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	Lab. Design Thesis	BALANCING OF MULTI - CYLINDER INTERNAL COMBUSTION ENGINES	

ROBERT COLLEGE
SCHOOL OF ENGINEERING

THESIS FOR THE M.S. DEGREE IN M.E.

"BALANCING OF MULTI-CYLINDER INTERNAL COMBUSTION ENGINES"

BY

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F O R E W O R D

After an introduction to the need and scope of balancing in machinery; single cylinder, in-line, V- , radial and opposed engines have been consider in succession and attempt has been made to show to what degree it is possibl to realize the condition of balance in internal combustion engines. An inde of unbalance, due to Prof. Necdet Eraslan, has been added to each engine, to show its state of balance. Repetition has been avoided wherever two engines are similar as far as balancing is concerned, and the first procedure is ref red to.

Many thanks are due to Prof. Necdet Eraslan for his kind direction and advices while this thesis was being written.

May, 1961 Istanbul

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I N D E X

PART I - GENERAL REMARKS Page

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PART I - GENERAL REMARKS

1. BALANCING OF AN ENGINE

Many moving parts of machines have reciprocating motion like that of the piston of an engine, or rotating motion such as that of the crank shaft of an engine.

If moving parts are not in perfect balance or if the parts have variable motion inertia forces are set up that tend to produce vibrations in the frame of the machine, and hence in the foundations to which the frame is attached. Such vibrations, particularly if they occur at high speeds may produce excessive noise, cause undue wear on the machinery and its supports. Furthermore if the natural period of vibration of any part of the supporting framework or foundation should happen to coincide with the period of vibration of the moving part, the disturbances set up could become dangerous. The purpose of balancing is to neutralize or minimize these unpleasant dangerous vibratory effects as far as may be pratical.

The cause of this undesirable motion lies interely in the masses and motion of the piston assemblies and connected rods. It is not due to the vibrations in gas pressure, exploisision etc., for on every firing stroke the force upward on the cylinder head is equal to the force downward on the piston, so that the net force on the engine assembly is zero.

An engine is said to be in balance, when operating at constant speed, if reactions at the support remain constant both in magnitude and direction. If it does not satisfy this condition the operation to make it balanced is called balancing.

Moving parts may be in (1) static or (2) dynamic balance.

Static balance exists if the center of gravity of all the moving parts remain in a fixed position relative to the frame of the machine regardless of the positions of the parts. A system is in dynamic balance when the inertia forces and couples exerted by the moving masses are in equilibrium among themselves.

2. DYNAMICALLY EQUIVALENT SYSTEM OF THE CONNECTING ROD.

A member is defined as dynamically equivalent to an actual member of a machine, and can hypothetically be regarded as replacing that actual member (for the purposes of dynamic analysis), if it will exhibit exactly the same response to given forces, and the same reactions to given accelerations, as the original.

The connecting rod in the simple engine mechanism is a member in which there is no fixed point; all points upon it have different motions.

Fig. 1a. shows any floating link, having distributed mass, of total value m. It has an acceleration a_G of its center of gravity G and an angular acceleration α, and thus inertia reactions $f=ma_G$ and $q=I_G \cdot \alpha$ Fig. 1b. shows a rigid frame of exactly the same shape, but weightless, except for the fact that it carries two masses m_1 and m_2 distant a, b from the point where G would be if it were super-imposed on the original. Now the latter

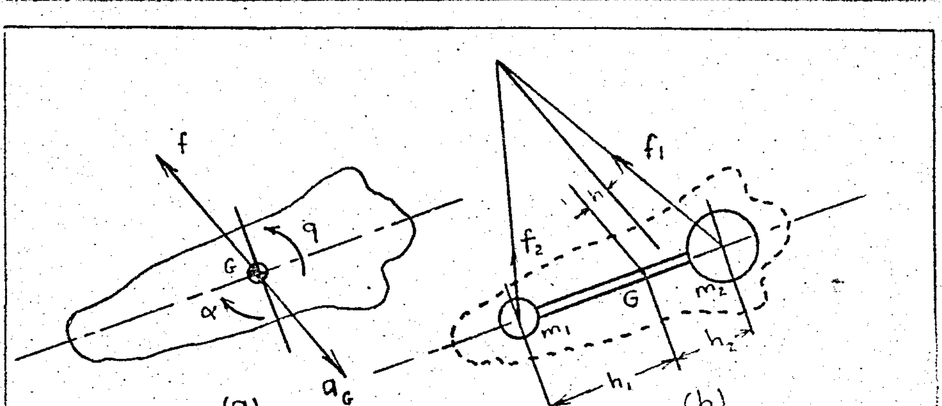

FIG. 1

will be equivalent to the former dynamically, if the vector sum of the two inertia forces f_1 and f_2 is f, both in direction and magnitude, and if the moment of f_1 and f_2 about G is q.

This will be accomplished if

1. The C.G. of m_1 and m_2 is at G.
2. The sum of m_1 and m_2 is still m
3. The moment of inertia of m_1 and m_2 about G is I_G.

These expressed mathematically are:

$$m_1 a = m_2 b \qquad (1)$$

$$m_1 + m_2 = m \qquad (2)$$

$$m_1 a^2 + m_2 b^2 = I_G = mk_G^2 \qquad (3)$$

where k_G is radius of gyration of the rod about center of gravity G. The third may be transformed as follows:

$$(m_1 a)a + (m_2 b)b = mk_G^2$$

Using equation (1)

$$(m_2 b)a + (m_1 a)b = mk_G^2$$

$$(m_1 + m_2)ab = mk_G^2$$

$$ab = k_G^2 \qquad (3a)$$

Equivalence therefore demands the satisfaction of three conditions, (1), (2) and (3a).

Lets find the dynamically equivalent system of an actual connecting rod, Fig. 2.

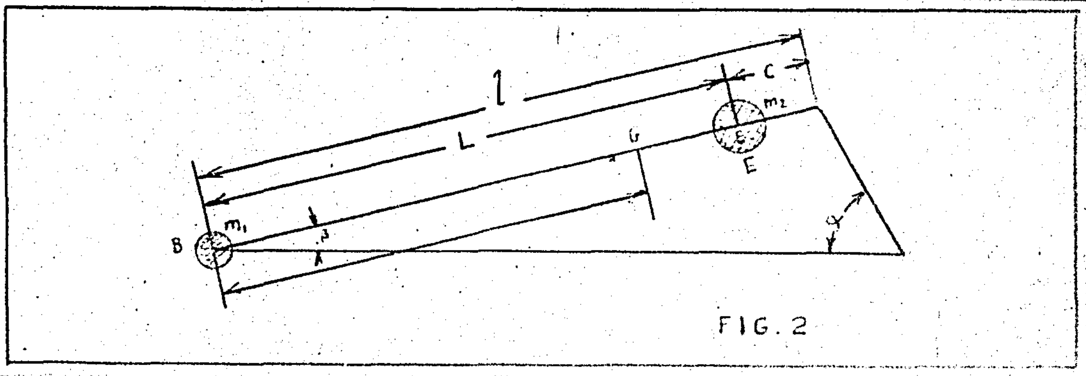

FIG. 2

Equation (3a) shows that if the distance a is fixed, b can not be chosen arbitrarily but is given by the relation:

$$b = \frac{K^2{}_G}{a}$$

If we replace m_1 at B, the position of m_2 is given by equation (3a) as at point E.

$$\overline{GE} = \frac{K^2{}_G}{\overline{BG}}$$

This point E is then the center of percussion of the connecting rod with respect to point B.

If we replace a mass M_1 at B and M_2 at A such that $M_1 + M_2 = m$, Fig.3. a new system will be obtained whose center of percussion may be found from

$$\overline{BG}.\ \overline{GA} = K'G$$

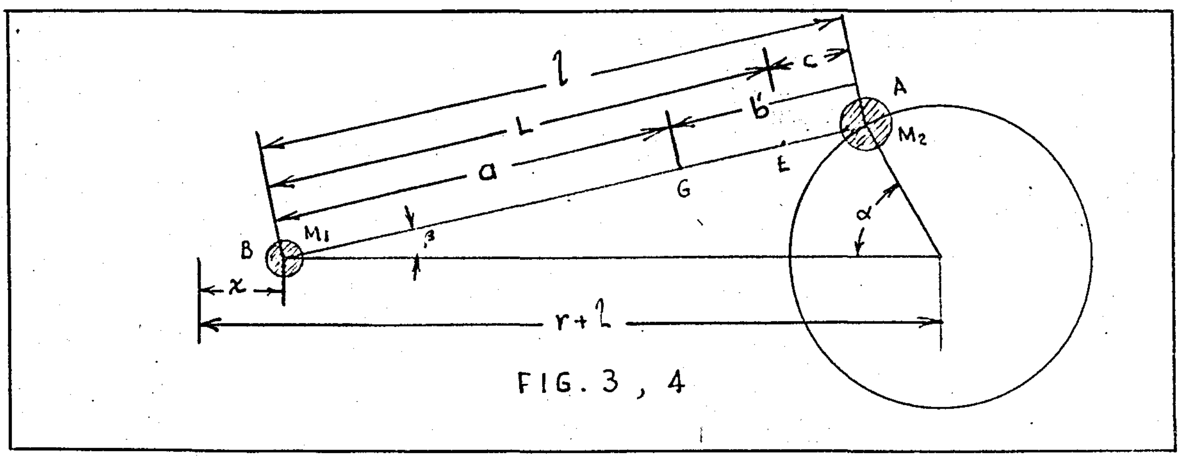

FIG. 3 , 4

Due to the form of the actual connecting-rod $K'G \rangle KG$ and $\overline{GA} \rangle \overline{GE}$. Therefore the new system is not exactly dynamically equivalent to the actual connecting-rod.

However we may consider the new system dynamically equivalent to the actul connecting rod if we introduce a corrective connecting-rod moment, Mc. Referring to Fig. 2.

$$m_1 + m_2 = m$$

$$m_1 a + m_2 b = o$$

$$m_2 = \frac{a}{L} m \, , \quad m_2 L = ma$$

Taking moment about point B:

$$M_B = - L \frac{d^2\beta}{dt^2} \cdot m_2 L = - Lam \frac{d^2\beta}{dt^2}$$

Referring to Fig. 3.

$$M_1 + M_2 = m$$

$$M_1 a + M_2 b' = o$$

$$M_2 = \frac{a}{l} m \, , \quad M_2 l = ma.$$

Again taking moment about B

$$M'_B = - l \frac{d^2\beta}{dt^2} M_2 l = - lam \frac{d^2\beta}{dt^2}$$

$$Mc = M'_B - M_B = - (l-L) \, am \frac{d^2\beta}{dt^2} = - cam \frac{d^2\beta}{dt^2}$$

In the triangle OAB, Fig. 3.

$$\frac{\sin\beta}{r} = \frac{\sin\alpha}{l}$$

$$\sin\beta = \frac{r}{l} \sin\alpha = \lambda \sin\alpha$$

Differentiating with respect to time and replacing $\frac{d\alpha}{dt}$ by ω.

$$\cos\beta \frac{d\beta}{dt} = \lambda\omega\cos\alpha$$

$$\frac{d\beta}{dt} = \frac{\lambda\omega\cos\alpha}{\cos\beta} = \frac{\lambda\omega\cos\alpha}{\sqrt{1-\sin^2\beta}} = \frac{\lambda m\cos\alpha}{\sqrt{1-\lambda^2\sin^2\alpha}}$$

Differentiating once more,

$$\frac{d^2\beta}{dt^2} = \frac{-\lambda(1-\lambda^2)\omega^2\sin\alpha}{(1-\lambda^2\sin^2\alpha)^{3/2}}$$

$$Mc = \frac{(1-L)\ am\ \ (1-\ ^2)\ \ ^2\ \sin}{(1-\ ^2\ \sin^2\ \ )\ 3/2}$$

Mc is called corrective - connecting - rod moment

3. INERTIA EFFECT OF THE RECIPROCATING MASSES IN THE ENGINE MECHANISM.

Referring to Fig. 4., the piston displacement is given by:

$$x = r + 1 - r\cos\alpha - 1\cos\beta \tag{1}$$
$$= r\left[1 - \cos\alpha + \frac{1}{\lambda}(1-\cos\beta)\right]$$

Using $\cos\beta = \sqrt{1-\lambda^2\sin^2\alpha}$

$$x = r\left[1 - \cos\alpha + \frac{1}{\lambda}\left(1 - \sqrt{1-\lambda^2\sin^2\alpha}\right)\right]$$

Expanding the term under radical by binomial series and neglecting the ter after λ^2 (since λ is usually about $\frac{1}{4}$, the terms beyond the square may be dropped without appreciable error), we get:

$$x = r\left(1 - \cos\alpha + \frac{\lambda}{2}\sin^2\alpha\right) \tag{2}$$

Differentiating with respect to time,

$$\frac{dx}{dt} = r\omega(\sin\alpha + \frac{\lambda}{2}\sin 2\alpha) \qquad (3)$$

where ω has been written for $\frac{d\alpha}{dt}$

Differentiating a second time, assuming ω to be constant.

$$\frac{d^2x}{dt^2} = r\,\omega^2(\cos\alpha + \lambda\cos 2\alpha) \qquad (4)$$

The magnitude of the inertia force due to a mass m having this acceleratio

$$F = m\frac{d^2x}{dt^2} = mr\omega^2\ (\cos\alpha + \lambda\ \cos 2\alpha) \qquad (5)$$

This expression for F may be written

$$F = mr\omega^2\ \cos\alpha + mr\lambda\omega^2\ \cos 2\alpha \qquad (6)$$

The term $F_I = mr\omega^2 \cos\alpha$ is called the inertia force of first order and $F_{II} = mr\omega^2\lambda\cos 2\alpha$ is called the inertia force of second order.

The above inertia forces may be determined graphically for any positic of the crank as follows:

First let a suitable force scale be chosen and then, referring to Fig. 5. let two concentric circles be drawn with a radius $OA_1 = mr\omega^2$ and the other

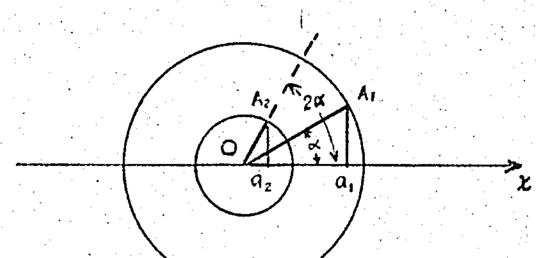

FIG. 5

with $OA_2 = mr\omega^2\lambda$. Now let the crank angle α be laid off as shown and let the line OA_2 be laid off making an angle 2α with the horizontal line ox. Then if the points A_1 and A_2 are projected on the horizontal line ox, the lines oa_1 and oa_2 will represent F_I, the primary and F_{II}, the secondary inertia forces respectively.

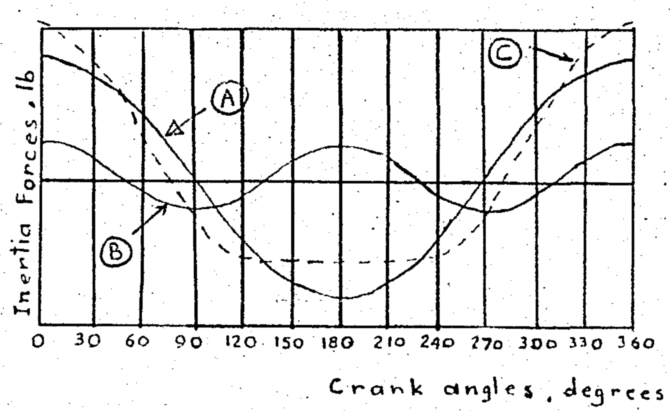

FIG. 6

The primary inertia force, F_I for a single cylinder has been calculated for each 30 - deg. crank position and plotted with a suitable force scale as the harmonic curve A in Fig. 6. Similarly the secondary inertia force F_{II} has been calculated and plotted for $\lambda = \frac{1}{4}$ as the harmonic curve B. Total inertia force, F, the algebraic sum of F_I and F_{II} is plotted as the curve C.

4. FUNDAMENTAL CONDITIONS OF BALANCE

The balancing of an engine requires the satisfaction of the following conditions:

a - Balancing of inertia forces and couples of rotating masses (consisting of the mass of crank pin plus the rotating part of the connecting-rod).

b - Balancing of first order inertia forces and couples of reciprocating masses (consisting of the piston, rings, wrist pin plus the reciprocating part of the connecting-rod).

c - Balancing of second order inertia forces and couples of reciprocating masses.

d - Balancing of corrective connecting-rod moment, Mc.

PART II - THE SINGLE CYLINDER ENGINE

5. BALANCING OF SINGLE CYLINDER ENGINE.

We will consider the conditions of article 4. successively and show if it is possible to realize them.

a - Rotating masses

It is seen from Fig. 7. that without counterweights $Fr \neq o$, but can be balanced completely by counter weights.

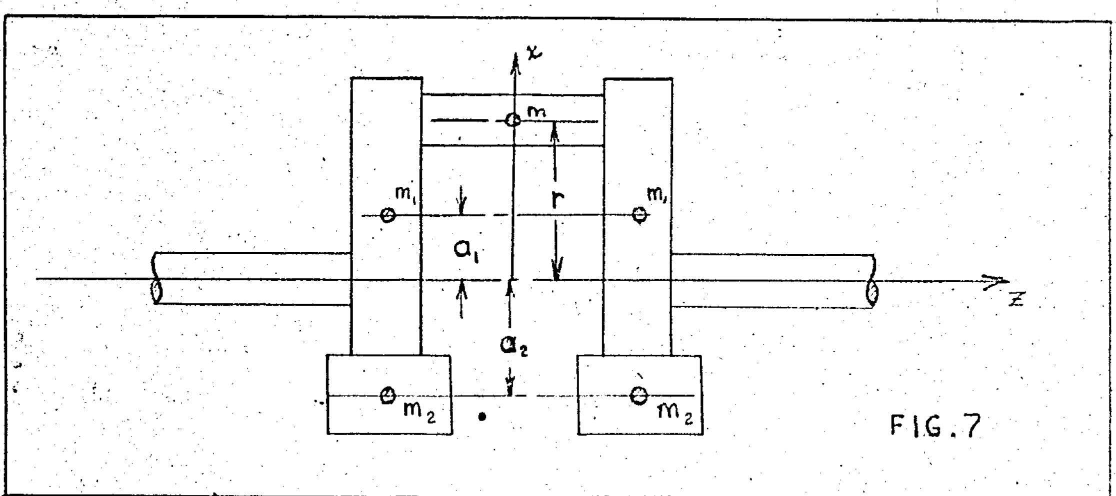

FIG. 7

m = mass of crank pin plus rotating part of connecting-rod (which is given by $\Delta m = \frac{a}{l}$ Mc.R.)

m_1 = mass of the crank arm

m_2 = mass of the counterweight to be added.

$Fr = mr + 2\, m_1 a_1$

Mass product to balance it is:

$$m_2 a_2 = \frac{mr + 2m_1 a_1}{2}$$

The system is symmetrical with respect to the transverse plane through center of gravity therefore dynamic balance is automatically satisfied.

$$Mr = o$$

b - Inertia forces of first order

Inertia force of first order is given by:

$$F_I = M_p r\omega^2 \cos\alpha$$

where M_p is the mass of the piston assembly plus reciprocating part of connecting rod $(1 - \frac{a}{l})\, M_{C.R.}$.

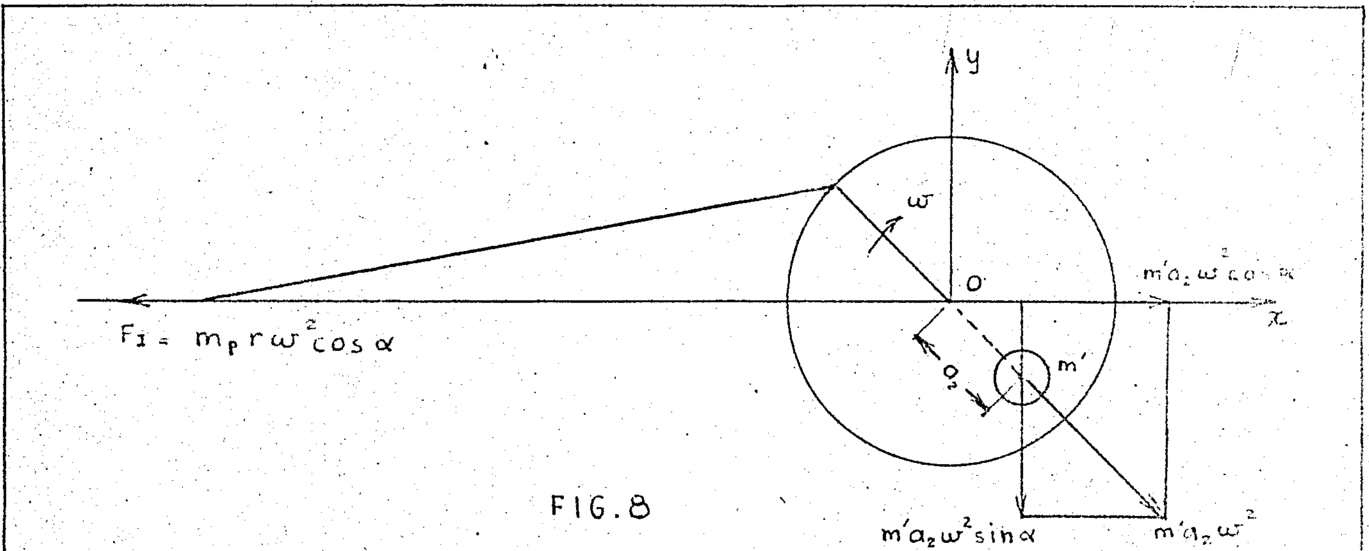

FIG. 8

We may balance this force by a counterweight m'_2 added at the extension of crank arms at a distance a_2 such that

$$m'_2 a_2 = m_p r$$

Then

$$- m_p r\omega^2 \cos\alpha + m'_2 a_2 \omega^2 \cos\alpha = o.$$

But this is not satisfactory. Because though we can balance $\overline{ox}$ component of unbalance, we introduce an unbalance in the vertical plane with magnitude:

$$-m'_2 a_2 \omega^2 \sin\alpha$$

We may have a better condition of balance if the maximum values of horizontal and vertical components of unbalance is equal to each other. This implies:

$$-m_p r\omega^2 + m'_2 a_2 \omega^2 = -m'_2 a_2 \omega^2$$

and

$$m'_2 a_2 = \frac{m_p r}{2}$$

This condition is known as half balancing.

Inertia moment of first order is zero due to symmetry.

$M_I = o$ Automatically.

c - <u>Inertia forces of second order</u>

Inertia force of second order is given by:

$$F_{II} = -m_p r\omega^2 \lambda \cos 2\alpha$$

This force can not be balanced by a counteweight at the extension of crank arms, because the $\overline{ox}$ component of the force due to the counteweight has the component $\cos\alpha$, whereas inertia forces of the second order depend upon the component $\cos 2\alpha$. Counterweight must rotate with a speed of 2ω to take care of the second order forces, which is not possible except for special constructions such as the Lanchester method. Therefore,

$$F_{II} \neq o$$

However we may apply the method of half balancing to total inertia forces due to the reciprocating masses.

$$F = F_I + F_{II} = - M_p r \omega^2 \,(\cos\alpha + \lambda \cos 2\alpha) \quad \text{on } \overline{ox} \text{ axis.}$$

Force due to the counterweight M_2 at a distance a_2,

$$M_2 a_2 \omega^2 \cos\alpha \qquad \text{on } \overline{ox} \text{ axis}$$

and

$$- M_2 a_2 \omega^2 \sin\alpha \qquad \text{on } \overline{oy} \text{ axis}$$

Equating maximum values of the $\overline{ox}$ and $\overline{oy}$ components of these forces, we get

$$- m_p r \omega^2 (1 + \lambda) + M_2 a_2 \omega^2 = - M_2 a_2 \omega^2$$

From which follows:

$$M_2 a_2 = \frac{M_p r \,(1 + \lambda)}{2} = r\,(0.5 + 0.5\lambda)\, m_p$$

General Motors company suggests the following formula for high speed engines:

$$M_2 a_2 = r\,(0.5 + 0.36\lambda)\, m_p$$

Inertia moment of second order is balanced automatically.

$$M_{II} = 0$$

d - <u>Connecting-rod moment.</u>

Corrective connecting-rod moment, which is given by:

$$Mc = (1 - L) m a \omega^2 \lambda (1 - \lambda^2) \frac{\sin\alpha}{(1 - \lambda^2 \sin^2\alpha)^{3/2}}$$

is not balanced and can not be balanced by counterweights.

Index of Unbalance

To compare the degree of balance of engines of different types and with different number of cylinders, an index of unbalance due to Prof. Necdet Eraslan, may be used. The method is to assign values to each item a, b, c, d, above when they are automatically balanced, completely balanced by counterweights, partially balanced by counterweights and when they can not be balanced by counterweights.

The values assigned will depend on the relative importance of the forces and couples involved as far as balancing is concerned.

This is done in the table below:

	STATE OF BALANCE	Fr Mr	F_I M_I	F_{II} M_{II}	Mc
1.	Automatically balanced	0	0	0	0
2.	Completely balanced by counterweights	2	2	1	0.5
3.	Partially balanced by counterweights	3	3	1.5	0.75
4.	Unbalance	4	4	2	1

Sum of these values in a certain engine give the index of unbalance which will be shown by n_u.

For a single cylinder engine we may form the index of unbalance as follows:

No. of cylinder	Crank angle	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_c	n_u
1	0°	2	3	2	0	0	0	1	8

PART III - MULTI - CYLINDER ENGINES

Engines with more than one cylinder may be arranged in different ways. The fundamental arrangements are:

A. - In-line engine
B. - V - Engine
C. - Radial engine
D. - Opposed engine

We will consider them in succession with different number of cylinders.

A. - IN-LINE ENGINE

6. THE TWO-CYLINDER ENGINE

1) Two - stroke cycle - In two-cylinder two stroke engine the two cranks are offset by the angle $\frac{360}{2} = 180^{o}$.

a - Rotating masses

Referring to Fig. 9.

m = mass of crank pin plus rotating part of connecting rod

m_1 = mass of crank arm

m_2 = counterweight

The system is symmetric with respect to center of gravity o. Therefore

$F_r = o$ Automatically

The system is not symmetric with respect to the transverse plane passing through o. Therefore

$M_r \neq o$

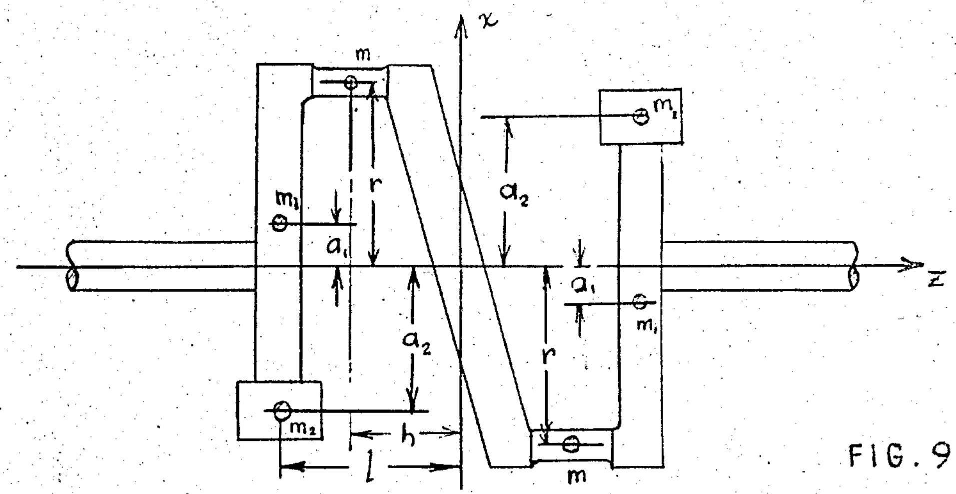

FIG. 9

Counterweights m_2 are added to the extension of crank arms to balance it dynamically.

$$2m_2a_2l - 2m_1a_1l - 2mrh = o$$

$$m_2 = \frac{mrh + m_1a_1l}{a_2\,l}$$

b - Inertia forces of first order

Since cranks are offset by 180^o, inertia force of first order is given by:

$$F_I = - m_p r\omega^2 \left[\cos\alpha + \cos(\alpha + \pi) \right]$$

$$F_I = o \quad \text{Automatically}$$

Inertia moment of first order is given by:

$$M_I = - m_p r\omega^2 h \left[\cos\alpha - \cos(\alpha + \pi) \right]$$

$$M_I = -2m_p r\omega^2 h \cos\alpha$$

This couple can be balanced by a counter couple produced by counterweights m'_2 placed at the extension of crank arms, Fig. 10.

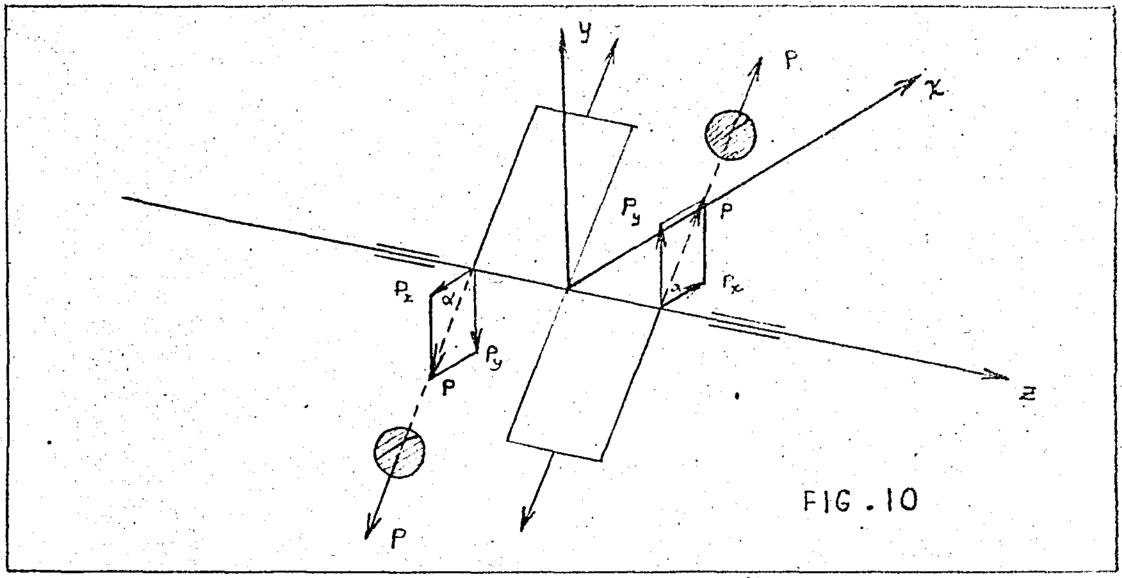

FIG. 10

Force produced by m'_2 is:

$$F = m'_2 r\omega^2$$

The components of this force on $\overline{ox}$ and $\overline{oy}$ axis are

$$Fx = m'_2 r\omega^2 \cos\alpha, \qquad Fy = m'_2 r\omega^2 \sin\alpha$$

The moment of F about y - axis can balance M_I if

$$-2m_p r\omega^2 h \cos\alpha + 4\, m'_2 r\omega^2 h \cos\alpha = o$$

or

$$m'_2 = \frac{m_p}{2}$$

But the moment of F about x - axis remain unbalanced. We use the method of half-balancing to have the same disturbance about x - and y - axes:

This implies:

$$-2m_p r\omega^2 h + 4m'_2 r\omega^2 h = -4\, m'_2 r\omega^2 h$$

AND

$$m'_2 = \frac{m_p}{4}$$

c - Inertia forces of second order

Total inertia force of second order is given by:

$$F_{II} = F_{II\,1} + F_{II\,2} = -m_p r \omega^2 \lambda \left[\cos 2\alpha + \cos 2(\alpha + \pi)\right]$$

$$F_{II} = -2 m_p r \omega^2 \lambda \cos 2\alpha$$

This force can not be balanced by counterweights, because these weights must revolve with a speed of (2ω) which is not possible without using special mechanisms, such as the Lanchester machine.

$M_{II} = 0$ Automatically.

d - Connecting-rod moment

$$M_c = a(1 - L)\,\lambda(1 - \lambda^2)\, m\omega^2 \left[\frac{\sin\alpha}{(1 - \lambda^2 \sin^2\alpha)^{3/2}} + \frac{\sin(\alpha + \pi)}{\left[1 - \lambda^2 \sin^2(\alpha + \pi)\right]^{3/2}}\right]$$

The term in parenthesis is zero, therefore

$M_c = 0$ Automatically

The force and Moment diagram for each item is shown in Fig. 11.

We form the index of unbalance:

No. of cylinder	crank angle	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_c	n_u
2	180^0	0	0	2	2	3	0	0	7

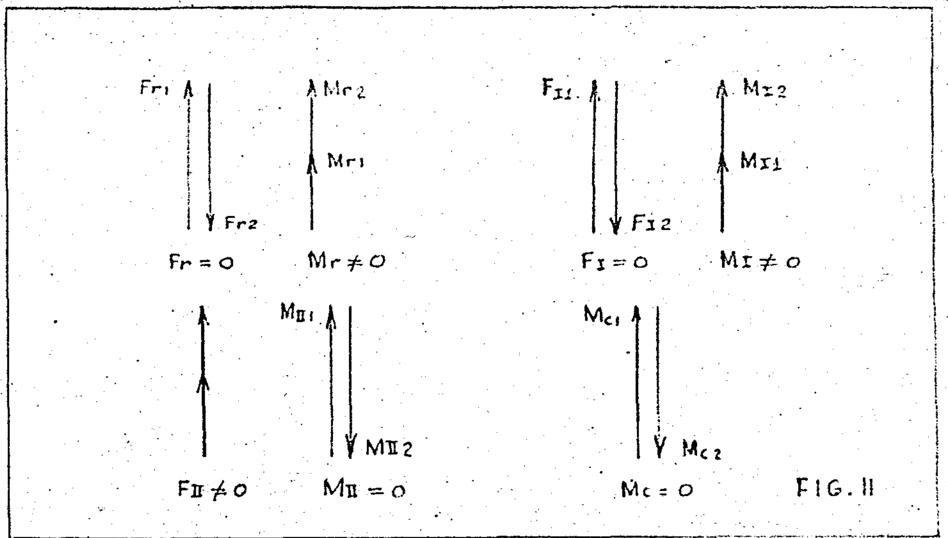

FIG. II

2) Four-stroke cycle - Since equal firing intervals require the cranks to be offset by the angle $\frac{720}{2} = 360^\circ$ or 0°, the conditions for inertia balancing are exactly those given for one-cylinder engine, with doubled values of masses involved. Index of unbalance then, is:

No. of cylinder	crank angle	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_c	n_u
2	360° or 0°	2	3	2	0	0	0	1	8

7. THE THREE-CYLINDER ENGINE

In the balancing of the three-cylinder engine it is unnecessary to distinguish between two-stroke and four-stroke types. For the two-stroke engine the crank-offset angle is $\frac{2\pi}{3} = 120^\circ$ and for the four-stroke $\frac{4\pi}{3} = 240^\circ$. The angles are equivalent to each other and result in similar (or mirror image) crank arrangements.

a - Rotating masses

To see if the rotating masses are in static and dynamic balance, ~~We~~ we draw the force and moment diagram using Dalby's method. We first compete the mass product and mass torque, shown in the table below:

No.	Mass	Radius	Mass Product	Distance From B	Mass Torque
1	m	r	mr	2a	2mra
2	m	r	mr	a	mra
3	m	r	mr	o	o

Force polygon is closed, therefore static balance prevails automatically:

$F_r = o$ Automatically

Moment polygon is not closed, therefore the system is dynamically unbalanced

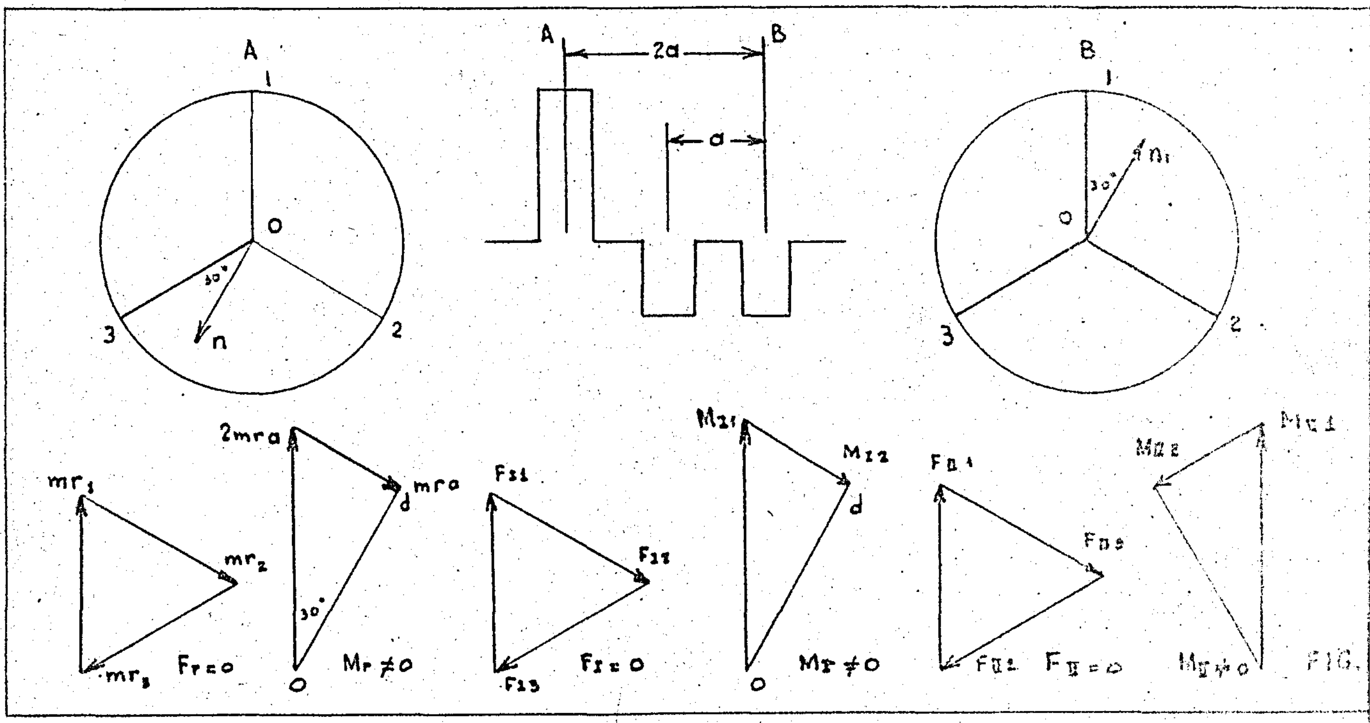

FIG.

However by two mass products on_1 and $on = - on_1$ replaced on B and A planes respectively, in the direction shown in Fig. 12, we may balance the moment of rotating masses.

$$on_1 = - on = \frac{od}{2a} = \frac{\sqrt{3}}{2}\ \frac{2mra}{2a} = \frac{\sqrt{3}}{2} mr$$

Counterweights can not be placed at the planes A and B, instead we place the at the neighbour crank arms at both sides of the planes as in Fig. 13.

Then

$$2m_1r_1 = \frac{\sqrt{3}}{2}mr$$

$$m_1r_1 = \frac{\sqrt{3}}{4}mr$$

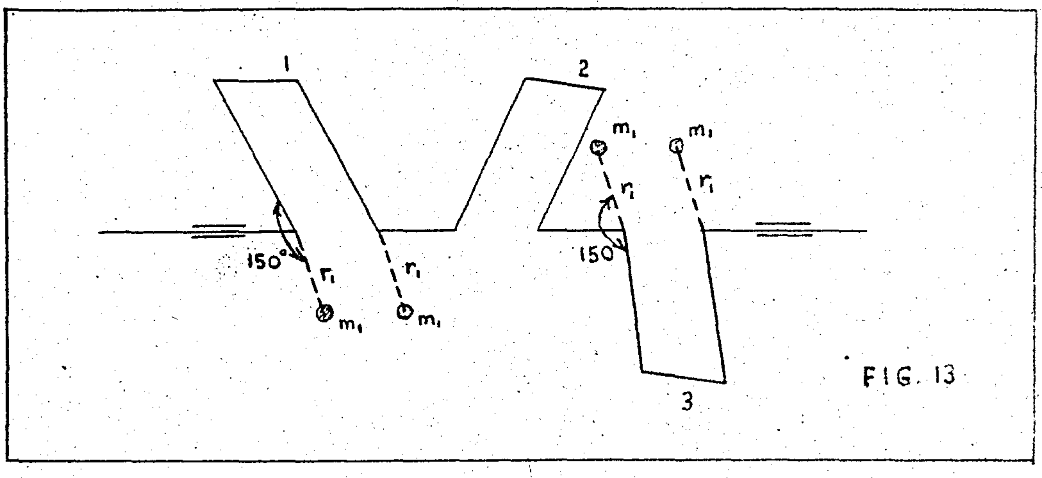

FIG. 13

b - Inertia forces of first order

Total inertia force of first order is given by:

$$F_I = -mr\omega^2\left[\cos\alpha + \cos\left(\alpha + \frac{2\pi}{3}\right) + \cos\left(\alpha + \frac{4\pi}{3}\right)\right]$$

$$F_I = -mr\omega^2\left[\cos\alpha + \left(-\frac{1}{2}\cos\alpha - \frac{\sqrt{3}}{2}\sin\alpha\right) + \left(-\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha\right)\right]$$

$F_I = 0$ Automatically

To find the unbalance of moments of first order, we proceed exactly as we did for rotating masses. The mass product to be added to m, r, ' s to balance the first order moment is given by:

$$m_2 r_2 = \frac{\sqrt{3}}{4} m_p r$$

where m_p includes reciprocating part of connecting rod. However this mass product introduces an unbalanced moment about the other axis. Again using the method of half balancing, the mass product to be added, reduces to

$$m_2 r_2 = \frac{\sqrt{3}}{8} m_p r$$

c - Inertia forces of second order

Inertia forces of second order are given by:

$$F_{II} = - m_p r \lambda \omega^2 \left[\cos 2\alpha + \cos 2 \left(\alpha + \frac{2\pi}{3}\right) + \cos 2 \left(\alpha + \frac{4\pi}{3}\right) \right]$$

$$F_{II} = - m_p r \lambda \omega^2 \left[\cos 2\alpha + \left(- \frac{1}{2} \cos 2\alpha + \frac{\sqrt{3}}{3} \sin 2\alpha\right) + \left(-\frac{1}{2} \cos 2\alpha - \frac{\sqrt{3}}{2} \sin 2\right.\right.$$

$F_{II} = o$ Automatically

Inertia moment of second order is not zero, and can not be balanced by counter weights.

$$M_{II} \neq o$$

d - Connecting-rod moment

Total connecting rod moment is given by:

$$M_c = (1 - L) a m \omega^2 \lambda (1 - \lambda^2) \left[\frac{\sin\alpha}{(1 - {}^2 \sin^2)^{3/2}} + \frac{\sin(\alpha + \frac{2\pi}{3})}{\left[1 - \lambda^2 \sin^2 (\alpha + \frac{2\pi}{3})\right]^{3/2}} \right.$$

$$\left. + \frac{\sin\left(\alpha + \frac{4\pi}{3}\right)}{\left[1 - \lambda^2 \sin^2 \left(\alpha + \frac{4\pi}{3}\right)\right]^{3/2}} \right]$$

Expanding $\dfrac{\sin\alpha}{(1-\lambda^2\sin^2\alpha)^{3/2}}$ in Fourier series and neglecting the terms with λ to the 5th and greater powers, we get

$$\frac{\sin\alpha}{(1-\lambda^2\sin^2\alpha)^{3/2}} = \left(1+\frac{9\lambda^2}{8}\right)\sin\alpha - \frac{3\lambda^2}{8}\sin 3\alpha - \cdots$$

The other two terms also are expanded in series, to give $\frac{9}{8}\lambda^2 \sin 3\alpha$ for the quantity in paranthesis. Then

$$M_c = (1-L)\, a m \omega^2 \, \frac{9}{8}\lambda^3 \sin 3\alpha$$

This moment can not be balanced.

The index of unbalance become.

No. of cylinder	crank angle	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_c	n_u
3	120^0	0	0	0	2	3	2	1	8

8. FOUR - CYLINDER ENGINE

1) Four-stroke cycle - The four-cylinder four stroke engine has a crank offset of $\frac{4\pi}{4} = 180^0$ and consequently, in respect to inertia forces and moment it is equivalent to two-cylinder two-stroke engines built together. Of the possible crank arrangements, the symmetric one shown in fig. 14 is by far the best since with it both static and dynamic balance of rotating parts are satisfied.

a - Rotating masses

As it is shown in Fig. 15

$F_r = 0$ Automatically

$M_r = 0$ Automatically

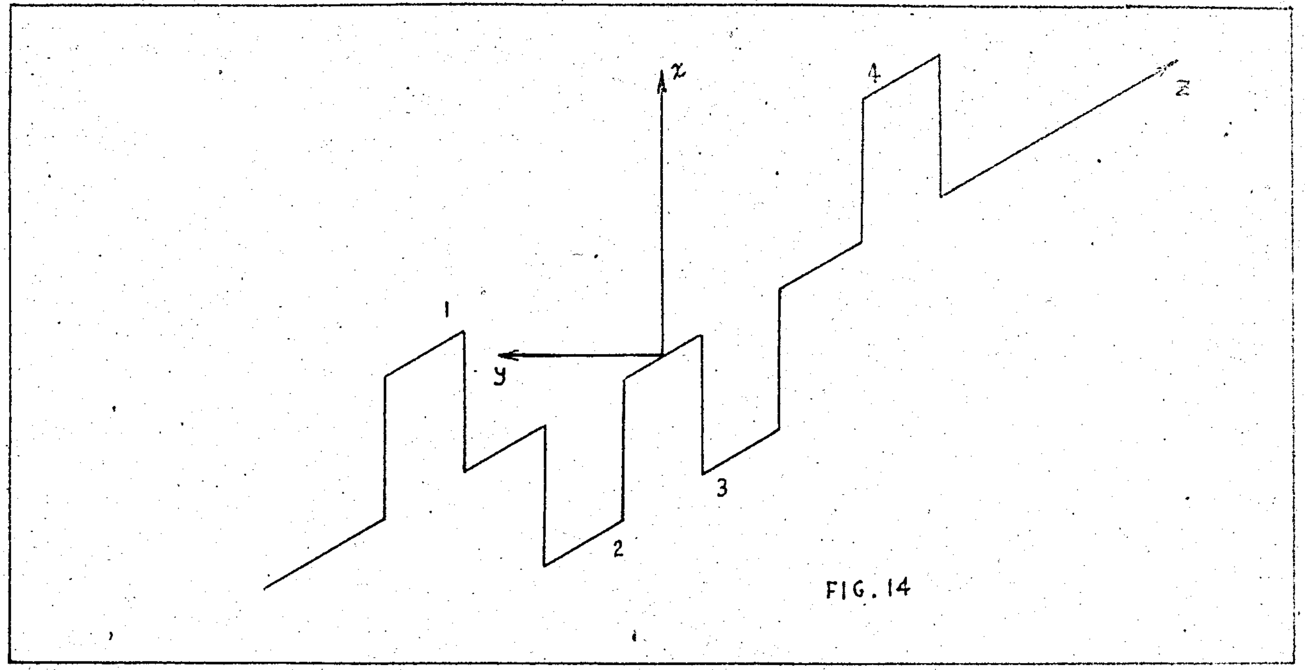

FIG. 14

b - Inertia forces of first order

Both first order force and moment polygons close.

$F_I = o$ Automatically

$M_I = o$ Automatically

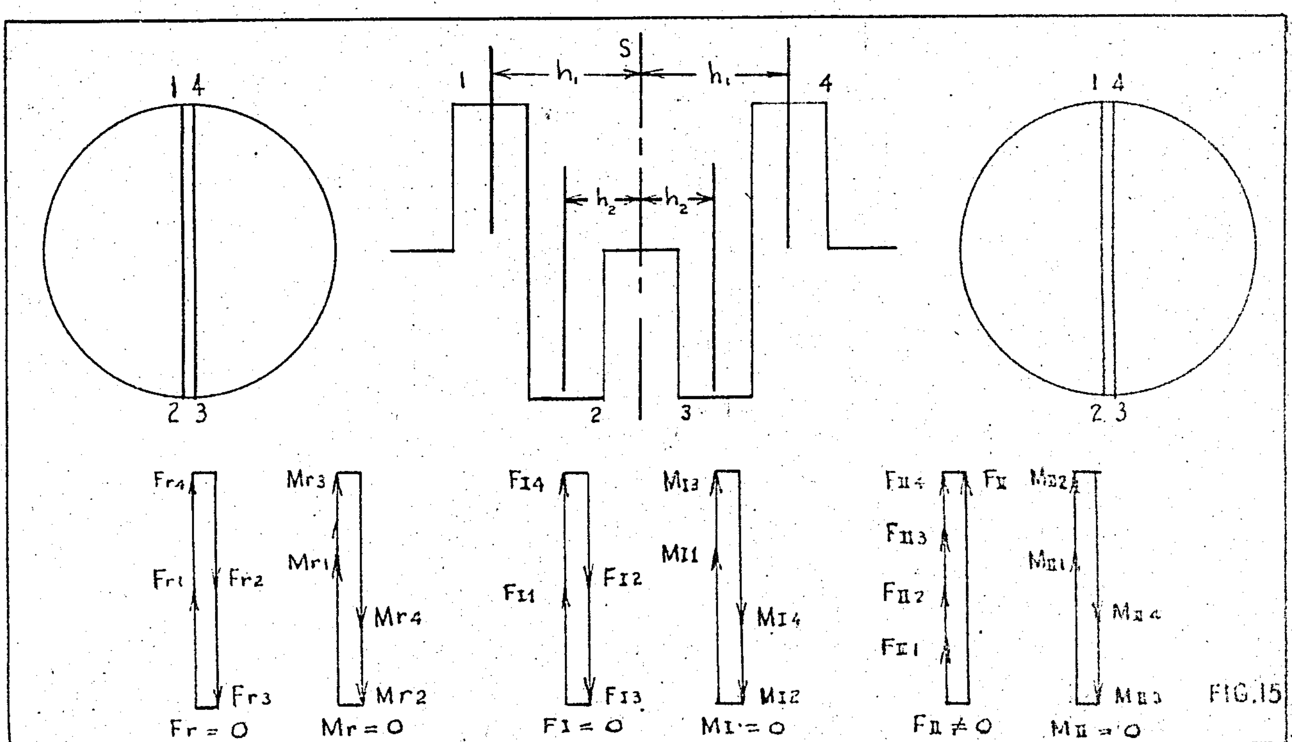

FIG.15

c - Inertia forces of second order

From Fig. 15. it is seen that the forces built up and the force polygon does not close.

$$F_{II} = - 4\, m_p r \omega^2 \lambda \cos 2\alpha$$

This force can not be balanced by counterweights, unless we use the Lanchest system

$$M_{II} = 0 \quad \text{Automatically}$$

d - Connecting-rod moment

Total connecting-rod moment for the system is:

$$M_c = (1 - L)\, a m \omega^2 \lambda (1 - \lambda^2) \left[\frac{2 \sin \alpha}{(1 - \lambda^2 \sin^2 \alpha)^{3/2}} \quad \frac{2 \sin (\alpha + \pi)}{\left[1 - \lambda^2 \sin^2 (\alpha + \pi)\right]^{3/2}} \right]$$

$$M_c = 0$$

We form the index of unbalance

No. of cylinder	crank angle	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_c	n_u
4	180^o	0	0	2	0	0	0	0	2

2) Two-stroke cycle - In the four-cylinder two-stroke engine the cranks of cylinder that fire consecutively are offset $\frac{2\pi}{4} = 90^o$.

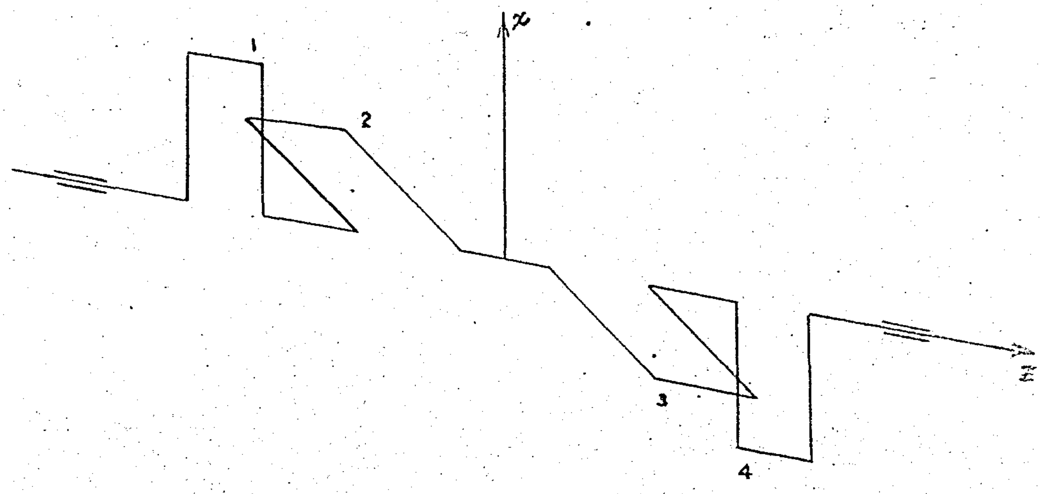

FIG. 16

The arrangement shown in Fig. 16 is the most frequently met in practice with the firing order 1 - 2 - 4 - 3 or 1 - 3 - 4 - 2.

a - <u>Rotating masses</u>

Force polygon closes, therefore

$F_r = 0$ Automatically

Moment polygon does not close

$$M_r = mr\omega^2 \sqrt{10}\, h$$

$$\theta = \tan^{-1} \frac{1}{3} = 18^\circ 25'$$

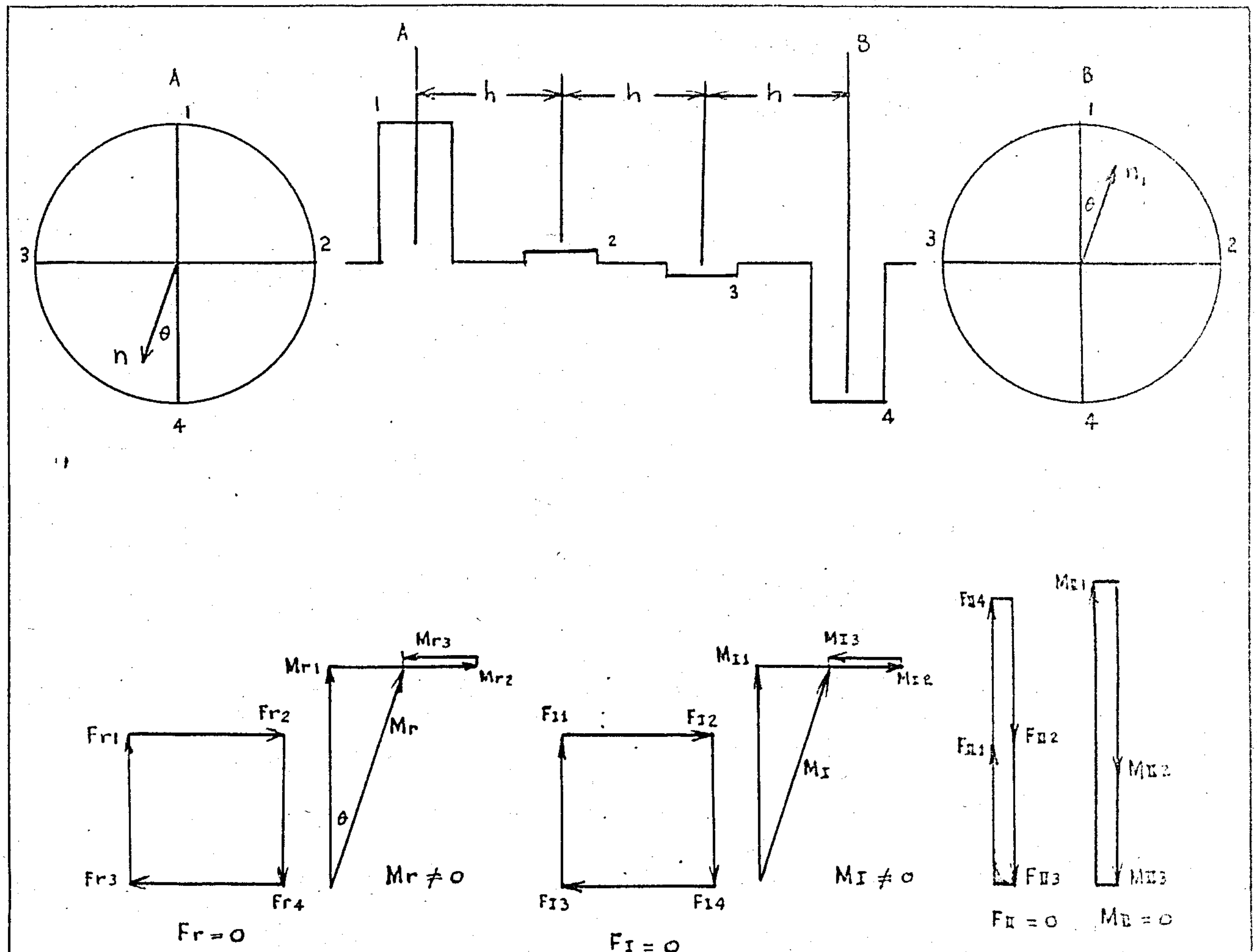

FIG. 17

The inertia moment due to rotating masses can be balanced completely by two mass products on = - on$_1$ with an angle θ to vertical and having the magnitude:

$$on = -on_1 = \frac{\sqrt{10}}{3}\ mr$$

on planes A on B as shown in Fig. 17. Instead of replacing masses on planes A and B, we replace them at the extension of crank arms 1 and 4, with

$$m_1 r_1 = \frac{\sqrt{10}}{6}\ mr$$

b - Inertia forces of first order

The force polygon closes as shown in Fig. 17

$$F_I = o$$

Moment polygon does not close and resultant moment is:

$$M_I = - m_p r\omega^2 \sqrt{10}\ h \cos\alpha$$

As before this moment can be balanced by m_2 counterweights added at the extension of crank arms 1 and 4 with the same angle θ. But as we had seen in previous sections these counterweights create an unbalance due to term $\sin\alpha$. Therefore we use the method of half balancing and chose counterweight

$$m_2 r_2 = \frac{\sqrt{10}}{12} mr$$

c - Inertia forces of second order

Force polygon closes as shown in Fig. 17. Therefore

$$F_{II} = o \quad \text{Automatically}$$

Moment polygon closes also. Therefore

$$M_{II} = o \quad \text{Automatically}$$

d - Connecting-rod moment

Connecting-rod moment is given by

$$M_c = (1 - L)\ am\omega^2\ \lambda(1 - \lambda^2)\left[\frac{\sin\alpha}{(1 - \lambda^2\sin^2\alpha)^{3/2}} + \frac{\sin(\alpha + \frac{\pi}{2})}{\left[1 - \lambda^2\sin^2(\alpha + \frac{\pi}{2})\right]^{3/2}} + \frac{\sin(\alpha + \pi)}{\left[1 - \lambda^2\sin^2(\alpha + \pi)\right]^{3/2}} + \frac{\sin(\alpha + \frac{3\pi}{2})}{\left[1 - \lambda^2\sin^2(\alpha + \frac{3\pi}{2})\right]^{3/2}}\right]$$

$$M_c = o \quad \text{Automatically}$$

The index of unbalance become:

No. of cylinder	Crank angle	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_o	n_u
4	90^o	0	0	0	2	3	0	0	5

9. **FIVE-CYLINDER ENGINE.**

Crank angle offset is $\frac{360}{5} = 72^o$. For four-stroke engines firing interval is $\frac{720}{5} = 144^o$ firing order is 1 - 4 - 2 - 5 - 3. Four two-stroke engines firing intervla is 72^o and firing order 1 - 5 - 4 - 3 - 2.

a - Rotating masses

Force polygon closes, therefore

$F_r = o$

crank no.	Mass	Radius	Mass product	Distance from B	Mass Torque
1	m	r	mr	a = 4d	mra
2	m	r	mr	b = 3d	mrb
3	m	r	mr	c = 2d	mrc
4	m	r	mr	d	mrd
5	m	r	mr	o	o

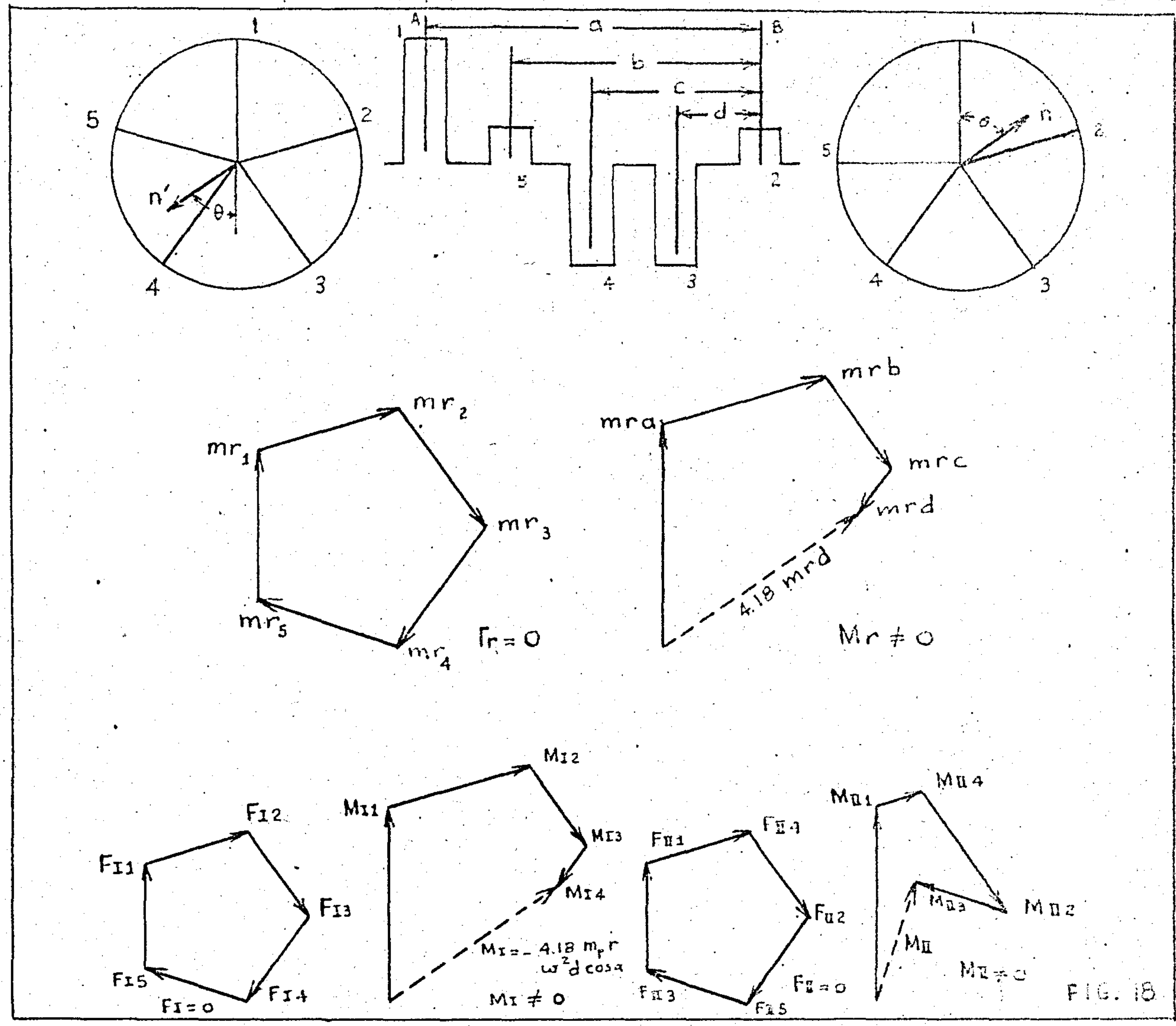

FIG. 18

Moment polygon does not close and has the magnitude:

$$M_r = 4.18 \text{ mrd}$$

$$\theta = \tan^{-1} \frac{3.44}{2.38} = 55^{\circ}24' \text{ with the vertical}$$

This moment can be balanced by two mass products on and on_1 placed θ degrees from vertical on A and B planes:

$$on_1 = - on = \frac{4.18 \text{ mrd}}{4d} = 1.0425 \text{ mr}$$

These counterweights can not be placed on A and B planes, but at the extension of first and last crank arms with

$$8\, m_1 r_1 d = 4.18\ mrd$$

$$m_1 r_1 = \frac{4.18}{8}\ mr$$

b - Inertia forces of first order

Force polygon closes, Fig. 18.

$$F_I = o$$

Moment diagram does not close. Resultant moment has the magnitude:

$$M_I = -\,4.18\ m_p r \omega^2 d \cos\alpha$$

This moment can be balanced by mass products on_2 and on'_2 on B and A planes respectively, with the magnitude

$$on'_2 = -\,on_2 = \frac{4.18}{4}\, m_p r$$

Then we introduce an unbalance in the vertical plane due to sine term. therefore we use half-balancing. Then

$$on'_2 = on_2 = \frac{4.18}{8}\, m_p r$$

c - Inertia force of second order

The force polygon closes, therefore

$$F_{II} = o$$

Moment polygon does not close

$$M_{II} \neq o$$

and can not be balanced by counter weights.

d - Connecting-rod moment

Connecting rod moment is given by:

$$M_c = K\left[\frac{\sin\alpha}{(1-\lambda^2\sin^2\alpha)^{3/2}} + \frac{\sin(\alpha+\frac{2\pi}{5})}{\left[1-\lambda^2\sin^2(\alpha+\frac{2\pi}{5})\right]^{3/2}} + \frac{\sin(\alpha+\frac{4\pi}{5})}{\left[1-\lambda^2\sin^2(\alpha+\frac{4\pi}{5})\right]^{3/2}} + \frac{\sin(\alpha+\frac{6\pi}{5})}{\left[1-\lambda^2\sin^2(\alpha+\frac{6\pi}{5})\right]^{3/2}} + \frac{\sin(\alpha+\frac{8\pi}{5})}{\left[1-\lambda^2\sin^2(\alpha+\frac{8\pi}{5})\right]^{3/2}}\right.$$

where $K = (1-L)\, a\, m\omega^2\lambda(1-\lambda^2)$

If we expanded $\dfrac{\sin\alpha}{(1-\lambda^2\sin^2\alpha)^{3/2}}$ in series

and neglect the terms with λ to the 5th and greater powers we get:

$$\frac{\sin\alpha}{(1-\lambda^2\sin^2\alpha)^{3/2}} = \left(1+\frac{9\lambda^2}{8}\right)\sin\alpha - \frac{3\lambda^2}{8}\sin 3\alpha \;;$$

The other terms too can be expended in series to give the value

$$\left(1+\frac{9\lambda^2}{8}\right)\left[\sin\alpha + \sin\left(\alpha+\frac{2\pi}{5}\right) + \sin\left(\alpha+\frac{4\pi}{5}\right) + \sin\left(\alpha+\frac{6\pi}{5}\right) + \sin\left(\alpha+\frac{8\pi}{5}\right)\right]$$

$$-\frac{3\lambda^2}{8}\left[\sin 3\alpha + \sin\left(3\alpha+\frac{6\pi}{5}\right) + \sin\left(3\alpha+\frac{12\pi}{5}\right) + \sin\left(3\alpha+\frac{18\pi}{5}\right) + \sin\left(3\alpha+\frac{24\pi}{5}\right)\right]$$

for the quantity in paranthesis

Using the identity:

$$\sin a + \sin(a+b) + \sin(a+2b) + \dots + \sin\left[a+(n-1)\,b\right] = \frac{\sin(n\frac{b}{2})}{\sin(\frac{b}{2})}\,\sin\left(a+\frac{n-1}{2}\,b\right)$$

$$M_c = K\left[\left(1+\frac{9\lambda^2}{8}\right)\frac{\sin\pi}{\sin\frac{\pi}{5}}\sin\left(\alpha+\frac{4\pi}{5}\right)-\frac{3\lambda^2}{8}\,\frac{\sin 6\pi}{\sin\frac{3\pi}{5}}\sin\left(3\alpha+\frac{12\pi}{5}\right)\right]$$

Both terms are zero, therefore:

$$M_c = 0$$

The index of unbalance become:

No. of cylinder	Crank angle	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_c	n_u
5	72°	o	o	o	2	3	2	o	7

10. SIX - CYLINDER ENGINE

In the six-cylinder four-stroke engine the cranks of cylinder that fire consecutively are offset $\frac{4\pi}{6} = 120^\circ$. In each plane 120° from each other two cranks are placed.

Force and moment diagrams are shown in Fig. 19.

a - Rotating masses

Both force and moment diagrams close, therefore,

$F_r = o$ Automatically

$M_r = o$ Automatically

b - Inertia force of first order

Both force and moment diagrams close, therefore

$F_I = o$ Automatically

$M_I = o$ Automatically

c - Inertia force of second order

Both force and moment diagrams close, therefore:

$F_{II} = o$ Automatically

$M_{II} = o$ Automatically

d - Connecting-rod moment

Connecting-rod moment is given by:

$$M_c = K\left[\frac{2\sin\alpha}{(1-\lambda^2\sin^2\alpha)^{3/2}} + \frac{2\sin(\alpha+\frac{2\pi}{3})}{\left[1-\lambda^2\sin^2(\alpha+\frac{2\pi}{3})\right]^{3/2}} + \frac{2\sin(\alpha+\frac{4\pi}{3})}{\left[1-\lambda^2\sin^2(\alpha+\frac{4\pi}{3})\right]^{3/2}}\right.$$

The term in paranthesis, if expanded in Fourier series and the terms with to the 5th and greater powers are neglected M_c becomes:

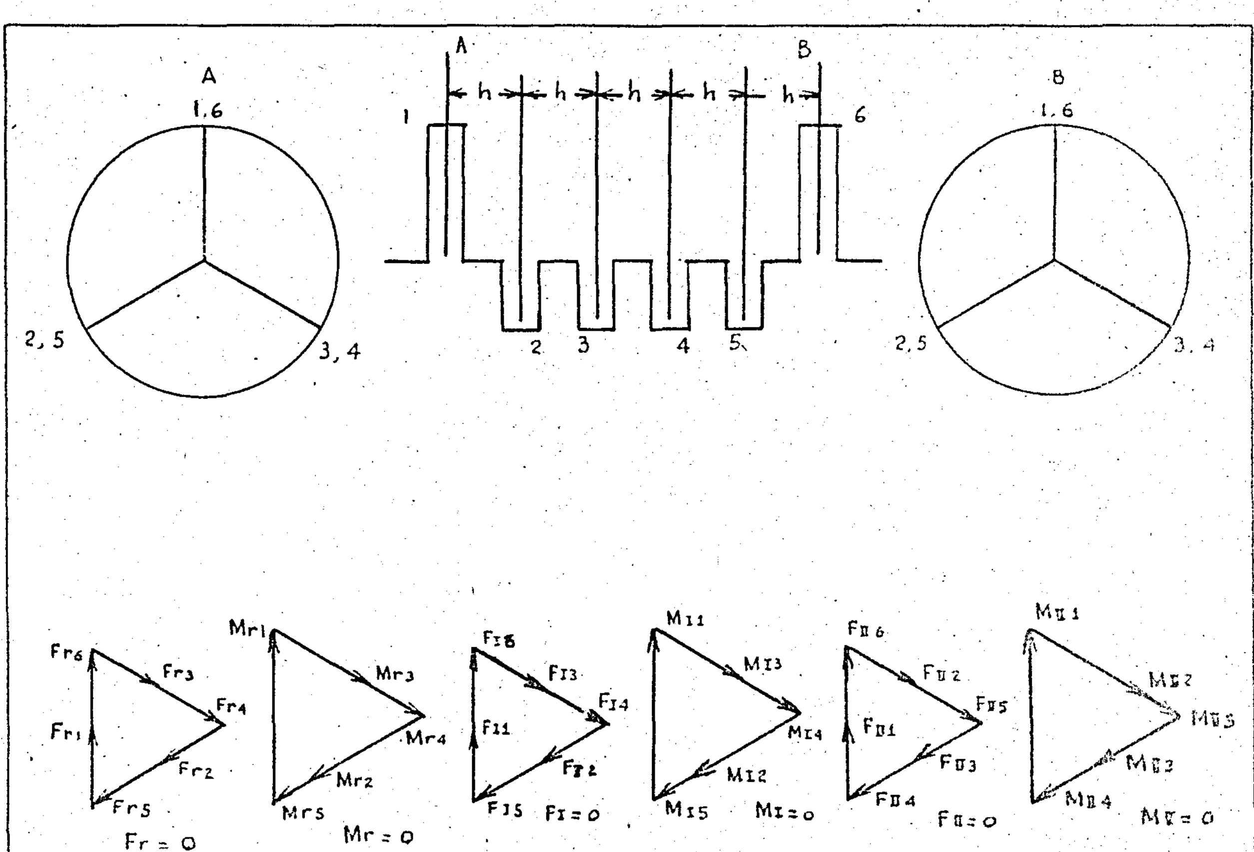

FIG. 19

$$M_c = (1 - L)\, a\, m\omega^2\, \frac{9}{4} \lambda^3 \sin 3\alpha$$

It can not be balanced by counterweights unless by the Lanchester system where diameter of second gear must be chosen such that when crank rotates with angular speed ω, it will rotate with 3ω. However due to λ^3, its magnitude is not appreciable.

The index of unbalance is shown in the table below:

No. of cylinder	crank angle	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_c	n_u
6	120^0	o	o	o	o	o	o	1	1

B - V-ENGINES

11. V-8 PLANE CRANK - SHAFT ENGINE

In this types cranks are in the same plane as shown in Fig. 20

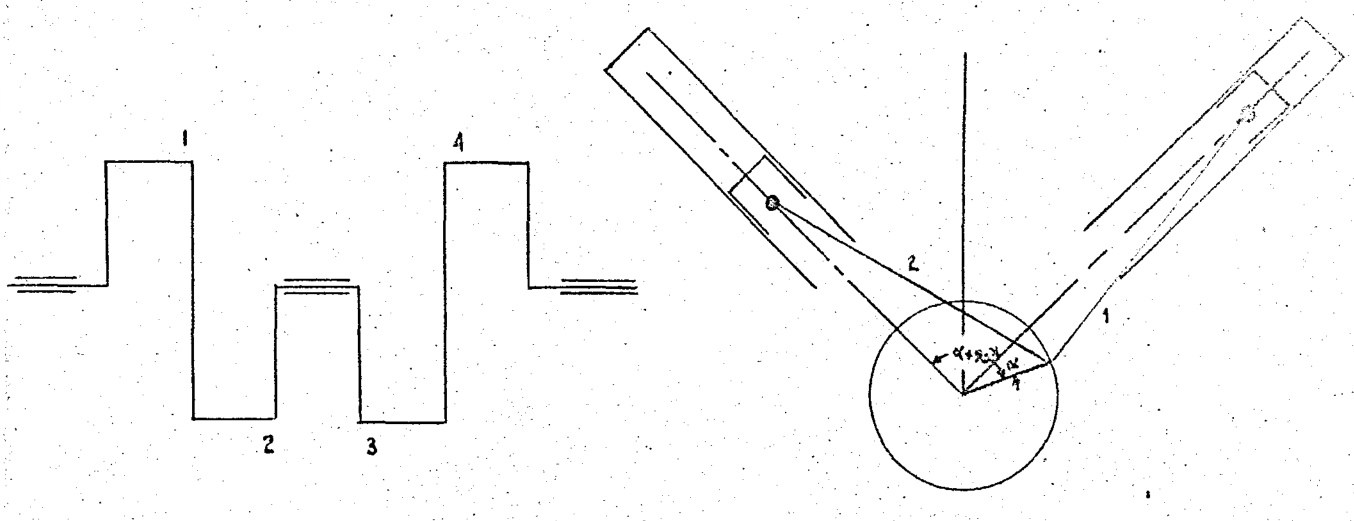

FIG. 20

a - Rotating masses

Balancing of rotating masses is the same as that of in-line engines with the exception that mass of the rotating parts is increased m includes mass of crank plus rotating parts of two connecting-rod.

As it is seen from Fig.15., inertia forces and moments are balanced automatically.

$F_r = o$ Automatically

$M_r = o$ Automatically

b - Inertia force of first order

For four-stroke cycle, firing interval is $\frac{4\pi}{8} = 90^o$. Therefore the angle between banks must also be 90^o.

Taking $A = -M_p r \omega^2$, we can write the following equations for each bank.

Right bank	Left bank
$F_{I1} = A \cos \alpha$	$F'_{I1} = A \cos (\alpha + \frac{\pi}{2})$
$F_{I2} = A \cos (\alpha + \pi)$	$F'_{I2} = A \cos (\alpha + \frac{3\pi}{2})$
$F_{I3} = A \cos (\alpha + \pi)$	$F'_{I3} = A \cos (\alpha + \frac{3\pi}{2})$
$F_{I4} = A \cos (\alpha + 2\pi)$	$F'_{I4} = A \cos (\alpha + \frac{5\pi}{2})$
$F_{I1} = F_{I2} + F_{I3} + F_{I4} = o$	$F'_{I1} + F'_{I2} + F'_{I3} + F'_{I4} = o$

Therefore:

$$F_I = o$$

Likewise each bank has, due to symmetry about the transverse plane from cent of gravity, dynamic balance, resulting total inertia moment to vanish

$$M_I = o$$

c - Inertia forces of second order

Denoting $- m_p r \omega^2 \lambda$ by B, and setting equation of second order forces for each bank, we got the following result:

Right bank	Left bank
$F_{II1} = B \cos 2\alpha$	$F'_{II1} = B \cos 2 (\alpha + \frac{\pi}{2})$
$F_{II2} = B \cos 2 (\alpha + \pi)$	$F'_{II2} = B \cos 2 (\alpha + \frac{3\pi}{2})$
$F_{II3} = B \cos 2 (\alpha + \pi)$	$F'_{II3} = B \cos 2 (\alpha + \frac{3\pi}{2})$
$F_{II4} = B \cos 2\alpha$	$F'_{II4} = B \cos 2 (\alpha + \frac{5\pi}{2})$

$F_{II1} + F_{II2} + F_{II3} + F_{II4} = 4B\cos 2$ $\quad F'_{II1} + F'_{II2} + F'_{II3} + F'_{II4} = -4B\cos 2$

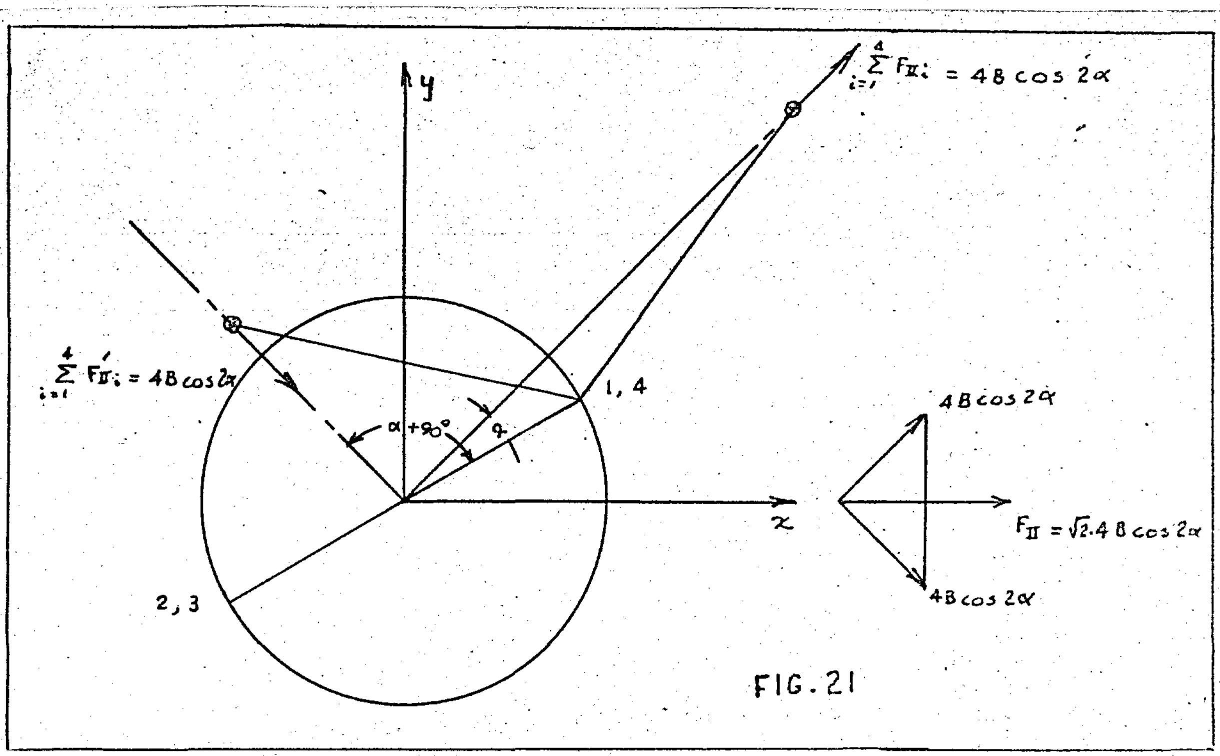

FIG. 21

When these two forces summed up vectorially their resultant become:

$$F_{II} = -4\sqrt{2}\, m_p r \omega^2 \lambda \cos 2\alpha$$

This force is along x - axis, Fig. 21 and can be balanced by the Lanchester method only. Inertia moment of second order vanishes for both banks, as it is shown for four-cylinder, four - stroke in line engine, Fig. 15. Therefore

$M_{II} = 0$ Automatically

d - <u>Connecting-rod moment</u>

Total connecting-rod moment is given by:

$$M_c = 2K\left[\frac{\sin\alpha}{(1-\lambda^2\sin^2\alpha)^{3/2}} + \frac{\sin(\alpha+\frac{\pi}{2})}{[1-\lambda^2\sin^2(\alpha+\frac{\pi}{2})]^{3/2}} + \frac{\sin(\alpha+\pi)}{[1-\lambda^2\sin^2(\alpha+\pi)]^{3/2}} + \frac{\sin(\alpha+\frac{3\pi}{2})}{[1-\lambda^2\sin^2(\alpha+\frac{3\pi}{2})]^{3/2}}\right]$$

where K stands for $(1 - L)\, a\, m\omega^2 \lambda (1 - \lambda^2)$.

The term in paranthesis is zero, therefore:

$$M_c = 0$$

The index of unbalance become:

No. of cylinder	Crank angle	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_c	n_u
V-8 Plane shaft	$180^\circ, \theta = 90^\circ$	0	0	2	0	0	0	0	2

12. V-8 FORD ENGINE

In this arrangement, Fig.22 cranks lie in two perpendicular planes. Car engines are usually of this type. In order that firing interval for four-stroke engines be $\frac{2\pi}{8} = 90^\circ$, the angle between banks, θ, must also be 90°

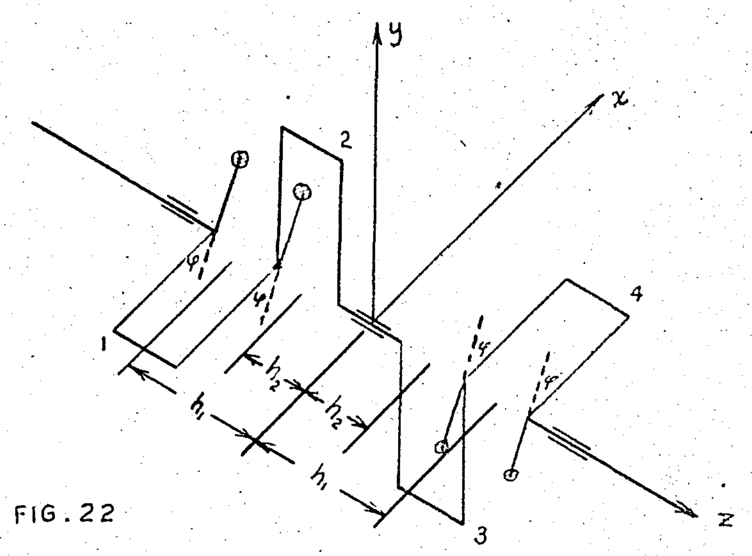

FIG.22

a - Rotating masses

Due to symmetry about center of gravity the inertia force of rotating masses is balanced automatically, Fig. 17.

$F_r = o$ Automatically

In considering the inertia moment of rotating masses we divide M_r into two parts Mrx and Mry, moments of rotating masses about x - and y - axis respectively.

$$Mrx = 2\ mr\omega^2 h_2 \text{ clockwise}$$

$$Mry = 2\ mr\omega^2 h_1 \text{ counter clockwise}$$

These moments can be balanced completely by counterweights in the y - z and x - z planes, replaced at the extensions of crank arms. Therefore though $M_r \neq o$, it is possible to make it so by counter weights.

b - Inertia forces of first order

Denoting $-m_p r\omega^2$ by A, we write the inertia forces of first order for each bank, referring to Fig. 23.

Right bank	Left bank
$F_{I1} = A \cos\alpha$	$F'_{I1} = A\cos(\alpha + \frac{\pi}{2})$
$F_{I2} = A\cos(\alpha + \frac{\pi}{2})$	$F'_{I2} = A\cos(\alpha + \pi)$
$F_{I4} = A\cos(\alpha + \pi)$	$F'_{I4} = A\cos(\alpha + \frac{3\pi}{2})$
$F_{I3} = A\cos(\alpha + \frac{3\pi}{2})$	$F'_{I3} = A\cos(\alpha + 2\pi)$
$F_{I1} + F_{I2} + F_{I3} + F_{I4} = o$	$F'_{I1} + F'_{I2} + F'_{I3} + F'_{I4} = o$

Therefore,

$F_I = o$ Automatically.

To find M_I, we refer to Figs. 22 and 23, and take moments about x-axis.

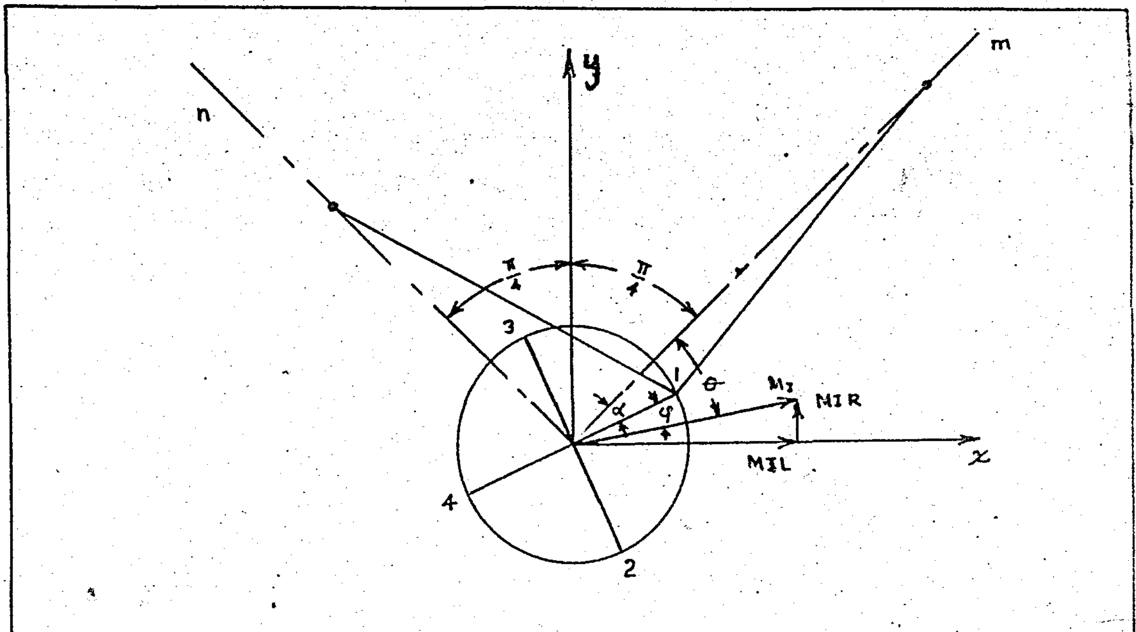

FIG. 23

Right bank

$M_{I1} = Ah_1 \cos\alpha$

$M_{I2} = -Ah_2 \sin\alpha$

$M_{I4} = Ah_1 \cos\alpha$

$M_{I3} = -Ah_2 \sin\alpha$

Left bank

$M'_{I1} = -Ah_1 \sin\alpha$

$M'_{I2} = -Ah_2 \cos\alpha$

$M'_{I4} = -Ah_1 \sin\alpha$

$M'_{I3} = -Ah_2 \cos\alpha$

$M_{IR} = -2A\,(h_1 \cos\alpha - h_2 \sin\alpha)$

$M_{IL} = -2A\,(h_1 \sin\alpha + h_2 \cos\alpha)$

Their vectorial sum will give M_I. If we call θ, the angle M_I forms with Om - axis then:

$$\tan\theta = \frac{MIL}{MIR} = \frac{h_1 \sin\alpha + h_2 \cos\alpha}{h_1 \cos\alpha - h_2 \sin\alpha}$$

denoting $\frac{h_2}{h_1}$ by $\tan\varphi$ and dividing both denominator and numerator by $h_1 \cos\alpha$, we get:

$$\tan\theta = \frac{\tan\alpha + \tan\varphi}{1 - \tan\alpha\tan\varphi} = \tan(\alpha + \varphi)$$

or

$$\theta = \alpha + \varphi$$

The magnitude of M_I is given by:

$$M_I = \sqrt{(MIR)^2 + (MIL)^2} = 2\, m_p r \omega^2 \sqrt{h_1^2 + h_2^2}$$

Therefore if we replace counter weights at the extension of crank arms at an angle $\varphi = \tan^{-1}\frac{h_2}{h_1}$ and with a magnitude such that their moment will balance M_I, the inertia moment of first order will be balanced completely.

c - <u>Inertia forces of second order</u>

The polygon of inertia forces of second order, Fig.17, closes showing that they are automatically balanced.

$$F_{II} = 0$$

Likewise the moment polygon for each bank closes, therefore:

$$M_{II} = 0 \quad \text{Automatically.}$$

Total connecting-rod moment is given by:

$$M_c = 2K\left[\frac{\sin\alpha}{(1-\lambda^2\sin^2\alpha)^{3/2}} + \frac{\sin\left(\alpha+\frac{\pi}{2}\right)}{\left[1-\lambda^2\sin^2\left(\alpha+\frac{\pi}{2}\right)\right]^{3/2}} + \frac{\sin\left(\alpha+\frac{3\pi}{2}\right)}{\left[1-\lambda^2\sin^2\left(\alpha+\frac{3\pi}{2}\right)\right]^{3/2}} + \frac{\sin(\alpha+2\pi)}{\left[1-\lambda^2\sin^2(\alpha+2\pi)\right]^{3/2}}\right]$$

where $K = (1 - L)\, a m \omega^2 \lambda (1 - \lambda^2)$

The quantity in paranthesis vanishes, therefore:

$$M_c = 0$$

We form the index of unbalance

No. of cylinder	Crank angle	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_c	n_u
V-8 Ford	90°, $\theta = 90^\circ$	0	0	0	2	2	0	0	4

13. V-12 ENGINE

For four-stroke engines the firing interval is $\frac{720}{12} = 60^o$, therefore the angle between the banks, θ, is 60^o. Each bank of V-12 engine is exactly like 6-cylinder in-line engines. Therefore the balance condition of rotating masses, first order inertia forces and second order inertia forces are exactly the same as that of 6-cylinder in-line engine. That is:

a - Rotating masses

$F_r = o$ Automatically

$M_r = o$ Automatically

b - Inertia force of first order

$F_I = o$ Automatically

$M_I = o$ Automatically

c - Inertia force of second order

$F_{II} = o$ Automatically

$M_{II} = o$ Automatically

d - Connecting-rod moment

Total connecting-rod moment is:

$$M_c = 2K\left[\frac{\sin\alpha}{(1-\lambda^2\sin^2\alpha)^{3/2}} + \frac{\sin(\alpha+60)}{\left[1-\lambda^2\sin^2(\alpha+60)\right]^{3/2}} + \frac{\sin(\alpha+120)}{\left[1-\lambda^2\sin^2(\alpha+120)\right]^{3/2}}\right.$$

$$\left. + \frac{\sin(\alpha+180)}{\left[1-\lambda^2\sin^2(\alpha+180)\right]^{3/2}} + \frac{\sin(\alpha+240)}{\left[1-\lambda^2\sin^2(\alpha+240)\right]^{3/2}} + \frac{\sin(\alpha+300)}{\left[1-\lambda^2\sin^2(\alpha+300)\right]^{3/2}}\right]$$

WHERE $K = (1 - L)\, a\, m\omega^2 \lambda (1-\lambda^2)$

The term in paranthesis is zero, therefore:

$$M_c = 0$$

We form the index of umbalance

No. of cylinder	θ	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_c	n_u
V-12	60°	0	0	0	0	0	0	0	0

C. THE RADIAL ENGINE

The radial engine consists of z identical reciprocating mechanisms making equal angles $\theta = \frac{2\pi}{z}$ with one another. In Fig.24 a radial engine of five cylinder is shown.

The radial engine with an even number of cylinders is suitable only for two-stroke use. The more important four-stroke engine requires an odd number of cylinders in order to have constant firing interval.

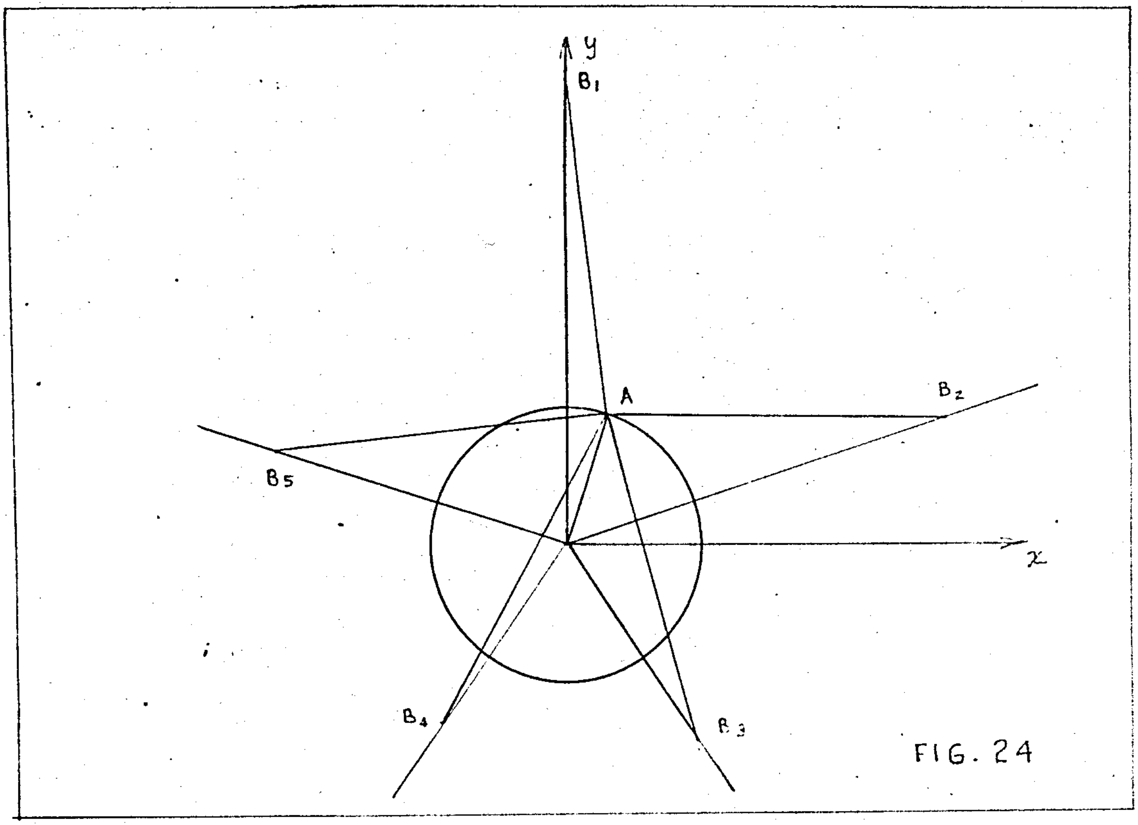

FIG. 24

14. THREE - CYLINDER ENGINE

The angle θ is $\theta = \frac{360}{3} = 120^{\circ}$ and firing order 1 - 2 - 3 for clockwise rotation.

a - Rotating masses

Inertia force due to rotating mass is not zero, but can be made so by counterweights by exactly the same method we used in the single cylinder engines. The only change here is that the mass of the rotating parts is increased due to three connecting rods.

$$F_r \neq 0$$

Inertia moment of rotating masses is zero due to symmetry about the transverse plane from C.G. of the shaft

$$M_r = 0 \quad \text{Automatically}$$

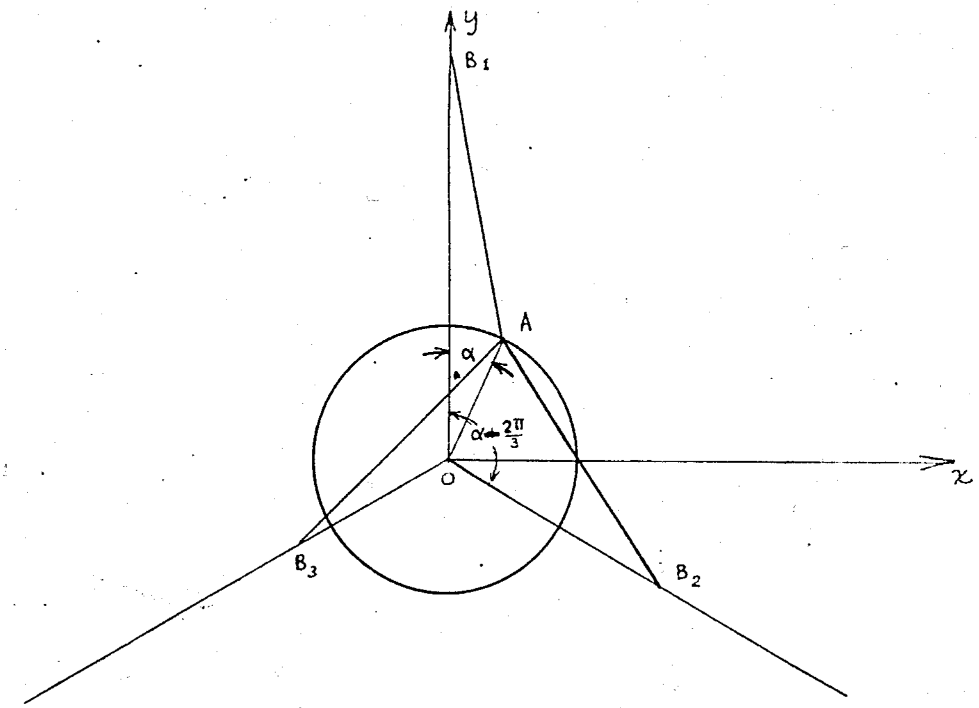

FIG.25

b - <u>Inertia force of first order</u>

We write the inertia forces of first order for z cylinders in general and find their components on two perpendicular axis as y - and x - axis of Fig. 26.

$$F_{I1} = -m_p r \omega^2 \cos \alpha$$

$$F_{I2} = -m_p r \omega^2 \cos \left(\alpha - \frac{2\pi}{z}\right)$$

$$F_{I3} = -m_p r \omega^2 \cos \left(\alpha - \frac{4\pi}{z}\right)$$

- - - - - - - - - - -

$$F_{Iz} = -m_p r \omega^2 \cos \left[\alpha - \frac{2(z-1)}{z}\pi\right]$$

The components of these forces on Oy - axis are:

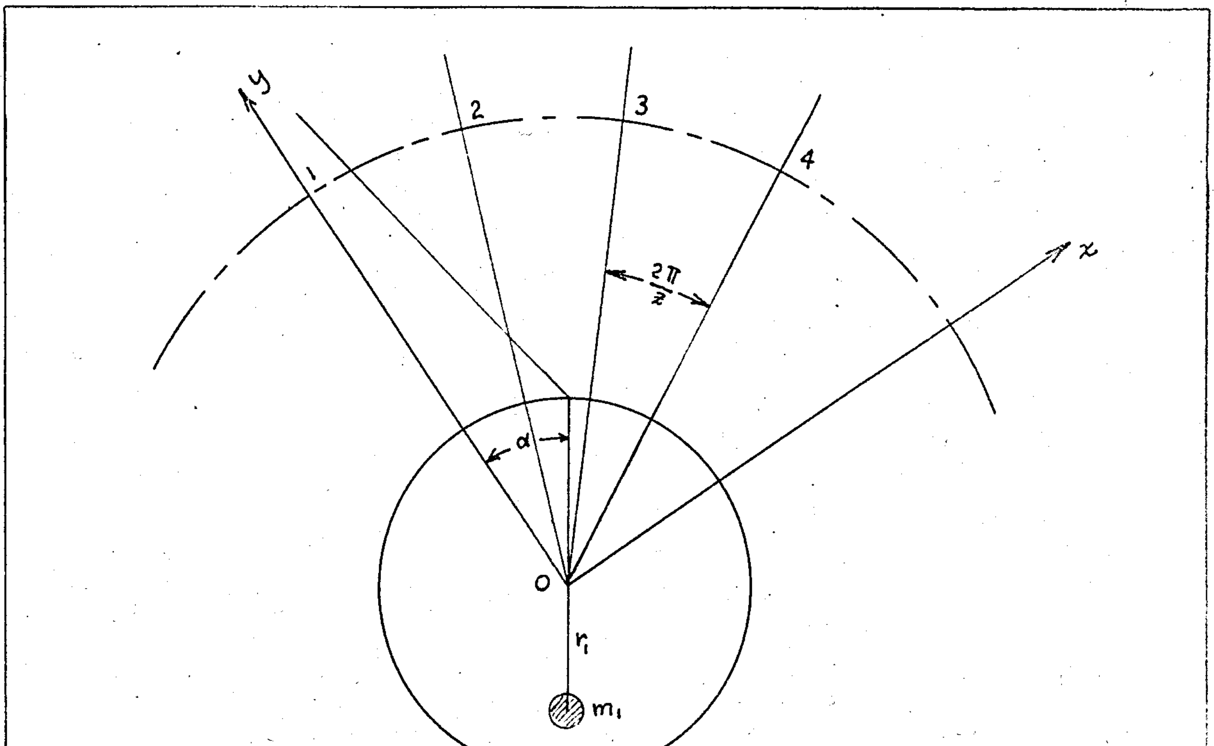

FIG. 26

$$F_{Iy} = -m_p r \omega^2 \left[\cos\alpha + \cos\left(\alpha - \frac{2\pi}{z}\right)\cos\frac{2\pi}{z} + \cos\left(\alpha - \frac{4\pi}{z}\right)\cos\frac{4\pi}{z} + \right.$$
$$\left. + \cos\left(\alpha - \frac{2(z-1)\pi}{z}\right)\cos\frac{2(z-1)\pi}{z} \right]$$

Using identity:

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$F_{Iy} = -m_p r \omega^2 \; \frac{1}{2}\left[z\cos\alpha + \cos\alpha + \cos\left(\alpha - \frac{4\pi}{z}\right) + - - - - + \right.$$
$$\left. + \cos\left(\alpha - \frac{4(z-1)\pi}{z}\right) \right]$$

Again using the identity:

$$\cos a + \cos(a+b) + \cdots + \cos\left[a + (n-1)b\right] = \frac{\sin \frac{n}{2} b}{\sin \frac{b}{2}} \cos\left[a + \frac{(n-1)}{2} b\right]$$

with $a = \alpha$, $b = -\frac{4}{z}$ and $n = z$

$$F_{Iy} = -\frac{1}{2} m_p r \omega^2 \left[z \cos\alpha + \frac{\sin 2\pi}{\sin \frac{2\pi}{z}} \cos\left(\alpha - \frac{z-1}{z} 2\pi\right)\right.$$

The second term in paranthesis is zero for $z > 2$, resulting

$$F_{Iy} = -\frac{z}{2} m_p r \omega^2 \cos\alpha$$

The Ox component of the first order forces likewise can be found to be:

$$F_{Ix} = -\frac{z}{2} m_p r \omega^2 \sin\alpha$$

The resultant of F_{Iy} and F_{Ix} is at the crank arm direction and its magnitude is given by:

$$F_I = -\frac{z}{2} m_p r \omega^2$$

This force can be balanced by counterweights, completely. In case of 3-cylinders, the counterweight is:

$$m_1 r_1 = \frac{3}{2} m_p r$$

The inertia moment is zero automatically.

$$M_I = 0$$

c - <u>Inertia force of second order</u>

Inertia forces of second order are found for Z cylinders in general:

$$F_{II1} = -m_p r \lambda \omega^2 \cos 2\alpha$$

$$F_{II2} = -m_p r \lambda \omega^2 \cos\left(2\alpha - \frac{4\pi}{Z}\right)$$

$$F_{II3} = -m_p r \lambda \omega^2 \cos\left(2\alpha - \frac{8\pi}{Z}\right)$$

- - - - - - - - - - - - - - -

$$F_{IIZ} = -m_p r \lambda \omega^2 \cos\left(2\alpha - \frac{Z-1}{Z} 4\pi\right)$$

Taking the Oy components of these forces:

$$F_{IIy} = -m_p r \lambda \omega^2 \Big[\cos 2\alpha + \cos\left(2\alpha - \frac{4\pi}{2}\right) \cos \frac{2\pi}{Z} + - - - -$$
$$+ \cos\left(2\alpha - \frac{Z-1}{Z} 4\pi\right) \cos \frac{2(Z-1)\pi}{Z}\Big]$$

By the same procedure, used in the previous section, we are led to the result:

$$F_{IIy} = -\frac{1}{2} m_p r \lambda \omega^2 \Big[\frac{\sin \pi}{\sin \frac{\pi}{2}} \cos\left(2\alpha - \frac{Z-1}{Z}\pi\right)$$
$$+ \frac{\sin 3\pi}{\sin \frac{3\pi}{Z}} \cos\left(2\alpha - \frac{Z-1}{Z} 3\pi\right)\Big]$$

The first term is zero for $Z > 1$, the second term is zero for $Z > 3$. If the same procedure is repeated for F_{IIx}, we get the same result, namely $F_{II\,x} = 0$ for $Z > 3$.

Therefore the inertia forces of second order in radial engines is balanced automatically for $Z > 3$

Now we investigate the limit of the second term for $z = 3$, which has the form $\frac{0}{0}$.

$$F_{IIy} = -m_p r\lambda\omega^2\left[\cos 2\alpha + \cos(2\alpha - \frac{4\pi}{3})\cos\frac{2\pi}{3} + \cos(2\alpha - \frac{8\pi}{3})\cos\frac{4\pi}{3}\right]$$

$$F_{IIy} = -m_p r\lambda\omega^2\left[\cos 2\alpha + \frac{1}{2}\cos(2\alpha - \frac{2\pi}{3}) + \frac{1}{2}\cos(2\alpha - 2\pi) + \frac{1}{2}\cos(2\alpha - \frac{4\pi}{3}) + \frac{1}{2}\cos(2\alpha - 4\pi)\right]$$

$$F_{IIy} = -m_p r\lambda\omega^2\left[2\cos 2\alpha + \frac{1}{2}(-\cos 2\alpha)\right]$$

$$F_{IIy} = -\frac{3}{2}m_p r\lambda\omega^2\cos 2 \quad = -\frac{3}{8}m_p r\lambda(2\omega)^2\cos 2\alpha$$

Likewise F_{IIx} is found to be:

$$F_{IIx} = -\frac{3}{2}m_p r\lambda\omega^2\sin 2\alpha = -\frac{3}{8}m_p r\lambda(2\omega)^2\sin 2\alpha$$

and

$$F_{II} = -\frac{3}{8}m_p r\lambda(2\omega)^2$$

This force can be balanced by mass product:

$m_2 r_2 = \frac{3}{8}m_p r\lambda$ revolving with a speed of 2ω.

d - <u>Connecting-rod moment</u>

Again considering z cylinders, we write resultant connecting-rod moment

$$M_c = K\left[\frac{\sin\alpha}{(1-\lambda^2\sin^2\alpha)^{3/2}} + \frac{\sin(\alpha - \frac{2\pi}{z})}{\left[1-\lambda^2\sin^2(\alpha - \frac{2\pi}{z})\right]^{3/2}} + \frac{\sin(\alpha - \frac{z-1}{z}2\pi)}{\left[1-\lambda^2\sin^2(\alpha - \frac{z-1}{z}2\pi)\right]^{3/2}}\right.$$

If $\sin\alpha$ is denoted by x and $(1-\lambda^2x^2)^{-\frac{3}{2}}$ is expanded by binomial series,

$$\frac{x}{(1-\lambda^2x^2)^{3/2}} = x + \frac{3}{2}\lambda^2x^3 + \frac{15}{8}\lambda^4x^5 + \cdots$$

If we neglect λ to the 5th power or greater and using the identity

$$\sin^3\alpha = \frac{3}{4}\sin\alpha - \frac{1}{4}\sin 3\alpha$$

$$\frac{\sin\alpha}{(1-\lambda^2\sin^2\alpha)^{3/2}} = (1+\frac{9\lambda^2}{8})\sin\alpha - \frac{3\lambda^2}{8}\sin 3\alpha +$$

Expressing other terms also in the same manner:

$$M_c = K\left\{ (1+\frac{9\lambda^2}{8})\left[\sin\alpha + \sin(\alpha - \frac{2\pi}{z}) + \cdots + \sin(\alpha - \frac{z-1}{z}2\pi)\right] - \frac{3\lambda^2}{8}\left[\sin 3\alpha + \sin(3\alpha - \frac{6\pi}{z}) + \quad + \sin(3\alpha - \frac{z-1}{z}6\pi)\right]\right\}$$

if the quantities in brackets are changed by their identities we get the following form for the brace.

$$(1+\frac{9\lambda^2}{8})\frac{\sin\pi}{\sin\frac{\pi}{z}}\sin(\alpha - \frac{z-1}{z}\pi) - \frac{3\lambda^2}{8}\frac{\sin 3\pi}{\sin\frac{3\pi}{z}}\sin(3\alpha - \frac{z-1}{z}3\pi)$$

The first term is zero for any z, the second is zero for $z > 3$

For $z = 3$

$$M_c = K\left\{(1+\frac{9\,^2}{8})\left[\sin\alpha + \sin(\alpha - \frac{2\pi}{3}) + \sin(\alpha - \frac{4\pi}{3})\right] - \frac{3\lambda^2}{8}\left[\sin 3\alpha + \sin(3\alpha - 2\pi) + \sin(3\alpha - 4\pi)\right]\right\}$$

$$M_c = K\left(\frac{-3\lambda^2}{8}\right) 3 \sin 3\alpha$$

$$M_c = -\frac{9}{8}(1 - L)\, a m \omega^2 \lambda^3 \sin 3\alpha$$

The index of unbalance becomes:

No. of cylinder	θ	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_c	n_u
3	120^o	2	2	2	o	o	o	1	7

15 - SEVEN - CYLINDER ENGINE

The angle between banks is $\frac{360}{7} = 51.429^o$.

a - Rotating masses

Inertia force due to rotating masses is not zero, but can be made so by counterweights.

$$F_r \neq o$$

Inertia moment is balanced automatically.

$$M_r = o$$

b - Inertia force of first order

The inertia force of first order was found to be:

$$F_I = -\frac{z}{2}\, m_p r \omega^2$$

where z is the number of cylinder, $z = 7$. This force can be balanced by counterweights, given by:

$$m_2 r_2 = \frac{7}{2} m_p r$$

$M_I = o$ Automatically

c - Inertia forces of second order

$F_{II} = o$ Automatically

$M_{II} = o$ Automatically

d - Connecting-rod moment

$M_c = o$ Automatically

We now from the index of unbalance

No. of cylinder	θ	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_c	n_u
7	$\frac{360^\circ}{7}$	2	2	o	o	o	o	o	4

16. NINE-CYLINDER ENGINE

The angle between cylinder axis is $\theta = \frac{360}{9} = 40^\circ$

a - Rotating masses

Inertia force due to rotating masses is not zero but can be made so by counter weights.

$F_r \neq o$,

Inertia moment due to rotating masses is zero

$M_r = o$ Automatically

b - Inertia forces of first order

$F_I = -\frac{z}{2} m_p r \omega^2$

(For $z = 9$)

$F_I = -\frac{9}{2} m_p r \omega^2$

This force can completely be balanced by mass product

$m_2 r_2 = \frac{9}{2} m_p r$

$M_I = o$ Automatically

c - Inertia forces of second order:

$F_{II} = o$ Automatically

(for $z \neq 3$)

$M_{II} = o$ Automatically

d - Connecting-rod moment

$M_c = o$ Automatically

We form the index of unbalance:

No. of cylinder	θ	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_c	n_u
9	40	2	2	0	0	0	0	0	4

17. DOUBLE-ROW RADIAL ENGINE

In double-row radial engines, crank is similar to the two-cylinder in-line engine, Fig.9. The angle between cylinder axis is $\frac{2\pi}{z}$, firing interval $\frac{4\pi}{2z} = \frac{2\pi}{z}$. Firing order is one from the first row, the next from the second row. Referring to Fig. 27, A and B are the cranks of first and second row respectively. The firing order is, for $z = 5$:

1 - V - 3 - II - 5 - IV - 2 - I - 4 - III

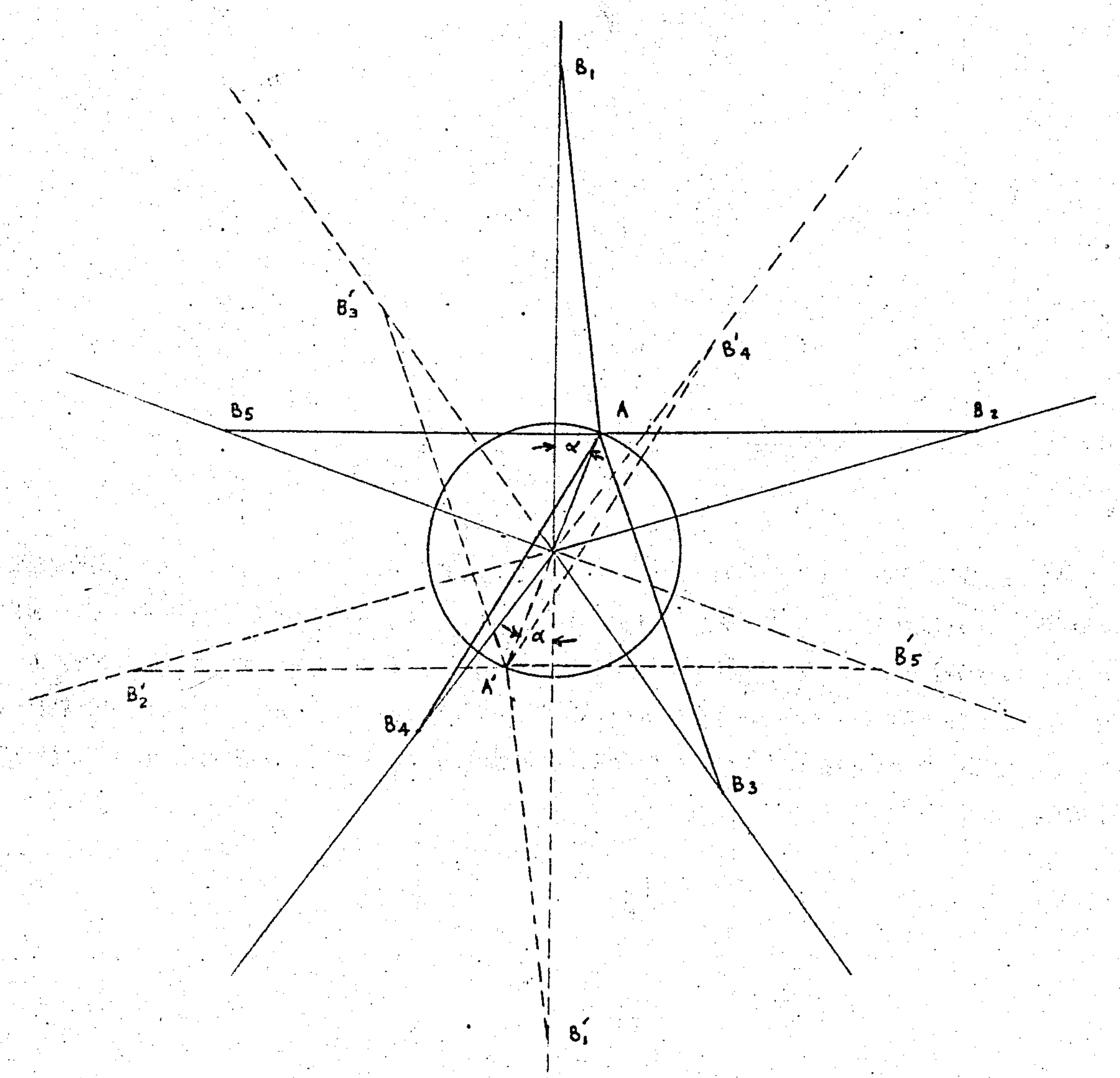

FIG. 28

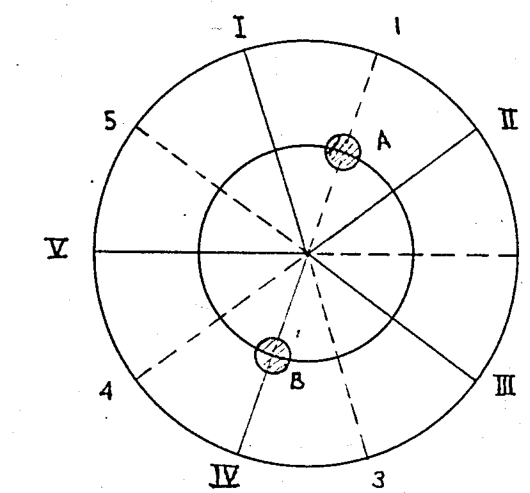

FIG. 27

Fig. 28 shows the schematic position of two rows at any instant when cra A makes an angle α with the first crank axis. First row is drawn with solid lines and second row is shown with dotted lines. Though the figure is drawn for five cylinders, we will consider the general case with Z cylinders in eac row and find the inertia forces and moments due to rotating and reciprocating -masses.

a - Rotating masses

Inertia force due to rotating masses is zero, because two cranks are 180° apart from each other and their polygon closes, Fig.29.

$$F_r = 0$$

Inertia moment due to rotating masses is not zero, as seen from the moment diagram and equals:

$$M_I = F_r \times 2\,h \qquad \text{(Fig. 9)}$$

This moment can be balanced completely by counterweights as has been done in article 6 for two-cylinder in-line engine.

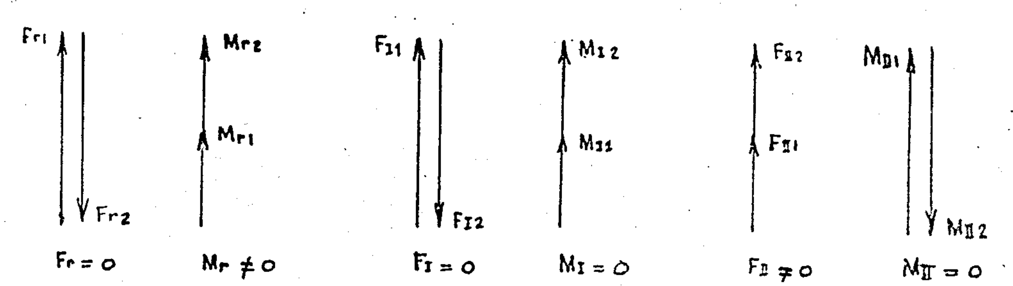

FIG. 29

b - Inertia force of first order

We had proved for simple radial engines that

$$F_{I1} = -\frac{z}{2}\, M_p\, r\, \omega^2$$

in the direction of crank arms. The same force is along the second crank arm having 180° with the first one. Therefore their sum is zero

$$F_I = 0$$

Inertia moment of the first order is not zero, and is given by

$$M_I = -\frac{z}{2}\; m_p r \omega^2 x\; 2h \quad \text{(Fig. 9)}$$

This moment can completely be balanced by counterweights.

c - Inertia force of second order

For simple radial engines, F_{II} has been found to be:

$$F_{II1} = 0 \qquad (\text{for } z > 3)$$

$$F_{II1} = -\frac{3}{2} m_p r \lambda \omega^2 \qquad (\text{for } z = 3)$$

For double-row radial engines:

$$F_{II} = 0 \qquad (\text{for } z > 3)$$

$$F_{II} = 2\left(-\frac{3}{8}\right) m_p r \lambda (2\omega)^2 \qquad (\text{for } z = 3)$$

Inertia moment of second order is zero for any z.

$$M_{II} = 0$$

d - Connecting-rod moment

For simple radial engines, M_c has been found to be:

$$M_c = 0 \qquad (z > 3)$$

$$M_c = (1 - L) a m \omega^2 \frac{3\lambda^3}{8}\; 3 \sin 3\alpha \quad (Z = 3)$$

For double-row radial engine:

$$M_c = 0 \qquad (z > 3)$$

$$M_c = K \left[\sin 3\alpha + \sin 3(\alpha + \pi) \right] \qquad (z = 3)$$

where $K = (1 - L)\, am\omega^2 \dfrac{9\lambda^3}{8}$

The term in bracket is zero, therefore

$$M_c = 0$$

Now we consider double-row radial engines with different number of cylinders.

18. THREE CYLINDER ENGINE

The angle between cylinder axis for each row, $\theta = 120^o$. Firing order is 1 - I - 2 - II - 3 - III, Fig.30.

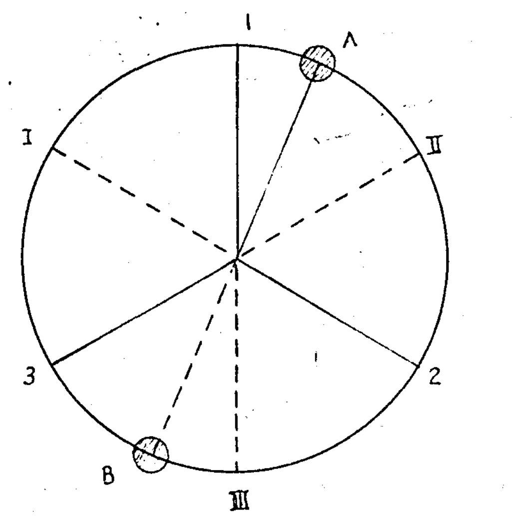

FIG.30

a - Rotating masses

$F_r = o$

$M_r \neq o$ (can be balanced completely)

b - Inertia force of first order

$F_I = o$

$M_I \neq o$ (can be balanced completely)

c - Inertia force of second order

$F_{II} \neq o$

$M_{II} = o$

d - Connecting-rod moment

$M_c = o$

Index of unbalance becomes

No. of cylinder	θ	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_c	n_u
3	120^o	o	o	2	2	2	o	o	6

19. SEVEN-CYLINDER ENGINE

The angle between cylinder axis for each row is $\theta = \frac{360^o}{7}$.

a - Rotating masses

$F_r = 0$

$M_r \neq 0$ (can be balanced completely)

b - Inertia force of first order

$F_I = 0$

$M_I \neq 0$ (can be balanced completely)

c - Inertia force of second order

$F_{II} = 0$

$M_{II} = 0$

d - Connecting-rod moment

$M_c = 0$

The index of unbalance is:

No. of cylinder	θ	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_c	n_u
7	$360^o/7$	0	0	0	2	2	0	0	4

20. NINE-CYLINDER ENGINE

The angle for each row is $\theta = \frac{360}{9} = 40^o$.

a - Rotating masses and first order inertia force

$F_I = 0$, $F_r = 0$

$M_I \neq 0$ (can be balanced completely). , $M_r \neq 0$

c - Inertia force of second order

$F_{II} = o$

$M_{II} = o$

d - Connecting-rod moment

$M_c = o$

The index of unbalance becomes:

No. of cylinder	θ	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_c	n_u
9	40^o	o	o	o	2	2	o	o	4

D. OPPOSED ENGINES

21. SINGLE-CYLINDER ENGINE

This engine has the form as shown in Fig. 31. Two cranks are 180^o from each other. One of the pistons is called "dummy piston", because it is used to improve the state of balance.

a - Rotating masses

Force polygon closes, therefore

$F_r = o$

The system is symmetrical with respect to transverse plane passing from its center of gravity.

$M_r = o$

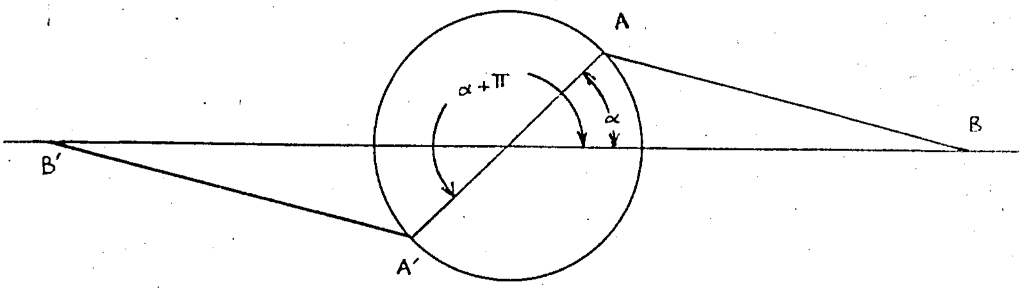

FIG.31

b - Inertia force of first order

$$F_I = 0$$

$$M_I = 0$$

c - Inertia force of second order

Force polygon does not close and resultant force is given by:

$$F_{II} = -2m_p r\omega^2\lambda\cos 2\alpha$$

This force can not be balanced except by the Lanchester method

$$M_{II} = 0$$

d - Connecting-rod moment

$$M_c = K\left[\frac{\sin\alpha}{(1-\lambda^2\sin^2\alpha)^{3/2}} + \frac{\sin(\alpha+\pi)}{\left[1-\lambda^2\sin^2(\alpha+\pi)\right]^{3/2}}\right]$$

where $K = (1 - L)\, am\,\omega^2\lambda(1-\lambda^2)$ and we assume the two connecting - rods similar to each other.

$$M_c = 0$$

We form the index of unbalance

ENGINE	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_c	$\bar{n}_u$
SINGLE-CYLINDER OPPOSED	0	0	2	0	0	0	0	2

22 - DOUBLE-CYLINDER ENGINE (JUNKERS ENGINE)

In this engine two pistons labeled 1 and 2 are connected to cranks 1' and 2' by means of connecting rods 1" and 2". As cranks rotate about Z - axis pistons do reciprocating motions along the cylinder, Fig. 32.

a - Rotating masses

The cranks are in the same plane, therefore:

$$F_r = 0$$

$$M_r = 0$$

b - Inertia force of first order

Inertia force of first order is given by:

$$F_I = - m_p r \omega^2 \cos\alpha - m'_p r' \omega^2 \cos(\alpha + \pi)$$

$$F_I = -(m_p r - m'_p r') \omega^2 \cos\alpha$$

if $m_p r = m'_p r'$, this force vanishes

$$F_I = 0$$

$$M_I = 0$$

c - Inertia force of second order

Inertia force of second order is given by:

$$F_{II} = - m_p r \lambda \omega^2 \cos 2\alpha - m'_p r' \lambda' \omega^2 \cos(2\alpha + 8\pi)$$

$$F_{II} = -(m_p r \lambda + m'_p r' \lambda') \omega^2 \cos 2\alpha$$

Thsi force is not zero and can not be balanced by counter weights.

$$M_{II} \neq 0$$

d - Connecting-rod moment

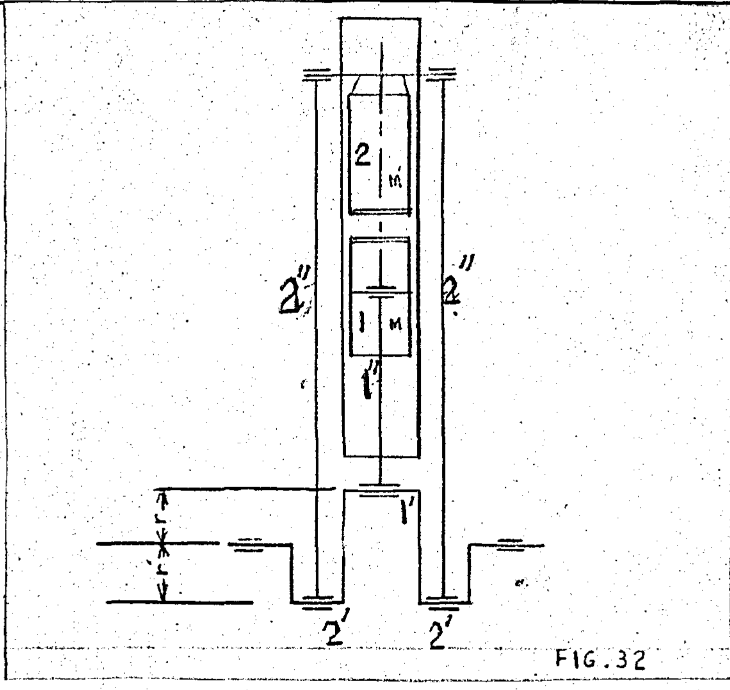

FIG. 32

$$M_c = K \frac{\sin\alpha}{(1-\lambda^2\sin^2\alpha)^{3/2}} + K' \frac{\sin(\alpha+\pi)}{\left[1-\lambda^2\sin^2(\alpha+\pi)\right]^{3/2}}$$

$$M_c = (K - K')\frac{\sin\alpha}{(1-\lambda^2\sin^2\alpha)^{3/2}}$$

where $K = (1 - L)\, am\omega^2 \lambda(1-\lambda^2)$

and $K' = (1' - L')\, a'm'\omega^2\lambda'(1-\lambda'^2)$

$M_c = o$ if $(1 - L)\, am = (1' - L')\, a'm'$

$Mo \neq$ $(1 - L)\, am \neq (1' - L')\, a'm'$

In general two connecting-rods are not similar, therefore $M_c \neq o$

We form the index of unbalance

ENGINE	F_r	F_I	F_{II}	M_r	M_I	M_{II}	M_c	n_u
TWO-PISTON OPPOSED	o	o	2	o	o	o	1	3

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