TRANSVERSE VIBRATION OF AN AXIALLY ACCELERATING STRING

by

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LIST OF SYMBOLS

A	:	Cross sectional area of the string or strip
E	:	Youngs modulus
F	:	Applied axial force
f	:	frequency of vibrations
L	:	Length of the string between supports
P_o	:	Initial tension
$P(v)$	:	Tensile force
$q_i(t)$	:	generalized displacement
R	:	Residual in Galerkins method
t	:	time variable
T	:	Kinetic energy
$u(x,t)$	:	relative longitudional displacement in the string or strip
v_o	:	maximum amplitude of the axial velocity
$v(t)$	:	axial velocity
V	:	Potential energy
W	:	Weighting function
w_o	:	frequency of axial velocity variation
$y(x,t)$	:	Displacement
x	:	spatial variable
ρ	:	density of the string or strip
Δ	:	total longitudional displacement of the system
η	:	pulley support constant $0<\eta<1$
K	:	$1-\eta$

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Ö Z E T

Eksen doğrultusunda ivmelenen ipin enine titreşimleri incelenmiştir. Hareket denklemi Hamilton prensibinden çıkarılmıştır. Ortaya çıkan kısmi diferansiyel denklem Galerkin metodu ile adi diferansiyel denklemlere dönüştürülmüştür. Galerkin yaklaşımında bir terim alındığında çözümü çok iyi bilinen Mathieu denklemi ortaya çıkmaktadır. İki terim yaklaşımında ortaya çıkan iki denklem sistemi nümerik metodlarla çözülmüştür. Bir terim ve iki terim yaklaşımlarının sonuçları karşılaştırılmış ve neticede bir terim yaklaşımının eksen doğrultusunda ivmelenen ipin temel enine mekanizmasını elde etmede yeterli olmadığı görülmüştür.

ABSTRACT

The transverse vibration of an axially accelerating string is investigated. The equation of motion is developed using Hamilton's principle. The resulting partial differential equations are discretized using Galerkin's method. Retaining one-term in Galerkin's approximation leads to a Mathieu equation, the solution of which is well known. In the two-term approximation the resulting coupled equations are solved by numerical methods. Results of the one-term and two-term approximations are compared, and it is concluded that the one-term approximation is not adequate for capturing the basic transverse instability mechanisms of the axially accelerating string.

I-INTRODUCTION

The problem of axially moving materials is a subject of technological interest since many such materials are observed in manufacturing industries. High speed fiber winding, magnetic tape systems, threadlines, band-saw blades, belts and pipes transporting fluids all belong to this class. Numerous researchers have examined the dynamic response of such materials. The early research in this area includes those by Skucth [1] and Sack [2]. The work done up to 1978 has been reviewed by Ulsoy and Mote |3|. Among these studies reviewed in [3] are several investigations of parametric instability due to axial tension variations and periodic edge loads in strings and band moving axially at constant velocity. More recently Ulsoy and Mote [4] have used an axially moving plate model to investigate the role of in-plane stresses in the transverse vibration of band saw blades. The coupled vibrations of the belt and tensioner in automotive accessary drive systems has also been experimentally and analytically investigated [5]. Chonan [6] studied the steady state response of an axially moving thick beam subjected to a concentrated constant lateral force.

In all these works the velocity of the moving system was taken to be constant. However, the actual systems are subject to accelerations and decelerations, which in fact may seriously change the vibration behaviour. The equation of motion for the transverse vibration of an axially accelerating string was derived by Miranker [7] but he did not present a

solution. In the present study the general case, in which velocity is not constant but a prescribed function of time is treated. The partial differential equations governing the motion are derived. Discretization of the differential equations by Galerkin's method gives a system of n ordinary differential equations. In the present analysis only the first and second terms are considered. (i.e. n=1 and n=2). The time dependant axial velocity function v(t) is assumed to be sinusoidal. Taking only one term this reduces to the standard Mathieu equation, the solution of which is well known. Taking two terms the resulting two ordinary differential equations are coupled and periodic. They are solved by numerical methods since analytical solutions are very difficult to obtain. A stability analysis for each solution is made, and the results of the two solutions are compared. In the two-term solution the coupling effects of the equations lead to different results from those of the one-term solution.

II- EQUATIONS OF MOTION

In this chapter the equations of motion are derived. The physical model considered, shown in figure 1, is a continuous string or strip passing over two pulleys at a transport velocity v(t). First the stationary string equations will be derived (i.e., v=0). Then the constant velocity case will be examined. The chapter will be concluded by treating the most general case, that is the variable velocity case.

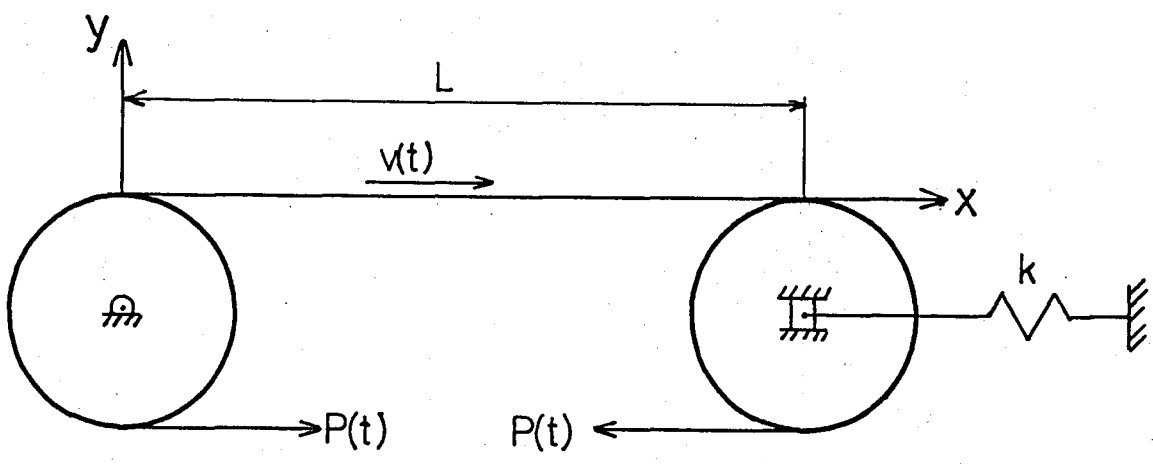


Figure 1. Coordinates and geometry

2-1. TRANSVERSE VIBRATIONS OF A STATIONARY STRING

According to Hamilton's principle the variation of the functional

$$\int_{t_1}^{t_2} (T-V)\, dt \tag{2.1}$$

is zero where T and V are, respectively, the kinetic and potential energies of the system [8],

$$\delta \int_{t_1}^{t_2} (T-V) = 0 \tag{2.2}$$

Because of zero velocity in the x direction the kinetic energy is only due to velocity in the y direction, hence

$$T = \int_{o}^{L} \frac{1}{2} \rho A \left(\frac{\partial y}{\partial t}\right)^2 dx \tag{2.3}$$

The potential energy v is obtained in the following way

$$dV = P(ds-dx) \tag{2.4}$$

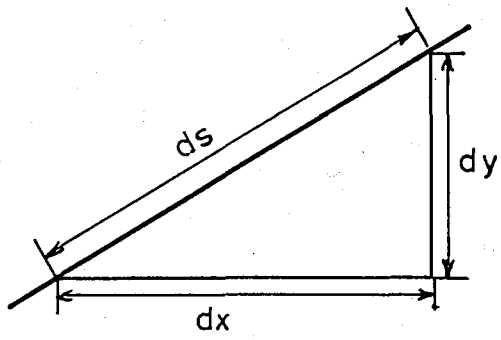


Figure 2- A differential element of the vibrating string

From figure 2

$$ds = (dx^2+dy^2)^{\frac{1}{2}} \tag{2.5}$$

Substituting (2.5) into (2.4) and factoring $(dx)^2$ gives,

$$dV = P\left(\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{1}{2}} dx - dx \right) \tag{2.6}$$

Expanding the square root term in binomial form and neglecting higher order terms and finally integrating over the length of the string yields

$$V \simeq \int_o^L \frac{1}{2} P\left(\frac{\partial y}{\partial x}\right)^2 dx \tag{2.7}$$

Substituting the expressions for T and V from equations (2.3) and (2.7) into equation (2.2) we get,

$$\delta \int_{t_1}^{t_2} \int_o^L \left\{ \frac{1}{2} \rho A(\dot{y})^2 dx - \frac{1}{2} P(y')^2 dx \right\} dt = 0 \tag{2.8}$$

Taking the variation gives

$$\int_{t_1}^{t_2} \int_o^L (\rho A\dot{y}\delta\dot{y} - P\, y'\delta y')\, dxdt = 0 \tag{2.9}$$

Using integration by parts we obtain

$$\int_{t_1}^{t_2} \int_o^L \{(\rho A\ddot{y} - P\, y'')\delta y\}\, dxdt - \rho A\dot{y}\delta y\Big|_{t_1}^{t_2} - Py'\delta y\Big|_o^L = 0 \tag{2.10}$$

For equation (2.10) to be zero with arbitrary δy the integrand and the boundary terms must all be zero. Thus,

$$\rho A \frac{\partial^2 y}{\partial t^2} = P \frac{\partial^2 y}{\partial x^2} \tag{2.11}$$

which is the equation of transverse motion of a stationary string. The second term in eq.(2.10) leads to initial conditions at t_1 and also states that variations must vanish at t_2.

$$\dot{y}(t_1) = \dot{y}(t_2) = 0$$

or

$$\delta y(t_1) = \delta y(t_2) = 0 \tag{2.12}$$

The third term in eq.(2.10) leads to boundary conditions

$$y'(0) = y'(L) = 0$$

or

$$\delta y(0) = \delta y(L) = 0 \tag{2.13}$$

Equations (2.11-2.13) are well known, and their solution has been extensively studied [9]

2-2. TRANSVERSE VIBRATIONS OF A STRING MOVING WITH CONSTANT VELOCITY V.

The expressions for the kinetic and potential energies in the case of a string moving with constant velocity are,

$$T = \int_0^L \frac{1}{2} \rho A\left[v^2+(\dot{y}+vy')^2\right] dx \tag{2.14}$$

$$V = \int_0^L \frac{1}{2} P\left(\frac{\partial y}{\partial x}\right)^2 dx \tag{2.15}$$

Substituting (2.14) and (2.15) into equation (2.2) one obtains,

$$\delta \int_{t_1}^{t_2} \int_0^L \{\rho A\left[v^2+(\dot{y}+vy')^2\right] - \frac{1}{2} P(y')^2\} dxdt = 0 \tag{2.16}$$

Taking the variation gives,

$$\int_{t_1}^{t_2} \int_0^L \{\rho A(\dot{y}+vy')\ (\delta\dot{y}+v\delta y') - Py'\delta y'\}\ dxdt = 0 \tag{2.17}$$

Expanding the products and making use of the relations

$$\delta\dot{y} = \frac{\partial}{\partial t}(\delta y) \qquad\qquad \delta y' = \frac{\partial}{\partial x}(\delta y) \tag{2.18}$$

Equation (2.17) can be written as

$$\int_{t_1}^{t_2} \int_0^L \rho A\ \left[\dot{y}\ \frac{\partial}{\partial t}(\delta y)+v\dot{y}\cdot\frac{\partial}{\partial x}(\delta y)+vy'\ \frac{\partial}{\partial t}(\delta y)+\right.$$

$$\left. v^2y'\frac{\partial}{\partial x}(\delta y)\right] - Py'\ \frac{\partial}{\partial x}(\delta y)\}\ dxdt = 0 \tag{2.19}$$

Using integration by parts yields

$$\int_{t_1}^{t_2}\int_{o}^{L}\{[\rho A(\ddot{y}+v\dot{y}'+v\dot{y}'+v^2y'')-Py'']\delta y\}\,dxdt -$$

$$\rho A\,\{\dot{y}\delta y\,\Big|_{t_1}^{t_2}+v\dot{y}\delta y\,\Big|_{o}^{L}+vy'\delta y\Big|_{t_1}^{t_2}+v^2y'\delta y\,\Big|_{o}^{L}\}+$$

$$Py'\delta y\,\Big|_{o}^{L}=0 \qquad (2.20)$$

Setting the coefficient of δy to zero we finally obtain the equations of motion.

$$\frac{\partial^2 y}{\partial t^2}+2v\frac{\partial^2 y}{\partial x\partial t}+\left(\frac{\rho Av^2-P}{\rho A}\right)\frac{\partial^2 y}{\partial x^2}=0 \qquad (2.21)$$

The initial conditions at t_1, and the conditions at t_2 are

$$\dot{y}(t_1)=\dot{y}(t_2)=0$$

or

$$\delta y(t_1)=\delta y(t_2)=0 \qquad (2.22.A)$$

$$v(t_1)=v(t_2)=0$$

or

$$y'(t_1)=y'(t_2)=0 \qquad (2.22.B)$$

or

$$\delta y(t_1)=\delta y(t_2)=0$$

and the boundary conditions are,

$$v(0)=v(L)=0$$

or

$$\dot{y}(0)=\dot{y}(L)=0 \qquad (2.23.A)$$

or

$$\delta y(0)=\delta y(L)=0$$

$$v(0) = v(L) = 0$$

or

$$y'(0) = y'(L) = 0 \qquad (2.23.B)$$

or

$$\delta y(0) = \delta y(L) = 0$$

$$y'(0) = y'(L) = 0$$

or

$$\delta y(0) = \delta y(L) = 0 \qquad (2.23.C)$$

Note that when v=0, equations (2.21), (2.22), (2.23) reduce to equations (2.11), (2.12) (2.13) respectively.

2-3. TRANSVERSE VIBRATIONS OF AN AXIALLY ACCELERATING STRING

The aim here is to derive the equations for the axially accelerating string and then introduce some assumptions to simplify the equations. The string or strip has a great flexibility in the transverse direction and we will also assume that the stiffness is very large in the longitudional direction. Due to longitudional displacement in the string we add a term u in the kinetic and potential energy relations. The velocity is not constant but assumed to be a prescribed function of time.

We begin the analysis by writing the kinetic and potential energies.

$$T = \frac{1}{2} \rho A \int_0^L [(\dot{y}+vy')^2 + (\dot{u}+v)^2] \, dx \qquad (2.24)$$

$$V = \int_0^L P\varepsilon dx + \frac{1}{2} EA \int_0^L \varepsilon^2 dx + \int_0^L F\Delta dx \tag{2.25}$$

The first integral in (2.25) is due to the tension force P, the second term is due to axial deformation and the third term is the work done by the net force F which causes the motion with Δ representing the total longitudional displacement of the system.

The strain, ε, can be written as

$$\varepsilon = \left[(1+u')^2 + (y')^2\right]^{\frac{1}{2}} - 1 \tag{2.26}$$

Inserting (2.26) into (2.25), then (2.24) and (2.25) into equation (2.2) we obtain

$$\delta \int_{t_1}^{t_2} \int_0^L < \frac{1}{2} \rho A\{\dot{y}^2+2\dot{y}vy'+v^2(y')^2+\dot{u}^2+v^2+2\dot{u}v\}$$

$$-P\{\left[(1+u')^2+(y')^2\right]^{\frac{1}{2}} -1\}- \frac{1}{2} EA \{(1+u')^2+(y')^2+1-$$

$$2\left[(1+u')^2+(y')^2\right]^{\frac{1}{2}} \} - F\Delta > dxdt = 0 \tag{2.27}$$

Taking the variation with respect to Δ, u,v, and y

$$\int_{t_1}^{t_2} \int_0^L < \frac{1}{2} \rho A\{2\dot{y}\delta\dot{y}+2vy'\delta\dot{y}+2\dot{y}y'\delta v+2\dot{y}v\delta y'$$

$$+2v(y')^2\delta v+2v^2y'\delta y'+2\dot{u}\delta\dot{u}+2v\delta v+2\dot{v}\delta\dot{u}+2\dot{u}\delta v \}$$

$$-P \{\frac{1}{2} \left[(1+u')^2+(y')^2 \right]^{-\frac{1}{2}} \cdot \left[2(1+u')\delta u' + 2y'\delta y' \right] \}$$

$$- \frac{1}{2} EA \{2(1+u')\delta u'+2y'\delta y' - 2 \frac{1}{2} \left[(1+u')^2+(y')^2 \right]^{-\frac{1}{2}}$$

$$\left[2(1+u')\delta u' + 2y'\delta y'\right] \} - F\delta\Delta > dxdt = 0 \qquad (2.28)$$

Rearranging the terms and writing $\delta\dot{\Delta}$ instead of δv

$$\int_{t_1}^{t_2}\int_{o}^{L} < \rho A \{\dot{y}y' + v(y')^2 + v + \dot{u}\} \delta\Delta - F\delta\Delta > dxdt +$$

$$\int_{t_1}^{t_2}\int_{o}^{L} < \rho A \{\dot{u}\delta\dot{u} + v\delta\dot{u}\} - P\frac{(1+u')\ \delta u'}{((1+u')^2+(y')^2)^{\frac{1}{2}}} - EA\{(1+u')\ \delta u' -$$

$$\frac{(1+u')\ \delta u'}{((1+u')^2+(y')^2)^{\frac{1}{2}}}\} > dxdt + \int_{t_1}^{t_2}\int_{o}^{L} < \rho A \{\dot{y}\delta\dot{y} + vy'\delta\dot{y} + \dot{y}v\delta y'$$

$$+v^2y'\delta y'\} - \frac{Py'\delta y'}{((1+u')^2+(y')^2)^{\frac{1}{2}}} - EA\{y'\delta y' - \frac{y'\delta y'}{((1+u')^2+(y')^2)^{\frac{1}{2}}}\}$$

$$> dxdt = 0 \qquad (2.29)$$

For equation (2.29) to be zero the three integrals should be equal to zero seperately, thus the integrals lead to 3 different equations which will be examined explicitly.

Applying integration by parts to the first integral and writing all the terms as a multiplication of $\delta\Delta$ and equating this factor to zero we obtain

$$\rho A \{\ddot{y}y' + \dot{y}\dot{y}' + \dot{v}(y')^2 + 2vy'\dot{y}' + \dot{v} + \ddot{u}\} - F = 0 \qquad (2.30)$$

Neglecting the second order terms and assuming a high stiffness in the x direction $\ddot{u}$ will be zero and the equation reduces to

$$\rho A\dot{v} = F \qquad (2.31)$$

The driving force F is arbitrary so v is also arbitrary and equation (2.30) does not restrict the choice of v.

We now take the second integral in equation (2.29) and apply integration by parts to each term. Equating this expression to zero we obtain the second equation of motion,

$$\rho A \{\ddot{u}+\dot{v}\} - P \left\{\frac{u''}{[(1+u')^2+(y')^2]^{\frac{1}{2}}} - \frac{(1+u')[(1+u')u''+y'y'']}{[(1+u')^2+(y')^2]^{3/2}}\right\}$$

$$-EA \left\{\frac{u''}{[(1+u')^2+(y')^2]^{\frac{1}{2}}} - \frac{(1+u')[(1+u')u''+y'y'']}{[(1+u')^2+(y')^2]^{3/2}}\right\} = 0 \qquad (2.32)$$

Lets check the equation for the limiting cases such as y very small. The factor multiplying P will be zero and we are left with the terms

$$\rho A \{\ddot{u}+\dot{v}\} - EAu'' = 0 \qquad (2.33)$$

or rearranging

$$\ddot{u}+\dot{v} = \frac{E}{\rho} u'' \qquad (2.34)$$

Equation (2.34) is obviously the equation of longitudional vibration of a string. When we make the assumption that the stiffness is high enough in the x direction equation (2.34) can be neglected.

Now we take the third integral from equation (2.29) and applying integration by parts we finally obtain,

$$\rho A \{\ddot{y} + \dot{v}y' + 2v\dot{y}' + \dot{y}v' + 2vv'y' + v^2y''\} -$$

$$P \left\{\frac{y''}{[(1+u')^2+(y')^2]^{\frac{1}{2}}} - \frac{y'[(1+u')u''+y'y'']}{[(1+u')^2+(y')^2]^{3/2}}\right\} -$$

$$EA \left\{y'' - \frac{y''}{[(1+u')^2+(y')^2]^{\frac{1}{2}}} - \frac{y'[(1+u')u''+y'y'']}{[(1+u')^2+(y')^2]^{3/2}}\right\} \qquad (2.35)$$

One can now make the assumption that the stiffness in the longitudional direction is very high, so that the u terms can be neglected. Also v' is zero because velocity is a function of time only. Second and higher order terms are also neglected and equation (2.35) finally reduces to

$$\rho A \left\{\frac{\partial^2 y}{\partial t^2} + \frac{\partial v}{\partial t}\frac{\partial y}{\partial x} + 2v\frac{\partial^2 y}{\partial x \partial t}\right\} + (\rho A v^2 - P)\frac{\partial^2 y}{\partial x^2} = 0 \qquad (2.36)$$

This equation has also been derived in [7]. However, its solution has not been studied, and that will be the goal of the following chapters. To review the formulation, we obtain three important equations namely equations (2.30), (2.32) and (2.35). With the assumptions made as previously mentioned

equation (2.30) reduces to axial force is equal to the mass multiplied by axial acceleration. The force being arbitrary gives us the flexibility to choose a desired axial velocity function v(t). Equation (2.32) vanishes for high longitudional stiffness and we are left with only equation (2.35). This also reduces to equation (2.36). The following analysis will be based on equation (2.36) and the associated initial conditions at $t=t_1$ and conditions at $t=t_2$ and boundary conditions are,

$$\dot{y}(t_1) = \dot{y}(t_2) = 0$$

or

$$\delta y(t_1) = \delta y(t_2) = 0 \qquad (2.37.A)$$

$$v(t_1) = v(t_2) = 0$$

or

$$y'(t_1) = y'(t_2) = 0 \qquad (2.37.B)$$

or

$$\delta y(t_1) = \delta y(t_2) = 0$$

$$\dot{y}(0) = \dot{y}(L) = 0$$

or

$$v(0) = v(L) = 0 \qquad (2.38.A)$$

or

$$\delta y(0) = \delta y(L) = 0$$

$$v(0) = v(L) = 0$$

or

$$y'(0) = y'(L) = 0 \qquad (2.38.B)$$

or

$$\delta y(0) = \delta y(L) = 0$$

$$y'(0) = y'(L) = 0$$

or

$$\delta y(0) = \delta y(L) = 0 \qquad (2.38.C)$$

III- DISRETIZATION OF THE EQUATION OF MOTION

The aim in this chapter is to discrete the differential equation given by (2.36) using Galerkin's method [10]. For convenince we write equation (2.36) again.

$$\rho A \left\{ \frac{\partial^2 y}{\partial t^2} + \frac{\partial v}{\partial t}\frac{\partial y}{\partial x} + 2v\frac{\partial^2 y}{\partial x \partial t} \right\} + (\rho A v^2 - P)\frac{\partial^2 y}{\partial x^2} = 0 \qquad (3.1)$$

3.1 GALERKIN METHOD FOR N TERMS

We first choose the trial function as

$$y(x,t) = \sum_{i=1}^{n} q_i \sin\frac{i\pi x}{L} \qquad (3.2)$$

where $\sin\frac{i\pi x}{L}$ are the eigen functions of the stationary string, and the generalized displacements $q_i(t)$ are merely a function of time. Taking the appropiate derivatives

$$\dot{y} = \sum_{i=1}^{n} \dot{q}_i \sin\frac{i\pi x}{L} \qquad y' = \sum_{i=1}^{n} \frac{i\pi}{L} q_i \cos\frac{i\pi x}{L}$$

$$\ddot{y} = \sum_{i=1}^{n} \ddot{q}_i \sin\frac{i\pi x}{L} \qquad y'' = -\sum_{i=1}^{n} \frac{i^2\pi^2}{L^2} q_i \sin\frac{i\pi x}{L} \qquad (3.3)$$

$$\dot{y}' = \sum_{i=1}^{n} \frac{i\pi}{L} \dot{q}_i \cos\frac{i\pi x}{L}$$

Inserting the relations (3.3) into equation (3.1) to find the residual

$$R = \sum_{i=1}^{n} \rho A \ddot{q}_i \sin \frac{i\pi x}{L} + 2\rho A v \frac{i\pi}{L} \dot{q}_i \cos \frac{i\pi x}{L} + \rho A \dot{v} \frac{i\pi}{L}$$

$$q_i \cos \frac{i\pi x}{L} + (P-\rho A v^2) \frac{i^2\pi^2}{L^2} q_i \sin \frac{i\pi x}{L} \tag{3.4}$$

The Galerkin's method requires,

$$\int_0^L R \cdot W \, dx = 0 \tag{3.5}$$

The weighting functions W(x) are also the stationary string eigen functions [11]

$$W_j = \sin \frac{j\pi x}{L} \tag{3.6}$$

Inserting equations (3.4) and (3.6) into equation (3.5)

$$\int_0^L \sum_{i=1}^{n} \left[\rho A \ddot{q}_i + (P-\rho A v^2) \frac{i^2\pi^2}{L^2} q_i\right] \sin \frac{i\pi x}{L} \sin \frac{j\pi x}{L} +$$

$$\left[2\rho A v \frac{i\pi}{L} \dot{q}_i + \rho A \dot{v} \frac{i\pi}{L} q_i\right] \cos \frac{i\pi x}{L} \sin \frac{j\pi x}{L} = 0 \tag{3.7}$$

Taking the integral

$$\sum_{i=1}^{n} \left[\rho A \ddot{q}_i + (P-\rho A v^2) \frac{i^2\pi^2}{L^2} q_i\right] \frac{1}{2} \left[\frac{\sin (i-j)\frac{\pi}{L} x}{(i-j)\frac{\pi}{L}} - \right.$$

$$\left. \frac{\sin (i+j)\frac{\pi}{L} x}{(i+j)\frac{\pi}{L}}\right]_0^L - \left[2\rho A v \frac{i\pi}{L} \dot{q}_i + \rho A \dot{v} \frac{i\pi}{L} q_i\right]$$

$$\frac{1}{2}\,\text{(b)}\left[\frac{\cos\,(j+i)\,\frac{\pi}{L}\,x}{(j+i)\,\frac{\pi}{L}} + \frac{\cos\,(j-i)\,\frac{\pi}{L}\,x}{(j-i)\,\frac{\pi}{L}}\right]_{o}^{L} \qquad (3.8)$$

Equation (3.8) is valid when $i \neq j$. When $i=j$ the paranthesized terms (a) and (b) will be replaced by the following,

$$\text{(a)}\left[\frac{x}{2} - \frac{1}{4\frac{i\pi}{L}}\sin\frac{2i\pi x}{L}\right]_{o}^{L}$$

$$\text{(b)}\left[\frac{\sin^2\frac{i\pi x}{L}}{\frac{2i\pi}{L}}\right]_{o}^{L} \qquad (3.9)$$

3.2 ONE - TERM APPROXIMATION

For the one-term approximation take $i=1$, $j=1$ in equation (3.8) while keeping in mind that for the terms (a) and (b) relation (3.9) is valid. The equation takes the form

$$\left[\rho A\ddot{q}_1 + (P-\rho A v^2)\,\frac{i^2\pi^2}{L^2}\,q_1\right].\left[\frac{x}{2} - \frac{L}{4\pi}\sin\frac{2\pi x}{L}\right]_{o}^{L} \qquad (3.10)$$

which after some rearrangement, can be written as,

$$\ddot{q}_1 + \left(\frac{P}{\rho A} - v^2\right)\frac{\pi^2}{L^2}\,q_1 = 0 \qquad (3.11)$$

As will be shown later, equation (3.11), with velocity being a sinusoidal function of time, can be easily put into the form of a standard Mathieu equation, the solution of which is known [12],[13].

3.3 TWO-TERM APPROXIMATION

When we take two terms in the Galerkin's method we obtain a system of coupled differential equations. First equation is obtained by setting i=1,2 j=1 in equation (3.8)

$$\left[\rho A\ddot{q}_1 + (P-\rho Av^2)\frac{\pi^2}{L^2}q_1\right]\frac{L}{2} - \left[2\rho Av\frac{2\pi}{L}\dot{q}_2 + \rho A\dot{v}\frac{2\pi}{L}q_2\right]$$

$$\cdot\frac{1}{2}\cdot\left(\frac{4L}{3\pi}\right) = 0 \qquad (3.12)$$

Dividing the equation by $\frac{\rho AL}{2}$ we obtain

$$\ddot{q}_1 + \left(\frac{P}{\rho A} - v^2\right)\frac{\pi^2}{L^2}q_1 - \frac{16v}{3L}\dot{q}_2 - \frac{8\dot{v}}{3L}q_2 = 0 \qquad (3.13)$$

To get the second equation one must set i=1,2 j=2 in equation (3.8), yielding,

$$-\left[2\rho Av\frac{\pi}{L}\dot{q}_1 + \rho A\dot{v}\frac{\pi}{L}q_1\right]\frac{1}{2}\left(-\frac{8L}{3\pi}\right) + \left[\rho A\ddot{q}_2 + (P-\rho Av^2)\right.$$

$$\left.\frac{4\pi^2}{L^2}q_2\right]\frac{L}{2} = 0 \qquad (3.14)$$

Dividing by $\frac{\rho AL}{2}$ and rearranging gives

$$\ddot{q}_2 + \left(\frac{P}{\rho A} - v^2\right)\frac{4\pi^2}{L^2}q_2 + \frac{16v}{3L}\dot{q}_1 + \frac{8\dot{v}}{3L}q_1 = 0 \qquad (3.15)$$

Finally writing equations (3.13) and (3.15) together using matrix notation gives,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-16v}{3L} \\ \frac{16v}{3L} & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} (\frac{P}{\rho A} - v^2) & \frac{\pi^2}{L^2} & \frac{-8\dot{v}}{3L} \\ \frac{8\dot{v}}{3L} & (\frac{P}{\rho A} - v^2) & \frac{2\pi^2}{L^2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = 0 \quad (3.16)$$

Note that these equations introduce features of the problem which were not evident in the one-term approximation. Namely the skew symmetric gyroscopic matrix multiply the generalized velocities, and the skew symmetric coupling terms in the stiffness matrix. Note also that both the gyroscopic and stiffness matrices contain time varying parameters due to the presence of the velocity v(t).

Therefore the axial velocity can be written as,

$$v(t) = v_o \sin w_o t \tag{4.1}$$

The tension force in the band varies with velocity according to the following relation [14],[15].

$$P = P_o + \eta\rho A v^2 \tag{4.2}$$

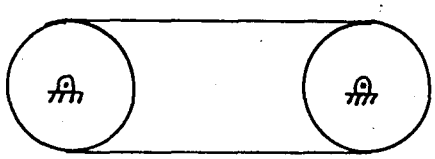


Figure 4-A- Constant displacement mechanism

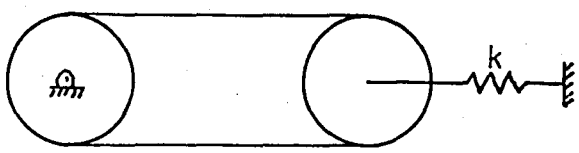


Figure 4-B- Spring supported mechanism

When the position of the wheels do not change relative to each other as in the case of figure 4-A we take $\eta=0$ because velocity has no influence on the tension in the strip. As in the case of figure 4-B the velocity influences the tension and when $k=0$ we have $\eta=1$. For values of the pulley support stiffness

$0 \leqslant k \leqslant \infty$ the parameter $1 \geqslant \eta \geqslant 0$. We also define the nondimensional pulley support constant $K = 1-\eta$.

4.1- ONE-TERM APPROXIMATION

We rewrite equation (3.11) which we obtained by taking one term in the Galerkin's method.

$$\ddot{q}_1 + \left(\frac{P}{\rho A} - v^2 \right) \frac{\pi^2}{L^2} q_1 = 0 \tag{4.3}$$

Inserting equations (4.1) and (4.2) into (4.3) and using the new variable K

$$\ddot{q}_1 + \left(\frac{P_o}{\rho A} - K\, V_o^2 \sin^2 W_o t \right) \frac{\pi^2}{L^2} q_1 = 0 \tag{4.4}$$

Equation (4.4) can be put in the form of a standard Mathieu equation

$$\ddot{q}_1 + \left(\frac{2P_o}{\rho A} - K\, v_o^2 + K\, v_o^2 \cos 2w_o t \right) \frac{\pi^2}{2L^2} q_1 = 0 \tag{4.5}$$

where we have used the identity

$$\sin^2 w_o t = \frac{1-\cos 2w_o t}{2} \tag{4.6}$$

Defining $t' = w_o t$ leads to,

$$\ddot{q}_1 = \frac{d^2 q_1}{dt^2} = w_o^2 \frac{d^2 q_1}{dt'^2} \tag{4.7}$$

Thus, Eq (4.5) can now be written as,

$$w_o^2 \frac{d^2 q_1}{dt'^2} + \left[\left(\frac{2P_o}{\rho A} - Kv_o^2 \right) \frac{\pi^2}{2L^2} + \frac{2\,K\,v_o^2\,\pi^2}{4L^2} \cos 2t' \right] q_1 = 0 \quad (4.8)$$

or

$$\frac{d^2 q_1}{dt'^2} + \left[\left(\frac{2P_o}{\rho A} - K\,v_o^2 \right) \frac{\pi^2}{2w_o^2 L^2} + 2\,\frac{Kv_o^2\,\pi^2}{4w_o^2 L^2} \cos 2t' \right] q_1 = 0 \quad (4.9)$$

This equation is now compared to the standard Mathieu equation

$$\frac{d^2 q_1}{dt'^2} + (\delta + 2\varepsilon \cos 2t')\, q_1 = 0 \qquad (4.10)$$

Thus we see that for our problem we have,

$$\delta = \left(\frac{2P_o}{\rho A} - K\,v_o^2 \right) \frac{\pi^2}{2w_o^2 L^2} \qquad \varepsilon = \frac{K\,v_o^2 \pi^2}{4w_o^2 L^2} \qquad (4.11)$$

The solution of the Mathieu equation is given in [12]. For convenience we simply present the results using the strutt diagram in figure 5. The solution leads to stability and instability regions, where the shaded areas in the figure represent the stable regions. The aim therefore in a design should be to stay in the stable region during operation. Note that the unstable areas intersect the δ axis at points given by the relation,

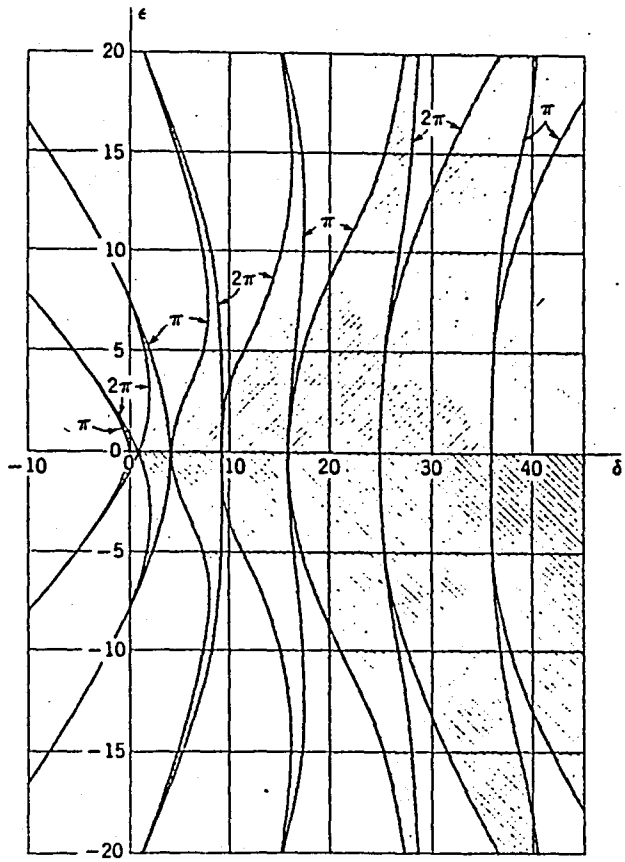


Figure 5- Mathieu stability,instability areas. (from [12]).

$$\delta = n^2 \qquad n = 0,1,2... \tag{4.12}$$

Examining the definitions of δ and ε from (4.11) we see that P_o and v_o are the important parameters influencing δ and ε. v_o appears in both δ and ε hence increasing v_o increases ε and decreases δ. So we conclude that the likelihood of instability increases at high speeds. P_o influences δ only, and increasing P_o increases δ without any change in ε. So increasing P may be useful for stability.

The constant K is another important parameter which alters the magnitude of δ and ε. In constant displacement mechanisms such as tape bands K can be taken as 1. In constant tension mechanisms K = 0. Now we apply the formulation (4.11) we have obtained to two working mechanisms

Example I- (Band Saw Blades)

The following measurements are taken from a small size typical band saw [4].

$P_o = 76.22$ N, $\rho = 7754$ kg/m^3 , $A = 0.5202\ 10^{-5}$ m^2

$v_o = 15$m/s , $K = 0.22$, $L = 0.3681$ m, $W_o = 0.2$ rad/s

From equations (4.11)

$$\delta = \left(\frac{2P_o}{\rho A} - K\, v_o^2 \right) \frac{\pi^2}{2w_o^2 L^2} = \left(\frac{2\ 76.22}{7754\ 0.5202\ 10^{-5}} - 0.22\ 15^2 \right) \frac{\pi^2}{2\ 0.2^2\ 0.368}$$

$$= 3395900$$

$$\varepsilon = \frac{Kv_o^2 \pi^2}{4w_o^2 L^2} = \frac{0.22\ 15^2 \pi^2}{4\ 0.2^2\ 0.3681^2} = 22535$$

$$\frac{\delta}{\varepsilon} \simeq 150$$

Note that $1842^2 < 3395900 < 1843^2$ and we are in between the two intersection points and very near to δ axis. Therefore we are in the stable region.

Example-II-Tape Band

A standard VHS video band is considered with data taken from the company RAKS

$P = 0.5N$, $\rho = 1050\ kg/m^3$, $A = 2.34\ 10^{-7}\ m^2$

$K = 1$ $v_o = 23.39\ mm/s$, $L = 14.5\ cm$, $w_o = 0.1\ rad/s$

From equations (4.11)

$$\delta = \left(\frac{2P_o}{\rho A} - K\, v_o^2 \right) \frac{\pi^2}{2w_o^2 L^2} = \left(\frac{2\ 0.5}{1050\ 2.34\ 10^{-7}} - 1\ 0.02339^2 \right) \frac{\pi^2}{2\ 01^2\ 0.145^2}$$

$$= 95\ 527\ 526$$

$$\varepsilon = \frac{K\, v_o^2 \pi^2}{4 w_o^2 L^2} = \frac{1\ 0.02339^2 \pi^2}{4\ 0.1^2\ 0.145^2} \simeq 6.4$$

$$\frac{\delta}{\varepsilon} \simeq 14\ 926\ 176$$

Once again $9773^2 < 95527526 < 9774^2$ and we are in between the two intersection points and very near to δ axis. Therefore we are in the stable region.

These two examples are important because they are working mechanisms without any instability problem. The criteria we derived confirms this expected result.

As a conclusion in these low working speeds instability due to axial acceleration is not a problem. However, in the future new designs may come out with extremely high speeds which may make it necessary for us to use the criteria we have derived above, namely eq. (4.11)

4.2 COMPUTER PROGRAM FOR TWO TERM APPROXIMATION

We rewrite equation (3.16) which is obtained by taking two terms in Galerkin method.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-16v}{3L} \\ \frac{16v}{3L} & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} (\frac{P}{\rho A} - v^2)\frac{\pi^2}{L^2} & \frac{-8v}{3L} \\ \frac{8v}{3L} & (\frac{P}{\rho A} - v^2)\frac{2\pi^2}{L^2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = 0 \tag{4.13}$$

The analytical solution of coupled equations (4.13) is very difficult to obtain, thus a numerical solution is preferred. Our aim is to write the equation in the form of

$$\dot{X} = AX \tag{4.14}$$

by reducing the two second order differential equations to four first order equation. Defining new variables as

$$\begin{aligned} x_1 &= q_1 & x_3 &= \dot{q}_1 \\ x_2 &= q_2 & x_4 &= \dot{q}_2 \end{aligned} \tag{4.15}$$

We finally obtain

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -(\frac{P}{\rho A} - v^2)\frac{\pi^2}{L^2} & \frac{8v}{3L} & 0 & \frac{16v}{3L} \\ \frac{-8v}{3L} & -(\frac{P}{\rho A} - v^2)\frac{2\pi^2}{L^2} & \frac{-16v}{3L} & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} \tag{4.16}$$

Inserting the expressions of v and P from equation (4.1) and (4.2) into equation (4.16) we get

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\left(\frac{2P_o}{\rho A} - Kv_o^2(1-\cos 2w_o t)\right)\frac{\pi^2}{2L^2} & \frac{8v_o w_o \cos w_o t}{3L} & 0 & \frac{16 v_o \sin w_o t}{3L} \\ \frac{-8v_o w_o \cos w_o t}{3L} & -\left(\frac{2P_o}{\rho A} - Kv_o^2(1-\cos 2w_o t)\right)\frac{\pi^2}{L^2} & \frac{-16 v_o \sin w_o t}{3L} & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} \tag{4.17}$$

Equation (4.17) is solved numerically by a computer program written in FORTRAN. The program uses a subroutine from the IMSL library which solves the differential equations by applying the Runge Kutta method of fifth or sixth order. The program is given in the appendix for reference.

We then check the program for the limiting cases, especially the constant velocity case. From [17] we know the solution for this case which is,

$$f_n = \frac{n(P - \rho A v_o^2)}{2L(\rho A P)^{\frac{1}{2}}} \tag{4.18}$$

Substituting the expression (4.1) and (4.2) for v and P

$$f_n = \frac{n(P_o - K\rho A v_o^2)}{2L(\rho A(P + (1-K)\rho A v_o^2))^{\frac{1}{2}}} \tag{4.19}$$

Running the program with different velocities we obtain the corresponding frequencies which are tabulated in table 1.

The results from computer are very close to those from equation (4.19).

V_o (axial velocity)	f (frequency of q_1)
0	59
10	57
20	52
30	45
40	37
50	29
60	22
70	14
80	8
92.67	0

Table 1- Axial velocity versus frequency of q_1

Note that 92.67 is the critical velocity for which frequency comes out to be zero. It is found by equating equation (4.19) to zero.The data of table 1 is shown graphically in figure 6.

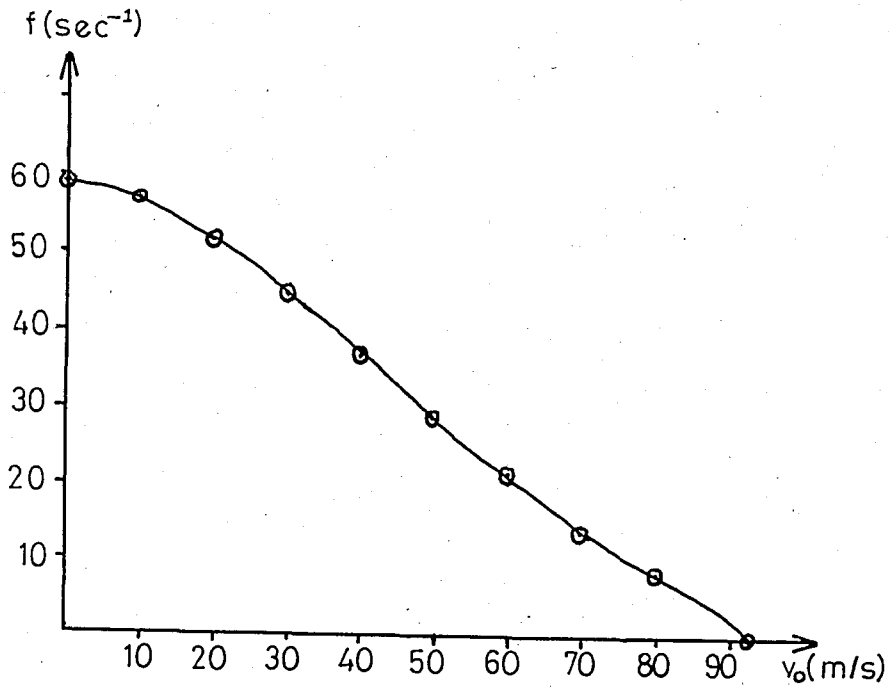


Figure 6- Fundamental Natural Frequency versus Axial Velocity.

4-3. RESULTS FOR TWO TERM APPROXIMATION

Equation (4.17) will be solved numerically using the parameters given in [4], where these parameters are shown in table 2.

Standard Parameters	
P_o	76.22 N
ρ	7754 kg/m^3
A	0.5202×10^{-5} m^2
K	0.22
L	0.3681

Table 2- Standard parameters for numeriacal analysis, [4]

We run the program for different axial velocity and frequency values and decide whether the system is stable or unstable for that values. In order to compare the results with the one term case we proceed in the following way. Rewriting equations (4.11) which are the definitions of δ and ε

$$\delta = \left(\frac{2P_o}{\rho A} - Kv_o^2 \right) \frac{\pi^2}{2w_o^2 L^2} \qquad \varepsilon = \frac{Kv_o^2 \pi^2}{4w_o^2 L^2} \tag{4.20}$$

These expressions can be solved for w_o and v_o, yielding

$$V_o = 2\sqrt{\frac{P_o \varepsilon}{(\delta+2\varepsilon)\rho AK}} \qquad w_o = \pi\sqrt{\frac{P_o}{\rho AL^2(\delta+2\varepsilon)}} \tag{4.21}$$

For specific δ and ε values we are now able to find the corresponding v_o and w_o values and running the program with the values obtained we find whether the point is stable or unstable. The data obtained in this way is presented in figure 7. Comparing figure 7 with figure 5 we can easily state that two solutions differ greatly from each other. As can be seen from figure 7 all the points except one came out to be stable. We then concentrate our study in the small shaded area in figure 7. This small area corresponds to high frequency values and velocity values that are close to the critical speed. The area is shown in detail in figure 8. As can be seen we obtained three instability regions. Computer drawn graphs corresponding to some of these points are obtained. Figures (9-18) are the response plots of stable points and figures

(19-28) are the response plots of unstable points. In stable points we observe that the amplitude doesn't increase. When the velocity is too low, the graph of q_1 becames sinusodial and amplitude of q_2 is rather low as shown in figure 13. Examing the unstable points we see that they differ from each other. The amplitudein figures 19,25 and 27 increase slowly whereas for the others the increase is rather fast.

As mentioned earlier, the analytical solution of equation (4.13) is rather comlex. However in [16] a solution is presented for a special case. In this reference the equation to be solved is of the form,

$$\frac{d^2x}{dt^2} + \varepsilon C(t) \frac{dx}{dt} + (B^{(o)} + \varepsilon B(t)) x = 0 \tag{4.22}$$

where $\varepsilon > 0$ is a small parameter. Comparing (4.13) with (4.22) we define the parameter ε as

$$\varepsilon = \frac{V_o}{L} \tag{4.23}$$

and the matrices are defined as

$$C(t) = \begin{bmatrix} 0 & \frac{-16}{3} \\ \frac{16}{3} & 0 \end{bmatrix} \sin w_o t \tag{4.24}$$

$$B^{(o)} = \begin{bmatrix} \left(\frac{2P_o}{\rho A} - Kv_o^2\right)\frac{\pi^2}{2L^2} & 0 \\ 0 & \left(\frac{2P_o}{\rho A} - Kv_o^2\right)\frac{\pi^2}{L^2} \end{bmatrix} \tag{4.25}$$

$$B(t) = \begin{bmatrix} \frac{Kv_o\pi^2}{2L} & \frac{-8w_o}{3} \\ \frac{8w_o}{3} & \frac{Kv_o\pi^2}{L} \end{bmatrix} \cos 2w_o t \tag{4.26}$$

According to [16] for small ε instability <u>may</u> occur at excitation frequencies in the neighborhoods of

$$\frac{w_i + w_j}{s} \quad , \quad \frac{w_j - w_i}{s} \qquad i,j,s = 1,2 \tag{4.27}$$

In our case from eq.(4.23) v_o should be small enough and that will lead to points very close to the δ axis. w_1^2 and w_2^2 are equal to the terms in the diagonal of $B^{(o)}$ matrix. From equation (4.25)

$$w_1^2 = \left(\frac{2P_o}{\rho A} - Kv_o^2\right)\frac{\pi^2}{2L^2} \tag{4.28}$$

$$w_2^2 = \left(\frac{2P_o}{\rho A} - Kv_o^2\right) \frac{\pi^2}{L^2} \tag{4.29}$$

For low velocities and frequencies given in (4.27) the corresponding δ and ε values are calculated and given in Table 3.

s	w_o	δ	ε
	1050	0.125	0.001
1	742	0.25	0.001
	448	0.686	0.001
	154	5.8	0.001
	525	0.5	0.001
2	371	1.0	0.001
	224	2.74	0.001
	77	23.2	0.001

Table 3- δ and ε values where instability may occur.

Results from analytical solutions are compared with the numerical solutions and we see that they agree with each other. From figure 8 we see that there is an instability area beginning from $\delta = 0.5$ and arising to the left. Also a narrow band is observed in the upper part of $\delta = 1$. We also wondered if the instability area at the top has a connection with the points just above $\delta = 2.74$. The numerical analysis in that region show that there is no link in between.

To summarize the chapter first we can state that the analytical solution given in [16] shows reasonable aggrement with our numerical solution. We also see that the results of two-term approximation is different from that of one-term approximation. The coupling terms may be effective in making the system more stable in the two term approximation case. Hence to depend only on the results of one-term approximation may lead to wrong results.

V- SUMMARY AND CONCLUSIONS

We investigated the transverse vibrations of an axially accelerating string. First equations of motion are derived.The partial differential equation governing the motion is discretized by Galerkin's method. First we take one term in Galerkins method and finally obtain the Mathieu equation. Then we take two terms and solve the resulting coupled ordinary differential equations by numerical methods. The results for both cases are presented.

Comparing the results of the two cases we conclude that they differ significantly. In the one term case the solution reduces to Mathieu stability and instability areas as shown in figure 5. The solution of the two-term case as shown in figures 7 and 8 are different. The coupling terms in the two term approximation may cause the system to be more stable. However our numerical solution gives approximately same results with the analytical solution presented in[16].Hence using two-terms in Galerkins method gives better results. In general we observe that instability occurs at high frequency and velocities that are close to critical speed. These values are high when compared with that of the working mechanisms such as band-saws and tape bands. However in future new design may come out with extremely high speeds and frequencies which may make it necessary to use the results obtained in this study.

Additional work can be done on the subject. For example we have chosen a sinusoidal function for axial velocity. Different axial velocity functions can be investigated in the same way. In the study, Galerkins method is applied for n-terms but only the one and two term approximations are examined. The solutions of higher order approximations can also be made and results can be compared with those of one-term and two-term approximations. Experimental studies would,of course, also be desirable.

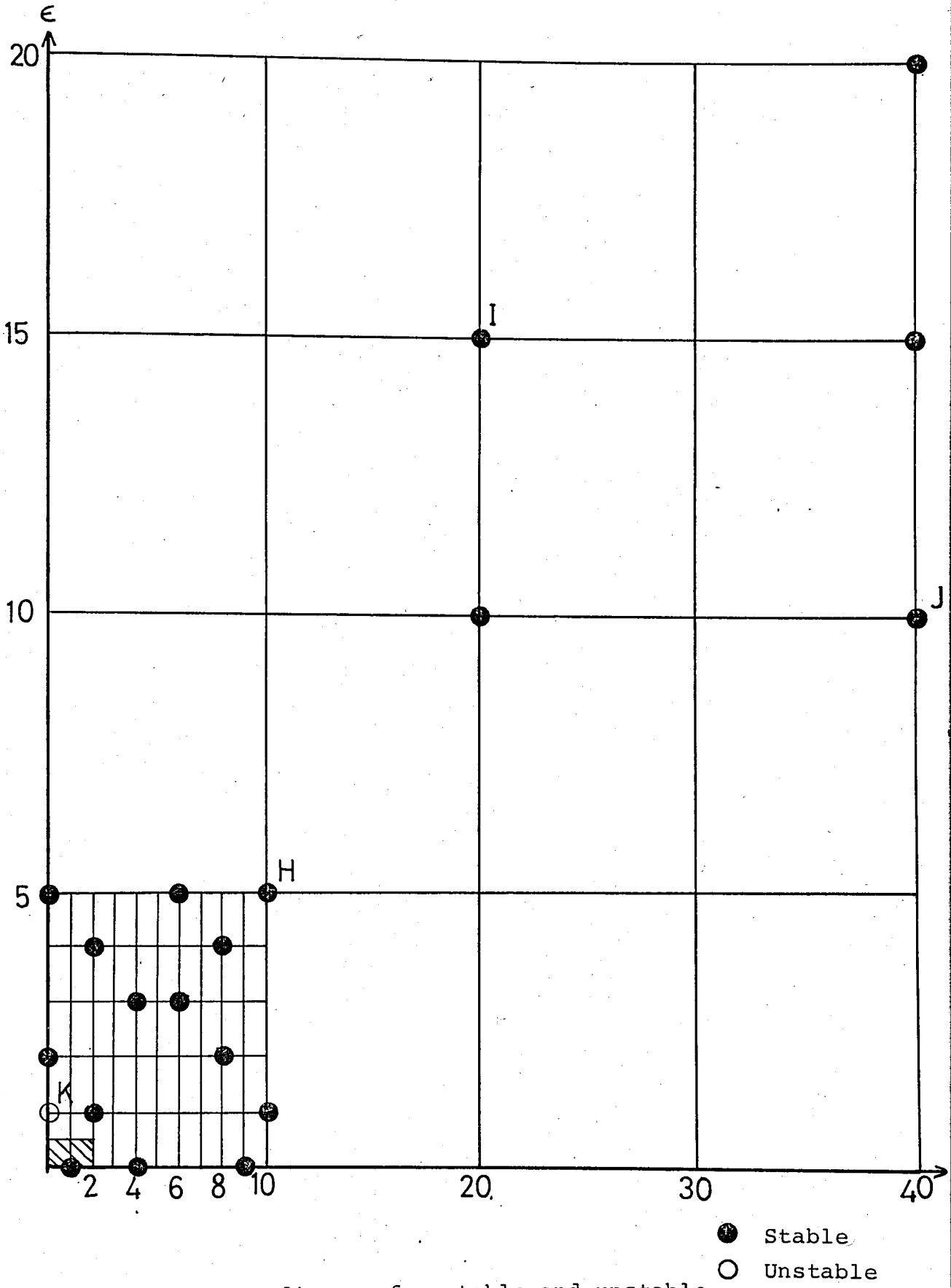
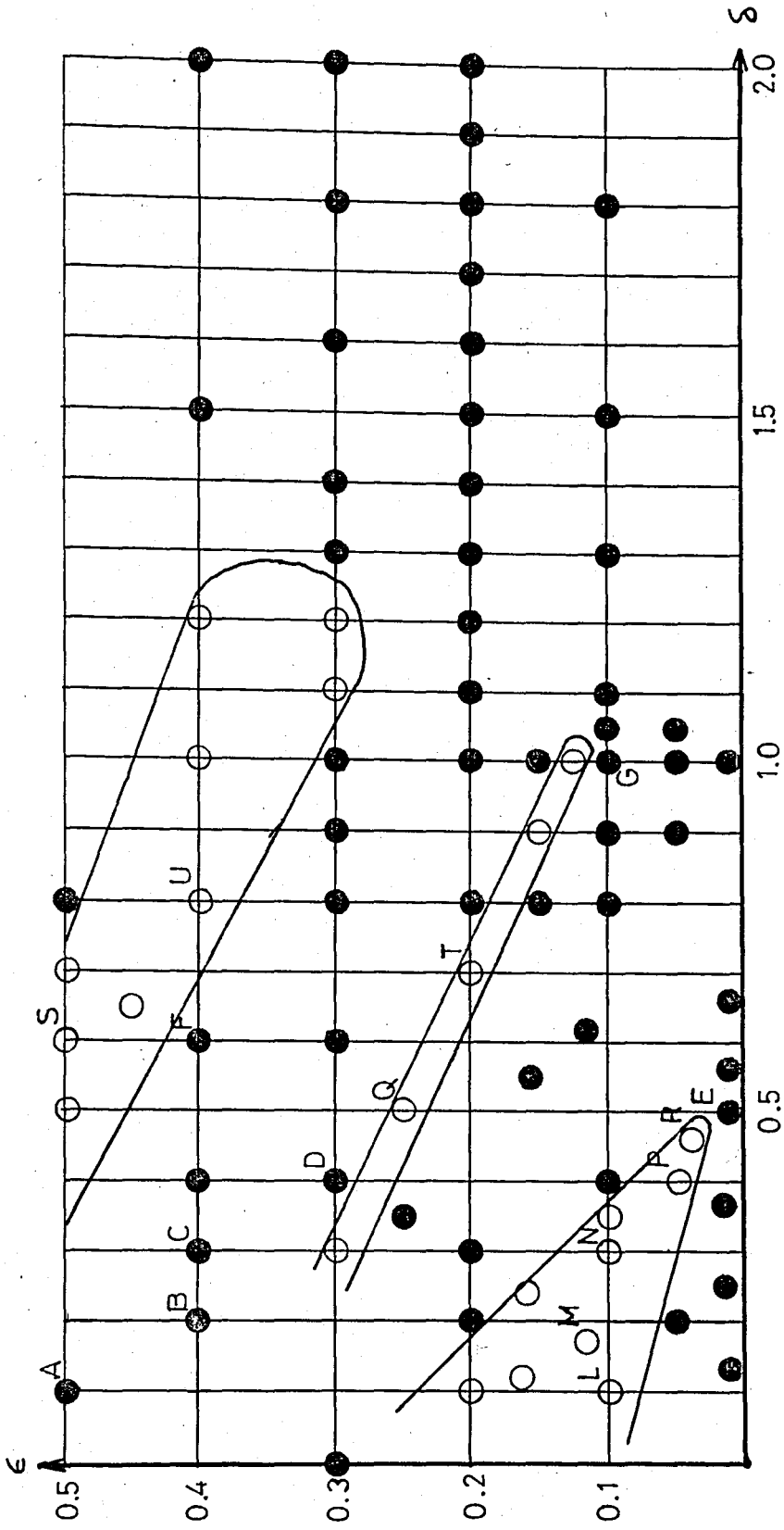


Figure 7- strutt diagram for stable and unstable points.



● Stable

○ Unstable

Figure 8- Strutt diagram for stable and unstable points.

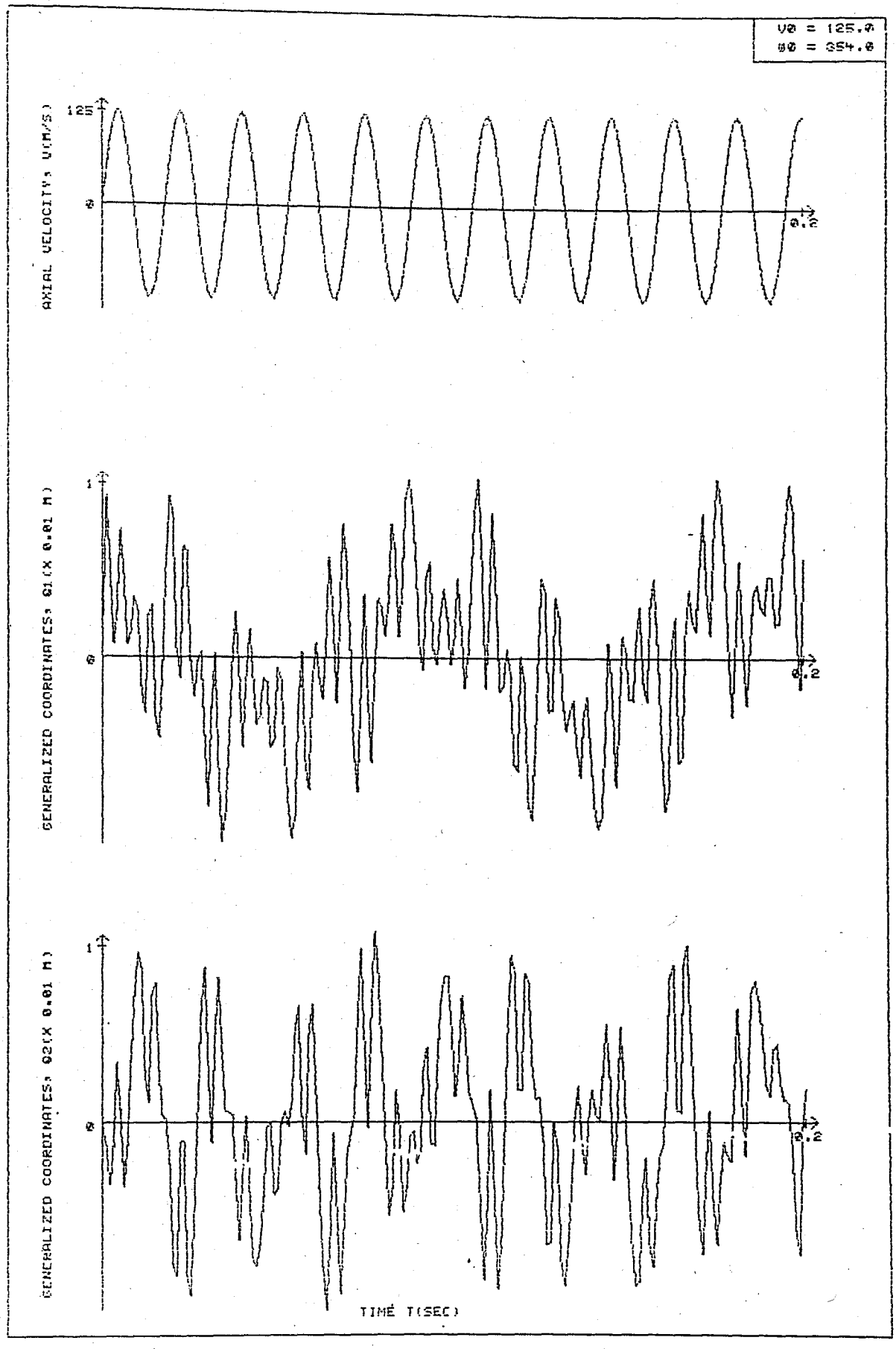


Figure 9- Generalized coordinates and velocity functions for point A(Stable)

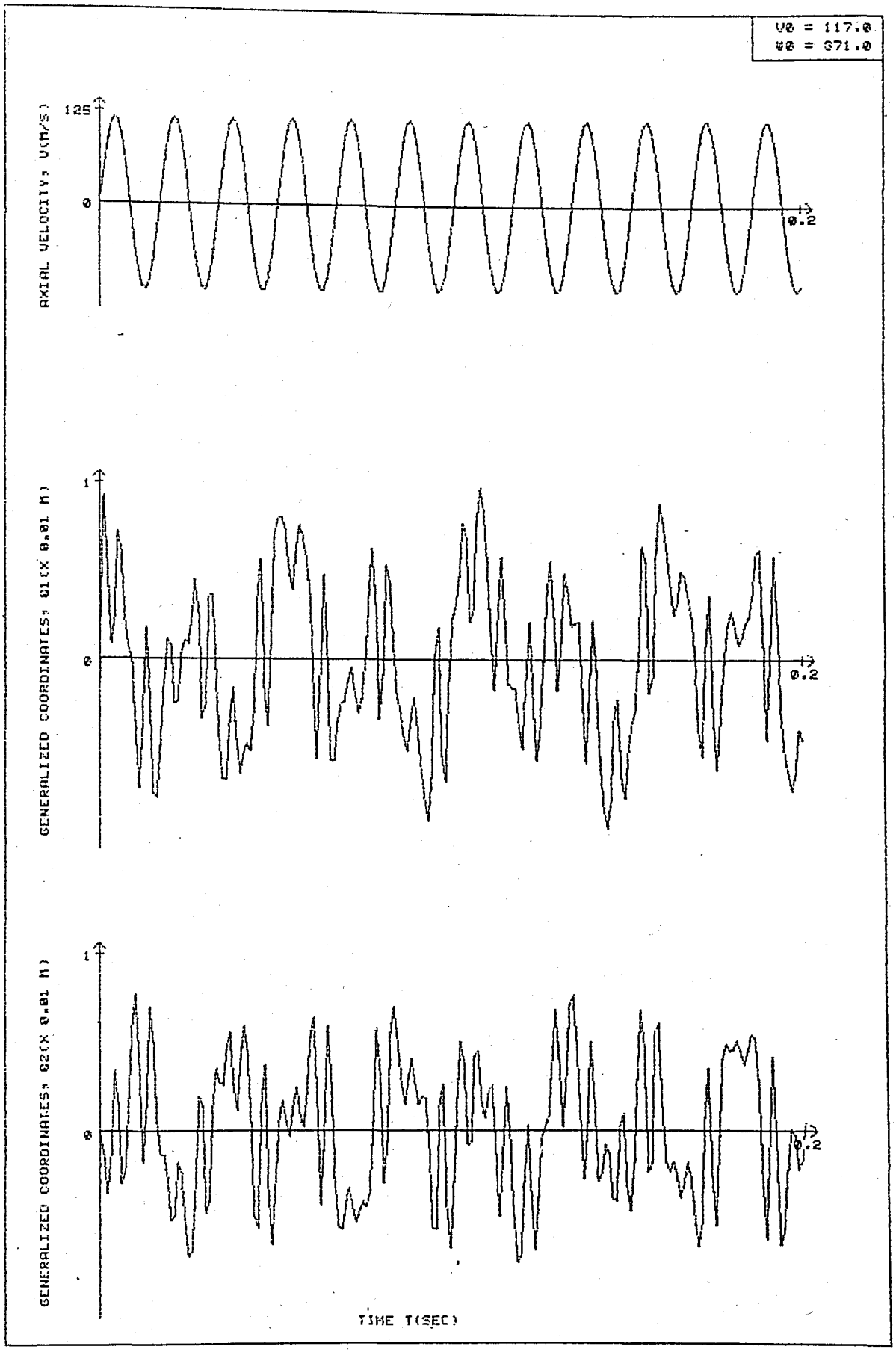


Figure 10- Generalized coordinates and velocity functions for point B(Stable)

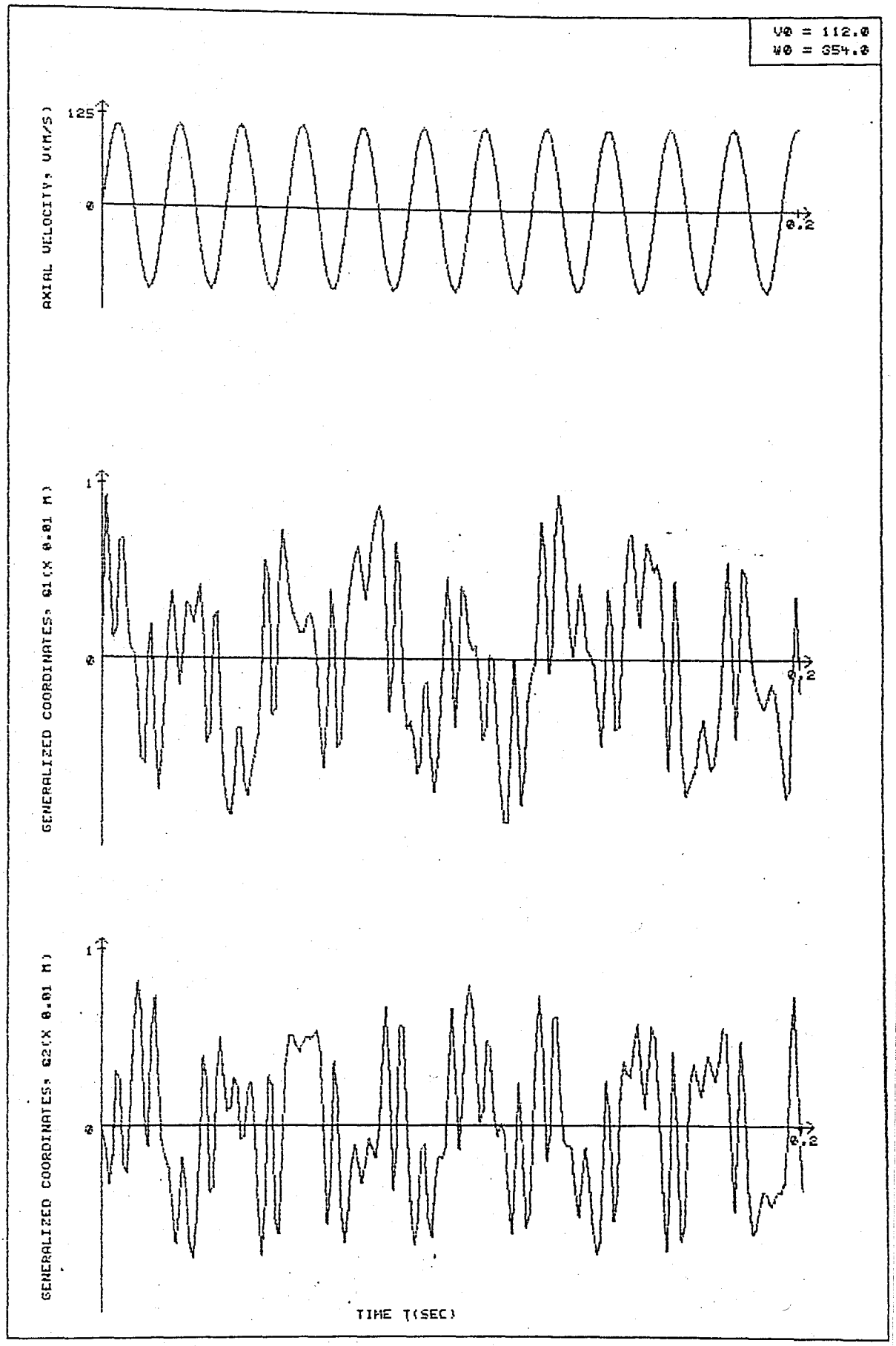


Figure 11- Generalized coordinates and velocity functions for point C(Stable)

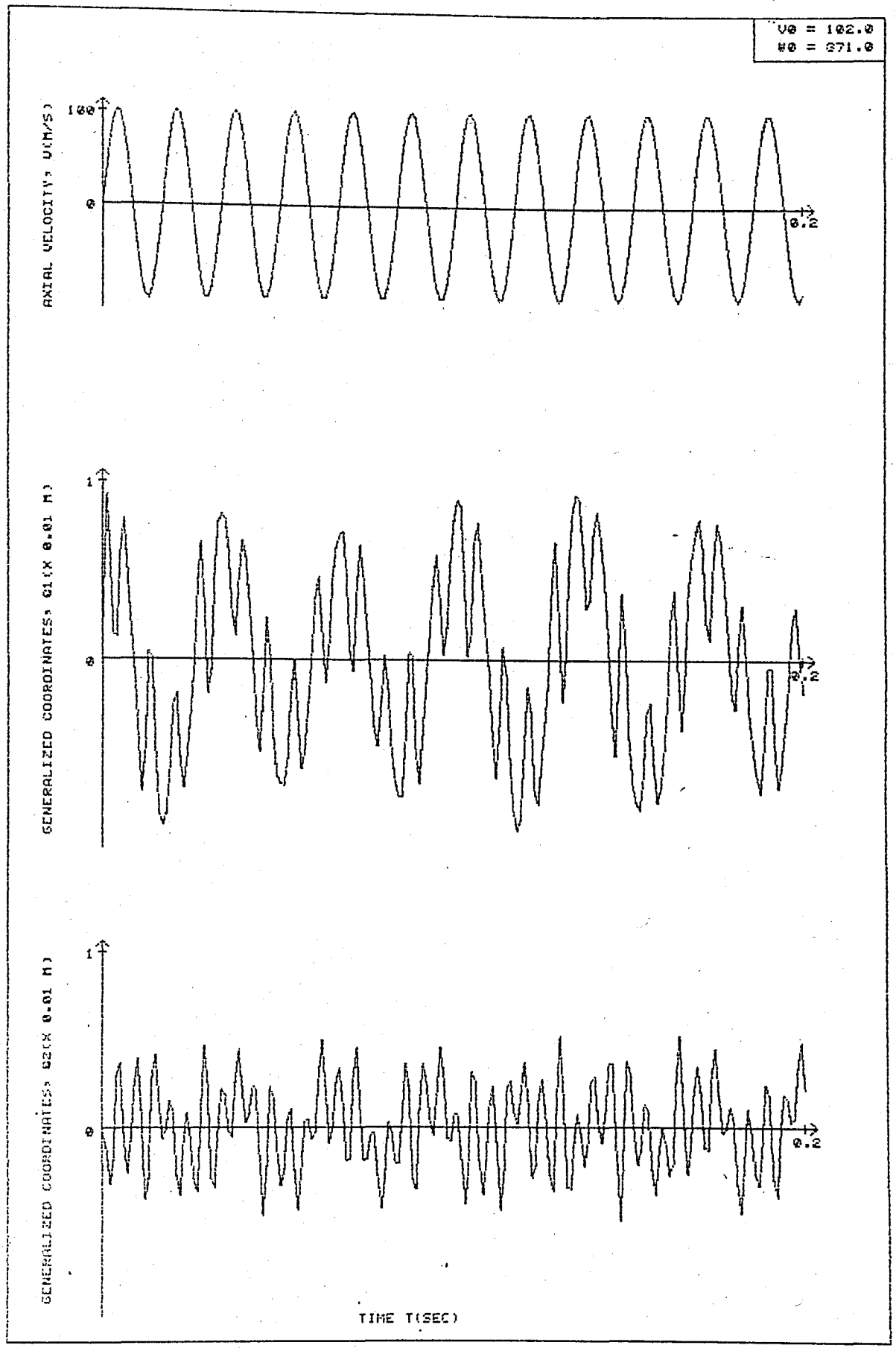


Figure 12- Generalized coordinates and velocity functions for point D(Stable)

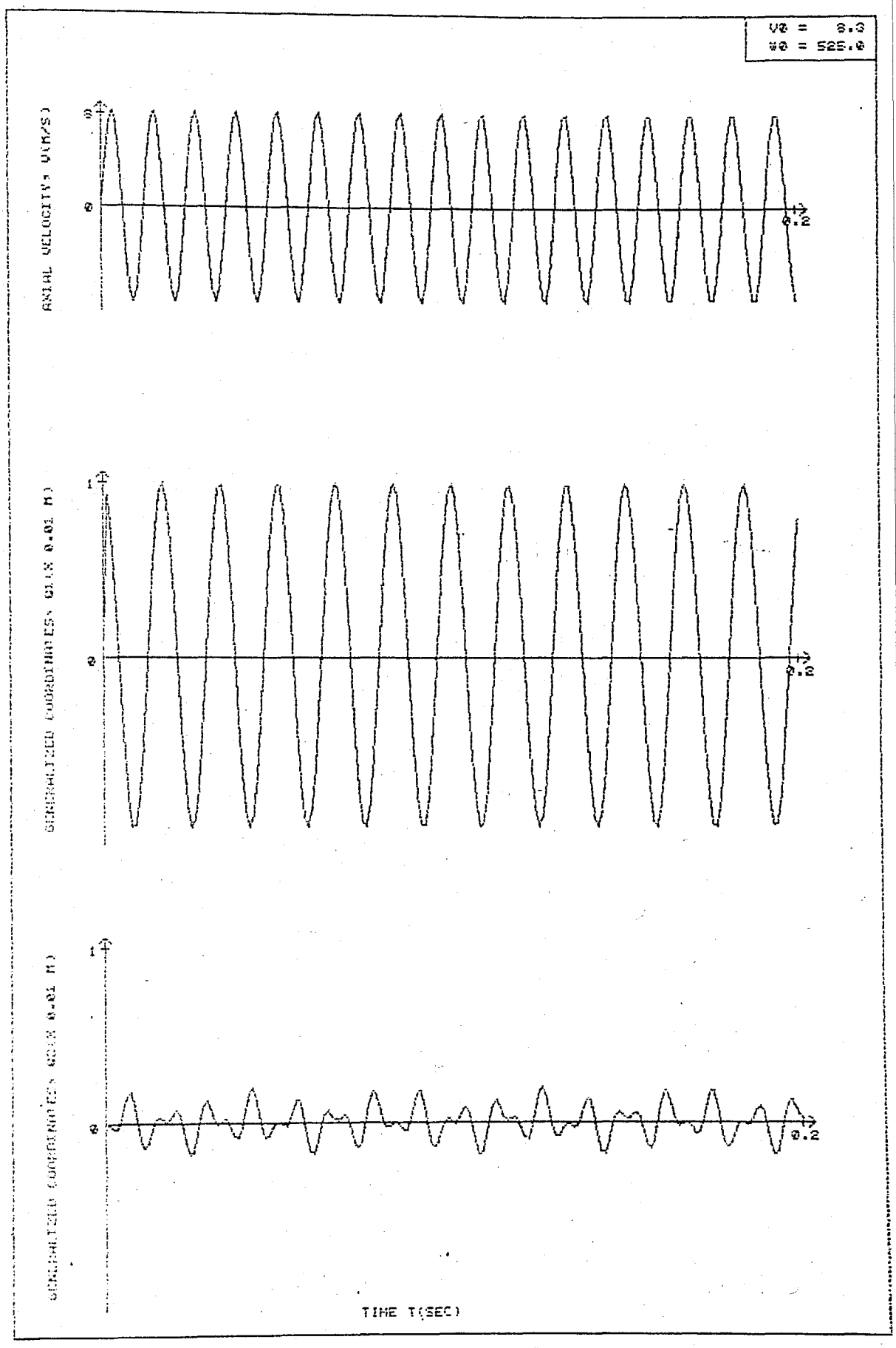


Figure 13- Generalized coordinates and velocity functions for point E(Stable)

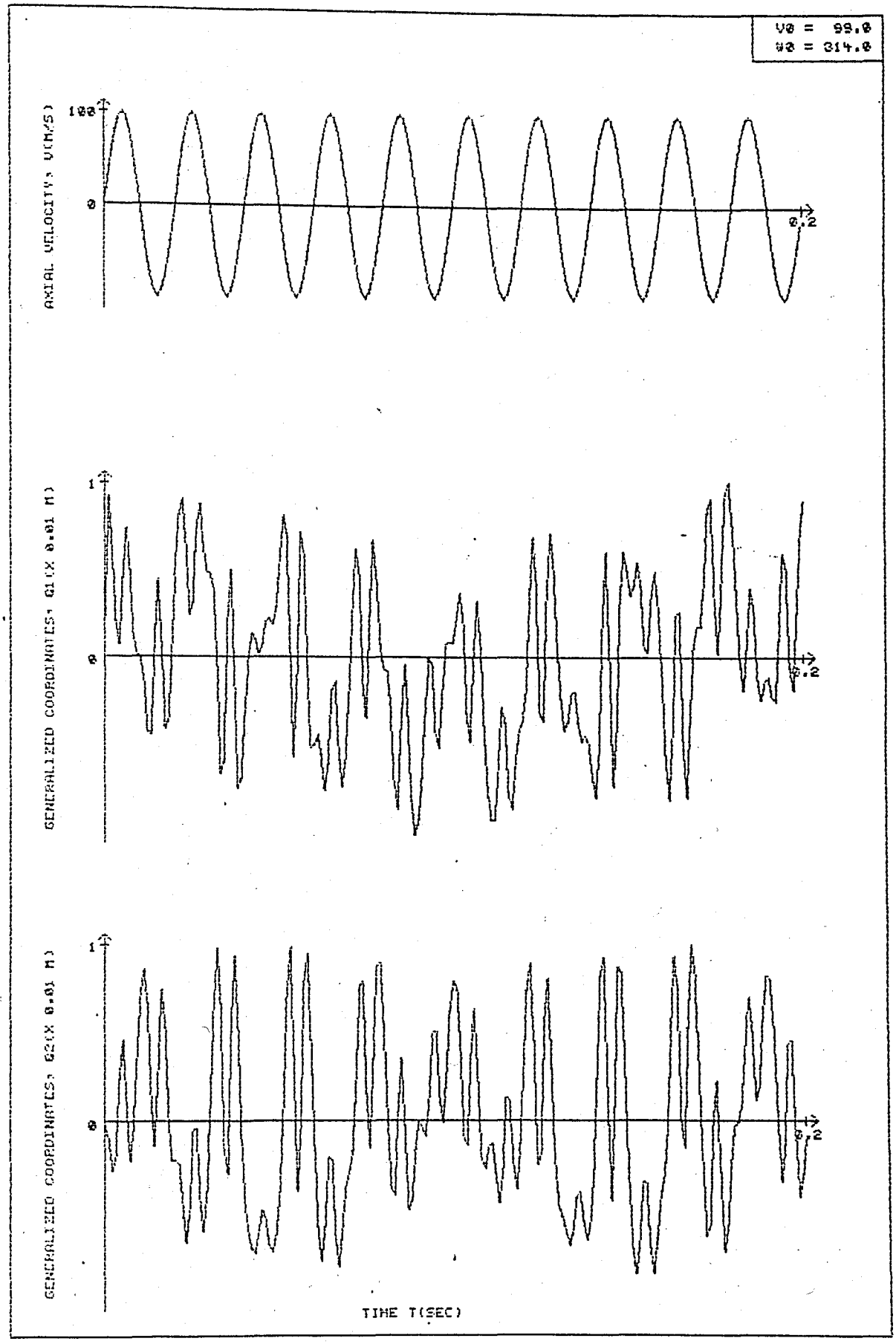


Figure 14- Generalized coordinates and velocity functions for point F(Stable)

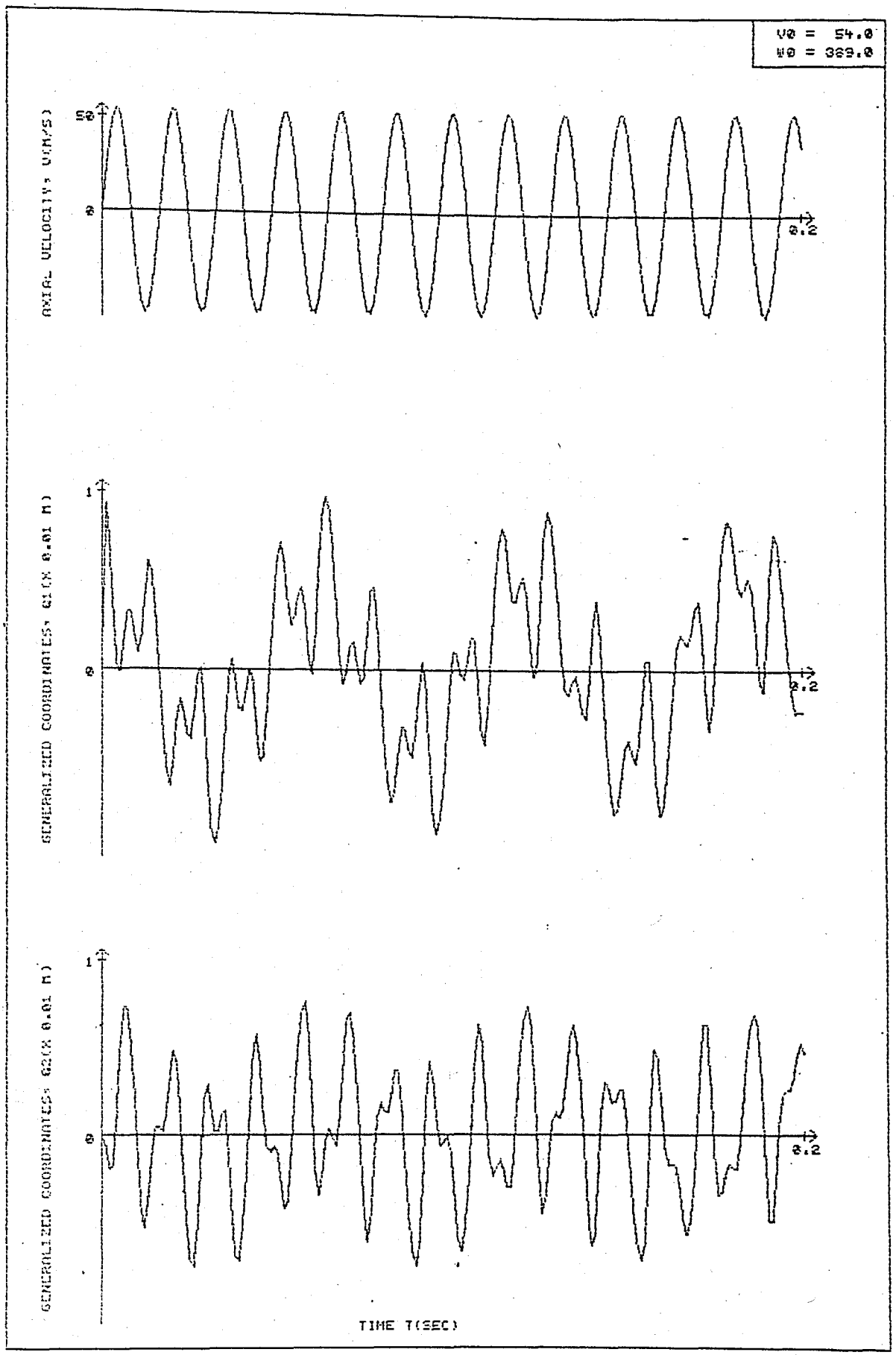


Figure 15- Generalized coordinates and velocity functions for point G(Stable)

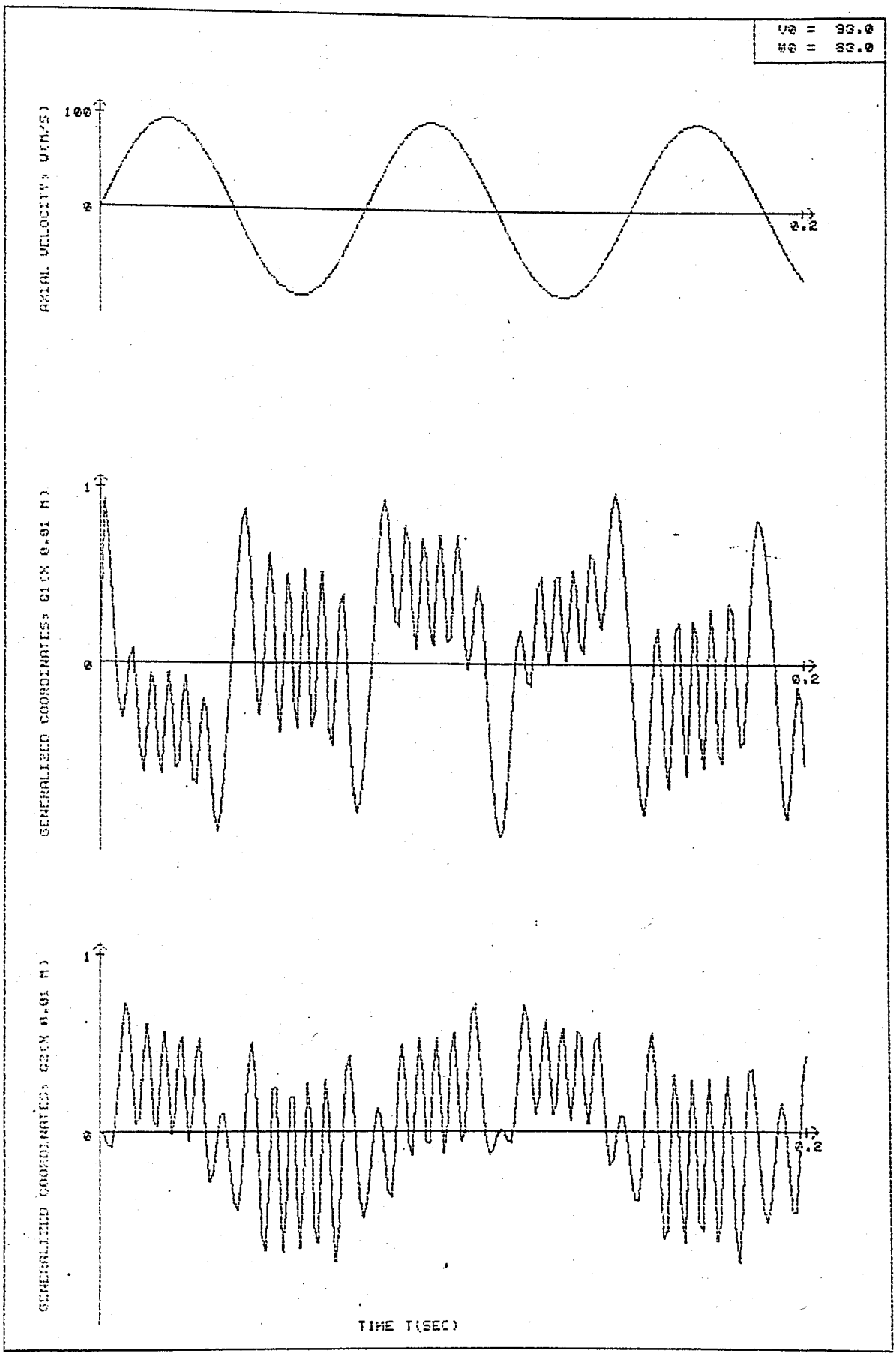


Figure 16- Generalized coordinates and velocity functions for point H(Stable)

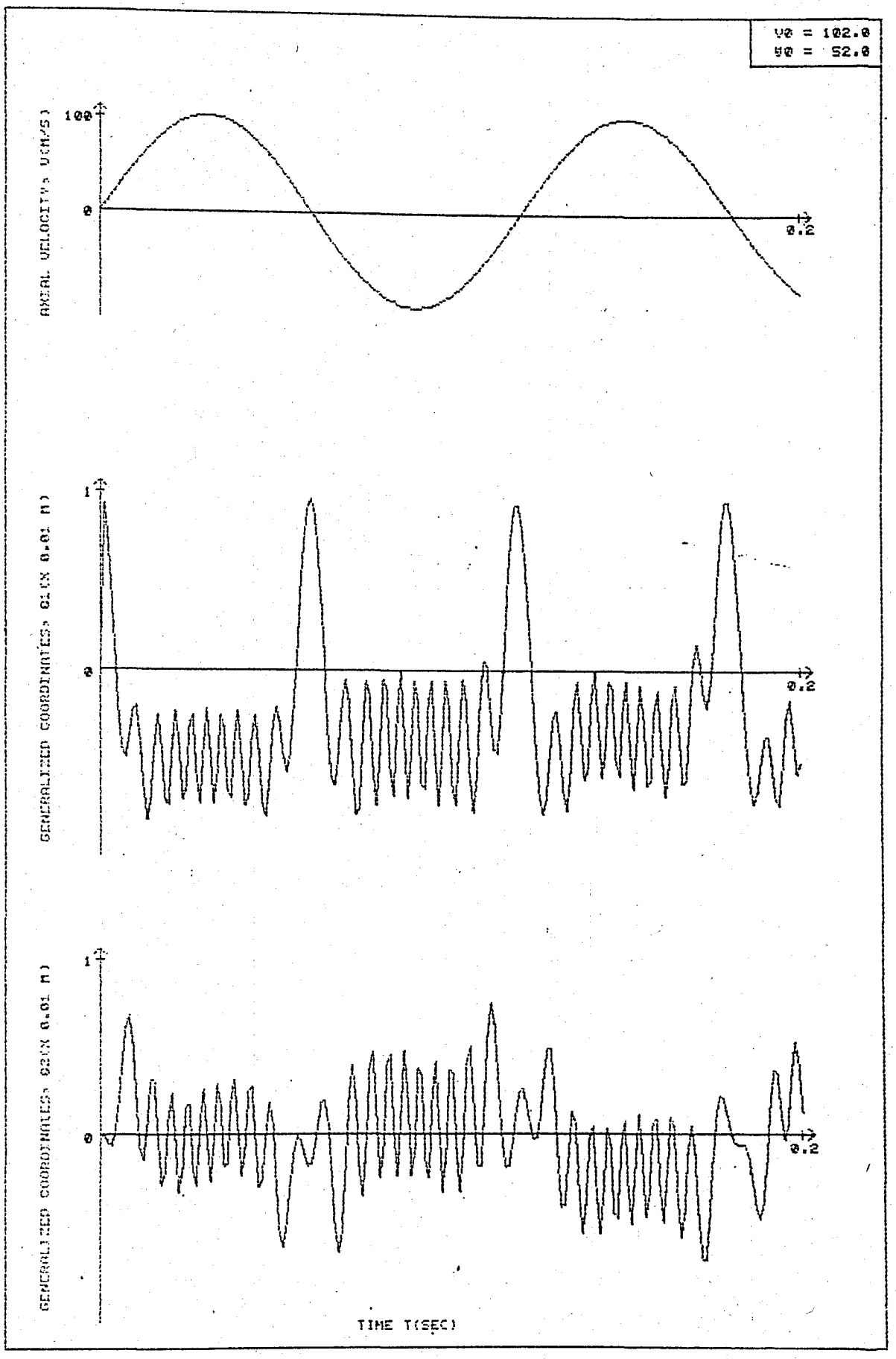


Figure 17- Generalized coordinates and velocity functions for point I(Stable)

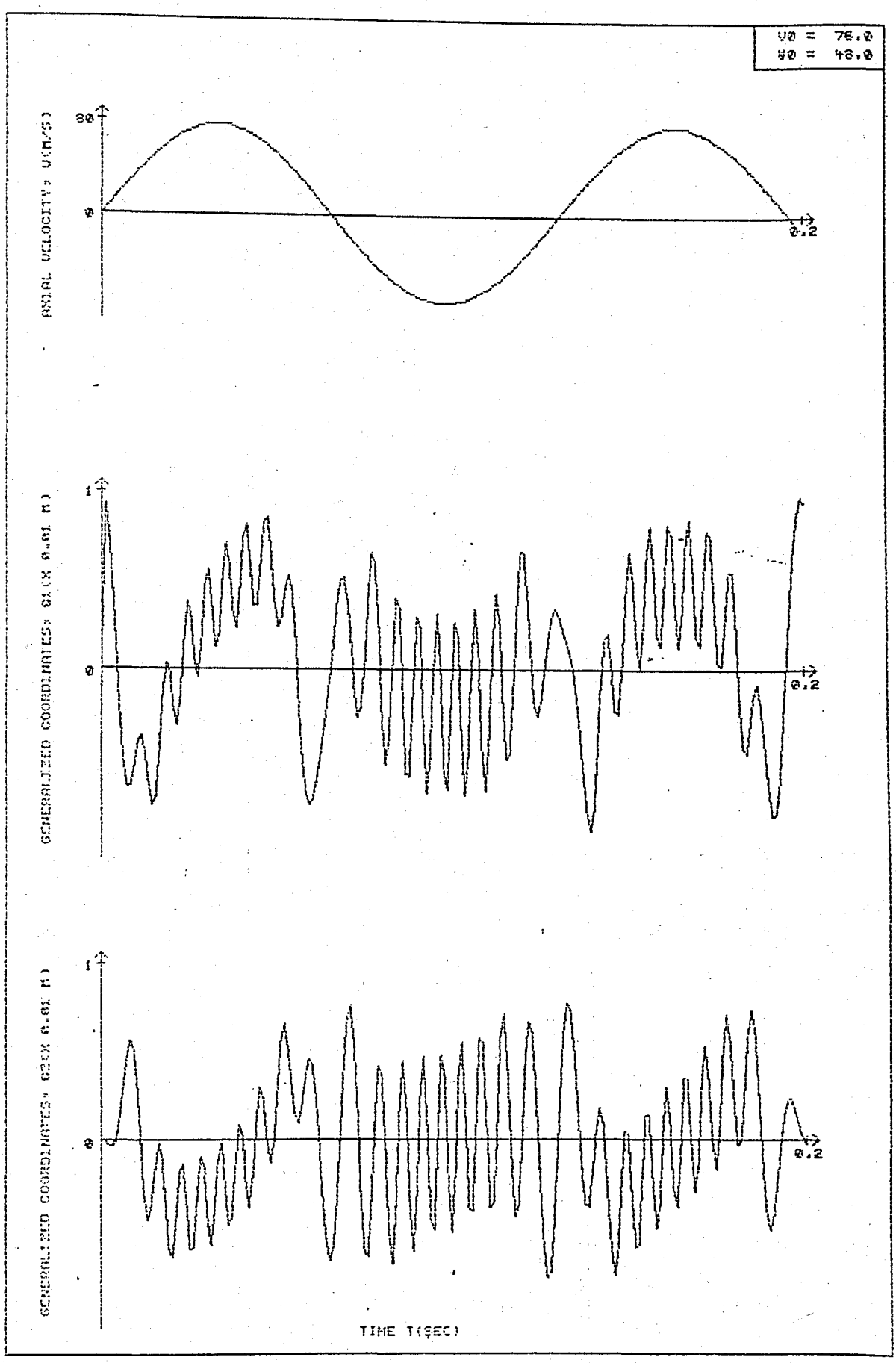


Figure 18- Generalized coordinates and velocity functions for point J(Stable)

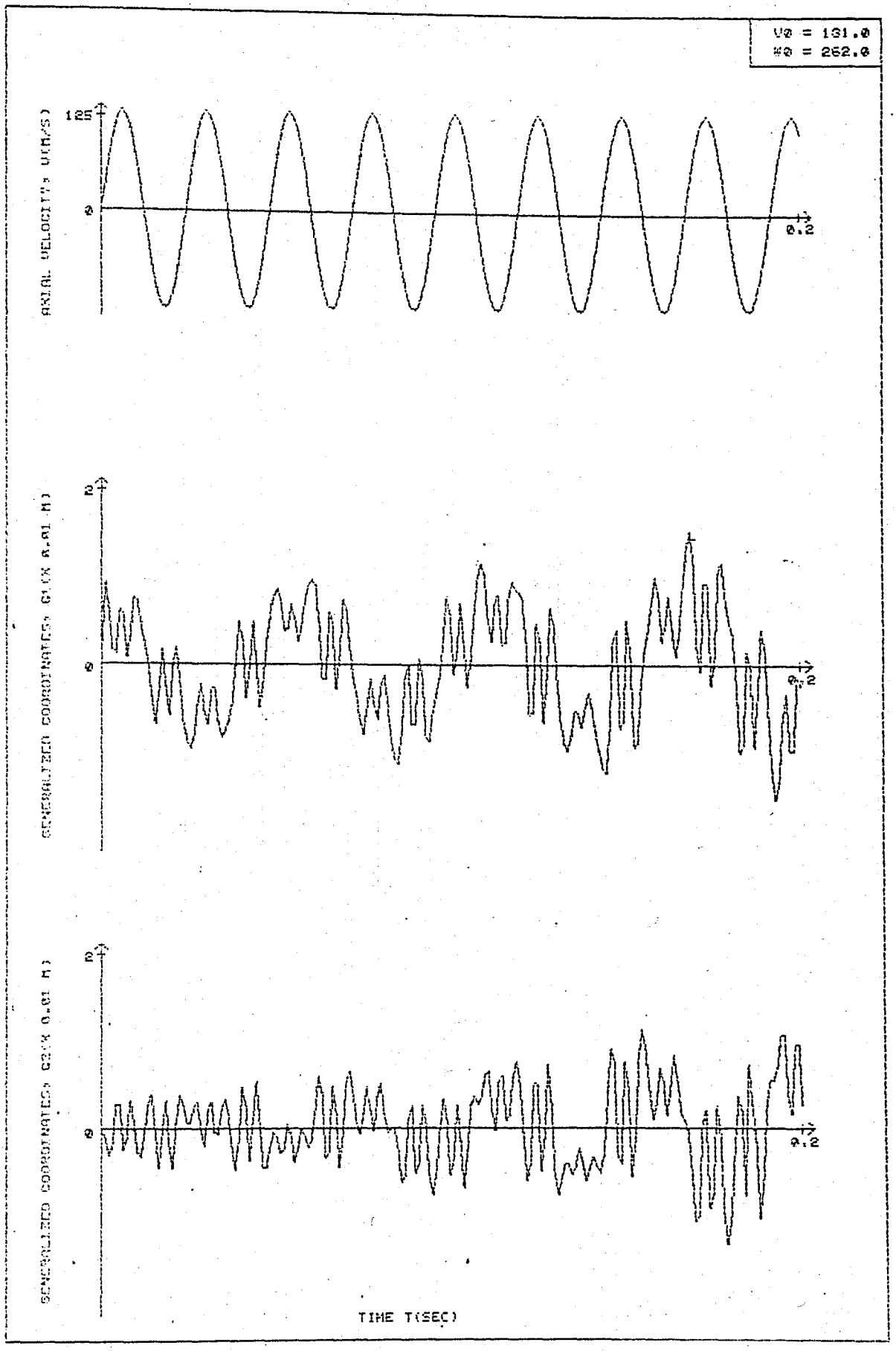


Figure 19- Generalized coordinates and velocity functions for point K(Unstable)

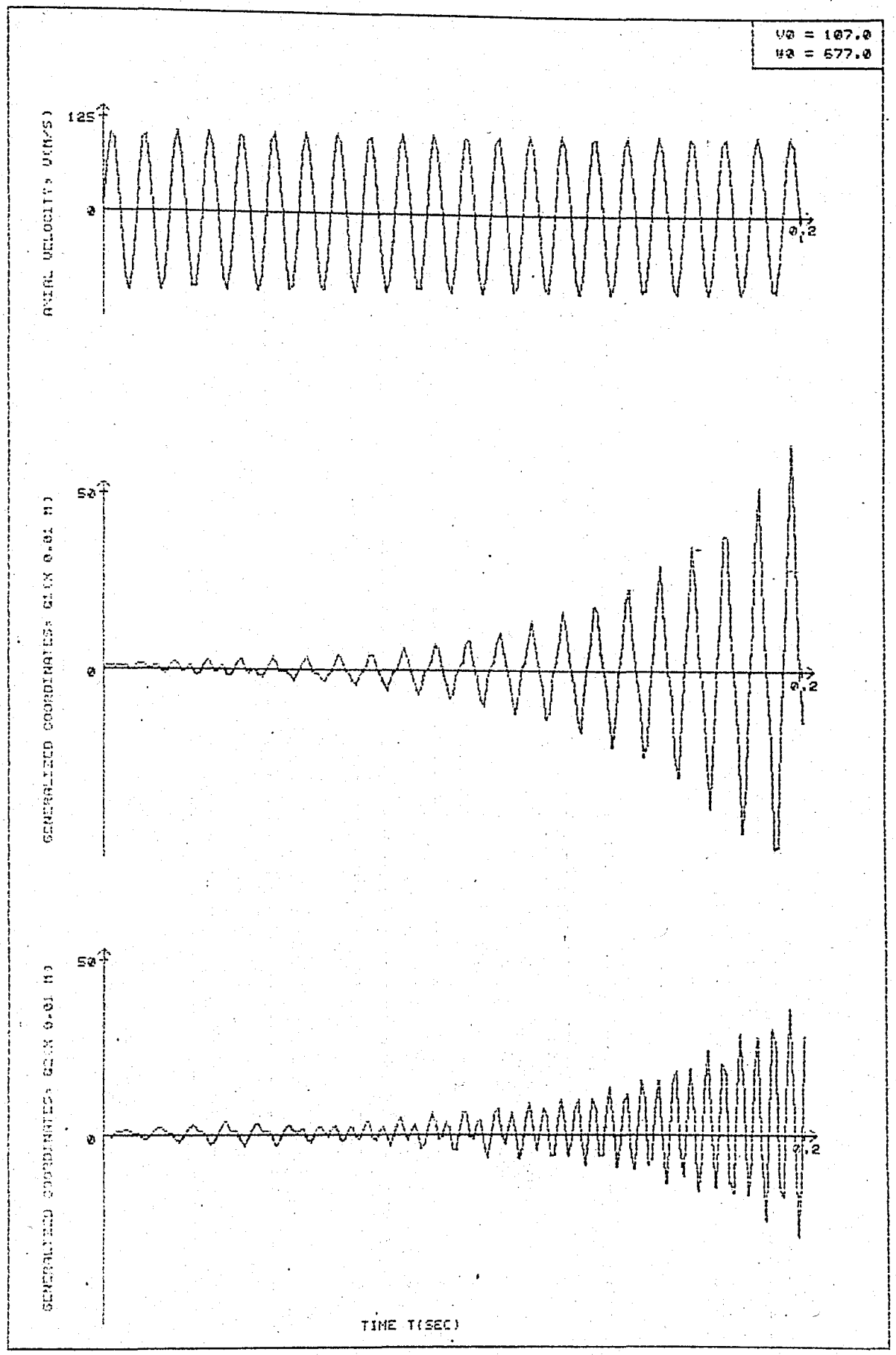


Figure 20- Generalized coordinates and velocity functions for point L(Unstable)

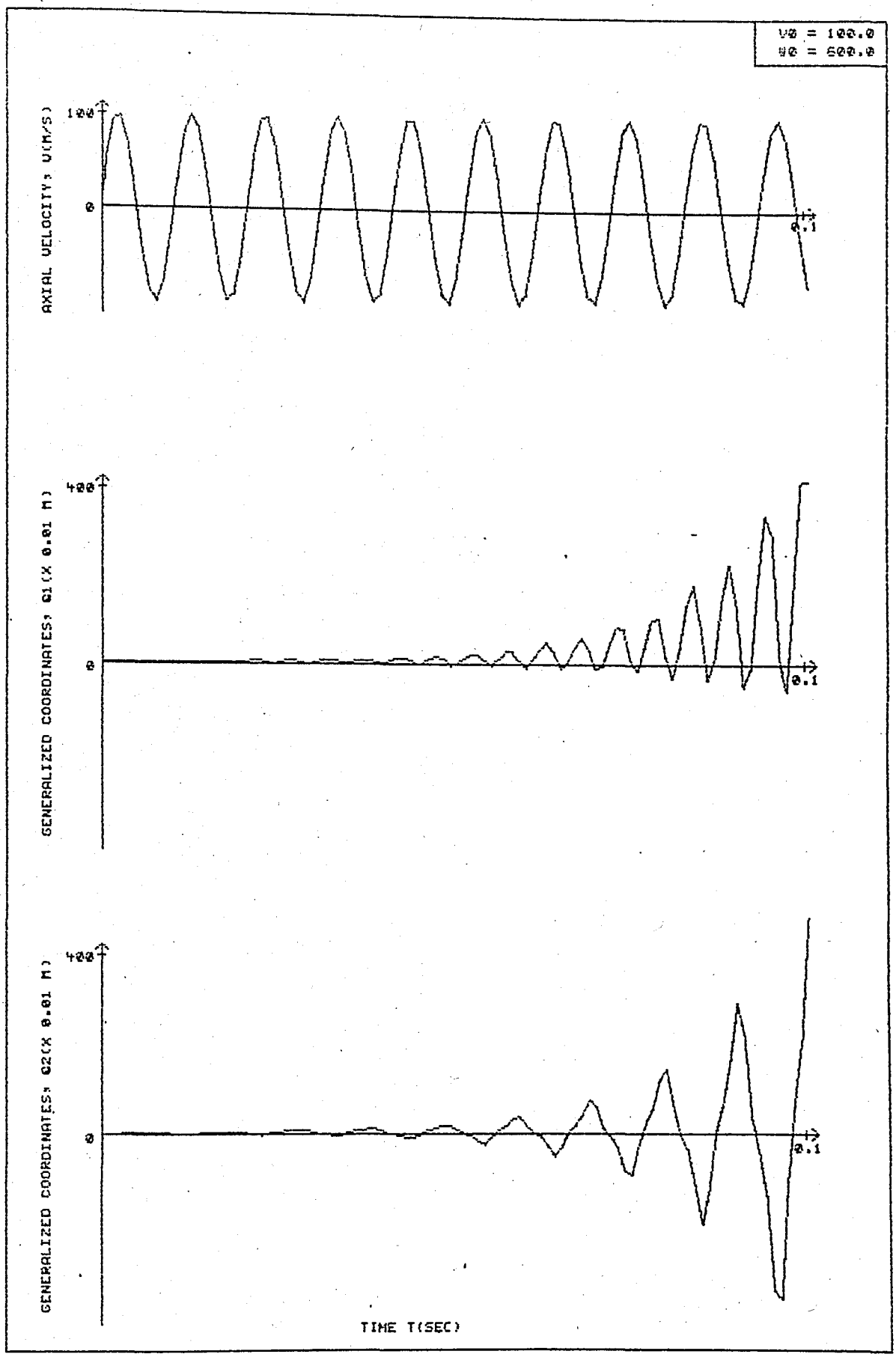


Figure 21- Generalized coordinates and velocity functions for point M(Unstable)

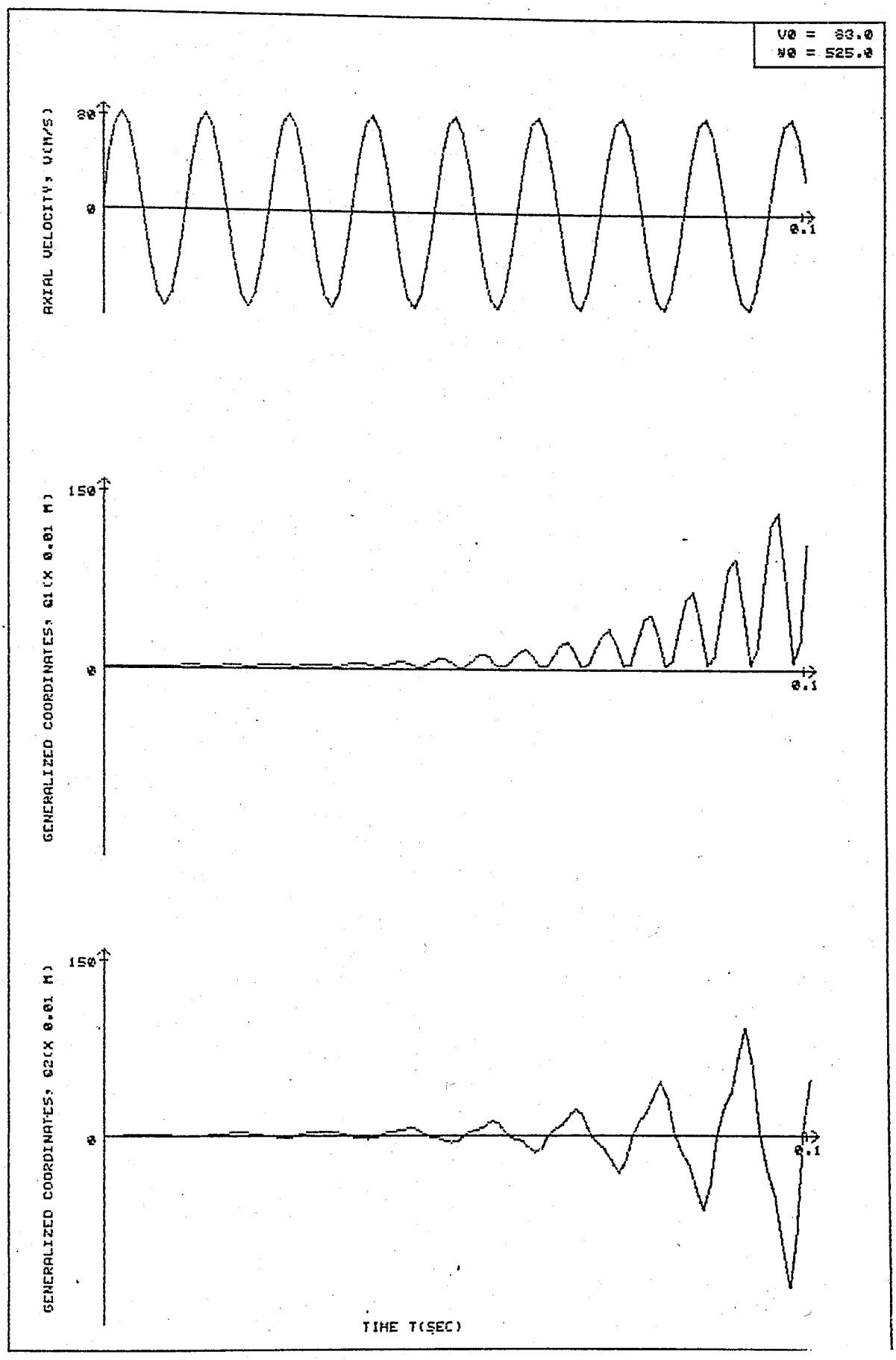


Figure 22- Generalized coordinates and velocity functions for point N(Unstable)

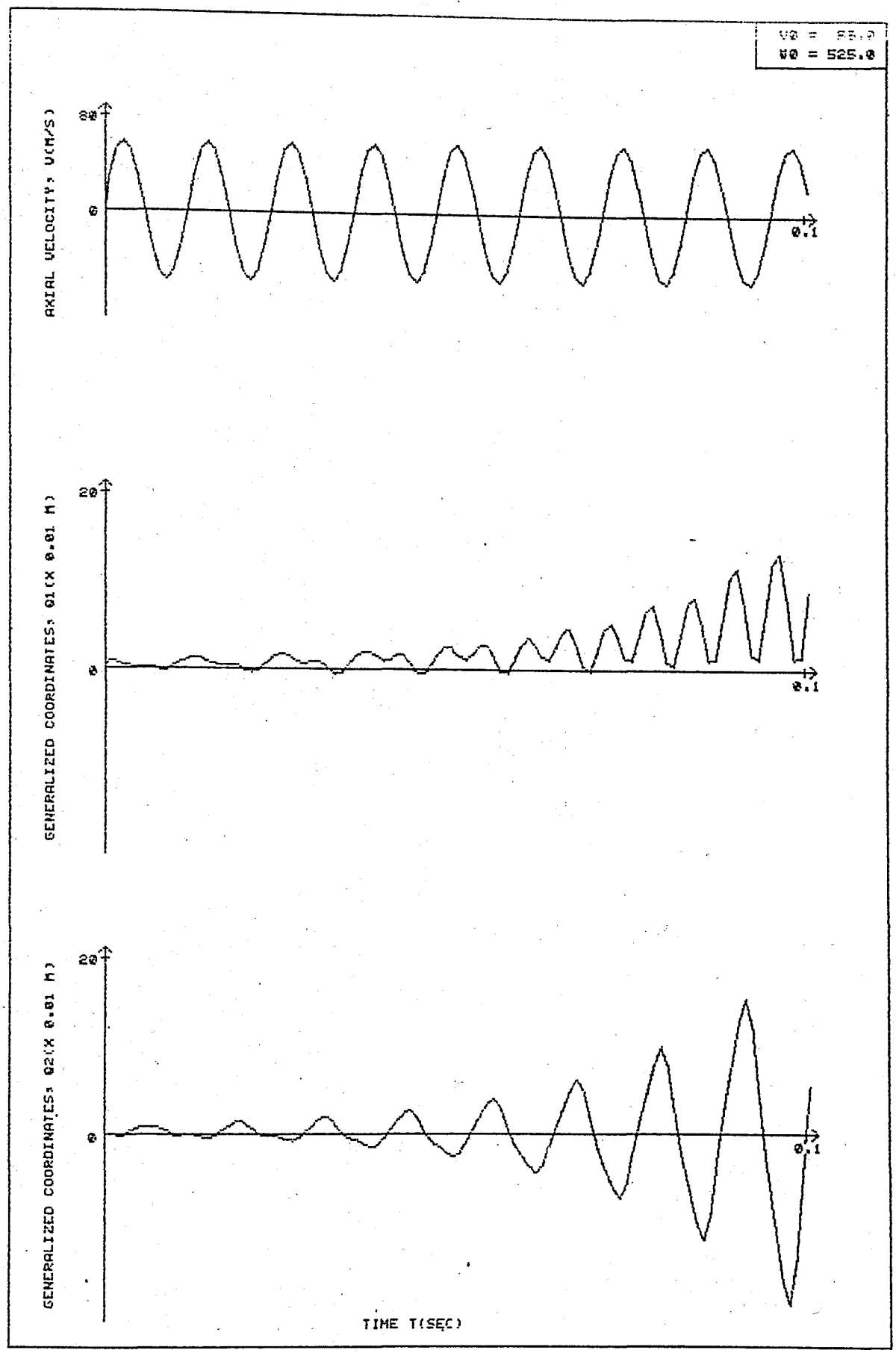


Figure 23- Generalized coordinates and velocity functions for point P(Unstable)

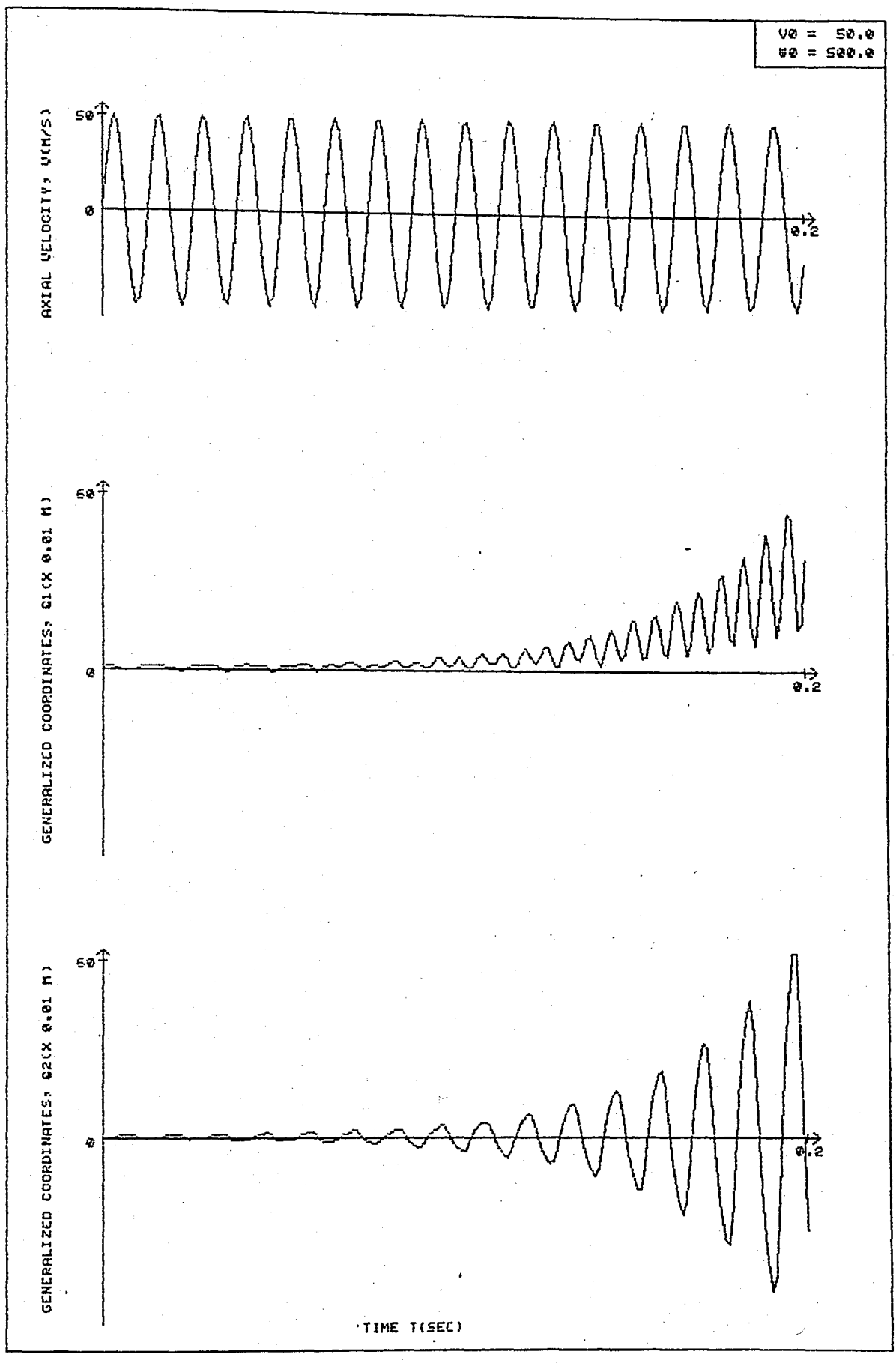


Figure 24- Generalized coordinates and velocity functions for point Q(Unstable)

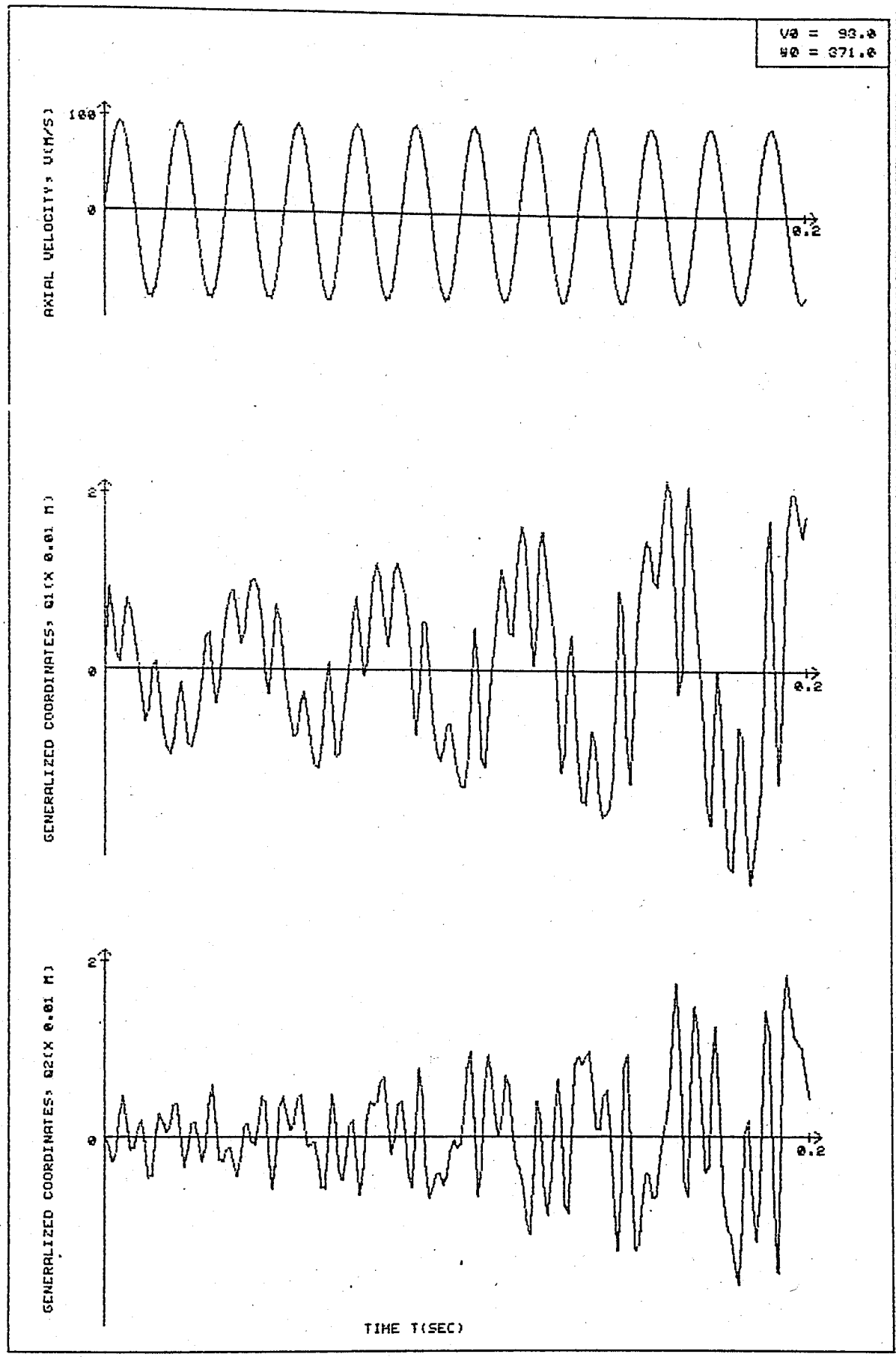


Figure 25- Generalized coordinates and velocity functions for point R(Unstable)

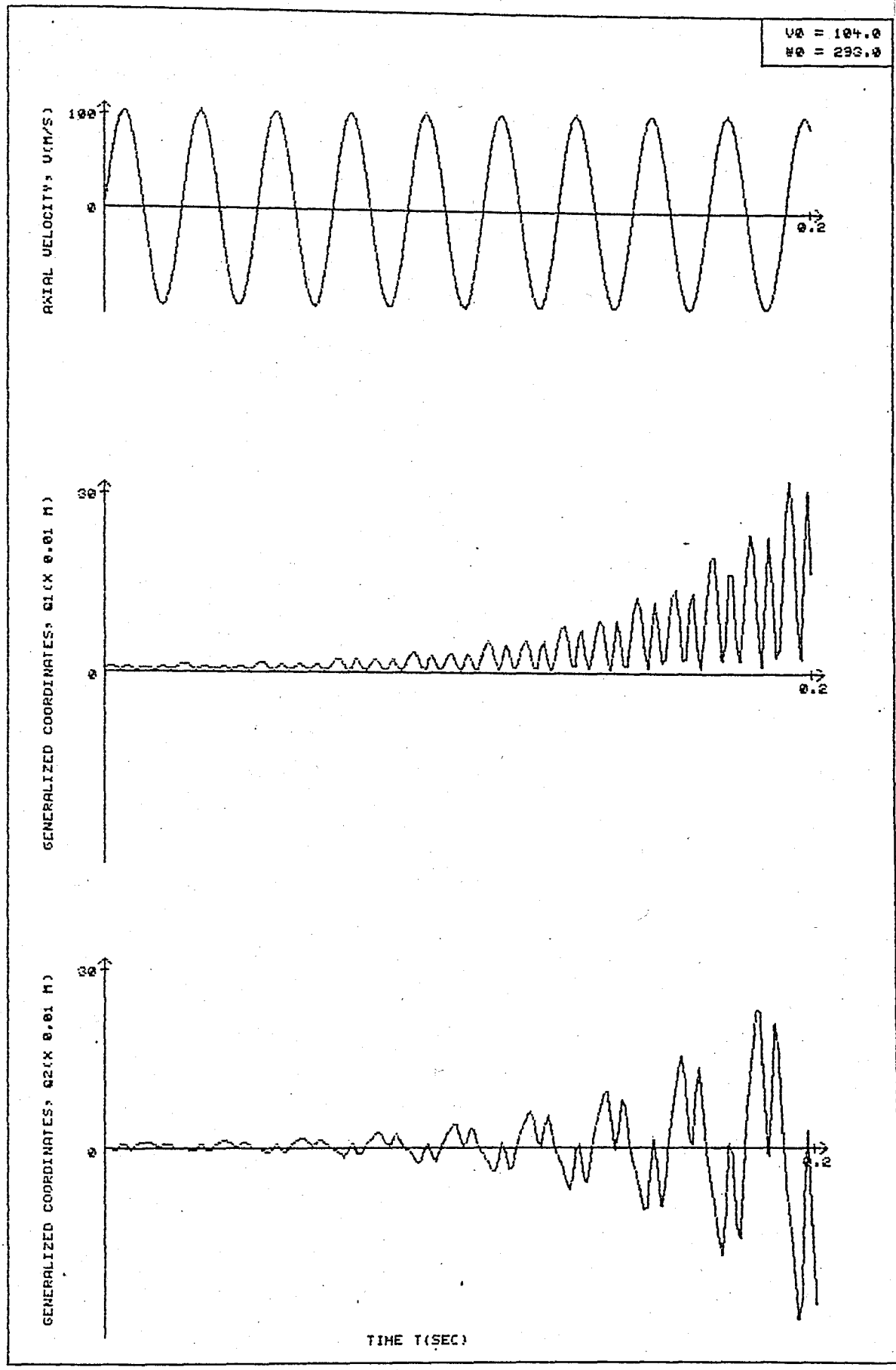


Figure 26- Generalized coordinates and velocity functions for point S(Unstable)

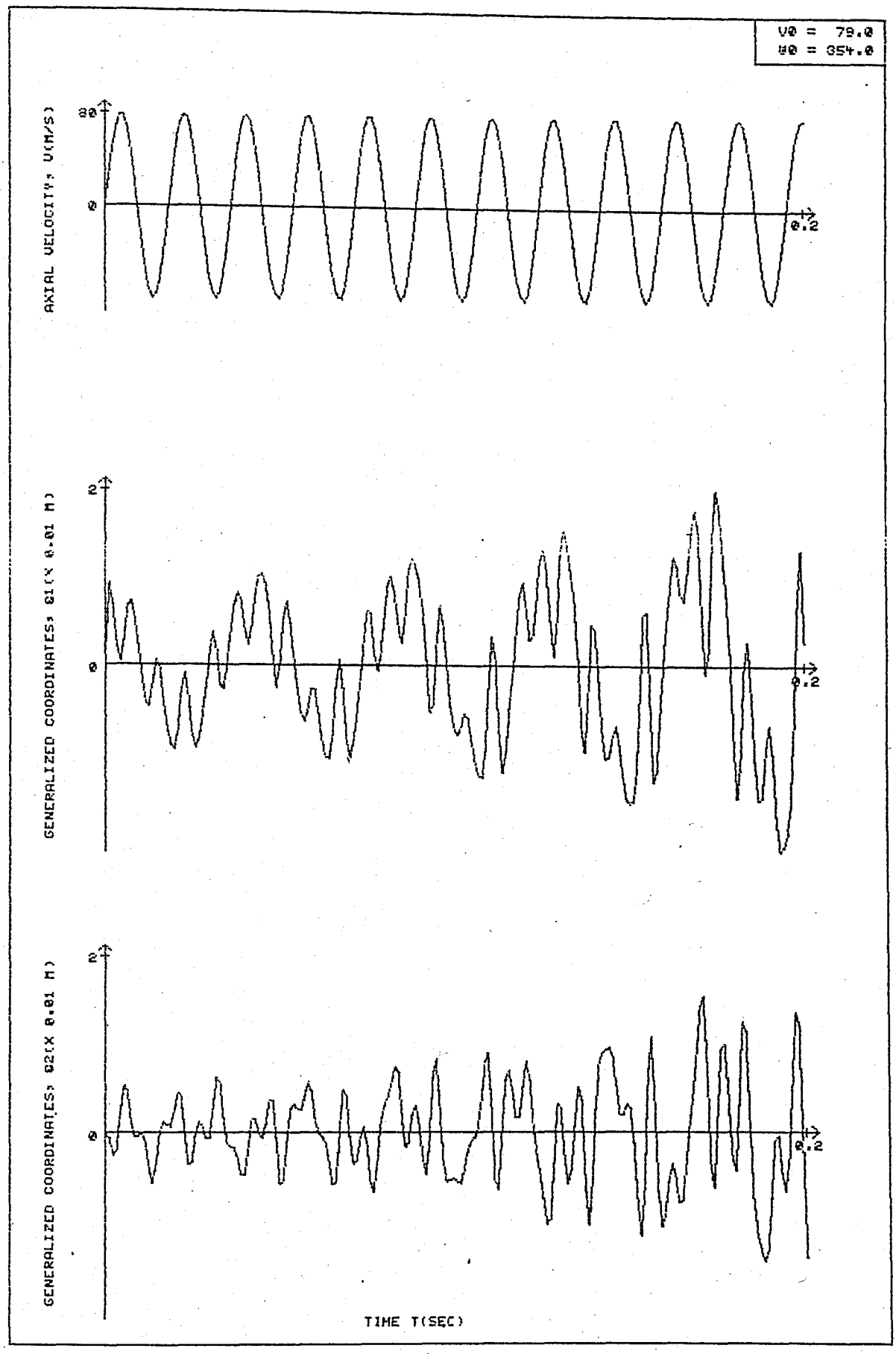


Figure 27- Generalized coordinates and velocity functions for point T(Unstable)

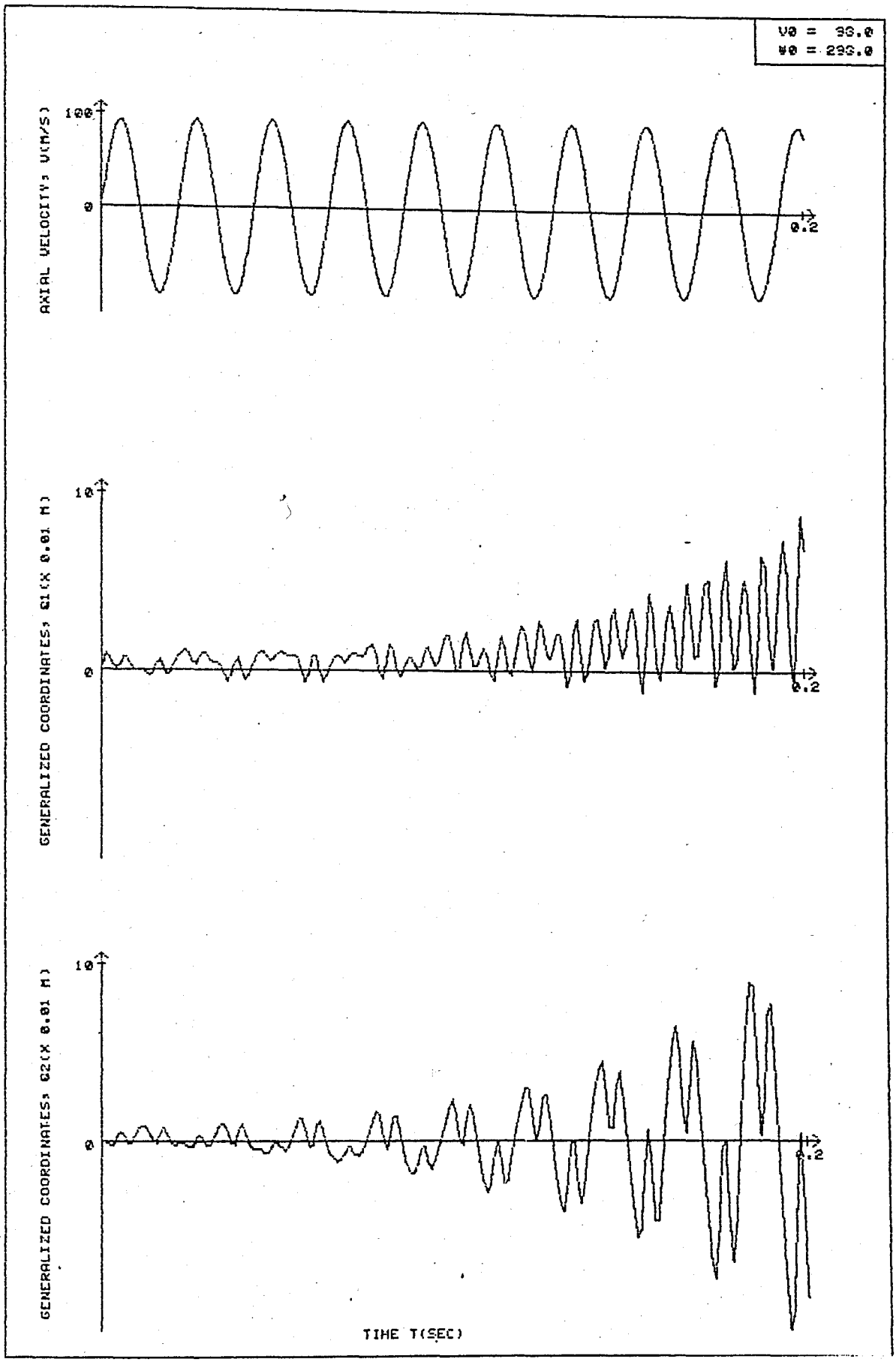


Figure 28- Generalized coordinates and velocity functions for point U(Unstable)

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APPENDIX

(COMPUTER PROGRAM)

```
C     THE PROGRAM IS WRITTEN TO SOLVE THE DIFFERENTIAL EQUATION GIVEN IN
C                 SUBROUTINE   FCNI
      PROGRAM DIFF(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      REAL X(4),C(24),W(4,100),T,TOL,TEND,TL
      INTEGER N,IND,NW,IER,K
      COMMON/OWER/P,D,A,V0,W0,TL,CK,PI
      EXTERNAL FCNI
      READ*,L0
C     SPECIFIC VALUES GIVEN FOR WHICH THE SOLUTION IS SEARCHED
      P=76.22
      D=7754.
      A=.5202E-5
      V0=0.
      W0=0.
      TL=0.3681
      CK=0.22
      READ*,V0
      READ*,W0
      WRITE(6,11)P,D,A,V0,W0,TL,CK
   11 FORMAT(6X,'STANDARD PARAMETERS USED',/,12X,'P=',F7.2,/,12X,'D=',          E8.4
     *F5.0,/,12X,'A=',E10.4,/,12X,'V0=',F8.3,/,12X,'W0=',F5.2,/,12X,'L='        2X,'
     *,F6.4,/,12X,'K=',F4.2)
      PI=3.141592654
      T=0.
      NW=4
      N=4
C     INITIAL VALUES FOR THE DIFFERENTIAL EQUATION
      X(1)=1.
      X(2)=0.
      X(3)=0.
      X(4)=0.
      TOL=0.00001
      DT=0.001
      IND=1
      WRITE(6,2)
      DO 1 K=1,L0
      TEND=FLOAT(K)*DT
C     A SUBROUTINE FROM IMSL LIBRARY FOR SOLVING DIFFERENTIAL EQUATIONS BY
C                    RUNGE-KUTTA METHOD
      CALL DVERK(N,FCNI,T,X,TEND,TOL,IND,C,NW,W,IER)
C     PRINTING OUT THE RESULTS
      WRITE(6,3)TEND,(X(I),I=1,2)
    1 CONTINUE
    2 FORMAT(6X,'T',12X,'X(1)',13X,'X(2)',/)
    3 FORMAT(3X,F7.4,2(4X,F13.8))
      STOP
      END
C     THE SUBROUTINE CONTAINS THE DIFFERENTIAL EQUATION TO BE SOLVED
C             IT IS USED BY SUBROUTINE DVERK
      SUBROUTINE FCNI(N,T,X,XPRIME)
      INTEGER N
      REAL X(N),XPRIME(N),T
      COMMON/OWER/P,R,A,V0,W0,TL,CK,PI
      A1=(2*P/(R*A)-CK*V0**2*(1-COS(2*W0*T)))*PI**2/(2*TL**2)
      A2=8*V0*W0*COS(W0*T)/(3*TL)
      A3=16*V0*SIN(W0*T)/(3*TL)
      XPRIME(1)=X(3)
      XPRIME(2)=X(4)
      XPRIME(3)=-A1*X(1)+A2*X(2)+A3*X(4)
      XPRIME(4)=-A2*X(1)-2*A1*X(2)-A3*X(3)
      RETURN
      END
```