COMPUTER MODELS FOR ESTIMATING THE PERFORMANCE OF A GIVEN AXIAL-FLOW GAS TURBINE STAGE AND FOR OPTIMIZING THE EFFICIENCY

by

A. Kamuran Kadıpaşaoğlu

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A. Kamuran Kadıpaşaoğlu

ABSTRACT

The present thesis involves the study of the aerodynamic performance of an axial-flow gas turbine stage. Existing studies have been reviewed and based on the most developed performance evaluation and efficiency optimization techniques, two computer programs have been developed. Given the geometry of a turbine stage, the inlet gas conditions and the rotational speed of the moving blades, the first program calculates the total-to-total isentropic efficiency of the stage and determines the gas conditions at the inlet and outlet of the blade rows. The second program, on the other hand, solves the inverse problem: For specified inlet gas conditions and blade rotational speed, the program determines the geometry of the turbine stage for which the isentropic efficiency is an optimum.

Both programs are applied to turbine stages operating at various inlet gas conditions and blade rotational speeds. The isentropic stage efficiency is observed to depend primarily on the overall pressure ratio, the initial mass flow parameter and the dimensionless speed of the stage. The results which are summarized in tabular and graphical forms are in conformity with those reported by the original authors. Finally, recommendations are given for improving the precision of the performance prediction method used.

ÖZET

Bu tezde eksenel akışlı bir gaz türbini kademesinin aerodinamik performansı incelenmiştir. Konuyla ilgili çalışmalar araştırılmış, en gelişmiş performans değerlendirme ve verim eniyileştirme teknikleri esas alınarak iki bilgisayar programı hazırlanmıştır. Birinci program, boyutları, akışkan giriş şartları ve dönme hızı verilen bir türbin kademesinin tersinir adiabatik verimini ve kanat dizinlerine giriş ve çıkış noktalarındaki gaz özelliklerini hesaplamaktadır. İkinci program ise ters problemi çözmekte yani, akışkan giriş şartları ve dönme hızı verilen bir türbin kademesinin boyutlarını tersinir adiabatik verimin eniyi olmasını sağlayabilecek şekilde hesaplamaktadır.

Her iki programın da değişik akışkan giriş şartları ve dönme hızları ile işleyen türbin kademelerine uygulaması yapılmıştır. Sonuçta tersinir adiabatik verimin özellikle giriş ve çıkıştaki basınçların oranı, girişteki gaz akımı parametresi, ve basamağın hız parametresine bağımlı olarak değişiklikler gösterdiği gözlenmiştir. Sonuçlar, çizelge ve grafik biçiminde toplanmış ve bunların daha önce yayınlanmış metodlarda görünen sonuçlarla uyuştuğu saptanmıştır. Son olarak da performans tahmininin hassasiyetini arttırabilecek bazı değişiklikler tavsiye edilmiştir.

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A	Gas flow area, measured normal to flow direction
A_n	Annulus area
A_t	Blade passage throat area
B	Constant defined by Eq. (13) and Eq. (14)
C_L	Tangential lift coefficient
C_p	Specific heat of the working fluid at constant pressure
c	Blade chord
c_a	Axial chord
D	Mean (reference) diameter of the turbine annulus
D_e	Equivalent diffusion ratio
D_p	Diffusion factor for the pressure surface
D_s	Diffusion factor for the suction surface
D_t	Total diffusion factor for a blade passage
e	Mean radius of curvature of upper blade surface between the throat and the trailing edge
$\vec{G}$	Vector formed by the constraint equations of the optimization problem
H	Total enthalpy
$H(R_{op})$	Objective function of the one-dimensional maximization problem
h	Annulus height
ID	Inner diameter of the turbine annulus
i	Gas incidence
i_s	Stalling gas incidence

J_{in}	Jacobian matrix
k	Radial tip clearance of the rotor blades
L	Blade camber length
M	Mach number
M_{ijk}	Three dimensional matrix used in the differentiation of the penalty function
$\dot{m}$	Turbine inlet mass flow rate
N	Rotational blade speed
n	Number of blades in a row
OD	Tip (outer) diameter of the turbine annulus
o	Blade opening or throat
P	Total gas pressure
P_c	Total critical gas pressure
$P(\vec{X},r_k)$	Interior penalty function
R	Degree of reaction of the stage
R_a	Gas constant
Re	Reynolds number
Re_h	Reynolds number based on throat mean hydraulic diameter
R_{op}	Optimal step size of the one-dimensional optimization of $H(R_{op})$
r_k	Non-negative resequensing factor defined by Eq. (101)
s	Blade pitch or spacing
T	Total gas temperature
ΔT	Total temperature drop across the stage
t	Maximum blade thickness
t_e	Blade trailing edge thickness, measured normal to the camber line at the trailing edge

U — Linear velocity of the rotor blades measured at the reference diameter

V — Total gas velocity

V_a — Axial component of the absolute gas velocity

V_{p-min} — Minimum gas velocity over the pressure surface

V_{s-max} — Maximum gas velocity over the suction surface

V_t — Tangential component of the absolute gas velocity

ΔW — Specific work output of the stage

$\vec{X}$ — Vector formed by the design variables of the optimization problem

x — Distance along the camber line to the leading edge of the blade

Y_e — Endwall loss coefficient

Y_k — Clearance loss coefficient

Y_p — Profile loss coefficient

$Y_{p(\beta_i=o)}$ — Profile loss coefficient of a nozzle blade ($\beta_i=o$) of same gas inlet angle and of same pitch-per-chord ratio as the actual blade

$Y_{p(\beta_i=-\alpha_o)}$ — Profile loss coefficient of a zero-reaction blade of same gas inlet angle and of same pitch-per-chord ratio as the actual blade

$Y_{p(i=o)}$ — Profile loss at zero incidence

Y_s — Secondary loss coefficient

Y_t — Overall mean stagnation pressure loss coefficient

Y_{te} — Trailing edge loss coefficient

Z — Blade loading parameter

Greek Symbols

α	Gas flow angle, measured relative to the axial direction
α_c	Included angle of divergence of the turbine annulus walls
β	Blade (metal) angle, measured relative to the axial direction
$\vec{\nabla}$	Gradient vector operator
γ	Ratio of the specific heats of the working fluid
δ_m	Momentum thickness
δu	Blade loading factor
ε	Gas deflection
ξ	Blade stagger angle
η	Overall isentropic efficiency of the stage
λ	Empirical function correlating the secondary losses to annulus geometry
λ_N	Stator total loss coefficient based on temperature drop
λ_R	Rotor total loss coefficient based on temperature drop
μ	Dynamic viscosity of the working fluid
μ_a	Acceleration ratio
ν	Kinematic viscosity of the working fluid
ϕ	Flow coefficient
ρ	Density of the working fluid
ψ	Temperature drop coefficient
$\overline{\omega}$	Mean loss of the total pressure through a blade row due to friction and flow separation

Subscripts

i	Inlet to a stage
is	Isentropic
m	Mean
N	Nozzle
o	Outlet from a stage
R	Rotor
s	Static
1	Inlet to the stator row
2	Outlet from the stator row
3	Inlet to the rotor row
4	Outlet from the rotor row

I. INTRODUCTION

Since the early days of the development of gas turbine engines and their progressive use in industrial, vehicular, railroad, marine, avaiation, nuclear and spatial applications, the necessity for high efficiency required for economic operation has become more and more dominant. In recent years, turbines of considerable size and capacity have been produced with many stages. Along with the demand for larger gas turbines, factors like efficiency and component weight have become major design considerations.

Compared to that of the turbine used in aerospace applications, the design of the turbine for power plants does not necessitate the accomplishment of the lightest weight condition, which is, in most of the cases, incompatible with the attainment of the best efficiency. The premium placed on performance almost invariably result in the employment of a multistage construction. In land applications, turbine efficiency is thereby considerably improved over the level which can be obtained with light-weight, single-stage turbines ordinarily used in aircraft units. In return, for the case of stationary power turbines, a low axial velocity is desirable in order to avoid high leaving losses which are proportional to the residual kinetic energy contained in the gas leaving the turbine at the last stage.

On contemporary aero-engines, every effort is made to reduce the number of stages and turbine weight to a minimum. Within the non-aeronautical field, R @ D on axial flow gas turbines has two principal aims. The first is to explore the gas flow processes within a turbine stage, which would lead to a better understanding of the aerodynamics and to the determination of the associated aerodynamic efficiencies. The second is the achievement of relaiable operation at progressively higher turbine inlet gas temperature closely related to metallurgical problems, leading to improvements in specific power output and specific fuel consumption.

Not considering the metallurgical aspect, if a turbine designer is to select the best design for his purpose, he must be able to predict the design point performance of a wide range of possible turbines, as well as the performance of his designs over a wide range of operation conditions. Moreover, he would like to be able to design a turbine stage with the objective of minimizing aerodynamic losses.

However, tests conducted on turbines have often revealed efficiencies considerably inferior to those predicted during the preliminary design stages. For this reason, aerodynamic research into the performance of turbines has been much accelerated.

Prediction of the aerodynamic efficiency of gas turbines requires the accurate calculation of aerodynamic losses. These losses are due to gas friction, vorticities, gas leakages boundary layer development on blade passages and adverse pressure gradients encountered through the blade passages of the turbine, and they constitute the main cause of the drop in total efficiency.

Since the flow of gas within the turbine at high pressure and temperature is quite a complicated process, it is nearly impossible to bring about a wholly analytical solution. Results of experimental work conducted by companies, research centers and universities, as well as theoretical and analytical considerations of aerodynamic concepts are therefore used together for the formation of generalized loss-geometry correlations.

With appropriate design input information such as rotative speed, mass flow rate, inlet state conditions of the working fluid, pressure ratio or power desired and with specified values for the significant dimensions of the blade profiles such as axial width, maximum thickness, pitch and chord lengths, blade inlet, exit and stagger angles, the use of aerodynamic equilibrium conditions in conjunction with the available loss correlations can lead to an accurate prediction of the design point efficiency of a turbine.

In the present thesis, two principal aims are considered: The first comprises the estimation of performance of an axial flow gas turbine stage of predetermined geometry, under different conditions of pressure ratio, inlet fluid conditions and machine rotative speed. The second is the determination of the geometry of a gas turbine stage for which the aerodynamic losses would be a minimum. Two separate computer programs are developed for each of the purposes cited above. The first program adopts the loss system of Ainley and Mathieson [1] with the suggested modification of Dunham and Came [2] as its basis. This loss system is chosen because it is one of the best known and

and most completely documented of the gas turbine loss evaluation systems so far described in the literature. The second computer program is based on the optimization method developed by Rao and Gupta [3], the aerodynamic losses to be minimized being mostly those predicted in [1] and [2], although the evaluation of the stage efficiency is carried out in a slightly different manner.

II. LITERATURE SURVEY

In the past thirty years, a number of loss evaluation systems as well as concrete surveys on design practices for gas turbines power elements have been described in the literature. Results of experimental works together with theoretical and analytical considerations have been published and served as the greatest contributors to the improvements achieved in gas turbine efficiency, dependability and low first and operating costs. Without doubt, the analytical and numerical procedures used in designing such turbomachines have now undergone a major revision. The empirical relations used in design process are being replaced by more sophisticated methods of analysis.

The purpose of the literature survey presented here is to review the present day basis of turbine performance calculations and turbine stage design, indicating where improvements have been made in recent years and where further work is needed.

Expressions, correlations and schematic representation of concepts related to the works presented in the following paragraphs can be found in the Appendix.

2.1 LITERATURE ON PERFORMANCE PREDICTION

According to Horlock [4], there are three principles sources of turbine cascade data:

i) The results of Zweifel [5] and miscellaneous turbine cascade results which have been brought together by Soderberg [6] in a simple correlation.

ii) British cascade data (the results of Todd [7], Reeman [8], and Dunsby [9]) which have been brought together in correlations by Ainley and Mathieson [1].

iii) Early data from steam turbine cascades, principally that of the Steam Nozzle Research Committee reported in reference [10].

The first two sets of data are reviewed and to some extent compared here. In addition, some more recent studies are reported.

An efficient method for the calculation of pressure loss is to represent the pressure gradient in flow direction by choosing the adequate cascade parameters. An early approach to this method was presented by Zweifel [5], in 1945. From a series of cascade tests, Zweifel suggested that the lift coefficient based on the tangential loading of the blade (aerodynamic blade loading coefficient, ψ_Z, can be expressed as a function of pitch-to-chord ratio and of the gas inlet and outlet angles. But, to arrive to this conclusion, Zweifel had made the highly simplified assumption that the actual velocity distribution on the suction and pressure sides of the blade could be approximated to a distribution independent of the distance from the leading edge, along the camber line. Zweifel concluded that the inter-relation between cascade total loss coefficient, Y_t, and aerodynamic blade loading coefficient, ψ_z, showed that the losses decreased to a minimum for the value of $\psi_z = .8$. Therefore, for given gas inlet and outlet angles (deflection), an optimum pitch-to-chord radio could be obtained.

This suggestion, although based on an oversimplified flow model, has been confirmed to a large degree, by later investigations.

Soderberg [6] in 1949, correlated the losses on a basis of pitch-to-chord ratio, Reynolds number, aspect ratio (based on axial chord), thickness-to-chord ratio and blading geometry. Soderberg used the work of Zweifel [5] to obtain the optimum pitch-to-chord ratio for a given change of direction through a cascade. Then, based on this optimum, he formulated the "nominal" loss coefficient, for turbine rows operating at a Reynolds number of 10^5 and with an aspect ratio of 3:1 as well as the "total" loss coefficient for aspect ratios other than 3:1 and for Reynolds numbers (based on throat hydraulic mean diameter) other than 10^5. Soderberg included a correction on the efficiency for clearance losses simply by multiplying the final calculated stage efficiency by the ratio of blade area to total area including leakage space.

Soderberg's correlation implies that the effect of profile shape is limited. Although the correlation is affected by thickness-to-chord ratio, no effect of trailing edge thickness is included. Further, it is implied that degree of reaction (or stagger) is unimportant, so long as the optimum pitch-to-chord ratio is chosen. The neglect of these parameters renders Sodenberg's correlation open to criticism. Further, the aspect ratio alone cannot be expected to be the only important parameter in the correction for secondary loss, since the entry boundary layer and the blade geometry must be vital parameters. However, it is understood that the correlation gives turbine efficiencies to within 3 percent over a wide range of Reynolds number and aspect ratio, and it is useful for obtaining quick estimates of turbine performance.

Stewart, Whitney and Wong [11], in 1960, using experimental data of Dunavant and Erwin [12], offered a relationship between the boundary layer momentum thickness and the total diffusion factor. The boundary layer momentum thickness is on important parameter in calculating the growth of the momentum loss due to boundary layer formation on blade surface. The total diffusion factor, on the other hand, is related to the blade surface velocity distribution. As a result, therefore, the work of [11] correlated the losses in a turbine cascade to the type of velocity distribution over the blade surface. The relationship showed an increasing momentum thickness for increasing total diffusion ratio, indicating also that for diffusion ratios greater than .6, flow separation would be imminentStewart, Whitney and Wong adopted a surface pressure distribution similar to that proposed by Zweifel [5], in order to interrelate surface velocities with cascade geometry. They assumed that the maximum suction surface velocity was equal to the cascade exit velocity and that the minimum pressure surface velocity had a value which produced an actual tangential force corresponding to the aerodynamic blade loading coefficient of [5].

These assumptions then led, through the use of aerodynamic blade loading coefficient and the assumed surface velocities, to an interrelation between the total diffusion factor and the pitch-to-chord ratio, for given gas inlet and exit angles. The momentum loss and hence the energy loss may then be obtained as a function of pitch-to-chord ratio for the given gas angles.

This is a useful correlation but it can only be a guide to the choice of velocity distribution on the blade surface. The blade shape to give a required velocity distribution must be separately determined.

In 1968, Balje and Binsley [13] reported that the correlations of Stewart, Whitney and Wong [11] did not represent a description of the real situation because of the fact that for nozzle blades operating at small gas exit angles it would be possible to get negative values for the total diffusion factor. Moreover, for an aerodynamic blade loading coefficient chosen to be greater than 1, the total diffusion factor became imaginary. According to Balje and Binsley, the reason for the failure of this flow model was related to the inability to obtain a satisfactory expression for the pressure distribution in terms of cascade geometry.

The model that Balje and Binsley used in [13] as an alternative was based on the assumption that the surface velocity varied linearly with camber line, from the leading edge to the trailing edge. Using this linear variation of the free stream velocity in Truckenbradt [14] expression for the momentum thickness at a point in a turbulent boundary layer with pressure gradient, and integrating over the total camber length, they obtained a relation for the momentum thickness in terms of cascade inlet and exit angles. This relation led to the calculation of the profile loss coefficient through the use of the correlation of Stewart, Whitney and Wong [11].

In another paper published in 1968, Balje [15] correlated the pressure losses (except for secondary losses) of a cascade to three key parameters. These parameters were the aerodynamic load coefficient of Zweifel, the accelerating rate formed by taking the ratio of gas exit and inlet velocities and the dimensionless boundary layer momentum thickness formed by dividing boundary layer momentum thickness to the camber length of the blade.

In a report presented to NGTE in 1951, Ainley and Mathieson [1] offered a method for estimating the performance of an axial flow gas turbine. Ainley and Mathieson calculated the profile loss at zero incidence first and then corrected the resulting loss coefficient according to the actual incidence on the blade, through a number of experimentally driven correlations. The clearance and secondary losses were correlated to the square of the lift coefficient, based on the work by Carter [16]. A method of calculation of absolute gas conditions depending on the total loss was also shown and the expression of the total-to-total aerodynamic efficiency was deriven. The method was illustrated by a worked example.

This method constitutes one of the most complete and accurate surveys among the loss evaluation systems so far published.

The most notable refinement on the Ainley and Mathieson system was the one due to Dunham and Came [2], which was published in 1970. In this paper, the additional profile losses, incurred when a blade row chokes due to the development of shock waves have been accounted for by the introduction of an arbitrary correction factor incorporating a numerical constant chosen to fit available efficiency data, the correction being applied only for Mach numbers greater than unity. Also, instead of correcting the final efficiency by a Reynolds number correction factor as was done by Ainley and Mathieson [1], an optional correction has been applied directly to the profile and secondary losses using the Reynolds number appropriate to the particular blade row.

Concerning the secondary losses, the blade loading parameter of Ainley and Mathieson [1] was adopted since it represented the effect of blade loading satisfactory, but the empirical constant of Ainley and Mathieson [1] used in the derivation of secondary losses has been replaced by a more realistic expression including the chord-to-blade height ratio and the ratio of gas outlet angle to cascade inlet angle. The final expression for tip clearance loss coefficient, on the other hand, contained the .78th power of clearance-to-chord ratio in the revised system [2].

As already stated, the loss model of Ainley and Mathieson [1] together with the corrections proposed by Dunham and Came [2] constitutes the basis of the first part of the present thesis. For this reason, both methods have been explained in detail in the following chapter.

2.2 LITERATURE ON TURBINE DESIGN

Research on conventional aerodynamic design of gas turbines and considerations on material limitations have a considerably short history which dates back to the early 1940's. At that time, the design of gas turbines was much influenced by previous experiences on steam turbines for which, neither the problem of high component efficiencies for reasonable economic operation nor that of new material which would withstand high stresses at much higher temperatures had ever been encountered. The necessity of more efficient operation at higher temperatures and with lighter stage weight, however, has soon led gas turbine engineers to review the design considerations both from an aerodynamic and a mechanical standpoint.

The first person to conceive that the blade design should allow the gas stream to preserve a constant angular momentum, instead of completely neglecting the centrifugal forces upon the swirling gas, was the Air-Commodore Sir Frank Whittle. Sir Whittle explained his theory in 1945 [17] and designed the turbines in his jet,propulsion engines accordingly. This "free vortex" assumption of Whittle was followed by a number of other assumption of flow model, since in theory, the flow can take, in a blade passage an infinite number of radial forms as long as the centrifugal forces tending to throw the gas outwards are always equally balanced by an opposing radial static pressure gradient. As a widely used example to these proposed flow models, the one in which the gas mass flow per unit area of annulus is maintained constant over the flow area at any point may be cited. Resultingly, the old distinction between reaction and impulse turbines, as used by the steam engineers lost a great deal of its meaning, because the calculated reaction of a turbine stage began to vary along the blade height, the point of least reaction occurring at the root of the blade.

In 1948, Ainley wrote a paper on the performance of axial flow turbines [18]. In the paper Ainley discussed the various flow models proposed up to his time and made the comparison between the impulse steam-turbine blading with constant sections at all radii and the typical gas turbine stage providing impulse (least reaction) conditions at the root of the blades. Ainley discussed also in this paper a technique developed by the NGTE on turbine testing, giving regard to power output, rotational speed, gas flow (aerodynamic losses), inlet and outlet gas pressures (both total and static), inlet and outlet total head temperatures, instrumentation and safety precautions. The paper may be considered as one of the key works which constituted a basis to the system [1] he developed later with Mathieson.

Again in 1948, Reeman [19] outlined the mathematical approach to the design of gas turbines for jet engines and surveyed very comprehensively the mechanical problems that are involved, drawing attention to the balance that must be sought between aerodynamic efficiency and material limitations. He stated out that low axial velocity and high total-to-static efficiency as required for the turbocharger turbine was not a necessary requirement in the aircraft gas turbine, for the leaving velocity was used in the final nozzle. His conclusions were as follows: "Larger axial velocities lead to smaller annulus areas and to lower centrifugal stresses the latter constituting the most important factor in fixing the nozzle angle. With nozzle angle fixed, a compromise has to be made between reaction, swirl, disc stresses and weight. Increasing the reaction lowers the Mach number relative to the blade but raises the disc stresses and blade root temperature.

Increasing the swirl on the other hand, decreases the disc stresses but increases the Mach number relative to blade also increasing the blade root temperature". Reeman finally suggested an optimum in which there was little root reaction and an 10-16^{o} outlet swirl. The preliminary design given in Reeman's discussion was relevant to single stage turbines in which the blade loading coefficient [5] was of order 2 at the root where the reaction approached zero.

Emmert [20] in 1950, supported the flow model in which the mass of flow through the stage is to be evenly distributed over the stage annulus and he stated that a value of .6 or greated was required for maximum blading efficiency for the reaction type of blading.

Miser and Stewart [21] presented in 1961, full details of the design of two stage air-cooled turbine for a jet engine. Their report covered the choice of velocity triangles and the blade design. Velocity distributions, both radial and blade-to-blade were also calculated.

In 1970, Horlock [4] outlined an example to the preliminary design study of an axial flow turbocharger. Choosing the pressure ratio, the total entry temperature, the design mass flow and the target total-to-total efficiency from the start, he calculated the reaction of the stage and the flow coefficient, defined by the ratio of axial velocity through the stage to the blade speed at reference diameter. From the velocity triangles, he made the choice of the required blade speed and he calculated the outlet gas angle and Mach numbers. Following the change of the blade height and annulus area having regard to allowable blade stress, root reaction, Mach number and nozzle and rotor deflections, Horlock calculated finally the pitch-to-chord ratio through the use of Zweifel's blade loading criterion [5].

In general, the problem under discussion comprises mostly the calculation techniques using vector diagrams which allow the fluid dynamics requirements to be satisfied throughout the stage. The basic premise of the correlations adopted in most of the works in the literature is that the shape of the velocity triangles at the mean radius is the principal factor which determines the efficiency of the turbine stage. The determination of the velocity triangles will provide the designer the flow outlet angle from the nozzle, flow inlet and outlet angle to and from the rotor and related relative and absolute velocities. With given velocity triangles, the efficiency will, of course, vary with blade geometry and, in particular, with incidence. The aerodynamic design problem becomes therefore, that of selecting a suitable blade profile and blade number.

The blade profile design methods are discussed in detail by Ainley [18]. Ainley presents the results of tests he conducted at NGTE, by comparing aerofoil and conventional blade profiles. He concludes, for most of the cases, that the aerofoil blading gives a less value for especially the profile loss coefficient. In accordance, Rao [3] proposes to superimpose a standard aerofoil section, such as A_3K_7, after a camber line is predicted from aerodynamic and mechanical considerations. The number of blades in a row, or blade pitch, would be determined either from the results of cascade experiments or directly from the use of Zweifel's loading coefficient [5].

Very little work has been reported so far, concerning the application of optimization techniques in the design of turbines. In 1966, George [22] reported about the optimization of a rocket engine turbine using a differential calculus approach. Balje and Binsley [13], in 1968 applied optimization techniques in the design of axial turbines using Wood's Direct Search Technique [23]. The optimizing variables were those geometric ratios which may be selected independently in order to achieve a maximum or optimum efficiency. These were chosen as the nozzle outlet angle, rotor inlet angle, blade height-to-mean diameter ratio, nozzle and rotor chord and pitch, mean diameter ratio and the degree of admission which varied from values just above 0 to values just below 1 in order to avoid indeterminate forms in the equations.

At the end of this work, Balje and Binsley concluded that the attainable efficiency might be significantly affected by the compressibility effects. It was noted also that the optimum turbine performance at low specific speeds was much more sensitive to the Reynolds number than performance at high specific speeds. In any case the optimum performance was very sensitive to chord length, i.e., blade number, especially at lower Reynolds numbers where large chords, hence, low blade numbers are recommended for optimum performance.

In 1980, Rao and Gupta [3] optimized the design of an axial gas turbine stage using the penalty function method of Fiacco and Mc Cormick [24]. They formulated the problem as a nonlinear mathematical programming problem with the objective of minimizing aerodynamic losses and mass of the stage. The aerodynamic losses were calculated according to the loss system of Ainley and Mathieson [1] as corrected by Dunham and Came [2]. The analysis of stress, deflection and natural frequencies of vibration of doubly tapered and twisted cantilever beams (blades) was performed by finite element methods. At the end of the paper, discussion of optimum variables is made and a sensitivity analysis is presented.

The method of Rao and Gupta (3) constitutes the second part of this thesis. It will therefore be treated in detail in the following chapter.

III. ANALYSIS

The thesis is formed of two parts. In the first part, the performance evaluation of a gas turbine stage is realized, using the method developed by Ainley and Mathieson [1] with the suggested refinements by Dunham and Came [2]. In the second part, some important parameters of a gas turbine stage are optimized in order to get maximum aerodynamic efficiency from the stage. The loss correlations as given by Ainley and Mathieson [1], the performance evaluation technique and constraints of the optimization problem as given by Rao and Gupta [3] and the sequential unconstrained minimization technique as given by Fiacco and Mc Cormick [24] are adopted as the basis. The analysis is accordingly subdivided into two parts each of which concerning the related part of the thesis.

3.1 GAS TURBINE STAGE PERFORMANCE EVALUATION

The importance and durability of the Ainley and Mathieson loss system [1] results from the fact that it provides a relatively high accuracy compared to works that followed during the last 30 years, and that it achieves a remarkable level of simplicity, reducing the number of variables appearing in the correlations to a minimum. The system has become a foundation worthy of subsequent refinement, the most notable of which was published in 1970 by Dunham and Came [2].

The procedure of the method of Ainley and Mathieson is formed of two parts. The first is the determination of pressure loss and gas efflux angle for each blade in the turbine under a wide range of inlet conditions. The second is to fix the gas inlet conditions at a given turbine speed and to follow the course of a fixed mass of gas from one blade row to the next, calculating the new gas conditions at each step. Finally, stage and overall pressure ratios, stage and overall efficiencies can be drawn out from the general picture.

Throughout the method, the following assumptions are made:

i) The flow path through each stage is considered at one diameter only - this diameter being termed the "reference diameter".

ii) The pressure loss coefficients in each blade row are not influenced by the gas Mach numbers.

iii) The gas outlet angles from a blade are not influenced by the incidence angle.

It is to be noted, throughout the following analysis, that gas conditions written without the subscript, s, refer to total conditions and that primed quantities imply absolute inlet conditions to the next stage.

3.1.1 Determination of Gas Outlet Angle from a Blade Row

It is assumed [1] that gas outlet angle, α_o, from any row remains constant over the gas outlet Mach number, M_o, range $0. < M_o < .5$ and given over this range by

$$\alpha_o = [-1.17\cos^{-1}(o/s) + 12.58] - 4. * (s/e) \qquad (1)$$

where

o = blade opening or throat

s = blade pitch

e = mean radius of curvature of upper blade surface between throat and trailing edge.

The parameters used in Eq. (1) are also illustrated in Fig. 3.1.

At a gas outlet Mach number unity, it is assumed that the outlet angle is given by [1]

$$\alpha_o = -\cos^{-1}[(A_t(1 - k/h) + \pi.OD.k)/A_{n,o}] \qquad (2)$$

where, as also shown in Fig. 3.2,

A_t = passage throat area

$A_{n,o}$ = annulus area downstream of the row

k = radial tip clearance

h = annulus height

OD = tip (outer) diameter of the turbine.

The throat area, A_t and the annulus area $A_{n,o}$ are calculated as follows:

$$A_t = o \times n \times (OD - ID)/2 \quad , \tag{2a}$$

and

$$A_{n,o} = (OD^2 - ID^2) \times \pi/4 \quad , \tag{2b}$$

where,

ID = inner diameter of the turbine,

and

n = blade number of the row.

For the Mach number range $.5 < M_o < 1.$, a linear variation of α_o may be assumed [1] with reasonable accuracy.

At low Mach numbers, α_o of Eq. (1) may be corrected as a function of radial clearance as [1]

$$\alpha_{o,corr} = \tan^{-1}\{[1 - 1.35(k/h)(\cos\beta_i/\cos\alpha_o)] * \tan\alpha_o$$
$$+ 1.35(k/h)(\cos\beta_i/\cos\alpha_o) * \tan\alpha_o \tag{3}$$

where

β_i = blade angle at inlet, as shown in Fig. 3.1.

3.1.2 Determination of Pressure Losses in a Blade Row

For a gas flow through a blade passage, the overall mean stagnation pressure loss coefficient, Y_t, can be expressed as:

$$Y_t = (P_i - P_o)/(P_o - P_{o,s}) \tag{4}$$

where, as illustrated in Fig. 3.3,

P_i = inlet total pressure to the stage

P_o = outlet total pressure from the stage

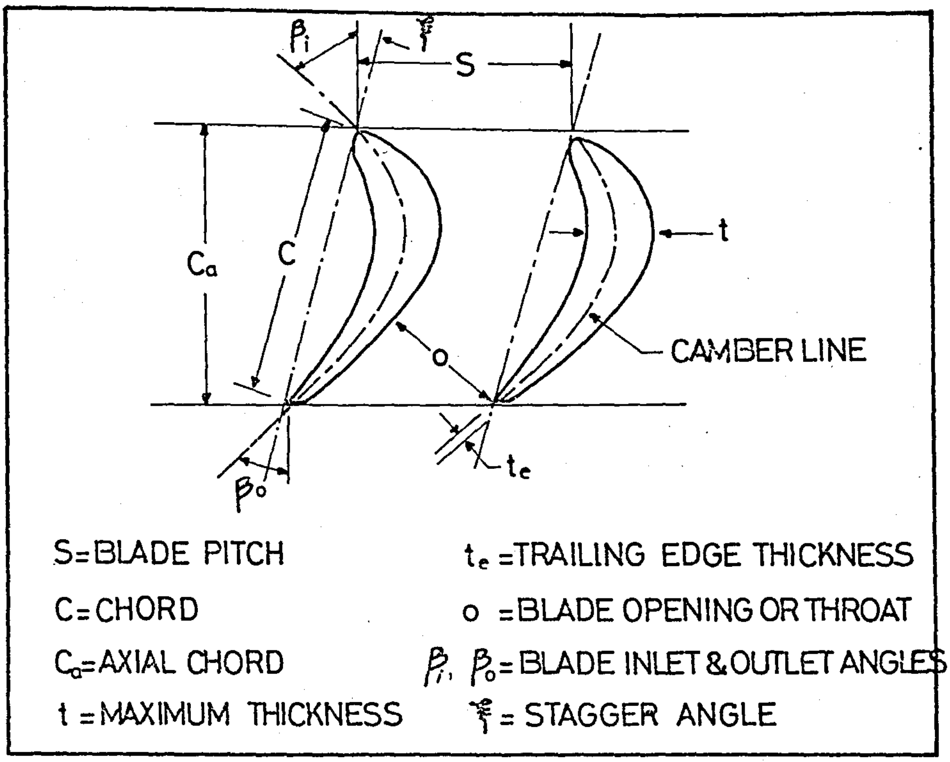

FIGURE 3.1 - Turbine blade nomenclature.

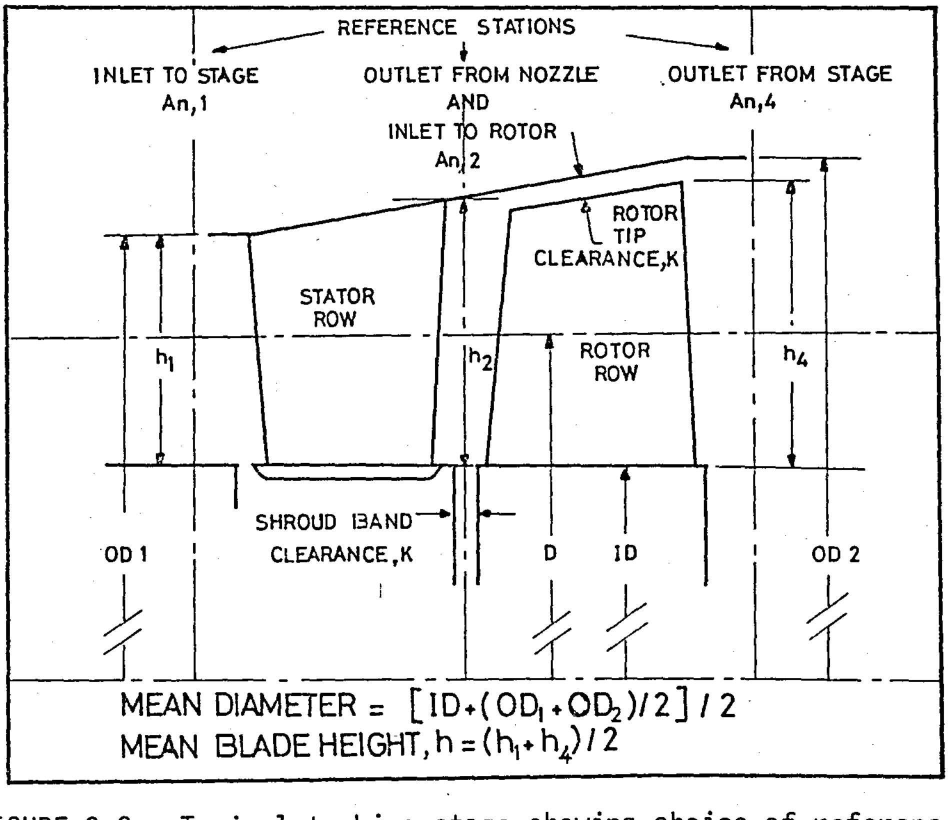

FIGURE 3.2 - Typical turbine stage showing choice of reference stations and reference heights.

and

$P_{o,s}$ = outlet static pressure from the stage

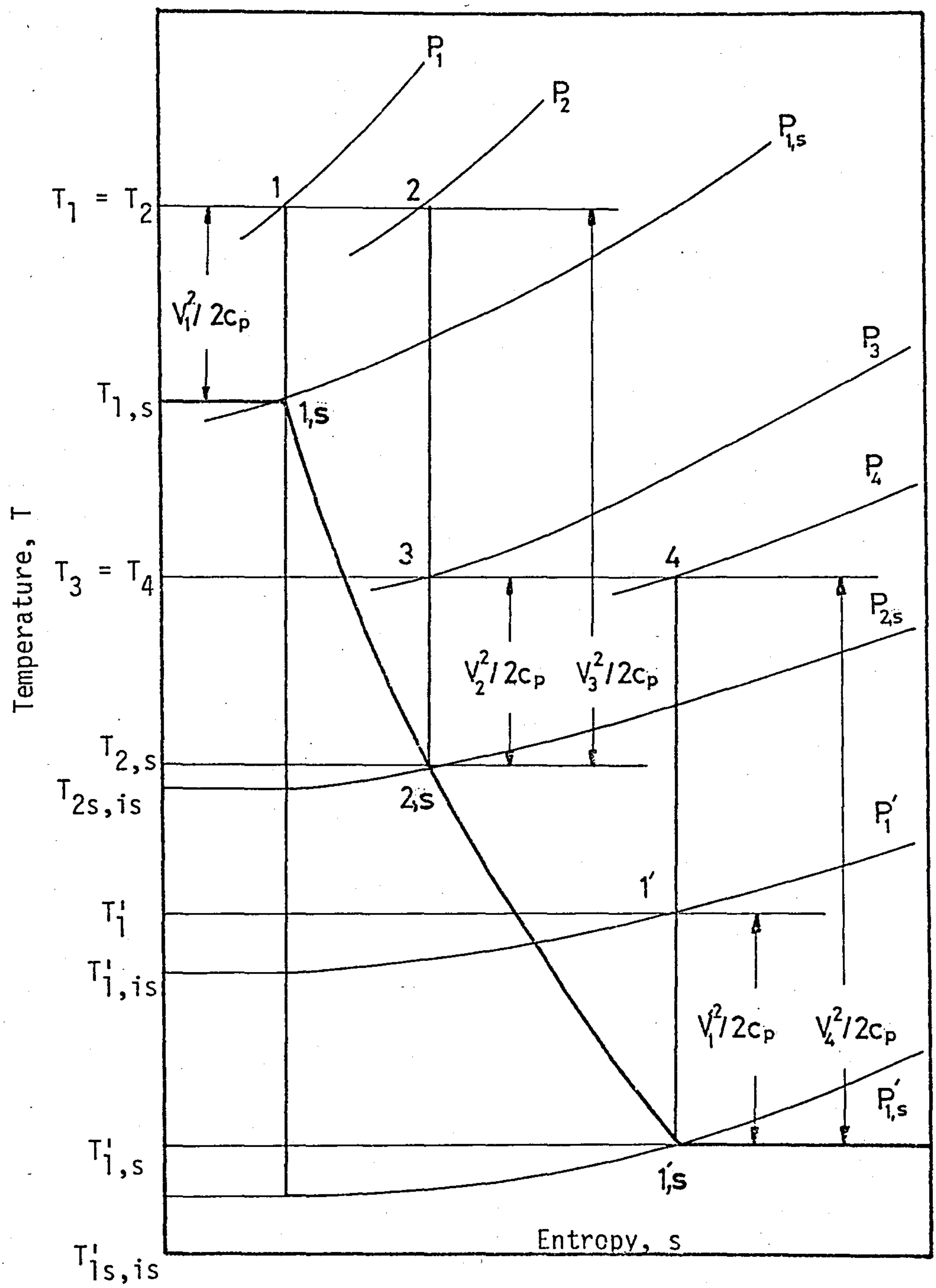

FIGURE 3,3 - Temperature-entropy diagram for a reaction stage.

The overall mean stagnation pressure loss coefficient, Y_t, may be split down into

i) The profile loss coefficient, Y_p, associated with boundary layer growth over the blade profile,

ii) The secondary loss coefficient, Y_s, associated with the turning of the boundary layer on the end walls of the blade and the related separation of flow

iii) The loss coefficient associated with blade tip clearance, Y_K,

and iv) The trailing edge loss coefficient, Y_{te}, caused by the trailing edge thickness of the blade.

3.1.2.1 Determination of Profile Loss Coefficient

In a turbine blade passage, friction losses are caused by the interaction of the flowing medium and the blade walls. A thin sheet of decelerated fluid, the boundary layer, is interposed between the blade surface and the flowing medium and its thickness increases with flow path length. Thus, the average flow in the cascade is slightly accelerated due to the boundary layer displacement thickness and a momentum loss results. The growth of the displacement thickness and the resulting momentum loss is strongly affected by the adverse pressure gradients in direction of flow. Strong adverse pressure gradients are typical for the suction surface of decelerating cascades. Actually a critical pressure gradient exists which causes boundary layer separation. As long as the flow is not separated, it is possible to calculate the boundary layer displacement thickness and hence the momentum (profile) loss, either from known pressure gradient or experimentally, from cascade data.

In the Ainley and Mathieson loss system [1], the profile loss coefficient, Y_p, is given as a function of the incidence, i, of the blade, expressed as:

$$i = -(\beta_i - \alpha_i) \tag{5}$$

where

α_i = gas inlet angle to blade, as shown in Fig. 3.4,

also,it is a function of the gas outlet angle, α_o, from the blade.

First, the profile loss is determined at zero incidence, then, the stalling incidence, i_s, of the blade row is determined; stalling incidence being defined as the incidence at which the profile loss is twice the loss at zero incidence.

Profile loss at zero incidence, $Y_{p(i=0)}$, is given by, [1],

$$Y_{p(i=0)} = \{Y_{p(\beta_i=0)} + (\beta_i/\alpha_o)^2[Y_{p(\beta_i=-\alpha_o)} - Y_{p(\beta_i=0)}]\} \\ *((t_e/c)/.2)^{-\beta_i/\alpha_o} \tag{6}$$

where

$Y_{p(\beta_i=0)}$ = profile loss coefficient of a nozzle blade (β_i=0) of same α_o and pitch-per-chord ratio, (s/c),as the actual blade (Fig. 3.5)

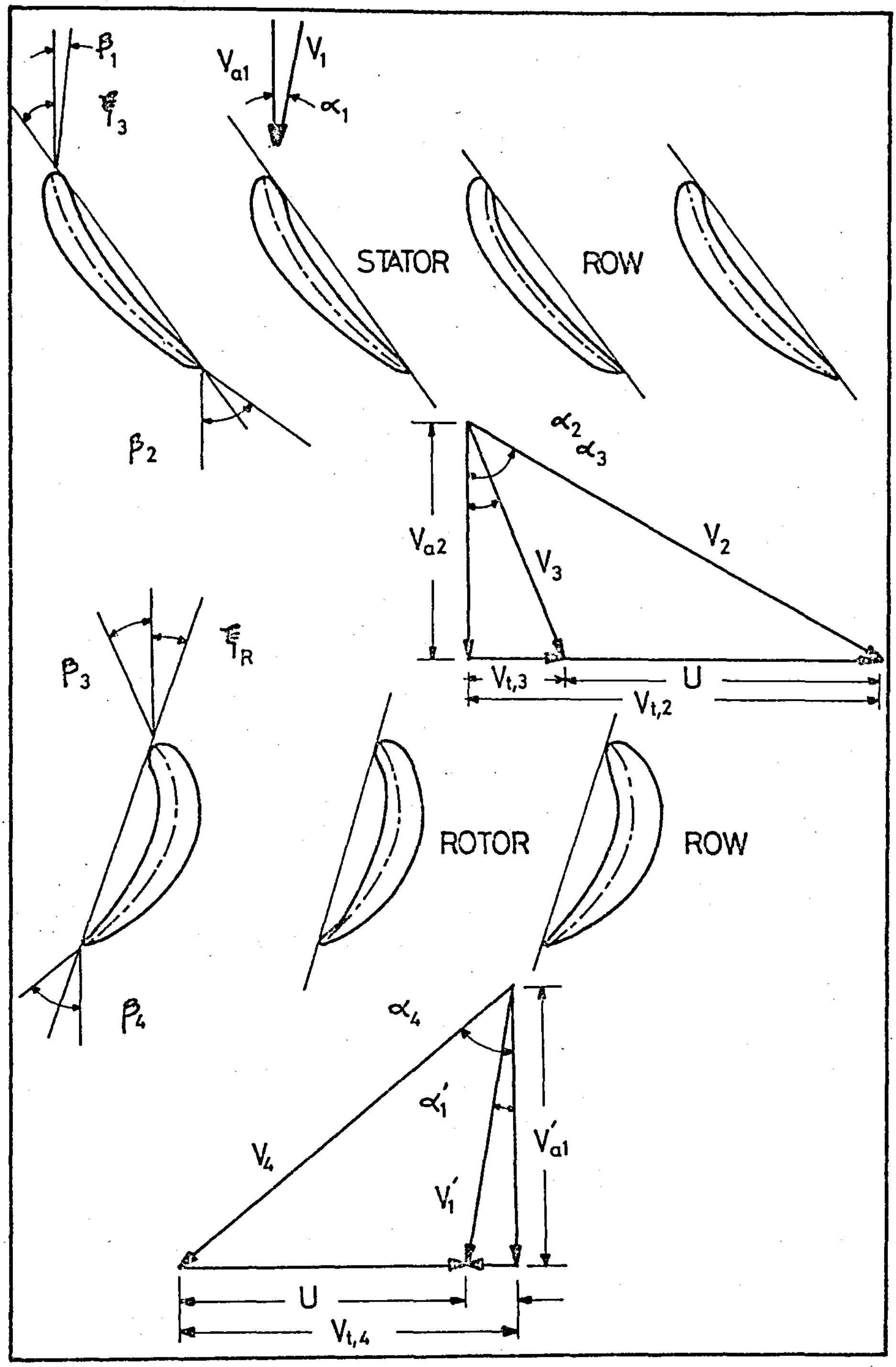

FIGURE 3.4 - Velocity triangles for an axial-flow turbine stage.

$Y_{p(\beta_i=-\alpha_o)}$ = profile loss coefficient of a zero-reaction blade (impulse blade) of same α_o and s/c as the actual blade (Fig. 3.6)

c = blade chord.

Once the loss coefficient at zero incidence is determined, the value of the stalling incidence, i_s, is needed is order to calculate the actual value of the profile loss coefficient. It is assumed in [1] that the ratio of profile loss at any incidence to profile loss at zero incidence, $Y_p/Y_{p(i=0)}$, is a function of the ratio of actual incidence to stalling incidence, i/i_s. This relationship is given in Fig. 3.7.

The stalling incidence can be determined as follows, [1]

i) First, the outlet angle for a blade having s/c = .75 is determined. Knowing the actual outlet angle, α_o, and the actual s/c, [1] gives

$$\alpha_{o(s/c=.75)} = \alpha_o/[-.35(s/c) + 1.25] \qquad (7)$$

ii) The stalling incidence for a blade of s/c = .75, $i_{s(s/c=.75)}$, is determined. This is a function of α_o and $\beta_i/\alpha_{o(s/c=.75)}$, as shown in Fig. 3.8 [1].

iii) To calculate the actual stalling incidence of the blade, Δi_s is needed. This is a function of s/c and α_o [1], and is given in Fig. 3.9.

The actual stalling incidence can be derived from

$$i_s = i_{s(s/c = 0.75)} + \Delta i_s \qquad (8)$$

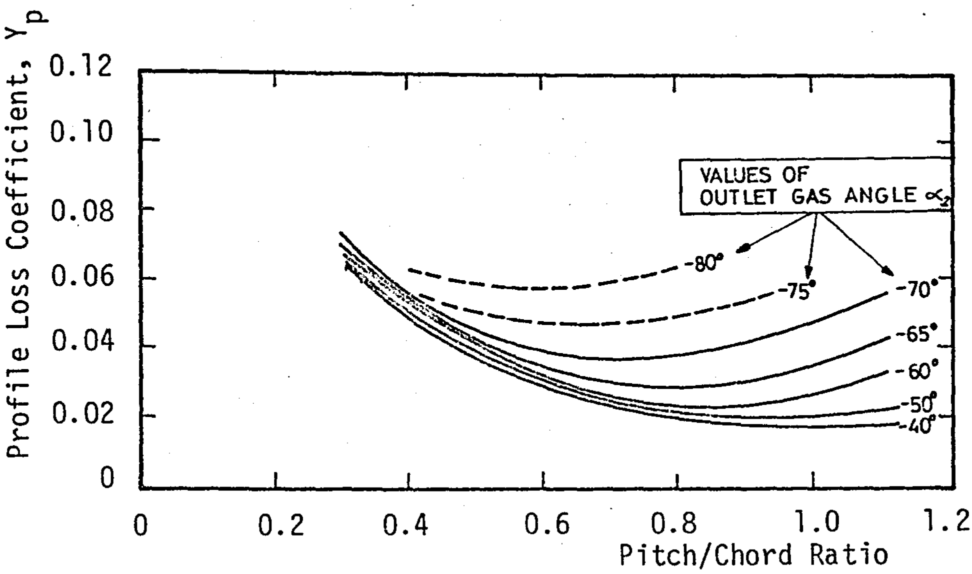

FIGURE 3.5 - Profile loss coefficient for nozzle blades (β_i = 0).

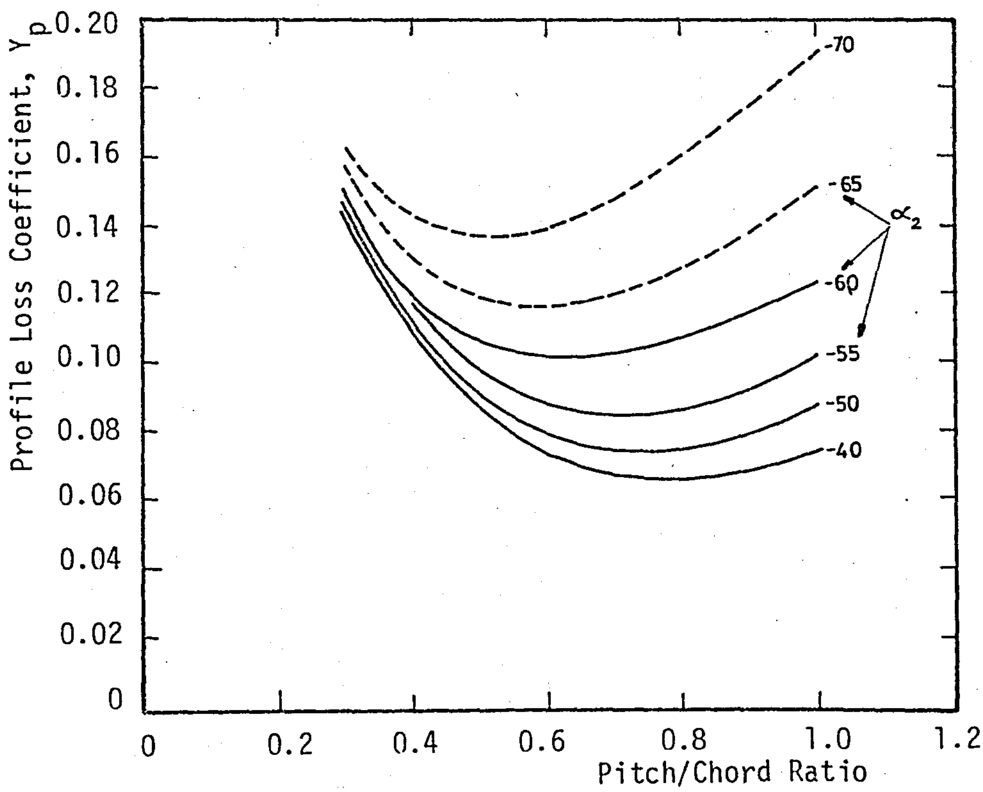

FIGURE 3.6 - Profile loss coefficients for impulse blades (β_i = $-\alpha_o$).

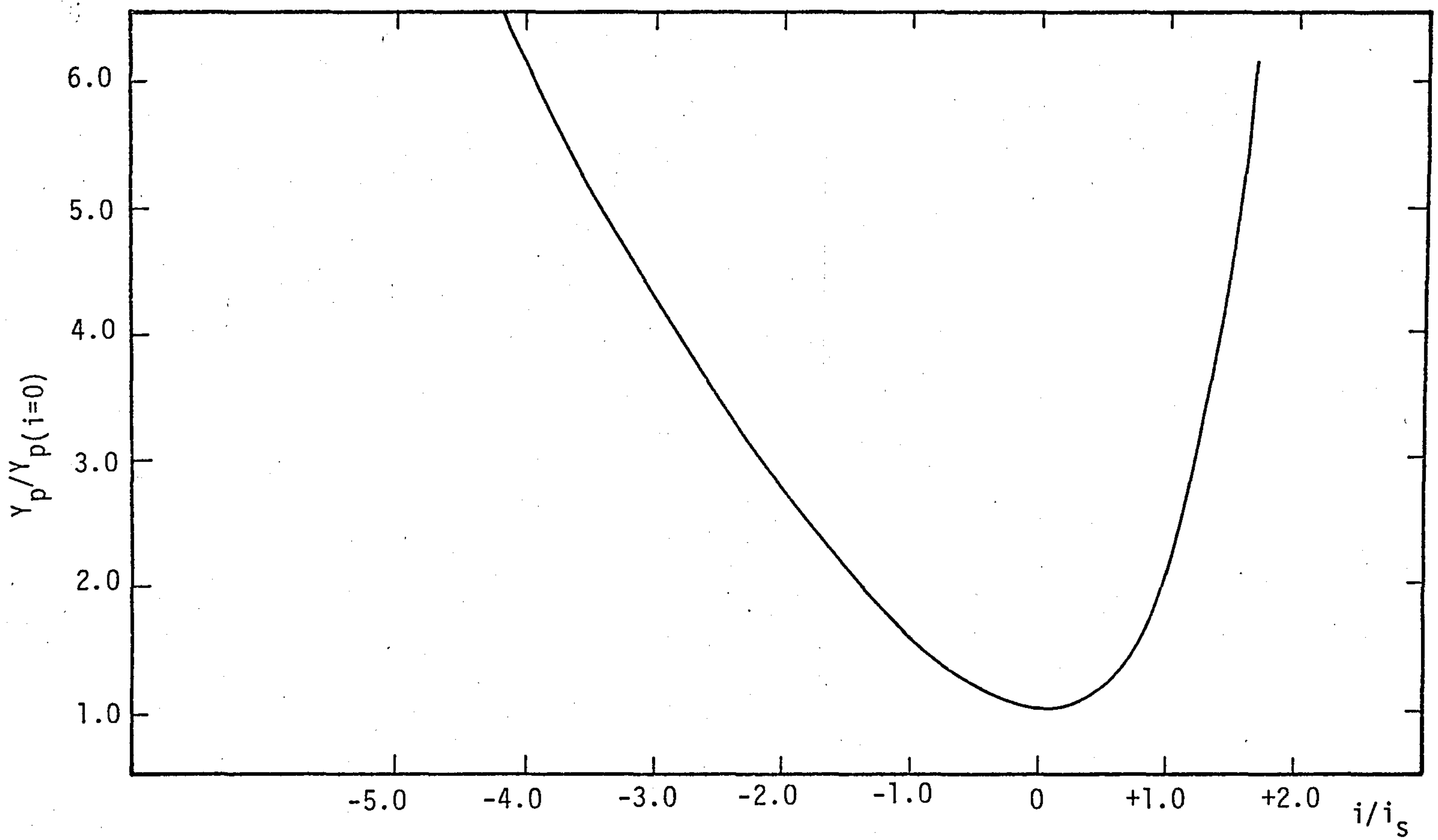

FIGURE 3.7 - Variation of the profile loss with incidence.

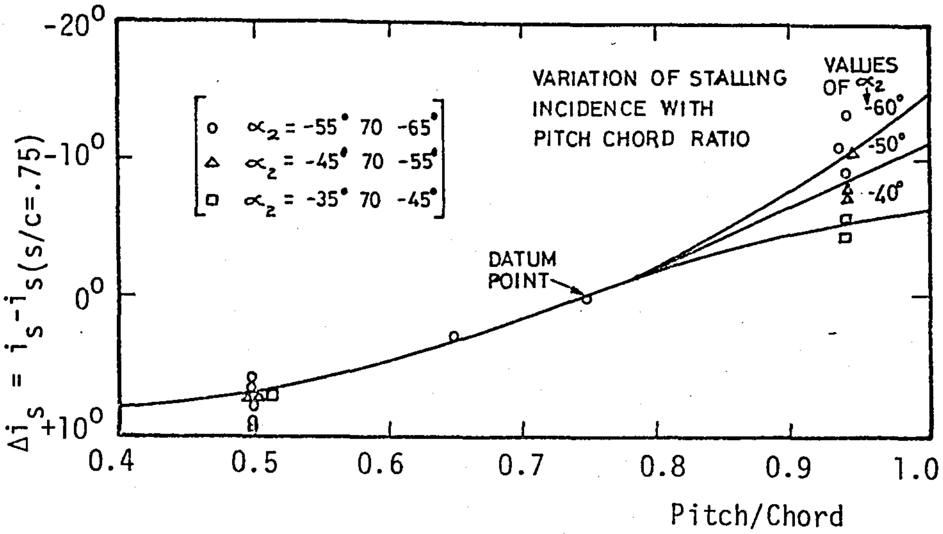

FIGURE 3.8 - Variation of the stalling incidence with pitch-per-chord ratio.

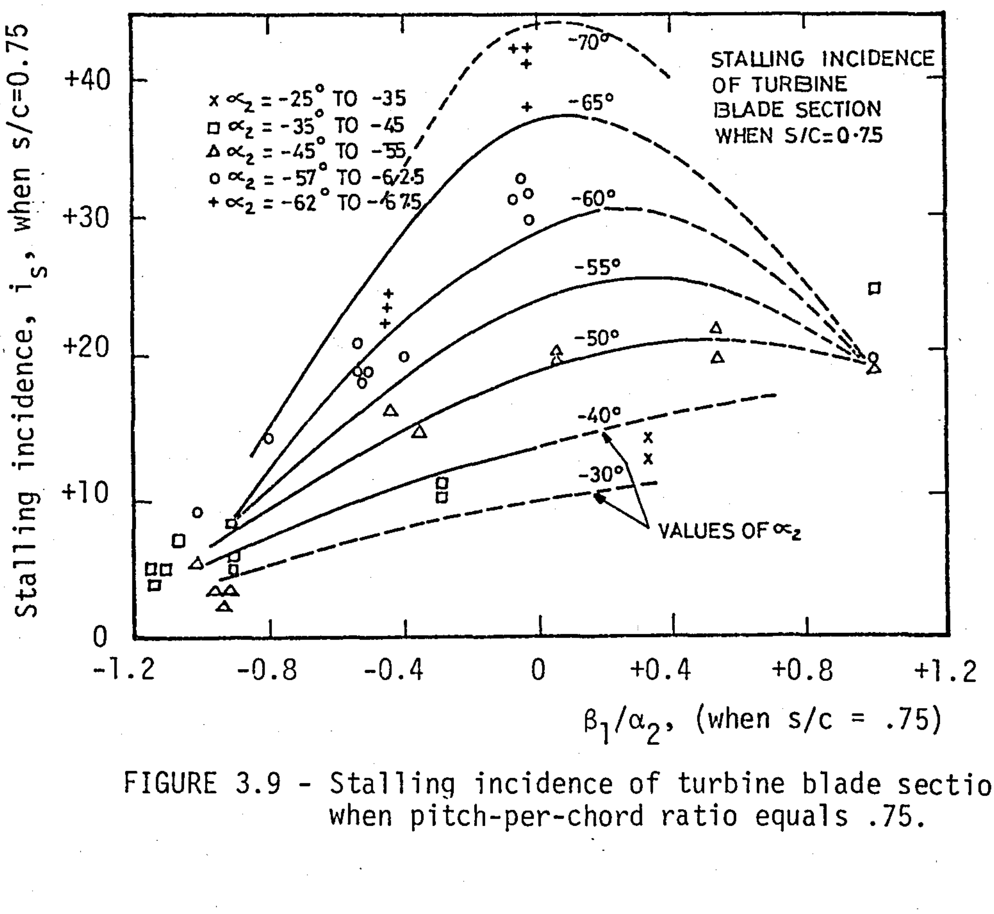

FIGURE 3.9 - Stalling incidence of turbine blade sections when pitch-per-chord ratio equals .75.

iv) Having now determined i_s and $Y_{p(i=0)}$, the profile loss coefficients over a wide range of incidence can be deduced from Fig. 3.7 [1].

3.1.2.2 Determination of Secondary and Tip Clearance Loss Coefficients

The secondary losses arise from the interaction between the 2-dimensional flow through the blade passage and the endwall boundary layer. The boundary layer at the annulus walls is deflected through a blade row and a secondary vortex is superimposed on the two-dimensional flow through the passage. This secondary flow is directed from the pressure side to the suction side of the blade near the top and bottom walls and in the opposite direction in the middle of the channel. The secondary losses, therefore depend on the blade loading, the blade shape (aspect ratio) and the endwall boundary layer.

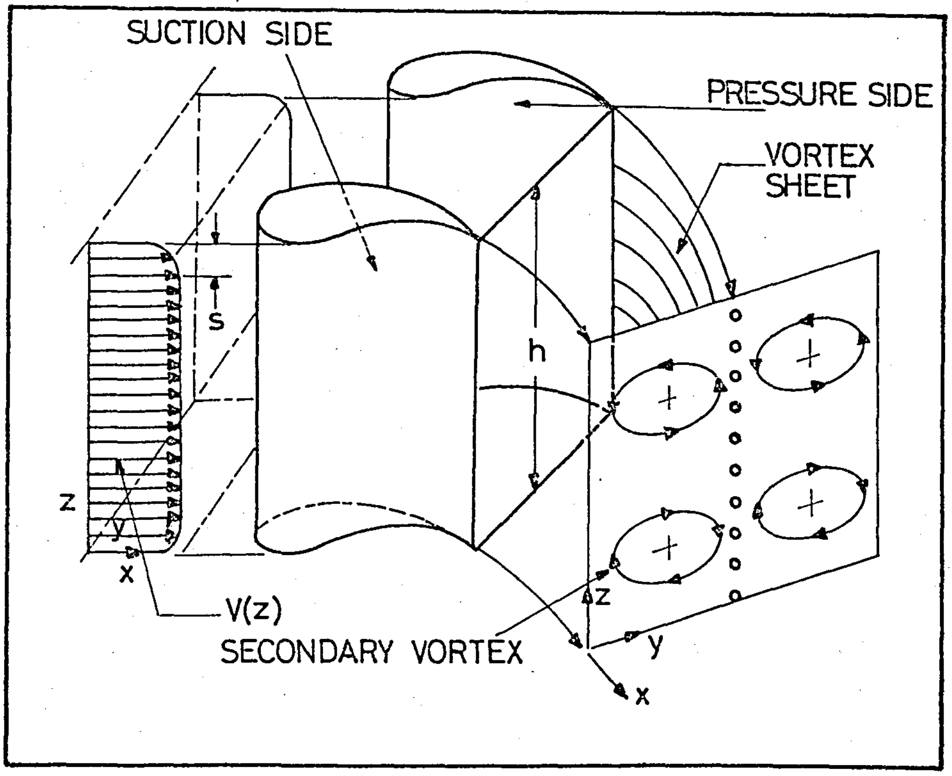

FIGURE 3.10 - Formation of secondary flow in a turbine blade passage.

Several empirical formulas have been suggested for the effect of blade loading (Sec. 2), one that fits best to experimental data being that due to Ainley and Mathieson [1].

The blade loading parameter of Ainley and Mathieson [1], Z, is in the form:

$$Z = \left(\frac{C_L}{s/c}\right)^2 (\cos^2\alpha_o/\cos^3\alpha_m) \tag{9}$$

where

$$C_L/(s/c) = 2(\tan\alpha_i - \tan\alpha_o)\cos\alpha_m ,$$

and

$$\alpha_m = \tan^{-1}\{(\tan\alpha_i + \tan\alpha_o)/2\} .$$

Ainley and Mathieson relate the secondary loss coefficient, Y_s, to the above defined blade loading parameter, Z, as [1]

$$Y_s = \lambda Z , \tag{10}$$

where λ is an empirical function given as [1]

$$\lambda = f\{\left(\frac{A_{n,o}}{A_{n,i}}\frac{\cos\alpha_o}{\cos\beta_i}\right)^2/(1 + ID/OD)\} , \tag{11}$$

where

$A_{n,i}$ = annulus area upstream of row, as shown in Fig. 3.2.

The functional relationship of λ, expressed in Eq. (11) is shown in Fig. 3.11.

Equation (11) has been corrected later by Dunham and Came [2]. Their correlation represents a significant improvement over that of Ainley and Mathieson [1], particularly for blades of low aspect ratio

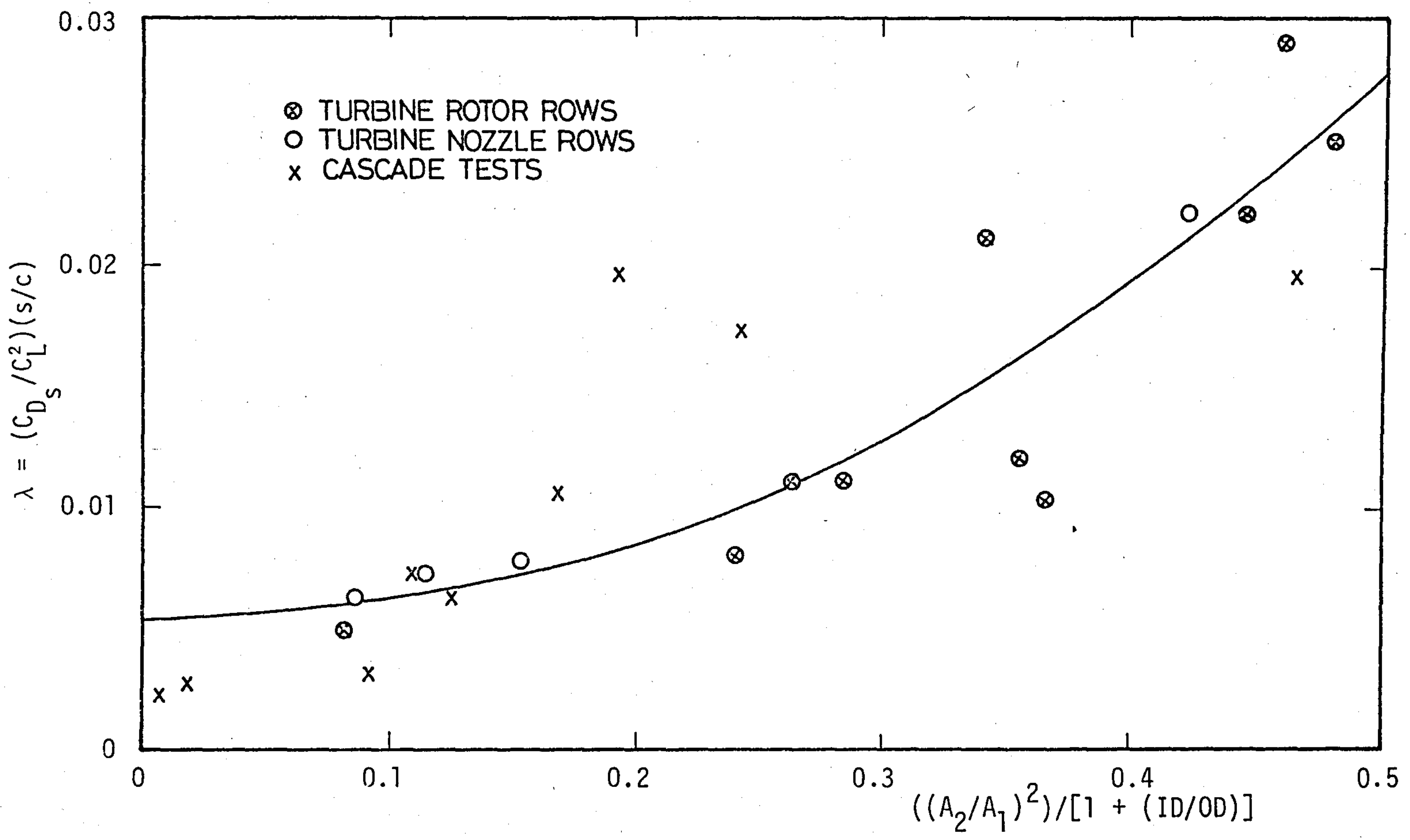

FIGURE 3.11 - Secondary losses in turbine blade rows.

(c/h), as encountered in small turbines. Although for more advanced methods of performance prediction, it would be necessary to calculate the wall boundary layer development, Dunham and Came [2] avoided this complication by the adoption of a single constant, in place of λ,

$$Y_s = 0.0334(c/h)(\cos\alpha_0/\cos\beta_i)Z \tag{12}$$

where, as shown in Fig. 3.1 and Fig. 3.2,

c/h = aspect ratio.

If there is a clearance between the tip of the blade and the side wall, there occurs a flow through the clearance directed from the pressure side towards the suction side. The intensity of this leakage flow through the clearance is proportional to the pressure difference between both sides of the blade, and thus proportional to the deflection [25]. The tip clearance loss coefficient, Y_k, depends consequently on the blade loading [1] and the size and nature of the clearance. Examination of both cascade and turbine data confirmed that Ainley and Mathieson's [1] blade loading parameter, Z, represents the effect of blade loading on clearance losses, satisfactorily.

In the method of Ainley and Mathieson, Y_k is expressed as [1],

$$Y_k = B(k/h)Z \tag{13}$$

where the constant, B, accounts for the type of the clearance and

B = 0.5 for plain tip clearance

B = 0.25 for shrouded tips.

This correlation has been corrected by Dunham and Came [2], according to Hubert's cascade data [26], in the following way

$$Y_k = B(c/h)(k/c)^{.78}Z \tag{14}$$

where $B = 0.47$ for plain tip clearance

$B = 0.37$ for shrouded tips.

3.1.2.3 Determination of Total Loss Coefficient and the Effect of Trailing Edge Thickness

In the Ainley and Mathieson Loss System [1], the profile, secondary and clearance loss coefficients are simply added to yield the total pressure loss coefficient, Y_t

$$Y_t = Y_p + Y_s + Y_k \tag{15}$$

This procedure applies to "conventional" blades [18], having a trailing edge thickness, t_e, roughly equal to 2% of the blade pitch, s. If the ratio of trailing edge thickness to blade pitch, t_e/s, differs from 0.02, then the total loss coefficient should be corrected by a factor, Y_{te}, [1]

$$Y_t = [Y_p + Y_s + Y_k] * Y_{te} \tag{16}$$

where, the trailing edge loss coefficient, Y_{te}, is defined as

$$Y_{te} = 21.6(t_e/s)^2 + 3.86(t_e/s) + 0.91 \quad , \tag{17}$$

and

t_e = blade trailing edge thickness, as shown in Fig. 3.1.

3.1.3 Determination of Gas Conditions

Once the total loss coefficient related to a particular blade is determined, the performance of the stage at particular entry flow conditions and at particular blade speed is determined. The calculations are then repeated for different values of these parameters and the overall performance map is built up. The method explained in this section is completely based on the system of Ainley and Mathieson [1]. It is assumed that the turbine is single stage and that the nozzle operates at zero incidence. The method can easily be adopted for use in a multistage turbine, the incidence onto the nozzle of any stage being calculated from the flow at exit from the previous stage.

3.1.3.1 Determination of Gas Flow Conditions at Outlet from a Nozzle Row

The procedure for determining the outlet gas conditions from the known inlet conditions and the calculated loss coefficient, Y_t, is outlined as follows.

i) Since the Mach number at outlet, M_2, is initially unknown, a preliminary guess for α_2 is obtained from Eq. (1) on the basis of geometric parameters.

ii) The non-dimensional inlet mass flow, $\dot{m}\sqrt{T_1}/A_2 P_1$, is calculated from inlet conditions, where

$\dot{m}$ = turbine inlet mass flow

T_1 = absolute total gas temperature at inlet to first nozzle row

P_1 = absolute total gas pressure at inlet to first nozzle row

$A_2 = A_{n,2} \cdot \cos \alpha_2$

iii) It is assumed that the maximum non-dimensional mass flow that can be passed through a specified blade passage, $(\dot{m}\sqrt{T_i}/A_o P_i)_{max}$, is a function of the total loss coefficient, Y_t. This relationship is plotted in Fig. 3.12.

The mean pressure drop within the blade passage is expressed in terms of a non-dimensional quantity, $\overline{\omega}/P_i$, where

$\overline{\omega}$ = mean loss of total pressure through a blade row due to friction and flow separation.

The value of $\overline{\omega}/P_i$ at the point where the non-dimensional mass flow reaches its maximum value, is termed the critical pressure drop ratio, $(\overline{\omega}/P_i)_{crit}$, and is a function of the total loss coefficient, Y_t. This relation is plotted in Fig. 3.13.

Therefore, the values of $(\dot{m}\sqrt{T_1}/A_2 P_1)_{max}$ and $(\overline{\omega}/P_1)_{crit}$ for given Y_t is found by referring to Fig. 3.12 and Fig. 3.13, respectively.

iv) Over the range of outlet Mach number $0 < M_o < 1.2$ and total loss coefficient $0 \leq Y_t < 1$, the actual ratio of pressure loss to inlet pressure, $\overline{\omega}/P_i$, can be related to the actual non-dimensional mass flow, $\dot{m}\sqrt{T_i}/A_o P_i$, by a single curve,plotting $(\overline{\omega}/P_i)/(\overline{\omega}/P_i)_{crit}$ against $(\dot{m}\sqrt{T_i}/A_o P_i)/(\dot{m}\sqrt{T_i}/A_o P_i)_{max}$. This relationship is given in Fig. 3.14.

Thus, from the above determined values of $(\overline{\omega}/P_1)_{crit}$, $(\dot{m}\sqrt{T_1}/A_2P_1)_{max}$ and $\dot{m}\sqrt{T_1}/A_2P_1$, one can find the corresponding value of actual $\overline{\omega}/P_1$ and from this, the absolute total pressure, P_2, of gas at outlet of the stator can be found from:

$$P_2 = P_1 - \overline{\omega} \quad . \tag{18}$$

Then, the non-dimensional mass flow at outlet of stator, $\dot{m}\sqrt{T_2}/A_2P_2$, can be calculated, knowing that for an uncooled nozzle, $T_2 = T_1$, where T_2 is the total absolute gas outlet temperature from nozzle (Fig. 3.3).

v) From the value of $\dot{m}\sqrt{T_2}/A_2P_2$, the value of M_2 and $V_2/\sqrt{T_2}$, where V_2 is the absolute gas outlet velocity from stator, can be deduced. The curves represented on Fig. 3.15 and Fig. 3.16, respectively, should be used for this purpose.

vi) It is stated in Section 3.1.1 that the gas outlet angle from a blade row, α_0, is a known function of outlet Mach number, M_0. Therefore, one can compare the value of α_2 corresponding to M_2 found in (v) and the initial value of α_2 guessed in (i). If the two values differ from each other by more than a given tolerance, the new value of α_2 is taken as the improved guess and the procedure is repeated.

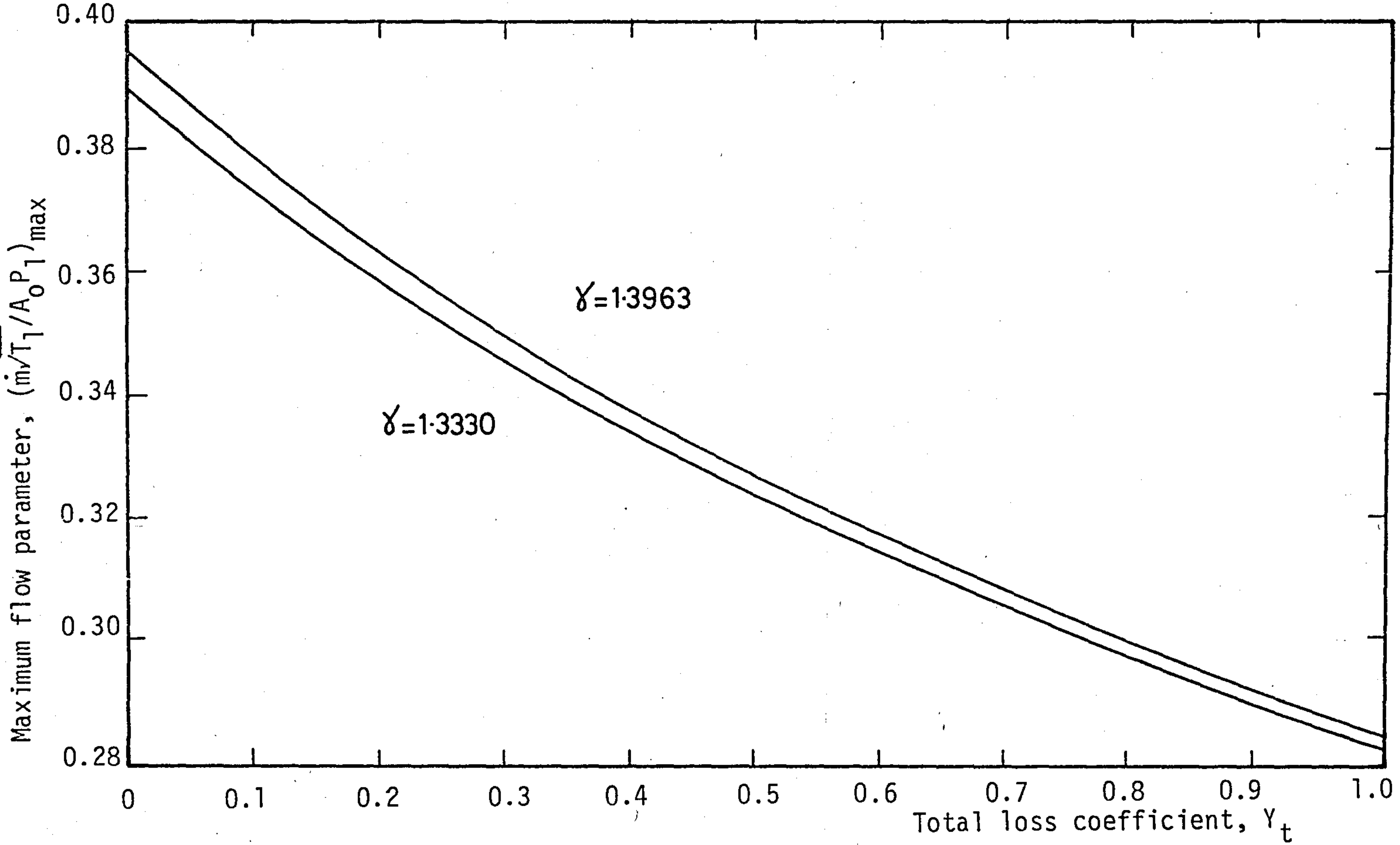

FIGURE 3.12 - Fundamental relation for compressible adiabatic flow.

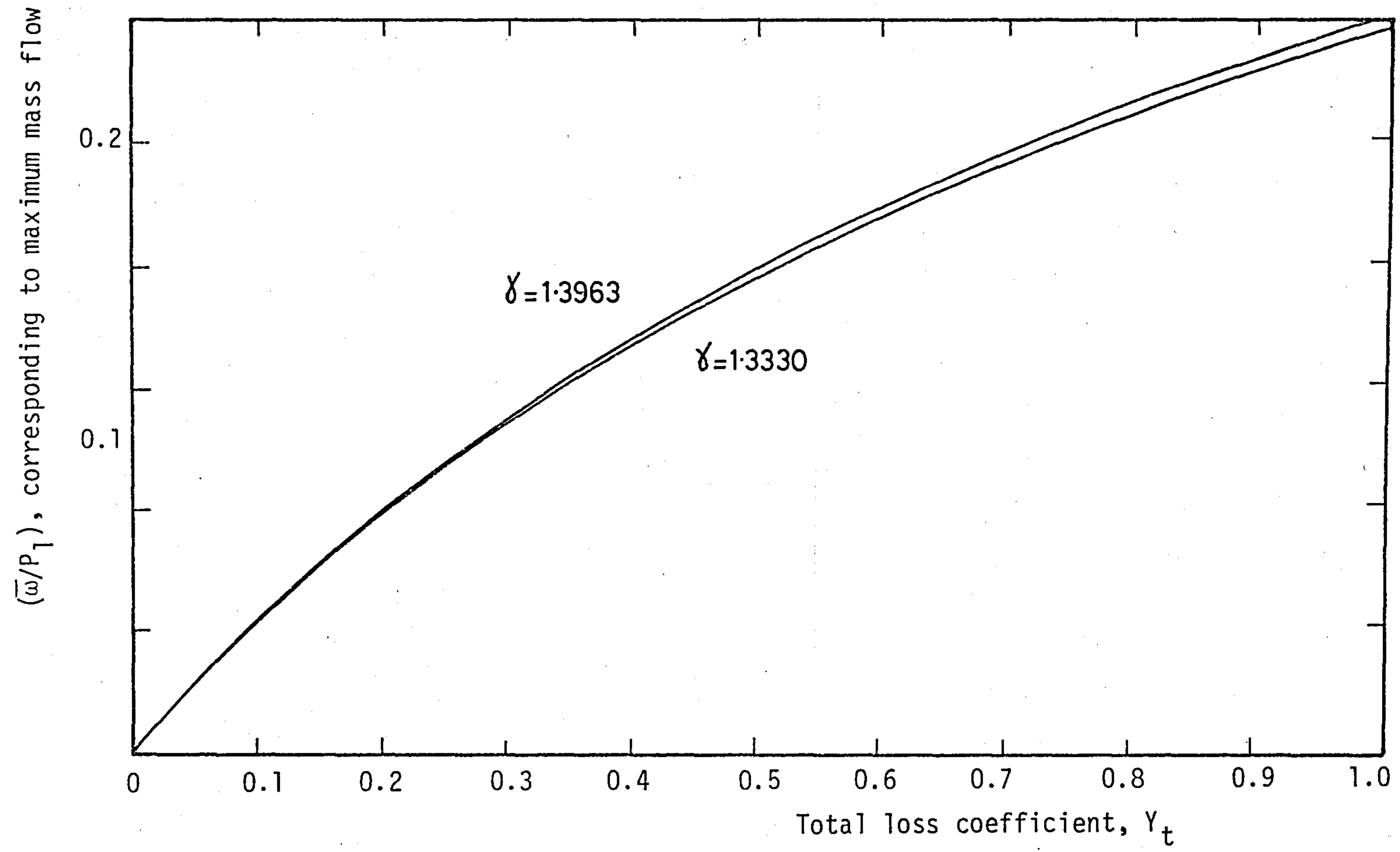

FIGURE 3.13 - Fundamental relation for compressible adiabatic flow.

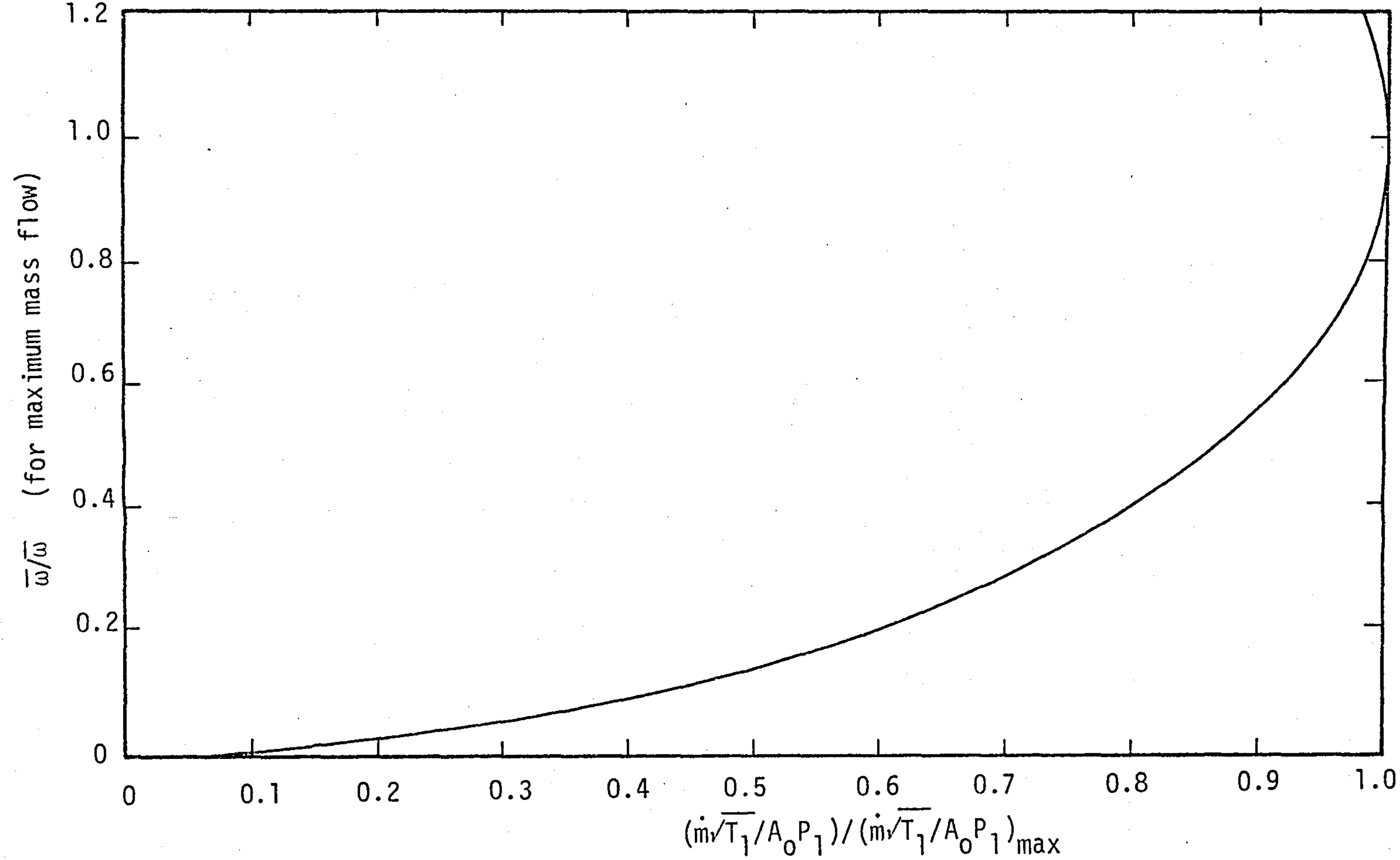

FIGURE 3.14 - Fundamental relation for compressible adiabatic flow.

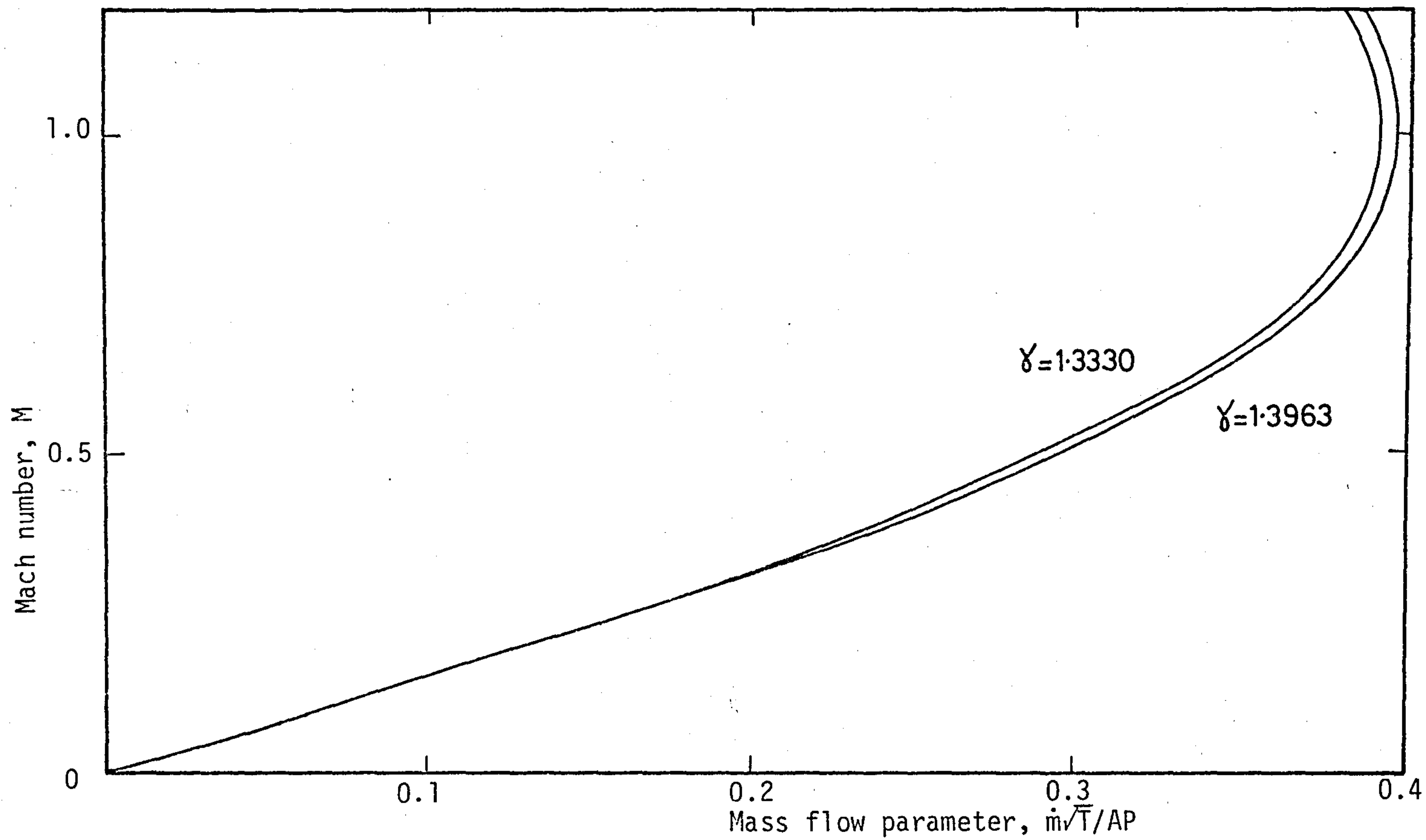

FIGURE 3.15 - Fundamental relation for compressible isentropic flow.

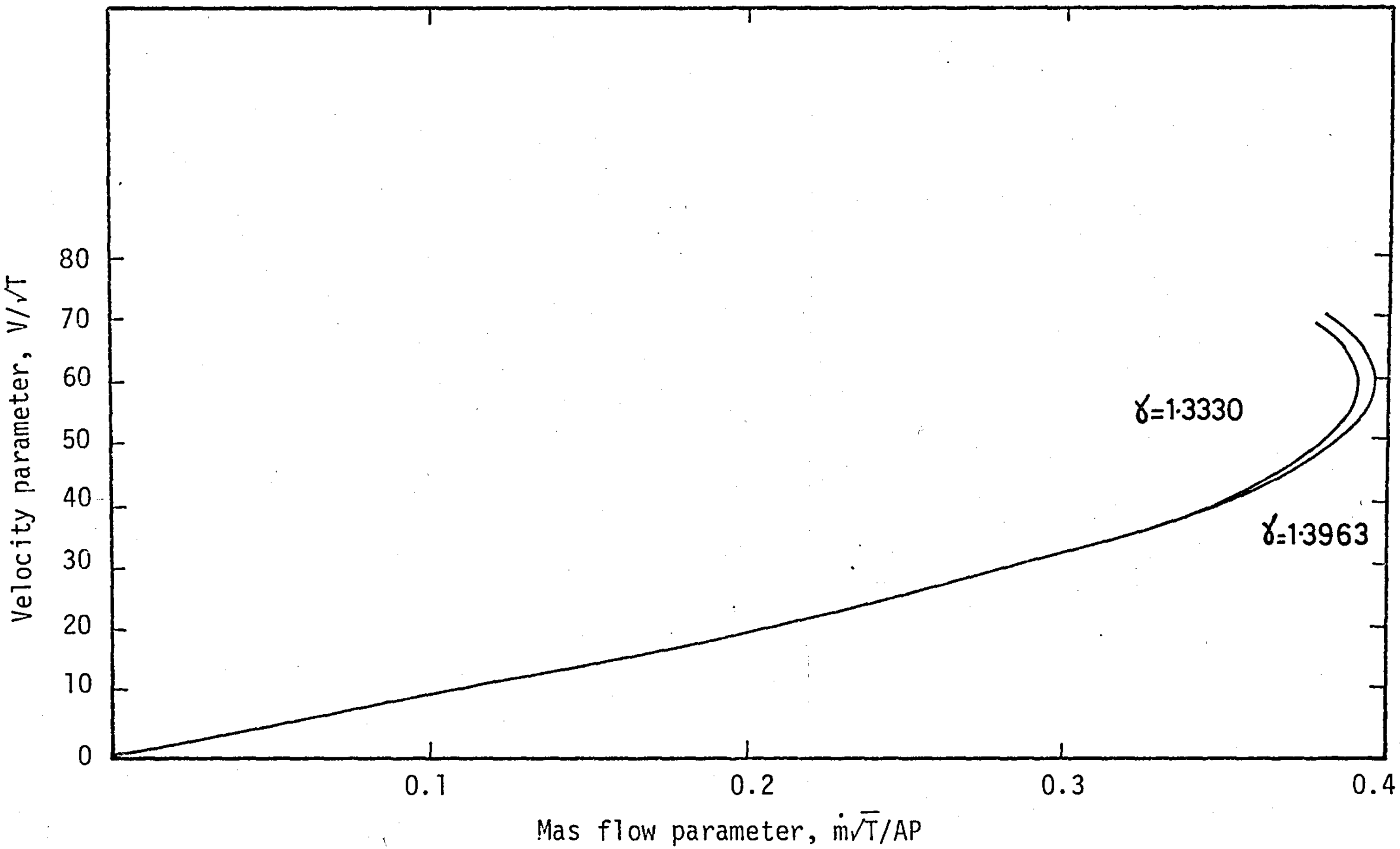

FIGURE 3.16 - Fundamental relation for compressible isentropic flow.

3.1.3.2 Determination of Gas Flow Conditions Relative to Rotor at Inlet

Knowing the gas velocity, flow angle from the nozzle row and the blade speed at the reference diameter, the gas conditions relative to the inlet of the rotor can be determined from the velocity triangles (Fig. 3.4)

$$V_{a,3} = V_2 \cos\alpha_2 \quad , \tag{19}$$

and

$$\alpha_3 = \tan^{-1}[(U/V_{a,3}) - \tan\alpha_2] \; , \tag{20}$$

where

$V_{a,3}$ = axial component of gas velocity at rotor inlet,

U = rotor blade speed at reference diameter,

α_3 = relative gas inlet angle to rotor.

Also from velocity triangles,

$$V_3 \cos\alpha_3 = V_2 \cos\alpha_2 \quad . \tag{21}$$

Thus,

$$V_3/V_2 = \cos\alpha_2/\cos\alpha_3 \; , \tag{21.a}$$

where

V_3 = relative gas inlet velocity to rotor.

The energy equation gives:

$$T = T_s + V^2/2C_p \tag{22}$$

where

C_p = specific heat of gas at constant pressure.

From Eq. (22) and from the fact that the static temperature at exit from the nozzle is equal to the static temperature relative to the rotor entry (i.e. $T_{s,2} = T_{s,3}$) the total head temperature, T_3, relative to the rotor inlet can be found:

$$T_2 = T_{s,2} + V_2^2/2C_p \tag{23}$$

$$T_3 = T_{s,3} + V_3^2/2C_p \tag{24}$$

$$T_3 = T_{s,2} + V_3^2/2C_p \tag{25}$$

From Eqs. (23) and (25)

$$T_2 - T_3 = (V_2^2 - V_3^2)/2C_p \tag{26}$$

Combining Eqs. (26) and (21.a) and rearranging,

$$T_3 = T_2 + V_2^2[1 - (\cos^2\alpha_2/\cos^2\alpha_3)]/2C_p \tag{27}$$

Having found T_3, total head pressure relative to rotor at inlet, P_3, may be deduced from:

$$P_3 = P_2[1 - (\frac{T_1 - T_2}{T_1})]^{\gamma/\gamma-1} \quad , \tag{28}$$

where

γ = ratio of specific heats for the gas.

3.1.3.3 Determination of Gas Flow Conditions Relative to Rotor at Outlet

The flow is calculated along an axial line at constant diameter, and no heat is transferred to or from the blades. Therefore, there must be no changes of total gas temperature relative to the rotor, i.e. $T_4 = T_3$, where T_4 is the total head temperature relative to rotor at outlet.

The procedure to be followed is given as:

i) From known rotor inlet gas angle, α_3 (Eq. (20)), and inlet blade angle, β_3, the value of the incidence on rotor can be determined from Eq. (5). Using the incidence, rotor total loss coefficient, $Y_{t,rotor}$, can be found as explained in Section 3.1.2.

ii) The value of $\dot{m}\sqrt{T_3}/A_4P_3$ is calculated. This is the non-dimensional mass flow at inlet to rotor, where

$$A_4 = A_{n,4}\cos\alpha_4 \qquad (29)$$

where

$A_{n,4}$ = Annulus area downstream the rotor row

α_4 = relative gas outlet angle from rotor.

Since the value of rotor outlet Mach number, M_4, is initially unknown, a preliminary guess for rotor outlet gas angle, α_4, is obtained from Eq. (1), on the basis of the geometric parameters.

iii) From the value of $Y_{t,rotor}$, values of $\dot{m}\sqrt{T_3}/A_4P_3)_{max}$ and $\overline{\omega}/P_3)_{crit}$ may be found from Fig. 3.12 and Fig. 3.13, respectively.

iv) The ratio $(\dot{m}\sqrt{T_3}/A_4P_3)/(\dot{m}\sqrt{T_3}/A_4P_3)_{max}$ is calculated, the corresponding value for $(\overline{\omega}/P_3)/(\overline{\omega}/P_3)_{crit}$ is found from Fig. 3.14 and finally from the known $(\overline{\omega}/P_3)_{crit}$, the actual $\overline{\omega}/P_3$ is deduced.

Since P_3 is known, the total gas pressure relative to rotor at outlet, P_4, can be derived from:

$$P_4 = P_3 - \overline{\omega} \tag{30}$$

The magnitude of rotor outlet non-dimensional mass flow, $\dot{m}\sqrt{T_4}/A_4P_4$, can now be calculated,and from Fig. 3.16 and Fig. 3.15, the corresponding values of $V_4/\sqrt{T_4}$ and M_4 may be found, where,

V_4 = gas velocity relative to rotor at outlet,

and M_4 = outlet Mach number from rotor.

v) As before, if the value of α_4 corresponding to the value of M_4 found in (iv), differs from the value approximated in (i), the process is repeated using the latest value of α_4.

3.1.3.4 Determination of Absolute Gas Flow Conditions at Outlet from Rotor

From the velocity triangles shown in Fig. 3.4

$$V_{a,4} = V_4\cos\alpha_4 \tag{31}$$

$$\alpha_1' = \tan^{-1}[(U/V_{a,4}) - \tan\alpha_4] \tag{32}$$

where

$V_{a,4}$ = axial component of gas velocity relative to rotor at outlet,

and α_1' = absolute gas outlet angle from rotor, or absolute gas inlet angle to the next stator.

From the energy equation (Eq. (22)),

$$T_1' = T_4 - V_4^2[1 - (\cos^2\alpha_4/\cos^2\alpha_1')]/2C_p \quad (33)$$

and

$$P_1' = P_4[1 - (\frac{T_3 - T_4}{T_3})]^{\gamma/\gamma-1} \quad (34)$$

where, as shown in Figure 3.3,

T_1' = absolute total gas temperature at rotor outlet, or absolute total gas temperature at inlet to next stator

P_1' = absolute total gas pressure at rotor outlet, or absolute total gas pressure at inlet to next stator.

3.1.4 Determination of Turbine Overall Characteristics

The overall stage pressure ratio, P_1'/P_1 and the temperature drop ratio, $(T_1 - T_1')/T_1$, corresponding to the initially selected value of non-dimensional mass flow at inlet, $\dot{m}\sqrt{T_1}/P_1$, may now be determined.

The overall stage isentropic efficiency, η, is then given by

$$\eta = [(T_1 - T_1')/T_1]/[1 - (P_1'/P_1)^{\gamma-1/\gamma}] \quad (35)$$

The magnitudes of pressure and temperature drop ratio and the isentropic efficiency for different values of non-dimensional mass flow can be calculated to construct a performance characteristic.

The process may be extended by choosing several blade speeds for the turbine and by constructing a constant speed characteristic curve for each speed chosen.

3.1.5 Performance of a Choked or Nearly Choked Stage

Choking is assumed to occur in a turbine row when the non-dimensional mass flow, $\dot{m}\sqrt{T_i}/P_oA_o$, in nozzle or rotor, attains its maximum value for the given total loss coefficient, Y_t, and blade outlet angle, α_o. The flow upstream of the choked row then remains unchanged as the overall pressure ratio is increased, the flow downstream becoming supersonic and the gas direction changing slightly to preserve continuity. The flow may choke again downstream as the pressure ratio is further increased.

Ainley and Mathieson [1] suggest the following procedure for calculating the flow through a choked row, assuming that the flow quantity, $\dot{m}\sqrt{T_o}/P_o$, and the loss coefficient, Y_t, remain constant as the pressure ratio is reduced below that required to choke the blade passage.

i) The value of outlet Mach number, M_o, is arbitrarily selected.

ii) Knowing M_o, the values of non-dimensional flow parameter at blade outlet, $\dot{m}\sqrt{T_o}/A_oP_o$, and the total to static pressure ratio, $P_{o,s}/P_o$, can be found referring to Fig.3.15 and Fig. 3.17 respectively.

iii) Now, from the definition of total loss coefficient (Eq. (4)), from known total to static pressure ratio, $P_{o,s}/P_o$, and from total loss coefficient, Y_t, which is unaltered, one gets

$$P_i - P_o = Y_t[P_o - P_{o,s}] \tag{36}$$

Dividing throughout by P_o, and using Eq. (18), one gets,

$$\overline{\omega}/P_o = Y_t[1 - P_{o,s}/P_o] \quad , \tag{37}$$

and

$$\overline{\omega}/P_i = (\overline{\omega}/P_o)/(1 + \overline{\omega}/P_o) \quad , \tag{38}$$

so that, finally,

$$P_o = P_i - \overline{\omega} \ .$$

Knowing that $T_o = T_i$, one can find from $\dot{m}\sqrt{T_o}/A_oP_o$, the necessary value of A_o. Since

$$A_o = A_{n,o}\cos\alpha_o \quad ,$$

the required value of blade outlet gas angle, α_o, can be determined. Having thus determined the new outlet conditions relative to the row, the absolute conditions at the outlet can be calculated as before. This procedure is repeated until the outlet Mach number is equal to unity. Only those results for which the mass flow attains its maximum value and the outlet Mach number reaches unity are taken into consideration. Although the intermediate steps are necessary for predicting the exact value of the pressure ratio for which the turbine fully chokes (when $M_o = 1$) and although they will appear on the output of the computer program, the data points corresponding to those intermediate steps will not be shown on the final performance curves.

3.1.6 Effect of Reynolds Number

In reference [1], it is stated that the magnitude of the efficiency and turbine performance calculated using the data presented, would correspond to a mean operating Reynolds number of approximately $2x10^5$, the Reynolds number being defined by blade chord, outlet gas velocity (relative to the row), outlet gas density and outlet gas viscosity.

Thus, it is recommended in [1] that an approximate correction be applied to the overall characteristics by assuming that the turbine overall efficiency is affected by Reynolds number variations as

$$1 - \eta_{corrected} = (1 - \eta) + \left(\frac{Re_{,m}}{2x10^5}\right)^{-0.2} \qquad (39)$$

where

$Re_{,m}$ = Mean Reynolds number of stator and nozzle.

In the revised method of Dunham and Came [2], however, an optional correction is applied directly to the profile and secondary losses, using the Reynolds number appropriate to the particular blade row:

$$(Y_p + Y_s)_{corrected} = (Y_p + Y_s)\left(\frac{Re}{2x10^5}\right)^{-0.2} \qquad (40)$$

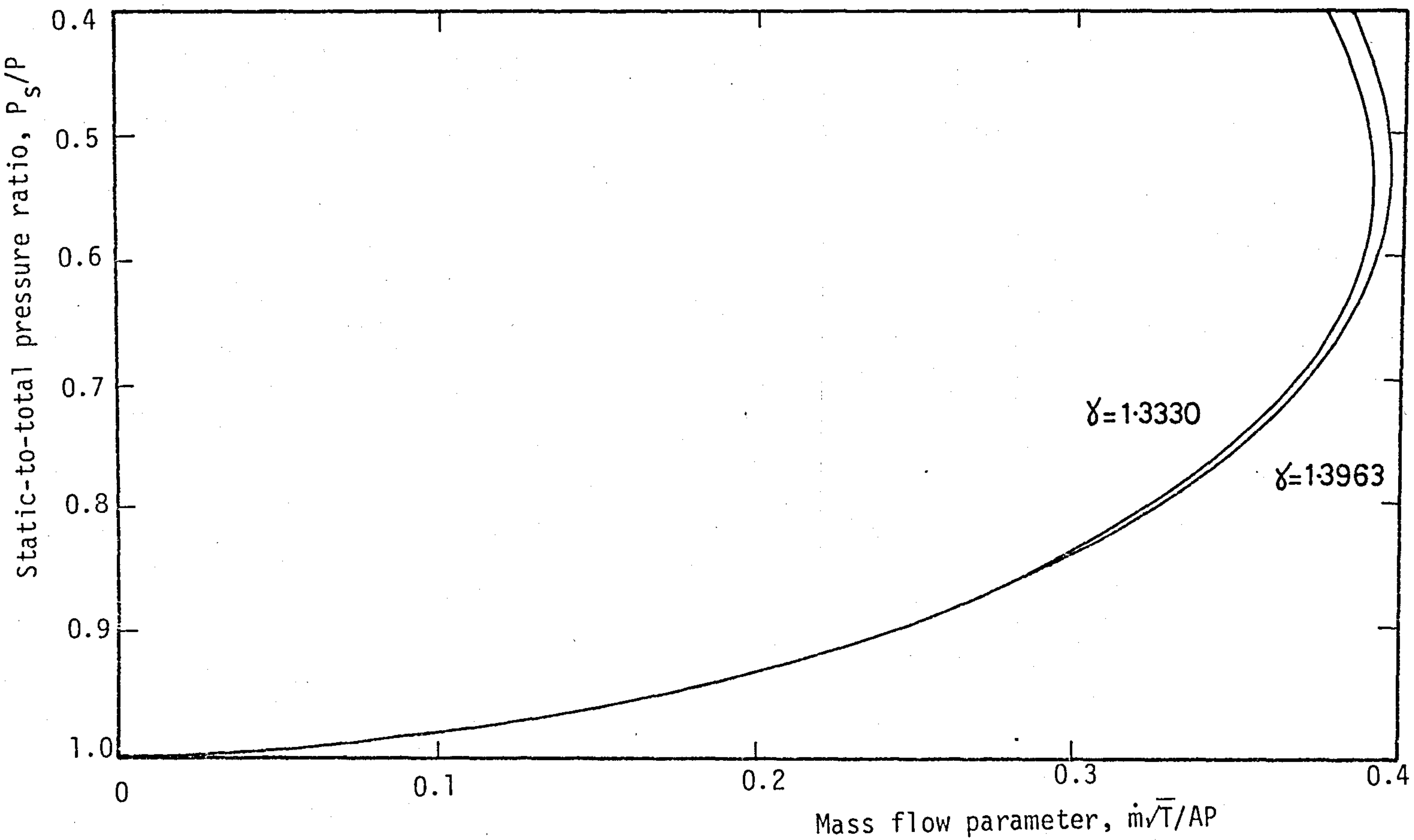

FIGURE 3.17 - Fundamental relation for compressible isentropic flow.

3.2 OPTIMUM DESIGN OF A GAS TURBINE STAGE

The second part of the thesis involves the design of a gas turbine stage in which the aerodynamic losses are minimized (or, the aerodynamic total-to-total efficiency is maximized). The analysis may be split down into two parts: the calculation of the efficiency and the optimization of the efficiency.

3.2.1 Calculation of the Efficiency of the Stage

The efficiency calculation is performed according to the method given by Rao and Gupta [3]. The method consists in relating the efficiency to the most relevant turbine parameters, namely, the turbine mean diameter, the chord and pitch lengths of the stator and rotor blades, the rotor blade inlet and exit angles and the axial velocity through the stage. The remaining parameters of the problem are expressed in terms of these 8 design variables. Initial values are given to the design variables at beginning of the analysis and these are modified through the iterative process explained in Section 3.2.2.

In the present design method, it is assumed that the total pressure and temperature at the inlet to the stage, the mass flow rate across the stage and the speed of the rotor are known parameters. Further, it is assumed that the properties of air such as the specific heat at constant temperature, the gas constant, the viscosity and the ratio of the specific heats are also known.

Before discussing the procedure, it must be noted that the effects of incidence and deviation of the gas stream onto and away the rotor blades are completely neglected in the present method, i.e., the gas angles at the inlet and outlet of the rotor blades are assumed to be equal to the respective blade angles. Consequently, the independent choice of the rotor inlet and outlet gas angles consitutes the main difference between the procedures of the first and the second parts of the thesis. In contrast to the method of Ainley and Mathieson [1] in which the outlet gas angles from the blade rows are determined through a trial and error process after having obtained the respective total losses of the rows, in the procedure of Rao and Gupta [3], the trial and error process is imposed directly on the total loss of the stator and on the total-to-total efficiency.

The performance estimation method of [3] may be outlined as follows:

First, the static gas conditions at the stator inlet are determined using the known inlet conditions. From the energy equation (Eq. (22)) it is deduced that:

$$T_{1s} = T_1 - V_a^2/2C_p \tag{41}$$

where the inlet gas velocity to the (first) stator row is assumed axial (and of an assigned magnitude since V_a is a design variable).

For an isentropic expansion as shown in Fig. 2.3, the static pressure can be found from the thermodynamic relation,

$$P_{1s} = P_1(T_{1s}/T_1)^{\gamma/(\gamma-1)} \tag{42}$$

and from the perfect gas low, the inlet gas density, ρ_1, is determined as:

$$\rho_1 = P_{1s}/(R_a T_{1s}) \tag{43}$$

where R_a is the gas constant.

The geometry of the turbine at inlet can now be determined from the consideration of the continuity equation:

$$A_{n,1} = \dot{m}/(\rho_1 V_a) \quad , \tag{44}$$

where, as also shown in Fig. 2.2,

$A_{n,1}$ = annulus area at inlet to stator.

The annulus height at turbine inlet, h_1, is given by

$$h_1 = A_{n,1}/\pi.D \quad , \tag{45}$$

where D, the mean diameter, is a design variable of preassigned value.

Since the rotor inlet gas angle, α_3, is also a design variable with an assigned magnitude, the velocity triangles shown in Fig. 3.4 can be used to determine the gas angle at the outlet of the stator. Thus,

$$\alpha_2 = \tan^{-1}(\tan\alpha_3 + V_a/U) \tag{46}$$

where the blade speed, U, at reference diameter, D, is determined from the known rotational blade speed, N, as

$$U = \pi.D.N \tag{47}$$

The gas outlet velocity from the stator is also determined from the velocity triangles as

$$V_2 = V_a/\cos\alpha_2 \tag{48}$$

Now, the static pressure, P_{2s}, and the temperature, T_{2s}, at the stator outlet must be determined. The energy equation (Eq. 22) gives T_{2s} easily. Hence,

$$T_{2s} = T_1 - V_2^2/2c_p \tag{49}$$

However, the evaluation of P_{2s} is not straight forward and requires the use of the stator total loss coefficient, λ_N, based on the temperature drop (or, the enthalpy loss). Horlock [4] defines λ_N as

$$\lambda_N = \frac{T_{2s} - T_{2s,is}}{T_1 - T_{2s}} \tag{50}$$

where $T_{2s,is}$ is the absolute isentropic static temperature at the stator outlet, as shown in Fig. 2.3.

It is noted that Eq. (50) contains two unknowns, namely, λ_N and $T_{2s,is}$. Therefore, this step requires a trial and error process to be carried out. Accordingly, a first trial value is assigned to λ_N, and then, combining Eq. (49) and Eq. (50), the value of $T_{2s,is}$ is obtained:

$$T_{2s,is} = T_1 - \frac{V_2}{2c_p}(1 - \lambda_N) \tag{51}$$

The absolute static pressure at the stator outlet, P_{2s}, can now be determined from the thermodynamic equation for isentropic expansion as follows:

$$P_{2s} = P_1/(T_1/T_{2s,is})^{\gamma/(\gamma-1)} \tag{52}$$

The gas density at the rotor inlet is given from the ideal gas law:

$$\rho_2 = P_{2s}/(R_a T_{2s}) \tag{53}$$

and as before the flow area at the rotor inlet is determined from the continuity equation:

$$A_{n,2} = \dot{m}/(\rho_2 V_a) \tag{54}$$

The annulus height at rotor inlet, h_2, is given by:

$$h_2 = A_{n,2}/(\pi . D) \tag{55}$$

In order to determine the gas conditions at the rotor outlet, an important aerodynamic parameter is used. This is the stage total temperature drop, ΔT, which is defined as:

$$\Delta T = T_1 - T_1' \tag{56}$$

where

T_1' = total absolute gas temperature leaving the stage.

However, another expression for ΔT can be obtained through the following analysis. For adiabatic flow in a turbine stage, the specific work output in the rotor blades, ΔW, is equal to the change in specific total enthalpy, i.e.,

$$\Delta W = H_1 - H_1' \tag{57}$$

where H_1 and H_1' are the total enthalpies of the gas entering and leaving the stage, respectively. Eq. (57) can also be expressed in terms of the total temperatures at the entry and exit of the stage. Therefore,

$$\Delta W = C_p(T_1 - T_1')$$
$$= C_p \Delta T \tag{58}$$

On the other hand, considering the momentum equation applied to a control surface enclosing the rotor, the specific work output, ΔW, can be expressed as the total change of momentum of gas per unit flow, i.e.,

$$\Delta W = U(V_{t_3} + V_{t_4}) \tag{59}$$

where, as shown in Fig. 3.4.

V_{t_3}, V_{t_4} = tangential components of absolute gas velocities at rotor inlet and outlet, respectively.

Again considering the velocity triangles in Fig. 3.4, Eq. (59) can be modified as follows

$$\Delta W = UV_a(\tan\alpha_3 + \tan\alpha_4) \tag{60}$$

Finally, combining Eq. (58) and Eq. (60), a second expression for the total temperature drop across the stage is obtained:

$$\Delta T = UV_a(\tan\alpha_3 + \tan\alpha_4)/C_p \tag{61}$$

and the absolute total gas temperature at the rotor outlet follows directly from Eq. (56)

$$T_1' = T_1 - \Delta T \tag{62}$$

The determination of the total gas pressure at the outlet of the rotor, P_1', is not as easy as in the case of the stator, because it involves the expression of the total-to-total stage efficiency, η_s. Horlock [4] defines the stage efficiency as follows:

$$\eta = \frac{T_1 - T_1'}{T_1 - T_{1,is}'} \tag{63}$$

where $T_{1,is}'$ is the total absolute isentropic temperature at the stage outlet. On the other hand, the thermodynamic equation for isentropic expression gives,

$$P_1' = P_1\left(\frac{T_{1,is}'}{T_1}\right)^{\gamma/(\gamma-1)} \tag{64}$$

Combining Eqs. (62), (63) and (64),one gets

$$P_1' = P_1\left(1 - \frac{\Delta T}{\eta T_1}\right)^{\gamma/(\gamma-1)} \tag{65}$$

It is noted that, again, there are two unknowns in Eq. (65) namely, P_1' and η. Therefore, the procedure must involve here the assumption of a trial value for the efficiency, η. This guessed value is replaced by the more accurate value of η at the end of the first iteration, and the iterative process is continued until the values of the stage efficiency in two consecutive iterations are sufficiently close to each other.

From the velocity triangles at the rotor outlet (Fig. 3.4) and using the assigned value of the design variable, α_4, the absolute gas outlet angle from the rotor (or, the absolute gas angle at the inlet to the next stator row), α_1',can be determined as

$$\alpha_1' = \tan^{-1}(\tan\alpha_4 - V_a/U) \quad , \tag{66}$$

and the absolute gas velocity at the outlet of the rotor row is given by

$$V_1' = V_a/\cos\alpha_1' \quad . \tag{67}$$

The absolute static temperature at the rotor outlet is determined from the energy equation (Eq. (22)):

$$T_{1,s}' = T_1' - (V_1'^2/2c_p) \quad , \tag{68}$$

whereas the absolute static pressure at the same station is determined from the thermodynamic equation for isentropic expansion:

$$P_{1,s}' = P_1'(T_{1,s}'/T_1')^{\gamma/(\gamma-1)} \tag{69}$$

The density of air leaving the rotor is found from the ideal gas law. The annulus area $A_{4,n}$ and the annulus height, h_4, are determined from the continuity equation as before:

$$\rho_4 = P_{4s}/(R_a T_{4s}) \tag{70}$$

$$A_{4,n} = \dot{m}/(\rho_4 V_a) \tag{71}$$

$$h_4 = A_{4,n}/(\pi . D) \tag{72}$$

After the gas conditions at each station are evaluated, the pressure loss coefficients of the stator and rotor, based on guessed blade outlet angles are calculated according to the procedure of Ainley and Mathieson [1] as corrected by Dunham and Came [2] (Section 3.1.2). The pressure loss coefficients for stator (Y_N) and for rotor

(Y_R) lead to the improved value of λ_N and λ_R and thereafter to the improved value of η. The correlations of pressure loss coefficients, Y, of Ainley and Mathieson [1] and enthalpy drop coefficients, λ, of Hoclock [4] are given by Rao and Gupta [3] as follows:

$$\lambda_{N,\text{improved}} = Y_N \frac{T_{2s,is}}{T_1} \tag{73}$$

and

$$\lambda_{R,\text{ improved}} = Y_R \frac{T_{4s,is}}{T_4} \tag{74}$$

where, $T_{4s,is}$ is the absolute static isentropic temperature at the rotor outlet and can be determined from the thermodynamic relation for isentropic expansion as,

$$T_{4s,is} = T_{2s}/(\frac{P_{2s}}{P_{4s}})^{(\gamma-1)/\gamma} \tag{75}$$

and

$$T_4 = T_{4s} + V_1'^2/2c_p \tag{76}$$

The expression for η, as given by Horlock [4] is as follows:

$$\eta = \frac{H_1 - H_1'}{H_1 - H_{1,is}'} \tag{77}$$

where

$H_{1,is}'$ = absolute total isentropic entropy at rotor outlet.

Rearranging and noting that the total enthalpy drop is equal to the static enthalpy drop for the rotor blades, Eq. (77) may be put in the conventional form which is,

$$\eta = \frac{1}{1 + (\lambda_R(V_1'^2/2c_p) + (T_{3s}/T_{2s})\lambda_{N_{improved}}(V_2^2/2c_p))/(T_1 - T_1')}$$

(78)

The final step of the procedure is the evaluation of the Reynolds number, Re, of the stage:

$$Re = (\rho_2 V_2 C_N + \rho_4 V_1' C_R)/(2\mu) \qquad (79)$$

where μ is the viscosity of the fluid.

The correction for Re applied to η is realized as suggested in [1]:

$$\eta_{,Re} = 1 - (1 - \eta)\left(\frac{2x10^5}{Re}\right)^{0.2} \qquad (80)$$

If $\eta_{,Re}$ differs from η assumed in Eq. (65), then the value of $\eta_{,Re}$ and $\lambda_{N,improved}$ are taken as the improved guesses to be used in Eq. (65) and Eq. (51) respectively, and the procedure is repeated until convergence.

3.2.2 Optimization of the Efficiency of the Stage

Inspection of the efficiency equations together with the loss relations presented earlier shows that analytical optimization is not possible due to the high non-linearity of the expressions (Sec. 3.2.1). It is therefore, necessary to resort to a numerical optimization technique.

It is stated in Section 3.2.1 that the isentropic efficiency, η, of the stage is considered to be a function of the eight design variables, including the mean diameter, rotor & stator chord & pitch lengths, rotor inlet and outlet blade angles and the axial velocity. The initial values of the design variables assigned at the beginning of the analysis are modified during the iterative optimization process. The remaining data of the problem are either given at the start or can be expressed in terms of the design variables during the analysis.

Grouping the design variables as a vector quantity, $\vec{X}$, the optimization problem in hand can be stated in the standard form:

$$\text{Find } \vec{X} = \{D,\ C_R,\ C_N,\ S_N, S_R,\ \alpha_3,\ \alpha_4,\ V_a\} \quad \text{which maximizes } \eta(\vec{X}) \tag{81}$$

However, a practical turbine design, in most cases, is the result of a multitude of compromises. The maximum efficiency to be obtained would turn out to correspond to a geometry which poses either fabrication problems such as excessive rotative speeds, huge diameters, etc., or violates the aerodynamic requirements such as excessive Mach numbers, choking, etc. In order to avoid such undesired results, it is therefore necessary to meet the requirements rising from aerodynamic, vibrational and strength considerations. In the present thesis, only those requirements which are related to aerodynamic considerations are considered. The constraints of the numerical problem are as visualized in Ref. [3] and they are stated in a non-dimensional form as follows:

1. The rotational velocity of the rotor should be within some upper and lower bounds. In the present study it will be required that

$$\frac{50}{U} - 1.0 \leq 0$$
$$\frac{U}{400} - 1.0 \leq 0 \quad (82)$$

2. The aspect ratio(height/chord) of rotor and nozzle blades should be within some specified upper and lower bounds. The bounds used in the present study are as follows:

$$(h_R/8D) - (C_R/D) \leq 0$$
$$(C_R/D) - (h_R/2D) \leq 0 \quad (83)$$
$$(h_N/10D) - (C_N/D) \leq 0$$
$$(C_N/D) - (h_N/2D) \leq 0$$

where h_N and h_R correspond to the average nozzle and rotor annulus heights, respectively and are expressed as follows:

$$h_N = (h_1 + h_2)/2 .$$
$$h_R = (h_2 + h_4)/2. \quad (84)$$

3. The pitch-to-chord ratio of the rotor and nozzle blades should lie within certain limits. The present analysis will require that

$$0.5 < S_N/C_N < 1.0$$
$$0.5 < S_R/C_R < 1.0 \quad (85)$$

4. The relative gas velocity angles at inlet and outlet of the rotor blades should be within some specified bounds. Here, it will be assumed that

$$0.01 < \alpha_3 < 1$$
$$40/57 < \alpha_4 < 1.4 \tag{86}$$

5. The axial velocity of flow should lie within some upper and lower bounds which will be given in the present analysis as

$$(50/V_a) - 1 \leq 0$$
$$(V_a/400) - 1 \leq 0 \tag{87}$$

6. The actual pressure ratio across the nozzle blades should be below the critical pressure ratio:

$$(P_1/P_{2s}) - (P_1/P_c) \leq 0 \tag{88}$$

where the critical pressure, P_c, across the nozzle is found from frictionless, adiabatic flow approximation [27] applied to the throat section for critical conditions, and is given as follows:

$$P_c = P_1 \left(\frac{2}{\gamma + 1}\right)^{\gamma/(\gamma+1)} \tag{89}$$

7. The Mach number at the exit from the stage should be less than a specified maximum value. Here, the maximum value will be assumed to be unity.

$$M_4 - 1 \leq 0 \tag{90}$$

where, M_4 is given in [3] as

$$M_4 = \frac{V_1'}{\sqrt{\gamma R_a T_{3s}}} \tag{91}$$

8. The included angle of divergence, α_c, of the turbine annulus walls should not exceed some specified upper limit taken in the present study as

$$(12\alpha_c/\pi) - 1.5 \leq 0 \tag{92}$$

where, α_c is defined as follows [3]

$$\alpha_c = \tan^{-1} \frac{h_4 - h_1}{2(C_N + 1.25C_R)} \tag{93}$$

9. The flow coefficient, ϕ, as defined by the ratio of axial velocity to blade speed (V_a/U) and the stage temperature drop coefficient, ψ, defined as:

$$\psi = 2c_p\Delta T/U^2 \tag{94}$$

should lie within certain limits. The present study will assume that these limits are given by

$$0.25 < \phi < 2 \tag{95}$$

$$0.5 < \psi < 6 \tag{96}$$

10. The degree of reaction at mean radius must be within some specified upper and lower bounds. Here,

$$0.3 < R < 0.7 \tag{97}$$

where the reaction is defined as [3]

$$R = \frac{\phi}{2}(\tan\alpha_4 - \tan\alpha_3) \tag{98}$$

Grouping the constraints given by Eq. (82) through Eq. (98) as a vector quantity, $\vec{G}$, and arranging so that

$$\vec{G} \leq 0 \tag{99}$$

the nonlinear programming model of Eq. (81) can be modified as

$$\begin{aligned} &\text{Find } \vec{X} \\ &\text{which maximizes } \eta(\vec{X}) \\ &\text{subject to } \vec{G}(\vec{X}) \end{aligned} \tag{100}$$

However, no analytical or numerical solution to the constrained case of a nonlinear programming problem has so far been offered in the literature. Therefore, it is necessary to convert the problem of Eq. (100) into an unconstrained optimization problem. The transformed problem can then be solved using the Steepest Ascent Method [29].

The transformation is realized through the widely used nonlinear programming method of Fiacco and Mc Cormick [24,28,29] known as the Sequential Unconstrained Maximization Technique (SUMT). The procedure is more or less similar to the use of the Lagrange multipliers method [29,30] and involves the transformation of the objective function into an interior penalty function, P, augmented by a penalty term consisting of the constraints.

This concept, applied to Eq. (100) results in the new function:

$$P(\vec{X},r_k) = \eta(\vec{X}) + r_k \sum_{i=1}^{10} \frac{1}{G_i(\vec{X})} \qquad (101)$$

where r_k is a non-negative parameter, called the resequencing factor.

It is assumed that the optimization of Eq. (101) is equivalent to the optimization of Eq. (100). To clarify the idea, the following reasoning is given.

First, the constraint equations, G_i, as given in Eqs. (82) through (98), are convex [29]. It follows that $1/G_i$ is concave as η also is. Consequently, P is concave and thus possesses a unique maximum.

Furthermore, the optimization of P is performed for a decreasing sequence of r_k, i.e.,

$$0 < r_{k+1} < r_k \, . \qquad (102)$$

This means that, in the limit, when r_k is sufficiently small,

$$\lim_{r_k \to 0} P(\vec{X},r_k) = \eta(\vec{X}) \qquad (103)$$

Therefore, at the end of the procedure, the contribution of the constraints to the penalty function will be minimized and the resulting optimum of $P(\vec{X},r_k)$ will correspond, with approximately no error, to an optimum efficiency, $\eta(\vec{X})$.

The algorithm is initiated by arbitrarily selecting an initial non-negative value for r_k. In the present work, this initial r_k is chosen, as suggested in Ref. [3], to be

$$r_1 = 4\text{x}10^{-4} \tag{104}$$

and the decreasing sequence imposed on r_k is realized through:

$$r_{k+1} = r_k/10 \tag{105}$$

An initial point, $\vec{X}_0$, is selected as the first trial solution. Since the method of [24] is essentially an "Interior Point Unconstrained Maximization", $\vec{X}_0$ must be an interior point, that is it must not lie on or beyond the boundaries of the solution space. The vaule of $\vec{X}_0$ given in [3] as calculated according to the constraint equations, is adopted in the present thesis.

Given the value of r_1 and $\vec{X}_0$, the Steepest Ascent Method (SAM) [29] is used to determine the corresponding optimal solution of $P(\vec{X},r_k)$. The object of the S.A.M. is to generate $\vec{X}_{n+1}$ once $\vec{X}_n$ is given. Logically, the next trial point, $\vec{X}_{n+1}$, should be selected in the direction of the fastest rate of improvement in the objective value. This direction is determined by using the information provided by the gradient of the objective function, $\vec{\nabla}P(\vec{X}_n,r_n)$, at the current trial point, $\vec{X}_n$. The relation between the next and the actual trial points is given as:

$$\vec{X}_{n+1} = \vec{X}_n + R_{op,n}\vec{\nabla}P(\vec{X}_n,r_n) \tag{106}$$

where R_{op} is known as the optimal step size.

Following Eq. (106), in order to determine the next trial point, it is therefore necessary to determine both a feasible direction, $\vec{\nabla}P$, and the optimal step size in this direction, R_{op}. In the present thesis, the determination of $\vec{\nabla}P$ and R_{op} at any point is performed in the following way:

Referring to Eq. (101), the gradient, $\vec{\nabla}P$, of the penalty function can be given as:

$$\vec{\nabla}P(\vec{X},r_k) = \vec{\nabla}\eta(\vec{X}) + \vec{\nabla}[r_k \sum_{i=1}^{10} \frac{1}{G_i(\vec{X})}] \tag{107}$$

or

$$\vec{\nabla}P(\vec{X},r_k) = \vec{\nabla}\eta(\vec{X}) - r_k\vec{\nabla}[\sum_{i=1}^{10} \frac{1}{G_i^2(\vec{X})}] \tag{108}$$

As presented in Eqs. (41) through (80), the efficiency equations expressed in terms of the design variables, $\vec{X}$, are implicit. Therefore a straight analytic differentiation would be quite cumbersome. The same is also valid for the determination of $\vec{\nabla}(1/G_i^2(\vec{X}))$. Consequently, the general chain rule using the Jacobian Matrix evaluation is applied to this problem. The general chain rule states that [31],the Jacobian Matrix, J_{in}, for a multivariable, continuously differentiable function, u, can be expressed as:

$$\begin{aligned} J_{in} &= \frac{\partial u}{\partial z_m} \\ &= \Sigma \frac{\partial u}{\partial v_i} \frac{\partial v_i}{\partial \omega_j} \cdots \frac{\partial x_k}{\partial y_\ell} \frac{\partial y_\ell}{\partial z_m} \end{aligned} \tag{109}$$

where

$u = u(v_i)$ $\qquad$ $i = 1,n_1$

$v_i = v_i(\omega_j)$ $\qquad$ $j = 1,n_2$

$\vdots$ $\qquad$ $\vdots$

$$\vdots$$

$$x_k = x_k(y_\ell) \qquad k = 1,n_3 \;\; ; \;\; \ell = 1,n_4$$

$$y_\ell = y_\ell(z_m) \qquad m = 1,n_5$$

The differentiation of $\eta(\vec{X})$ and $1/G_i^2(\vec{X})$ is thus performed by the use of equation (109). R_{op} in Eq. (106) can be determined as follows. Since the objective is to determine $\vec{X}_{n+1}$ which gives the largest improvement in the value of $P(\vec{X},r_k)$, R_{op} is determined as the optimal value for which

$$H(R_{op}) = P(\vec{X}_n + R_{op}\vec{\nabla}P(\vec{X}_n,r_k) \qquad (110)$$

is maximized over R_{op}.

Noting that, in Eq. (110) both $\vec{X}_n$ and $P(\vec{X}_n,r_k)$ are known and fixed for the moment, H is a function of R_{op} only. Therefore, the value of R_{op} which maximizes $H(R_{op})$ can be determined by a numerical trial and error process. This process is started from $R_{op} = 0$ and continued until, for some X_i,

$$X_{n+1,i} > \text{Upper bound of } X_i$$

or (111)

$$X_{n+1,i} < \text{Lower bound of } X_i.$$

Once the optimal step size, R_{op} of Eq. (106) is determined from Eq. (110), this optimal step size is fed into Eq. (106) and the new starting trial point $\vec{X}_{n+1}$ is found. Then, the gradient of P at this new point is determined and the procedure is repeated until the values of P corresponding to two consecutive iterations are approximately the same. This process is called the "one-dimensional maximization" of P.

Note that at the point where one-dimensional maximization is terminated, the optimum design vector, $\vec{X}$, and the corresponding optimum efficiency, η, are expressed for the relevant value of r_k. Therefore, once an optimum η is found for some r_k, r_k is updatedaccording to Eq. (105) and the whole procedure is repeated, until for two successive values of r_k, the corresponding optimum efficiencies are approximately the same.

Numerical applications of the above procedure have indicated that for $r_k < 4\text{x}10^{-6}$ further iterations result in negligible improvements in η. For computer time saving purposes, therefore, the calculations are stopped beyond this value of r_k. Consequently, for each run with new input conditions, three sequential maximization steps are performed in the range,

$$4\text{x}10^{-6} < r_k < 4\text{x}10^{-4}.$$

IV. RESULTS AND DISCUSSIONS

Based on the analysis given in Chapter 3, two computer programs have been developed. Input and output descriptions together with the program listings are presented in Appendices B and C. The application of the programs to turbine stages with various inlet gas conditions and blade rotational speeds are discussed in this chapter and the numerical results are presented in graphical and tabular form.

4.1 RESULTS ON PERFORMANCE EVALUATION

As presented in Chapter 3, the program developed is based on the study of Ainley and Mathieson [1] with the suggested modification of Dunham and Came[2]

The variation of the blade outlet gas angles with the outlet Mach number is plotted in Fig. 4.1. The curve obtained for the case of the stator is in agreement with the results of Ref. [1], but a deviation ($\sim 1.5^{0}$) in the outlet angle is observed for the case of the rotor, especially for Mach numbers exceeding 0.5. It should be remembered that the calculations of Ref. [1] are hand-made. Therefore, the computerized calculations of the present work can be considered as more precise.

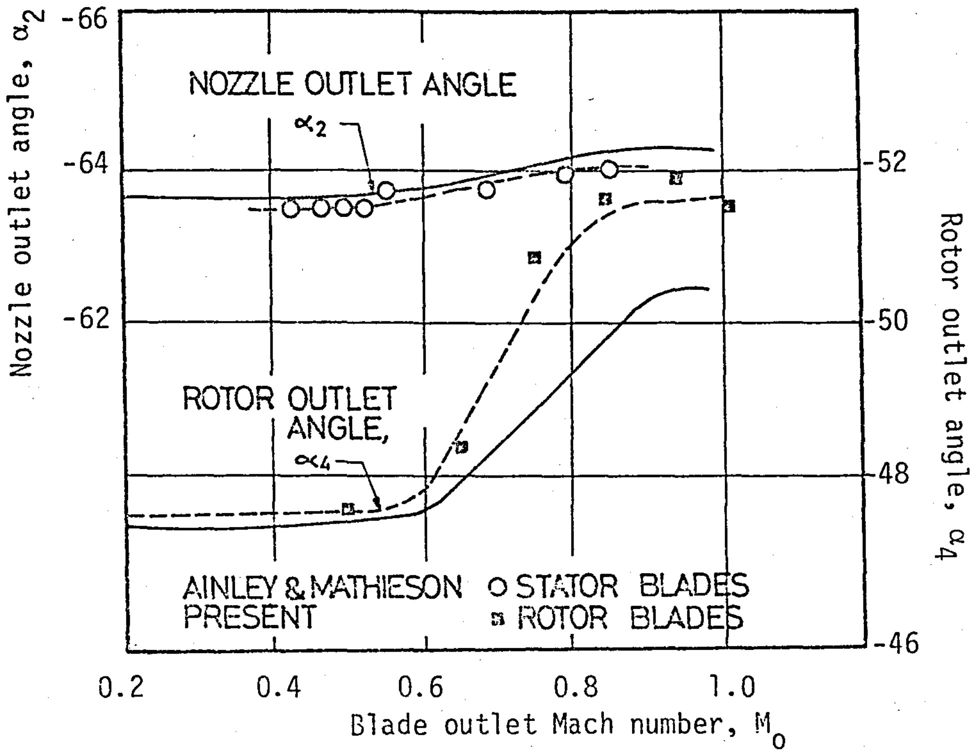

FIGURE 4.1 - Variation of outlet gas angle with outlet Mach number.

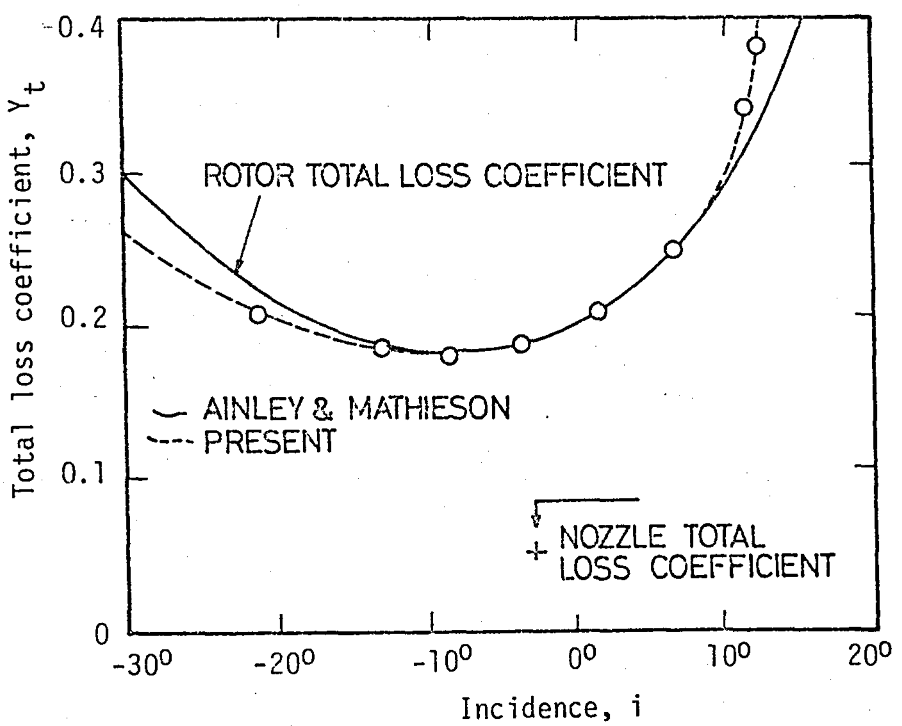

FIGURE 4.2 - Variation of total loss coefficient with incidence.

The variation of the rotor loss coefficient with respect to the incidence is given in Fig. 4.2. The deviation of the results from those of the original method [1], especially for incidences lower than -15^{o} and higher than $+10^{o}$ are due to the fact that secondary and tip clearance losses are calculated in this thesis, according to the suggested modifications of Dunham and Came to the original Ainley and Mathieson method. Since Dunham and Came's correlations have proven, over the last three decades, to give more adequate results, especially for the case of the turbines with low aspect ratio, the outputs of the present thesis may be found more reliable than those reported in Ref. [1].

The total-to-total aerodynamic efficiency, η, of the stage is plotted in Fig. 4.3, against the overall pressure ratio. A plot of the initial mass flow parameter is also given on the same figure. Three curves, corresponding each to a different value of the speed parameter, $N/\sqrt{T}$, are shown on the figure. It is observed, in general, that the efficiency decreases slightly for decreasing values of the speed parameter. The same is also valid for the mass flow parameter, which decreases from 10.268 (corresponding to $N/\sqrt{T}$ = 435, as given by Ref. [1]) first to 10.159 (for $N/\sqrt{T}$ = 424) and then to 10.113 (for $N/\sqrt{T}$ = 393) at the point where the rotor is choked. For both cases, the pressure ratio for which the stage completely chokes (i.e. Mach number equals unity) is correctly predicted as 0.51.

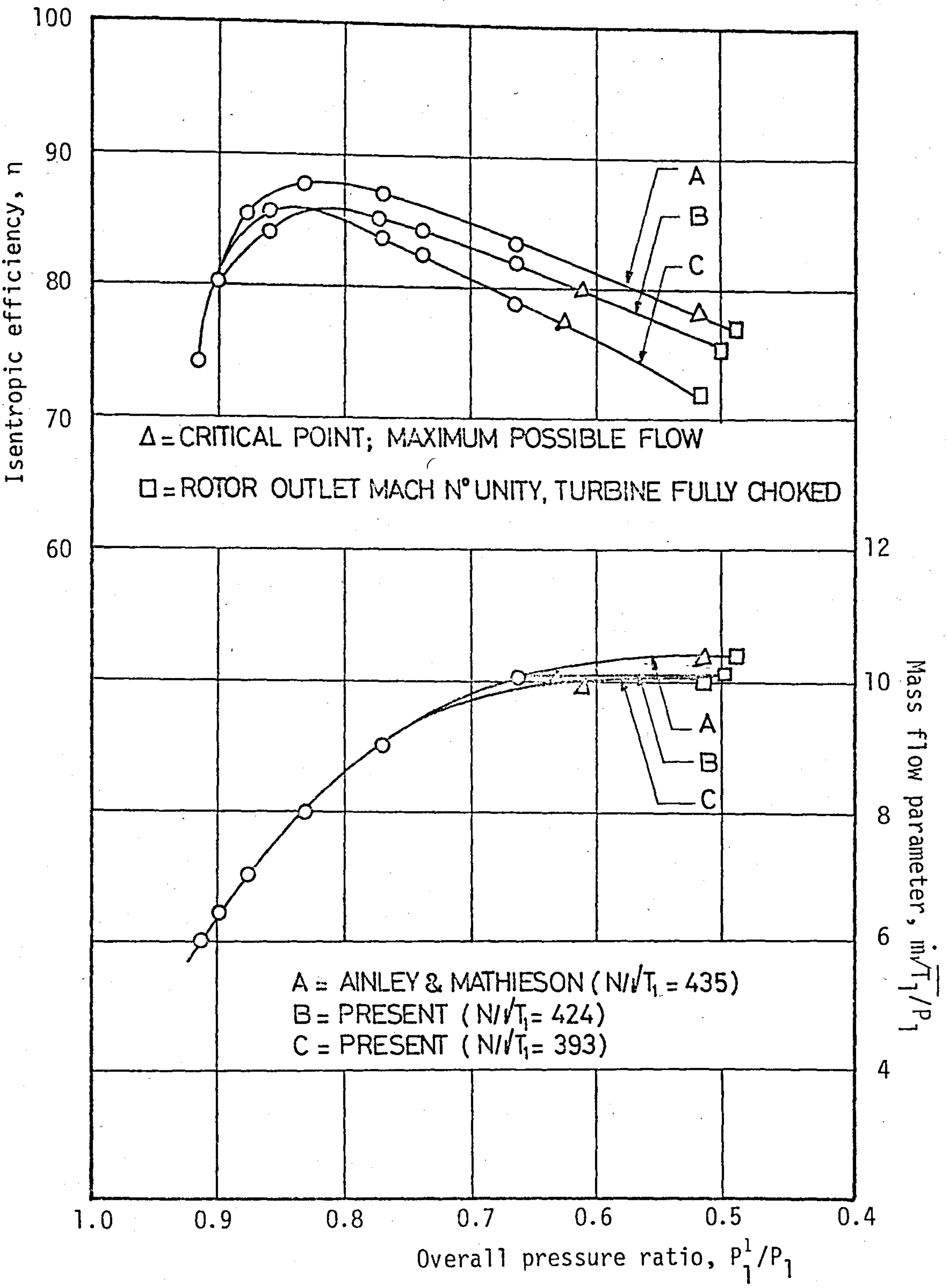

FIGURE 4.3 - Variation of the isentropic efficiency and the non-dimensional mass flow with overall pressure ratio.

4.2 RESULTS ON OPTIMUM STAGE DESIGN

A computer program is developed, based on the turbine stage performance evaluation model suggested by Rao and Gupta [3] and on the optimization method of Fiacco and Mc Cormick [24]. Table 4.1 summarizes the results obtained by the application of the program to various inlet gas conditions and to various blade speeds. The results are also presented in graphical form within Fig. 4.4 through Fig. 4.9.

Fig. 4.4 is a plot of the overall efficiency to the overall pressure ratio. The data points are grouped under two different sets: regions corresponding to three ranges of the mass flow parameter based on annulus diameter, $\dot{m}\sqrt{T_1}/P_1D^2$, and various lines corresponding to constant values of the turbine speed parameter, $ND/\sqrt{T_1}$. Inspection of Fig. 4.4 indicates that along constant $ND/\sqrt{T_1}$ lines, the efficiency tends to increase for increasing values of the pressure ratio. Compared to the corresponding pressure ratio range of Fig. 4.3, this efficiency variation is in accordance with the results of Ainley and Mathieson [1]. It is also observed from Fig. 4.4 that the mass flow parameter being held constant, the efficiency tends to decrease as the pressure ratio is increased.

The relation between the mass flow parameter, the pressure ratio and the efficiency is also plotted on Fig. 4.5 in an altered form. The mass flow parameter is shown on the ordinate axis versus the pressure ratio and the data points are grouped for different ranges of the efficiency. The general trend of the figure indicates that higher efficiencies tend to be associated with lower pressure

TABLE 4.1 - Aerodynamic Variables Corresponding to the Optimum Efficiency, as calculated by the computer program RAO.

Point # \ Variable	$\frac{\dot{m}\sqrt{T_1}}{P}\times 10^5$	$\frac{\dot{m}\sqrt{T_1}}{P_1 D^2}\times 10^2$	$ND/\sqrt{T_1}$	P'_1/P_1	$Re \times 10^{-3}$	$\frac{\Delta T}{T_1}\times 10$	η
1	166	1.77	2.77	0.562	268	1.12	0.837
2	193	2.07	2.37	0.631	199	0.082	0.840
3	232	1.97	1.77	0.729	246	0.061	0.806
4	237	1.53	1.86	0.654	422	0.079	0.784
5	237	1.69	2.36	0.596	429	0.100	0.826
6	248	3.94	2.65	0.599	275	0.105	0.874
7	263	2.23	2.18	0.619	256	0.093	0.821
8	290	2.18	2.35	0.672	253	0.081	0.853
9	322	2.75	1.77	0.734	156	0.061	0.818
10	387	3.06	2.29	0.684	240	0.079	0.868
11	387	2.89	1.89	0.730	308	0.065	0.854

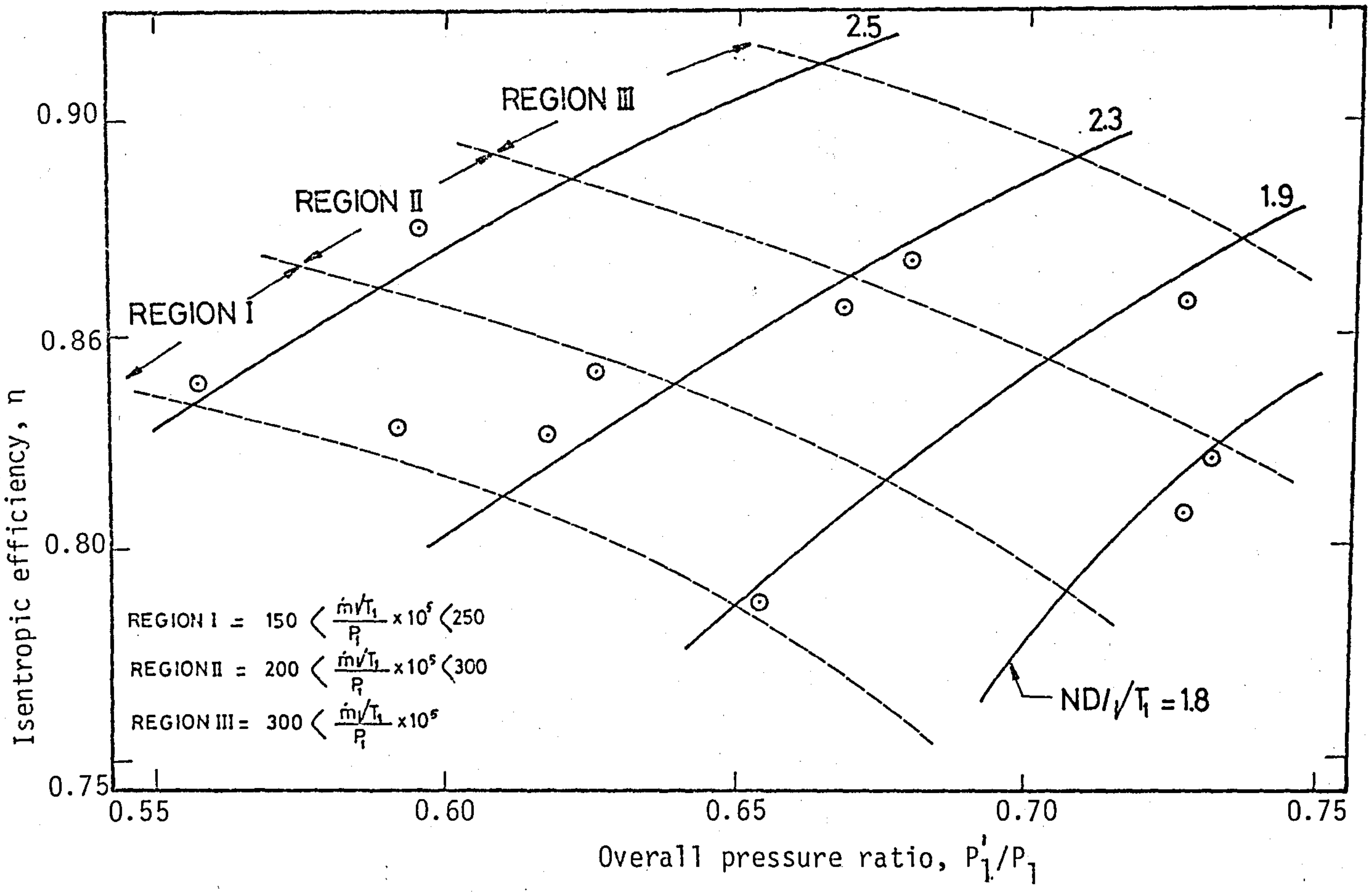

FIGURE 4.4 - Variation of the isentropic efficiency with overall pressure ratio.

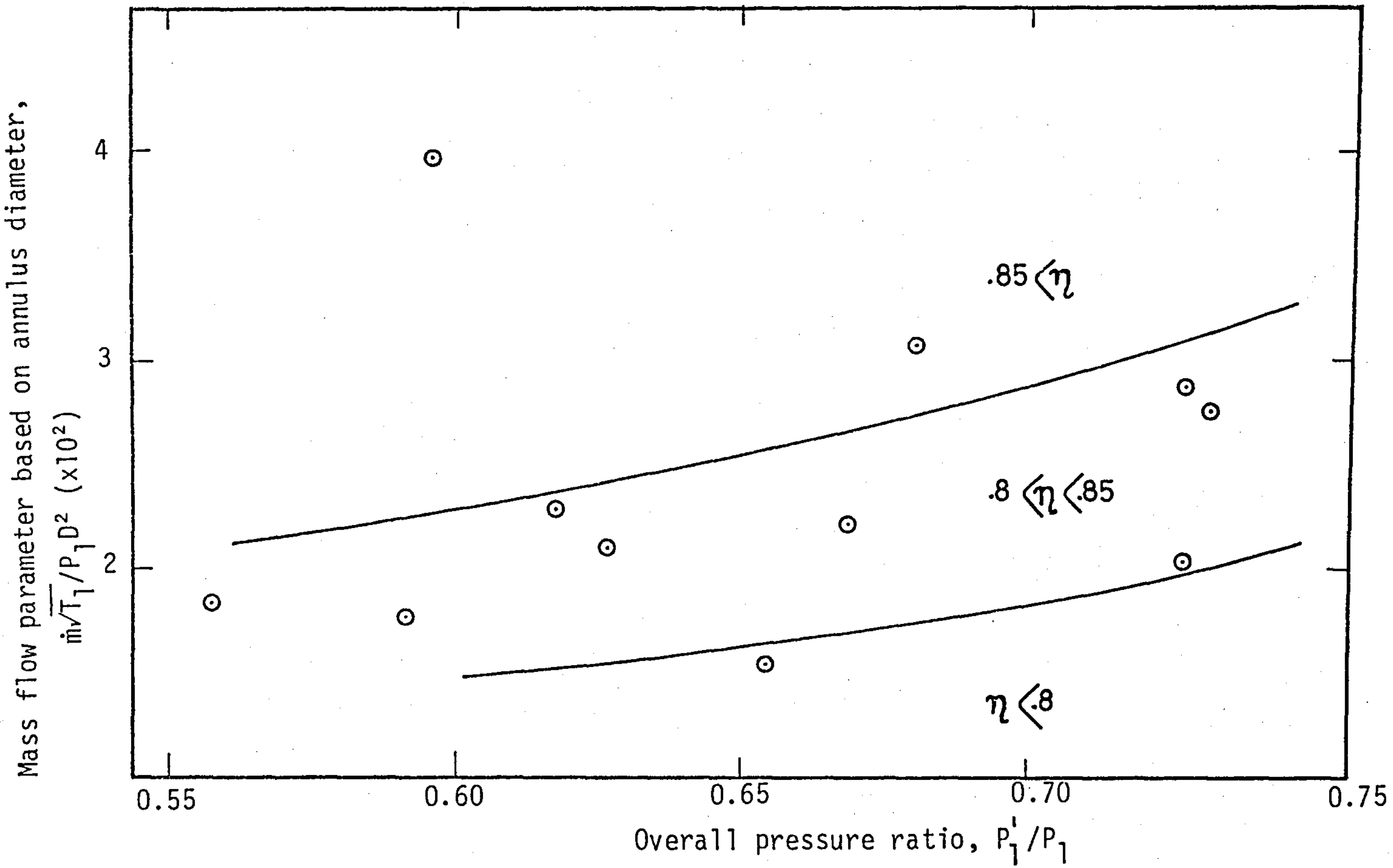

FIGURE 4.5 - Variation of the non-dimensional flow parameter with the overall pressure ratio.

ratios, since the mass flow parameter required is smaller.

The isentropic efficiency is plotted in Fig. 4.6 as a function of the speed parameter. It is observed from the figure that along (approximately) constant speed parameter lines, and for increasing mass flow rates, the efficiency also tends to increase. However, the rate of increase in the efficiency is lowest at higher pressure ratios. This trend is in conformity with the results appearing in Fig. 4.3.

Finally, the temperature drop coefficient, $\Delta T/T_1$ is plotted against the overall pressure ratio and the speed parameter in Fig. 4.7 and 4.8, respectively. It is interesting to note from Fig. 4.7 that the curve goes through a minimum, corresponding to a range of pressure ratio such as, $0.6 < P_1'/P_1 < 0.7$. Recalling that the temperature drop, ΔT, is directly related to the work output, ΔW, of the stage through Eq. (58), it can be deduced that an optimization of the specific work output can be realized by controlling the back pressure of the turbine. Fig. 4.8, on the other hand, points out that the specific work output of the stage tends to increase with increasing blade rotative speed.

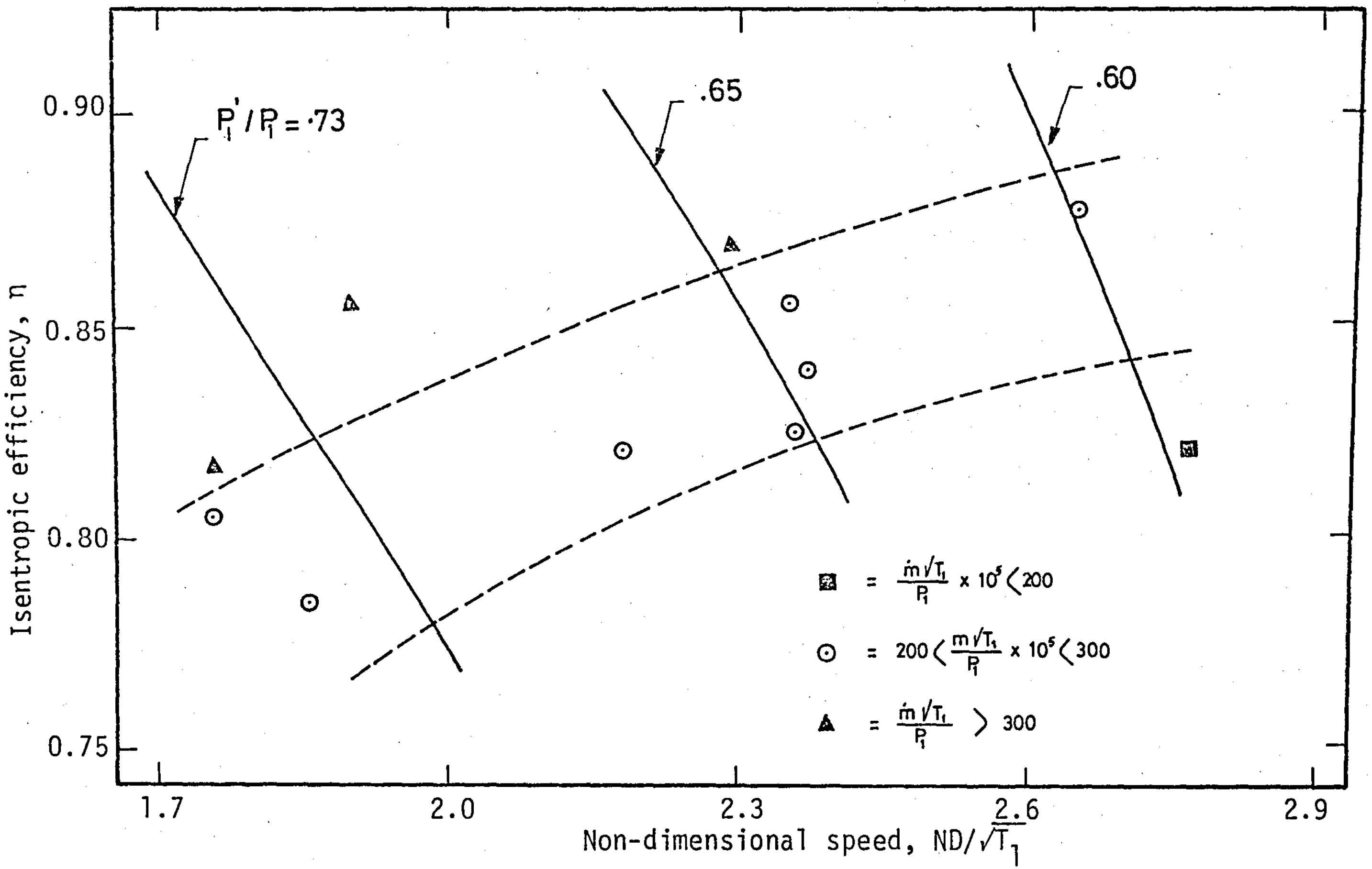

FIGURE 4.6 - Variation of the isentropic efficiency with the non-dimensional speed parameter.

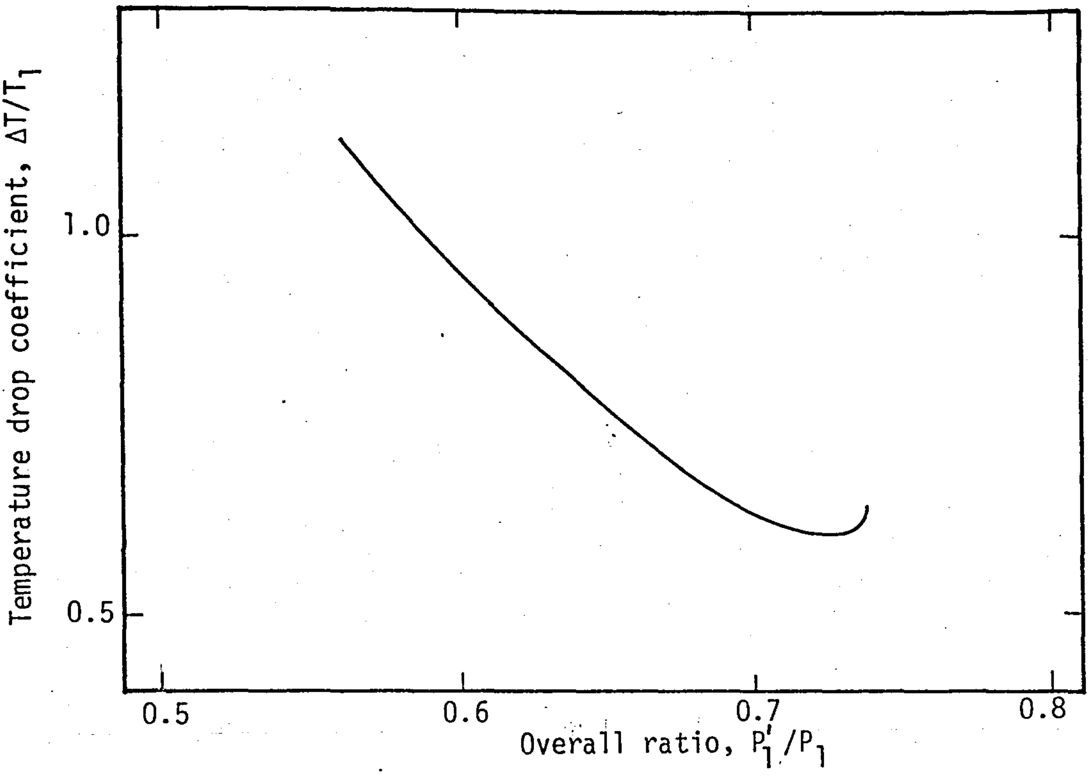

FIGURE 4.7 - Variation of the temperature drop coefficient with the overall pressure ratio.

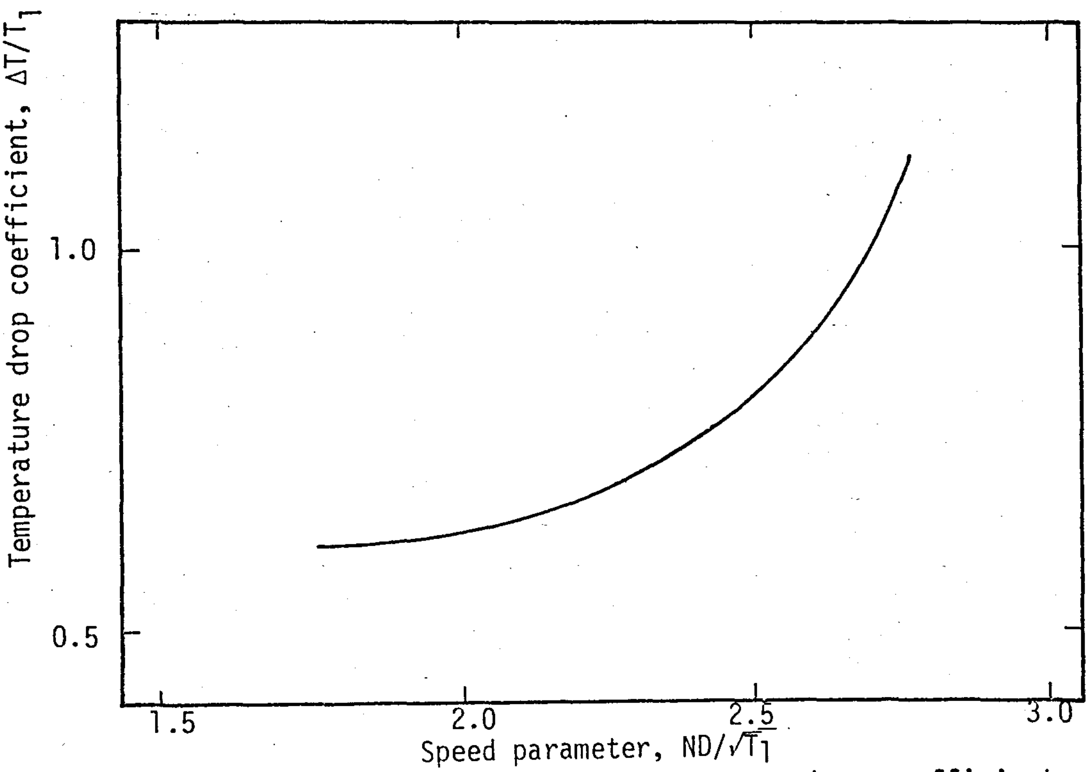

FIGURE 4.8 - Variation of the temperature drop coefficient with the non-dimensional speed parameter.

V. CONCLUSIONS AND RECOMMENDATIONS

1. Two computer programs have been developed to calculate the performance of an axial flow gas turbine stage and to predict the geometry of a turbine stage which would develop the maximum efficiency.

2. The first program is based on the loss evaluation and performance prediction method of Ainley and Mathieson [1], as modified by Dunham and Came [2] and the second program, on the efficiency evaluation method of Rao and Gupta [3] and on the optimization technique of Fiacco and Mc Cormick [24]. It might be understood that the contribution of the present thesis is not in working out a new method of its own, but rather, in combining the existing ones and computerizing them in order to offer a useful tool for the researchers as well as for the designers working on gas turbine aerodynamics.
Indeed in the method presented in Ref. [1], it took about six to eight man-hours to calculate a single performance map (curves A, for example, of Fig. (4.3)) of a given turbine. However in the related computer program the performance map

containing three different curves (curves A, B and C of Fig. 4.3) together with the gas conditions at each stage, are obtained in not more than three seconds. The second computer program, on the other hand, provides an efficient means for the user, especially in the preliminary design stage of the turbine (The computer program of Rao and Gupta's method [3] was not available). The original method of Ref. [3] is slightly altered to incorporate the Sequential Unconstrained Maximization Technique of Fiacco and Mc Cormick [24]. The use of this technique allowed a considerable computer time saving and the 61 minutes of computer time on an IBM 7044 computer in the original method were reduced down to 320 CDC seconds (cpu time).

3. The performance characteristics are studied by making use of the dimensionless turbine parameters, and this non-dimensional analysis enlarged the scope of the observations, allowing the prediction of the behaviour of any aerodynamic variables when the others remain fixed.

4. The following conclusions have been obtained from the application of the programs to various gas turbine data.

   a) The isentropic efficiency is a function of the inlet mass flow parameter (which can be expressed by various groups such as $\dot{m}\sqrt{T_1}/P_1$, $\dot{m}\sqrt{T_1}/P_1A_1$ or $\dot{m}\sqrt{T_1}/P_1D^2$). This dependence is in conformity with the results given by

Ainley and Mathieson [1]. The effect of the rotative speed parameter, $N/\sqrt{T_1}$, is also observed and it is found that the efficiency increases with increasing rotative speeds.

b) The functioning of the programs are seen to be largely restricted by the interpolation subroutines in that for very small and for very large mass flows, some parameters fall out of the interpolation domain, making further calculations impossible.

c) The results of the optimization program are in good agreement with those of the performance prediction program. The values of the design variables corresponding to optimum efficiency lie within acceptable bounds. It is noted that the efficiency depends least on the axial velocity, C_a, since the final value of this variable shows very small difference from the initial value. This consequence is due to the fact that the axial velocity is related to the physical variables as given by Eq. (46) and Eq. (47).

5. The mathematical model, the interpolation technique and the optimization method need to be improved in the following areas.

a) The present gas flow model is 2-D and does not describe the flow adequately, especially in stages having small hub-to-tip radius ratios. In such cases a 3-D solution is required.

b) The interpolation methods used are quite restrictive and do not allow calculations beyond the limits of the interpolation domain. To overcome this difficulty the capacity of the related subroutine may be extended to allow extrapolation, or the interpolation domain may be slightly enlarged.

c) The computer program AINLEY may be modified such that the correlations of the blade outlet angles and of the stage losses as well as the presentations of the final performance maps can automatically be obtained in a graphical form. Also the velocity triangles corresponding to a fixed value of the inlet gas conditions and of the rotative speed can be displayed together with the final results, in order to provide a considerable help for the design engineer, especially in obtaining quick estimates of the turbine performance.

d) The variation of the stage efficiency with respect to small perturbations of the design variables can be obtained by introducing a sensitivity analyses to the present optimization technique. Such an improvement would give information about the relative importance of the stage parameters during the preliminary design of the turbine. The sensitivity analyses can be carried out by means of a control variable which decides whether the program will optimize or will directly give the efficiency related to the input design vector.

APPENDICES

APPENDIX A

TURBINE AERODYNAMIC LOSS ANALYSES

A.1 CORRELATION OF ZWEIFEL

The lift coefficient, C_L, based on tangential blade loading is given by Zweifel [5] as:

$$C_L = 2(s/c_a)(\tan\alpha_o + \tan\alpha_i)\cos^2\alpha_o \tag{A.1}$$

where

c_a = axial chord.

A.2 CORRELATION OF SODERBERG

The nominal stage aerodynamic loss coefficient, Y_t, correspon-ding to an aspect ratio $h/c_a = 3$ and to a Reynolds number, $Re = 10^5$, is given by Soderberg [6] as a function of the gas deflection, ε, that may be expressed as

$$\varepsilon = \alpha_o + \alpha_i \ . \tag{A.2}$$

The functional relationship of Y_t and ε is given in Fig. A.1.

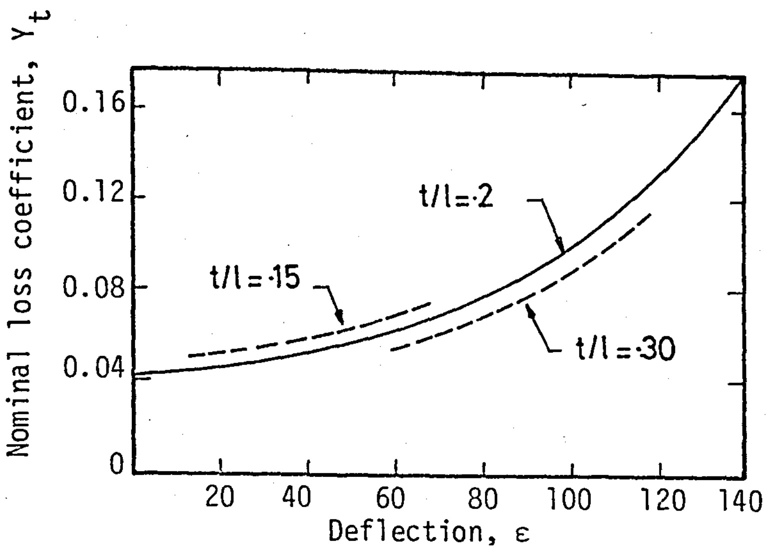

FIGURE A.1 - Soderberg's correlation of the total loss coefficient.

In Fig. A.1, the variable t, is the maximum blade thickness (Fig. 2.1).

If the "corrected" total loss coefficient, $Y_{t,corr}$, of a blade row operating with an aspect ratio other than 3 and at a Reynolds number other than 10^5, is desired, the following correlation suggested by Soderberg can be used:

$$Y_{t,corr} = \left(\frac{10^5}{Re_h}\right)^{1/4}[(1 + Y_t)(0.975 + 0.075(C_a/h))] \qquad \text{(A.3)}$$

where

Re_h = Reynolds number based on throat mean hydraulic diameter,

and Y_t = Stage total aerodynamic loss coefficient as given in Fig. A.1.

A.3 CORRELATION OF STEWART, WHITNEY AND WONG

According to Stewart et.al [11] the diffusion factor for the suction surface, D_s, is given as:

$$D_s = 1 - (V_o/V_{s,max}) \quad , \qquad \text{(A.4)}$$

where $V_{s,max}$ = maximum gas velocity over the suction surface, and the diffusion factor for the pressure surface, D_p, can be expressed as:

$$D_p = 1 - (V_{p,min}/V_i) \qquad \text{(A.5)}$$

where $V_{p,min}$ = minimum gas velocity over the pressure surface.

A total diffusion factor, D_t, for a blade passage can therefore be defined using Eq. (A.4) and Eq. (A.5) as follows:

$$D_t = D_s + D_p \qquad \text{(A.6)}$$

or

$$D_t = (1 - (V_o/V_{s-max})) + (1 - (V_{p-min}/V_i)) \qquad \text{(A.7)}$$

The total diffusion factor, D_t, can be related to a dimensionless momentum thickness parameters, δ_m/ℓ, as [11],

$$\delta_m/\ell = 0.003/(1 - 1.4D_t) \qquad \text{(A.8)}$$

where

δ_m = momentum thickness

ℓ = camber length of the blade.

This relationship is also given in Fig. A.2.

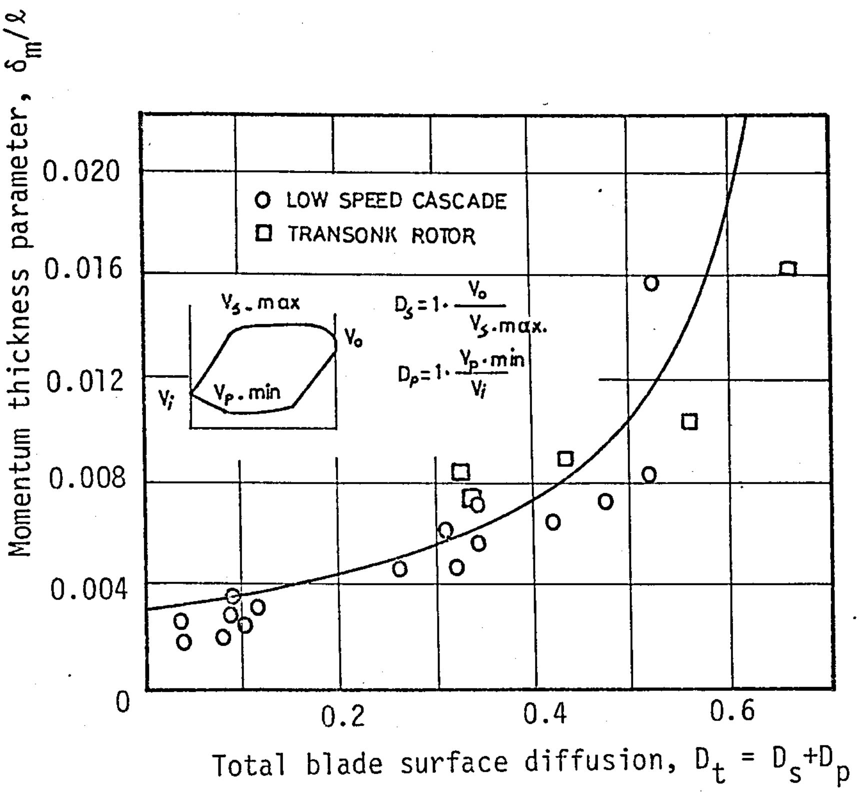

FIGURE A.2 - Correlation of the momentum thickness with the total blade surface diffusion factor.

The total diffusion factor, D_t, can also be related to the tangential lift coefficient, C_L, by the following analysis [12]: By definition, C_L is given by:

$$C_L = \frac{[(P_o - \frac{1}{2}\rho V^2_{p\text{-min}}) - (P_o - \frac{1}{2}\rho V^2_{s\text{-max}})]}{(1/2)\rho V_o^2} \tag{A.9}$$

But, assuming a constant suction surface gas velocity through the blade passage, i.e., assuming

$$V_{s\text{-max}} = V_o \quad , \tag{A.10}$$

one can rearrange Eq. (A.9) as follows:

$$C_L = 1 - V^2_{p-min}/V^2_o \tag{A.11}$$

On the other hand, combining Eq. (A.7) and Eq. (A.10), D_t can be expressed as:

$$D_t = 1 - \frac{V_{p-min}}{V_i} \tag{A.12}$$

Considering the velocity triangles as shown in Fig. 3.4, it is seen that

$$(V_i/V_o)^2 = (\cos\alpha_o/\cos\alpha_i)^2 \tag{A.13}$$

and finally knowing that

$$(\frac{V_{p-min}}{V_o})^2 = (\frac{V_{p-min}}{V_i})^2(V_i/V_o)^2 \quad , \tag{A.14}$$

one can obtain, by the combination of Eqs. (A.11), (A.12), (A.13) and (A.14), a correlation of C_L and D_t, in terms of blade inlet and outlet gas angles as:

$$C_L = 1 - (\cos\alpha_o/\cos\alpha_i)^2(1 - D_t)^2 \tag{A.15}$$

A.4 CORRELATION OF BALJE and BINSLEY

Balje and Binsley [13] assumed a velocity profile of the form:

$$V = V_o[\frac{\sin\beta_o}{\sin\beta_i} + \frac{x}{\ell}(1 - \frac{\sin\beta_o}{\sin\beta_i})] \quad , \tag{A.16}$$

where

x = distance along the camber line

β_o = blade outlet angle.

This velocity profile is inserted in the Truckenbrodt expression [14], which is given as:

$$\delta_m = \left(\frac{V_o \delta_{m,o}}{\nu_o}\right)^{1/n} = V_o^{-3-(2/n)}\ a \int_{x=o}^{x=\ell} V^{3+(2/n)}\, dx \qquad \text{(A.17)}$$

where

$\delta_{m,o}$ = momentum thickness at blade outlet,

ν = kinematic viscosity of the gas,

and a,n = constants.

Resultingly, a relation for the momentum thickness is obtained:

$$\frac{\delta_m}{\ell} = 0.0021\left[\frac{1 - \mu_a^{4.5}}{1 - \mu_a}\right]^{.8} \quad , \qquad \text{(A.18)}$$

where, the acceleration ratio, μ_a, is defined as:

$$\mu_a = \sin\beta_o / \sin\beta_i \qquad \text{(A.19)}$$

A.5 CORRELATION OF BALJE

Balje, in Ref. [15], defines the blade loading, δu, as:

$$\delta u = \cot\alpha_o - \cot\alpha_i \quad , \qquad \text{(A.20)}$$

and the equivalent diffusion ratio, D_e, as

$$D_e = \left[1 + 0.9(t/c) + \frac{0.3 C_L \mu_a^2}{\sqrt{1 + \cot^2\alpha_m}} \frac{\sin\xi}{\sin\alpha_m}\right] \frac{1.025}{\mu_a} \qquad \text{(A.21)}$$

where, also shown in Fig. 3.1,

ξ = blade stagger angle.

In Ref. [15], the relation between the momentum thickness and the equivalent diffusion ratio is given as follows:

$$\frac{\delta_m}{\ell} = \frac{0.013}{2.62 - D_e} - 0.004 \qquad \text{(A.22)}$$

Using Eqs. (A.20), (A.21) and (A.22), Balje gives the profile clearance and endwall loss coefficients as follows

$$Y_p = \frac{1.93(c/s)(\delta_m/\ell)(\ell/c)(1 + W^2)^{1.5}}{1 - 1.08(c/s)(\delta_m/\ell)(\ell/c)(1 + W^2)^{1.5}} \qquad \text{(A.23)}$$

where

$$W = \cot\alpha_m + (\delta u/2) ,$$

$$Y_k = 0.19(k/h)^{\frac{1}{2}}(c/h)^{\frac{1}{2}}\delta u(1 + W\delta/\sqrt{W}) \qquad \text{(A.24)}$$

and

$$Y_e = Y_p(c/h)C_L \frac{1 + W^2}{2\delta uW} \qquad \text{(A.25)}$$

where

Y_e = endwall loss coefficient.

APPENDIX B

COMPUTER PROGRAM AINLEY

B.1 GENERAL DESCRIPTION

The computer program AINLEY calculates total aerodynamic losses for nozzle and rotor separately, combines the results to determine the total stage loss and also prints the gas flow conditons at each station (inlet and outlet of blade rows), following a fixed mass of gas throughout its trajectory.

The data fed into the program may be divided into two parts:

i) the data representing the geometrical variables of the turbine stage in hand and the properties of the gas (usually air) which passes through. These data are grouped under the data file TURBO1 and must be fed separately by the user,

ii) the data corresponding to the figures and graphs presented in Chapter 3, which are used to carry out the necessary interpolations by means of the library subroutines ICS1VE, ICS2VE and ICS1VU. These data form the datafile TUBRO2.

The program AINLEY first calculates gas outlet angle values corresponding to a range of Mach numbers at blade outlet from M = 0 to M = 1. This is done conformly to the analysis given in Section 3.1.1 and by calling the subroutine SUB1. If the value of the control variable, G, which appears in the argument list of SUB1 is set equal to 1, SUB1 performs the calculations for the stator row, but if G equals 2, then rotor outlet gas angles vs rotor outlet Mach numbers are evaluated. Another correlation is established in the subroutine SUB2, between the stator outlet gas angles and the corresponding stator total loss coefficient, following the theory explained in Section 3.1.2.

Following the choice of an initial guess for the stator outlet gas angle, the stator total loss coefficient and the corresponding value of the non-dimensional mass flow parameter are determined. Then the gas conditions at the outlet of the stator are calculated (Section 3.1.3.1), a check being imposed upon the actual inlet and exit non-dimensional mass flows, for the case in which they would exceed the maximum allowable flow for the stator (chocking of the row). If such a situation occurs, the subroutine CHOKE is called, in order to perform the procedure explained in Section 3.1.2. Subroutine CHOKE returns the results to the calling program which uses them as absolute gas conditions at the inlet to the next rotor row.

Once the absoulte outlet gas conditions from the stator are determined and the guessed outlet angle checked, an initial value for the rotor outlet gas angle must be chosen and the related rotor total loss coefficient calculated (as for the stator case, subroutine SUB31 calculates the value of the rotor total loss coefficient corresponding to an outlet gas angle). The maximum non-dimensional mass flow corresponding to this loss coefficient is easily obtained by interpolation. Similarly to the case of stator, the non-dimensional mass flows at inlet and outlet of the rotor are calculated (Section 3.1.3.3) and checked against the maximum flow for any possible choking of the row: if the gas has overexpanded in the row, the subroutine CHOKE is called again.

According to the gas flow conditions at the rotor outlet, the absolute gas conditions at the inlet to the next stage and the total-to-total stage aerodynamic efficiency are calculated. A final correction is applied for the stage mean Reynolds number.

At the final part of the first iteration, the magnitudes of gas properties at each station, the stage efficiency and the efficiency corrected for Reynolds number are printed, the non-dimensional flow quantity is augmented by an increment and a new set of calculations is started for the new value of the mass flow. This procedure is repeated until choking occurs in one of the blade rows.

Once the rotor (or stator) chokes, the subroutine CHOKE takes into account the reasoning given in Section 3.1.5: When choking occurs in a turbine blade passage, no greater quantity of gas can pass through the passage throat. Therefore, the mass flow upstream must remain constant, and the non-dimensional mass flow at turbine inlet is no more incremented in the next loop. In return, the subroutine CHOKE starts to increase the Mach number at the outlet of the choked row until, the outlet Mach number reaches unity. This point represents the fully choked state of the blade passage and the calculations are stopped.

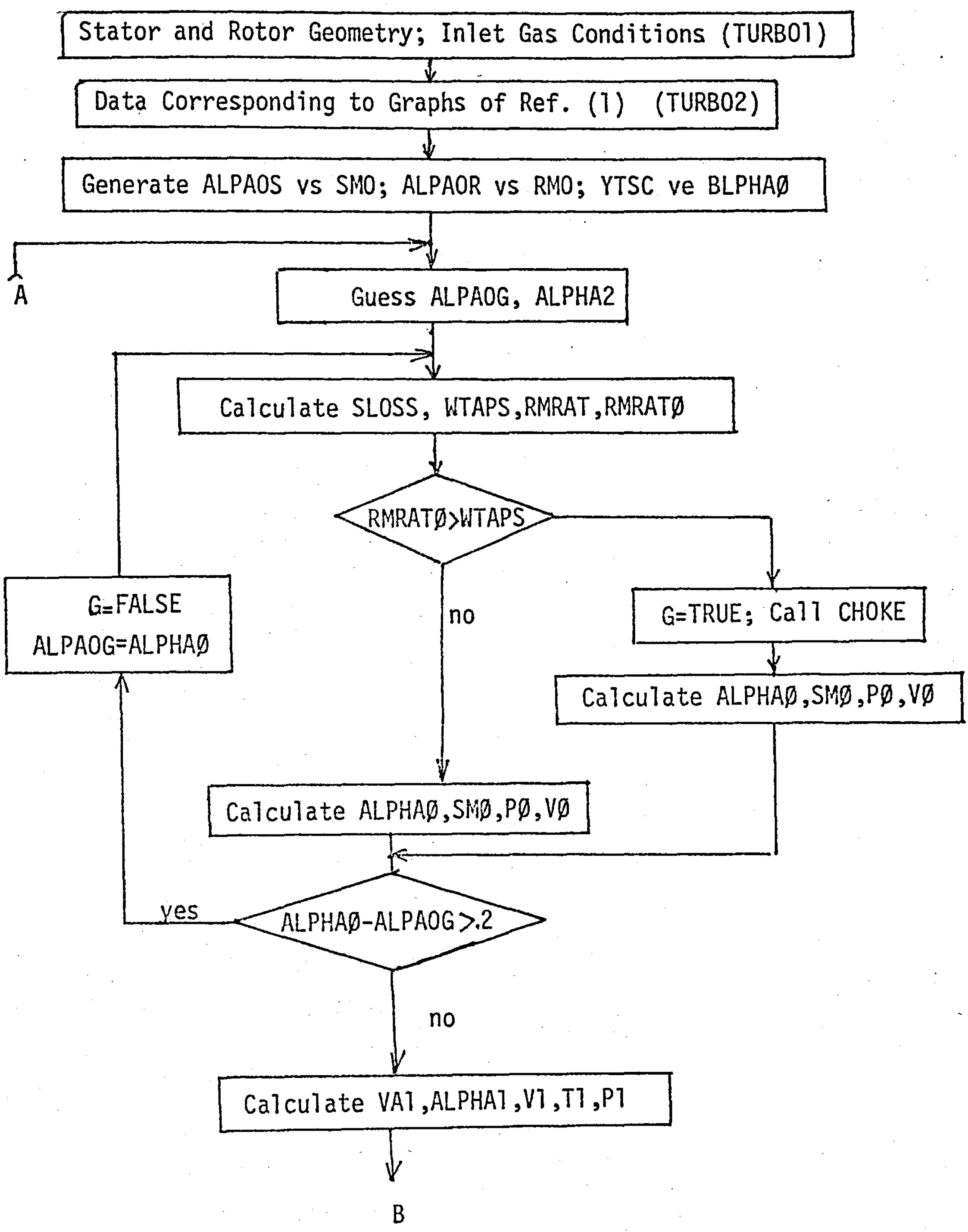

FIGURE B.1 - Flow chart of the computer program "AINLEY"

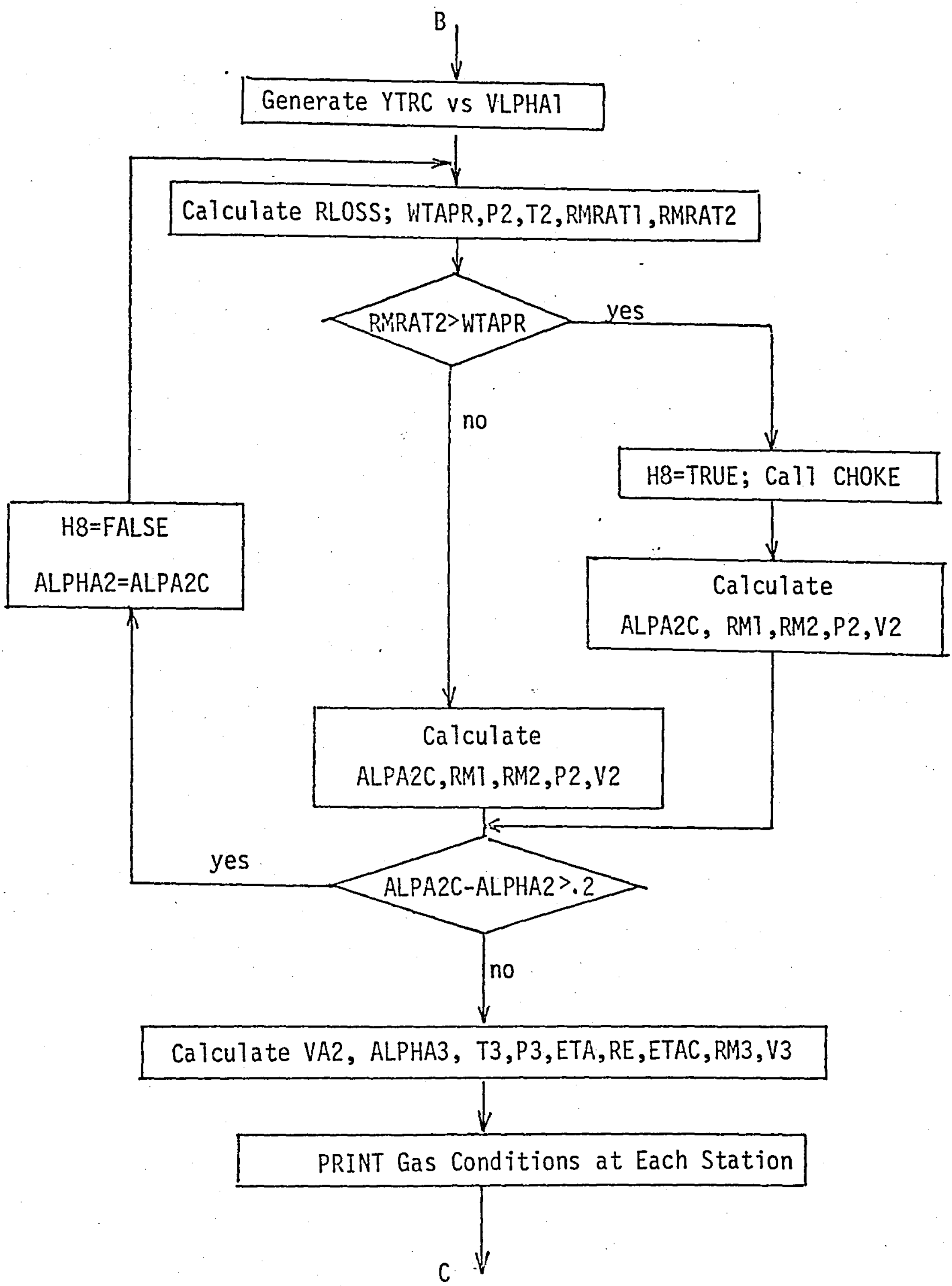

FIGURE B.1 (continued).

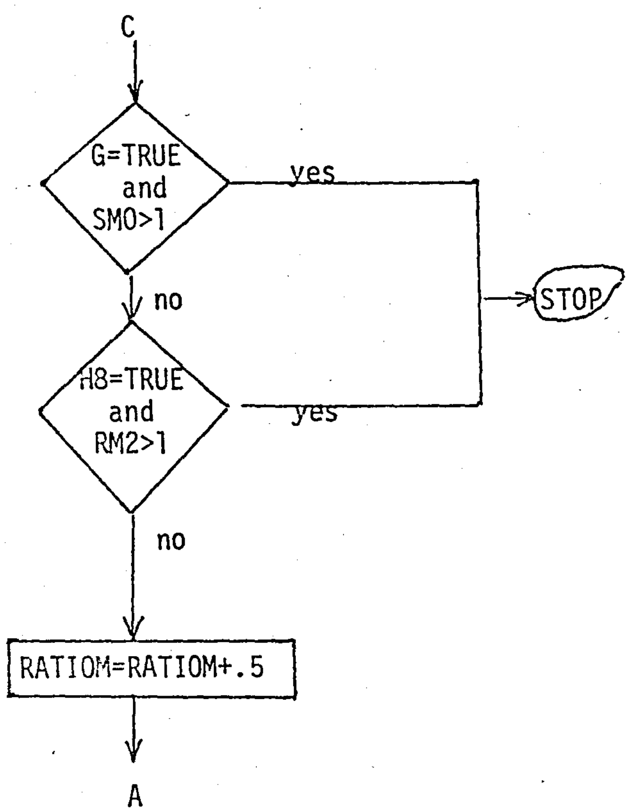

FIGURE B.1 (continued).

In the output, the value of the non-dimensional mass flow for which a row chokes is designated. Also, the overall pressure ratio corresponding to a Mach number of unity at the outlet of the choked row is printed as the limit pressure ratio of the turbine.

B.2 LIST OF THE IMPORTANT VARIABLES OF THE PROGRAM AINLEY

RATIOM	:	Mass flow parameter, $\dot{m}\sqrt{T_1}/P_1$
ALPAOS	:	Vector formed by the stator outlet gas angle values, calculated in SUB1
SMO	:	Vector formed by the stator outlet Mach number values, ranging from zero to unity
RMO	:	Vector formed by the rotor outlet Mach number values, ranging from zero to unity
ALPAOR	:	Vector formed by the rotor outlet gas angle values, calculated in SUB1
YTSC	:	Vector formed by the stator total loss coefficient values, calculated in SUB2
BLPHAØ	:	Vector formed by the stator outlet gas angle values, created in SUB2
ALPAOG	:	Initial guess for α_2
ALPHA2	:	Initial guess for α_4
SLOSS	:	Stator total loss coefficient, Y_N
WTAPS	:	Maximum value of the mass flow parameter, $(\dot{m}\sqrt{T}/AP)_{max}$, that can be passed through the stator
RMRAT	:	Mass flow parameter, $\dot{m}\sqrt{T_1}/P_1A_1$, at inlet to the stator

RMRATØ	:	Mass flow parameter, $\dot{m}\sqrt{T_2}/P_2A_2$, at outlet from the stator
G	:	Logical control variable TRUE, when stator choked FALSE, when stator not choked.
ALPHAØ	:	Stator outlet gas angle, α_2
SMØ	:	Stator outlet Mach number, M_2
PØ	:	Total gas pressure, P_2, at outlet of the stator
VØ	:	Absolute gas velocity, V_2, at outlet of the stator
VA1	:	Axial component of V_2
ALPHA1	:	Gas angle, α_3, relative to the rotor at inlet
V1	:	Gas velocity, V_3, relative to the rotor at inlet
T1	:	Total gas temperature, T_3, at inlet to the rotor
P1	:	Total gas pressure, P_3, at inlet to the rotor
YTRC	:	Vector formed by the rotor total loss coefficient values, calculated in SUB31.
VLPHA1	:	Vector formed by the rotor inlet gas angle values, created in SUB31
RLOSS	:	Rotor total loss coefficient, Y_R
WTAPR	:	Maximum value of the mass flow parameter, $(\dot{m}\sqrt{T}/PA)_{max}$, that can be passed by the rotor
T2	:	Total gas temperature, T_4, at outlet from the rotor
P2	:	Total gas pressure, P_4, at outlet from the rotor
RMRAT1	:	Mass flow parameter, $\dot{m}\sqrt{T_3}/P_3A_3$, at inlet to the rotor

RMRAT2	:	Mass flow parameter, $\dot{m}\sqrt{T_4}/P_4A_4$, at outlet from the rotor
H8	:	Logical control variable TRUE , when rotor choked FALSE, when rotor not choked
ALPA2C	:	Gas angle, α_4, relative to the rotor at outlet
V2	:	Gas velocity, V_4, relative to the rotor at outlet
RM1	:	Mach number, M_3, relative to the rotor at inlet
RM2	:	Mach number, M_4, relative to the rotor at outlet
VA2	:	Axial component of V_4
T3	:	Total temperature, T_1', at inlet to the next stage
P3	:	Total pressure, P_1', at inlet to the next stage
V3	:	Absolute velocity, V_1', at inlet to the next stage
RM3	:	Mach number, M_1', at inlet to the next stage
ETA	:	Total-to-total isentropic efficiency, η, of the stage
RE	:	Mean Reynolds number, Re, of the stage
ETAC	:	Total-to-total isentropic efficiency corrected for the Reynolds number.

B.3 SUBROUTINE DESCRIPTION

The functional relationship among the subroutines of the program AINLEY is given below:

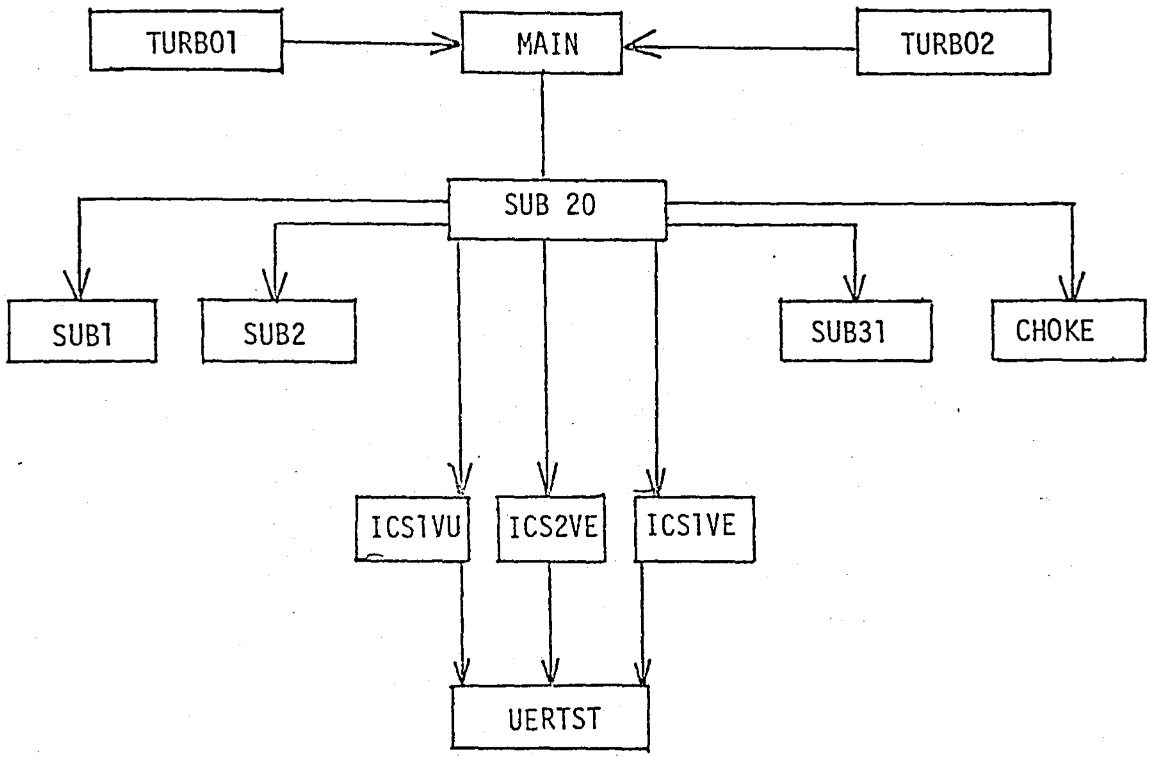

FIGURE B.2 - Functional relationship among the subroutines of the program "AINLEY".

A summary describing the function of each subroutine used in AINLEY is given as follows:

MAIN : The main program reads the datafiles TURBO1 and TURBO2 and calculates the blade speed, annulus areas down and upstream of the blade rows, throat areas of the blade passages and works out the non-dimensional geometrical variables of the stage such as the pitch-to-chord ratio, etc. It then transmits the data to SUB20 by means of the "common" declarations.

SUB20 : Subroutine SUB20 has no argument list, the interconnection with the main program is realized by means of "common declarations". It is SUB20 that perform the procedure described in Section B.1. This subroutine acts as the principal execution control consol.

SUB1 : Subroutine SUB1 generates the two vectors SMO and ALPAOS which represent the vector of the outlet Mach numbers from a blade and the vector of the related outlet gas angles, respectively. The value of the control variable, G, which appears on the argument list decides whether the blade is a rotor or stator blade. For G = 1, SUB1 performs the calculations for a stator blade and returns the vectors ALPAOS and SMO to SUB20. But if G = 2, rotor values are calculated and vectors ALPAOR and RMO are returned to the calling program.

The calculations performed in SUB1 are as explained in Section 3.1.1.

SUB2 : Subroutine SUB2 calculates the stator total loss coefficient as a function of the blade outlet angle, and returns two vectorial quantities to SUB20:BLPHAØ, the set of outlet angles ranging from -70^{0} to -45^{0} and YTSC, the set of total loss coefficients, each corresponding to one element of BLPHAØ.

In SUB2, the calculations are performed according to the theory explained in Section 3.1.2 and the results are returned to SUB20 by means of a common declaration.

SUB31 : The rotor total loss coefficient, Y_R, is calculated in the subroutine SUB31. The important input quantity is the rotor outlet gas angle as guessed in SUB20. The subroutine SUB31 generates rotor inlet gas angles stored in the vector VLPHA1 and calculates the corresponding total loss coefficients stored in the vector YTRC. Then both vectors are returned to SUB20 by means of a common declaration. The calculation of the rotor total loss coefficient is realized through the analysis outlined in Section 3.1.2. Accordingly, if the rotor outlet angle guessed at the beginning differs from that calculated at the end of the calling program, SUB20 uses this latest value of the outlet angle and, by interpolation out of the vector YTRC, finds the corresponding total loss coefficient.

CHOKE : The subroutine CHOKE performs the calculations which are valid only for a choked blade row. Within the subprogram SUB20, if it is found that for any blade row the non-dimensional flow quantity equals or exceeds the maximum allowable non-dimensional flow, then CHOKE is called. The input quantities in the argument list are the total loss coefficient of the row, the inlet pressure and temperature and the non-dimensional mass flow. According to the analysis explained in Section 3.1.5, the performance of the choked row is calculated and the values of the corrected gas outlet angle, Mach number, the outlet gas pressure and velocity related to the blade are returned to SUB20.

ICS1VE, ICS1VU, ICS2VE: These subroutines are available in the Boğaziçi University Computer Center library and they perform the interpolations (one or two dimensional, equally or unequally distributed data) from the figures of Ref. [1] or from the vector quantities generated in SUB1, SUB2 and SUB31. Their usage, function and parameter listings are explained in detail in Section B.5.

B.4 INPUT DESCRIPTION

An explanation of the usage of the program AINLEY is given. The five cards composing the data file TURBO1 must be fed in the following sequence:

Card 1 : RA, GAMA, CP

RA : Gas constant (ft. pdls/lb/^{o}C)

GAMA : Ratio of the specific heats

CP : Specific heat at constant pressure (ft.pdls/lb/^{o}C)

Card 2 : TID, TOD, RK

TID : Turbine inner diameter (in)

TOD : Turbine outer diameter (in)

RK : Clearance of the rotor blades (in)

Card 3 : OPS, PITS, RCS, TMAXS, CHOS, TETS, BETTAI, BNS

OPS : Opening,or throat,of the stator blades (in)

PITS : Pitch length of the stator blades (in)

RCS : Mean radius of curvature of the upper stator blade surface between throat and trailing edge (in)

TMAXS : Maximum thickness of the stator blades (in)

CHOS : Chord length of the stator blades (in)

TETS : Trailing edge thickness of the stator blades (in)

BETTAI: Inlet angle of the stator blades (deg)

BNS : Number of the stator blades

Card 4 : OPR, PITR, RCR, TMAXR, CHOR, TETR, BETTA1, BNR, HEIR, BR

OPR : Opening or throat of the rotor blades (in)

PITR : Pitch length of the rotor blades (in)

RCR : Mean radius of curvature of the upper rotor blade surface between throat and trailing edge (in)

TMAXR : Maximum thickness of the rotor blads (in)

CHOR : Chord length of the rotor blades (in)

TETR : Trailing edge thickness of the rotor blades (in)

BETTA1: Inlet angle of the rotor blades (deg)

BNR : Number of the rotor blades

HEIR : Annulus height at reference station (in)

BR : Constant of Eq. (14)

Card 5 : ALPHA1, TI, PI, RPM, EMRAH

ALPHA1: Gas inlet angle to the stator (deg)

TI : Absolute total gas inlet temperature to the stator (^{o}K)

PI : Absolute total gas inlet pressure to the stator (lbf/in^2)

RPM : Blade rotative speed (rev.per min)

EMRAH : Inlet non-dimensional mass flow-initial value ($\dot{m}\sqrt{T_i}/P_i$)

The five cards are all in free format. A sample data for forming TURBO1 is given below:

Card 1 : 3073., 1.333, 12300.

Card 2 : 9.5, 13.0, .030

Card 3 : .429, .9817, 3.516, .266, 1.33, .0196, 0., 36.

Card 4 : .447, .7113, 2.000, .143, 0.95, .0071, 36., 50., 1.75, .5

Card 5 : 0., 1100., 40., 14000., 7.

B.5 OUTPUT DESCRIPTION

The description of the output is automatically printed by the computer. The output consists of the results corresponding to each value of the non-dimensional flow parameter, $\dot{m}\sqrt{T_i}/P_i$. The program stops when that value of the pressure ratio for which the stage completely chokes is obtained.

A sample output of the program AINLEY, operating with the data given above is as follows:

*** ROTOR INCIDENCE=-21.19162305433

STATOR TOTAL LOSS COEFFICIENT, L(S)= .05762254
****

ROTOR TOTAL LOSS COEFFICIENT, L(R)= .21572196
****

****
* ABSOLUTE STATOR OUTLET GAS CONDITIONS *
****

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-63.519246	39.721	1100.0000	884.22546	.422657

****
* INLET & OUTLET GAS CONDITIONS (RELATIVE TO ROTOR) *
****

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
14.206377	36.226	1074.9781	467.81255	.266595

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-47.575304	35.629	1074.9781	604.99123	.290666

****
* ABSOLUTE GAS CONDITIONS AT INLET TO THE NEXT STAGE *
****

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
-30.923724	35.068	1073.2249	460.35010	.223650

** UNCORRECTED EFFICIENCY= .8650591626409

****
* THE EFFICIENCY OF THE STAGE AS GIVEN BY AEN IS ETA(STAGE)= .84033520 *
****

NON-DIMENSIONAL MASS FLOW W*SQRT(T1)/P1 = 7.0000000

OVERALL PRESSURE RATIO P3/P1 = .8760888

*** ROTOR INCIDENCE=-13.8109688474 3

STATOR TOTAL LOSS COEFFICIENT, L(S)= .05702254

ROTOR TOTAL LOSS COEFFICIENT, L(R)= .19160632

ABSOLUTE STATOR OUTLET GAS CONDITIONS

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-63.519295	39.682	1100.0000	963.55392	.462466

INLET & OUTLET GAS CONDITIONS (RELATIVE TO ROTOR)

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
22.189011	35.659	1071.0109	464.00828	.314104

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-47.575304	35.233	1071.0109	661.85119	.318402

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-47.575304	35.233	1071.0109	661.85119	.318402

ABSOLUTE GAS CONDITIONS AT INLET TO THE NEXT STAGE

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
-23.986541	34.178	1062.9126	464.06725	.232437

** UNCORRECTED EFFICIENCY= .8750214164211

THE EFFICIENCY OF THE STAGE AS GIVEN BY AGN IS ETA(STAGE)= .8547492

NON-DIMENSIONAL MASS FLOW M*SQRT(T1)/P1 = 7.5000000

OVERALL PRESSURE RATIO P3/P1 = .8544550

*** ROTOR INCIDENCE=-7.5769785710095

STATOR TOTAL LOSS COEFFICIENT, L(S)= .05702254
****

ROTOR TOTAL LOSS COEFFICIENT, L(R)= .16241634
****

****
* ABSOLUTE STATOR OUTLET GAS CONDITIONS *
****

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-53.525449	39.622	1100.0000	1051.02913	.567072

****
* INLET & OUTLET GAS CONDITIONS (RELATIVE TO ROTOR) *
****

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
28.423621	35.026	1066.6334	532.77063	.344692

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-47.575334	34.534	1066.6334	724.75845	.390427

****
* ABSOLUTE GAS CONDITIONS AT INLET TO THE NEXT STAGE *
****

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
-17.294170	33.169	1055.9407	506.28671	.244103

** UNCORRECTED EFFICIENCY= .876414657533

****
* THE EFFICIENCY OF THE STAGE AS GIVEN BY AEN IS ETA(STAGE)= .85695356 *
****

NON-DIMENSIONAL MASS FLOW W*SQRT(T1)/P1 = 8.0000000
OVERALL PRESSURE RATIO P3/P1 = .8292301

```
*** ROTOR INCIDENCE=-2.32519826303016

          STATOR TOTAL LOSS COEFFICIENT,  L(S)= .05702254
     *********************************************************

          ROTOR TOTAL LOSS COEFICIENT,  L(R)= .16969657
     *********************************************************
```

```
*************************************************
*     ABSOLUTE STATOR OUTLET GAS CONDITIONS      *
*************************************************
```

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-63.580743	39.556	1100.0000	1147.07506	.553944

```
**************************************************************
*     INLET & OUTLET GAS CONDITIONS (RELATIVE TO ROTOR)      *
**************************************************************
```

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
33.674602	34.333	1061.8024	613.28575	.377738

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-47.575304	33.745	1061.8024	794.37820	.385899

```
**************************************************************
*     ABSOLUTE GAS CONDITIONS AT INLET TO THE NEXT STAGE     *
**************************************************************
```

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
-10.656783	32.052	1048.2364	542.10456	.259554

```
** UNCORRECTED EFFICIENCY= .874126761526?

***********************************************************************************
*     THE EFFICIENCY OF THE STAGE AS GIVEN BY AEH IS     ETA(STAGE)= .85895399    *
***********************************************************************************
```

```
NON-DIMENSIONAL MASS FLOW     W*SQRT(T1)/P1  = 8.5000000
OVERALL PRESSURE RATIO              P3/P1  =  .9013103
```

*** ROTOR INCIDENCE=2.411986932934

STATOR TOTAL LOSS COEFFICIENT, L(S)= .05719393

ROTOR TOTAL LOSS COEFFICIENT, L(R)= .21055816

ABSOLUTE STATOR OUTLET GAS CONDITIONS

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-63.645719	39.469	1100.0000	1262.97977	.613612

INLET & OUTLET GAS CONDITIONS (RELATIVE TO ROTOR)

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
33.411987	33.514	1055.9657	715.45332	.415835

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPRATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-47.575394	32.723	1055.9657	877.34413	.428393

ABSOLUTE GAS CONDITIONS AT INLET TO THE NEXT STAGE

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
-3.627630	30.666	1033.9799	556.93296	.264793

** UNCORRECTED EFFICIENCY= .3637167401973

THE EFFICIENCY OF THE STAGE AS GIVEN BY AEM IS ETA(STAGE)= .55035416

NON-DIMENSIONAL MASS FLOW W*SQRT(T1)/P1 = 9.0000000

OVERALL PRESSURE RATIO P3/P1 = .7666563

*** ROTOR INCIDENCE=6.3000682048E

STATOR TOTAL LOSS COEFFICIENT, L(S)= .05718393
****

ROTOR TOTAL LOSS COEFICIENT, L(R)= .24443306
****

* ABSOLUTE STATOR OUTLET GAS CONDITIONS *

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-63.723794	37.347	1100.0000	1391.58431	.679431

* INLET & OUTLET GAS CONDITIONS (RELATIVE TO ROTOR) *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
42.300069	32.630	1049.4818	832.92254	.459163

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-47.575304	31.547	1049.4818	976.09657	.450175

* ABSOLUTE GAS CONDITIONS AT INLET TO THE NEXT STAGE *

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
2.894677	29.669	1028.4233	625.08576	.320081

** UNCORRECTED EFFICIENCY= .65071l1441153

* THE EFFICIENCY OF THE STAGE AS GIVEN BY AEN IS ETA(STAGE)= .6394220 *

NON-DIMENSIONAL MASS FLOW W*SQRT(T1)/P1 = 9.5000000

OVERALL PRESSURE RATIO P3/P1 = .7272169

*** ROTOR INCIDENCE=10.55017639105

STATOR TOTAL LOSS COEFFICIENT, L(S)= .05744417

ROTOR TOTAL LOSS COEFFICIENT, L(R)= .31536661

ABSOLUTE STATOR OUTLET GAS CONDITIONS

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-53.645019	39.214	1100.0000	1589.75521	.765917

INLET & OUTLET GAS CONDITIONS (RELATIVE TO ROTOR)

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
46.550178	31.264	1039.4707	1015.97027	.523644

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-47.920993	29.413	1039.4707	1154.46119	.574113

ABSOLUTE GAS CONDITIONS AT INLET TO THE NEXT STAGE

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
12.367542	26.297	1010.7962	757.35719	.389255

** UNCORRECTED EFFICIENCY= .815252230223

THE EFFICIENCY OF THE STAGE AS GIVEN BY AEN IS   ETA(STAGE)= .80719551

NON-DIMENSIONAL MASS FLOW   W*SQRT(T1)/P1 =10.0000000

OVERALL PRESSURE RATIO   P3/P1 = .6574291

```
*** ROTOR INCIDENCE=12.25442201438

          STATOR TOTAL LOSS COEFFICIENT,  L(S)= .05744417
      **********************************************************

          ROTOR TOTAL LOSS COEFFICIENT,  L(R)= .3582554b
      **********************************************************
```

```
                         *************************************************
                         *    ABSOLUTE STATOR OUTLET GAS CONDITIONS      *
                         *************************************************
```

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-53.911616	39.156	1100.0000	1695.51261	.943548

```
                   ***********************************************************
                   *    INLET & OUTLET GAS CONDITIONS (RELATIVE TO ROTOR)    *
                   ***********************************************************
```

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
46.254422	30.541	1034.1186	1119.35588	.560687

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-48.265683	28.217	1034.1188	1272.24139	.638683

```
                   ***********************************************************
                   *    ABSOLUTE GAS CONDITIONS AT INLET TO THE NEXT STAGE   *
                   ***********************************************************
```

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
17.260830	24.698	1003.2725	850.92153	.440742

```
** UNCORRECTED EFFICIENCY= .7999506260921

             ***************************************************************************
             *    THE EFFICIENCY OF THE STAGE AS GIVEN BY (EX IS    ETA(STAGE)= .7734601L   *
             ***************************************************************************
```

```
NON-DIMENSIONAL MASS FLOW      W*SQRT(T1)/P1  =10.1551964
OVERALL PRESSURE RATIO               P3/P1  =  .6174556

THE MAXIMUM NON-DIMENSIONAL FLOW WHICH PASSES THROUGH THIS STATOR IS EQUAL TO      10.2114399

THE MAXIMUM NON-DIMENSIONAL MASS FLOW WHICH PASSES THROUGH THIS ROTOR IS EQUAL TO      10.1351964
```

*** CORRECTED BLADE OUTLET ANGLE =38.56165193591
*** ROTOR INCIDENCE=12.25442201448

STATOR TOTAL LOSS COEFFICIENT, L(S)= .05744417

ROTOR TOTAL LOSS COEFFICIENT, L(R)= .35829546

ABSOLUTE STATOR OUTLET GAS CONDITIONS

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-53.911616	39.106	1100.0000	1695.51281	.343846

INLET & OUTLET GAS CONDITIONS (RELATIVE TO ROTOR)

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
48.254422	30.541	1034.1168	1119.85968	.560687

OUTLET GAS ANGLE	OUTLET GAS PRESSURE	OUTLET GAS TEMPERATURE	OUTLET GAS VELOCITY	OUTLET MACH NUMBER
-51.420062	26.219	1034.1188	1905.34477	1.000000

ABSOLUTE GAS CONDITIONS AT INLET TO THE NEXT STAGE

INLET GAS ANGLE	INLET GAS PRESSURE	INLET GAS TEMPERATURE	INLET VELOCITY	INLET MACH NUMBER
34.039794	20.309	970.0954	1322.66779	.722066

** UNCORRECTED EFFICIENCY= .7577515029438

THE EFFICIENCY OF THE STAGE AS GIVEN BY AGM IS   ETA(STAGE)= .75949621

NON-DIMENSIONAL MASS FLOW   MASS*SQRT(T1)/P1 =10.1551984
OVERALL PRESSURE RATIO   P3/P1 = .5074976

ROTOR MACH = EXCEEDS UNITY BEYOND PRESSURE RATIO=   .5074976

B.6 PROGRAM LISTING OF "AINLEY"

```
      PROGRAM AINLEY(INPUT,OUTPUT,TURBO1,TAPE8=TURBO1,
     #                              TURBO2,TAPE5=TURBO2,
     #                              RESULT,TAPE6=RESULT)
      COMMON/D55/VWOP(51,2),VWTAP(51,2),RMN(80,2),VWOW(100),VVOT(100)
      COMMON/S11/RA,RINCRE,VO
      COMMON/S12/TI,NF
      COMMON/S13/OOSS,SOES
      COMMON/S14/ANO,AN2,ATS,ATR
      COMMON/S17/B
      COMMON/S18/OOSR,SOER
      COMMON/S19/RK,RKOH
      COMMON/S99/TID,TOD
      COMMON/S21/YP1(17,8),YP2(15,6),STAIN(21,9),DELSIS(31,4),
     #            YPOYPS(100),SLAM(100)
      COMMON/S22/BETTAI,ALPHAI
      COMMON/S23/TOCS,SOCS,TOSS,SR,CS
      COMMON/S24/BLPHAO(100),YTSC(100),SAS,M,MF
      COMMON/S15/ALPAOS(500),SMO(500)
      COMMON/S31/SOCR,TOCR,TOSR,BR,RR,CR
      COMMON/S32/VLPHA1(500),YTRC(500),BETTA1
      COMMON/S33/K,KF
      COMMON/S34/ALPHA2,A2,EE
      COMMON/S55/PIN
      COMMON/D56/RPM,U,ALPAOG,SLOSS,PI,CP
      COMMON/S77/GAMA
      COMMON/S88/VMU(100),VRO(100)
      COMMON/C11/CWTAP(500),OPOP(500),OVOT(500),ORMN(500),BWTAP(500)
      COMMON/C12/RK(100),H(5000)
      COMMON/E13/EMRAH
      COMMON/F14/SINCRE
      COMMON/P1/CHOS,HEIR,CHOR
      READ(8,*)RA,GAMA,CP
      READ(8,*)TID,TOD,RK
      READ(8,*)OPS,PITS,RCS,TMAXS,CHOS,TETS,BETTAI,BNS
      READ(8,*)OPR,PITR,RCR,TMAXR,CHOR,TETR,BETTA1,BNR,HEIR,BR
      READ(8,*)ALPHAI,TI,PI,RPM,EMRAH
      READ(5,*)M,P,K,SAS,RINCRE,VO
      READ(5,1) ((YP1(I,J),I=1,17),J=1,8)
    1 FORMAT(8(16F5.3,/),8F5.3)
      READ(5,2) ((YP2(I,J),I=1,15),J=1,6)
    2 FORMAT(5(16F5.3,/),10F5.3)
      READ(5,3) ((STAIN(I,J),I=1,21),J=9,1,-1)
    3 FORMAT(9(20F4.1,/),9F4.1)
      READ(5,4) (YPOYPS(I),I=1,30)
    4 FORMAT(20F4.2,/,10F4.2)
      READ(5,5) ((DELSIS(I,J),I=1,31),J=1,4)
    5 FORMAT(18F4.2,/,13F6.2,/,16F5.2,/5(13F6.2,/),12F6.2)
      READ(5,*)(SLAM(I),I=1,41)
      READ(5,*)((RMN(I,J),I=1,80),J=1,2)
      READ(5,*)((VWTAP(I,J),I=1,51),J=1,2)
      READ(5,*)((VWOP(I,J),I=1,51),J=1,2)
      READ(5,*)(VVOT(I),I=1,80)
      READ(5,*)(VWOW(I),I=1,51)
      READ(5,*)(VMU(I),I=1,34)
      READ(5,*)(VRO(I),I=1,34)
      READ(5,*)(CWTAP(I),I=1,60)
      READ(5,*)(OPOP(I),I=1,60)
      READ(5,*)(OVOT(I),I=1,60)
      READ(5,*)(ORMN(I),I=1,60)
      READ(5,*)(BWTAP(I),I=1,50)
      PIN=ATAN(1.)*4.
      OOSS=OPS/PITS
      SOES=PITS/RCS
      TOCS=TMAXS/CHOS
      SOCS=PITS/CHOS
      TOSS=TETS/PITS
      OOSR=OPR/PITR
      SOER=PITR/RCR
      TOCR=TMAXR/CHOR
      SOCR=PITR/CHOR
      TOSR=TETR/PITR
      SR=TID/TOD
      RR=TID/TOD
      RKOH=RK/HEIR
      ALPAOG=-1.17*ACOS(OOSS)*180./PIN+12.58-4.*SOES
      CS=CHOS/12.
      CR=CHOR/12.
      B=BETTAI*PIN/180.
      ANO=(TOD**2.-TID**2.)*PIN/4.
      AN2=ANO
```

```
      U=-1.*PIN*(TID+TOD)*RPM/1440.
      ATS=12.*OOSS*SOCS*CS*BNS*(TOD-TID)/2.
      ATR=12.*OOSR*SOCR*CR*BNR*(TOD-TID)/2.
      CALL SUB20
      STOP
      END
C
C
C
C
C
      SUBROUTINE SUB1(G,ALPAOS,SMO)
      DIMENSION ALPAOS(500),SMO(500)
      COMMON/S11/RA,RINCRE,VOO
      COMMON/S12/TI,NF
      COMMON/S13/OOSS,SOES
      COMMON/S14/ANO,AN2,ATS,ATR
      COMMON/S17/B
      COMMON/S18/OOSR,SOER
      COMMON/S19/RK,RKOH
      COMMON/S99/TID,TOD
      COMMON/S55/PIN
      COMMON/S77/GAMA
      ALPHA(A1,A2)=-1.17*ACOS(A1)*180./PIN+12.58-4.*A2
      CO=(GAMA*RA*TI)**(.5)
      IF(G.EQ.1.) THEN
      E=-ACOS(ATS/ANO)
      ALPAO2=E*180./PIN
      ALPAO1=ALPHA(OOSS,SOES)
      END IF
      IF(G.EQ.2.) THEN
      D=ALPHA(OOSR,SOER)*PIN/180.
      F=ATAN((1.-1.35*RKOH*COS(B)/COS(D))*T[illegible]
     #  *TAN(B))
      ALPAO1=F*180./PIN
      ATR=ATR*(1.-RKOH)+PIN*TOD*RK
      ALPAO2=-/COS(ATR/AN2)*180./PIN
      END IF
      SMO(1)=0.
      DO 22 I=1,500
    1 IF(SMO(I).GT.1.)GOTO 8
      IF(SMO(I).GT..5)GOTO 3
      ALPAOS(I)=ALPAO1
      GO TO 7
    3 IF(SMO(I).EQ.1.)GOTO 4
      ALPAOS(I)=2.*(ALPAO2-ALPAO1)*SMO(I)+2.*ALPAO1-ALPAO2
      GO TO 7
    4 ALPAOS(I)=ALPAO2
    7 SMO(I+1)=SMO(I)+.01
   22 CONTINUE
    8 NF=I-1
    9 RETURN
      END
C
C
C
C
C
      SUBROUTINE SUB2
      COMMON/S21/YP1(17,8),YP2(15,6),STAIN(21,9),DELSIS(31,4),
     #           YPOYPS(100),SLAN(100)
      COMMON/S22/BETTAI,ALPHAI
      COMMON/S23/TOCS,SOCS,TOSS,SF,CS
      COMMON/S14/ANO,AN2,ATS,ATR
      COMMON/S24/BLPHAO(100),YTSC(100),SAS,M,NF
      COMMON/S55/PIN
      COMMON/S17/B
      COMMON/P1/CHOS,HEIR,CHOR
      DIMENSION WK(100),YPBIO(200),YPBIA(200),ALPASC(100),SISCS(100),DEL
     #TIS(90),STINS(90),YPS(90),G(90),YPSIO(90),AMS(100),AOS(100),YSS(10
     #0),AO(100),SF(100),YTS(100),Y(100)
      TEFAC=21.6*TOSS**2.+3.86*TOSS+.91
      AIS=ALPHAI*PIN/180.
      AI=ANO*COS(B)
      BLPHAO(1)=SAS
      GO TO 10
    9 BLPHAO(1)=SAS+1.
   10 XO=SOCS
      DO 11 I=1,M
      IF(BLPHAO(I).GT.-45.)GOTO 14
```

```
      Y(I)=BLPHAO(I)
      CALL ICS2VE(YP1,X0,.30,1.10,-80.,-45.,17,8,1,17,WK,Y(I),IER)
      YPBIO(I)=Y(I)
      Y(I)=BLPHAO(I)
      CALL ICS2VE(YP2,X0,.30,1.00,-70.,-45.,15,6,1,15,WK,Y(I),IER)
      YPBIA(I)=Y(I)
      YPSIO(I)=(YPBIO(I)+((BETTAI/(BLPHAO(I))**2)*(YPBIA(I)-YPBIO(I)))*(
     #        (TOCS/.2)**(-BETTAI/BLPHAO(I))))
      ALPASC(I)=(-.35*SOCS+1.25)*BLPHAO(I)
      Y(I)=BLPHAO(I)
      X0=BETTAI/ALPASC(I)
      CALL ICS2VE(STAIN,X0,-1.0,1.0,-70.,-30.,21,9,1,21,WK,Y(I),IER)
      SISCS(I)=Y(I)
      X0=SOCS
      Y(I)=BLPHAO(I)
      CALL ICS2VE(DELSIS,X0,.40,1.0,-70.,-40.,31,4,1,31,WK,Y(I),IER)
      DELTIS(I)=Y(I)
      STINS(I)=SISCS(I)+DELTIS(I)
      G(I)=(ALPHAI-BETTAI)/STINS(I)
      CALL ICS1VE(YPOYPS,-4.2,1.6,0.,0.,0.,0.,30,1,G(I),PSCOND,IER)
      YPS(I)=G(I)*YPSIO(I)
      AOS(I)=BLPHAO(I)*PIN/180.
      AMS(I)=ATAN((TAN(AIS)+TAN(AOS(I)))/2.)
      AO(I)=ANO*COS(AOS(I))
      SF(I)=(AO(I)/AI)**2./(1.+SK)
      IF(SF(I).GT.0.5)GOTO 9
      IF(SF(I).LT.0.0)GOTO 12
      CALL ICS1VE(SLAM,0.0,0.5,0.,0.,0.,0.,41,1,SF(I),PSCOND,IER)
      DCF=.0334*(CHOS/HEIR)*(COS(AOS(I))/COS(B))
      YSS(I)=DCF*(2.*(TAN(AIS)-TAN(AOS(I))))*COS(AMS(I)))**2.*COS(AOS(I))
     #        **2./COS(AMS(I))**3.
      YTS(I)=YPS(I)+YSS(I)
      IF(TOSS.EQ..02) GOTO 8
      YTSC(I)=YTS(I)*TEFAC
      GO TO 11
    8 YTSC(I)=YTS(I)
   11 BLPHAO(I+1)=BLPHAO(I)+1.
      GO TO 14
   12 WRITE(6,13)
   13 FORMAT(//,20X,'ERROR IN FORMULATION OF SF(I)')
   14 MF=I-1
      RETURN
      END
C
C
C
C
C
      SUBROUTINE SUB31
      COMMON/S21/YP1(17,8),YP2(15,6),STAIN(21,9),DELSIS(31,4),
     #           YPOYPS(100),SLAM(100)
      COMMON/S31/SOCR,TOCR,TOSR,BR,RR
      COMMON/S14/ANO,AN2,ATS,ATR
      COMMON/S32/VLPHAL(500),YTRC(500),BETTA1
      COMMON/S33/K,KF
      COMMON/S34/ALPHOR,A2,FE
      COMMON/S55/PIN
      COMMON/S18/OOSR,SOER
      COMMON/S19/RK,PKOH
      COMMON/P1/CHOS,HEIR,CHOR
      DIMENSION  RINS(500),E(500),YPR(500),F(500),AMR(500),YSKR(500),
     #           YTR(500),KK(500)
      TEFAC=21.6*TOSR**2.+3.86*TOSR+.91
      ALPSCR=(-.35*SOCR+1.25)*ALPHOR
      B=BETTA1*PIN/180.
      A1=ANO*COS(B)
      D=(A2/A1)**2./(1.+RR)
      A01=ALPHOR
      A02=ALPHOR
      A03=ALPHOR
      A05=ALPHOR
      A04=BETTA1/ALPSCR
      CALL ICS2VE(YP1,SOCR,.30,1.10,-80.,-45.,17,8,1,17,WK,A01,IER)
      CALL ICS2VE(YP2,SOCR,.30,1.00,-70.,-45.,15,6,1,15,WK,A02,IER)
      CALL ICS2VE(STAIN,A04,-1.0,1.0,-70.,-30.,21,9,1,21,WK,A03,IER)
      CALL ICS2VE(DELSIS,SOCR,.40,1.0,-70.,-40.,31,4,1,31,WK,A05,IER)
      CALL ICS1VE(SLAM,0.0,0.5,0.,0.,0.,0.,41,1,D,PSCOND,IER)
      YPPIO=(A01+(BETTA1/ALPHOR)**2.*(A02-A01))*(TOCR/.2)**(-BETTA1/
     #      ALPHOR)
      STINR=A03+A05
```

```
      VLPHA1(1)=BETTA1-4.2*STINR
      GO TO 2
    1 VLPHA1(1)=VLPHA1(1)+.2
    2 DO 10,I=1,K
      RINS(I)=-(BETTA1-VLPHA1(I))
      F(I)=RINS(I)/STINR
      IF(F(I).LT.-4.2)GOTO 1
      IF(F(I).GT.1.6)GO TO 11
      CALL ICSIVF(YPOYPS,-4.2,1.6,0.,0.,0.,0.,30,1,F(I),PSCOND,IER)
      ASAF=F(I)
      YPR(I)=F(I)*YPRIO
      F(I)=VLPHA1(I)*PIN/180.
      AMR(I)=ATAN((TAN(F(I))+TAN(EE))/2.)
      DCFR=.0334*(CHOR/HEIR)*(COS(EE)/COS(B))
     #      +BR*(CHOR/HEIR)*(RR/CHOR)**.76
      YSKR(I)=DCFR*(2.*(TAN(F(I))-TAN(EE))*COS(AMR(I)))**2.*(COS(EE))
     #        **2./(COS(AMR(I)))**3.
      YTK(I)=YPR(I)+YSKR(I)
      IF(TOSR.EQ..02)GOTO 7
      YTRC(I)=YTK(I)*TEFAC
      GOTO 10
    7 YTRC(I)=YTK(I)
   10 VLPHA1(I+1)=VLPHA1(I)+1.
   11 KF=I-1
      RETURN
      END
C
C
C
C
C
      SUBROUTINE SUB20
      DIMENSION ALPAOS(500),SMO(500),ALPAOR(500),RMO(500)
      COMMON/D55/VWOP(51,2),VWTAP(51,2),RMN(60,2),VWOW(100),VVOT(100)
      COMMON/S11/EA,FINCRE,VO
      COMMON/S12/T1,NF
      COMMON/S13/OUSS,SOES
      COMMON/S14/ANO,AN2,ATS,ATR
      COMMON/S17/B
      COMMON/S18/OOSR,SOER
      COMMON/S19/RK,RKOH
      COMMON/S99/TIO,TOD
      COMMON/S21/YP1(17,8),YP2(15,6),STAIN(21,9),DELSIS(31,4),
     #            YPOYPS(100),SLAH(100)
      COMMON/S22/BETTAI,ALPHAI
      COMMON/S23/TOCS,SOCS,TOSS,SR,CS
      COMMON/S24/BLPHAO(100),YTSC(100),SAS,N,MF
      COMMON/S31/SOCR,TOCR,TOSR,BR,RR,CR
      COMMON/S32/VLPHA1(500),YTRC(500),BETTA1
      COMMON/S33/K,KF
      COMMON/S34/ALPHA2,A2,EE
      COMMON/S55/PIN
      COMMON/D56/RPM,U,ALPAOG,SLOSS,PI,CP
      COMMON/S77/GAMA
      COMMON/S88/VRU(100),VRO(100)
      COMMON/C11/OWTAP(500),OPOP(500),OVOT(500),ORMN(500),BWTAP(500)
      COMMON/C12/WK(100),H(5000)
      COMMON/C13/G,H1,H2,PUS,H3,H4,H8
      COMMON/C25/THDAT
      COMMON/E13/EMRAH
      COMMON/E14/SINCRE
      LOGICAL G,H1,H2,PUS,H3,H4,H6,H7,H8,H9,H10,H11
      II=0
      JJ=0
      KK=0
      LL=0
      PUS=.FALSE.
      G=.FALSE.
      H1=G
      H6=G
      H7=G
      H9=G
      H10=G
      H11=G
      RATIOM=EMRAH
C
C
C     ****   GENERATE  BLADE OUTLET ANGLES VS. MACH NUMBERS   ****
C
C
```

```
      CALL SUB1(1.,ALPAOS,SMO)
      CALL SUB1(2.,ALPAOR,RMO)
C
C
C     ****   GENERATE  STATOR  LOSS VS. OUTLET ANGLES  ****
C
C
      CALL SUB2
C
C
C      **** GUESS ROTOR OUTLET ANGLE FROM THE START ****
C
C
      AA=ACOS(OOSR)
      ALPHA2=-1.17*AA*180./PIN+12.49-4.*SOER
C
C
      ALPAOG=-1.17*ACOS(OOSS)*180./PIN+12.58-4.*SOES
    6 SLOSS=ALPAOG
      CALL ICS1VU(YTSC,PLPHAO,MF,1,SLOSS,H,IER)
      STALOS=SLOSS
      WTAPS=GAMA
C
C
C     **** FIND MAX. NON-DIM. FLOW THROUGH STATOR  ****
C
      CALL ICS2VE(VWTAP,SLOSS,0.,1.,1.333,1.3963,51,2,1,51,WK,WTAPS,IER)
       AXFL=WTAPS
C
C
      BB=ALPAOG*PIN/180.
      AO=ANO*COS(BB)
      GO TO 290
C
  275 RATIOM=RATIOM-.05
      GO TO 290
C
C
  280 RATIOM=RATIOM-SINCRE
  290 TMOAT=RATIOM*PI/SQRT(TI)
      RMRAT=TMOAT*SQRT(TI)/(AO*PI)
      WOWS=RMRAT/WTAPS
C
C
C     **** CHECK INLET MASS FLOW TO STATOR ****
C
C
      IF(WOWS.GT.1.) THEN
      II=II+1
      GO TO 275
      END IF
C
C
C
C
      IF(G) GO TO 325
  300 WOPCS=GAMA
      CALL ICS2VE(VWOP,SLOSS,0.,1.,1.333,1.3963,51,2,1,51,WK,WOPCS,IER)
      CALL ICS1VE(VWOW,0.,1.,0.,0.,0.,0.,51,1,WOWS,PSCOND,IER)
      WBARS=WOPCS*WOWS*PI
      PO=PI-WBARS
      RMRATO=TMOAT*SQRT(TI)/(AO*PO)
C
C
C     **** CHECK CHOKE ON STATOR ****
C
C
      W1=ABS(RMRATO/WTAPS-1.)
      IF(W1.GT..01)GO TO 330
      G=.TRUE.
      IF(LL.NE.0) GO TO 324
      SMAXFL=RATIOM
  324 GO TO 331
  325 H8=.FALSE.
      CALL CHOKE(SLOSS,PI,TI,AO,ANO,ALPHAO,SMO,RATIOM,PO,VO)
       IF(PUS.OR.H2) STOP
      AGAHM=SMO
      AGAHA=ALPHAO
      GO TO 326
C
```

```
      C
      C     **** CHECK OUTLET MASS FLOW FROM STATOR ****
      C
      C
        330 IF(RMRATO.GT.WTAPS) THEN
             JJ=JJ+1
             DIFFER=1.-(WTAPS/RMRATO)
             RATION=RATION-DIFFER
             GO TO 290
            END IF
      C
      C
        331 SMO=RMRATO
            CALL ICS1VU(GRMN,BWTAP,50,1,SMO,H,IER)
            VOTO=SMO
            CALL ICS1VE(UVOT,.02,1.2,0.,0.,0.,0.,60,1,VOTO,PSCOND,IER)
            VELOCI=VOTO
            ALPHAO=SMO
            CALL ICS1VU(ALPAOS,SMO,NF,1,ALPHAO,H,IER)
      C
      C
            DIFF1=ABS(ALPAOG-ALPHAO)
            IF(DIFF1.LT..1)GOTO 7
            ALPAOG=ALPHAO
            G=.FALSE.
            GO TO 6
          7 VO=VOTO*SQRT(TI)
      C
      C
      C     ****  START CALCULATIONS ON ROTOR GAS CONDITIONS ****
      C
      C
        326 CC=ALPHAO*PIN/180.
            VA1=VO*COS(CC)
            DD=ATAN((U/VA1)-TAN(CC))
            ALPHA1=DD*180./PIN
            V1=VA1/COS(DD)
            T1=TI-(VO**2.*(1.-(COS(CC)/COS(DD))**2.)/(2.*CP))
            T2=T1
            P1=PO*(T1/TI)**(GAMA/(GAMA-1.))
            IF(H7) GO TO 825
          1 EE=ALPHA2*PIN/180.
            A2=AN2*COS(EE)
      C
      C
      C     **** GENERATE ROTOR LOSS VS. INCIDENCE ANGLE ****
      C
      C
            CALL SUB31
          2 FLUSS=ALPHA1
            CALL ICS1VU(YTKC,VLPHA1,KF,1,RLOSS,H,IER)
      C
      C
            IF(IER.NE.129) GO TO 2600
            RATION=RATION+.05
            GO TO 290
      C
      C
       2600 ROTLOS=RLOSS
            WTAPR=GAMA
      C
      C
      C     **** FIND MAX. NON-DIM. FLOW THROUGH ROTOR ****
      C            ***AS A FUNCTION OF INCIDENCE***
      C
      C
            CALL ICS2VE(VWTAP,RLOSS,0.,1.,1.333,1.3963,51,2,1,51,WK,WTAPR,IER)
             BOXEL=WTAPR
            IF(HL) GO TO 825
      C
      C
      C     **** CHECK INLET MASS FLOW TO ROTOR ****
      C
      C
            WONR=(TMDAT*SQRT(T1)/(A2*P1))/WTAPR
            IF(WONR.GT.1.)THEN
            KK=KK+1
            GO TO 275
            END IF
      C
```

```
      C
  800 CALL ICS1VE(VNOW,0.,1.,0.,0.,0.,0.,51,1,WOWR,PSCOND,IER)
      WOPCR=GAMA
      CALL ICS2VE(VNGP,RLOSS,0.,1.,1.333,1.3963,51,2,1,51,WK,WOPCR,IER)
      WBARR=WOWR*WOPCR*P1
      P2=P1-WBARR
      RMRAT1=TMDAT*SQRT(T1)/(A2*P1)
      RMRAT2=TMDAT*SQRT(T2)/(A2*P2)
      C
      C
      C     **** CHECK CHOKE ON ROTOR ****
      C
      C
      W2=ABS(RMRAT2/WTAPR-1.)
      IF(W2.GT..01) GO TO 830
      H1=.TRUE.
      GO TO 831
  825 H8=.TRUE.
      CALL CHOKE(RLOSS,P1,T2,A2,AN2,ALPHA2,RM2,RATIOM,P2,V2)
       IF(POS.OR.H2) STOP
      GO TO 826
      C
      C
      C
      C
      C     **** CHECK OUTLET MASS FLOW FROM ROTOR ****
      C
  830 IF(RMRAT2.GT.WTAPR) THEN
      LL=LL+1
      DIFFES=1.-(WTAPR/RMRAT2)
       RATIOM=RATIOM-DIFFES
      G=.FALSE.
      GO TO 290
      END IF
      C
      C
  831 RM1=RMRAT1
      RM2=RMRAT2
      CALL ICS1VU(ORMN,BWTAP,50,1,RM1,H,IER)
      CALL ICS1VU(ORMN,BWTAP,50,1,RM2,H,IER)
      ALPA2C=RM2
      CALL ICS1VU(ALPAOF,RMO,NF,1,ALPA2C,H,IER)
      C
      C
      DIFF2=ABS(ALPHA2-ALPA2C)
      IF(DIFF2.LT..2) GO TO 12
      ALPHA2=ALPA2C
      H1=.FALSE.
      GO TO 1
   12 VOT2=RM2
      CALL ICS1VE(OVOT,.02,1.2,0.,0.,0.,0.,60,1,VOT2,PSCOND,IER)
      V2=VOT2*SQRT(T2)
  826 GG=ALPHA2*PIN/180.
      VA2=V2*COS(GG)
      HH=ATAN((U/VA2)-TAN(GG))
      ALPHA3=HH*180./PIN
      T3=T2-V2**2.*(1.-(COS(GG)/COS(HH))**2.)/(2.*CP)
      P3=P2*(T3/T2)**(GAMA/(GAMA-1.))
      ETA=((T1-T3)/T1)/(1.-(P3/P1)**((GAMA-1.)/GAMA))
      C
      C     ****   CALCULATE STAGE MEAN REYNOLDS NUMBER ****
      C
      C
      C
      STMU=T1
      ROMU=T2
      ROST=T1
      RORO=T2
      CALL ICS1VE(VMU,100.,2500.,0.,0.,0.,0.,34,1,STMU,PSCOND,IER)
      CALL ICS1VE(VMU,100.,2500.,0.,0.,0.,0.,34,1,ROMU,PSCOND,IER)
      CALL ICS1VE(VRO,100.,2500.,0.,0.,0.,0.,34,1,ROST,PSCOND,IER)
      CALL ICS1VE(VRO,100.,2500.,0.,0.,0.,0.,34,1,RORO,PSCOND,IER)
      RENST=CS*VG*ROST*100000./STMU
      RENRO=CR*V2*RORO*100000./ROMU
      RENM=(RENST+RENRO)/2.
      C
      C
      C
      ETAC=1.-((1.-ETA)/((RENM/200000.)**(.2)))
      A3=AN2*COS(HH)
```

```
      VOT3=TMDAT*SQRT(T3)/(A3*P3)
      RM3=GAMA
      CALL ICS2VL(RMN,VOT3,0.,.395,1.333,1.3963,80,2,1,80,WK,RM3,IER)
      IF(IER.EQ.129) GO TO 1001
      CALL ICS1VL(VVOT,0.,.4,0.,0.,0.,0.,80,1,VOT3,PSCOND,IER)
      IF(IER.EQ.129) GO TO 1001
      V3=VOT3*SQRT(T3)
C
C
C
C
 1001 RIN=-(BETTA1-ALPHA1)
      WRITE(6,*)  '    *** ROTOR INCIDENCE=',RIN
C
C
      WRITE(6,4)STALOS,ROTLOS
    4 FORMAT(8(/),20X,'STATOR TOTAL LOSS COEFFICIENT,  L(S)=',F10.8,/,15
     #X,57('*'),///,20X,'ROTOR TOTAL LOSS COEFICIENT,  L(R)=',F10.8,/,15
     #X,57('*'),8(/))
      WRITE(6,5)
    5 FORMAT(///,50X,47('*'),/,50X,'*',4X,'ABSOLUTE STATOR OUTLET GAS CO
     #NDITIONS',4X,'*',/,50X,47('*'),/)
      WRITE(6,8)
    8 FORMAT(////,5X,'OUTLET GAS ANGLE',4X,'OUTLET GAS PRESSURE',6X,'OUT
     #LET GAS TEMPERATURE',7X,'OUTLET GAS VELOCITY',4X,'OUTLET MACH NUMB
     #ER',//)
      WRITE(6,9)ALPHA0,P0,T1,V0,SMO
    9 FORMAT(/,7X,F10.6,16X,F7.3,17X,F9.4,16X,F10.5,11X,F10.6,///)
      WRITE(6,13)
   13 FORMAT(/,45X,60('*'),/,45X,'*',4X,'INLET & OUTLET GAS CONDITIONS (
     #RELATIVE TO ROTOR)',5X,'*',/,45X,60('*'),5(/),5X,'INLET GAS ANGLE'
     #,7X,'INLET GAS PRESSURE',7X,'INLET GAS TEMPERATURE',7X,'INLET VELO
     #CITY',10X,'INLET MACH NUMBER',//)
      WRITE(6,14) ALPHA1,P1,T1,V1,RM1
   14 FORMAT(//,7X,F10.6,16X,F7.3,17X,F9.4,16X,F10.5,11X,F10.6)
      WRITE(6,40)
   40 FORMAT(////,5X,'OUTLET GAS ANGLE',4X,'OUTLET GAS PRESSURE',6X,'OUT
     #LET GAS TEMPERATURE',7X,'OUTLET GAS VELOCITY',4X,'OUTLET MACH NUMB
     #ER',//)
      WRITE(6,16) ALPHA2,P2,T2,V2,RM2
   16 FORMAT(/,7X,F10.6,16X,F7.3,17X,F9.4,16X,F10.5,11X,F10.6,///)
      WRITE(6,18)
   18 FORMAT(///,45X,60('*'),/,45X,'*',4X,'ABSOLUTE GAS CONDITIONS AT IN
     #LET TO THE NEXT STAGE',4X,'*',/,45X,60('*'),//)
      WRITE(6,20)
   20 FORMAT(//5X,'INLET GAS ANGLE',7X,'INLET GAS PRESSURE',7X,'INLET GA
     #S TEMPERATURE',7X,'INLET VELOCITY',10X,'INLET MACH NUMBER',//)
      WRITE(6,19)ALPHA3,P3,T3,V3,RM3
   19 FORMAT(/,7X,F10.6,16X,F7.3,17X,F9.4,16X,F10.5,11X,F10.6,//)
      WRITE(6,*) '** UNCORRECTED EFFICIENCY= ',ETA
      WRITE(6,17) ETAC
   17 FORMAT(///,39X,81('*'),/,39X,'*',4X,'THE EFFICIENCY OF THE STAGE A
     #S GIVEN BY AEM IS     ETA(STAGE)=',F10.8,4X,'*',/,39X,81('*'),///)
      PRATIO=P3/PI
      WRITE(6,2000) RATIOM,PRATIO
 2000 FORMAT(5(/),10X,'NON-DIMENSIONAL MASS FLOW',5X,'W*SQRT(T1)/P1',2X,
     #'=',F10.7,//,10X,'OVERALL PRESSURE RATIO',12X,'P3/PI',2X,'=',F10.7
     #,//)
C
C
C
C
      IF(H3.AND.H4) GO TO 5000
       IF(H6) GO TO 6001
C
C
       IF(G) THEN
      WRITE(6,320) SMAXFL
  320 FORMAT(//,10X,'THE MAXIMUM NON-DIMENSIONAL FLOW WHICH PASSES THROU
     #GH THIS STATOR IS EQUAL TO',5X,F10.7,//)
      H6=.TRUE.
      END IF
C
C
 6001 IF(H7) GO TO 7001
C
C
      IF(H1) THEN
      WRITE(6,820) RATIOM
  820 FORMAT(//,10X,'THE MAXIMUM NON-DIMENSIONAL MASS FLOW WHICH PASSES
```

```
      #THROUGH THIS ROTOR IS EQUAL TO',5X,F10.7,//)
       H7=.TRUE.
       END IF
C
C
 7001  IF(H9)GO TO 8001
C
C
       IF(H3) THEN
       WRITE(6,7002) PRATIO
 7002  FORMAT(//,10X,'STATOR MACH # EXCEEDS UNITY AT PRESSURE RATIO=
      #',2X,F10.7,//)
       H9=.TRUE.
       END IF
C
C
 8001  IF(H10) GO TO 9001
C
C
       IF(H4) THEN
       WRITE(6,8002)PRATIO
 8002  FORMAT(//,10X,'ROTOR MACH # EXCEEDS UNITY BEYOND PRESSURE RATIO= '
      #,2X,F10.7,//)
       H10=.TRUE.
       END IF
C
C
 9001  IF(H10.OR.H9) GO TO 5000
C
C
       IF((H6.AND.H7).AND.(SMAXFL.GT.RATIOM)) THEN
       G=.FALSE.
       GO TO 6
       END IF
C
C
       IF(H7) GO TO 6
C
C
       IF(G)  GO TO 325
C
C
       IF((II.NE.0).OR.(KK.NE.0)) THEN
       RATIOM=RATIOM+.02
       GO TO 290
       END IF
C
C
       RATIOM=RATIOM+.5
       GO TO 6
 5000  RETURN
       END
C
C
C
C
C
        SUBROUTINE CHOKE(LOSS,PM,TM,AM,ANN,ALPANC,MACH,RATIOM,PN,VN)
       COMMON/C11/OWTAP(500),OPOP(500),OVOT(500),ORMN(500),BWTAP(500)
       COMMON/C12/WR(100),H(5000)
       COMMON/C13/G,H1,H2,PUS,H3,H4,H8
       COMMON/S77/GAMA
       COMMON/S55/PIN
       COMMON/C25/TMDAT
       REAL LOSS,MACH,NWTAP,NVOTN
       LOGICAL G,H1,H2,H3,H4,PUS,H8
       MACH=MACH+.1
C
C
       IF(MACH.GT.1.2) THEN
       PUS=.TRUE.
       GO TO 9
       END IF
C
C
       IF(H3.OR.H4)GO TO 1
       IF(MACH.GT.1.) GO TO 5
     1 NWTAP=MACH
       CALL ICSIVE(OWTAP,.02,1.20,0.,0.,0.,0.,60,1,NWTAP,PSCOND,IER)
       PNSOPN=MACH
```

```
      CALL ICS1VE(DPOP,.02,1.20,0.,0.,0.,0.,60,1,PNSOPN,PSCOND,IER)
      NVOTN=MACH
      CALL ICS1VE(OVOT,.02,1.20,0.,0.,0.,0.,60,1,NVOTN,PSCOND,IER)
      WBOPN=LOSS*(1.-PNSOPN)
      WBOPN=WBOPN/(1.+WBOPN)
      PN=PN-WBOPN*PN
      ANC=(1./NVTAP)*TMDAT*SQRT(TM)/PN
      WRITE(6,*) ' *** CORRECTED BLADE OUTLET ANGLE =',ANC
      FF=ACOS(ANC/ANN)
      ALPANC=-(FF*180./PIN)
      VN=NVOTN*SQRT(TM)
      GO TO 9
C
C
    5 IF(.NOT.H3) THEN
      H3=.TRUE.
      MACH=1.
      GO TO 1
      ELSE
      H4=.TRUE.
      MACH=1.
      GO TO 1
      END IF
C
C
    9 RETURN
      END
C
C
C
C
C
C     SUBROUTINE ICS1VE (F,A,B,U1,D1,UN,DN,N,M,G,PSCOND,IER)
C
C-ICS1VE---------S--------LIBRARY 2----------------------------------------
C
C   FUNCTION            - CUBIC SPLINE ONE-DIMENSIONAL INTERPOLATION -
C                           EQUALLY SPACED DATA
C   USAGE               - CALL ICS1VE(F,A,B,U1,D1,
C                           UN,DN,N,M,G,PSCOND,IER)
C   PARAMETERS   F      - VECTOR OF N EQUALLY SPACED
C                           FUNCTIONAL VALUES F(1),F(2),...,
C                           F(N), WHERE F(I) IS THE
C                           FUNCTIONAL VALUE AT
C                           X=A+(I-1)*H AND H=(B-A)/(N-1)
C                A      - VALUE OF THE ABSCISSA FOR F(1)
C                B      - VALUE OF THE ABSCISSA FOR F(N)
C                U1     - BOUNDARY RELATED PARAMETER
C                           (SEE ELEMENT DOCUMENTATION)
C                D1     - BOUNDARY RELATED PARAMETER
C                           (SEE ELEMENT DOCUMENTATION)
C                UN     - BOUNDARY RELATED PARAMETER
C                           (SEE ELEMENT DOCUMENTATION)
C                DN     - BOUNDARY RELATED PARAMETER
C                           (SEE ELEMENT DOCUMENTATION)
C                N      - NUMBER OF DATA POINTS
C                M      - NUMBER OF ANSWERS SOUGHT
C                G      - VECTOR G(1),G(2),...,G(M) OF ANSWERS
C                           WHERE G(I) IS THE I-TH ABSCISSA
C                           UPON ENTRY AND THE I-TH
C                           ORDINATE UPON COMPLETION.
C                PSCOND - PSEUDO CONDITION NUMBER FOR BOUNDARY
C                           PARAMETERS. PSCOND.GT.1.E5
C                           INDICATES VALUES RETURNED IN G(I)
C                           MAY BE UNRELIABLE.
C                IER    - ERROR PARAMETER
C                         TERMINAL ERROR = 128+N
C                           N = 1 INDICATES G(I) NOT IN INTERVAL (A,B)
C                             FOR SOME I=1,2,...,M
C                           N = 2 INDICATES UNIQUE SOLUTION NOT POSSIBLE
C                             FOR GIVEN BOUNDARY VALUES
C   PRECISION           - SINGLE
C   REQ'D IMSL ROUTINES - UERTST
C   LANGUAGE            - FORTRAN
C--------------------------------------------------------------------------
C   LATEST REVISION     - FEBRUARY 3,1971
C
      SUBROUTINE ICS1VE  (F,A,B,U1,D1,UN,DN,N,M,G,PSCOND,IER)
C
      DIMENSION          F(1),G(1)
```

```
      EQUIVALENCE        (IDET,DET)
      REAL               LAM,LAMN
      DATA               S3,LAM/1.7320508,-.26794919/
      IER=0
      H=(B-A)/(N-1)
      LAMN=LAM**(N-1)
C                                  CALCULATE LAM AND TAU
      GAM=F(1)
      TAU=F(N)
      N1=N+1
         DO 5 I=2,N
         J=N1-I
         GAM=GAM*LAM+F(I)
         TAU=TAU*LAM+F(J)
    5    CONTINUE
      GAM=3.*GAM
      TAU=3.*TAU
      C1=1.-U1*LAM
      CT=1.-U1/LAM
      C2=CT*LAMN
      C3=D1*H/(2.*S3)+(CT*TAU-S3*((1.+6.*U1)*F(1)-U1*F(2)))/H
      CT=1.-UN/LAM
      C4=CT*LAMN
      C5=1.-UN*LAM
      C6=DN*H/(2.*S3)+(CT*GAM-S3*((1.+6.*UN)*F(N)-UN*F(N-1)))/H
      DET=C1*C5-C2*C4
      IF (IDET.EQ.0) GO TO 55
C                                  CALCULATE BOUNDARY PARAMETERS B1,BN
      B1=(C2*C6-C3*C5)/DET
      BN=(C1*C6-C4*C3)/DET
C                                  CALCULATE PSCOND
      C1=ABS(C1)
      C2=ABS(C2)
      C4=ABS(C4)
      C5=ABS(C5)
      PSCOND=AMAX1(C1,C2,C4,C5)/ABS(DET)
      KP=0
      N2=N-1
C                                  CALCULATE G(I)'S
         DO 50 I=1,M
         XI=A
         IF(G(I).LT.A) GO TO 15
            DO 10 J=1,N2
            XI=XI+H
            IF(G(I).GT.XI) GO TO 10
            K=J
            GO TO 20
   10       CONTINUE
         IF(G(I).GT.B) GO TO 15
         K=N2
         GO TO 20
   15    IER=129
         GO TO 9000
   20    X=(G(I)-(A+(K-1)*H))/H
         IF(K.EQ.KP) GO TO 45
         GAMK=F(1)
         TAUK=F(N)
         IF(K.LE.1) GO TO 30
            DO 25 J=2,K
            GAMK=GAMK*LAM+F(J)
   25       CONTINUE
   30    K1=N-K
         IF(K1.LE.1) GO TO 40
            DO 35 J=2,K1
            JJ=N1-J
            TAUK=TAUK*LAM+F(JJ)
   35       CONTINUE
   40    GAMK=3.*GAMK
         TAUK=3.*TAUK
         KP=K
   45    TX=(1.-(LAM+2.)*X+(LAM+1.)*X*X)*X
         Y=1.-X
         TY=(1.-(LAM+2.)*Y+(LAM+1.)*Y*Y)*Y
         G(I)=GAMK*TX+TAUK*TY+F(K)*Y**3+F(K+1)*X**3+B1*H*TX*LAM**(K-1)
     1      -BN*H*TY*LAM**(N-K-1)
   50    CONTINUE
      GO TO 9005
   55 IER=130
 9000 CONTINUE
      CALL UERTST(IER,'ICSIVE')
```

```
 9005 RETURN
      END
C
C
C
C
C
C     SUBROUTINE ICS2VE (F,X0,A,B,C,D,I,J,N,IF,WK,Y,IER)
C
C-ICS2VE---------S--------LIBRARY 2-------------------------------------
C
C   FUNCTION            - BICUBIC SPLINE TWO-DIMENSIONAL INTERPOLATOR -
C                           EQUALLY SPACED DATA
C   USAGE               - CALL ICS2VE(F,X0,A,B,C,D,I,J,N,IF,WK,Y,IER)
C   PARAMETERS   F      - MATRIX OF I*J FUNCTIONAL VALUES F(1,1),F(2,1),
C                           F(3,1),...,F(1,2),...,F(I,J). THE VALUE
C                           F(I1,J1) IS THE FUNCTIONAL VALUE AT X(I1),
C                           Y(J1) WHERE X(I1)=A+(I1-1)*HX,Y(J1)=C+(J1-1)
C                           *HY, HX=(B-A)/(I-1), AND HY=(D-C)/(J-1)
C                X0     - X-DIRECTION ABSCISSA FOR WHICH INTERPOLATES
C                           ARE SOUGHT
C                A      - LOWER LIMIT OF THE INTERVAL (A,B)
C                B      - UPPER LIMIT OF THE INTERVAL (A,B)
C                C      - LOWER LIMIT OF THE INTERVAL (C,D)
C                D      - UPPER LIMIT OF THE INTERVAL (C,D)
C                I      - NUMBER OF MESH POINTS IN THE FIRST COORDINANT,
C                           X, DIRECTION
C                J      - NUMBER OF MESH POINTS IN THE SECOND COORDINANT
C                           Y, DIRECTION
C                N      - NUMBER OF INTERPOLATED VALUES DESIRED
C                IF     - FIRST DIMENSION OF F IN CALLING PROGRAM
C                WK     - WORK AREA OF DIMENSION J
C                Y      - VECTOR Y(1),Y(2),...,Y(N) WHERE Y(J1) IS THE
C                           J1-TH ABSCISSA UPON ENTRY AND THE INTERPO-
C                           LATED FUNCTIONAL VALUE AT COORDINANT
C                           (X0,Y(J1)) UPON COMPLETION
C                IER    - ERROR PARAMETER
C                         TERMINAL ERROR = 128+N
C                           N = 1 INDICATES X0 NOT IN INTERVAL (A,B)
C                           N = 2 INDICATES Y(J1) NOT IN INTERVAL (C,D)
C                             FOR SOME J1=1,2,...,N
C   PRECISION           - SINGLE
C   REQ'D IMSL ROUTINES - ICS1VE,UERTST
C   LANGUAGE            - FORTRAN
C-----------------------------------------------------------------------
C   LATEST REVISION     - JANUARY 18, 1971
C
      SUBROUTINE ICS2VE  (F,X0,A,B,C,D,I,J,N,IF,WK,Y,IER)
C
      DIMENSION          F(IF,1),Y(1),WK(1)
C                                  INTERPOLATE IN Y DIRECTION AT X
         DO 5 I1=1,J
         WK(I1)=X0
         CALL ICS1VE(F(1,I1),A,B,0.,0.,0.,0.,I,1,WK(I1),PSCOND,IER)
         IF(IER.NE.0) GO TO 9000
    5    CONTINUE
      CALL ICS1VE (WK,C,D,0.,0.,0.,0.,J,N,Y,PSCOND,IER)
      IF(IER.NE.0) GO TO 10
      GO TO 9005
   10 IER=130
 9000 CONTINUE
      CALL UERTST(IER,'ICS2VE')
 9005 RETURN
      END
C
C
C
C
C
C     SUBROUTINE ICS1VU (F,X,N,M,G,H,IER)
C
C-ICS1VU---------S--------LIBRARY 2-------------------------------------
C
C   FUNCTION            - CUBIC SPLINE ONE-DIMENSIONAL INTERPOLATION -
C                           UNEQUALLY SPACED DATA
C   USAGE               - CALL ICS1VU(F,X,N,M,G,H,IER)
C   PARAMETERS   F      - VECTOR OF N UNEQUALLY SPACED
C                           FUNCTIONAL VALUES F(1),F(2),...,
C                           F(N), WHERE F(I) IS THE FUNCTIONAL
C                           VALUE AT X(I)
```

```
C                    X       - VECTOR OF N ABSCISSA X(1),X(2),...,X(N)
C                    N       - NUMBER OF DATA POINTS
C                    M       - NUMBER OF ANSWERS SOUGHT
C                    G       - VECTOR G(1),G(2),...G(M) OF ANSWERS
C                                WHERE G(I) IS THE I-TH ABSCISSA
C                                UPON ENTRY AND THE I-TH ORDINATE
C                                UPON COMPLETION.
C                    H       - WORK AREA OF DIMENSION 8*N+M
C                    IER     - ERROR PARAMETER
C                              TERMINAL ERROR = 128+N
C                                N = 1 INDICATES G(I) NOT IN INTERVAL
C                                  (X(1),X(N)) FOR SOME I=1,2,...,M
C                                N = 2 INDICATES THAT CONVERGENCE WAS
C                                  NOT OBTAINED IN 5*N ITERATIONS
C   PRECISION           - SINGLE
C   REQ'D IMSL ROUTINES - UERTST
C   LANGUAGE            - FORTRAN
C-----------------------------------------------------------------------
C   LATEST REVISION     - MARCH 5, 1971
C
      SUBROUTINE ICS1VU(F,X,N,M,G,H,IER)
C
      DIMENSION          F(1),X(1),G(1),H(1)
      DATA               EPSLN,OMEGA/1.E-6,1.0717968/
C                                  SET UP WORK AREAS
      I2=N
      I3=N+N
      I4=I3+N
      I5=I4+N
      I6=I5+N
      I7=I6+N
      I8=I7+N
      I9=I8+N
      NT=I6
      IER = 0
      N1 = N-1
C                                    DERIVATIVES,H(J6), USING CENTRAL
C                                    DIFFERENCES
      DO 5  I=1,N1
         J2=I2+I
         H(I)=X(I+1)-X(I)
         H(J2)=(F(I+1)-F(I))/H(I)
    5 CONTINUE
      DO 10  I=2,N1
         J2=I2+I
         J3=I3+I
         J4=I4+I
         J5=I5+I
         J6=I6+I
         J7=I7+I
         H(J3)=H(I-1)+H(I)
         H(J4)=.5*H(I-1)/H (J3)
         H(J5)=(H(J2)-H(J2-1))/H(J3)
         H(J6)=H(J5)+H(J5)
         H(J7)=H(J6)+H(J5)
   10 CONTINUE
      H(I6+1)=0.
      J6=I6+N
      H(J6)=0.
C                                  BEGIN ITERATION ON SECOND DERIVATIVES
      KCOUNT=0
   15 ETA=0.
      KCOUNT=KCOUNT+1
      DO 25  I=2,N1
         J4=I4+I
         J6=I6+I
         J7=I7+I
         W=(H(J7)-H(J4)*H(J6-1)-(.5-H(J4))*H(J6+1)-H(J6))*OMEGA
         IF (ABS(W).LE.ETA) GO TO 20
         ETA=ABS(W)
   20    H(J6)=H(J6)+W
   25 CONTINUE
      IF(KCOUNT.GT.NT) GO TO 75
      IF (ETA.GE.EPSLN) GO TO 15
C                                  CONVERGENCE OBTAINED
      DO 30 I=1,N1
         J6=I6+I
         J8=I8+I
         H(J8)=(H(J6+1)-H(J6))/H(I)
   30 CONTINUE
```

```
          DO 65 J=1,M
             I=1
             J9=I9+J
             IF (G(J)-X(1)) 70,60,35
   35        IF (G(J)-X(N)) 45,50,70
   40        IF (G(J)-X(I)) 55,60,45
   45        I=I+1
             GO TO 40
   50        I=N
   55        I=I-1
C                                           COMPUTE G(J)
   60        J6=I6+I
             J2=I2+I
             J8=I8+I
             HT1=G(J)-X(I)
             HT2=G(J)-X(I+1)
             PROD=HT1*HT2
             H(J9)=H(J6)+HT1*H(J8)
             DELSQS=(H(J6)+H(J6+1)+H(J9))/6.
             G(J)=F(I)+HT1*  H(J2)+PROD*DELSQS
   65 CONTINUE
      GO TO 9005
   70 IER=129
      GO TO 9000
   75 IER=130
 9000 CONTINUE
      CALL UERTST(IER,'ICS1VU')
 9005 RETURN
      END
C
C
C
C
C
C     SUBROUTINE UERTST (IER,NAME)
C
C-UERTST----------------LIBRARY 2-----------------------------------------
C
C   FUNCTION            - ERROR MESSAGE GENERATION
C   USAGE               - CALL UERTST(IER,NAME)
C   PARAMETERS   IER    - ERROR PARAMETER. TYPE + N  WHERE
C                            TYPE= 128 IMPLIES TERMINAL ERROR
C                                   64 IMPLIES WARNING WITH FIX
C                                   32 IMPLIES WARNING
C                            N    = ERROR CODE RELEVANT TO CALLING ROUTINE
C                NAME   - INPUT SCALAR (DOUBLE PRECISION ON DEC)
C                            CONTAINING THE NAME OF THE CALLING ROUTINE
C                            AS A 6-CHARACTER LITERAL STRING.
C   LANGUAGE            - FORTRAN
C-------------------------------------------------------------------------
C   LATEST REVISION     - OCTOBER 1,1975
C
      SUBROUTINE UERTST(IER,NAME)
C
      DIMENSION          ITYP(3,4),IBIT(4)
      CHARACTER*6        ITYP
      INTEGER            WARN,WARF,TERM,PRINTR
      EQUIVALENCE        (IBIT(1),WARN),(IBIT(2),WARF),(IBIT(3),TERM)
      DATA               ITYP/'WARNIN','G     ','      ',
     *                        'WARNIN','G(WITH',' FIX) ',
     *                        'TERMIN','AL    ','      ',
     *                        'NON-DE','FINED ','      '/,
     *                   IBIT/ 32,64,128,0/
      DATA               PRINTR/ 6/
      IER2=IER
      IF (IER2 .GE. WARN) GO TO 5
C                                           NON-DEFINED
      IER1=4
      GO TO 20
    5 IF (IER2 .LT. TERM) GO TO 10
C                                           TERMINAL
      IER1=3
      GO TO 20
   10 IF (IER2 .LT. WARF) GO TO 15
C                                           WARNING(WITH FIX)
      IER1=2
      GO TO 20
C                                           WARNING
   15 IER1=1
C                                           EXTRACT 'N'
```

```
   20 IER2=IER2-IBIT(IER1)
C                                          PRINT ERROR MESSAGE
      WRITE (PRINTR,25) (ITYP(I,IER1),I=1,3),NAME,IER2,IER
   25 FORMAT(' *** I M S L(UERTST) ***  ',3A6,2X,A6,2X,I2,
     1    ' (IER = ',I3,')')
      RETURN
      END
```

APPENDIX C

COMPUTER PROGRAM RAO

C.1 GENERAL DESCRIPTION

The computer program RAO calculates the geometry and aerodynamic properties of a gas turbine stage for which the aerodynamic efficiency is an optimum.

The input data of RAO is grouped under two data files:

i) As for the case of the program AINLEY (App. B) data are necessary to evaluate the interpolations encountered on loss calculations. These data are fixed and automatically fed into the program under the data file TURBO4.

ii) The variable data of the program is fed by the user. These constitute the datafile TURBO3. The content of TURBO3 is explained in detail in Section C.4.

The program RAO calculates first the efficiency, ETASRE, of the stage according to the given input trial vector, $\vec{X}_0$, The loss calculations required for the efficiency evaluation are performed with the help of the interpolation subroutine ICS2VE, making use of the data provided by TURBO4. The calculation of the sum, CONST, of the inverse of the constraint equations allows then,the evaluation of the first value of the penalty function, PENFUN.

The next step is the evaluation of the gradients of ETA and CONST. A three dimensional matrix, M, is created. Each slice of M corresponds to a two-dimensional matrix. The first slice, M_1, for example, is formed of the partial derivatives of Eq. (78) combined with Eq. (80), simply with respect to the present variables, i.e.,

$$\begin{aligned} M(1,1,1) &= \partial\eta/\partial\lambda_R \\ M(1,1,2) &= \partial\eta/\partial\lambda_{N\ \text{improved}} \\ M(1,1,3) &= \partial\eta/\ \partial T_{2s} \\ &\vdots \\ M(1,1.8) &= \partial\eta/\ \partial Re \end{aligned} \tag{C.1}$$

The remaining elements of M(1,i,j) are zero.

The second slice, M(2,i,j) is formed of the partial derivatives of λ_R, $\lambda_{N,i}$, T_{2s},...,Re, with respect to their own components. That is,

$$\begin{aligned} M(2,1,1) &= \partial\lambda_R/\partial Y_R \\ M(2,1,5) &= \partial\lambda_R/\partial T_4 \end{aligned} \tag{C.2}$$

or

$$M(2,2,2) = \partial\lambda_{N,i}/\partial Y_N$$

Finally, the dependent variables of the problem are eliminated and the partial derivative of the efficiency with respect to each one of the design variables can be obtained. This procedure can be explained as follows:

The first slice, M_1, of the M vector, is multiplied with the next slice, M_2, such that $M_{1,2}$ is obtained.

$$M_{1,2} = \left[\frac{\partial\eta}{\partial\lambda_R}, \frac{\partial\eta}{\partial\lambda_N}, \ldots, \frac{\partial\eta}{\partial Re}\right] \times \begin{vmatrix} \frac{\partial\lambda_R}{\partial Y_R} & \frac{\partial\lambda_R}{\partial Y_N} & \cdots & \frac{\partial\lambda_R}{\partial C_R} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ \frac{\partial Re}{\partial Y_R} & \frac{\partial Re}{\partial Y_N} & \cdots & \frac{\partial Re}{\partial C_R} \end{vmatrix}$$

$$M_{1,2} = \left[\frac{\partial\eta}{\partial Y_R}, \ldots, \frac{\partial\eta}{\partial C_R}\right] \tag{C.3}$$

Then $M_{1,2}$ is multiplied with the next slice, to obtain $M_{1,3}$, i.e.

$$M_{1,3} = \left[\frac{\partial\eta}{\partial Y_R}, \ldots, \frac{\partial\eta}{\partial C_R}\right] \times \begin{vmatrix} \frac{\partial Y_R}{\partial Y_{p,r}} & \cdots\cdots\cdots & \frac{\partial Y_R}{\partial C_R} \\ \frac{\partial C_R}{\partial Y_{p,r}} & \cdots\cdots\cdots & \frac{\partial C_R}{\partial C_R} \end{vmatrix}$$

$$M_{1,3} = \left[\frac{\partial\eta}{\partial Y_{p,r}}, \ldots, \frac{\partial\eta}{\partial C_R}\right] \tag{C.4}$$

such that, when the resulting vector is multiplied with the last, nth slice, one is left with:

$$M_{1,n} = \left[\frac{\partial\eta}{\partial d} \frac{\partial\eta}{\partial C_R} \frac{\partial\eta}{\partial C_N} \cdots \frac{\partial\eta}{\partial V_a}\right] \tag{C.5}$$

which is recognized as $\nabla\eta$, or DETA in the program. The multiplication of each slice of M with the following slice is performed by means of the subroutine MULTIP. The gradient of the constraint equation, DCONST, is performed in the same way. When DETA and DCONST are evaluated, the gradient, DPEN, of the penalty function is evaluated according to Eq. (108).

The next step, is the determination of the maximizing step length, R_{op}. The idea is explained in Eq. (111). For each variable, the upper bound, XUP(i) and the lower bound XLOW(i) are initially given in the datafile TURBO3. Therefore, the next value to be assigned to that variable cannot exceed these limits. Thus, replacing XUP(i) by $X_{n+1}(i)$ in Eq. (106) one gets

$$R_{op,n}(i) = \frac{XUP(i) - X(i)}{\nabla P(i)} \qquad i = 1,8 \qquad (C.6)$$

Note that XUP(i) must be replaced by XLOW(i) in Eq. (C.6) for the case where $\nabla P(i)$ is negative.

Therefore, eight values of R_{op} for which each design variable would violate its bounds is determined. The lowest value of R_{op} is chosen among those eight possible R_{op}'s and, this value is divided by 50 to find the final value of R_{op} to be used in Eq. (110). Consequently, the search for the maximum of $H(R_{op})$ can be carried out safely for 50 increments of R_{op}.

The maximum value of $H(R_{op})$ is called PENMAX in the program. Once PENMAX is determined, the corresponding values of the design vector X(i), ETASRE, pressure ratio,PRATIO, Mach number,MC3,and Reynolds number,RE,are assigned to XMAX(i),ETAMAX, PRMAX, MAKMAX and REMAX respectively. If PENMAX turns out to be the absolute maximum of the objective function, PENFUN, these maxima are printed in the output.

In the opposite case, they are changed with new values corresponding to the latest values of PENMAX.

After PENMAX is obtained, the program calculates a new DPEN, using the values of XMAX(i) and comes up with a new PENMAX. If the last value is greater, the procedure is repeated. If not, a counter, PCOUNT is increased by 1, and the procedure is repeated. When, at any point, PCOUNT is greater than 2, the procedure is stopped, for the renewal of the resequencing factor, RF. Then the second sequential maximization step starts and a new PENMAX is found corresponding to the new value of RF. RF is updated three times, and as indicated in Eq. (114), the procedure is terminated for

$$RF < 4x10^{-6} .$$

The absolute maxima of each design variable and those of ETASRE, PRATIO, MC3, RE, PENFUN and the other aerodynamic variables are printed in the output.

C.2 LIST OF THE IMPORTANT VARIABLES OF THE PROGRAM RAO

X(I)	:	Vector formed by the design variables
FPENF	:	Final value (corresponding to the optimum) of the penalty function
LO1	:	Logical control variable
LO2	:	Logical control variable
ETASRE	:	Total-to-total isentropic efficiency, η, of the stage (corrected for Reynolds number)

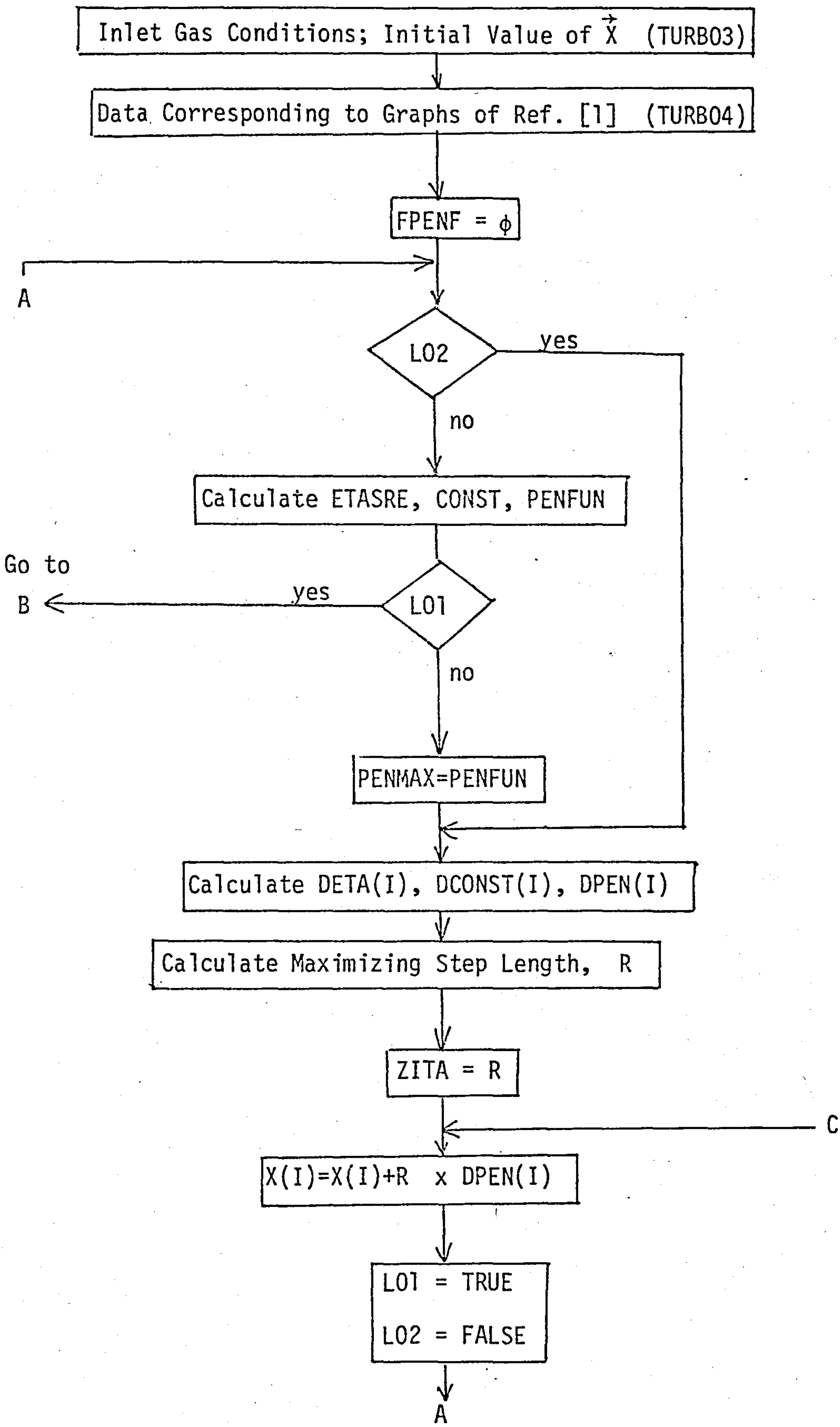

FIGURE C.1 - Flow chart of the computer program "RAO".

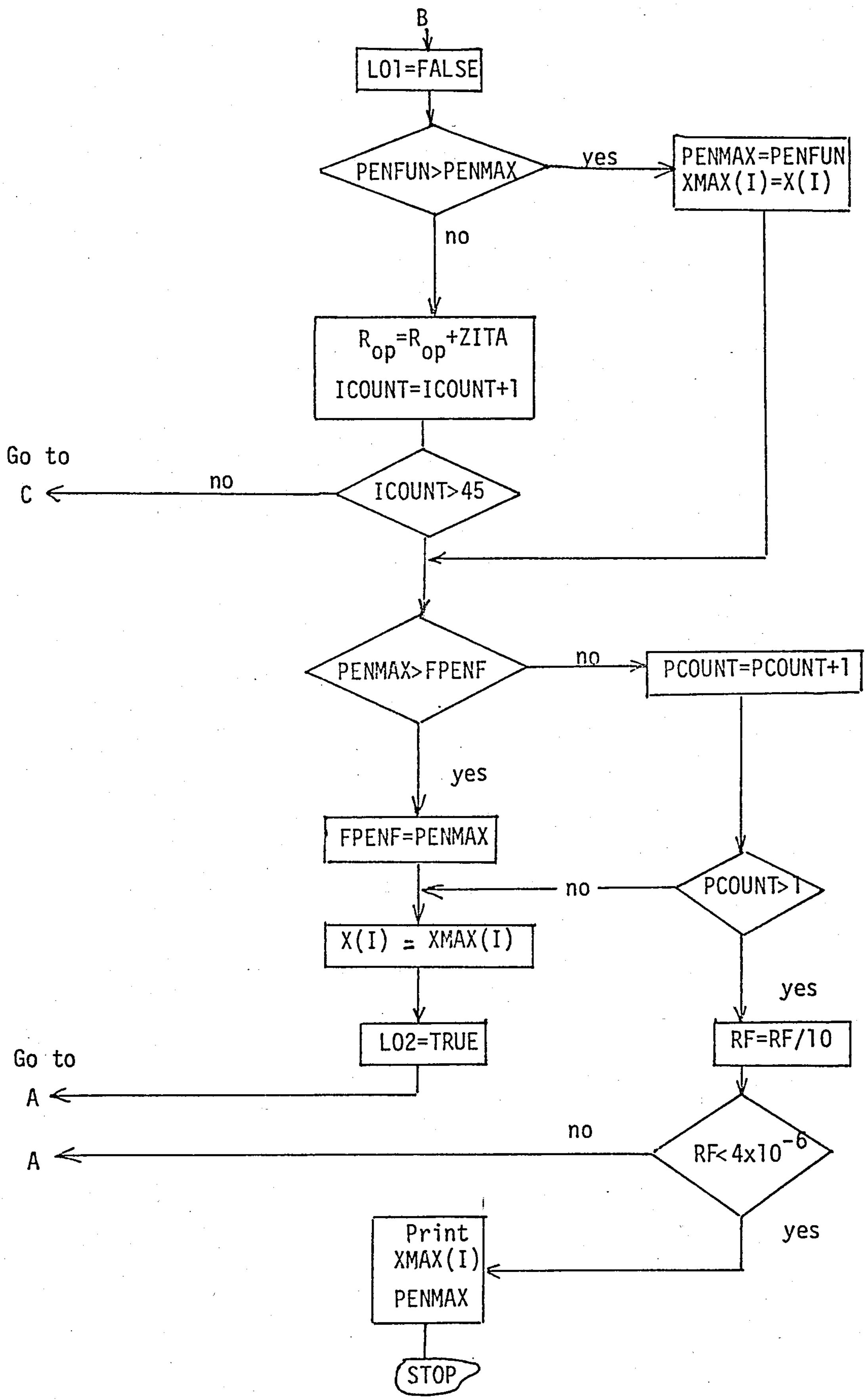

FIGURE C.1 (continued).

CONST	:	Sum of the constraint equations
PENFUN	:	Penalty function
PENMAX	:	Local maximum of the penalty function
DETA(I)	:	Gradient of the efficiency
DCONST(I)	:	Gradient of the constant equations
DPEN(I)	:	Gradient of the penalty function
R	:	Maximizing step length
ZITA	:	Increment
XMAX(I)	:	Absolute maxima of the design variables
ICOUNT	:	Counter
PCOUNT	:	Counter
RF	:	Resequencing factor

C.3 SUBROUTINE DESCRIPTION

The functional relationship between the subroutines of the program RAO is given below.

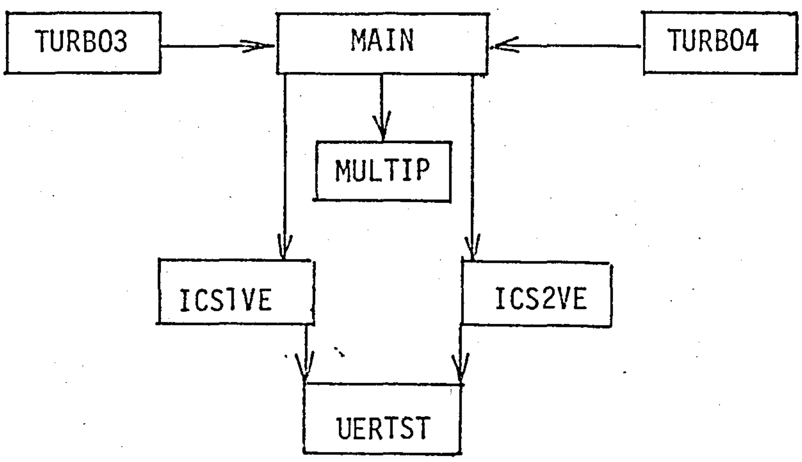

FIGURE C.2 - Functional relationship among the subroutines of the program "RAO".

The functions of the main program and of the subroutines are as follows:

MAIN : The main program reads the datafiles TURBO3 and TURBO4 and executes the job explained is Section C.1

MULTIP : The subroutine MULTIP performs the matrix multiplications. The argument list comprises the names of the two matrices to be multiplied and their dimensions. This subroutine is called by the statement,

CALL MULTIP(A,B,M,N)

The first matrix A is an 1xM matrix and the second matrix B is an MxN matrix. The resulting vector is thus an 1xN matrix and returned to the calling program under the name A.

ICS1VE, ICS2VE, UERTST: These subroutines are explained in Section B.2.1.

C.4 INPUT DESCRIPTION

The datafile TURBO4 is automatically fed to RAO. The datafile TURBO3, to be fed by the user, must be prepared as follows:

Card 1 : XI(I), I = 1,8

XI : Vector of the first trial values of the design variables, where

XI(1) : Mean diameter of the turbine (m)

XI(2) : Chord length of the rotor blades (m)

XI(3) : Chord length of the stator blades (m)

XI(4) : Pitch length of the stator blades (m)

XI(5) : Pitch length of the rotor blades (m)

XI(6) : Gas inlet angle to the rotor (rad)

XI(7) : Gas outlet angle from the rotor (rad)

XI(8) : Axial gas velocity, constant throughout the stage (m/s)

Card 2 : RPM, MDAT, P01, T01

RPM : Blade rotational speed (rev/s)

MDAT : Gas mass flow rate at inlet to the stage (kg/s)

P01 : Absolute total pressure at inlet to the stage (N/m^2)

T01 : Absolute total temperature at inlet to the stage (^oK)

Card 3 : AL1, B, CLEAR

AL1 : Inlet gas angle to the stage (deg)

B : Constant of Eq. (14)

CLEAR : Tip clearance of the rotor blades (m)

Card 4 : ILAMN, IETAS

ILAMN : Initial guess for λ_N, to be used in Eq. (51)

IETAS : Initial guess for η, to be used in Eq. (65)

Card 5 : RESFAC, MSL, EPS

RESFAC: First value of the resequencing factor of Eq. (101)

MSL : First value of the maximizing step length of Eq. (106)

EPS : Tolerance to be used in the iterative efficiency estimation.

Card 6 : CP, GAMA, RA

CP : Specific heat at constant pressure ($N.m/kg^oK$)

GAMA : Ratio of the specific heats

RA : Gas constant ($N.m/kg^oK$)

Card 7 : G

G = 0 , optimize

G $\neq$ 0 , calculate the efficiency, only.

The seven input cards must be given in free format. A sample data set for TURBO3 is given below:

Card 1 : .430, .024, .017, .015, .018, .358, .953, 270.

Card 2 : 250., 20., 400 000., 1100.

Card 3 : 0., .5, .000154

Card 4 : .0570, .80

Card 5 : $4x10^{-4}$, 0., .001

Card 6 : 1160., 1.333, 287.

Card 7 : 0.

C.5 OUTPUT DESCRIPTION

On the output of the computer program RAO, appears first the initial trial of the design vector, $\vec{X}_0$. Then, the chosen upper and lower limits of the design variables are shown and the initial gas conditions and the selected rotative speed are printed.

The results of the program, then, are given: The values of the design variables corresponding to the calculated optimum of the efficiency are printed; also, various aerodynamic parameters calculated at optimum conditions are shown, in the following order:

i) Efficiency, η, of the stage

ii) The relevant value of the penalty function, P.

iii) The inlet non-dimensional mass flow, $\dot{m}\sqrt{T_1}/P_1$.

iv) The pressure ratio, P_1'/P_1.

v) Rotor outlet Mach number, M_4

vi) Stage mean Reynolds number, Re

vii) The inlet mass flow parameter based on the optimum annulus diameter, $\dot{m}\sqrt{T_1}/P_1D^2$(DQMMAX)

viii) The overall temperature drop ratio, $\Delta T/T_1$ (DQTMAX)

ix) The speed parameter, $ND/\sqrt{T}$ (DQMAMX)

x) The work output parameter of the stage, $C_p\Delta T/N^2D^2$ (DQM2MX)

xi) The flow coefficient, ϕ (FIMAX)

xii) The temperature drop coefficient, ψ (PSIMAX)

A sample output of the program RAO is given below:

```
*** THE FIRST TRIAL OF THE DESIGN VECTOR IS **

.432
.024
.0173
.0151
.0181
.356
.953
272.

*** THE CHOSEN UPPER & LOWER BOUNDS ARE AS FOLLOWS **
.064 < X(1) < .512
.00128 < X(2) < .04096
.00064 < X(3) < .04096
.000512 < X(4) < .04096
.000256 < X(5) < .04096
.0105 < X(6) < .9948
.696 < X(7) < 1.396
50. < X(8) < 400.
```

```
*** INPUT CONDITIONS ARE AS FOLLOWS **

**  REV. PER SECOND = 250.
**  GAS TEMPERATURE = 1100.
**  GAS PRESSURE    = 400000.
**  GAS MASS FLOW   = 20.




** VALUES OF DESIGN VARIABLES CORRESPONDING TO OPTIMUM EFFICIE

** X(1)  = .2517366221649
** X(2)  = .03189277987009
** X(3)  = .02273274830175
** X(4)  = .01700489146619
** X(5)  = .009887143687339
** X(6)  = .3579559169991
** X(7)  = .9690816785905
** X(8)  = 272.0000124397

** VARIOUS OPTIMUM PARAMETERS ARE **

** EFFICIENCY = .7980291526246  **
** PENALTY FUNCTION = .7963460573962  **
** INLET NON-DIM MASS FLOW =.001658312395178 **
** PRESSURE RATIO =.663191088576  **
** ROTOR MACH NUMBER=.5540689163215  **
** REYNOLDS NUMBER= 305084.2288297 **

** DQMMAX= .02616818165689
** DQTMAX= .07781462381428
** DQMAMX= 1.89753618571
** DQWMAX= 24.85300627392
** DQM2MX= 3.95790509403
** FIMAX= 1.375728207634
** PSIMAX= 5.036272025487
```

C.6 PROGRAM LISTING OF "RAO"

```
      PROGRAM RAO(INPUT,OUTPUT,TURBO3,TAPE7=TURBO3,
     #                            TURBO4,TAPE5=TURBO4)
C
C
C
      DIMENSION M(17,25,25),G(13,11,11),H(13,9,9),O(10,10,10),
     #          SIZE1(17,2),SIZE2(13,2),SIZE3(13,2),SIZE4(10,2),
     #          DETA(30),MD2(30,30),GG(31),DCONST(50),
     #          C(31,8),D3(30),D4(30),D5(30),D6(30),D19(30),
     &          X(8),DPEN(8),XMAX(8),ABSX(8),DIFF(8),DIFFX(8),
     &          YP1(17,8),YP2(15,6),VMU(34),WK(500),
     #          XLOW(8),XUP(8),XI(8),RMAX(8),RFI(10)
C
C
      INTEGER SIZE1,SIZE2,SIZE3,SIZE4
      REAL    M,MD2,LAMN,LAMF,LAMCN,LAMCR,LAMNI,LAMRI,MDAT,MUAVG,MSL,
     #        ILAMN,IETAS,MC3,MAKMAX
      LOGICAL L01,L02,L03,L04,L05,LL1,LL2
C
C
      DATA ((SIZE1(I,J),J=1,2),I=1,17) /1,8,8,17,17,20,20,24,
     #                                  24,23,23,19,19,16,16,17,
     #                                  17,20,20,19,19,17,17,14,
     #                                  14,11,11,10,10,9,9,8,8,8/
C
      DATA ((SIZE2(I,J),J=1,2),I=1,13) /1,3,3,4,4,5,5,5,5,7,
     #                                  7,9,9,10,10,11,11,8,
     #                                  8,7,7,9,9,8,8,8/
C
      DATA ((SIZE3(I,J),J=1,2),I=1,13) /1,3,3,4,4,5,5,5,5,7,
     #                                  7,6,6,6,6,7,7,7,7,6,
     #                                  6,9,9,8,8,6/
C
      DATA ((SIZE4(I,J),J=1,2),I=1,10) /1,4,4,5,5,6,6,8,8,9,
     #                                  9,8,8,10,10,6,6,7,7,8/
C
      READ(5,*) (VMU(I),I=1,34)
      READ(5,*) (XLOW(I),I=1,8)
      READ(5,*) (XUP (I),I=1,8)
      READ(5,1) ((YP1(I,J),I=1,17),J=1,8)
    1 FORMAT(8(16F5.3,/),8F5.3)
      READ(5,2) ((YP2(I,J),I=1,15),J=1,6)
    2 FORMAT(5(16F5.3,/),10F5.3)
C
C
      READ(7,*) (XI(I),I=1,8)
      READ(7,*) RPM,MDAT,P01,T01
      READ(7,*) ALL,B,CLEAR
      READ(7,*) ILAMN,IETAS
      READ(7,*) RESFAC,MSL
      READ(7,*)  EPS
      READ(7,*) CP,GAMA,RA
      READ(7,*) CONTY
C
C
      PRINT*,' *** THE FIRST TRIAL OF THE DESIGN VECTOR IS ** '
      PRINT*,' '
      DO 3 I=1,8
    3 PRINT*,        XI(I)
      PRINT*,' '
      PRINT*,' *** THE CHOSEN UPPER & LOWER BOUNDS ARE AS FOLLOWS **'
      DO 4 I=1,8
    4 PRINT*,  XLOW(I),' < X(',I,') < ',XUP(I)
      PRINT*,' '
      PRINT*,' *** INPUT CONDITIONS ARE AS FOLLOWS **'
      PRINT*,' '
      PRINT*,' **  REV. PER SECOND = ',RPM
      PRINT*,' **  GAS TEMPERATURE = ',T01
      PRINT*,' **  GAS PRESSURE    = ',P01
      PRINT*,' **  GAS MASS FLOW   = ',MDAT
      PRINT*,' '
C
                ODMN=1.
                SMS=1.
                II=1
                RFI(1)=RESFAC
                FPENF=0.
                ICOUNT=0
                PIN=ATAN(1.)*4.
C
```

```
                LO4=.FALSE.
C
C     *** SET THE RESPONSE FACTOR FOR SEQUENTIAL MINIMIZATION ***
C
C
                 RF=RESFAC
C
C
C
    5             LO1=.FALSE.
                  LO2=.FALSE.
                  LO3=.FALSE.
                  LO5=.FALSE.
                  LL1=.FALSE.
                  LL2=.FALSE.
                  L=0
C
    7             DO 6 I=1,8
    6             X(I)=XI(I)
C
C
C     *** GIVE INITIAL SET OF VARIABLES ,X(I) ***
C
C
   10                 D=       X(1)
                      CR=      X(2)
                      CN=      X(3)
                      SS=      X(4)
                      SR=      X(5)
                      BET2=    X(6)
                      BET3=    X(7)
                      CA=      X(8)
C
C
                      LLL=0
                      TR=.2*CR
                      TN=.2*CN
C
C
C
C
C
C     **** CALCULATE EFFICIENCY WITH GIVEN SET OF VARIABLES ,X(I) ****
C
C
              RMRAT=MDAT*SQRT(T01)/P01
              ETAS=IETAS
              LAMN=ILAMN
              U=PIN*RPM*D
              FI=CA/U
              REAC=FI*(TAN(BET3)-TAN(BET2))/2.
              DTOS=U*CA*(TAN(BET2)+TAN(BET3))/CP
              PSI=2.*CP*DTOS/U**2.
              AL2=ATAN(TAN(BET2)+1./FI)
              IF(AL2.LT..768) THEN
              GO TO 2000
              END IF
              AL3=ATAN(TAN(BET3)-1./FI)
              CC2=CA/COS(AL2)
              CC3=CA/COS(AL3)
              V2=CA/COS(BET2)
              V3=CA/COS(BET3)
              CW2=CC2*SIN(AL2)
              CW3=CC3*SIN(AL3)
              T2=T01-CC2**2./(2.*CP)
C
C
C
C
   20         T2PR=T2-CC2**2.*LAMN/(2.*CP)
              P2=P01/(T01/T2PR)**(GAMA/(GAMA-1.))
              IF(P2.GT.10000000.) GO TO 2000
              PC=P01/((GAMA+1.)/2.)**(GAMA/(GAMA-1.))
              RO2=P2/(RA*T2)
              A2=MDAT/(RO2*CA)
              A2N=MDAT/(RO2*CC2)
              H2=A2*RPM/U
              CC1=CA
              T1=T01-(CC1**2./(2.*CP))
              P1=P01*(T1/T01)**(GAMA/(GAMA-1.))
```

```
      RO1=P1/(RA*T1)
      A1=MDAT/(RO1*CA)
      H1=A1*RPM/U
      TO3=TO1-DTOS
      T3=TO3-(CC3**2./(2.*CP))
      PO3=PO1*(1.-DTOS/(ETAS*TO1))**(GAMA/(GAMA-1.))
      PRATIO=PO3/PO1
      P3=PO3*(T3/TO3)**(GAMA/(GAMA-1.))
      RO3=P3/(RA*T3)
      A3=MDAT/(RO3*CA)
      H3=A3*RPM/U
      HR=(H2+H3)/2.
      HN=(H2+H1)/2.
      ALC=ATAN((H3-H1)/(2.*(CN+1.25*CR)))
C
C
      MC3=CC3/(GAMA*RA*T3)**.5
      T3DPR=T2/(P2/P3)**((GAMA-1.)/GAMA)
      LAMR=2.*CP*(T3-T3DPR)/V3**2.
      SOCS=SS/CN
      SOCR=SR/CR
      YPB20=-1.*BET3*180./PIN
      YPB2B3=YPB20
      YPA10=-1.*AL2*180./PIN
      YPA1A2=YPA10
      CALL ICS2VE(YP1,SOCR,.3,1.1,-60.,-45.,17,8,1,17,WK,YPB20,IER)
      CALL ICS2VE(YP1,SOCS,.3,1.1,-60.,-45.,17,8,1,17,WK,YPA10,IER)
      CALL ICS2VE(YP2,SOCR,.3,1.,-70.,-45.,15,6,1,15,WK,YPB2B3,IER)
      CALL ICS2VE(YP2,SOCS,.3,1.,-70.,-45.,15,6,1,15,WK,YPA1A2,IER)
      YPR=(YPB20+(BET2/BET3)**2.*(YPB2B3-YPB20))
     #      *(TR/(.2*CR))**(BET2/BET3)
      YPN=(YPA10+(AL1/AL2)**2.*(YPA1A2-YPA10))
     #      *(TN/(.2*CN))**(AL1/AL2)
      ALM=ATAN((TAN(AL2)-TAN(AL1))/2.)
      BETM=ATAN((TAN(BET3)-TAN(BET2))/2.)
      COSRS=COS(AL2)**2./COS(ALM)**3.
      COSRR=COS(BET3)**2./COS(BETM)**3.
      CLR=2.*SR*(TAN(BET3)+TAN(BET2))*COS(BETM)/CR
      CLN=2.*SS*(TAN(AL2)+TAN(AL1))*COS(ALM)/CN
      LAMCN=.0334*CN*COS(AL2)/(HN*COS(AL1))
      LAMCR=.0334*CR*COS(BET3)/(HR*COS(BET2))
      CLE=CR
      YSYKR=(LAMCR+B*CR*(CLEAR/CR)**.78/HR)
     #        *CR*CLR**2.*COSRR/SR
      YSYKN=LAMCN*CN*CLN**2.*COSRS/SS
      YN=YPN+YSYKN
      YR=YPR+YSYKR
      LAMNI=YN*T2PR/TO1
      TO3REL=T3+V3**2./(2.*CP)
      LAMRI=YR*T3DPR/TO3REL
      DENOM=(1.+(LAMRI*V3**2./(2.*CP)+T3*LAMNI*CC2**2.
     #       /(2.*T2*CP))/(TO1-TO3))**2.
      ETAS=1./SQRT(DENOM)
      BLADEN=PIN*D/SS
      BLADER=PIN*D/SR
      STMU=TO1
      ROMU=T2
      CALL ICS1VE(VMU,100.,2500.,0.,0.,0.,0.,34,1,STMU,PSCOND,IER)
      CALL ICS1VE(VMU,100.,2500.,0.,0.,0.,0.,34,1,ROMU,PSCOND,IER)
      MUAVG=(STMU+ROMU)/200000.
      RE=(RO2*CC2*CN+RO3*V3*CR)/(2.*MUAVG)
      REFAC=(2.*100000./RE)**.2
      ETASRE=1.-(1.-ETAS)*REFAC
      DOM=MDAT*SQRT(TO1)/(PO1*D**2.)
      DQT=DTOS/TO1
      DOMA=RPM*D/SQRT(TO1)
      DQU=CP*DTOS/(RPM*D)**2.
      DOM2=MDAT*RA*TO1/(PO1*RPM*D**3.)
C
C
      DIFFET=ABS(ETASRE-ETAS)
            IF(DIFFET.GT.EPS) THEN
              ETAS=ETASRE
              LAMN=LAMNI
              LLL=LLL+1
              IF(LLL.GT.5) GO TO 37
              GO TO 20
            END IF
C
C
```

```
C     **** CALCULATE SUM OF INVERSE CONSTRAINT EQUATIONS ****
C
C
   37         IF(CONTY.NE.0.) GO TO 4444
              CONST=0.
              GG(1)=1./(50./U-1.)
              GG(2)=1./(U/400.-1.)
              GG(3)=1./(HR/(8.*D)-CR/D)
              GG(4)=1./(CR/D-HR/(2.*D))
              GG(5)=1./(HN/(10.*D)-CN/D)
              GG(6)=1./(CN/D-HN/(2.*D))
              GG(7)=1./(.5-SS/CN)
              GG(8)=1./(SS/CN-1.)
              GG(9)=1./(.3-SR/CR)
              GG(10)=1./(SR/CR-1.)
              GG(11)=1./(.01-BET2)
              GG(12)=1./(BET2-1.)
              GG(13)=1./(40./57.-BET3)
              GG(14)=1./(BET3-1.4)
              GG(15)=1./(50./CA-1.)
              GG(16)=1./(CA/400.-1.)
              GG(17)=1./(P01/P2-P01/PC)
              GG(18)=1./(MC3-1.)
              GG(19)=1./(12.*ALC/PIN-1.5)
              GG(20)=1./(.25-FI)
              GG(21)=1./(FI-2.)
              GG(22)=1./(.5-PSI)
              GG(23)=1./(PSI-6.)
              GG(24)=1./(.3-REAC)
              GG(25)=1./(REAC-.7)
              GG(26)=1./(40./57.-AL2)
              GG(27)=-1./D
              GG(28)=-1./CR
              GG(29)=-1./CN
              GG(30)=-1./SS
              GG(31)=-1./SR
C
                DO 51  I=1,31
   51           CONST=CONST+GG(I)
C
C
          IF(CONST.GT.0.) THEN
          GO TO 1140
          END IF
C
C            **** CALCULATE PENALTY FUNCTION ****
C
C
            PENFUN=ETASKE+RF*CONST
C
C
            IF(PENFUN.LT.0) GO TO 1140
C
C
          IF(LO1)  GO TO 5000
C
C
C
            PENMAX=PENFUN
 4444       ETAMAX=ETASKE
            CONMAX=CONST
            FIMAX=FI
            PSIMAX=PSI
            REMAX=RE
            PRMAX=PRATIO
            MAKMAX=MC3

            PSIMAX=PSI
            FIMAX=FI
            DOMMAX=DOM
            DOTMAX=DOT
            DOMAMX=DOMA
            DONMAX=DON
            DOM2MX=DOM2
            DO 53 I=1,6
   53       XMAX(I)=X(I)
            IF(CONTY.NE.0.) GO TO 4445
C      *** CALCULATE DERIVATIVE OF PENALTY FUNCTION ***
C
C
```

```
52   DENUM=(1.+(LAMRI*V3**2./(2.*CP)+T3*LAMNI*CC2**2.
   #         /(2.*T2*CP))/(TO1-TO3))**2.
     CLE=CR
     REFAC=(2.*100000./RE)**.2
     COSRR=(COS(BET3))**2./(COS(BETM))**3.
     COSRS=COS(AL2)**2./COS(ALM)**3.
     M(1,1,1)=(-1.)*REFAC*V3**2./(2.*CP*DENOM*(TO1-TO3))
     M(1,1,2)=(-1.)*REFAC*T3*CC2**2./(T2*2.*CP*DENOM*(TO1-TO3))
     M(1,1,3)=REFAC*T3*CC2**2.*LAMNI/(T2**2.*2.*CP*DENOM
   #          *(TO1-TO3))
     M(1,1,4)=(-1.)*REFAC*CC2**2.*LAMNI/(T2*2.*CP*DENOM*(TO1-TO3))
     M(1,1,5)=(-1.)*REFAC*(LAMRI*V3**2./(2.*CP)+LAMNI*T3*CC2**2.
   #          /(T2*2.*CP))/((TO1-TO3)**2.*DENOM)
     M(1,1,6)=(-1.)*REFAC*T3*CC2*2.*LAMNI/(T2*2.*CP*DENOM*(TO1-TO3))
     M(1,1,7)=(-1.)*REFAC*V3*2.*LAMRI/(2.*CP*DENOM*(TO1-TO3))
     M(1,1,8)=.2*(200000.)**.2*(1.-1./SQRT(DENOM))/FL**1.2
       M(2,1,1)=T3DPR/TO3REL
       M(2,1,5)=(-1.)*YR*T3DPR/TO3REL**2.
       M(2,1,6)=YR/TO3REL
       M(2,2,2)=T2PR/TO1
       M(2,2,7)=YN/TO1
       M(2,3,9)=(-1.)*CC2/CP
       M(2,4,4)=1.
       M(2,4,10)=(-1.)*CC3/CP
       M(2,5,3)=-1.
       M(2,5,4)=1.
       M(2,6,6)=1./COS(AL2)
       M(2,6,9)=1.
       M(2,6,12)=CA*SIN(AL2)/COS(AL2)**2.
       M(2,7,8)=1./COS(BET3)
       M(2,7,13)=CA*SIN(BET3)/COS(BET3)**2.
       M(2,7,11)=1.
       M(2,8,9)=RO2*CN/(2.*MUAVG)
       M(2,8,11)=RO3*CR/(2.*MUAVG)
       M(2,8,14)=CC2*CN/(2.*MUAVG)
       M(2,8,15)=V3*CR/(2.*MUAVG)
       M(2,8,16)=RO2*CC2/(2.*MUAVG)
       M(2,8,17)=RO3*V3/(2.*MUAVG)
     M(3,1,1)=1.
     M(3,1,2)=1.
     M(3,2,3)=1.
     M(3,2,4)=1.
     M(3,3,5)=CA*(TAN(BET2)+TAN(BET3))/CP
     M(3,3,6)=U*(TAN(BET2)+TAN(BET3))/CP
     M(3,3,7)=U*CA/(CP*(COS(BET2))**2.)
     M(3,3,8)=U*CA/(CP*(COS(BET3))**2.)
     M(3,3,9)=1.
     M(3,4,9)=-1.
     M(3,5,10)=1.
     M(3,5,11)=V3/CP
     M(3,6,12)=(P3/P2)**((GAMA-1.)/GAMA)
     M(3,6,13)=(GAMA-1.)*T2/(GAMA*(P2**((GAMA-1.)/GAMA))*(P3**(1./GAMA
   #          )))
     M(3,6,14)=(1.-GAMA)*T2*(P3**((GAMA-1.)/GAMA))*(P2**((1.-2.*GAMA)/
   #          GAMA))/GAMA
     M(3,7,12)=1.
     M(3,7,15)=-1.*LAMN*CC2/CP
     M(3,8,6)=1.
     M(3,9,15)=1.
     M(3,9,6)=1./COS(AL2)
     M(3,9,16)=CA*SIN(AL2)/(COS(AL2)**2.)
     M(3,10,6)=1./(COS(AL3))
     M(3,10,17)=CA*SIN(AL3)/(COS(AL3))**2.
     M(3,11,6)=1./COS(BET3)
     M(3,11,11)=1.
     M(3,11,8)=CA*SIN(BET3)/(COS(BET3))**2.
     M(3,12,7)=1./((1.+(TAN(BET2)+1./FI)**2.)*COS(BET2)**2.)
     M(3,12,18)=-1./((1.+(TAN(BET2)+1./FI)**2.)*FI**2.)
     M(3,12,16)=1.
     M(3,13,8)=1.
     M(3,14,14)=1./(RA*T2)
     M(3,14,12)=-1.*P2/(RA*T2**2.)
     M(3,15,10)=-1.*P3/(RA*T3**2.)
     M(3,15,13)=1./(RA*T3)
     M(3,16,19)=1.
     M(3,17,20)=1.
      M(4,1,1)=2.*BET2*(YPB2B3-YPB20)/BET3**2.
      M(4,1,2)=-2.*BET2**2.*(YPB2B3-YPB20)/BET3**3.
      M(4,2,2)=(LAMCR+B*CR*(CLEAR/CR)**.78/HR)*CLR**2.*CR*(-2.)
   #  *COS(BET3)*SIN(BET3)/(SR*COS(BETM)**3.)
```

```
      M(4,2,3)=(LAMCR*CLR**2.*(COS(BET3))**2./(SR*(COS(BETM))**3.))+
     # 1.22*B*CLEAR**.78*CLR**2.*CR**.22*(COS(BET3))**2./(HR*SR
     # *COS(BETM)**3.)
      M(4,2,4)=CLR**2.*(COS(BET3))**2.*CR/(SR*(COS(BETM))**3.)
      M(4,2,5)=-1.*B*(CLEAR/CR)**.78*CLR**2.*(COS(BET3))**2.*CR/
     # (HR**2.*SR*(COS(BETM))**3.)
      M(4,2,6)=(LAMCR+B*CR*(CLEAR/CR)**.78/HR)*2.*CLR*CR*COSRR/SR
      M(4,2,7)=(LAMCR+B*CR*(CLEAR/CR)**.78/HR)*(-1.)*(CLR/SR)**2.
     # *CR*COSRR
      M(4,2,8)=(LAMCR+B*CR*(CLEAR/CR)**.78/HR)*CLR**2.*CR*COSRR
     # *SIN(BETM)*3./(SR*COS(BETM))
      M(4,3,9)=-2.*AL1**2.*(YPA1A2-YPA10)/AL2**3.
      M(4,4,10)=CLN**2.*CN*COSRS/SS
      M(4,4,11)=2.*LAMCN*CLN*CN*COSRS/SS
      M(4,4,12)=LAMCN*CLN**2.*COSRS/SS
      M(4,4,13)=-1.*LAMCN*CLN**2.*COSRS*CN/SS**2.
      M(4,4,9)=-2.*LAMCN*CLN**2.*COS(AL2)*CN*SIN(AL2)/(SS*(COS(ALM))**
     #3.)
      M(4,4,14)=3.*LAMCN*CLN**2.*CN*COSRS*SIN(ALM)/(SS*COS(ALM))
      M(4,5,17)=1.
      M(4,5,15)=PIN*RPM
      M(4,6,16)=1.
      M(4,7,1)=1.
      M(4,8,2)=1.
      M(4,9,17)=M(3,3,5)
      M(4,9,16)=M(3,3,6)
      M(4,9,1)=M(3,3,7)
      M(4,9,2)=M(3,3,8)
      M(4,10,18)=1.
      M(4,10,19)=M(2,4,16)
      M(4,10,22)=1.
      M(4,11,16)=M(2,7,8)
      M(4,11,2)=M(2,7,13)
      M(4,12,26)=M(2,3,9)
      M(4,13,21)=(T3/T03)**(GAMA/(GAMA-1.))
C
      M(4,13,22)=(GAMA/(GAMA-1.))*P03*T3**(1./(GAMA-1.))/T03**(GAMA/
     # (GAMA-1.))
      M(4,13,18)=(GAMA/(1.-GAMA))*P03*T3**(GAMA/(GAMA-1.))*T03**((2.*
     # GAMA-1.)/(1.-GAMA))
      M(4,14,23)=(GAMA/(GAMA-1.))*P01*T2PR**(1./(GAMA-1.))*T01**(GAMA/
     # (1.-GAMA))
      M(4,15,20)=1.
      M(4,15,16)=M(2,6,9)
      M(4,15,9)=M(2,6,12)
      M(4,16,1)=M(3,12,7)
      M(4,16,24)=M(3,12,18)
      M(4,16,9)=1.
      M(4,17,2)=1./((1.+(TAN(BET3)-1./FI)**2.)*COS(BET3)**2.)
      M(4,17,24)=1./((1.+(TAN(BET3)-1./FI)**2.)*FI**2.)
      M(4,18,16)=1./U
      M(4,18,17)=-1.*CA/U**2.
      M(4,18,24)=1.
      M(4,19,12)=1.
      M(4,20,3)=1.
      M(5,1,1)=1.
      M(5,2,2)=1.
      M(5,3,3)=1.
      M(5,4,3)=.0334*COS(BET3)/(HR*COS(BET2))
      M(5,4,4)=-.0334*COS(BET3)*CR/(HR**2.*COS(BET2))
      M(5,4,2)=-.0334*SIN(BET3)*CR*COS(BET3)/(HR*COS(BET2))
      M(5,4,1)=.0334*CR*SIN(BET2)*COS(BET3)/(HR*(COS(BET2)**2.))
      M(5,5,8)=.5
      M(5,5,9)=.5
      M(5,6,1)=2.*SR*COS(BETM)/(CR*COS(BET2)**2.)
      M(5,6,2)=2.*SR*COS(BETM)/(CR*COS(BET3)**2.)
      M(5,6,3)=-2.*SR*(TAN(BET2)+TAN(BET3))*COS(BETM)/CR**2.
      M(5,6,10)=2.*(TAN(BET2)+TAN(BET3))*COS(BETM)/CR
      M(5,6,11)=-2.*(TAN(BET2)+TAN(BET3))*SR*SIN(BETM)/CR
      M(5,7,10)=1.
      M(5,8,1)=-2./((4.+(TAN(BET2)-TAN(BET3))**2.)*COS(BET2)**2.)
      M(5,8,2)=2./((4.+(TAN(BET3)-TAN(BET2))**2.)*COS(BET3)**2.)
      M(5,8,11)=1.
      M(5,9,6)=1.
      M(5,9,1)=M(3,12,7)
      M(5,9,12)=M(3,12,18)
      M(5,10,5)=.0334*COS(AL2)/(HN*COS(AL1))
      M(5,10,6)=-.0334*SIN(AL2)*CN /(HN*COS(AL1))
      M(5,10,7)=-.0334*COS(AL2)*CN /(HN**2.*COS(AL1))
      M(5,11,13)=2.*(TAN(AL1)+TAN(AL2))*COS(ALM)/CN
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      M(5,11,14)=-2.*(TAN(AL1)+TAN(AL2))*SS*SIN(ALM)/CN
      M(5,11,5)=-2.*SS*(TAN(AL1)+TAN(AL2))*COS(ALM)/CN**2.
      M(5,11,6)=2.*SS*COS(ALM)/(CN*COS(AL2)**2.)
      M(5,12,5)=1.
      M(5,13,13)=1.
      M(5,14,14)=1.
      M(5,14,6)=2./((4.+(TAN(AL2)-TAN(AL1))**2.)*COS(AL2)**2.)
      M(5,15,15)=1.
      M(5,16,16)=1.
      M(5,17,23)=1.
      M(5,17,15)=PIN*RPM
      M(5,18,17)=-1.
      M(5,18,19)=1.
      M(5,19,16)=M(3,10,6)
      M(5,19,18)=M(3,10,17)
      M(5,19,20)=1.
      M(5,20,16)=M(3,9,6)
      M(5,20,6)=M(3,9,16)
      M(5,20,22)=1.
      M(5,21,17)=P01*GAMA*(1.-DTOS/(ETAS*T01))**(1./(GAMA-1.))/((1.-GAM
     *A)*ETAS*T01)
      M(5,22,19)=1.
      M(5,22,20)=-1.*CC3/CP
      M(5,23,21)=1.
      M(5,23,22)=LAMN*CC2*(-1.)/CP
      M(5,24,12)=1.
      M(5,24,16)=1./U
      M(5,24,23)=M(4,18,17)
         M(6,1,1)=1.
         M(6,2,2)=1.
         M(6,3,3)=1.
         M(6,4,4)=.5
         M(6,4,5)=.5
         M(6,5,6)=1.
         M(6,6,12)=1.
         M(6,6,1)=M(3,12,7)
         M(6,6,7)=M(3,12,18)
         M(6,7,8)=.5
         M(6,7,4)=.5
         M(6,8,9)=RPM/U
         M(6,8,10)=-1.*RPM*A2/U**2.
         M(6,8,4)=1.
         M(6,9,10)=-1.*RPM*A3/U**2.
         M(6,9,11)=RPM/U
         M(6,9,5)=1.
         M(6,10,13)=1.
         M(6,11,1)=M(5,8,1)
         M(6,11,2)=M(5,8,2)
         M(6,12,14)=1./U
         M(6,12,10)=M(4,18,17)
         M(6,12,7)=1.
         M(6,13,15)=1.
         M(6,14,12)=M(5,14,6)
         M(6,15,16)=1.
         M(6,16,14)=1.
         M(6,17,17)=1.
         M(6,17,10)=M(3,3,5)
         M(6,17,14)=M(3,3,6)
         M(6,17,1)=M(3,3,7)
         M(6,17,2)=M(3,3,8)
         M(6,18,2)=M(4,17,2)
         M(6,18,7)=M(4,17,24)
         M(6,18,18)=1.
         M(6,19,17)=-1.
         M(6,20,14)=M(3,10,6)
         M(6,20,18)=M(3,10,17)
         M(6,21,19)=M(2,3,9)
         M(6,22,14)=M(3,9,6)
         M(6,22,12)=M(3,9,16)
         M(6,22,19)=1.
         M(6,23,16)=M(5,17,15)
         M(6,23,10)=1.
       M(7,1,1)=1.
       M(7,2,12)=1.
       M(7,3,3)=1.
       M(7,4,4)=M(6,8,9)
       M(7,4,5)=M(6,8,10)
       M(7,5,6)=M(6,9,11)
       M(7,5,5)=M(6,9,10)
       M(7,6,7)=1.
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M(7,7,8)=1./U
M(7,7,5)=M(4,18,17)
M(7,7,12)=1.
M(7,8,9)=RPM/U
M(7,8,5)=-1.*RPM*A1/U**2.
M(7,9,10)=-1.*MDAT/(RO2**2.*CA)
M(7,9,8)=-1.*MDAT/(RO2*CA**2.)
M(7,9,4)=1.
M(7,10,5)=1.
M(7,10,15)=PIN*RPM
M(7,11,6)=1.
M(7,11,8)=-1.*MDAT/(RO3*CA**2.)
M(7,11,11)=-1.*MDAT/(RO3**2.*CA)
M(7,12,1)=M(3,12,7)
M(7,12,12)=M(3,12,13)
M(7,12,16)=1.
M(7,13,13)=1.
M(7,14,8)=1.
M(7,15,14)=1.
M(7,16,15)=1.
M(7,17,5)=M(3,3,5)
M(7,17,8)=M(3,3,6)
M(7,17,1)=M(3,3,7)
M(7,17,2)=M(3,3,8)
M(7,18,2)=M(4,17,2)
M(7,18,12)=M(4,17,24)
M(7,19,8)=M(3,9,6)
M(7,19,16)=M(3,9,16)
   M(8,1,1)=1.
   M(8,2,2)=1.
   M(8,3,3)=1.
   M(8,4,4)=M(7,9,10)
   M(8,4,5)=M(7,9,8)
   M(8,5,6)=PIN*RPM
   M(8,5,14)=1.
   M(8,6,7)=M(7,11,11)
   M(8,6,5)=M(7,11,8)
   M(8,7,8)=1.
   M(8,8,5)=1.
   M(8,9,9)=-1.*MDAT/(CA*RO1**2.)
   M(8,9,5)=-1.*MDAT/(RO1*CA**2.)
   M(8,10,10)=1./(RA*T2)
   M(8,10,11)=-1.*P2/(RA*T2**2.)
   M(8,10,4)=1.
   M(8,11,12)=1./(RA*T3)
   M(8,11,13)=-1.*P3/(RA*T3**2.)
   M(8,11,7)=1.
   M(8,12,5)=1./U
   M(8,12,14)=M(4,18,17)
   M(8,12,17)=1.
   M(8,13,15)=1.
   M(8,14,16)=1.
   M(8,15,6)=1.
   M(8,16,1)=M(3,12,7)
   M(8,16,17)=M(3,12,13)
M(9,1,1)=1.
M(9,2,2)=1.
M(9,3,3)=1.
M(9,4,4)=M(8,10,10)
M(9,4,5)=M(8,10,11)
M(9,5,6)=1.
M(9,6,7)=1.
M(9,7,8)=M(8,11,12)
M(9,7,9)=M(8,11,13)
M(9,8,10)=1.
M(9,9,11)=1./(RA*T1)
M(9,9,12)=-1.*P1/(RA*T1**2.)
M(9,10,13)=M(4,14,23)
M(9,10,4)=1.
M(9,11,14)=-1.*CC2/CP
M(9,11,5)=1.
M(9,12,15)=M(4,13,21)
M(9,12,9)=M(4,13,22)
M(9,12,16)=M(4,13,18)
M(9,12,8)=1.
M(9,13,9)=1.
M(9,13,17)=-1.*CC3/CP
M(9,13,16)=1.
M(9,14,7)=PIN*RPM
M(9,14,20)=1.
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      M(9,15,18)=1.
      M(9,16,19)=1.
      M(9,17,6)=1./U
      M(9,17,20)=M(4,18,17)
        M(10,1,1)=1.
        M(10,2,2)=1.
        M(10,3,3)=1.
        M(10,4,4)=M(4,14,23)
        M(10,5,5)=M(9,11,14)
        M(10,5,14)=1.
        M(10,6,6)=1.
        M(10,7,7)=1.
        M(10,8,8)=M(4,13,21)
        M(10,8,9)=M(4,13,22)
        M(10,8,10)=M(4,13,18)
        M(10,9,9)=1.
        M(10,9,10)=1.
        M(10,9,11)=M(9,13,17)
        M(10,10,12)=1.
        M(10,11,13)=(GAMA/(GAMA-1.))*P01*T01**(GAMA/(1.-GAMA))*T1**(1.
     #              /(GAMA-1.))
        M(10,12,6)=-1.*CA/CP
        M(10,12,13)=1.
        M(10,13,14)=1.
        M(10,13,4)=1.
        M(10,13,5)=M(5,23,22)
        M(10,14,6)=M(3,9,6)
        M(10,14,15)=M(3,9,16)
        M(10,14,5)=1.
        M(10,15,16)=M(5,21,17)
        M(10,15,8)=1.
        M(10,16,10)=1.
        M(10,16,16)=-1.
        M(10,17,6)=M(3,10,6)
        M(10,17,17)=M(3,10,17)
        M(10,17,11)=1.
        M(10,18,18)=1.
        M(10,19,19)=1.
        M(10,20,7)=PIN*RPM
      M(11,1,1)=1.
      M(11,2,2)=1.
      M(11,3,3)=1.
      M(11,4,4)=1.
      M(11,4,5)=M(5,23,22)
      M(11,5,5)=1.
      M(11,5,6)=M(3,9,6)
      M(11,5,7)=M(3,9,16)
      M(11,6,6)=1.
      M(11,7,8)=1.
      M(11,8,9)=M(5,21,17)
      M(11,9,10)=1.
      M(11,9,11)=M(5,22,20)
      M(11,10,10)=1.
      M(11,10,9)=-1.
      M(11,11,6)=M(3,10,6)
      M(11,11,12)=M(3,10,17)
      M(11,11,11)=1.
      M(11,12,13)=1.
      M(11,13,6)=M(10,12,6)
      M(11,14,5)=M(9,11,14)
      M(11,14,4)=1.
      M(11,15,1)=M(3,12,7)
      M(11,15,14)=M(3,12,18)
      M(11,15,7)=1.
      M(11,16,15)=M(3,3,5)
      M(11,16,6)=M(3,3,6)
      M(11,16,1)=M(3,3,7)
      M(11,16,2)=M(3,3,8)
      M(11,16,9)=1.
      M(11,17,2)=M(4,17,2)
      M(11,17,12)=1.
      M(11,17,14)=M(4,17,24)
      M(11,18,16)=1.
      M(11,19,17)=1.
       M(12,1,1)=1.
       M(12,2,2)=1.
       M(12,3,3)=1.
       M(12,4,4)=M(9,11,14)
       M(12,5,4)=1.
       M(12,5,5)=M(3,9,6)
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      M(12,5,6)=M(3,9,16)
      M(12,6,5)=1.
      M(12,7,6)=1.
      M(12,7,1)=M(3,12,7)
      M(12,7,7)=M(3,12,18)
      M(12,8,8)=1.
      M(12,9,9)=M(3,3,5)
      M(12,9,5)=M(3,3,6)
      M(12,9,1)=M(3,3,7)
      M(12,9,2)=M(3,3,8)
      M(12,9,10)=1.
      M(12,10,10)=-1.
      M(12,11,5)=M(3,10,6)
      M(12,11,11)=M(3,10,17)
      M(12,12,11)=1.
      M(12,12,2)=M(4,17,2)
      M(12,12,7)=M(4,17,24)
      M(12,13,12)=1.
      M(12,14,7)=1.
      M(12,14,5)=1./U
      M(12,14,9)=M(4,18,17)
      M(12,15,9)=1.
      M(12,15,8)=M(10,20,7)
      M(12,16,13)=1.
      M(12,17,14)=1.
    M(13,1,1)=1.
    M(13,2,2)=1.
    M(13,3,3)=1.
    M(13,4,4)=M(3,9,6)
    M(13,4,5)=M(3,9,16)
    M(13,5,4)=1.
    M(13,6,5)=1.
    M(13,6,1)=M(3,12,7)
    M(13,6,6)=M(3,12,18)
    M(13,7,6)=1.
    M(13,7,4)=1./U
    M(13,7,7)=M(4,18,17)
    M(13,8,8)=1.
    M(13,9,7)=1.
    M(13,9,8)=M(10,20,7)
    M(13,10,7)=M(3,3,5)
    M(13,10,4)=M(3,3,6)
    M(13,10,1)=M(3,3,7)
    M(13,10,2)=M(3,3,8)
    M(13,11,2)=M(4,17,2)
    M(13,11,6)=M(4,17,24)
    M(13,12,9)=1.
    M(13,13,10)=1.
    M(13,14,11)=1.
      M(14,1,1)=1.
      M(14,2,2)=1.
      M(14,3,3)=1.
      M(14,4,4)=1.
      M(14,5,1)=M(3,12,7)
      M(14,5,5)=M(3,12,18)
      M(14,6,5)=1.
      M(14,6,4)=1./U
      M(14,6,6)=M(4,18,17)
      M(14,7,6)=1.
      M(14,7,7)=M(10,20,7)
      M(14,8,7)=1.
      M(14,9,8)=1.
      M(14,10,9)=1.
      M(14,11,10)=1.
    M(15,1,1)=1.
    M(15,2,2)=1.
    M(15,3,3)=1.
    M(15,4,4)=1.
    M(15,5,4)=1./U
    M(15,5,5)=M(4,18,17)
     (15,6,5)=1.
    M(15,6,6)=M(10,20,7)
    M(15,7,6)=1.
    M(15,8,7)=1.
    M(15,9,8)=1.
    M(15,10,9)=1.
      M(16,1,1)=1.
      M(16,2,2)=1.
      M(16,3,3)=1.
      M(16,4,4)=1.
```

```
                 M(16,5,5)=M(10,20,7)
                 M(16,6,5)=1.
                 M(16,7,6)=1.
                 M(16,8,7)=1.
                 M(16,9,8)=1.
                M(17,1,6)=1.
                M(17,2,7)=1.
                M(17,3,2)=1.
                M(17,4,8)=1.
                M(17,5,1)=1.
                M(17,6,3)=1.
                M(17,7,5)=1.
                M(17,8,4)=1.
C
C
C                    ***** OBTAIN VECTOR  DETA(I) ********
C
C
C
                      DO 91 J=1,SIZE1(1,2)
       91             DETA(J)=M(1,1,J)
C
                      DO  93   K=2,17
                      DO  92  KK=1,SIZE1(K,1)
                      DO  92 KKK=1,SIZE1(K,2)
       92             MD2(KK,KKK)=M(K,KK,KKK)
C
C
C
C
                      CALL MULTIP(DETA,MD2,SIZE1(K,1),SIZE1(K,2))
C
C
C
C
C
       93             CONTINUE
C
C
C
                G(2,1,1)=1.
                G(2,2,2)=1.
                G(2,3,3)=.5
                G(2,3,4)=.5
                   G(3,1,1)=1.
                   G(3,2,2)=1.
                   G(3,3,3)=M(6,8,9)
                   G(3,3,4)=M(6,8,10)
                   G(3,4,5)=M(6,9,11)
                   G(3,4,4)=M(6,9,10)
                G(4,1,1)=1.
                G(4,2,2)=1.
                G(4,3,3)=M(7.9,10)
                G(4,3,4)=M(7,9,8)
                G(4,4,1)=M(7,10,15)
                G(4,5,5)=M(7,11,11)
                G(4,5,4)=M(7,11,8)
                   G(5,1,1)=1.
                   G(5,2,2)=1.
                   G(5,3,4)=M(8,10,10)
                   G(5,3,5)=M(8,10,11)
                   G(5,4,3)=1.
                   G(5,5,6)=M(8,11,12)
                   G(5,5,7)=M(8,11,13)
                G(6,1,1)=1.
                G(6,2,2)=1.
                G(6,3,3)=1.
                G(6,4,4)=M(9,10,13)
                G(6,5,5)=M(9,11,14)
                G(6,6,6)=M(4,13,18)
                G(6,6,7)=M(4,13,21)
                G(6,6,8)=M(4,13,22)
                G(6,7,6)=1.
                G(6,7,9)=M(9,13,17)
                G(6,7,8)=1.
                   G(7,1,1)=1.
                   G(7,2,2)=1.
                   G(7,3,3)=1.
                   G(7,4,4)=M(5,23,22)
                   G(7,4,5)=1.
```

```
      G(7,5,3)=H(3,9,6)
      G(7,5,6)=H(3,9,16)
      G(7,5,4)=1.
      G(7,6,7)=-1.
      G(7,6,8)=1.
      G(7,7,7)=H(5,21,17)
      G(7,8,8)=1.
      G(7,8,9)=H(9,13,17)
      G(7,9,3)=H(3,10,6)
      G(7,9,10)=H(3,10,17)
      G(7,9,9)=1.
    G(8,1,1)=1.
    G(8,2,2)=1.
    G(8,3,3)=1.
    G(8,4,3)=H(3,9,6)
    G(8,4,5)=H(3,9,16)
    G(8,4,4)=1.
    G(8,5,4)=H(9,11,14)
    G(8,6,5)=1.
    G(8,6,6)=H(3,12,7)
    G(8,6,7)=H(3,12,18)
    G(8,7,8)=H(3,3,5)
    G(8,7,3)=H(3,3,6)
    G(8,7,6)=H(3,3,7)
    G(8,7,9)=H(3,3,8)
    G(8,7,10)=1.
    G(8,8,10)=-1.
    G(8,9,3)=H(3,10,6)
    G(8,9,11)=H(3,10,17)
    G(8,10,7)=H(4,17,24)
    G(8,10,9)=H(4,17,2)
    G(8,10,11)=1.
      G(9,1,1)=1.
      G(9,2,2)=1.
      G(9,3,3)=1.
      G(9,4,3)=H(3,9,6)
      G(9,4,6)=H(3,9,16)
      G(9,5,6)=1.
      G(9,5,4)=H(3,12,7)
      G(9,5,7)=H(3,12,18)
      G(9,6,4)=1.
      G(9,7,7)=1.
      G(9,7,3)=1./U
      G(9,7,8)=H(4,18,17)
      G(9,9,5)=1.
      G(9,10,8)=H(3,3,5)
      G(9,10,3)=H(3,3,6)
      G(9,10,4)=H(3,3,7)
      G(9,10,5)=H(3,3,8)
      G(9,11,5)=H(4,17,2)
      G(9,11,7)=H(4,17,24)
      G(9,8,8)=1.
      G(9,8,1)=H(12,15,8)
        G(10,1,1)=1.
        G(10,2,2)=1.
        G(10,3,3)=1.
        G(10,4,4)=1.
        G(10,5,5)=1.
        G(10,6,4)=H(3,12,7)
        G(10,6,6)=H(3,12,18)
        G(10,7,6)=1.
        G(10,7,3)=1./U
        G(10,7,7)=H(4,18,17)
        G(10,6,7)=1.
        G(10,8,1)=H(12,15,8)
      G(11,1,1)=1.
      G(11,2,2)=1.
      G(11,3,8)=1.
      G(11,4,6)=1.
      G(11,5,7)=1.
      G(11,6,8)=1./U
      G(11,6,9)=H(4,18,17)
      G(11,7,9)=1.
      G(11,7,1)=H(12,15,8)
        G(12,1,1)=1.
        G(12,2,2)=1.
        G(12,3,3)=1.
        G(12,4,4)=1.
        G(12,5,5)=1.
        G(12,6,6)=1.
```

```
          G(12,7,7)=1.
          G(12,8,8)=1.
          G(12,9,1)=M(12,15,8)
        G(13,1,1)=1.
        G(13,2,2)=1.
        G(13,3,3)=1.
        G(13,4,4)=1.
        G(13,5,5)=1.
        G(13,6,6)=1.
        G(13,6,6)=1.
        G(13,7,7)=1.
        G(13,8,8)=1.
C
C
C
       H(2,1,1)=1.
       H(2,2,2)=1.
       H(2,3,3)=.5
       H(2,3,4)=.5
         H(3,1,1)=1.
         H(3,2,2)=1.
         H(3,3,3)=RPM/U
         H(3,3,4)=M(7,8,5)
         H(3,3,5)=M(7,4,4)
         H(3,3,4)=M(7,4,5)
       H(4,1,1)=1.
       H(4,2,2)=1.
       H(4,3,4)=M(8,9,9)
       H(4,3,3)=M(8,9,5)
       H(4,4,1)=M(8,5,6)
       H(4,5,3)=M(7,9,8)
       H(4,5,5)=M(7,9,10)
       H(5,2,2)=1.
       H(5,1,1)=1.
       H(5,3,3)=1.
       H(5,4,4)=M(9,9,11)
       H(5,4,5)=M(9,9,12)
       H(5,5,6)=M(8,10,10)
       H(5,5,7)=M(8,10,11)
       H(6,1,1)=1.
       H(6,2,2)=1.
       H(6,3,3)=1.
       H(6,4,4)=M(10,11,13)
       H(6,5,3)=M(10,12,6)
       H(6,5,4)=1.
       H(6,6,5)=M(4,14,23)
       H(6,7,6)=M(9,11,14)
         H(7,1,1)=1.
         H(7,2,2)=1.
         H(7,3,3)=1.
         H(7,4,3)=M(10,12,6)
         H(7,5,4)=1.
         H(7,5,5)=M(5,23,22)
         H(7,6,5)=1.
         H(7,6,3)=M(3,9,6)
         H(7,6,6)=M(3,9,16)
       H(8,1,1)=1.
       H(8,2,2)=1.
       H(8,3,3)=1.
       H(8,4,4)=M(9,11,14)
       H(8,5,4)=1.
       H(8,5,3)=M(3,9,6)
       H(8,5,5)=M(3,9,16)
       H(8,6,5)=1.
       H(8,6,6)=M(3,12,7)
       H(8,6,7)=M(3,12,18)
         H(9,1,1)=1.
         H(9,2,2)=1.
         H(9,3,4)=1.
         H(9,4,4)=M(3,9,6)
         H(9,4,5)=M(3,9,16)
         H(9,5,5)=1.
         H(9,5,3)=M(3,12,7)
         H(9,5,6)=M(3,12,13)
         H(9,6,3)=1.
         H(9,7,6)=1.
         H(9,7,4)=1./U
         H(9,7,7)=M(4,18,17)
             H(10,1,1)=1.
             H(10,2,2)=1.
```

```
        H(10,3,3)=1.
        H(10,4,4)=1.
        H(10,5,3)=M(3,12,7)
        H(10,5,5)=M(3,12,18)
        H(10,6,5)=1.
        H(10,6,4)=1./U
        H(10,6,6)=M(4,18,17)
        H(10,7,6)=1.
        H(10,7,1)=M(8,5,6)
      H(11,1,1)=1.
      H(11,2,3)=1.
      H(11,3,6)=1.
      H(11,4,8)=1.
      H(11,5,8)=1./U
      H(11,5,9)=M(4,18,17)
      H(11,6,9)=1.
      H(11,6,1)=M(8,5,6)
        H(12,1,1)=1.
        H(12,2,2)=1.
        H(12,3,3)=1.
        H(12,4,4)=1.
        H(12,5,5)=1.
        H(12,6,6)=1.
        H(12,7,7)=1.
        H(12,8,8)=1.
        H(12,9,1)=M(8,5,6)
      H(13,1,1)=1.
      H(13,2,2)=1.
      H(13,3,3)=1.
      H(13,4,4)=1.
      H(13,5,5)=1.
      H(13,6,6)=1.
      H(13,7,7)=1.
      H(13,8,8)=1.
C
C
C
      O(2,1,1)=1.
      O(2,2,2)=1.
      O(2,3,3)=M(7,8,9)
      O(2,3,4)=M(7,8,5)
      O(2,4,5)=M(6,9,11)
      O(2,4,4)=M(6,9,10)
    O(3,1,1)=1.
    O(3,2,2)=1.
    O(3,3,3)=M(8,9,5)
    O(3,3,5)=M(8,9,9)
    O(3,4,4)=M(8,5,6)
    O(3,5,3)=M(8,6,5)
    O(3,5,6)=M(8,6,7)
      O(4,1,3)=1.
      O(4,2,2)=1.
      O(4,3,4)=1.
      O(4,4,1)=1.
      O(4,5,5)=M(9,9,11)
      O(4,5,6)=M(9,9,12)
      O(4,6,7)=M(9,7,8)
      O(4,6,8)=M(9,7,9)
    O(5,1,1)=1.
    O(5,2,2)=1.
    O(5,3,3)=1.
    O(5,4,4)=1.
    O(5,5,5)=M(10,11,13)
    O(5,6,5)=1.
    O(5,6,4)=M(10,12,6)
    O(5,7,6)=M(4,13,21)
    O(5,7,7)=M(4,13,22)
    O(5,7,8)=M(4,13,18)
    O(5,8,7)=1.
    O(5,8,8)=1.
    O(5,8,9)=M(9,13,17)
      O(6,1,1)=1.
      O(6,2,2)=1.
      O(6,3,3)=1.
      O(6,4,4)=1.
      O(6,5,4)=M(10,12,6)
      O(6,6,5)=M(5,21,17)
      O(6,7,6)=1.
      U(6,7,7)=M(9,13,17)
      U(6,8,6)=1.
```

```
            D(6,8,5)=-1.
            D(6,9,4)=M(3,10,6)
            D(6,9,8)=M(3,10,17)
            D(6,9,7)=1.
          D(7,1,1)=1.
          D(7,2,2)=1.
          D(7,3,3)=1.
          D(7,4,4)=1.
          D(7,5,7)=M(3,3,5)
          D(7,5,4)=M(3,3,6)
          D(7,5,5)=M(3,3,7)
          D(7,5,6)=M(3,3,8)
          D(7,5,8)=1.
          D(7,6,8)=-1.
          D(7,7,4)=M(3,10,6)
          D(7,7,9)=M(3,10,17)
          D(7,8,9)=1.
          D(7,8,6)=M(4,17,2)
          D(7,8,10)=M(4,17,24)
            D(8,1,1)=1.
            D(8,2,2)=1.
            D(8,3,3)=1.
            D(8,4,6)=1.
            D(8,5,4)=1.
            D(8,6,5)=1.
            D(8,7,7)=1.
            D(8,7,1)=M(10,20,7)
            D(8,8,7)=M(3,3,5)
            D(8,8,6)=M(3,3,6)
            D(8,8,4)=M(3,3,7)
            D(8,8,5)=M(3,3,8)
            D(8,9,5)=M(4,17,2)
            D(8,9,8)=M(4,17,24)
            D(8,10,8)=1.
            D(8,10,6)=1./U
            D(8,10,7)=M(4,18,17)
          D(9,1,1)=1.
          D(9,2,2)=1.
          D(9,3,3)=1.
          D(9,4,4)=1.
          D(9,5,5)=1.
          D(9,6,6)=1.
          D(9,7,7)=1.
          D(9,8,7)=M(4,18,17)
          D(9,7,1)=M(10,20,7)
          D(9,8,6)=1./U
            D(10,1,1)=1.
            D(10,2,2)=1.
            D(10,3,3)=1.
            D(10,4,6)=1.
            D(10,5,7)=1.
            D(10,6,8)=1.
            D(10,7,1)=M(10,20,7)
C
C         ****  OBTAIN PARTIAL DERIVATIVE(S) OF INVERSE CONSTRAINT EQ'NS
C
C
          DM3DT3=-.5*CC3/((GAMA*RA)**.5*T3**1.5)
          DM3DC3=1./(GAMA*RA*T3)**.5
          PDC19=1./(1.+((M2-M1)/(2.*(CN+1.25*CR)))**2.)
          DC22DT=2.*CP*(U/(.5*U**2.-2.*CP*DTOS))**2.
          DC22DU=-1.*DC22DT*DTOS/U**2.
          DC23DT=-2.*CP*(U/(2.*CP*DTOS-6.*U**2.))**2.
          DC23DU=-2.*DC23DT*DTOS/U**2.
C
          C17FAC=(PC/(PC-P2))**2./P01
          C17FAD=(1.+LAMN)*CA**2.*SIN(AL2)/(-2.*CP*COS(AL2)**3.)
          C18FAC=-1./(MC3-1.)**2.
          C18FAD=M(3,10,6)+M(3,10,17)*M(4,16,24)*M(4,18,16)
          C18FAE=M(3,10,17)*M(4,17,24)*M(4,18,17)*PIN*RPM
          C19FAC=-12.*PIN/(12.*ALC-1.5*PIN)**2.
          C24FAC=1./(.3-R)**2.
          C25FAC=-1./(R-.7)**2.
          C26FAC=(57./(40.-57.*AL2))**2.
C
C
      C(1,1)=50.*PIN*RPM/(50.-PIN*RPM*D)**2.
C
        C(2,1)=-400.*PIN*RPM/(PIN*RPM*D-400.)**2.
C
```

```
                 G(1,1,1)=8./(HR-8.*CR)
                 G(1,1,2)=64.*D/(HR-8.*CR)**2.
                 G(1,1,3)=-8.*D/(HR-8.*CR)**2.
C
C
C                 *****OBTAIN   VECTOR C(3) ******
C
                      DO 201 J=1,SIZE2(1,2)
       201            D3(J)=G(1,1,J)
C
                      DO 401   K=2,13
                      DO 301  KK=1,SIZE2(K,1)
                      DO 301 KKK=1,SIZE2(K,2)
       301            MD2(KK,KKK)=G(K,KK,KKK)
                      CALL MULTIP(D3,MD2,SIZE2(K,1),SIZE2(K,2))
       401            CONTINUE
C
                      DO 402 I=1,8
       402            C(3,I)=D3(I)
C
C
                  G(1,1,1)=2./(2.*CR-HR)
                  G(1,1,2)=-4.*D/(2.*CR-HR)**2.
                  G(1,1,3)=2.*D/(2.*CR-HR)**2.
C
C
C                 *****OBTAIN   VECTOR C(4) ******
C
                      DO 501 J=1,SIZE2(1,2)
       501            D4(J)=G(1,1,J)
C
                      DO 701   K=2,13
                      DO 601  KK=1,SIZE2(K,1)
                      DO 601 KKK=1,SIZE2(K,2)
       601            MD2(KK,KKK)=G(K,KK,KKK)
                      CALL MULTIP(D4,MD2,SIZE2(K,1),SIZE2(K,2))
       701            CONTINUE
C
C
                      DO 702 I=1,8
       702            C(4,I)=D4(I)
C
C
C
                   H(1,1,1)=10./(HN-10.*CN)**2.
                   H(1,1,3)=-10.*D/(HN-10.*CN)**2.
                   H(1,1,2)=100.*D/(HN-10.*CN)**2.
C
C                 *****OBTAIN   VECTOR C(5) ******
C
                      DO 801 J=1,SIZE3(1,2)
       801            D5(J)=H(1,1,J)
C
                      DO 1001  K=2,13
                      DO 901  KK=1,SIZE3(K,1)
                      DO 901 KKK=1,SIZE3(K,2)
       901            MD2(KK,KKK)=H(K,KK,KKK)
                      CALL MULTIP(D5,MD2,SIZE3(K,1),SIZE3(K,2))
      1001            CONTINUE
C
C
                      DO 1011 I=1,8
      1011            C(5,I)=D5(I)
C
                    H(1,1,1)=2./(2.*CN-HN)
                    H(1,1,2)=-4.*D/(2.*CN-HN)**2.
                    H(1,1,3)=2.*D/(2.*CN-HN)**2.
C
C                 *****OBTAIN   VECTOR C(6) ******
C
                      DO 1002 J=1,SIZE3(1,2)
      1002            D6(J)=H(1,1,J)
C
                      DO 1004  K=2,13
                      DO 1003  KK=1,SIZE3(K,1)
                      DO 1003 KKK=1,SIZE3(K,2)
      1003            MD2(KK,KKK)=H(K,KK,KKK)
                      CALL MULTIP(D6,MD2,SIZE3(K,1),SIZE3(K,2))
      1004            CONTINUE
C
```

```
C
               DO 1077 I=1,8
 1077          C(6,I)=D6(I)
C
             C(7,3)=-1.*SS/(.5*CN-SS)**2.
             C(7,4)=CN/(.5*CN-SS)**2.
C
            C(8,3)=SS/(SS-CN)**2.
            C(8,4)=-1.*CN/(SS-CN)**2.
C
            C(9,2)=-1.*SR/(.3*CR-SR)**2.
            C(9,5)=CR/(.3*CR-SR)**2.
C
           C(10,2)=SR/(SR-CR)**2.
           C(10,5)=-1.*CR/(SR-CR)**2.
C
            C(11,6)=1./(.01-BET2)**2.
            C(12,6)=-1./(BET2-1.)**2.
C
            C(13,7)=1./(40./57.-BET3)**2.
            C(14,7)=-1./(BET3-1.4)**2.
C
          C(15,8)=50./(50.-CA)**2.
          C(16,8)=-400./(CA-400.)**2.
C
        C(17,1)=C17FAC*M(4,14,23)*C17FAD/(FI*D*(1.+TAN(AL2)**2.))
        C(17,6)=C17FAC*M(4,14,23)*C17FAD/(COS(BET2)**2.*(1.+TAN
     # (AL2)**2.))
        C(17,6)=C17FAC*M(4,14,23)*(1.+LAMN )*(U*SIN(2.*AL2)-2.*CA)
     # /(2.*CP*COS(AL2)**2.)
C
         C(18,1)=C18FAC*(DM3DC3*C18FAE+DM3DT3*(M(2,4,10)*C18FAE
     #          -M(3,3,5)*PIN*RPM))
         C(18,6)=-1.*C18FAC*DM3DT3*M(3,3,7)
         C(18,7)=C18FAC*(DM3DC3*M(3,10,17)*M(4,17,2)+DM3DT3*(M(2,4,10)
     #          *M(3,10,17)*M(4,17,2)-M(3,3,8)))
         C(18,8)=C18FAC*(C18FAD*(DM3DC3+M(2,4,10)*DM3DT3)-DM3DT3
     #          *M(3,3,6))
C
C
C
           O(1,1,1)=-2.*C19FAC*PDC19*(H3-H1)/(2.*CN+2.5*CR)**2.
           O(1,1,2)=O(1,1,1)*2.5/2.
           O(1,1,3)=-1.*C19FAC*PDC19/(2.*CN+2.5*CR)
           O(1,1,4)=-1.*O(1,1,3)
C
C          *****OBTAIN   VECTOR C(19) ******
C
               DO 1005 J=1,SIZE4(1,2)
 1005          D19(J)=O(1,1,J)
C
               DO 1007  K=2,10
               DO 1006  KK=1,SIZE4(K,1)
               DO 1006 KKK=1,SIZE4(K,2)
 1006          MD2(KK,KKK)=O(K,KK,KKK)
               CALL MULTIP(D19,MD2,SIZE4(K,1),SIZE4(K,2))
 1007          CONTINUE
C
C
               DO 1088 I=1,8
 1088          C(19,I)=D19(I)
C
           C(20,1)=-1.*PIN*RPM*CA/(.25*U-CA)**2.
           C(20,8)=PIN*RPM*D/(.25*U-CA)**2.
           C(21,1)=PIN*RPM*CA/(CA-2.*U)**2.
           C(21,8)=-1.*PIN*D*RPM/(CA-2.*U)**2.
C
         C(22,1)=(DC22DU+DC22DT*M(3,3,5))*PIN*RPM
         C(22,6)=DC22DT*U*CA/(CP*COS(BET2)**2.)
         C(22,7)=DC22DT*U*CA/(CP*COS(BET3)**2.)
         C(22,8)=DC22DT*U*(TAN(BET2)+TAN(BET3))/CP
C
         C(23,1)=(DC23DU+DC23DT*M(3,3,5))*PIN*RPM
         C(23,6)=DC23DT*U*CA/(CP*COS(BET2)**2.)
         C(23,7)=DC23DT*U*CA/(CP*COS(BET3)**2.)
         C(23,8)=DC23DT*U*(TAN(BET2)+TAN(BET3))/CP
C
         C(24,1)=C24FAC*(TAN(BET3)-TAN(BET2))*M(4,18,17)*PIN
     #          *RPM/2.
         C(24,8)=(TAN(BET3)-TAN(BET2))*C24FAC/(2.*U)
```

```
               C(24,6)=-1.*FI*C24FAC/(2.*COS(BET2)**2.)
               C(24,7)=FI*C24FAC/(2.*COS(BET3)**2.)
C
                C(25,1)=C(24,1)*C25FAC/C24FAC
                C(25,8)=C(24,8)*C25FAC/C24FAC
                C(25,6)=C(24,6)*C25FAC/C24FAC
                C(25,7)=C(24,7)*C25FAC/C24FAC
C
              C(26,1)=C26FAC*M(3,12,18)*M(4,18,17)*M(10,20,7)
              C(26,6)=C26FAC*M(3,12,7)
              C(26,8)=C26FAC*M(3,12,18)/U
C
C
                 C(27,1)=1./D**2.
                 C(28,2)=1./CR**2.
                 C(29,3)=1./CN**2.
                 C(30,4)=1./SS**2.
                 C(31,5)=1./SR**2.
C
C
C
C    *** CALCULATE SUM OF DERIVATIVE OF CONSTRAINT EQ'NS ***
C
C
               DO 1030 I=1,8
  1030         DCONST(I)=0.
               DO 1100 MMM=1,8
               DO 1090 NNN=1,31
  1090         DCONST(MMM)=DCONST(MMM)+C(NNN,MMM)
  1100         CONTINUE
C
C
C      **** OBTAIN FINAL FORM OF DERIVATIVE OF PENALTY FUNCTION ****
C
C
C
                 DO 1110   I=1,8
                 DPEN(I)=DELTA(I)+RF*DCONST(I)
  1110           CONTINUE
C
C
C
C      *** SET MAXIMIZING STEP LENGTH, R ***
C
C
  9999               RTOT=ASL
                     XLOW(5)=.31*XI(2)
                     XLOW(4)=.31*XI(3)
                     DO 1027 I=1,8
                     IF(DPEN(I).EQ.0.) GO TO 1027
                     IF(DPEN(I).GT.0.) THEN
                     RMAX(I)=(XUP(I)-XI(I))/DPEN(I)
                     ELSE
                     RMAX(I)=(XLOW(I)-XI(I))/DPEN(I)
                     END IF
C
                     IF(RMAX(I).LT.RTOT) THEN
                     RTOT=RMAX(I)
                     END IF
  1027               CONTINUE
                     R=RTOT/51.
                     ZITA=R
C
                     IF(DPEN(5).LT.0.) THEN
                     X2FIN=XI(2)+45.*R*DPEN(2)
                     XLOW(5)=.31*X2FIN
                     RMAX(5)=(XLOW(5)-XI(5))/DPEN(5)
                     IF(RMAX(5).LT.RTOT) THEN
                     RTOT=RMAX(5)
                     R=RTOT/51.
                     ZITA=R
                     END IF
                     GO TO 1120
                     END IF
C
C
  1120               ICOUNT=ICOUNT+1
                     IF(ICOUNT.GT.45) GO TO 2000
C
C     *** CALCULATE NEW SET OF VARIABLES ***
```

```
C
C
            DO 1121 I=1,8
            X(I)=XI(I)+K*DPEN(I)
              IF(X(I).LT.XLOW(I)) THEN
            LO2=.TRUE.
              END IF
 1121       CONTINUE
C
            IF(LO2) GO TO 2000
C
            LO1=.TRUE.
            LO2=.FALSE.
            GO TO 10
C
C
C
 5000       LO1=.FALSE.
C
              IF(LL1) THEN
              LL1=.FALSE.
              GO TO 1024
              END IF
C
              IF(LL2) THEN
              LL2=.FALSE.
              L=L+1
              PENLES=PENFUN
              LL1=.FALSE.
              GO TO 1140
              END IF
C
C
 1022 IF(PENFUN.GT.PENMAX) THEN
                   IF(PENFUN.GT.1.) THEN
                   MLK=0
                   GO TO 1140
                   END IF
                 PENMAX=PENFUN
                 ETAMAX=ETASRE
                 CONMAX=CONST
                 REMAX=RE
                 PRMAX=PRATIO
                 MAKMAX=MC3
                 PSIMAX=PSI
                 FIMAX=FI
                 DONMAX=DON
                 DOTMAX=DOT
                 DOMAMX=DOMA
                 DOWMAX=DOW
                 DOM2MX=DOM2
                 DO 1130 I=1,8
 1130            XMAX(I)=X(I)
                 GO TO 1140
            ELSE
                    L=L+1
                 PENLES=PENFUN
                 LL1=.TRUE.
                 GO TO 1140
              END IF
C
C
C
 1024             IF(PENLES.GT.PENFUN) THEN
C
                        IF(MLK.EQ.0) THEN
                         PENLES=PENFUN
                            L=L+1
                            IF(L.GT.100) GO TO 2000
                            LL1=.TRUE.
                            GO TO 1140
                        ELSE
                            MLK=0
                            L=L+1
                            PENLES=PENFUN
                            LL1=.TRUE.
                            GO TO 1140
                          END IF
C
                  ELSE
```

```
                              IF(MLK.EQ.0) THEN
                               MLK=MLK+1
                               L=0
                               KLM=KLM+1
                               IF(KLM.GT.50) GO TO 2000
                               GO TO 1025
                              ELSE
                                GO TO 1025
                              END IF
C
                      END IF
 1025                   IF(PENFUN.GT.PENMAX) THEN
                         IF(PENFUN.GT.1.) THEN
                         MLK=0
                         GO TO 1140
                         END IF
                         PENMAX=PENFUN
                         MLK=0
                         ETAMAX=ETASRE
                         CONMAX=CONST
                         REMAX=RE
                         PRMAX=PRATIO
                         HAKMAX=HC3
                         PSIMAX=PSI
                         FIMAX=FI
                         DQMMAX=DQM
                         DQTMAX=DQT
                         DQMAMX=DQMA
                         DQWMAX=DQW
                         DQM2MX=DQM2
                         DO 1026  I=1,8
 1026                    XMAX(I)=X(I)
                         GO TO 1140
                        ELSE
                         PENLES=PENFUN
                         LL1=.TRUE.
                         GO TO 1140
                        END IF
C
C
 1140     R=R+ZITA
          GO TO 1120
C
C
 2000    DO 2001  I=1,8
 2001    XI(I)=XMAX(I)
         L02=.FALSE.
         L01=.FALSE.
         DIFPEN=PENMAX-FPENF
         FPENF=PENMAX
         IF(DIFPEN.LT..0001) THEN
         PCOUNT=PCOUNT+1.
         IF(PCOUNT.GT.1.) THEN
         KK=II+1
         RFI(KK)=RFI(II)/10.
         PRINT*,' '
         PRINT*,' '
         PCOUNT=0.
         L05=.TRUE.
         DQMN=1.
         GO TO 2006
         END IF
         END IF
         DQMN=DQMN+1.
 2006    LL1=.FALSE.
         LL2=.FALSE.
         KLM=0
         MLK=0
         L=0
         RTOT=MSL
         ICOUNT=0
         IF(L05) GO TO 2100
         GO TO 7
C
C
C
 2100     L05=.FALSE.
          RF=RF/10.
 4445     IF(CONTY.NE.0.) RF=.0000004
          IF(RF.LT..000004) THEN
```

```
      PRINT*,' ** VALUES OF DESIGN VARIABLES CORRESPONDING TO OPTIMUM
     #          EFFICIENCY ARE AS FOLLOWS **'
      PRINT*,' '
      DO 2004 I=1,8
 2004 PRINT*,' ** X(',I,') = ',XI(I)
      PRINT*,' '
      PRINT*,' ** VARIOUS OPTIMUM PARAMETERS ARE **'
      PRINT*,' '
      PRINT*,' ** EFFICIENCY = ',ETAMAX,'  **'
      PRINT*,' '
      PRINT*,' ** PENALTY FN.= ',PENMAX,'  **'
      PRINT*,' '
      PRINT*,' *** INLET NON-DIM MASS FLOW =',RMRAT,' **'
      PRINT*,' '
      PRINT*,' **  PRESSURE RATIO =',PRMAX,'  **'
      PRINT*,' **  ROTOR MACH #   =',MAKMAX,'  **'
      PRINT*,' '
      PRINT*,' ** REYNOLDS # = ',REMAX,' **'
      PRINT*,' '
      PRINT*,' ** DOMMAX= ',DOMMAX
      PRINT*,' **DQTMAX= ',DQTMAX
      PRINT*,' ** DQMAMX= ',DQMAMX
      PRINT*,' ** DOWMAX= ',DOWMAX
      PRINT*,' ** DOM2MX= ',DOM2MX
      PRINT*,' ** FIMAX= ',FIMAX
      PRINT*,' ** PSIMAX= ',PSIMAX
      GO TO 2200
      END IF
      SMS=SMS+1.
      GO TO 5
 2200 STOP
      END
C
C
C
C
      SUBROUTINE MULTIP(A,B,M,N)
      DIMENSION A(30),B(30,30),C(30)
C
C
C
      DO 1 I=1,N
      T=0.
      DO 2 J=1,M
C
C
    2 T=T+B(J,I)*A(J)
    1 C(I)=T
C
C
      DO 12 I=1,N
   12 A(I)=C(I)
C
       RETURN
       END
C
C
C      SUBROUTINE ICS1VE (F,A,B,U1,D1,UN,DN,N,M,G,PSCOND,IER)
C
C-ICS1VE--------S-------LIBRARY 2---------------------------------------
C
C   FUNCTION             - CUBIC SPLINE ONE-DIMENSIONAL INTERPOLATION -
C                            EQUALLY SPACED DATA
C   USAGE                - CALL ICS1VE(F,A,B,U1,D1,
C                            UN,DN,N,M,G,PSCOND,IER)
C   PARAMETERS   F       - VECTOR OF N EQUALLY SPACED
C                            FUNCTIONAL VALUES F(1),F(2),...,
C                            F(N), WHERE F(I) IS THE
C                            FUNCTIONAL VALUE AT
C                            X=A+(I-1)*H AND H=(B-A)/(N-1)
C                A       - VALUE OF THE ABSCISSA FOR F(1)
C                B       - VALUE OF THE ABSCISSA FOR F(N)
C                U1      - BOUNDARY RELATED PARAMETER
C                            (SEE ELEMENT DOCUMENTATION)
C                D1      - BOUNDARY RELATED PARAMETER
C                            (SEE ELEMENT DOCUMENTATION)
C                UN      - BOUNDARY RELATED PARAMETER
C                            (SEE ELEMENT DOCUMENTATION)
C                DN      - BOUNDARY RELATED PARAMETER
C                            (SEE ELEMENT DOCUMENTATION)
C
```

```
C                   N       - NUMBER OF DATA POINTS
C                   M       - NUMBER OF ANSWERS SOUGHT
C                   G       - VECTOR G(1),G(2),...,G(M) OF ANSWERS
C                               WHERE G(I) IS THE I-TH ABSCISSA
C                               UPON ENTRY AND THE I-TH
C                               ORDINATE UPON COMPLETION.
C                   PSCOND  - PSEUDO CONDITION NUMBER FOR BOUNDARY
C                               PARAMETERS. PSCOND.GT.1.E5
C                               INDICATES VALUES RETURNED IN G(I)
C                               MAY BE UNRELIABLE.
C                   IER     - ERROR PARAMETER
C                             TERMINAL ERROR = 128+N
C                               N = 1 INDICATES G(I) NOT IN INTERVAL (A,B)
C                                 FOR SOME I=1,2,...,M
C                               N = 2 INDICATES UNIQUE SOLUTION NOT POSSIBLE
C                                 FOR GIVEN BOUNDARY VALUES
C   PRECISION               - SINGLE
C   REQ'D IMSL ROUTINES     - UERTST
C   LANGUAGE                - FORTRAN
C-----------------------------------------------------------------------
C   LATEST REVISION         - FEBRUARY 3,1971
C
      SUBROUTINE ICSIVE  (F,A,B,U1,D1,UN,DN,N,M,G,PSCOND,IER)
C
      DIMENSION          F(34),G(1)
      EQUIVALENCE        (IDET,DET)
      REAL               LAM,LAMN
      DATA               S3,LAM/1.7320508,-.26794919/
      IER=0
      H=(B-A)/(N-1)
      LAMN=LAM**(N-1)
C                                  CALCULATE LAM AND TAU
      GAM=F(1)
      TAU=F(N)
      N1=N+1
         DO 5 I=2,N
         J=N1-I
         GAM=GAM*LAM+F(I)
         TAU=TAU*LAM+F(J)
    5    CONTINUE
      GAM=3.*GAM
      TAU=3.*TAU
      C1=1.-U1*LAM
      CT=1.-U1/LAM
      C2=CT*LAMN
      C3=D1*H/(2.*S3)+(CT*TAU-S3*((1.+6.*U1)*F(1)-U1*F(2)))/H
      CT=1.-UN/LAM
      C4=CT*LAMN
      C5=1.-UN*LAM
      C6=DN*H/(2.*S3)+(CT*GAM-S3*((1.+6.*UN)*F(N)-UN*F(N-1)))/H
      DET=C1*C5-C2*C4
      IF (IDET.EQ.0) GO TO 55
C                                  CALCULATE BOUNDARY PARAMETERS B1,BN
      B1=(C2*C6-C3*C5)/DET
      BN=(C1*C6-C4*C3)/DET
C                                  CALCULATE PSCOND
      C1=ABS(C1)
      C2=ABS(C2)
      C4=ABS(C4)
      C5=ABS(C5)
      PSCOND=AMAX1(C1,C2,C4,C5)/ABS(DET)
      KP=0
      N2=N-1
C                                  CALCULATE G(I)'S
         DO 50 I=1,M
         XI=A
         IF(G(I).LT.A) GO TO 15
            DO 10 J=1,N2
            XI=XI+H
            IF(G(I).GT.XI) GO TO 10
            K=J
            GO TO 20
   10       CONTINUE
         IF(G(I).GT.B) GO TO 15
         K=N2
         GO TO 20
   15    IER=129
         GO TO 9000
   20    X=(G(I)-(A+(K-1)*H))/H
         IF(K.EQ.KP) GO TO 45
```

```
         GAMK=F(1)
         TAUK=F(N)
         IF(K.LE.1) GO TO 30
            DO 25 J=2,K
            GAMK=GAMK*LAM+F(J)
   25       CONTINUE
   30    K1=N-K
         IF(K1.LE.1) GO TO 40
            DO 35 J=2,K1
            JJ=N1-J
            TAUK=TAUK*LAM+F(JJ)
   35       CONTINUE
   40    GAMK=3.*GAMK
         TAUK=3.*TAUK
         KP=K
   45    TX=(1.-(LAM+2.)*X+(LAM+1.)*X*X)*X
         Y=1.-X
         TY=(1.-(LAM+2.)*Y+(LAM+1.)*Y*Y)*Y
         G(I)=GAMK*TX+TAUK*TY+F(K)*Y**3+F(K+1)*X**3+B1*H*TX*LAM**(K-1)
     1      -BN*H*TY*LAM**(N-K-1)
   50    CONTINUE
      GO TO 9005
   55 IER=130
 9000 CONTINUE
      CALL UERTST(IER,'ICS1VE')
 9005 RETURN
      END
C
C
C
C
C     SUBROUTINE ICS2VE (F,X0,A,B,C,D,I,J,N,IF,WK,Y,IER)
C
C-ICS2VE--------S--------LIBRARY 2-------------------------------------
C
C   FUNCTION            - BICUBIC SPLINE TWO-DIMENSIONAL INTERPOLATOR -
C                           EQUALLY SPACED DATA
C   USAGE               - CALL ICS2VE(F,X0,A,B,C,D,I,J,N,IF,WK,Y,IER)
C   PARAMETERS   F      - MATRIX OF I*J FUNCTIONAL VALUES F(1,1),F(2,1),
C                           F(3,1),...,F(1,2),...,F(I,J). THE VALUE
C                           F(I1,J1) IS THE FUNCTIONAL VALUE AT X(I1),
C                           Y(J1) WHERE X(I1)=A+(I1-1)*HX,Y(J1)=C+(J1-1)
C                           *HY, HX=(B-A)/(I-1), AND HY=(D-C)/(J-1)
C                X0     - X-DIRECTION ABSCISSA FOR WHICH INTERPOLATES
C                           ARE SOUGHT
C                A      - LOWER LIMIT OF THE INTERVAL (A,B)
C                B      - UPPER LIMIT OF THE INTERVAL (A,B)
C                C      - LOWER LIMIT OF THE INTERVAL (C,D)
C                D      - UPPER LIMIT OF THE INTERVAL (C,D)
C                I      - NUMBER OF MESH POINTS IN THE FIRST COORDINANT,
C                           X, DIRECTION
C                J      - NUMBER OF MESH POINTS IN THE SECOND COORDINANT
C                           Y, DIRECTION
C                N      - NUMBER OF INTERPOLATED VALUES DESIRED
C                IF     - FIRST DIMENSION OF F IN CALLING PROGRAM
C                WK     - WORK AREA OF DIMENSION J
C                Y      - VECTOR Y(1),Y(2),...,Y(N) WHERE Y(J1) IS THE
C                           J1-TH ABSCISSA UPON ENTRY AND THE INTERPO-
C                           LATED FUNCTIONAL VALUE AT COORDINANT
C                           (X0,Y(J1)) UPON COMPLETION
C                IER    - ERROR PARAMETER
C                         TERMINAL ERROR = 128+N
C                           N = 1 INDICATES X0 NOT IN INTERVAL (A,B)
C                           N = 2 INDICATES Y(J1) NOT IN INTERVAL (C,D)
C                             FOR SOME J1=1,2,...,N
C   PRECISION           - SINGLE
C   REQ'D IMSL ROUTINES - ICS1VE,UERTST
C   LANGUAGE            - FORTRAN
C----------------------------------------------------------------------
C   LATEST REVISION     - JANUARY 18, 1971
C
      SUBROUTINE ICS2VE  (F,X0,A,B,C,D,I,J,N,IF,WK,Y,IER)
C
      DIMENSION          F(IF,1),Y(1),WK(1)
C                                  INTERPOLATE IN Y DIRECTION AT X
C
         DO 5 I1=1,J
         WK(I1)=X0
         CALL ICS1VE(F(1,I1),A,B,0.,0.,0.,0.,I,1,WK(I1),PSCOND,IER)
         IF(IER.NE.0) GO TO 9000
    5    CONTINUE
```

```
      CALL ICS1VE (WK,C,D,0.,0.,0.,0.,J,N,Y,PSCOND,IER)
      IF(IER.NE.0) GO TO 10
      GO TO 9005
   10 IER=130
 9000 CONTINUE
      CALL UERTST(IER,'ICS2VE')
 9005 RETURN
      END
C
C
C
C
C
C     SUBROUTINE ICS1VU (F,X,N,M,G,H,IER)
C
C-ICS1VU---------S--------LIBRARY 2--------------------------------------
C
C   FUNCTION            - CUBIC SPLINE ONE-DIMENSIONAL INTERPOLATION -
C                           UNEQUALLY SPACED DATA
C   USAGE               - CALL ICS1VU(F,X,N,M,G,H,IER)
C   PARAMETERS   F      - VECTOR OF N UNEQUALLY SPACED
C                           FUNCTIONAL VALUES F(1),F(2),....,
C                           F(N), WHERE F(I) IS THE FUNCTIONAL
C                           VALUE AT X(I)
C                X      - VECTOR OF N ABSCISSA X(1),X(2),...,X(N)
C                N      - NUMBER OF DATA POINTS
C                M      - NUMBER OF ANSWERS SOUGHT
C                G      - VECTOR G(1),G(2),...G(M) OF ANSWERS
C                           WHERE G(I) IS THE I-TH ABSCISSA
C                           UPON ENTRY AND THE I-TH ORDINATE
C                           UPON COMPLETION.
C                H      - WORK AREA OF DIMENSION 8*N+M
C                IER    - ERROR PARAMETER
C                         TERMINAL ERROR = 128+N
C                           N = 1 INDICATES G(I) NOT IN INTERVAL
C                             (X(1),X(N)) FOR SOME I=1,2,...,M
C                           N = 2 INDICATES THAT CONVERGENCE WAS
C                             NOT OBTAINED IN 5*N ITERATIONS
C   PRECISION           - SINGLE
C   REQ'D IMSL ROUTINES - UERTST
C   LANGUAGE            - FORTRAN
C-----------------------------------------------------------------------
C   LATEST REVISION     - MARCH 5, 1971
C
      SUBROUTINE ICS1VU(F,X,N,M,G,H,IER)
C
      DIMENSION          F(1),X(1),G(1),H(1)
      DATA               EPSLN,OMEGA/1.E-6,1.0717968/
C                                  SET UP WORK AREAS
      I2=N
      I3=N+N
      I4=I3+N
      I5=I4+N
      I6=I5+N
      I7=I6+N
      I8=I7+N
      I9=I8+N
      NT=I6
      IER = 0
      N1 = N-1
C                                  DERIVATIVES,H(J6), USING CENTRAL
C                                  DIFFERENCES
      DO 5  I=1,N1
         J2=I2+I
         H(I)=X(I+1)-X(I)
         H(J2)=(F(I+1)-F(I))/H(I)
    5 CONTINUE
      DO 10  I=2,N1
         J2=I2+I
         J3=I3+I
         J4=I4+I
         J5=I5+I
         J6=I6+I
         J7=I7+I
         H(J3)=H(I-1)+H(I)
         H(J4)=.5*H(I-1)/H (J3)
         H(J5)=(H(J2)-H(J2-1))/H(J3)
         H(J6)=H(J5)+H(J5)
         H(J7)=H(J6)+H(J5)
   10 CONTINUE
```

```
          H(I6+1)=0.
          J6=I6+N
          H(J6)=0.
C                                           BEGIN ITERATION ON SECOND DERIVATIVES
          KCOUNT=0
       15 ETA=0.
          KCOUNT=KCOUNT+1
          DO 25  I=2,N1
             J4=I4+I
             J6=I6+I
             J7=I7+I
             W=(H(J7)-H(J4)*H(J6-1)-(.5-H(J4))*H(J6+1)-H(J6))*OMEGA
             IF (ABS(W).LE.ETA) GO TO 20
             ETA=ABS(W)
       20    H(J6)=H(J6)+W
       25 CONTINUE
          IF(KCOUNT.GT.NT) GO TO 75
          IF (ETA.GE.EPSLN) GO TO 15
C                                           CONVERGENCE OBTAINED
          DO 30 I=1,N1
             J6=I6+I
             J8=I8+I
             H(J8)=(H(J6+1)-H(J6))/H(I)
       30 CONTINUE
          DO 65 J=1,M
             I=1
             J9=I9+J
             IF (G(J)-X(1)) 70,60,35
       35    IF (G(J)-X(N)) 45,50,70
       40    IF (G(J)-X(I)) 55,60,45
       45    I=I+1
             GO TO 40
       50    I=N
       55    I=I-1
C                                           COMPUTE G(J)
       60    J6=I6+I
             J2=I2+I
             J8=I8+I
             HT1=G(J)-X(I)
             HT2=G(J)-X(I+1)
             PROD=HT1*HT2
             H(J9)=H(J6)+HT1*H(J8)
             DELSQS=(H(J6)+H(J6+1)+H(J9))/6.
             G(J)=F(I)+HT1*  H(J2)+PROD*DELSQS
       65 CONTINUE
          GO TO 9005
       70 IER=129
          GO TO 9000
       75 IER=130
     9000 CONTINUE
          CALL UERTST(IER,'ICS1VU')
     9005 RETURN
          END
C
C
C
C
C         SUBROUTINE UERTST (IER,NAME)
C
C-UERTST-----------------LIBRARY 2----------------------------------------
C
C   FUNCTION            - ERROR MESSAGE GENERATION
C   USAGE               - CALL UERTST(IER,NAME)
C   PARAMETERS   IER    - ERROR PARAMETER. TYPE + N  WHERE
C                           TYPE= 128 IMPLIES TERMINAL ERROR
C                                  64 IMPLIES WARNING WITH FIX
C                                  32 IMPLIES WARNING
C                           N    = ERROR CODE RELEVANT TO CALLING ROUTINE
C                NAME   - INPUT SCALAR (DOUBLE PRECISION ON DEC)
C                           CONTAINING THE NAME OF THE CALLING ROUTINE
C                           AS A 6-CHARACTER LITERAL STRING.
C   LANGUAGE            - FORTRAN
C-------------------------------------------------------------------------
C   LATEST REVISION     - OCTOBER 1,1975
C
          SUBROUTINE UERTST(IER,NAME)
C
          DIMENSION            ITYP(3,4),IBIT(4)
          CHARACTER*6          ITYP
          INTEGER              WARN,WARF,TERM,PRINTR
```

```
      EQUIVALENCE        (IBIT(1),WARN),(IBIT(2),WARF),(IBIT(3),TERM)
      DATA               ITYP/'WARNIN','G     ','      ',
     *                    'WARNIN','G(WITH',' FIX) ',
     *                    'TERMIN','AL    ','      ',
     *                    'NON-DE','FINED ','      '/,
     *                   IBIT/ 32,64,128,0/
      DATA               PRINTR/ 6/
      IER2=IER
      IF (IER2 .GE. WARN) GO TO 5
C                                  NON-DEFINED
      IER1=4
      GO TO 20
    5 IF (IER2 .LT. TERM) GO TO 10
C                                  TERMINAL
      IER1=3
      GO TO 20
   10 IF (IER2 .LT. WARF) GO TO 15
C                                  WARNING(WITH FIX)
      IER1=2
      GO TO 20
C                                  WARNING
   15 IER1=1
C                                  EXTRACT 'N'
   20 IER2=IER2-IBIT(IER1)
C                                  PRINT ERROR MESSAGE
      WRITE (PRINTR,25) (ITYP(I,IER1),I=1,3),NAME,IER2,IER
   25 FORMAT(' *** I M S L(UERTST) ***  ',3A6,2X,A6,2X,I2,
     1   ' (IER = ',I3,')')
      RETURN
      END
```

REFERENCES

1. Ainley, D.G., and Mathieson, G.C.R., "A Method of Performance Evaluation for Axial Flow Turbines", Aeronautical Research Council, R&M 2974 (1957).

2. Dunham, J., and Came, P.M., "Improvements to the Ainley-Mathieson Method of Turbine Performance Prediction", J. Eng'g. for Power, July (1970), 252.

3. Rao, S.S., and Gupta, R.S., "Optimum Design of Axial Flow Gas Turbine Stage", J. Eng'g. for Power, V 102, Oct.(1980), 782.

4. Horlock, J.H., Axial Flow Turbines, Robert E. Krieger Publishing Company, Huntington, NY (1973).

5. Zweifel, O., "The Spacing of Turbomachine Blading, Especially with Large Angular Deflection", Brown Boveri Rev., V32 (Dec. 1945), 32.

6. Soderberg, C.R., "Unpublished Notes", Gas Turbine Laboratory, MIT (1949).

7. Todd, K.W., "Practical Aspects of Cascade Wind Tunnel Research", Proc. Instn. Mech. Engrs., V157 (1947), 482.

8. Reeman, J., "Performance of Cascades of Aerofoils at Positive Stagger", Aeronautical Research Council, Rep. 10829 (1943).

9. Dunsby, J.A., "High Speed Tests on a Turbine Cascade of Aerofoil Blading Having a Maximum Profile-Thickness of 20% Chord", Aeronautical Research Council, Rep. 13088 (1949).

10. ASME. Symposium on the Effects of Reynolds Number on Turbomachinery, 1963.

11. Stewart, W.L., Whitney, W.J., and Wong, R.Y., "A Study of Boundary Layer Characteristics of Turbomachine Blade Rows and Their Relation to Overall Blade Loss", J. Bas. Eng'g., V82-D, (1960), 588.

12. Dunavant, J.C., and Erwin, J.R., "Investigation of a Related Series of Turbine Blade Profiles in Cascades", NACA Tech. Note 3802 (1956).

13. Balje, O.E., and Binsley, R.L., "Axial Turbine Performance Evaluation", J. Eng.g for Power, V90 (Oct. 1968), 341.

14. Schlichting, H., Boundary Layer Theory, McGraw-Hill, 4th Ed., (1960).

15. Balje, O.E., "Axial Cascade Technology and Application to Flow Path Designs", J. Eng'g for Power, (Oct. 1968), 309.

16. Carter, A.D.S., "Three Dimensional Flow Theories for Axial Compressors and Turbines", Proc. Instn. Mech. Engrs., V159 (1948) 41.

17. Whittle, F., "The Propulsion Jet Engine", Proc. Instn. Mech. Engrs., V152 (1945), 419.

18. Ainley, D.G., "Performance of Axial Flow Turbines", Proc. Instn. Mech. Engrs., V159 (1948), 230.

19. Reeman, J., "The Turbine for Simple Jet Propulsion Engine", Proc. Instn. Mech. Engrs., V153 (1945), 495.

20. Emmert, H.D., "Current Design Practices for Gas Turbines Power Elements", Trans. ASME, V72 (1950), 189.

21. Miser, J.C., and Stewart, W.L., "Investigation of Two-Stage Air-Cooled Turbine Suitable for Flight at Mach Number of 2.5", Part I - "Velocity Diagram Study", NACA Res. Memo E56H14 (1961), Part II "Blade Design", NACA Res. Memo E56K06 (1961).

22. George, B.B., "Rocket Engine Turbine Optimization", Aerojet General Corporation, TM4901 (1966), 66.

23. Wood, C.F., "Recent Development in Direct Search Techniques", Westinghouse Research Laboratories, Research Report 62.159.522 R1 (1962).

24. Fiacco, A., and McCormick, G., Nonlinear Programming, Sequential Unconstrained Minimization Techniques, NY, Wiley (1968).

25. Schlichting, H., and Das, A., "Recent Research on Cascade Flow Problems", J. Bas. Eng.g, (March 1966), 221.

26. Hubert, G., "Problems in Secondary Flows of Axial Turbomachines", (in German), VDI Forschungsheft 496.

27. Streeter, V.L., and Wylie, E.B., Fluid Mechanics , McGraw Hill (1979).

28. Fiacco, A., and Mc Cormick, G.P., "The Sequential Unconstrained Minimization Technique for Nonlinear Programming, A Primal-Dual Method", J. of Management Science, V10 (Jan. 1964), No.2.

29. Hamdi, A.T., Operations Research - An Introduction, Mc Millan, NY (1971).

30. Spiegel, M.R., "Theory and Problems of Advanced Mathematics for Engineers and Scientists", Schaum's Outline Series in Mathematics, Mc Graw Hill, (1971).

31. Kaplan, W., and Lewis, D.J., Calculus and Linear Algebra, J. Wiley, U.S.A. (1971).