ULTIMATE STRENGTH DESIGN OF REINFORCED CONCRETE MEMBERS BY COMPUTERS

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CONTENTS

Appendixes

ABSTRACT

The Ultimate Strength design formulas of the "Building Code Requirements for Reinforced Concrete (ACI 318-63) are transformed into such a form that, the reinforcement can be directly computed, once the basic loads, geometrical data, and material properties are known.

At first, the stress resultants of each member for different load cases are combined in any prescribed way, in order to obtain the critical condition. However, the members are designed for each case of the combined stress resultants.

In beams, both the flexural and the shear reinforcements are calculated, not only at the ends of the member but also at midpoints and quarter points.

In case the beam section is found to be inadequate to carry the bending moment, the depth of the section is increased to a minimum satisfactory value, and a new reinforcement is computed. The dimensions of the cross-section are not altered if the section is inadequate against the shear. However, a warning message is printed out to this effect.

Web reinforcement is calculated for an inch length of the beam. This allows the designer to choose the spacing of stirrups and/or bent-up bars in accordance with the common practice.

Columns are designed for both uniaxial and biaxial bending cases. In the formulation phase of columns subject to uniaxial bending three kinds of formulas are studied: 1-Exact formulation recommended by the Code ACI 318-63 ; 2-Empirical equations of ACI 318-63 ; 3-Equations derived by considering strain compatibility (ACI Publication SP-7, "Ultimate Strength

Design of Reinforced Concrete Columns."

All of the above formulas are transformed into a form most suitable for the straightforward computation of reinforcement areas. After a comparative test of these different formulas in the computer, it has been found that they produce almost identical results for a wide range of data variation. Therefore, the empirical column formulas of the Code (ACI 318-63) are adapted for use in the programming as they are the simplest.

For biaxial bending, the approximate method, in which the biaxial bending of a column is related to its uniaxial resistance through a factor β, is used.

A comprehensive Fortran program for the automatic design of reinforced concrete beams and columns is developed in a way that, "Building Code Requirements for Reinforced Concrete (ACI 318-63)" Ultimate Strength Design requirements are completely satisfied.

The IBM 1620 machine of the Computer Center of Robert College has been used.

I N T R O D U C T I O N

The first published ultimate load theory was that of Koenen's. In 1886, he assumed a straight line distribution of concrete stress and a neutral axis at middepth. Since that time about thirty theories have been published, and many different distributions of stress in the concrete compression zone have been suggested. But recent experimental and analytical investigations proved that the "equivalent rectangular stress block" yields sufficiently accurate results, and at the same time it leads to considerable simplification of design calculations. In 1904, for the first time, von Emperger proposed the use of a rectangular concrete compression stress block, and thereafter, several other engineers reccommended it.

Extensive ACI investigation in the early thirties, several papers on stress distribution and ultimate strength design published in Europe during the 1930's, the studies of ultimate strength by Whitney, a study of the ultimate strength of eccentrically loaded columns reported by Hognestad in 1951, and additional experimental evidence as to the parameters of the concrete stress block presented during the Symposium on the Strength of the Concrete Structures, London, May, 1956, all agreed upon the maximum concrete stress to be $0.85f'_c$, and confirmed the use of a rectangular concrete compression stress block having a width of $0.85f'_c$.

In recent years various tests carried out on plain concrete specimens by Portland Cement Association with sand-gravel and lightweight aggregates, by Rusch at Munich Institute of Technology helped to determine the magnitude and position of the internal concrete force at ultimate strength by the application of statistical methods.

By analytical investigations, assuming that concrete stress, f, is some function of strain, ϵ, and is given by $f=F(\epsilon)$, it was deduced from a study of concrete stress-strain curves obtained in the Portland Cement Association that the relationship between concrete stress at extreme compressive fiber, f_u, and the cylinder strength, f'_c, can be expressed as $f_u=5f'^{0.8}_c$. However, $f_u=0.85f'_c$, a conservative straight-line relationship was proposed for design purposes.

In 1940 the Joint Committee Reccommendation, and in 1941 the ACI Code, adopted a modified ultimate strength specification for axially loaded columns. In October 1955, the Report of the ASCE-ACI Joint Committee on Ultimate Strength Design allowed the use of "a rectangle, trapezoid, parabola or any other shape which results in ultimate strength in reasonable agreement with tests " without specifying an exact shape for the compressive stress distribution. In 1956, the ACI added Art.601b, "The ultimate strength method of design may be used for the design of reinforced concrete members." The ACI Building Code (ACI 318-63) incorporates detailed rules of ultimate strength design.

Ultimate strength design has certain advantages. In ultimate strength design a lower load factor is used for loads that are definitely known such as dead load, and a higher load factor for loads that are less certain. Thus the factor of safety becomes more consistent with the type of loading. Certain inconsistencies in the design of members carrying axial load and bending are eliminated by ultimate strength design applied to all members.

Elastic or straight line theory does not give a true picture when ultimate strength is reached, thus at ultimate load the actual factor of safety is left uncertain. Ultimate strength design method eliminates this discrepancy since it gives a more realistic picture at failure.

The ultimate strength design gives a better balanced design that may result from the general straight line theory.

The economy depends upon the load factors used,so extensive tests are made to decrease the load factors.

The Ultimate Strength Design based on a theory giving a better picture of the reinforced concrete section at failure is more reasonable than the Working Stress Design Method.The rectangular stress compression block is relatively easy to apply where need be with stress-strain compatibility. With the accumulation of test results the load factors are getting less and less , thus allowing more economical design.No wonder with ultimate strength design gaining more importance day after day.

BASIC ASSUMPTIONS - The standard assumptions associated with ultimate strength procedures are taken from the ACI 318-63 Code.

These are:

1-At ultimate strength, concrete stress is not proportional to strain. The diagram of compressive concrete stress distribution may be assumed to be a rectangle, trapezoid, parabola, or any other shape which results in predictions of ultimate strength in reasonable agreement with the results of comprehensive test.

The above requirements may be considered satisfied by the equivalent rectangular concrete stress distribution. At ultimate strength a concrete stress intensity of $0.85f'_c$ is assumed uniformly distributed over an equivalent compression zone bounded by the edges of the cross-section and a straight line located parallel to the neutral axis at a distance a k_1c from the fiber of maximum compressive strain. The distance c from the fiber of maximum strain to the neutral axis is measured in a direction perpendicular to that axis. The fraction k_1 is taken as 0.85 for concrete strengths, f'_c, up to 4000 psi and is reduced continuously at a rate of 0.05 for each 1000 psi of strength in excess of 4000 psi.

This is expressed as,

$$k_1 = 0.85 \qquad \text{for} \quad f'_c < 4000 \text{ psi}$$

$$k_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) \qquad \text{for} \quad f'_c > 4000 \text{ psi}$$

2-Tensile strength of the concrete is neglected in flexural computations.

3-At ultimate strength linear strain distribution in the concrete is assumed.

4-The maximum strain at the extreme compression fiber at ultimate strength is assumed equal to 0.003.

5-Elastic-perfectly plastic stress-strain relationship for steel in tension and compression is assumed. Stress in reinforcing bars below the yield strength, f_y, for the grade of steel used is taken as 29,000,000 psi times the steel strain.

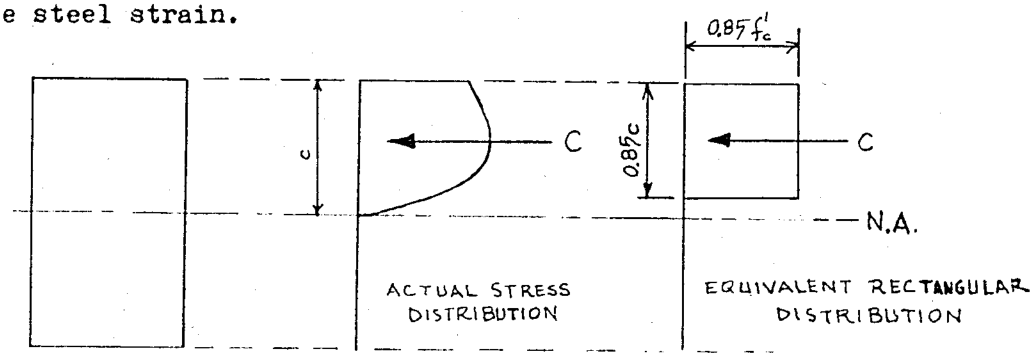


Fig. 1-Equivalent stress block for rectangular section

Actually, due to the linear strain distribution accross the section, the rectangular stress block may be considered to be equivalent to replacing the actual stress-strain curve for concrete by an equivalent rectangular stress-strain curve as in Fig.2.

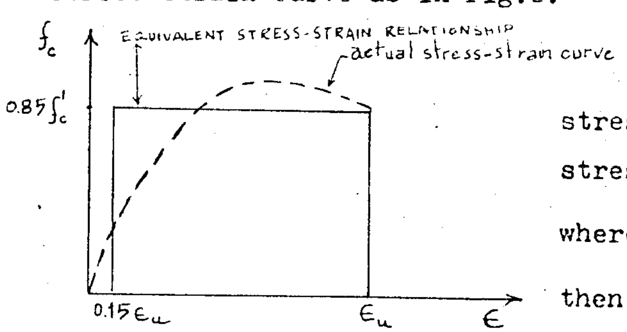


Fig.2-Equivalent stress-strain curve for concrete .

For this assumed, fictitious stress-strain relationship, the concrete stress is taken zero up to a strain of $0.15\,\epsilon_u$ where ϵ_u is the ultimate concrete strain, and then assumes a constant value of $0.85\ f'_c$ until failure occurs. (ref-9)

Rectangular Beams with tension reinforcement only- The formula given in the Code (ACI 318-63) is,

$$M_u = \phi \left[A_s f_y (d-a/2) \right] \qquad \text{(Eq.16-1)in the Code .} \qquad \text{Eq.1}$$

where $a = A_s f_y / 0.85 f'_c\ b$ Eq.2

putting the value of a from eq.2 into 1 and solving for As one gets,

$$A_s = \varkappa / f_y \left[1 - \sqrt{1 - 2M_u / \phi \varkappa d} \right] \qquad \text{Eq.3}$$

(for this derivation see appendix eq.A1).

In Eq.3 $\varkappa = 0.85 f'_c\ bd$

One directly gets the reinforcement from the equation 3,but the ACI Code states that the reinforcement ratio shall not exceed 0.75 of the reinforcement ratio which produces balanced conditions at ultimate strength. Thus,if the computed reinforcement ratio is greater than 0.75 of the balanced ratio,or when the term under the square root in Eq.3 is negative, we have to change our design that is either we have to increase the depth of our beam or put compression reinforcement .

Increase in depth of the Rectangular Beams - When the applied moment is excessive,or when we are beyond the limitations of the Code,we increase the depth of the beam.The formula given in the(ACI 318-63)Code is,

$$M_u = \phi \left[bd^2\ f'_c\ q(1-0.59\ q) \right] \qquad \text{(Eq.16-1)} \qquad \text{Eq.4}$$

$$q = p\ f_y / f'_c \qquad \text{Eq.5}$$

At the limit the reinforcement ratio may be equal to 0.75 of the balanced reinforcement ratio, p_b . Therefore,

$$p = 0.75 p_b = 0.75 \left[0.85 k_1\ f'_c / f_y \right] \left[87{,}000 / 87{,}000 + f_y \right]$$

$$\text{(Eq.16-2)} \qquad \text{Eq.6}$$

When the value of p is substituted from eq.6 to eq.5 ,and q is eliminated between the equations 5 and 4 ,we get the new depth as

$$d = \sqrt{M_u / \phi \, b f'_c q (1-0.59\, q)} \qquad \text{Eq.7}$$

where

$$q = .6375\, k_1 \left(87{,}000/87{,}000+f_y\right) \qquad \text{Eq.8}$$

(for the derivation of equations 7 and 8 see appendix A2) .

RECTANGULAR BEAMS WİTH COMPRESSİON REİNFORCEMENT - When the applied moment is excessive,or when the calculated reinforcement is beyond the limitations of the Code,instead of increasing the depth we add compression reinforcement.The ultimate design resisting moment in rectangular beams with compression reinforcement is given by:

$$M_u = \phi \left[(A_s - A'_s) f_y (d-a/2) + A'_s f_y (d-d') \right] \qquad \text{(Code 16-3)} \qquad \text{Eq.9}$$

where $a = (A_s - A'_s) f_y / 0.85 f'_c b$ Eq.10

The equation (16-3) is only valid when the compression steel reaches the yield strength,f_y ,at ultimate strength.This is satisfied when:

$$p - p' \geq 0.85\, k1 f'_c d'/f_y d \quad 87{,}000/87{,}000-f_y \qquad \text{(Code 16-4)} \qquad \text{Eq.11}$$

If (p-p') is less than the value given by Eq.11,it means that the compression steel stress is less than the yield strength,f_y,or the effects of compression steel are neglected.Then the calculated ultimate moment shall not exceed that given by Eq.16-1 (ACI Code 1602-c).

The Code gives further

$$p - p' \leq 0.75 p_b = 0.75 \left(0.85 k_1 f'_c / f_y\right) \left(87{,}000/87{,}000+f_y\right) \qquad \text{Eq.12}$$

Calling $(p-p') = F$ Eq.13

we can get $(A_s - A'_s) = Fbd$ Eq.14

At the limit

$$F = 0.75p_b \qquad \text{Eq.15}$$

Putting (14) and (15) into (10) we get

$$a = 0.75p_b bdf_y / 0.85f'_c b \qquad \text{Eq.16}$$

Elimination of a between equations (9) and (16) gives for A'_s ,

$$A'_s = \left[M_u/\phi \; -0.75p_b bdf_y\left(d - \frac{0.75p_b df_y}{1.7f'_c}\right)\right] / f_y(d-d') \qquad \text{Eq.17}$$

where $0.75p_b$ is as defined in (12).

Tension reinforcement becomes

$$A_s = 0.75p_b bd - A'_s \qquad \text{Eq.18}$$

See appendix ,equation A3 for this derivation.

FLEXURAL COMPUTATİONS ULTİMATE STRENGTH DESİGN I- AND T- SECTIONS-

When the flange thickness is less than $1.18qd/k_1$ the ultimate moment is given by:

$$M_u = \phi\left[(A_s - A_{sf})f_y(d-a/2) + A_{sf}f_y(d-0.5t)\right] \qquad \text{Code(16-5)} \qquad \text{Eq.19}$$

in which A_{sf}, the steel area necessary t o develop the compressive strength of overhanging flanges is:

$$A_{sf} = 0.85(b-b')tf'_c / f_y \qquad \text{Code(16-6)} \qquad \text{Eq.20}$$

and

$$a = (A_s - A_{sf})f_y / 0.85f'_c b' \qquad \text{Eq.21}$$

Now if in the equation (19) we make the following substitutions

$$A_s - A_{sf} = X \qquad \text{and} \qquad a = Xf_y / 0.85f'_c b'$$

we will get a quadratic equation for X and its solution will give

$$X = \frac{\chi}{f_y}\left\{1 - \sqrt{1 - \frac{2}{\chi d}\left[\frac{M_u}{\phi} - A_{sf}f_y(d-0.5t)\right]}\right\} \qquad \text{Eq.22}$$

See appendix A equation 4 for this derivation.

Where $\chi = 0.85f'b'd$ and A_{sf} is taken from Eq.20.

Now having X we may get total reinforcement as

$$A_s = A_{sf} + X \qquad \text{Eq.23}$$

INCREASE IN DEPTH OF THE I- AND T- SECTIONS - If we get a negative quantity under the square root of the equation (22), or if $(p_w - p_f)$, the difference between the web and the flange reinforcement ratios, exceeds $0.75p_b$ (defined by eq.6), we will increase the depth of our section.

At the limit

$$p_w - p_f = 0.75p_b = L \qquad \text{Eq.24}$$

$$\frac{A_s}{b'd} - \frac{A_{sf}}{b'd} = L \qquad \text{Eq.25}$$

$$X = A_s - A_{sf} = Lb'd \qquad \text{Eq.26}$$

Eliminating X between equations 22 and 26 we get a quadratic equation for d, which is:

$$d - 2\alpha d + \beta = 0 \qquad \text{Eq.27}$$

Solution of this gives

$$d = \alpha\left[1 - \sqrt{1 - (\beta/\alpha^2)}\right] \qquad \text{Eq.28}$$

where

$$\beta = \frac{0.85f'_c b\left[2M_u/\phi + A_{sf}f_y t\right]}{Lb'f_y\,(Lb'f_y - 1.7f'_c b)} \qquad \text{Eq.29}$$

$$\alpha = \frac{0.85f'_c b f_y A_{sf}}{Lb'f_y\,(Lb'f_y - 1.7f'_c b)} \qquad \text{Eq.30}$$

(See appendix A equation 5).

COMBİNED AXIAL COMPRESSİON AND BENDING

COLUMN FORMULAS

UNIAXIAL BENDING OF COLUMNS - Short columns with symmetrical reinforcement in two faces are considered. The ultimate strength is given by:

$$P_u=\phi\left[0.85f'_c ba-A'_s f_y-A_s f_s\right] \quad \text{(ACI Code 19-1)} \qquad \text{Eq.31}$$

$A_s=A'_s$ because of symmetrical reinforcement

The balanced load, P_b becomes

$$P_b=\phi\, 0.85f'_c ba_b \qquad \cancel{\text{Eq.32}}$$

$$a_b=k_1c_b \;\; k_1d(87{,}000/87{,}000+f_y) \quad \text{(Code 1902-b)} \qquad \text{Eq.33}$$

Thus $$P_b=\phi\, 0.85f'_c b\; k_1(87{,}000)/(87{,}000-f_y) \qquad \text{Eq.34}$$

When P_u is less than P_b, the ultimate capacity of the member is controlled by tension. For symmetrical reinforcement in two faces the ACI Code gives the expression (19-5) which is:

$$P_u=\phi\left[0.85f'_c bd\left\{-p+1-e'/d+\sqrt{(1-e'/d)^2+2p\left[m'(1-d'/d)+e'/d\right]}\right\}\right] \qquad \text{Eq.35}$$

Solution of this equation for p gives a quadratic equation

$$p-2\alpha p+\varphi=0 \qquad \text{Eq.36}$$

where

$$\alpha=\frac{f_y}{0.85f'_c}\left(1-\frac{d'}{d}\right)-\frac{P_u}{0.85f'_c bd}+\frac{d'}{d} \qquad \text{Eq.37}$$

$$\varphi=\frac{P_u}{0.85f'_c bd}\left(\frac{P_u}{0.85f'_c bd}+\frac{2e'}{d}-2\right) \qquad \text{Eq.38}$$

$$e'=M_u/P_u+\left(t/2-d'\right) \qquad \text{Eq.39}$$

We compute total reinforcement from $A_{st}=2pdb$ Eq.40

See appendix A6 for the derivation of eq.36.

From the equation (36) the steel percentage is computed as:

$$p=\alpha\left(1-\sqrt{1-q/\alpha^2}\right) \qquad \text{Eq.41}$$

When P_u is greater than P_b the ultimate capacity of the member is controlled by compression. In this case, the ultimate load is assumed to decrease linearly from P_o to P_b as the moment is increased from zero to the balanced moment, M_b, where

$$P_o=\phi\left[0.85f'_c(A_g-A_{st})+A_{st}f_y\right] \qquad \text{(ACI Code 19-7)} \qquad \text{Eq.42}$$

When compression governs the ultimate capacity is given by two different equations, one of them is the exact equation and the other is the empirical one.

1-Exact equation- The ultimate load is given by:

$$P_u=P_o-(P_o-P_b)M_u/M_b \qquad \text{(ACI Code 19-9)} \qquad \text{Eq.43}$$

$$P_b=\phi\left[0.85f'_c b a_b\right] \qquad \text{Eq.44}$$

$$M_b=\phi\left[0.85f'_c b a_b(d-d''-a_b/2)+A'_s f_y(d-d'-d'')+A_s f_y d''\right] \qquad \text{Eq.45}$$

(ACI Code 19-3)

$$A_s=A'_s$$

$$a_b=k_1\ 87{,}000\ d/87{,}000+f_y$$

Taking P_o from Eq.42 and putting all of the above values of P_b, M_b, A'_s, a_b into the Eq.43 and solving for A_s we get a quadratic equation in A_s

$$\alpha A_s^2-2\beta A_s+\gamma=0 \qquad \text{Eq.46}$$

where

$$\alpha=2\phi f_y(f_y-0.85f'_c)(d-d') \qquad \text{Eq.47}$$

$$\beta=-M_u(f_y-.85f'_c)-P_u f_y(d-d')/2+.5\phi\,.85f'_c btf_y(d-d')+\phi.85f'_c ba_b(f_y-.85f'_c)(d-d''-.5a_b$$

$$\gamma = 0.85^2 \phi f'_c b^2 t a_b (d-d''-.5a_b) + .85 f'_c M_u b(a_b - t) - 0.85 f'_c b a_b (d-d''-.5a_b) P_u$$

$$A_{st} = 2A_s = 2\,\beta/\alpha \left(1 - \sqrt{1 - \gamma\alpha/\beta^2}\right) \qquad \text{Eq.48 a}$$

$$A_{st} = 2\left(\beta/\alpha\right)\left(1 - \sqrt{1 - \frac{(\gamma/\alpha)}{(\beta/\alpha)^2}}\right) \qquad \text{Eq.48 b}$$

2-Empirical equation- The ultimate load is given by:

$$P_u = \phi \left[\frac{A'_s f_y}{\frac{e}{d-d'} + 0.5} + \frac{b t f'_c}{(3te/d^2) + 1.18} \right] \qquad \text{(ACI Code 19-10)}$$

Eq.49

Solution of the above equation for A gives

$$A_{st} = 2A_s = 2 \left[\frac{P_u}{\phi} - \frac{b t f'_c}{(3te/d^2) + 1.18} \right] \frac{(e/d-d') + 0.5}{f_y} \qquad \text{Eq.50}$$

See appendix A parts 7 and 8 for the derivations of the equations 48 and 50 .

CIRCULAR SHORT COLUMNS WITH BARS CIRCULARLY ARRANGED

The reinforcement is computed using the empirical equations given in the ACI Code 318-63 .

1-When tension controls:

$$P_u = \phi \left\{ 0.85 f'_c D^2 \left[\sqrt{\left(\frac{0.85e}{D} - 0.38\right)^2 + \frac{p_t m D_s}{2.5D}} - \left(\frac{0.85e}{D} - 0.38\right) \right] \right\} \qquad \text{(ACI Code 19-11)}$$

Eq.51

Solving equation 51 for p_t and multiplying it by A_g we get A_{st} as

$$A_{st} = p_t\, A_g = \frac{2.5 P_u D}{\phi\, D_m^2 D_s f'_c 0.85} A_g \left(\frac{P_u}{0.85 f'_c D^2} + \frac{1.7e}{D} - 0.76 \right) \qquad \text{Eq. 52}$$

See appendix A9 for the derivation of the formula.

2-When compression controls :

$$P_u = \phi \left[\frac{A_{st} f_y}{\frac{3e}{D_s} + 1} + \frac{A_g f'_c}{\frac{9.6\, De}{(0.8D + 0.67 D_s)^2} + 1.18} \right] \qquad \text{(ACI Code 19-12) Eq. 53}$$

Solving for A_{st} we get

$$A_{st} = \frac{\frac{3e}{D_s} + 1}{f_y} \left[\frac{P_u}{\phi} - \frac{A_g f'_c}{\frac{9.6De}{(0.8D + 0.67 D_s)^2} + 1.18} \right] \qquad \text{Eq. 54}$$

See appendix A 10 for the derivation of the above formula.

SQUARE SHORT COLUMNS WITH BARS CIRCULARLY ARRANGED -

The reinforcement is computed using the equations given in the ACI Code, these are empirical equations.

1- When tension controls:

$$P_u = \phi \left\{ 0.85 b t f' \left[\sqrt{\left(\frac{e}{t} - 0.5\right)^2 + 0.67 \frac{D_s}{t} p_t m} - \left(\frac{e}{t} - 0.5\right) \right] \right\} \qquad \text{(ACI Code 19-13) Eq. 55}$$

Solving equation 55 for p_t, and multiplying it by A_g we get A_{st} as

$$A_{st} = p_t A_g = \frac{t \quad P_u}{0.67 D_s m \phi \, 0.85 t^2 f'_c} \left[\frac{P_u}{0.85 t^2 f'_c} + \frac{2e}{t} - 1 \right] A_g \qquad \text{Eq.56}$$

See appendix A11 for the above equation.

2-When compression controls:

$$P_u = \phi \left[\frac{A_{st} f_y}{\frac{3e}{D_s} + 1} + \frac{A_g f'_c}{\left(\frac{12te}{t+0.67D_s}\right)^2 + 1.18} \right] \qquad \text{(ACI Code 19-14)} \qquad \text{Eq.57}$$

Solving equation 57 for A_{st} we get

$$A_{st} = \frac{\frac{3e}{D_s} + 1}{f_y} \left[\frac{P_u}{\phi} - \frac{A_g f'_c}{\left(\frac{12te}{t+0.67D_s}\right)^2 + 1.18} \right] \qquad \text{Eq.58}$$

See appendix A 12 for the derivation of the above equation.

UNIAXIAL BENDING OF COLUMNS

The formulas satisfying the conditions of equilibrium and compatibility of strains are derived in the following pages.The basic assumptions are those given in the Code ACI 318-63.

The column sections having symmetrical reinforcement in two faces are considered.

RECTANGULAR CONCRETE CROSS SECTIONS

The expressions giving the ultimate load and moment carried by rectangular concrete sections are derived from the fig.3 where the dimensions and variables related to the section are shown.

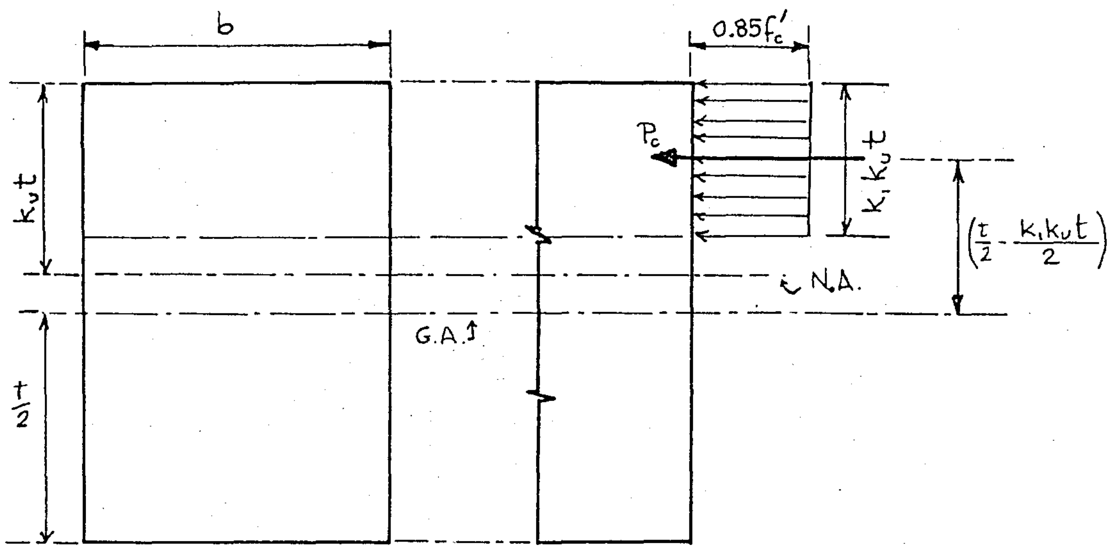


Fig.3

By summing the forces in the compressed portion of the section, one gets

$$P_c = 0.85f_c'k_1k_ubt \qquad \text{Eq.59}$$

If the moment of P_u is taken about the gravity axis of the section, the resulting expression will be

$$M_c = 0.85f_c'k_1k_utb(t/2-k_1k_ut/2) = 0.85f_c'bt^2k_1k_u(1-k_1k_u)/2 \qquad \text{Eq.60}$$

When P_c is divided by $0.85f_c'bt$, and M_c by $0.85f_c'bt^2$, the dimensionless expressions are obtained as

$Q_c = 0.85k_1k_u$ for $k_1k_u < 1.0$ Eq.61

$Q_c = 0.85$ for $k_1k_u > 1.0$ Eq.62

$R_c = 0.85k_1k_u\left(\frac{1-k_1k_u}{2}\right)$ for $k_1k_u < 1.0$ Eq.63

$R_c = 0.0$ for $k_1k_u > 1.0$ Eq.64

We found the expressions for the load and moment carried by a rectangular concrete section. Now we have to compute the load and moment carried by the reinforcing steel from the conditions of equilibrium, and the compatibility of strains. Then we will add the load and moment carried by the concrete to the one carried by steel.

RECTANGULAR COLUMNS WITH REINFORCEMENT ON TWO FACES

CASE 1, Fig.4.

Case 1 considers a rectangular section where the strain in the outermost reinforcement is greater than the strain at yield in the outermost reinforcement $(\epsilon_s > \epsilon_y)$ and where the strain in the outermost compression reinforcement is less than the strain at yield in the outermost reinforcement $(\epsilon'_s < \epsilon_y)$.

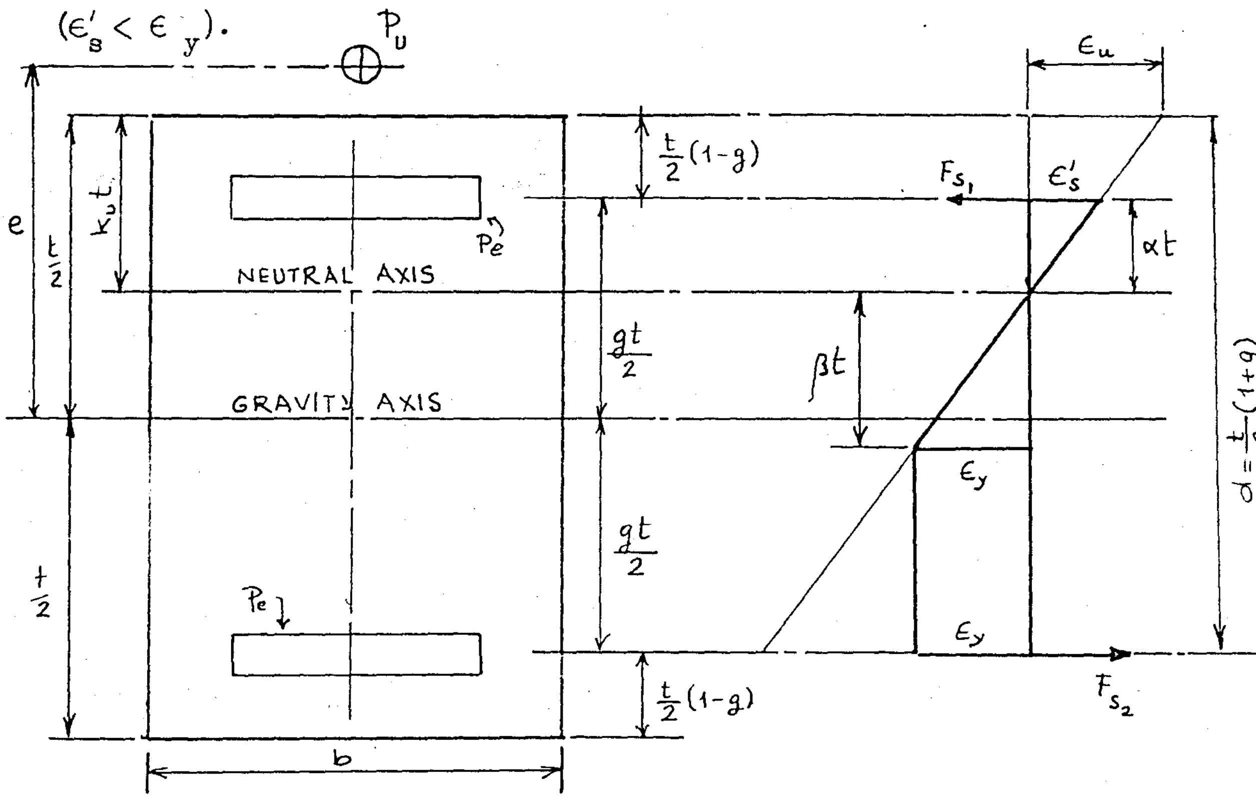


Fig.4

From the fig.4

$$d = gt/2 + t/2 = (t/2)(1+g) \quad \text{Eq.65}$$

$$d' = t-d = (t/2)(1-g) \quad \text{Eq.66}$$

From similar triangles $\epsilon_y / \beta t = \epsilon_u / k_u t$

$$\beta = \frac{\epsilon_y}{\epsilon_u} k_u \qquad \text{Eq.67}$$

$$\alpha t = k_u t - (t/2)(1-g)$$

$$\alpha = (2k_u - 1 - g)/2 \qquad \text{Eq.68}$$

$$F_{s_1} = (p_e bt)\left[(f_s) - 0.85 f'_c\right]$$

$$F_{s_2} = (p_e bt) f_y$$

$$F = \text{Force in end steel} = (p_e bt)\left[f_s - 0.85 f'_c - f_y\right] = F_{s_1} - F_{s_2} \qquad \text{Eq.69}$$

From the similar triangles $\epsilon_y / \beta t = \epsilon'_s / \alpha t$

$$\epsilon'_s = \epsilon_y \, \alpha / \beta \qquad \text{Eq.70}$$

$$f_s = E_s \epsilon_s \qquad \text{Eq.71}$$

$$f_y = E_s \epsilon_y \qquad \text{Eq.72}$$

Eliminating ϵ'_s and ϵ_y between the equations (70),(71),and (72) we get

$$f_s = f_y \, \alpha / \beta \qquad \text{Eq.73}$$

Putting Eq.(73) into (69)

$$F = (p_e bt)\left[\left(f_y \, \alpha/\beta\right) - 0.85 f'_c - f_y\right] \qquad \text{Eq.74}$$

factoring out $0.85 f'_c$ and multiplying it by the capacity reduction factor ϕ we have

$$P_{es} = f'_c bt 0.85 p_e \left[\left(m \, \alpha/\beta\right) - 1 - m\right] \qquad \text{Eq.75}$$

This is the ultimate load carried by end steel,we have to add to it the load carried by the concrete taking it from the eq.(61).

$$P_u = \phi f'_c bt \left\{ 0.85 p_e \left[(m \alpha/\beta) - 1 - m \right] + 0.85 k_1 k_u \right\} \qquad \text{Eq.76}$$

Taking moments about the plastic centroid which coincides with the gravity axis in this case, we get

$$M_{es} = (0.85 p_e)\ f'_c bt (gt/2) \left[(m\, \alpha/\beta) - 1 + m \right] \qquad \text{Eq.77}$$

in equation 77 $(gt/2)$ is the lever arm.

$$M_c = f'_c bt^2\ 0.85\ k_1 k_u \frac{(1 - k_1 k_u)}{2} \qquad \text{Eq.78}$$

Thus we have the ultimate moment carried by the steel and the concrete, adding them we get the ultimate moment carried by the section,

$$M_u = (M_{es} + M_c) = \phi f'_c bt^2 \left\{ 0.85 p_e (g/2) \left[m\, \alpha/\beta - 1 + m \right] + 0.85 k_1 k_u (1 - k_1 k_u)/2 \right\}$$

Eq.79

We put β and α from equations 67 and 68 into (76) and (79), then eliminating p_e between (76) and (79) and solving for k_u we get a cubic equation

$$A k_u^3 + B k_u^2 + C k_u + D = 0 \qquad \text{Eq.80}$$

where

$$A = -0.85 k_1^2 \left[m(\epsilon_u - \epsilon_y) - \epsilon_y \right]$$

$$B = \left\{ 0.85 k_1 \left[-\epsilon_u m(g-1) + \epsilon_y (g-1) - \epsilon_y m(g+1) \right] - (0.85 k_1^2/2)(\epsilon_u m)(g-1) \right\}$$

$$C = \left\{ \frac{P_u g}{\phi f'_c bt} (\epsilon_u m - \epsilon_y - \epsilon_y m) - \frac{2M_u}{\phi f'_c bt^2} (\epsilon_u m - \epsilon_y - \epsilon_y m) - \frac{0.85}{2} k_{c1} \epsilon_u m (g-1)^2 \right\}$$

$$D = \frac{\epsilon_u m(g-1)}{\phi\ f' bt} \left[\frac{P_u g}{2} - \frac{M_u}{t} \right]$$

Thus having k_u we compute p_e from one of the equations (76) or (79), and get the reinforcement as

$$A_{st} = 2 p_e bd$$

(see appendix B 1 for further detail) .

CASE 2, Fig.5.

Case 2 considers a rectangular section where the strain in the outermost reinforcement is greater than the strain at yield in the outermost reinforcement ($\epsilon_s > \epsilon_y$) and where the strain in the outermost compression reinforcement is greater than the strain at yield in the outermost reinforcement ($\epsilon'_s > \epsilon_y$).

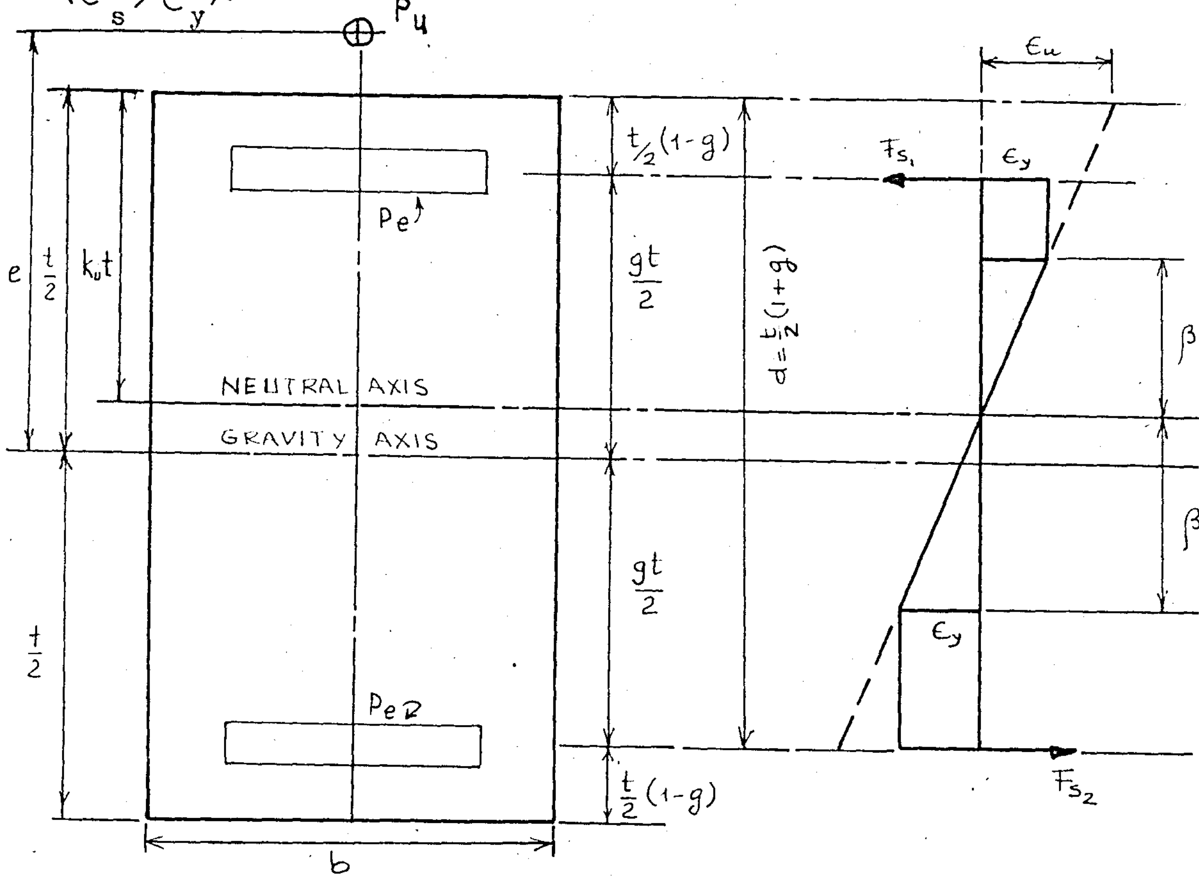


Fig.5

For this case from the figure we get the limits of k_u :

$$\left[(1-g)/2\right] t + \beta t \leq k_u t \leq \left[(1+g)/2\right] t - \beta t$$

$$F_{s_1} = (p_e b t)\left[f_y - 0.85 f'_c \right]$$

$$F_{s_2} = p_e bt\, f_y$$

$$F = \text{Force in end steel} = F_{s_1} - F_{s_2} = -p_e bt 0.85 f'_c \qquad \text{Eq.81}$$

Adding to the load carried by steel the load carried by concrete, and multiplying by capacity reduction factor ϕ ,we get

$$P_u = \phi\ f'_c bt \left[-0.85 p_e + 0.85\ k_1 k_u \right] \qquad \text{Eq.82}$$

(In eq.81 we factor out $0.85f'_c$, and take the load carried by concrete from the equation 61 .)

Taking moment about the plastic centroid which coincides with the gravity axis in this case,we get

$$M_{es} = (gt/2) \left[F_{s_1} + F_{s_2} \right] = (gt/2) p_e bt \left[f_y - 0.85 f'_c + f_y \right]$$

$$M_{es} = p_e bt^2 (g/2) \left[2f_y - 0.85 f'_c \right] \qquad \text{Eq.83}$$

Factoring out $0.85f'_c$ and multiplying by ϕ ,we obtain the ultimate moment carried by the reinforcement,then if we add to it the moment carried by the concrete section alone from the Eq.60,we will get the ultimate moment carried by this section.

$$M_{es} = f'_c bt^2 (g/2) \left[(0.85p)(2m-1) \right]$$

$$M_c = f'_c bt^2 \left[0.85\ k_1 k_u (1 - k_1 k_u)/2 \right]$$

$$M_u = \phi (M_{es} + M_c) = \phi\, f'_c bt^2 \left[(0.85 p_e)(g/2)\ (2m-1) + 0.85 k_1 k_u (1 - k_1 k_u)/2 \right] \qquad \text{Eq.84}$$

Eliminating p_e between (82) and (84) ,a quadratic equationis obtained for k_u ,

$$k_u^2 - 2\alpha\, k_u + \beta = 0 \qquad \text{Eq.85}$$

$$k_u = \alpha \left[1 - \sqrt{1 - \beta/\alpha^2} \right] \qquad \text{Eq.86}$$

where

$$\alpha = \frac{g(2m-1)}{2k_1} + \frac{1}{2k_1}$$

and

$$\beta = \frac{gt(2m-1)P_u + 2M_u}{\phi\, 0.85 f_c' b t^2 k_1^2}$$

Thus having k_u we compute p_e from one of the equations (82) or (84), and get the reinforcement area as

$$A_{st} = 2p_e bd$$

(see appendix B2 for further detail).

CASE 3, Fig.6.

Case 3 considers a rectangular section where the strain in the outermost reinforcement is less than the strain at yield in the outermost reinforcement ($\epsilon_s < \epsilon_y$) and where the strain in the outermost compression reinforcement is greater than the strain at yield in the outermost reinforcement ($\epsilon_s' > \epsilon_y$).

From the figure 6 we get the limits of k_u as

$$(t/2)(1+g) - \beta t \leqslant k_u t \leqslant (t/2)(1+g)$$

From the similar triangles (fig.6.)

$$\epsilon_s / (\Phi\ t) = \epsilon_y / (\beta\ t) \qquad \therefore \qquad \epsilon_s = \epsilon_y \Phi / \beta \qquad \text{Eq.87}$$

From the fig.6. noticing $\Phi t + k_u t = (t/2)(1+g)$

$$\Phi = \frac{1+g}{2} - k_u \qquad \text{Eq.88}$$

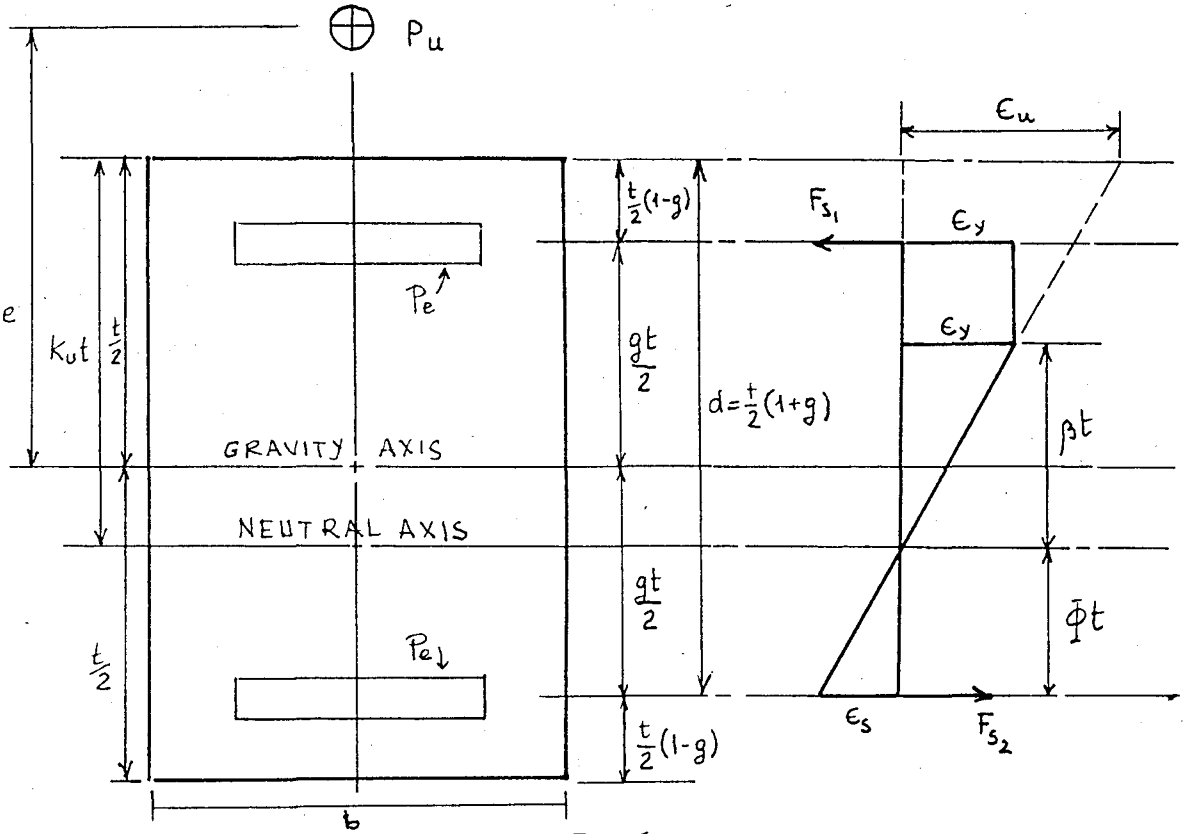


Fig.6 .

$$F_{s_1} = (p_e bt)\left[f_y - 0.85f'_c\right] \qquad \text{Eq.89}$$

$$F_{s_2} = (p_e bt)\, f_s \qquad \text{Eq.90}$$

$$f_s = E_s\, \epsilon_s \qquad \text{Eq.91}$$

$$f_y = E_y\, \epsilon_y \qquad \text{Eq.92}$$

From equations (87),(91),(92) we derive

$$f_s = f_y\, \Phi/\beta \qquad \text{Eq.93 a}$$

Putting eq.93 into eq.90 F_{s_2} becomes

$$F_{s_2} = (p_e bt)\; f_y\; \Phi/\beta \qquad \text{Eq.93}$$

Force in end steel becomes

$$F = F_{s_1} - F_{s_2} = (p_e bt)\left[f_y - 0.85f'_c - f_y\, \Phi/\beta\right] \qquad \text{Eq.94}$$

In eq.94 factoring out $0.85f'_c$,adding to this load carried by reinforcement the load carried by concrete taken from Eq.61, and multiplying Eq.94 by ϕ we get the ultimate load carried by this section as

$$P_u = \phi\,(F + F_c) = \phi\, f_c' bt\left[(0.85p_e)(m-1-m\,\Phi/\beta) + 0.85k_1k_u\right] \qquad \text{Eq.95}$$

Taking moments about the plastic centroid which coincides in this case with the gravity axis, we get

$$M_{es} = (gt/2)\,(F_{s_1} + F_{s_2}) = p_e bt\,(gt/2)\left[f_y - 0.85f_c' + f_y\,\Phi/\beta\right] \qquad \text{Eq.96}$$

Factoring out $0.85f_c'$ and multiplying by

$$M_{es} = f'bt^2\; 0.85p_e(g/2)\left[m-1+m\,\Phi/\beta\right] \qquad \text{Eq.97}$$

$$M_c = f_c'bt^2\; 0.85\, k_1k_u(1-k_1k_u)/2 \qquad \text{Eq.60}$$

The ultimate load carried by the sectionis obtained by summing the moments carried by reinforcement and concrete separately

$$M_u = \left[M_{es} + M_c\right]\phi = \phi\, f_c'bt^2\left[0.85p_e(g/2)(m-1+m\,\Phi/\beta) + 0.85k_1k_u(1-k_1k_u)/2\right] \qquad \text{Eq.98}$$

Eliminating p_e between Eq.95 and Eq.98, we get an equation of third degree in k_u,

$$A\,k_u^3 + B\,k_u^2 + C\,k_u + D = 0 \qquad \text{Eq.99}$$

where

$$A = \left[-0.85k\;(\epsilon_y m - \epsilon_y + \epsilon_u m)\right]$$

$$B = \left[-0.85k_1 g(\epsilon_y m - \epsilon_y - \epsilon_u m) + 0.85k_1(\epsilon_y m - \epsilon_y + \epsilon_u m) + \frac{0.85k_1^2}{2}\,\epsilon_u m(g+1)\right]$$

$$C = \left[\frac{P_u}{\phi f_c'bt}g(\epsilon_y m - \epsilon_y - \epsilon_u m) - \frac{0.85}{2}k_1 gm\,\epsilon_u(1+g) - \frac{2M_u}{\phi f_c'bt^2}(\epsilon_y m - \epsilon_y + \epsilon_u m) - \frac{0.85}{2}\,k_1\,\epsilon_u m(g+1)\right]$$

$$D = \frac{m\,\epsilon_u(1+g)}{\phi f'_c bt}\left[\frac{P_u g}{2} + \frac{M_u}{t}\right]$$

Thus having k_u, the depth of the rectangular concrete compression zone, we may compute p_e from one of the equations (95) or (98), and get the reinforcement area as

$$A_{st}\ 2p_e bd$$

(see appendix B3 for the above derivations) .

CASE 4, Fig.7

Case 4 considers a rectangular section with compression throughout; the neutral axis falling outside the cross section; and the strain in the outermost compression reinfocement is greater than the strain at yield in the outermost reinforcement $(\epsilon'_s > \epsilon_y)$

In this case $\frac{(1+g)}{2}\,t \leqslant k_u t \leqslant (1/k_1)t$

From the Fig.7

$$\eta = k_u - \left(\frac{1+g}{2}\right) \qquad \text{Eq.100}$$

From similar triangles (Fig.7)

$$\epsilon_s/\eta t = \epsilon_y/\beta t \quad ; \quad \epsilon_s = \epsilon_y(\eta/\beta) \qquad \text{Eq.101}$$

$$f_s = E_s \epsilon_s \qquad \text{Eq.102}$$

$$f_y = E_y \epsilon_y \qquad \text{Eq.103}$$

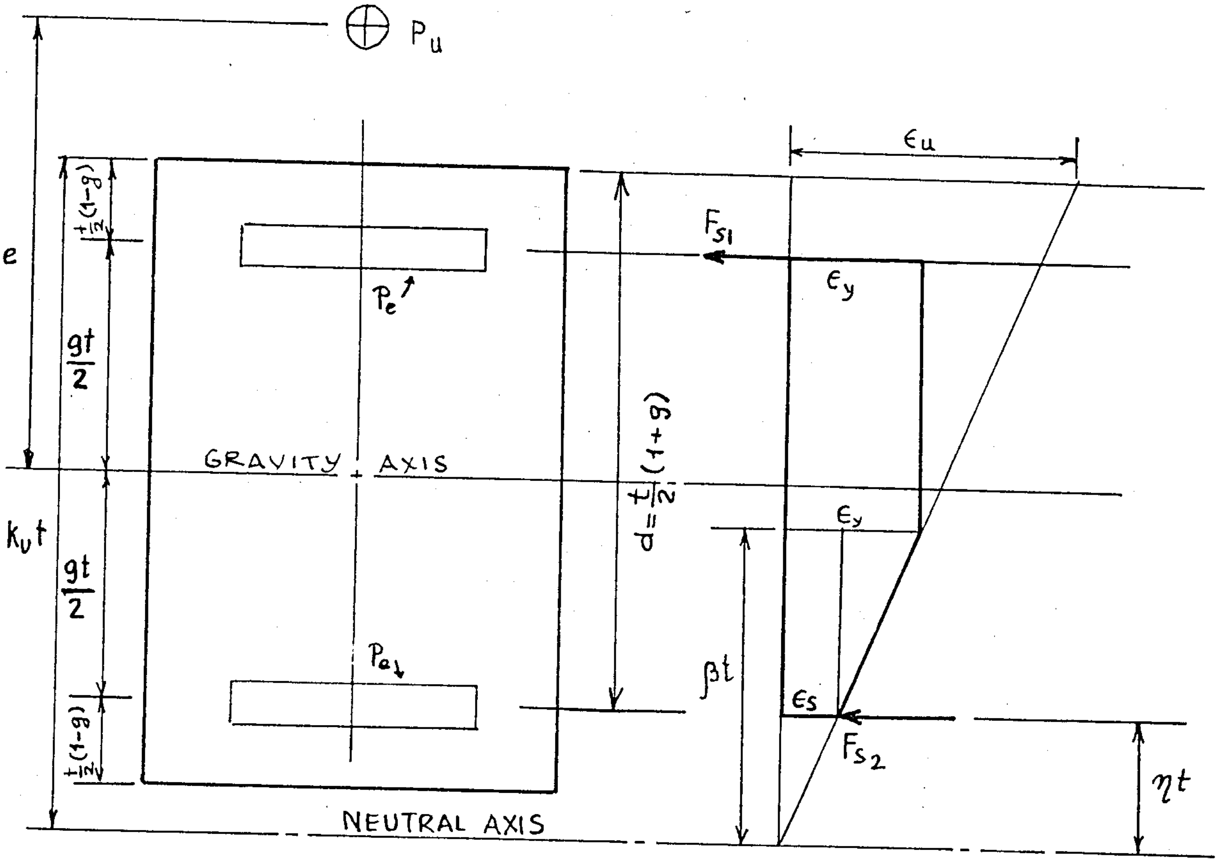


Fig.7

From equations (101),(102), and (103) we derive

$$f_s = f_y \, \eta/\beta \qquad \text{Eq.104}$$

$$F_{s_1} = p_e bt \left(f_y - 0.85\, f'_c\right)$$

$$F_{s_2} = p_e bt \left(f_s - 0.85 f'_c\right)$$

Substituting f_s from(104) into F_{s_2} we have

$$F_{s_2} = (p_e bt)\left[\, f_y \, \eta/\beta \; - 0.85 f'_c\right]$$

Force in the end steel becomes

$$F = F_{s_1} + F_{s_2} = (p_e bt)\left[\, (f_y - 0.85 f'_c) + (f_y \, \eta/\beta - 0.85 f'_c)\right] \qquad \text{Eq.105}$$

Factoring out $0.85f'_c$, multiplying by ϕ the eq.105 ,and adding to it the load carried by concrete taken from the eq.61, we get the ultimate load carried by the section as

$$P_u = \phi[F + F_c] = \phi f'_c bt\left\{0.85p_e\left[(m-1)+(m\,\eta/_\beta\ -1)\right]+0.85k_1k_u\right\} \qquad \text{Eq.106}$$

Taking moments about the elastic centroid which coincides in this case with the gravity axis, we get

$$M_{es} = (gt/2)\left(F_{s_1} - F_{s_2}\right) = p_e bt(gt/2)\left[(f_y - 0.85f'_c) - (f_y\,\eta/_\beta\ - 0.85f'_c)\right]$$

Factoring out $0.85f'_c$ and multiplying by ϕ

$$M_{es} = f'_c bt^2\ 0.85p_e\ (g/2)\left[(m-1)-(m\,\eta/_\beta\ -1)\right]$$

$$M_c = f'_c bt^2\ 0.85\ k_1k_u\ (1-k_1k_u)/2 \qquad \text{Eq.60}$$

The ultimate moment carried by the section is obtained by summing the moments carried by reinforcement and concrete seperetely

$$M_u = [M_{es} + M_c]\phi = \phi f'_c bt^2\left\{0.85p_e(g/2)\left[(m-1)-(m\,\eta/_\beta\ -1)\right] + 0.85k_1k_u\frac{(1-k_1k_u)}{2}\right\}$$

Eq.107

Eliminating p_e between the equations (106) and (107), we get an equation of third degree in k_u,

$$A\,k_u^3 + B\,k_u^2 + C\,k_u + D = 0 \qquad \text{Eq.108}$$

$$A = \left[-0.85k_1^2\ (\epsilon_y m - 2\,\epsilon_y + \epsilon_u m)\right]$$

$$B = \left[-0.85k_1 g(\epsilon_y - \epsilon_u) + 0.85k_1(\epsilon_y m - 2\epsilon_y + \epsilon_u m) + \frac{0.85k_1^2}{2}\epsilon_u m(1+g)\right]$$

$$C = \left\{\frac{P_u g}{\phi f'_c bt}(\epsilon_y - \epsilon_u)m - \frac{0.85k_1}{2}\ \epsilon_u m(g+1)^2 - \frac{2M_u}{\phi f'_c bt^2}\ (\epsilon_y m - 2\,\epsilon_y + \epsilon_u m)\right\}$$

$$D = \left\{\frac{\epsilon_u m(1+g)}{\phi f'_c bt}\left[(P_u/2)g + M_u/t\right]\right\}$$

Thus knowing k_u, the depth of the rectangular concrete compression zone, we may compute p_e from one of the equations (106) or (107), and get the reinforcement area as $A_{st} = 2p_e bd$

See appendix B4 for the above derivations.

CASE 5.

Case 5 considers a rectangular section with compression throughout; the neutral axis falling outside the cross section;and the strain in the outermost compression reinforcement is greater than the strain at yield in the outermost reinforcement ($\epsilon'_s > \epsilon_y$) .

$k_u = 1/k_1$ $\qquad F_c = 0.85f'_c bt$ $\qquad$ from eq.62

$k_u = 1/k_1$ $\qquad M_c = 0$ $\qquad$ from eq.64

The ultimate load and moment carried by the reinforcement is the same as in the case 4.

$$F = f'_c bt \; 0.85p_e \left[(m-1)+(m\,\eta/\beta \; -1)\right]$$

Adding the concrete ultimate load we get

$$P_u = \phi \; f'_c bt\left\{(0.85p_e)\left[(m-1)+(m\,\eta/\beta \; -1)\right]+0.85\right\} \qquad \text{Eq.109}$$

The ultimate moment carried by the section is equal to the one carried by the reinforcement alone,since the concrete does not carry any moment,eq.64.

$$M_u = f'_c bt^2\phi\left\{0.85p_e(\varepsilon/2)\left[(m-1)-(m\,\eta/\beta \; -1)\right]\right\} \qquad \text{Eq.110}$$

Eliminating p_e between equations (109) and(110),then solving for k_u,we get:

$$k_u = \frac{-\epsilon_u m(1+g)\,\frac{g}{2}\left(\frac{P_u}{\phi f'_c bt} - 0.85\right) - \frac{M_u}{\phi f'_c bt^2}}{mg(\epsilon_y - \epsilon_u)\left(\frac{P_u}{\phi f'_c bt} - 0.85\right) - \frac{2M_u}{\phi f'_c bt^2}(\epsilon_y m - 2\epsilon_y - \epsilon_u m)} \qquad \text{Eq.111}$$

Then we may solve for p_e from Eq. (109) or (110) and get the reinforcement area as:

$$A_{st} = 2p_e bd$$

See appendix B 5 for the above derivations.

RECTANGULAR BEAMS UNDER MOMENT AND SMALL AXIAL LOAD

In the computation of ultimate load and moment for rectangular beams usually the small axial forces are neglected. However, winds, earthquakes, etc.., create axial loads. This load may be taken into account considering the beam under the effect of an eccentric force which creates moment. The axial eccentric force being smaller we get at ultimate load the case where tension controls (P_u P_b)

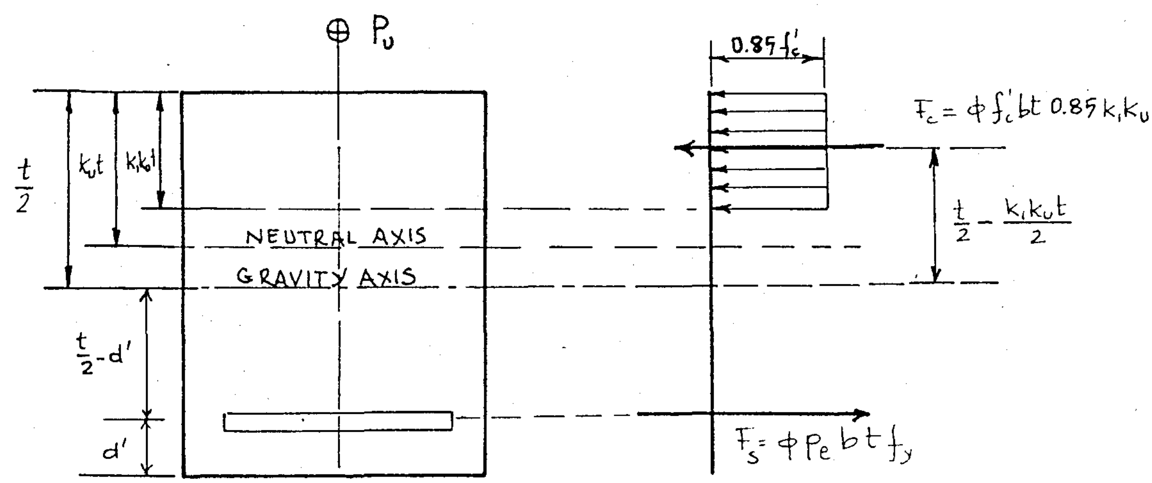


Fig.8

From the fig.8

Summing internal and external forces

$$P_u = \phi\ 0.85 k_1 k_u f'_c bt - \phi\ p_e btf_y \qquad \text{Eq.112}$$

Taking moment around the gravity axis

$$M_u = \phi\ 0.85 k_1 k_u f'_c bt\ \left(\frac{t}{2} - \frac{k_1 k_u t}{2}\right) + \phi\ p_e btf_y\left(\frac{t}{2} - d'\right) \qquad \text{Eq.113}$$

Eliminating p_e between Eqs. (112) and (113), and solving for k_u, we get

$$k_u^2 - 2\alpha k_u + \beta = 0 \qquad \text{Eq.114}$$

where

$$\alpha = \frac{1}{k_1}(1-\frac{d'}{t}) = d/k_1 t \quad ; \quad \beta = \frac{2\left[M_u + P_u(t/2 - d')\right]}{0.85 k_1^2 f'_c b t^2} \qquad \text{Eq.114}$$

$$k_u = \alpha\left[1 - \sqrt{1 - \beta/\alpha^2}\right] \qquad \text{Eq.115}$$

Having k_u, we easily get p_e from one of the equations (112) or (113), then we compute A_{st}

$$A_{st} = p_e bt \qquad \text{Eq.116}$$

$$A_{st} = \left\{ \chi / f_y \left[1 - \sqrt{1 - \frac{2\left[M_u + P_u \ (t/2 - d')\right]}{\phi \chi \quad d}} \right] - \frac{P_u}{\phi f_y} \right\} \qquad \text{Eq.117}$$

where

$$\chi = 0.85 f'_c bd$$

(See appendix B 6 for further detail)

COLUMNS SUBJECT TO BIAXIAL BENDING

Where beams and girders frame into the column in the direction of both walls and transfer their end moments into the column in two perpendicular planes, we have axial compression accompanied by simultaneous bending about both principal axes of the column section. This happens in corner columns, but similar situations can occur with respect to interior columns, especially in irregular column layouts, and in a great variety of other structures.

We may compute the ultimate load of a biaxially eccentric column on the basis of the general basic assumptions of ultimate strengt design. There are mainly two different methods, one developed by A.H.Mattock, L.B.Kriz, and Eivind Hognestad (ref.8), and the other by Boris Bresler (ref.1).

1-The method developed by Mattock, Kriz, and Hognestad- When a section is under the action of an axial load and bending about both principal axes, the neutral axis at ultimate load takes an individual position caused by only one combination of axial load and moments acting on this section. If the magnitude of the axial load or either bending moment is changed, the position of the neutral axis will also be changed. Therefore, considering all possible positions of the neutral axis, we may cover all possible combinations of axial load and bending moments that cause the section to reach the ultimate load. In this method the position of the bottom of the compressive concrete stress block is defined by its coordinates in x and y directions. Then knowing the concrete compression zone area, the resultant concrete force may be computed and located at the centroid of the concrete compression zone. These quatities may be computed relatively easily since expressions for the area of concrete compression zone, A_c, and the coordinates of its centroid, x_c, and y_c, have been derived by Mattock and Kriz for different positions of the neutral axis.

The resutant tensile and compressive steel forces, F_{st} and F_{sc} and their centers of action are calculated by statics. Then from the conditions of equilibrium, the ultimate axial load is found from summing internal and external forces and its eccentricities from the center of the section are found by summing moments about the extreme compressive concrete fiber.

2- The Method developed by Parme, Nieves, and Gouwens based on the equation suggested by Boris Bresler- The biaxial bending resistance of of an an axially loaded column may be represented in three dimensions as a failure surface formed by a series of interaction curves drawn radially from the P_u axis. At constant values of P_u we get different load contours. (Fig.9).

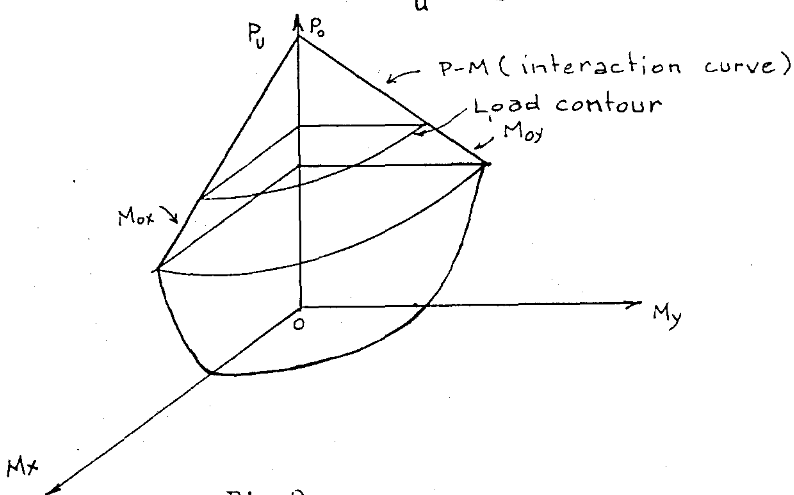


Fig.9

When the bending resistance is plotted in terms of the dimensioless parameters P_u/P_o, M_x/M_{ox}, and M_y/M_{oy}, the ultimate capacity surface generated takes the typical shape shown in Fig.10 .

The contours can be approximated by the expressions

$$(Mx/M_{ox})^n+(M_y/M_{oy})^n = 1 \qquad \text{Eq.118}$$

and

$$\left(\frac{M_x}{M_{ox}}\right)^{\frac{\log 0.5}{\log}} + \left(\frac{M_y}{M_{oy}}\right)^{\frac{\log 0.5}{\log}} = 1 \qquad \text{Eq.119}$$

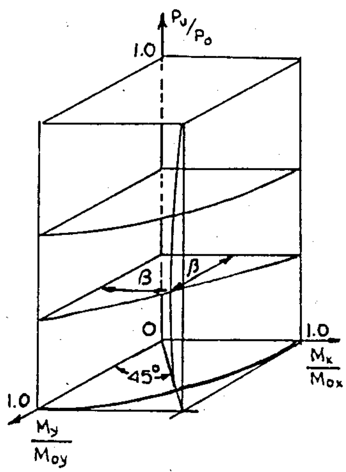
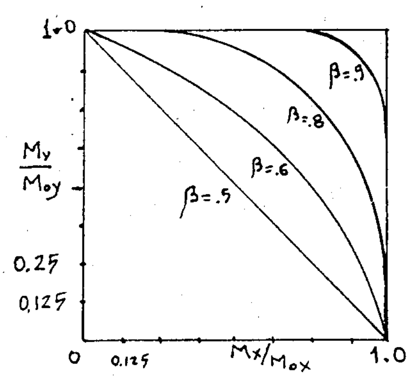


Fig.10

These interaction equations are suggested by Bresler (ref.1).

β is a factor which relates the biaxial bending of a column to its uniaxial resistance. When β=0.5, the lower limit of β, equation(119) describes a straight line joining the points at which the relative moments equal one at the coordinate planes. When β=1.0, its upper limit, equation (119) describes two lines each of which is parallel to one of the coordinate planes. For intermediate values of β equation (119) describes curves which have been sometimes called sub- and superellipses, Fig.11 .



M_x = Applied Moment about x-axis.
M_y = Applied Moment about y-axis.

M_{oy} = Uniaxial bending capacity about y-axis.
M_{ox} = Uniaxial bending capacity about x-axis.

Fig.11

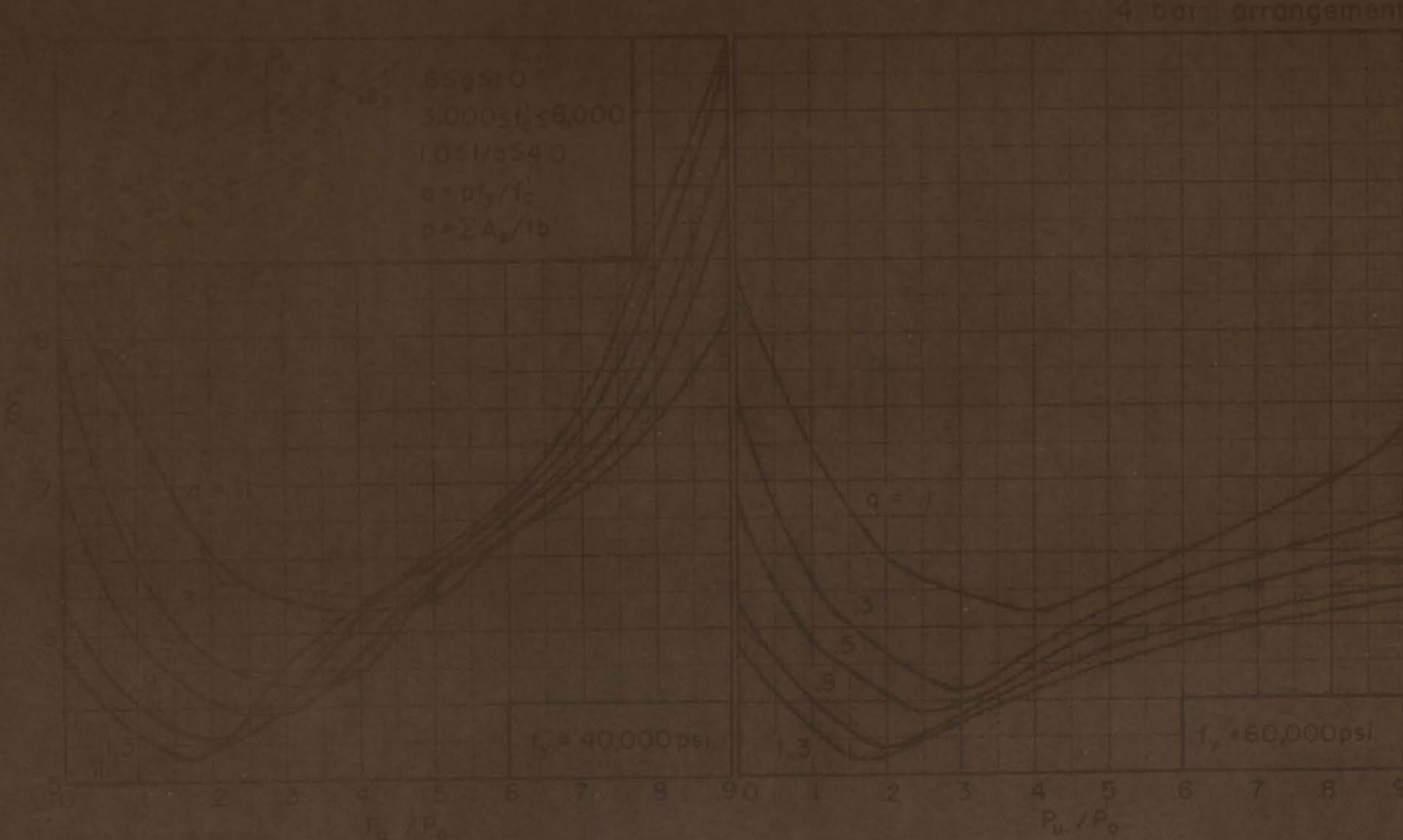


Fig. 5 — Biaxial Bending Design Constants

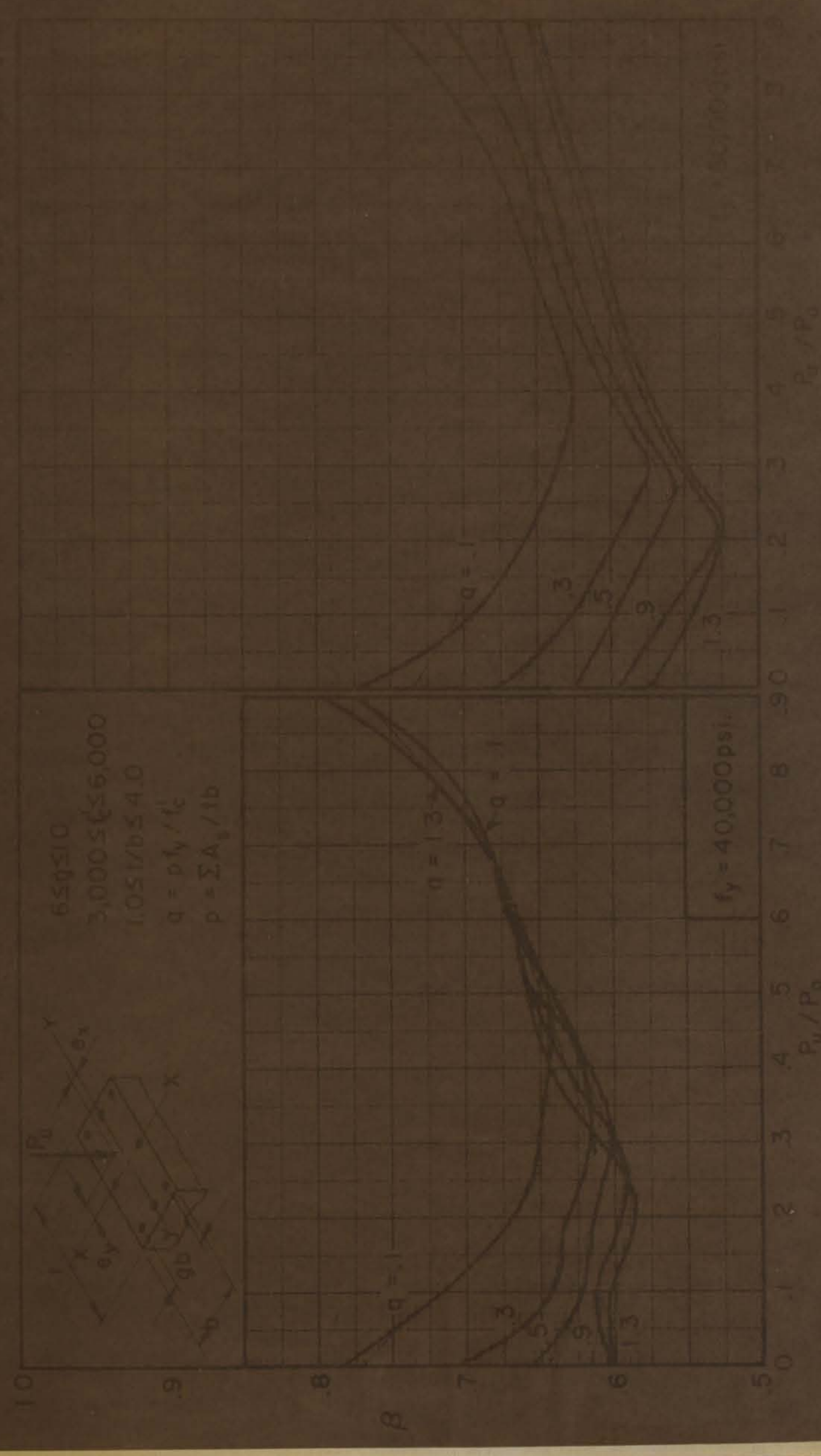


Fig. 6 – Biaxial Bending Design Constants

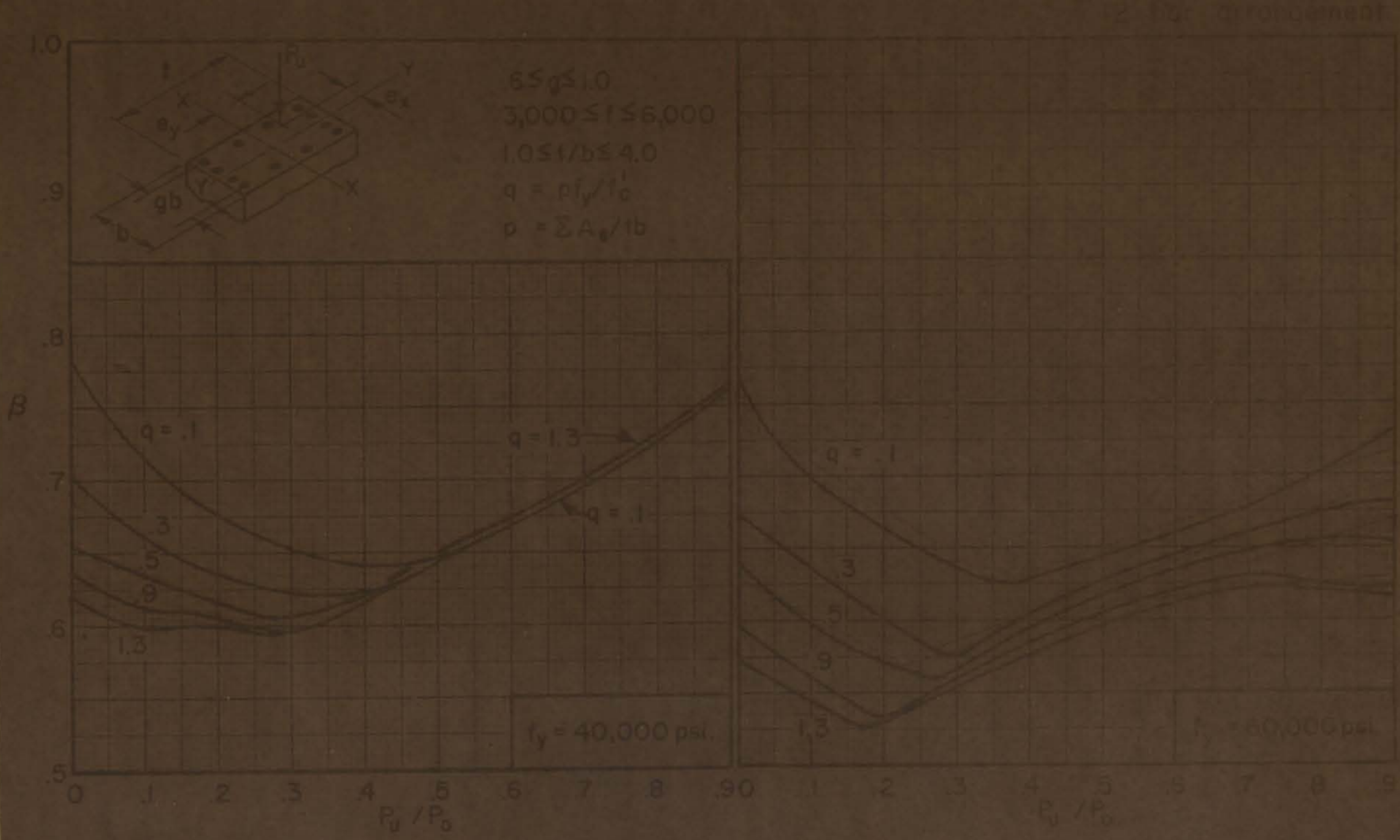


Fig. 7 — Biaxial Bending Design Constants

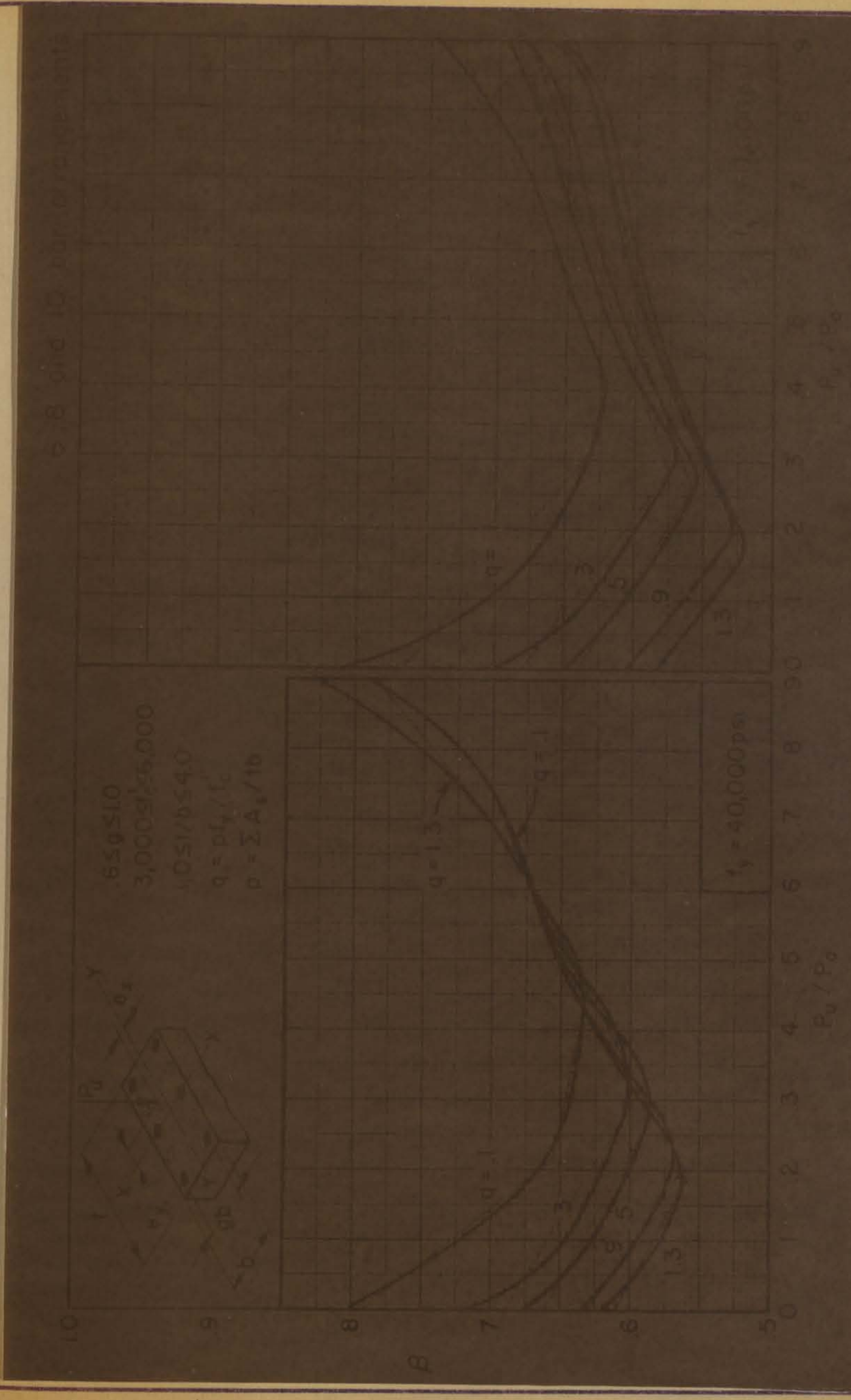


Fig. 8 - Biaxial Bending Design Constants

β is a function of the amount ,distribution,and location of the reinforcement, the dimensions of the column and of the strength and elastic properties of the steel and concrete.

β is dependent on the ratio P_u/P_o ,the bar arrangement, the reinforcement index,q,and the strength of the reinforcement.The envelopes of β values are given in figures 12 to 15. (They are taken from ref.9) .

For design purposes we may make our first approximations as straight lines instead of exponential contour.

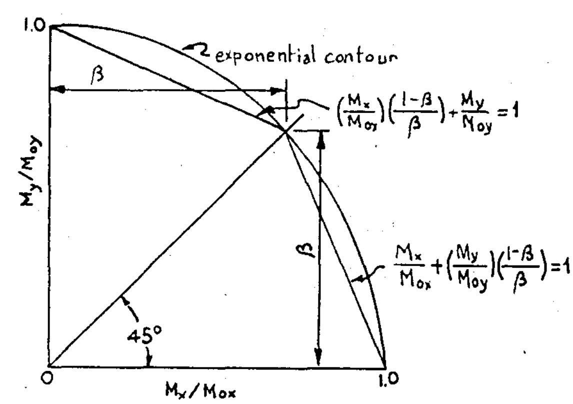


Fig.16

From the figure 16 ,by simple geometry the equation of the lines are:

when $M_y/M_{uy} > M_x/M_{ux}$

$$\frac{M_y}{M_{uy}} + \frac{M_x\,(1-\beta)}{M_{ux}\;\beta} = 1 \qquad \text{Eq.120}$$

Eq.120 may be written as

$$M_y + M_x\left(M_{uy}/M_{ux}\right)\ (1-\beta)/_{\beta} = M_{uy} \qquad \text{Eq.121}$$

For rectangular sections with reinforcement equally distributed on all faces, Eq.121 can be approximated by:

$$M_y + M_x \left(\frac{b}{t}\right)\frac{(1-\beta)}{\beta} \cong M_{Uy} \qquad \text{Eq.122}$$

And similarly :

when $M_y / M_{uy} < M_x / M_{ux}$

$$\frac{(1-\beta)}{\beta}\frac{M_y}{M_{uy}} + \frac{M_x}{M_{ux}} = 1 \qquad \text{Eq.123}$$

$$M_y \frac{M_{ux}}{M_{uy}} \frac{(1-\beta)}{\beta} + M_x = M_{ux} \qquad \text{Eq.124}$$

$$M_y \left(\frac{t}{b}\right)\frac{(1-\beta)}{\beta} + M_x \cong M_{ux} \qquad \text{Eq.125}$$

In the computer programming the second method is used, since it is more suitable to design than the first method in which we have to know reinforcement, its exact location, compressive concrete zone depth to get the ultimate load and moment. In the second case a uniaxial moment is computed from the equations 122 and 125, assuming 0.65 (since this is a good average value). Then a section is designed by empirical column equations, and from the idealized β, P_u / P_o, q charts (fig.12 to 15) a new β value is read. If this β value is different from the old one, we will compute new M_{ux} or M_{uy} from the equation 119 , then we will find new reinforcement and repeat the above iteration up to a point where we will get a β which will be in close agreement with the preceding one.

COMPUTER PROGRAM DESCRIPTION

Two programs are prepared, one for beam design (appendix C), and the other for column design (appendix C). Due to the memory capacity limitation of IBM 162o at Robert College, the two programs are debugged separately. However, a combined mainline of beams and column design is also given. It may be used in bigger computers.

Beam Analysis Program Description - Number of members, number of load cases, number of load combinations, participation and over stress factors, the concrete strength, the steel yield strength, and the end forces acting on the members are read. The factor k_1 is computed. The depth of the beam is checked against the ACI Code (318-63) section 910, and a message is given if the beam is deep.

Then the subroutine MPV is called. This subroutine computes the stress resultants of each member for different load cases, combining them in any prescribed way in order to obtain the critical condition. Howevér, the members are designed for each case of the combined stress resultants. The combined stress resultants are calculated not only at the ends of the member but also at midpoints and quarter points. The flexural and the shear reinforcements are calculated at each of these points.

In subroutine rectangular beam the reinforcement is computed using the refined expression that takes the effect of small axial loads into consideration. The maximum percentage of reinforcement can not exceed $0.75p_b$, this is checked. If the computed reinforcement gives a greater percentage than this maximum allowed value, or if the term under the square root of the derived equation is negative, a message saying "insufficient depth, depth is increased is given, and a new depth is computed by the Eq.7. This new depth is on the limit and to avoid any further complication in the reinforcement computation it is increased 10 %, then a new reinforcement is calculated, and the results are printed out.

Subroutine Shear-This subroutine determines the shear reinforcing steel area at five points of any member per inch length of the beam. Thus the designer may choose the spacing of stirrups and/or bent-up bars in accordance with the common practice.

The shear stress permitted on an unreinforced web is computed by equation (17-2) of ACI Code 318-63, taking into account the axial load in addition to shear and flexure. The modified bending moment given by eq.17-3 is used in eq.17-2. The nominal ultimate shear stress is computed by eq.17-1 (ACI 318, 63). For I- and T- sections b' is substituted for b in eq.17-1. By ACI 1705-b $v_u = 10\phi\sqrt{f'_c}$, if this condition is not met, a message saying "inadequate section for shear" is printed out. The computed allowable shear on unreinforced sections is checked against $3.5\phi\sqrt{f'_c(1+0.002\ N/A_g)}$ (ACI 1701-e). The longitudinal reinforcement is computed by eq.(17-4), of the ACI Code. Then the computed reinforcement is checked against 0.15 % of the area "bs" and set equal to it in case when it is smaller than this value (ACI Code 318-63, 1706-b) .

Subroutine T- Beam - The reinforcement is computed by eq.23. The T-beam is considered as a rectangular beam at the end points because it carries negative moment and the flange is under tension.

For interior points, if the flange thickness exceeds $1.18qd/k_1$ we design the beam as a rectangular beam (ACI Code 1603-a). When the flange thickness is less than $1.18\ qd/k_1$, The formula derived for I- and T- sections is used to get the reinforcement. If we get a negative quantity under the square root, or if (p_w-p_f) exceeds 0.75 of the balanced steel percentage, we increase the depth making use of the eq.28.

BEAMS INPUT DATA CARDS

Card No.	Variable	Format	Remarks
1 (one card)	Title of the run	80H	
2 (one card)	ME,NLØAD,NC	20I4	ME=number of members NLØAD= number of load cases. NC=number of combinations of different load cases.
3 (NC cards)	PARTIC(I,L),I-1,NLØAD,FAC(L)	8F10.0	Repeat this card as many as NC.
4 (one card)	FC,FY	8F10.0	FC=f'_c ,FY=f_y
	NLOAD times [(F(J,L),J-1,8]	8F10	Repeat this card as many as NLØAD
	I,S,B,BP,T,DP,FT,D1,D2,LBAR	I3,F7.0, 6F10.0, F8.0,I2	
	K [I,A,BD,W,LTYPE]	20I4 I4,3F6.2 14	Supply these cards for beams only.
	Repeat as many as ME, total number of members Repeat as many as NLØAD Repeat as many as K		

Column Analysis Program Description- In the column mainline number of members, number of load cases, number of load combinations, concrete strength, steel yield stress, and the column properties are read. The factor k_1 is computed. Minimum and maximum reinforcements (ACI Code 913-a) are calculated and the reinforcement is compared to them, a message being given when it is out of the range.

Subroutine Column- In this subroutine h/r ratios are computed. If $60 < (h/r) < 100$, the strength reduction factor R is calculated and the loads and the moments are divided by this factor. If h/r is greater than 100 a message saying "in this column h/r ratio is greater than 100 " is printed out. In ACI Code (article 916-a-1) it is advised : "If lateral displacements of the ends of the member is prevented and the ends of the member are fixed or definitely restrained such that a point of contraflexure occurs between the ends, no correction for length shall be made unless h/r exceeds 60." When compression controls, the above, most encountered, case is considered. On the other hand, when tension controls, the factor R is considered to vary linearly with axial load fromthe values given by equations 9-2, 9-3 (ACI Code) at the balanced conditions to a value of 1.0 when the axial load is zero, thus

$$R' = R + (1-R)\left(\frac{P_b-P}{P_b}\right)$$

The minimum eccentricities 0.05t for spirally reinforced columns, or 0.10t for tied columns about either principal axis are set in this subroutine. The maximum moment accompanying these eccentricities are calculated.

The minimum ratio of spiral reinforcement, p_s, is computed from

$$p_s = 0.45\,(A_g/A_c - 1)\,f'_c/f_y \qquad \text{eq.9-1 (ACI Code)}$$

Subroutine Circle- The reinforcing steel areas of circular columns with bars circularly arranged, and square columns with bars circularly arranged are computed according the transformed empirical equations of the ACI Code .

Subroutine ACIUNI - The total reinforcement of the rectangular columns under uniaxial bending is computed by the transformed equations of the ACI Code 318-63. These empirical equations are given for symmetrical reinforcement in single layers parallel to the axis of bending, the ones used in the programming are equations 40 and 50 .

Subroutine BIAX - In this subroutine the uniaxial moment about one of the axes, M_{uy}, or M_{ux} is computed from one of the equations 122 or 125 , after the comparison between M_x/M_{ux} and M_y/M_{uy} having been made. Then the predominating equation is chosen, and a reinforcement is computed by uniaxial design methods. Total steel area gives $q=p_t f'_c/f_y$; P_o, P_u/P_o are also calculated. From the BETTA subroutines containing straight line idealizations of β envelopes, knowing q , P_u/P_o a new β is computed, then this new β is compared with the old one, and if these two values differ more than a certain tolerated percentage, a new M_{ux} or M_{uy} is computed from the equation 119. A new reinforcement is computed with new moment, and the above procedure is repeated up to the point where two succeeding β's are in close agreement.

Subroutine BETTA - This subroutine contains the straight line idealizations of β envelopes .

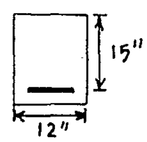
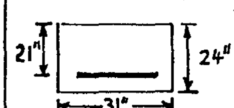
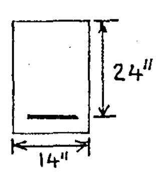
COLUMNS INPUT DATA CARDS

Card No.	Varible	Format	Remarks
1 (one card)	Title of the run	80H	
2 (one card)	ME,NLØAD,NC	20I4	ME=number of members. NC=number of combinations of different load cases.
3 (one card)	FC,FY	8F10	NLØAD=number of load cases. $FC=f'_c$, $FY=f_y$
4 (ME cards)	I,S,B,BP,T,DP,FT,LBAR	I3,F7.0, 6F10.0, F8.0,I2	
	F(J,I),J-1,8	8F10.0	
	Repeat as many as NLØAD		
	Repeat as many as ME .		

For the combined mainline input data cards given for beams are used.

EXAMPLES

RECTANGULAR BEAM EXAMPLES

Example No.	Source	Given	A_s in^2	M_u k'	A_s (Karaca) in^2
1	"Theory and problems of Reinforced Concrete Design" Everard and Tanner p.85 prob.4-14	 f_y=40 ksi f'_c= 3 ksi M = 55 k'	2.52	91.0	2.24
2	"same" p.86 prob.4-16	 f_y = 40ksi f'_c= 3 ksi M = 250 k'	8.20	415.0	7.21
3	"same" p.88 prob.4-24	 f_y =40 ksi f'_c= 3 ksi M = 167.0 k'	5.90	276.0	4.25

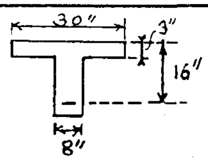
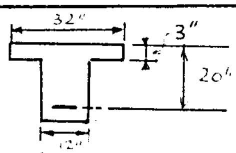
These examples are designed by working stress design method in the "Theory and Problems of Reinforced Concrete Design" ,that is why to get the ultimate strength design moment we multiplied the moment by 1.65, an average of 1.5 and 1.8 (ACI 318-63,1506 a).

Example 4 - Given: a rectangular beam $b = 18''$, $t = 36''$, $d' = 3''$

$f'_c = 3$ ksi, $f_y = 40$ ksi

M_u k'	P_u k	V_u k	A_s in^2	A_{sv} in^2
290.0	20.0	105.0	2.97	0.040
360.0	20.0	5.0	3.55	0.000
460.0	51.0	110.0	4.30	0.045
650.0	71.0	205.0	6.41	0.123
760. 0	20.0	105.0	8.44	0.040
1,390.0	20.0	105.0		
INSUFFICIENT DEPTH-DEPTH IS INCREASED t- 42.5"				
1,390.0	20.0	105.0	13.80	0.027

T- BEAM EXAMPLES

Example No.	Source	Given	M_u k'	A_{s_2} in^2	A_s (Karaca) in^2
1	" Everard and Tanner " (ref-17) p.82 prob.4-8	 f'_c=3ksi f_y= 40 ksi M_u = 50 k'	82.5	2.0	1.95
2	"Everard and Tanner " p.87 prob.4-19	 f'_c= 3 ksi f_y= 40 ksi M_u= 150 k'	247	5.07	4.90

In Everard and Tanner the design is done by working stress design method, that is why we multiplied the moment by 1.65 to get the ultimate strength design moment.

COMPARİSON OF THE NEW BEAM FORMULA

Given : A rectangular beam which has $b = 18''$, $t = 36''$, $d' = 3''$.

$f'_c = 3$ ksi and $f_y = 40$ ksi.

$M_u = 1{,}000$ k' and $P_u = 100$ k .

Solutions:

1- The load being smaller than the balanced load, tension controls. From the column formulas of the ACI Code , equations 40, and 41. the reinforcement is computed as 15.4 in^2.

2- From the beam formula ,equation 3, we get the reinforcement as 12.1 in^2.

3- From the formula taking into account small axial loads derived for use in the design of beams (equation 117) , we compute the reinforcement as 11.23 in^2.

As seen in the above examples economy is achieved by the refined expression when we consider the effect of small axial loads in the design of beams .

UNIAXIAL BENDING OF COLUMNS

Computation of k_u from the cubic equation is made by using Newton-Raphson method. Here are presented the solution of equations for different cases derived before.

In all examples below $f_y = 40$ ksi and $f'_c = 3$ ksi .

EXAMPLE 1.

Given: b= 17" , t = 25" , d' = 2.5 " , M_u=275k' , P_u=66 k

Solution: The cubic equation for k_u is

$$- 10.7050\, k_u^3 - 13.5192\, k_u^2 - 2.7619\, k_u - 0.8073 = 0 \qquad \text{Eq.80}$$

this equation gives for k_u = .1569 which falls into the region defined by the Case 1.

Then we compute the reinforcement area which is $A_{st} = 9.14\ in^2$

The empirical equation of ACI 318-63 gives $A_{st} = 9.09\ in^2$

EXAMPLE 2 .

Given : b = 18" , t=36 " , d' = 3 "

M_u= 1000.0 k' P_u= 250.0 k

Solution: The equation giving k_u is

$$k_u^2 - 2 \times 15.4767\, k_u^2 + 8.3693 = 0 \qquad \text{Eq.85}$$

this equation gives k_u=0.2727 which falls into the region defined by case 2 .

Then the reinforcement area is computed as $A_{st}= 20.39\ in^2$.

The empirical equation of ACI Code 318-63 gives A_{st}= 2o.39 in^2.

EXAMPLE 3.

Given: $b=17''$, $t=25''$, $d'=2.5''$, $M_u=431.5k'$, $P_u= 519\ k$.

Solution: The cubic equation for k_u is from eq.99

$$-29.971k_u^3 + 65.349\ k_u^2 - 71.624\ k_u + 28.536 = 0$$

this equation gives for $k_u= 0.7043$ which falls into the region defined by the Case3 .

Then the reinforcement area is computed $A_{st}= 13.94\ in^2$.

The exact equation of ACI Code gives $A_{st}= 13.78\ in^2$.

The empirical equation of ACI Code (eq.19-10) gives $A_{st}= 13.07\ in^2$.

EXAMPLE 4.

Given: $b=17''$, $t=25''$, $d'= 2.5''$, $M_u=600\ k'$, $P_u= 1{,}440\ k$.

The cubic equation for k_u from equation 108 is

$$-29.357\ k_u^3 + 64.048\ k_u^2 - 94.575\ k_u + 59.450 = 0$$

This equation gives **for** $k_u= 0.9944$ which falls into the region defined by Case 4 .

Then the reinforcement area is computed as $A_{st}= 52.87\ in^2$.

The exact equation of ACI Code (Eq.19-9) gives $A_{st}= 53.90\ in^2$.

The emprical equation of ACI Code (Eq.19-10) gives $A_{st}= 52.25\ in^2$.

COMPARATIVE EXAMPLES OF DIFFERENT COLUMN FORMULAS

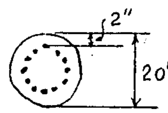
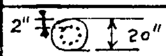
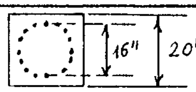
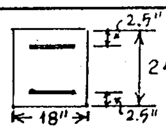
FOR AXIAL LOAD AND BENDING

Example	b in.	t in.	d' in.	M_u k'	P_u k	ACI Code 318-63 Eqs. Empirical A_{st} (in^2)	Exact A_{st} (in^2)	Equations derived taking strain compatibility into account A_{st} (in^2)
1	18	36	3	1000	250	20.39		20.39
2	17	25	2.5	374	225.4	9.10		9.10
3	17	25	2.5	431.5	519	13.07	13.78	13.94
4	"	"	"	344	829	19.45	20.07	18.84
5	"	"	"	289.8	87	8.99		9.13
6	"	"	"	600	1440	52.25	53.90	52.87
7	"	"	"	265	53	9.13		9.15
8	"	"	"	320	128	9.01		9.01

For all of the above examples $f'_c = 3$ ksi and $f_y = 40$ ksi

From the above examples we conclude that for design purposes we may use any of the column formulas we transformed, since they give almost the same steel area .

Uniaxial design of Columns

Example	Source	Given	$A_{st}(in^2)$	A_{st}(Karaca) (in^2)
1	Everard Tanner p.186 prob.10.9	 f'_c=3 ksi f_y=60 ksi M_u=240 k'; P_u=720 k	15.71	17.81
2	p.188 prob.10.16	 M_u=208 k' ; P_u=139.7 k	7.22	7.17
3	p.188 prob.10.18	 M_u= 306 k' ; P_u=183.5 k	9.43	9.44
4	p.188 prob.10.15	 M_u=370 k' ; P_u=738 k	12.00	12.03

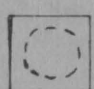
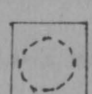
The above examples are designed by ultimate strength design method in Everard and Tanner. In the first example the difference is due to the fact that in this circular column the design is made according to strain compatibility conditions. But the empirical equation of the ACI Code gave a steel area 11.8% on the safe side.

In all of the above examples f'_c=3.0 ksi

f_y= 60.0 ksi

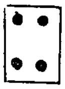
COLUMNS UNDER UNIAXIAL BENDING

EXAMPLES

Example	Given	A_{st} (Karaca) in^2
1	 D=t = 30" ; d' = 3" ; D_s = 24 M_u= 1,000.0 k' ; P_u=513.6k	36.23
2	 t = 30", d' = 3, D_s = 24" M_u= 1,000.0 k' ; P_u= 567.3k	26.09
3	 D = 30", d' = 3", D_s = 24" M_u= 250.0 k' P_u= 1,000 k	7.06
4	 t = 30", d' = 3", D_s = 24" M_u= 250.0 k' ; P_u= 1,000 k	9.00

In all of the above examples f'_c = 3.0 ksi
f_y = 40.0 ksi

COLUMNS UNDER BIAXIAL BENDING

Example	Source	Given	A_{st} (in^2)	Loads for USD	A_{st} (in^2) (Karaca
1	Czerniak	 b = 14" t = 18" d' = 2.6" f'_c = 4ksi f_y=40 ksi M_x=40k' M_y= 30k' P_u = 200 k	4.00 5.08	M_x = 66 M_y = 49.5 P_u = 330.0	2.52
2	"	Same as in 1. M_x=40 M_y=20 P_u=120	4.00 5.08	M_x=66 M_y=33 P_u=197	2.52
3	"	M_x = 40 M_y = 19.7 P_u = 120 Same as in 1.	4.00 5.08	M_x = 66 M_y = 32.5 P_u = 197	
4	"	Same as in 1. M_x = 40 M_y = 19.7 P_u = 162	4.00 5.08	M_x=66 M_y=32.5 P_u=266	2.52
5	"	Same as in 1. M_y=40 M_x=19.7 P_u=250	4.00 5.08	M_x=66 M_y=32.5 P_u=410	2.52

From a Xerox copy

In the source from where the above examples are taken, the WSD (working stress design) method is used that is why to get the ultimate strength design moment we multiplied this moment by 1.65, an average of 1.5 and 1.8 (ACI Code 318-63, 1506 a).

We investigate the behaviour of the examples given above under a variation of bending moments about the two axes.

Example I

Given: b=14", t=18", d'=2.6", f_y = 40 ksi, f'_c = 3 ksi

M_x(k')	M_y(k')	P_u (k)	A_{st} (in^2)
80.0	50.0	200.0	3.03
80.0	50.0	270.0	3.36
80.0	60.0	200.0	3.67
80.0	70.0	200.0	7.36
80.0	80.0	200.0	7.12
50.0	80.0	200.0	4.72
100.0	100.0	200.0	13.30
100.0	100.0	270.0	11.60

Example II

Given: b=14", t=18", d'=2.6", f_y = 40 ksi, f'_c = 4 ksi .

M_x (k')	M_y(k')	P_u (k)	A_{st}(in^2)
50.0	100.0	200.0	4.82
49.5	90.0	330.0	4.02
70.0	50.0	410.0	2.52
90.0	49.5	330.0	2.52
90.0	80.0	330.0	5.56

M_x (k')	M_y (k')	P_u (k)	A_{st} (in^2)
100.0	33.0	197.0	2.52
100.0	50.0	200.0	3.10
100.0	70.0	410.0	5.13

COLUMNS UNDER BIAXIAL BENDING

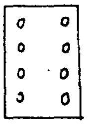
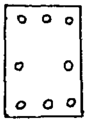
Given: b=17" , t=25" , d'=2.5", f_y= 40.0 ksi, f'_c= 3.0 ksi, 8.bar arrangement.

	M_x (k-ft)	M_y (k-ft)	P_u (k)	A_{st} (in^2)	A_{st}(Karaca) (in^2)
1	374.0	20.0	225.4	6.28	11.66
2	344.0	20.0	829.0	"	27.14
3	289.0	20.0	87.0	"	11.05
4	74.0	300.0	225.4	"	18.00
5	106.5	106.5	255.0	"	4.25
6	58.6	117.2	140.5	"	4.25
7	142.0	53.0	85.1	"	8.78
8	17.5	73.0	3.5	"	4.25

Source for the above example is reference No.7 pp.151-153 .

BIAXIAL BENDING OF COLUMNS -

FURTHER EXAMPLES

Example No.	Source	Given		A_{st} in^2	A_{st} (Karaca) in^2
1	Fleming ref.11 p.335	b=11.3", t=11.3" 4-bar arrangement f'_c= 3ksi f_y= 30.0 ksi M_x=45k' M_y=17.5k' P_u=70.0k	b=t=12", d'=2.5" 4-bar arrangement f'_c= 3 ksi f_y= 40 ksi M_x= 45 M_y= 17.5 P_u= 70.0 k	2.89	3.93
2	Fleming ref.11 p.335	b=14", t=14" 4-bar arrangement f'_c= 3 ksi f_y=60 ksi M_x= 46.6 M_y= 58.5 P_u=175.0	b=14", t=14", d'=2.6 4-bar arrangement f'_c = 3 ksi f_y = 60 ksi M_x = 46.6 M_y= 58.5 P_u=175.0	2.64	3.41
3	Parme ref.9 p.919	b=16", t=16" 8-bar arrangement f'_c= 3 ksi f_y=40 ksi M_x= 180.0 k' M_y= 80.0 k' P_u= 220.0 k	b=16", t=16", d'=2.5" 8-bar arrangement f'_c= 3 ksi f_y= 40 ksi M_x= 180.0 k' M_y= 80.0 K' P_u= 220.0 k	8.00	15.81
4	Parme ref.9 p.921	 b=12" t=24" f'_c= 3ksi f_y= 40 ksi M_x=200.0k' M_y= 80.0k' P_u=300.0 k	 b=12" t=24" d'=2.5" f'_c= 3 ksi f_y= 40 ksi M_x=200.0k' M_y= 80.0k' P_u=300.0 k	4.81	8.46

D I S C U S S I O N

In the Reinforced Concrete Building Code (ACI 318-63), all of the formulas are ih such a form that the ultimate loads and moments are given in function of the other variables. Therefore the designer assumes everything in order to check whether or not the assumed section is satifactory. However, while designing a reinforced cocrete section, in many instances the aim is to find the reinforcement That is why the ideal thing would be to have formulas which directly give the reinforcement once the loads, geometrical data, and material properties are known.

Formulas for rectangular beams, I- and T- sections are transformed into such a form that the reinforcement is directly computed. A refined formula taking into account the small axial loads in rectangular beams has also been developed from the strain-compatibility considerations. It is seen, from the example p.53 , that a small economy is made when we have tension.

From the percentage steel limitations of the Code , a formula at the limit giving the new increased depth when a section is found to be inadequate to carry the moment, is developed. In addition to this, for rectangular beams, an equation satisfying the given steel percentage requirements of the ACI Code, is derived, when a section can not carry a given moment, to compute the compression steel area. The comparisons made with beams designed by Working Stress Design method (p.50) showed that Ultimate Strength Design gives less reinforcement, but it is necessary to note that the loads factors used have an important effect on the reinforcement .

When a section is inadequate for shear the decision is left to the designer.

The empirical and exact column equations of the ACI Code are transformed into a form suitable to design purposes, that is the formulas giving the reinforcement directly are developed. The equations considering strain-compatibility are also derived. These three sets of equations gave almost the same results for a wide range data variation as seen in the examples p.56 .

In the computer programming the empirical equations of the ACI Code are used, since they are the simplest. The examples compared with the designs made by Ultimate Strength Design method are in close agreement (p.57).

In biaxial bending the idealized β envelopes are given for different bar arrangements. The charts with bars arrangements symetrical in two faces coincide exactly with the derived design equations. In case we have steel in four faces, though the idealized β envelopes are programmed, care should be taken because the total reinforcement is approximated, but the error produced is on the conservative side.

Columns under biaxial bending designed by USD have less steel than the ones designed by WSD (p.59 Czerniak).

Comparison with examples taken from Fleming (p. 62) showed that our steel area is about 25 % higher. The examples are designed in Fleming by USD method making use of the charts developed by the method of Mattock.

Comparison with the examples of Parme (p.62) shows that our steel area is greater. But Parme does not consider capacity reduction factor, , and for 8-bar arrangement our formulas do not give exact solutions, they are derived for symmetrical reinforcement in two faces, thus our steel area is expected to be greater, and this is the case.

Comparison with Czerniak (p. 61) shows again that we get greater steel area. But again Czerniak does not take into consideration any capacity reduction factor, ϕ , When we designed the same examples with reduced moment and load we get less reinforcement (examples 5,6,7,8, p. 61).

When the eccentricities about either principel axis are smaller than the values required by the ACI Code 318-63 (article 1901-a), they are augmehted to these minimum values. Thus in biaxial bending design if the moment about one of the axis is smaller, it will automatically be increased, and this increase will affect the design a lot.

CONCLUSIONS.

The following conclusions are deduced after the solution of examples with our transformed formulas:

1-The design is simplified by the fact that the designer has to plug into the transformed formulas the known quantities to get the reinforcing steel directly.

2-The comparison with Working Stress Design method shows clearly the importance of load factor in Ultimate Strength Design to achieve economy.

3-In column design, the exact and empirical equations of the ACI Code, and the formulas derived by applying strain-compatibility considerations to symmetrically reinforced rectangular sections in two faces, all give almost the same reinforcement for wide range of data variation.

4-In the biaxial bending case when we have smaller eccentricities than the minimums required by the ACI Code, the design gives conservative results if the eccentricity, and therefore the moment is increased about one of the principal axes.

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APPENDIXES

APPENDIX AI

$$M_u = \phi[A_s f_y (d - a/2)] \qquad \text{Eq. 16-1 (ACI Code)} \qquad \text{Eq. 1}$$

$$a = A_s f_y / 0.85 f_c' b \qquad \text{Eq. 2}$$

putting this into (1)

$$M_u = \phi A_s f_y d - \phi f_y^2 A_s^2 / 1.7 f_c' b$$

rearranging

$$A_s^2 - 2 \frac{0.85 f_c' bd}{f_y} A_s + \frac{1.7 f_c' b}{\phi f_y^2} M_u = 0$$

$$A_s = \frac{0.85 f_c' bd}{f_y} - \sqrt{\left(\frac{0.85 f_c' bd}{f_y}\right)^2 - \frac{1.7 f_c' b M_u}{\phi f_y^2}}$$

$$A_s = \frac{.85 f_c' bd}{f_y} - \frac{.85 f_c' bd}{f_y} \sqrt{1 - \frac{2}{0.85} \frac{M_u}{\phi f_c' bd^2}}$$

replacing $\chi = 0.85 f_c' bd$

$$A_s = \frac{\chi}{f_y}\left[1 - \sqrt{1 - \frac{2 M_u}{\phi \chi d}}\right] \qquad \text{Eq. 3}$$

APPENDIX A 2

Increase in depth of the rectangular beams

$$M_u = \phi\left[b d^2 f_c' \, q\,(1-0.59\, q)\right] \qquad \text{Eq 16-1 (Code)} \qquad \text{Eq. 4}$$

Eq. (4) gives for d

$$d = \sqrt{M_u / \phi b f_c' q\,(1-0.59\, q)} \qquad \text{Eq. 7}$$

$$q = p f_y / f_c' \qquad \text{Eq. 5}$$

$$p = 0.75\, p_b = 0.75\left[0.85\, k_1 f_c' / f_y\right] \frac{87{,}000}{87{,}000 + f_y} \qquad \text{Eq. 6}$$

$$q = 0.75 \times 0.85\, k_1 \frac{87{,}000}{87{,}000 + f_y} = 0.6375\, k_1 \frac{87{,}000}{87{,}000 + f_y} \qquad \text{Eq. 8}$$

putting equation (8) in, equation (7) becomes

$$d = \sqrt{\frac{M_u}{\phi b f_c' \, 0.6375\, k_1 \frac{87{,}000}{87{,}000 + f_y}\left(1 - 0.59 \times 0.6375\, k_1 \frac{87{,}000}{87{,}000 + f_y}\right)}}$$

APPENDIX A 3

Rectangular Beams with compression reinforcement-

$$M_u = \phi \left[(A_s - A'_s) f_y \left(d - a/2\right) + A'_s f_y (d - d') \right] \qquad \text{(ACI Code 16 3)} \qquad \text{Eq. 9}$$

where

$$a = (A_s - A'_s) f_y / 0.85 f_c b \qquad \text{Eq. 10}$$

the Code gives

$$p - p' \leq 0.75\, p_b = 0.75 \left(0.85\, k_1 f'_c / f_y\right) \left(\frac{87{,}000}{87{,}000 + f_y}\right) \qquad \text{Eq. 12}$$

calling $(p - p') = F$ Eq. 13

we get $(A_s - A'_s) = F b d$ Eq. 14

at the limit $F = 0.75\, p_b = 0.75 \left(0.85\, k_1 f'_c / f_y\right)\left(\frac{87{,}000}{87{,}000 + f_y}\right)$ Eq. 15

eq 10. becomes after the elimination of $(A_s - A'_s)$ by means of equations (14) and (15)

$$a = 0.75\, p_b\, b d\, f_y / 0.85 f'_c b = 0.75\, b d\, k_1 \frac{87{,}000}{87{,}000 + f_y} \qquad \text{Eq. 16}$$

In equation (9) putting $(A_s - A'_s) = F b d$ and the value of a from eq. (16) we get

$$M_u = \phi \left[F b d f_y \left(d - \frac{0.75\, b d\, k_1}{2} \frac{87{,}000}{87{,}000 + f_y}\right) + A'_s f_y (d - d') \right]$$

solving the above equation for A'_s

$$A'_s = \frac{M_u/\phi - F b d f_y \left(d - \frac{0.75\, b d\, k_1}{2} \frac{87{,}000}{87{,}000 + f_y}\right)}{f_y (d - d')} \qquad \text{Eq. 17}$$

putting the value of F from Eq. 15, the expanded form of Eq. 17 becomes

$$A'_s = \frac{M_u/\phi - \left[0.75\, (0.85\, k_1 f'_c) \frac{87{,}000}{87{,}000 + f_y}\, b d \left(d - \frac{0.75\, b d k_1}{2} \frac{87{,}000}{87{,}000 + f_y}\right)\right]}{f_y (d - d')}$$

APPENDIX A 4

Flexural computations I- and T- sections-

When $t < 1.18\,qd/k_1$

$$M_u = \phi\left[(A_s - A_{sf}) f_y (d - a/2) + A_{sf} f_y (d - 0.5t)\right] \quad \text{(ACI Code Eq.16-5) Eq.19}$$

$$A_{sf} = 0.85(b-b')t f'_c / f_y \quad \text{(" " " 16-6) Eq. 20}$$

$$a = (A_s - A_{sf}) f_y / 0.85 f'_c b' \quad \text{Eq.21}$$

let us call $A_s - A_{sf} = X$ Eq. A41

$$A_{sf} = A_s - X \quad \text{Eq. A42}$$

putting equation A41 into the equation 19, we get

$$\frac{M_u}{\phi} = X f_y \left(d - \frac{a}{2}\right) + A_{sf} f_y (d - 0.5t) \quad \text{Eq. A43}$$

$$a = X f_y / 0.85 f'_c b' \quad \text{Eq. A44}$$

eliminating a between the equations A43 and A44

$$\frac{M_u}{\phi} = X f_y d - \frac{X^2 f_y^2}{1.7 f'_c b'} + A_{sf} f_y (d - 0.5t)$$

rearranging

$$\frac{f_y^2}{1.7 f'_c b'} X^2 - f_y d X + \left[\frac{M_u}{\phi} - A_{sf} f_y (d - 0.5t)\right] = 0$$

$$X^2 - \frac{1.7 f'_c b' d}{f_y} X + \frac{1.7 f'_c b'}{f_y^2}\left[\frac{M_u}{\phi} - A_{sf} f_y (d - 0.5t)\right] = 0 \quad \text{Eq. A45}$$

the solution of this quadratic equation gives

$$X = \frac{\chi}{f_y}\left\{1 - \sqrt{1 - \frac{2}{\chi d}\left[\frac{M_u}{\phi} - A_{sf} f_y (d - 0.5t)\right]}\right\} \quad \text{Eq. 22}$$

where

$$\chi = 0.85 f'_c b' d$$

APPENDIX A-5

Increase in depth of the I- and T- sections-

At the limit $\quad p_w - p_f = 0.75 p_b = L$ Eq. 24

this means $\quad (A_s - A'_s)/_{b'd} = L = (A_s - A_{sf})/_{b'd}$ Eq. 25

therefore $\quad (A_s - A_{sf}) = L b'd = X$ Eq. 26

eliminating X between equations (22) and (26)

$$L b'd = \frac{\chi}{f_y} \quad 1 - \sqrt{1 - \frac{2}{\chi d}\left[\frac{M_u}{\phi} - A_{sf} f_y (d - .5t)\right]}$$

$$\frac{L b'd f_y}{\chi} - 1 = -\sqrt{1 - \frac{2}{\chi d}\left[\frac{M_u}{\phi} - A_{sf} f_y (d - 0.5t)\right]} \qquad A51$$

Squaring A51 and simplifying we have

$$d^2 - 2\,\frac{0.85 f'_c\, b f_y A_{sf}}{\left[L b' f_y (L b' f_y - 1.7 f'_c b)\right]}\, d + \frac{0.85 f'_c b \left[\frac{2 M_u}{\phi} + A_{sf} f_y t\right]}{\left[L b' f_y (L b' f_y - 1.7 f'_c b)\right]} = 0 \qquad (A52)$$

calling $\alpha = \dfrac{0.85 f'_c b f_y A_{sf}}{\left[L b' f_y (L b' f_y - 1.7 f'_c b)\right]}$ and $\beta = \dfrac{0.85 f'_c b \left[\frac{2 M_u}{\phi} + A_{sf} f_y t\right]}{\left[L b f_y (L b' f_y - 1.7 f'_c b)\right]}$

Eq A52 becomes

$$d^2 - 2\alpha d + \beta = 0 \qquad \text{Eq. 27}$$

Solution of this equation gives for the increased depth,

$$d = \alpha\left[1 - \sqrt{1 - (\beta/\alpha^2)}\right] \qquad \text{Eq. 28}$$

APPENDIX A6

Uniaxial bending of columns, when tension controls -

$$P_u = \phi\left[0.85 f'_c bd\left\{-p+1-\frac{e'}{d}+\sqrt{\left(1-\frac{e'}{d}\right)^2+2p\left[m'\left(1-\frac{d'}{d}\right)+\frac{e'}{d}\right]}\right\}\right] \quad \text{(ACI Code Eq. 19-5)}$$

Eq. 35

let us call $\left(1-\frac{e'}{d}\right)=\beta$; $\frac{P_u}{\phi\, 0.85 f'_c bd}=\mathcal{K}$; $m'\left(1-\frac{d'}{d}\right)+\frac{e'}{d}=\gamma$

putting all these into eq. 35, we get,

$$-p+\beta+\sqrt{\beta^2+2\gamma p}=\mathcal{K} \qquad \text{A61}$$

$$\sqrt{\beta^2+2\gamma p}=(\mathcal{K}-\beta)+p \qquad \text{A62}$$

Squaring A62 $\beta^2+2\gamma p=(\mathcal{K}-\beta)^2+2(\mathcal{K}-\beta)p+p^2$

$$p^2+2[\mathcal{K}-\beta-\gamma]p+(\mathcal{K}-\beta)^2-\beta^2=0$$

$$p^2-2[\beta+\gamma-\mathcal{K}]+\mathcal{K}^2-2\mathcal{K}\beta=0 \qquad \text{A63}$$

calling $\beta+\gamma-\mathcal{K}=\alpha=\left(1-\frac{e'}{d}\right)+m'\left(1-\frac{d'}{d}\right)+\frac{e'}{d}-\mathcal{K}$

$$\alpha=1+m'\left(1-\frac{d'}{d}\right)-\mathcal{K} \qquad \text{A64}$$

and $\varphi=\mathcal{K}^2-2\mathcal{K}\beta=\mathcal{K}\left(\mathcal{K}-2+\frac{2e'}{d}\right)$ A65

A63 becomes $p^2-2\alpha p+\varphi=0$ A66

Solving A66 for p we get

$$p=\alpha\left(1-\sqrt{1-(\varphi/\alpha^2)}\right) \qquad \text{Eq. 41}$$

APPENDIX A 7

Exact column equation when compression controls -

$$P_u = P_o - (P_o - P_b)\, M_u/M_b \qquad \text{(ACI Code 19-9)} \qquad \text{Eq. 43}$$

Taking all the terms at one side we get the equation

$$P_u M_b + P_o M_u - P_b M_u - P_o M_b = 0 \qquad \text{A71}$$

$$P_b = \phi\left[0.85 f_c' b a_b\right] \qquad \text{Eq. 44}$$

$$M_b = \phi\left[0.85 f_c' b a_b\left(d - d'' - \frac{a_b}{2}\right) + A_s' f_y (d - d' - d'') + A_s f_y d''\right] \qquad \text{Eq. 45}$$

$$P_o = \phi\left[0.85 f_c'(A_g - A_{st}) + A_{st} f_y\right] \qquad \text{Eq. 42}$$

$$A_g = bt \quad ; \quad A_s = A_s' \quad ; \quad A_{st} = 2A_s$$

putting these into equations 42 and 45

$$M_b = \phi\left[0.85 f_c' b a_b\left(d - d'' - \frac{a_b}{2}\right) + A_s f_y (d - d' - d'') + A_s f_y d''\right] \qquad \text{Eq. 45a}$$

$$P_o = \phi\left[0.85 f_c'(bt - 2A_s) + 2A_s f_y\right] \qquad \text{Eq. 42a}$$

P_b, P_o, M_b from the equations 44, 42a, and 45a are inserted into the equation A71, terms are expanded, and an equation is obtained in terms of A_s,

$$\alpha A_s^2 - 2\beta A_s + \gamma = 0 \qquad \text{Eq. 46}$$

where

$$\alpha = 2\phi f_y (f_y - 0.85 f_c')(d - d')$$

$$\beta = -M_u(f_y - 0.85 f_c') - P_u f_y (d-d')/2 + .5\phi .85 f_c' bt f_y (d-d') + \phi .85 f_c' b a_b (f_y - .85 f_c')(d - d'' - .5a_b)$$

$$\gamma = 0.85^2 \phi f_c' b^2 t a_b (d - d'' - .5a_b) + 0.85 f_c' M_u b (a_b - t) - 0.85 f_c' b a_b (d - d'' - 0.5a_b) P_u$$

in these expressions $a_b = k_1 \dfrac{87{,}000\, d}{87{,}000 + f_y}$

solution of the equation 46 gives for A_s

$$A_s = \frac{\beta}{\alpha}\left(1 - \sqrt{1 - \frac{\gamma\alpha}{\beta^2}}\right)$$

APPENDIX A 8

Empirical column equation when compression controls-

$$P_u = \phi\left[\frac{A'_s f_y}{\frac{e}{d-d'} + 0.5} + \frac{b t f'_c}{\frac{3te}{d^2} + 1.18}\right]$$

Eq. 49
(ACI code 19-10)

$$A'_s = A_s$$

$$\frac{A_s f_y}{\frac{e}{d-d'} + 0.5} = \frac{P_u}{\phi} - \frac{b t f'_c}{\frac{3te}{d^2} + 1.18}$$

$$A_s = \left[\frac{P_u}{\phi} - \frac{b t f'_c}{\frac{3te}{d^2} + 1.18}\right]\frac{\frac{e}{d-d'} + 0.5}{f_y}$$

$$A_{st} = 2A_s$$

APPENDIX A 9

Circular columns with bars circularly arranged, when tension controls-

$$P_u = \phi\left\{0.85 f'_c D^2\left[\sqrt{\left(\frac{0.85e}{D} - 0.38\right)^2 + \frac{p_t m D_s}{2.5D}} - \left(\frac{0.85e}{D} - 0.38\right)\right]\right\}$$

(ACI Code 19-11) Eq.51

$$\sqrt{\left(\frac{0.85e}{D} - 0.38\right)^2 + \frac{p_t m D_s}{2.5D}} = \frac{P_u}{\phi\, 0.85 f'_c D^2} + \left(\frac{0.85e}{D} - 0.38\right)$$

Squaring this equation and cancelling

$$\frac{p_t m D_s}{2.5D} = \left(\frac{P_u}{\phi\, 0.85 f'_c D^2}\right)^2 + 2\frac{P_u}{\phi\, 0.85 f'_c D^2}\left(\frac{0.85e}{D} - 0.38\right)$$

$$p_t = \frac{2.5 D P_u}{m D_s \phi\, 0.85 f'_c D^2}\left[\frac{P_u}{\phi\, 0.85 f'_c D^2} + \frac{1.7e}{D} - 0.76\right]$$ Eq.A91

$$A_{st} = p_t A_g$$ Eq. 52

APPENDIX A 10

Circular columns with bars circularly arranged, when compression controls.

$$P_u = \phi \left[\frac{A_{st} f_y}{\frac{3e}{D_s} + 1} + \frac{A_g f'_c}{\frac{9.6 De}{(0.8D + 0.67D_s)^2} + 1.18} \right]$$ (ACI Code 19-12) Eq. 53

$$\frac{A_{st} f_y}{\frac{3e}{D_s} + 1} = \frac{P_u}{\phi} - \frac{A_g f'_c}{\frac{9.6 De}{(0.8D + 0.67D_s)^2} + 1.18}$$

$$A_{st} = \frac{\frac{3e}{D_s} + 1}{f_y} \left[\frac{P_u}{\phi} - \frac{A_g f'_c}{\frac{9.6 De}{(0.8D + 0.67D_s)^2} + 1.18} \right]$$ Eq. A101
Eq. 54

APPENDIX A 11

Square columns with bars circularly arranged, when tension controls-

$$P_u = \phi \left\{ 0.85\, b t f_c' \left[\sqrt{\left(\frac{e}{t} - 0.5\right)^2 + 0.67 \frac{D_s}{t} p_t m} - \left(\frac{e}{t} - 0.5\right) \right] \right\}$$

(ACI Code 19-13)
Eq. 55

$$\sqrt{\left(\frac{e}{t} - 0.5\right)^2 + 0.67 \frac{D_s}{t} p_t m} = \frac{P_u}{\phi\, 0.85 f_c' b t} + \left(\frac{e}{t} - 0.5\right) \qquad \text{Eq. A111}$$

Squaring A111 and cancelling terms

$$0.67 \frac{D_s}{t} p_t m = \left(\frac{P_u}{\phi\, 0.85 f_c' b t}\right)^2 + \frac{2 P_u}{\phi\, 0.85 f_c' b t} \left(\frac{e}{t} - 0.5\right)$$

$b = t$

$$p_t = \frac{P_u t}{0.67 D_s m \phi\, 0.85 f_c' t^2} \left[\frac{P_u}{\phi\, 0.85 f_c' t^2} + \frac{2e}{t} - 1 \right] \qquad \text{A 112}$$

$$A_{st} = p_t A_g = p_t t^2$$

APPENDIX A 12 -

Square columns with bars circularly arranged, when compression controls-

$$P_u = \phi \left[\frac{A_{st} f_y}{\frac{3e}{D_s} + 1} + \frac{A_g f_c'}{\frac{12te}{(t + 0.67D_s)^2} + 1.18} \right]$$

(ACI Code 19-14)

Eq. 57

$$\frac{A_{st} f_y}{\frac{3e}{D_s} + 1} = \frac{P_u}{\phi} - \frac{A_g f_c'}{\frac{12te}{(t + 0.67D_s)^2} + 1.18}$$

$$A_{st} = \frac{\frac{3e}{D_s} + 1}{f_y} \left[\frac{P_u}{\phi} - \frac{A_g f_c'}{\frac{12te}{(t + 0.67D_s)^2} + 1.18} \right]$$

Eq. 58

APPENDIX B 1

Rectagular columns ,derivation of equation by strain compatibility –

CASE 1–

From the fig.4 $$d = \frac{gt}{2} + \frac{t}{2} = \frac{t}{2}(1+g) \qquad \text{Eq.65}$$

$$d' = t - d = t - \frac{t}{2}(1+g) = \frac{t}{2}(1-g) \qquad \text{Eq. 66}$$

From similar triangles $$\frac{\epsilon_y}{\beta t} = \frac{\epsilon_u}{k_u t}$$

$$\beta = \frac{\epsilon_y}{\epsilon_u} k_u \qquad \text{Eq. 67}$$

From the fig. 4 $$\alpha t = k_u t - \frac{t}{2}(1-g)$$

$$\alpha = k_u - \frac{1-g}{2} = \frac{2k_u - 1 + g}{2} \qquad \text{Eq.68}$$

$$F_{s_1} = (p_e bt)\left[(f_s) - 0.85 f_c'\right]$$

$$F_{s_2} = (p_e bt)\left[f_y\right]$$

Total force in the steel $= F = F_{s_1} - F_{s_2} = p_e bt\left[f_s - 0.85 f_c' - f_y\right]$ Eq.69

From the similar triangles $$\frac{\epsilon_y}{\beta t} = \frac{\epsilon_s'}{\alpha t} \; ; \quad \epsilon_s' = \epsilon_y \alpha/\beta \qquad \text{Eq.B11}$$

$f_s = E_s \epsilon_s'$, $f_y = E_s \epsilon_y$ putting these two into B11

$$\frac{f_s}{E_s} = \frac{f_y}{E_s}\frac{\alpha}{\beta} \; ; \qquad f_s = f_y \frac{\alpha}{\beta} \qquad \text{Eq.73}$$

Eliminating f_s between 73 and 69

$$F = p_e bt\left[f_y \frac{\alpha}{\beta} - 0.85 f_c' - f_y\right] = p_e bt \; 0.85 f_c' \left[\frac{f_y}{0.85 f_c'}\frac{\alpha}{\beta} - 1 - \frac{f_y}{0.85 f_c'}\right]$$

but $\frac{f_y}{0.85 f_c'} = m$ $\therefore$ $P_{es} = f_c' bt \, 0.85 \, p_e\left[m\,\alpha/\beta - 1 - m\right]$

to this we add the ultimate load carried by concrete

$$P_c = f'_c bt\, 0.85 k_1 k_u \qquad \text{Eq. 59}$$

thus our ultimate load for this case is

$$P_u = \phi\left[P_{es} + P_c\right] = \phi f'_c bt\left[0.85 p_e\left(m\alpha/\beta - 1 - m\right) + 0.85 k_1 k_u\right] \qquad \text{Eq. 76}$$

Taking moments about the plastic centroid, noting lever arm $= \frac{gt}{2}$

Moment carried by steel $= M_{es} = f'_c bt\,(0.85 p_e)\,\frac{gt}{2}\left(m\frac{\alpha}{\beta} - 1 + m\right)$

Moment carried by concrete $= M_c = f'_c bt^2\, 0.85 k_1 k_u\,(1 - k_1 k_u)/2$ Eq. 60

The ultimate moment carried by the section is

$$M_u = \phi\left[M_{es} + M_c\right] = \phi f'_c bt^2\left[0.85 p_e \frac{g}{2}\left(m\alpha/\beta - 1 + m\right) + 0.85 k_1 k_u\,(1 - k_1 k_u)/2\right] \qquad \text{Eq. 79}$$

When α and β are replaced by their value in Eqs. 76 and 79

$$P_u = \phi f'_c bt\left\{0.85 p_e\left[\left(m\,\frac{2k_u - 1 + g}{2} \times \frac{\epsilon_u}{\epsilon_y k_u}\right) - 1 - m\right] + 0.85 k_1 k_u\right\} \qquad \text{Eq. B12}$$

$$M_u = \phi f'_c bt^2\left\{0.85 p_e \frac{g}{2}\left[\left(m\,\frac{2k_u - 1 + g}{2} \times \frac{\epsilon_u}{\epsilon_y k_u}\right) - 1 - m\right] + 0.85 k_1 k_u(1 - k_1 k_u)/2\right\} \qquad \text{Eq. B13}$$

$$\frac{P_u}{\phi f'_c bt} = 0.85 p_e\left[m\left(\frac{2k_u - 1 + g}{2}\right)\left(\frac{\epsilon_u}{\epsilon_y k_u}\right) - 1 - m\right] + 0.85 k_1 k_u \qquad \text{Eq. B14}$$

$$\frac{M_u}{\phi f'_c bt^2} = 0.85 p_e \frac{g}{2}\left[m\left(\frac{2k_u - 1 + g}{2}\right)\left(\frac{\epsilon_u}{\epsilon_y k_u}\right) - 1 + m\right] + 0.85 k_1 k_u\left(\frac{1 - k_1 k_u}{2}\right) \qquad \text{Eq. B15}$$

$$p_e = \frac{\left(P_u/\phi f'_c bt\right) - 0.85 k_1 k_u}{0.85\left[m\left(\frac{2k_u - 1 + g}{2}\right)\left(\frac{\epsilon_u}{\epsilon_y k_u}\right) - 1 - m\right]} \qquad \text{from B14} \qquad \text{Eq. B16}$$

$$p_e = \frac{\left(M_u/\phi f'_c bt^2\right) - 0.85 k_1 k_u\left(\frac{1 - k_1 k_u}{2}\right)}{0.85\ g/2\left[\left(\frac{2k_u - 1 + g}{2}\right)\left(\frac{\epsilon_u}{\epsilon_y k_u}\right)m - 1 + m\right]} \qquad \text{from B15} \qquad \text{Eq. B17}$$

Equating B16 to B17 and taking all the terms to one side of the equation we get

$$\left(\frac{P_u}{\phi f_c' bt} - 0.85 k_1 k_u\right)\left\{\frac{g}{2}\left[\left(\frac{2k_u - 1 + g}{2}\right)\left(\frac{\epsilon_u}{\epsilon_y k_u}\right) m - 1 + m\right]\right\} - \left[\frac{M_u}{\phi f_c' bt^2} - 0.85 k_1 k_u \left(\frac{1 - k_1 k_u}{2}\right)\right]\left[\left(\frac{2k_u - 1 + g}{2}\right)\frac{\epsilon_u}{\epsilon_y k_u} m - 1 - m\right] = 0$$

EQ. B18

Multiplying B18 by $2\epsilon_y k_u$ we get

$$\left(\frac{P_u}{\phi f_c' bt} - 0.85 k_1 k_u\right)\frac{g}{2}\left(2k_u(\epsilon_u m - \epsilon_y + \epsilon_y m) - \epsilon_u m + \epsilon_u m g\right] - \left[\frac{M_u}{\phi f_c' bt^2} - 0.85 k_1 k_u \left(\frac{1 - k_1 k_u}{2}\right)\right]\left[2k_u(\epsilon_u m - \epsilon_y - \epsilon_y m) - \epsilon_u m + \epsilon_u m g)\right] = 0$$

EQ. B19

Equation B19 gives the following cubic equation for k_u

$$A k_u^3 + B k_u^2 + C k_u + D = 0 \qquad \text{EQ. 80}$$

where

$$A = -0.85 k_1^2 \left[m(\epsilon_u - \epsilon_y) - \epsilon_y \right]$$

$$B = \left\{0.85 k_1 \left[-\epsilon_u m (g-1) + \epsilon_y (g-1) - \epsilon_y m (g+1)\right] - \frac{0.85 k_1^2}{2}(\epsilon_u m)(g-1)\right\}$$

$$C = \left\{\frac{P_u g}{\phi f_c' bt}(\epsilon_u m - \epsilon_y + \epsilon_y m) - \frac{2 M_u}{\phi f_c' bt^2}(\epsilon_u m - \epsilon_y - \epsilon_y m) - \frac{0.85 k_1}{2}\epsilon_u m (g-1)^2\right\}$$

$$D = \frac{\epsilon_u m (g-1)}{\phi f_c' bt}\left[\frac{P_u g}{2} - \frac{M_u}{t}\right]$$

APPENDIX B 2

CASE 2 -

From the fig.5 we get the limits of k_u

$$\left[\left(\frac{1-g}{2}\right)+\beta\right] t \leq k_u t \leq \left[\left(\frac{1+g}{2}\right)-\beta\right] t$$

$$F_{s_1} = p_e bt \left[f_y - 0.85 f'_c\right]$$

$$F_{s_2} = p_e bt f_y$$

$$F = \text{total force in steel} = F_{s_1} - F_{s_2} = p_e bt \left[f_y - 0.85 f'_c - f_y\right]$$

Factoring out $0.85 f'_c$

$$F = -0.85\, p_e bt f'_c = -f'_c bt\, 0.85\, p_e \quad ; \quad P_c = f'_c bt\, 0.85\, k_1 k_u \qquad \text{Eq. 59}$$

The ultimate load carried by the section

$$P_u = \phi \left[F + P_c\right] = \phi f'_c bt \left[-0.85\, p_e + 0.85\, k_1 k_u\right] \qquad \text{Eq. 82}$$

Taking moments about the plastic centroid
(the lever arm is $gt/2$)

$$M_{u_{steel}} = \frac{gt}{2}\left[F_{s_1} + F_{s_2}\right] = \frac{gt}{2}\left[f_y - 0.85 f'_c + f_y\right] p_e bt$$

$$= f'_c\, 0.85\, bt^2\, p_e\, g/2 \left[\frac{2 f_y}{0.85 f'_c} - 1\right] = f'_c bt^2\, 0.85\, p_e\, g/2 \left[2m-1\right]$$

$$M_{concrete} = f'_c bt^2\, 0.85\, k_1 k_u (1 - k_1 k_u)/2 \qquad \text{Eq. 60}$$

The ultimate moment carried by the section is

$$M_u = (M_{es} + M_c)\phi = \phi f'_c bt^2 \left[(0.85\, p_e)\, g/2 \left[2m-1\right) + 0.85\, k_1 k_u (1 - k_1 k_u)/2\right]$$

Eq. 84

From eq. 82
$$p_e = k_1 k_u - \frac{P_u}{0.85\, \phi f'_c bt} \qquad \text{Eq. B21}$$

From eq. 84
$$p_e = \frac{2 M_u}{\phi f'_c bt^2\, 0.85\, g(2m-1)} - \frac{2\, k_1 k_u}{g(2m-1)} \left(\frac{1 - k_1 k_u}{2}\right)$$

Eq. B22

Equating B21 to B22, and taking all the terms to one side of the equation we get

$$k_1 k_u - \frac{P_u}{\phi f'_c b t\, 0.85} - \frac{2M_u}{\phi f'_c b t^2\, 0.85\, g(2m-1)} + \frac{2\, k_1 k_u}{g(2m-1)}\left(\frac{1-k_1 k_u}{2}\right) = 0$$

$$\phi f'_c b t^2\, 0.85\, g(2m-1)\, k_1 k_u - g(2m-1)\, t P_u - 2M_u + \phi f'_c b t^2\, 0.85\, k_1 (k_u - k_1 k_u^2) = 0$$

expanding and arranging to the power of k_u

$$-\phi f'_c b t^2\, 0.85\, k_1^2\, k_u^2 + \left[\phi f'_c b t^2\, g(2m-1)\, 0.85\, k_1 + \phi f'_c b t^2\, 0.85\, k_1\right] k_u - t g(2m-1) - 2M_u = 0$$

Eq. B23

Eq. B23 is transformed finally

$$k_u^2 - 2\left[\frac{g(2m-1)}{2k_1} + \frac{1}{2k_1}\right] k_u + \frac{t g(2m-1) P_u + 2M_u}{\phi f'_c b t^2 k_1^2\, 0.85} = 0 \qquad \text{Eq. B24}$$

calling $\alpha = \dfrac{g(2m-1)+1}{2k_1}$ and $\beta = \dfrac{t g(2m-1) P_u + 2M_u}{\phi f'_c b t^2 k_1^2\, 0.85}$

Eq. B24 becomes

$$k_u^2 - 2\alpha k_u + \beta = 0 \qquad \text{Eq. 85}$$

solution of this quadratic equation for k_u gives

$$k_u = \alpha\left[1 - \sqrt{1 - \beta/\alpha^2}\right] \qquad \text{Eq. 86}$$

APPENDIX B.3

CASE 3 -

From the fig 6 the limits of k_u are

$$\frac{t}{2}(1+g) - \beta t \leq k_u t \leq \frac{t}{2}(1+g)$$

From similar triangles in fig. 6

$$\frac{\epsilon_s}{\Phi t} = \frac{\epsilon_y}{\beta t} \quad ; \quad \epsilon_s = \epsilon_y \, \Phi/\beta \qquad \text{EQ. 87}$$

$$f_s = E_s \epsilon_s \quad , \quad f_y = E_s \epsilon_y \qquad \text{EQ'S 91, 92}$$

Eliminating ϵ_s and ϵ_y between (87) and (91), (92)

$$f_s = f_y \, \Phi/\beta \qquad \text{EQ. 93a}$$

From the fig 6 $\quad \Phi t + k_u t = \frac{t}{2}(1+g)$

therefore $\quad \Phi = \frac{1+g}{2} - k_u \qquad$ EQ. 88

Forces in the steel are

$$F_{s_1} = (p_e bt)\left[f_y - 0.85 f'_c\right] \qquad \text{EQ. 89}$$

$$F_{s_2} = (p_e bt) f_s \qquad \text{EQ. 90}$$

replacing f_s by EQ 93a we obtain

$$F_{s_2} = (p_e bt) f_y \, \Phi/\beta \qquad \text{EQ 93}$$

Total force carried by steel is

$$F_{es} = F_{s_1} - F_{s_2} = (p_e bt)\left[f_y - 0.85 f'_c - f_y \Phi/\beta\right] \qquad \text{EQ. 94}$$

Factoring out $0.85 f'_c$ in EQ. 94 and replacing $\frac{f_y}{0.85 f'_c} = m$

$$F_{es} = 0.85 f'_c bt \, p_e \left[m - 1 - m \Phi/\beta\right]$$

The load carried concrete is $\quad P_c = f'_c bt \, 0.85 k_1 k_u \qquad$ EQ. 59

Thus the ultimate load carried by the section in the third case is

$$P_u = \phi\left[F_{es} + P_c\right] = \phi f_c' bt\left[0.85\, p_e\left(m-1-m\,\Phi/\beta\right) + 0.85\, k_1 k_u\right] \qquad \text{EQ.95}$$

Taking moments about the plastic centroid
(note - lever arm $= gt/2$)

Moment carried by steel $= M_{es} = \frac{gt}{2}\left(F_{s1} + F_{s2}\right) = \frac{gt}{2} p_e bt\left(f_y - 0.85 f_c' + f_y \Phi/\beta\right)$

EQ.96

Factoring out $0.85 f_c'$ in EQ.96 and putting $m = f_y/0.85 f_c'$, we obtain

$$M_{es} = f_c' bt^2\, 0.85\, p_e\left(g/2\right)\left[m - 1 + m\,\Phi/\beta\right] \qquad \text{EQ.97}$$

Moment carried by concrete is $M_c = f_c' bt^2\, 0.85 k_1 k_u (1 - k_1 k_u)/2$ EQ.60

The ultimate moment carried by the section is

$$M_u = \phi\left[M_{es} + M_c\right] = \phi f_c' bt^2\left[0.85\, p_e\left(\frac{g}{2}\right)\left(m - 1 + m\,\Phi/\beta\right) + 0.85\, k_1 k_u (1 - k_1 k_u)/2\right] \qquad \text{EQ.98}$$

When Φ and β are replaced in EQ'S 95 and 98, we get

$$P_u = \phi f_c' bt\left\{0.85\, p_e\left(m - 1 - m\left(\frac{1+g-2k_u}{2}\right)\times\frac{\epsilon_u}{\epsilon_y k_u}\right) + 0.85\, k_1 k_u\right\} \qquad \text{EQ.B31}$$

$$M_u = \phi f_c' bt^2\left\{0.85\, p_e\left(\frac{g}{2}\right)\left[m - 1 + m\left(\frac{1+g-2k_u}{2}\right)\times\frac{\epsilon_u}{\epsilon_y k_u}\right] + 0.85\, k_1 k_u (1 - k_1 k_u)/2\right\} \qquad \text{EQ.B32}$$

$$\frac{P_u}{\phi f_c' bt} = 0.85\, p_e\left[(m-1) - m\left(\frac{1+g-2k_u}{2}\right)\times\frac{\epsilon_u}{\epsilon_y k_u}\right] + 0.85\, k_1 k_u \qquad \text{EQ.B33}$$

$$\frac{M_u}{\phi f_c' bt^2} = 0.85\, p_e\, \frac{g}{2}\left[(m-1) + m\left(\frac{1+g-2k_u}{2}\right)\times\frac{\epsilon_u}{\epsilon_y k_u}\right] + 0.85\, k_1 k_u \frac{(1 - k_1 k_u)}{2} \qquad \text{EQ.B34}$$

$$p_e = \frac{\left(P_u/\phi f_c' bt\right) - 0.85\, k_1 k_u}{0.85\left[(m-1) - m\left(\frac{1+g-2k_u}{2}\right)\frac{\epsilon_u}{\epsilon_y k_u}\right]} \qquad \text{from B33} \qquad \text{EQ.B35}$$

$$p_e = \frac{\left(M_u/\phi f_c' bt^2\right) - 0.85\, k_1 k_u\left(\frac{1 - k_1 k_u}{2}\right)}{0.85\, \frac{g}{2}\left[(m-1) + m\left(\frac{1+g-2k_u}{2}\right)\frac{\epsilon_u}{\epsilon_y k_u}\right]} \qquad \text{from B34} \qquad \text{EQ.B36}$$

Equating B36 to B35 and taking all the terms to one side of the equation we get

$$\left(\frac{P_u}{\phi f_c' bt} - 0.85 k_1 k_u\right)\left(\frac{g}{2}\right)\left[(m-1) + m\left(\frac{1+g-2k_u}{2}\right)\frac{\epsilon_u}{\epsilon_y k_u}\right] - \left[\frac{M_u}{\phi f_c' bt^2} - 0.85 k_1 k_u\left(\frac{1-k_1 k_u}{2}\right)\right]\left[(m-1) - m\left(\frac{1+g-2k_u}{2}\right)\frac{\epsilon_u}{\epsilon_y k_u}\right] = 0$$

EQ. B37

Multiplying B37 by $2\epsilon_y k_u$, we get

$$\left(\frac{P_u}{\phi f_c bt} - 0.85 k_1 k_u\right)\frac{g}{2}\left[2k_u(\epsilon_y m - \epsilon_y - \epsilon_u m) + m\epsilon_u(1+g)\right] - \left[\frac{M_u}{\phi f_c' bt^2} - 0.85 k_1 k_u\left(\frac{1-k_1 k_u}{2}\right)\right]\left[2k_u(\epsilon_y m - \epsilon_y + \epsilon_u m) - \epsilon_u m(g+1)\right] = 0$$

EQ. B38

EQ. B38 gives the following cubic equation for k_u

$$A k_u^3 + B k_u^2 + C k_u + D = 0$$

EQ. 99

where

$$A = -0.85 k_1^2 (\epsilon_y m - \epsilon_y + \epsilon_u m)$$

$$B = \left[-0.85 k_1 g(\epsilon_y m - \epsilon_y - \epsilon_u m) + 0.85 k_1(\epsilon_y m - \epsilon_y + \epsilon_u m) + \frac{0.85 k_1^2}{2}(g+1)\epsilon_u m\right]$$

$$C = \left[\frac{P_u}{\phi f_c' bt} g(\epsilon_y m - \epsilon_y - \epsilon_u m) - \frac{0.85}{2} k_1 g m \epsilon_u (1+g) - \frac{2M_u}{\phi f_c' bt^2}(\epsilon_y m - \epsilon_y + \epsilon_u m) - \frac{0.85 k_1}{2}\epsilon_u m(g+1)\right]$$

$$D = \frac{m\epsilon_u(1+g)}{\phi f_c' bt}\left[\frac{P_u g}{2} + \frac{M_u}{t}\right]$$

APPENDIX B 4

CASE 4 -

From the fig.7 the limits of k_u are

$$\left(\frac{1+g}{2}\right) t \le k_u t \le \left(\frac{1}{k_1}\right) t$$

From the fig.7 $\eta = k_u - \left(\frac{1+g}{2}\right)$

similar triangles in fig.7 give

$$\frac{\epsilon_s}{\eta t} = \frac{\epsilon_y}{\beta t} \quad ; \quad \epsilon_s = \epsilon_y \, \eta/\beta \qquad \text{EQ. 101}$$

$$f_s = E_s \epsilon_s \quad ; \quad f_y = E_s \epsilon_y \qquad \text{EQ'S 102, 103}$$

Let us replace ϵ_s and ϵ_y in EQ. 101 by their values from 102 and 103

$$f_s = f_y \, \eta/\beta \qquad \text{EQ. 104}$$

Forces in the steel are, $F_{s_1} = (p_e bt)(f_y - 0.85 f'_c)$

$$F_{s_2} = p_e bt \, (f_s - 0.85 f'_c)$$

Substituting f_s from (104) into F_{s_2} we have

$$F_{s_2} = (p_e bt)(f_y \, \eta/\beta - 0.85 f'_c)$$

Total force carried by steel is

$$F_{es} = F_{s_1} + F_{s_2} = (p_e bt)\left[(f_y - 0.85 f'_c) + (f_y \, \eta/\beta - 0.85 f'_c)\right] \qquad \text{EQ. 105}$$

Factoring out $0.85 f'_c$ in EQ. 105 and substituting $m = f_y/0.85 f'_c$

$$F_{es} = f'_c bt \; 0.85 \, p_e \left[(m-1) + (m \, \eta/\beta - 1)\right]$$

The load carried by concrete is $P_c = f'_c bt \; 0.85 \, k_1 k_u$ EQ. 59

Thus the ultimate load carried by the section in the fourth case is

$$P_u = \phi\left[F_{es} + P_c\right] = \phi f'_c bt\left\{0.85\, p_e\left[(m-1) + \left(m\frac{\eta}{\beta} - 1\right)\right] + 0.85\, k_1 k_u\right\} \qquad \text{EQ.106}$$

Taking moments about the plastic centroid
(note that lever arm is $gt/2$)

Moment carried by steel $= M_{es} = \dfrac{gt}{2}\left(F_{s_1} - F_{s_2}\right)$

$$M_{es} = \frac{gt}{2}\, p_e bt\left[\left(f_y - 0.85 f'_c\right) - \left(f_y\frac{\eta}{\beta} - 0.85 f'_c\right)\right]$$

Factoring out $0.85 f'_c$, and substituting $m = f_y / 0.85 f'_c$

$$M_{es} = f'_c bt^2\; 0.85\, p_e\, \frac{g}{2}\left[(m-1) - \left(m\frac{\eta}{\beta} - 1\right)\right]$$

$$M_c = \text{Moment carried by concrete} = f'_c bt^2\, 0.85\, k_1 k_u\left(\frac{1 - k_1 k_u}{2}\right) \qquad \text{EQ.60}$$

The ultimate moment carried by the section is

$$M_u = \phi\left[M_{es} + M_c\right] = \phi f'_c bt^2\left\{0.85\, p_e\frac{g}{2}\left[(m-1) - \left(m\frac{\eta}{\beta} - 1\right)\right] + 0.85\, k_1 k_u\left(\frac{1 - k_1 k_u}{2}\right)\right\} \qquad \text{EQ.107}$$

When β and η are substituted in EQ's 106 and 107 by their value

$$P_u = \phi f'_c bt\left\{0.85\, p_e\left[m + m\left(\frac{2k_u - 1 - g}{2}\right)\times\frac{\epsilon_u}{\epsilon_y k_u} - 2\right] + 0.85\, k_1 k_u\right\} \qquad \text{EQ.B41}$$

$$M_u = \phi f'_c bt^2\left\{0.85\, p_e\left(\frac{g}{2}\right)\left[m - m\left(\frac{2k_u - 1 - g}{2}\right)\times\frac{\epsilon_u}{\epsilon_y k_u}\right] + 0.85\, k_1 k_u\left(\frac{1 - k_1 k_u}{2}\right)\right\} \qquad \text{EQ.B42}$$

$$\frac{P_u}{\phi f'_c bt} = 0.85\, p_e\left[m + m\left(\frac{2k_u - 1 - g}{2}\right)\frac{\epsilon_u}{\epsilon_y k_u} - 2\right] + 0.85\, k_1 k_u \qquad \text{EQ.B43}$$

$$\frac{M_u}{\phi f'_c bt^2} = 0.85\, p_e\left(\frac{g}{2}\right)\left[m - m\left(\frac{2k_u - 1 - g}{2}\right)\times\left(\frac{\epsilon_u}{\epsilon_y k_u}\right)\right] + 0.85\, k_1 k_u\left(\frac{1 - k_1 k_u}{2}\right) \qquad \text{EQ.B44}$$

$$p_e = \frac{\dfrac{P_u}{\phi f'_c bt} - 0.85\, k_1 k_u}{0.85\left[m + m\left(\dfrac{2k_u - 1 - g}{2}\right)\dfrac{\epsilon_u}{\epsilon_y k_u} - 2\right]} \qquad \text{from B43} \qquad \text{EQ.B45}$$

$$p_e = \frac{\dfrac{M_u}{\phi f'_c bt^2} - 0.85\, k_1 k_u\left(\dfrac{1 - k_1 k_u}{2}\right)}{0.85\left(\dfrac{g}{2}\right)\left[m - m\left(\dfrac{2k_u - 1 - g}{2}\right)\dfrac{\epsilon_u}{\epsilon_y k_u}\right]} \qquad \text{from B44} \qquad \text{EQ.B46}$$

Equating EQ's B45 and B46, and taking all the terms to one side of the equation we get

$$\left(\frac{P_u}{\phi f'_c bt} - 0.85k_1k_u\right)\left(\frac{g}{2}\right)\left[m - m\left(\frac{2k_u-1-g}{2}\right)\frac{\epsilon_u}{\epsilon_y k_u}\right] - \left[\frac{M_u}{\phi f'_c bt^2} - 0.85k_1k_u\left(\frac{1-k_1k_u}{2}\right)\right]\left[m+m\left(\frac{2k_u-1-g}{2}\right)\frac{\epsilon_u}{\epsilon_y k_u} - 2\right] = 0$$

EQ. B47

Multiplying B47 by $2\epsilon_y k_u$, we get

$$\left(\frac{P_u}{\phi f'_c bt} - 0.85k_1k_u\right)\left(\frac{g}{2}\right)\left[2k_u(\epsilon_y - \epsilon_u)m + m\epsilon_u(1+g)\right] - \left[\frac{M_u}{\phi f'_c bt^2} - 0.85k_1k_u\left(\frac{1-k_1k_u}{2}\right)\right]\left[2k_u(m\epsilon_y - 2\epsilon_y + m\epsilon_u) - m\epsilon_u(1+g)\right] = 0$$

EQ. B48

EQ. B48 gives the following cubic eqation for k_u

$$Ak_u^3 + Bk_u^2 + Ck_u + D = 0$$

EQ. 108

where

$$A = \left[-0.85\,k_1^2\,(\epsilon_y m - 2\epsilon_y + \epsilon_u m)\right]$$

$$B = \left[-0.85k_1 g\,(\epsilon_y - \epsilon_u)m + 0.85k_1(\epsilon_y m - 2\epsilon_y + \epsilon_u m) + \frac{0.85k_1^2}{2}\epsilon_u m(1+g)\right]$$

$$C = \left[\frac{P_u g}{\phi f'_c bt}(\epsilon_y - \epsilon_u)m - \frac{0.85k_1}{2}g\,\epsilon_u m(1+g) - \frac{2M_u}{\phi f'_c bt^2}(\epsilon_y m - 2\epsilon_y + \epsilon_u m) - \frac{0.85k_1}{2}\epsilon_u m(g+1)\right]$$

$$D = \frac{\epsilon_u m(1+g)}{\phi f'_c bt}\left(\frac{P_u g}{2} + \frac{M_u}{t}\right)$$

APPENDIX B 5

CASE 5 -

The limits of k_u are

$$t\left(\frac{1}{k_1}\right) \leq k_u t < \infty$$

The steel force is as in the case 4, now our concrete section carries a different load given by

$$P_c = 0.85 f'_c bt \qquad \text{for} \quad k_u > 1/k_1 \qquad \text{EQ. 62}$$

The ultimate Load is

$$P_u = \phi\left[F_{es} + P_c\right] = \phi f'_c bt\left\{0.85\, p_e\left[(m-1) + (m\,\eta/\beta - 1)\right] + 0.85\right\} \qquad \text{EQ. 109}$$

The moment carried by concrete section is equal to zero

$$M_c = 0 \qquad \text{for} \quad k_u > 1/k_1 \qquad \text{EQ. 64}$$

Therefore the ultimate moment carried by the section is equal to the moment carried by steel alone in the case 4, it is

$$M_u = \phi f'_c bt^2\ 0.85\, p_e\left(\frac{g}{2}\right)\left[(m-1) - (m\,\eta/\beta - 1)\right] \qquad \text{EQ. B51}$$

When η and β are eliminated, EQ'S 109 and B51 become

$$P_u = \phi f'_c bt\left\{0.85\, p_e\left[m + m\left(\frac{2k_u - 1 - g}{2}\right)\frac{\epsilon_u}{\epsilon_y k_u} - 2\right] + 0.85\right\} \qquad \text{EQ. B52}$$

$$M_u = \phi f'_c bt^2\left\{0.85\, p_e\left(\frac{g}{2}\right)\left[m - m\left(\frac{2k_u - 1 - g}{2}\right)\frac{\epsilon_u}{\epsilon_y k_u}\right]\right\} \qquad \text{EQ. B53}$$

$$p_e = \frac{\left(P_u / \phi f'_c bt\right) - 0.85}{0.85\left[m + m\left(\frac{2k_u - 1 - g}{2}\right)\frac{\epsilon_u}{\epsilon_y k_u} - 2\right]} \qquad \text{from EQ. 109} \qquad \text{EQ. B54}$$

$$p_e = \frac{(M_u/\phi f'_c b t^2)}{0.85\left(\frac{g}{2}\right)\left[m - m\left(\frac{2k_u - 1 - g}{2}\right)\frac{\epsilon_u}{\epsilon_y k_u}\right]}$$ from EQ. B53 EQ.B55

Equating B54 and B55, and taking all the terms to one side of the equation

$$\left(\frac{P_u}{\phi f'_c b t} - 0.85\right)\left(\frac{g}{2}\right)\left[2k_u(\epsilon_y - \epsilon_u)m + m(1+g)\epsilon_u\right] - \frac{M_u}{\phi f'_c b t^2}\left[2k_u(m\epsilon_y - 2\epsilon_y + \epsilon_u m) - m(1+g)\epsilon_u\right] = 0$$

EQ.B56

$$\frac{P_u}{\phi f'_c b t}(\epsilon_y - \epsilon_u) m g k_u - 0.85(\epsilon_y - \epsilon_u) m g k_u + m(1+g)\epsilon_u \frac{g}{2}\left[\frac{P_u}{\phi f'_c b t} - 0.85\right] -$$

$$- \frac{2M_u}{\phi f'_c b t^2}(\epsilon_y m - 2\epsilon_y + \epsilon_u m) k_u + \frac{M_u}{\phi f'_c b t^2} m \epsilon_u (1+g) = 0$$

EQ.B57

Solution of the equation B57 gives k_u as

$$k_u = \frac{-\left\{\epsilon_u m(1+g)\left[\frac{g}{2}\left(\frac{P_u}{\phi f'_c b t} - 0.85\right) + \frac{M_u}{\phi f'_c b t^2}\right]\right\}}{\left\{mg(\epsilon_y - \epsilon_u)\left(\frac{P_u}{\phi f'_c b t} - 0.85\right) - \frac{2M_u}{\phi f'_c b t^2}(\epsilon_y m - 2\epsilon_y + \epsilon_u m)\right\}}$$ EQ. 111

APPENDIX B 6

From the fig.8 $\qquad F_c = \phi f'_c bt\, 0.85 k_1 k_u$ EQ. 59

$F_s = \phi p_e\, bt f_y$ EQ. B61

The ultimate load is the sum of the loads carried by concrete and steel

$$P_u = F_c - F_s = \phi \; bt\,(f'_c 0.85 k_1 k_u - p_e f_y) \qquad \text{EQ. 112}$$

The ultimate moment taken about the gravity axis is

$$M_u = \phi f'_c bt\, 0.85 k_1 k_u\,(t/2 - k_1 k_u t/2) + \phi p_e\, bt f_y\,(t/2 - d') \qquad \text{EQ. 113}$$

From EQ. 112 $\qquad p_e = \dfrac{-P_u + \phi f'_c bt\, 0.85 k_1 k_u}{\phi bt f_y}$ EQ. B62

putting this value of p_e into EQ. 113

$$M_u = 0.85\phi f'_c bt k_1 k_u \left(t/2 - k_1 k_u t/2\right) + \phi \left(\frac{-P_u + \phi f'_c bt\, 0.85 k_1 k_u}{\phi bt f_y}\right) bt f_y \left(t/2 - d'\right)$$

$$M_u = (\phi f'_c bt^2 k_1 k_u 0.85)/2 - (\phi f' bt^2 k_1^2 k_u^2 0.85)/2 - P_u\left(\frac{t}{2} - d'\right) + \phi f'_c bt\, 0.85 k_1 k_u (t/2 - d')$$

$$M_u = -\phi f'_c bt^2 k_1^2\, 0.85\, k_u^2/2 + \left[+\phi f'_c bt\, 0.85 k_1 (t/2 - d') + \phi f'_c bt^2 k_1\, 0.85/2\right] k_u + P_u (t/2 - d')$$

Taking all the terms to one side and dividing by $(\phi f'_c bt^2 k_1^2\, 0.85)/2$

$$k_u^2 - 2\left[+\frac{1}{k_1 t}\left(\frac{t}{2} - d'\right) + \frac{1}{2k_1}\right] k_u + \frac{M_u + P_u(t/2 - d')}{(\phi f'_c bt^2 k_1^2\, 0.85)/2} = 0$$

$$k_u^2 - \frac{2}{k_1 t}\left[\frac{t}{2} - d' + \frac{t}{2}\right] k_u + \frac{2[M_u + P_u(t/2 - d')]}{\phi f'_c bt^2\, 0.85 k_1^2} = 0$$

$$k_u^2 - \frac{2d}{k_1 t} k_u + \frac{2[M_u + P_u(t/2 - d')]}{\phi f'_c bt^2 k_1^2\, 0.85} = 0$$

$$k_u = \frac{d}{k_1 t}\left(1 - \sqrt{1 - \frac{2[M_u + P_u(t/2 - d')]\; k_1^2 t^2}{\phi f'_c bt^2 k_1^2\, 0.85\, d^2}}\right)$$

therefore $\qquad k_u = \dfrac{d}{k_1 t}\left(1 - \sqrt{1 - \dfrac{2[M_u + P_u(t/2 - d')]}{\phi f'_c b\, 0.85\, d^2}}\right)$ EQ. B63

$$A_s = p_e bt \qquad \text{EQ B64}$$

Substituting B62 and B63 into B64 we get

$$A_s = \frac{-P_u bt}{\phi bt f_y} + \frac{\phi f_c' bt\, 0.85 k_1 bt}{\phi f_y bt} \frac{d}{k_1 t}\left(1 - \sqrt{1 - \frac{2[M_u + P_u(t/2 - d')]}{\phi f_c' b\, 0.85 d^2}}\right)$$

$$A_s = \left\{\frac{0.85 f_c' bd}{f_y}\left(1 - \sqrt{1 - \frac{2[M_u + P_u(t/2 - d')]}{\phi\; 0.85 f_c' bd\; d}}\right) - \frac{P_u}{\phi f_y}\right\} \qquad \text{EQ. B65}$$

calling $\chi = 0.85 f_c' bd$

$$A_s = \left\{\frac{\chi}{f_y}\left(1 - \sqrt{1 - \frac{2[M_u + P_u(\frac{t}{2} - d')]}{\phi \chi d}}\right) - \frac{P_u}{\phi f_y}\right\} \qquad \text{EQ. 11'}$$

APPENDIX C

IDEALIZED β ENVELOPES

1- 4 bar arrangement , f_y - 40.0 ksi -

$q = .1$	$0 < (P_u/P_o) < .22$	$\beta = -0.93 \frac{P_u}{P_o} + 0.84$
	$0.22 < (P_u/P_o) < 0.50$	$\beta = -0.0535 (P_u/P_o) + 0.653$
	$0.50 < (P_u/P_o) < 0.90$	$\beta = 0.425 (P_u/P_o) + 0.41$
$q = 0.3$	$0 < (P_u/P_o) < 0.18$	$\beta = -1.05 (P_u/P_o) + 0.77$
	$0.18 < (P_u/P_o) < 0.36$	$\beta = -0.0835 (P_u/P_o) + 0.599$
	$0.36 < (P_u/P_o) < 0.90$	$\beta = 0.464 (P_u/P_o) + 0.398$
$q = 0.5$	$0 < (P_u/P_o) < 0.19$	$\beta = -0.66 (P_u/P_o) + 0.675$
	$0.19 < (P_u/P_o) < 0.32$	$\beta = 0.55$
	$0.32 < (P_u/P_o) < 0.73$	$\beta = 0.45 (P_u/P_o) + 0.41$
	$0.73 < (P_u/P_o) < 0.90$	$\beta = 1.145 (P_u/P_o) - 0.102$
$q = 0.9$	$0 < (P_u/P_o) < 0.17$	$\beta = -0.47 (P_u/P_o) + 0.61$
	$0.17 < (P_u/P_o) < 0.24$	$\beta = 0.53$
	$0.24 < (P_u/P_o) < 0.35$	$\beta = 0.464 (P_u/P_o) + 0.42$
	$0.35 < (P_u/P_o) < 0.70$	$\beta = 0.415 (P_u/P_o) + 0.435$
	$0.70 < (P_u/P_o) < 0.90$	$\beta = 1.225 (P_u/P_o) - 0.135$
$q = 1.3$	$0 < (P_u/P_o) < 0.17$	$\beta = -0.41 (P_u/P_o) + 0.58$
	$0.17 < (P_u/P_o) < 0.66$	$\beta = 0.45 (P_u/P_o) + 0.434$
	$0.66 < (P_u/P_o) < 0.90$	$\beta = 1.08 (P_u/P_o) + 0.015$

2- 4-bar arrangement , f_y- 60 ksi -

$q=.1$ $\quad 0 < (P_u/P_o) < 0.20 \quad \beta = -0.90\,(P_u/P_o) + 0.825$

$\quad 0.20 < (P_u/P_o) < 0.41 \quad \beta = -0.167\,(P_u/P_o) + 0.678$

$\quad 0.41 < (P_u/P_o) < 0.90 \quad \beta = 0.224\,(P_u/P_o) + 0.518$

$q=0.3$ $\quad 0 < (P_u/P_o) < 0.13 \quad \beta = -1.08\,(P_u/P_o) + 0.75$

$\quad 0.13 < (P_u/P_o) < 0.28 \quad \beta = -0.333\,(P_u/P_o) + 0.653$

$\quad 0.28 < (P_u/P_o) < 0.90 \quad \beta = 0.202\,(P_u/P_o) + 0.504$

$q=0.5$ $\quad 0 < (P_u/P_o) < 0.10 \quad \beta = -0.85\,(P_u/P_o) + 0.675$

$\quad 0.10 < (P_u/P_o) < 0.24 \quad \beta = -0.416\,(P_u/P_o) + 0.631$

$\quad 0.24 < (P_u/P_o) < 0.90 \quad \beta = 0.182\,(P_u/P_o) + 0.496$

$q=0.9$ $\quad 0 < (P_u/P_o) < 0.20 \quad \beta = -0.400\,(P_u/P_o) + 0.600$

$\quad 0.20 < (P_u/P_o) < 0.90 \quad \beta = 0.179\,(P_u/P_o) + 0.484$

$q=1.3$ $\quad 0 < (P_u/P_o) < 0.16 \quad \beta = -0.375\,(P_u/P_o) + 0.575$

$\quad 0.16 < (P_u/P_o) < 0.90 \quad \beta = 0.179\,(P_u/P_o) + 0.484$

3 - 8-bar arrangement , f_y - 40 ksi -

$q = .1$ $\quad 0 < (P_u/P_o) < 0.26 \quad \beta = -0.462\,(P_u/P_o) + 0.77$
$0.26 < (P_u/P_o) < 0.46 \quad \beta = -0.05\,(P_u/P_o) + 0.663$
$0.46 < (P_u/P_o) < 0.70 \quad \beta = 0.167\,(P_u/P_o) + 0.562$
$0.70 < (P_u/P_o) < 0.90 \quad \beta = 0.45\,(P_u/P_o) + 0.365$

$q = 0.3$ $\quad 0 < (P_u/P_o) < 0.28 \quad \beta = -0.243\,(P_u/P_o) + 0.680$
$0.28 < (P_u/P_o) < 0.70 \quad \beta = 0.155\,(P_u/P_o) + 0.572$
$0.70 < (P_u/P_o) < 0.90 \quad \beta = 0.45\,(P_u/P_o) + 0.365$

$q = 0.5$ $\quad 0 < (P_u/P_o) < 0.30 \quad \beta = -0.150\,(P_u/P_o) + 0.645$
$0.30 < (P_u/P_o) < 0.70 \quad \beta = 0.200\,(P_u/P_o) + 0.540$
$0.70 < (P_u/P_o) < 0.90 \quad \beta = 0.450\,(P_u/P_o) + 0.365$

$q = 0.9$ $\quad 0 < (P_u/P_o) < 0.25 \quad \beta = -0.100\,(P_u/P_o) + 0.615$
$0.25 < (P_u/P_o) < 0.70 \quad \beta = 0.200\,(P_u/P_o) + 0.540$
$0.70 < (P_u/P_o) < 0.90 \quad \beta = 0.450\,(P_u/P_o) + 0.365$

$q = 1.3$ $\quad 0 < (P_u/P_o) < 0.23 \quad \beta = -0.065\,(P_u/P_o) + 0.605$
$0.23 < (P_u/P_o) < 0.70 \quad \beta = 0.200\,(P_u/P_o) + 0.540$
$0.70 < (P_u/P_o) < 0.90 \quad \beta = 0.450\,(P_u/P_o) + 0.365$

4- 8-bar arrangement , f_y - 60 ksi -

$q = 0.1$ $\quad 0 < (P_u/P_o) < 0.36 \quad \beta = -0.306\,(P_u/P_o) + 0.735$
$0.36 < (P_u/P_o) < 0.90 \quad \beta = 0.176\,(P_u/P_o) + 0.561$

$q = 0.3$ $\quad 0 < (P_u/P_o) < 0.29 \quad \beta = -0.310\,(P_u/P_o) + 0.665$
$0.29 < (P_u/P_o) < 0.90 \quad \beta = 0.205\,(P_u/P_o) + 0.516$

$q = 0.5$ $\quad 0 < (P_u/P_o) < 0.25 \quad \beta = -0.260\,(P_u/P_o) + 0.625$
$0.25 < (P_u/P_o) < 0.90 \quad \beta = 0.184\,(P_u/P_o) + 0.514$

$q = 0.9$ $\quad 0 < (P_u/P_o) < 0.21 \quad \beta = -0.310\,(P_u/P_o) + 0.595$
$0.21 < (P_u/P_o) < 0.45 \quad \beta = 0.250\,(P_u/P_o) + 0.477$
$0.45 < (P_u/P_o) < 0.90 \quad \beta = 0.133\,(P_u/P_o) + 0.530$

$q = 1.3$ $\quad 0 < (P_u/P_o) < 0.20 \quad \beta = -0.225\,(P_u/P_o) + 0.570$
$0.20 < (P_u/P_o) < 0.45 \quad \beta = 0.250\,(P_u/P_o) + 0.477$
$0.45 < (P_u/P_o) < 0.90 \quad \beta = 0.133\,(P_u/P_o) + 0.530$

5- 12-bar arrangement, f_y - 40 ksi -

$q=0.1$	$0<(P_u/P_o)<0.38$	$\beta=-0.370(P_u/P_o)+0.760$
	$0.38<(P_u/P_o)<0.90$	$\beta=0.270(P_u/P_o)+0.518$
$q=0.3$	$0<(P_u/P_o)<0.37$	$\beta=-0.176(P_u/P_o)+0.685$
	$0.37<(P_u/P_o)<0.90$	$\beta=0.270(P_u/P_o)+0.518$
$q=0.5$	$0<(P_u/P_o)<0.3$	$\beta=-0.183(P_u/P_o)+0.655$
	$0.30<(P_u/P_o)<0.9$	$\beta=0.270(P_u/P_o)+0.518$
$q=0.9$	$0<(P_u/P_o)<0.29$	$\beta=-0.121(P_u/P_o)+0.630$
	$0.29<(P_u/P_o)<0.9$	$\beta=0.270(P_u/P_o)+0.518$
$q=1.3$	$0<(P_u/P_o)<0.29$	$\beta=-0.052(P_u/P_o)+0.610$
	$0.29<(P_u/P_o)<0.90$	$\beta=0.270(P_u/P_o)+0.518$

6 - 12- bar arrangement, f_y - 60 ksi -

$q=0.1$	$0<(P_u/P_o)<0.35$	$\beta=-0.330(P_u/P_o)+0.740$
	$0.35<(P_u/P_o)<0.90$	$\beta=0.164(P_u/P_o)+0.568$
$q=0.3$	$0<(P_u/P_o)<0.27$	$\beta=-0.334(P_u/P_o)+0.670$
	$0.27<(P_u/P_o)<0.90$	$\beta=0.183(P_u/P_o)+0.531$
$q=0.5$	$0<(P_u/P_o)<0.23$	$\beta=-0.260(P_u/P_o)+0.630$
	$0.23<(P_u/P_o)<0.90$	$\beta=0.149(P_u/P_o)+0.536$
$q=0.9$	$0<(P_u/P_o)<0.19$	$\beta=-0.342(P_u/P_o)+0.595$
	$0.19<(P_u/P_o)<0.59$	$\beta=0.212(P_u/P_o)+0.490$
	$0.59<(P_u/P_o)<0.90$	$\beta=0.0485(P_u/P_o)+0.587$
$q=1.3$	$0<(P_u/P_o)<0.17$	$\beta=-0.264(P_u/P_o)+0.570$
	$0.17<(P_u/P_o)<0.59$	$\beta=0.212(P_u/P_o)+0.490$
	$0.59<(P_u/P_o)<0.90$	$\beta=0.0485(P_u/P_o)+0.587$

7- 6,8,10 bar arrangements, fy - 40 ksi -

$q=0.1$	$0 < (P_u/P_o) < 0.21$	$\beta = -0.645 (P_u/P_o) + 0.790$
	$0.21 < (P_u/P_o) < 0.46$	$\beta = -0.100 (P_u/P_o) + 0.676$
	$0.46 < (P_u/P_o) < 0.64$	$\beta = 0.278 (P_u/P_o) + 0.502$
	$0.64 < (P_u/P_o) < 0.90$	$\beta = 0.404 (P_u/P_o) + 0.422$
$q=0.3$	$0 < (P_u/P_o) < 0.31$	$\beta = -0.322 (P_u/P_o) + 0.690$
	$0.31 < (P_u/P_o) < 0.64$	$\beta = 0.278 (P_u/P_o) + 0.502$
	$0.64 < (P_u/P_o) < 0.90$	$\beta = 0.404 (P_u/P_o) + 0.422$
$q=0.5$	$0 < (P_u/P_o) < 0.28$	$\beta = -0.268 (P_u/P_o) + 0.660$
	$0.28 < (P_u/P_o) < 0.64$	$\beta = 0.278 (P_u/P_o) + 0.502$
	$0.64 < (P_u/P_o) < 0.90$	$\beta = 0.404 (P_u/P_o) + 0.422$
$q=0.9$	$0 < (P_u/P_o) < 0.20$	$\beta = -0.325 (P_u/P_o) + 0.630$
	$0.20 < (P_u/P_o) < 0.64$	$\beta = 0.278 (P_u/P_o) + 0.502$
	$0.64 < (P_u/P_o) < 0.90$	$\beta = 0.404 (P_u/P_o) + 0.422$
$q=1.3$	$0 < (P_u/P_o) < 0.18$	$\beta = -0.278 (P_u/P_o) + 0.610$
	$0.18 < (P_u/P_o) < 0.64$	$\beta = 0.278 (P_u/P_o) + 0.502$
	$0.64 < (P_u/P_o) < 0.90$	$\beta = 0.404 (P_u/P_o) + 0.422$

8 - 6,8,10 bar arrangements , f_y - 60 ksi -

$q=0.1$	$0 < (P_u/P_o) < 0.30$	$\beta = -0.434 (P_u/P_o) + 0.765$
	$0.30 < (P_u/P_o) < 0.41$	$\beta = -0.136 (P_u/P_o) + 0.676$
	$0.41 < (P_u/P_o) < 0.90$	$\beta = 0.224 (P_u/P_o) + 0.528$
$q=0.3$	$0 < (P_u/P_o) < 0.28$	$\beta = -0.375 (P_u/P_o) + 0.680$
	$0.28 < (P_u/P_o) < 0.90$	$\beta = 0.187 (P_u/P_o) + 0.523$
$q=0.5$	$0 < (P_u/P_o) < 0.26$	$\beta = -0.308 (P_u/P_o) + 0.635$
	$0.26 < (P_u/P_o) < 0.90$	$\beta = 0.188 (P_u/P_o) + 0.506$
$q=0.9$	$0 < (P_u/P_o) < 0.20$	$\beta = -0.375 (P_u/P_o) + 0.605$
	$0.20 < (P_u/P_o) < 0.53$	$\beta = 0.197 (P_u/P_o) + 0.491$
	$0.53 < (P_u/P_o) < 0.90$	$\beta = 0.135 (P_u/P_o) + 0.523$
$q=1.3$	$0 < (P_u/P_o) < 0.17$	$\beta = -0.353 (P_u/P_o) + 0.580$
	$0.17 < (P_u/P_o) < 0.53$	$\beta = 0.197 (P_u/P_o) + 0.491$
	$0.53 < (P_u/P_o) < 0.90$	$\beta = 0.135 (P_u/P_o) + 0.523$

APPENDIX D

N O T A T I O N

a= depth of equivalent rectangular stress block, defined by Section 1503(g) ACI Code, it is equal to k_1c

a_b=depth of equivalent rectangular stress block for balanced conditions $-k_1c_b$

A_g=gross area of section

A_s=area of tension reinforcement

A'_s=area of compression reinforcement

A_{sf}=area of reinforcement to develop compressive strength of overhanging flanges in I- and T- sections

A_{st}=total area of longitudinal reinforcement

A_v=total area of web reinforcement in tension within a distance, s, measured in a direction parallel to the longitudinal reinforcement

b=width of compression face of flexural member

b'=width of web in I- and T- sections

c=distance from extreme compression fiber to neutral axis

c_b=distance from extreme compression fiber to neutral axis for balanced conditions=$d(87,000)/(87,000-f_y)$

d'=distance from extreme compression fiber to centroid of compression reinforcemnt

d=distance from extreme compression fiber to centroid of tension reinforcement

d''=distance from plastic centroid to centroid of tension reinforcement

D=over-all diameter of circular section

D_s=diameter of the circle through centers of reinforcement arranged in a circular pattern

e=eccentricity of axial load at end of member measured from plastic centroid of the section, calculated by conventional methods of frame analysis

e'=eccentricity of axial load at end of member measured from the centroid of the tension reinforcement, calculated by conventional methods of frame analysis

e_b=eccentricity of load P_b measured from plastic centroid of section

f'_c=compressive strength of concrete

f_s=calculated stress in reinforcement when less than the yield strength, f_y

f_y=yield strength of reinforcement

F= force end steel(total)

k_1=a factor defined in Section 1503(g) in ACI Code, and in p.

$k_u t$=the depth of the compressed area, measured from the extreme fiber on the compression side

$k_1 k_u t$=the depth of the equivalent rectangular stress block

$m= f_y /0.85f'_c$

$m'=m-1$

M=bending moment

M'=modified bending moment

M_b=moment capacity at simultaneous crushing of concrete and yielding of tension steel(balanced conditions)-$P_b e_b$

M_c=moment carried by concrete section alone

M_{es}=moment carried by end steels

M_u=moment capacity under combined axial load end bending

N=load normal totto the cross section, to be taken as positive for compression negative for tension, and to include the effects of tension due to shrinkage and creep

$p=A_s /bd$

$p'= A'_s /bd$

p_b=reinforcement ratio producing balanced conditions at ultimate strength

$p_f= A_{sf} /b'd$

$p_t= A_{st} /A_g$

$p_w= A_s /b'd$

p_s=ratio of volume of spiral reinforcement to total volume of core (out to out of spirals) of a spirally reinforced concrete or composite column

P_b=axial load capacity at simultaneous crushing of concrete and yielding of tension steel (balanced conditions)

P_o=axial load capacity of actual member when concentrically loaded

P_u=P_u-axial load capacity under combined axial load and bending

r=radius of gyration of gross concrete area of column

R=a reduction factor for long columns

$q= A_s f_y / bdf'_c$

t=flange thickness in I- and T- sections

v_c=shear stress carried by concrete

v_u=nominal ultimate shear stress as a measure of diagonal tension

V=total shear at section

V_u=total ultimate shear

V'_u=ultimate shear carried by web reinforcement

ϵ_u=ultimate concrete compressive strain=0.003

ϵ_y=strain at yiel in outermost reinforcement

ϵ_s=strain in outermost tension reinforcement

ϵ_s=strain in outermost compression reinforcement

ϕ=capacity reduction factor

APPENDIX E

```
*06044
C      I. KARACA -S. TEZCAN  ULTIMATE STRENGTH DESIGN PER ACI-318/63
C      ULTIMATE STRENGTH DESIGN OF REINFORCED CONCRETE MEMBERS PER ACI-
       DIMENSION F(8,3),PARTIC(3,3),FAC(3),A(3,3),BD(3,3),W(3,3),
      1LTYPE(3,3),K(3),SR(2,5),R(8)
C  M A I N L I N E    O F    B E A M S
999    READ 102
102    FORMAT(80H
      1                         )
       PRINT 102
       READ 902,ME,NLOAD,NC
       PRINT 97 ,ME,NLOAD,NC
902    FORMAT(20I4)
97     FORMAT(//23H NUMBER OF MEMBERS(ME)=,I4/29H NUMBER OF LOAD CASES(N
      1OAD)=,I3,3X/33H NUMBER OF LOAD COMBINATIONS(NC)=,I3/)
       PRINT 975
975    FORMAT(/51H COMB. NO.    PARTICIPATION AND OVER STRESS FACTORS)
       DO 12 L=1,NC
       READ 908,(PARTIC(I,L),I=1,NLOAD),FAC(L)
12     PRINT 976,L,(PARTIC(I,L),I=1,NLOAD),FAC(L)
976    FORMAT(I5,5X,10F7.2)
       READ 908,FC,FY
       PRINT 910,FC,FY
       FC=FC/1000.
       FY=FY/1000.
910    FORMAT(24H CONCRETE STRENGTH(FC) =,F8.0/  24H STEEL YIELD STRESS
      1Y)=,F8.0)
C      READ MEMBER DATA
       DO 1000 I=1,ME
       PRINT 911,I
911    FORMAT(/////,I4,19X,23H END FORCES AND MOMENTS,19X,2HPY,8X,2HMY)
       DO 5 L=1,NLOAD
       READ 908,(F(J,L),J=1,8)
5       PRINT 909,(F(J,L),J=1,8),L
908     FORMAT(8F10.0)
909    FORMAT(5X,8F9.2,I3)
       READ 993,IX,S,B,BP,T,DP,FT,D1,D2
993    FORMAT(I3,F7.0,7F10.0)
       IF(I-IX) 804,803,804
804    PRINT 802,I,IX
802    FORMAT(30H MEMBER NUMBER IS OUT OF ORDER/2I5)
       GO TO 77
803    CONTINUE
       IF(DP) 3,3,4
3      DP=2.5
4      CONTINUE
       IF(BP) 45,45,46
46     PRINT 913,I
913     FORMAT(16H LOADS ON MEMBER,I4)
       DO 18 L=1,NLOAD
       READ 902,K(L)
       PRINT 91,K(L)
       KK=K(L)
       IF(K(L)) 18,18,13
13     DO 17 M=1,KK
```

```
      READ 11,I,A(M,L),BD(M,L),W(M,L),LTYPE(M,L)
      PRINT 91,I,A(M,L),BD(M,L),W(M,L),LTYPE(M,L),L
17    CONTINUE
18    CONTINUE
45    CONTINUE
11    FORMAT(I4,3F6.2,I4)
91    FORMAT(8X,I5,3F9.2,I4,I8)
      PRINT 912,I,S,B,BP,T,DP,FT,D1,D2
912   FORMAT(/I4,3H S=,F6.1,3H B=,F6.1,4H BP=,F6.1,3H T=,F6.1,4H DP=,F
     16.1,4H FT=,F6.1,4H D1=,F6.1,4H D2=,F6.1)
      G1=.85
      IF(FC-4.) 299,299,298
298   G1=.85+.05*(FC-4.)
299   CONTINUE
      IF(BP) 501,500,501
C BEAMS
501   IF(T-12.*S/20.) 325,326,326
325   PRINT 499,I
499   FORMAT(I4,6X,35HCHECK THE DEPTH AGAINST  ACI-TBL909)
326   IF(T-.4*12.*S) 500,500,498
498    PRINT 497
497   FORMAT(61H DESIGN THIS BEAM AS A DEEP BEAM,SINCE T IS GREATER TH
     1 0.4L////)
      GO TO 1000
500   CONTINUE
      DO 300 L=1,NC
      IF(NC-1) 897,897,898
898   PRINT 896,L
896   FORMAT(/72X,5HCOMB.,I3)
897   CALL MPV(SR,XM,YM,P,IS,R,S,A,BD,W,LTYPE,K,D1,D2,B,BP,L,NLOAD,F,
     1PARTIC,T,DP)
      IF(BP) 401,400,401
401   CONTINUE
      PRINT 890
890   FORMAT(6X,72HAT      MU        PU       VU         T          AS        TNEW
     1 ASNEW           ASV)
C BEAMS
      TH=T
       DO 800 MP=1,5
      T1=TH
      TNEW=0.
      ASNEW=0.
      ASV=0.
      XM1=SR(2,MP)
      VM=SR(1,MP)
      GO TO(396,396,397,397,397),MP
396   IF(B-BP) 385,385,387
387   BH=BP
      P=0.
      GO TO 384
385   BH=B
384   CONTINUE
      CALL RBEAM(XM1,P,FC,FY,BH,BP,TH,DP,FT,G1,AS,ASP,TNEW,ASNEW,I,IS,
     1)
      ASH=AS
      IF(TNEW-TH) 562,562,563
563   TH=TNEW
      ASH=ASNEW
562   CONTINUE
```

```
      CALL SHEAR(VM,XM1,P,FC,FY,B,BP,FT,TH,DP,ASH,ASV,I,MP,IS)
      GO TO 799
397    IF(B-BP) 301,396,301
301   CALL TBEAM(XM1,P,FC,FY,B,BP,TH,DP,FT,G1,AS,ASF,I,MP)
      CALL SHEAR(VM,XM1,P,FC,FY,B,BP,FT,TH,DP,AS,ASV,I,MP,IS)

799   PRINT 126,I,MP,XM1,P,VM,T1,AS,TNEW,ASNEW,ASV
800   CONTINUE
126   FORMAT(2I4,4F8.1,F8.2,F8.1,F8.2,F14.3)
C COLUMNS
300   CONTINUE
      GO TO 1000
400   CONTINUE
1000  CONTINUE
77    STOP
      END
```

```
*06044
      SUBROUTINE MPV(SR,XM,YM,P,IS,R,S,A,BD,W,LTYPE,K,D1,D2,B,BP,L,NLO
     1,F,PARTIC,T,DP)
C THIS SUBROUTINE  COMBINES LOAD CASES AND DETERMINES M,N,V AT  FIVE P
      DIMENSION R(8),A(3,3),BD(3,3),W(3,3),LTYPE(3,3),K(3),F(8,3),
     1PARTIC(3,3),SR(2,5)
      D=T-DP
      DO 390 J=1,8
390   R(J)=0.
      DO 33 KK=1,NLOAD
      DO 321 J=1,8
321   R(J)=R(J)+F(J,KK)*PARTIC(KK,L)
33    CONTINUE
      IS=1
      P1=R(1)+R(7)
      P2=R(4)-R(7)
      IF(ABSF(P1)-ABSF(P2)) 38,38,37
38     P=ABSF(P2)
      IF(P2) 35,35,32
32    IS=2
      GO TO 35
37    P=ABSF(P1)
      IF(P1) 34,34,35
34    IS=2
C  IS=1 MEANS COMPRESSION,   IS=2 MEANS TENSION
35    CONTINUE
      DO 391 JV=1,5
      DO 391 JL=1,2
391   SR(JL,JV)=0.
      SR(2,1)=-R(3)+D1*R(2)
      SR(2,2)=R(6)+D2*R(5)
C FOR BEAMS CALCULATE MNV AT INTERIOR POINTS, FOR COLUMNS  SKIP THIS
      IF(BP) 45,45,46
46    SR(1,1)=R(2)
      SR(1,2)=R(5)
      DO  100 M=3,5
      Z=M
      DEF=Z-2.
      DIS=DEF*S/4.
      SR(1,M)=R(2)
      SR(2,M)=-R(3)+R(2)*DIS
      DO 39 KK=1,NLOAD
      IT=K(KK)
      IF(IT) 39,39,810
810   DO 380 J=1,IT
      IF(A(J,KK)-DIS) 80,380,380
80    LT=LTYPE(J,KK)
      GO TO(71,72,73,380),LT
71    SR(1,M)=SR(1,M)-W(J,KK)*PARTIC(KK,L)
      SR(2,M)=SR(2,M)-W(J,KK)*(DIS-A(J,KK))*PARTIC(KK,L)
      GO TO 380
72    IF(M-3) 82,81,82
81    SR(1,1)=SR(1,1)-( D/12.+D1)*W(J,KK)*PARTIC(KK,L)
      SR(1,2)=SR(1,2)-(D/12.+D2)*W(J,KK)*PARTIC(KK,L)
      SR(2,1)=SR(2,1)-W(J,KK)*D1*D1*.5*PARTIC(KK,L)
      SR(2,2)=SR(2,2)-W(J,KK)*D2*D2*.5*PARTIC(KK,L)
82    SR(1,M)=SR(1,M)-W(J,KK)*(DIS-A(J,KK))*PARTIC(KK,L)
      AA=A(J,KK)
```

```
      BB=BD(J,KK)
      C=S-AA-BB
      IF((AA+C)-DIS) 61,62,62
61    ARM=DIS-(AA+.5*C)
      SR(2,M)=SR(2,M)-W(J,KK)*C*ARM*PARTIC(KK,L)
      GO TO 380
62    SR(2,M)=SR(2,M)-.5*W(J,KK)*(DIS-A(J,KK))*(DIS-A(J,KK))*PARTIC(KK
     1)
      GO TO 380
73    SR(2,M)=SR(2,M)-W(J,KK)*PARTIC(KK,L)
380   CONTINUE
39    CONTINUE
100   CONTINUE
      DO 101 M=1,5
      DO 101 MM=1,2
101   SR(MM,M)=ABSF(SR(MM,M))
45    CONTINUE
      XM=ABSF(SR(2,1))
      IF(ABSF(SR(2,1))-ABSF(SR(2,2))) 30,31,31
30    XM=ABSF(SR(2,2))
31    CONTINUE
      YM=ABSF(R(8))
      RETURN
      END
```

```
*06044
      SUBROUTINE RBEAM(XM,P,FC,FY,B,BP,T,DP,FT,G1,AS,ASP,TNEW,ASNEW,
     1I,IS,MP)
      ASP=0.
4     D=T-DP
      ASM=B*D*.2/FY
      GAPA=.85*FC*B*D
      SAYN=1.
      IF(IS-2) 66,67,67
67    SAYN=-1.
66    SAY=1.-2.*(12.*XM+SAYN*P*(.5*T-DP))/(.9*GAPA*D)
      IF(SAY) 5,6,6
5     PRINT 12,I,MP
12    FORMAT(2I4,4X,19H INSUFFICIENT DEPTH,4X,20HDEPTH IS INCREASED..)
      AS=0.
      GO TO 13
6     AS=GAPA*(1.-SQRTF(SAY))/FY-SAYN*P/(.9*FY)
C CHECK FOR LOCATION OF  N.A
       IF(P) 803,822,803
803   EPU=.003
      EPY=FY/29000.
      ALF=D/(G1*T)
      GU=ALF*(1.-SQRTF(SAY))
      BETA=EPY*GU/EPU
      ALT=0.
      UST=1.-(DP/T)-BETA
      IF (MP-3) 75,822,75
  75  IF (MP-5) 821,822,821
821   IF(UST-GU) 820,822,822
820   PRINT 823,I
823   FORMAT(18H DESIGN MEMBER NO=,I4,4X/76H AS A COLUMN, SINCE THE AX
     1L FORCE PUSHES THE N.A. BEYOND THE RANGE DEFINED/26H IN CASE 2
     2 THE ACI-SP7)
      GO TO 642
822   CONTINUE
C CHECK FOR MAXIMUM PERCENTAGE  PB
10      PB=.6375*G1*FC*87./(FY*(87.+FY))
      IF(AS-PB*B*D) 103,103,5
103   IF(AS-ASM)104,101,101
104   AS=ASM
      GO TO 101
C INCREASE DEPTH INCASE OF INADEQUACY
13     Q=.6375*G1*87./(87.+FY)
      DNEW=SQRTF(12.*XM/(.9*B*FC*Q*(1.-.59*Q)))*1.10
      TNEW=DNEW+DP
      GAPA=.85*FC*B*DNEW
      SAY=2.*XM*12./(.9*GAPA*DNEW)
      ASNEW=GAPA*(1.-SQRTF(1.-SAY))/FY
101   CONTINUE
642   RETURN
      END
```

```
*06044
      SUBROUTINE SHEAR(VM,XM1,P,FC,FY,B,BP,FT,T,DP,AS,AV,I,MP,IS)
C THIS SUBROUTINE DETERMINES THE SHEAR REINFORCING STEEL AREA AT 5 PTS.
      SAYN=1.
      IF(IS-2) 126,127,127
127   SAYN=-1.
126   CONTINUE
      D=T-DP
      VU=VM/(BP*D)
      VAL=8.5*SQRTF(FC*1000.)/1000.
      IF(VU-VAL) 2,2,3
3     PRINT 4,I,MP,VU
4     FORMAT(2I4,47H INADEQUATE SECTION FOR SHEAR(ACI/1705B)  (VU)=,F7
     1,1X,3HKSI)
      GO TO 87
2     AG=B*FT+(T-FT)*BP
      VALM=3.5*.85*SQRTF(FC*1000.*(1.+2.*P*1000.*SAYN/AG))/1000.
      X=VM*D
      IF(12.*XM1-X) 10,11,11
11    X=XM1*12.
10    DIN=BP*(X-P*SAYN*(.5*T-.125*D))
      VC=.85*(1.9*SQRTF(FC*1000.)+2500.*AS*VM/DIN)/1000.
      AV=0.
      IF(VC-VALM) 26,26,27
27    VC=VALM

26    IF(VU-VC) 36,36,37
37    IF(FY-60.) 46,46,47
 47   FY=60.
46    AV=(VM-VC*BP*D)/(.85*FY*D)
      IF(AV-.0015*BP) 56,36,36
56    AV=.0015*BP
36    CONTINUE
87    RETURN
      END
```

```
*06044
      SUBROUTINE TBEAM(XM,P,FC,FY,B,BP,T,DP,FT,G1,AS,ASF,I,MP)
      PB=.6375*G1*FC*87./(FY*(87.+FY))
      PH=0.
      D=T-DP
200   CONTINUE
      ASF=0.
      GAPA=.85*FC*B*D
      SAY=1.-2.*12.*XM/(.9*GAPA*D)
      IF(SAY) 5,6,6
 5    PRINT 12,I,MP
12    FORMAT(2I4,4X,19H INSUFFICIENT DEPTH)
      AS=0.
13    Q=.6375*G1*87./(87.+FY)
      D=SQRTF(XM*12./(.9*B*FC*Q*(1.-.59*Q)))*1.10
      T=D+DP
      GO TO 200
6     AS=GAPA*(1.-SQRTF(SAY))/FY
      Q=AS*FY/(B*D*FC)
      VAL=1.18*Q*D/G1
      IF(FT-VAL) 26,121,121
121   IF(AS-B*D*PB) 101,101,5
26    ASF=.85*(B-BP)*FT*FC/FY
      SAY=1.-2.*((XM*12./.9)-ASF*FY*(D-.5*FT))/(GAPA*D)
      IF(SAY) 13,61,61
61    X=GAPA*(1.-SQRTF(SAY))/FY
      IF(X) 74,75,75
74     X=0.
75     AS=X+ASF
      IF((AS-ASF)-BP*D*PB) 101,101,14
14    SH=.85*FC*B
      DIN=PB*BP*FY*(PB*BP*FY-2.*SH)
      ALF=SH*FY*ASF/DIN
      BET=SH*(2.*XM*12./.9+ASF*FY*FT)/DIN
      D=ALF*(1.-SQRTF(1.-BET/(ALF*ALF)))*1.1
      T=D+DP
      GO TO 200
101    AS=AS+ASF
      RETURN
      END
```

```
C     ULTIMATE STRENGTH DESIGN OF COLUMNS      I.KARACA - S.TEZCAN
C     C O L U M N S   M A I N L I N E
      DIMENSION CL(1),GL(1),HP(1)
999   READ 102
102   FORMAT(80H
     1                              )
      PRINT 102
      READ 902,ME,NLOAD,NC
      PRINT 97 ,ME,NLOAD,NC
902   FORMAT(20I4)
97    FORMAT(//23H NUMBER OF MEMBERS(ME)=,I4/29H NUMBER OF LOAD CASES(
     1OAD)=,I3,3X/33H NUMBER OF LOAD COMBINATIONS(NC)=,I3/)
      READ 908,FC,FY
      PRINT 910,FC,FY
      FC=FC/1000.
      FY=FY/1000.
910   FORMAT(24H CONCRETE STRENGTH(FC) =,F8.0/  24H STEEL YIELD STRESS
     1Y)=,F8.0)
908    FORMAT(8F10.0)
      G1=.85
      IF(FC-4.) 299,299,298
298   G1=.85+.05*(FC-4.)
299   CONTINUE
      DO 1000 I=1,ME
      AST=0.
      PS=0.
      LBAR=0
      READ 993,IX,S,B,BP,T,DP,FT,D1,D2,LBAR
      READ 993,IX,XM,YM,P
993   FORMAT(I3,F7.0,6F10.0,F8.0,I2)
      PRINT 912,I,S,B,BP,T,DP,FT,D1,D2
912   FORMAT(///I4,3H S=,F6.1,3H B=,F6.1,4H BP=,F6.1,3H T=,F6.1,4H DP=
     16.1,4H FT=,F6.1,4H D1=,F6.1,4H D2=,F6.1)
      PRINT 796
796   FORMAT(/2X,65HM  LBAR   MX       MY        PU       PB        R
     1ST      SPIRAL)
      XMF=XM
      YMF=YM
      XM=12.*XM
      YM=12.*YM
      CALL COLUMN(XM,YM,P,FC,FY,B,BP,T,DP,FT,G1,I,AST,S,PS,R,AG,PB,PBY
     1AXES,HP,MEM,NU1,NU2,NU3)
      IF(AXES-1.) 29,29,30
C     UNIAXIAL BENDING
29    CONTINUE
      IF(FT) 410,410,411
411   CONTINUE
      CALL CIRCLE(XM,P,PB,T,DP,B,FT,FC,FY,AST,G1,I,E,AG)
      GO TO 129
410   CONTINUE
      CALL ACIUNI(XM,P,PB,T,DP,B,FT,FC,FY,AST,G1,I,AG)
C     CALL SP7KU (XM,P,PB,T,DP,B,FT,FC,FY,AST,G1,I,AG,KU,UK)
C     IF(AST+100.) 128,210,128
C 128   CALL SP7AST(XM,P,PB,T,DP,B,FT,FC,FY,AST,G1,I,AG,KU,UK)
C      IF(AST+100.) 129,1000,129
129   CONTINUE
      GO TO 800
```

```
C  BIAXIAL BENDING
30     CONTINUE
       CALL BIAX(XM,YM,P,PB,PBY,T,DP,B,FT,FC,FY,AST,G1,I,AG,LBAR,BET,TO
800     CONTINUE
       IF(AST-.01*AG) 204,205,205
204     AST=.01*AG
       GO TO 210
205    IF(AST-.08*AG) 210,210,206
206    PRINT 209,I,AST
209    FORMAT(I4,10X,4HAST=,F8.2,4X,46HIS IN EXCESS OF 0.08 AGROSS,  IN
      1EASE SIZE...)
210    CONTINUE
       PRINT 126,I,LBAR,XMF,YMF,P,PB,R,AST,PS
126    FORMAT(2I4,4F8.1,2F8.2,F10.4)
1000   CONTINUE
       STOP
       END
```

```
      SUBROUTINE COLUMN(XM,YM,P,FC,FY,B,BP,T,DP,FT,G1,I,AST,S,PS,R,AG,
     1PB,PBY,AXES,HP,MEM,NU1,NU2,NU3)
      DIMENSION HP(1)
      NU1=0
      NU2=0
      NU3=0
C NU1=1 USE SEC916A1,  NU2=1 USE SEC916A2,  NU3=1 USE SEC916 EQ9.5
C NU1=0,NU2=0, NU3=0 MEANS  USE SEC916 EQ.9-4
      AXES=1.
      IF(YM) 28,26,28
28    AXES=2.
26    D=T-DP
      RGX=.25*T
      RGY=RGX
      IF(FT-1.) 5,6,5
5     RGX=.3*T
      RGY=.3*B
6      RMIN=RGX
      IF(RGY-RGX) 3,4,4
3     RMIN=RGY
4     R=1.
       SL=12.*S/RMIN
C     SH=12.*HP(I)/RMIN
      SH=SL
      PB=.7*.85*FC*B*D*G1*87./(87.+FY)
      PBY=.7*.85*FC*T*(B-DP)*G1*87./(87.+FY)
      IF(NU1) 702,702,701
701   IF(SH-60.) 14,14,8
 8    IF(SH-100.) 9,9,10
9     R=1.32-.006*SL
      GO TO 127
702   IF(NU2) 703,703,704
704   R=1.07-.008*SL
      GO TO 127
703    IF(NU3) 705,705,706
706   R=1.18-.009*SH
      GO TO 127
705   R=1.07-.008*SH
127   IF(R-1.) 120,120,121
121   R=1.
120   IF(P-PB) 12,12,14
12    R=R+(1.-R)*(PB-P)/PB
760   IF(R-1.) 14,14,15
15    R=1.
      GO TO 14
10     PRINT 17,I,SL
17    FORMAT(10H IN COLUMN,I4,3X,11H, H/R RATIO,F8.2,4X,19HIS GREATER
     1AN 100)
       GO TO 15
14    P=P/R
      IF(XM-.1*T*P) 21,22,22
21    XM=.1*T*P
22    IF(YM-.1*B*P) 23,24,24
23     YM=.1*B*P
24    CONTINUE
```

```
129     AG=B*T
      PS=0.
      IF(FT-1.) 201,202,201
202     AG=(3.14159*T*T)*.25
      AC=3.14159*(T-2.*DP)*(T-2.*DP)*.25
      PS=.45*(AG/AC-1.)*FC/FY
201   CONTINUE
200   RETURN
      END
```

```
      SUBROUTINE CIRCLE (XM,P,PB,T,DP,B,FT,FC,FY,AST,G1,I,E,AG)
      E=XM/P
      AL=P/(.85*.75*FC*T*T)
      DS=T-2.*DP
      EMS=FY*DS/(.85*FC)
      IF(FT-1.) 700,700,701
700   IF(P-PB) 607,607,608
607   AST=(2.5*AG*T/EMS)*AL*(AL+1.7*E/T-.76)
      GO TO 777
701   IF(P-PB) 606,606,608
606   AST=(T*AG/(.67*EMS))*AL*(AL+2.*E/T -1.)
      GO TO 777
608   FAC=((3.*E/DS)+1.)/FY
      IF(FT-1.) 900,900,901
900   DIN=9.6*T*E/((.8*T+.67*DS)**2)+(1.18)
      GO TO 899
901   DIN=12.*T*E/((T+.67*DS)**2) +(1.18)
899   AST=FAC*((P/.7) -AG*FC/DIN)
777   RETURN
      END
```

```
      SUBROUTINE ACIUNI(XM,P,PB,T,DP,B,FT,FC,FY,AST,G1,I,AG)
C SOLUTION BY ACI EQ.10 EMPRICAL FORMULA
      D=T-DP
      AL=.85*FC
      E=XM/P
      EP=E+.5*T-DP
410   IF(P-PB) 101,101,102
C TENSION GOVERNS
101   SA=P/(.7*AL*B*D)
      ALF=FY*(1.-DP/D)/AL+DP/D-SA
      SI=SA*(SA+2.*EP/D - 2.)
      CALL SECON(ALF,SI,R1,R2,I,1,MESAJ)
      IF(MESAJ-1) 197,299,197
197   AST=2.*(R1*B*D)
      GO TO 300
C  COMPRESSION GOVERNS
102    CONTINUE
      AST=(P/.7- (B*T*FC)/((3.*T*E/(D*D)) +1.18))* 2.*(E/(D-DP) +.5)/F
      GO TO 300
299   AST=-100.
300   RETURN
      END
```

```
      SUBROUTINE BIAX(XM,YM,P,PB,PBY,T,DP,B,FT,FC,FY,AST,G1,I,AG,LBAR,
     1BET,TOL)
      TOL=.03
      RX=T/B
      RY=B/T
      BET=.65
      Y1=YM+XM*RY*(1.-BET)/BET
      X1=XM+YM*RX*(1.-BET)/BET
999   CONTINUE
      XMH=X1
      PBH=PB
      TH=T
      BH=B
      IM=1
C  IM=1 MEANS  MX PREDOMINATING,    IM=2  MEANS   MY PREDOMINATING
      IF(YM-XM*Y1/X1) 3,3,4
4     XMH=Y1
      BH=T
      TH=B
       PBH=PBY
      IM=2
3     CONTINUE
      IF(FT) 410,410,411
411   CALL CIRCLE(XMH,P,PBH,TH,DP,BH,FT,FC,FY,AST,G1,I,E,AG)
      GO TO 98
```

```
410   CALL ACIUNI(XMH,P,PBH,TH,DP,BH,FT,FC,FY,ASTH,G1,I,AG)
129   CONTINUE
      IF (LBAR) 716,716,717
716   PRINT 715,I,IM,LBAR
      GO TO 327
717   CONTINUE
715   FORMAT(3I4,38H IMPROPER BAR ARRANGEMENT NUMBER(LBAR))
      GO TO(101,102,103,104),LBAR
101    AST=ASTH
      GO TO 98
102    AST=(8./7. )*ASTH
      GO TO 98
103    AST=(12./10.)*ASTH
      GO TO 98
104    IF(IM-1) 26,26,27
27    AST=ASTH
       GO TO 98
26    AST=(10./7.)*ASTH
98    PT=AST/(B*T)
      Q=PT*FY/FC
      PO=.7*(.85*FC*(AG-AST)+ FY*AST)
      PP=P/PO
      CALL BETTA(PP,FY,LBAR,Q,BETN)
      SA=LOGF(.5)/LOGF(BETN)
      IF(((ABSF(BETN-BET))/BETN) -TOL) 320,320,87
320   CONTINUE
      BET=BETN
      IF(IM-1) 261,261,271
271   XMK=X1*((1.- (YM/Y1)**SA)**(1./SA))
      IF(1.03*XMK-XM) 325,327,327
325    CONTINUE
      GO TO 87
261   YMK=Y1*((1.- (XM/X1)**SA)**(1./SA))
      IF(1.03*YMK-YM) 326,327,327
326    CONTINUE
87    CONTINUE
      BET=BETN
      RY=XM/X1
      RX=YM/Y1
      X1=XM/( (1.-RX**SA)**(1./SA) )
      Y1=YM/( (1.-RY**SA)**(1./SA) )
      GO TO 999
327   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SECON(AL,BE,R1,R2,I,J,MESAJ)
      MESAJ=0
      SAY=1.- BE/(AL*AL)
      IF(SAY) 6,5,5
6     PRINT 7,I,AL,BE,J
7     FORMAT(I4,5X,17H IMAGINARY ROOTS ,4X,4HALF=,F14.4,4X,4HBET=,F14.
     1I5)
      MESAJ=1
      GO TO 770
5     R1=AL*(1.-SQRTF(SAY))
      R2=AL*(1.+SQRTF(SAY))
770   RETURN
      END
```

```
      SUBROUTINE BETTA(PP,FY,LBAR,Q,B)
C   4 BAR ARRANGEMENT    LBAR=1        FY=40.
      B=0.
151   IF(Q-.2) 171,171,170
171   IF(PP-.22) 172,172,173
173   IF(PP-.5)174,174,175
175   IF(PP-.9) 176,176,500
172   BETA=-.93*PP+.84
      GO TO 77
174   BETA=-.0535*PP+.635
      GO TO 77
176   BETA=.425*PP+.41
      GO TO 77
170   IF(Q-.4) 181,181,180
181   IF(PP-.18) 182,182,183
183   IF(PP-.36)184,184,185
185   IF(PP-.9) 186,186,500
182   BETA=-1.05*PP+.77
      GO TO 77
184   BETA=-.0835*PP+.599
      GO TO 77
186   BETA=.464*PP+.398
      GO TO 77
180   IF (Q-.7)191,191,190
191   IF(PP-.19) 192,192,193
193   IF(PP-.32) 194,194,195
195   IF (PP-.73) 196,196,197
197   IF(PP-.90) 198,198,500
192   BETA=-.66*PP+.675
      GO TO 77
194   BETA=.55
      GO TO 77
196   BETA=.45*PP+.41
      GO TO 77
198   BETA=1.145*PP-.102
      GO TO 77
190   IF(Q-1.1) 211,211,200
211   IF(PP-.17)212,212,213
213   IF(PP-.24) 214,214,215
215   IF(PP-.35) 216,216,217
217   IF(PP-.70) 218,218,219
219   IF(PP-.90) 220,220,500
212   BETA=-.47*PP+.61
      GO TO 77
214   BETA=.53
      GO TO 77
216   BETA=.464*PP+.42
      GO TO 77
218   BETA=.415*PP+.435
```

```
      GO TO 77
220   BETA=1.225*PP-.135
      GO TO 77
200   IF(Q-1.8) 231,231,1500
231   IF(PP-.17) 232,232,233
233   IF(PP-.66) 234,234,235
235   IF(PP-.90) 236,236,500
232   BETA=-.41*PP+.58
      GO TO 77
234   BETA=.45*PP+.434
      GO TO 77
236   BETA=1.08*PP+.015
      GO TO 77
500   PRINT 1497,PP
1497  FORMAT(4X,54H PU/PO GREATER THAN .9  NOT ALLOWED IN PARMER,S CHAI
     1S,2F8.2)
      GO TO 77
1500   PRINT 1496,Q
1496  FORMAT(4X,48H Q IN PARMER,S CHARTS CAN NOT BE LARGER THAN 1.6,6X
     1F8.2)
77    CONTINUE
      B=BETA
      RETURN
      END

      SUBROUTINE BETTA(PP,FY,LBAR,Q,B)
C   8 BAR ARRANGEMENT   LBAR=2      FY=40.
      B=0.
 141  IF(Q-.2) 291,291,290
 291  IF(PP-.26) 292,292,293
 293  IF(PP-.46) 294,294,295
 295  IF(PP-.70) 296,296,297
 297  IF(PP-.90) 298,298,500
 292  BETA=-.462*PP+.77
      GO TO 77
 294  BETA=-.05*PP+.663
      GO TO 77
 296  BETA=.167*PP+.562
      GO TO 77
 298  BETA=.45*PP+.365
      GO TO 77
 290  IF(Q-.4) 301,301,300
 301  IF(PP-.28) 302,302,303
 303  IF(PP-.70) 304,304,305
 305  IF(PP-.90) 306,306,500
 302  BETA=-.243*PP+.68
      GO TO 77
```

```
 304   BETA=.255*PP+.572
       GO TO 77
 306   BETA=.45*PP+.365
       GO TO 77
 300   IF(Q-.7) 311,311,310
 311   IF(PP-.30) 312,312,313
 313   IF(PP-.70) 314,314,315
 315   IF(PP-.90) 316,316,500
 312   BETA=-.15*PP+.645
       GO TO 77
 314   BETA=.20*PP+.54
```

```
       GO TO 77
 316   BETA=.45*PP+.365
       GO TO 77
 310   IF(Q-1.1) 321,321,320
 321   IF(PP-.25) 322,322,323
 323   IF(PP-.70) 324,324,325
 325   IF(PP-.90) 326,326,500
 322   BETA=-.10*PP+.615
       GO TO 77
 324   BETA=.20*PP+.54
       GO TO 77
 326   BETA=.45*PP+.365
       GO TO 77
 320   IF(Q-1.8) 331,331,1500
 331   IF(PP-.23) 332,332,333
 333   IF(PP-.70) 334,334,335
 335   IF(PP-.90) 336,336,500
 332   BETA=-.065*PP+.605
       GO TO 77
 334   BETA=.20*PP+.54
       GO TO 77
 336   BETA=.45*PP+.365
       GO TO 77
500    PRINT 1497,PP
1497   FORMAT(4X,54H PU/PO GREATER THAN .9  NOT ALLOWED IN PARMER,S CHA
      1S,2F8.2)
       GO TO 77
1500    PRINT 1496,Q
1496   FORMAT(4X,48H Q IN PARMER,S CHARTS CAN NOT BE LARGER THAN 1.6,6X
      1F8.2)
77     CONTINUE
       B=BETA
       RETURN
       END
```

```
      SUBROUTINE BETTA(PP,FY,LBAR,Q,B)
C 12  BAR ARRANGEMENT   LBAR=3       FY=40.
      B=0.
 131  IF(Q-.2) 391,391,390
 391  IF(PP-.38) 392,392,393
 393  IF(PP-.90) 394,394,500
 392  BETA=-.370*PP+.76
      GO TO 77
 394  BETA=.27*PP+.518
      GO TO 77
 390  IF(Q-.4) 411,411,410
 411  IF(PP-.37) 412,412,413
 413  IF(PP-.90) 414,414,500
 412  BETA=-.176*PP+.685
      GO TO 77
414   BETA=.27*PP+.518
      GO TO 77
410    IF(Q-.7) 421,421,420
421   IF(PP-.3) 422,422,423
423   IF(PP-.9) 424,424,500
422   BETA=-.183*PP+.655
      GO TO 77
424   B=.27*PP+.518
       GO TO 77
 420  IF(Q-1.1) 431,431,430
 431  IF(PP-.29) 432,432,433
 433  IF(PP-.90) 434,434,500
 432  BETA=-.121*PP+.63
      GO TO 77
 434  BETA=.27*PP+.518
      GO TO 77
430   IF(Q-1.8) 441,441,1500
441   IF(PP-.29) 442,442,443
443   IF(PP-.90) 444,444,500
442   B=-.052*PP+.61
       GO TO 77
444   B=.27*PP+.518
      GO TO 77
500   PRINT 1497,PP
1497  FORMAT(4X,54H PU/PO GREATER THAN .9  NOT ALLOWED IN PARMER,S CHA
     1S,2F8.2)
      GO TO 77
1500   PRINT 1496,Q
1496  FORMAT(4X,48H Q IN PARMER,S CHARTS CAN NOT BE LARGER THAN 1.6,6X
     1F8.2)
77    CONTINUE
      B=BETA
      RETURN
       END
```

```
      SUBROUTINE BETTA(PP,FY,LBAR,Q,B)
C 10  BAR ARRANGEMENT   LBAR=4       FY=40.
      B=0.
 121  IF(Q-.2) 511,511,510
 511  IF(PP-.21) 512,512,513
 513  IF(PP-.46) 514,514,515
 515  IF(PP-.64) 516,516,517
 517  IF(PP-.90) 518,518,500
 512  BETA=-.645*PP+.790
      GO TO 77
 514  BETA=-.10*PP+.676
      GO TO 77
 516  BETA=.278*PP+.502
      GO TO 77
 518  BETA=.404*PP+.422
      GO TO 77
 510  IF(Q-.4) 521,521,520
 521  IF(PP-.31) 522,522,523
 523  IF(PP-.64) 524,524,525
 525  IF(PP-.90) 526,526,500
 522  BETA=-.322*PP+.69
      GO TO 77
 524  BETA=.278*PP+.502
      GO TO 77
 526  BETA=.404*PP+.422
      GO TO 77
 520  IF(Q-.7) 531,531,530
 531  IF(PP-.28) 532,532,533
 533  IF(PP-.64) 534,534,535
 535  IF(PP-.90) 536,536,500
 532  BETA=-.268*PP+.66
      GO TO 77
 534  BETA=.278*PP+.502
      GO TO 77
 536  BETA=.404*PP+.422
      GO TO 77
 530  IF(Q-1.1) 541,541,540
541   IF(PP-.20) 542,542,543
 543  IF(PP-.64) 544,544,545
 545  IF(PP-.90) 546,546,500
 542  BETA=-.325*PP+.63
      GO TO 77
 544  BETA=.278*PP+.502
      GO TO 77
 546  BETA=.404*PP+.422
      GO TO 77
 540  IF(Q-1.8) 551,551,1500
 551  IF(PP-.18) 552,552,553
 553  IF(PP-.64) 554,554,555
 555  IF(PP-.90) 556,556,500
 552  BETA=-.278*PP+.61
      GO TO 77
```

```
 554  BETA=.278*PP+.502
      GO TO 77
 556  BETA=.404*PP+.422
      GO TO 77
500   PRINT 1497,PP
1497  FORMAT(4X,54H PU/PO GREATER THAN .9  NOT ALLOWED IN PARMER,S CHA
     1S,2F8.2)
      GO TO 77
1500   PRINT 1496,Q
1496  FORMAT(4X,48H Q IN PARMER,S CHARTS CAN NOT BE LARGER THAN 1.6,6X
     1F8.2)
77     CONTINUE
      B=BETA
      RETURN
       END
```

```
      SUBROUTINE BETTA(PP,FY,LBAR,Q,B)
C  4  BAR ARRANGEMENT   LBAR=1      FY=60.
      B=0.
152   IF(Q-.2) 241,241,240
241   IF(PP-.20) 242,242,243
243   IF (PP-.41) 244,244,245
245   IF(PP-.90) 246,246,500
242   BETA=-.9*PP+.825
      GO TO 77
244   BETA=-.167*PP+.678
      GO TO 77
246   BETA=.244*PP+.518
      GO TO 77
240   IF(Q-.4) 251,251,250
251   IF(PP-.13) 252,252,253
253   IF(PP-.28) 254,254,255
255   IF(PP-.90) 256,256,500
252   BETA=-1.08*PP+.75
      GO TO 77
254   BETA=-.333*PP+.653
      GO TO 77
```

```
256    BETA=.202*PP+.504
       GO TO 77
250    IF(Q-.7) 261,261,260
261    IF(PP-.10) 262,262,263
263    IF(PP-.24) 264,264,265
265    IF(PP-.90) 266,266,500
262    BETA=-.85*PP+.675
       GO TO 77
264    BETA=-.416*PP+.631
       GO TO 77
266     BETA=.182*PP+.496
       GO TO 77
260    IF(Q-1.1) 271,271,280
271    IF(PP-.2) 272,272,273
273    IF(PP-.9) 274,274,500
272    BETA=-.4*PP+.60
       GO TO 77
274    BETA=.179*PP+.484
       GO TO 77
 280   IF(Q-1.8) 281,281,1500
 281   IF(PP-.16) 282,282,283
 283   IF(PP-.90) 284,284,500
 282   BETA=-0.375*PP+0.575
       GO TO 77
 284   BETA=.179*PP+.484
       GO TO 77
500    PRINT 1497,PP
1497   FORMAT(4X,54H PU/PO GREATER THAN .9  NOT ALLOWED IN PARMER,S CHA
      1S,2F8.2)
       GO TO 77
1500    PRINT 1496,Q
1496   FORMAT(4X,48H Q IN PARMER,S CHARTS CAN NOT BE LARGER THAN 1.6,6X
      1F8.2)
77     CONTINUE
       B=BETA

       RETURN
       END
```

```
      SUBROUTINE BETTA(PP,FY,LBAR,Q,B)
C   8 BAR ARRANGEMENT   LBAR=2        FY=60.
      B=0.
202   IF(Q-.20) 341,341,340
 341  IF(PP-.36) 342,342,343
 343  IF(PP-.90) 344,344,500
 342  BETA=-.306*PP+.735
      GO TO 77
 344  BETA=.176*PP+.561
      GO TO 77
 340  IF(Q-.40) 351,351,350
351   IF(PP-.29) 352,352,353
 353  IF(PP-.90) 354,354,500
 352  BETA=-.310*PP+.665
      GO TO 77
 354  BETA=.205*PP+.516
 350  IF(Q-.7) 361,361,360
 361  IF(PP-.25) 362,362,363
 363  IF(PP-.90) 364,364,500
 362  BETA=-.26*PP+.625
      GO TO 77
 364  BETA=.184*PP+.514
      GO TO 77
 360  IF(Q-1.1) 371,371,370
 371  IF(PP-.21) 372,372,373
 373  IF(PP-.45) 374,374,375
 375  IF(PP-.90) 376,376,500
 372  BETA=-.31*PP+.595
      GO TO 77
 374  BETA=.25*PP+.477
      GO TO 77
 376  BETA=.133*PP+.53
      GO TO 77
 370  IF(Q-1.8) 381,381,1500
 381  IF(PP-.20) 382,382,383
 383  IF(PP-.45) 384,384,385
 385  IF(PP-.90) 386,386,500
 382  BETA=-.225*PP+.57
      GO TO 77
 384  BETA=.25*PP+.477
      GO TO 77
 386  BETA=.133*PP+.53
      GO TO 77
500   PRINT 1497,PP
1497  FORMAT(4X,54H PU/PO GREATER THAN .9  NOT ALLOWED IN PARMER,S CHA
     1S,2F8.2)
      GO TO 77
1500   PRINT 1496,Q
1496  FORMAT(4X,48H Q IN PARMER,S CHARTS CAN NOT BE LARGER THAN 1.6,6X
     1F8.2)
77    CONTINUE
      B=BETA
      RETURN
      END
```

```
      SUBROUTINE BETTA(PP,FY,LBAR,Q,B)
C 12  BAR ARRANGEMENT   LBAR=3        FY=60.
      B=0.
 132  IF(Q-.2) 451,451,450
 451  IF(PP-.35) 452,452,453
 453  IF(PP-.90) 454,454,500
 452  BETA=-.33*PP+.74
      GO TO 77
 454  BETA=.164*PP+.568
      GO TO 77
 450  IF(Q-.4) 461,461,460
 461  IF(PP-.27) 462,462,463
 463  IF(PP-.9) 464,464,500
 462  BETA=-.334*PP+.67
      GO TO 77
 464  BETA=.183*PP+.531
      GO TO 77
 460  IF(Q-.7) 471,471,470
 471  IF(PP-.23) 472,472,473
 473  IF(PP-.9) 474,474,500
 472  BETA=-.26*PP+.63
      GO TO 77
 474  BETA=.149*PP+.536
      GO TO 77
 470  IF(Q-1.1) 481,482,480
 481  IF(PP-.19) 482,482,483
 483  IF(PP-.59) 484,484,485
 485  IF(PP-.90) 486,486,500
 482  BETA=-.342*PP+.595
      GO TO 77
 484  BETA=.212*PP+.490
      GO TO 77
 486  BETA=.0485*PP+.587
      GO TO 77
 480  IF(Q-1.8) 491,491,1500
 491  IF(PP-.17) 492,492,493
 493  IF(PP-.59) 494,494,495
 495  IF(PP-.90) 496,496,500
 492  BETA=-.264*PP+.57
      GO TO 77
 494  BETA=.212*PP+.490
      GO TO 77
 496  BETA=.0485*PP+.587
      GO TO 77
500   PRINT 1497,PP
1497  FORMAT(4X,54H PU/PO GREATER THAN .9  NOT ALLOWED IN PARMER,S CHA
     1S,2F8.2)
      GO TO 77
1500   PRINT 1496,Q
1496  FORMAT(4X,48H Q IN PARMER,S CHARTS CAN NOT BE LARGER THAN 1.6,6X
     1F8.2)
77    CONTINUE
      B=BETA
      RETURN
       END
```

```
      SUBROUTINE BETTA(PP,FY,LBAR,Q,B)
C 10  BAR ARRANGEMENT   LBAR=4        FY=60.
       B=0.
 122  IF(Q-.2) 561,561,560
 561  IF(PP-.30) 562,562,563
 563  IF(PP-.41) 564,564,565
 565  IF(PP-.90) 566,566,500
 562  BETA=-.434*PP+.765
      GO TO 77
 564  BETA=-.136*PP+.676
      GO TO 77
 566  BETA=.224*PP+.528
      GO TO 77
 560  IF(Q-.4) 571,571,570
 571  IF(PP-.28) 572,572,573
 573  IF(PP-.90) 574,574,500
 572  BETA=-.375*PP+.68
      GO TO 77
 574  BETA=.185*PP+.523
      GO TO 77
 570  IF(Q-.7) 581,581,580
 581  IF(PP-.26) 582,582,583
 583  IF(PP-.90) 584,584,500
 582  BETA=-.308*PP+.635
      GO TO 77
 584  BETA=.188*PP+.506
      GO TO 77
 580  IF(Q-1.1) 591,591,590
 591  IF(PP-.20) 592,592,593
 593  IF(PP-.53) 594,594,595
 595  IF(PP-.90) 596,596,500
 592  BETA=-.375*PP+.605
      GO TO 77
 594  BETA=.197*PP+.491
      GO TO 77
 596  BETA=.135*PP+.523
      GO TO 77
 590  IF(Q-1.8) 601,601,1500
```

```
 601   IF(PP-.17) 602,602,603
 603   IF(PP-.53) 604,604,605
 605   IF(PP-.90) 606,606,500
 602   BETA=-.353*PP+.58
       GO TO 77
 604   BETA=.197*PP+.491
       GO TO 77
 606   BETA=.135*PP+.523
       GO TO 77
500    PRINT 1497,PP
1497   FORMAT(4X,54H PU/PO GREATER THAN .9  NOT ALLOWED IN PARMER,S CHA
      1S,2F8.2)
       GO TO 77
1500    PRINT 1496,Q
1496   FORMAT(4X,48H Q IN PARMER,S CHARTS CAN NOT BE LARGER THAN 1.6,6X
      1F8.2)
77     CONTINUE
       B=BETA
       RETURN
        END
```

```
      SUBROUTINE ACIUNI(XM,P,PB,T,DP,B,FT,FC,FY,AST,G1,I,AG)
C    SOLUTION BY ACI EQ.19/7,8,9       EXACT EQUATIONS IN COMPRESSION
      D=T-DP
      AL=.85*FC
      E=XM/P
      EP=E+.5*T-DP
410   IF(P-PB) 101,101,102
C TENSION GOVERNS
101   SA=P/(.7*AL*B*D)
      ALF=FY*(1.-DP/D)/AL+DP/D-SA
      SI=SA*(SA+2.*EP/D - 2.)
      CALL SECON(ALF,SI,R1,R2,I,1,MESAJ)
      IF(MESAJ-1) 197,300,197
197   AST=2.*(R1*B*D)
      GO TO 300
C  COMPRESSION GOVERNS
102    CONTINUE
      AB=G1*D*87./(87.+FY)
      ALF=1.4*FY*(FY-AL)*(D-DP)
      BE=XM*(FY-AL) +.5*P*FY*(D-DP)-.35*AL*B*T*FY*(D-DP)
     1 - .35*AL*B*AB*(FY-AL)*(T-AB)
      GA=.35*AL*AL*B*B*T*AB*(T-AB)- .5*AL*B*AB*(T-AB)*P+ XM*AL*B*(AB-T
      ALFA=BE/ALF
      BE=GA/ALF
      CALL SECON(ALFA,BE,R1,R2,I,2,MESAJ)
      IF(MESAJ-1) 193,299,193
193   COZ=R1
      IF(R1) 30,31,31
30    IF(R2) 99,40,40
31    IF(R2) 50,51,51
51    IF(R1-R2) 50,50,40
99    PRINT 98,I,MP
98    FORMAT(2I4,5X,36H NEGATIVE AST FROM EQ. ACI(19/7,8,9))
      AST=2.*COZ
      GO TO 300
40    COZ=R2
50    AST=2.*COZ
      GO TO 300
299   AST=-100.
300   RETURN
      END
```

```
      SUBROUTINE SP7KU (XM,P,PB,T,DP,B,FT,FC,FY,AST,G1,I,AG,KU,UK)
      DIMENSION ROOT(3)
      EM=FY/(.85*FC)
      E=.003*EM*29000./FY
      EYU=(FY/29000.)/(.003)
      PU=P/(.7*FC*B*T)
       XU=XM/(.7*FC*B*T*T)
      G=(T-2.*DP)/T
      DO 104   KU=1,5
      SAYN=1.
      GO TO(201,202,203,204,205),KU
201   GIS=1.-G
      Y=E-EM-1.
      Z=E+EM-1.
      SAYN=-1.
      ALT=(.5 -.5*G)* .99
      UST=(.5*(1.-G)/(1.-EYU))*1.01
      GO TO 801
202   AK=.5*(G*(2.*EM-1.)+1.)/G1
      BK=(G*(2.*EM-1.)*PU +2.*XU)/(.85*G1*G1)
      CALL SECON(AK,BK,UK,R2,I,3,MESAJ)
      PRINT 555,I,KU,AK,BK,CK,DK,UK,R2
555   FORMAT(2I4,6F12.4)
      IF(MESAJ) 104,776,104
776   ALT=(.5*(1.-G)/(1.-EYU)) *.99
      UST=(.5*(1.+G)/(1.+EYU)) *1.01
      GO TO 770
203   Y=EM-1.+E
      Z=EM-1.-E
      GIS=1.+G
      ALT=(.5*(1.+G)/(1.+EYU))* .99
      UST=(.5*(1.+G))* 1.01
      GO TO 801
204   Y=EM-2.+E
      Z=EM-E
      GIS=1.+G
      ALT=(.5*(1.+G))* .99
      UST=(1./G1 )* 1.01
801    AK=-.85*G1*G1*Y
      BK=.85*G1*(Y-G*Z+.5*G1*GIS*E)
      CK=G*Z*PU-2.*Y*XU-.425*G1*GIS*GIS*E
      DK=GIS*E*(SAYN* .5*G*PU+XU)
      PRINT 555,I,KU,AK,BK,CK,DK,UK
      GO TO 777
205    DK=(1.+G)*E*(.5*G*PU+XU-.425*G)
      CK=G*Z*PU -2.*Y*XU -.85*G*Z
      UK=-DK/CK
      ALT=(1./G1 )* .99
      UST=9999999.
      PRINT 555,I,KU,AK,BK,CK,DK,UK
      GO TO 770
C     ROOTS OF CUBIC EQUATION
777   R1=ALT+.02
      TOL=.005
      NCYCLE=20
      CALL CUBIC(TOL,NCYCLE,R1,R2,R3,AK,BK,CK,DK,IM,MESAJ,KU)
```

```
      IF(MESAJ) 104,769,104
769   CONTINUE
      LIM=3
      ROOT(1)=R1
      IF(IM-1) 718,717,718
718   ROOT(2)=R2
      ROOT(3)=R3
      GO TO 766
717   LIM=1
766   DO 768  JJ=1,LIM
       PRINT 555,I,KU,ROOT(JJ)
      GO TO(761,762,762),JJ
762   IF(IM-1) 761,104,761
761   IF(ROOT(JJ)-ALT) 768,37,38
38    IF(ROOT(JJ)-UST) 37,37,768
768   CONTINUE
      GO TO 104
770   IF(UK-ALT) 103,700,98
98    IF(UK-UST) 700,700,103
103    IF(R2-ALT) 104,699,698
698    IF(R2-UST) 699,699,104
699   UK=R2
      GO TO 700
104    CONTINUE
      AST=-100.
      PRINT 306,I
306   FORMAT(I4,4X,43H ACI-SP7 FORMULAE DID NOT SUPPLY A SOLUTION)
      GO TO 200
37    UK=ROOT(JJ)
      PRINT 555,I,KU,AK,BK,CK,DK,UK
700   CONTINUE
200   CONTINUE
      PRINT 555,I,KU,ALT,UST,UK
      RETURN
      END
```

```
      SUBROUTINE SP7AST(XM,P,PB,T,DP,B,FT,FC,FY,AST,G1,I,AG,KU,UK)
      EM=FY/(.85*FC)
      E=.003*EM*29000./FY
      PU=P/(.7*FC*B*T)
      SA=.85*G1*UK
      G=(T-2.*DP)/T
      GO TO (611,612,613,614,615),KU
611   AST=(PU-SA)/(.85*(E-E*(1.-G)/(2.*UK)-1.-EM))
      GO TO 616
612   AST=G1*UK-PU/.85
      GO TO 616
613   AST=(PU-SA)/(.85*(E-E*(1.+G)/(2.*UK)-1.+EM))
      GO TO 616
614   AST=(PU-SA)/(.85*(E-E*(1.+G)/(2.*UK)-2.+EM))
      GO TO 616
615   AST=(PU-.85)/(.85*(E-E*(1.+G)/(2.*UK)-2.+EM))
616   AST=2.*AST*B*T
      PRINT 555,I,KU,UK,AST
555   FORMAT(2I4,4F14.3/)
      RETURN
      END
```

```
      SUBROUTINE CUBIC(TOL,NCYCLE,X,R2,R3,A,B,C,D,IM,MESAJ,KU)
      IM=0
      MESAJ=0
      DO 10 I=1,NCYCLE
      FX=A*X*X*X+ B*X*X + C*X +D
      FP=3.*A*X*X +2.*B*X +C
      XN=X-FX/FP
      IF(I-1) 27,125,27
27    IF(ABSF(XN-X)-DIF) 125,125,25
125   DIF=ABSF(XN-X)
      IF(ABSF((XN-X)/XN)-TOL) 20,20,10
10    X=XN
25    PRINT 17,KU
17    FORMAT(32H FUNCTION KU NOT CONVERGED, CASE,I4/)
      MESAJ=1
      GO TO 200
20    CO=.5*(B/A)+.5*X
      UN=CO*CO+D/(A*X)
      IF(UN) 26,28,28
28    R2=-CO-SQRTF(UN)
      R3=-CO+SQRTF(UN)
      GO TO 200
26    PRINT 18,X
18     FORMAT(8X,32H ROOTS X2, X3 ARE IMAGINARY, X1=,F16.5)
      IM=1
200   RETURN
      END
```