), textbooks and other lecture materials?

In Table 4.2, all standard and non-standard contextual presentations of the equal sign taken from Capraro *et al.* (2011) are presented. The frequencies of various equal sign presentations of the 2nd and 6th grade textbooks of Turkey and the USA (central Texas) are also taken from Capraro *et al.*'s study (2011). The reason for adding the results of Turkey and the USA (central Texas) textbooks is to see the changes in the frequencies of different equal sign usage from 2nd grade to 6th grade sequentially. Also, those previous findings in 2nd and 6th grades textbooks are comparable with the findings of 4th and 5th grades textbooks in the current study because 4th grade materials are in Turkish, while 5th grades materials are in English. Moreover, Pearson International Mathematics Book 1 is directly taken as an English data that is used by the 5th grade sample of the current study. Therefore, it is important to examine both the 4th and 5th grade textbooks findings together with Capraro *et al.*'s 2nd and 6th grades textbook findings (2011).

Table 4.2. Percentages of equal sign presentations in fourth and fifth grade course materials compiled with previous data from second and sixth grade Turkish and USA (central Texas) textbooks.

Equal Sign Presentations	**Capraro *et al.*'s study results Grade 2****		**Current study's results**				**Capraro *et al.*'s study results Grade 6****	
			Grade 4		**Grade 5**			
	Turkey	USA (central Texas)	Course Booklets	Sadik Uygun Workbook	Course Booklets	Pearson International Math 1 Book	Turkey	USA (central Texas)
Standard Contextual Presentation	**39.6**	**54.29**	**32.31**	**63.72**	**37.69**	**16.1**	**26.86**	**17.84**
Operation on left side only (e.g., 3 + 15 = _)	29.37	27.6	18.61	37.34	30.4	12.47	21.84	14.92
Equivalency bar (e.g., $\begin{array}{r}11\\+3\\\hline 14\end{array}$)	10.23	26.69	13.7	26.38	7.29	3.63	5.02	2.92
Nonstandard Contextual Presentation	**60.4**	**45.71**	**67.69**	**36.28**	**62.31**	**83.9**	**73.14**	**82.17**
Name part of the operation (e.g., 4_4 = 8; place a + sign on the line)	0.88	1.25	0	0	0.61	0	0.4	0
Using arrow to connect (e.g., 7⟶3 + 4)	15.61	2	9.2	4.21	0.3	0	2.75	4.84
Filling missing numbers (e.g., 5 + _ = 9)	24.6	17.91	6.34	6.28	2.89	0.69	0.73	0.96
No explicit operations on either side (reflexive) (e.g., 12 inches = 1 foot; 150 =_)	8.2	2.64	35.79	12.21	39.06	16.11	27.59	22.09
Operation on right side only (e.g., _ = 7 + 9)	0.18	2.89	1.02	1.07	1.37	3.24	9.14	7.63
Operations on both sides (e.g., 6 + _ = 7 + _)	0.97	1.86	1.43	0.59	0.76	7.65	12.14	2.05
Use/Insert relational symbols (e.g., 6 _9; insert <, >, or =)	2.2	7.63	6.14	3.44	11.25	1.76	4.21	4.7
Verbal representation (e.g., 7 + 3 is the same as_; 2 x 5 - possible solution)	5.56	6.6	1.43	1.54	3.19	2.5	1.46	5.1
Without equal sign (e.g., 7 + 3; match to an equivalent quantity)	2.2	2.93	2.86	0.18	0.3	0	14.72	34.8
Without equal sign type-b* (e.g., 9.08 + 12.16) (given without any instruction)	NA[a]	NA[a]	3.48	6.76	1.52	50.43	NA[a]	NA[a]
Running equal sign* (e.g., _+3 = 5 + 7 = _)	NA[a]	NA[a]	0	0	1.06	1.52	NA[a]	NA[a]

Note. *These categories are added to the other categories taken from Capraro *et al.*'s study (2011).

[a] These categories were not presented in Capraro *et al.*'s study (2011).

** The findings of these two grades are taken from Capraro *et al.*'s study (2011).

While looking at Table 4.2, it can be seen that 4th grade course booklets include more non-standard contextual presentations of the equal sign (67.69%) than standard contextual presentations (32.31%) in total. On the contrary, in Sadik Uygun Workbook, there are more standard presentations (63.72%) than non-standard ones (36.28%). However, in 5th grade materials, the course booklets have 37.69% standard contextual presentations and 62.31% non-standard contextual presentations of all equal sign instances detected in the course booklets. Likewise, Pearson International Mathematics 1 Book has fewer standard presentations (16.10%) than non-standard presentations (83.90%) of all equal sign instances. On the other hand, according to Capraro *et al.*'s study (2011), in Turkish textbooks, 2nd grade course material has less standard contextual presentations (39.60%) than non-standard contextual presentations (60.40%) of all equal sign instances, while 6th grade material has also less standard contextual presentations (26.86%) than non-standard contextual presentations (73.14%). However, in the USA (central Texas), 2nd grade course material has fairly more standard (54.29%) than non-standard presentations (45.71%), while 6th grade course material has a very low percentage of standard presentations (17.84%). According to those data, in both countries, the amount of standard contextual presentations of the equal sign decreases, while the amount of non-standard contextual presentations increases from 2nd grade to 6th grade.

There are two types that are given for the standard contextual presentations of the equal sign which are: (a) operation on the left side only (i.e., $5 + 8 = \_$) and (b) equivalency bar (i.e., $\frac{\substack{11\\+3}}{14}$). Operation on the left side only type is the most common standard equal sign representation in all grades. It has the highest percentage in grade 4 Sadik Uygun Workbook which is 37.34%, although it is seen as 18.61% of the total amount of the equal sign representations in 4th grade course booklets. Also, operation on the left side only usage of the equal sign has the highest percentage in 2nd grade textbooks which are 29.37% in Turkey and 27.60% in the USA (central Texas). However, while reaching grade 6, its usage decreases, 26.86% in Turkey and 17.84% in the USA (central Texas) textbooks.

When it comes to the non-standard contextual presentations, there are eleven different versions of the equal sign usage. The first nine versions of the equal sign presentations are taken from Capraro *et al.*'s (2011) study. The last two of them are formed while coding 4th and 5th grade course materials in the current study. The first type of the equal sign representations is naming part of the operation (i.e., 4 _ 4 = 8). Although this type is not mostly used in the course materials, it is mostly seen in 2nd grade Turkey and USA (central Texas) course materials with 0.88% and 1.25% respectively. On the other hand, it is never used (0.00%) in 4th grade materials, 5th grade Pearson International Math 1 Book, and 6th grade USA (central Texas) textbook. It is barely seen in 5th grade course booklets (0.61%) and 6th grade Turkey textbook (0.40%). Overall, it can be said that the usage of naming part of the operation presentation is very few.

The second type of equal sign representation is using an arrow to connect (i.e., 7 $->$ 3 + 4), and the third type of equal sign representation is filling missing numbers (i.e., 5 + _ = 9). They both are highly used 2nd-grade textbooks; then, their usage dramatically decreased from 2nd grade to 6th grade.

The fourth type of the equal sign representations is no explicit operations on either side (reflexive) (i.e., 160 = 160, 1 km = 1000 m, 2000 mL = 2 L, 1 h = 60 min.). This presentation type is dramatically high in 4th and 5th grade course booklets, 35.79%, and 39.06% respectively. These are the highest amounts of equal sign usage in those categories. Moreover, in 6th grade, it is also used as a big amount of the total equal sign presentations, 27.59% in Turkey, and 22.09% in USA (central Texas) textbooks.

The last type of nonstandard presentations taken from Capraro *et al.* (2011) is without an equal sign. This type has never been seen (0.00%) in 5th grade Pearson International Math 1 Book, thus without equal sign type-b part is added to the categories and it occupies a huge place as 50.43% in the book. Also, without equal sign type is mostly seen in 6th grade textbooks (14.72% in Turkey and 34.80% in the USA [central Texas]) while it is rarely seen in other grades.

According to the 4th and 5th grade course materials, which are course booklets, Pearson International Mathematics 1 Book and Sadik Uygun Mathematics Workbook, two more presentation types of the equal sign are added to the nonstandard contextual presentations:(a) without equal sign type-b and (b) running equal sign. Without equal sign type-b presentation represents the equations given without any instruction (e.g., 9.08 + 12.16) and running equal sign presentation represents equations given sequentially and connected (Knuth *et al.*, 2006) (e.g., $\frac{3}{5} = \frac{3x5}{5x5} = \frac{15}{25}$, $2 + 3 = 5 + 7 = 12$). The reason for including those newly occurring non-standard contextual presentations while coding course materials is that they occupy remarkable places in the course materials, which are 10.24% in 4th grade and 54.53% in 5th grade in total. Those percentages cannot be neglected because they affect how to interpret data obtained from the course materials.

In general, according to the results of coded materials in the current study, in Pearson International Math 1 Book, without equal sign type-b is mostly used as 50%. In grade 5 booklets, the most used type of equal sign is no explicit operations on either side (reflexive) type with 39%, and operation on the left side only type follows with 30%. By looking at all grade 4 booklets, no explicit operations on either side (reflexive) usage of the equal sign has the highest score with 36%, and operation on the left side only type follows with 19%. In 4th grade, Sadik Uygun Workbook, usage of the equal sign as an operation on the left side only type has the highest amount with 37%, and equivalence bar usage follows with 26%. In both fourth and fifth grade booklets, that are developed by the teachers at the participant school in the study, the reflexive type of the equal sign presentation (35.79 % and 39.06% respectively) has the highest amount. Overall, it can be seen that non-standard contextual presentations of the equal sign increased dramatically from 2nd grade to 6th grade.

5. CONCLUSION AND DISCUSSION

This chapter consists of discussions of the research findings, implications, limitations of the study, and recommendations for further studies. The research findings will be discussed in light of the research questions and relevant literature reviews. Educational implications will follow. Then, the limitations of the current study and some recommendations for further studies will be presented.

5.1. Discussion of Research Findings

There are some significant points to discuss in the light of the findings of the research. First of all, in Table 4.2, the fourth and fifth grade students' common incorrect answers to the eight questions of the EST are given. The reasons behind these specific errors were discussed during the semi-structured interviews. According to the interview transcriptions, students expressed that they had common misunderstandings about the equal sign concept, mostly because of being exposed to the operational view that overshadows the relational meaning of the equal sign (Baroody & Ginsburg, 1983; Denmark *et al.*, 1976; Kieran, 1981; McNeil & Alibali, 2005; McNeil, 2014; Renwick, 1932). Also, running equal sign misconception (Capraro *et al.*, 2011; Knuth *et al.*, 2006) is captured in students' common mistakes in questions 5, 8, and 9-a during the interviews. Therefore, it may be concluded that there are specific error-patterns related to students' inadequate knowledge about the equal sign concept that most students displayed in the EST.

According to the revised mathematics curriculum (MoNE, 2018), there are two objectives about the equal sign concept added to the fourth-grade content (see Table 1.1). In Turkey, when some objectives are added to the revised curriculum, they should also be involved in the textbooks that the ministry of national education gives permission to be published because the content of textbooks should meet all the objectives of the relevant curriculum (TTKB, 2019). When the objectives take place in the textbooks, it means that they will be discussed and taught in class as well. According to these newly added objectives to the fourth grade, it can be said that fourth graders have become familiar with mostly the relational view of the equal sign concept in the fall semester of the 2018-2019 academic year. However, according to the equal sign test (EST) results, the average score of fourth grade students was 8.13 out of 13 which is moderate.

The fourth-grade students' achievements in the EST may emerge from that teachers were not informed about the revised curricula, and so they encountered some challenges in the application process (Duru & Korkmaz, 2010). Çiftçi, Akgün, and Deniz (2013) argued that teachers may tend to apply the old-fashioned teaching methods as they could not interiorize new revisions in the content. In spite of the new curricula's requirements, they keep applying the traditional education approach that does not fit to the revised objectives. In this sense, in Çiftci and Tatar's study (2015) the teachers stated that the revised curriculum was not introduced to them sufficiently; thus, they emphasized that the content of the updated curriculum should be explained to teachers in detail. Otherwise, teachers could not implement the adapted curricula within the determined requirements (Tekbıyık & Akdeniz, 2008).

Another perspective on the issue why teachers cannot teach in the intended way is the absence of teachers' guidebooks. The 2009 mathematics curriculum included the teachers' guidebook that explains each objective by giving some examples of how to teach them (MoNE, 2009). Afterward, the guide part of the curriculum was eliminated from the revised curricula in the later years (i.e., 2013 and 2018). Instead of publishing a guidebook, some videos about the 2013 revised curriculum were published on the internet in order to explain the changes to teachers (e.g., Çetin, 2015). Some research (e.g., Ubuz, Erbaş, Çetinkaya, & Ozgeldi, 2010; Bozkurt & Aslanargun, 2015) indicated that the guidebook sounds mandating to teachers not only what to teach but also how to teach and to what extent, and so it prevented teachers from being more creative. Further, providing examples for each objective potentially makes teachers consider curriculum as a ready-to-apply guide. However, Ubuz *et al.* (2010) indicated that curriculum guidebooks were very important resources for both teachers and textbook writers to implement the revised curriculum properly. Therefore, because of the absence of teachers' guidebook in the 2018 mathematics curriculum, the newly added objectives about the equal sign concept might not be stressed adequately in fourth grade.

In the revised curriculum (2018), not only the fourth grade but also the second and seventh grade have newly added objectives about the equal sign concept (see Table 1.1). The objectives in the curriculum are usually built as complementary to each other within a holistic approach (MoNE, 2018). For instance, the fourth-grade objectives are built on the third-grade content. Similarly, the objectives about the equal sign in fourth grade are constructed upon the second-grade equal sign objective whereas the seventh-grade objectives follow the fourth-grade ones. In this regard, fourth grade students did not encounter the newly added equal sign objective (i.e., Students will be able to realize the balance meaning of the equal sign between two mathematical equations (M.2.1.3.5.)) when they were in second grade, although the objective is the building block of students' equal sign conception. Thus, the change in fourth grade may not be beneficial as it was planned. Also, curriculum modifications may not be applied as fast as they were planned because of some deficiencies such as inadequate information about newly added objectives and lack of stationeries in schools (Duru & Korkmaz, 2010). It is also important to notice that the current study is conducted in the transition year of the revised curriculum. Therefore, it makes sense that fourth graders were not very successful as they expected in the EST.

According to the findings of coding course materials, there is a vast difference between the amounts of the standard and non-standard presentations of the equal sign in fourth grade booklets and Sadik Uygun Workbook. Fourth grade booklets have 67.69% non-standard presentations, while the workbook has only 36.28%. This difference may emerge from the booklets prepared by four teachers who work with fourth graders with the help of more than one resource whereas the workbook is created by a publishing company and it is questionable in terms of its up-to-datedness. Also, Sadik Uygun Workbook includes mostly standard contextual presentations of the equal sign (63.72%) that does not support students' understanding of the relational meaning of the equal sign (Baroody & Ginsburg, 1983; Carpenter *et al.*, 2003; Molina *et al.*, 2009). This situation shows that the workbook was not updated according to the revised curriculum within a holistic approach. Thus, it does not support students' equal sign understanding. To that extent, it can be said that curriculum modifications may not be enough to put the newly added objectives into practice due to the outdated textbooks and course materials. Moreover, this suggests that by continuing to expose the students to the same standard usage of the equal sign to the fourth graders, students could not improve their way of thinking about the equal sign concept according to their EST results.

In Table 4.2, equivalency bar and filling missing numbers usage of the equal sign decreases from second to sixth grade according to the given mathematical objectives. In second grade, students start to learn numbers and arithmetic operations, and they mostly focus on learning these topics in that year, and their densities decrease year by year. Towards sixth grade, they have more complicated objectives to learn such as algebraic expressions, data analysis, angles, area, circle, and so on. Therefore, this situation may be due to the increased and varied mathematical objectives applied in sixth grade, and most of the time, sixth grade students work on non-standard presentations. Also, according to the finding from coded course materials and the EST results, it can be argued that while working mostly on arithmetic operations due to the content 'order of operations', the fifth-grade students should have comprehended the relational meaning of the equal sign well.

No explicit operations on either side (reflexive) type is one of the most presented non-standard equal sign presentation types in fourth and fifth grade booklets, 35.79%, and 39.06% respectively. It can be due to the density of the content. Since in fourth and fifth grade students learn standard units of measurement and their conversions (i.e., meter, liter, and hour), their course booklets contain lots of exercises about converting those measurements into each other. Therefore, this circumstance increases the percentages of no explicit operation on either side (reflexive) type of equal sign usage in both grades.

5.2. Implications

The findings of the current study provide some educational implications. First of all, the results may offer beneficial information for mathematics teachers who would like to differentiate their instructions in terms of emphasizing the relational view of the equal sign in order to support students' algebraic knowledge in further grades. It is also suggested that assessing students' prior knowledge about the equal sign concept before teaching pre-algebra may be helpful for identifying students' misunderstandings of the equal sign and further preventing these misunderstandings from becoming deep-rooted as known as misconceptions (Byrd *et al.*, 2015).

On the other hand, the traditional arithmetic approach keeps emphasizing the operational view of the equal sign that prohibits students from learning algebra (McNeil & Alibali, 2005; McNeil, 2014). In order to support students' algebraic thinking in further grades, curriculum changes should be reinforced in terms of constructing a better understanding of mathematical equivalence. Several studies in the literature (e.g., Carpenter *et al.*, 2003; Hattikudur & Alibali, 2010; McNeil, 2008) have also promoted curriculum revisions in order to reinforce students' understanding of the equal sign concept. To that extent, the revised Turkish mathematics curriculum (2018) should also be explored through as a study in which a more detailed analysis of the reflections of the curriculum onto teaching and learning in order to investigate whether it is effective for improving algebraic thinking in the future.

Lastly, in the coding course materials process of the current study, there were two more non-standard equal sign presentations-without equal sign type-b and running equal sign-added to the eleven different presentations taken from Capraro *et al.* (2011) (see Table 4.2). With the newly added types of equal sign contextual presentation form, researchers may have an extensive perspective about the equal sign concept in further studies.

5.3. Limitations of the Study and Recommendations for Further Studies

The current study has some limitations as well. These limitations may have affected the results of the study; thus, they need to be mentioned. First of all, the study is limited to one school that had only one fifth grade and two fourth grade classes in the 2018-2019 academic year. Total participants of the study were 53 students-23 fifth grade students and 30 fourth grade students-that is not enough to generalize to all fourth and fifth grade students in Turkey. Further studies may be conducted with a larger sample from various schools in order to provide more representative results.

The second limitation is related to the grades of the participants. According to the revised Turkish mathematics curriculum (MoNE, 2018), the objectives about the equal sign concept were added to the second, fourth, and seventh grades (see Table 1.1). However, the study was conducted only with fourth and fifth grade students to see whether there is a difference between their equal sign conceptions. As a recommendation, in further studies, the sample may be chosen from second grade students who were exposed to the 2018 mathematics curriculum and third graders who did not encounter with the newly added objective in second grade in order to examine the difference between their equal sign understandings.

The third limitation is related to in-service teacher training, support materials, and guidebooks. In the literature, some studies (e.g., Çiftçi *et al.*, 2013; Duru & Korkmaz, 2010; Tekbıyık & Akdeniz, 2008) claim that since teachers are not informed about revised curricula and the absence of teachers' guidebooks, they encounter some challenges in the application process. So, they could not interiorize the revisions of the updated curricula. As a recommendation, teachers can be supported by giving in-service training included changes in the revised curricula.

The final limitation of the current study concerns how to determine students' conceptions of the equal sign. In order to examine students' equal sign understandings in-depth, the equal sign test developed by Capraro *et al.* (2011) was applied, the semi-structured interviews with selected students were conducted, and the course materials were checked in terms of various equal sign usage. However, the number of interviewees and the durations of the interviews may be increased in order to attain a deeper understanding of students' equal sign knowledge in further studies.

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APPENDIX A: THE EQUAL SIGN INSTRUMENT

Table A.1. The Equal Sign Instrument.

<table>
<tr><td colspan="2">Name-Surname= Grade= Student ID=</td></tr>
<tr><td colspan="2">a) Which of the following is the result of 27 + 46?
63 613 73 713</td></tr>
<tr><td colspan="2">b) Match the equations with the correct results given below.
72
5 x 8 9
14 - 6 16
9 + 7 40
8 x 9 8
63</td></tr>
<tr><td colspan="2">Fill in the blanks with appropriate numbers by showing your work step by step in the boxes next to them.</td></tr>
<tr><td>1) 13 + 51 = 51 +</td><td></td></tr>
<tr><td>2) 6 + 3 + 7 = 5 +</td><td></td></tr>
<tr><td>3) 8 + _ = _ + 7</td><td></td></tr>
<tr><td>4) 160 =</td><td></td></tr>
<tr><td>5) 15 - 7 = + 5</td><td></td></tr>
<tr><td>6) 6 x _ = 40 - _</td><td></td></tr>
<tr><td>7) 47 + = 63</td><td></td></tr>
<tr><td>8) 15 - 7 = 5 +</td><td></td></tr>
<tr><td>9) _+ 3 = 5 + 7 = _</td><td></td></tr>
<tr><td>10) _+ 5 = 2 x 8</td><td></td></tr>
<tr><td>11) 13 + 51 = 24 +</td><td></td></tr>
<tr><td>12) 18 = - 8</td><td></td></tr>
<tr><td>E/K</td><td>Ü/O/A</td></tr>
</table>

APPENDIX B: THE EQUAL SIGN INSTRUMENT (TURKISH)

Table B.1. The Equal Sign Instrument (Turkish).

<table>
<tr><td colspan="2">İsim-Soyisim= Sınıf= Öğrenci No=</td></tr>
<tr><td colspan="2">a) 27 + 46 işleminin sonucu aşağıdakilerden hangisidir?
63 613 73 713</td></tr>
<tr><td colspan="2">b) Aşağıdaki işlemlerle sonuçlarını eşleştiriniz.
72
5 x 8 9
14 - 6 16
9 + 7 40
8 x 9 8
63</td></tr>
<tr><td colspan="2">Aşağıdaki işlemleri yanlarındaki kutucuklara yaparak boşluklara gelecek sayıları bulunuz.</td></tr>
<tr><td>1) 13 + 51 = 51 +</td><td></td></tr>
<tr><td>2) 6 + 3 + 7 = 5 +</td><td></td></tr>
<tr><td>3) 8 + _ = _ + 7</td><td></td></tr>
<tr><td>4) 160 =</td><td></td></tr>
<tr><td>5) 15 - 7 = + 5</td><td></td></tr>
<tr><td>6) 6 x _ = 40 - _</td><td></td></tr>
<tr><td>7) 47 + = 63</td><td></td></tr>
<tr><td>8) 15 - 7 = 5 +</td><td></td></tr>
<tr><td>9) _+ 3 = 5 + 7 = _</td><td></td></tr>
<tr><td>10) _+ 5 = 2 x 8</td><td></td></tr>
<tr><td>11) 13 + 51 = 24 +</td><td></td></tr>
<tr><td>12) 18 = - 8</td><td></td></tr>
<tr><td>E/K</td><td>Ü/O/A</td></tr>
</table>

APPENDIX C: INTERVIEW PROTOCOL

Table C.1. Interview questions

Questions formed according to Ashlock's -2010 suggestions for a diagnostic interview	• What does the equal sign tell us? • How did you get the answer to this question? I may have missed something. • Why do you think that your answer is correct? • If someone said your answer is not correct, how would you explain that it is correct? Could you explain it another way? • If you had to teach your friend/a third-grade student to do this, how would you do it? • What do you say to yourself as you do this? Say it out loud so I can understand the way you think, too.
Common questions asked to the selected participants	• Do you know what this sign (while pointing the equal sign) is called in mathematics? • What does the equal sign mean for you? • In which situations in mathematics do we use the equal sign? • Is there any other meaning of the equal sign than which you mentioned before? • Would you like to use pattern blocks to complete this question and explain your solution to me out loud?
Other questions emerged during the interviews	• Why did you add all the numbers up in question 4 (6 + 3 + 7 = 5 + _)? Do you think that using the equal sign in the middle of this equation makes a difference in the result? • Which numbers would you write in the blanks of these two questions? What is the difference between these two questions? (i.e., 15 - 7 = _ + 5 and 15 - 7 = 5 + _) • What is your strategy to solve question 14 (18 = _ - 8)? Does it look f amiliar to you somehow? • Why did you write 1 and 2 in the blanks of question 5 (8 + _ = _ + 7) respectively? • Are there any other answers for question 5 (8 + _ = _ + 7) and question 8 (6 x _ = 40 - _)? • How did you find 160 = 80? Can you redo it step by step for me? • Why do you think that you can write only 160 in question 6 (160 = _)? • Why do you think that question 14 (18 = _ - 8) is harder than question 12 (_ + 5 = 2 x 8)? • Why do you think that you did all questions wrong in the test? • Why do you think that you are not good enough in math?

APPENDIX D: INTERVIEW PROTOCOL (TURKISH)

Table D.1. Interview Protocol (Turkish)

Ashlock (2010) görüşme önerilerine göre şekillenen sorular	• Eşittir işareti bize ne anlatır? • Bu sorunun cevabını nasıl aldınız? Bir şeyi kaçırmış olabilirim. • Neden cevabınızın doğru olduğunu düşünüyorsunuz? • Birisi cevabınızın doğru olmadığını söylese, bunun doğru olduğunu nasıl açıklarsınız? Bunu başka bir şekilde açıklayabilir misin? • Arkadaşınıza / üçüncü sınıf öğrencisine bunu yapmayı öğretmek zorunda olsaydınız, bunu nasıl yapardınız? • Bunu yaparken kendinize ne söylersiniz? Yüksek sesle söyle ki senin düşünceni de anlayabileyim.
Seçilen katılımcılara sorulan ortak sorular	• Bu sembolün (eşittir işaretini göstererek) adını biliyor musun? • Eşittir işareti senin için ne anlam ifade ediyor? • Matematikte hangi durumlarda eşittir işaretini kullanırız? • Sence eşittir işaretinin söylediğinden başka bir anlamı var mı? • Bu soruyu çözerken onluk taban bloklarını kullanarak çözüm yöntemini bana yüksek sesle açıklayabilir misin?
Görüşmeler esnasında ortaya çıkan sorular	• 4. Soruda (6 + 3 + 7 = 5 + _) neden tüm sayıları topladın? Sence aradaki eşittir işareti sonucu değiştirir mi? • Bu sorularda boşluklara hangi sayıları yazarsın? Bu iki soru arasındaki fark nedir? (sorular 15 - 7 = _ + 5 ve 15 - 7 = 5 + _) • 14. Soruyu (18 = _ - 8) çözerkenki stratejin nedir? Bu soru sana bir yerden tanıdık geliyor mu? • Neden 5. Soruda (8 + _ = _ + 7) boşluklara sırasıyla 1 ve 2 yazdın? • Soru 5 (8 + _ = _ + 7) ve soru 8'deki (6 x _ = 40 - _) boşluklara başka sayılar gelebilir mi? • Şu sonucu nasıl buldun; 160 = 80? Benim için adım adım tekrar yapar mısın? • Neden 6. Sorudaki (160 = _) boşluğa yalnızca 160 yazılacağını düşünüyorsun? • Neden 14. Sorunun (18 = _ - 8) 12. sorudan (_ + 5 = 2 x 8) daha zor olduğunu düşünüyorsun? • Neden testteki tüm soruları yanlış cevapladığını düşünüyorsun? • Neden matematikte yeterince iyi olmadığını düşünüyorsun?

APPENDIX E: 15-POINT CHECKLIST FOR A GOOD THEMATIC (TA)

Table E.1. 15-Point Checklist for a Good Thematic Analysis (TA).

Process	**No.**	**Criteria**
Transcription	**1**	The data have been transcribed to an appropriate level of detail, and the transcripts have been checked against the tapes for 'accuracy'
Coding	**2**	Each data item has been given equal attention in the coding process
	3	Themes have not been generated from a few vivid examples (an anecdotal approach), but instead, the coding process has been thorough, inclusive and comprehensive
	4	All relevant extracts for all each theme have been collated
	5	Themes have been checked against each other and back to the original dataset
	6	Themes are internally coherent, consistent, and distinctive
Analysis	**7**	Data have been analysed - interpreted, made sense of - rather than just paraphrased or described
	8	Analysis and data match each other - the extracts illustrate the analytic claims
	9	Analysis tells a convincing and well-organised story about the data and topic
	10	A good balance between analytic narrative and illustrative extracts is provided
Overall	**11**	Enough time has been allocated to complete all phases of the analysis adequately, without rushing a phase or giving it a once-over-lightly
Written report	**12**	The assumptions about, and specific approach to, thematic analysis are clearly explicated
	13	There is a good fit between what you claim you do, and what you show you have done - i.e., described method and reported analysis are consistent
	14	The language and concepts used in the report are consistent with the epistemological position of the analysis
	15	The researcher is positioned as active in the research process; themes do not just emerge?

APPENDIX F: INTERVIEW TRANSCRIPTIONS CITED IN THE RESULTS

Table F.1. Interview Transcriptions Cited in the Results (Turkish).

	Ahmet: Yani öğretmenim kesin yanlış olabilir ama neyse. 51 13 toplarım
1	hocam. Cevabı 64. Sonra hocam sonra bunları toplarım. 64 ile 51'i toplarım... Öyle bulabilirim yani.
2	Ahmet: Hmm...8...21... 13 gelebilir... 15'le 7 yi çıkardım. Sonra 5 i topladım 13 çıktı... 10 17 5.... Gene aynı konu söz konusu olabilir. 15 7... O zaman şey olur bu...8 oluyor.
3	Gamze: 15'ten 7 çıkınca 8. Bu 8'le de 5'i toplicam. Sonra buraya başka bir şey yazacağımızı düşünmüyorum.
	Araştırmacı: Peki 8'le 5'i toplarsan kaç olur?
	G: 13.
	A: 13'ü nereye yazacaksın peki burada başka boşluk vermemiş bana.
	Ne yapacağım?.
	G: ...Kendim yazarım.
	A: Peki. 10. soruya bakalım ne diyor.
	G: Hmm...Önce bunları toplayacağız... Ay çıkaracağız... 5 ile de 5'i toplayınca 8 çıkıyor, o da 3
4	Ebru: İlk önce 51 ile 13 ü toplarım. 4, 6, 64 buldum. E şey, sonra bu sayıya 51 eklerim... 11, 115.
5	Ebru: İlk önce yine aynı şekilde 7, 6, 3, eşittir. 7 ile 6'yı topladım. ...13 ediyor, bir 3 daha eklersek 16 oluyor. Sonra şu 5'i de ekliyim 16'ya. Bu sefer de 21 oluyor.
6	Ahmet: Ben şunları şöyle toplarım. Cevap 16. Sonra gene aynı şekilde. Sonra 21? Öğretmenim burada 6. 6'yla 3'ü topla diyor. Sonra bulduğun sayıyla 7'yi topla. Sonra bir de 5'i topla diyor.
7	Ebru: Bu soruda, ıı şey, ilk önce şu iki sayıyı birbirinden çıkartıyoruz yani 15 - 7 yapıyoruz 14,13,12,11,10,9,8. 8 kalıyor. Daha sonra 8 + 5 = 13 oluyor? Ee şey, burada yine aynı şekilde ilk önce 15'le 7'yi çıkartıyoruz çünkü yine ? işareti var. 10 kaldı. 14,15,... 10,9,8...Bir dk aynı soruyu ben arka sayfada yapmadım mı?.. Tıpatıp aynısı, aynı sorular... Yine 13.
8	Remzi: ...15'ten 7'yi çıkarıp 5'le toplicaz sonucu... 13 ediyor 8'le 5'i topladığımızda.
9	Ebru: Iı şey, burada ilk önce 8 ve 2 çarparız çünkü çarpma işareti var. 8 çarpı 2 eşittir 16. Sonra burda 6 ve 5 olduğu için 16yla 5i toplamalıyım... 11 elde var 1. 21 oluyor. 21 buranın cevabı bence.

Table f.1. Interview Transcriptions Cited in the Results (Turkish) (cont.).

10	Araştırmacı: Bir de şu son soruyu merak ediyorum. Önce bir sesli oku ne soruyormuş.
	Nazlı:18 eksi bir sayı eşittir 8.
	A: Burası eşittir. Tekrar okur musun?
	N: 18 eşittir kare eksi 8
	A: Şimdi o boşluğun karenin içine ne gelecek acaba?
	N: 10
	A: Peki yaz 10'u. Şimdi oku bakalım sesli bi şekilde.
	N: 18 eşittir 10 eksi 8. Olmuyor!
	A: Ne yapacağız peki?
	N:...
	A: Değiştirebilirsin cevabını, silebilirsin?
	N: 18 olsun eşit olsun.
	A: Şimdi oldu mu? Sesli bir şekilde okur musun?
	N: 18 eşittir 18 eksi 8... 10.
11	Ali: Çünkü orda 5 olduğu için 2. Öyle buldum.
12	Nazlı: 2 ile 3'ü toplayınca 5 oluyor. 5 ile 7'yi toplayınca da 12 oluyor.
13	Deniz: Evet bu sefer 5 ile 7'yi toplayacağız önce. 12 oluyor, ondan sonra? 12'den de 3'ü çıkaracağız 9.
	Araştırmacı: Peki neden 12'den 3'ü çıkardın?
	D: Burada olabilecek sayıyı bulmak için, çünkü 3 ile 9'un toplamı
	5 ile 7'nin toplamına eşit.
14	Erdem: 5 + 7 12. Iıı... Orası 12 olduğu için ıı... 12 - 3 9. Burası 9. Burası da 12.
15	Emre: 5+ 7 12. 9'la 3'ü toplayınca 12 oluyor. O zaman bu da 9 olur.
16	Melek: O zaman burada da 15'ten 7'yi çıkarıp 8 yani buranın da 3 olması gerekiyor ki bunu birbirinden çıkarınca 8, bunları birbirine toplayınca 8 olması gerekiyor. O yüzden eşit... Araştırmacı: Peki 10.soru? M: Aa 10. soru da yine aynı. Yani burası çıkarınca 8. Bununla da toplayınca 3 ile toplayacağım, 8... Birbirine eşit.

Table f.1. Interview Transcriptions Cited in the Results (Turkish) (cont.).

17	Engin: Ee...Öğretmenim eşittir...Şu şöyle ya, işte aynısı olarak tam tersi. Yani sonucu aynı çıkıyor... Örneğin bunlar bu kadar ya topluyoruz ya bir tane sayı çıktı, ve eşittir? Yerlerini değiştirsek aynı sonuç çıkar... Öğretmenim cevap olarak şey vermiştim, 13 koymuştum oraya.
18	Engin: Öğretmenim ikinci soruda da bunların toplamı... Bu 5 ile neyi toplarsak ikisi eşit olur yani sonuçları... Buraya 11 geliyor... Öğretmenim şunları toplayıp hani 16 çıkıyor ya, işte eşitlemek için 5'ten çıkarıyorum buluyorum.