TEACHERS’ CONTENT KNOWLEDGE IN TEACHING SLOPE OF A LINE

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ABSTRACT

TEACHERS' CONTENT KNOWLEDGE IN TEACHING SLOPE OF A LINE

The purpose of this study was to investigate teachers' content knowledge - subject matter knowledge (SMK) and pedagogical content knowledge (PCK)-during teaching the slope of line in eighth grade. The study focused on two pre-service, two novice, and two experienced primary mathematics teachers'content knowledge in instruction. The study included a semi-structured pre-interview with the participants on the ways they were planning to teach, observation and video recording of lessons on slope of line, and finallya semi-structured post-interview. Instructions and interviews were transcribed and coded by open coding. The video recorded data were analyzed in terms of the units of a framework, Knowledge Quartet (KQ), and triangulated with the interview data. Findings were reported in four sections. The sections in which the findings were reported in terms of the units of the analytical framework were (i) the pre-service teachers (no official teaching experience), (ii) the novice teachers (1-3 years of teaching experience), (iii) the experiencedteachers (3-5 years of teaching experience), and finally (iv) the comparison of teachers' content knowledge. Findings were reported in terms of the four units of the framework with a comparison table among the participants.The study of the slope of lines provided a rich source to interpret teachers' content knowledge in this mathematical concept and its teaching (SMK and PCK). The study provided that foundational knowledge was significantly observed in teachers' instruction. In addition, this type of knowledge was considerablysignificant in experienced teachers' instructions. Findings indicated also that as teachers become more experienced, they may have more robust content knowledge (SMK and PCK). As experienced teachers differed from their novice and pre-service colleagues, not much significant differences were observed between groups of novice and pre-service teachers in terms of their content knowledge of teaching slope of a line based on KQ.

ÖZET

ÖĞRETMENLERİN DOĞRUNUN EĞİMİNİN ÖĞRETİMİ İLE İLGİLİMESLEKİ ALAN BİLGİSİ

Bu araştırmanın amacı matematik öğretmenlerinin ders sırasındaki mesleki alan bilgilerinin incelenmesidir. Araştırma beş yılı aşmamış olmak kaydı ile farklı sürelerde öğretmenlik tecrübesine sahip altı öğretmen ile gerçekleştirilmiştir. Araştırmacı, katılımcılarla derslerini nasıl anlatmayı planladıklarını öğrenme amaçlı yarı-yapılandırılmış ders öncesi mülakat, doğrunun eğimi konusunun anlatımı sırasında video kayıt ve gözlem raporları ve en son yarı-yapılandırılmış ders sonrası mülakat gerçekleştirmiştir. Elde edilen verilen çözümlenmiş ve açık kodlama ile kodlanmıştır.Ders kayıt verileri Bilgi Dörtlüsü (Knowledge Quartet) ünitelerine göre analiz edilmiş elde edilen verilerin mülakatlardan elde edilen verilerle üçlenerek geçerliği ve güvenirliği artırılmıştır. Bulgular dört grupta sunulmuştur. Bu gruplar sırası ile öğretmen adayları (öğretmenlik tecrübesi olmayanlar), mesleğe yeni başlamış öğretmenler(1-3 yıl öğretmenlik tecrübesi olanlar), deneyimli öğretmenler (3-5 yıl öğretmenlik tecrübesi olanlar) ve en son olarak grupların derslerindeki mesleki alan bilgisi karşılaştırması şeklinde verilmiştir. Veriler her grup için modelin üniteleri bağlamında sunulmuş ve karşılaştırma tabloları verilmiştir. Araştırmada doğrunun eğiminin çalışılmış olması bu konu hakkındaki öğretmen bilgisini ve konunun öğretimi hakkında önemli ve çeşitli bilgileri ortaya koymuştur. Bu çalışma modelin ilk ünitesi olan Temel ünitesinin öğretmenlerin ders işleyişlerindeki önemini göstermiştir.Ayrıca bu çalışmada bu ünite açısından bakıldığında öğretmenlerin belli bir dönemi aşan deneyimiyle, bilgilerinin artışı arasında bir ilişki olduğu ortaya konmuştur.Öğretmenlikte deneyim süresine bağlı olarak hem alan bilgisinin hem de bu alanın nasıl öğretileceğine dair bilgilerin artışı yine bu çalışmanın bulguları arasında yer almaktadır. Mesleğe yeni başlamış öğretmenler ve öğretmen adaylarını açısından ise oldukça paralel bulgular elde edilmiştir.

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LIST OF SYMBOLS

m	Slope
ℕ	Natural NumbersSet
ℝ	Real NumbersSet
Δ**Y**	Change in y- coordinates
ΔX	Change in x- coordinate

LIST OF ACRONYMS /ABBREVIATIONS

CCS	Common Core State Standards
CK	Content Knowledge
KQ	Knowledge Quartet
MEB	Milli Eğitim Bakanlığı
MKT	Mathematical Knowledge for Teaching
NCTM	National Council of Teachers of Mathematics
PCK	Pedagogical Content Knowledge
SMK	Subject Matter Knowledge

1. INTRODUCTION

Even though, George Bernard Shaw contrasted by: “He, who can, do. He, who cannot, teaches”, the great philosopherAristotle said that “Those that know, do. Those that understand, teach”. Also, some argue that the best way to *completely* learn something is to teach it (Leikin and Zazkis, 2010). Neither first nor the second claim can be *proven* or *falsified* scientifically. However, they inform us the way teaching is perceived in two extreme ways. Shulman (1986) suggested that Shaw’s negative view on teaching may result from the lack of knowledge that teachers ought to know. And, among the long list of knowledge kinds that teachers should know, the lack of content-specific knowledge maybe a reason in deserving Shaw’s accusation to teachers.

Researchindicate that content-specific knowledge of teachers is imperative in reaching an effective teaching. Teachers should know human psychology, communication, management, learning theories and many others. They should also know the content they will teach, the reasoning behind this content, the cases it is guaranteed, the effective ways of representing the content so that it becomes easier to understand (Shulman, 1986).

The work of Shulman (1986) and several subsequent studies haveindicated that teachers’ content knowledge is a prerequisite for effective teaching(Ball*et al*.,2008; Rowland, 2010). Teachers need to know a great deal of knowledge in terms of subject matter knowledge. Transferring this knowledge into pedagogically powerful forms -which is pedagogical content knowledge in general terms-, is also necessary in teaching.Many mathematics teacher education studies, hence, focused solely on mathematics teachers’ knowledge for teaching (e.g. Ball *et al*.,2008). These studies suggested the amount and kind of knowledge that teachers ought to know.However, how this knowledge specifically is visible in mathematics teachers’ practice has not been investigated in a setting where teaching is in action.

Curiosity is the first source of motivation for learning. I curiously wanted to learn the kinds of content-specific knowledge which areobservable in teachers' practice of teaching.To reach a sound answer to this question, the investigation should focus not on the attributes of teachers but the teaching. Since teaching is dynamic and situated (Fennema and Franke, 1992; Hodgen, 2011; Rowland and Ruthven, 2011; Wagner*et al*.,2007) this investigation would be limited by a paper-pencil test. It would not *assess* teachers but *observe*and *analyze* their instruction. In addition, the investigation should take placein the act of teaching, not outside of teaching practice. The discussion on the nature of teachers' knowledge yield a consensus that the knowledge needed in teaching is dynamic, bettervisible via practice, and should be studied in actual classroom setting (Fennema and Franke, 1992; Hodgen, 2011; Rowland and Ruthven, 2011; Wagner *et al*., 2007).

The ideasmentioned above motivated me to analyze mathematics teachers' act of practice with a framework which can elaborate the content-specific knowledge that appearsin teaching. The Knowledge Quartet (Rowland*et al*.,2005) may be effective to be used as an analytical framework to investigate teachers' knowledge during instruction. Hence, the purpose of this study is to investigatemathematical content knowledge of mathematics teachers, with varying teaching experience, during teaching a mathematical concept, the slope of a line.

2. LITERATURE REVIEW

Researchers and policy makers have an ongoing interest on improving students' learning. New standards, assessments, and curricula have been initiated, however, they do not automatically enhance student learning: teachers must use these resources at classrooms (Cohen*et al.*,2003).

One of the most important influences on students' learning is their teachers (Darling-Hammond and Ball, 1998; Even, 1993; National Commission on Teaching and America's Future, 1996). Researchers argue that it is naturally true that better learning will result primarily from better teaching (Darling-Hammond and Rustique-forrester, 2005). Sullivan (2008) suggesteda number of features of effective teaching. Many of teacher-related characteristics such as teacher education, classroom management and quality assessment would play substantial role on teaching effectively. Another feature that merit scrutiny is teachers' content knowledge since teachers' effectiveness is influenced by the knowledge they possess (Aslan-Tutak, 2009; Gilbert and Gilbert, 2011; Wagner *et al.*, 2007).

In addition to its effect on students' learning, teachers' knowledge is influential in shaping their practices (Borko and Putnam, 1996; Calderhead, 1991, 1996; Even, 1993; Even and Trosh, 1995; Fennema and Franke, 1992; Rowland and Ruthven, 2011; Sherin, 2002; Shulman, 1986).Teachers consult to various types of knowledge when they plan and implement instruction. Hence, the importance of teacher knowledge raises essential discussions on the nature of this knowledge (Wagner *et al.*, 2007).

The nature of teachers' knowledge has been studied in several perspectives. In contrast to its extensive use and focus in educational research, there is a considerably less agreement on the components of teachers'knowledge. A number of essential features of teachers' knowledge have been agreed upon but a complete agreementhas not been reached among teacher education researchers (Ball *et al.*,2001; Fennema and Franke, 1992; Silverman and Thompson, 2008; Thompson, 1992). Shulman and his colleagues proposed an analysis of the kinds of teachers' knowledge. Shulman (1987) proposed seven categories of teacher knowledge.

- General pedagogical knowledge, such as classroom management principles and strategies;
- Knowledge of learners' characteristics;
- Knowledge of educational contexts;
- Knowledge of educational ends, purpose and values and their philosophical and historical grounds;
- Subject matter knowledge;
- Curricular knowledge;
- Pedagogical content knowledge (p.8).

According to Shulman (1986),teacher evaluation criteria in the United States, focused more on general characteristics of effective teachers in 1980's. Shulmandrew attention that researchers at the time were studying more generic aspects of teaching, such as classroom management and student motivation (Shulman, 1986). He claimed a missing paradigm in studying teachers' knowledge as a blind spot with respect to the content to be taught. Shulman suggested that content-specific knowledge is essential and should be included in any list of teachers' knowledge. Shulman's conceptualization became essential and widely cited in studying teacher education (Ball*et al.*,2008; Rowland *et al.*, 2007; Wagner *et al.*, 2007; Petrou and Goulding, 2011).

Among seven categories, the first four were related to content-free and the last three ones were on content-specific knowledge (Rowland and Turner, 2007). The last three categories -subject-matter knowledge, curricular knowledge and pedagogical content knowledge,-constitute the content-specific knowledge that teachers have (Shulman, 1986).

Shulman defined subject matter knowledge (SMK) as the knowledge of the subject and its structural organization. It refers to "the amount and organization of knowledge per se in the mind of the teacher" (Shulman, 1986, p.9). Shulman further divided subject matter knowledge into substantive knowledge and syntactic knowledge. The substantive structures are the key facts, basic concepts and principles, and the way they integrate to result in other facts. On the other hand, the syntactic structure of a subject refers to the family of ways in which truth or falsehood, validity or invalidity is built.Shulman advocated that SMK of a teacher should be at least equal to the person who is merely majoring in subject matter. Furthermore, the teacher's comprehension should exceed the knowledge of subject matter. In addition to that, the teacher must have in depth understanding of the reasoning behind the subject matter, the cases it is guaranteed, or weakened and ignored. This provides that a teacher should be able to both define the accepted or accumulated truths in the domain and also explain to students why a proposition is warranted, why it is worth learning, and how it connects to other propositions. Also, the teacher should understand and be able to articulate the reasons that they make a concept more central whereas making others more peripheral (Shulman, 1986).

Another category of content knowledge is curricular knowledge. Shulman (1986) claimed that teachers should be knowledgeable about the curriculum of the subject matter they teach. A teacher should know all available alternatives in curriculum, the way topics are arranged and be able to relate this knowledge to other classes and grade levels. Curricular knowledge constitutes a necessary amount of knowledge needed by teachers.

> The full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances (p. 10).

Shulman's last category of content knowledge is pedagogical content knowledge (PCK) (Shulman, 1986). PCK is "the special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (p. 9). It goes beyond knowledge of subject to a dimension of subject matter knowledge for teaching. It is "the ways of representing and formulating the subject that make it comprehensible to others" (p. 9). It includes "the most useful forms of representation of ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations, in a word, the ways of representing the subject what makes it comprehensible to others". Since more than one way may effectively represent the matter, teachers must have alternative forms of representations derived from both research and practice (Shulman, 1986).

PCK also includes knowing the reasons that make a topic easy or difficult to understand. Teachers should know age and background-related conceptions and preconceptions students bring to the educational settings. Misconceptions, for instance, come into these former conceptions in which teachers should be veteran in editing the preconceptions or altering them with correct ones (Shulman, 1986). The research on these issues are important since the findings provide essential information to the researcher on the kind of representations, illustrations, attempts which are more effective than others (Ball *et al.*, 2008; Shulman, 1986).

Shulman's study suggested that content-specific knowledge is essential in studying teachers' knowledge. Its importance comes from the relationship between content-specific knowledge and content-free knowledge. Content-free knowledge cannot remedy any lack in content-specific knowledge. Furthermore, solitary content knowledge would be useless pedagogically as content-free skills (Shulman, 1986).

Shulman's categorization of teachers' content knowledge (SMK and PCK) received attention in several subject specific studies. Specifically, Shulman's model led some important research in content knowledge of mathematics teachers' which yield to new models to study mathematics teachers' content knowledge (Ball *et al.*, 2008; Fennema and Franke, 1992; Rowland *et al*, 2005). In addition to Shulman's work, those current models of mathematics teachers' knowledge claim that content knowledge is necessary for effective teaching (Ball, 1990b; Fennema and Franke, 1992).

2.1. Mathematics Teachers' Knowledge

Shulman's conceptualization on teachers' knowledge has been used heavily by mathematics education researchers. Researchers have been studying mathematics teachers' knowledge from several perspectives. While some researchers study on pre-service teachers' understanding of various concepts in mathematics (Ball, 1990a; Rowland *et al*., 2000), others have focused on investigating the relationship between SMK and PCK and teaching (Even, 1993; Hill*et al*., 2005; Rowland*et al*., 2005).Following sections will summarize two important approaches in investigating mathematics teachers' knowledge.

2.1.1. Mathematical Knowledge for Teaching

A group of researchers at the University of Michigan studied both the teaching of mathematics and the mathematics used in teaching (Ball *et al*., 2008; Hill *et al*., 2005). The aim of the research was to develop a practice-based theory of content knowledge needed for *assessing* mathematics teachers. The team used qualitative methods to collect and analyze data in order to investigate what teachers do as they teach mathematics, and the mathematical knowledge and skills required to teach mathematics effectively. The study led to a model,Mathematical Knowledge for Teaching (MKT),and observation-based instrument. The studyis a result of the attempt to validate Shulman's conceptualization by developing reliable and valid measures of mathematical knowledge for teaching (Ball *et al*., 2008).

The MKT modelproposesa practice-based categorization system, which is supposed to identifycritical components of knowledge for teaching (Hill*et al*., 2007;Hill *et al*., 2008). It suggests three sub-domains for Shulman's SMK. First of all, common content knowledge (CCK) refers to the kind of mathematical knowledge and skills that may be used in any setting, not necessarily for teaching.It includes individual's ability to solve mathematical problems and find answers correctly. Second, specialized content knowledge (SCK)is the mathematical knowledge used in teaching, but not taught to students and not typically used by the people who are outside of teaching (e.g., knowing the way to represent ideas andsuggest explanations). Finally, horizon content knowledge includes teachers' knowledge of mathematical topics inter-relationships over the span of mathematics included in the curriculum (Ball *et al.*, 2008).

Another group of domains in MKT model are related to Shulman's PCK (Ball *et al.*, 2008). One of them is knowledge of content and students (KCS) which is described as the interaction of knowledge of mathematics and students' mathematical conceptions. KCS is composed of knowledge about students and knowing mathematics. Teachers should predict students' difficulties and impediments, act accordingly to students' thinking and responses, and choose appropriate examples, representations and problems during teaching. A second category in PCK is knowledge of content and teaching (KCT). Researchers described it as the interaction of knowledge of mathematics and teaching methods. KCT refers to the knowledge that guides teachers on the lesson's sequencing. It also refers to the knowledge of the possible advantages and disadvantages of using various representations (Ball *et al.*, 2008). The last category is knowledge of content and curriculum which includes the necessary knowledge about the curriculum which is needed by teachers.To conclude, MKT suggests that content knowledge is a central component for teaching. In addition, it raises attention to the study of mathematics education as a basis for theorizing what teachers should know to teach effectively (Hill *et al.*, 2004).

The model has been adapted and usedwidely in mathematics teacher education studies. However, there exist a number of important discussions on the nature of teachers' content knowledge which also suggests questions on the MKT model. For example, beliefs about the nature of mathematics (Goulding *et al.*, 2002) and emotions (Hodgen and Askew, 2007) may also be critical in the way teachers approach to mathematics teaching. MKT model does not include the effect of teachers' beliefs on teaching mathematics.

Fennema and Franke (1992) claimed that the nature of teachers' knowledge is "a large, integrated, and functioning system where its components are difficult to isolate" (p. 148).In contrast, MKT model provides further sub-categories to the categories Shulman suggested, such as SCK or KCS. This perspective may hinder understanding the complex and interacting dimensions of teacher knowledge(Askew, 2008; Aubrey, 1997; Sherin, 2002). Teachers might not think just in terms of their subject matter knowledge or their pedagogical content knowledge for instruction; instead, they tend to call to both types of knowledge(Sherin, 2002). In brief, there may be larger elements of teacher knowledge which is not possible to be categorized as subject matter knowledge or pedagogical content knowledge.

Fennema and Franke (1992) proposed that the knowledge needed in teaching is interactive and dynamic in nature. Similarly, Hodgen (2011) claimed that mathematics knowledge is situated. The discussion on the nature of teachers' knowledge yield a consensus that the knowledge needed in teaching is dynamic, better visible in practice, and should be studied in its actual setting (Fennema and Franke, 1992; Hodgen, 2011; Rowland and Ruthven, 2011; Wagner *et al*., 2007). Assessing or developing a mathematical knowledge for teaching would unlikely be successful unless it carefully takes the classroom context of teachers' professional work into account (Hodgen, 2011; Rowland and Ruthven, 2011; Sherin, 2002). To conclude, focusing on the use of teacher knowledge in the practice of teaching mathematics may be more informative in studying teachers' knowledge (Hodgen, 2011; Rowland and Turner, 2007).

It should be noted that MKT is a practice-based model in which researchers used lesson videos, students' works and other useful instructional sources of classroom teaching. However, implementation of the instrument is currently on a paper-pencil test. As a conclusion, even though the MKT model suggests essential indications in studying teachers' knowledge itmay not be an appropriate selection as an analytical framework in studying mathematics teachers' knowledge *during teaching*. This has also been indicated by the scholars of the MKT model(LMT, 2006, p. 3):

> Researchers have developed open-ended, interview, and multiple-choice assessments ofmathematical knowledge for teaching. However, none of these methods is satisfactory in one critical way, in that none can actually measure the quality of the mathematics in actual classroom instruction. Teachers' performance on pencil-and-paper assessments (or oral interview tasks) may or may not correlate with what they can actually do with real-life content, materials, and students… (p. 3).

The MKT have also directed the study of teachers' content knowledge to in-class teaching. In other words, the scholars claimed that they were currently involved in validating their pencil-and-paper measures of teachers' mathematical knowledge for teaching through observing and video recording of the assessed teachers' instruction (LMT, 2006).MKT is limited in studying mathematics teachers' content knowledge during instruction so another model for mathematics teachers' model mightserve better in studying mathematics instruction. One framework which may provide studying teachers' content knowledge during instruction is the Knowledge Quartet(Rowland *et al*., 2005).

2.1.2. The Knowledge Quartet

One of the studies which investigate mathematics teachers' knowledge in the act of practice is attempted by a group of researchers in the United Kingdom. Rowland, Huckstep and Thwaites, (2005) investigated British pre-service primary mathematics teachers'content knowledge in their practice of teaching. The rationale to the research was that both pre-service teachers and their mentors in the UK, generally, focused more on content-free knowledge during school practicum. They, rarely, consider content-specific knowledge and its *display* in actual classroom setting (Rowland *et al*., 2005). The aim of the research was to develop an empirically-based conceptual framework for pre-service teachers' lesson analysis with an emphasis on the content of the lesson and the effect of pre-service teachers' content knowledge on their teaching. Hence, such a framework would provide a number of important ideas and factors about content-related knowledge within a small number of categories (Rowland *et al*., 2005).

The researchers used a grounded theory to investigate pre-service teachers' content knowledge by observing and video recording participating teachers' mathematics lessons. It took place in the context of a one-year internship course in UK. Six trainees who chose to focus on early-year mathematics of ages 3-8 and another six trainees who focused on primary year mathematics of ages 7-11 were randomly selected from 149 trainees. For each selected trainees, their two mathematics instructions were observed and video recorded. The researcher wrote a summary of the lesson he observed based on his memory and field notes (Rowland *et al.*, 2005).

The second step in the development of the Knowledge Quartet (KQ) was identification of the aspects of trainees' lessons that may provide information on content knowledge, -SMK and PCK-, of the pre-service teachers. The codes for video records were first invented than rationalized and reduced by negotiation and agreement among the members of the research team (Rowland *et al.*, 2005).

Researchers revisited each video record after generating codes. They identified and tagged significant episodes with one of these codes offering also the rationale for tagging and analysis of the role of trainees' content knowledge. The last step was to reach a framework which is easy to use. Researchers claimed that understanding of the four units of the KQ would better serve than knowing all eighteen codes. The research team concluded on four broad units: (i) foundation, (ii) transformation, (iii) connection, (iv) contingency(Rowland*et al.*,2005). Codes for each unit will be mentioned in data analysis section.

The first unit of the KQ is rooted in the *foundation* of teachers' knowledge, beliefs and understanding of mathematics and its teaching (Rowland *et al.*,2005). It encompasses teachers' knowledge, understanding and recourse to their learning. Since "it is about knowledge possessed, irrespective of whether it is being put to purposeful use" (p. 112), it differs from the remaining three units. For example, a teacher may know what it means to divide a number by zero. Or, the teacher may know the sources where he can ask for information about it.

Researchers claimed that the remaining three units originate from a foundational underpinning. They provided that teachers' knowledge fundamentally determines the way they teach which also makes foundation as the most essential unit of the framework. The key components of foundation are:knowledge and understanding of mathematics per se; knowledge of significant guidance of the literature on how to teach it; and beliefs about mathematics, the reason and the way of learning mathematics (Rowland *et al*.,2005).

There is compelling finding that teachers' beliefs and instructional practices are correlated (Rowland *et al*.,2005; Thompson, 1992). Beliefs component of the unit has been taken in three different ways in the framework. It is composed of teachers' beliefs about (i) the nature of mathematics itself and various philosophical perspectives on the nature of mathematical knowledge, (ii) purposes of mathematics education and the reason for studying mathematics topics in school, and (iii) conditions under which pupils will learn mathematics best (Rowland *et al*.,2005). The focus on beliefs indicates Shulman's arguments on the syntactic knowledge since this kind of knowledge deals with the way teachers perceive mathematics as a discipline, and these perceptions relate to substantive knowledge within a specific content knowledge (Murphy, 2012).

The second unit of the KQ is *transformation*.Teachers' own meanings and descriptions are transformed and presented in ways aiming students to learn it(Rowland *et al*.,2005). For instance, a teacher's use of a 100 square as a model or representation of the sequence of two-digit positive integers provides important information on the transformation. The unit mainly corresponds to the Shulman's model of transformation and pedagogical reasoning. In his conceptualization, Shulman (1987) emphasized the transformation of a teacher's knowledge of a subject into pedagogical content knowledge and consequent pedagogical actions by ''taking what he or she understands and making it ready for effective instruction'' (p. 14).

*Connection*concerns the depth, breadth and coherence of relationships observed in teaching. The teacher unifies the subject matter and draws out coherencewithin a single lesson, or across a series of lessons (Rowland *et al.*,2005).The rationale given for this unit is that intellectual depth and breadth “is a matter of making connections’’ (Ma, 1999, p. 121). Coherence which includes the arrangement of topics of instruction, tasks and exercises within and across lessons is a key component in this unit (Rowland *et al.*,2005). To illustrate, a teacher’s sequencing of concepts throughout lessonssuch as building functions on relations, and describing relations through Cartesian product may illustrate the way connection surfaces in the practice of teaching.

The fourth unit of the framework, c*ontingency*,is described as a teacher’s ways to “think on students’ feet” and respond appropriately to the contributions made by students during instruction(Rowland *etal*.,2005). It can be seen in the teacher’s willingness to deviate from her own agenda when to develop a student’s unanticipated contribution. It involves responding appropriately to the content specific events and ideas which occur during instruction. In brief, it is about contingent action of teachers in the classroom (Rowland *et al.*,2005). A teacher may shift the lesson’s scope form the way she planned to the way students’ suggestions direct during interaction in the classroom. To illustrate, a third grade student’s claim thatsix is both odd and even gives an opportunity to visit odd and even number concepts, grouping, matching, and comparison.

Table2.1.The Knowledge Quartet.

Unit	Nature	Description
Foundation	Knowledge in propositional form	Teachers' propositional knowledge, beliefs and understanding of mathematics and its teaching.
Transformation	Knowledge-in-action	Teachers' own meanings anddescriptions are transformed and presented in ways aiming students to learn it.
Connection	Knowledge-in action	The depth, breadth and coherence of relationships observed in teaching.Within a single lesson, or across a series of lessons, the teacher unifies the subject matter and draws outcoherence.
Contingency	Knowledge-in-interaction	Teacher ways to 'think on her feet' and respond appropriately to thecontributions made by students during instruction. It can be seen in the teacher's willingness todeviate from her own agenda when to develop a student'sunanticipated contribution.

Byanalyzing pre-service mathematics teachers' lessons through the KQ in various studies, researchers suggested also that the framework is comprehensive as a tool for thinking about the ways that mathematics teachers' content knowledge becomes visible in the classroom (Rowland *et al.*,2005; Rowland *et al.*,2007; Turner, 2012).

The KQ has been grounded in classroom practice, and the findings have been open to enhancement and revision according to any new instructional data. In this respect, KQ can be effectively used to investigate the way teachers' content knowledge enacts during instruction.

2.2. The Teaching and Learning Slope of a Line

Starting from very early grades, the curricula for school mathematics include a number of important topics which are regarded as crucial in students' mathematical development. Functions are one of these topics which is emphasized in mathematics curricula universally (Cooney *et al.*,2010).

Mathematics education reforms and curricula put substantial emphasis on the concept of function. Chazan, Yerushalmy, and Leikin, (2008) suggested that sign of this attentionis visible in shifts in the curricular approach to teaching algebra from equation-based to function-based. In addition, NCTM (1989, 2000) suggests that functions should be focused throughout students' school years.

Curriculum, curricular materials, and textbooks are influential in shaping teachers' mathematical understanding of slope and their pedagogical content knowledge of teaching it (Stump, 1999). Curricular documents as well as a large body of research in mathematics education suggest that the concept of function is an essential objective of mathematics teaching and learning before the study of mathematics in high school.Though whether mentioned explicitly as function in curricula, students are expected to understand linear functional relations, proportional relationships, lines, slope and its relationship to equation and graph (CCS, 2010; MEB, 2009; NCTM, 2000).

Common Core State Standards for Mathematics is a document defining the concepts and their scope thatstudents in the U.S. should learn in their study of mathematics. This document has also emphasized the importance of teaching and learning functions by providing that linear function is one of the critical areas of eighth grade mathematics. Students should understand connections between proportional relationship, line, and linear equation. Students are also expected to graph proportional relationship and recognize the unit rate as the slope of afunction (CCS, 2010).

Turkish National Mathematics Curriculum (MEB, 2009)does not include any objectives which requireteaching functionuntil grade nine. However, it suggests that proper attainment of some of the objectives at earlier grades will provide important skills and knowledge needed in learning functions in the future. To illustrate, it is advised that students should generalize global rulesin number patternsand express them algebraically. Pattern generalizations will then be related to equations with two variables in which one variable changes as the other is changed. These relationships will provide a way to learn functions meaningfully and conceptually in the future (MEB, 2009, p 98). Line, in general, also provides important opportunities to understand function. It involves an important aspect of function concept, particularly functional relations of the form y=mx+b in which x and y are called as variables and m and b are called as constant coefficients.As a result, the curriculum suggests addressing algebraic properties of line which basically represents linear functions. However, it should also be concerned that if textbooks do not relate slope and lines to concepts such as rate of change, covariation, linear functions, and proportionality then teachers may have serious difficulties in recognizing the required connections.

The crucial reason for emphasizing the teaching of functions at early grades is that it is a powerful organizing concept in understanding algebra and many concepts such as the concept of variable. In addition, transition from arithmetic to algebra is challenging for most of the students (Brenner *et al.*,1997) and linear functions serve as a means to understand algebra. Furthermore, functions are prerequisite for further advanced mathematical concepts such as limit (Leinhardt*et al.*,1990). In brief, learning linear functions prepares students to the study of function and study of functions prepares students for further study in mathematics.To conclude, functions should be focused throughout students' school years (Leinhardt *et al.*,1990; NCTM, 2000; Usiskin, 1999).

2.2.1. Understanding Functions

The concept of function can be viewed in two main ways; (i) a correspondence between elements of two sets or (ii) a covariation between two types of quantities (Smith, 2003). The modern set-theoretic conception of function requires to regard functions as collections of isolated ordered-pair matching. This static description, which is called as correspondence viewsuggests that functions do not need to be defined by any specific expression, follow akind of regularity, or be described by a graph with a smooth shape (Cooney *et al.*,2010; Coulombe, 1997). This is also called as the arbitrarinessof function (Even, 1993; Falcade *et al.*,2007). In brief, a function can be build with or without a pattern between elements of given two sets (Cooney *et al*.,2010; Stein*et al*.,1990).

Regarding functions as isolated ordered-pair matches has an utmost importance, but, it may not suffice in building a conceptual understanding of function.One crucial aspect ofunderstanding function -which also makes it more desirable to teach-,is the notion of covariation between two related variables of function (Falcade *et al*.,2007; Hines, 2002; NCTM, 1989). Behaving functions in this way iscalled as “covariation perspective” (Cooney *et al*.,2010; Coulombe, 1997) and will be described in more detail. It should be noticed that this perspective already involves the correspondence viewand it is more powerful thereby, it deserves more attention (Confrey and Smith, 1995; Smith, 2003).

Confrey and Smith (1995) explained covariation perspective of function as “the juxtaposition of two sequences, each of which is generated independently through a pattern of data values” (p. 67). According to researchers, the idea is realized through proceeding between successive values of a variable and then coordinating this pattern with proceeding between corresponding successive values of the second variable. In addition to providing the notion of isolated ordered-pair matching, this relation refers to a systematic pattern between two variables in which the value of one variable can be computed by applying the rule to the value of the second (Brenner *et al*.,1997). Main concern in this perspective is on “how changes in one variable relate to changes in another variable” (Cooney *et al*., 2010, p. 24).Cooney and colleagues (2011) distinguished covariation perspective as a focus on how outputs and inputs (or varying quantities) change in relation to each other. Coulombe and Berenson (1998) suggested the path to reach covariation as:

> (a) the identification of two data sets, (b) the coordination of two data patterns to form associations between increasing, decreasing, and constant patterns, (c) the linking of two data patterns to establish specific connections between data values, and (d) the generalization of the link to predict unknown data values (p. 88).

This section involved a brief discussion on the description of function concept. Literature indicates two fundamental and interrelated perspectives to define function which are correspondence and covariation. While correspondence view is absolutely important for students to comprehend, covariation perspective suggests it already. As a result, a teacher should be competent in elaborating both perspectives in teaching functions conceptually. The following will cover another essential issue which is necessary especially in conceptual understanding of function.

2.2.2. Representations of a Function

Most of the concepts in mathematics may appear in different ways. They may be displayed bydifferent labels, notations and representations. In addition, comprehension of a concept through one representation does not necessarily lead an understanding it via another representation (Even, 1990). To illustrate, Kaput's study (1992) proposed that many undergraduate students were not able to view graph of a function as a way to describe covariation between the variables of the function.

Conceptual understanding of a mathematical concept requires a synthesis of a number of different mathematical ideas and representations,and meaningful switchbetween each other (CCS, 2010; Even, 1990; Hines, 2002; MEB, 2009; NCTM, 2000;Rasslan and Vinner, 1995; Sherin, 2002; Stein *et al*.,1990).To illustrate, Schoenfeld, Smith and Arcavi (1993) suggested that competence in understanding line concept requires seeingit as a graph in the plane, as an equation,or numerically in tables.

Different representations highlight different characteristics of mathematical concepts. A representation may be more useful than others depending on the context or the purpose of teaching (Cooney *et al*.,2010).To illustrate, Saldanha and Thompson (1998) suggested using tables for showing the successive states of a variation if the aim is to present covariation as the coordination of sequences.A proper understanding of a concept should also include the reason and cases where using a representation serves wellthan the other. As a conclusion, mathematics teachers should know and be able to teach these notions to students.

Issues related to representations have been studied extensively by mathematics education community. Lobato and Bowers (2000)classified these studies as multi-representational perspective since these studies examine students' efforts to combine these conventional mathematical representations in general. Focus on representations of mathematical concepts provides a construct which is called as representational fluency. It has been studied among mathematics education scholars extensively hence, it is almost impossible to cite even a small number of these studies (Nathan *et al.*,2011).

Conventionally, functions are represented as algebraically, graphically, numerically in tables (tabular), or by verbal descriptions in school mathematics(CCS, 2010; Lobato and Bowers, 2000; NCTM, 1989). However, among those representations,theconnection betweenthe graphical and the algebraic representation of a function is central (Leinhardt*et al.*, 1990; Presmeg, 2006; Stein *et al.*,1990).

2.2.3. Linear Functions and Slope

Students'typical exposition to functions starts with linear ones (Cooney *et al.*,2010). Though linear functions are only a kind of functions they serve as a suitable starting point in understanding functions in general.Lines, unless vertical,are the graphical representations of linear functions. Geometrically, a function is called as linear if it can be represented by a straight line (Cooney *et al.*,2010). Regarding linear functions and lines together, slope is a fundamental mathematical concept (Anton *et al.*,2002; Rasslan and Vinner, 1995). Notion of slope is crucial in understanding the behavior of line, its graphical representation, and the functional relationship between the quantities (Stump, 1999).

Before to describe slope algebraically (and in functions perspective), geometrical meaning of slope will be elaborated.First of all, slope is a measure of a line's steepness and this numerical measure can take positive, negative, or zero values (Anton *et al*.,2002).Slope is defined for non-vertical lines. If a line is drawn in coordinate plane then computing slope is described asthe change in the y-coordinates divided by the change in the x-coordinates (of a line). The formula given for slope is as the ratio of changes in vertical to horizontal and formulated as$\frac{\Delta \mathbf{y}}{\Delta \mathbf{x}}$in which $\Delta \mathbf{y}$ stands for vertical changes and$\Delta \mathbf{x}$stands for horizontal changes between two points of a line. It is also given as the ratio of rise to run. The key condition in using the quotient form is that the axis should be scaled homogeneously (Zaslavsky *et al*.,2002). The reason is that slope is an algebraic entity of the line hence does not depend on the coordinate plane in which it is drawn (Rasslan and Vinner, 1995).

At geometrical perspective, examination of lines in coordinate plane suggests a number of essential mathematical ideas. For example, a line which has a positive slope rises to the left and a line which has a negative slope value fall to the right (Lobato and Bowers, 2000). This relationship indicates that a line which inclines to the right has positive and aline inclined to left has negative slope.

Slope is an attribute of line, though the procedures such as computing slope are done on line segments (Rowland, 2010). Conventionally, slope of a line is reached by isolating a segment of the line through choosing two distinct points on a line since any isolated segment of line provides the same slope. It does not matter which two points are chosen on a line, as long as they are distinct. In addition, once the points are chosen, there is not anorderrestriction in using points to the formula. One may use similarity of slope triangles to explain forslope being same between any two distinct segments on a line in the coordinate plane(Anton *et al*.,2002; Cooney *et al*.,2010).

Though slope has its roots in geometry, slope should also be regarded as a rate of changewhich in turn suggests meaning in "formulae, tables, physical situations, and verbal descriptions" (Stump, 1999, p. 125).Ifslope is regarded in algebraic terms, it suggests the notion of rate of change. A rate of change means "how one variable quantity changes with respect to another… rate of change describes the covariation between two quantities"(Cooney *et al*.,2010, p. 23). Slope can be taken as a ratio of the variation of one quantity to the associated variation of another quantity. This description suggests a covariation between two quantities and it is a crucial step in learning functions. Similarly, slope of a function is the rate of change of dependent variable with respect to changes in the independent variable (Anton *et al*.,2002; Lobato*et al*.,2003; Stump, 1999).

If examination of slope in functions is limited to linear functions, then the rate of change is unchanging which indicates a constant covariation.A constant covariation between dependent and independent variables suggests that each unit of increase in the independent variable results in the same change in the dependent variable. Linearity can be understood as the constant covariation. Hence, slope is a constant number in linear functions and linear function are represented by lines in coordinate plane. In brief, the unit rate of a linear function which is constantmay be interpreted as the slope of its graph(Cooney *et al*.,2010).

Linear functions are characterized by a constant rate of change between variables. Alternatively, if there is a constant rate of change between two variables then this relationship is characterized by linear functions. These ideas may be proven by reasoning on similarity of slope triangles whose hypotenuses lie on function's graph (which is a line by definition), and legs are built through vertical and horizontal lines (Cooney*et al*.,2010).

Linear functionscan be expressed by line equations. An equation, y=mx for a line that passes through the origin, and the equation y=mx+b for a line which intercepts y-axis at b can used for algebraic representation of a line. These equations are reached by using a slope triangle on a graphed line through (i) choosing an arbitrary point (x,y) and y-intercept of the line, (ii) calculating horizontal (x) and vertical distances (y-b), (iii) writing it as$m = \frac{(y-b)}{x}$since the operation is same as calculating slope of the line (Cooney *et al*., 2010). Hence, linear functions may be expressed by $y = mx + b$ or $ax + by = c$ forms. In these forms, slope is represented with parameters as m or $-\frac{a}{b}$ respectively (Stump, 1999).

Another important conjecture to be justified is how it is possible to assert that the rate of change of a function is m if it is given by $f(x)=mx+b$ form where m and b are constants. This may be shown by plugging arbitrary two elements of domain of f, say s and t, to the function and computing rate of change from s to t (Cooney *et al.*,2010). As a result, slope of a linear function can be computed algebraically by reaching these parameters (Rasslan and Vinner, 1995; Zaslavsky*et al.*,2002).

Regarding slope in algebraic perspective provides several implications. For instance, a line with positive slope value would suggest that both quantities in the function are either increasing or decreasing at the same time. Similarly, a negative value for a slope would indicate that there is an increase in one quantity as the other decreases. These relations indicate that slope provides essential information about the characteristics of a function in addition to providing the measure of *steepness* defined for lines.

Comprehension of slope through algebraically, geometrical or tabularrepresentations as well as the connections between them requiresconceptualizing an important set of concepts. To illustrate, Schoenfeld, Smith and Arcavi (1993) proposed Cartesian Connection which means a point must be on the graph of a line if and only if its coordinates satisfy the algebraic expression of the line. Similarly PARCC (2011) suggested that students' study on proportional relationships, unit rates and graphing may encourage to connect these ideas and help in recognizing that any point (x, y) on a non-vertical line is a solution to the equation $y = mx + b$.

Slope is a deep and multi-faceted concept(Stump, 1999). Learning slope requires proper comprehension of important concepts such as ratio, rate of change, proportionality, covariation, and synthesis of different representations.Hence, understanding slope of a line is crucial in early grades especially for future learning.PARCC (2011) suggested these ideas as:

> Students build on previous work with proportional relationships, unit rates and graphing to connect these ideas and understand that the points (x, y) on a non-vertical line are the solutions of the equation y = mx + b, where mis the slope of the line as well as the unit rate of a proportional relationship (in the case b = 0). Students also formalize their previous work with linear relationships by working with functions- rules that assign to each input exactly one output. (p. 36)

In addition to conceptual understanding of slope, procedural attainment of slope is also multi-faceted. It includes using formal language and symbol systems effectively as well as applying algorithms and rules to calculate slope (Stump, 1999). For instance, slope may be calculated through several ways. In addition, itmay be computed in various cases such as when the line's equation, its graph, or two points of a line is given.

Slope is first introduced at eighth grade mathematics in Turkey. Turkish students should be able to (i) describe slope by models, and (ii) define the relationship between slope and equation of a line. It is stated in the MEB (2009) that students should be able to see that the constant number m in the equation of the form y = mx + n is the slope of the corresponding line. However, the curriculum does not clearly suggest what other notions are to be visited during the instruction on the relationship between slope and equation of a line. To illustrate, Turkish curriculum does not provide connections between covariation and slope but it leaves to teachers. In sum, the presentation of slope in the curriculum is mainly in geometrical perspective. Time to be spent for these two aforementioned objectives is three hours for each and six class hours in total.

2.2.4. Studies on Teaching and Learning the Concept of Slope

Slope is a fundamental but a conceptually complex concept for students in learning algebra (Lobato *et al.*,2003; Stump, 1999; 2001). Rasslan and Vinner (1995) investigated that majority of the nine gradersdid notrealize that "the slope is an algebraic invariant of the line and therefore does not depend on the coordinate system in which the line is drawn"(p. 264). The reason is that function does not depend on the value selected for the scale of the axis (Lobato *et al*.,2003).Similarly, Zaslavsky andthe colleagues (2002) examined students' knowledge onslope conceptin a case where scale was non-homogeneous. Findings of the study supported the results of Rasslan and Vinner's (1995) study.The studies also showed that many students were not able to use the relevant data to compute slope (Cheng, 2010; Schoenfeld*et al*.,1993).In addition, Saldanha and Thompson (1998) suggested that it may be challenging for students to understand "graphs as representing a continuum of states of covarying quantities"(p. 7). Similarly, Rowland (2010) provided that very few primary school students realize that proportional relationship holds in any segment of a line. The National Assessment of Educational Progress (2005) found that students' knowledge of slope is insufficient in many respects.The studies also pointed that comprehension of the concept has a depth and complexity.

Lobato and Bowers (2000) provided that students have various difficulties in learning quantitative complexity of slope. For example, majority of the participants in thestudy showed difficulty in understanding the role of change in rise and run on steepness of a line. In addition, students have difficulties in regarding slope as a ratio (Bell and Janvier, 1981; Leinhardt *et al*.,1990; Lobato *et al*.,2003).

The previous studies showed that there exist a number of misconceptionsamong learners on slope concept. Some of the incorrect ideas mentioned in the literature were: the quadrant where the line is located is related to slope value, changing slope alters y-intercept of the line, slope is the scale of the x-axis, and slope is the difference in y-axis (Lobato*et al*.,2003). Among them, slope-height confusion is a misconceptionwhich is observed very often.Being aware of those misconceptions is crucial to teach slope correctly.

A robust understanding of slope requires a number of proficiency. Learners should know all important interpretations of slope, make logical connections between the interpretations, and decide on the interpretation that best applies to a particular problemsituation. Stump (1999) indicated seven sub-constructs of slope as:

> Slope as a geometric ratio“rise over run”
> Slope as an algebraic ratio or formula “change in y over change in x”
> Slope as a physical property “steepness”
> Slope as a functional property “rate of change”
> Slope as a parametric coefficient, e.g., the m in the equation, y = mx + b
> Slope as a trigonometric ratio, that is, the tangent of the angle that a linear graph makes with the x-axis
> Slope as the derivative of a function (p. 129).

One of the comprehensive studies of slope was conducted by Sheryl Stump. Stump (1999) investigated pre-service and in-service mathematics teachers' concept definitions, mathematical understanding, and appreciation of various representations of slope. Based on teachers' responses to surveys and interview questions, she claimed that though teachers expressed concern with students' understanding of the meaning of slope, the specific student difficulties they identified focused on procedures rather than conceptual aspects of slope.However, as Lobato and the colleagues (2003) suggested, the teaching of slope should shift from finding a slope value to the formation ofslope as a conceptual entity. Stump (1999) also proposed that, slope as a geometric ratio was dominant among multiple perspectives. She suggested behaving slope as a fundamental concept through emphasizing its connection to the concept of function.

Rowland (2010), based on a pre-service teacher's instructional episode, made two important suggestions to be cared while introducing the slope concept to the primary school students. Depending on the video record of a teacher's instruction, he claimed that some form of PCK specific to teaching slope is crucialwhile teaching the concept. First, some segments of a given line may servewell than others in students' determining the differences in x and y-coordinate. This, in turn, it may lead to an easier access to calculation of slope. Second, the increase in x-coordinate should be simple (such as 1) so that the calculation of ratio is facilitated for the students who may have computational difficulties. To conclude, Rowland's (2010) suggestions focused more on procedural aspects of slope and its teaching.

Previous studies showed that both students and teachers may have inadequate comprehensionon function in general, and on the slope concept,specifically.

However, Shulman (1987) claimed that teaching starts with understanding. Hence, Stein and her colleagues' (1990) study isessential in providing evidence of how a teacher's limited SMK had narrowed her teaching.These studies, in general investigated the areas that teachers' SMK is insufficient in several respects hence result in a limited teaching. In brief, the literature suggested that limited and poorly organized teacher knowledge often leads to instruction that is weak in conceptual connections, powerful representations, and increased over routinized student responses. However, the studieseither (i) do not suggest much information about teachers'overall content-specific knowledge such as focusing both SMK and PCK or, (ii) investigate teachers' content specific knowledge visible duringinstruction. Over all, studies mentioned above did not scrutinize content knowledge of teachers and studied them in the act of teaching, at the same time. The situation bolsteredmeto study the in-class investigation of teachers' CK on the slope concept since it is missing in the mathematics education research.

Research focusing on Turkish mathematics teacher knowledge is also limited in several ways. First, there are only a few studies on mathematics teacher education and PCK (e.g. Karahasan, 2010). In these existing studies, the researchers examined PCK in contexts such as patterns, derivative, number, arithmetic operations, and shape(Ubuz *et al*.,2011). Besides, majority of the studieswere quantitatively conductedoutside of classroom environment (Ubuz*et al*.,2011) though the literature on mathematics teacher education stresses necessity of in-class investigation of teachers' content-specific knowledge. Hence, research investigating the way Turkish mathematics teachers' content knowledge during their practice is limited. In addition, more qualitative and in-depth studies are needed to reach a fruitful description of mathematics teacher knowledge of slope in Turkey.

Mathematics education studies overly suggest that function is one of the essential and integral parts of mathematics teaching. Especially in middle school years, students should comprehend linear function which in turn necessitates learning line. Lines (non-vertical ones) are graphical representations of linear functions hence;they are prerequisite for the robust learning of functions. Lastly, this implies that slope is absolutely important in learning functions since slope of a line is a measure to lines' steepness as well as indicating a functional relationship between two quantities (Stump, 1999). Considering the connections between slope, line, and function, it was hypothesized that studying teachers' content knowledge (SMK and PCK)in teaching slope would be an essential contribution to research in mathematics education.

Studies indicate that comprehension of the slope concept islimited.However, it has an utmost importancein mathematics learning. The concept provides opportunities to work in the concrete such as lines and in the abstract such as proportional relationship.In addition, the slope of line has relationship to various concepts in school mathematics. These aspects provided that it is essential to investigateteachers' content knowledge ofslope during teaching. As a conclusion, a detailed understanding of mathematics teachers' implementation of slope of a line is fundamental and fruitful in various respects.

In contrast to the large body of literature on students' learning and performance on slope, there are relatively few studies which focused on teachers' content knowledge of teaching slope (e.g. Rowland, 2010 and Stump, 1999). Among other related studies, most of them focused on teachers' knowledge of functions in general and provided indirect results for the concept of slope (e.g. Even, 1993, Stein *et al.*,1990; Wilson, 1994). The studies, in general, suggest that a significant number ofpre-service as well as in-service teachers lack a deep understanding of the concept and rely too heavily on rote procedures such as emphasizing algorithms to compute slope. Some research has reported that many of the same misconceptions and naïve conceptions identified in students are also prevalent among mathematics teachers.

3. SIGNIFICANCE OF THE STUDY

Today, much is known on the body of the content-free knowledge that teachers ought to know and skills that they will needin managing their teaching effectively. However, a critical issue in studying teachers' knowledge and their teaching is kinds of teachers' content-specific knowledge (SMK and PCK)that they resort during instruction. In other words, what is this knowledge that is used during instruction is missing in the literature (Sherin, 2002). The instruction may be defined as any act that teachers do to support the students' learning, the interactive work of teaching in classroom, and all the tasks that arise in lessons. This study will provide information on mathematics teachers' content specific knowledge in instruction by using the KQ for analytical approach.

The teaching and learning the concept of slope necessitate a number of essential mathematical ideas. The objective as the relationship between slope and equation of a line was purposefully chosen since it requires the study of line in various perspectives such as exploring its equation, graph and the relationships. Hence, it was aimed to study teachers' content knowledge in teaching slope of line.

Mathematics education research strongly suggests that teaching function is one of the essential parts of mathematics teaching. Especially in middle school years, students should comprehend linear function which in turn necessitates learning line. Lines (non-vertical ones) are graphical representations of linear functions hence; they are prerequisite for the robust learning of functions. Lastly, slope is absolutely important in learning functions since research indicates that slope is a mathematical concept where various importation concepts cross each other.

Conducting a research with pre-service, novice, and experienced teachers is educationally fruitful. Studying with teachers with varying professional experience may provide important informationespecially on the transition to being a more proficient and knowledgeable teacher who is assumed to be versatile in teaching and have more robust knowledge. To conclude, considering all aspects together it was assumed that investigating teachers' (with varying teaching experience) content knowledge in teaching slope of a line would significantly contribute to the mathematics education research community, mathematics teacher educators, mathematics teachers, and other people who are concerned with mathematics education.

Kinds of content-specific knowledge which are observable in teachers' practice of teaching is essential. Since teaching is dynamic and situated (Fennema and Franke, 1992; Hodgen, 2011; Rowland and Ruthven, 2011; Wagner *et al.*,2007) this investigation would observe and analyze teachers' actual teaching practice. The discussion on the nature of teachers' knowledge yield a consensus that the knowledge needed in teaching is dynamic, better visible via practice, and should be studied in actual classroom setting (Fennema and Franke, 1992; Hodgen, 2011; Rowland and Ruthven, 2011; Wagner *et al.*,2007).

Analyzing mathematics teachers' act of practice with a framework can possibly elaborate the content-specific knowledge that appears in teaching. The Knowledge Quartet (Rowland*et al.*,2005) may be effective to be used as an analytical framework to investigate teachers' knowledge during instruction. Hence, the study may make significantly contribution in investigating mathematical content knowledge of mathematics teachers, with varying teaching experience, during teaching a mathematical concept, the slope of a line.

4. STATEMENT OF THE PROBLEM

The study aimed to describe in class investigation of content knowledge (SMK and PCK)of pre-service and in-service teachers who has at most five-years of experience. Therefore, this study investigates the following questions:

- What is pre-service mathematics teachers' content knowledge in teaching slope of a line?

- What is novice (who has less than three years of experience) mathematics teachers' content knowledge in teaching slope of a line?

- What is experienced (who has three-five years of experience) mathematics teachers' content knowledge in teaching slope of a line?

5. METHOD

The study focused on video records of two pre-service, twonovice, and twoexperiencedprimary mathematics teachers' instructions. Icollected data from publicschool teachers who wereteachingin a metropolitan city of Turkey in March 2012.

The aim of the studywas to investigate teachers' content knowledge (SMK and PCK)in their teaching.Therefore, before instruction,I conducted pre-interviews with teachersontheir plans of teaching and perspectiveson teaching and learning of the slope of line. It took around twenty minutes and was video recorded. Then, their instruction wasvideo recorded. Either one or two lessons were video recorded depending on the duration spent for the investigated mathematical topic.Finally, after instructions, I conducted twenty minutes post-interviews with teachers to discuss their instruction and video recorded them.

5.1. Setting and Participants

The participants in this study were from three groups who have teaching experience up to five years. One group of the participants waspre-service primary mathematics teachers in their senior year of their formal educationin a public university. Second and third groups of the participants werefrom in-service primary mathematics teachers who graduated from the same university. Totally, six teachers, two from each group,voluntarily participated to the study.

The selection of participants was purposefully limited to teachers who have at most five years of experience. The decision was based on findings of the large scale studies. First of all, it is provided through cross-sectional and longitudinal studies that the relationship between teachers' experience and effectiveness is significant especially during the first few years of teaching. In addition, research indicated that even a short duration of experience indicates better conclusions when compared to inexperienced teachers. Lastly, this significance reaches its peak generally after three-five years of teaching (Clotfelter *et al*.,2007; Ladd, 2008; Rivkin*et al*.,2005).As a conclusion, the study included three groups of teachers which are described as; (i) pre-service teacher who has no formal teaching experience, (ii) novice teacher whose experience is up to three years, and (iii) experienced teacher who has a three-five years of teaching experience.

The participating pre-service, novice and experienced teachers have a number of important common characteristics. Firstly, all of the participants graduated or will graduate from the same undergraduate program of a public university. The purpose for selecting the graduates of the same university was to eliminate the effect of differences in teacher education.Secondly, all of the teachers teach at the same grade.In addition, participants are working or interns in public schools in Istanbul. This is considerably important since public and private public schools may have some important differencesin Turkey in terms of student profile, parental expectations on students, school management, facilities and administrative aspects such as budgeting and control on teachers' performance. Lastly, all of the participants taught the same mathematical concept. The study included the video records of the participating teachers' instruction on the relationship betweenline equation and slope.

Among the participants, all of the in-service teachers were regular classroom teachers and observations took place during their regular class hours. On the other hand, pre-service teachers were observed while they were practicing in their internship schools. It is important to notice that pre-service teachers had a significant level of familiarity with the classroom they have instructed since they had been observing the classes during a year with their mentor teacher.

Following teacher names are pseudonyms.Among the participants, two of them, Cansu and Akif were pre-service mathematics teachers at the time of the study. The number of female and male students in Cansu's classroom was 14 and 13 respectively. There were 15 female and 13 male students in Akif's classroom. Erkin and Yasemin were novice mathematics teachers. Erkin has two years teaching experience at 6-8 grade mathematics in Istanbul. The observation took place in one of his eighth grade class and there were 14 male and 14 female students in the class at the time of observation. Yasemin has been teaching for one year. She also has a teaching experience at 6-8 grade mathematics. The observation took place in one of his eighth grade class. There were 20 male and 17 female students in the class at the time of observation.The first experienced teacher, Müge has been teaching mathematics for four years. There were 12 female and 12 male students in her classroom. Another teacher, Öznur has also a four years teaching experience. There were 12 female and 15 male students during the data collection period.

Lastly, the undergraduate education in teaching primary mathematics will be summarized. In Turkey, primary mathematics teachers are expected to teach for the grades 6 to 8. The current nation-wide undergraduate program of teaching primary mathematics is mainly composed of three categories. These are mathematical content courses for 50-60%, pedagogical courses for 25-30% and the remaining course work for cultural courses and electives (YÖK, 2007). Some of the mathematics courses are calculus, geometry, abstract algebra, probability and statistics, linear algebra, and number theory. Mathematics related pedagogical courses contain mathematics teaching methods, school experience and practicum. Some of the general pedagogical courses are classroom management, guidance, educational psychology and introduction to education. Offered elective courses depend on the interest and academic background of faculty members. Some of the elective courses widely offered are problem solving, teaching geometry or mathematics courses. The remaining courses such as history, Turkish, computer literacy constitute the others category which is indicated in the Table5.1 (YÖK, 2007).

Table 5.1. Summary of courses in primary mathematics education program.

Courses	Number of Courses	Credits
Mathematics	19	64
Pedagogical courses of mathematics	7	23
General pedagogical courses	6	15
Elective	6	16
Others(such as Turkish, language, computer)	13	28
Total	51	146

The national curriculum for undergraduate primary mathematics education requires senior students to complete a practicum before graduation. This course is the only course in which senior students have teaching experience in actual classroom setting. It is composed of primary school teaching experience and mentorship of those experiences at the university. The senior students are assigned to a school and a mentor teacher to follow for fourteen weeks. Each week is associated with a task to complete such as observing a teacher's day, analyzing teachers' questioning, and assessment. The students should also plan and implement two lessons during their practicum. Planning, implementation and analysis of those lessons are guided by both their mentor teachers in the school and professors in the university.

5.2. Data Collection

I collected data in three phases. They were pre-interviews, video records of the instructions, and the post-interviews. The main source of the data was the video records of classroom instruction which was supported by the data from the individual pre and post-interviews.Data were collected throughout March 2012 with the written permission of governmental authority (Appendix A). The permission included two interviews with each participant and the full-video record of their instruction throughout a week.

Slope is first introduced at eighth grade mathematics in Turkey. Turkish students should be able to (i) describe slope by models, and (ii) define the relationship between slope and equation of a line. Time to be spent for these two aforementioned objectives is three hours for each and six class hours in total. It was decided to focus only on the second objective since (i) increasing the duration of recording would risk the depth of knowledge to be assumed to have after data analysis, and (ii) assuming that it would be more effective since a focus on the second objective will also provide data indirectly for the first objective.

5.2.1. Pre-Interviews

I conducted semi-structured interviews with each participating teacher. The interviews, which were video-recorded and latertranscribed, took approximatelytwenty minutes.Aim of the individual pre-interviews was to explore teachers' plans of teaching the relationship between the slope and equation of a line. In the interview, I asked questions such as "How are you planning to teach the subject?" In this interview, teachers described the way they will follow during instruction. The interview protocol is provided in the Appendix B. The interview transcripts wereused for analysis.

5.2.2. Video Records of Classroom Instruction

The purpose of the study was to investigate content knowledge in teaching slope of a line. Hence, it is important to capture the instruction, namely, teaching in the classroom. The aim of the video recording of the instructions was to explore teachers' instruction from the perspective of content knowledge in teaching. Video recording is a powerful tool to provide such insight (Burgess, 2008). Hence, the main source of the data was the video records of mathematics lessons of the participating teachers.

Instructionswere video recorded. I placed a camera at the back of the classroom to capture primarily the teacher's actions in teaching. During the instructions I also kept observation notes. During video recordingIgave special attention to not to disrupt the flow of the instruction. Table5.2 summarizes the time line of the lesson video records.

Table 5.2.Summary of the time line for lesson video records.

Teacher	Date	Duration(Lesson Hours)
Müge (Experienced)	March 5	2
Öznur (Experienced)	March 12	2
Yasemin (Novice)	March 14	2
Cansu(Pre-service)	March 16	1
Akif (Pre-service)	March 20	1
Erkin (Novice)	March 22	1

5.2.3. Post-Interview

The last step of the data collection was semi-structured post-interviews with teachers. I interviewed with each teacher following their instruction. The interviews, which I video-recorded (and latertranscribed), took approximately twenty minutes. The main purpose of the teacher interviews was to learn the way teachers thought about their lesson after instruction. The post-interview questions are provided in the Appendix C.

The post-interviews were crucial inincreasing triangulation because the literature supports that a researcher's interpretations may be limited in describing teachers' instructional decisions (Burgess, 2008). The post-interviewsprovided whether the researcher's interpretations were coherentto teachers' aims and attentions.

A comparison of the datafrom three sources was considered to be helpful in interpreting the findings. The combination of these three sources and their integration during the analysis phase was assumed to provide strong inferences and produce a more thorough understanding of participants' content knowledge during their teaching.

5.3. Data Analysis

The main source of data wasthe video records of the instructions.Hence, I first examined video recorded lessons, aiming to formulate the initial analysis concerning teachers' content knowledge (SMK and PCK)in their instruction. Subsequently, I attemptedto triangulatefindings with interview data. The analytical framework of the study was the Knowledge Quartet (Rowland *et al*.,2005).The KQ is a model that can be used in studyingcontent knowledge in instruction. It was developed inductively through observing classroom instruction. In this study,I will deductively use the KQ to analyzedatain terms of the unitsofit.

The KQis composed offour units which are foundation, transformation, connection, and contingency. Foundation is teachers' knowledge, beliefs and understanding of mathematics and its teaching. Transformation concerns knowledge-in-action as demonstrated in the act of teaching itself and it includes the kind of representation and examples used by teachers, as well as, teachers' explanations and demonstrationsdirected to students. Third unit, connection is the knowledge of teachers to link series of lessons, between multiple mathematical ideas and the different parts in a lesson. Connection also includes the sequencing of activities for instruction, and knowledge about students' difficulties and obstacles against understanding different mathematical topics and tasks. Finally, contingency is related to teachers' ability to respond to students' ideas, to respond appropriately to students' wrong answers and to deviate for their lesson plan. In other words, it concerns teachers' readiness to react to situations that are almost impossible to plan for.

Rowland and his colleagues (2007) conceptualization of the four members of the KQis summarized in Table 5.3 and Table 5.4. It should be reminded that the KQdoes not suggest an explanation or example for any code. The explanations are based on literature review and understanding of the codes. The reason for describing the codes is to inform readers about the way I usethe codes and units of KQ in data analysis.

Table 5.3.Codes of foundation and transformation.

Unit	Codes	Explanation-Example
Foundation	Awareness of purpose	Awareness of objectives, aims and goals of teaching math.
	Identifying errors	Ability to identify mathematical errors that students, textbook, or any learning material may suggest.
	Overtsubject knowledge	Critical understanding ofcontent to be taught. For example, why slope is undefined in vertical lines.
	Theoretical underpinning of pedagogy	Perception of teaching mathematics on the conditions under which pupils will best learn mathematics.
	Use of terminology	Treatment of mathematical language during instruction.
	Use of textbook	Use of textbook materialsfor the instruction.
	Reliance on procedures	Use ofconventional and important proceduresduring instruction such as computing slope through conventional procedure
Transformation	Choice of examples	Decisions of using/choosing examples for instructional purposes. For example choice of using y=3x+2 (instead of y=2x+2)
	Teacher demonstration	Way of using demonstrations to explain procedures, rules, algorithms…etc.
	Choice of representations	Decisions of using various representations for instructional purpose: representations of slope included in teachers' in-class instruction.

Table 5.4.Codes of connection and contingency.

Unit	Codes	Explanation-Example
Connection	Making connections between procedures	Act of building connections between multiple procedures during instruction.
	Making connections between concepts	Act of building conceptual connections between mathematical concepts such as making connections among slope, ratio and rate.
	Anticipation of complexity	Awareness of students' obstacles against understanding different mathematical topics and tasks.
	Decisions about sequencing	Teacher's ordering of topics, tasks or other units of instruction such as examples within and between lessons.
	Recognition of conceptual appropriateness	Awareness of the relative cognitive demands of learning mathematical concepts, relations, etc.
Contingency	Responding to students' ideas	The way a teacher attends to, interpret, and handle students' ideas.
	Use of opportunities	Using an unanticipated contribution as an instructional opportunity.
	Deviation from agenda	Ability to extent teaching to other aspects of mathematical content.

Aim of the study was to explore teachers' content knowledge of teaching slope in the identified episodes. After the transcription of the video records, I used open coding. Initial data analysis resulted in themes for the open coded data. Then, I assigned codes for each theme that emerged. To illustrate, disregarding a student's incorrect proposition theme emerged which was then assigned to responding to students' ideas of contingency unit.I provided justification to the codes and the rationale for groupingthem into one of the four units. I also compared the codesI investigated and the 18codes of the KQ to check whether any new codes arise.The language of instruction was Turkish so I completed all of these analyses steps in Turkish. Quotes in findings section will be givenin English which were translated from Turkish by the researcher.

Any single moment or episode in an instruction is open to a number of content free and content-related analyses. Different people may see differentphenomena in a lesson, and may hold different ideas (LMT, 2006). However, purpose of a study and the analytical framework would be influential in the way a video record of a lesson is analyzed. In this study, the purpose of the study was to investigate teachers' content knowledge of teaching slope of a line by using KQ as a framework.Therefore,the focus of analysis was not on the individual episodes but the whole instruction. Instead of analyzing a single episode of an instruction from various KQ units, this study focused on identifying different types of teacher knowledge during instruction.In other words, the purpose was not to provide how many or in what degree different codes are visible in a single episode but to identify the types of knowledge that teachers address during instruction.

It is important to notice that any instant of an instruction or any single event is almost open to discussion in more than one code or unit. Rowland and Turner (2007) suggested, for example, that a contingent response to a student's suggestion might help to connect ideas visited earlier. Furthermore, it could be argued that the application of subject knowledge in the classroom always rests on foundational knowledge.

Episodes which are open to be discussed by all the units may also be given. For example, analysis of an episode may provide that in order to*respond to students' ideas*an observed teacher may *use* an incorrect *terminologyto make connections between concepts*through*a demonstration*. To illustrate, in one of the observed episodes students computed slope of a graphed line in an incorrect way. This episode may be analyzed in terms of teacher's identifying errors. The episode suggests that the teacher realized students' errors and act on it without any reference to slope formula. The episode may also be viewed in terms of the codes in the transformation unit since there is a choice of explanation to the errors students made. Furthermore, the episode gives clues of connection unit since it shows how the teacher sequences examples, explanations or relationships throughout her lesson. The teacher did not mention about the relationship between the inclination of line and the corresponding slope until students make mistake while computing slope. Lastly, one can also analyze the episodes through the codes of the fourth unit, the contingency. Teacher preferred to talk more on new mathematical relations since the incorrect answers provide an opportunity for teachers to introduce new mathematical relations. In brief, while I may distinguish many examples, I will present some selected episodes. If an episode is presented in a certain unit, it does not mean that the episode is not coded for another unit.

As mentioned earlier, the KQ is composed of four units. The model suggests studying mathematical content knowledge, not specific/limited to any concept in mathematics. It does not provide the knowledge teachers ought to knowfor any mathematical topic in any grade (Rowland *et al.*,2005). Instead, it provides types of content knowledge which are part of instructions for any mathematical topic. To be clear, the framework can be used for content analysisthough their description is*free of content*. In sum, this modelcontrasts to the MKT model in the sense that it does not suggest any initial data to be used for the units.

Suggesting a hypothetical description of units for teaching slope of a line might be useful to illustrate during data analysis since the purpose was to identify teachers'content knowledge (SMK and PCK) in teaching. However, descriptions should not be used as a checklist for whether these descriptions are observed ininstructions but to give an idea during analysisof data.

The following will be thedescription of the units in terms of the slope of a line based on literature about teaching and learning slope. It isimportant to notice that the descriptions were not attained from the research data. Besides, they were not directly used during data analysis but they guided the researcher during data analysis. In addition, due to the situated nature of teaching there might be slight changesin describing theunits and codes for teaching slope of a line. As a result, the description of the units for the concept stands as hypothetical even though all of these descriptions were reached from the review of literature, thetextbook,and the Turkish mathematics curriculum. For triangulation, the researcher discussed the descriptions with two mathematics teacher educators. One of them has been working as an instructor in secondary mathematics teaching more than three years. The other one was teaching as an assistant professor in primary mathematics teaching and has ten years of teaching experience.

Considering all the codes of the units, description of the unit suggest that slope is a complex and demanding concept to be taught. In addition, it is always possible to include more descriptions to the unit.

Foundation: It is teachers' knowledge, beliefs and understanding of mathematics and its teaching (Rowland *et al.*,2005). It is basically composed of teachers' knowledge, understanding and recourse to their learning.

As discussed in previous sections, slope and lines are central concepts in mathematical learning. Slope of a line provides opportunities to visit concepts such as ratio, proportionality, and covariation. Proper comprehension of these concepts will provide important skills and knowledge to learn functions meaningfully and conceptually in the future (MEB, 2009, p. 98). Lineinvolves an important aspect of function concept, particularly functional relations of the form y=mx+b in which x and y are called as variables and m and b are called as constant coefficients. Students are expected to understand linear functional relations and the relationship between slope and equation of a line (CCS, 2010; MEB, 2009; NCTM, 2000). It presents connections between algebraically represented linear functions and their corresponding graphical representations. As a conclusion, a meaningful teaching of the concept should enable students to see that slope is a central concept and it includes substantial role in further mathematical learning. Slope and line concepts are commonly used in the middle school curriculum as a route to transition from arithmetic to algebra and to develop function concept. Hence, teachers should be aware of the purpose in teaching slope concept.

Identifying errors is one of the important and professional works of teachers in instruction. Teachers should be able to identify students' errors during instruction. However, identification of errors requires a critical perspective, a robust understanding of mathematics, its teaching and learning, and many other qualifications. As discussed in previous sections students may have slope-height confusion, use irrelevant data to compute slope, or represent ordered pairs (x,y) incorrectly on coordinate plane. Teachers should be careful in identifying all types of errors which impede the learning of the concept.

Slope has its references both in geometry and algebra. In geometrical terms, slope is an entity of lines which informs about how steep a line is(Stump, 2001). Teachers may present that parallel lines always have the same slope though their coordinate points differ and if two different lines have the same slope, then they become parallel (Rowland, 2010).

In algebraic way, slope is the ratio of the change in the dependent variable with respect to changes in the independent variable (Lobato *et al.*,2003). In algebraic understanding of slope, the creation of a ratio of the variation of one quantity to the associated variation of another quantity is very important.Hence, the ideas suggest a need to see slope as a measure of steepness and rate of change at the same time (Stump, 2001).

Teaching slope and the relationship between slope and equation of line requires a conceptual understanding of the subject matter. To illustrate, an instruction on the concepts may possibly indicate why two points are necessary and sufficient to draw a graph, the rationale that *any* two points of a line possibly gives its slope,how it is possible to express any line by y=mx+n, and why slope is undefined for vertical lines. MEB (2009) recommend teachers to construct conceptual aspects of the relationship between slope and equation of a line.

Theoretical underpinning of pedagogy is described as a teacher’s perception of teaching mathematics and on the conditions under which pupils will learn best. The methods and techniques used in teachers’ instruction may suggest teachers’ perspective of their theoretical underpinning of pedagogy. To illustrate, a teacher may have a perspective that the most crucialelement influencing learning is what studentsalready know. Hence, she may organize and implement her instruction based on students' previous learning.

The mathematical terminology that teachers use in teaching slope of line is an indicator of teachers' knowledge in foundation unit. Teachers should use mathematical language and definitions in an appropriate way. An instruction on slope, slope and lines, or the relationship between slope and equation of a line will most probably include mathematical terms such as vertical change, horizontal change, ratio, rate of change, vary, covariation, steepness, intercepts, coordinate points, graph of a line, linear, and line equation.

Reliance on procedures is crucial. To illustrate, graphing a line is extremely important in learning slope of a line. Students should realize that if two distinct coordinate points of a line is given, then rest of the data can be reached. This may be proved by the fact that a given line has two unknowns (the coefficient of x and the constant coefficient) and the points given are two knowns. Students may compute slope of the line and write its equation. Besides, teachers may explore the way to use an equation to graph a line by finding some points on the line before graphing it. So it is important for students to realize that two points are needed before they can graph a line. However, finding the points on a line may be a demanding work for most of the students especially in earlier grades. Teachers may addresshow to find these points and the rationale behind the procedures.

Slope is an attribute of line, not a line segment. A line has a unique slope and the slope of a line does not change as the chosen line segment is changed due to linearity. Hence, teachers should know and make students recognize that slope of a line may be easily reached graphically by choosing a segment of line since any isolated segment in a line provides the same slope (Rowland, 2010). In computing slope of a line two points are chosen on the line to compute slope as long as they are distinct. In addition, once the points are chosen, there is not a restriction of orders in using points in the formula.

Transformation: All of the examples, explanations, demonstrations, illustrations and analogies used during teaching are a number of important sources which provides evidence of teachers' transformation of knowledge. Teachers are expected to be deliberate and conscious in their choices. Choice of examples, choice of representations and teacher demonstrations in an instruction are important indicators of transformation unit.

Examples used in an instruction provideimplications for teachers' content knowledge (SMK and PCK) in transformation unit. Teachers should use their mathematical knowledge to generate examples. In determining the examplesfor instruction teachers may consider including a group of lines whose slope, equation, coefficients of equation, and graph vary. The slope values of lines may include positive-negative integers and positive-negative fractions. The chosen examples should include both kinds of lines which pass through the origin and which does not.Providing both types of lines in an instruction might be particularly helpful. It may help in conjecturing that the relationship between slope and equation of line exist in either case. It may also help students to see the role of a constant coefficient in a line equation and recognize that it does not alter slope of a line. Besides, algebraic representation of a line should not be limited to $y=mx+n$ form since there are additional forms with slight variations which also indicate lines.Some of the important forms may be $ax+by=c$ or $ax+by+c=0$. Presenting various forms for line equation may help students in their conceptual learning. Exploring vertical and horizontal lines may also be very helpful during an instruction on slope of a line.Students mayexplore these lines in terms of slope concept and its algebraic representation.

Choice of examples is a work to be done by teachers. For example, an equation such as $y=x+1$ may not be an appropriate selection for introduction of the relationship between slope and equation of a line since both the constant coefficient and the coefficient of x is 1 in the example. This sameness may impede learning the relationship between slope and equation of a line since it may create an ambiguity of the role of slope on its equation.

Selection of representations is also crucial. Teachers should plan and act in a way that important representations are visited. To illustrate, a teacher may prefer to start lesson with a numerically represented tabular data which models a real-life situation, then plot a graph and then reach algebraic representation. During the instruction teacher may organize materials in way that students see that (i) the rate of change of variables is fixed in tabular data, (ii) this rate of change corresponds to slope of line which is steepness on graph, and (iii) the slope is a coefficient in algebraic form which provides the covariation between two sets of data.

The concept of slope is open to multiple representations. While it is defined in one way, it may appear by different labels, notations and representations. The students (depending on the grade level) need to recognize the relationships among these representations of slope (algebraic, geometric-graphical, and numerical), to understand slope conceptually. To conclude, conceptual understanding of the slope of a line requires a synthesis of a number of different mathematical ideas and representations and meaningful switch between each other (CCS, 2010; Even, 1990; Hines, 2002; MEB, 2009; NCTM, 2000; Rasslan and Vinner, 1995; Sherin, 2002; Stein*et al*.,1990)

Schoenfeld, Smith and Arcavi (1993) suggested that competence in the understanding the concept may include; to be able to see lines in the plane, in the algebraic way or in a tabular form. In addition, a proper understanding may include the reason and cases where using a representation is educationally suitable than the other.

Connection: Coherence and making connectionsduring instruction are indicators of connection unit. The instruction should enable to make connection between procedures and make connections between concepts. In addition, teachers should anticipate the complexity of teaching a mathematical concept. Sequencing mathematics teaching during a lesson and across lessons is also crucial. Lastly, teachers should recognizewhether the material is conceptually appropriate for students to learn.

The procedures should be connected. To illustrate, teachers may present the relationship between slope and equation of a line in multiple ways. They may explore slope of a line by creating numerically represented tabular data, find slope of a line after reaching a y=mx+n form, or compute slope by graphing it on coordinate plane.

Teachers should make connections between slope and other mathematical concepts. Inclination of a line, steepness, ratio,proportion, rate of change, and covariation are important concepts that are needed to be addressed during teaching slope and line.In particular, teachers' appreciation of a connection between the concepts of slope and function, and slope as a rate of change would be a positive indicator of teachers' content knowledge. In brief, teachers should be knowledgeable about the essential understanding of these concepts and their relationship between each other.

Teachers' awareness of students' obstacles against understanding different mathematical topics and tasks such as seeing the difficulties students may have during learning slope, equation and graph of a line may guide teachers for both in planning and implementing the lesson.

Sequencing lessons is also essential. For instance, students may find it easier to construct the relationship between the slope of a line and its equation first from the lines which pass through origin. In addition, teachers should be aware of the relative cognitive demands of mathematical learning.The connections to be reached and their depth are fundamentally shaped by the context, objective, and the grade level that slope of a line is introduced.

All in all, the connections unit relates the concept to many others in mathematics depending on teachers' knowledge, belief and expertise. In addition, both lesson plan and the instruction should look coherent since mathematics is a discipline famous for its coherence.

Contingency: The unit relates to teachers' contingent action in instruction. Responding to students' ideas, using them as an opportunity and deviating from the lesson plan indicate teachers' expertise on contingency.

Teachers may not predict the classroom events beforehand. Students' solutions or suggestions cannot be predicted completely in advance. Teachers may prefer listening students' ideas actively andproceed accordingly (Sherin, 2002). It is almost impossible to include any specific example of contingency which may occur during the instruction on slope of a line. However, a NAEP report might be used as an illustration for the unit.

Fourth NAEP nation-wide study for the U.S. investigated the responses given to a task on graphing lines (Lobato and Bowers, 2000). In the study, the students were asked to draw a line passing through the origin which was parallel to a given line.Secondly, they were asked to reach the equation of it. Researchers assumed that the most likely way of solving the problem would indicate these phases:

(i) The m in the linear equation of the form y=mx+bstands for the slope,

(ii) Slope of lines would besame since they are parallel lines,

(iii) b represents the y-intercept of the graph of the line,

(iv) b for the new line will be 0 because the line will pass through the origin (Lobato and Bowers, 2000).

The study is, in fact, analogues to studying contingency unit. Four steps provided above may be taken as a typical of teachers' plan for the instruction on graphing lines. However, the research indicated that 84% of the 12^{th} graders were not able to answer both parts of the question in an appropriate way including the way that the researchers assumed. If it would be the case during a classroom instruction the teacher had to deviate from the agenda since the assumptionsthat teacher had did not work in the classroom.

This section concludes the method part of the research. The following will be on detailed report of findings. Findings will also be provided in terms of teacher groups and the units of the KQ.

6. FINDINGS

In this section, analysis of video records and the interviewdata will be presented. Findings of the study will beprovided in four sections. First three sections will start with the summary of teachers' instruction. Then, findings will be presented in terms of the groups, namely, the pre-service, novice, experienced, and lastly the comparison of groups. Both the language of instruction and the language used in interviews were in Turkish. The quoted data are the researcher's translation to English.

In the following sections, the first task will be individual summary of participating teachers' instructions. Then, types of different content knowledge findings for the teachers will be provided in terms of the units of the KQ. Findings will provide an opportunity to identify types of teachers' content knowledge on teaching slope of a line.

6.1. Pre-service Mathematics Teachers

Cansu and Akif were pre-service mathematics teachers in their last year of formal undergraduate education in a public university. They both had not had a formal teaching experience in a real classroom until this study. Both teachers' stated objective in the lesson planswas as students will be able to explain the relationship between slope and equation of a line. Their lesson plansdid not indicate an attempt to introduce slope of a line through a covariation perspective. Teachers devoted one lesson hourfor theinstructionof teaching slope of a line and the algebraic relationship between slope and equation of a line.

The first observed pre-service teacher, Cansudivided the lesson into three main phases. She talkedabout slope concept with a two-minute introduction. She reminded that students had learned how to find slope on plane hence they can also follow the same work for slope of lines. She used a notebook as a model for plane and indicated that any side of notebook can be regarded as a model for lines or line segments. She claimed that a flat surface has no slope whereas the slope increases as the angle between the notebook and the ground gets larger. She told that any line may have a slope when the procedure demonstrated for notebook (for plane) is applied for lines.

Cansu's main activity started with a story attached to a numerically represented tabular data. The tabular data gave the number of pokes that comes to Esraon facebook each day. This functional situation asked to use the tabular data in order to reach a graph, equation, and slope of the line. The activity aimed to; (i) write line equation in slope-intercept form, (ii) graph it, (iii) compute slope on graph, and (iv) observe that slope and the coefficient of x in the equation are same. Cansu, by referring the tabular data (1,3), (2,6), (3,9) claimed that the number of pokes will be 3x in the x^{th} day and this relationship may be expressed by a y=3x relationship.She made this generalization by indicating that in each day the number of pokes is 3 times the day. Then, she calculated the slope by working on the graphshe drew for y=3x.Then, Cansu indicated the relationship between equation and the slope of a line. She concluded that the coefficient of x in the line equation is same as the slope of that line.

The lesson followed by a series of exercises in which students were asked to match line equations and graphs of lines. The line equations to be matched to the given graphs were: y=-x+2, y=-6x, $y=\frac{2}{3}x$, and y=4x+4. The teacher reached the slope of lines from the graphs and related these slopes to the given equations by referring to the aforementioned relationship (For brevity, relationship will stand for relationship between slope and equation of a line unless stated differently).

The last phase in Cansu's lesson was finding slope of a number of lines by applying the relationship to the given equations of lines directly. Lines given for the exercise were y=3x, y=4x+5, 3y=4x+9, 2y+5x=8, y=13, y=6, and x=6. Significant amount of time was devoted to the discussion on the last two examples, slope of horizontal and vertical lines.

The second observed pre-service teacher, Akif, also segmented his lesson into three phases. In the first phase, Akifremindedthe conceptof slope. The teacher talked about slope on a plane example in which riding bicycle in rampswith different steepness was discussed. To visualizedifferent cases, he drew a figure on the board and named each ramp cases with a number such as I, II and III as in Figure 6.1. Akif emphasized that both vertical and horizontal distances have a major role in the slope concept. He also made explicit claims that measure of angle in a plane, and the quotient of vertical distance to horizontal distance determines the steepness of a ramp, thereby, the magnitude of slope.

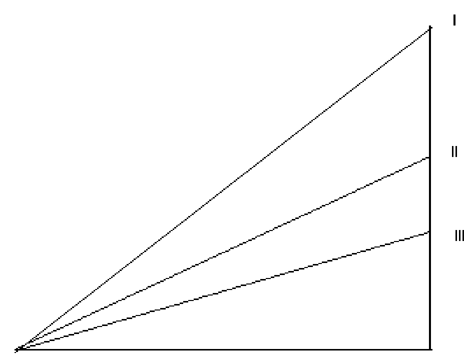


Figure 6.1.Ramps modeled in a right triangle.

The second phase lasted with an activitywhich included a scenario and a numerically represented tabular data which provided number of goods manufactured in a factory for a day. The story asked to make two graphs based on the data given in tabular form. For the first graph, Akif asked students to select x axis as the number of employees and y axis as the number of goods manufactured in that day.For the second one, students were required to name axis in reverse order; number of goods in x axis and number of employees in y axis. The activity aimed to; (i) graph a line, (ii) write its equation in slope-interceptform, (iii) compute slope on graph, and (iv) investigate that slope and the coefficient of x are same in slope-intercept form.

The lesson followed by a computation on a series of exercises in which students were asked to match line equations and graphs of lines in coordinate plane.

The line equations given for the exercise were as follows: y=8-4x, y=2x-2, y=3x+6, and y=-x+2. Akif preferred first to find slope of lines on the equations. Then, he reached the slope of lines on the graphs and related these slopes to the equations by resorting to the relationship discussed. The teacher talked about the inclination of lines and its relationship to slope value during solving exercise questions.

6.1.1. Foundation

The unit is concerned with teachers' knowledge, beliefs and understanding. The KQ provides seven codes in foundation unit. Six of these codes were identified in pre-service mathematics teachers' instructions and interviews. These codes were awareness of purpose,identifying errors, overt subject knowledge, theoretical underpinning of pedagogy, use of terminology and reliance on procedures. The data suggested no use of textbook.

Awareness of purpose

Awareness of purpose was described as teachers' awareness of objectives, aims and goals of teaching mathematics. The code wasobservable in both teachers' data. Teachers stated their objective in their lesson plans. Besides, they also expressed it to students in the classroom. Both teachers referred finding relationship between slope and equation of a line. This objective was explicitly told throughout the lessons. For example, Akif's activity provided a y=2x relationship. He started the lesson as telling that "Today, we will use line equations in calculating slope".

In addition, the purpose was observable throughout the instructions. Teachers tried to reach the objective through an activity. Starting with the same numerical representation, use of tabular data, Cansu's tabular data indicated a y=3x relationship between the variables whereas Akif's data provided a y=2x relationship. Through the activity, Cansuclaimed that the coefficient of x would be the slope of line if it is in y=mx+n form. She repeated the objective again as follows:

> Good okay what was the objective… it was the investigation of the relationship between equation and the slope of a line.

In addition to teachers' statements, the whole instructions suggested pre-service teachers' focus. Data suggested that both of the teachers' instruction was strictly adhered in investigating the mentioned relationship. To be clearer, the data above as well as the remaining instructions were devoted to use algebraic representation of lines to reach slope of these lines. The below episode clearly indicates this interpretation.

> Akif: Okay, we know that the lines have different slopes [he indicates the slope of y=2x and 2y=x] how would we understand that these lines have different slopes if we had no graphs but the equations. Okay, I will erase the graphs [he erases the graphs of lines] and have only equations… we have no table or graph… think algebraically, can we see the same things [indicates the slope of lines] on the equations of lines.

A similar episode which indicates the purpose may be illustrated by another episode in Cansu's classroom.As provided in Cansu's lesson, the teacher proposed to find the slope of lines y=13, y=6, and x=6. , Although she may use these lines to introduce very important concepts, Cansupreferred to talk more on the algebraic relationship between slope and equation of a line. She attempted to show that a horizontal line may also be expressed as y=0x+b hence, slope of any horizontal line is 0. Similarly, a vertical line may be written as 0y=ax+b hence slope is undefined in vertical lines. I agree that these are absolutely valuable deductions and supports to understandthe relationship between slope and equation of a line in a clearer way. However, they also inform teacher's purpose in a way.

The analysis of video records showed that teachers also emphasized to graph a line, compute slope on the graphs, relate the slope to a trigonometric ratio and inclination of lines. However, the analysis showed that all of these episodes were given in an attempt to introduce the relationship.Activities as well as exercise questions were centered on teaching the relationship.In brief, data suggested that pre-service teachers' purpose was to introduce the algebraic relationship between slope and equation of a line, namely, the coefficient of x would be the slope of that line if it is written in slope-intercept form (y=mx+b).

To conclude, data analysis suggested that there is one dominant purpose in these lessons. Teachers behaved slope as a parametric coefficient of line equation, y = mx + b. Even though both curriculum and participating teachers (through lesson plan or activities) suggests itas the relationship between slope and equation of a line, the purpose of these lessons was to teach how to *compute* slope of a line from its equation. Acting the relationship in a narrowed way (focusing on procedural attainment of slope concept) lead to conclusion that the awareness of the purpose in teachers' instructions was not versatile. Lastly, teachers' knowledge of the relationship will be discussed further in overt subject knowledge code.

Identifying errors

Identifying errors is one of the crucial and professional works of teachers. Teachers should be able to identify errors during instruction. However, identification of errors requires a critical perspective, a robust understanding of the content, its teaching and learning, and many other qualifications.In reporting findings for this code, the focus is not on teachers' action after identifying errors but their capabilities to be able to identify an error. Identifying errors is a code of the foundation unit and provides important data on teachers' foundational knowledge. On the other hand, responding to students' responses and errors belongs to last unit of the KQ.

There was limited number of episodes in teachers' instruction which could be labeledas identification of errors. Two episodes of Cansu's lesson may be addressed in terms of this code. On the other hand, analysis of Akif's lesson did not suggest any episode for the code. The below example is from Cansu's classroom.

> Cansu: Look, we were looking the change in x coordinates and the change in y coordinates you [since students tend to take the x-intercept as the horizontal change]cannot take this point [pointing to the x-intercept of the line].

Another episode in Cansu's classroom will be illustrated. The students in Cansu's class had difficulty in getting a slope value from the given graph below (Figure 6.2.).

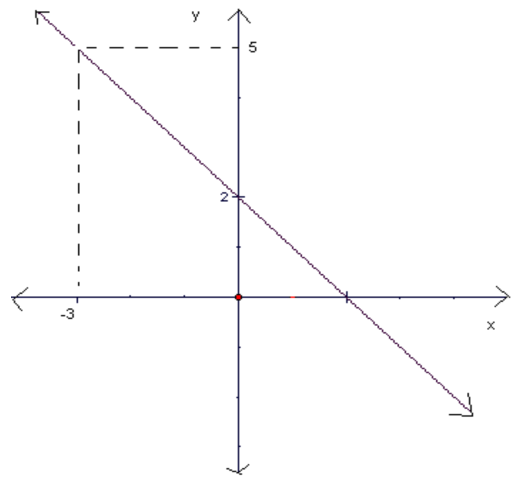


Figure 6.2. Graph of a line in Cansu's instruction.

Student: Here is 1 [student indicates the x-intercept of the graphed line in Figure 6.2.]
Cansu: Is it 1, is it written as 1?
Student: Okay it is not
Cansu: Here is 1 to 1[teacher indicates the point (1,1) on coordinate plane]
[After student makes further attempts which are not correct]
Cansu: you chose this [she points to distance between (-3,0) and (1,0)] and this [she points to distance between (0,2) and (0,5) since a student used the points to compute slope] and divided them. Is there a relationship or can we use them to compute slope?
Student: Okay we can not

Often, students were not able to use relevant data to compute slope. The above are common errors and Cansu was able to identify these errors during her lesson.These episodes show that Cansu is familiar with using the necessary data to compute slope and types of errors students may face with in computing slope. Thus, these episodes may be interpreted positively in terms of Cansu's foundational knowledge.In other words, Cansu's foundational knowledge enabled her to identify student errors during instruction.

Overall, data analysis suggested that teachers' instruction did not suggest many instances in which teachers identified student errors.There was only a few case in Cansu's classroom in which students provided their solution strategy to an exercise.In most of the times, student did not provide their ideas and work. Students' contribution to instruction will be discussed later in terms of the other units.

Overt subject knowledge

Teachers' depth of subject knowledge in teaching mathematics was also visible in data analysis. Both teachers' instructions suggested that they have overt subject knowledge on slope concept. For example, the way Akif introduced proportionalityor Cansu indicated the properties of slope of a line support their proficiency in slope concept.

Analysis of teachers' instruction has indicated also that they had deficiencies either inunderstandingor representing line. Both teachers' main activities presented similar tabular data which in fact do not represent lines (the reason will be explained). On the contrary, they claimed that students would reach graph of lines by connecting the points got from the presented tabular data. The below is the way Cansu represented the numerically represented tabular data.

> Cansu:Look for the first day the number of pokes is 3 [she indicates to the tabular data that in the first day there were 3 pokes]
> Students: Yes
> Cansu: These make a point and the second day make a point with 6[she indicates to the tabular data that in the second day there were 6 pokes] and all these make a line isn't it?

Similarly, Akif claimed that:

> Do we reach a line if we combine these points [the points that were extracted from the tabular data] on the coordinate plane?
> Students: Yes
> Akif: Yes, we'll reach a line like this [he draws a line passing through the points (0,0), (1,2), (2,4), and others].

Both teachers used numerically represented tabular data to graph a line. However,the dataintended for the lineswere tabular form of functions whichwere defined only on integers. What may be reached from the given data is ordered pairs of natural numbers such as (1,2), or (2,6). The scenarios in teachers' activities cannot include, for example, half days. It indicates that functions were defined on $\mathbb{N}$. Thus,the data suggested through the activities cannot have the kind of continuous variation which is a prerequisite to be a line. Graphically, the tabular data would only permit to draw a graph in which the positive numbers are defined and the function is undefined between any two positive integers. In other words, y=mx or y=mx+n provides a freedom since they are defined on ℝ. Figure 6.3 illustrates Akif's interpretation of his data (left) and the actual outcome (right).

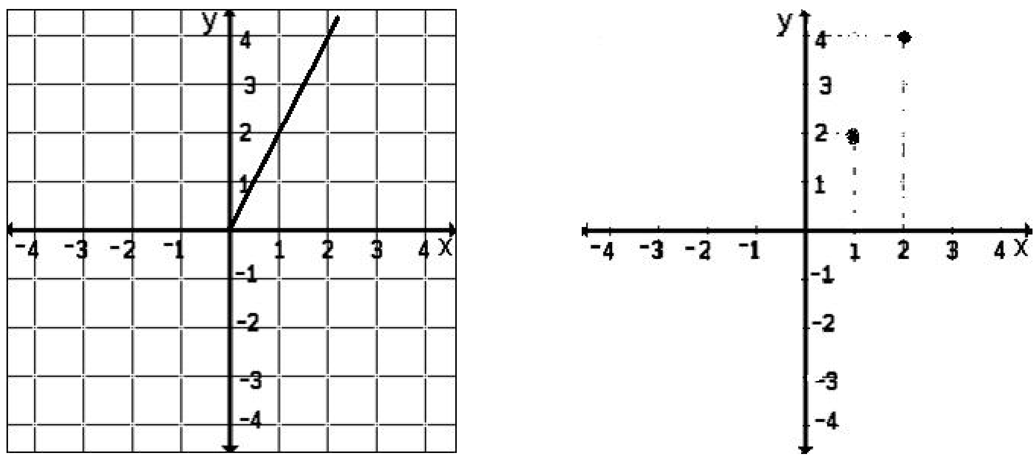


Figure 6.3. Interpretation of the data (left) and actualoutcome (right).

Teachers should be aware of the properties of the line concept. A line, irrespective of the way it is represented, holds some common properties. For example, teachers should know that an equation with at most two variables of first degree is called as lineequation. A linear equation which is graphed as a straight line indicates a type of continuous function. Hence, lines have infinitely many solutions inℝ.

To conclude, data suggested that teachers were not able to realize that the scenarios in their activities couldnot represent line but indicate only linearity. This may be interpreted as a negative aspect in terms of teachers' subject knowledge. Teachers should have overt subject knowledge of line to teach slope effectively. This requires the consciousness to distinguish examples and non-examplesof a conceptwhichwere unavailable in pre-service teachers' case.

Theoretical underpinning of pedagogy

The code is described as a teacher's perception of teaching mathematics and on the conditions under which students will learn best. There were episodes that teachers' perspective of teaching and learning mathematic was observed. Teachers' use of y=mx form (y=2x and y=3x) and tabular data for modeling is one of the essential cases that teachers' position on the theoretical underpinning of pedagogy is more observable. During the post-interviews,teachers claimed that tabular data would be more helpful for students to understand the main idea. This may be interpreted as using tables in order to showthe successive states of a variationstep by step in an effective way. Providing data in a tabular form may highlight the relationship between variables in a more convenient way.

Even though teachers did not emphasize the concept of covariation, theirway of using the first graphsand equation of line (the episodes will be included in other codes in detail), and the way slope is computed was more based on covariationand rate of change. It may reason from the fact that numerical representation is helpful in juxtaposing the data columns. Instead of suggesting the relationship at the beginning of the lesson, teachers preferred to use tabular of the lines passing through origin by using properties of direct proportionality.They emphasized the proportional relationship between the quantities by asking the relationship. This showed that teachers tried to introduce the slope by showing that there is a ratio between the quantities.

Findings supported that teachers tended to use inductive approaches to assist concept formation of the mathematical concepts and relations explored in the study. By using the word inductive, I mean that the relationship has been explored through examples of linesat first and thengeneralized to all lines. Based on observing the outcome in these examples, teachers concluded the relationship between slope and equation of a line in general.

Teachers claimed in the post-interviews that the relationship between slope and equation of line would be understood first on a line which passes through origin since it is simpler. Using lines in y = mx form as an introduction to relationship is more straightforward in reaching the objective. When compared to lines in y = mx+b form, studying with lines of y = mx may be less problematic and would be advantageous. Teachers told that a constant (such as a b value in the line equation y = mx+b) would make it difficult to understand the main idea. Cansu described her expectation in the post-interview as “A value for b may complicate students to recognize the ratio”.

In sum, teachers’ made a conscious decision to start with a y=mx relationship and generalize the results to y=mx+n relationship. In addition, they preferred to introduce the relationship between two variables via a tabular form. These two important decisions may implytheoretical underpinning of pedagogy and teachers’ perceptions of teaching the relationship in overall designing and implementing instruction. To conclude, teachers attempted to conduct an instruction which aimed to help students in understanding and prevent the difficulties they may encounter during learning the concept of slope. In other words, instructions suggested that a mathematics teacher’s mission is to provide a concept in its simpler and basic form so that students get familiar with the idea without much difficulty.

Use of terminology

The mathematical terminology that teachers use in instruction is an indicator of teachers’ knowledge in foundation unit. Teachers should use mathematical language and definitions in an appropriate way.The code, use of terminology, appeared as a code in both teachers’ instruction.

When Cansu introduced slope concept for lines in coordinate plane she warned students it is not a correct way to take coordinates of points or the distance between two points in computing slope of a line. She claimed that the crucial step in computing slope requires first the calculation of vertical and horizontal change and then dividing vertical change bythe horizontal. She consistently used the word *change* to describe slope as saying “We were looking the change in x coordinates and the change in y coordinates”.

Furthermore, Akif introduced slope through comparing various ramps by using steepness.

> Akif: Which increase are we looking for slope what would increase in the ramp… the ramp's steepness… as the ramp gets steeper… what are the differences among these ramps[he indicates to three different ramp models]..each have different steepness.

Akif's use of terminology provides important information on the way he presents slope. In addition to its proper use in the context, use of the word steepness as a measure seems to indicate that Akif introduced slope through its physical property.Akif's introduction to slope concept suggested a concentration on geometrical perspective.However, in further moments of Akif's instruction suggested that he addressed slope throughproportionality.

> Akif: What is the relationship between x and y? [pointing to the coordinate axis (1,2), (2,4), (3,6)]
> Students: y's increase as the x's increase.
> Akif: y's increase as the x's increase [he repeats to students' reply], good then how are they increasing
> Students: 2y
> Student: Proportionally
> Student: 2 times y
> Students: y increases 2 by 2
> Student: It will increase as they are directly proportional isn't it? [the students suggest their response to class]
> Akif: There is a direct proportionality… what is the ratio between x and y, is it 1 over 2… you said there is a direct proportionality, the ratio between x and y is 1 over 2 then we can express it as x over y is equal to 1 over 2. [He wrote x/y=1/2]

In addition to its proper use in the context, use of direct proportionalityseems to indicate that Akifpresented the algebraic relationship through proportionality.Akif, in further moments implied that the unit rate of this proportional relationship is same asthe coefficient m in the equation and it will be the slope of the line at the same time.

Data did not suggest whether students understood the mathematical work in the activity. In addition, data also did not suggest whether teacher planned to present these ideas or they arose by chance. However, these issues are not focused in this study since the purpose in this study was to investigate teachers' content knowledge in teaching. In brief, it suggested that Akif was able to convey his knowledge through an appropriate mathematical terminology and use of terminology positively supported Akif's foundational knowledge in slope concept and the algebraic relationship between slope and equation of a line. To conclude, the episodes as well as the whole instructions indicate that Cansu and Akifwere able to convey their mathematical ideas through an appropriate terminology.

Use of textbook

The only code which did not arise is use of textbook. Akif and Cansu did not mention using textbook in pre-interviews, lesson plans, and implementation of instructions. It was also verified by the researcher in post-interview that teachers did not use textbooks in any part of their instruction. These teachers were pre-service teachers who had mentor teachers in their practicing schools. The reason for not using a textbook may be because their teachers also did not use textbooks as the main source in their instruction. In addition, teachers might consider using textbook inconvenient since the previous lessons as well the following lessons are not implemented by them.

Reliance on procedures

The code is described as teacher's use of conventional and essential procedures during instruction.Exercise questions in teachers' instructions required to compute slope which requires using a well-known procedure. Data suggested that teachers had deficiencies in applying the procedure. Instead of applying the procedure, Cansu and Akif encouraged students to employ additional steps in reaching slope of a line. These are checking the measure of angle or inclination of line. Anepisodemay be given in Cansu's instruction for illustration.

Cansu: Is there anyone who found the slope
Students: Yes 4 over negative 1[they chose the x and y intercepts of the graphed line]
Cansu: Is 4over minus [means negative] 1 is minus 4
Students: Yes
Cansu: Okay do you remember what we talked we said that if a line inclinesto right then its (slope) is always positive.

Students: Then it is 4
Cansu: Yes it is 4 since the angle here is an acute angle okay
Student: Hıımm
Cansu: Remember if it (the line) is inclined to right it is positive (slope) if it (the line) is inclined to left it is negative (slope) [demonstrated by drawing a figure] the teacher [refers to the mentor teacher, the actual classroom teacher] has already talked about it… a graph like this [she draw a right inclined line on the coordinate plane and indicated the angle] what am I doing there is an angle here I want the tangent of that angle opposite is four adjacent is one and since it is inclined to right it is plus 4 over 1 resulting as 4[wrote +4/1=4] okay are we done with it.

While computing slope students take the quotient of interceptsof the line (the line crosses x axis at (-1,0) and y axis at (0,4))which is obvious from students' reply since 4 and negative 1 were y and x intercepts, respectively.The above episode clearly shows that Cansu does not rely on the procedure. Instead, she suggests considering the measure of angle or the inclination of line on coordinate plane to compute slope throughout the instruction. Data showed that she used additional relationships such as relation of inclination of a line to its numerical value. She used the relationship both at first and at the end of computing slope.

Cansu: There is an angle here now since the line is inclined to left I added a minus sign here there is minus sign in my slope and I also look to the change in vertical [pointing from (0,2) to (0,5)] here is 2 and reached 5 how much change.
Students: 3
Cansu: A change of 3 my vertical change is 3 since it changed from 0 to negative 3[pointing from (0,2) to(-3,2)]
Students: 3
Cansu: Again three I put a 3 to down [means denominator] minus 3 over 3 gives minus 1 slope is minus 1

Similar cases were also observable in Akif's classroom. The teacher asked students to calculate the slopes of lines from given graphs. He told that slope is the ratio of vertical distance to horizontal distance. Immediately after, he added the information on the measure of angle to the procedure.

Akif: What did we say for slope [waits a second] the vertical distance divided by horizontal distance how do we call the angle between the line and the x-axis
Students: Obtuse angle
Akif: Obtuse angle …how does it stand since it is an obtuse angle it is to the left it is inclined to left isn't it we, in this obtuse angle we call them as left-inclined the slope in left-inclined lines is always negative we have to know this then we can reach this result.

The data indicated that teachers do not refer to core idea, change, behind computing slope graphically. They suggest students to consider whether the angle is obtuse or acute since a line with an acute angle would have a positive slope and an obtuse angle will lead to a negative slope. Besides, teachers also suggest considering the inclination of lines on coordinate plane. Geometrically if a line inclines to right on coordinate plane then it has a positive slope.

In middle school grades, the conventional procedure suggested for computing slope of a graphed line is to calculate change in y-coordinates divided by the change in the x-coordinates. It requires first to select any two points on a line and then take the distances in vertical and horizontal dimensions in the same order. The last step is to take quotient of changes in vertical to changes in horizontal. Though it is not introduced to eighth grade students, the idea behind computing slope is based on calculating $\frac{y_2-y_1}{x_2-x_1}$where (x_1, y_1) and (x_2, y_2) stands for the coordinate points selected arbitrarily on a line. It should be noticed that$\frac{y_1-y_2}{x_1-x_2}$also gives the slope of a line. The order in which the coordinate points areplugged does not matter, as long as one subtracts the x-values in the same order as he subtracts the y-values.Slope computing procedure does not require any further algorithms or relationship. Hence, a robust understanding of the procedure is substantially necessary.

To conclude, data indicated that teachers were not able to rely on the key (graphical) procedure during their instruction. Instead of following the conventional procedure, teachers preferred to apply an algorithm which needed additional geometrical facts such as the identities of tangent function and inclination of lines. Teachers' dependency on a geometrical perspective of slopeduringtheir instruction has implications on foundation unit. Findings implied that teachers did not apply to definition of slopeconcept in computing slope. Instead, they chose to use identities of lines and the relationship between slope and graph of a line.

All codes together

Overall, findings suggest that both teachers have an idea of the way slope of a line could be taught and the critical knowledge needed to teach it. They combined these ideas in their instruction. Though their fundamental knowledge is remarkable, data suggested that instructions suggested some limitations in foundational knowledge and its teaching. Both teachers preferred to stress graphical representation of slope through procedural ways even though their instructions suggested diverse meaning of slope. This is especially significant since they used proportionality and change terminology for only explanations. In particular, data did not provide teachers' appreciation of a connection between line equation and function, and slope as a rate of change.

As stated, teachers had strengths and weaknesses in terms of foundation knowledge. The strengths were visible through use of terminology. Both teachers' instruction provided an appropriate use of mathematical language. On the other hand, weaknesses appeared as data suggested findings on teachers' reliance on procedures.This purpose seem to focus more on procedural attainment of the concept and does not address conceptual learning of slope such as relating slope of a line to steepness or rate of change.

Teachers had the same purpose which was investigation of the relationship between slope and equation of a line represented algebraically as y=mx+n. In other words, the purpose was to reach slope of a line represented algebraically. For the purpose, teachers used a numerically represented tabular data. In addition, teachers consciously preferred to start with a y =mx form to explore the relationship. These indicated teachers' perception of theoretical underpinning of pedagogy. Using a tabular data also suggested important implications in teachers' weaknesses in terms of overt subject knowledge.

The pre-service teachers' instruction provided almost all of the codes in foundation unit. While use of textbook was missing in pre-service teachers' instruction, the remaining codes were observed in varying degrees. For instance, identifying errors varied among teachers. While data analysis suggested that Cansu identified errors during her instruction, Akif's instruction did not suggest any episode that may be categorized as identifying errors.

Table 6.1.Summary of the foundation unit for pre-service teachers.

		Teachers	
Unit	Codes	Cansu	Akif
Foundation	Awareness of purpose	*Compute* slope of a line from its equation	*Compute* slope of a line from its equation
	Identifying errors	Few cases: students errors in computing slope	Not observed
	Overt subject knowledge	Strength ex: properties of slope of a line, Deficiency in:inadequate representation of line	Strength ex: proportionality, Deficiency in: inadequate representation of line
	Theoretical underpinning of pedagogy	Use of; tabular data in numerical representation, inductive approach, step by step and straightforward instruction	Use of tabular data in numerical representation, inductive approach, step by step and straightforward instruction
	Use of terminology	Using *change* to define slope of a line in coordinate plane	Using *steepness* as a measure and *direct proportion* as a functional property
	Use of textbook	Not observed	Not observed
	Reliance/concentration on procedures	Deficiency in reliance on procedures: computation of slope only fromgeometricperspective	Deficiency in reliance on procedures: computation of slope only from geometric perspective

6.1.2. Transformation

The unit is described as teachers' capacity in transforming the content knowledge into pedagogically powerful forms. It concerns the way mathematics is communicated to students. The unit is observed through example, demonstration, and representation that a teacher uses during teaching.

The KQ suggests three codes in the transformation unit; choice of examples, choice of representation and teacher demonstration. Teachers' selection of examples, demonstration, and representations provide a considerable amount of information the way teachers' content knowledge is in effect during instruction. Hence, as suggested in the framework, findings will be provided in terms of these codes.

Choice of examples

Teachers' choice and use of examples is a rich source that reflects teachers' content knowledge in teaching.Bearing in mind that they arise from a foundational underpinning, teachers' choice of examples will be discussed first. The aim is to get a familiarity of the findings that show the way teachers transformed their knowledge to teaching the algebraic relationship between slope and equation of a line.

Teachers started with similar scenario and tabular data that resulted in y=3x in Cansu's whereas Akif's students reached to y=2x and 2y=x as a starting point to discuss the algebraic relationship between slope and equation of a line. These lines, as an example, have a number of characteristics which will be discussed.

Examples are important means to comprehend mathematical relationships. In addition, examples may help to concept formation on the condition that they are carefully chosen. One of theadvantages of choosing line examples of y=mx, over others is that computing slope and graphing linesare considerably less confusing since calculation of horizontal and vertical change to be used in reaching slope is not problematic. Slope of lines were reached by isolating an appropriate segment of the graph of line. Both lines pass through the origin which enabled students to take (0,0) as one end of the line segments and (1,m) as the second. For instance, Cansu selected (0,0) and (1,3) to calculate slope. She also showed that another pairs of points on the line would also work. To illustrate, she chose (0,0) and (2,6) to calculate slope. In sum, teachers' first choice of examples were helpful in (i) recognizing the direct proportionality between the variables of the tabular data, (ii) demonstrating the procedure to compute slope, and (iii) reaching slope and graphs of lines.

The purpose of teachers' instruction was to show the algebraic relationship between slope and equation of a line. Teachers claimed in the post-interviews that the relationship between slope and equation of line would be examined first on a line which passes through origin since it is easier for students to understand. In addition to interview data, video records of the instructions also suggested that the examples provided an opportunity for students to observe the relationship in a clear way. The below is quoted data in which a student deducts the relationship.

Akif: Okay, let's do not consider how we computed slope on graphs. Think on the equations.
Student: The coefficient in front of x is equal to the slope.
Akif: … can we reach that result if we only think the equations [The video record suggested that teacher tries to be sure that student derived her conclusion from the equation of line.]
Student: When we look to equations we see that the slope is obvious if y is written alone in one side of the equation [Since there were two equations student makes a distinction. She gives the condition the relationship is valid.] the coefficient in front of x is equal to the slope.

Up to this point, findings indicated that both teachers' work on selection of exercises were helpful in many respects. Teachers' examples supported their aims considerably. However, when compared to Cansu's example, Akif's first examplewas more effective since the line examples enabled Akif to effectively transform the content knowledge into pedagogically powerful forms. Introducing y=2x and 2y=x at the same time might help students to conceptualize the constraints (the conditions when it is valid) of the relationship in a clear way. Algebraic relationship between slope and equation of a line is applied directly only when y stands alone in one side of a line equation. Students, who give less attention to the condition, may accidentally transfer this relation to lines which are not in slope-intercept form. This may result in a misconception such as taking the coefficient of x as slope in lines $ax + by + c = 0$. Akif's decision on including both example and nonexample is an advantage at this point.Akif'sexamples enabled to recognize the relationship and its conditions. Hence, it is visible through his selection of example that Akif transformed his knowledge effectively to confront and resolve common misconceptions.

Lastly, Cansu also mentioned the conditions of using the relationship to calculate slope. In addition to warnings such as "the coefficient of x is slope only when y stands separately in one side of the equation", Cansu included a number of line examples in which slope is to be computed algebraically. As included in Cansu's lesson summary, solving these examples (such as 3y=4x+9, 2y+5x=8)suggested the conditions. In addition, these examples enabled to see how to approach a line equation if it is not given in slope-intercept form.

To conclude, analysis indicated that the choice of examples is a rich source to explore teachers' content knowledge in terms of PCK. Teachers transformed their foundational knowledge by choosing and using appropriate examples. Introductory examples were helpful especially in (i) investigating a direct proportionality between the variables that is given tabular form, (ii) demonstrating procedure in computing slope, and (iii) providing slope and graph of a line. Lastly, teachers emphasized that the algebraic relationship between slope and equation of a line is in effect when it is in slope-intercept form.

Teacher Demonstration

Teacher demonstration is an important component of teaching. It was described as teachers' way of using demonstrations to explain procedures, rules, and other important components of learning in mathematics. Cansu and Akif did not use any specific materials to demonstrate a process. However, they applied the procedures needed to graph a line, calculate slope of it through algebraically and graphically. These were not categorized in terms of this code. The reason is that most of these instances indicated more in depth discussion in terms of other codes such as reliance on procedure. Hence, this code has been reported as not observed in pre-service teachers' lessons.

Choice of representation

The code is described as teachers' decisions of using various representations for instructional purpose.Representations are important means in mathematics instruction hence teachers may use a number of representations for the concepts to be learned. For instance, representation of a line is not unique. Lines are mainly displayed as graphically, numerically, or algebraically. Besides, different representations highlight different characteristics of mathematical concepts.A representation of line may be more helpful than others depending on the contextthe concept is taken into consideration.

Pre-service teachers started their main activities with numerically represented tabular data. The numerical representation in teachers' lesson indicated a direct proportional relationship. Hence, use of numerically represented tabular data will be discussed in terms of choice of representation code.

Teachers claimed in post-interviews that students would be able to realize the relationship between pairs of quantities without much difficulty. Akif claimed that “They can easily see the relationship between the variables” and Cansu suggested that “In this way I think that they can generalize, generalize conveniently to reach the equation”.

Cansureached the algebraic form which was y=3x based on the generalization of the numerical representation. She told that the relationship between an input (the days in her story) and an output (the number of pokes) is same as saying three times the input is output. In addition, Cansu used the numerical representation again to reach graphical representation. In achieving the relationship, she used two row data on the table which were (1,3) and (2,6).Akif also suggested using tabular data for both writing the equation and graphing. Students in Akif’s lesson continued to create graphical representation from numerical representation by using (1,2), (2,4), (3,6), (4,8), and (5,10). Lastly, they worked on the tabular data again to reach algebraic representation.

Data indicated that teachers used numerically represented tabular data as a tool in order to reach algebraic and graphical representation. The difference on using representations was that Cansu’s instruction did not suggest the use of tabular data to indicate the rate of change. While Akif lead a whole discussion on ratio between the variables by using tabular data, Cansu benefitted from tabular data in order to write the equation of a line and graph it on coordinate plane.

To conclude, choice of representation is important to effective teaching. Findings supported that teachers’ choice of representation was helpful in conveying the mathematical ideas in both teachers’ instruction. In addition to choice of representations, using representations appropriately and effectively is also important. The code of this unit suggested that even though teachers included the same representations in their lesson, Akif's instruction suggested an effective use of tabular data because it enabled to discuss important concepts such as ratio, rate of change, and proportion.

All codes together

Putting all findings in the unit together, data suggested that pre-service teachers were able to form their content knowledge in a way so that mathematical knowledge in learning slope is communicated to students.Teachers' both choice of examples and representations indicated the strength of their knowledge in transforming the content.

Data analysis suggested that there exist a number of similarities between teachers' instruction in terms of the codes in transformation unit. The interviews with teachers suggested that teachers were not in contact with each other during the planning of their practice. It is remarkable that Akif's introductory line examples were pedagogically more powerful in presenting the relationship. In addition, even though both teachers used same representations, the students in Akif's classroom reached equation of line through investigating the relationship by ratio, rate of change, and proportion. These indicated that he was able to transform his content knowledge into teaching more effectively.

Table 6.2. Summary of the transformation unit for pre-service teachers.

		Teachers	
Unit	**Codes**	**Cansu**	**Akif**
Transformation	Choice of examples	Appropriate and inductive: y=3xas a starting point	Appropriate,inductive,and more powerful: y=2x, 2y=x as a starting point
	Teacher demonstration	Not observed	Not observed
	Choice of representation	Numerical representation of tabular data to reach line equation	Numerical representation of tabular data to reach line equation through rate of change, ratio and proportion

6.1.3. Connection

Codes of the unit are making connections between procedures, making connections between concepts, anticipation of complexity, decision about sequencing, and recognition of conceptual appropriateness. The unit concerns the coherence of the planning and instruction across a series of lessons, during an individual lesson, or through an important episode. Teachers in this category have been observed during one hour since they allocated one class hour for teaching the concept. Hence, the data suggests the findings on connection unit during one class hour.

Making connections between procedures

The code is described as teachers' act of building procedural connections between multiple procedures during instruction. Findings suggested that teachers did not provide additional procedures throughout the instructions.To calculate slope, Both Cansu and Akif proposed to form right triangles in which the hypotenuse of triangle lies on the line.Teachers proposed to reach a ratio by dividing the vertical side to horizontal side. Then, they used either the measure of angle between x-axis and the line or the inclination of line to decide whether slope is negative. The unavailability of another procedure may suggest that teachers' current state of mathematics knowledge and its teaching is not multidimensional in terms of making connections between procedures.

Making connections between concepts

The code is described by the act of building conceptual connections between mathematical concepts during instruction.Effective teaching requiresmaking connections. The first connection that is raised but not advanced in detail in pre-service teachers' instructions was the relationship between slope of a line and tangent of the angle that a linear graph makes with the positive x-axis.

Connections between the concepts has taken place in Akif's classroom by his remind that "In the previous lesson you learned trigonometry for example tangent…what is the relationship between tangent and slope…first of all what is slope". The teacher rearranged his talk and did not go further in exploring the relationship between these two mathematical concepts.Cansu also followed a similar trend since she did not want to talk more on the relationship between the trigonometric ratio and the slope. Here, the conversation follows:

> Cansu: How is slope calculated?
> Students: Tangent
> Student: Opposite over adjacent. [Students raised their idea randomly]
> Cansu: Change in vertical. You can also say it as opposite over adjacent
> Students:Opposite over adjacent
> Cansu: Let's do not take it as tangent now you can also say tangent alpha is equal to the slope of that line.

It is not possible to assert the reason for sure why teachers immediately ended talking about slope as a trigonometric ratio, that is, the tangent of the angle that graph of a line makes with the x-axis.One of the possible answers is that tangent of the angles is defined in terms of the ratios of length of sides of right triangles in eighth grade. Hence, tangent of an angle is always positive in right triangles though slope of a line is not restricted to positive numbers. Besides, using tangent of the angle that a line makes with x-axis requires defining directed angles. All in all, teachers may think that connecting tangent of an angle and slope of a line would not be appropriate at this grade. Or, their content knowledge might be insufficient to build connections of slope as a trigonometric ratio of an angle.

There exist additional essential concepts which were attempted to be connected to the mathematical concept of slope. Both Cansu and Akif indicatedrelationship between slope and the inclination of lines.

> Cansu: … we said that if a line inclines to right then its (slope) is always positive. [Students reached negative 4 for slope but it was 4 actually]
> Students: Then it is 4[Students changed the result according to teacher's immediate explanation]
> Cansu: Yes it is 4 since the angle here is an acute angle okay remember if it (the line) is inclined to right it is positive (slope) if it (the line) is inclined to left it is negative (slope) [demonstrated by drawing a figure] … a graph like this [she draw a right inclined line on the coordinate plane and indicated the angle] … since it is inclined to right it is plus 4 over 1 resulting as 4[wrote +4/1=4] okay are we done with it.

Akif also talked about this relationship in his instruction. He told the relationship by making the below statement.

> "…this line makes an obtuse angle and if we look how does it look like it looks like as if it inclines to left isn't it we say that these lines have obtuse angles and called them as left inclined and left inclined lines have always negative slope we have to know this."

Data analysis suggested that teachers used relationship between inclination and slope value in a limited way during computing slope. Findings suggested two important indications. Firstly, the relationship is used in a limited way. Secondly, instead of introducing relationship as a main objective,it was emphasized only when students have difficulties in computing slope.

Teachers' use of inclination of lineswas limited. Both teachers' explanation spanned two important but incomprehensive properties of the relationship. They are "if the line is inclined to right it is positive (slope) if it (the line) is inclined to left it is negative (slope)" as in Cansu's wording. However, there are additional properties which are also very important since they enable to see inclination and corresponding slope value in a dynamic way. Graphically, slope of a horizontal line is zero. Slope of a right inclined line increases as the line gets steeper (when it is moved to left) and it is undefined when it is a vertical line. Besides, a left inclined line has negative slope. Negative slope gets close to zero valueas it inclines more to the left.In addition to graphical meaning, inclination also provides algebraic properties of a line. For example, a positive slope of a line is represented by a right inclined line since a right inclination indicates that both quantities/variables increase at the same time with a constant rate of change. To conclude, the issue may be interpreted in two ways. Firstly, connections that teachers made have deficiencies in their depth of knowledge. Secondly, findings may imply that teachers' objective was not to address inclination but to use it as a tool in computing slope. Hence, teachers did not provide a rich connection.

Teachers' use of inclination was also limited.Findings showed that teachers did not explore the relationship between inclination of a line and its relation to the value of slope until some critical instances. Data suggested that both teachers did not mention inclination of a line or the measure of an angle until students calculate slope incorrectly. In Cansu's episode, the students took the x-intercept as the horizontal change and reached an incorrect answer (since x-intercept may not be necessarily equal to horizontal distance). In Akif's episode students took only the distance in vertical and horizontal dimensions. It gave incorrect results since the distance between two points is already nonnegative though the change between two points might have a negative value.

Instances which forced teachers to talk on the relationship were the times where students had difficulty in calculating slope. Findings suggest evidence that teachers' foundational knowledge is highly related to their teaching. Episodes suggest that procedures given by teachers to compute slope was inadequate in some cases which was discussed in findings in the previous unit, foundation. As a result, teachers proposed the relationship between slope of a line and its relation to graph. To conclude, teachers used inclination as a means to compute slope.

Teachers also made some weak connections to similar triangles and ratio during their instruction. Cansu told that slope of a line is unique hence, it is not related to the segment of line that slope is calculated. She reminded students that the triangle to be used in computing slope is not unique, meaning that more than one right triangle works in calculating slope of line sinceselected triangles would be similar. Akif also explored the relationship through a discussion on ratio. The numerical representation suggested a ratio between the variables. Considering the students' comments, he was able to write $\frac{x}{y} = \frac{1}{2}$ which indicates a proportion.To conclude, teachers aim in connecting different concepts was to use them to compute slope. Interviews and instructions of these teachers indicated that making connections between conceptswere unable to serve as a means for in-depth exploration of slope concept.

Anticipation of complexity

Anticipation of complexity may be defined by teachers' awareness of students' obstacles against understanding different mathematical topics and tasks. Teachers' anticipation of students' difficulties in understanding mathematical topics and tasks deserve special attention. Data indicated that both teachers were aware of the complexity of teaching slope of a line. The organization of lesson plans, the materials included in them, and the flow of lesson provided that students did not havemuch obstacles. However, students had difficulties in applying the formula to compute slope especially the lines which have negative slope. Findings suggested that both teachers were not able to anticipate the difficulties students may face during computing slope.Since those episodes were reported extensively in previous section, they will not be reported again. The post-interview data to be provided indicates the way Cansu explained the difficulty she felt during computing slope of a graphed line (Figure 6.2).

> Cansu: Computing slope on the changes was difficult for students. I thought that students would find slope from the changes[she meant vertical and horizontal change between two points] I expected from students to form triangles [to compute slope] not necessarily on x-axis, but may be in upper side.
> Researcher: Yes
> Cansu: but [waits a second] but they could not. Then I showed it but some of them understood and some did not.Teachers show only to form triangles whose side corresponds to x-axis but I tried but they were not able to get it. It was difficult for students.

The above statement clearly illustrates that the teacher was not able to anticipate students' complexity in computing slope of a line which is graphed with a slight difference. Teachers' reaction to one of the pre-interview item had also indicated teachers' position. The item asked as "How will students think mathematically to your exercises, activities, and materials?" Both teachers' responses to this question did not indicate any anticipation of difficulty in computing slope.Anticipating students learning is one of professional work of teaching. Pre-service teachers might not anticipate the complexity of learning a concept in advance due to lack of experience. Besides, they might focus more on their teaching than students' learning.

Decision about sequencing

The code may be defined by teachers' ordering of topics, tasks or other units of instruction such as examples within and between lessons. It should be noticed that this unit concerns the coherence of the planning and instruction across a series of lessons, during an individual lesson, or through an episode. Since teachers in this category were observed during one lesson hour, the data suggests the sequence inside this one lesson.

The video records of the lessons indicated that teachers had made an important attempt in deciding the sequence of lessons. Ordering of the tasks and the exercises as well as the step-by-step movement in activities showed that they were deliberate on their sequencing. Teachers sequenced the questions in their plan in a way that students may explore and realize the relationship between equation and the slope of line. Both Akif and Cansu asked to reach the graphs and the equation of line from the tabular data, and compared equation and slope found from the graph. Very similarly, teachers moved to exercise questions immediately after reaching the relationship.

The additional activity that took place in Cansu's classroom was solving exercise problems. In these exercises, Cansu proposed to compute slope algebraically, meaning that, using the fact that the coefficient of x is equal to slope if a line is in slope-intercept form. Line equations and their ordering indicated the way the teacher sequenced exercises. Examples of lines given by Cansu were $y=3x$, $y=4x+5$, $3y=4x+9$, $2y+5x=8$, $y=13$, $y=6$, and $x=6$. The sequencing indicates that she prepared the materials for her lesson bearing in mind that she should sequence these materials according to a criterion. She gave attention that students reach the results based on what they have learned in previous exercise. In addition, their lesson plans indicated their decision of sequencing. To conclude, teachers caredin sequencing the activities and other material during this one lesson hour.

Recognition of conceptual appropriateness

Awareness of the relative cognitive demands of learning mathematical concepts, relations, and others is related to teachers' recognition of conceptual appropriateness of teaching mathematics. Teachers should be aware of the relative cognitive demands of different topics and tasks in learning mathematics. Instructional findings did not suggest any episode which may be specifically interpreted in terms of teachers' recognition of conceptual appropriateness of the subject matter. It may originate from the context of the study. In other words, Turkey's nation-wide mathematics curriculum suggests which concepts to be presented, the level, and even the durationto be spent for teaching. In addition, both teachersclaimed in the pre-interviews that teaching slope at eighth grade is appropriate decision. The reasoning Akif had was as "Students had already learned trigonometric ratio of angles in a right triangle such as tangent of an angle. They [refers to eighth grade students] may transfer what they have learned to slope. In addition, we may use ratio to introduce the relationship between variables of a line equation. I think learning slope would not be a problem for them, if we can introduce it in an interesting way".Cansu also referred to curriculum and said that "Slope is first introduced at eighth grade. I feel that the stuff up to this concept prepares students, they are cognitively prepared. Students already learned ratio, proportion and patterns between numbers. The only problem is that students may have difficulties in connecting slope and tangent function".

All codes together

Considering all the codes together, the data suggested that constructing a meaningful connection is one of the challenges for pre-service teachers. It may imply that teachers' professional knowledge needs to provide more depth and breadth in order to connect mathematical ideas and present them meaningfully. In contrast, teachers carefully sequenced their lesson which may be clearly drawn from the data.

Teachers were not able to provide additional procedures. Besides their connections with the conventional procedures in learning the relationship between slope and equation of a line was deficient. On the other hand, they indicated some limited connections between of the concepts during exercise problems. The episodes which included these connections provided to assert that teachers made connections for the sake of their proposed procedure for computing slope. Findings suggest that both teachers were careful about the sequencing the lessons and anticipating the complexities of learning the relationship.

Table 6.3. Summary of the connection unit forpre-service teachers.

		Teachers	
Unit	Codes	Cansu	Akif
Connection	Making connections between procedures	Not observed	Not observed
	Making connections between concepts	Limited and weak connections for procedures: tangent,inclination of lines, similar triangles	Limited and weak connections for procedures: tangent, inclination of lines, angle, ratio
	Anticipation of complexity	Almost anticipated but not able to predict the difficulty in computing slope	Almost anticipated but not able to predict the difficulty in computing slope
	Decisions about sequencing	Deliberate and step-by-step movement to objective	Deliberate and step-by-step movement to objective
	Recognition of conceptual appropriateness	Verified during interview	Verified during interview

6.1.4. Contingency

Contingency is related to the way a teacher reacts to unpredictable or deviant ideas and comments of students. The unit has three main codes. They are responding to students' ideas, use of opportunities, and deviation from agenda.

Responding to students' ideas

The first finding will be provided in terms of teachers' act of responding to students' ideas.It was described as the way a teacher attends to, interpret, and handle students' ideas. The data indicated asignificant variation between Cansu and Akif in terms of responding to student ideas.

In contrast to findings in preceding codes, findings in this code will be reported separately for Cansu and Akif. The reason is that, even though there exist similarities between Cansu and Akif in terms of their responses to students' ideas, there is a variety between teachers' instruction.The analysis of classroom video records suggested that classroom environment in both teachers' lesson were not discouraging for students to express their ideas. Findings will be provided for Cansu at first and then findings in Akif's classroom will be presented.

There were very few episodes in Cansu's instruction in which students raised their ideas. The teacher statedalso in the post-interview that "very few students collaborated and actively participated to the lesson".The data suggested that even in these scarce cases Cansu did not spend much time to respond to students' ideas. The below episode is an illustration of her mode of reaction to students' comments.

> Cansu: How is slope calculated?
> Students: Tangent
> Student [another]: Opposite over adjacent [students expresses their comments in random]
> Cansu: Change in vertical … you can also say it asopposite over adjacent[wrote m=vertical/horizontal]

Cansu's question such as "How is slope calculated" may be regarded as an invitation for students to express their ideas. However, the data, especially her reaction to students' suggestions, showed that she did not explore students' own proposals in detail. Even if she comprehendeda student's response, she did not work for remaining students to understand what the student proposes. The only positive feedback that Cansu suggested to the contributor is to announce whether the suggestion is correct such as by saying "you can also say it as ..." The missing, on the other hand, is that Cansu did not provide the reason she did not use a student's mathematically correct and appropriate suggestion in proceeding the lesson. As a conclusion, it is hard to say that Cansu's responses to students' ideas were beneficial to the one who raised his idea or the others in the classroom.

Cansu's reactions suggested that she was unable to get familiarity with students'ideas. By student ideas, it is meantsuch as theirstrategy to solve exercise problems, or the ways that students make sense of the slope concept.As the above episode indicates, responses to a question such as "how is slope calculated" were not elaborated by the teacher.This may result from the teacher's indecision of giving importance to student ideas. Alternatively, Cansu may be skeptical in utilizing students' comments to improve the instruction. In either way, she was not able to attend students' ideas, partially due to her current state of content knowledge.

Findings suggested that Akif encouraged students to raise their ideas and as a result students propose a number of ideas during the instruction. His instructional data suggested that he was able to respond students' ideas notably as well. To illustrate, as discussed in earlier units, Akif asked students to realize and explain the relationship between slope and equation of a line. The students were able to reach the explored relationship *on their own*. Akif examined the relationship step by step in the classroombased onstudents' ideas by their own wording.

> Akif: What is the relationship between x and y? [Pointing to the coordinate axis (1,2), (2,4), (3,6)]
> Students: y's increase as the x's increase.
> Akif: y's increase as the x's increase, good then how are they increasing.
> Students: 2y
> Student: Proportionally
> Student: 2 times y
> Students: y increases 2 by 2
> Student: It will increase as they are directly proportional isn't it?
> Akif: There is a direct proportionality.

Though the above is a short episode it clearly indicates the way Akif responded to students' ideas. A student's idea of "y's increase as the x's increase" was used by the teacher. He was also eager to use another comment in which the variation between the variables was described as "proportionally" by a student. The above episode as well as the whole instruction suggested that the teacherresponded students' comments during instruction.This is what has emerged after the analysis of data hence; the following will be an interpretation of the data in terms of teacher's content knowledge in teaching the relationship between slope and equation of a line.

The episode illustratedAkif's eagerness in responding to students' ideas. In addition, it indicated that he was successful in responding students' ideas on the relationship between the given variables. Hence, the findings provided to probe whetherthe teacher's content knowledge (SMK and PCK) on teaching the relationship between slope and proportionality was remarkable as well since he was able to make use of proportionality effectively to scaffold slope concept.Another episode in Akif's classroom will be provided.

> Akif: …we said that the coefficient of x would be the slope, okay the coefficient of x since y stands separately [in one side of the line equation]
> Student: if there would be a number instead of y
> Akif: if there would be a number [waits a second] you mean no y [he meant the case in which no y appears in line equation]
> Student: Yes
> Akif: Okay, now when we write an equation [points to the graph of line of 2y=x] we need to write in terms of y and in terms of x.

Akif followed the algebraic relationship between slope and the coefficient of x in slope-intercept form of linein computing slope. Then, a student raisedan important idea. This episode indicated that even though he tried to respond student's idea in multiple ways, the teacher was not able to suggest the correct response. In other words, the teacher was not able to recognize that a line equation does not need to include y. For example, a vertical line may be expressed as x= a where a is a number. The data suggested that he made an explanation which was dependent on his activity. According to his activity, a case such as the student proposed would not be possible. However, a line may be algebraically expressed without a y if it is a vertical line. It seems that the teacher did not remembered vertical lines. The explanation made by the teacher seemed to satisfy the student even though the explanation does not respond the student's question. To conclude, the data suggests that Akif responded to students' ideas willingly even though his responses may occasionally address the question.

It was observed that responding to students' ideas requires synthesizing a teacher's knowledge to students' ideas in varying degrees. For Cansu, it was one of the most difficult tasks. On the other hand, Akif was able to explore and probe students' responses confidently. The availability as well as the quality of responses to students' ideas is undoubtedly related to, at least in part, by the knowledge that is available to the teacher.

Use of opportunities

The code is described as teachers' use of unanticipated contributions as an instructional opportunity. As discussed earlier, students' comments and questions may provide valuable instructional opportunities. Findings indicated an episode that Cansuwas not able to use an opportunity. On the other hand, Akif's instruction provided both success and failure of using opportunities.

Akif was able to use opportunities partially. As given before, he was able to explore slope through proportionality and ratio. Since the concepts were raised by students and the teacher was able to discuss on them it may be interpreted as a positive indicator of Akif's knowledge. On the other hand, the following is a different episode in Akif's classroom. A student in Akif's class wondered a case in lines and asked about it.

> Student: if there would be a number instead of y
> Akif: if there would be a number [waits a second] you mean no y [he meant the case in which no y appears in line equation]
> Student: Yes.

The student wonders if there would be no y in the equation. Akif replied that such a case would be impossible. According to his explanation there has to be a relationship between x and y, always and without y in the equation it is impossible to find that relationship. He was not able to use this opportunity to discuss the lines in the form x=a.

> Akif: There is something like that while writing such an equation…should not we write the equation by x and y... Think the forming of a line where its x and y intercepts are given what do we do x over a plus y over b is equal to 1[the students do not remember the formula]… if there was no y how could it be possible to determine the number of goods from the number of employees.
> Student: Right
> Student: Absent [to the last question teacher asked]
> Akif: No good for these two people how can I determine it as a result there is a dependency of one variable to the other there is a combination so I cannot neglect any one of them both of them should exist at the same time so we write like that.

The first explanation the teacher suggested was on the reminder of writing the equation of line from its x and y intercepts. However, the student's question already excluded that case since there would be no y values which mean there is no y intercept. The second trial for explanation referred to teacher's own activity. The teacher explained that there is a relationship between the variables and *no y* is impossible. To conclude, Akif was not able to use the opportunity to investigate the slope issue in vertical lines. It may arise due to teacher's lack of knowledge on lines.

Similarly, the students in Cansu's class had difficulty in getting a slope value from a given graph (Figure 6.2.).Cansuinvited a student to show his solution strategy for finding slope from the given graph. The student tried to find slope by attempting to reach horizontal and vertical change for the line segment he has chosen. However, the student was not able to fix two points on the line. This resulted in an inconclusive attempt and the teacher stopped her after the struggle. This episode may be interpreted as an opportunity for teacher to provide more in-depth explanation for computing slope.

> Student: Here is minus 2 since here is 5. [the student points the y-intercept of the line] [put it as -2/5]
> Student: Why did not you add 5 and 2
> Cansu: You made a ratio of this and this[showing between 3 and 1 for change in x , showing between 5 and 0 for y, aiming to show that students were not able form a right triangle to compute slope] is there a relationship between each other.
> Student: Iıı oh no
> Student: Isn't he add by 2
> Student: We will add 5 and 7 and do we add minus 3 with 1
> Cansu: There may be an easier way… okay let's skip this item.

Cansu preferred to skip students' comments. She did not explore the reasons for students' selection of incorrect numbers. Overall, Cansu decided to visit the exercise in later moments without further investigating students' thinking. It seems that she did not consider the case as an opportunity to discuss slope. Findings indicate that she was too set on her own course to explore the ideas offered by the students.

To conclude, pre-service teachers' instruction suggested that there were episodes which opened a way to use them as an opportunity by the teachers. However, Cansu was not able to use these instances to more in-depth exploration of the learning material. On the other hand, Akif's instruction suggested a partially better case for Akif in using opportunities. To illustrate, Akif's instruction suggested combining proportionality and slope but he was not successful in exploring slope in vertical lines.

Deviation from agenda

The code was described as ability to extent teaching to further learning. Findings suggested there were not much radical distractions from both teachers' planned agenda for the lesson. Hence, the data suggested almost no cases which may be regarded in terms of the code. Findings indicated that both teachers were almost too set on their own course even though Akif was more comfortable in reshaping his instruction with slight changes.

Instructions as well as the lesson plans were highly structured in pre-service teachers' case. Almost all of the materials in their instruction served as a means to reach the objective. Teachers designed and implemented a more straightforward lesson plan. The implementation of the lesson plans suggested a linear movement by easy-to-identify steps. This supports that pre-service teachers may have a tendency to teach mathematics in a more straightforward way.

All codes together

Overall, the data suggested that teachers were open to hear students' ideas. On the other hand, while Cansu was not able to respond to students' ideas, Akif gave special attention to students' ideas. The data also suggested that Cansu might not think to use students' responses as an opportunity. In contrast, Akif was able to use opportunities partially. The issue raised to probe that teachers may able to use students' responses depending on their current state of content knowledge. Lastly, the findings suggested that teachers' deviation from agenda was almost unavailable, most probably because of their dependence on their own agenda.

Table 6.4. Summary of thecontingency unit forpre-service teachers.

		Teachers	
Units	Codes	Cansu	Akif
Contingency	Responding to students’ ideas	Weaknesses in responding	Willing to listen and respond students’ ideas
	Use of opportunities	Almost not observed: unable to address slope formula in detail	Opportunity to discuss ratio-proportion, unable to address slope in vertical lines
	Deviation from agenda	Not observed	Not observed

6.2. Novice Mathematics Teachers

In this section I will, first, summarize the instructions of observed novice mathematics teachers and then present findings in terms of the units in the KQ.

Erkinallocated one lesson hour to the concept investigated here. He introduced the slope concept with a whole class discussion on the everyday examples where slope is faced. A ramp has been used as a model for introducing slope. The teacher compared various right triangles as models of ramps and reached that as the angles and the sides vary slope changes. Triangles he used were similar to the ones in the Figure 6.4.

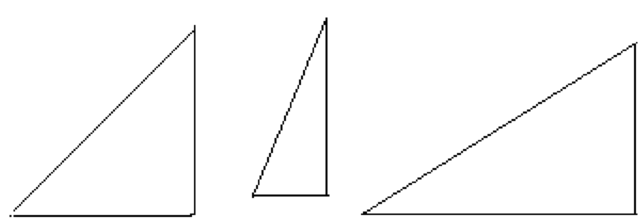


Figure 6.4.Ramps modeled by right triangles.

Erkin stressed that slope can also be regarded as a ratio since there is a ratio of two quantities. He indicated that it is reached by the quotient of the legs of a right triangle. The teacher made a remark that slope can also be defined by the tangent of angle. Though the slope may be defined as tangent, the difference, he told that he would not use the formula that is used for tangent function. He told that he would not describe slope as opposite side divided by adjacent side. He claimed to use vertical change over horizontal change to describe slope.

The teacher indicated that students will need a right triangle and the sides of it to calculate slope. To illustrate his ideas, he gave two exercise questions in which slope is explored. The first one was an air plane model in which an air plane takes off. The second exercise was a model of a kite on the air. In computing slope, he drew a line segment between two lattice points. Then, he completed a right triangle, using endpoints of the line segment, to show the horizontal and vertical increments. Then, he showed how to calculate slope as getting a ratio between vertical to horizontal distances. Erkin generalized that in both of the examples the planes are inclined to right hence resulted in positive slopes. He told that it is also possible to get negative slopes. He introduced it with another example in which slope is explored in a given right triangle.

After mentioning slope in general, Erkin continued on slope of a line in coordinate plane. He used the equation $y=2x+2$, to graph the line.The teacher, with students' comments reached values of x and y that satisfy the equation. Then, he found the slope of the line by applying the same procedure, the ratio of vertical distance to horizontal distance. The second equation of the line, which was plotted was $y=3x$. As a conclusion to these two lines, Erkin indicated that the slope of a line is the coefficient aof x if the line equation is in $y=ax+b$ form.

Erkin spent much of the remaining time to exercise questions to apply the algebraic relationship. Exercise questions he selected were $y=4x+10$, $y=-2x+3$, $3y=5x+3$, $2y+4x-30=0$ respectively. The students tried to find slope of the lines by applying the algebraic relationship. Students chose the coefficient of x in line equations as to be slope value. The teacher warned students that third and fourth examples were not in the $y=ax+b$ form so he showed the procedure to reach to this form. After all, the teacher introduced a new formula for finding slope. The teacher claimed that if a line equation is given in the form of $ay+bx+c=0$, then the slope m is as, m= $-\frac{b}{a}$. At the end of the lesson, Erkin reminded the relationship between the inclination of lines and their value, namely, if the line is inclined to right then it has a positive slope. The teacher told that students, first, should determine the angle and form a right triangle. Then, they could decide on the sign of the slope by looking at inclination of the line. Overall, the teacher attempted to show how to compute slope of a line algebraically and graphically.

The second teacher observed in this category was Yasemin. The teacher, as she reported, introduced the slope concept in previous lessons and allocated almost two hours of instruction to the mathematical relationship investigated here. Yasemin started to lesson with a two-sentence reminding on slope. Then, she plotted the graph of a line passing through the origin. She preferred to introduce the concept by an arbitrary line. By using the slope formula, she reached the equation y=mx from the graphed line. She told that the coefficient of x is the slope of the line if line passes through origin and its equation is in y=mx form. This has been exemplified through plotting y=3x and finding its slope graphically. The teacher preferred to talk on the relationship between the inclination of the lines and the corresponding slope value. She explained that the angle determines the numerical slope value. She, erroneously, summarized that the lines passing in first and third quadrants has positive whereas the lines passing in second and fourth quadrant has negative slope.

The teacher plotted the graph of a line which did not pass through the origin. By using the slope formula, she reached the equation y = mx- ma. The teacher told that -ma is a constant and she would prefer to plug n for it. Hence, the coefficient of x is the slope of the line of the form y=mx+n is reached. This has been exemplified by plotting y=3x+6 and finding the slope on the coordinate plane. The teacher told that they need to locate two points on the line to graph a line. After graphing and finding slope,she realized that the slope has no direct relationship with the quadrants that lines pass through. She explained the sign of slope by a trigonometric ratio, tangent. After this explanation, she calculated slope in two steps. First, she found horizontal and vertical distances and then, looked for the angle or inclination of line to determine whether the slope is positive or negative.

The teacher introduced the line equation 3x+2y+12=0 as a further example of how to use the relationship. Then, she indicated that slope can be expressed by the formula, $m=-\frac{\text{coefficient of x}}{\text{coefficient of y}}$ if the line equation is given in ax+by+c=0 form. She spent the remaining time to the exercise questions from students' exercise book. The examples selected for the lesson were 3y – x+5=0, x – y+11=0, and 3x+y – 1=0 in order. The students preferred to manipulate line equations in an attempt to reach y=mx+n form. Besides, the teacher reminded that the same results will be reached if the students had used $m=-\frac{\text{coefficient of x}}{\text{coefficient of y}}$. In addition to the line equations given in algebraic form, the teacher made an illustration of a building and sun light coming to it. Students were asked to calculate slope and plug it to the equation. At the very end of the lesson, the teacher asked students to compare slopes of the lines given in algebraic form. They were y=x, y=-x, $y=\frac{1}{1250}x$. The last exercise remained unsolved since the time for the lesson finished. To conclude, the instruction suggested how to compute slope of a line algebraically and graphically.

6.2.1. Foundation

Foundation is teachers' knowledge, beliefs and understanding that they constructed during preparing for their profession. Findings for novice teachers' data will be provided in terms of codes suggested for the unit of the KQ. The framework provides seven codes in foundation unit.

Awareness of purpose

The first code of foundation, awareness of purpose, will provide teachers' attitude toward teaching the mathematical relationship. Teachers' beliefs and the objectives they had for these lessons were visible through data analysis. The instructions suggested that teachers had similar aims of teaching the algebraic relationship between slope and equation of a line. After computing slope of a line by graphing, teachers told that they will observe a *quicker* way to reach slope. It seemed that they intended to encourage their students to realize a quicker wayin calculating slope.

> Erkin: Then we can find slope of a line in this way we draw figure I mean I give you equation of a line you plot it in this way and find slope however we will do its quicker ways.

Similarly, Yasemin also informed her students that they can find slope easily by applying what they learned from the relationship to the equation given. She told that "What should I remember when I hear slope I will think in a practical way".As given in the summary of lessons, the data suggested that both teachers spent considerable amount of time to represent lines graphically. Besides, they emphasized that slope is a trigonometric ratio, namely, tangent of an angle in a right triangle. In addition, slope of a line was related to its inclination. However, the analysis showed that teachers regarded the objective of lessons as presenting a quicker way to compute slope. Instructions were centered more on computing slope algebraically. Both teachers provided that the algebraic relationship between slope and equation of a line is an alternative way of computing slope of a line from its equation. In this respect, it seems that teachers' purpose in their instructions was different than the way national curriculum suggests.

Teachers gave special attention in computing slope of a line in ax+by+c=0 form. They claimed that students do not necessarily need to get a y=mx+n since there is a short cut for getting slope if a line is in ax+by+c=0 form. They motivated students to find slope quickly from the given line in order to save time. Teachers emphasized that students should use this method whenever possible. The data supports that teachers view the relationship between slope and equation of line as a practical way of finding slope. Yasemin's closing remarks of her instruction clearly shows teachers perspective.She summarized her instruction by claiming that "We learn how to compute slope of a line which may be given in three different forms [She wrote on the board $y = mx$, $y = mx + n$, $ax + by + c = 0$, $m = -\frac{a}{b}$]"

In brief, data analysis suggested that there is one dominant purpose in these lessons. Teachers behaved slope as a parametric coefficient of line equation, y = mx + b. Even though it is defined as the relationship between slope and equation of a line, the purpose of these lessons was to compute slope of a line from the given algebraic form.

Identifying errors

Teachers identified students' errorsrarely. This has been likely when students made comments or displayed their solution strategies to the exercisequestions. Similar cases were observed in teachers' identification of students' errors.

In Yasemin's classroom, after a student found two intercept points by plugging 0 values for x and y respectively in a line equation, he tried to combine these intercepts to produce a new point. Then, the teacher intervened and plotted graph of line by connecting the intercept points. Similarly, Erkin identified a student's error in plotting a coordinate point.

> Erkin: Yes who can show 0 to 2 on coordinate plane the first point in coordinate plane yes
> Student:[the student started to draw a line from (0,2) to (0,0)]
> Erkin: no[the teacher opposes] make this more visible [teacher points to (0,2) but the student points both (0,2) and (0,0)] no not there look what does it mean to have 0 to 2 where is the plane where x is 0… yes the plane where 2 is equal to 2 is here [the teacher points the line y=2] the intersection of these two points is here make it visible.

Findings indicated only one episode for each teacher.The only observed identification of errors was related to students' errors in locating points of a line on a plane. The data suggested that both teachers' instructions were more teacher-centered. Students, rarely, suggested their way of think. Hence, there was less opportunity for teachers to identify student errors. It may be claimed that teachers were not able to provide instructions in a way so that they might identify students' errors. It may also be argued whether teachers could identify errors if they existed. As a result, findings in this category is limited in understanding teachers' current state of teaching slope of a line even though it indicates evidence on teachers' perspective and belief on teaching mathematics.

Overt subject knowledge

Findings suggested that both Erkin and Yasemin provided essential mathematical ideas during teaching slope of a line. For example, teachers emphasized that two coordinate points would be sufficient and necessary to graph a line. In addition, teachers provided slope as a trigonometric ratio, namely, the tangent of the angle that a graph of line with the horizontal axis. In brief, data suggested that teachers visited essential ideas of teaching slope of a line especially in graphical representation.

There are also other episodes of instructions which may indicate a kind of limitation in terms of overt subject knowledge. Regarding Yasemin and Erkin' classroom data, findings suggested a similar outcome for the teachers. Both teachers made a distinction between slope in general and slope in lines.The subject knowledge in teaching slope of a line includes a number of key ideas. A key idea in teaching the concept includes that slope has a unique definition. However, teachers indicated that the lesson will proceed by a new slope concept. Teachers claimed that slope will be introduced in a different perspective. This view has been told by Yasemin as claiming that "Today we will show slope on coordinate plane. We will find slope of a line".Similarly, Erkin, after a ten minute of instruction on slope in planes, stated that:

> Erkin: Now, let'sfinish this slope. We use slope, in fact, more on coordinate plane. ..I mean we will look slope of a line. There is a topic as slope of a line we will do differently in those. You can write it as slope of a line it as a subtitle if you wish.

The quotes are examples of the way teachers behave slope in general and slope in lines. The quotes indicate that teachers may have a fragmented perspective in teaching slope. In other words, teachers might assume that slope is described in different ways in plane and lines. In contrast, slope is defined in one way since slope is a property of line.

Teaching the relationship between slope and equation of a line is not limited to computingit. The relationship may also be helpful in understanding other essential concepts such as ratio, proportionality, covariation and function. However, the data did not suggest this kind of focus during instructions. As a result, teachers might not be aware of the purpose of teaching this certain concept or relationship which is a prerequisite or fundamental step in learning other advanced concepts. Equally, teachers might focus more on procedural investigation of slope of a line in their lessons due to the fact that the Turkish curriculum is limited in regarding slope as rate of change.However, in any way,covariation perspective is missing in instructions. As discussed in data analysis sections the focus in the study is not on what is missing in instructions. In contrast, the focus in analyzing teachers' content knowledge is more on what is included during teaching.

Theoretical underpinning of pedagogy

Theoretical underpinning of pedagogy is described as a teacher's perception of teaching mathematics and on the conditions under which pupils will learn best. Methods and techniques used in teachers' instruction were based more on explaining and practice. Teachers tended to reveal the algebraic relationship as soon as possible. Then, both teachers practiced the relationship through exercise questions. However, teachers followed a different path in reaching the relationship. While Erkin preferred an inductive approach, Yasemin's instruction was based more on a deductive approach.

Erkin tended to use an inductive approach to assist concept formation of the relationship explored in the study. By using the word inductive, I mean that he explored the relationship through examples of lines and then generalized. Based on observing the outcome in lines $y=2x+2$and $y=3x$ the teacher concluded the relationship between slope and equation of a line in general. On the other hand, Yasemin introduced the relationship through an arbitrary line which through the origin. Then, she showed that the relationship works for $y=3x$. As a second step, she showed the relationship on an arbitrary line which do not pass through the origin. Then, it has been exemplified by plotting $y=3x+6$.Novice teachers' instruction suggested that practicing was an important component of learning slope.

Use of terminology

The other aspect of foundation concerns teachers' use of terminology. The data suggested that this code significantly indicates teachers' foundational knowledge in teaching slope of a line. Data analysis revealed that both teachers used words such as angle, calculating slope, right/left inclined line, tangentof an angle, vertical/horizontal distance, right triangle, and coefficient extensively. The mathematical language observed in instructions suggestedteachers' foundational knowledge in teaching the relationship. The instructions focused on slope as the distance in vertical dividedby distance in horizontal, the parametric coefficient, e.g., the m in the equation, $y = mx + b$, and as the tangent of the angle that a linear graph makes with the x-axis.As a conclusion, findings related to the use of terminology indicated that teachers focused on procedural attainment of slope concept in graphical and algebraic representations.

Use of textbook

Findings in this category suggested that both teachers did not adhere to textbook much. While Yasemin chose exercise questions from a textbook, Erkin did not make any explicit reference to textbook. As a result, the data did not suggest much information on teachers' content knowledge through the code.

Reliance/concentration on procedures

The final code that will be provided is teachers' reliance on procedures. The code is described as a teacher's use of conventional and important procedures during instruction.The data showed that teachers were not able to use the conventional procedure to compute slope. It also suggested that teachers benefitted from an algorithm which does not correspond to the conventional procedure. Thisprocedure suggests calculating slope as the change in y-coordinates divided by the change in x-coordinates. Instead, teachers depended on a procedure in which slope is calculated through a two-steps algorithm. Teachers calculated vertical and horizontal *distance* which is always nonnegative, and then check the measure of angle to determine sign of the slope. This calculation needed two steps in either order. When teachers introduced the algorithm they also needed right triangles. They, very often stated that students *had to* form right triangles. They used this algorithm for all exercise questions.

> Yasemin: I need a right triangle… opposite over adjacent [pointing to the distance between origin and (0,6), and the distance between origin and (-2,0)] …the tangent of this angle or
> Student: 6 over negative2
> Yasemin:Vertical distance over horizontal distance like our friend says, by y over [she wrote as m=6/-2] x [she pauses] … While computing y over x is this line inclined to right what did we say about the right inclined lines
> Students: Positive
> Yasemin: Positive yes I can see it here also [pointing to the equation] we will reach positive 3… how is this angle
> Student: Acute
> Yasemin: …Acute angle acute angle what is the tangent of acute angle if it is an acute angle slope is positive isn't it if it is an obtuse angle it is negative.
> Erkin: We form right triangles while finding slope.Yes I will form a right triangle here choose two points here [pointing to the segment of line above the x-axis] I am building a right triangle by using the points you have chosen. Vertical over horizontal
> Students: Yes
> Erkin:What would I put in front of it.
> Students: Negative
> Erkin:Why..since it is to this way since the angle is obtuse angle.

Both teachers explained the relationship between slope value and the inclination of a line. They made explicit links with angle and the value of slope during exercise questions. Erkin told that slope is related to the angle. He stated that an angle measure smaller than 90 degrees leads to a positive slope and the line will incline to right.

The use of procedures in teachers' instructions suggested that they had difficulties in practicing the conventional procedure. Data suggests that both teachers were not able to present slope concept which is free of a trigonometric ratio. Hence, as the quoted episodes indicate teachers' foundational knowledge in teaching slope is dominated by geometrical perspective. In other words, teachers were not able to follow the procedure due to the fact that their content knowledge of slope suggestsregarding slopeonly as a trigonometric ratio.

All codes together

Most of the findings did not vary between teachers. As stated, teachers had both strengths and weaknesses in terms of foundation knowledge which was observable through data analysis. Overall, data suggested that teachers' foundation knowledge was not multi-dimensional which was observable through the codes of the unit. In particular, data did not provide any attempt to introduce slope as a rate of change. Teachers' awareness of purpose, identifying errors, overt subject knowledge, use of terminology, and reliance on procedures indicated that the instructions were dominated by the procedural attainment of the slope concept through a geometrical-trigonometric ratio perspective. In addition, the only codes in which teachers' data indicated differences were use of textbook and theoretical underpinning of pedagogy.

Table 6.5. Summary of the foundation unit fornovice teachers.

		Teachers	
Unit	**Codes**	**Erkin**	**Yasemin**
Foundation	Awareness of purpose	m is slope in y=mx+n, quicker way	m is slope in y=mx+n, quicker way
	Identifying errors	Few case: plotting points	Few case: plotting points
	Overt subject knowledge	Focus on graphical aspects. Fragmented view of slope.	Focus on graphical aspects. Fragmented view of slope.
	Theoretical underpinning of pedagogy	Explaining and practicing, inductive	Explaining and practicing, deductive
	Use of terminology	Emphasis on procedural knowledge: graphical and algebraic representation	Emphasis on procedural knowledge: graphical and algebraic representation
	Use of textbook	Not observed	For exercise questions
	Reliance/concentration on procedures	Procedure that needs trigonometric ratio properties	Procedure that needs trigonometric ratio properties

6.2.2. Transformation

The unit is described as teachers' capacity in transforming the content knowledge into pedagogically powerful forms. It concerns the way mathematics is communicated to students which is observed through example, analogy, demonstration, representation, and illustrations that a teacher uses during teaching.

The KQ suggests three codes in the transformation unit; choice of examples, choice of representation and teacher demonstration. Teachers' choice of examples, demonstration, and representations provide a considerable amount of information the way teachers' content knowledge is in effect during teaching.

Choice of examples

The choice of examples deserves special attention in understanding teachers' content knowledge during instruction. Teachers have chosen a number of equations as a representation for lines. The line equations chosen for graphing in Erkin's classroom were y=2x+2 and y=3x respectively. Yasemin drew arbitrary lines y=mx and y=mx+n respectively. She also graphed the linesy=3x and y=3x+6.

The sequence of line examples that teachers chose to follow may be explored in terms of teachers' decision of sequencing and I will revisit it connection unit again. The result of data analysis which concerns choice of examples indicates that Erkin did not make a deliberate decision on selection ofexamples. The reason is that the example chosen was open to ambiguity since the first equation of line had the same value, namely 2, for both slope and y-intercept. A line equation whose constants are different, say for example y=2x+3, or y=x-4 may serve better to explore relationship in a more clear way. The second which was also the last example chosen to graph by Erkin was y=3x.

Erkin's use of examples provided that he had two main aims in choosing y=2x+2 and y=3x. First of all, Erkin reminded the procedure to graph a line and indicated the slight variation in graphing lines which pass through origin and which do not. In brief, he demonstrated that putting 0 values for both and x and y gives two distinct points of a line while it results in the same point in the second line (lines passing through origin). Secondly, he indicated the algebraic relationship between slope and equation of a line. Though Erkin's choice of examples were beneficial for the first aim, it was not that beneficial in indicating the second.

Yasemin did not started with a line example but she chose to show relationship on general form of line (y=mx, y=mx+n) as in Figure 6.5. The teacher, consciously or not, regarded those lines on the coordinate plane as having positive slope. Both of the graphs inclined to right. She graphed the lines y = 3x and y = 3x+6 to indicate the way the relationship is valid. As in Erkin's case, the teacher did not provide an example of line equation which had negative slope.

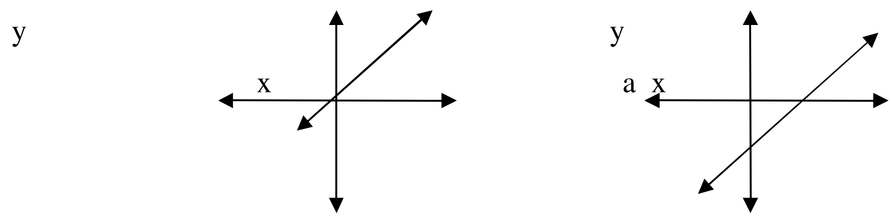


Figure 6.5. Graph of lines y=mx and y=mx+n.

All in all, the line examples chosen by teachers indicate that teachers gave special attention to emphasize that the relationship between slope and equation of a line is valid in either cases; in lines which pass through the origin and the lines which do not. It also provides opportunities for students to conjecture the roles of coefficients even though teachers did not show an interest on it. On the other hand, teachers did not provide any line which has negative slope. It is important to include lines with negative slope since students may have serious difficulties in both graphing lines and computing slope on it. To conclude, teachers' examples chosen for instructional purposes had both strengths and weaknesses in terms of helping students to learn the relationship.

Teacher demonstration&Choice of representation

Teacher demonstration is an important component of teaching. It was described as teachers' way of using demonstrations to explain procedures, rules, and other important components of learning in mathematics. Teachers in this category did not use any specific materials to demonstrate the concept. However, they demonstrated the procedures needed to graph a line or calculate slope of a line.

Teachers introduced and relied on checking measure of angle throughout the lessons. To them, students should check their results by the measure of angle.

> Yasemin: We look only to angles in determining the positive and negative slope.
> Yasemin:Since the slope will be negative in obtuse angles we put a negative sign in front of the ratio we found.
> Erkin: How will I find slope which angle I should look and find slope… obtuse angle acute angle look in acute angle and it is positive obtuse angle in fact the angle which looks to positive side of x is obtuse angle and in an obtuse angle the result is negative.

In computing slope, teachers used $\frac{\text{vertical distance}}{\text{horizontal distance}}$ and checked the measure of angle. The use of the algorithm to find slope of lines on their graphs necessitated the formation of right triangles. Teachers located a right triangle between the line and the x-axis whenever possible. Erkin explained that:

> Erkin: I am trying to form a right triangle here. I am forming a right triangle by any points you choose.
> Erkin: I am okay but what should I put in front of it.
> Students:negative
> Erkin: why... it is to this side since the angle is obtuse.
> Yasemin: I did determine the tangent of this angle what is the angle let's the angle would be A. what is tangent A angle opposite right side divided by adjacent right look from [forming a right triangle]a right triangle is created…I need a right triangle to compute slope.
> Yasemin: it is an obtuse angle and the slope is negative we put a negative sign in front of the ratio.

The algorithm used by teachers may indicate the way teachers understand the slope of a line. It seemed from the data that teachers depend on angles and inclination in computing slope. They use right triangles to find slope of a line. In addition, it may also indicate an assumption that the algorithm that they suggested was a good choice to find slope when it is compared to work to be done for using $\frac{\Delta y}{\Delta x}$.However, since the change gives also the sign, the formula of change in the y-coordinates divided by the change in the x-coordinates is particularly important. Hence, one does not need to check the result with by magnitude of angle or inclination of line.

Findings indicated that teachers demonstrated the procedures and the algebraic relationship through practicing directly on exercise questions. Throughout the instructions teachers showed that there is a direct relationship between slope value and its geometrical representation. Through solving exercise questions, teachers stressed that students should consult to graphical representation to understand whether slope is nonnegative. Data also indicated that teachers primarily focused on graphical and algebraic representation of slope of a line. All graphical examples in teachers' lesson were related to trigonometric properties.

All codes together

To conclude, data indicated that teachers transformed the necessary knowledge as their foundational knowledge permitted. It was remarkable to observe that (i) findings in the previous unit and in this unit are congruent. In sum, teachers' choice of examples indicated that they gave importance to show that the algebraic relationship between slope and equation of line is valid which does not depend on whether line crosses the origin or not. In addition, both teacher demonstration and choice of representation supported the previous findings that teachers behave slope as a purely trigonometric ratio perspective.

Table 6.6. Summary of the transformation unit for novice teachers.

		Teachers	
Unit	**Codes**	**Erkin**	**Yasemin**
Transformation	Choice of examples	Introducing slope by a limiting example, diversity in exercises.	Use of y=mx and y=mx+n before examples, diversity in exercises.
	Teacher demonstration	Demonstration by practicing through exercise questions.	Demonstration by practicing through exercise questions.
	Choice of representation	Geometrical representation by using right triangles.	Geometrical representation by using right triangles.

6.2.3. Connection

Connection unit concerns the coherence of the planning and instruction across a series of lessons, during an individual lesson, or through an episode. The codes in the connection unit are making connections between procedures, making connections between concepts, anticipation of complexity, decision about sequencing, and recognition of conceptual appropriateness. The findings will be reported in terms of these codes.

Making connections between procedures

The code is described as teachers' act of building procedural connections between procedures during instruction. Both teachers' instruction suggested two main procedures. These are (i) computing slope of a graphed line and (ii) computing slope of an algebraically represented line.Findings suggested that teachers depend on their unique ways and did not provide additional connections to other procedures throughout the instructions.

To compute slope graphically, both Erkin and Yasemin proposed to form right triangles in which the hypotenuse of triangle lies on the line. By dividing the vertical side to horizontal side teachers reached a nonnegative ratio. Then, teachers used either the measure of angle between x-axis and the line or the inclination of line in order to decide whether slope is positive or negative. The unavailability of another procedure may suggest that teachers' current state of content knowledge and its teaching is not multidimensional in terms of making connections between procedures in graphical computation of slope.

The line equations of ax+by+c =0 form took special attention in both of the classes. Teachers claimed that students did not need to convert an equation of ax+by+c=0 form to y=mx+n form in order to find the slope. According to teachers, there exists a quick way of finding slope directly from any form of equation.

> Erkin: By just knowing this formula [pointing to the y=mx+n] in fact not a formula but easy way, we can solve problems but there exist an easier way also.
> Yasemin: In ax+by+c= 0 [wrote as $m = -\frac{\text{the coefficient of x}}{\text{the coefficient of y}}$] you can find slope directly.

Teachers preferred to introduce the formula after giving the slope intercept form in order to find slope. Both teachers displayed the connection between two procedures through an exercise. Based on the solution of the exercise, students were asked to explore if there could be an alternative way. Teachers told that the new formula provides the slope of a line from its algebraic representation in an easier and quicker way.

> Yasemin: …the coefficient of x divided by the coefficient of y if the equation is in ax+by+c=0 form.
> Erkin: If we had an equation like that [wrote to board as ay+bx+c=0] how can I find the slope how did it turn out as a simpler way look to the example the coefficient of x then I divided by the coefficient of y… then I put a negative sign.

Teachers attempted to indicate that the procedure needed to get the slope of a line in ax+by+c=0 form is derived from the procedure in slope-intercept form. Both Erkin and Yasemin justified the reason for using the second procedure. As a conclusion, it may be claimed that teachers aimed to make a meaningful connection between these two procedures.

Making connections between concepts

There exist a number of connections between mathematical concepts and teachers should make those essential mathematical ideas connected.The code is described by act of building conceptual connections between mathematical concepts during instruction.Effective teaching requires making connections.Data suggested teachers' attempts to make connections between some trigonometric concepts throughout their instruction.

> Erkin: Then I will use tangent when I find slope it turns out as positive isn't it then the angle is.
> Students: Acute.
> Erkin:Acute angle then if it becomes acute angle how is slope.
> Students: Positive.

Yasemin followed a very similar path to Erkin in introducing the connections between the concepts. She told that "A slope of a line is the tangent of an angle to the positive direction of x coordinate. If the angle is acute then slope is positive and if the angle is obtuse then slope is negative".

Both teachers presented the connection between tangent of an angle and slope of a line. In these episodes of instructions for this connection, teachers preferred to make references to right triangles, and measure of angle. Another connection which was emphasized by teachers was the inclination of lines and the corresponding slope value.

> Yasemin: …Vertical distance over horizontal distance like our friend says, by y over [she wrote as m=$\frac{6}{-2}$] x [she pauses] … While computing y over x is this line inclined to right what did we say about the right inclined lines
> Students: Positive
> Yasemin: Positive yes I can see it here also [pointing to the equation] we will reach positive 3…

Yasemin used inclination of a line as verification for her solution. She recognizes that her suggestion as m=$\frac{6}{-2}$is incorrect since the graphed line was a right inclined one.Connections made by the teachers during their instruction suggest that these connections either helpedteachers in verifying the solutions or they were prerequisite knowledge to be able to apply the algorithms for computing slope. In other words, the aim in introducing the relationship that the angle is in effect in learning whether slope is a positive number was not to provide a rich meaning for slope. Teachers, in a way, needed to provide the connections which clearly indicate their trigonometry dependent knowledge of slope. In addition, it is also not possible to claim from the data that students understoodwhy angle or inclination of a line determines whether slope is nonnegative.

Findings in the previous codes and this code seem to indicate a correspondence. For the transformation unit, it was concluded that teachers relied more on geometric representation of slope. Similarly, the data indicated that teachers' choice of representation required teachers to emphasize these connections. To conclude, the findings suggested that teachers regard slope totally a geometrical entity which is a trigonometric ratio of an angle, tangent. As a conclusion, the connections that both teachers preferred to present were more on geometrical ones. Teachers' ways of behaving slope concept during their instruction are relatedto the connections that they aimed to provide. ,

Anticipation of complexity

Anticipation of complexityis described as teachers' awareness of students' obstacles against understanding mathematical topics and tasks. Data indicated that teachers were cautious more on the challenge that students may have during computing slope of a line. Both teachers warned students that they should carefully check whether the results are correct.

> Erkin: Since the line is to left then I put a negative to the value we have found [the teacher indicates the nonnegative ratio he got from the exercise question]...should I write it as a remark, when we make it negative
> Students: Yes
> Erkin: The angle should be obtuse or our hill [indicates the exercise question] should be to the reverse side.
> Yasemin: [after reaching the slope value] But we should be careful about to check the angle. I start with the x axis and the positive angle to the line is necessary look here it is right [angle] and then I increased this angle and we call the angles bigger than right angles as obtuse angle. If the angle is obtuse then slope is negative. Let's write it.

Teachers tried to resolve the deficiency of their algorithm by indicating the above relationship. Findings, at least, show that teachers are aware of the difficulty that students may face in using the algorithm to compute slope. Hence, data indicate that Yasemin and Erkin are concerned with student learning. The following episodes –first Erkin's and then Yasemin's- also show teachers' anticipation of complexity in learning slope of a line and its algebraic representation.

> Erkin: Here there is y and here 5x plus 3 over 3 now can I split to fractions [he meant $\frac{5x}{3}+\frac{3}{3}$].
> Students: Yes
> Erkin: You might not remember it
> Student: 5x over 3
> Erkin: 5x over 3
> Student: 3 over 3
> Erkin: …Now what is the coefficient of x.
> Students: 8
> Erkin: 5 over
> Students: 5 over 3

Erkin:…okay you do not need to split like that every time we already see that [indicates to $\frac{5x+3}{3}$] it is $\frac{3}{3}$ but I did this for the first time to make it understandable for you.

Yasemin: [After reaching the solution as y=$\frac{3}{2}$x + 5] You did everything right and had an equation like that but you observe that there is not an equation in the choices
Student: Then we try it through another way
Yasemin: There is already a line but it is not included in the choices
Student: Then the question is not correct
Yasemin: No everything is okay the question is also right how can we write the equation differently
Student: Do you mean triangles or the equations like 8, 12, 6, [Student's comment was not understood by the researcher, but data indicated that Yasemin seemed to understand student's comment]
Yasemin: No nothere is no relationship between what you say and the thing here. I just want you to manipulate the equation and write it in this way [she indicates the ax+by+c=0]…okay you will have same denominators … what I need is to have the same denominators if I had an equation with fractions [indicates when a coefficient comes up as a fraction].

It is remarkable that both episodes illustrate almost a similar issuewhich is handling fractions in line equations. Both teachers' instruction suggested that they anticipated that students may have difficulties especially when they deal with fractions in writing the equation of a line. As a conclusion, data indicate that teachers' expertise as well as their current state of content knowledge may motivate them to act more on those lines. All in all, it is clear that there were episodes in both teachers' instruction which may indicate positively in terms of the code, thereby, in terms of the unit.

Decision about sequencing

The code is described as teachers' ordering of topics, tasks or other units of instruction such as examples within and between lessons. It should be noticed that this unit concerns the coherence of the planning and instruction across a series of lessons, during an individual lesson, or through an episode. Since teachers in this category were observed during either one or two lesson hours, the data suggests the sequence during these durations.

Teachers shouldsequence instructions within and between lessons, including the ordering of tasks and exercises. It was observed through the data that both teachers had ideas in sequencing their instructions even though the decision of sequencing was more evident in Yasemin's instruction.

The main reason to claim that teachers had sequenced their instruction is because they introduced slope by positive examples. Teachers presented slope concept through positive ones and then generalized to negative slope values. Starting with positive slope may be regarded as an appropriate starting point in teaching slope of a line especially because it indicates a direct proportionality. In addition, computing a positive slope is often easier than reaching a negative slope value especially for the students who have difficulty in applying conventional procedure.

Teachers' choice of introductory examples differed. Yasemin introduced the algebraic relationship between slope and equation of a line on a line passing through origin.The second step in Yasemin's instruction was showing that the relationship holds even if lines do not cross origin. Sheexplained the idea by reminding that lines which do not cross origin may be produced from the lines that pass through origin by shifting them. Yasemin's attempt shows how she was careful on sequencing.On the other hand, Erkin followed a different path. He preferred to introduce the algebraic relationship on a line not passing through origin. Erkin introduced the relationship through y=2x+2 first and y=3x after.

Lastly, teachers'inclusion of a new slope-equation relation also indicates positively to their decision of sequencing. Both of the teachers preferred to talk about the new relationship after some exercise questions in which slope was calculated by using slope-intercept form of lines. Teachers asked students to investigate the algebraic relationship in ax+by+c=0 form after students get acquainted with using y=mx+n form to compute slope. These episodes indicated that both teachers gave importance to show that computing slope of a line in ax+by+c=0 form is nothing new because it is derived from what students already learned. In addition, the order of exercise questions (line equations) in both teachers' instructions suggest teachers' way of thinking in learning slopes of an algebraically represented line.All in all, data indicated both positively and negatively on teachers' act of sequencing during teaching slope. As a result, teachers' decision of sequencing indicated that both teachers' individual state of content knowledge and its image in instruction.

Recognition of conceptual appropriateness

Recognition of conceptual appropriateness is the last code of the unit. Awareness of the relative cognitive demands of learning mathematical concepts, relations, etc. is related to teachers' recognition of conceptual appropriateness of teaching mathematics. Teachers should be aware of the relative cognitive demands of different topics and tasks in learning mathematics. Data did not indicate any episode for Erkin which is coded as his recognition of conceptual appropriateness. As it was the case for pre-service teachers, the result may originate from the context of the study. In other words, Turkey's nation-wide mathematics curriculum suggests which concepts to be presented, the level, and even the time to be spent for teaching. Though the issue is also valid for Yasemin, data suggested an additional episode which may be treated as an indicator of Yasemin's recognition of conceptual appropriateness.

An episode in Yasemin's instruction suggested whether she recognizes what is conceptually appropriate in teaching slope concept. Upon recognizing her mistake of making an incorrect generalization, Yasemin tried to correct her idea by suggesting another explanation. However, the explanation she made included an advanced relationship that 8 grade students were not expected to know. Before analyzing whether teacher recognizes that her explanation is convenient it should be better to learn the incorrect generalization she made.

Yasemin tried to relate the slope value to line's graphical inclination. She told that if a line makes an obtuse angle to the x-axis in the positive direction then its slope is negative and if it makes an acute angle then the slope is positive. To justify the conjecture she said that the reason for this idea is related to quadrants of coordinate plane. In brief, she claimed that if a line passes through first or third quadrant it has a positive slope and if it passes through the second and fourth quadrant it will have a negative slope. However, Yasemin, through observing a contradictory example, recognized that such a generalization would not be correct and she tried to correct it by another explanation.

> Yasemin: We made a mistake here, let's correct it…There is information like. you have learned trigonometry but it is not given in eighth grade.. the tangent of angles which complete each other to 180 degrees have the same value but there is a difference in their sign.

Yasemin's explanation includes knowledge of tangent for obtuse angles.Yasemin recognized that her explanation was not conceptually appropriate. She did not insist on her explanation. She preferred to explain by another example through drawing a new line. The episode indicates that Yasemin is familiar with whether her explanation is conceptually appropriate though she felt some problem at the beginning. She knows depth and breadth of trigonometry knowledge that eighth graders ought to know or cancomprehend without much problem. In addition, the episode also shows how she was involved on her students' learning.

All codes together

Putting all the codes of the unit together, instructions did not suggest considerable and meaningful connections between slope and other important concepts in mathematics. It may imply that either novice teachers need to have more depth and breadth of content knowledge in teaching slope or they need to a focus to make connection during teaching. In addition, findings for the codes indicated a dominant focus on procedural attainment of slope concept which is especially taken as a trigonometric ratio.

Teachers were not able to provide additional procedures. They indicated some limited connections between the concepts such as connecting slope to tangent of an angle. The episodes which included these connections provided also that teachers made connections for the sake of their proposed procedure for computing slope. Findings suggest that Yasemin was more careful on sequencing the lessons.

Table 6.7. Summary of the connection unit for novice teachers.

		Teachers	
Unit	**Codes**	**Erkin**	**Yasemin**
Connection	Making connections between procedures	Connections were not observed in graphical procedures, but in algebraic procedures	Connections were not observed in graphical procedures, but in algebraic procedures
	Making connections between concepts	Connections to trigonometry needed for using procedures	Connections to trigonometry needed for using procedures
	Anticipation of complexity	On computing slope graphically, and in dealing with fractions in equations	On computing slope graphically, and in dealing with fractions in equations
	Decisions about sequencing	Needs more attention in introduction but exemplary in exercise questions	Exemplary in both introduction and exercises
	Recognition of conceptual appropriateness	Not observed	A trigonometry case

6.2.4. Contingency

Contingency is concerned with the way a teacher reacts to unpredictable or deviant ideas and comments of students. The unit has three main codes which are responding to students' ideas, use of opportunities and deviation from agenda. Findings provided limited number of episodes on teachers' contingent actions during their instruction. The data related to unit indicated that the interaction of ideas was limited and one-way. The only code which was visible after data analysis was teachers' responses to students' ideas.

Responding to students' ideas

The code is described as the way a teacher attends to, interpret, and handle students' ideas. The analysis of classroom video records suggested that the classroom environment in both teachers' lesson were not discouraging for students to express their ideas. However, teachers did not prefer to ask open-ended questions very often during the instruction. As a conclusion, there were almost no episodes available to be interpreted in terms of the code.

There was only oneshort episodethat took place in Yasemin's instruction in which a student raised his idea. The episodesuggested that even in this scarce caseYasemin spent little time to respond to the student's idea. The below episode is an illustration of her mode of reaction to students' comments.

> Yasemin: No everything is okay the question is also right how can we write the equation differently? [Teacher's aim was to show that a line in slopeintercept formmay alsobe written in ax+by+c=0 form]
> Student: Do you mean triangles or the equations like 8, 12, 6, [it is not understood by the researcher]
> Yasemin: No no there is no relationship between what you say and the thing here. I just want you to manipulate the equation and write it in this way [she indicates the ax+by+c=0]…okay you will have same denominators … what I need is to have the same denominators if I had an equation with fractions [indicates when a coefficient comes up as a fraction].

Yasemin's question may be regarded as an invitation for students to express their ideas. However, the data, especially her reaction to the student's suggestion, showed that she did not explore students' own proposals in detail. She comprehended the student's recommendationbut did not worked for remaining students to understand what the student proposed. The only feedback that the teacher suggested to the contributor is to announce whether the suggestion is correct. The missing, on the other hand, is that Yasemin did not provide the reason or the way that student's response is not appropriate to be used by the teacher.

Both teachers' aim in asking questions was not to get familiarity with students' ideas. As the above episode indicates, students' responses were not elaborated by the teacher. This may result from the teacher's point of view that giving importance to student ideas will be whether beneficial.

It was observed that responding to students' ideas requires synthesizing a teacher's knowledge to students' ideas in varying degrees. For novice teachers, it was one of the tasks that they need to effort. The availability as well as the quality of responses to students' ideas is undoubtedly related to, at least in part, by the content knowledge (both SMK and PCK) that is available to the teacher.

Use of opportunities& Deviation from agenda

Use of opportunities is described as teachers' use of unanticipated contributions as an instructional opportunity. As discussed earlier, students' comments and questions may provide valuable instructional opportunities. The findings did not indicate any episode for the code. The other code, deviation from agenda, is described as ability to extent teaching to further learning. Analysis of lesson video records suggested that teachers followed their predetermined agenda strictly during their instruction with no radical deviation from agendas.

Findings indicated that both teachers were almost too set on their own course. Hence, it may result in the unavailability of the examples of the codes. Besides, it may also result from teachers' perspective on teaching mathematics. The way teachers lead the instructions were unable to produce any opportunity for teachers to deviate from their predetermined agenda. As claimed in foundation unit teachers' instruction suggested that they explained and practiced during instruction. As a result, students' influence was almost negligible in teaching slope of a line.

All codes together

To conclude, novice teachers structured and sequenced their instructions in a way where students' contribution was limited. Data indicated that questioning strategies used by the teachers were unable to enhance this contribution. Almost no episodes were taken as an indicator of teachers' contingent actions during teaching.

Table 6.8. Summary of thecontingency unit for novice teachers.

		Teachers	
Units	**Codes**	**Erkin**	**Yasemin**
Contingency	Responding to students' ideas	Not observed	Observed, limited
	Use of opportunities	Not observed	Not observed
	Deviation from agenda	Not observed	Not observed

6.3. Experienced Mathematics Teachers

In this group I will provide findings from two experienced teachers' instructions. The teachers Müge and Öznur, introduced the slope concept in the previous two lesson hours. They spent two lesson hours to graph a line, compute slope of it on its graph, and explore the relationship between slope and equation of a line.

Müge started the instruction with the graph of y=2x and compute slope by first forming a right triangle on the coordinate plane and second by creating a table in which x and corresponding y values are inserted from the points got from graph (Figure 6.6). She showed that the slope is same as the coefficient of x in the equation.

x	y
0	0
1	2
2	4

Figure 6.6. Line in tabular form.

As a second phase, the teacher provided four lines in the same coordinate plane all of them passing through the origin. She formed a whole class discussion on the inclination of the lines and the *sign* of the slope. She used slopes of lines graphed (m=1, m=4, m=-4, m=-1) to help students compare lines. The teacher summarized findings. Then, instruction followed by computing slope of lines given in graphical forms.

For the next phase, Müge asked students to find the equation of a line given graphically. The teacherrepeated that the slope value and the coefficient of x are same. Then, shedirectedadditional line equations in order to apply the newly learned relationship. The equations were y=3x, y=-2x+1, y=12- $\frac{3}{4}$x, 2y=12x+6, 5y=10-7x, 3x+4y – 8=0 and 8x+2y=9.Students attempted to compute slope values by using the algebraic relationship in these examples.

Lastly, Müge asked to compute slope of a line in which only two coordinate points were given, (1,2) and (3,6). Students had difficulties in reaching the intercept points and equation of the line. The teacher reminded to students that they should remember how to write equation of a line from its intercept points. The teacher used the formula eventhough the students did not remember it. The last exercise was on finding slope of a line which passes through (-2,4) and (-1,-5) and write the equation of the line by using the points and calculated slope.

The second teacher, Öznur, used a Power Point presentation in her lessons. She told, at the beginning, that she would use the presentation program in addition to exercise questions in the exercise book. She started the instruction with an exercise on finding slope of a plane in which a ski slope is modeled on coordinate plane. The slope of same ski plane has been asked again by iterating the plane on the coordinate plane without any change in horizontal and vertical dimensions. By using this physical situations, the teacher discussed the parallel lines-slope relationship. The next exercise asked students to find the slope of a castle’s walls (Actually, it was asking slope of the hill in which castle was founded)whose several points were indicated on coordinate plane (Figure 6.7). She asked whether there is a relationship between the slope computed and the equation, y=2x-1, of the line.

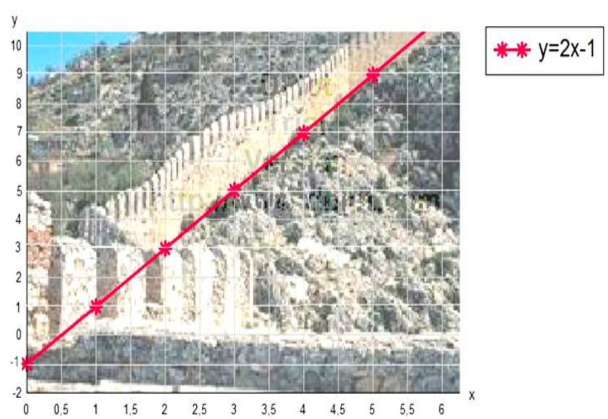


Figure 6.7. Castle figure on coordinate plane.

The lesson followed by solving exercise questions from the textbook such as graphing y=2x+4. Öznur stressed the algebraic relationship again. Then, she discussed the slopes of the lines y=4 and x=-2. Then, a significant amount of time has been allocated to the discussion on two concept cartoons. In these cartoons two conjectures were given. They were (i) "Slope is calculated by dividing the change in y to change in x." and (ii) "Slope is the coefficient a of the equation y=ax+b".

Öznurdirectedexercise questions on computing slope where two points that satisfy the equation of line is given. These questions were chosen from students' exercise book. The teacher reminded that slope is the coefficient of x in y=ax+b though students preferred to find slope by plotting the graph of lines. She discussed different strategies of calculating slope. At the end, she reminded that slope is the coefficient a when the line equation is in y=ax+b form.

6.3.1. Foundation

Foundation is teachers' knowledge, beliefs and understanding that they constructed during preparing for their profession. Data analysis of experienced teachers' lessons suggested teachers' foundational knowledge inteaching slope. Findings will be presented in terms of the codes of the unit.

Awareness of purpose

The code is described as a teacher's awareness of objectives, aims and goals of teaching mathematics. Findings in this category will provide teachers' attitude toward teaching slope concept and the algebraic relationship between slope and equation of a line.

Teachers' beliefs and the objectives they had for these lessons were visible through data analysis. Müge and Öznur indicated in the interviews that they had introduced the slope concept in various contexts in previous lessons. According to teachers, the aim of the observed lesson would be on learning the relationship between slope and the equation of a line. The lessons went more on to graph lines and calculate slope of those lines. This has also been indicated in the interviews. Teachers, while talking about their lesson plans, made these statements.

> Müge: I will talk about lines, what it means to be m in y=mx+n. I will show slope of a line in these lessons.
> Öznur: In the previous lessons we talked about slope on plane in this lesson I will talk about slope of a line we will explore equation and graph of a line.

The objectives of the lessons were also told to students. Öznur started to instruction by stating that they will explore the slope in lines. Similarly, Müge wrote a title on the board as slope of lines.

Both teachers gave importance to draw graph of lines. In addition, they spent considerable part of their instruction to compute slope graphically. Teachers indicated the relationship between slope and equation of a line through exercise questions in several times.The purposes in teachers' instruction were (i) to draw graph of lines, (ii) compute slope graphically, (iii)compute slope of a line from its equation. Classroom data did not suggest any attempt to introduce ratio, proportion or covariationthough teachers used these concepts implicitly in exploring the nature of slope formula. As a conclusion, data suggested that teachers took slope concept as a ratio such as *change in y over change in x* and as a parametric coefficient which is m in the equation, $y = mx + b$

Identifying errors

The code is described as a teacher's ability to identify mathematical errors that students, textbook, or any learning material may suggest during learning mathematics. The data indicated teachers' content knowledge in teaching slope concept through the episodes in which they identified students' errors.

There wereepisodesin which students' ideas and way of thinking were more observable. Teachers were able to distinguish students' ideas by hearing students' comments or strategiesandthen to correct students' errors. This has been achieved, generally, when students suggested a solution strategy to the exercise questions. The data showed that students may have difficulties in understanding how to plot points of a line on plane. In addition, students may have errors in behaving coordinate points. To illustrate students in Müge's class often did not pay much attendance to the order of coordinates.

Müge: The coordinate of these points
Students: 2 and 1[students took y value first and x as the second]
Müge: 1 and 2. Don't make this error x is the first right.

A more complicated error has been identified by Öznur during her instruction when she asked to graph the line 3y-x+5=0. A student suggested an irrelevantstrategy to graph the line.

Öznur: What are you going to do tell us please?
Student: We will find the vertical distance 3 [pointing to 3 in the written equation 3y-x+5=0]
Öznur: All right
Student: Then, I would find 5 then [She pointed to the constant coefficient of the equation and started to locate (5,3) on coordinate plane]
Öznur: It would be very easy in that way but we do not do in this way.

Both examples suggest that students may have serious misunderstandings and errors. It is evident that both teachers carefully identified students' errors in graphing a line which indicate positively on teachers' foundational knowledge. Below episodes are two examples of student errors during in computing slope.

Öznur: Please can you explain how you reached negative 1 [The student reached the slope of line passing through (0,0), and (-1,1) and the teachers questions student's solution]
Student: The coordinates are -1 to 1 hence if we divide -1 to 1
Öznur: Are you dividing the coordinate points or you do something different [she checks whether the student makes a mistake]

This short episode is provided for a reason. It is remarkable that Öznur successfully identified a very common error in computing slope even though the student's answer to the exercise question was correct. Hence, it may be claimed that Öznur's instruction suggested good examples of the code. The teacher's instruction indicated how she was careful in identifying errors, thereby indicating positively on her foundational knowledge.

Student: I did like you said [the student implies that used $\frac{y_2-y_1}{x_2-x_1}$ to compute slope] but I did a mistake in somewhere.
Müge: Look your friends also followed that rule and reached a correct answer you should follow the same pattern I mean if you take one's y first then you should take its x first okay…

This episode is an example in Müge's instruction in which she identified the student's error. Her response to student's inquiry suggests that she was sure where the student's error is in using the formula. This and other similar types of episodes in teachers' instruction suggest that they were successful in identifying students' mistakes during teaching slope.As a conclusion, findings in this code are a positive indicator of teachers' foundational knowledge. In other words, teachers were able to identify students' content-related errors during teaching slope of a line.

Overt subject knowledge

The code is described as teachers' critical understanding of content to be taught.Teachers' depth of subject knowledge in teaching mathematics was visible through data analysis. Findings suggested that both Müge and Öznur provided essential mathematical ideas during teaching slope of a line. To illustrate, teachers emphasized that two distinct coordinate points would be sufficient and necessary to graph a line. In addition, teachers provided slope as a trigonometric ratio, namely, the tangent of the angle that a graph of line with the horizontal axis.

Müge gave attention to the inclination of lines and its relationship to slope value. Though she summarized the finding as claiming that right inclined lines have positive and left inclined lines have negative slope, the episode indicated the dynamic relationship between graphical and algebraic representation of slope. By plotting four graphs in coordinate plane at the same time she indicated that a flat surface or a horizontal line has no slope and slope will increase if it is moved to counter clock wise. As a conclusion, she showed how slope gets bigger and smaller as the inclination of line is changed.

On the other hand, Öznur followed a different way in reaching slope of lines in two critical cases. Horizontal and vertical lines are critical since the slope is zero for the first and undefined for the second. The teacher preferred to introduce theserelations on two examples of lines. Öznur asked students to find the slopes of lines y=4 and x=-2 as in the Figure6.8.

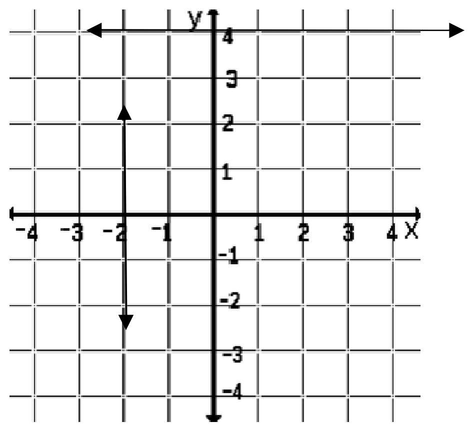


Figure 6.8.Vertical and horizontal lines.

Öznur's selection of y=4 and x=-2 to explore slope in vertical and horizontal lines is essential in several respects. The episode will also be analyzed in terms of other codes (such as choice of examples, anticipation of complexity) in multiple ways.

The conceptual understanding of slope requires that slope is an attribute of non-vertical lines. In addition, calculation of slope in any segment of line also gives the slope of that line due to similarity between the right triangles. Besides, slope of lines plotted in coordinate plane may be calculated by the change in the y-coordinates divided by the change in the x-coordinates. The data showed that teacher made reference to *change* terminology during their instruction.

Öznur:[while computing slope a line which passes through (-1,-3) and (1,3)] Let's calculate the vertical distance this 3
Students: -3
Öznur: How much distance
Students: 6

Öznur: Okay is everybody all right
Students: Yes
Öznur: Okay we will look at the change in horizontal it was 1 and it became -1 did it reduced yes we will look at this decrease in a directional way. We cannot say it 2 it is negative 2.

Both teachers made reference to the above ideas in several times. As given in the summary of instructions teachers told that students should carefully determine the vertical and horizontal change between two points of the given line.

Experienced teachers' instruction suggested additional episodes which also positively indicate to teachers' overt subject knowledge. For instance, Müge and Öznur emphasized that any point chosen on a line should satisfy its equation.This particular knowledge seems to be essential since it indicates the connection between graphical and algebraic representation of lines. Bearing in mind all of the episodes of this code, data suggests that teachers' overt subject knowledge is remarkably high especially in graphical sense.

Though data suggested episodes in which experienced teachers' overt subject knowledge was remarkable in graphical perspective, Müge's instruction suggested further dimensions of her knowledge in terms of the code. Among them, Müge's attempts to introduce line equation through functional dependencymay be given as an example.

Müge:[teacher's attempts to write the equation of line given in tabular form]Let's write the equation of this line what is the equation of this line…in other words how can we write y in terms of x how can we define this relationship .... Here [indicates to x and y values on the table] we look to relationship between each other how can we get 2 from 1 how can we get 4 from 2 I always multiply by 2 then hence x goes to 2x hence y=2x okay …I also want to say another idea look the equation is y=2x and the slope is also 2 there is a relationship..

The episode is essential in showing the way Müge handled line equation. It indicates that Müge introduced slope as a functional property. She implied a relationship between the variables in the table and indicated this dependency as an equation.

Müge did not explicitly tell that the slope is the rate of change of the dependent variable with respect to change of independent variable. However, she overly said that they would investigate how y and x changes in computing slope. As a result, she is familiar with the meaning of slope in functional perspective which is rate of change.

Theoretical underpinning of pedagogy

Theoretical underpinning of pedagogy is described as a teacher's perception on how to teach mathematics and on the conditions under which pupils will learn best. There were episodes where teachers' perspective of teaching and learning mathematic was observable.

> Öznur: [while computing slope a line whose several coordinate points are provided] Do you agree with Nesli's strategy she have chosen two points to compute slope does it make sense
> Students: Yes she is right.
> Student: I choose the first and the last coordinate point [another student proposes his strategy]
> Öznur: Then what you do is also to select two points…Now here selecting (any) two points really works and we can prove it let's select these two points and another two points and see if the slope is same…

The episode in Öznur's instruction as well as the remaining part of both teachers' instruction suggested that they seek to understand students' thinking.Experienced teachers tried to publicize students' suggestions and explore whether these suggestions make sense to whole class. Teachers' instruction suggested that lessons were situated based on the reflections got form the students. Teachers usedthese feedbacks to determine the direction of their instruction.To conclude, Müge and Öznur followed a path in which students' current state of knowledge, understanding, and suggestions were in effect.

Use of terminology

The mathematical terminology that teachers use during instruction is an indicator of teachers' knowledge in foundation unit. Use of terminology is described as teachers' treatment of mathematical language during instruction.Teachers are expected to use mathematical language in an appropriate way.

Findings related to the code suggested two main indications for experienced teachers' content knowledge in teaching. First of all, experienced teachers were proficient in using the mathematical terminology. Secondly, the variety of vocabulary in the instructions suggested the way they presented slope to students.

Majority of the essential concepts or relations have been used fluently by teachers. The concepts such as vertical and horizontal change, inclination of lines, graphing lines, coordinate plane, ratio, right triangle, coefficient and the increase/decrease are some of terminology which took place in experienced teachers' instruction.

> Öznur: [while waiting for students to compute slope of the line y=4] you said no slope
> Students: Yes there is no slope
> Öznur: You think that in a very smooth surface there is no slope if there is quantitatively nothing how do you express is …0 or 1
> Students: zero
> Öznur: Then how can we find it through the formula is there a vertical distance
> Students: No
> Öznur: Then the nominator is 0 what about the horizontal
> Students: Four
> Öznur: If you divide 0 by 4 you have 0…now let's turn to other line[while students compute slope of the line x=-2]
> Students: …5 over 0
> Öznur: Is 5 over 0 defined or undefined
> Students: …5 over 0 is undefined
> Öznur: Yes if there is 0 in denominator then it is undefined. Hence the slope is undefined in this line.

As the above episode indicates teachers' instruction and the majority of the mathematical terminologysuggested a more concentration on geometrical-graphical perspective. The mathematical language observed in instructions suggested teachers' foundational knowledge in teaching the relationship. The instructions focused on graphing lines, slope as the change in y over change in x, and slope as the parametric coefficient, e.g., the m in the equation, $y = mx + b$.

Data suggested that teachers were able to convey the fundamental knowledge through an appropriate mathematical terminology.Use of terminology positively supported experienced teachers' foundational knowledge in slope concept and the algebraic relationship between slope and equation of a line. To conclude, teachers were able to convey their mathematical ideas through an appropriate terminology.

Use of textbook

It is described as teachers' use of textbook materials for the instruction. Data indicated no adherence to a textbook by Müge. Müge did not refer to any single textbook during instruction. She told the researcher in the pre-interview that she uses her individual materials in addition to exercise questions that she selected from other books. On the other hand, Öznur announced to students that she would address some of the exercise questions from students' workbook. Overall, teachers did not adhere to textbook very often. They implemented the lessons through their own knowledge and individual materials probably because they are familiar with the way textbook presents slope in lines.

Reliance on procedures

The final code that will be provided is teachers' reliance on procedures. The code is described as teacher's use of conventional and essential procedures during instruction.Data showed that teachers were able to use the conventional procedure to compute slope.

> Müge: [Teacher asks to compute slope of the line y=2x which passes through (0,0) and (1,2). She first forms an appropriate right triangle and computes slope than she plugs the coordinate points on the formula] Changes in y
> Student: Increased by 2
> Müge: What is the change in x's
> Students: Increases by 1
> Müge: …since there is an increase I can write these in the formula as positive 2 divided by positive 1and slope is positive 2…I advise you to compute slope by using triangle and then check with this formula if we talk about the slope of a line we should take this formula to the account.

Müge and Öznur consistently applied the same procedure to compute slope. In addition, teachers used the procedure in order to show some other ideas in slope of lines. To illustrate, Öznur showed that slope in horizontal lines is 0 an slope in vertical lines is undefined. This particularly important idea is reached through the procedure to compute slope. Öznur also showed that any two arbitrary points in a line results in the same slope. Similarly, Müge explored a student's important suggestion through the procedure. The reason for not including these episodes is that they are provided either in previous codes of the unit or will be provided in further units.

To conclude, teachers adhered to the procedure throughout their lesson. In addition, they effectively used the procedure to explore some other facts related to slope concept. Findings suggest that experienced teachers' current state of knowledge in following and using the conventional procedures enabled to observe that their foundational knowledge is robust.

All codes together

Findings indicated a rich source of data to be reported in terms of the codes in the foundation unit. Experienced teachers' instruction suggested that they were able to explain/explore multiple important ideas through some basic facts and procedures such as slope formula. In addition, their foundational knowledge enabled them to discuss slope in multiple ways especially in graphical perspective.

To summarize, experienced teachers purpose was more than computing slope algebraically. Graphing lines and computing slope graphically were also important objectives of the instructions. Their overt subject knowledge enabled them to consider students' suggestions as important means in organizing the instructions, thereby, giving also teachers' perspective on the theoretical underpinning of pedagogy.

Teachers mostly identified students' errors in graphical sense which was consistent with the results gained for their overt subject knowledge and use of terminology. Lastly, teachers' reliance on procedures indicated positively to their foundational knowledge observable through data analysis.

Table 6.9. Summary of the foundation unit for experienced teachers.

		Teachers	
Unit	**Codes**	**Müge**	**Öznur**
Foundation	Awareness of purpose	To draw graph of lines, compute slope graphically, compute slope algebraically	To draw graph of lines, compute slope graphically, compute slope algebraically
	Identifying errors	In graphing a line and computing slope	In graphing a line and computing slope
	Overt subject knowledge	In graphical perspective: inclination, definition of slope. Rate of change	In graphical perspective: vertical-horizontal lines, definition of slope
	Theoretical underpinning of pedagogy	Publicizing students' suggestions, proceeding according to students' understanding	Publicizing students' suggestions, proceeding according to students' understanding
	Use of terminology	Fluency in usage and variety especially in graphical perspective	Fluency in usage and variety especially in graphical perspective
	Use of textbook	Not observed	Usage for exercise questions
	Reliance/concentration on procedures	Using procedures consistently, utilizing them to explore other important conjectures	Using procedures consistently, utilizing them to explore other important conjectures

6.3.2. Transformation

The unit is described as teachers' capacity in transforming the content knowledge into pedagogically powerful forms. It concerns the way mathematics is communicated to students. The unit is observed through example, analogy, demonstration, representation, and illustrations that a teacher uses during teaching.

The KQ suggests three codes for the transformation unit; choice of examples, choice of representation and teacher demonstration.Teachers' selection of examples, demonstration, and representations provide a considerable amount of information the way teachers' content knowledge is in effect in instruction.

Choice of examples

Teachers' choice and use of examples is a rich source that reflects teachers' content knowledge in teaching. The aim in focusing teachers' choice of examples is to get a familiarity of the findings that show the way teachers transformed their knowledge to teaching the algebraic relationship between slope and equation of a line.

Teachers' choice of examples is crucial. Examples may serve as important means to comprehend mathematical relationships. In addition, the examples may help to concept formation on the condition that they are carefully selected.

Öznurintroduced slope of lines by an exercise on finding slope of a plane in which a figure of bird skies on coordinate plane. The slope of the same ski plane has been asked again by iterating the plane on the coordinate plane without any change in horizontal and vertical dimensions. Students were able to calculate slope of line easily since the vertical and horizontal differences were small enough to count. Then, Öznur gave another real-life example of slope. The line equation which modeled this real life case was $y=2x-1$ (Figure 6.7). These three exercises indicated that Öznur aimed to illustrate that slope is a concept which might be encountered in real-world situations. In addition, these exercises provided students an opportunity to observe that slope of a line in coordinate plane is nothing new since they already explored it on plane. As a result, data suggests that Öznur tried to scaffold a meaningful slope concept on what students already learned in previous lesson.

Öznur's real-life examples werefollowed by another line equation, $y=2x+4$ for students to graph. The teacher then selected lines which also have negative slope. The given line equation $y=-3x$ seems to be important since it has negative slope and the line passes through origin.

The teacher gave a number of additional lines to graph or to explore the algebraic relationship between slope and equation of a line. However, two of these lines seem to be considerably important to be reported. Öznurdiscussed the slope of vertical and horizontal lines both algebraically and graphically through an exercise question (What is slope of $y=4$ and $x=-2$). She introduced that if a line is *flat*which means no vertical change then slope is zero and slope is undefined for vertical lines since there cannot be any horizontal change in vertical lines. The inclusion of these lines suggested that Öznur was able to use slope formula effectively and demonstrateher students the way it works.

Selection of examples by Müge also indicated that she included all essential forms of lines. As Öznur, she started with a line which has positive slope, y=2x. Müge indicated the relationship between slope and equation of a line on y=2x. The lines given by Müge included the types of lines such as the lines which pass through origin or do not, and has positive or negative slope. Müge provided a number of line equations in order to apply the relationship that the slope of a line is the coefficient of x if the line is in y=mx+n form. To summarize, Müge provided examples such as(i) graphs of lines in order to compute slope, (ii) graph of a line to compute slope, express it algebraically, and verify whether the algebraic relationship holds, (iii) several equations of line to apply algebraic relationship, (iv) two coordinate points of a line to compute slope, (v) two coordinate points of a line to compute slope and write its equation.

Considering teachers' choice and use of examples all together, findings indicated that both teachers included almost all essential examples though there were slight varieties between each other. Teachers used these examples effectively forconcept formation and practice. An episode will be suggested to show how Müge presented the relationship between graphical and numerical image of slope through an example.

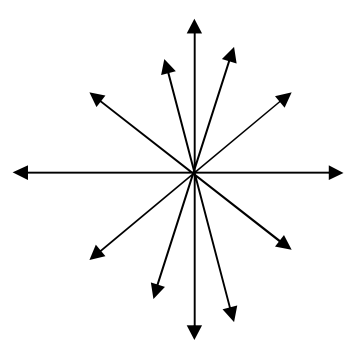


Figure 6.9. Four lines in the same plane.

Müge: [points to the plane and the lines on it] we will find slopes then we will look whether there exist a relationship between slopes and their standing.
Student: the slope of d_1 is 1…
Müge: …[students computed slope of lines in order. The teacher carefully demonstrates the way slope of the last two lines is reached. After reaching all slope values 1,4,-4, and -1] let's talk a little bit how do they have a relationship what is the slope in this way [indicates to x-axis]
Students: zero
Müge: Let's increase [she moves a notebook to the line y=x]
Student: It would increase
Müge: It is smaller than 1 it is between 0 and 1 but it increases in this way…
Student: …[after some time passes in the investigation] here it increases up to a perpendicular position and then it again increases to the left…

Müge spent a considerable amount of time to show the dynamic relationship between inclination and slope value.She used the exercise question effectively in order to show that right inclined lines have positive and left inclined lines have negative slope. To conclude, data indicated that experienced teachers were able to choose appropriate examples for the instruction. In addition, they effectively used these examples in many ways throughout teaching. As a result, choice of examples indicated positively to teachers' content knowledge in teaching.

Teacher demonstration

Teacher demonstration is an important component of teaching. It was described as teachers' way of using demonstrations to explain procedures, rules, and other important components of learning in mathematics. As discussed in the findings of the previous code, teachers used the examples effectively for demonstration. For example, slope in vertical and horizontal lines were demonstrated through exercise questions.

Öznur, as discussed in a different way earlier, asked to compute slope of y=4 and x=-2. The lines have been represented by both algebraically and graphically. The lines were graphed in coordinate plane where the plane was drawn on a checker plane (Figure 6.8).

Öznur: 0 but how will we write them in the slope formula does it has a vertical distance
Students: No
Öznur: If not the nominator is 0 then what is horizontal distance for example
Students: 4

Öznur: If you divide 0 by 4 then the conclusion is also 0 then what is its slope
Students: 0
Öznur: what about the other's slope [Öznur points to vertical line]
Student: 5 divided by 0
Student: Undefined
Öznur: 5 divided by yes is it something defined or not have we ever deal with such kind of fractions
Student: 5 divided by 0 is undefined.
Öznur: 5 divided by 0 is undefined yes a zero for denominator is undefined then what is the slope of that line
Students: undefined.

Exploring slope in vertical and horizontal lines may be a demanding work for both students and teachers. However, the episode indicated that Öznur used line examples effectively for demonstrating the way slope formula is used for horizontal and vertical lines. Instructional data showed that students did not have much difficulty in comprehending the slope in vertical and horizontal case. This implies the way Öznur was successful in demonstrating especially through the use of examples and previous learning such as students' knowledge of fractions.

Analysis of data did not focus on a comparison between experienced teachers' content knowledge. However, Müge's reaction to a pre-interview question indicatedthat while Müge was hesitant in dealing with the slope in vertical lines, Öznur was able to explore slope concept in vertical lines. In the pre-interview, it was asked to Müge whether she would explore slope in vertical and horizontal lines. She replied that eighth grade students could not comprehend how slope becomes undefined in vertical lines hence she would not demonstrate it. In contrast, Öznur was able to show that slope is undefined in vertical lines by using knowledge of fractions.

Choice of representations

The code is described as teachers' decisions of using various representations for concepts.Representations are important means in mathematics instruction hence teachers may use a number of representations for the concepts to be learned. Teachers' choice of representation had also emerged from the data.

Teachers used graphical and algebraic representations of slope and line. It should be noticed that teachers' choice of representations are aligned with the findings in foundation unit. Both teachers gave importance to show slope as analgebraic ratio which is change in y divided by change in x. They also indicated that this formula is a coefficient, m in the equation, y = mx + b.

In addition to both teachers' recourse to algebraic and graphical representation of slope and line, Müge's use of numerical representation was noticed. Müge frequently used numerically represented tabular data in order to compute slope.

> Müge: look [pointing the points (-1,4) and (0,0)] I will apply the change in y's divided by the changes in x's look if I would write two points I will form the table [he creates a table as in the following]

x	y
-1	4
0	0

> The changes in y's … from 4 to 0 …it reduced by 4 isn't it since a decrease I put it as -4… now let's look to changes in x's…from negative 1 to 0 it increased up by 1an increase in 1 [wrote it as m=- 4/1] negative4 over positive1.

Numerically represented tabular data was helpful in computing slope. It enabled to observe the change in y with respect to x values from this numerical representation. In addition, within this representation students are not required to remember any further explanations or relationships such as the relationship between inclination of lines and slope value or checking the measure of angle. Hence, Müge's instruction suggested that she was able to transform her content knowledge into pedagogically powerful forms through use of appropriate representations.

All codes together

To conclude, findings related to the unit suggested that teachers were able to communicate mathematics to students without much difficulty. Their choice of representation, demonstration, and representations indicated that they were shaped by their foundational knowledge. In addition, teachers' preferences related to this unit served effectively to their predetermined aims of teaching slope of a line.

Table 6.10. Summary of the transformation unit for experienced teachers.

		Teachers	
Unit	**Codes**	**Müge**	**Öznur**
Transformation	Choice of examples	Diversity in graphical and algebraic examples of lines	Diversity in graphical and algebraic examples of lines. Real life examples of lines that connect slope of a plane and line
	Teacher demonstration	Demonstration through exercise questions	Demonstration through exercise questions, demonstrating slope in vertical lines
	Choice of representation	Algebraic and graphical, use of tabular in slope calculation	Algebraic and graphical

6.3.3. Connection

The unit concerns the coherence of the planning and instruction across a series of lessons, during an individual lesson, or through an episode. The codes in the connection unit are making connections between procedures, making connections between concepts, anticipation of complexity, decision about sequencing, and recognition of conceptual appropriateness. Findings will be reported in terms of these codes. Teachers in this category have been observed during two lesson hours hence, data suggests the findings on connection unit during these hours.

Making connections between procedures

The code is described as teachers' act of building procedural connections between multiple procedures during instruction. Findings indicated that teachers made connections between procedures during their instruction. Teachers' attempts to connect two different ways of computing slope will be given as an example.

Öznur: [she asks to compute slope of the line given algebraically, x-y+11=0] Calculate the slope okay [she waits for a minute] without drawing its graph
Student: I have drawn its graph.
Öznur: Any other method
Student: I got numbers from the equation
Öznur: Points (in coordinate plane)
Student: Yes then I calculated the difference [refers to change in verticaland change in horizontal]
Öznur: Can we try this…we had a cartoon if you remember and it claimed that slope is a when the line is y =ax+b how can we use it ..

Student: we give a number for y
Öznur: You think like that
Student: No we give a number for x
Öznur: Actually we will change this [indicates to x-y+11=0] to this [[indicates to y =ax+b] form we will write y in one side and the others in another side [Then, Öznur shows writing an equation in slope intercept form step by step and reaches y=x+11]…there is not an a here then what is the number in front of x
Students: 1
Öznur: We can say that slope is 1 what was your result
Students: 1
Students: It is easier to compute slope
Öznur: Actually, (the statement in the) cartoon was correct. Let's turn the previous exercise [she applies the same procedure in order to show that algebraic procedure produces the same result]

Öznur indicated that slope of a line can be computed either graphically oralgebraically. Then, she investigated the algebraic relationship between slope and equation of a line several times through exercise questions. In other words, she showed that slope of a line is the coefficient m, of x when line is expressed by y= mx+b. She also provided this relationship through a concept cartoonas given in the summary of Öznur's instruction.

Müge: Let's revisit the previous exercise [indicates to exercise question in which two points of a line was given and it was asked to compute slope] it crosses x at 4 and y at 2 let's have its equation we were writing like that [forms a table and puts (0,2) and (4,0)]…we should remember this here last year we learned that we can write an equation with respect to its intercept points…we write x over 4 plus y over 2 do you remember
Students: No
Müge: I showed you last year…we can multiply each side by 4[she explains that she would multiply by 4 and reaches x+2y=4] then I have 2y=4-x then I need to have only y in one side
Student: 2 minus x over 2
Müge: 2 minus x over 2…why did I strive for can you say the coefficient of x here…
Student:…negative 1 over 2
Müge: Okay what was the slope of this line
Student: negative 1 over 2
Müge: negative 1 over 2
Student: Both are same
Müge: Yes both are same both slope and the coefficient of x is negative 1 over 2 … this is not a coincidence the coefficient of x is same as slope of the line.

Müge also presented the algebraic relationship between slope and equation of a line several times through exercise questions. In other words, she showed that slope of a line is the coefficient of x when line is expressed in slope-intercept form. However, instructional data indicated that her students were unable to get accounted with using algebraic procedure until the above exercise.The remaining of Müge's instruction suggested that students were able to use algebraic procedure since Müge directed a number of line equations in order to practice the procedure. These lines were given in the summary of her instruction.

There exist a number of procedures to compute slope. However, students at eighth grade are expected to compute slope of a line basically by two ways. The first is computing slope of aline which is graphed. The conventional procedure rests on (i) choosing any two distinct points of a graphed line and then (ii) reaching "the change in y divided by change in x" to compute slope. Secondly, slope is extracted from its equation; slope is the coefficient of x when a line is expresses in slope-intercept form. Episodes suggested that teachers followed a similar path in connecting different procedures to calculate slope of a line. They used these procedures and presented the connections between each other. In other words, they emphasized that slope may be computed in either way.

Data suggested that Müge and Öznur are familiar with the connections between the procedures mentioned above. In addition, their instruction suggested a number of episodes in which teachers tried to show that in either way slope is same. These trials were basically achieved through exercise questions.The attempts took a considerable amount of time during instruction which suggests that both teachers gave importance to show this procedural connection. However, it is almost impossible from the data to assert that students were able to connect these procedures or the degree they achieve.

Making connections between concepts

The code is described by act of building conceptual connections between mathematical concepts during instruction.Effective teaching requires making connections. Multiple connections between concepts may be built based on teachers' depth and breadth of content knowledge in teaching.

> Öznur: [a student graphs a line and compute its slope] Is there a right triangle here please show it [the students point the coordinate points (0,0),(2,0) and (2,4)] okay then

how is it interpreted in trigonometry is it sine or cosine or what elsehow can we express slope in trigonometry
Student:Sine over cosine
Öznur: We had talked about it in previous lesson
Student: Tangent
Öznur:Tangentokay in which angle we are looking for the tangent…
Students: …this [the student shows the angle between x axis and the line in positive direction]
Öznur: Yes that angle the tangent of that angle there are three angles here and we take the tangent of this angle it gives the slope.

Öznur presented the relationship between slope of a line and tangent through a line exercise. The episode indicated that Öznur only reminds the connection most probably because she explored it in her previous instructions. Öznur's overall instruction suggested no further attempts to connect slope to tangent or other mathematical concepts.

Müge's instruction also suggested attempts to connect slope to some graphical entities. For instance, Müge gave importance to connect slope to its graphical reflection. She emphasized that slope and steepness has a relationship. The episodes that show the way Müge connected slope to steepness and inclination will not be provided here since the episodes were given in previous sections to illustrate the findings of the other codes.

Results for both teachers are consistent with the findings of foundation unit for experienced teachers. It was reached that experienced teachers' overt subject knowledge is remarkably high especially in graphical sense. As a conclusion, teachers attempted to make connections between slope and the concept or measures in graphical representation.

It was reached in foundation unit that Müge introduced line equation through functional dependency. Findings indicated she provided a relationship between the variables in a table and indicated this relationship as an equation. As a result, it was claimed for Müge that she was familiar with the meaning of slope in functional perspective which is rate of change. However, she did not explicitly tell that the slope is the rate of change of the dependent variable with respect to change of independent variable. As a conclusion, it is not possible to claim that Müge successfully connect (i) slope concept to proportionality and rate of change or (ii) equation of a line to the concept of function. This implies that teachers did not aim to connect slope to concepts such as ratio or covariation. In other words, teachers' connections included only the graphical-geometrical aspects of slope.

Anticipation of complexity

Anticipation of complexity is described as teachers' awareness of students' obstacles against understanding different mathematical topics and tasks. Teachers' anticipation of students' difficulties on understanding different mathematical topics and tasks deserve special attention.

Müge anticipated that students might have difficulty in applying the procedure to compute slope of a line which have negative slope. She preferred to explain the procedure more explicitly in solving exercise questions. However, the instructions did not suggest extreme problems that students faced with in computing slope. On the other hand, Öznur claimed that the previous learning objectives prepared students to comprehend the idea behind slope concept. In addition, she also said that her instruction on slope concept would not cause problems for students. To conclude, both teachers' data did not suggest any episode specific to the code. The result may originate from several reasons. One of it is the advantage of experiential knowledge that teachers accumulated during their professional work. Teachers might organize their instruction in a way so that students are not negatively challenged.

Decisions about sequencing

Decision about sequencing is outlined as ordering of topics, tasks or other units of instruction such as examples within and between lessons. It should be noticed that this unit concerns the coherence of the planning and instruction across a series of lessons, during an individual lesson, or through an episode. Since teachers in this category were observed during two lesson hour, the data suggests the sequence during this duration.

Teachers' organization of the instruction, such as ordering examples throughout the lesson, may indicate teachers' decision of sequencing. Both lessons have similarities in the introduction of the lesson. Both Müge and Öznur gave graphs of lines in which slope is calculated on the graph. The step followed by teachers was giving the relationship that slope of a line is the coefficient of x if the line is represented as y=mx+n form.

Episodes will be suggested in both teachers' instruction in order to observe teachers' decision of sequencing better. Öznur's first part of her instruction and Müge's ordering line examples may be helpful. As it was mentioned in the summary of lessons, Öznur started the instruction by exploring the slope of a ski plane depicted on the coordinate plane. Then, she examined the slope of the same plane which was iteratedthrough y and x axis. This sequencing showed that parallel lines have same slope as well as showing the reason (2 units of vertical and 3 units of horizontal change in both cases). In the next step, Öznur investigated another real life case which was illustrated in Figure 6.7. In contrast to previous two cases, representation of the line in the new exercise enabled to compute slope both graphically and algebraically. In addition, the line was helpful in exploring whether *any* two points on it gives the same slope. Data showed that Öznur's decision on ordering suggested a linear movement to the aim of the lesson. Similarly, Müge directed several line equations to reach slope algebraically. The order of lines (as given in the summary of her lesson) showed that the algebraic relationship between slope and equation of a line can be used either directly or after completing a few computations.

Müge told that all the methods used to find slope is in fact same as using change in the y-coordinates divided by the change in the x-coordinates. When Müge introduced $\frac{y_2-y_1}{x_2-x_1}$as an alternative way to compute slope, she told that the reason for giving this formula at the end of the lessonconcerns that students may have confusions unless they get acquainted with the first procedure.

To conclude, experienced teachers sequenced their instruction in a way that they were able to reach the objective of the lessons. Findings indicated that teachers' content knowledge in teaching slope of a line enabled a linear movement during instruction.

Recognition of conceptual appropriateness

Awareness of the relative cognitive demands of learning mathematical concepts, relations, etc. is related to teachers' recognition of conceptual appropriateness of teaching mathematics. Teachers should be aware of the relative cognitive demands of different topics and tasks in learning mathematics.

Experienced teachers' instruction did not suggest any episode specific to the code. As discussed for pre-service teachers, the result may be related to the context of the study. Turkey's nation-wide mathematics curriculum proposes teachers the concepts to be presented, the level, and even the time to be spent for teaching.

All codes together

Putting all codes of the unit together, experienced teachers' instruction suggested that teachers were concerned with students learning. Their attempts to make connections and the coherence of the lessons indicated they effectively used their content knowledge and expertise in teaching slope of a line. Findings for the unit also suggested the way teachers' handled the slope concept in a better way since the depth and breadth of the knowledge in instructions suggested an emphasis on graphical and geometrical aspects of slope concept.

Table 6.11. Summary of the connection unit for experienced teachers.

		Teachers	
Unit	Codes	Müge	Öznur
Connection	Making connections between procedures	Focus on the connections between graphical and algebraic procedures to compute slope, additional exercise questions	Focus on the connections between graphical and algebraic procedures to compute slope
	Making connections between concepts	A strong connection to inclination and steepness.	Limited connection between tangent and slope
	Anticipation of complexity	Emphasis on computing (negative) slope	Not observed
	Decisions about sequencing	Ordering exercise questions from y=mx to any form of lineequation	Presenting slope from planes to lines, from simple to complex
	Recognition of conceptual appropriateness	Not observed	Not observed

6.3.4. Contingency

Teachers' contingent actions are the last unit in investigating their content knowledge during instruction. Teachers' instruction provided a rich source of data to be regarded in terms of the codes in the unit. The codes in this unit are responding to students' ideas, use of opportunities and deviation from agenda. Findings will be provided in terms of the codes.

Responding to students' ideas

The code was described as the way a teacher attends to, interpret, or handle students' ideas. Students in these teachers' classroom were given opportunities to raise questions and offer alternative views. Even though the focus of analysis was on teachers' instruction, video records of the instructions also enabled to observe that students were eager to share their ways of thinking during instructions.

The number of episodes which informs their responses to students' ideas is considerably high in experienced teachers' instruction. Most of these episodes indicated the availability of teachers' responses. In addition, teachers' responses provided that their content knowledge in teaching slope of a line is considerably robust.

There were very few cases in which teachers did not reply students' comments. In some of these cases, data did not indicate whether teachers could not hear students' comments or preferred not to respond them consciously. One episode will be given for illustration.

> Student: I have done some calculation I put 6 for y in the previous example [it was y=2x+4] and I put 3 for x and b was left over…hence the statement is wrong.
> Öznur: …[addressing to all students' suggestions] In fact what all of you claim are not incorrect however, both statements are correct we cannot claim that they are wrong what we need is more information.

A student in Öznur's classroom reflected on the concept cartoons. She claimed that the statement "Slope is the coefficient a of the equation y=ax+b" is not correct since her calculations showed that b was left over. Öznur's response was not directed to the student's comment even though her response indicated a serious misunderstanding. The student was not aware that she cannot plug an arbitrary ordered pair (which is (3,6) for this case) to the equation of a line. Similarly, instructional data suggested that a student in Müge's classroom tried to combine slope and area of triangles that were formed to compute slope. She claimed that though the slope of lines (y=4x and y=-4x) are not same they have triangles of same size and area.

Teachers did not provide a response to their students' comments which may imply that they (i) could not comprehend students' comments, (ii) thought that the commentswere not worth to discuss, or (iii)were not able to suggest why students' suggestion werenot true.All in all, teachers may have very good reasons for not responding to students' some of the ideas.

Often, teachers re-organized their instruction by considering students' comments. Öznur usually followed students' comments such as saying that "Do you agree with her method she said she selected two points is it reasonable… is it sufficient to choose two points to compute slope let's try".Similarly, Müge gave importance to respond students' comments during teaching slope of a line.

> Student: Isn't the squares of lines same [he was not able to express his suggestion]

Müge: Which are same?
Student: For example things of the parallel beneath this line[he was not able to express his suggestion]
Müge: What?
Student (Another): slopes
Student: Yes slope
Müge: Slopes are same yes very good
Student: Then, can we write them [means the coordinate points of any parallel line] on the table
Müge: Hımm [thinks]
Student: For instance what about 0 to negative 2 [(0,-2) does not satisfy the graphed line]
Müge: No, I am writing the points that this line passes what you suggest is the points of the parallel line all right.
Student: Okay.

The episode as well as the remaining episodes in both teachers' instruction suggested that experienced teachers gave great importance to answer to students' comments. As the above episode indicates, Müge was able to discuss the properties of parallel lines through the student's suggestion. In other words, she was able to provide the content related correct answer.

Teachers' responding to students' ideas may be interpreted in several ways. For instance, it may be related to the perception of being a teacher or beliefs about teaching mathematics. However, the focus in this research is mainly concerned with teachers' content-specific responses to students' content-specific ideas. As a conclusion, teachers' instruction suggested that teachers were able to synthesize their content knowledge to students' ideas. In addition, majority of these episodes indicated that the focus in these instructions were basically on computing slope, graphing lines, and exploring the role of coefficients of y=mx+n.

Use of opportunities

The code is described as teachers' use of unanticipated contributions as an instructional opportunity. As discussed earlier, students' comments and questions may provide valuable instructional opportunities. Both Müge and Öznur were able to use unanticipated contributions as opportunities during their instruction.

Students'ideas provided opportunities to discuss concepts, relations or other important issues in learning slope concept.For example, Öznur asked students to graph the line 3y-x+5=0. When she recognizedthe student's incorrect strategy, she was able to use that case as an opportunity.

Öznur: What are you going to do tell please?
Student: We will find the vertical distance 3 [pointing to 3 in the written equation 3y-x+5=0]
Öznur: All right
Student: I would find 5 then [started to locate (5,3) on coordinate plane]
Öznur: It would be very easy in that way but we do not do in this way… you pointed 3 right how is it written I mean what is its x and y coordinates its x is 0 and its y is 3 [the teacher put (0,3) on the board] now let's think for a while whether it satisfies the equation let's put zero for x and write 3 for y let's look at the result [the teacher put those values to the equation] … is 16 equals to 0
Students: No
Öznur: And this point[by pointing (0,3)] does not satisfy.

Öznur used a student's incorrect response as an opportunity. She showed that a line passes through the points where these points also satisfy its equation.The episode also indicated that her response suggested an important foundational knowledge in learning connection between algebraic and graphical representation of lines.

In another phase of her lesson, Öznur asked the way to compute slope of a line on its graph.Based on a student's response, the teacher clarified that slope of line remains unchanged in its any segment.

Öznur: Our friend claims that it is enough to choose two points to compute slope.
Students: I think so.
Students: I choose the starting and ending points.
Öznur: Then it also turns out to be two points [to another student] you think we should choose 3 points you think it would be betternow here it is enough to choose two points and we can prove it let's choose two points and choose another two points and see what will be slope.

Öznur was able to demonstrate an important identity of line through students' suggestions. In other words, she used students' comments to explore lines and slope in detail. Öznur was able to address slope of a line due to this unanticipated contribution. Her demonstration suggested that two arbitrary points on a line may possibly give its slope which suggests that it is not restricted to behave a point say A, as the first point and B as the second. In either order, the slope will be same. The order in which the points arelisted does not matter, as long as one subtracts the x-values in the same order as he subtracts the y-values.Öznur was able to present these ideas due to the opportunity. Mügealso visited these important ideas but in a different way. She told that among two selected points, one should take the point as first if its x-value is smaller than the other point's (Figure 6.10) in computing slope though it is not necessary.

> Müge: Your friend asked a good question he said that if I would write (3,0) at first what would be the conclusion he said that he found the inverse hey kids if you recognize I start writing with smaller x values from this side [pointing from (0,2) to right side] in writing the change I start with the smaller x values.

x	y
0	2
3	0

Figure 6.10. The table for coordinate points.

A student in Müge's classroom objected Müge's solution that it would not be necessary to select a point as first or second. This promptedMüge to engage in a mathematical investigation.

> Student:It does not matter.
> Müge:Let's try [She thinks a few seconds] let's try [she tries and observes that the slope is same in either way] good okay the same is reached in this way too.

Müge usedstudents' response as an opportunity to see and show that there is not a hierarchy or condition in behaving points to calculate slope of lines. This also seems to be an important foundational knowledge in learning slope of a line. The episode indicates that though the teacher did not suggest the rationale explicitly, she showed it procedurally on an exercise question. In brief, both Müge and her students investigated animportant characteristics of slope of a line through considering the opportunity.

To conclude, both teachers' instruction suggested that the number of episodes which may be put into the code was high. In addition, data showed teachers' eagerness as well as ability to use the unanticipated opportunities during instruction. Findings also showed that teachers' current state of content knowledge did not limit but helped them to use these opportunities effectively. As a result, these opportunities were of special benefit to the individual who started the action such as raising her idea. In addition, experienced teachers often suggested a platform that was a particularly fruitful avenue of inquiry for other members of the classrooms.

Deviation from agenda

The code was described as ability to extent teaching to further learning. Data supported that experienced teachers did not much deviated from their predetermined agenda.The video records of the lesson indicated that teachers made slight changes during their instructions. While Öznur's instruction did not provide any deviation, there were two episodes in Müge's instruction which may be interpreted in terms of deviation.

> Student: Can we say like this n+2 over n+1 [the teacher asks students the slope of line from its coordinate points given in tabular form]
> Müge: [thinks carefully] n+2 over n+1 how did you find such a thing [perplexed] which n
> Student: If x would be a number then slope might be n+1
> Müge: [refuses] we may talk it with you we may talk when the class ends okay let's do not confuse others it is not like what you say. Here [indicates to x and y values on the table] we look to relationship between each other.

As evident from Müge's response, she did not want to deviate from her agenda. Student's suggestion seems to indicate that she seeks or defines a pattern for the variables. However, Müge did not go on more most probably because she does not see any benefit to discuss it more. The episode shows that she is almost sure what is educationally beneficial or appropriate for students to explore.

I will provide an episode which seems to indicate a deviation from Müge's plan. Müge gave two points of a line and asked students to find slope of the line. As a result, students found that slope of the line should be -1. Then, Müge asked students to find the equation of the line. One of the students was able to see that the coefficient of x would be -1 if the equation is written in y=mx+n form since the slope is -1.

> Student: We know from calculating slope that the coefficient of x is -1
> Müge: The coefficient of x is -1 if I set y alone the coefficient of x is -1 we know it it is very good and important information isn't it
> Student:y equals to negative x but there is something else negative x plus something [the student clarifies her thinking]
> Müge: y equals to …very nice..Congratulations…
> Müge: I have something that I do not know how we can call it
> Students: let's say a
> Müge: Let's call it as a if I find a then I can reach equation of line and say here is the equation of the line.

The episode suggests that Müge deviated from her plan. This is evident from the episode that she was amazed by a student's strategy. In addition, she used another way to write the equation of line in the penultimate exercise. Hence, it may be claimed that she was planning to write the equation of the line different than the student's strategy. As a conclusion, she felt no indecision in deviating from her plan. Hence, she was able to explore another way of writing equation of a line.

All codes together

Findings indicated that teachers' knowledge in interaction did not limit them during their instruction. In contrast, findings of the unit suggested that contingent events in the classroom helped teachers to explore students' thinking in a better way.In most cases, both the student who raised his idea as well as the other members of the classroom benefitted from these occasions. Lastly, findings indicated that studying teacher' actions to unanticipated classroom events are very beneficial in speculating their content knowledge in teaching.

Table 6.12. Summary of thecontingency unit for experienced teachers.

		Teachers	
Units	**Codes**	**Müge**	**Öznur**
Contingency	Responding to students' ideas	Willingly responding to students' ideas	Willingly responding to students' ideas
	Use of opportunities	Ability to use opportunities and reach a beneficial platform for all, including the teacher	Ability to use opportunities and reach a beneficial platform for students
	Deviation from agenda	Limited: extending to express line equations by a second way	Not observed

6.4. Comparison

In this section, the aim will be to compare teacher groups' content knowledge (SMK and PCK)in teachingslope and the algebraic relationship between slope and equation of a line. The comparison will follow in terms of the units of the analytical framework, the KQ.

6.4.1. Foundation

Data suggested that there exist variationsamong groups in terms of the first unit. These variations indicated that both pre-service and novice teachers may have deficiencies in terms of the codes in the foundation unit. In contrast, data displayed that experienced teachers were more comfortablewith their foundational knowledge during instruction.

Findings indicated that the purpose in teaching slope and the relationship between slope and equation of a line varied among teachers.While pre-service teachers confined their purpose on investigating the relationship between slope and equation of a line, novice teachers' were more concerned with the practical use of the relationship, namely, using the relationship to compute slope. In other words, pre-service teachers aimed to show that slope of a line is the coefficient of x when a line is in y=mx+n form. On the other hand, novice teachers behaved this algebraic relationship in their instruction as a means to find slope of a line by claiming that itsaves time and requires less work. Lastly, experienced teachers did not limit themselves on exploring the algebraic relationship.Drawing graph of lines seemed as also another purpose in experienced teachers' instruction though the teachers did not neglect to introduce the relationship. In addition, computing slope of a line on its graph was equally emphasized when compared to emphasis given to compute slope algebraically.In sum, experienced teachers' purpose for teaching slope and the algebraic relationship between slope and equation of a line was morecomprehensive. This can be illustrated by one of the episodes in Müge's classroom in which she and her students used the algebraic relationship to write equation of a line.

> Student: We know from calculating slope that the coefficient of x is -1
> Müge: The coefficient of x is -1 if I set y alone the coefficient of x is -1we know it it is very good and important information isn't it
> Student: y equals to negative x but there is something else negative x plus something [the student clarifies her thinking]
> Müge: y equals to …very nice.. Congratulations…
> Müge: I have something that I do not know how we can call it
> Students: let's say a
> Müge: Let's call it as a if I find a then I can reach equation of line and say here is the equation of the line.

One of the students was able to see that the coefficient of x would be -1 if the equation is written in y=mx+n form because slope value was -1. The claim is that using the algebraic relationship to compute slope does not guarantee a robust understanding. A learner should be able to transfer the newly learned knowledge to another place. The student's suggestion in Müge's classroom indicates that the student was able to use what is meant by the algebraic relationship between slope and equation of a line. She effectively transferred her mathematical knowledge to a new case which is benefitting from the relationship in order to write equation of the line. As a result, using the algebraic relationship between slope and equation of a line to write equation of it indicated that the purpose in teaching was more comprehensive.

Identifying errors emerged as a code for almost all participating teachers. Except Akif, all teachers identified some students' errors especially in computing slope and graphing a line.Identification of errors and the identified errors indicated teachers' focus on teaching the concept, which were basically on procedural attainment of the concept of slope. Data did not indicate a significant difference among the groups of teachers in terms of the types of errors that teachers identified. However, experiencedteachers were able to differentiate student errors better when compared the ones in novice and pre-service group. As it was given in experienced teachers' data, they did not feel any difficulty in identifying students' errors. In addition, they were able to identify students' errors even in cases where students' suggestions or results to exercise questions were correct.Besides, as discussed in contingency units, experienced teachers were qualified to use those student ideas as new opportunities for whole class learning.

Comparison of data in terms of overt subject knowledge code indicated that pre-service and novice teachers lacked some kind of essential subject knowledge. To illustrate, while pre-service teachers' instructions suggested an insufficient representation of line, novice teachers' instruction suggested deficiency in computing slope. However, data indicated that all teachers' subject knowledge in slope was overt especially in graphical aspects. The only teachers who indicated an implicit relation between slope and functions were Akif and Müge. While Akif's instruction suggested a link between proportion and slope, Müge indicated a covariation between variables. As a conclusion, introducing slope as a functional concept was rare among teachers. In other words, majority of teachers did not present slope as a rate of change between two variables.

Theoretical underpinning of pedagogy is another code in which variation between groups of teachers was observable. Data suggested that pre-service teachers were more concerned with the material that is supposed to be beneficial for students to understand mathematics easily and with less challenge. On the other hand, novice teachers preferred an explain and practice method. Lastly, experienced teachers focused more on students' comments, understanding as well as their current state of mathematical knowledge that is assumed to be helpful in understanding the learning material. Though it was difficult to imply how teachers' content knowledge is consistent with their theoretical underpinning of pedagogy, it can be concluded thatteachers' theoretical underpinning of pedagogy was influential in deciding what kind of knowledge would be better to include in teaching slope of a line.

The code use of terminology indicated not much difference among groups of teachers in terms of the variety. All group of teachers used mathematical words especially in graphical and algebraic representation. For all teachers, use of terminology indicated more emphasis on teachingthe concept of slope in procedural way. In addition, it was remarkable to observe that teachers' use of terminology is an important indicator of their depth of content knowledge and focus on teaching a certain mathematical concept (which is slope in this study).

Use of textbook was not recognized for four teachers. While none of the pre-service teachers indicated a use of textbook, one for each group of novice and experienced teachers used textbooks. Yasemin from novice group and Öznur from experienced group used textbooks for only exercise questions. Use of textbook was either nonexistent or very limited in participating teachers' instructions. It was one of the codes of the KQ in which not much information on teachers' content knowledge of teaching slope was derived through the code.

Reliance on procedures was one of the most important codes which emerged as significantly in the comparison phase. According to the national curriculum and textbooks, teachers are expected to compute slope of a line by the change in y-coordinates divided by the change in x-coordinates of line. The formula may also be given by$\frac{\Delta \mathbf{y}}{\Delta \mathbf{x}}$, $\frac{\text{Rise}}{\text{Run}}$,or $\frac{y_2 - y_1}{x_2 - x_1}$. However, in either way the idea relies oncomputing *change* between any two distinct points of a line. The conclusion for the experienced teachers was that they were aware of the formula and used it successfully. In addition, they consulted to the formula and the idea behind it when it was necessary in any second of the lesson. However, both novice and pre-serviceteachers were not able to use it correctly. Even though some of the teachers occasionally gave the correct definition, the data showed that teachers were not able to use the formula effectively.

Two episodes, one for experienced, and one for pre-service teacher, illustrates the use of slope formula. Both Öznur and Cansu asked students to find the slope of vertical and horizontal lines. Öznur asked the slope of lines y=4 and x=-2 which were graphically given. Similarly, Cansu asked students to find the slope of y=13 and x=6 given algebraically. As examples suggest equation of lineshadthe same characteristics. In computing slope, Öznur applied the conventional formula. She asked students two select two points on the lines and compute slope by calculating vertical and horizontal change between the points. In brief, she presented the results by the definition of slope. Cansu preferred to explain the solution in a different way. She preferred to explain that the slope of horizontal lines is 0 by suggesting that o horizontal line may be algebraically written as y=0x+b.

All codes together

Overall, the data showed that experienced teachers' knowledge in the unit weredistinctive duringtheir instruction when compared to other teachers. It was observed that there was little or no evidence to support that novice teachers' foundational knowledge was more robust and versatile than their pre-service counterparts.

Table 6.13.Comparison of groups in terms of foundation.

	Teachers					
Codes	**Cansu**	**Akif**	**Erkin**	**Yasemin**	**Müge**	**Öznur**
Awareness of purpose	*Compute* slope of a line from its equation	*Compute* slope of a line from its equation	m is slope in y = mx + n, quicker way	m is slope in y = mx + n, quicker way	To draw graph of lines, compute slope graphically, compute slope algebraically	To draw graph of lines, compute slope graphically, compute slope algebraically
Identifying errors	Few cases: students errors in computing slope	Not observed	Few case: plotting points	Few case: plotting points	In graphing a line and computing slope	In graphing a line and computing slope
Overt subject knowledge	Strength ex: properties of slope of a line, deficiency in: Inadequate representation of line	Strength ex: proportionalit y, Deficiency in:Inadequate representation of line	Focus on graphical aspects. Fragmented view of slope.	Focus on graphical aspects. Fragmented view of slope.	In graphical perspective: inclination, definition of slope. Rate of change	In graphical perspective: vertical-horizontal lines, definition of slope
Theoretical underpinning of pedagogy	Use of tabular data in numerical representation , inductive approach, step by step and straightforwar d instruction	Use of tabular data in numerical representation , inductive approach, step by step and straightforwar d instruction	Explaining and practicing, inductive	Explaining and practicing, deductive	Publicizing students' suggestions, proceeding according to students' understanding	Publicizing students' suggestions, proceeding according to students' understanding
Use of terminology	Using *change* to define slope of a line in coordinate plane	Using *steepness* as a measure and *direct proportion* as a functional property	Emphasis on procedural knowledge: graphical and algebraic representation	Emphasis on procedural knowledge: graphical and algebraic representation	Fluency in usage and variety especially in graphical perspective	Fluency in usage and variety especially in graphical perspective
Use of textbook	Not observed	Not observed	Not observed	For exercise questions	Not observed	For exercise questions
Reliance/concentrat ion on procedures	Deficiency in reliance on procedures: computation of slope only from geometric perspective	Deficiency in reliance on procedures: computation of slope only from geometric perspective	Procedure that needs trigonometric ratio properties	Procedure that needs trigonometric ratio properties	Using procedures consistently, utilizing them to explore other important conjectures	Using procedures consistently, utilizing them to explore other important conjectures

6.4.2. Transformation

The transformation unit includes teachers' capacity in transforming the content knowledge into pedagogically powerful forms. Transformation of knowledge is majorly related to teachers' knowledge in foundation unit. The codes of the unit are choice of examples, teacher demonstration, and choice of representation.

The comparison of teachers' instruction provided that both pre-service and experienced teachers' *choice of examples*in the introductory part was deliberate. On the other hand, selection of examples by novice teachers seemed to lack care. For example, Erkin chose only two line equations for introduction to slope of a line and both of them had positive slopes. Yasemin also suggested three lines for graphing, all having positive slope.

The variation among teachers' choice of examples was more visible.Findings suggested that both pre-service and experienced teachers chose examples which assisted in exploring the relationship between slope and equation of a line as discussed in transformation unit. In addition, experienced teachers were able to use their selected examples in more productive way. The reason for excluding novice teachers is that though they also used examples for more than one objective (and it will be covered in teacher demonstration) use of examples could not suggest effective use for each aim. To illustrate, while Erkin was very successful in indicating the difference in graphing y=2x+2 and y=3x, these exercises were not appropriate enough in showing the relationship between slope and equation of a line.

The remaining examples in all teachers' instructions suggested that teachers were able to include essential forms of lines in algebraic and graphical representations. This indicated that the work done for choice of examples was careful and to the aim of the instructions.

Teacher demonstration is the second code of the unit. None of the teachers used any concrete material or technology (such as a graphic software) to demonstrate mathematical processes or prove propositions. Both novice and experienced teachers depended on exercises in demonstration and in many times those demonstrations were on procedural and graphical aspects of slope. On the other hand, pre-service teachers' instruction did not provide enough episodes to discuss on their demonstration. This does not mean that pre-service teachers did not use any demonstration but may support to claim that demonstration was not focused.

The analysis of data in terms of the selection of representations indicated miscellaneous results. Both of the pre-service teachers and an experienced teacher (Müge) preferred to provide numerical representation in addition to representations in algebraic and graphical. Use of numerical representation was helpful in many respects for both concept formation and procedure to compute slope. It showed teachers' strength in transforming content with less challenge to students.

All codes together

Novice teachers included algebraic and geometrical representations. It was remarkable that novice teachers referred very often to trigonometric ratio meaning of slope. Teachers' excessive use of right triangles suggested that teachers either have a more dependence on trigonometric ratio meaning of slope or they see it as more appropriate in teaching slope. On the other hand, pre-service teachers avoided in connecting slope to trigonometric ratio of angles. Similarly, Öznur from experienced group talked about the tangent of an angle once. In short, teachers emphasized algebraic representation of slope as it is normally expected. In addition, they also used graphical representation of line and slope.

Table 6.14. Comparison of groups in terms of transformation.

	Teachers					
Codes	**Cansu**	**Akif**	**Erkin**	**Yasemin**	**Müge**	**Öznur**
Choice of examples	Appropriate and inductive: y=3x as a starting point	Appropriate, inductive, and more powerful: y=2x, 2y=x as a starting point	Introducing slope by a limiting example, diversity in exercises.	Use of y=mx and y=mx+n before examples, diversity in exercises.	Diversity in graphical and algebraic examples of lines	Diversity in graphical and algebraic examples of lines. Real life examples of lines that connect slope of a plane and line.
Teacher demonstration	Not observed	Not observed	Demonstrati on by practicing through exercise questions	Demonstrati on by practicing through exercise questions	Demonstrati on through exercise questions	Demonstrati on through exercise questions, demonstrati ng slope in vertical lines
Choice of representation	Numerical representati on of tabular data to reach line equation	Numerical representati on of tabular data to reach line equation through rate of change, ratio and proportion	Geometrical representati on by using right triangles	Geometrical representati on by using right triangles	Algebraic and graphical, use of tabular in slope calculation	Algebraic and graphical

6.4.3. Connection

Making connections and building coherence throughout the instruction summarizes the codes of the unit. Data showed that pre-service and novice teachers provide one graphical procedure for computing slope. Eventhough Cansu told that slope is computed by finding the change in vertical and horizontal, she preferred to use right triangles in calculating slope. She showed the way to form a right triangle between a line and the x-axis. Similarly, Akif also suggested using right triangles in computing slope of lines in coordinate plane. Novice teachers provided the same procedure for computing slope of a line, namely, right triangles. They told the students that students need right triangles to calculate slope.

When compared to their pre-service and experienced counterparts, novice teachers' instruction suggested that they gave more attention to algebraic procedures. As discussed in their findings part, teachers emphasized that slope of a line may be reached directly from both forms. They showed how to calculate slope of a line in y=ax+b and ax+by+c=0 form. This supported the findings that novice teachers emphasized how to compute slope as quick as possible. As a result they suggested connections which are specifically more straightforward.

The experienced teachers, on the other hand focused on the connections between graphical and algebraic procedures to compute slope.Teachers proposed to choose two arbitrary points on the graph of a line. Hence, teachers showed that (i) there is no need to choose more than two points in calculating slope, two points would work (ii) the selection of these points on the graph of a line has no criteria, any two points would work, and (iii) the points are plugged to the formula in any order.

Teachers told that slope may also be calculated by using tangent function. However, the procedure suggested by all of these teachers was no more than the procedure that teachers applied during the instruction. Using right triangles to compute slope, in fact, originates from the definition of tangent of an angle in right triangles though teachers preferred to announce it as another procedure in computing slope.

Findings suggested that both pre-service and novice teachers did not satisfy with giving the algorithms solely. They also tried to connect these procedures since it is *needed* to follow the algorithms. This indicated that teachersmade connections not for enriching the teaching of the slope conceptbut for a healthy use of procedure.This dataindicate that teachers' foundational knowledge and beliefs might encourage teachers to make connections between concepts or procedures.

The connections that all of the teachers tried to introduce was the inclination of lines and its relation to slope value. All teachers preferred to show that if a line inclines to right then its slope is negative and if it inclines right then it has a positive slope. Müge was the only teacher who presented the relationship between a numerical slope value to its graphical image in a dynamic way.

Considering all of the instructions together it was seen that much of the connections between slope and other concepts were more on geometrical and graphical ones. All teachers' were more concerned with showing the relationship between slope and its graphical representation. Data implies that presentation of the concept was dominantly on its graphical meaning.Another connection which was introduced was the ratio-slope relationship. Akifconnected slope to ratio and proportion. Akif designed his activities in a way that students were able to claim that slope is a ratio between two quantities.

Teachers' anticipation of complexity indicated that pre-service teachers were unable to predict the difficulty in computing slope. It was remarkable that pre-service teachers either had no ideas or did not think of the complexity in computing slope. In contrast, both novice teachers and Müge gave special attention to students' difficulties on computing slope of a graphed line.This may indicate that teaching experience even in a limited duration may help teachers to predict better on students' learning.

Teachers' decision of sequencing emerged as a significant code of connection unit. Video records of the instructions as well as the pre-interview data showed that the pre-service teachers sequenced the activities, exercise questions and connections in a deliberate way. In other words, their instruction suggested a deliberate and step-by-step movement to objective in which steps were identifiable. In addition to indicating teachers' organization of slope concept for teaching, sequencing also showed the way pre-service teachers' preparation to their in-class duty. They were almost determined in where and how to start with, how to move from an episode to other, and how to reach an end during teaching slope.

Sequencing in novice and experienced teachers' instructions,on the other hand, weremore complicated when compared with pre-service teachers. However, the only instruction in which sequencing needed to be re-organized was Erkin's.Though his instruction suggested good examples of sequencing (such as the sequencing of line equations)in further moments it lacked ordering especially in introductory part. In brief, the comparison indicated that pre-service teachers gave more importance to sequencing which may also be related to the anxiety of teaching in a real classroom.

Recognition of conceptual appropriateness is the last code of the unit. Compared to other codes of the unit, data did not suggest a significant finding. It was reported as not observed for almost all participating teachers. This conclusion is interpreted in two ways. Firstly, theremight be episodes of the code which were interpreted through other codes since they suggest more valuable findings. Secondly, the result may be related to the context of the study. Turkey's nation-wide mathematics curriculum proposes teachers the concepts to be presented, the level, and even the time to be spent for teaching. In other words, teachers in Turkey are not required to think on whether teaching slope is conceptually appropriate. The only remarkable feedback was gathered from the interview with the pre-service teachers. They claimed that teaching slope is conceptually appropriate by listing a number of concepts which are to be learned by students before learning slope.

All codes together

To conclude, comparing teachers in terms of the codes of the unit suggested that all participating teachers cared students' learning. However, the efficiency of their planning and implementation of the instructions were consistent to their current state of content knowledge (SMK and PCK). While teachers' knowledge was helpful in implementing a coherent mathematics instruction the reverse was also observable in some episodes.

Table 6.15. Comparison of groups in terms of connection.

	Teachers					
Codes	**Cansu**	**Akif**	**Erkin**	**Yasemin**	**Müge**	**Öznur**
Making connections between procedures	Not observed	Not observed	Connections were not observed in graphical procedures, but in algebraic procedures	Connections were not observed in graphical procedures, but in algebraic procedures	Focus on the connections between graphical and algebraic procedures to compute slope, additional exercise questions	Focus on the connections between graphical and algebraic procedures to compute slope
Making connections between concepts	Limited and weak connections for procedures: tangent, inclination of lines, similar triangles	Limited and weak connections for procedures: tangent, inclination of lines, angle, ratio	Connections to trigonometry needed for using procedures	Connections to trigonometry needed for using procedures	A strong connection to inclination and steepness.	Limited connection between tangent and slope
Anticipation of complexity	Almost anticipated but not able to predict the difficulty in computing slope	Almost anticipated but not able to predict the difficulty in computing slope	On computing slope graphically, and in dealing with fractions in equations	On computing slope graphically, and in dealing with fractions in equations	Emphasis on computing (negative) slope	Not observed
Decisions about sequencing	Deliberate and step-by-step movement to objective	Deliberate and step-by-step movement to objective	Needs more attention in introduction but exemplary in exercise questions	Exemplary in both introduction and exercises	Ordering exercise questions from y=mx to any form of line equation	Presenting slope from planes to lines, from simple to complex
Recognition of conceptual appropriateness	Verified during interview	Verified during interview	Not observed	A trigonometry case	Not observed	Not observed

6.4.4. Contingency

Teachers' contingent actions demonstrate their content knowledgein teaching. Findings suggested that the episodes of contingency unit suggest very important data on teachers' knowledge in other units also and its relationship to teaching.

Findings suggested that all of the participating teachers were willing to hear students' responses. Comparison of the instructions in terms of the first code may be summarized that Akif in the pre-service group and both of the experienced teachers were able to respond students' ideas. On the other hand, data also suggested episodes in which teachers lacked in understanding students' comments. Experienced teachers seemed not to worry too much even if they did not understand students' responses at first. In addition, experienced teachers used students' responses as an opportunity. However, it was not possible for all teachers to use those student responses as an opportunity.

Akif and experienced teachers were able to use opportunities during teaching slope. In other words, especially experienced teachers were successful in using students' suggestions as an opportunity. Handling students' comments seriously helped teachers as well as students in their classroom to observe mathematical content in a critical way.

There was a common finding all over the groups. Teachers did not deviate much from their agenda. It was almost impossible to match any episode to the category of deviation from agenda in pre-service and novice teachers' instruction. Müge, on the other hand, spent some time by a deviation when compared to other teachers.

All codes together

To conclude, teachers were too set on their agenda. Deviation from the agenda was almost not observed which shows teachers' readiness and willingness to expand the aims and borders of teaching mathematics. Besides, almost half of the teachers were able to respond students' ideas and use opportunities. In addition to indicating teachers' beliefs and perception on teaching in general, being a teacher, and teaching mathematics, findings suggested how experienced teachers' and one of the pre-service teachers' knowledge were observable during teaching.

Table 6.16. Comparison of groups in terms of contingency.

	Teachers					
Codes	**Cansu**	**Akif**	**Erkin**	**Yasemin**	**Müge**	**Öznur**
Responding to students' ideas	Weaknesses in responding	Willing to listen and respond students' ideas	Not observed	Observed, limited	Willingly responding to students' ideas	Willingly responding to students' ideas
Use of opportunities	Almost not observed: unable to address slope formula in detail	Opportunity to discuss ratio-proportion, unable to address slope in vertical lines	Not observed	Not observed	Ability to use opportunities and reach a beneficial platform for all, including the teacher	Ability to use opportunities and reach a beneficial platform for students
Deviation from agenda	Not observed	Not observed	Not observed	Not observed	Limited: extending to express line equations by a second way	Not observed

In this section I provided the research data in a comparative way. In the comparison, I presented the data in a way that the group of teachers' instruction may be compared in terms of the units of the quartet. The comparison between the groups showed that there were slightvariations among the pre-service and novice teachers in terms of the units of analyses. However, the comparative analyses suggested differenceof experienced teachers better. Besides, the differences reported here was not limited to one or two units but the differences were observable throughout all units of the quartet.

7. CONCLUSIONS AND DISCUSSION

The aim of the study was to investigate mathematics teachers' content knowledge during teaching. Teachers' content knowledge in teaching slope of a line was specifically focused in the study.Findings of the study were provided in four sections. Pre-service, novice, and experienced teachers' findings and a comparison section were provided in the previous section. Discussion of the findings will follow a similar pattern.

7.1. Pre-service Teachers' Content Knowledge

Pre-service teachers emphasized slope as a graphical-trigonometric concept. In addition, theyfocusedmore on teaching procedural aspects.This suggests that pre-service teachers were not able to enrich teaching and learning slope of a line during their instruction. In other words, teachers were not able to provide an instruction in which slope is introduced with its multidimensional conceptual meaning.

Teachers' lack of focus on conceptual learning of slopedoes not indicate that teachers' knowledge of slope is limited to procedural knowledge.The missing was lack of focus in teaching slope conceptually which is an almost general problem. Studies have shown that teachers generally do not tend to introduce slope as a rate of change. As a conclusion, teaching conceptual notions of slope is almost missing in teachers' teaching the concept of slope(Stump, 1999).

Pre-service teachers' instruction suggested that they feel more challenged in transforming what they already know. In other words, they may have a great deal of knowledge in terms of SMK. However, transforming this knowledge into pedagogically powerful forms -which is PCK in general terms-, may be difficult for them especially in their first experiences.

Pre-service teachers also needs professional assistance in terms of connections to be made. This is consistent with Livingston and Borko's (1990) hypothesis that pre-service teachers were impeded by a lack of necessary connections to be built in their content knowledge. First of all, it was seen that teachers may not recognize the importance of making connections between related concepts. Second, pre-service teachers need to spend more time on investigating(i) the mathematical concepts to be connected to slope, (ii) the depth and breadth of the connections, (iii) the means that arepedagogically powerful to be successful for reaching connections during the instruction.

It was mentioned in the findings section that the pre-service teachers designed a more straightforward lesson plan. The implementation of those lesson plans with little changes during instruction suggested a linear movement by easy-to-identify steps. This supports Ambrose (2004) that pre-service teachers may believe that teaching mathematics is mostly straightforward. The flexibility to reshape instructions according to students' ideas is necessary to be effective in teaching while this is one of the issues that pre-service teachers may feel uncomfortable. Rowland and the colleagues (2005) claimed that pre-service teachers' lack of confidence in their SMK may result in avoiding from risky situations such as responding to children's unexpected questions.

7.2. Novice Teachers' Content Knowledge

Novice teachers' also focused on procedural aspects of slope concept.Data showed that novice teachers took slope solely as a trigonometric ratio through procedures. Teachers' SMK on slope may be summarized as seeing slope as the tangent of the angle between a line and x axis. Hence, it was hypothesized that novice teachers were not able to interpret slope as a rate of change. In addition, teachers' foundational knowledge in teaching the relationship between slope and equation of a line was procedural.

Knowing a concept through a single dimension may limit instruction in several ways. Novice teachers' knowledge of slope which is dominated by the trigonometric ratio may narrow its teaching. Though it is true that slope of a line is described by the tangent of an angle, depending only on this description may not lead the conceptual comprehension of slope. For example, this definition does not indicate that slope characterizes a constant rate of change in a situation where two variables covary. All in all, novice teachers need to re-organize the concept of slope through its various meaning and interpretation.

Novice teachers' instruction suggested that they had more action in transforming what they already know. Depending only on trigonometric ratio meaning limited them. Transforming this knowledge into pedagogically powerful forms, -which is PCK in general terms-, was difficult for them. Theyneed additional study of making connections. Teachers need to know the necessary knowledge of conceptual connections between the areas which they aim to teach. Otherwise, they will continue to teach mathematics as disconnected topics or perceive relationships as rules (Ball, 1990a). Behaving the relationship between slope and equation of a line as a practical way of computing slope suggest that teachers are not aware, for example, of the importance in representing lines in slope-intercept form.

Teachers' lack of connections with slope and rate of change, and connections between line, equation of a line, and linear function may result a failure to teach slope conceptually. Findings were similar to the Even's (1993) study that understanding connections between concepts to be taught could not be taken for granted for participating teachers.

7.3. Experienced Teachers' Content Knowledge

The experienced teachers focused on teaching procedural aspects of slope. However, data indicated that experienced teachers' SMK was significant even if teaching slope is on its procedural attainment. In other words, teachers' success in presenting the procedures suggested that they arise from a conceptual underpinning.

Data suggested that experienced teachers' foundational knowledge was significant. Analysis of teachers' instruction suggested to observe that they have a robust understanding of the procedural aspects of slope concept. The versatility of their knowledge was especially visible through the codes of the unit. To illustrate, their purpose of teaching slope was more comprehensive as addressed in findings unit. In addition, their reliance on procedures indicated the way they applied their foundational knowledge in responding student ideas.

In addition, teachers' practices suggested that they already have an idea of how to teach slope. Teachers' knowledge of students' thinking was also helpful in enhancing and proceeding the instruction on slope. Teachers effectively used questioning strategies, responded students' ideas and used them as a new opportunity. Experienced teachers asked questions, raised discussions, and suggest different points of views of the mathematical content to students. It was observed that these activities and decisions require teachers to have a sufficient level of subject matter knowledge, pedagogical content knowledge, and teaching experience (Even, 1990).

To conclude, experienced teachers' content knowledge in teaching was remarkable. However, the extent and the quality of it might be better visible when their knowledge is compared to their less-experienced colleagues. Hence, this will end this section since their content knowledge will be provided in the following part.

7.4. Differences in Teachers' Content Knowledge

Shulman (1987) suggested that teaching is necessarily influenced by the teachers' understanding of what is to be learned (SMK) and how it is to be taught (PCK). The study indicated also that the main difference among groups of teachers may be classified in two groups.

Experienced teachers explored more than one concept or relationship via a single example or explanation. On the other hand, both novice and pre-service teachers presented concepts as a set of disconnected rules and algorithms. Even if these teachers succeeded in reaching their objective, it was a challenging duty for them. This may also explained by the fact that pre-service and novice teachers may know a great deal that they had not tried yet to articulate (Shulman, 1987).

The purpose of instruction for pre-service and novice teachers were limited when compared to their experienced counterparts. The relationship between equation and the slope of a line is the major concern for those instructions. However, the experienced teachers enlarged the purpose to draw graph of lines and compute slope on the graphs.

It was remarkable to observe that novice teachers who participated to this study demonstrated almost no advantage over the pre-service teachers in terms of their content knowledge (SMK and PCK).The results presented in this study have provided an opportunity to consider the way pre-service and novice teachers' content knowledge is related to strengths or limitations inthe acts of their instruction. For example, data showed that pre-service and novice teachers may have limited knowledge in teaching the slope of line and it was observable during instruction. There is evidence that pre-service and novice teachers have a similar reliance on procedures in teaching the slope of a line.

Data supported Even's (1990) claims that teachers' content knowledge may be consistent to their ways of teaching. Teachers who have strong mathematical knowledge may better help their studentsunderstand the mathematical subject matter more meaningfully (Even, 1990). On the other hand, when teachershave a narrow content knowledge, their knowledge might limit their ability to present subject matter in appropriate ways, give helpful explanations, and conduct discussions(Even and Tirosh, 1995; Fennema and Franke, 1992).As a result, this may indicate negatively on teachers' pedagogical thinking.

Analyzing instructions of various teachers who vary in terms of experience (pre-service, novice, and experienced teachers) was helpful.It suggested that the experiences that teachers gain with teaching have a potential to give rise to meaningful changes in their practice (Leikin and Zazkis, 2010)Teachers continue to learn as they experience to teach. Teachers may increase their knowledge in both subject matter, which is the concept of slope in this study, and the pedagogy of that subject matter through teaching. This is consistent with two kinds of research findings: first is that teachers may better understand the subject they teach with experience (Leikin and Zazkis, 2010; McDiarmid and Wilson, 1991) and second is that teaching mathematics is something to be learned especially by practicing (Ball, 1990; Leikin and Zazkis, 2010; Putnam and Borko, 2000; Shulman, 1987).

The participant teachers' had the same undergraduate education. If prior learning experiences such as undergraduate education were an accurate indication for teaching practices, there should not be so much difference among teachers' knowledge and instruction (Jong, 2009). However, the study indicated differences among teachers' knowledge in teaching. This supported the claim that there is a relationship between teachers' effectiveness in teaching and their experiences.

7.5. Using the KQ toExploreTeachers' Content Knowledge

The study indicated the importance of mathematical knowledge in teaching especially in foundation unit. The study supported Rowland and the colleagues (2005) that the other three units are based on a foundational underpinning. The knowledge which may be categorized in foundation unit is influential on other units in various ways.

Listening to students has arisen as important characteristics of teachers. Teachers' reactions to content-related inquiries during their instruction were in the group of significant episodes. The KQ enabled to see that experienced teachers were open to hear student ideas without any hesitation. It supports the findings that tension of responding to students' inquiries reduce and the ability to re-shape the lesson agenda increases with experience (Brown and Wragg, 1993).

Responding to students' ideas, first of all, requires active and careful listening of students' content related expressions (Davis, 1997; NCTM, 2000). The study supports Sherin (2002) that students' behaviors such as elaboration of their ideas may be regarded as a chance for teachers to revise their content knowledge. Experienced teachers may effectively use those episodes to increase their and students' knowledge (Sherin, 2002). Besides, these episodes enabled to see teachers' content knowledge in teaching in a better way. The results are consistent with the claims made by Rowland, Thwaites, and Jared (2011) that teachers' contingent action provide valuable information on the effect of teachers' content knowledge in teaching.

Rowland and the colleagues (2005) claimed the use of quartet as a framework for lesson observation. However, it was used as an analytical framework in the study. The framework was helpful in exploring the episodes of instruction in terms of content knowledge in teaching.

The use of framework brought some implications on the nature of teachers' content knowledge. There were episodes of instructions which was not possible to be put into any unit. In some cases it was difficult to select a unit for an episode. The issue is also supported by Rowland and the colleagues (2007) that many moments or episodes within a lesson can be explored in terms of two or more of the units of the quartet. Sherin (2002), in a similar perspective, claimed that there may be larger elements of teacher knowledge which can be categorized as neither subject matter knowledge nor pedagogical content knowledge. Teachers may tend to call on both subject matter knowledge and pedagogical content knowledge at the same time. Hence, the study supported to research teachers' knowledge in a holistic way.

8. LIMITATIONS AND RECOMMENDATION FOR FURTHER RESEARCH

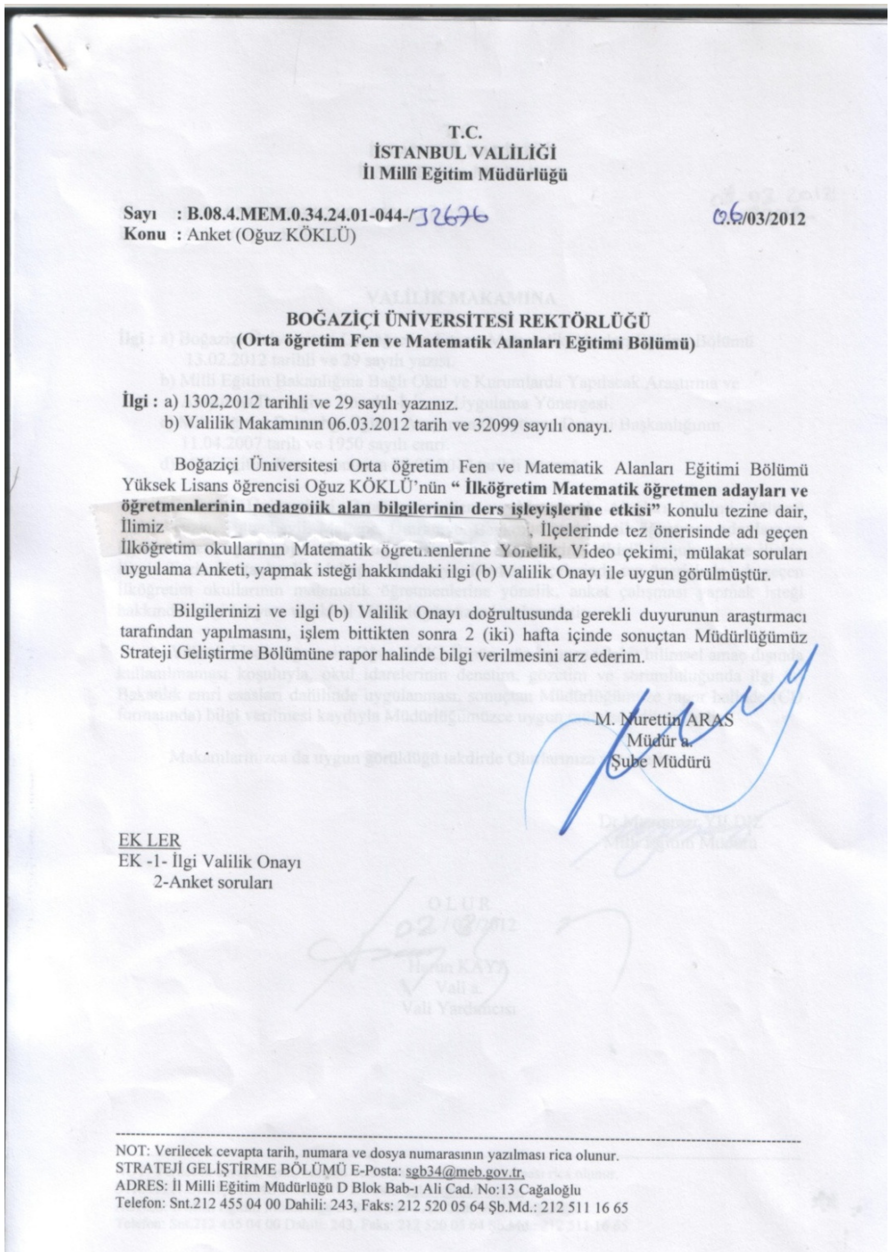
The section provides the limitations of the study and highlights several areas in which future research on mathematics teacher education can be extended. This study investigated teachers' content knowledge in their instruction and it included the teaching slope. However, it is not guaranteed to say that the findings apply to teaching of all mathematical concepts. The research may be conducted for other mathematical concepts also.

The study included six teachers and their instruction during two lesson hours. Purposefully, all of the participants were chosen from the graduatesof a public university. In addition, the selection of participants was intentionally limited to teachers at most five years of teaching. Further studies may be more inclusive by studying with graduates of other universities. In addition, the literature may be improved by designing studies that extend the experienced teacher participants beyond teaching five years.

A cross-sectional research design was chosen due to the time constraints. Longitudinal studies are also appropriate in finding answers to complex questions about teachers' knowledge and instruction; the study might provide more space to comparison among teachers' knowledge. Conducting a longitudinal study may provide a thorough examination of pre-service, novice, and experienced teachers' knowledge in their teaching.

Purposefully, the number of participants for the study was six teachers in total.Studying with six teachers enabled to have in-depth knowledge in terms of teachers' content knowledge in teaching. However, it also does not enable to make large-scale comparisons and reach generalizations based on those comparisons. Overall, the aim of the study was not to make generalizations. Having the vision that the research in mathematics education is also cumulative, I can claim that the data provided in this study providesan opportunity for researchers to make comparisons and reach more valid generalizations in the future. The studies similar to this research in terms of method and content may be collected to increase the power of its generalizability. Besides, the study is also open to meta-analysis.

APPENDIX A: PERMISSION FORM



T.C.
İSTANBUL VALİLİĞİ
İl Millî Eğitim Müdürlüğü

Sayı : **B.08.4.MEM.0.34.24.01-044-/32676** 06/03/2012
Konu : Anket (Oğuz KÖKLÜ)

BOĞAZİÇİ ÜNİVERSİTESİ REKTÖRLÜĞÜ
(Orta öğretim Fen ve Matematik Alanları Eğitimi Bölümü)

İlgi : a) 1302,2012 tarihli ve 29 sayılı yazınız.
b) Valilik Makamının 06.03.2012 tarih ve 32099 sayılı onayı.

Boğaziçi Üniversitesi Orta öğretim Fen ve Matematik Alanları Eğitimi Bölümü Yüksek Lisans öğrencisi Oğuz KÖKLÜ'nün **" İlköğretim Matematik öğretmen adayları ve öğretmenlerinin pedagojik alan bilgilerinin ders işleyişlerine etkisi"** konulu tezine dair, İlimiz , İlçelerinde tez önerisinde adı geçen İlköğretim okullarının Matematik öğretmenlerine Yönelik, Video çekimi, mülakat soruları uygulama Anketi, yapmak isteği hakkındaki ilgi (b) Valilik Onayı ile uygun görülmüştür.

Bilgilerinizi ve ilgi (b) Valilik Onayı doğrultusunda gerekli duyurunun araştırmacı tarafından yapılmasını, işlem bittikten sonra 2 (iki) hafta içinde sonuçtan Müdürlüğümüz Strateji Geliştirme Bölümüne rapor halinde bilgi verilmesini arz ederim.

M. Nurettin ARAS
Müdür a.
Şube Müdürü

EK LER
EK -1- İlgi Valilik Onayı
2-Anket soruları

NOT: Verilecek cevapta tarih, numara ve dosya numarasının yazılması rica olunur.
STRATEJİ GELİŞTİRME BÖLÜMÜ E-Posta: sgb34@meb.gov.tr,
ADRES: İl Milli Eğitim Müdürlüğü D Blok Bab-ı Ali Cad. No:13 Cağaloğlu
Telefon: Snt.212 455 04 00 Dahili: 243, Faks: 212 520 05 64 Şb.Md.: 212 511 16 65

APPENDIX B: PRE-INTERVIEW PROTOCOL

Mülakat____ **Katılımcı için**

kod:___________

Tarih:

Yer:

1. Bu konuyu nasıl öğretmeyi planladınız?
   a. Planladığınız bu dersin amacı nedir? Eğim kavramı neden öğretiliyor?
   b. Bunları ele almak için ne tür matematik etkinlikleri/problemleri hazırladınız?
      i. Temel matematik kavramları
      ii. Matematik kavramlarını bu problemlerde nasıl ele almayı düşünüyorsunuz?
      iii. Gösterimler, açıklamalar, örnekler, matematik soruları (matematik ve uygulama/pedagoji)
      iv. Öğrencilerin bu problemlerde/etkinliklerde matematiksel olarak nasıl düşüneceğini bekliyorsunuz?
   c. Öğrenme sürecini nasıl değerlendireceksiniz?

APPENDIX C: POST INTERVIEW PROTOCOL

Mülakat____ **Katılımcı için**

Kod:______________

Tarih:

Yer:

a. Ders planladığınızdan nasıl *farklılaştı*?

Boyutları:

i. Temel matematik kavramları

ii. Matematik etkinlikleri/problemleri

iii. Konu anlatımında/açıklama yaparken ya da problem içerisinde: Gösterimler, açıklamalar, örnekler, matematik soruları (matematik ve uygulama/pedagoji)

b. Öğrenciler bu konuyu nasıl öğrendi, sizin kazanımınıza/amacınıza ulaştılar mı? Buna nasıl karar verdiniz? (değerlendirme?) Planladığınız bu dersin amacı ne idi? Eğim kavramı neden öğretiliyor?

Farklılaşma konusunda neler değişti ve buna nasıl karar verdiniz? (yeni duruma nasıl adapte oldunuz?

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