DEVELOPMENT AND VALIDATION OF A SCALE FOR MEASURING STUDENTS' MATHEMATICS-RELATED BELIEFS

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Dedicated to

my beloved mom

for glorious childhood memories

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ABSTRACT

DEVELOPMENT AND VALIDATION OF A SCALE FOR MEASURING STUDENTS' MATHEMATICS-RELATED BELIEFS

This study aimed to develop and validate a scale based on a framework of students' mathematics related beliefs. Several models have been proposed to explain this rather extensive construct. The framework used in this study was based on the existing models in the literature to explain students' mathematics related beliefs. The framework included students' beliefs about mathematics education, students' beliefs about self and students' beliefs about social context. The categories of the framework were divided into dimensions for scale development. The scale was developed to cover the dimensions as students beliefs about (1) the nature of mathematics, (2) learning and problem solving, (3) teaching; (4) self efficacy, (5) control, (6) task-value, (7) goal-orientation, (8) social norms, (9) socio-mathematical norms. Data on 300 8^{th} grade students were analyzed to assess the psychometric qualities of the instrument, to assess model fit to the framework, to describe students' math related beliefs, and to investigate gender differences and the relationship between mathematics achievement and beliefs. The instrument was found to be reliable and valid. The data provided empirical evidence for a modified model on the framework. Students' beliefs indicated a desirable direction. There were no gender differences except on the first subscale of students' beliefs about the nature of mathematics. The relation between beliefs about social context and mathematics achievement were found to be statistically significant.

ÖZET

ÖĞRENCİLERİN MATEMATİKLE İLGİLİ İNANÇLARI: BİR ÖLÇEK GELİŞTİRME ÇALIŞMASI

Bu çalışmada öğrencilerin matematikle ilişkili inançları çerçevesinde bir ölçek geliştirme ve çerçevenin geçerliliğini kontrol etme hedeflenmektedir. Oldukça kapsamlı bir kavram olan "öğrencilerin matematikle ilişkili inançları" hakkında bir çok model sunulmuştur. Bu çalışmada kullanılan ve öğrencilerin matematikle ilişkili inançlarını açıklayan yapı literatürde var olan modellere dayanmaktadır. Bu çerçeveye göre öğrencilerin matematik eğitimiyle ilgili genel inançları, kendileriyle ilgili inançları ve sosyal ortamla ilgili inançları öğrencilerin matematikle ilişkili inançlarını oluşturur. Burada belirtilen kategoriler ölçek geliştirmek için boyutlara ayrılmıştır. Geliştirilen ölçekte yer alan boyutlar öğrencilerin (1) matematiğin doğası (2) öğrenme ve problem çözme (3) öğretme, (4) öz yeterlilik, (5) kontrol, (6) değer verme , (7) hedefe yönelme, (8) sosyal normlar, (9) matematiğe özgü normlar hakkında inançlarını kapsamaktadır. 300 sekizinci sınıf öğrencisinden toplanan veri ölçeğin psikometrik kalitesini değerlendirmek, oluşturulan modelin çerçeveye uygunluğunu anlamak, öğrencilerin matematikle ilişkili inançlarını tanımlamak, cinsiyet farklılıklarını ve inançlarla matematik başarısı arasındaki ilişkiyi incelemek için analiz edilmiştir. Analizler sonucunda, geliştirilen ölçek geçerli ve güvenilir bulunmuştur. Toplanan veri modelle çerçeve arasındaki uyumu ampirik olarak kanıtlamıştır. Öğrencilerin matematikle ilişkili inançları genel olarak beklenen yöndedir. Matematiğin doğası dışında diğer alt ölçekler için cinsiyete bağlı farklılıklar yoktur. Öğrencilerin matematik başarıları sosyal ortamla ilgili inançlarıyla manidar şekilde ilişkili bulunmuştur.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

AGFI	Adjusted Goodness of Fit Index
CFA	Confirmatory Factor Analysis
CFI	Comparative Fit Index
CMIN/DF	Ratio of Minimum Sample Discrepancy to Degrees of Freedom
DF	Degrees of Freedom
GFI	Goodness of Fit Index
MEB	Turkish Ministry of Education
MI	Modification Index
NCTM	National Council of Teachers of Mathematics
NFI	Normed Fit Index
RMSEA	Root Mean Square of Error of Approximation
SEM	Structural Equation Modeling
TLI	Tucker-Lewis Index

1. INTRODUCTION

There are many studies carried out with the purpose of understanding students' mathematics-related beliefs (Op't Eynde *et al.* 2002). These studies made crucial contributions in terms of conceptualization of the students' mathematics related beliefs and their relation to mathematics achievement or other educationally essential constructs such as gender and motivation (Kloosterman, 1996; McLeod, 1992; Pehkonen, 1995; Schoenfeld, 1983; Underhill, 1988). Despite the abundance of studies in relation to students' mathematics related beliefs, a theory of the construct still needs to be refined.

Different perspectives dealing with various facets of the construct "belief" account for the disagreement on its definition and its dimensions. These commonly used perspectives are motivational theories, psychological perspective and socio-cultural perspective. The motivational theories point at the affective domain as a superset of beliefs (McLeod, 1992). From the psychological perspective, the distinction between knowledge and beliefs must precede all else. Since, for several researchers belief is defined as subjective knowledge (Lester *et.al.*, 1989; Pehkonen, 1998). On the other hand, the socio-cultural perspective stresses the importance of social interactions in the formation of beliefs (de Abreu *et al.*, 1997). In other words, the differences on the definition of beliefs and its dimensions mostly arise from different perspectives.

From a macro perspective, in the field of education there has been a shift from behaviorism to constructivism. Constructivist views of learning have become dominant in the educational arena for more than two decades now. The interpretations of the constructivist philosophy created two distinct approaches to educational research. These are the psychological perspective and the social perspective. The psychological perspective and the social perspective are used to study learning and learning-related factors and outcomes (Anderson *et al.*, 2000). The psychological perspective focuses on an individual's construction of meaning whereas the social perspective draws attention to the social construction of meaning through social interactions.

According to Cobb's *emergent perspective*, the psychological perspective and the social perspective should not be understood as two opposing views. They should be considered as two complementary views (Cobb and Yackel, 1996). There are several researchers supporting and using Cobb's emergent perspective (Anderson *et al.*, 1997; Greeno, 1997). While studying the construct of beliefs, emergent perspective is thought to be an appropriate one for the nature of the construct. Beliefs are formed by the individual as a result of social interactions (de Abreu *et al.*, 1997). Learner's unique interpretation of these interactions by combining them with other social interactions in other social contexts is rooted in the individual (Pehkonen and Torner, 1996). The learner's beliefs are independent of neither the social interactions nor the individual.

The framework used in this study tries to combine the results of the studies from both psychological and social perspectives. It is in line with Cobb's emergent perspective. In other words, the self and the social contexts have been considered to be equally important concepts in studying beliefs.

Belief by its traditional definition includes the subject or the holder of the belief (self) and the object of the belief. Beliefs of an individual person (subject) consist of his/her understandings of self and the world around him/her (object) (Bem, 1970). A belief requires an object of belief. Furthermore, a belief is always about something.

Schoenfeld (1985), in his pioneering work on mathematical problem solving, emphasized the importance of beliefs in solving problems. The concepts, which are explained above, self (subject), object, and social context form the three categories stated in Schoenfeld's (1983) notion of beliefs. His categories are stated as: beliefs about (a) the task at hand (object i.e. mathematics education), (b) self, (c) the social environment (mathematics class).

Op't Eynde *et al.* (2002) by reviewing the studies on students' mathematics related beliefs concluded that there was a need for the clarification of conceptualization of the students' mathematics related beliefs. In their inclusive review, the studies from 1984 to 2000 on students' mathematics related beliefs were evaluated and summarized with a theoretical framework. The framework formed by Op't Eynde *et al.* fits the structure of three categories stated by Schoenfeld. This framework was a theoretical attempt to conceptualize students' mathematics related beliefs.

In the present study, an attempt was made to collect empirical evidence to conceptualize students' mathematics related beliefs. The purpose of this study was to develop and validate a scale to measure students' mathematics related beliefs based on the framework proposed by Op't Eynde *et al.* (2002).

2. LITERATURE REVIEW

2.1. Related Concepts

Students' mathematics related beliefs is a broad construct which is closely related with two important concepts such as "nature of beliefs" and "nature of mathematics as a domain" (Buehl *et al.*, 2002; Op't Eynde *et al.*, 2002). In order to simplify the construct, an analytical approach was taken in this study. In other words, the concepts that were thought to be essential in understanding students' mathematics related beliefs were explained one by one. First, the studies about the nature of beliefs, about the nature of mathematics and then about the nature of students' mathematics related beliefs were summarized. Then, the framework and its dimensions used in the present study were introduced with literature references.

2.1.1. Nature of Beliefs

The definitions of the term "belief" are given in different disciplines such as philosophy, psychology and sociology (Leder *et al.*, 2002). Each definition reveals another aspect of the construct. In order to understand what a belief is, several scholars made comparisons (Biddle, 1979; Pehkonen and Törner, 1996; Thompson, 1992). They distinguished between beliefs and other concepts for clarity. The two most important distinctions that are helpful in clarifying the nature of beliefs are "attitude versus belief" and "knowledge versus belief".

The definitions of attitude and belief are made in sociology and psychology to illuminate the nuances between the two constructs. For sociologists belief is a *covertly held description* whereas an attitude is a *diffuse, preferential conception* (Biddle, 1979). In other words, belief has a description side but attitude has a preference side. Beliefs of an individual may be unobservable, if s/he doesn't volunteer to share them. Belief statements according to the definition are general propositions about an event, domain, person or circumstance. An attitude is a choice and it is observable in appropriate occasions. For psychologists, attitudes are likes or dislikes. Beliefs of a person consist of his/her understandings of self and the world around him/her (Bem, 1970). Affect is emphasized in the definition of attitudes whereas cognition is emphasized in the definition of beliefs. Especially, for social psychologists *attitude is a learned predisposition to respond in a consistently favorable or unfavorable manner to a given object* (Biddle, 1979). Attitude is a tendency to react constantly in the same way to an event or domain, person or circumstance. The points put forward in the definitions of belief and attitude can be summarized as the former being a disposition filtered cognitively, whereas, the latter being an affective preference.

Although the definition of knowledge is not crystal-clear, researchers tried to discriminate belief from knowledge (Pehkonen and Törner, 1996). There are two important distinctions between knowledge and belief. The first distinction is that knowledge is socially accepted whereas belief is accepted individually (Furinghetti and Pehkonen, 2002). In order to express the difference between knowledge and belief, the authority who determines the truth value of a statement is underlined. Although objectivity is a required condition for knowledge to be true, this is not the case for a person's belief. The truth value of a belief is determined subjectively by the belief holder. The second distinction between knowledge and belief lies in the justification process. Knowledge is formed through a logical justification whereas a belief is formed through a quasi-logical justification (Pehkonen and Törner, 1996; Thompson, 1992). The quasi-logical justification refers to a person's unique subjective logic that is psychological. A person's quasi-logical justification steps for a belief might be irrational for another person.

The studies on the structure of beliefs supported the idea that beliefs form *belief systems* like knowledge systems (Schoenfeld, 1985). The dimensions of belief systems include (1) cluster structures; (2) quasi-logicalness and (3) psychological centrality as defined by Green (1971) and discussed by Furinghetti and Pehkonen (2002).

Beliefs exist in clusters because of their vertical and horizontal structures (Bem, 1970). The vertical structure of belief systems refers to the syllogistic chain of reasoning where one belief is both a conclusion and a premise in two sentences. Two or more vertical structures combine and form a horizontal structure if they reach a common conclusion. In other words, if a belief is the conclusion of more than one syllogistic chain of reasoning then it has a horizontal structure.

The quasi-logicalness (psycho-logic) dimension indicates the reasoning in the belief system that is more psychological than logical (Furinghetti and Pehkonen, 2002). While explaining the formation of beliefs as a result of repetitive experiences Snow *et al.* (1996) state:

> *'Human beings in general show tendencies to form and hold beliefs that serve their own needs, desires and goals; these beliefs serve ego-enhancement, self-proactive, and personal social control purposes and cause biases in perception and judgment in social situations as a result.'*

The psychological centrality dimension refers to the strength and endurance of beliefs. A belief which is in the centre of a cluster is not necessarily a strong belief. Having a broad vertical and horizontal structure does not guarantee psychological centrality. The importance of the belief for the holder defines its centrality. As stated by Bem (1970) "the underlying importance of the belief to other beliefs forms the centrality". Similarly, the stability of beliefs is always defined through its' centrality (Kaplan, 1991).

The stability of beliefs is emphasized in a study from analysis of the affective domain (Goldin and De Bellis, 2006). The definition of beliefs proposed by Goldin and De Bellis (2006) also highlights that beliefs are the highly cognitive but affect is immersed in belief structures and affect adds to the stability of beliefs. Goldin's (2002) consecutive studies formed a tetrahedral model of sub-domains in affect. The sub-domains in the affective domain are emotions, feelings, attitudes, beliefs and values. The sub-domains are compared according to their stability. Emotions are rapidly changing. Attitudes are moderately stable. Values are quite stable. Beliefs are highly stable. Moreover, Goldin's (2002) extensive work on affect and mathematical belief structures is in line with Snow *et al.*'s (1996) emphasis on the quasi-logical dimension of beliefs.

A widely discussed issue on beliefs, especially for epistemological beliefs, is whether beliefs are general or domain specific. Some claim that the beliefs held by a person about knowing are the same across different domains such as mathematics, music, physics, and literature according to one of the perspectives in belief research. But there is also another perspective which assumes that the beliefs held by an individual are different in different domains (Buehl *et al.*, 2002). Moreover, the studies on domain specificity or generality of beliefs indicate the existence of a general factor common across domains (Buehl *et. al.*, 2002; Hofer, 2000). Notwithstanding the existence of a general factor, the domain specific approach attracted more attention by the researchers (Op't Eynde, 2006). In this study a domain specific approach is taken to expose students' mathematics related beliefs.

2.1.2. Nature of Mathematics as a Domain

The nature of a domain is the essential component in understanding learning and teaching within that domain. Ernest (1998) underlines this idea by quoting from Rene Thom (1973): *"In fact, whether one wishes it or not, all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics."*. In other words, pedagogy of mathematics is closely related with the philosophy of mathematics (Ernest, 1998).

There are mainly two classifications of domains according to the products of the domain and according to the processes in the domain: The first classification is based on the product that is the body of knowledge in the domain. It includes two categories such as the absolutist view and the probabilistic view. This approach is not only general to mathematics but also works for the other domains. The shifts in different domains from absolute, one right solution approach to probabilistic and optimum solutions have been following one another. Gödel's work in mathematics in 1931, Heisenberg's principle of uncertainty, a radical contribution both in chemistry and physics, Schrödinger's work in quantum theory all changed the established views on nature of domains. The educational correspondent mathematics, chemistry and physics curricula followed these changes. Indisputably after a while, the transformations about the nature of domains took place almost simultaneously in the first half of the 20^{th} century. The accepted views on the nature of domains is always open to debate and modifications which are based on the developments within the domain and related domains.

Nature of mathematics as a domain has been open to debate as mathematics education has gone under radical changes. There are many classifications for the nature of mathematics. Ernest (1991) stated, while citing the work of Confrey in 1981, that the views on the nature of mathematics could be mainly classified based on an absolutist view or philosophy of conceptual change. Absolutist views of mathematics underline the certain, static mathematical knowledge whereas the conceptual change view puts the emphasis on mathematical knowledge as being a dynamic social product. The existence of these two views reflects the changes in the understanding of the nature of domains that started to take place in 1920s.

Secondly, the process oriented classifications focus on the processes of forming and verifying knowledge in the domain. The classification of domains according to the processes used includes categories as rational, empirical or metaphorical domains (Royce, 1978). There is no single right answer to the question of which category mathematics belongs to. There is a continuum of answers. On one pole mathematics is seen as a rational domain and on the other it is treated as belonging to an empirical domain. In other words, for mathematics, philosophical considerations of mathematical knowledge and knowing can be clustered into two main groups. One group of ideas stresses rational nature of mathematics and another focuses on empirical nature of the discipline.

According to the first group of ideas, mathematics is dominantly a rationalist domain rather than empirical or metaphoric (Royce, 1978). Ways of knowing, nature of mathematical knowledge and mathematical justification are dependent on the rational nature of mathematics. For the second group, mathematics is defined to be “the science of patterns” where the word science is used on purpose to denote the empirical nature of mathematics (Hoffman, 1989; Steen, 1988). Schoenfeld (1994), by using the definition of mathematics as science of patterns stated that mathematics became the study of all sort of regularities and it could be differentiated from other sciences or empiric endeavors because of the objects and tools of study.

Actually, the two groups of ideas are not contradictory. They just put emphases on different aspects of mathematics. This makes them complementary. The difference between applied and pure mathematics is relative and time dependent. Several topics listed under pure mathematics now, can be studied under the name applied mathematics in the future. An example from Steen (1988) can explain what is meant by time dependency: Number theory, which was thought to be a topic of pure mathematics, is now a starting point for applications in coding and data transmission. It is not unwise to think that today's topics of pure mathematics will probably find their places in applications in the future. It is also possible to think that applied mathematics can also create new areas in pure mathematics.

The philosophy of mathematics is an essential force behind curriculum reforms. At different stages in the history of education, sometimes the rational nature of mathematics was emphasized; at others the empirical nature was dominant. So, the rational and empirical natures of mathematics have never been represented equally in schools due to the curriculum trends, teachers and other educational actors.

2.2. Mathematics Related Beliefs

It is important to define mathematics related beliefs before trying to describe them. In defining students' mathematics related beliefs, many researchers used the concept of "belief systems" proposed by Schoenfeld (1985). This is clarified as follows:

> *'Belief systems are one's mathematical world view, the perspective with which one approaches mathematics and mathematical tasks. One's beliefs about mathematics can determine how one chooses to approach a problem, which techniques will used or avoided, how long and how hard one will work on it, and so on. Beliefs establish the context within resources, heuristics and control operate.'* (Schoenfeld, 1985)

There exist studies that are based on the idea of beliefs exist as systems. Lerch (2004) carried out an in-depth study of four students' beliefs while solving routine and non-routine problems. The results of the observations explained in the study were consistent with the dimensions proposed by Schoenfeld (1985) as a belief system.

The term belief is characterized by different aspects in a research study where the sample consisted of researchers in mathematics education (Furinghetti and Pehkonen, 2002). This study used definitions or descriptions of beliefs from the previous studies. Researchers were asked to indicate their agreement level on these definitions with a five-step scale. Based on the responses, this study advised to consider beliefs as subjective knowledge and pointed to consciously and unconsciously held beliefs. In mathematics classroom, as consciously held beliefs lead to a certain type of action pattern, so does the unconsciously held beliefs (Furinghetti, 1996).

The importance of mathematics related beliefs in solving problems is emphasized through several studies. Beliefs are defined to be more stable and strong. Understanding beliefs is vital because students' belief systems influence how they approach a problem (Lerch, 2004; Schoenfeld, 1985; 1992). The beliefs can even make students stop before starting to solve a problem (Zeitz, 1999). Some research studies supported that, beliefs about mathematics and problem solving determined how students approach to a problem (Lester *et al.*, 1989).

Although the researchers and educators did not reach a consensus on the definition of mathematics related beliefs, the review of the previous definitions guided the formation of a recent definition. In the light of the existing definitions, a definition was formed by Op't Eynde *et al.* (2002). That definition has been used in this study. According to this definition students' mathematics-related beliefs are the implicitly or explicitly held subjective conceptions students hold to be true about mathematics education, about themselves as mathematicians, and about the mathematics class context.

Contrary to the fact that there is a huge amount of research about students' mathematics related beliefs, there is no agreement on the conceptualization of students' beliefs and there is a need to clarify the dimensions of students' beliefs related with mathematics (Op't Eynde *et* *al.*, 2002). Op't Eynde *et al.* (2002) presented a framework of students' mathematics related beliefs based on several previous models (Kloosterman, 1996; McLeod, 1992; Pehkonen, 1995; Schoenfeld, 1983; Underhill, 1988) as follows:

- Beliefs about mathematics education
  - Beliefs about mathematics as a subject
  - Beliefs about mathematical learning and problem solving
  - Beliefs about mathematics teaching in general
- Beliefs about self
  - Self-efficacy beliefs
  - Control beliefs
  - Task-value beliefs
  - Goal-orientation beliefs
- Beliefs about the social context
  - Beliefs about social norms
  - Beliefs about socio mathematical norms

Although the definition of students' mathematics related beliefs is given. There are several important points in understanding the term "belief" used in this study. "Belief"

- belongs to subjective knowledge
- is highly related with affect
- is not likes, dislikes or preferences
- is a general proposition
- does not mean behavior.

2.3. The Framework

The framework was a product of extensive review of studies from 1984 to 2000 (Op't Eynde *et al.*, 2002). After the analysis of these studies according to their appropriateness, references cited by the relevant studies were also included in their study. The categories and models of students' mathematics related beliefs mentioned in these studies were analyzed and then a framework was derived on a theoretical basis by using the existing literature. The structure of the framework is in line with Schoenfeld's (1983) notion of beliefs as divided into three categories: beliefs about (a) the task at hand (object i.e. mathematics education), (b) the social environment (mathematics class), (c) self

Op't Eynde *et al.* (2002) presented that students' mathematics related beliefs formed three main categories. The three categories are (1) students' beliefs about mathematics education, (2) students' beliefs about self and (3) students' beliefs about the social context. Each main category is divided into sub categories in the framework. In the present study, the categories or subcategories introduced in the framework were divided into dimensions through analyzing the studies in the literature review and according to the researcher's understanding of the categories.

According to the framework; first main category; (1) students' beliefs about mathematics education covers three sub categories of beliefs: (a) beliefs about mathematics as a subject, (b) beliefs about mathematics learning and (c) beliefs about mathematics teaching. According to Op't Eynde *et al.* (2002), the students' beliefs of mathematics are not independent of their beliefs about mathematics learning and teaching.

First of all, the first subcategory of the first main category that is beliefs about mathematics education is related to the beliefs about mathematics as a subject. This subcategory is divided into dimensions. These dimensions are student's *beliefs about usefulness, effort and nature of mathematical knowledge. The Indiana Mathematics Belief Scales* included *effortful math* as one of its dimensions (Kloosterman and Stage, 1992). *Usefulness and effort* dimensions were also used in measuring epistemological beliefs and mathematical problem solving beliefs (Schommer-Aikins et al., 2005). Effort was used in measuring beliefs about other domains such as physics in *The Colorado Learning Attitudes about Science Survey* by Adams *et al.* (2004). Furthermore, the third dimension beliefs about mathematical knowledge was divided into five sub-dimensions as beliefs about certainty of knowledge, source of knowledge, structure of knowledge, control of knowledge acquisition, speed of knowledge acquisition by Schommer in her studies about epistemological beliefs (1990; 1998). In order to explain the relation between these dimensions and beliefs, in several research studies, the examples of students' beliefs are given. The study of Schommer (1994) defined certainty of knowledge as an epistemological belief and to be an indicator of "nature of disciplines". For instance, believing that "Formal mathematics has nothing to do with real thinking and problem solving" is related with beliefs about the nature of mathematics and certainty of knowledge in particular. De Corte *et al.* (1996) have studied the beliefs about mathematics, and found that one of the common unproductive beliefs of students was "mathematics is a domain of excellence" which was consistent with the belief as "Only geniuses are capable of discovering of creating mathematics". The last belief can be thought under the dimension of "control of knowledge acquisition" in terms of Schommer's dimensions.

The second subcategory of the first main category, which is students' beliefs about mathematics education, is beliefs about mathematical learning and problem solving. This subcategory is divided into two dimensions as learning and problem solving. In our understanding learning dimension refers to understanding concepts and problem solving refers to the role/function of problem solving in mathematics.

The third subcategory of the beliefs of the first main category, that is students' beliefs about mathematics education, is beliefs about mathematics teaching in general. The mathematics teaching is usually divided into two dimensions as mathematical knowledge and understanding of the teacher about what to teach, and pedagogical knowledge of teacher on how to teach. In mathematics teaching the harmony of mathematical knowledge and pedagogical knowledge is thought to be important and it is added as a third dimension under beliefs about mathematics teaching.

In brief, the first main category of the construct "students' mathematics related beliefs" is students' beliefs about mathematics education. This main category includes three sub categories in the framework. The three sub categories are beliefs about mathematics as a domain, beliefs about learning and beliefs about teaching. Each subcategory is defined to be composed of dimensions and if needed sub dimensions based on the literature review.

Among the three main categories in the framework, the second main category is students' beliefs about self. The category was divided into four sub categories in the framework: Self-efficacy beliefs, control beliefs, task-value beliefs and goal orientation beliefs. These sub categories were perfectly established on socio-cognitive model of motivation (Pintrich, 1989). So the four sub categories are the dimensions of the framework in the present study. In socio- cognitive model of motivation, there are three components which are expectancy, value and affect. Self-efficacy and control beliefs fall under expectancy component, whereas task-value and goal-orientation beliefs are under value component (Op't Eynde *et al.*, 2002). Students' self-efficacy beliefs are their beliefs about whether they can succeed in or not. Control beliefs refer to students' beliefs about where the locus of control for learning resides. Students' task- value and goal orientation beliefs indicate the beliefs about the value of the task.

In short, the dimensions for the main category; students' beliefs about self for the purposes of this study, were exactly the same with the subcategories that were based on the socio-cognitive model of motivation (Pintrich, 1989). The four dimensions for students' beliefs about self were beliefs about self-efficacy, beliefs about control, beliefs about task value and beliefs about goal orientation.

The last main category is students' beliefs about social context. Several studies emphasized the importance of the social context in the formation of beliefs (Cobb and Yackel, 1998; de Abreu *et al.*, 1997; Schoenfeld, 1992; Yackel and Cobb, 1996). The social context category in the framework includes social norms and socio-mathematical norms (Cobb and Yackel, 1998). Beliefs about social norms refer to beliefs about the role of teacher and role of students in a mathematics classroom. The notion of socio-mathematical norms is explained as the norms that are specific to mathematical activity in a social context (Yackel and Cobb, 1996). For example, the events such as explaining the correctness of a mathematics problem, justifying an answer in mathematics, choosing an efficient method to solve a problem are experienced many times in a mathematics classroom. Thus, students form beliefs about “good solutions”, “justification in mathematics” and “efficient methods” and these beliefs are called beliefs about socio-mathematical norms.

Briefly, the students’ beliefs about the social context were divided into dimensions by detailed reviews of the subcategories. The first sub category social norms included two dimensions role of teacher, role of students. The second sub category consisted of two dimensions as acceptable explanations and solutions in mathematics in line with findings of the studies explained in the previous paragraph.

The framework used in this study covered the three main categories and nine sub categories in its original form. The dimensions for the sub categories of the framework were used to clarify the meaning of each sub category based on the literature findings.

Table 2.1. The framework: students' mathematics related beliefs

Main categories	Sub Categories	Dimensions
Beliefs about mathematics education	Beliefs about mathematics as a domain	Beliefs about mathematical knowledge
		Beliefs about effort in mathematics
		Beliefs about usefulness of mathematics
	Beliefs about mathematics learning and problem solving	Beliefs about learning mathematics
		Beliefs about problem solving
	Beliefs about teaching mathematics in general	Beliefs about mathematical knowledge of the teacher
		Beliefs about pedagogical knowledge of the teacher
		Beliefs about interaction of mathematical knowledge and pedagogical knowledge
Beliefs about self	Self-efficacy beliefs	Self-efficacy beliefs
	Control beliefs	Control beliefs
	Task-value beliefs	Task-value beliefs
	Goal-orientation beliefs	Goal-orientation beliefs
Beliefs about social context	Beliefs about social norms	Beliefs about role of the teacher
		Beliefs about role of the students
	Beliefs about socio-mathematical norms	Beliefs about acceptable explanation in mathematics
		Beliefs about good solutions in mathematics

2.4. Mathematics Related Beliefs and Achievement

There is a widely accepted view shared by the educators about the relationship between students' mathematics related beliefs and their mathematics achievement. There was no consensus about the strength and the direction of the relation (Lester *et al.*, 1989; McLeod, 1992; Papanastasiou, 2000 ; Schoenfeld, 1992; Zimmerman, 1995). Valentine *et al.* (2004) carried out a meta-analysis study, to expose the influence of self beliefs on achievement, the strength of relation was small ($\beta = 0.08$; Cohen's threshold is $r = 0.10$ for small effect size).

The influence of beliefs on problem solving has been highlighted in several studies (Lerch, 2004; Schoenfeld, 1985; 1992; Zeitz, 1999). Problem solving, the life long necessary skill is one of the essential components of mathematics education. Therefore, the relationship between beliefs and problem solving implies the importance of the role of beliefs in mathematics achievement.

2.5. Mathematics Related Beliefs and Gender

As stated by Cai (2003), gender related differences in mathematics have become a popular research area in education. Gender differences in self related beliefs that focused on academic self efficacy have been explained in a study by Schunk and Pajares (2002). As stated in their study, self related beliefs of males have been found to be positive than those of females. They also explained that the possible underlying factors were previous achievement, differing moods in responding such as girls' being modest and boys' being self-congratulatory and the stereotypic beliefs in their cultures.

3. SIGNIFICANCE OF THE STUDY

Students' mathematics related beliefs are crucial in mathematics education for their commonly accepted influence on problem solving and learning of mathematical concepts. Mathematics education as a complex discipline with an abundance of variables interacting with each other is yet far away from well structured theories. Every single variable that is assumed to be related with learning requires extensive research to understand what it is and how it is related to other educational variables. The existing studies in the literature did not only indicate the importance of *beliefs as a hidden variable* in mathematics education, but also emphasized the differences in the conceptualization of beliefs.

Based on the previous studies, it was observed that the significance of beliefs on problem solving is more than a view. It has been empirically justified that beliefs are the key determinants of problem solving. Problem solving that is a life long fundamental skill is assumed to be one of the ultimate goals of mathematics education if not of education as a whole.

The present study seeks for empirical evidence for the theoretical framework of students' mathematics related beliefs based on the previous literature. It is hoped to make a contribution in the clarification of the construct of students' mathematics related beliefs.

4. STATEMENT OF THE PROBLEM

4.1. The Purpose of the Study

The framework presented by Op't Eynde *et al.* (2002) provides a comprehensive synthesis of the previous studies about students' mathematics related beliefs. The framework takes its power from the broad inclusion of existing research studies. The theoretical rather than empirical nature of the framework requires further experimental work to validate its structure. The purpose of this study was to develop and validate a scale to measure students' mathematics related beliefs based on the framework.

4.2. Research Questions

1. Is the scale for measuring students' mathematics-related beliefs valid?
2. Is the scale for measuring students' mathematics-related beliefs internally consistent?
3. Is there empirical evidence for the structural validity of students' mathematics-related beliefs?
4. What are the 8^{th} graders' typical mathematics-related beliefs?
5. Are there gender differences related to students' mathematics-related beliefs?
6. Is there a relation between mathematics achievement and constitutive dimensions of students' mathematics-related beliefs?

5. METHODOLOGY

5.1. Sample / Participants

In this study, 300 8th grade students were chosen on a convenience basis. The subjects were chosen from a private institution “dersane” located in different areas of Istanbul and the sample covered students attending to a number of different schools. 93% of the subjects were from public schools and 7% were from private schools. In general, the ratio of students in private schools is about one percent in Turkey according to the 2005-2006 statistics provided by Ministry of Education (MEB). Only 208 subjects specified their gender. According to this information, the sample was composed of 107 girls and 101 boys from Kartal, Ümraniye, Çekmeköy, Beylikdüzü, Beşiktaş and Kadıköy. The mothers of approximately 56% of subjects who participated in the study were housewives. 16% of the mothers were dentists, pharmacists, teachers, architects, engineers and were university graduates. 28% of subjects’ mothers were either retired or worked as traders or secretaries. Fathers of subjects in the sample were 99% employed or retired. Approximately 30% of the fathers were university graduates. As stated earlier, the subjects’ achievement level on mathematics tests carried out by "dersane" showed that the sample showed a normal distribution in mathematics achievement. These conveniently selected participants, although not the best representative of Turkish 8th graders, showed to some degree a similarity with the distribution of the target population. The elimination of subjects who didn’t respond to all items in the scale left 241 subjects in the sample for some of the analyses.

5.2. Procedure

5.2.1. Development of the Instrument

In this study, initially 222 items were generated to represent the dimensions of the construct "students' mathematics related beliefs" based on a framework proposed by Op't Eynde *et al.* (2002) and used in this study. The framework had its' roots in the literature. The findings of studies in the literature contributed to the formation of dimensions and sub-dimensions of the construct. The framework used in this study focused on three categories: beliefs about mathematics education, beliefs about self and beliefs about social context. The literature review was extended in three categories to form the dimensions and sub-dimensions of the construct.

222 items were then analyzed to find the items pointing to the same construct in each dimension. Of all the items related with the same construct, the most clear and appropriate one of the similar items was chosen. 84 items were eliminated during this phase. Thus, the first form of the instrument was formed which included 138 items.

5.2.2. Item Development

The Items, for the scale, were developed as 5-point Likert type items with choices from "strongly disagree" to "strongly agree". It was thought that the item format assessing the strength of agreement would be better than a format assessing frequency because of the nature of beliefs. Beliefs are assumed to be stable rather than instantaneous characteristics.

The method used during item development was The Domain Referenced Approach (Gable, 1986). Based on the dimensions of the framework, each dimension was associated with sample beliefs cited in the literature. For example "Usefulness" was one of the commonly mentioned domains in beliefs about mathematics as a discipline. The word "useful" was used to generate an item and then the transformations of the item were developed. The representative item was chosen from a set of items about "usefulness". The variations of items about a domain helped to increase the inclusion of many possible representative items in the first version of the instrument.

The items were generated to provide opportunity for the students to express their beliefs through impersonal, personal and social contexts because of the context dependent nature of beliefs (Schoenfeld, 1989; Snow *et al.*, 1996). The items with impersonal contexts were designed to expose students' general beliefs about mathematics, its' learning and teaching. The items developed to measure beliefs about self were written within personal contexts. Items including a mathematics classroom context and peers were to measure beliefs about the social contexts. In other words, through the development of items for measuring students' mathematics related beliefs, the wordings of items and the contexts which they implied were taken into consideration to guide the process.

The framework proposed by Op't Eynde *et al.* (2002) that included three main categories was divided into dimensions and sub-dimensions for item development. The existing studies were used to form the dimensions representing the categories of the framework. The items were developed for nine dimensions.

In the first category: students beliefs about mathematics education, the first sub category was students' beliefs about mathematics as a domain. This sub category included three sub-dimensions, such as, effort, usefulness and the nature of mathematical knowledge. The last dimension was divided into five sub-dimensions by Schommer (1990; 1998). These sub-dimensions were beliefs about certainty of knowledge, source of knowledge, structure of knowledge, control of knowledge acquisition and speed of knowledge acquisition. The dimensions used in the literature guided the development of items for the first category students' beliefs about nature of mathematics. Items for beliefs about mathematics as a domain were written for the dimensions effort in mathematics, usefulness of mathematics and nature of mathematics. A sample item for the usefulness dimension in the last version of the scale was "Mathematics improves thinking skills".

The second sub category of the first category that was students' beliefs about learning and problem solving was divided into two dimensions. These dimensions were learning and problem solving. Learning is a general term. In the present study it was defined as understanding mathematical concepts. A sample item, generated for this dimension was: "Understanding is important in learning mathematics". The problem solving dimension is about the function of problem solving in mathematics. One of the items developed for this dimension was "Problem solving is a significant tool in learning mathematics".

The third subcategory of the beliefs is students' beliefs about mathematics teaching in general. The dimensions of mathematics teaching were teacher's knowledge on subject matter, pedagogy of mathematics and the interaction of the two dimensions. The third dimension was thought to be important for effective teaching. A representative item of the last dimension was "To be able to teach mathematics, one must not only know mathematics, but also know how to teach it".

The second main category of the framework is students' beliefs about self. For this category, the subcategories in the framework were taken as the dimensions according to which the items were generated. The dimensions were rooted in the socio-cognitive model of motivation (Pintrich, 1989). They were self-efficacy beliefs, control beliefs, task-value beliefs and goal-orientation beliefs. A sample item given for the self efficacy dimension would be: "If I try hard, I can solve math problems". Control beliefs indicate the students' beliefs about the focus on learning. One of the items developed for this dimension was "If I miss a class, I can learn the topic from a book or the lesson notes"

The last main category was students' beliefs about the social context. This category included two sub categories. Each one of them was divided into two dimensions. Thus, there were four dimensions for students' beliefs about the social context. The first two dimensions consisted of the role of the teacher and the role of the students. They formed the students' beliefs about the social norms. The dimension, beliefs about the role of the teacher included items like: "While working on math as a class, our teacher is the person who guides us". The remaining two dimensions belonged to students' beliefs about socio-mathematical norms that were the norms specific to mathematics. The dimensions were beliefs about acceptable explanation and beliefs about good solution in mathematics. A sample item in beliefs about acceptable explanation was “In a math lesson for an answer to be satisfactory, the explanation of the answer must be clear to everyone”.

Each item in the scale was generated to fit the meaning of the dimension it belonged to, to lay the stress on either mathematics education, or self or the social context. Variations on each item were created to find the best representative items that the dimension explained. The item generation continued for more than a semester, thus 222 items were developed. After that, there was a saturation point; the new items generated became very similar to the existing ones. Hence, the development of items came to an end. Then, the elimination process began.

The elimination of items went through four phases. These were elimination of very close items by the researcher; elimination of items based on the expert judgment; elimination based on clarity of items; elimination and improvement based on language experts. These four phases are explained in detail in the following paragraphs.

After the items were generated, two university professors, three graduate students whose focus of study were mathematics education and two mathematics teachers were asked to match items with the dimensions of the construct and also to determine the degree of relevance for the item for that dimension by giving a score from 1 to 3. A two-step analysis was carried out on the reviews of the experts. First, the items were eliminated on the basis of disagreement about the dimension of the item. Second, the remaining items which were agreed upon were chosen according to the indicated degree of relevance. Data from 7 experts were used; the items with an average score of less than 2 were eliminated. Experts were interviewed after the completion of the reviews for further suggestions about the instrument. As a result, the items of the scale which were to be used for the pilot implementation were selected. There remained 65 items in this form of the scale.

In order to improve the clarity of the items, 10 mathematics teachers examined the scale for the relevance to the age level: They made suggestions on the wording of the items. Based on their answers and suggestions, several items were improved to ensure the clarity of the items. Thus, the instrument was developed.

5.2.3. Pilot Implementation for the Improvement of Items

The first implementation of the scale was carried out on a pilot sample which included three 7th grade classes. Two classes were from a public school and one class was from a private school. 16 students came from the private school and 50 students from a public school. The ratio of boys to girls was 28:38. The scale was administered during a mathematics lesson by the researcher. Subjects were asked first to read the instructions on the cover page, then to respond to the items. The reliability analysis of the data from the pilot study was observed to have a tentative understanding of the working items. From the pilot implementation data Cronbach's alpha coefficient for the scale was calculated as 0.77. By using the analysis derived from the pilot implementation, the items having item total correlations less than 0.15 were eliminated. The 65 item form of the instrument was thus reduced to 40 items.

The wording of the items were evaluated and corrected by two language experts to ensure that the items were clear and also that language would not form a barrier for the subjects to express their real beliefs. The experts were explained the meaning and the emphasis of each item. They made suggestions in the order of words to convey the exact meaning of each item.

5.2.4. Implementation of the Instrument

The instrument, that was developed to be in line with the framework proposed by Op't Eynde *et al.* (2002) and improved based on the pilot study, was administered to 300 8th graders from a private institution during Fall 2007. The teachers administered the instrument. In order to provide standard conditions for the implementations, teachers were given written instructions about the process prior to the implementation. They were requested to record the questions asked by the students and filled in a form for the evaluation of the implementation.

The forms of scales from classes were matched with these evaluation forms for control purposes.

5.3. Statistical Analysis

The data was analyzed to evaluate the validity and reliability of the scores and to investigate the research questions. Statistical evidences were tested for construct-related validity and for reliability in terms of internal consistency and consistency over time of the scores. The secondary goals of the present study were to analyze the gender related differences in students' mathematics related beliefs and to expose the relationship between beliefs and mathematics achievement.

5.3.1. Reliability Analysis of the Scale

5.3.1.1. Internal Consistency. Internal consistency of the instrument was evaluated by using Cronbach's alpha. Internal consistency coefficient of the scale was calculated as 0.77 in terms of Cronbach's alpha for the pilot implementation. George and Mallery (2003) indicated that coefficients greater than 0.60 are acceptable.

5.3.1.2. Test-retest Reliability. Reliability of scores over time is very critical in measuring beliefs which are assumed to be more stable. Test- retest reliability was calculated on data from a pilot group.

5.3.2. Validity Analysis of the Scale

5.3.2.1. Construct-Related Validity. Data were analyzed to validate the proposed structural model of students mathematics related beliefs. Structural equation modeling that is usually referred as SEM was used in data analysis as a confirmatory factor analysis.

5.3.2.2 Structural Equation Modeling. Structural equation modeling (SEM) is a statistical technique that uses multivariate regression and forms conceptual models based on the regression equations. In Structural Equation Modeling, there are mainly two types of variables; observed variables and latent variables. Observed variables are the variables that are measured, directly, whereas latent variables are hypothesized constructs/dimensions assumed to be measured, indirectly. Each observed variable is associated with an unobserved variable that is a latent variable and an error. For example, four questions in a mathematics achievement test are observed variables and mathematics achievement is the latent variable. As seen in Figure 5.1, observed or directly measured variables are modeled by using rectangles and latent variable is modeled by an ellipse or circle. The arrows are from the latent variable to the observed variables because the latent variable causes observed variables. In other words, latent variable is the underlying factor to have such observations. Single-headed arrows indicate impact of one variable on another, whereas double-headed arrows indicate covariance or correlation between variables. An estimated measurement error as represented by e's is linked to every observed variable. Residual is linked to the latent variable.

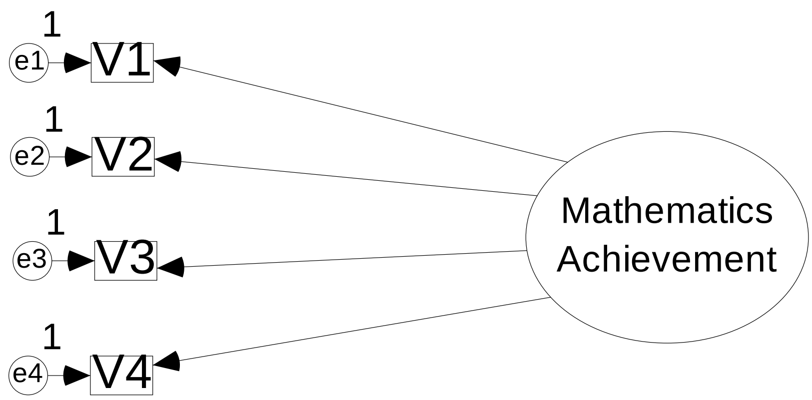

Figure 5.1. Sample of a Structural Equation Model

Confirmatory factor analysis (CFA) within the framework of SEM focuses on the factor loadings of observed variables. Statistically, CFA analyzes the covariance structures to calculate the regression coefficients (Byrne, 2001). In CFA, the researcher shapes a hypothetical model based on the findings in the literature or observations, and then tests for the goodness of fit between the hypothesized model and the data collected. In CFA, the null hypothesis states that the data fits the model, and when the researcher fails to reject the null hypothesis, it indicates that the model is not disconfirmed. It is important to understand there can be other models that the data fits. Based on calculating the chi-square, the result in CFA shows that either the data fits the model or not, but it does not provide a level of significance.

5.3.2.3. Advantages of Using CFA and Disadvantages of Using CFA. There are advantages and disadvantages of using Confirmatory Factor Analysis in model testing and scale validation. First of all, the separation of variables measured directly and indirectly is an advantage in social sciences. The existence of this separation warns researchers not to overestimate the relations among the variables or not to hold a misconception about the equivalence of the observed and latent variables.

Technical advantages of CFA over other multivariate statistical techniques are providing estimates of error and residuals. The goodness of fit between the collected data and the hypothetical model is calculated in CFA. Since perfect fit is almost impossible, there is a difference between the model and data and it is called *residual*. In CFA, calculation of the residuals may provide clues about the inconsistency between the data and model. *Residual* should not to be confused with measurement error that is the difference between reality and data. Measurement error is also estimated for every observed variable in CFA.

The important assumption about the data that can be analyzed through SEM CFA is that data distribution should possess normality (multivariate normality). Although there are studies using CFA and not reporting about the deviations from normality, the non-normal data distribution may inflate the chi-square value and this leads to rejecting the null hypothesis. In order to overcome this problem, there are two methods developed by statisticians to handle non-normal data distribution. Two methods are elimination of outliner cases and bootstrapping. In the elimination of outliner cases, the outliner scores which are statistically responsible for the non-normality are removed from the sample. The application of bootstrap technique is possible when the sample size large. In bootstrapping, the original sample data are used to derive multiple sub-sample data. Before applying one of these methods, data distribution should be tested to see whether it is normal or non-normal. There are statistical techniques to check the normality assumption for certain data. Kolmogorov-Simirnov/Lilliefor Test; Shapiro Wilk W test; D'Agostino kurtosis and skewness tests are commonly used tests for univariate normality. AMOS program assesses multivariate normality by calculating kurtosis value as *Mardia's coefficient*. Values greater than 1.96 indicate significant kurtosis leading to significant non-normality (Bryne, 2001). Mahalanobis distance from the centroid is used to find the outliner cases that interfere with normality. Calculated chi-square value is large in case of non-normality, resulting in rejection of fitting models (Hu *et al.*, 1992).

The disadvantage of using chi-square is partially solved. Researchers and statisticians using SEM developed several indices because chi-square value is very sensitive to sample size, number of variables in the study, non-normal data distribution. Each index is developed to control one or more of these factors.

The effect of sample size is a source of bias in rejecting models through CFA. The larger the sample size, the higher is the probability of rejecting a model. TLI and CFI were developed to solve the effect of large samples and solved this problem partially (Bentler, 1990; Marsh *et al.*, 1988).

The effect of the number of measured variables on the indices such as CFI, GFI, AGFI, TLI, NFI, Satorra-Bentley chi-square was studied by Chau and Hocevar (1995). Their study suggested that CFI, NFI and TLI were highly stable against item number.

Root mean square of error of approximation (RMSEA) is found to be relatively stable against sample size (Marsh and Balla; 1994). Actually, it is usually called a "badness of fit index". RMSEA values between 0.05-0.08 indicate reasonable fit and values less than 0.05 indicate good fit (Holmes-Smith *et al.*, 2004).

CMIN/DF is the ratio of chi-square to degrees of freedom. Byrne (1989) suggested that CMIN/DF greater than two is an indicator of inadequate fit whereas the ratios from two to five indicate reasonable fit (Marsh and Hocevar; 1985).

TLI and CFI control both for sample size and item number. It is thought that these indices with RMSEA and CMIN/DF were appropriate to check whether the data fit the model or not, where the number of measured variables in Mathematics-Related Beliefs Model was 40, and the sample size was 241.

5.3.3. Descriptive Statistics

The means and standard deviations for students' mathematics related beliefs were calculated. Furthermore, the scale was divided into subscale during statistical analysis. The subscales were the main categories of the framework: Beliefs about mathematics education, beliefs about self and beliefs about social context. Each subscale included only the items for each category. In other words, a subject's score on the scale was the sum of all scores obtained from the items in the scale; subject's score on a subscale was the sum of scores for the items in that subscale. The means and standard deviations were reported for sub scales that were beliefs about mathematics education, beliefs about self and beliefs about social context.

5.3.4. Further Analysis

Depending on the results of validity and reliability studies, further analysis was planned regarding the relationship between the dimensions of beliefs and math achievement. The correlation coefficients between the students' total scores on each subscale and the mathematics achievement index that was the net score in mathematics practice exam administered by the private institution were calculated. The significance of the correlation coefficients were tested with $\alpha = 0.05$.

Mean scores of girls and boys in the sample were compared by using t-test, in order to expose the gender related differences in beliefs about nature of mathematics, belief about self, and beliefs about social context. The mean scores of gender groups were calculated to compare the means for each subscale.

6. RESULTS

6.1. Reliability Analysis of the Scale

6.1.1. Internal Consistency

The scale including 40 items were implemented. The data was analyzed to have an understanding of the internal consistency of the scale. 6 items were eliminated because they had item-total correlations less than 0.15.

The internal consistency of the scale was calculated twice for 34 items after the elimination of items with item-total correlations less than 0.15. The first calculation was done for 241 cases and the Cronbach's alpha coefficient was calculated to be 0.88. The second calculation of Cronbach's alpha coefficient was carried out for 200 cases and it was 0.80.

After the elimination of items based on the confirmatory factor analysis, 21 items were left. Internal consistency (Cronbach's alpha) coefficient of the instrument was calculated as 0.75 for 200 subjects and 0.83 for 241 subjects for the implementation. Thus, the scale's internal consistency was at an acceptable level before and after the elimination subjects based on the normality tests of the data set. The scale had an acceptable internal consistency since it was above 0.60 (George and Mallery, 2003). Although the number of items has a great impact on alpha value, the first implementation of the scale with 65 items and last implementation of the scale with 21 items had very close values. The Cronbach's alpha value on the pilot sample implementation with 65 items was calculated as 0.77 and the ultimate value of Cronbach's alpha coefficient was 0.75.

The internal consistency coefficients of sub-scales were also calculated for 200 cases and 21 items. Cronbach's alpha coefficient for the subscale 'beliefs about mathematics education in general' that was the nature of mathematics with 7 items was 0.67. The alpha coefficient for the sub-scale 'beliefs about self' with 9 items was 0.68. The remaining 5 items formed the last sub-scale ‘beliefs about social context’ and Cronbach’s alpha coefficient for the last sub-scale was 0.39.

6.1.2. Test-Retest Reliability

Test-retest reliability was used in this study because of the nature of beliefs. As explained by Goldin (2002) one of the important characteristic of beliefs is to be highly stable Test- retest reliability was assessed on a pilot group of 68 subjects through two methods: Correlation coefficient and absolute agreement percentage. Subjects responded to the items in the scale twice within a time interval of 15 days. The correlation coefficient between the two implementations was 0.99.

Correlation was thought to be a sensitive measure that might amplify the degree of consistency between two tests. For example, if each subject increased their responses by 1 point then the correlation would be perfect. To avoid this, absolute agreement percentage which was generally used as a method of inter-rater reliability was thought to provide a better understanding of consistency over time for this scale. Each subject’s initial and final responds were analyzed. The same responses were counted. The percentage of absolute agreement was calculated by dividing the number of same responses to the number of responses and then the dividend was multiplied by hundred to obtain a percentage. The absolute agreement for the whole group was found by calculating the average of absolute agreement percentages for all subjects. The average of percentage scores of subjects in this group was found to be 92.6% where the range of percentages was between 71% and 100%. The subjects with 71% agreement responded to 15 items out of 21 items in exactly the same way in the test and retest situations. The maximum agreement of 100 % would indicate absolute intra-subject consistency over time. The range of agreement in terms of item numbers was between 15 and 21 out of 21 items.

6.2. Validity Analysis of the Scale

6.2.1. Confirmatory Factor Analysis and Model Test

Normal distribution of data is one of the important assumptions of using Confirmatory Factor Analysis. Non-normal distributions of data are responsible for inflated chi-square values. There are two techniques to avoid this problem. These are elimination of outliner cases and bootstrapping (Byrne, 2001). For this study, bootstrapping technique was not an appropriate one because it required a large sample to obtain derivatives of data. The method of elimination of the outliner cases was used to eliminate or reduce the non-normality condition.

At first, data was analyzed to assess multivariate normality of data distribution. Using the AMOS program data from the whole sample was tested and *Mardia's Coefficient* was found as 289.93. This indicated a high degree of non-normality. Mardia's coefficient values less than 1.96 indicate multivariate normality whereas greater values point out non-normality (Bryne, 2001). The collected data showed a non-normal distribution. The outliner cases were found by using the *Mahalanobis distance-squared* value indicating the distance from the centroid. 41 of the outliner subjects were eliminated to adjust for the normality assumption based on the *Mahalanobis distance-squared* values. Mahalanobis distances of the eliminated cases ranged from 108.57 to 55.74. Although the data with 200 cases did not meet the desirable normality conditions, the deviation from normality was reduced to an acceptable level.

6.2.2. Model I

The hypothetical model for students' mathematics related beliefs with the dimensions of mathematics education, self and social was constructed with 34 items as seen in Figure 6.1. The model was analyzed by using confirmatory factor analysis and checked for 200 cases. The data did not confirm the model. As seen in Table 6.1 departure of the data from the model was significant with a probability level 0.001. If the probability level is 0.05 or less, then data fails to confirm the model.

Table 6.1. Model I with 34 items

	Chi Square Probability of Significance	**CMIN/DF**	**TLI**	**CFI**	**RMSEA**
Sample size N=200	Chi-square = 752.817 Prob.level=0.001	1.439	0.719	0.738	0.047

Even though data failed to confirm the first hypothetical model, the analysis by using SEM provided clues about the unsuitable items in the scale. Confirmatory factor analysis has an advantage of checking the appropriateness of items by examining their Modification Indices (abbreviated as MI). The analysis of items with high Modification Indices is used to figure out the deficiencies in the model. The conventional threshold value for MI is 10.00 to find out the inappropriate items (Byrne, 2001).

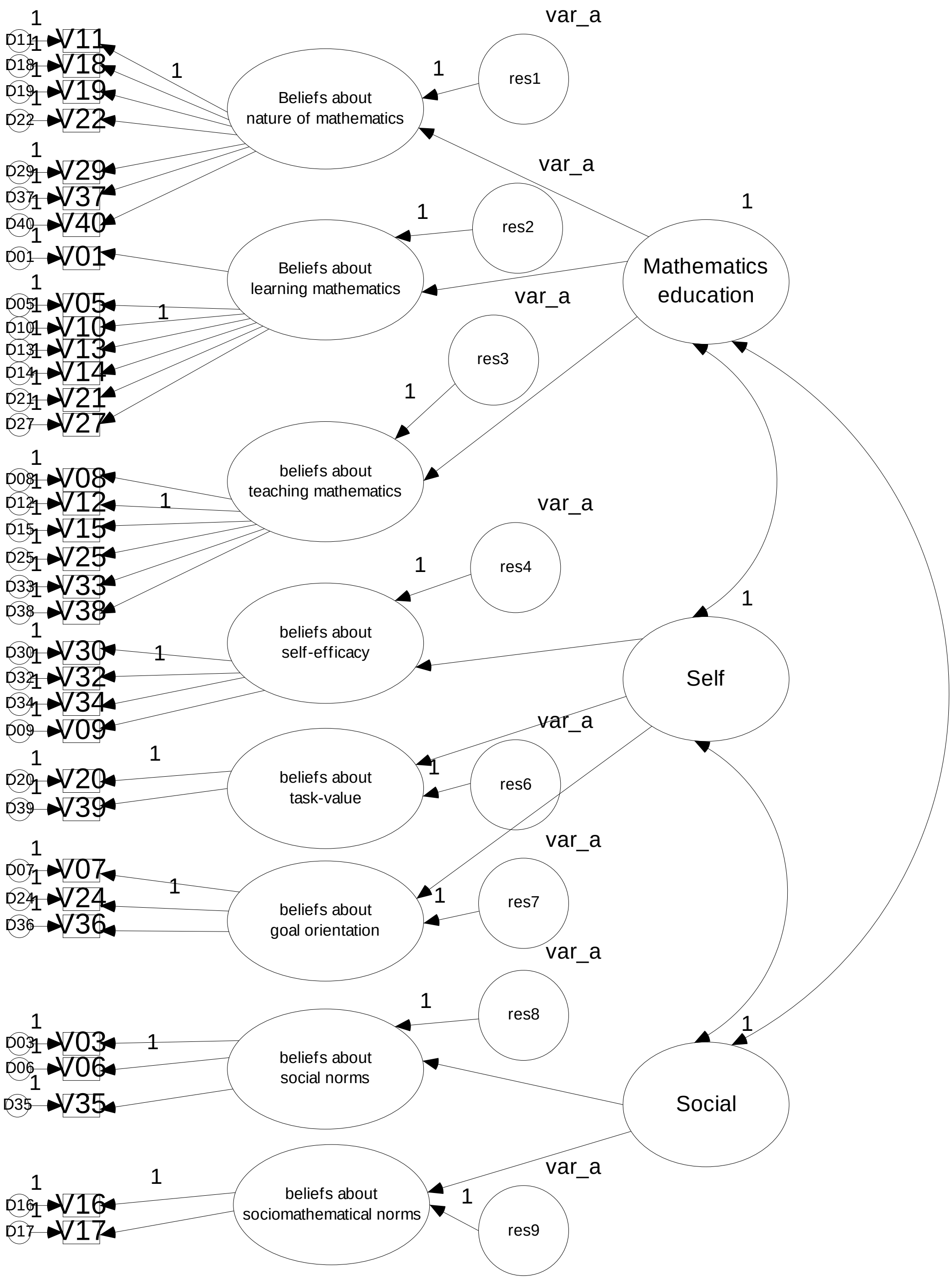

Figure 6.1. Model I

6.2.3. Model II

Based on the first hypothetical model, items of the scale those with high MI values (greater than 10) were eliminated. The eliminated items were #1, #5, #8, #10, #12, #13, #14, #15, #21, #25, #27, #33 and #38. These items belonged to the dimensions of beliefs about learning mathematics and beliefs about teaching mathematics.

The difference of Model II from Model I was essentially related with the dimensions under mathematics education. In the first hypothetical model, the dimension of mathematics education included beliefs about the nature of mathematics, beliefs about learning mathematics and beliefs about teaching mathematics. In the second model, the elimination of items with high MI values changed the constitutive dimensions of mathematics education. Mathematics education included students' beliefs about the nature of mathematics as a discipline and what it meant to make sense in mathematics.

Although the first hypothetical model included learning and teaching as two additional dimensions combined with nature of mathematics, the collected data failed to confirm these two dimensions. This also came up as the experts classified the items for the dimensions. Most of the disagreements during the collection of expert judgments took place in teaching and learning dimensions, and these items were classified under social dimension by several experts. Even though the items in these dimensions were carefully eliminated according to the disagreements, the collected data confirmed the confusion in these dimensions.

The items under learning and teaching dimension were eliminated because the data did not confirm these dimensions. In line with the expert judgments, it is reasonable to think that students' beliefs about teaching and learning belong to the social dimension instead of beliefs about mathematics since all experiences about teaching and learning are in a social setting. Learning and teaching mathematics always take place in social context so these items did not form a dimension with the nature of mathematics.

The items under learning and teaching mathematics were eliminated rather than placing them under self or social dimensions. Because these items were formulated to be context independent by the use of an impersonal style in wording but the items under self or social dimension were not. In other words, these items were not placed under self or social dimensions because of conceptual reasoning. The items did not have an emphasis on being personal or contextual as the items in self and social dimensions had. Contrary to the meaning conveyed by the use of different styles such as context dependent versus context independent, the data showed that the subjects' responses to the items of beliefs about learning and teaching mathematics were usually personal and contextual.

Model II was formed by the elimination of items with high MI values as seen in Figure 6.2. Model II included 21 items from the scale developed. As the Model II was analyzed, the chi-square value had a probability level of 0.113. This probability value indicated that the departure of data from the model was not significant.

The analysis was extended by using several indices of model fit as seen in Table 6.2. Since the chi-square value is sensitive to sample size, number of variables in the study, and non-normal data distribution, a number of indices were developed by statisticians. For this study, the indices CMIN/DF, TLI, CFI and RMSEA were thought to be appropriate for the analysis because CMIN/DF controls for degrees of freedom; TLI and CFI are quite stable against sample size and item number; RMSEA is stable against sample size (Chau and Hocevar, 1995; Marsh and Hocevar, 1985). Shortly, all indices CMIN/DF, TLI, CFI and RMSEA satisfied the lower and upper limits for model acceptance.

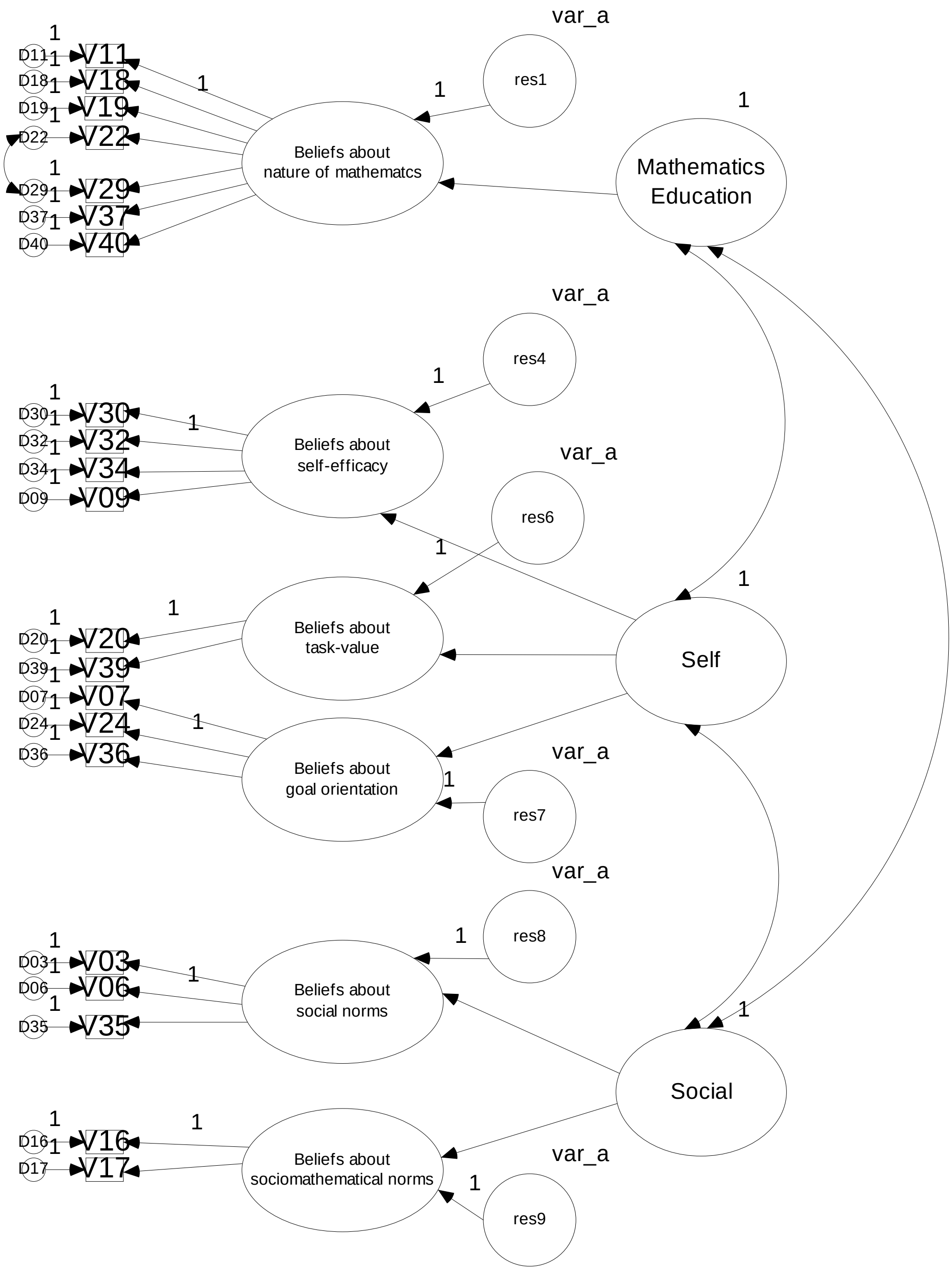

Figure 6.2. Model II

“Students’ mathematics related beliefs” construct had three dimensions as beliefs about mathematics education, beliefs about self and beliefs about social context according to the collected data. This is in general consistent with the literature findings and the model proposed by Op't Eynde *et al.* (2002).

Table 6.2. Model II with 21 items

	Chi Square Probability of Significance	**CMIN/DF**	**TLI**	**CFI**	**RMSEA**
Sample size N=200	Chi-square = 207.474 Prob.level=0.113	1.128	0.945	0.952	0.025

The final scale developed in this study consisted of 21 items; 7 items for the dimension nature of mathematics, 9 items for the dimension self and 5 items for the dimension social.

6.3. Descriptive Statistics

The means for subscales; beliefs about nature of mathematics, self and social were calculated for all 200 cases in the sample. The mean and the standard deviation for "the nature of mathematics" subscale that was composed of 7 items were found as 28.30 and 3.98. For the subscale "self" with 9 items, the mean and standard deviation were 36.10 and 4.99. The mean and the standard deviation for the last subscale that was "social" with 5 items were calculated as 20.61 and 2.74.

Table 6.3. Descriptive statistics

	N	Min.	Max.	Mean	Std. Dev.	Skewness		Kurtosis	
						value	Std. Err.	value	Std. Err.
Nature of math	200	16	35	28.30	3.976	-0.590	0.172	0.236	0.342
Self	200	23	45	36.10	4.998	-0.473	0.172	-0.229	0.342
Social	200	12	25	20.61	2.736	-0.494	0.172	0.021	0.342
Valid N (list wise)	200								

The distributions of the scores from each subscale were examined for 200 cases. Skewness is the degree of asymmetry of the distribution according to its' mean. If the deviation of a distribution is to the right, it is called negatively skewed and the skewness value is negative. For all subscales; beliefs about nature of mathematics, beliefs about self and beliefs about social context, the distributions were negatively skewed (see Table 6.3). In other words, students mostly held positive beliefs about all three dimensions.

Kurtosis value is the peaked-ness of the distribution according to normal distribution. If the value is positive, then the distribution is relatively peaked. The negative value indicates the flatness of the distribution with respect to normal distribution. The kurtosis values of the data distributions for the subscales are given in Table 6.3. The positive value of kurtosis for the subscale of nature of mathematics means the distribution is peaked, whereas its' negative value for the subscale of self shows a rather flat distribution. The kurtosis value very close to 0 for the subscale of social context is an indicator of a distribution that has a similar steepness to normal distribution.

The means for the three subscales were calculated separately for gender. The means for each sub-scale across groups were very close to each other. Several cases were excluded from the analysis because of these subjects' not providing information about their genders. Descriptive statistics according to gender was calculated for a total of 180 cases including 92 girls and 88 boys.

The means and standard deviations for the nature of mathematics sub-scale were found as 28.98 and 3.65 for 92 girls; 27.62 and 4.27 for 88 boys, respectively. Although the means were very close to each other, the t-test for independent samples was used to check the statistical significance of the mean differences. The value of t was found to be 2.28. The independent samples t- test indicated that there was a statistical difference between girls and boys in mean scores of beliefs about nature of mathematics with $p=0.024$ where was $\alpha=0.05$.

The means and standard deviations for the sub-scale self were 36.68 and 4.80 for 92 girls; 35.39 and 5.31 for 88 boys, respectively. Independent samples t-test was used to calculate the t value and it was found as 1.72 with $p=0.087$. The gender difference between the means was not statistically significant at $\alpha=0.05$ for the subscale self.

The means and standard deviations for the sub-scale social context were 20.60 and 2.93 for 92 girls and 20.70 and 2.68 for 88 boys, respectively. The calculation of the t value as 0.26 with p=0.799 showed that the difference between girls and boys was not statistically significant at α=0.05.

In spite of very close values of means and standard deviations in some of the subscales for males and females, the research question: "Are there gender differences related to students' mathematics-related beliefs?" was elaborated by using t-test. The t values for all the subscales were calculated to seek the statistical significance of the difference. None of the t values indicated any significant difference between males and females except the subscale beliefs about nature of mathematics. There was a statistical difference between the gender groups in terms of their beliefs about the nature of mathematics.

Table 6.4. Gender analysis

Subscales	**Gender**	**Mean**	**Standard Deviation**	**t**	**Significance α-level**
Nature of mathematics (7 items) Possible range 7-35	Girls (n= 92)	28.98	3.65	2.28	p=0.024* α=0.05
	Boys (n=88)	27.62	4.27		
Self (9 items) Possible range 9-45	Girls (n= 92)	36.68	4.80	1.72	p=0.087 α=0.05
	Boys (n=88)	35.39	5.31		
Social (5 items) Possible range 5-25	Girls (n= 92)	20.60	2.93	0.26	p=0.799 α=0.05
	Boys (n=88)	20.70	2.68		

The mean for each item in the scale was calculated. As seen in Table 6.4, responses for each item ranged from 1 to 5 except the items V11 and V36. The values of mean for items of the scale were mostly around or above 4.00. In other words, the students' mathematics related beliefs measured by the scale were mostly in the desired direction.

Table 6.5. Item based descriptive statistics

Item number	N	Minimum	Maximum	Mean	Std. Deviation
V3	200	1	5	3.51	1.22
V6	200	1	5	4.08	1.15
V7	200	1	5	4.41	0.82
V9	200	1	5	3.84	1.20
V11	200	2	5	4.47	0.74
V16	200	1	5	4.49	0.81
V17	200	1	5	4.35	0.82
V18	200	1	5	3.56	1.07
V19	200	1	5	3.81	1.11
V20	200	1	5	4.14	1.04
V22	200	1	5	4.34	0.89
V24	200	1	5	3.45	1.21
V29	200	1	5	4.34	0.99
V30	200	1	5	3.09	1.37
V32	200	1	5	4.27	0.88
V34	200	1	5	4.14	1.05
V35	200	1	5	4.19	1.02
V36	200	2	5	4.53	0.74
V37	200	1	5	3.56	1.05
V39	200	1	5	4.24	1.00
V40	200	1	5	4.21	0.99
Valid N (listwise)	200				

6.4. Further Analysis

Students' beliefs have been assumed to be one of the underlying factors in explaining their mathematics achievement (Papanastasiou, 2000). The correlation coefficients were calculated between total scores of students for the scale, each sub-scale and their mathematics achievement and general achievement. A total score for the scale was calculated adding the scores obtained from each sub-scale. The scores for subscales were the sum of item scores in the subscale. The total scores for the scale conceptually represented the students mathematics related beliefs. The total scores for each subscale corresponded to students' beliefs about the nature of mathematics, students' beliefs about self, and students' beliefs about social context, respectively.

Mathematics achievement was defined as the net score on mathematics section of practice exams "deneme sınavı" administered by the private institutions called "dersane". The general achievement was defined as the total net score in all sections of the practice exams that included mathematics, science, social sciences and Turkish literature.

There is a consensus on the influence of students' mathematics related beliefs to their mathematics achievement but the strength and the direction of the influence or relation are controversial (Lester *et al.*, 1989; McLeod, 1992; Papanastasiou, 2000; Schoenfeld, 1992; Zimmerman, 1995). In the present study, students' mathematics related beliefs were not significantly correlated with mathematics achievement or general achievement level. The correlation coefficient between students mathematics related beliefs and mathematics achievement was 0.13.

Students' beliefs about the nature of mathematics were thought to be related with their mathematics achievement but the statistical analysis did not verify this foresight. The non-significant correlation coefficient between students' beliefs about nature of mathematics and mathematics achievement was 0.10.

In the study at hand, the correlation coefficient between beliefs about self and mathematics achievement was found as 0.06 and it was not significant. In a meta-analysis study, Valentine *et al.* (2004) found the influence of self beliefs on achievement although the magnitude of relation was small.

As seen in Table 6.6, the highest correlation coefficient was found between students' beliefs about social context and mathematics achievement. The correlation coefficient between students' beliefs about social context and mathematics achievement was 0.17 and it is significant at the 0.05 level. The beliefs about the social context were composed of beliefs about the role of teacher, beliefs about the role of students, beliefs about socio-mathematical norms.

Table 6.6. Correlation coefficients between sub scales and mathematics achievement

		Math Achievement	**General Achievement**
Beliefs Scale Total	Pearson Correlation	0.125	0.079
	Sig. (2-tailed)	0.094	0.292
	N	180	180
Nature of Math Sub Scale	Pearson Correlation	0.095	0.075
	Sig. (2-tailed)	0.204	0.316
	N	180	180
Self Sub Scale	Pearson Correlation	0.055	0.053
	Sig. (2-tailed)	0.460	0.477
	N	180	180
Social Sub Scale	Pearson Correlation	0.166	0.049
	Sig. (2-tailed)	0.026	0.510
	N	180	180

In the analysis of students mathematics related beliefs and subscales in relation to mathematics achievement the correlation coefficients were non-significant except the coefficient between beliefs about social context and mathematics achievement. Since the correlation coefficients between mathematics achievement and other dimensions were not significant, no further analysis was carried out to expose the partial effects of constitutive dimensions of the model for students' mathematics related beliefs on mathematics achievement.

7. DISCUSSION & SUGGESTIONS

7.1. Discussion

The need for the conceptualization of the construct "students' mathematics related beliefs" was raised in several studies (Eisenhart *et al.*, 1988; Furinghetti and Pehkonen, 1999; Op't Eynde *et al.*, 2002). Beliefs became an essential variable in mathematics education, so the number of studies and models proposed to explain students' mathematics related beliefs increased (Furinghetti and Pehkonen, 1999; Schoenfeld, 1992). The direct or indirect relation between beliefs and learning mathematics or mathematics achievement was pointed at in the previous studies (Lerch, 2004; Schoenfeld, 1985; 1992; Zeitz, 1999).

The psychological and social research approaches studied the construct from different perspectives. *Emergent perspective* that was proposed by Cobb (1996) tried to combine these perspectives while studying learning in the classroom context. The findings about the formation of beliefs in the literature guided this study to use emergent perspective as a frame for this study. The studies indicated the importance of both the individual and the social context in belief formation (Pehkonen and Torner, 1996).

In this study, an attempt was made to develop a scale for measuring students' mathematics-related beliefs according to the framework proposed by Op't Eynde *et al.* (2002). The framework was based on the literature findings and the models proposed by Kloosterman (1996); McLeod (1992); Pehkonen (1995) and Underhill (1988). This study aimed to make a contribution to the clarification of the construct of students' mathematics related beliefs. It was thought to be necessary because there was no single scale to measure the range of dimensions in the framework.

The theoretical construct of mathematics related beliefs was operationally defined by developing a scale in this study. The structural validity of the students' mathematics related beliefs scale was tested empirically. Data supported the framework proposed by Op't Eynde *et al.* (2002) for the dimensions; beliefs about self and beliefs about social context. But, the data failed to confirm the sub dimensions of the first dimension that was beliefs about mathematics education, completely. Data confirmed the sub dimension; beliefs about nature of mathematics, but failed to confirm the sub-dimensions; beliefs about learning mathematics and beliefs about teaching mathematics.

Although the findings of the study did not confirm the framework altogether as a whole, they were nevertheless, consistent with the framework. The learning and teaching sub dimensions were found to be immersed in other sub dimensions. In other words, the beliefs about learning and beliefs about teaching mathematics were not two disjoint sub dimensions among the other dimensions but were intertwined in sub dimensions, “beliefs about self” and “beliefs about social context”.

A similar study was carried out by Op’t Eynde and De Corte (2003) to test the structural validity of students’ mathematics related beliefs framework that was the same as the one used in this study. Their study exposed a four-factor structure of students’ mathematics related beliefs explaining 38.3% of the variance. The four factors were “beliefs about social context”; “certain beliefs about self” and two factors for beliefs about mathematics. There were similarities and differences between their study and this study.

The study of Op’t Eynde and De Corte (2003) and this study included several similarities. The dimensions that were validated in both studies were “students’ beliefs about social context” and “beliefs about self”. These factors were validated through factor analysis in their study. The “beliefs about self” and “beliefs about social context” were also identified as two separate factors in this study by using confirmatory factor analysis. Fortunately, two studies used different statistical methods but the factors that were validated were mainly similar.

The remaining two factors in the study of Op’t Eynde and De Corte (2003) actually fit the nature of mathematics dimension in this study. Although the names of the remaining two factors in their study were different from the name that was used in the present study, the elaboration of their items indicated that factors were similar. The last two factors relating to beliefs about mathematics were named as "mathematics as a social activity" and "mathematics as a domain of excellence". According to Op't Eynde and De Corte (2003) as explained in their article, former refers to usefulness of mathematics; latter refers to the nature of the discipline. The sample items of the scale for the last two factors were given in their article are "anyone can learn mathematics" and "there is only one way to find the correct answer on a mathematics problem". The examples helped to grasp the meaning of these factors. These factors are very similar to what is meant by the "beliefs about nature of mathematics" in this study. In this study, the dimension "beliefs about mathematics" that included "beliefs about the nature of mathematics", "beliefs about learning mathematics" and "beliefs about teaching mathematics" was not validated. Thus, the model was modified to have only beliefs about nature of mathematics under the first dimension beliefs about mathematics education.

There were also differences between two studies. Definition of belief and the literature that two studies rise on are not much different, but in practice, the several items developed to measure "belief" in their study was very difficult to think as a belief from the perspective of the present study. There were items generated for measuring students beliefs about self efficacy and task value such as "I like mathematics" and "I'm interested in mathematics". These items do not fit into the definition of belief when defined as subjective knowledge, a person's understandings of self and the world around him/her (Bem, 1970). In the present study the attitudes were defined to be likes or dislikes or preferences, these items rather fit into the attitude category. The incompatible items can be thought as empirical evidences to understand the disagreement on the definition and conceptualization of "students' mathematics related belief".

Besides the consistency and practical differences between the two studies, it is noteworthy to keep in mind that the statistical technique used in this study; confirmatory factor analysis does not reject the existence of other better fitting models. It only tests the fit between data and the hypothesized model, so there may be better explaining models for the students' mathematics related beliefs.

Gender is an important variable in education and especially in mathematics education. The relation of students' beliefs about mathematics and gender differences was analyzed for the scale and for all three subscales. Students' mathematics related beliefs were examined according to gender and the analysis exposed a statistically significant mean difference between the scores of girls and boys on nature of mathematics sub-scale (α=0.05). Girls' mean score was higher than boys' mean score on nature of mathematics. The difference was in favor of girls and this is not consistent with literature findings (Ernest, 1995). The age level might be a factor underlying this difference because the differences between boys and girls in their beliefs are developmental (Schunk and Pajares, 2002).

It is noteworthy that the findings of this study indicated the beliefs of boys and girls are not statistically different in general. This result should be cautiously used because the sample was chosen from a private institution which prepares students for nationwide high school entrance exams. One might conclude that the education of the students in the sample is a priority for their parents because of their decision in registering their children to a private institution. Girls' scores on their beliefs about nature of mathematics was higher that boys' scores on the same subscale. In other words girls tend to hold more positive beliefs about the nature of mathematics which is usually labeled as a male domain.

From a macro perspective, it is a fact that the gender differences in education are high in Turkey. There are basic gender differences in the access to education. This information was based on 2005-2006 statistics provided by Ministry of Education (MEB). The findings of this study indicated that when the conditions were somewhat similar for boys and girls, there were not any gender differences in favor of boys in math related beliefs. Furthermore, girls held more affirmative beliefs about the nature of mathematics. The gender differences in students mathematics related beliefs was not the primary goal of this study, further studies are needed to understand the gender differences in beliefs.

The relation of students' beliefs and mathematics achievement was also tapped. The widespread agreement on the influence of beliefs on mathematics achievement was not confirmed in this study. The correlations between the scale, sub scales and mathematics achievement were evaluated. One of the findings of this study was a significant relation between beliefs about social context and mathematics achievement. Social context referring to the role of teacher, the role of students and beliefs about socio-mathematical norms meaning the norms related with mathematics such as acceptable explanation and justification in mathematics. Students' beliefs about social context and their mathematics achievement had a significant correlation. The importance of hidden rules, norms of mathematics classroom was highlighted in NCTM (1989). The classroom practices forming the social context of mathematics education have vital importance in students' understanding of mathematics. Several researchers used the term "classroom culture"; to explain norms that attracted frequent attention in mathematics education (Cobb and Yackel, 1998). The significant correlation between beliefs about social context and mathematics achievement once again indicated the importance of classroom culture in mathematics lessons.

There were several limitations of the present study. At times, preference on practicality caused some of the limitations, at other times nature of the variables were responsible. The use of a self report instrument, the operational definition of mathematics achievement and not controlling for the variables that might influence beliefs or achievement were among the factors which might have created some limitations.

Self report instruments although widely used may create limitations for the studies. Using self–report instruments is usually criticized because of the social-desirability factor that interferes with the responses of subjects. Individuals hold beliefs either consciously or unconsciously. Self report instruments form a barrier for individuals to express their unconscious beliefs. These instruments are often used because of the practicality in their implementation. Self-report instruments provide advantages when hundreds of subjects are involved in the study. Therefore, the use of Likert type items in the scale is one of the limitations of this study.

The way chosen to define mathematics achievement in the present study was solely for practicality purposes. Actually, exposing the relation between achievement and beliefs was not the main goal of the study. Mathematics achievement was defined to be the net score on the mathematics questions in the practice exam given by the private institution. The type of understanding necessary to solve the questions in the practice tests might be one the factor that contributed to the low correlation coefficients. Skemp (1972) named two poles of understanding as relational understanding and instrumental understanding. According to this categorization, understanding relationally might be harder and takes longer time. The understanding appreciated in the private institution whose primary goal was to prepare students for the high school entrance exams might be instrumental. Therefore, the definition of mathematics achievement may be one of the underlying factors for the low correlations between mathematics achievement and students' mathematics related beliefs which is not compatible with the majority of studies in the literature.

Moreover, the relation between beliefs and mathematics achievement could be better explained when controlled for aptitude such as mathematical aptitude or intelligence. The study of Gagne and St. Pere (2002) explained the relation between motivation and achievement while controlling intelligence defined as I.Q as an important variable. A similar study might be a milestone to understand beliefs and their influences on achievement.

The statistical limitation of model testing by confirmatory factor analysis must be understood well. In this study, a scale was developed for a multi dimensional construct that is students' mathematics-related beliefs. The higher the number of dimensions constitutes a construct, the higher is the complexity of the construct. Therefore, model testing with a complex construct creates numerical boundaries in structural equation modeling. Despite the limitation, the model was validated. It is important to underline that the validation of model does not necessarily imply there are no better models to explain students' mathematics related beliefs. So, the findings in this study are merely a starting point in understanding the construct of students' mathematics related beliefs. Hence, the results should be interpreted cautiously. For more generalizable results, further studies with large samples are needed.

Another limitation of the study is the sample size that didn't allow the use of bootstrapping technique. The elimination of cases that decreased the deviation of the distribution from normality was a proper technique but because of the barely enough size of the sample, the elimination was limited. If the sample size was large enough, it would be possible to create normally distributed data from the derivatives of actual data. Thus, coping with the statistical barriers of a non-normal distribution would be facilitated.

The limitations of this study can be coped with better research designs and enriching the scale with adding projective measurement components. The theoretical development of the construct students' mathematics related beliefs is still in its first stages, it needs to be elaborated more.

7.2. Suggestions for Further Studies

The study found empirical evidence in partially supporting the structural validity of the framework proposed by Op't Eynde *et al.* (2002) about students' mathematics related beliefs. The validated model in this study should be tested with larger samples.

The dimensions about teaching and learning of mathematics when asked to students did not form separate dimensions. It was thought that learning and teaching were so immersed in their social life; it was difficult to consider these three as separate dimensions. Measurement of students' beliefs about learning and teaching mathematics can be studied in detail to understand the interaction among these dimensions.

There are attempts all over the world to change classroom practices for improvement. From a global perspective, the change in students' beliefs about nature of mathematics with respect to reform curricula must be monitored by longitudinal studies. From a local perspective, the effectiveness of the recent Turkish mathematics curriculum on creating positive beliefs about mathematics as a domain, about self and about social context should be assessed. Following global curriculum trends does not necessarily contribute to the formation of positive beliefs towards mathematics because beliefs are not culture free as explained by Cobb and Yackel (1998).

Gender related differences of beliefs in social context must be related with the social experiences in school, family, friends and how education can create such differences must be examined with further studies.

The relation between mathematics achievement and beliefs can be better examined by defining mathematics achievement carefully and creating an instrument according to its definition. In addition to all different views on nature of mathematics can be used to create subscales for mathematics achievement. For example, mathematics as science of patterns can guide development of a subscale for mathematics achievement. Mathematics as a body of absolute facts and theories can guide the development of another subscale. The relations between the subscales of mathematics achievement and beliefs can be an informative study. Furthermore, the relation of mathematics aptitude or I.Q. and students' mathematics related beliefs can be a useful study. In addition to all, the scale with appropriate adjustments according to the age level can be used to have an understanding of mathematicians' mathematics related beliefs.

The future studies will bring the constitutive dimensions of students' mathematics related beliefs and its importance in mathematics learning in light. This study was a modest step in this way.

APPENDIX A: INSTRUCTIONS FOR TEACHERS

Öğretmen Yönergesi

Sevgili Öğretmenler,

Bu çalışmanın uygulanmasına verdiğiniz destek ve işbirliğiniz için sizlere teşekkür ederiz.

Anketin uygulamasında standart koşulları sağlayabilmede ve çalışmanın başarıya ulaşmasında işbirliğinize ihtiyaç duyuyoruz.

Uygulama başlarken,

- Lütfen “Öğrenci Yönergesini” öğrencilere okuyunuz.

Uygulama esnasında,

- Öğrencilere cevapsız madde bırakmamalarını hatırlatmak faydalı olabilir.

Uygulama sonunda,

- Öğrencilerden optik formları ve anket kağıtlarını lütfen toplayınız.

İşbirliğiniz için tekrar teşekkür ederiz.

APPENDIX B: INSTRUCTIONS FOR STUDENTS

Öğrenci Yönergesi

Sevgili Öğrenciler,

Bu çalışmada yer aldığınız için sizlere teşekkür ederiz.

Formda sizden istenen bilgileri eksiksiz yazınız.

- Lütfen size verilen cümleleri dikkatlice okuyup, her bir cümle için katılım derecenizi işaretleyiniz.

  A: kesinlikle katılmıyorum, E: Kesinlikle katılıyorum
  A şıkkından E şıkkına doğru katılım derecesi artmaktadır.

- Her satır için yalnızca bir işaretleme yapınız.
- Hiç bir sorunun kesin doğru ya da yanlış cevabı yoktur.

- **Değerlendirmelerinizi içtenlikle yapacağınıza güveniyoruz.**

APPENDIX C: THE FIRST SCALE

1) Matematik öğrenmenin önemli nedenlerinden biri problem çözme becerisini arttırmaktır.
2) Matematikte bir konuyu ilk karşılaştığımızda anlayamazsak daha sonra hiç anlayamayız.
3) Matematik dersinde öğrenciler konuyla ilgili tartışarak matematiksel doğrulara ulaşırlar.
4) Matematik estetik anlayışımızı geliştirir.
5) İsteyen herkes matematik öğrenebilir.
6) Sınıfça matematikle uğraşırken öğretmenimiz bize rehberlik eder.
7) Matematikte başarılı bir öğrenci olmak için çalışırım.
8) Matematik öğretmeni bildiklerimizden yola çıkarak yeni konunun kavramlarını bize buldurur.
9) Matematik dersinde öğrenci konuyu anlamamışsa sorumlusu çoğunlukla öğretmendir.
10) Matematik en iyi, konuyu bilen birinden öğrenilir.
11) Matematik düşünmeyi geliştirir.
12) Matematik öğretmeni ders boyunca kafamızı karıştırmadan herşeyi adım adım anlatmalıdır.
13) Problem çözme matematik öğrenmede önemli bir araçtır.
14) Matematik öğrenmede anlamak önemlidir.
15) Matematik öğretmeni matematiğin anlaşılır bir alan olduğunu hissettirmelidir.
16) Matematik dersinde cevabın yeterli olması için herkes tarafından anlaşılacak şekilde açıklanması gerekir.
17) Matematik dersinde sonuç veren çözüm yollarını bulmak, sonuca ulaşmak kadar önemlidir.
18) Matematik insanların düşüncelerine tutarlılık getirir.
19) Matematik teknolojinin gelişmesine katkıda bulunur.
20) Matematik dersinde yaptığım ödevler beni geliştirir.
21) Matematiği anlamak öğrendiklerimizi ilişkilendirmektir.
22) Matematik kendine ait sembolleri ve dili olan bir alandır.

23) Okula gidemediğim gün matematik dersinde öğrenilenleri, kitaptan ya da defterden çalışıp anlayabilirim.

24) Bazen öğretmenin verdiği ödev ve çalışmalardan daha fazlasını yaparım.

25) Matematik öğretmeni doğru cevaplar üzerinde durduğu gibi yanlış cevaplar üzerinde de durmalı ve açıklamalıdır.

26) Matematikte farklı düşünmeye yer yoktur.

27) Problem çözme konuyu tekrar etmek dışında bir şey kazandırmaz.

28) Matematikte bir konuyu belli bir sürede öğrenemeyen o konuyu hiç öğrenemez.

29) Öyle ya da böyle, insanlara mutlaka matematik gereklidir.

30) Matematikte diğer derslerde olduğum kadar başarılı olamam.

31) Matematik dersinde öğrenilmesi gereken sınıfta anlatılanlardan ibarettir.

32) Matematik problemlerini uğraşırsam çözebilirim.

33) Matematik öğretmek için matematiği bilmenin ötesinde matematik öğretmeyi de bilmek gerekir.

34) Matematikte zorlandığımda çalışarak üstesinden gelebilirim.

35) Sınıfça matematikle uğraşırken öğretmenimiz sınıfın başvurduğu kişidir.

36) Konuyu öğrenmek için matematik dersini dikkatle dinlerim.

37) Matematik ortak bir düşünme dilidir.

38) Matematik öğretmeni matematiği iyi bilmelidir.

39) Matematik verdiğim emeğe değer.

40) Matematik düzenli ve belli kurallar çerçevesinde düşünmeyi öğretir.

APPENDIX D: THE SCALE VALIDATED BY THE MODEL

1) Matematik dersinde öğrenciler konuyla ilgili tartışarak matematiksel doğrulara ulaşırlar.
2) Sınıfça matematikle uğraşırken öğretmenimiz bize rehberlik eder.
3) Matematikte başarılı bir öğrenci olmak için çalışırım.
4) Matematik dersinde öğrenci konuyu anlamamışsa sorumlusu çoğunlukla öğretmendir.
5) Matematik düşünmeyi geliştirir.
6) Matematik dersinde cevabın yeterli olması için herkes tarafından anlaşılacak şekilde açıklanması gerekir.
7) Matematik dersinde sonuç veren çözüm yollarını bulmak, sonuca ulaşmak kadar önemlidir.
8) Matematik insanların düşüncelerine tutarlılık getirir.
9) Matematik teknolojinin gelişmesine katkıda bulunur.
10) Matematik dersinde yaptığım ödevler beni geliştirir.
11) Matematik kendine ait sembolleri ve dili olan bir alandır.
12) Bazen öğretmenin verdiği ödev ve çalışmalardan daha fazlasını yaparım.
13) Öyle ya da böyle, insanlara mutlaka matematik gereklidir.
14) Matematikte diğer derslerde olduğum kadar başarılı olamam.
15) Matematik problemlerini uğraşırsam çözebilirim.
16) Matematikte zorlandığımda çalışarak üstesinden gelebilirim.
17) Sınıfça matematikle uğraşırken öğretmenimiz sınıfın başvurduğu kişidir.
18) Konuyu öğrenmek için matematik dersini dikkatle dinlerim.
19) Matematik ortak bir düşünme dilidir.
20) Matematik verdiğim emeğe değer.
21) Matematik düzenli ve belli kurallar çerçevesinde düşünmeyi öğretir.

APPENDIX E: ELIMINATED ITEMS

1. Matematik öğrenmenin önemli nedenlerinden biri problem çözme becerisini arttırmaktır.

5. İsteyen herkes matematik öğrenebilir.

8. Matematik öğretmeni bildiklerimizden yola çıkarak yeni konunun kavramlarını bize buldurur.

10. Matematik en iyi, konuyu bilen birinden öğrenilir.

12.Matematik öğretmeni ders boyunca kafamızı karıştırmadan herşeyi adım adım anlatmalıdır.

13.Problem çözme matematik öğrenmede önemli bir araçtır.

14.Matematik öğrenmede anlamak önemlidir.

15.Matematik öğretmeni matematiğin anlaşılır bir alan olduğunu hissettirmelidir.

21. Matematiği anlamak öğrendiklerimizi ilişkilendirmektir.

25. Matematik öğretmeni doğru cevaplar üzerinde durduğu gibi yanlış cevaplar üzerinde de durmalı ve açıklamalıdır.

27. Problem çözme konuyu tekrar etmek dışında bir şey kazandırmaz.

33. Matematik öğretmek için matematiği bilmenin ötesinde matematik öğretmeyi de bilmek gerekir.

38. Matematik öğretmeni matematiği iyi bilmelidir.

APPENDIX F: FIRSTLY ELIMINATED SIX ITEMS

First deleted 6 items according to item total correlations. (34 left)

2. Matematikte bir konuyu ilk karşılaştığımızda anlayamazsak daha sonra hiç anlayamayız.
4. Matematik estetik anlayışımızı geliştirir.
23. Okula gidemediğim gün matematik dersinde öğrenilenleri, kitaptan ya da defterden çalışıp anlayabilirim.
26. Matematikte farklı düşünmeye yer yoktur.
28. Matematikte bir konuyu belli bir sürede öğrenemeyen o konuyu hiç öğrenemez.
31. Matematik dersinde öğrenilmesi gereken sınıfta anlatılanlardan ibarettir.

APPENDIX G: THE PILOT SCALE

1) Problem çözme becerisini artırmak matematik öğrenmenin önemli nedenlerinden biridir.
2) Matematikte bir konuyu ilk karşılaştığımızda anlayamazsak zamanla hiç anlayamayız.
3) Matematik öğretmeni bir örneği çözerken neden belli yöntemlerin kullanıldığını vurgulamalıdır.
4) Matematik dersinde öğrenci konuyu anlamamışsa sorumlusu çoğunlukla öğrencidir.
5) Matematik öğretmek için bilmek yeterlidir.
6) Matematik dersinde öğrenciler konuyla ilgili tartışarak matematiksel doğrulara ulaşırlar.
7) Matematikte bir konu anlaşılmışsa, unutulsa da tekrar keşfedilebilir.
8) Matematik estetik anlayışımızı geliştirir.
9) İnsanlar, bilgisayarı icat ettikleri gibi matematiği de icat etmişlerdir.
10) İsteyen herkes matematiği öğrenir.
11) Sınıfça matematikle uğraşırken öğretmenimiz sınıfa rehberlik eden kişidir.
12) Matematikte başarılı bir öğrenci olmak için çalışırım.
13) Matematik insanların ortak çabalarıyla oluşturulmuştur.
14) İnsanlar, Amerika kıtasını keşfettikleri gibi matematiği de keşfetmişlerdir.
15) Matematik öğretmeni bildiklerimizden yola çıkarak yeni konunun kavramlarını öğrencilere buldurur.
16) Matematik dersinde öğrenci konuyu anlamamışsa sorumlusu çoğunlukla öğretmendir.
17) Matematik öğretmeni konuyu anlatmadan bize soru sormamalıdır.
18) Matematik en iyi konuyu bilen birinden öğrenilir.
19) Matematik düşünmeyi geliştirir.
20) Matematikte bugün doğru olan gelecekte de doğru olacaktır.
21) Bir konuyu kendi başıma çalışarak öğrenebilirim.
22) Matematik öğretmeni ders boyunca herşeyi adım adım anlatmalı, karışıklık yaratmamalıdır.
23) Problem çözme matematik öğrenmede önemli bir araçtır.
24) Matematik öğrenmede anlamak önemlidir.

25) Matematik öğretmeni matematiğin anlaşılır bir alan olduğunu hissettirmelidir.

26) Matematik genel olarak akılda tutmaya bağlıdır.

27) Ben problem çözerken harcadığım zamanı kayıp olarak görmüyorum.

28) Matematik öğretmeni çözümünü bilmediği bir soruyu yaptığımızda çözümümüzün doğruluğunu değerlendiremez.

29) Matematik dersinde cevabın yeterli olması için herkes tarafından anlaşılacak şekilde açıklanması gerekir.

30) Matematik dersinde sonuç veren çözüm yollarını bulmak cevaba ulaşmak kadar önemlidir.

31) Matematik insanların düşüncelerine tutarlılık getirir.

32) Matematik hayata anlam kazandırır.

33) Matematik teknolojinin gelişmesine katkıda bulunur.

34) Matematik dersinde yaptığım ödevler beni geliştirir.

35) Matematiği anlamak ilişkileri bilmektir.

36) Matematik öğretmenden öğrenilir.

37) Matematik kendine ait sembolleri ve dili olan bir alandır.

38) Matematik mantık kullanılarak öğrenilir.

39) Okula gidemediğim gün matematik dersinde öğrenilenleri kitaptan ya da defterden çalışıp anlayabilirim.

40) Bazen öğretmenin verdiği ödev ve çalışmalardan daha fazlasını yaparım.

41) Matematik öğretmeni doğru cevaplar üzerinde durduğu gibi yanlış cevaplar üzerinde durmalı, açıklamalıdır.

42) Matematikte farklı düşünmeye yer yoktur.

43) Problem çözme konuyu tekrar etmek dışında bir şey kazandırmaz.

44) Matematikte konuyu anlamam için öğretmenin çok iyi anlatması gerekir.

45) Matematikte bir konuyu belli bir sürede öğrenmeyen o konuyu öğrenemez.

46) Öyle ya da böyle insanlara mutlaka matematik gereklidir.

47) Matematikte diğer derslerde olduğum kadar başarılı olamam.

48) Matematik öğretmeni bir örneği çözerken cevabın en kolay nasıl bulunacağını göstermelidir.

49) Bulduğum bir sonuçtan farklı bir sonuç bulunmuşsa açıklamalarını isterim, belki de onların sonucu yanlıştır.

50) Matematik dersinde öğrenilmesi gereken sınıfta anlatılanlardan ibarettir.

51) Matematik dersinde öğretmenin çözüm yoluna bağlı kalmakta fayda vardır.

52) Matematik öğretmeni her öğrenciyi matematik öğrenmesi için zorlamalıdır.

53) Matematikte ilk kez karşılaştığım bir soruyu çözebilirim.

54) Uğraşırsam matematik problemlerini çözebilirim.

55) Matematik öğretmek için matematiği bilmenin ötesinde matematik öğretmeyi bilmesi gerekir.

56) Matematikte zorlandığımda çalışarak üstesinden gelebilirim.

57) Matematik öğretmenimizin bazı çözümleri neden farklı bulduğunu anlamıyorum, aynı cevabı buluyoruz.

58) Sınıfça matematikle uğraşırken öğretmenimiz sınıfın başvurduğu kişidir.

59) Problem çözme, daha önce çözülen soruların farklı sayılarla tekrarlanması değildir.

60) Konuyu öğrenmek için matematik dersini dikkatle dinlerim.

61) Matematik düşünme biçimi olarak ortak bir dil sunar.

62) Matematik öğretmeni matematiği iyi bilmelidir.

63) Matematik verdiğim emeğe değer.

64) Sınıfça matematikle uğraşırken öğretmenimiz de sınıfın öğrenciler gibi bir üyesidir.

65) Matematik düzenli ve belli kurallar çerçevesinde düşünmeyi öğretir.

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