COMPARATIVE STUDY OF METHODS OF ANALYSIS OF SHEET PILE WALLS

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Scope of Study: A comparative study of different methods of analysis of sheet pile retaining walls is presented in this work. The methods selected are: Free Earth, Fixed Earth Methods, Graphical Method, Equivalent Beam Method, Newmark's Numerical Method, Rowe's Method, Tschebotarioff's Method and Danish Rule. The derivations of equations for all methods are presented in a form suitable for the use in practical design. The emphasis is placed on practical considerations influencing the choice of method or type of construction and on practical design.

Findings and Conclusion: The numerical example illustrates the various methods of analysis of sheet pile retaining walls. Relative merits and suitability of these methods are given in the last chapter.

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The writer, in completing the final phase of his work for his Master's Degree, wishes to express his gratitude to the following persons:

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M. S. B.

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CHAPTER I

HISTORICAL DEVELOPMENT OF THE ANALYSIS OF SHEET PILE RETAINING WALLS

Up to 70 years ago, sheet walls were made up almost exclusively f timber, and were designed by means of certain empirical rules, hich had proved satisfactory in practice.

In the beginning of present century, increasing water depths ere demanded, and, at the same time, new materials were made vailable, reinforced concrete and steel sheet piles. The question rose of how these new types of anchored sheet walls were to be esigned.

In most countries where such design methods were attempted, the heet walls considered as vertical beams, subjected to active and ossive earth pressures calculated by means of Coulomb's or Rankine's ethods. However, when ordinary allowable stresses for the materials n question were used, very heavy sheet walls were found to be ecessary, so that the new materials could not compete economically ith empirically designed timber sheet walls.

The first to find a practical solution to this problem was the anish engineer, Rud. Christiani, in 1906. He started by making heck computations of a number of existing timber sheet walls, according to the above-mentioned method. He thereby found nominal stresses n the timber, which were 3-4 times as great as the allowable stresses ormally used. He concluded then that some redistribution of the arth pressures must take place making the actual moments considerably maller than the calculated ones, and he assumed, finally, that the ame would apply to sheet walls of steel or reinforced concrete. herefore, he decided to design such sheet walls for the calculated moments, but with "allowable" stresses 3-4 times as great as those ordinarily used.

After Christiani's introduction of the empirical calculation principle described above, other Danish engineers tried to solve the same problem in a different way, by changing the earth pressure distribution assumed in Coulomb's theory. This resulted in the empirical methods known as the Danish Rules. The Danish Rules for the design of the anchored sheet walls in sand were first published by the Danish Society of Civil Engineers in 1923.

In 1930, H. Blum of Germany suggested Equivalent Beam Method. By this method, Blum succeeded in establishing a theoretical relationship between angle of internal friction and the distance x from the dredge time to the point of contraflexure.

In the United States, methods of the design of the bulkheads on the basis of classical earth pressure theories were developed and introduced into practice of structural design by A. P. Penneyor, in 1933.

Some of the most careful and comprehensive model tests ever made with anchored, flexible sheet walls were carried out by Tschebotarioff, between the years of 1944 and 1951, at Princeton University. In most of these tests the backfill was deposited successively and no dredging took place in front of the wall. The results of the model tests with flexible bulkheads showed that full restraint of the lower portion of the bulkhead was effective when the ratio D/H equaled to 0.43, where the point of contraflexure approximately corresponded to the dredge-line elavation for normal backfilling operations.

During the period from 1947 to 1951 a larger number of tests were made, by R. W. Rowe in Scotland, on model sheet pile walls of varying heights, stiffness and materials. Other variables investigated were the effects of surcharge, dredge level, tie rod locations. The results of these tests, proved that the bending moment and tie-rod loads of sheet pile wall depend principally on the flexibility of the wall and the density of the subsoil.

Anchored Bulkhead design by Numerical Method was first developed by Moran, Proctor, Mueser, and Rutledge, Consulting Engineers, in 1953. The use of Newmark's numerical methods provides a rapid means for determining the bending moments and deflections of the bulkhead due to the soil loads.

CHAPTER II

SHEET PILING

Sheet piling consists of special shapes of interlocking piles, -iven so as to form a continuous wall, which may be a permanent -taining wall, a cofferdam, a sea wall or a pier. The applications sheet piling to other uses, such as for lining trenches, are only riations of the same design problem that arises with river walls.

Sheet piles may be of timber, reinforced concrete or steel. ie choise will depend not only on the relative cost of the materials, it on the suitability of a particular material for the intended use id its durability.

Reinforced concrete has displaced timber to large extent because s greater durability, particularly where timber would be subject to ttack by marine borers. Reinforced concrete is also cheaper in the irst cost in areas where suitable timber is not readily available.

Steel sheet piling is usually somewhat more expensive than reinforced concrete for permanent construction, but can be driven rough highly resistant strata, and it is in general use for temporary work because it can be extracted and re-used a number of times, nd has at the finish a salvage value. Where water-tightness is ecessary, as in the case of cofferdams, steel sheet piling is in eneral use.

The advantages of sheet piling over other types of walls are peed of construction, economy of material, and the ommission of xcavation.

IMBER SHEET PILES

Sheet piling of timber is used for short spans and light lateral oads. In recent years timber sheet piling has been used much less n permanent construction than formerly reinforced concrete having aken its place owing to its greater durability, but for temporary orks timber sheet-piles are still used because of their lightness and he consequent lightness of the pile-driving equipment required.

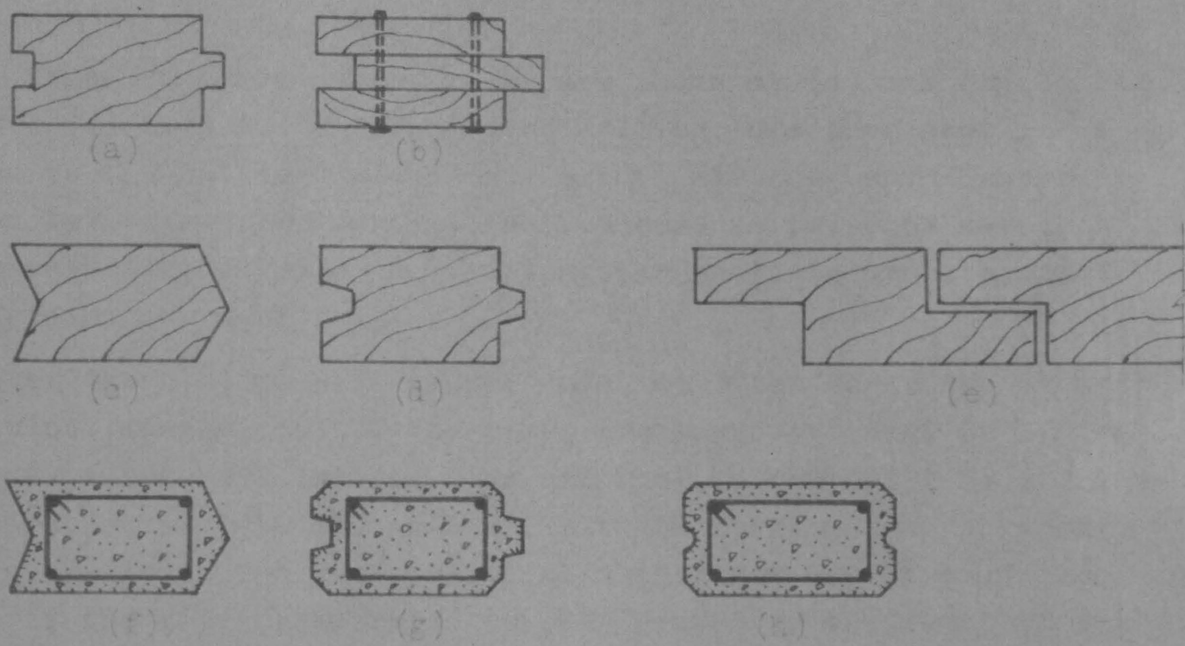


Fig. 2-1 Typical Sections of Timber and Concrete Sheet-Piles

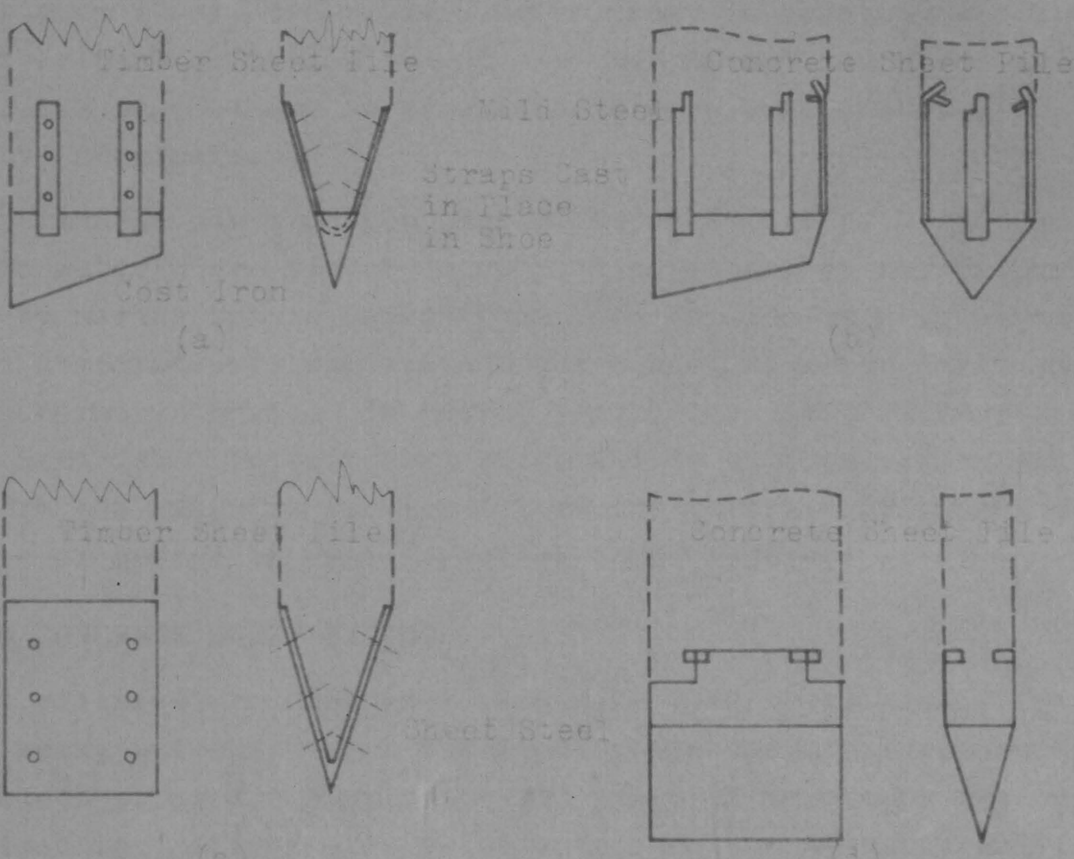


Fig. 2-2 Types of Shoes for Sheet-Piles

Two types of timber sheet piles are shown at (a) and (b) in Fig. 2-1. Type (b) know as Wakefield sheet-piling, has been used for a long time in the United States. It is both stronger and chearer than type (a) as well as having less tendency to twist or warp. Timber sheet piles of plain and rectangular sections are also used for cofferdams of small height.

Timber sheet piles with square ends may often be driven in soft soils without damage, but if the piles are large or long, or if the soil necessitates hard driving, the end must be protected by a covering of sheet metal 1/16 in. or 1/8 in. thick forming a cutting edge as in Fig. 2-2(c). The sheet metal also prevents cracking of the timber. If the pile penetrates compact gravel or a stratum of shale, a cast iron shoe as in Fig. 2-2(a) may be necessary. A shoe with one sloping face only is usual for piles lining a trech excavation.

For timber piles, the resistance to decay is greatest with teak. On the other hand softwoods is not very strong and lacks an efficient interlock, although it can be very durable when impregnated with preservative chemicals.

For permanent construction only a few varieties of timber are suitable in water where timber is subject to attack by marine borers. As attack by marine borers varies from time to time in given localities, only comparatively expensive timbers such as green heart and teak are generally immune. In recent years, the use of timber sheet piling is restricted to temporary works and to work on rivers and canals where the water is too low for marine borers. Softwoods should be pressure creasoted if used for other than temporary work.

REINFORCED CONCRETE SHEET PILING

Sheet piling of reinforced concrete is used occasionally for permanent work, but only where the lengths are comparatively short. Concrete sheet piles are simply precast piles of square or rectangular cross section, driven side by side to form a continuous wall. To keep the piles in line, some form of interlock is needed, such as the tongue and groove joints shown in Fig. 2-1(g) and 2-1(h).

It is both usual and the best practice to reduce the heads of einforced concrete sheet-piles as shown in Fig 2-3(a) to take a riving helmet, unless the driving is easy and a head-packing only s used. The section can in that case be uniform throughout to the ead. The objections are, however, that the piles are then not so asy to guide and there is more to cut away to expose the main bars or concreting into the coping or capping beam. If the head is reduced n section and the driving is very hard, the shoulders should be loped.

With concrete sheet-piles to be driven in soft soil a metal shoe s unnecessary. If the piles have to be driven to a set, say in compact gravel, or driven through any hard soil a cast-iron shoe, as show in Fig. 2-2(b) is used. When driving, the sloping edge of the shoe is on the far side from the pile already driven. The shoe of the pile first driven, however, is symmetrical, and is generally slightly sloped on both edges.

For the detail design of reinforced concrete sheet-piles all the considerations affecting the design of reinforced concrete compression members apply, except that with hard driving the piles may be subject to stresses approaching the crushing strength of the concrete. The stresses during driving are dealt with elsewhere. The area of the main reinforcement bars in slender piles should be not less than 2 percent of the cross-sectional area of the pile, or 1 1/2 percent if the slenderness ratio is between 30 and 40, and not less than 1 1/4 percent for stiffer piles.

The main bars may be tied together by diagonal wire ties at intervals as shown, but, to prevent them coming inwards during concreting, pressed-steel forks may be used in pairs holding apart diagonally opposite bars. The only objection to the pressed-steel spacers is that when cross-cracking of piles takes place, either by heavy or eccentric driving, the cracks generally occur at the points where the diagonal spacers are placed. Whatever method is used for fixing the main reinforcement bars it is important that the main bars are not permitted to be too close to the surface because of risks of spalling

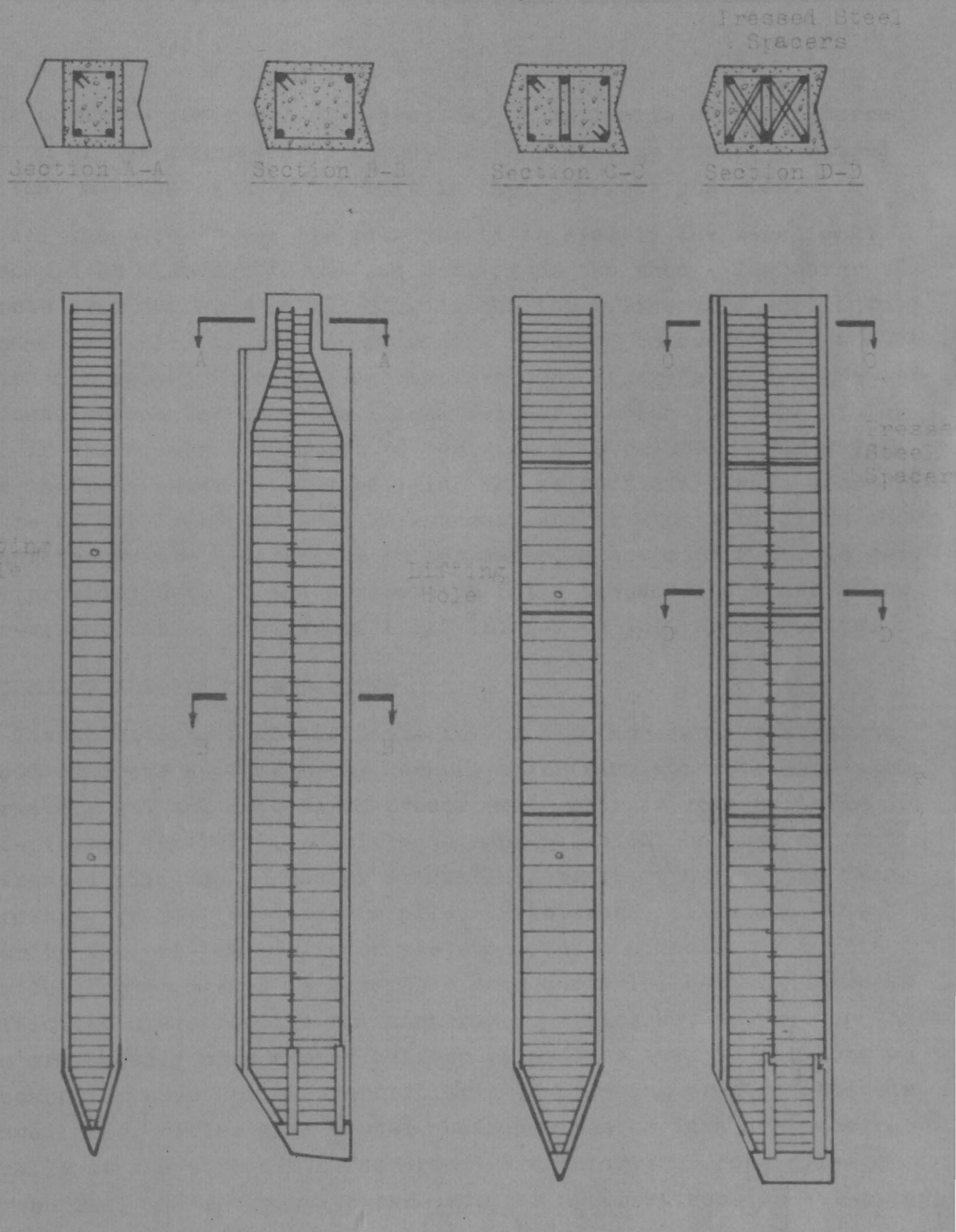


Fig. I-3 Typical Details of Reinforced Concrete
Sheet Piles

of the concrete cover and of corrosion, or be inside of their correct position of the considerable reduction in resisting moment compared with that calculated with the bars in their correct position.

All the bars in any one pile should be exactly the same length, and should be placed with one end bearing on the shoe. The cover of concrete over the ends of all bars at the top of the pile should be the same, say, 2 in. The amount of the cover of concrete at the sides of the main bars is a compromise between cost, therefore, the greater the cost, unless the bars are placed further in from the face of the pile, in which case the moment of resistance is reduced. For sheet-piles in fresh water a cover of 1 in. may be sufficient with a concrete mixture of 1:1 1/2:3 and good workmanship and driving conditions which are not too severe. For piles in sea-water, a cover of 2 in. is generally provided but, if the concrete is fully compacted and has a low water-cement ratio, a cover of 1 1/2 in. may be just as effective.

PRESTRESSED CONCRETE SHEET-PILES

The ability to take advantage of the high compressive strength of good concrete and the great durability of such concrete even when alternately wet and dry, makes prestressed concrete very suitable for sheet-piles. Generally, possibly thinner piles can be used so that a prestressed pile may be easier to handle, transport and driven than an ordinary reinforced concrete pile. Prestressed piles are made either by the pre-tensioning or post-tensioning methods. For thin piles the former method is generally more suitable, because there is insufficient space for the end anchorages. Piles having pre-tensioned wires are usually more economical for piles less than 30 ft. long as is common for sheet piles, particularly if existing prestressed beds are available. Piles with post-tensioned wires or bars are made economically at the site or in factores, particularly if long piles are reauired: Some typical cross-sections of prestressed sheet-piles are given in Fig. 2-4.

Concrete for prestressing and the permissible stresses are described in British and American Codes, but it is often common to require concrete having a crushing strength at twenty-eight days of 5000 lb. per square inch if determined by cylinders, or 6000 lb. per square inch if by cubes. The water-cement ratio should be between 0.38 and 0.45, and the concrete should be compacted to its greatest density. In the case of concrete in contact with water, the cover of concrete over the wires should be not less than about 1 in.

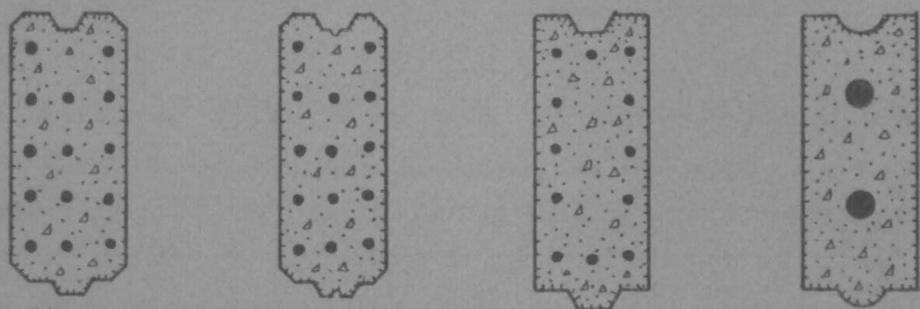


Fig. 2-4 Cross-sections of Prestressed Sheet-Piles

The diameter of pre-tensioned wires is generally 0.12 in. to 0.2 in., larger wires not being generally suitable for sheet-piles as a high degree of bond is desirable to withstand the driving. Wire ropes (strands) are used extensively in America and are becoming more common in Great Britain. The average total initial prestress is preferably about 800 to 1000 lb. per square inch after allowing for losses for creep and shrinking which are greater with pre-tensioning than with post-tensioning. In some cases other stresses are used, but it must not be over-looked that the tensile stress induced in the concrete during handling, transporting, and pitching of prestressed piles often determines the lower limit of the prestress.

Although it is not often necessary to lengthen sheet-piles, prestressed piles with post-tensioned steel can be easily lengthened and mild-steel reinforcement can be cast in the head to bond to the extension if required. Should the piles need to be shortened the end anchors are removed. If the grouting is effective, and as is usually the case, the driving has not been severe enough to destroy the bond, the strength in bending will not be impaired. In the case of post-tensioned bars, it is easy to remove and unstress the end anchor to avoid the tendency of the conrete to fly when cutting away the head, but with some other system of post-tensioning this risk has to be taken. Piles with post-tensioned high-tensile bars can be extended by using couplers to attach an additional bar, and subseauently stressing them in the usual way, but this is not often needed owing to the small bending moments near the top of the pile. For the same reason piles with pre-tensioned steel can be shortened easily by cutting, and can be lengthened by adding concrete reinforced with mild steel.

In both types of pile, mild steel is generally only reauired for bonding into the coping or capping beam; it is only in exceptional cases that question of extending sheet-piles may arise. It is desirable to provide links at the top and bottom of the pile as in a reinforced concrete pile.

STEEL SHEET PILING

Steel is, by far, the most freauently used material for piling. It is extremely used not only for temporary construction such as cofferdams, but also for permanent structures such as bulkheads, sea walls, wharves and piers. This wide application of steel sheet piling for heavy lateral loads and long spans can be attributed to two factors: First, it is generally recognized that steel, if properly protected, is well adapted for subsurface and marine constructi in both fresh and salt waters. Numbers of steel sheet-piling structures exist today, built twenty and thirty years ago, which testify to this. Secondly, the simplicity, strength and economy of steel sheet piling are very considerable.

Steel Sheet Piling has characteristics which render it for a wide variety of applications. It has the toughness and resilience inherent in steel, enabling it to withstand rough handling and battering during construction followed by the stressed and impacts of service. It is a finished product ready for use as shipped by the manufacturer. It may be stored, handled, set up and driven in comparatively small units simplifying problems of erection and equipment. Its installation is less dependent on favorable temperatures or weather conditions and may be independent of other construction operations. When interlocked and driven, the resulting wall is continuous, earth-tight, and virtually water-tight to any depth to which the sheet piling can be driven. Allowance for corrosion is generally made by providing excess metal thickness in the steel section. Further protection may be obtained by coating with heavy bitumastic paint. In the zone "between wind and wave", where corrosion is most severe, a concrete jacket is sometimes cast around the pile.

All makers of sheet-steel piling make special sections to form corners and T-connections. Standard details for splicing on additional lengths are available, but, unless the connection has the same moment of inertia as the plain section, the ability to transmit the full moment is not obtained, so that it is always desirable to avoid splicing. If, however, splices must be provided, say, because the sheet-piles cannot be handled or pitched in one length, the joint should either be placed at a position of small bending moment or the necessary strength obtained by increasing the number of bolts. Welded connections, although more expensive as the welds must be made at the site, are preferable to riveted or bolted connections for obtaining the full strength of the section for permanent work, but bolted joints are used where the top length is to be subsequently recovered.

The sections entitled "United States Steel" are typical of American piles, and in many cases the sections rolled by the Bethlehem Steel Company and the Inland Steel Company are identical. Sections similar to those of the Appleby-Frodingham Steel Company, Limited are rolled on the Continent, for example by the Belval Company in Luxemburg, which Company, together with the Lorraine-Escaut Company in France, roll sections suitable for cellular cofferdams.

In continental Europe similar sections of piles are in use and in fact several types were introduced into Great Britain from there. The Frodingham type is known in Germany as Hoesch; Larssen is known by the same name while the American sections, as MZ38 and ZP38, have equivalents in the Klöckner types.

The section modulus of sections marked MP is based on separated piles neglecting any interaction due to friction of the interlocks.

With the European sections it is frequently assumed that the full section modulus is developed, and the correctness of this assumption under normal conditions has been claimed to have been proved by practical experience in deep cofferdams and by tests. Tests in which the actual section modulus has been determined by careful measurement of the deflection of loaded piles are reported to have shown that even when the interlocks of the piles are quite free and have been oiled to reduce friction, something like 60 percent of the combined section modulus is developed. A small amount of loose sand or other material, which helps to develop friction, may be expected to increase this figure to nearly loo percent. In a case in which, for example, the piling is driven through a stratum of mud and then rests on top of hard rock which it cannot penetrate, it is advisable to make a reduction of as much as 25 percent in the combined section modulus of the sheeting. Alternatively, adjacent piles can be welded to one another so as to ensure that no sliding takes place. It is also advisable to make a reduction when the piles cantilever and filling is placed on one side of the sheet-pile wall after driving. There is no reduction of efficiency if the interlocks are on both faces of the wall.

CHAPTER III

ANCHORED BULKHEADS

Anchored bulkheads serve the same purpose as retaining walls. However, in contrast to retaining walls whose weight always represents an appreciable fraction of the weight of the sliding wedge, bulkheads consist of a single row of relatively light sheet piles

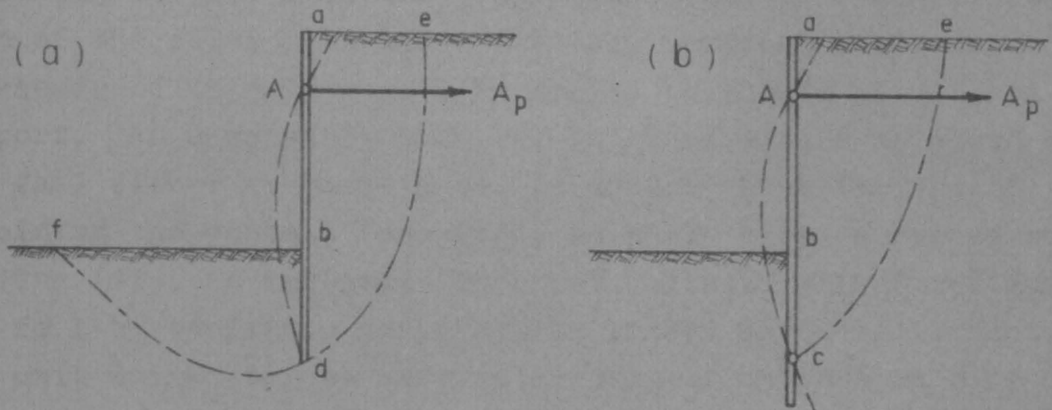


Fig. 3-1 Anchored bulkhead with (a) free and (b) fixed earth support. Dashdotted lines indicate potential surfaces of sliding.

of which the lower ends are driven into the earth. The active earth pressure is taken up partly by anchor rods which are tied to the sheet piles at A in Fig. 3-1 at a short distance below the upper edge (a) of the bulkhead and partly by the passive resistance of the soil located on the left side of the lower part of the sheet piles. The anchor rods are held in place by anchors which are buried in the backfill at a considerable distance from the bulkhead.

In further contrast to retaining walls, bulkheads are flexible. On account of the anchorage of the uppermost part and the passive resistance of the soil adjoining the lowest part of the bulkhead, the upper and the lower edges of a bulkhead are partically fixed. Therefore, a bulkhead yields only by bending in a hori-antal direction and the maximum deflection occurs approximately at midheight of the bulkhead.

If the sheet piles have been driven only to a shallow depth, the deflection of the bulkhead is somewhat similar to that of a vertical elastic beam whose lower end d is simply supported without being fixed, as shown in Fig. 3-1(a). Bulkheads which satisfy this condition are called bulkheads with free earth support. On the other hand, if sheet piles have been driven to a considerable depth, as indicated in Fig. 3-1(b), the lower end of the bulkhead is practically fixed in its position, because the resistance of the sand adjoining the end does not permit more than an insignificant deviation of the piles from their initial, vertical position. Therefore, bulkheads of this type will be called bulkheads with fixed earth support. An adequately anchored bulkhead with free earth support can fail either by bending or, on account of a failure of the sand adjoining the contact face (bd), by shear along a curved surface of sliding (df). A securely anchored bulkhead with fixed earth support can fail by bending. In laterally loaded sheet piling total length and unit weight can be materially reduced by anchorage of top or some other part of the near top. Anchorage may consist of concrete blocks, hori-antal beams, piles or short panels of sheet piles. The cantilevered wall has limited applications, as the introduction of single row of ties reduces maximum bending moment to less than 50 percent that of the cantilever.

THEORETICAL AND REAL PRESSURES ON ANCHORED BULKHEADS

Although many theories have been put forward to demonstrate the form and magnitude of the active and passive earth pressures on sheet-pile walls, they are generally influenced by the desire to obtain pressure diagrams which simplify the determination of the maximum necessary depth of penetration. These simplifications are generally agreed to be justified, since the results obtained cannot be more reliable than the information available about the characteristics of the soil which vary in course of time and may vary along the length of the wall.

Fig. 3-2(a) represents a bulkhead with free earth support. The inner face of the bulkhead is assumed to be acted upon by the active Coulomb pressure (pressure triangle, abc). The force that resists the outward movement of the buried part, eb, is represented by the shaded triangle, bef.

Fig. 3-2(b) shows a bulkhead with fixed earth support. The triangle, abc, represents the active earth pressure on the inner face. The assumed resistance against the outward movement of the buried part of the bulkhead is indicated by the pressure area, edb. Since the elastic line of the sheet piles forming an anchored bulkhead with fixed earth support is assumed to pass below a certain point b onto the right-hand, or inner, side of the original position of the bulkhead, it is also assumed that the earth pressure on the inner face changes at the elevation of point b from active to passive (line cc_1,

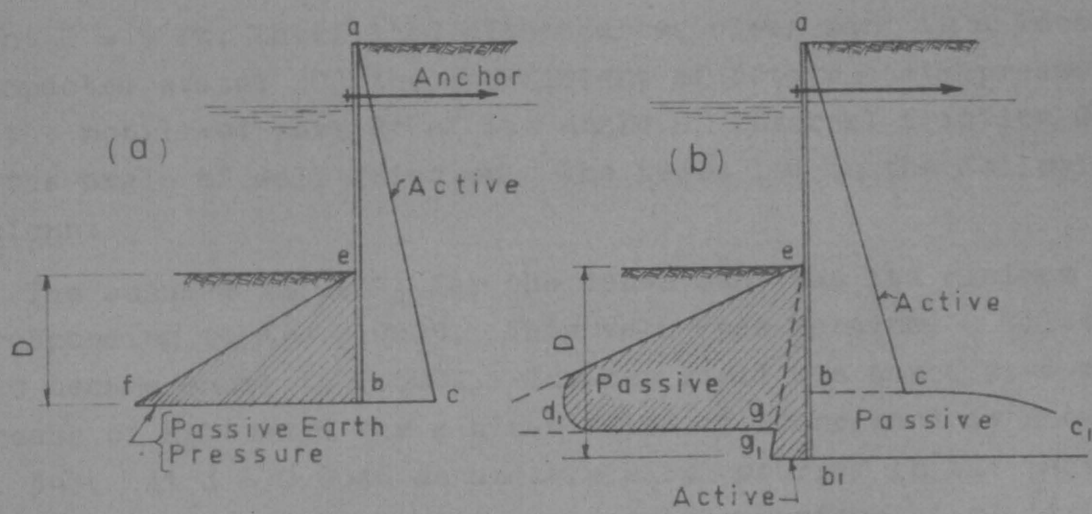


Fig. 3-2 Classical conceptions concerning distribution of earth pressure on bulkheads with (a) free and (b) fixed earth support.

Fig. 3-2(b), whereas the pressure on the outer face changes at the same point from passive to active (line gg_1). The depth of penetration of the sheet piles (D) is computed in such a manner that the elastic line of the sheet piles satisfies the condition of fixed earth support. The computation is performed either by trial and error or on the basis of supplementary simplifying assumptions.

At the time when the assumptions illustrated by Fig. 3-2 were originated, the physical conditions for the validity of these assumptions were still known. Since that time, many observational data have been accumulated which are incompatible with the original assumptions. Nevertheless, the methods of bulkhead design remained practically unchanged.

EFFECT OF WALL MOVEMENTS ON EARTH PRESSURE

The design methods illustrated in Fig. 3-2 involve the assumption that an infinitesimal yield of a lateral support is sufficient to reduce the intensity of the earth pressure to its minimum value and that a further yield has no influence on this pressure. This assumption is incompatible with the results of large-scale earth pressure tests which furnished accurate information concerning relations among (1) the yield ratio $d = Y/H$ of a vertical wall (Fig. 3-3(b) with height $H = 5$ ft, backfilled with coarse, clean sand in a loose or in a compacted state; (2) the coefficient of active earth pressure K_A; (3) the mobilized part $\emptyset'$ of the angle of internal friction $\emptyset$; and (4) the angle of wall friction. The tests led to the following conclusions:

The assumed value K_A for the dense sand was its minimum value corresponding to $d = 0.0005$. This value was retained until the yield ratio became equal to 0.0002. Further yield was associated with an increase of K_A toward the minimum value of loose sand as shown in Fig. 3-3. At $d = 0.0046$ an audible slip occurred in the backfill. Along the line of intersection between the surface of sliding and the surface of the backfill a low fault scarp appeared.

The value of K_A for loose sand decreased from 0.4 to 0.30 while the yield ratio d increased from zero to 0.0003. Further yield was associated with a less important decrease of K_A (Fig. 3-3(a)). At a yield ratio of $d = 0.0007$, corresponding to the maximum distance through which the model retaining wall could be advanced, the value of K_A was still considerably greater than the minimum value for the dense sand (0.23), and no slip had yet occurred.

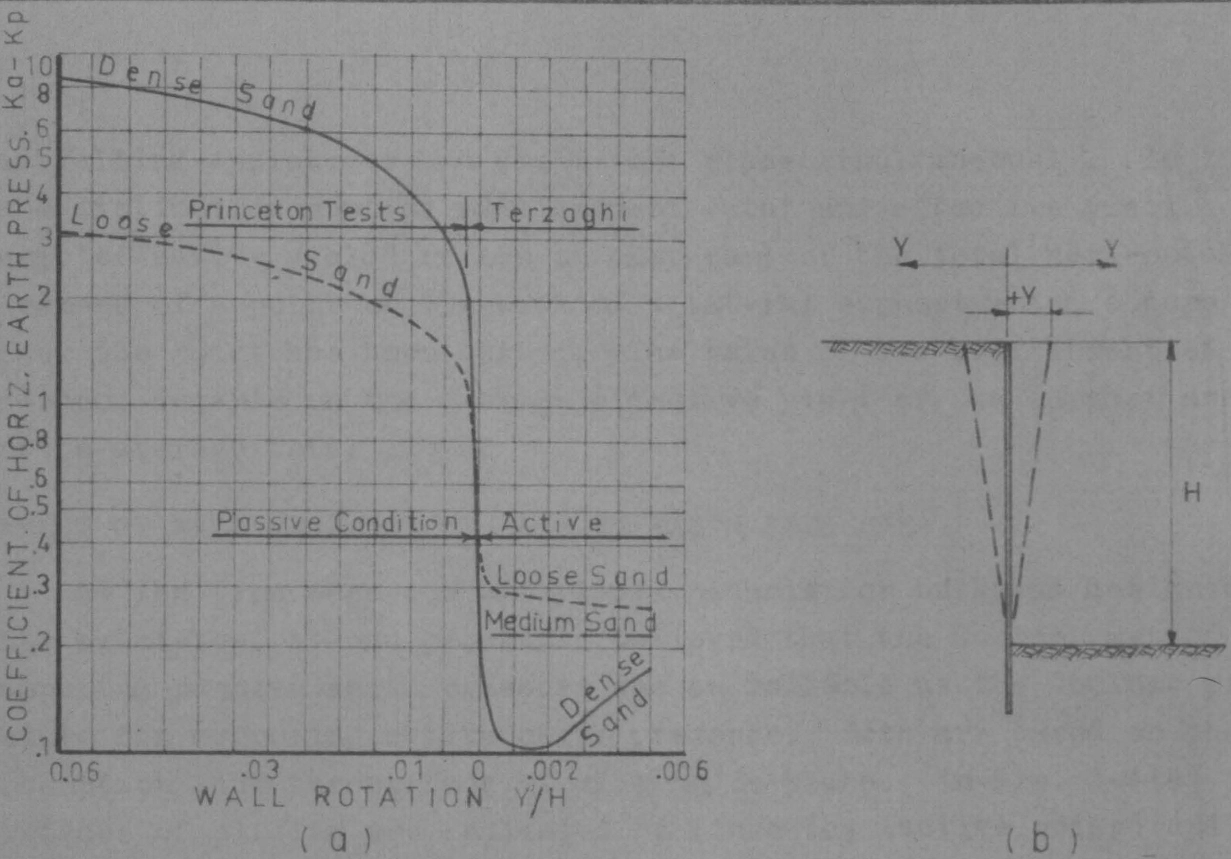


Fig. 3-3 Effect of Wall Movement on Coefficient of Hori-antal Earth Pressure

The angle of wall friction δ assumed its full value before the nternal friction was completely active. On the other hand, when the all advanced toward the backfill over a distance of 0.0002 H, the all friction was still much smaller than its maximum value.

At any stage of the tests, as soon as the wall ceased to yield, oth the angle of internal friction $\emptyset'$ and the angle of wall friction δ escreased slightly at a decreasing rate. Part of the decrease was robably caused by the fact that the backfill of the model retaining all was subject to intermittent vibrations caused by passing trains.

The backfill of bulkheads is almost never compacted by artificial ethods, and the average yield of the bulkhead hardly exceeds a raction of 1% of the height of the bulkhead. Therefore, it is unikely that the lateral pressure of a sand backfill on an anchored ulkhead is as low as the active earth pressure of the fill material.

In the tests illustrated in Fig. 3-3, the wall was not allowed o yield until the entire backfill had been placed. In practice, backfilling operations and yield take place simultaneously. In this case distinction must be made between total and effective yield. The term "effective yield" refers to that part of the total hori-ontal movement of a point on the back of a lateral support which occurs after the point has been buried. The value of the coefficient of eart pressure depends on the average effective yield of the support and not on the average total yield.

EFFECT OF WALL FRICTION ON PASSIVE EARTH PRESSURE

At the time when the analytical methods for bulkhead design came into existence, it was generally believed that the Coulomb method for computing passive earth pressure was as reliable as the Coulomb procedure for computing active earth pressure. Both are based on the assumption that the surface of sliding is plane. In Fig. 3-4(a) the surfaces of sliding are indicated by lines bc_1 (active wedge) and bd_1 (passive wedge).

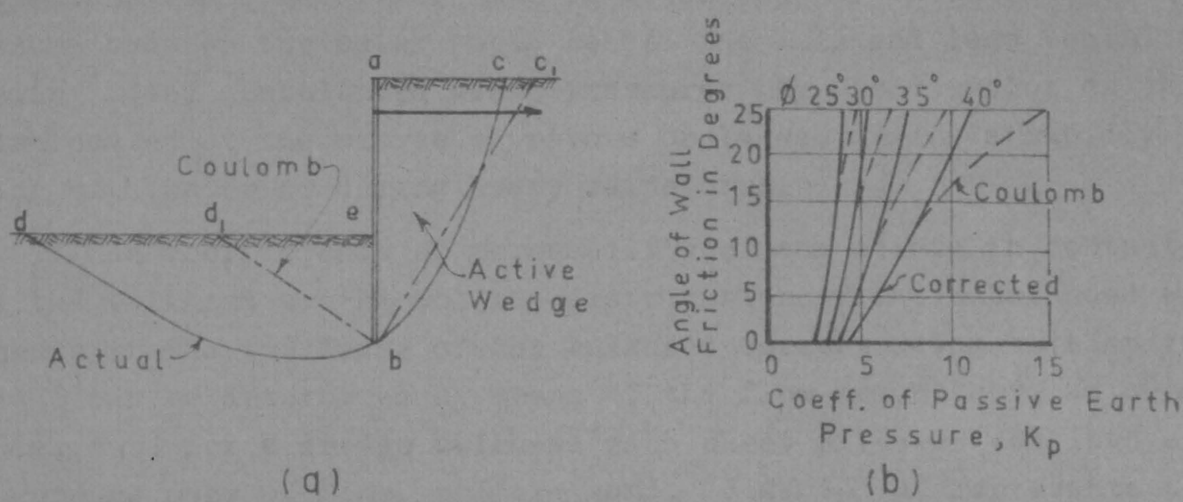


Fig. 3-4 Diagrams illustrating errors involved in Coulomb's assumption that the passive failure takes place along a plane surface of sliding.

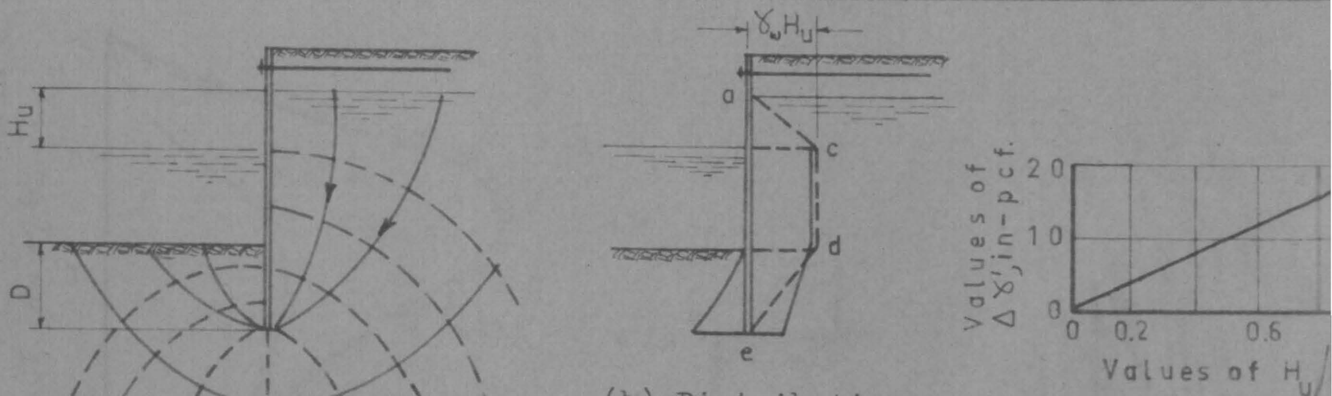
If the Coulomb assumption represented by line bd_1 were justified the coefficient of passive earth pressure K_p, of a sand with an angle of internal friction Ø, would increase with increasing angle of wall friction δ , as shown in Fig. 3-4(b) by dashed lines. Subsequent theoretical investigations have shown that the based of the active wedge (line bc, Fig. 3-4(a)) really is almost a plane. Therefore, the Coulomb minimum value for the active earth pressure is almost correct. However, in connection with the passive earth pressure, both the theory of plasticity and various experimental investigations led to the conclusion that the base of the passive wedge is not even approximately a plane. It consists of one curved section and one plane section, as indicated in Fig. 3-4(a) by line bd. On the basis of the knowledge of the actual shape of the surface of sliding, it was found by computation that the coefficient K_p of the passive earth pressure on a vertical lateral support (ratio between horizontal and vertical pressure at any depth below the surface) increases in accordance with the solid lines, and not the dashed lines, in Fig. 3-4(b).

UNBALANCED WATER PRESSURE

If an anchored bulkhead is located at the seashore, the earth pressure on the inner face of the sheet piles is a maximum at low tide. At the same time, the inner face is acted upon by an unbalanced water pressure because the water table behind the bulkhead lags behind the receding tide. Unbalanced water pressures may also develop at bulkheads located at the shores of rivers or lakes, during a rapidly receding high water or during heavy rainstorms.

If the coefficients of permeability of the strata in contact with the bulkhead are known, the distribution of the unbalanced water pressure on the two faces of the bulkhead corresponding to a hydraulic head H_u can be determined by means of the flow net method, as shown in Fig. 3-5, for a dredge bulkhead with sheet piles driven into a homogeneous mass of fine, uniform sand. Fig. 3-5(a) represents the flow net and Fig. 3-5(b), the corresponding distribution of the unbalanced part of the water pressure over the faces of the bulkhead.

If the permeability of all the soil strata in contact with the bulkhead is practically the same, it can be assumed, without serious error, that the inner face of the bulkhead at any depth between the surface line and the outside water level is acted upon by an unbalanced



(a) Flow net

(b) Distribution of unbalanced water pressure

(c) Average red tion of eff tive unit weight of p sive wedge to seepage ssure exert the upward flow of wat

Fig. 3-5 Unbalanced water pressure

water pressure, in which γ_w is the unit weight of water. Below the

$$P_u = \gamma_w H_u \qquad \text{(Eq. 3-1)}$$

the dredge line, P_u decreases from H_u to zero at the lower edge of th sheet piles, as indicated in Fig. 3-5(b) by the straight line, de.

If the permeability varies in vertical directions between wide limits, the determination of the distribution of the unbalanced water pressure may reouire the construction of a flow net.

When the water table in the backfill is above the free water level, the water percolates through the backfill in a downward direction, flows around the lower edge of the sheet piles, and rises beyond the outer face, as indicated in Fig. 3-5(a). The seepage pressure exerted by the rising ground water reduces the effective unit weight of the soil in contact with the outer face of the bulkhead and, as a consequence, it reduces the passive earth pressure. If i is the average hydraulic gradient in the soil adjoining the outer face, the corresponding reduction of the submerged unit weight γ' of the soil is

$$\Delta\gamma' = i\,\gamma_w \qquad \text{(Eq. 3-2)}$$

Under the conditions illustrated in Fig. 3-5(a), the average value of i is somewhat smaller than $\frac{H_u}{3D}$. Hence, the effective unit weight

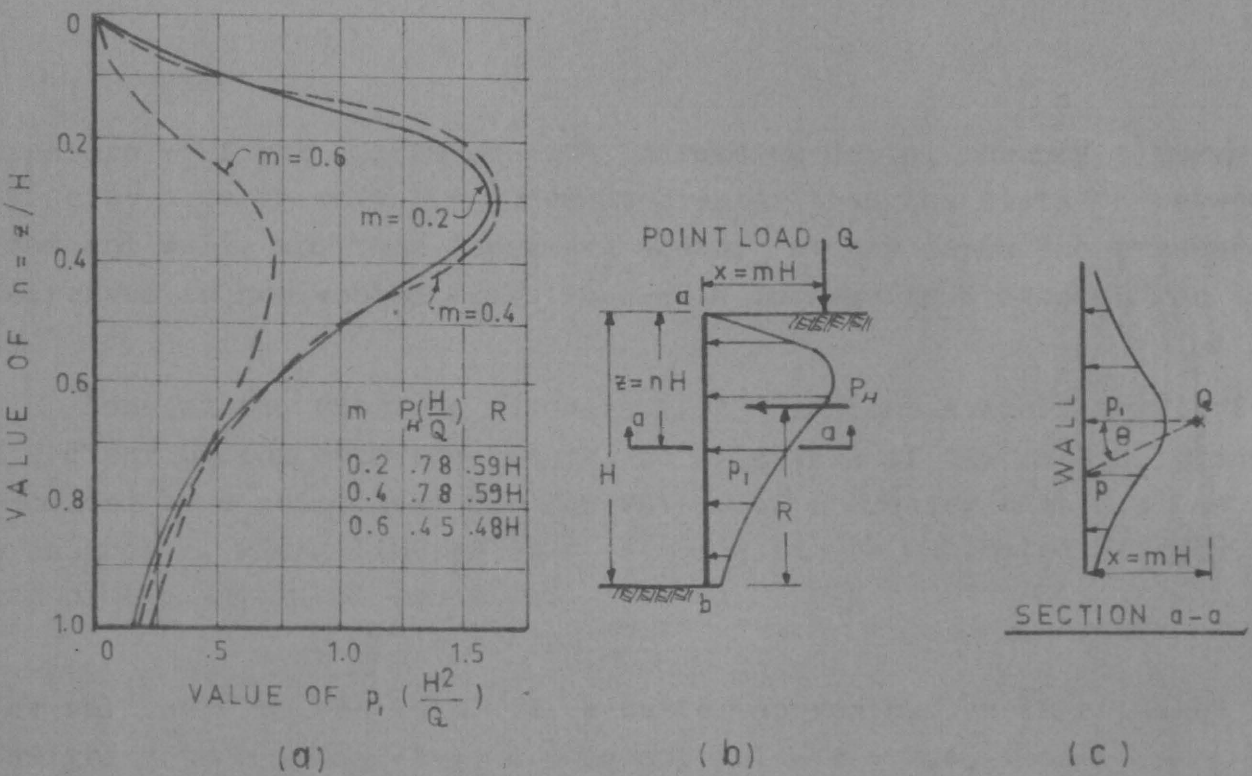


Fig. 3-6 Lateral Pressures Due to Point Loads

of the soil in contact with the outer face of the bulkhead will be slightly greater than

$$\gamma' - \Delta\gamma' = \gamma' - \gamma_w \frac{H_w}{3D} \qquad (Eq. 3\text{-}3)$$

The relationship between $\Delta\gamma'$ and H_w/D is shown in Fig. 3-5(c). Withi the backfill the water percolates in a downward direction and it produces an increase of the effective unit weight of the fill by $\Delta\gamma''$. However, $\Delta\gamma''$ is very much smaller than $\Delta\gamma'$ and does not require con sideration.

LATERAL PRESSURE RESULTING FROM POINT LOADS

Intensity and distribution of the lateral pressure resulting fro point loads were investigated by Gerber and Spangler. The test results were practically identical.

Fig. 3-6 shows the distribution of the lateral pressure over the back of the wall. The pressure is greatest along the line of intersection, ab, between the wall and a vertical plane through the center of the load at right angles to the wall. Along this line, the unit pressure p_1 first increases with increasing depth, assumes a maximum value at a depth which is somewhat greater than the distance between load and wall, and then decreases again. At any depth the pressure decreases in horizontal directions with increasing distance from line ab.

None of the existing theories (1953) account satisfactorily for the distribution over the inner face of a wall of the lateral pressure produced by a point load Q. For values of m greater than 0.4 the unit pressures p_1 along line ab (Fig. 3-6(b)) can be estimated roughly by use of the empirical equation:

$$p_1\left(\frac{H^2}{Q}\right) = 1.77\,\frac{m^2n^2}{(m^2+n^2)^3} \qquad \text{(Eq. 3-4)}$$

For values of m less than 0.4, a better approximation is obtained by assigning to m in Eq. 3-4, a constant value m = 0.4, thus

$$p_1\left(\frac{H^2}{Q}\right) = \frac{0.28\,n^2}{(0.16+n^2)^3} \qquad \text{(Eq. 3-5)}$$

The intensity of the lateral pressure on the back of the wall on both sides of the line ab in Fig. 3-6(b) is a complicated function of the depth below the crest of the wall and the horizontal distance from the line ab. If the point load Q is located at a distance m H from the wall, an upper limiting value for the unit pressure p at a depth n H below the surface of the fill and at a horizontal distance m H θ (Fig. 3-6(c)) from the vertical line through point a can be obtained by use of the empirical equation:

$$p = p_1 \cos^2(1.1\,\theta) \qquad \text{(Eq. 3-6)}$$

LATERAL PRESSURE RESULTING FROM LINE LOADS

According to the data shown in Fig. 3-7 the information furnished by the Coulomb theory concerning the intensiry and distribution of the lateral pressure resulting from line loads is incompatible with the experimental data. More satisfactory is the agreement between the measured pressure and the theory of Boussinesq. According to this theory the horizontal unit pressure σ_x on a vertical section, ac, in Fig. 3-7(b) through a semi-infinite elastic medium, at a depth n H below the surface caused by a surface load Q per unit of length, acting on a line at a distance mH from the vertical section is equal to

$$\sigma_x = \frac{2}{\pi}\,\frac{Q}{H}\,\frac{(m^2 n)}{(m^2+n^2)^2} \qquad (Eq.\ 3.7)$$

However, the application of the line tends to produce a lateral deflection of the vertical section, and the flexural rigidity of the bulkhead interferes with that deflection. In order to obtain the lateral pressure on a relatively rigid diaphragm, ab, in Fig. 3-7(b), located at the site of the vertical section, a second and equal line load Q must be applied at a distance m H on the right-hand side of point a. This second line load doubles the unit pressure; therefore, the unit pressure on the wall at depth n H below the surface is

$$p = 2\sigma_x = \frac{4Q}{\pi H}\,\frac{m^2 n}{(m^2+n^2)^2}$$

and

$$p\,\frac{H}{Q} = \frac{4}{\pi}\,\frac{m^2 n}{(m^2+n^2)^2} \qquad (Eq.\ 3\text{-}8)$$

For values of m greater than about 0.4 the agreement between theory and observation is fair. However, for values smaller than 0.4, the discrepancy between observed and computed values increases with decreasing values of m. For such values of m (less than 0.4) it was found by trial and error, that the observed pressure distribution has greater similarity to the computed distribution for m = 0.4 which is determined by the equation:

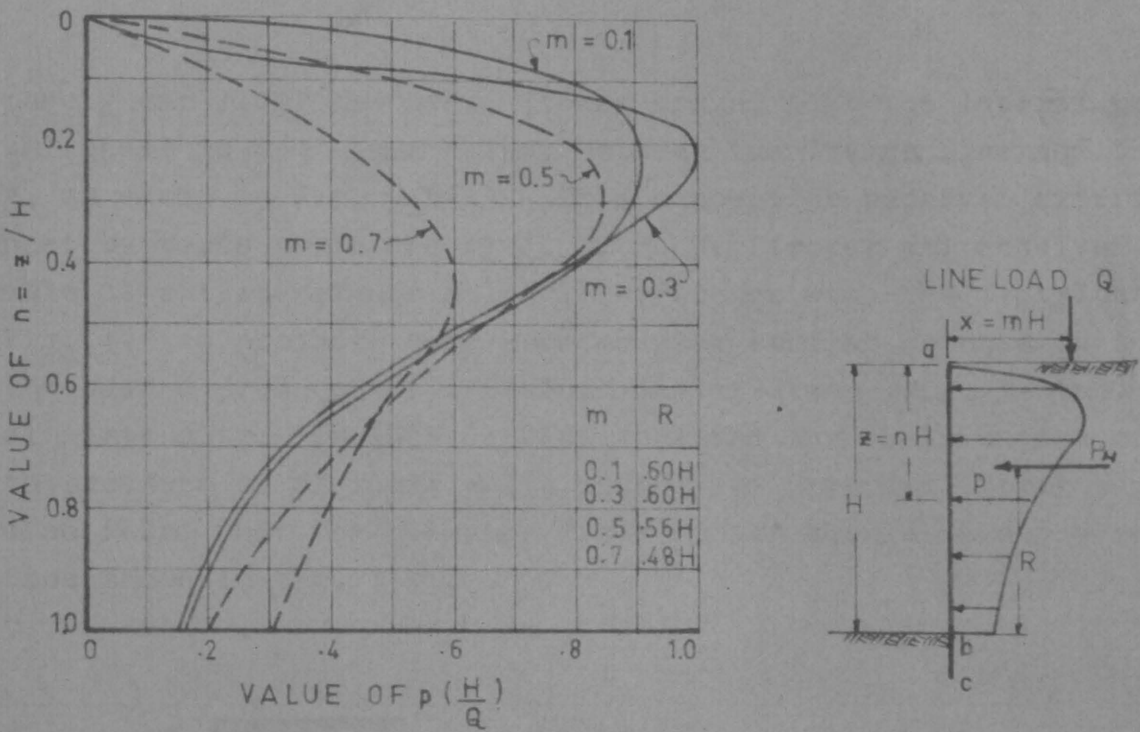
$$p\,\frac{H}{Q} = \frac{0.203\,n}{(0.16+n^2)^2} \qquad (Eq.\ 3\text{-}9)$$

For values of m greater than 0.4

$$P_H = \frac{0.64\,Q}{m^2+1} \qquad (Eq.\ 3\text{-}10)$$

For values of m smaller than 0.4

$$P_H = 0.55\,Q \qquad (Eq.\ 3\text{-}11)$$



(a) (b)

Fig. 3-7 Lateral pressure due to line loads

The observational data shown in Fig. 3-7 eliminated the Coulomb theory as a source of information concerning the lateral pressure produced by line loads, and they made it possible to establish empirical equations for estimating upper limiting values of the pressures produced by such loads.

DISTRIBUTION OF EARTH PRESSURE

Theory and observation have shown that the distribution of the pressure on a lateral support is by no means necessarily in accordance with the Coulomb theory because it depends largely on the type of yield. This fact is illustrated by Fig. 3-8 which represents the distribution of the lateral pressure on the back of a lateral support for three different types of yield. The effective yield at any depth below the surface is indicated by the width of the shaded area at that depth.

In connection with anchored bulkheads, the validity of the Coulomb theory was questioned for the first time in 1906 by Danish engineers on purely empirical grounds. It was argued that the lateral pressure on bulkheads is a minimum midway between the dredge line and the anchor line, as shown in Fig. 3_6(c). This conception received experimental support by tests performed by J. P. R. N. Stroyer and received considerable attention because a lateral pressure with the distribution shown in Fig. 4-8(c) produces very much smaller bending moments in the sheet piling than a pressure with Coulomb distribution (Fig. 3-8(a)) and equal total intensity. In 1938 J. Ohde computed the distribution of the earth pressure on flexible walls with fixed upper and lower edges and he also found that the pressure distribution should have the characteristics shown in Fig. 3-8(c).

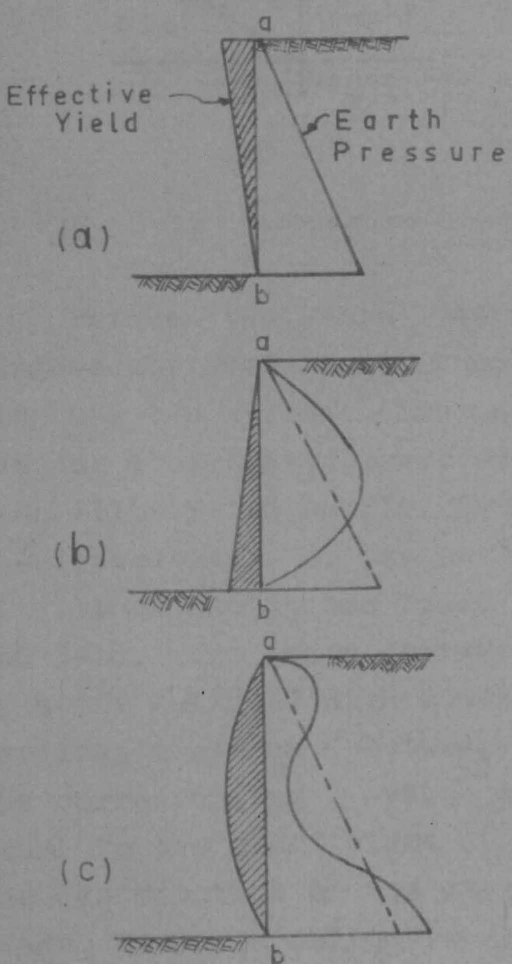


Fig. 3-8 Influence of Movement Type on Pressure Distribution

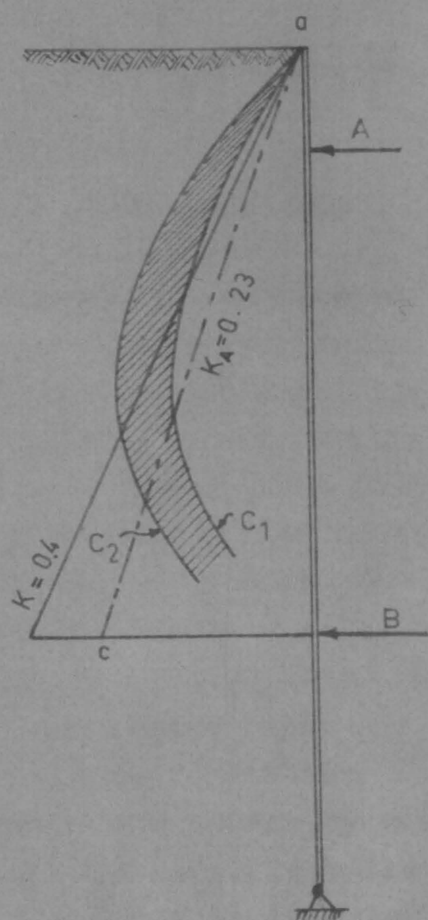


Fig. 3-9 Intensity and Distribution of active earth pressure on inner face of relatively stiff model bulkhead

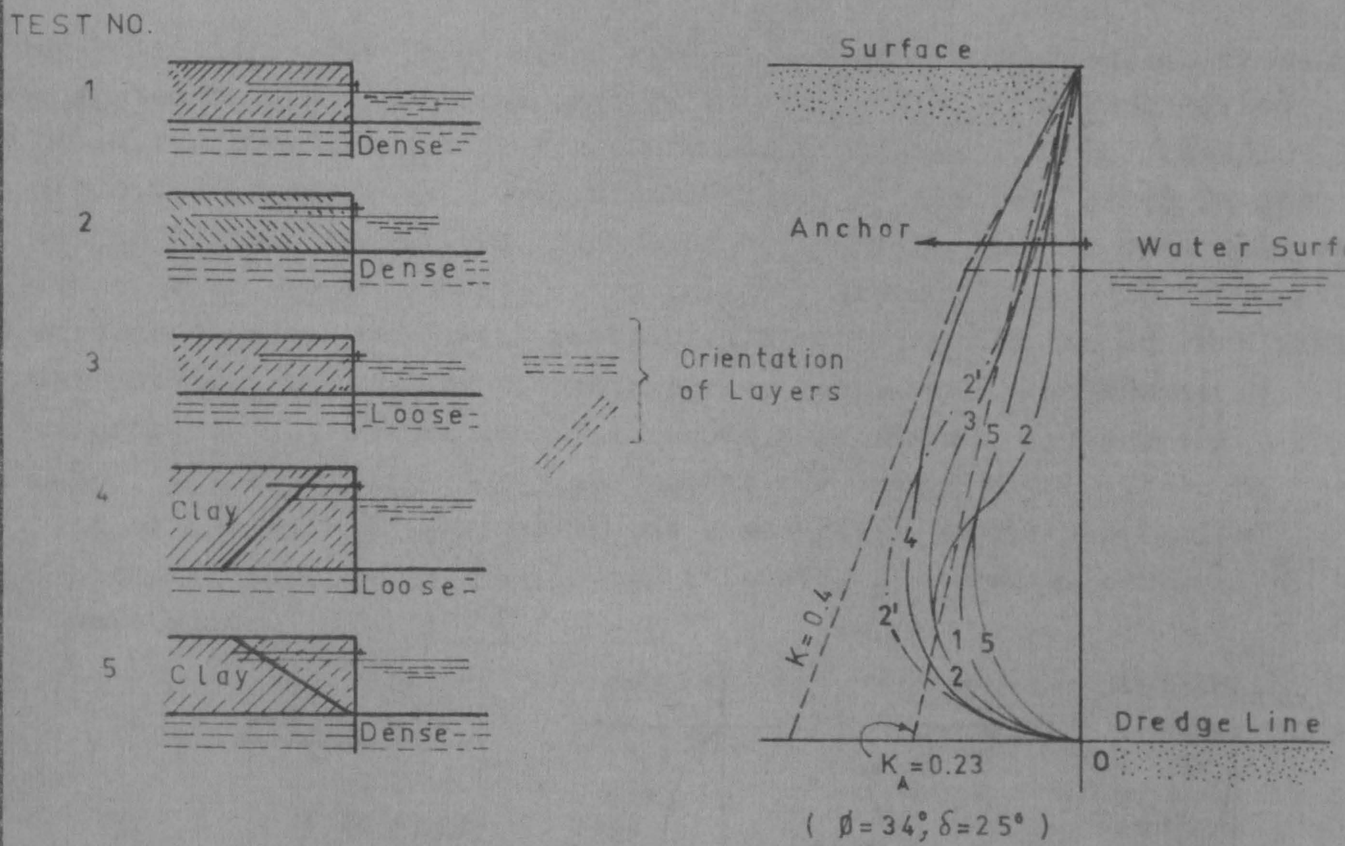


Fig. 3-10 Pressure Distribution on Flexiable Model Bulkheads

Between the years 1944 and 1951 G. P. Tschebotarioff, performed large-scale tests on bulkhead models. He measured the lateral deflection and the extreme fiber stresses in the bulkheads at different elevations above their lower edge and he computed the pressure distribution illustrated by Fig. 3-8(c) for bulkheads of the dredge type only. The distribution of the earth pressure on the inner face of the model of fill bulkheads was found to be similar to that shown in Figs. 3-9 and 3-10. The tests represented by Fig. 3-9 were made with a relativ ly stiff bulkhead with a maximum deflection eoual to about 0.1% of th vertical distance H between the anchor line and the dredge line. The corresponding K_A-value was nearly 0.4, which is approximately eoual to the coefficient of earh pressure at rest. The intensiry and the distribution of the active earth pressure on more flexible bulkheads, having a deflection ratio of about 0.5%, is shown in Fig. 3-1C The corresponding K_A- value approximated that of the active earth pressure. Curve 2' represents Test No. 2 after compaction of the fil by vibration. The line marked $K_A=0.23$ represents the Coulomb pressure computed on the assumption that $\emptyset=34^o$ and $=25^o$. The line marked K=0.4 represents the earth pressure was a maximum at some elevation above the dredge line. They also showed that the real pressure distribution depends on factors, such as the method of placing the fill, whi do not receive any consideration in earth pressure theory. Hence, the agreement between the real pressure distribution and the Coulomb pressure distribution is by no means perfect because the intensity of the lateral earth pressure is a maximum at some elevation above the dredge line (Fig.3-9 and Fig.3-10) and not at the dredge line. The computation of the bending moments in the sheet piles on the basis of a Coulomb pressure with equal total intensity involves an error on the unsafe side.

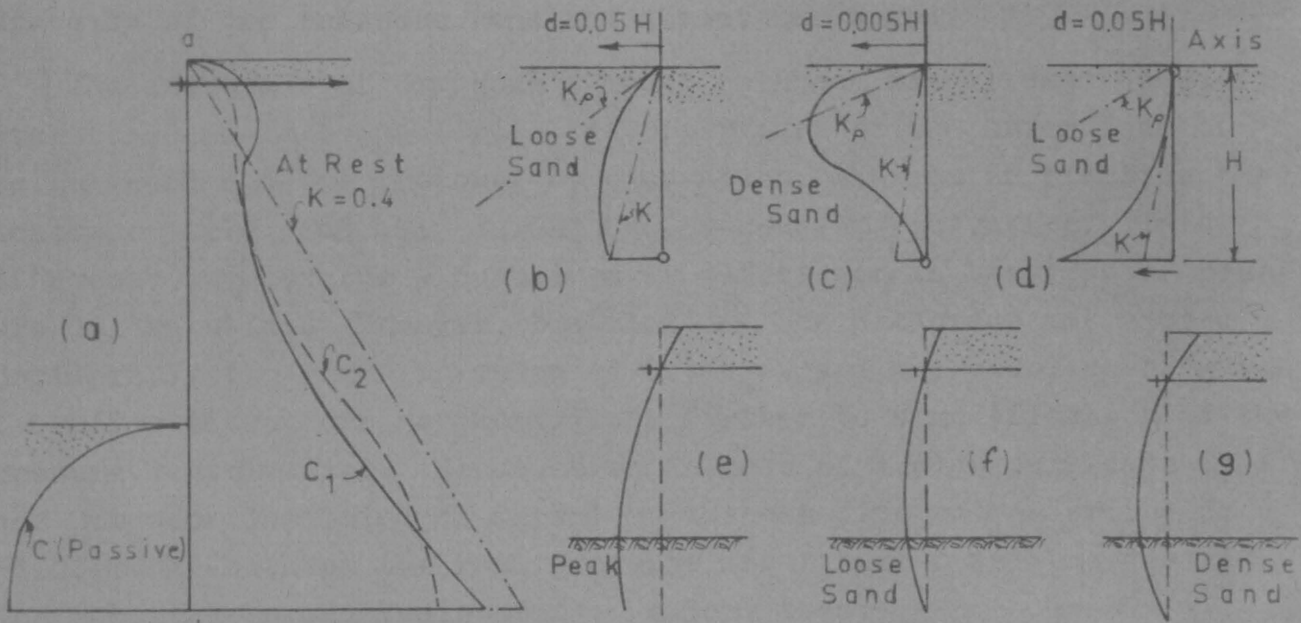


Fig. 3-11 Diagrams illustrating the effect of subsoil conditions on distribution of passive earth pressure and on type of bulkhead deflection.

Mr. Tschebotarioff also experimented with composite backfills, Fig.3-1 Tests No. 4 and 5. The lateral pressure exerted by sand fills backed by clay fills in Test No. 4 was found to be slightly greater than the lateral pressure of a continuous sand backfill with identical properti

The lateral pressure exerted by sand fills located between the bulkhead and a clay slope as in Test No. 5 was slightly smaller than that exerted by the fill in Test No. 4. P. W. Rowe determined the distribution of the earth pressure on a flexible wall directly by means of pressure cells in 1952. In agreement with Mr. Tschebotarioff's findings, he obtained the pressure distribution for dredge bulkheads as shown in Fig. 3-8(c). However, he found that an anchor yield of 0.1% of the height of the bulkhead is sufficient to change the pressure distribution into one that agrees fairly closely with the Coulomb theory. The test results are illustrated in Fig. 3-11(a), In Fig.3-11 curve C_1 represents the distribution of the earth pressure on the inner face of the model bulkhead prior to yielding of the anchorage and C_2 represents distribution after the bulkhead had yield. The total intensity of the pressure remained almost unchanged.

The anchorage of Mr. Rowe's model bulkhead was allowed to yield after the sand in contact with the upper part of the outer face of the bulkhead had been removed by excavation, whereas in practice the anchorage yield gradually during the process of excavation. This difference may involve a considerable difference in the type of pressure distribution. However, the yield of the anchorage may exceed considerably the limiting value of 0.001 H, and the pounding of waves of traffic vibrations may contribute further to a modification of the pressure distribution. Hence, even in case of dredge bulkheads it does not seem justified to depend on the benefits to be derived from a difference between the real pressure distribution and the distribution computed on the basis of the Coulomb theory.

Curve C (Passive), Fig. 3-11(a), shows the results of the measurement of the passive earth pressure that acted upon the buried part of the model bulkhead. In order to obtain supplementary information concerning the effect of the type of wall movement on the distribution of the passibe earth pressure, Mr. Rowe experimented with a 3/8 i steel plate that was buried to a depth of 2 ft. in clean sand. The plate could be advanced toward the sand by rotation about a hori-ontal axis. The passive earth pressure on the wall was measured by use of seven pressure cells, space 3 in. on centers along the vertical axis of the area acted upon by the passive earth pressure. The test results are shown in Figs. 3-11(b), (c) and (d).

The pressure distribution represented by curve C (passive) in Fig. 3-11(a) is intermediate between those shown in Fig. 3-11(c) and 3-11(d), but none of them involves an increase of the passive earth pressure in simply proportion to the depth below the dredge line. With increasing flexibility of the buried part of the bulkhead, the movement of this part changes from a displacement almost parallel to the original position of the buried part into a movement by rotation about the lower edge of the bulkhead so that the distribution of the passive earth pressure becomes increasingly similar to that shown in Fig. 3-11 (b), or Fig. 3-11(c).

INFLUENCE OF FLEXURAL RIGIDITY ON BENDING MOMENT

According to the theories of bulkhead analysis as illustrated in Fig. 3-2, the conditions of end support and the maximum bending moment in the sheet piles are independent of the flexural rigidity of the sheet piles. According to the same theories, the maximum bending moment decreases with increasing depth of sheet pile penetration, whatever the flexural rigidity may be. In fact, if the sheet piles were perfectly rigid, the maximum moment would increase with increasing depth of pile penetration. However no observational data were available concerning these important relationships until the results of Mr. Rowe's experimental investigations were published in 1952. The results of his measurements are shown in an abbreviated and simplified form in Figs. 3-12 and 3-13, which illustrate the effect of pile flexure and soil density on the bending moment of the pile. It can be seen how critical these factors are, and how wide a range of bending moments can occur under varying conditions.

A survey of Mr. Rowe's results shows that for a pile of average stiffness in a soil of average density (Fig. 3-12(b)) the bending moment is of the same order of magnitude as the bending moment computed by the conventional fixed earth support method. If a very stiff pile has a toe embedment in a loose soil (Fig. 3-12(a)), the bending moment may be considerably larger. For a very flexible pile in a dense soil (Fig. 3-13(c)) the bending moment may be smaller than that computed by the fixed earth support method.

Fig. 3-12 Bending-moment diagrams for bulkheads in very loose soil.

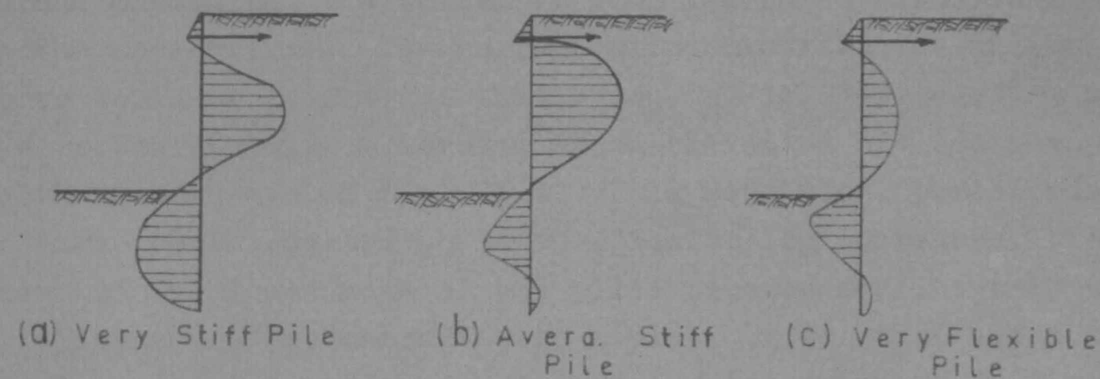


Fig. 3-13 Bending-moment diagrams for bulkheads in dense soil.

It should be noted that in the following arbitrary pile classification (according to varying degrees of stiffness), it is assumed that (1) piles of average stiffness include steel sheet piles in the lengths normally used for anchored sheet-pile bulkheads; (2) very stiff piles include reinforced concrete sheet piles and short lengths of the heavier sections of steel sheet pile; and (3) very flexible piles include timber sheetings and longer-than-average lengths of steel sheet piles.

For very stiff bulkheads, the maximum bending moment M in the sheet piles is practically independent of the flexibility number and is equal to the value M (max.) computed on the assumption of free earth support, Fig.3-2(a). However, if the flexibility number ρ exceeds a certain value, the maximum bending moment M decreases with increasing values of ρ and finally approaches a value approximately equal to one third of M (max.). The critical flexibility ρ_c at which the maximum bending moment starts to drop below the value of M (max.) increases with descreasing relative density of the sand, Rowe's Method of design is described herein under a separate subheading.

The fact illustrated by figure shown in Rowe's Method - that the bending moment in sheet piles decreases with increasing flexibility of the piles - is chiefly the result of the interdependence between the type of deflection of the buried part of the sheet piles and the corresponding distribution of the passive earth pressure. If the sheet piles, with a depth penetration D, were perfectly rigid and the anchorage unyilding, the buried part of the sheet piles would rotate about the anchor line. The corresponding distribution of the earth pressure would be similar to that shown in Fig. 3-11(d), and the center of the pressure would be located at an elevation of less than D/3 above the lower edge of the piles. This condition corresponds to the ideal "free earth support". As the flexibility increases, the outward movement of the lower edge of the piles becomes smaller and smaller. The yield assumes the character of a yield by rotation about the lower edge, involving a pressure distribution as shown in Figs. 3-11(b) and 3-11(c). The elevation of the center of the passive pressure increases to more than D/2, whereby the "free span" -equal to the distance between anchor line and center of the passive pressure - decreases, and the maximum bending moment decreases with the third power of the span. Finally, if the piles are extremely flexible, the lowest part of the sheet piles will neither advance nor rotate. In other words, the lower ends of the sheet piles will be "fixed" as shown in Fig. 3-2(b).

The critical value ρ_c of the flexibility number increases with increasing compressibility of the soil because the resistance of the soil against tilt and outward movement of the buried part of the sheet piles decreases. This interdependence is illustrated in Figs. 3-11(e) to 3-11(g). If sheet piles are driven into peat (Fig. 3-11(e)), they receive "free earth support" even if they are made of a flexible material such as wood.

II-DESIGN OF ANCHORED BULKHEADS

The experimental investigations described in Part I have made it possible to eliminate the most serious misconceptions associated with the customary methods of bulkhead design. On the basis of the findings, one can reliably estimate the forces exerted on anchored bulkheads by homogeneous layers of soil with known physical properties. Hence, the uncertainties involved in the design of bulkheads no longer result from inadequate knowledge of the fundamental principles involve They are caused only by the fact that the structure of natural soil deposits is usually complex, whereas the theories of bulkhead design inevitably presuppose homogenous materials. Not even the backfills composed of excavated and transported soils can be considered homogenous. Because of local variations of the soil properties within the borrowpit area and segregation according to grain si*e during the process of underwater deposition, the characteristics of the backfill may change from place to place and its properties cannot be reliably determined by tests in advance of construction.

Because of these conditions the most economical and expedient procedure consists in estimating the constants and coefficients - such as the unit weights and the coefficients of earth pressure - on the basis of the results of exploratory borings and of routine tests performed on representative samples, and compensating for the uncertainties involved in this procedure by an adequate margin of safety.

FREE EARTH SUPPORT METHOD

This method is based on the assumption that the soil into which the lower end of the sheet piling is driven is incapable of producing effective restraint of the sheet piling to the extent necessary to induce negative bending moments. The pressure distribution in a granular soil which corresponds to this assumption is shown in Fig. 3-14 for a minimum depth of embedment D' compatible with equilibrium, that is, when the factor of safety in respect to the limit value of the passive resistance of the soil in front of the bulkheads is equal to unity (G_s=1.00).

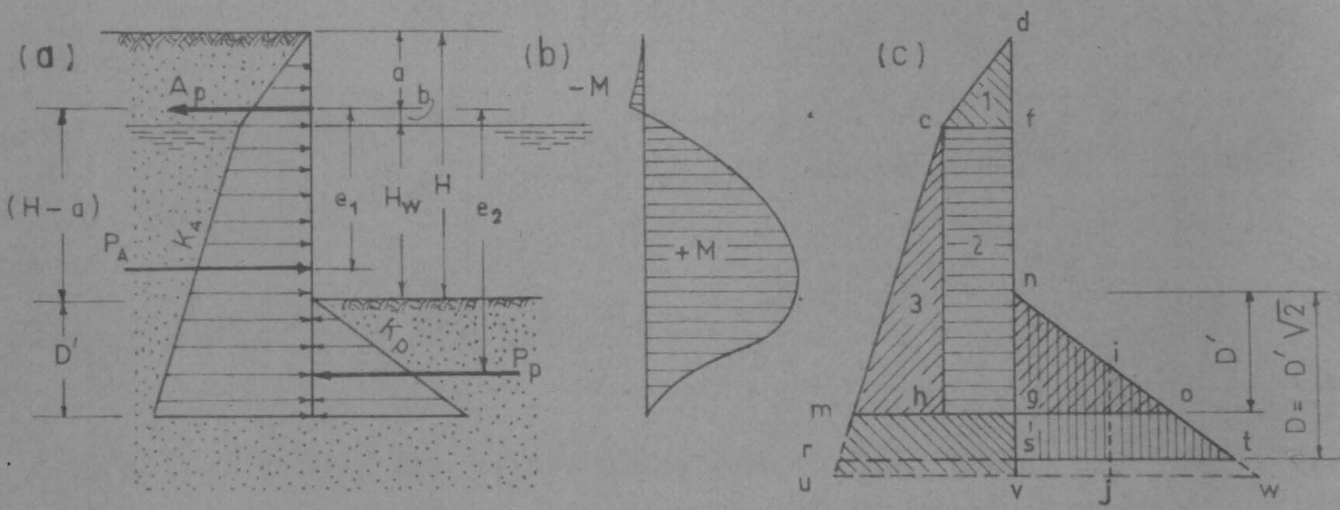


Fig. 3-14 The free-earth support method of anchored-bulkhead design in sand.

Figure 3-14(a) refers to a condition where the depth of embedment is just sufficient for limit equilibrium, assuming that the maximum possible passive resistance is fully mobilized. The anchor pull can be determined from the condition that the sum of the horizontal forces is equal to zero:

$$A_p = P_A - P_p \qquad \text{Eq. 3-}$$

where

$$P_A = [\tfrac{1}{2}(a+b)^2 \gamma K_A] + [\gamma (a+b) K_A (H_W + D')] + [\tfrac{1}{2} \gamma' (H_W + D')^2 K_A] \qquad \text{Eq.}$$

and

$$P_p = \tfrac{1}{2} \gamma' D'^2 K_p \qquad \text{Eq. 3-1}$$

In the above equations γ refers to the unbuoyed and γ' to the buoyed unit weight of the granular soil; all the other symbols are as shown in Fig. 3-14(a). The first bracketed expression in Eq. (3-13) represents the area (def) of the lateral-earth-pressure diagram Fig. 3-14(c), the second represents the area (efgh), and the third represent the area (emh). Equation (3-14) represents the area (ngo). The depth of embedment D', which will correspond to the limit equilibrium, can be determined from the condition that the sum of the moments around the point where the anchor connects with the bulkhead must equal zero:

$$P_A e_1 = P_p e_2 \qquad \text{Eq. 3-15}$$

where

$$P_A e_1 = -\left[\tfrac{1}{2}(a+b)^2 \gamma K_A \left(\tfrac{a+b}{3} - b\right)\right] + \left[\gamma'(a+b)K_A (H_W + D')\left(\tfrac{H_W + D'}{2} + b\right)\right] + \tfrac{1}{2}\gamma'(H_W + D')^2 K_A \left[\tfrac{2}{3}(H_W + D') + b\right] \qquad \text{Eq. 3-16}$$

and

$$P_p e_2 = \tfrac{1}{2}\gamma' D'^2 K_p \left(H_W + b + \tfrac{2}{3}D'\right) \qquad \text{Eq. 3-17}$$

In actual design all values contained in Eqs. "3-16) and (3-17) are selected and are numerically known, with the one exception of the depth of embedment D'. Thus after insertion of the corresponding figures into these equations, Eq. (3-15) may be given the form

$$C_1 D'^3 + C_2 D'^2 - C_3 D' - C_4 = 0 \qquad \text{Eq. 3-18}$$

where C_1, C_2, C_3, and C_4 are numerical coefficients, so that the value of D' can be determined.

Once D' is known, P_a and P_p are determined from Eqs. (3-13) and (3-14) and the anchor pull A_p from Eq. (3-12). The numerical values of all forces acting upon the bulkhead are then known. The bending moments induced by these forces can be determined in accordance with the customary procedures of mechanics of materials. Figure 3-14(b) indicates the shape of the bending moment curve which will result from such computations.

The actual depth of embedment D is made greater than D' to obtain the desired factor of safety against rupture of the soil in front of the bulkhead and against failure of the entire structure. A factor of safety of two (G_s=2.00) is sometimes chosen to that end. One of the frequently used methods of computation assumes that the effectively mobilized portion of the passive earth pressure has the shape of a trapezoid nvji, the area of which is equal to one-half of the maximum theoretically possible triangular resistance area nwv, as shown in Fig. 3-14(c). This assumption complicates the relevant computations to an extent which does not appear justified, since so many other approximations are involved in the basic assumptions of this design method. For that reason the actual depth of embedment D from

$$D = \sqrt{2}\,D' = 1.414\,D' \qquad \text{Eq. 3-19}$$

This relationship appears to be based on the fact that, as shown in Fig. 3-14(c), the area of the triangle ngo will be one-half of the area of the triangle nst if ng is made equal to D' and ns to D, as indicated by Eq. 3-19. This leaves out of consideration the theoretical increase of the total active pressure by the area mgsr. Therefore, Eq. 3-19 would provide an approximate factor of safety G_s=1.7, and not 2.0, as assumed. If G_s=2.0 is desired, then D should be made equal to

$$D = 1.7\,D \qquad \text{Eq. 3-20}$$

GRAPHICAL METHOD OF SOLUTION OF FREE EARTH SUPPORT

When it is required to find the minimum penetration the graphical method in Fig. 3-15 is applicable. For this case, the total pressure on the front of the wall plus the tension in the ties must equal to the total pressure on the back of the wall. There is no pressure on the back of the wall near the bottom if the penetration is just sufficient to prevent the wall moving forward.

At the outset of our investigation the depth D at which point (d) is located is unknown. Hence we are obligated to make a first guess, represent the valve D=bd in Fig. 3-15(a).

Divide the diagram into any convenient number of strips, 1, 2, 3, etc. and calculate the total pressure corresponding to the area of each strip. Plot the total pressure along the base line of the moment vector diagram (Fig. 3-15 f) to any convenient scale and mark points 1, 2, 3, etc., corresponding to strips 1, 2, 3, etc. join these points to the pole.

Project horizontal lines through the center of gravity of each strip and number the spaces these lines 1-11. Then we replace the continuous system of forces 1-11 as shown in Fig. 3-15(b). In spaces 1-11, draw lines parallel to lines pole 1, pole 2, etc. in the vector diagram down to point O. The lines drawn are part of the bending moment curve, Fig. 3-15(c).

By taking moments about O, determine the value of A_p. Complete the lower part of pressure diagram and complete the vector diagram, and bending moment curve as already described, check the calculated valve of A_p with the value obtained from vector diagram.

Produce the line in space O to intersect the line of the tie-bar. A line joining this point to the foot of the bending moment curve is the base line of the bending moment diagram.

DANISH METHOD

This empirical method for the design of anchored bulkheads was developed by Danish engineers in 1926. Although numerous dredge and fill bulkheads designed from these rules have been constructed in water depths of up to 40 ft., no failures have been recorded except for a few cases of excessive anchor yield.

Although certain details in the Danish Rules can be criticized, it is a fact that, by this method, a great number of sufficiently,

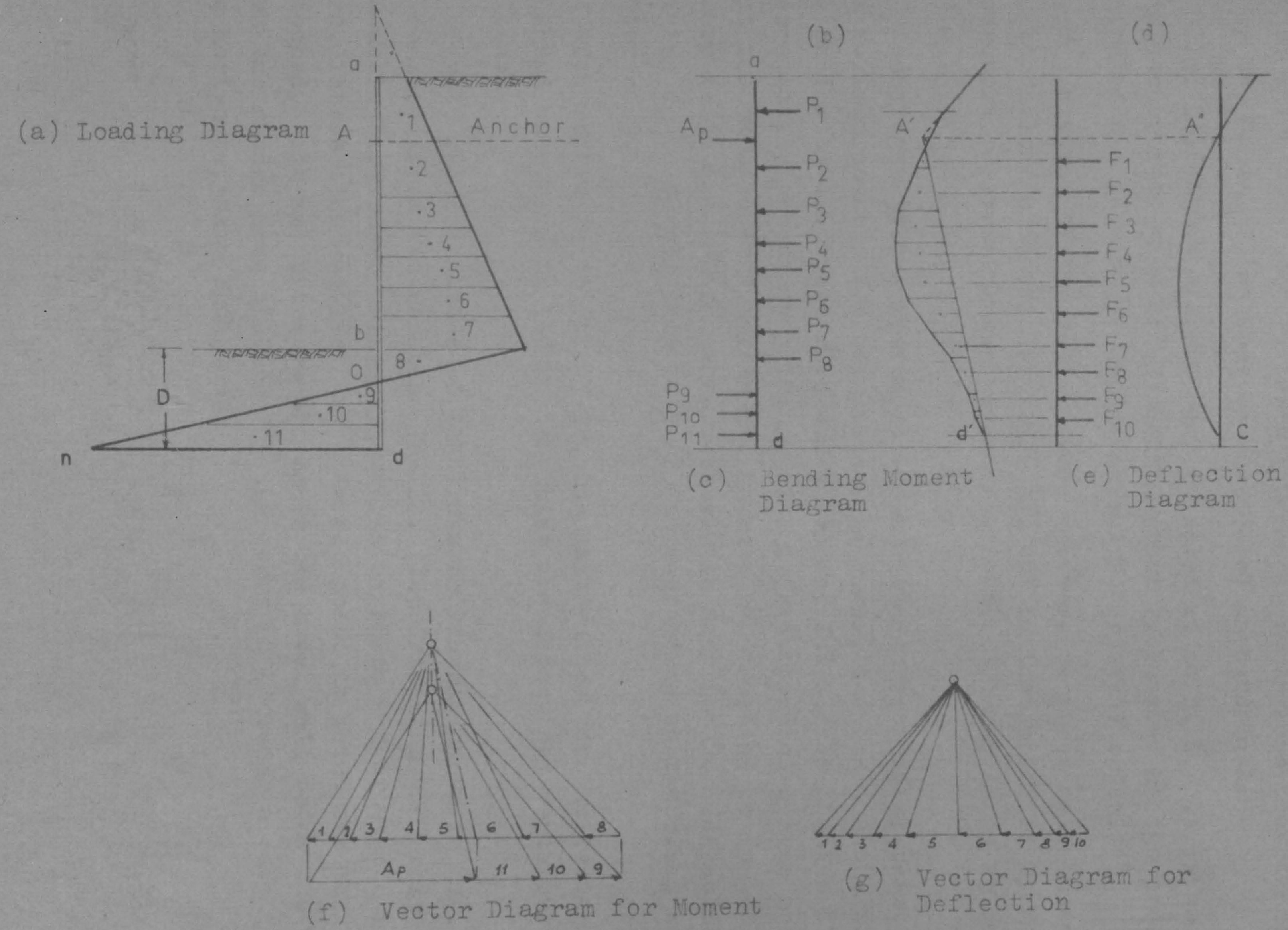


Fig. 3-15 Graphical Method of Solution of Free Earth Support

safe and economical bulkheads have been constructed. On the basis of this extensive practical evidence, it is maintained that any design method which consistently leads to greater dimensions than those requi ed by the Danish Rules must be considered uneconomical.

Fig. 3-16 shows a sheet wall anchored at point A. It is assumed simply supported here and at anchor point B, located at the pressure centre of the passive pressure necessary for equilibrium.

The diagram of the active earth pressure is first calculated accor ing to Coulomb's thoery (with $\delta_a = o$), but is then modified by means of a parobola, decreasing the pressure in the middle of AB by an amount q and increasing the pressure at A by 1.5 q.

Assuming simple supports at A and B the corresponding reactions and maximum moments are calculated. The wall and the anchors are then designed with allowable stresses which are 25% higher than usual. The theoretically necessary driving depth d is determined by the condition that the passive earth pressure, calculated according to Coulomb's theory (with $\delta_p = 1/2\ \phi$), should equal the reaction B. The actual driving depth should then be $d\sqrt{2}$, corresponding to a nominal safety factor of 2.

With reference to Fig. 3-16, the main values for the design of a bulkhead according to the Danish rules are determined empirically as follows:

First, the ordinate q of the pressure diagram shown in Fig. 3-16 is computed from the equation (3-21). The trapezium ADFB represents the active pressure, using any formula by which a hydrostatic pressure distribution results, and A is the point to which the tie is attached. The net pressure acting on the sheet wall below the tie is then the shaded area A'ADFBM'A', the curve BM'A' being a parabola with the horizontal axis such that

$$MM' = q = \frac{k[4 + (10h/L)]}{5 + (10h/L)}\, p_m \qquad \text{Eq. 3-21}$$

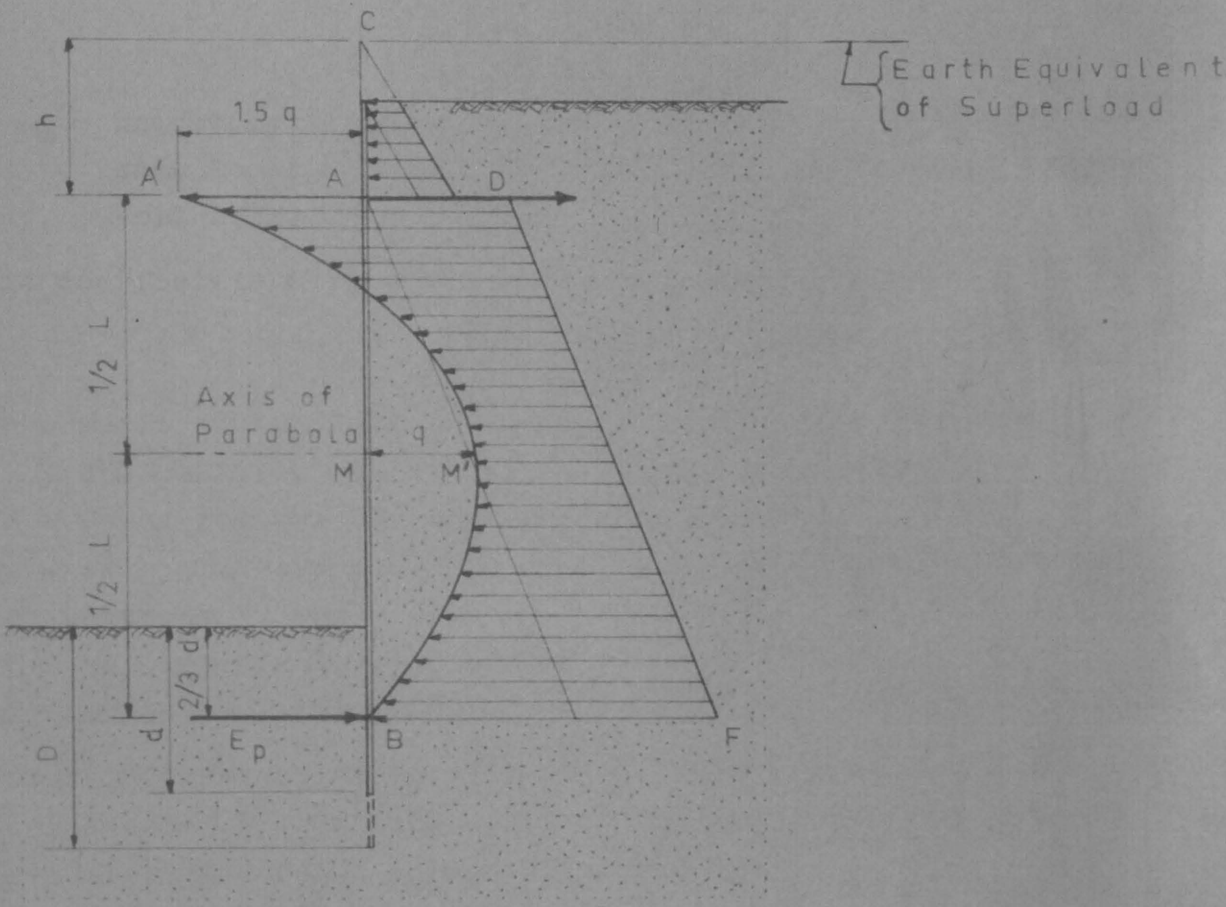


Fig. 3-16 Danish Method of Modifying Pressure Diagram for Flexible Walls in Sand

where p_m is the equivalent uniformly-distributed unit pressure on the wall between A and B, that will give with simple supports at A and B the same bending momentas the load area ADFBM'.

The pressure diagram ADFB is always very nearly a triangle; mathematically the moment at a level 1/2 L (Fig. 3-16) is always 1/8 PL for a triangular, rectangular or any trapezoid shape of loading diagram between these limits, and

$$p_m = 1/2\ (AD + FB)$$

where q = distance MM' in Fig. 3-16

L = distance AB in Fig. 3-16

h = height of surcharge and of the soil above the anchor level, transformed to correspond to the unit weight of the soil above the anchor level

p_m = uniformly distributed unit load which will produce the same bending moment in the sheeting as the load trapezoid ADFB.

The coefficient k is given by the empirical formula

$$k = \frac{1}{1 + \frac{0.01}{\sin\phi}\sqrt{\frac{(1+n)Ea}{L\sigma}}} \qquad \text{Eq. 3.22}$$

where n = ratio of the negative bending moment at anchor level to the positive bending moment of the span below it

E = Young modulus for sheet-pile material

a = wall thickness or, for steel sheeting, distance between extreme fibers

σ = permissible bending stress

ϕ = angle of internal friction.

For reinforced concrete piles k is generally between 0.7 and 0.85. For steel sheet-piles the combinations of other values of E and a often result in the same, or slightly greater.

The theoretical minimum depth of penetration d is estimated as being equal to 0.30 or 0.35 of the depth H_w from water level to the dredge line. The actual depth of penetration D for a factor of safety G_s=2.0 is made $D=\sqrt{2}.d$.

Once the pressure-distribution diagram has been established as indicated above, the bending moments and the anchor pull A_p are determined for estimation purposes

$$+M_2 = M_0 - \frac{M_1}{2} - \frac{17}{192} qL^2 \qquad \text{Eq. 3-23}$$

$$A_p = A_0 + A_1 - \frac{1}{12} qL \qquad \text{Eq. 3-24}$$

$$B = E_p = B_0 - \frac{M_1}{L} - \frac{1}{3} qL \qquad \text{Eq. 3-25}$$

where $+M_2$ = maximum positive bending moment to be used for design of sheet piling

M_o, A_o, B_o = bending moment and the two reactions of a beam of span L freely supported at A and B and loaded with the trapezoid ADFB

M_1 = negative bending moment at anchor level
A_1 = reaction component equal to the area enclosed by pressure curve above anchor level
q = pressure-reduction ordinate computed from Eq. 3-21.

Tschebotarioff made tests on large model flexible walls at Princeton and found agreement with the Danish rules in so far that the reduction in pressure due to arching occurred almost entirely below the level of the tie. However, the distribution of pressure for clean sand was very different from, and almost the contrary to, that assumed by the Danish rules. It was concluded that it appeared unadvisable to rely on arching effects in sand for a reduction of the bending moments, but rather the contrary if increased pressure by horizontal arching is possible. This distribution is valid only for a completely unyielding anchor support. Such a condition is not likely to arise in the field, except for sheet-pile bulkheads on the land side of relieving platforms. In addition, the method is valid only for minimum embedment D producing a condition on the verge of failure and it involves very complicated computation.

FIXED EARTH SUPPORT METHOD

The lower end of a bulkhead is assumed to be fixed if the depth of penetration of the sheeting is sufficient to produce an elastic line of the type represented in Fig. 3-17 by the dashed line E. The S-shape of this line represents the combined result of the flexibility and of the deep penetration of the sheet piles. Owing to the fact that the active earth pressure produced bending between the anchorage and the earth support, the bulkhead yields toward the left. As a result, passive earth pressure sufficient to maintain the equilibrium of the system is mobilized in the sand adjoining the upper part (bc) of the section (bd) of the sheet piles.

On the other hand, at a greater depth, below some point (c) located between (b) and (d) the sheet piles must deflect to the right of their original position, because if the sheet piles were long enough the elastic line E would ultimately become identical with the straight vertical line (an). This asymptotic approach to (an) can only be accomplished by successive deviations of the elastic line to the left and to the right of (an), which gradually die out with depth. A deflection of the elastic line toward the left of its original position

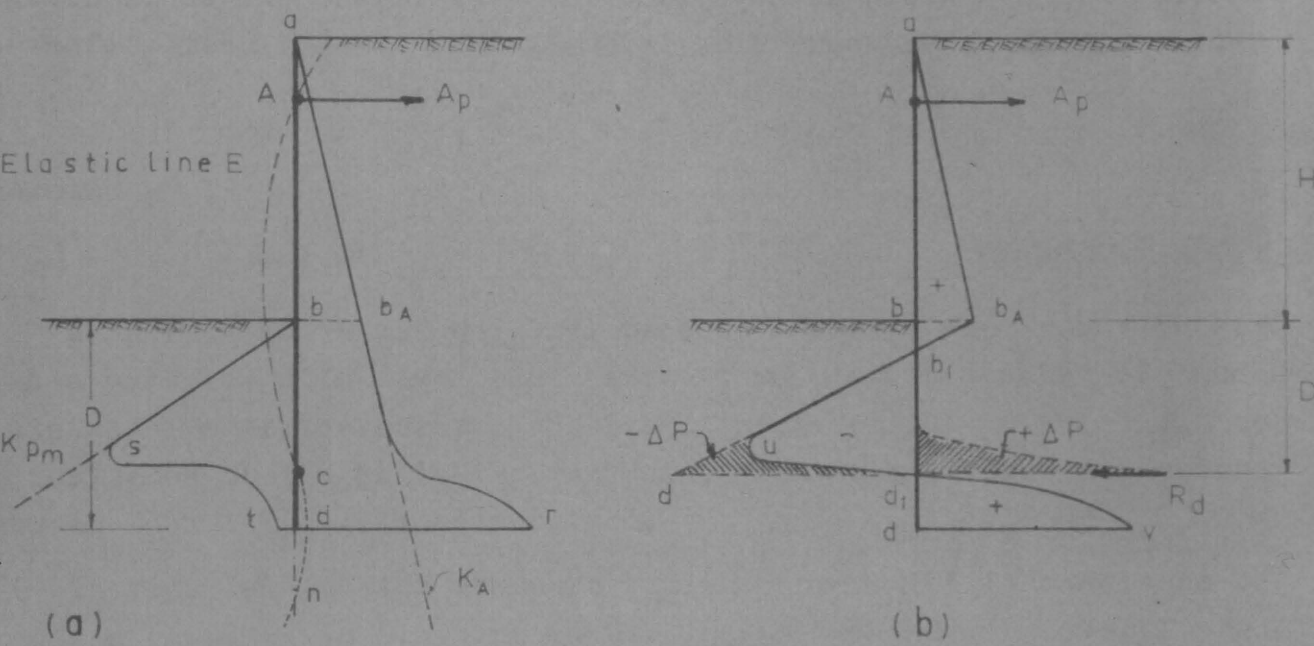


Fig. 3-17 (a) Real and (b) assumed distribution of horizontal pressures on the two sides of a bulkhead with fixed earth support.

tion involves the mobilization of the passive earth pressure on the left side of the sheet piles while the right face is acted upon by the active earth pressure. A deflection toward the right has the opposite effect. In order to ascertain the extreme boundaries for the values which the earth pressure can assume at different depths below the surface the designers reasoned in the following manner. The smallest value which the earth pressure on the right-hand face of the bulkhead can assume is equal to the active earth pressure. The horizontal component of this pressure is represented by the pressure area (anK_A), Fig. 3-17(a). The hori-ontal pressure exerted by the buried part bd of the bulkhead on the adjoining soil should nowhere exceed the horizontal component P_{pn} of the passive earth pressure divided by the safety factor G_s. At any depth z' below point b the pressure P_{pn} is

$$p_{pn} = \gamma z' k_p \qquad \text{Eq. 3-26}$$

wherein K_p is the coefficient of passive earth pressure. In order to the safety requirement the hori-ontal unit pressure should not exceed

$$p_{pm} = \frac{1}{G_s} p_{pn} = \gamma z' K_{pm} \qquad \text{Eq 3-27}$$

wherein

$$K_{pm} = \frac{1}{G_s} K_p \qquad \text{Eq 3-28}$$

As a rule it is assumed $G_s=1$ because there is hardly any danger that a bulkhead with fixed earth support may fail on account of inadequate passive earthpressure. However, the numerical value of G_s has no influence on the following analysis. Therefore, the distinction between K_p and K_{pm} will be retained.

In Fig. 3-17(a) the pressure P_{pm} is represented by abscissas of the line (bK_{pm}). No limiting values are needed for the lateral pressure exerted by the section (cd) of the bulkhead, because this pressure is always very small compared with what the soil can stand.

The effect of the deflection of the bulkhead on the intensity and distribution of the hori-ontal pressure on the two sides of the bulkhead is shown in Fig. 3-17(a) by the pressure area located between the bulkhead and the line (ab_Ard) for the right-hand side and by the pressure area (bstd) for the left-hand side. The resultant pressure per unit of area of the bulkhead is given by the abscissas of the line (ab_Auv) in Fig. 3-17(b) with reference to the vertical line (ad). These abscissas are equal to the algebraic sum of the abscissas of the lines (ab_Ar) and (bst) in Fig. 3-17(a).

Our first problem is to determine the depth to which the piles must be driven in order to get an elastic line similar to that shown in Fig. 3-17(a). In order to simplify this problem we add to the pressures indicated by the abscissas of the curve ($b_A uv$) (Fig. 3-17(b)) two equal and opposite pressures represented by shaded areas and we replace the entire positive pressure which acts on the lower end of the bulkhead, including the pressure + P represented by a shaded area, by its resultant R_d per unit of length of the bulkhead. The point of application of the resultant pressure R_d is located approximately at point (d_1). Thus we replace the two real pressure areas ($b_1 ud_1$) and ($d_1 vd$) with curved boundaries by one triangular area ($b_1 d_R d_1$) located on the left-hand side of the bulkhead section (bd) and one concentrated force R_d acting on the right-hand side of this section. The section ($d_1 d$) of the bulkhead located below the point of application of R_d is assumed to be fixed.

EQUIVALENT BEAM METHOD

The equivalent beam method represents a simplification of the elastic line method described in the preceding article. It involves a considerable saving in time and labor at a small sacrifice of accuracy. As the depth of the sheet piling increases the point of contraflexure will move up, and for the most economical depth, this point of contraflexure will almost coincide with the point of equal active and passive pressure intensities immediately under the lower surface. For computation purposes, therefore, the upper portion of the constrained sheet-piling wall, where the maximum bending moment occurs, can be replaced by a beam simply supported at the point of anchorage and the first point of zero pressure intensity. The shears and moments of this equivalent beam will approximate very closely the shears and moments in the constrained beam.

Fig. 3-18(a) and 3-18(b) illustrate the principle on which this method is based. Fig. 3-18(a) represents a loaded beam of which one end (b) is fixed and the other end (a) is simply supported. The corresponding moment curve is shown in Fig. 3-18(b). The point of inflection of the elastic line is at c. If we cut the beam at point c and provide a free support at c for the left section ac of the beam, the bending moments in the section ac remain unaltered. The beam ac is called the equivalent for the section ac of the beam ab.

The application of this reasoning to the design of bulkheads with fixed earth support is illustrated by Figs. 3-18(c) and 3-18(f). Fig. 3-18(c) represents the system of forces which act on the bulkhead according to Fig. 3-18(a) and Fig. 3-18(f) shows the corresponding moment curve.

In order to apply the equivalent beam method to our problem we must ascertain the location of the point at which the bending moment in the sheet piles is equal to zero. This point is identical with the point B in Fig. 3-18(f) at which the closing line intersects the moment curve. It is located at a certain depth x below the original surface of the ground. The bulkhead analysis by means of the elastic line method (Fig. 3-18) furnishes the following values of x for different values of Ø:

ϕ	=	$20°$	$30°$	$40°$
x	=	$0.25H$	$0.08H$	$-0.007H$

The angle of internal friction Ø of sandy bakcfills is approximately 30° corresponding to a value of x of about 0.1H. Hence, if both the backfill and the earth on the left-hand side of bd_1 in Fig. 3-18(c) are sandy we are entitled to assume $x = 0.1H$ without risking a serious error. This assumption makes it possible to solve our problem by means of the equivalent beam method, illustrated by Figs. 3-18(a) and (b). After tracing the boundaries $ab_A d_B d_1$ of the pressure areas, we cut the sheet piles at B (Fig. 3-18(c)) at a depth $x = 0.1H$ below point b. We replace the shearing force at B by a reaction R_B per unit of length of the bulkhead and replace the earth pressure acting on aB by a system of individual force 1 to 6 as shown in Fig. 3-18(d). We then construct the polygon of forces (Fig. 3-18(e)) and the corresponding funicular polygon (Fig. 3-18(f)) with the closing line AB. By tracing the ray OB in the polygon of forces (Fig. 3-18(e)) parallel to the closing line AB in Fig. 3-18(f) we obtain the magnitude of the reaction R_B as well as that of the anchor pull A_p as shown in Fig. 3-18(e). The maximum bending moment M_o in the sheet piles is determined by the funicular polygon (fig. 3-18(f)).

The lower part Bd_1 of the sheet piles (Fig. 3-18(c)), with a depth $D_1 - x$, is acted upon by the upper reaction R_B, by the earth pressure represented by the pressure area located between $B_R d_R$ and Bd_1 (Fig. 3-18(c)), and by the lower reaction R_d. The moments about any point, for instance point d_1 in Fig. 3-18(c) must be equal to zero. This condition requires that

$$\frac{(D_1-x)^2}{2}(K_{pm}-K_A)\frac{(D_1-x)}{3}\gamma' + \left[(K_{pm}-K_A)x - K_A H)\right]\frac{(D_1-x)^2}{2}\gamma = R_B(D_1-x) \qquad \text{Eq. 3-29}$$

Solving this equation we obtain

$$D_1 = \frac{3}{2}H\frac{K_A}{K_{pm}-K_A} - \frac{x}{2} + \sqrt{\frac{6R_B}{(K_{pm}-K_A)\gamma'} + \frac{9}{4}\left(x - H\frac{K_A}{K_{pm}-K_A}\right)^2} \qquad \text{Eq. 3-30}$$

The second term under the root is very small compared with the first one and can be neglected. Hence we can write

$$D_1 = \frac{3}{2}H\frac{K_A}{K_{pm}-K_A} - \frac{x}{2} + \sqrt{\frac{6R_B}{(K_{pm}-K_A)\gamma}} \qquad \text{Eq. 3-31}$$

The value D_1 is obtained by means of computations shown above. For practical purposes it is admissable to assume without any further investigations that the sheet piles should be driven to a depth $D = 1.2D_1$.

GRAPHICAL METHOD

The following graphical procedure (which is approximate, but practical) has been found to be in reasonable agreement with more

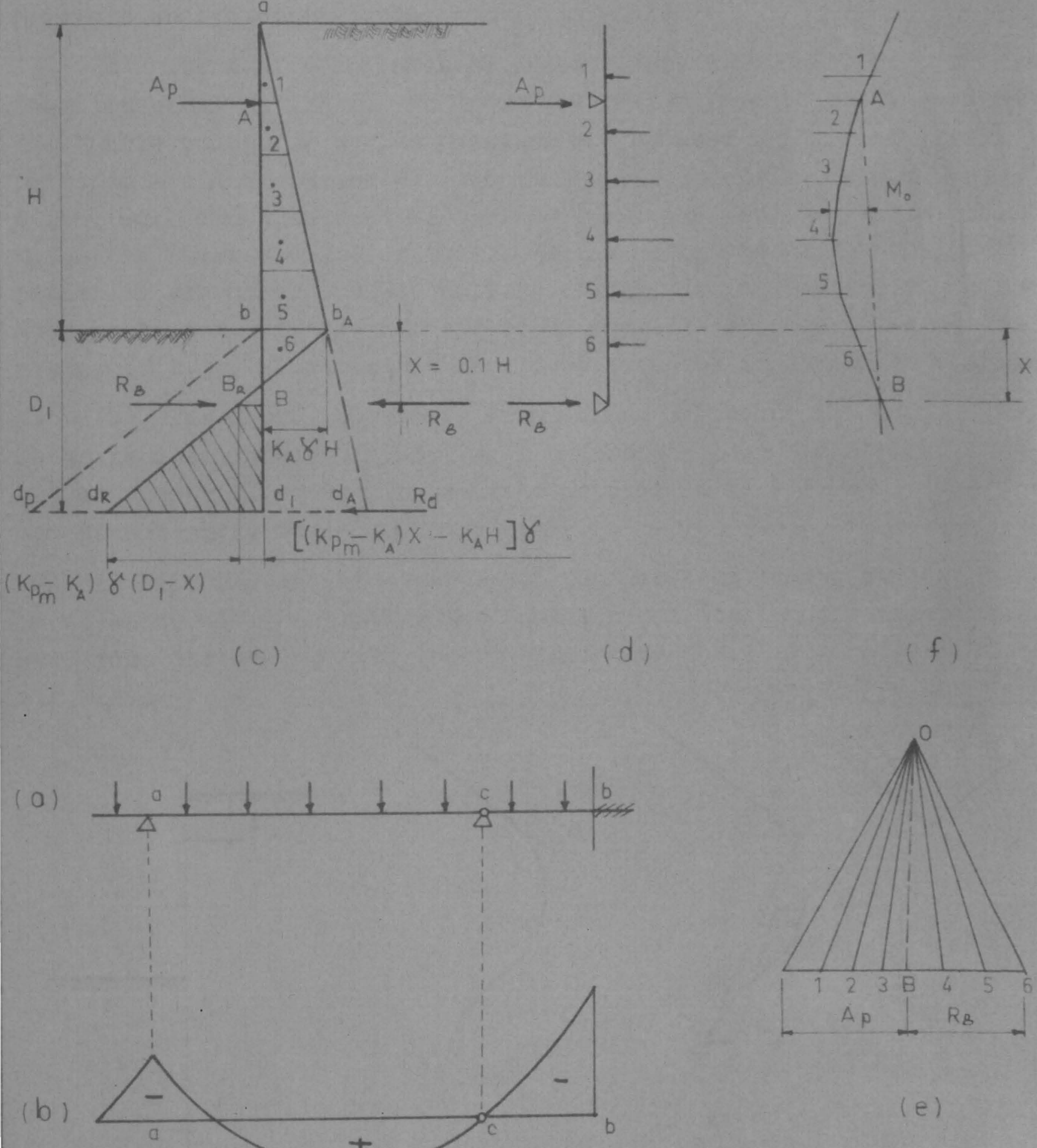


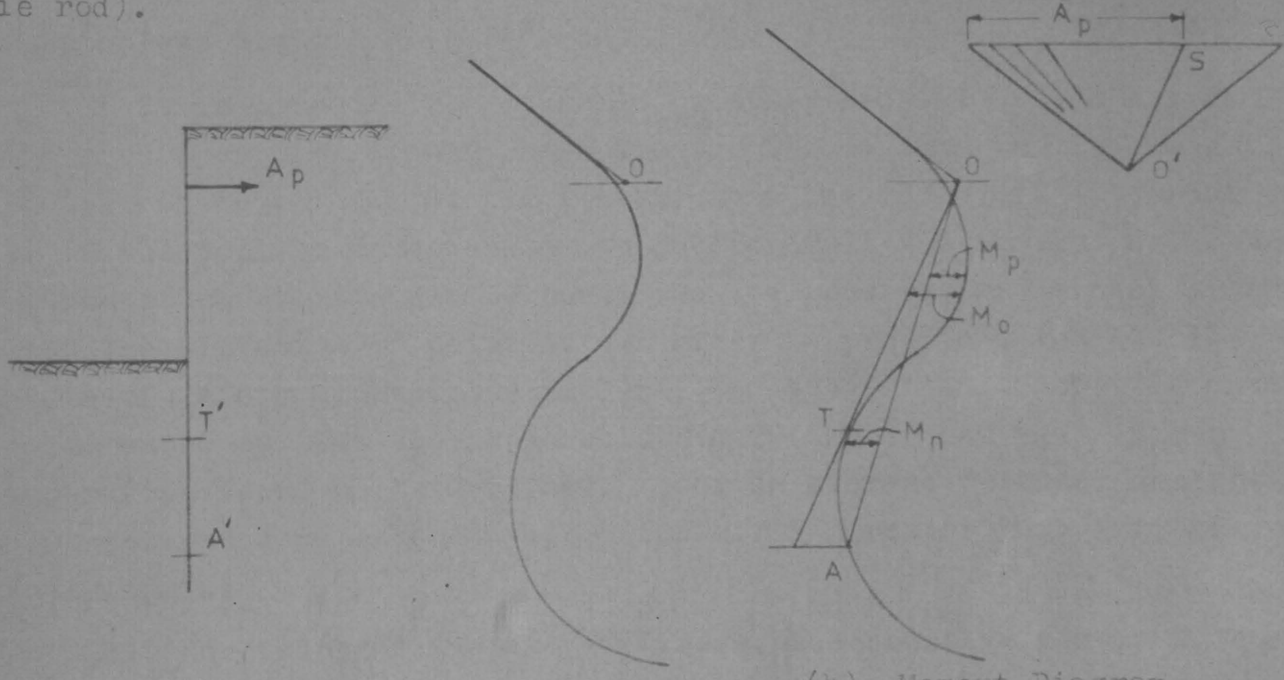
Fig. 3-18 Equivalent beam method for computing anchor pull and maximum bending moment in anchored bulkhead with fixed earth support.

rigorous analytical methods:

(a) The active and passive pressure are computed using full wall friction equal to 2/3 Ø. To avoid arithmetical work in the computation the Rankie values of active pressure can be used - neglecting wall friction - and the error will be small. In computing passive pressures, wall friction must be included, and the passive pressure can be twice the Rankine value or more. Values of K_p can be obtained from tables or graphs to include wall friction. It is also important that account be taken of the effects of hydrostatic or pore pressures as a result of tidal fluctuations or ground water on the pressure diagram.

(b) The force polygon is drawn from the vector diagram. There is no need to locate the closing line precisely by the elastic line method, however, since its position depends on the degree of the toe, and is extremely variable.

In Fig. 3-19(a), the force polygon has been drawn, and the closing line, which will constitute the base line of the bending moment diagram will pass through point O (point O lines on the line of action of the tie rod).



(a) Force Polygon (b) Moment Diagram

Fig. 3-19 Graphical Analysis

The base line will actually lie somewhere in between line OT (which is tangential to the force polygon), and line OA (which is drawn so that $M_n = M_p$). If there is no fixity at the toe, the maximum value will be M_p (which is equal to M_n).

(c) The tie-rod pull is obtained from the vector diagram by drawing a vector O'S parallel to line OT, and measuring the resultant force. The value of the tie-rod pull obtained by this procedure (assuming that there is no toe fixity) is a maximum. In most cases where toe fixity occurs, the tie-rod pull is less, but by using the maximum figure an adequate safety factor is automatically introduced into the design. No safety factor is required other than the use of a normal working stress for the tie-rod - 20,000 lb per sq in. if structural steel is used.

(d) The required penetration of the toe of the pile is determined after the amount of toe fixity has been estimated. The toe penetration depends on (1) the density of the soil and (2) the section modulus of the sheet piling. Sand and gravel can generally be considered to be dense, unless it is from a recently deposited bed or in a loose, cohesionless state.

If it is proposed for economic or other reasons to use a sheet pile with a section modulus adequate for the maximum bending moment, the toe can be cut off at the elevation of the point T'. Alternatively, if the soil is dense enough to develop full toe fixity, a lighter section of sheet pile can be used; the toe penetration is then extended to the elevation of point A. In order to provide a reasonable margin of safety against toe failure, the actual length of pile must be increased so that the penetration is 20% more than the minimum penetration found by this method. (For an average bulkhead design this requires a pile length approximately 5% greater than the theoretical length.)

(d) For piles of average stiffness, the positive moment M_p can be used to obtain the required section modulus (assuming that M_n equals M_p, as shown in Fig. 3-19(b).

For very flexible sheet piling, the actual bending moments will be smaller than M_n or M_p, but it is not advisable to make any further reduction in the section modulus of the sheet pile without a very thorough investigation of soil properties and a rigorous analysis of earth pressures and moments for the particular pile section that it is proposed to use.

For very stiff piles or for piles embedded in a loose soil, the bending moment can be taken as the full positive value without end fixity (M_o in Fig. 3-19(b). In the case in which the density of the soil is doubtful, value of M_n equal to 1/2 M_p can be used.

For determining the bending moment the graphical method is the most satisfaction and accurate. It is also possible to determine the depth to which the sheet piling must be driven in order to utilize to the maximum the constraining action of the earth and thereby develop the least maximum bending moment for a given lateral load. In this method it is necessary to construct a deflection diagram, which develops into a very intricate problem. For a simple problem, the analytical method would be quicker and easier. For more complicated conditions, the graphical method is simple and more accurate and has the further advantage of being more amenable to analysis if the tie-rod elevation has to be adjusted or the sheet piling reinforced. However, in most cases the analytical method is sufficiently accurate for use in practice and for a check on the graphical solution.

TSCHEBOTARIOFF'S METHOD

Fig. 3-20(a) illustrates the lateral-earth-pressure diagram which is proposed for the design of flexible anchored bulkheads in sand. It corresponds to the results of the model tests with flexible anchored bulkheads at Princeton University, which showed that full restraint of the lower portion of the bulkhead was effective when the ratio D/H equaled 0.43, where the point of contraflexure approximately corresponded to the dredge-line elevation (x = 0) for normal backfilling operations. This depth of embedment D provides a factor of safety of at least $G_s = 2.0$, since tests performed with a ratio D/H = 0.27 still showed full stability of the bulkhead.

The proposed procedure represents a simplification of Blum's equivalent-beam method. A depth of embedment D = 0.43H is selected, and a hinge is assumed at dredge-line elevation. The active pressure p_h above dredge-line elevation are computed with the help of the equations

$$K_A = \left(1 - \frac{a}{f'H}\right) 0.33 f'' \qquad \text{Eq. 3-32}$$

$$p_h = K_A \gamma h \qquad \text{Eq. 3.33}$$

In the above equations h is measured from the top of the backfill downward. γ is the unit weight of the backfill, and the product γh represents the weight of the overburden at the depth h. The coefficient f''' is intended to express the effect of wall friction on the reduction of the active earth pressure. It can be taken into account the uncertainty concerning the relative importance of the passive earth pressures above anchor level and the tensile strength of the sand layer saturated by capillarity above the water level.

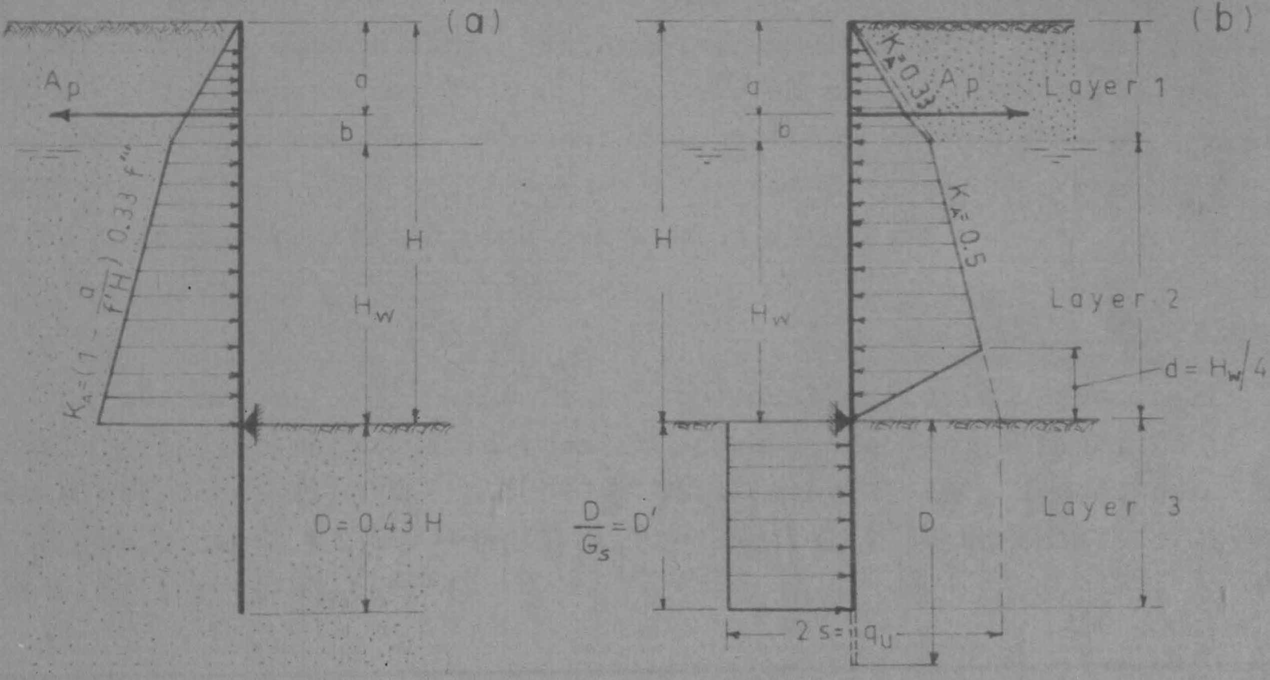


Fig. 3-20 The simplified equivalent-beam method for the design of anchored bulkheads. (a) in clean sand (b) in clay

This coefficient may vary between the values f' = 1.5 and f'' = 3.5. It is recommended that until further studies and observations are made, the value of this coefficient be taken for design purposes at f' = 3.5.

The maximum positive bending moment computed with the help of Fig. 3-20(a) and Eq. 3-32 can be used for design purposes in conjunction with a 33 percent increase in the permissible unit stresses in the steel from 18,000 psi to 24,000 psi. The factor of safety of $G_s = 1.4$ which is thus obtained in respect to the yield point of the steel is believed to be sufficient to take care of vibration and of other possible unfavorable effects where sheet piling is used in clean sand, in view of the ductility of the piling material and of the observed decrease of the active lateral pressures which takes place with an increased flexibility of the bulkhead. Many earth structures, including dams and embankments, often do not have a factor of safety against rupture of the soil higher than $G_s = 3.0$. To equalize the strength of these parts of the structure, some designers have suggested an increase of the permissible unit stress in the sheet piling to the elastic limit. Danish designers appear to have actually done so in practice.

No corresponding decrease of factors of safety is permissible in the design of the anchors and of their supports and connections, especially if there is some uncertainty concerning the properties of the soil beneath the dredge line. Any excessive yielding of the soil in front of the bulkhead may be counteracted, in part, by an increased resistance of the anchor. For that reason it is recommended to increase in all cases, the anchor-pull values computed from the diagram of Fig. 3-20(a) by dividing them by the expression

$$\left(1 - \frac{a}{f'H}\right) f'' \qquad \text{Eq. 3-34}$$

The coefficient f'' can be taken to equal unity in cases where reliable granular material is located beneath the dredge line. Any uncertainty concerning the nature of this material and, hence, concerning the safe depth of embedment, may be partially compensated for by decreasing the value of f''.

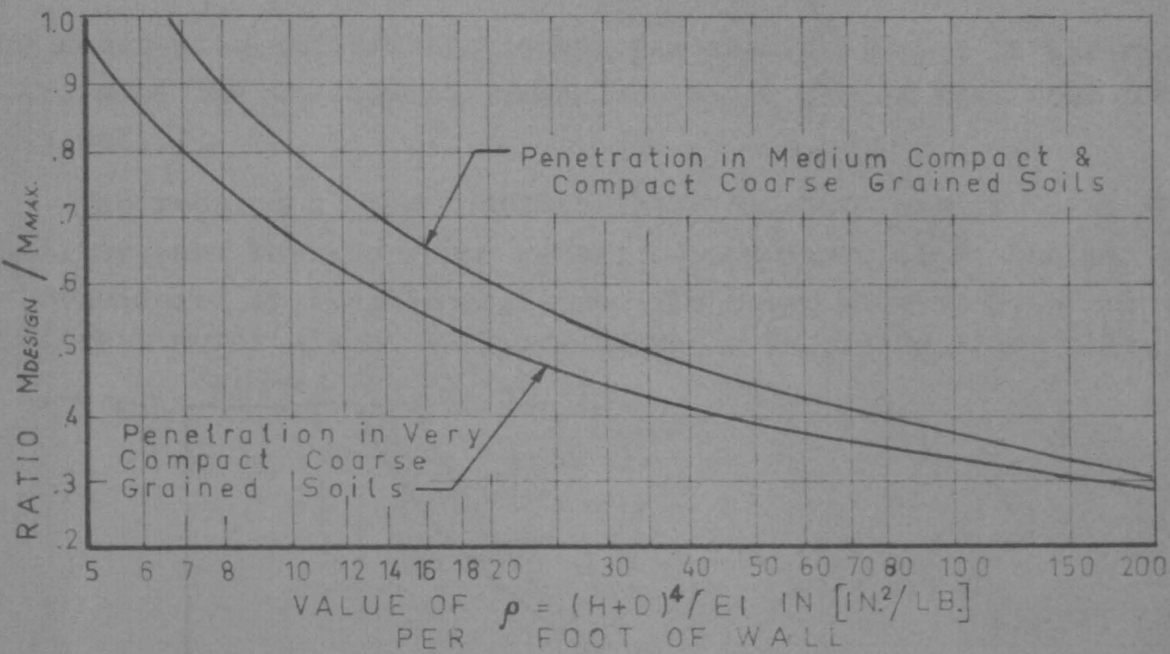
Fig. 3-20(b) suggests a lateral pressure diagram for the design of anchored bulkheads in clay soils. The proposed depth of embedment D is based on the ultimate strength of the clay below the dredge line and a factor of safety $G_s = 2.0$. Therefore, D = 2D', is established by taking moments around the point of application of the anchor pull A_p. If both the layers 2 and 3 are composed of soft clay, the distanc d in Fig. 3-20(b) should be made equal to zero, and the clay of both layers in front and behind the bulkhead may be strengthened by means of sand piles or drains.

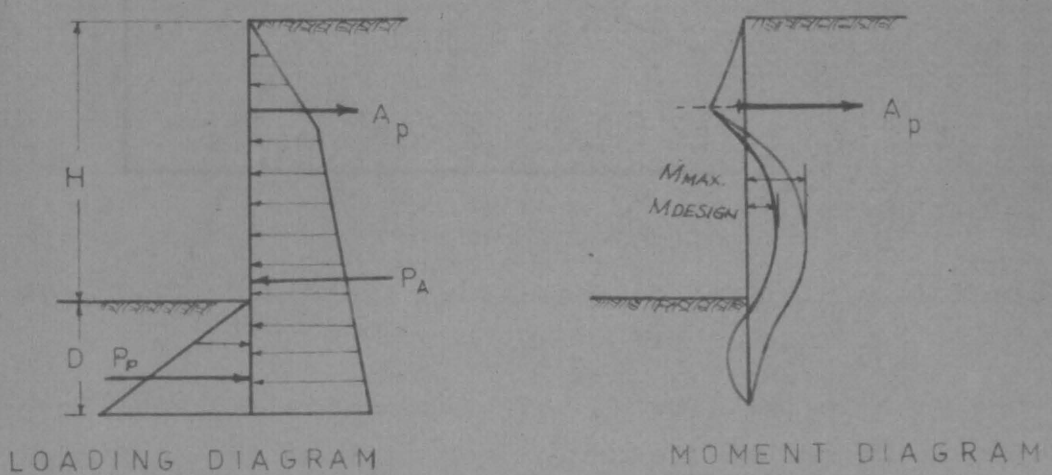
ROWE'S METHOD

The investigations of Mr. Rowe have shown that no tangible benefits can be obtained by driving the sheet piles deeper than the depth required to assure an adequate margin of safety with respect to a failure resulting from an outward movement of the buried part of the sheet piles, and a sufficiently small horizontal displacement of the lower edge of the sheet piles.

If the profile of the soil into which the sheet piles will be driven is erractic or if no reliable data concerning the details of the soil profile are available, the maximum bending moment in the sheet piles should be computed on the assumption of free earth suppor The maximum bending moment can be determined by analytical or graphic methods in the usual manner.

Briefly a stiff wall will be subject to free-earth-support conditions of pressure. If the wall is made more flexible, the pressure distribution will approach that assumed by the fixed-earth-support pressure diagram, and at very high flexibilities the pressure distribu tion will approach that corresponding to complete fixity. The change is a continuous process, independent of all variables in the design other than the relative density of the subsoil. The results of the investigations conducted in Scotland are shown in the two empirical moment-reduction curves. In Fig. 3-21 (H+D) is the total height of the sheet piling in inches; E denotes the modulus of elasticity of





Notes

1. M_{design} is obtained by successive trials of sheeting si-e until max. bending stress in sheeting equals allowable bending stress.

2. No reduction in $M_{max.}$ is permitted for penetration in fine grained soils or loose or very loose coarse grained soils.

3. Flexibility number is computed on the basis of lubricated interloc

Fig. 3-21 Reduction in bending moments in anchored bulkhead due to wall flexibility.

the sheet-pile material in pounds per square inch; I is the moment of inertia of the sheet-pile cross section in inches ^{4} per foot of sheet-pile wall.

The results of the Scotland investigations signify that the flexibility, and therefore the nature of the material of the piling must be considered in the design. Specific moment-flexibility curves for different materials of wall are shown in Fig. 3-22 which allow the

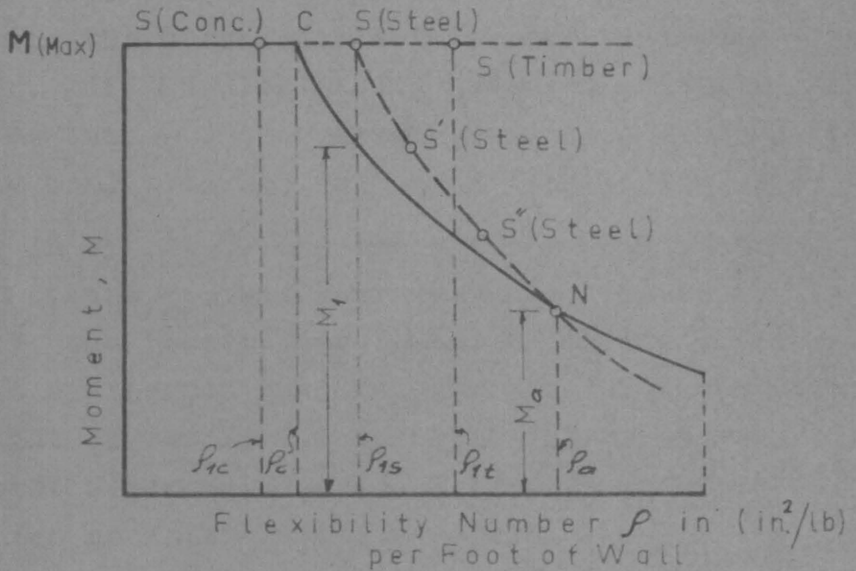


Fig. 3-22 Graphical procedure for determining tolerable moment reduction.

choice of the most economical type of wall and size of section. In Fig. 3-22 M is the maximum bending moment on the sheet-pile wall in pound inches per foot of wall length. Curve A was plotted from the maximum free-earth support moment and by interpolating from Fig. 3-21 for sand with medium density will be considered.

The first step consists of computing the maximum bending moments for free earth support, M(max), and of the cross section of the piles required to withstand M(max). The flexibility number ρ of these piles is determined by Eq.

$$\rho = (H+D)^4 / EI \qquad \text{Eq 3.35}$$

At a specified value of M(max) the moment of inertia I and the corresponding flexibility number ρ depend on the value assigned to the allowable fiber stresses and on the construction material. Hence, at specified value of M(max), very different values of ρ may be obtained such as ρ_{1t} for timber piles, ρ_{1s} for steel piles, and ρ_{1c} for reinforced concrete piles.

The next step consists in plotting the moment-reduction curve, which is shown as the heavy curve in Fig. 3-22. This curve is obtained by multiplying the ordinates of the curve for sand with medium density in the Rowe diagram (Fig. 3-21) by M(max). The abscissa ρ_c of point C at which the curve starts to descend represents the critical flexibility number. If the flexibility number of the pile required by M(max) is smaller than ρ_c, the maximum bending moment in this pile is determined by the condition of free earth support and is equal to M(max).

In Fig. 3-22 the sheet piles with flexibility numbers ρ, such as ρ_{t}, ρ_{1c}, and ρ_{1s}, are represented by points S. At a given value of M(max), the flexibility number of timber sheet piles with the required moment of inertia has the greatest value-designated as point S (timber)- and that of reinforced concrete piles the smallest value-designated as point S(concrete). Point S(steel), representing steel sheet piles, occupies an intermediate position.

If point S is located on the left-hand side of C, no moment reduction can be tolerated. A position of point S on the right-hand side of C indicates that the pile represented by the point is stronger than necessary because the maximum bending moment in the pile M_1 will be less than M(max). In order to select a more economical profile for the sheet piles, the allowable bending moments M', M",... and the corresponding flexibility numbers ρ', ρ'', ... for various weaker profiles are computed. In Fig. 3-22 these weather sheet piles are represented by points such as S'(steel) with the ordinate M' and the abscissa ρ'. All these points are located in proximity of a curve that intersects the moment-reduction curve at N, with abscissa ρ_a and ordinate M_a. The corresponding moment reduction is M(max)-M_a. However, because of the rudimentary state of present knowledge of the performance of bulkheads under field conditions, the computed bending moment M(max) should be reduced by not more than 1/2 (M(max)-M_a).

If the sheet piles are to be driven into a homogeneous of dense or medium silty sand, Mr. Rowe's moment-reduction curves for medium and loose sand should be used instead of those for dense and medium sand. Sheet piles to be driven into loose, silty sand should be dimensioned for free earth support because the copressibility of such sands may be very high.

ANCHORED SHEET PILE DESIGN BY NUMERICAL METHOD

Dr. Newmark's numerical method for solving beam problems was used and direct solutions were obtained rapidly and easily by this method.

The following is a brief description of the numerical procedure for analyzing beam problems which was developed by Dr. Newmark. The advantage of this procedure is that almost any beam problem can be reduced rather easily to a problem of simple arithmetic. Fig. 3-23 shows a cantilever beam carrying a load which has a random distribution of intensity. If we cut the load at various intervals along the span and apply the load within each interval to a simple supported auxiliary beam, the reactions required to support each end of the simple beam can be computed. If the length of the simple beam is designated by the term "h", it is obvious that if "h" is very small, a simple trapezoidal load distribution will be good approximation for the actual distributi

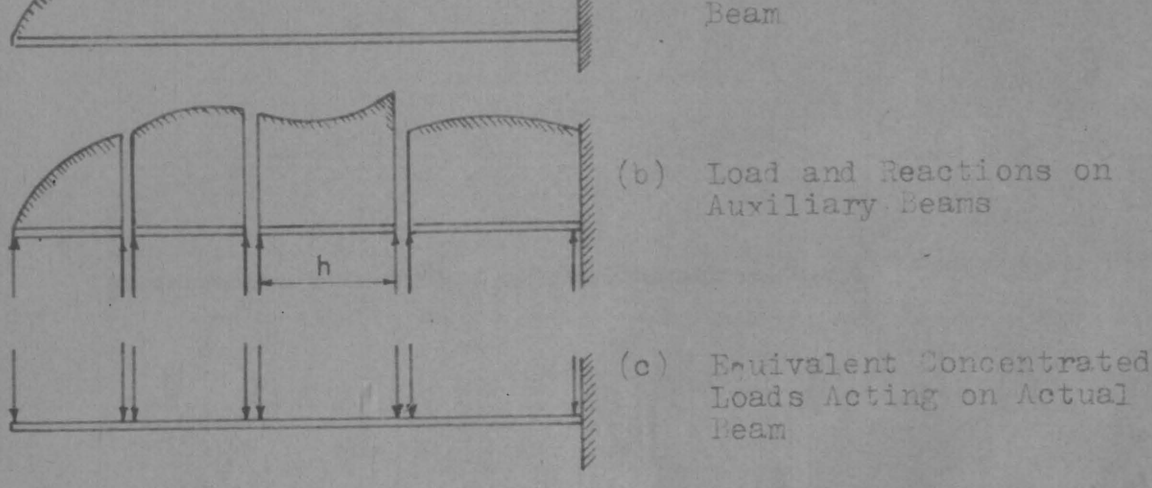
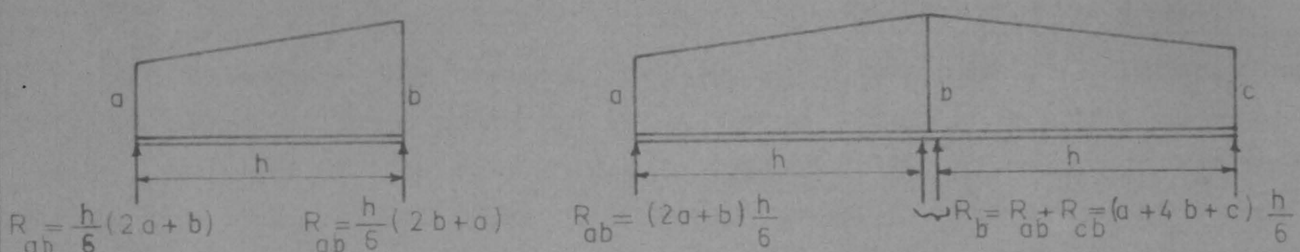


Fig. 3-23 Method of replacing a distributed load by concentrated loads on a beam.

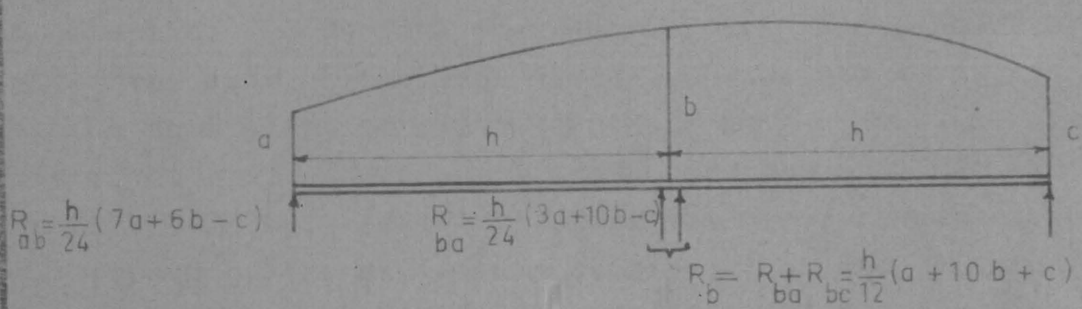
over this interval. As the length h increases, the trape-oidal loading will probably be more inaccurate. In order to make a better approximation to a curved loading diagram, a parabola may be passed through any given three points.

Fig. 3-24 gives the expressions for the reactions at the ends of the auxiliary beams which carry trape-oidal or parabolic loads. For a given problem either set of reactions may be used and the results will nearly be identical if "h" is taken very small. For larger valves of "h", the straight or curved loading diagram will be chosen, whichever fits the conditions better.

After end reactions for the auxiliary beams have been computed, these reactions are now applied to the actual beam. We have replaced the distributed load on the beam by a set of concentrated loads which are statically equivalent to the original load. This condition is illustrated in Fig. 3-23(c).



(a) Trape-oidal Loading



(b) Parabolic Loading

Fig. 3-24 Expressions for the Reactions at the Ends of the Auxiliary Beams.

The sign convetion described in Dr. Neward's paper is as follows:

+ loads act upwards;
+ shears act upward on the left side of a cut section;
+ moments produce compression in the top fibers of the beam;
+ deflection is downward; and
+x is to the right.

The deflections and slopes are assumed to be small so that

$$\frac{d^2y}{dx^2} = \frac{d^2y/dx^2}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}} = -\frac{M}{EI}$$

and the ordinary beam relations hold, that is

$$y = \text{deflections}$$

$$\frac{dy}{dx} = \theta = \text{slope}$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = \alpha = \text{angle change}$$

$$\frac{d^3y}{dx^3} = -\frac{dM}{dx}\frac{1}{EI} = -\frac{V}{EI} \quad \text{or} \quad EI\frac{d^3y}{dx^3} = V = \text{shear}$$

$$-EI\frac{d^4y}{dx^4} = \frac{d^2M}{dx^2} = \frac{dV}{dx} = p = \text{distributed load}$$

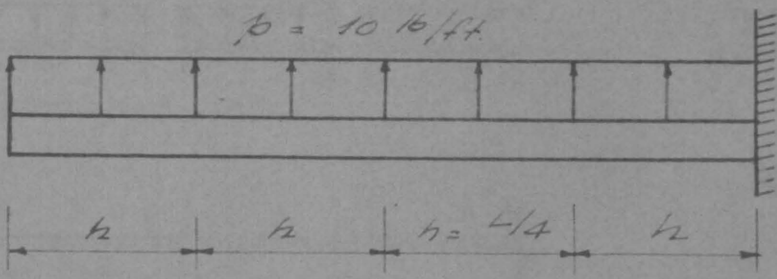
To determine the shear at any point in the beam, it is only neces sary to start at some point of known shear (usually a point of zero shear), and add up the concentrated loads as the computation progresses across the beam. Fig. 3-25 is a sample problem consisting of a canti-lever beam carrying a uniform load of 10 lb per ft. The equivalent concentrated loads are determined by use of the equations in Fig. 3-24 (since the loading diagram is a straight line) and are designated as $\bar{p}$ in Step No. 2. To find the shear at any point, start from the left end (free end) of the beam where the shear is zero, and add the values of the equivalent concentrated loads as the computation proceeds toward the right end. The shear diagram consists of a series of steps as shown in Fig. 3-25(b). If the value of the shear in each auxiliary beam is superposed on the step-shear diagram, the exact diagram is obtained. For our purpose, however, it is sufficient to note that the step shear diagram coincides with exact value at mid-span of each h-interval. Consequently, in Step 3, the value of shear is written in the middle of the h-interval. When it becomes necessary to evaluate the shear at the end of the h-interval, the concentrated load acting at the right end of the h-length must be computed and added to the value of the average shear over this h-length. That is, at a point b, as shown on Fig. 3-24, R_{ba} must be computed and added to the value of average shear over the length ab to give the correct shear at point b.

In order to evaluate the moments, it is necessary to multiply the value of shear by its moment arm to the point in question. Ordinary it is easier to solve these problems by assuming all values of h to be equal. Then to find moment, the shears are simply added from left to right and the common multiplication factor is carried as a common term in the right hand margin of the problem. In Fig. 3-25(a) h/6 is the common term for the concentrated loads and shear. The moment requires multiplication by another h so the common term now becomes $h^2/6$. The value of moment is exact at each h point.

The distributed angle change, $=M/EI$, consitutes the load on the conjugate beam. Since the moment diagram is curbed the concentrated reactions are computed from Fig. 3-24(b). The term h/12 becomes a factor in the common term. The concentrated angle changes correspond to the concentrated loads of Step 2. For Step 7, to determine the slope (or the shear on the conjugate beam), start from the right end of the beam. When proceeding from the right end of the beam to the left, subtract the next number. Thus for zero slope at the right end, the concentrated angle change is -2430, so the slop at mid-h is +2430, etc., until the left is reached, where the slope is $+7680 \dfrac{h^3}{72\,E\,I}$. The slope at this point is consistent with the sign convention mentioned previously.

For deflections, again start at the right (fixed) end and subtract the values of slope as computation proceeds toward the left end. The value of deflection at the left (free) end is found to be $-23{,}000 \dfrac{h^4}{72\,EI}$, or $y = \dfrac{pL^4}{8EI}$.

This answer is identical to the exact answer because the loadin diagrams p, and α, consist of straight lines and parabolic curves,



STEP NO.										Common Terms	Units
1. Distributed Load p	10		10		10		10		10		#/ft
2. Equivalent Concentrated Load P	30		60		60		60		30	$h/6$	lb
3. Shear V	(0)	30		90		150		210	(240)	$\frac{h}{6}$	lb
4. Moment M	0		30		120		270		480	$\frac{h^2}{6}$	ft.lb
5. Angle Change $\alpha = M/EI$	0		-30		-120		-270		-480	$\frac{h^2}{6EI}$	$\frac{1}{ft}$
6. Concentrated Angle Change α	-30		-420		-1500		-3300		-2430	$\frac{h}{12}\frac{h^2}{6EI}$	—
7. Slope θ	(7680)	7650		7230		5730		2430	(0)	$\frac{h}{12}\cdot\frac{h^2}{6EI}$	—
8. Deflection y	-23040		-15390		-8160		-2430		0	$\frac{h^2}{12}\cdot\frac{h^2}{6EI}$	ft

$$y_{max.} = \frac{-23040\,h^4}{72EI} = -\frac{1.25\,L^4}{EI} = -\frac{pL^4}{8EI} \quad \text{(Upward Deflection)}$$

NOTES. 1. Trapezoidal loading expressions used to get equivalent concentrated loads from distributed loads.

2. Parabolic loading expressions used to get equivalent concentrated angle changes from distributed angle changes.

COMPUTATION OF THE DEFLECTION OF A CANTILEVER BEAM CARRYING A UNIFORM LOAD

FIG 3-25a

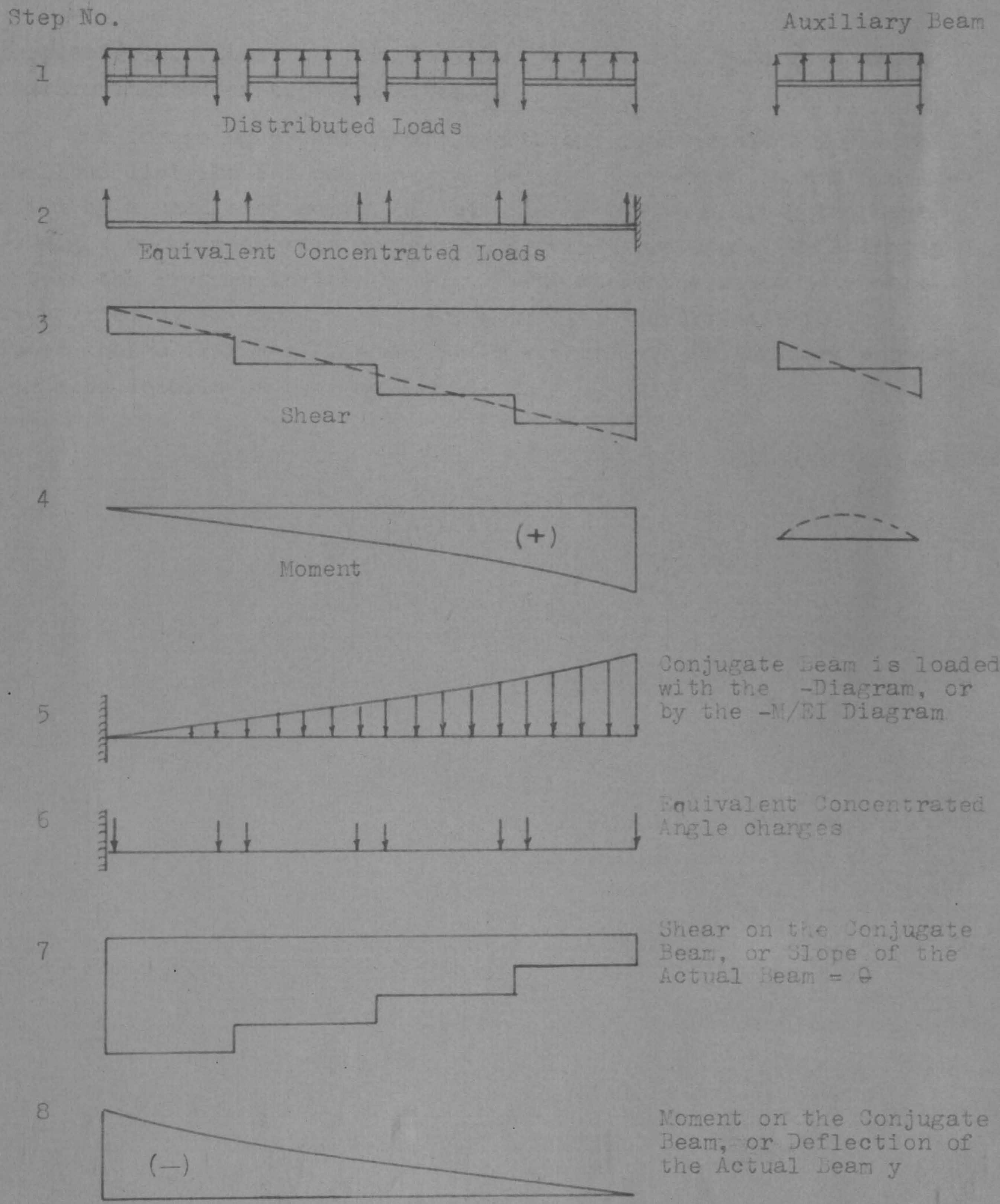


Fig. 3-25(b) Diagrams of the Values Computed in (a) respectively. Thus, the expressions given in Fig. 3-24 gave exact answers instead of approximations.

The foregoing procedure applies to any beam problems for which the load distribution can be established. The method is not complicated by a change of moment of intertia of the beam along the span as might occur when reinforced pile sections are used. This simply enters the picture in Step 5 (Fig. 3-25) where the value of moment is divided by the moment of inertia to give the ordinate to the angle change diagram. The reader is referred to Dr. Newmark's paper for more details on this method.

CHAPTER IV

CANTILEVER SHEET PILE WALLS

The simple case of a sheet pile wall is where sheet piles are driven to a sufficient depth to retain the earth without any support at the top of piles. Cantilever sheet piling wall is a wall which has no anchorage or bracing, but owes its stability exclusively to the fact it is supported by earth on both sides.

Cantilever walls may be used to advantage on occasion but should generally be avoided where an anchored wall can be used to high beam strength reouirements for corresponding lateral loads. In permanent cantilever walls the piles are apt to creep out of alignment due to lack of uniformity in the supporting soil at the toe and due to frost action and hydrostatic head in the retained soil. Cofferdam walls of sheet piling are freouently cantilevered for some distance down to the first row bracing this construction does not, however, depend on soil alone for support as in a true cantilever.

Cantilever walls may be applicable for some structures, such as harbor protection walls in shallow water, jetties, and wing walls for bridge abutments. Lateral stiffness and added resistance to pressures may be gained by so called buttress piles: that is, two, three or more sheet piles driven at right angles to the wall at suitable centers and connected to the sheet pile wall by tee connections. Welding the tops of these buttress piles greatly increases their supporting value.

PRESSURE AND LOADING DIAGRAMS

Cantilevered sheet piling may be used to protect an excavation against lateral earth pressure and water during the process of excavation and of building the foundation or structure that goes in it. In almost all cases, the sheet piling is removed when no longer needed. Sometimes the excavation inside the sheet piling is not unwatered, but generally one of the most important functions of this temporary structure is to facilitate conduct of the work "in the dry". One simple type of sheet piling is shown in Fig. 4-1(a). It consists of a single wall of steel sheet piling which encloses an area and which is driven into the ground below the bottom of the proposed excavation far enough to enable the earth at BD to hold the piling in place. The lateral pressure of the ground above D bends the piling as a beam that is cantilevered above this vicinity. The supporting soil is not rigid hence the upper resultant reaction is below D. The lateral pressure is assumed to be hydrostatic in character. However, the most uncertai part of such a sheet piling is the resistance of the soil below D.

Fig. 4-1 shows idealized pressure diagrams for a cantilever bulkhead. Fig. 4-1(b) indicates minimum penetration of the sheeting as required for stability. This bulkhead is subject to progressively increasing deflection as the soil at the surface of the ground yields under pressure and with time. The deflection can be reduced by increasing penetration of sheet piles as shown in Fig. 4-1(c). For

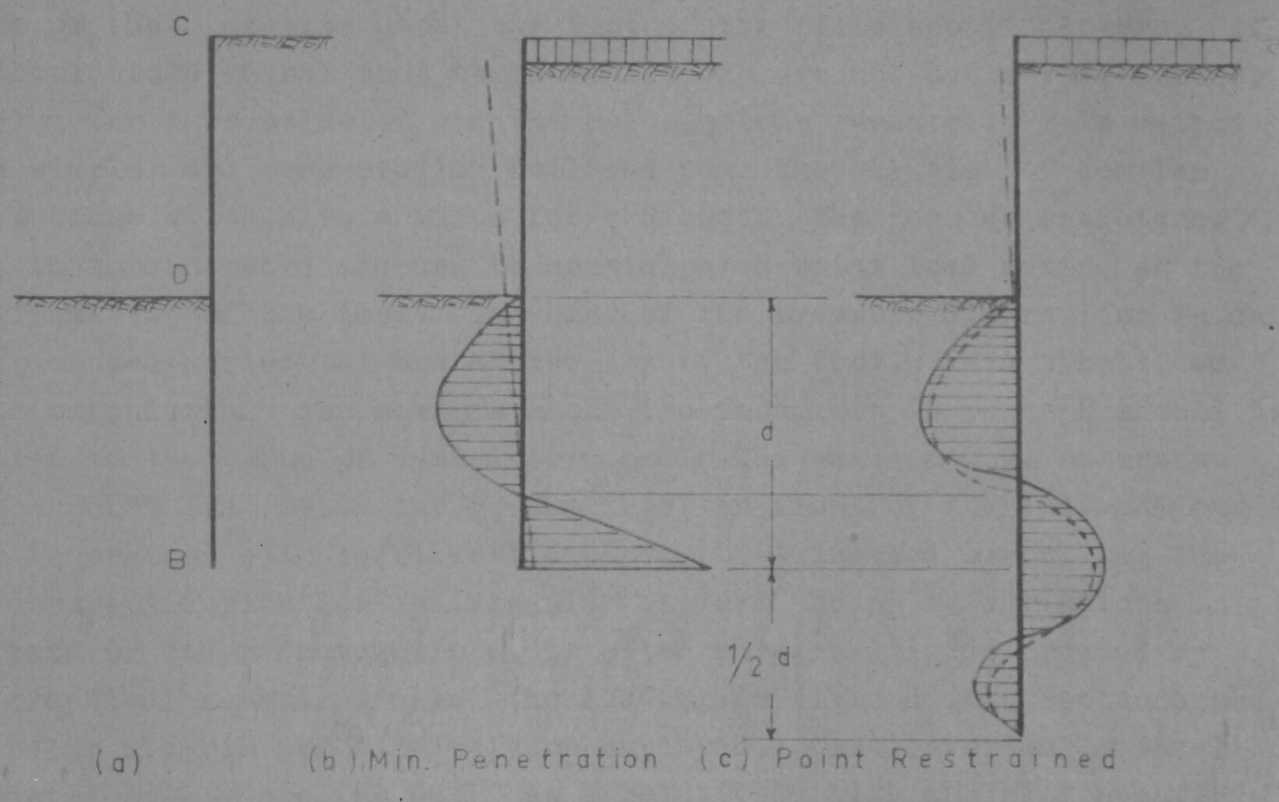


Fig. 4-1 Pressure Diagrams

bulkheads where alinement and safety are important, increased penetration above the minimum reouired for stability is essential. Cantilever bulkheads should be of limited height and should be restricted to use in consolidated or compacted soils.

DESIGN OF CANTILEVER SHEET PILING

The type of calculations for a cantilever sheet pile wall are indicated below. In this example the loading is simple, but the method can be readily applied to submerged and surchanged conditions. Fig. 4-2(a) shows the derivation of the pressure diagram and Fig. 4-2 (c) the probable combined diagram together with an approximation consisting of straight lines. The depth d of penetration for stability can be determined by trial and error. An approximate value for d is selected (Fig. 4-2(a)) and lateral forces at the back and front of the wall are determined from the pressure diagram (Fig. 4-2(c)). The algebraic sum of these forces should be zero, and the algebraic sum of their moments about the foot of the piles should be zero. If calculations reveal that these conditions are not satisfied, another value for d is selected and the calculations repeated. This method is simpler and more readily followed than the solution of complex equations which give a value for d direct. The passive resistance P_3 is concentrated and can be considered a point load acting at the extreme tip of the foot. The base of the pressure diagram for P_2 can be assumed horizontal and at the tip of the foot. This simplifies the calculations for moments about the foot, but 20 percent should be added to the value of d so determined. The maximum B.M. occurs at some point just below the ground level in front of the wall and can be determined with sufficient accuracy by trial and error. If the pressure distribution is irregular in form, owing to variations in strata or other factors, B.M. is often more readily determined by a graphical method in which the loading is divided into sections and a polar diagram and link polygon employed. Cantilever walls should be adopted only where the soil has a relatively high angle of shearing resistance.

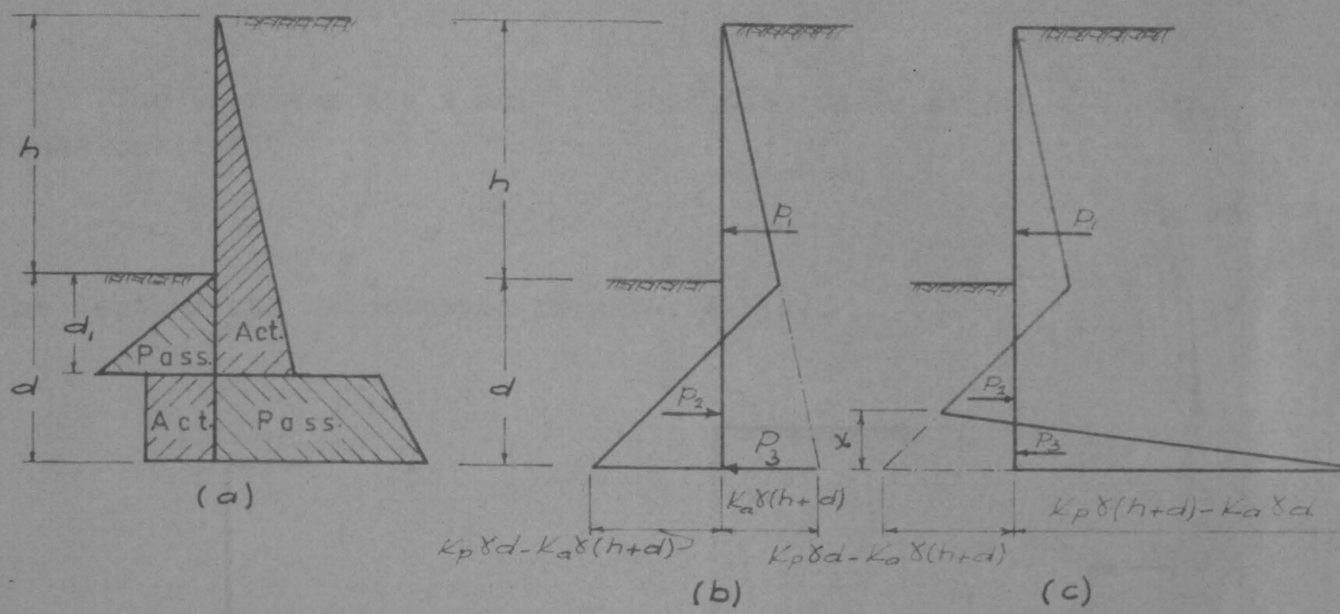


Fig. 4-2 Cantilevered Sheet Retaining Wall

The classical method of computation involves a number of arbitrary simplifying assumptions. The passive resistance of the soil behind the wall at its lowest tip is replaced by a concentrated force P_3, as shown in Fig. 4-2(b).

Active pressure at right-passive pressure at left + the dotted line area = 0

$$\frac{1}{2}K_A\gamma(h+d)^2 - \frac{1}{2}K_p\gamma d^2 + \frac{1}{2}x\gamma\left[K_p d - K_a(h+d) + K_p(h+d) - K_a d\right] = 0$$

or

$$x = \frac{K_p d^2 - K_a(h+d)^2}{(K_p - K_a)(h+2d)} \qquad \text{Eq. 4.1}$$

Taking moment at the base of the wall:

$$K_a(h+d)^3 - K_p d^3 + x^2(K_p - K_a)(h+2d) = 0 \qquad \text{Eq 4.2}$$

The unknowns are x and d. Substituting Equation (4-1) in Equation (4-2)

$$\frac{1}{6}K_p \gamma d^3 - \frac{1}{6}K_a \gamma (h+d)^3 = 0 \qquad \text{Eq. 4-2a}$$

The depth d can be obtained from Eq. 4-2(a).

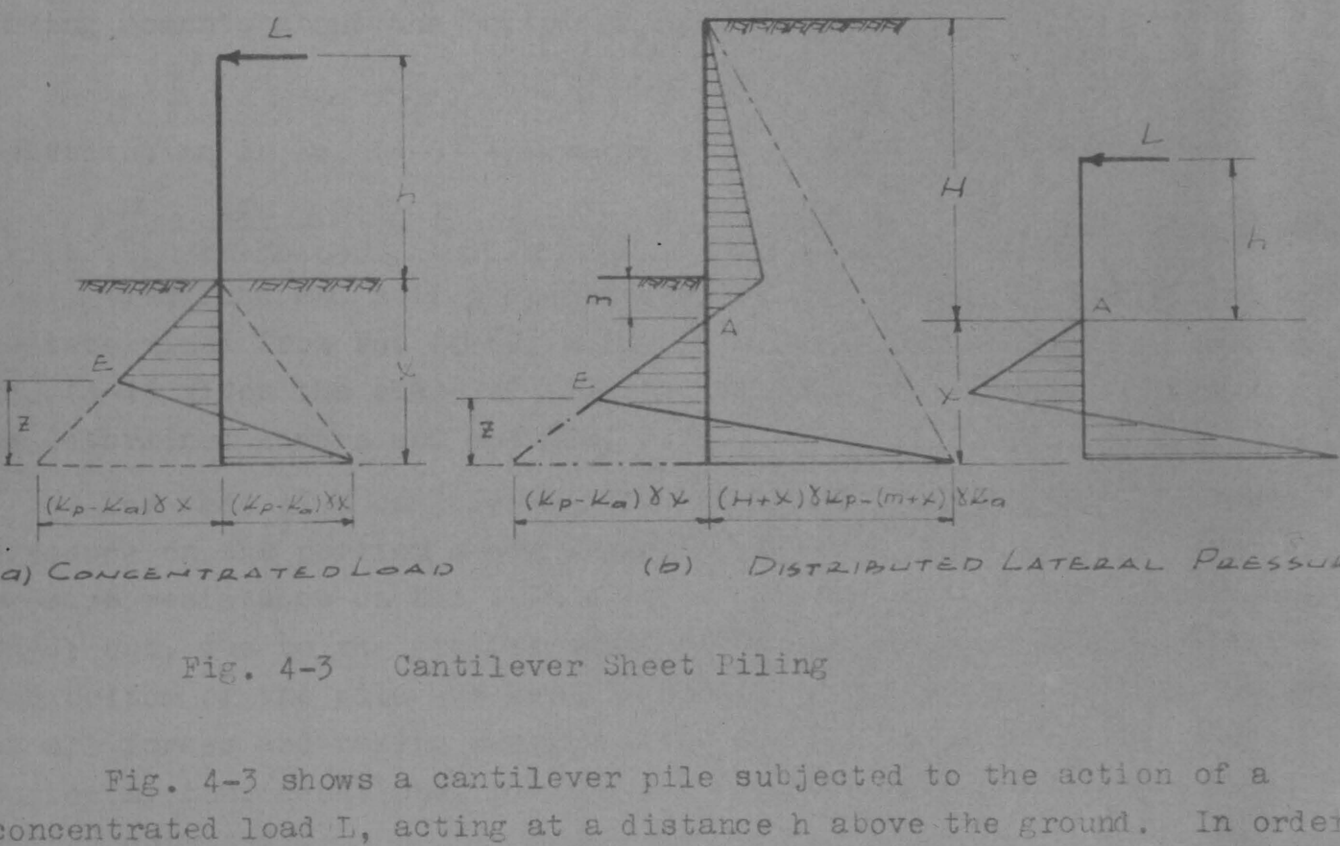


Fig. 4-3 Cantilever Sheet Piling

Fig. 4-3 shows a cantilever pile subjected to the action of a concentrated load L, acting at a distance h above the ground. In order to maintain equilibrium, lateral earth pressures must act in opposite directions just below this ground surface and in the same direction farther down. These pressures are brought about by movements of the pile against the earth; the resulting pressures can therefore be evaluated as the difference between passive and active pressure. It is now assumed that this passive resistance will be fully mobilized down to point E, which is a distance z from the bottom; and also fully mobilized in the opposite direction at the bottom of the pile. The change in pressure between point E and the bottom is assumed to vary according to a straight line.

Equating to zero, the sum of all forces gives

$$L - \frac{1}{2}\gamma(K_p - K_a)x^2 + \gamma(K_p - K_a)xZ \qquad \text{Eq 4-3}$$

or

$$Z = \frac{\gamma(K_p - K_a)x^2 - 2L}{2\gamma(K_p - K_a)x} \qquad \text{Eq. 4-4}$$

Taking moments about the bottom of pile gives

$$L(h+x) - \frac{1}{6}\gamma(K_p - K_a)x^3 + \frac{1}{3}(K_p - K_a)xZ^2 = 0 \qquad \text{Eq 4-5}$$

Substituting in Eq. (4-5) the value of z from Eq. (4-4) gives

$$x^4 - \frac{8L}{\gamma(K_p - K_a)}x^2 + \frac{12Lh}{\gamma(K_p - K_a)}x - \left[\frac{2L}{(K_p - K_a)\gamma}\right]^2 = 0 \qquad \text{Eq 4-6}$$

With load and soil properties known, the depth of penetration can be determined from Eq. (4-6), which is solved quite readily by trial. Eq. (4-4) gives the shape of the load diagram from which, in turn, can be determined shears and moments.

The case of a cantilever pile subjected to distributed lateral pressure on the portion above ground is shown in Fig. 4-3(b). The passive resistance on the left side is the same as for the concentrated load; but, due to the greater overburden, the passive resistance at the bottom of the pile has been increased. By equating to zero the sum of all forces and taking moments about the bottom of the pile, the following four equations, revised for the case of distributed load, can be obtained:

$$L - \frac{x^2}{2}\gamma(K_p - K_a) + \frac{Z}{2}\left[(H+2x)\gamma K_p - (m+2x)\gamma K_a\right] = 0 \qquad \text{Eq 4-3a}$$

$$Z = \frac{\gamma(K_p - K_a)x^2 - 2L}{2\gamma(K_p - K_a)x + H\gamma K_p - m\gamma K_a} \qquad \text{Eq. 4-4a}$$

$$L(h+x) - \frac{x^3}{6}\gamma(K_p - K_a) + \frac{Z^2}{6}\left[(H+2x)\gamma K_p - (m+2x)\gamma K_a\right] = 0 \qquad \text{Eq 4-5a}$$

and, eliminating z,

$$6L(h+x) - x^3(K_p - K_a)\gamma + \frac{[(K_p - K_a)\gamma x^2 - 2L]^2}{2(K_p - K_a)\gamma x - H\gamma K_p - m\gamma K_a} = 0 \qquad \text{Eq. 4-6a}$$

In these last equations the symbol L represents the resultant of all forces above the point A where the pressure intensity becomes zero, h denotes the distance from the resultant to this point, and H is the distance from the top surface to the point A.

It should be noted that the difference in penetration, which results from the application of either Eqs. (4-3) to (4-6) or Eqs. (4-3a) to (4-6a) to the case of distributed load, is very small; and, inasmuch as the former are simpler and always on the safe side, they are often applied to all cantilever piles regardless of manner of loading.

CHAPTER V

ILLUSTRATIVE NUMERICAL EXAMPLE

The classical design methods for anchored bulkheads involve a consideration of the stability of the bulkhead under the imposed soil loads without consideration of the bending flexibility of the piles. Since their development by R. A. Pennoyer, these methods have been used continuously in design offices for the selection of sheet pile lengths and section properties. The three methods commonly used are (a) the "Free Earth Support", (b) the "Elastic Line", and (c) the "Equivalent Beam" methods: each method has been discussed fully in this paper. Even with the introduction of more elaborate methods, which take into account the flexibility of the pile, the classical methods give the preliminary design which is then modified in accordance with the pile flexibility effect. P. W. Rowe's method considers the influence of pile flexibility in modifying the bending moment established by the free earth support method.

In the present methods for calculating the loads and moments resulting from earth pressures, the free earth support method involves the solution of a cubic algebraic equation, and the elastic line method utilizes the methods of graphic statics for moments and deflections. Both types of solutions are straightforward but time consuming. The author has found that the numerical method developed by N. M. Newmark provides a simple and rapid solution by each of the three classical bulkhead design methods.

In the examples, the bulkhead has been rotated into a horizontal position to simplify the arrangement of the computations. The upper free end of the bulkhead is placed at the left margin of the sheet.

The anchored bulkhead is held in equilibrium by the loads of the soil retained by the bulkhead, and by resistance provided by the anchor and the soil pressure on the embedded length of the bulkhead. Soil pressures are determined as active or passive Coulomb pressures depending upon whether the bulkhead is moving away from or into the soil mass

ANCHORED BULKHEAD DESIGN BY NUMERICAL METHOD

Design Based on Free Earth Support - Free earth support represents the limiting resistance reached when the entire embedded portion of the bulkhead moves outward. Horizontal soil loads are developed on both the loading and restraining surfaces of the bulkhead because of the bulkhead movement and the overburden weight, as shown in Fig. 5-2(a). If the bulkhead is considered to be fixed at a particular depth, z, the resultant forces produced by the soil loads above this depth may be designated by P_P for the passive force and by P_A for the active earth pressure force. The point of application of each of these forces is at $\bar{z}_P$ and $\bar{z}_A$, respectively, as shown on Fig. 5-2(b). Stability considera- tion require that the summation of the moments of these forces about the anchor point be zero, or

$$M_{A_P} = P_A \bar{z}_A - P_P \bar{z}_P = 0 \qquad (5\text{-}1)$$

As explained in Chapter 3, the Newmark procedure permits a rapid evaluation of the total shear $\bar{V}_z$, above a given evaluation z, and of the total moment, $\bar{M}_z$, of the soil forces about the point z. If we ignore the anchor force temporarily, as shown on Fig. 5-2(b), the moment of the soil forces at the depth z is given by

$$\bar{M}_z = \bar{P}_A(z - \bar{z}_A) - \bar{P}_P(z - \bar{z}_P) = (\bar{P}_A - \bar{P}_P)z - \bar{P}_A\bar{z}_A + \bar{P}_P\bar{z}_P$$

or since $\bar{V}_z = \bar{P}_A - \bar{P}_P$,

$$\bar{M}_z = \bar{V}_z z - M_{A_P} \qquad (5\text{-}2)$$

When Eq. 5-1 is satisfied, Eq. 5-2 becomes

$$\bar{M}_z = \bar{V}_z z \qquad (5\text{-}3)$$

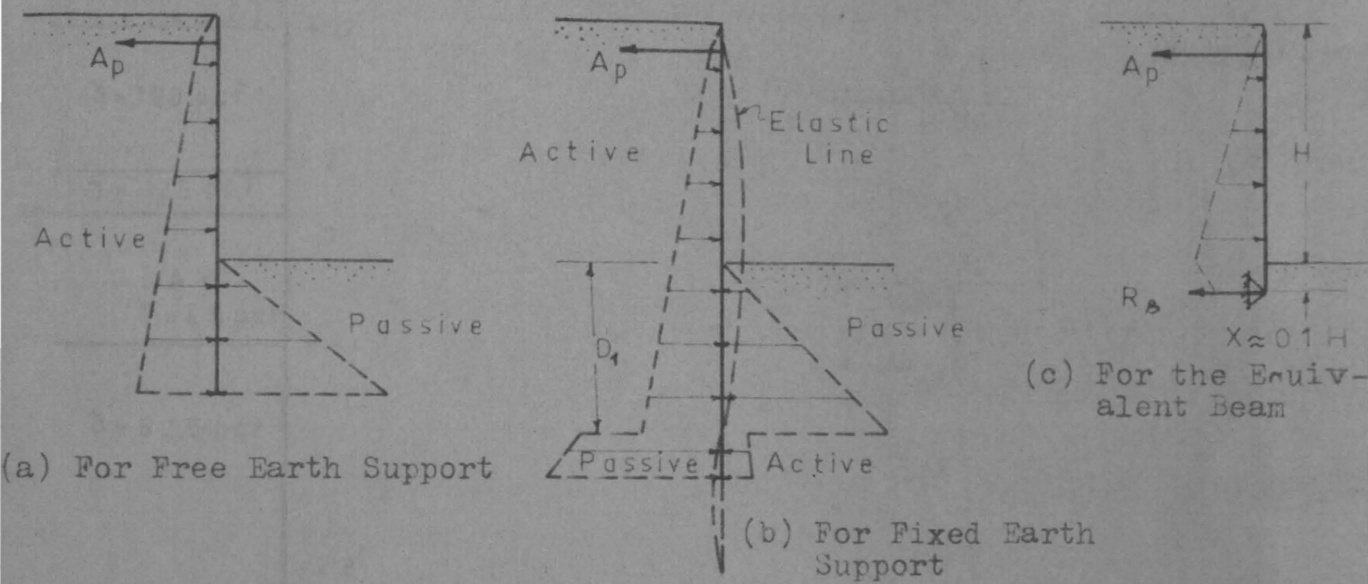


Fig. 5-1 Limiting Soil Pressure Distribution of the Anchored Bulkhead

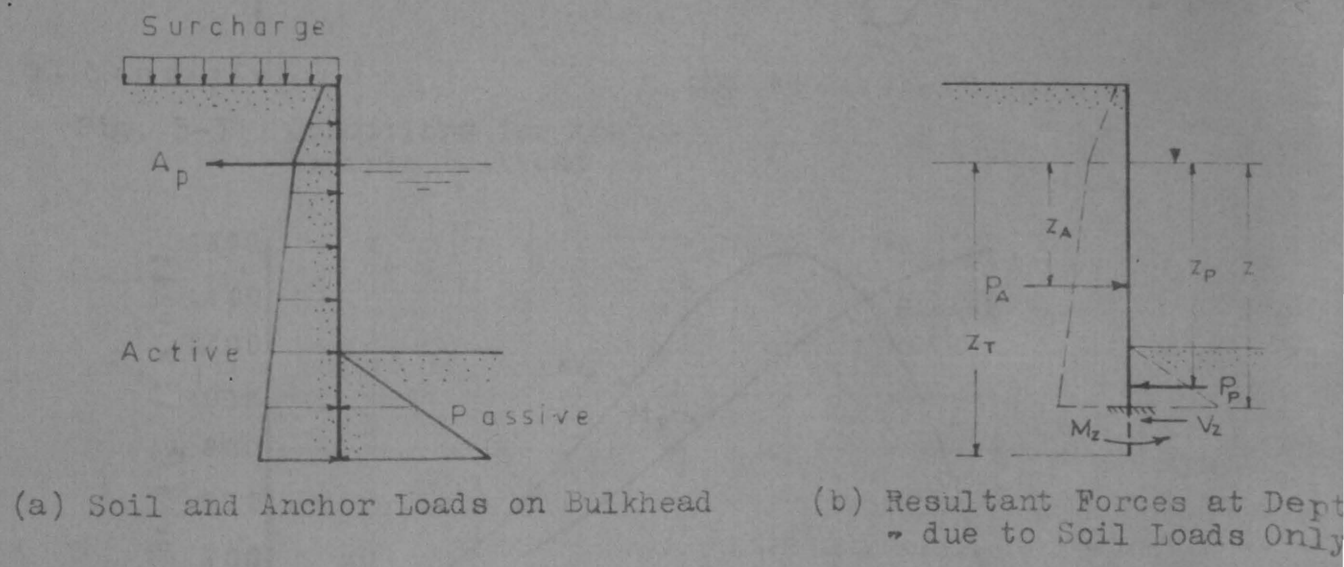
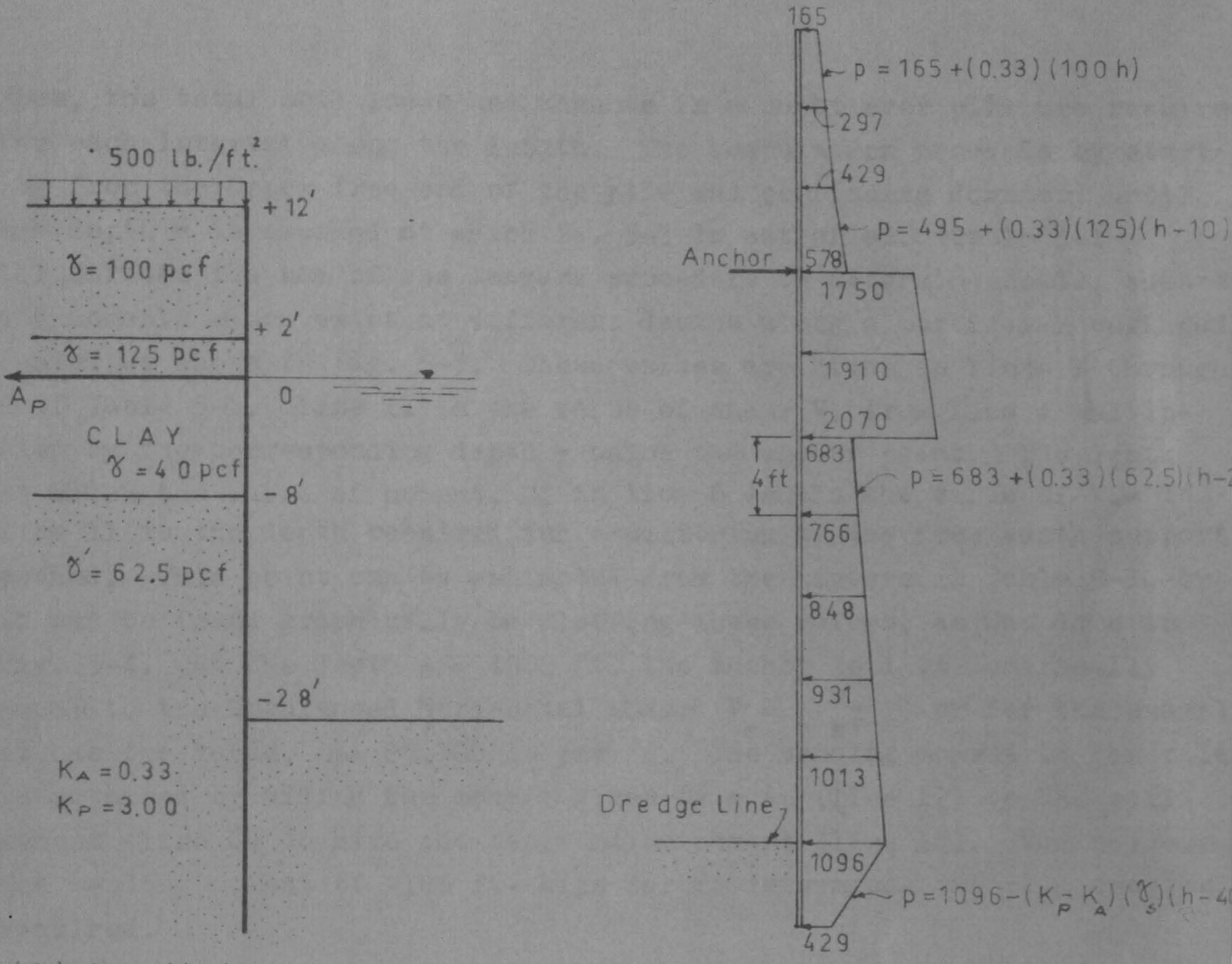


Fig. 5-2 Bulkhead with Free Earth Support



(a) Conditions (b) Pressure Diagram

Fig. 5-3 Conditions for the Example Bulkhead

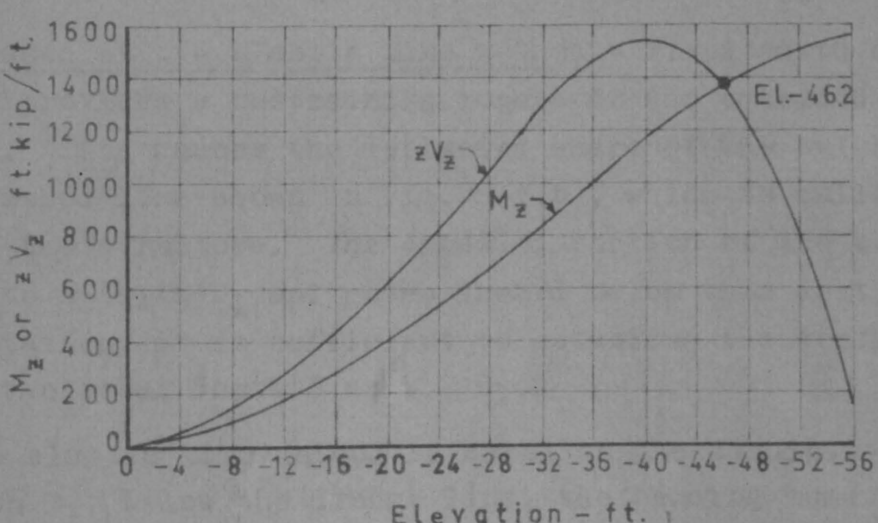


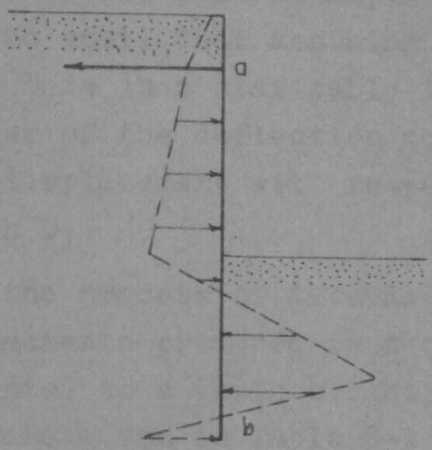
Fig. 5-4 Graphical Solution for the Required Depth of Embedment of Bulkhead by the Free Earth Support

Thus, the total soil loads and moments in a cantilever pile are required for each interval along the length. The computation proceeds by starting from the upper free end of the pile and continuing downward until the depth z is reached at which Eq. 5-3 is satisfied. Table 5-1 illustrates the use of the Newmark procedure to determine loads, shears, and moments which exist at different depths along a particular bulkhead loaded as shown in Fig. 5-3. These values are given in lines 1 through 6 of Table 5-1. Line 11 is the value of shear V_z from Line 4 multiplied by the corresponding depth z below the anchor point. The depth at which the value of moment, M_z in line 6 equals the value of $V_z z$ in line 11 is the depth required for equilibrium by the free earth support method. This point can be estimated from the numbers in Table 5-1, or it can be found graphically be plotting these values, as was done in Fig. 5-4. At the depth z = 46.2 ft. the anchor load is numerically equal to the unbalanced horizontal shear ($V_z = \frac{M_z}{z}$) or for the example it has the value, A_p= 29,700 lb per ft. The bending moment in the pile is obtained by adding the moment given by $z A_p$ (line 12) to the soil moment (line 6) to give the net bending moment (line 13). The maximum net bending moment of -186 ft- kips per ft determines the pile section required.

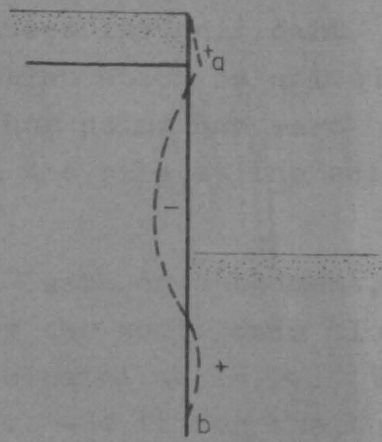
Thus, a solution for the free earth support condition requires only the computations on lines 1 through 6, and 11 through 13 in Table 5-1.

Design Based on the Elastic Line Method - Fixed earth support of the bulkhead provides a restraining moment on the embedded portion of the bulkhead. This causes the deflected shape of the bulkhead to correspond to the dashed line shown in Fig. 5-1(b), which is called the "elastic line" of the structure. The embedded portion of the bulkhead moves outward to a depth D_1 and moves inward below this depth. For purposes of computation, it is sufficient to establish the depth D_1 and to compute the total depth D as 1.2 D_1.

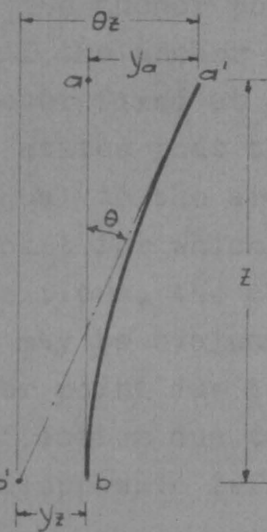
The elastic line method is based on the assumptions that (a) at the depth D_1, below the dredge line, the bending moment is zero, the slope of the pile is zero (vertical) and the outward deflection is



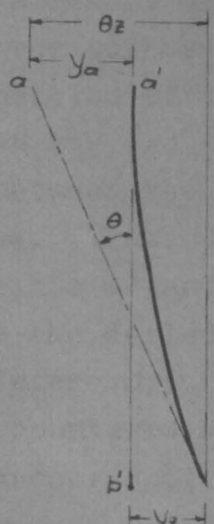
(a) Soil pressures on Bulkhead



(b) Bending Moment in Bulkhead due to Soil Loads and Anchor Force



(c) Deflection of Bulkhead due to Soil Loads Only (Anchor Removed)



(d) Deflection of Bulkhead due to Soil Loads Only - Considering Fixity on the Anchor Point

Fig. 5-5 Bulkhead with Fixed Earth Support

-ero, and (b) that the ourward deflection of the anchor_point is -ero. The slope of the pile at depth D_1 will not actually be -ero, but it will be so small that assuming it to be -ero causes insignificant errors. This is a statically indeterminate problem which is solved by making use of the deflection condition that anchor point has -ero lateral displacement with respect to a point on the pile at the embedded depth D_1.

In the process of determining the re-uired depth of embedment, th bending moments produced in a cantilever beam by the soil loads alone are computed to a depth slightly below the anticipated depth D_1. Thes moments are given in Table 5-1 to a depth of El. -64 ft for the bulkhead shown in Fig. 3. Then the outward deflection of the anchor point caused by the soil loads alone, can be determined by utili-ing the moment-area method of structural mechanics. Figs. 5-5(a) and (b) show the soil loads and bending moments, respectively, acting on the bulkhead with fixed earth support. Fig. 5-5(c) illustrates the outward movement of the anchor point which would occur as a result of these soil loads if the anchor restraint is removed and the bulkhead acts as a cantilever fixed at depth -. The first proposition of the moment area method states that the change in slope between any two points in a beam is equal to the area under the M/EI curve between the two point about the point for which the deflection is desired. Thus, by using these propositions, the change in slope between points a' and b in Fig. 5-5(c) may be evaluated as the angle θ. Also the deflection y_a at the anchor point due to the soil loads may be determined. This value of deflection due to the soil loads must be counterbalanced by an equal and opposite deflection provided by the anchor pull.

In the conventional graphical procedure for determining the required depth of embedment, a depth z (corresponding to an embedment depth D_1) is guessed from inspection of the M_z diagram (shown in Fig. 5-4), and the bulkhead is considered to be fixed at that elevation. The outward deflection of the anchor point due to the soil loads is evaluated graphically and is compared to the inward deflection which results from the concentrated anchor load of value, $A_p = \frac{M_z}{-}$ applied to the cantilever structure. If these deflections are not approximately of equal magnitude, the process must be repeated. Usually several trials are required.

By use of the Newmark procedure, the deflection of the bulkhead due to soil loads is most conveniently calculated by using the anchor point as a reference and working downward toward the pile tip. By considering the bulkhead to be vertical (slope = 0) and to have no outward deflection at the anchor point, the deflection $y_{\bar{z}}$ at any depth can be computed directly as shown in Table 5-1. The resulting deflection curve is illustrated by Fig. 5-5(d). Since the change in slope between any two points along the length a' b is established by the area of the M/EI diagram between these two points, it is evident that the same elastic curve exists between a' and b in Fig. 5-5(c) and (d). The curve of Fig. 5-5(d) may be obtained by rotating the curve of Fig. 5-5(c) through an angle of θ. Thus, to determine the anchor point deflection, y_a, by means of the numerical procedure which defines $y_{\bar{z}}$ and $\theta_{\bar{z}}$ at successive depths we can use the relation

$$y_a = \theta_{\bar{z}}\bar{z} - y_{\bar{z}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5\text{-}4)$$

This deflection must be equal to that produced by the anchor pull, or

$$y_{Ap} = \frac{A_p \bar{z}^3}{3\,E\,I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5\text{-}5)$$

The anchor pull, A_p, must produce a moment at depth $\bar{z}$ which is equal in magnitude and opposite in sense to the soil load moment $M_{\bar{z}}$ from line 6 of Table 5-1, or

$$A_p = \frac{M_{\bar{z}}}{\bar{z}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5\text{-}6)$$

After substituting Eq. 5-6 into Eq. 5-5 and setting Eqs. 5-4 and 5 equal to each other, the final expression to be satisfied is:

TABLE 5-1 Computation of Moments and Deflections for Bulkhead Loaded as shown in Fig 5-3

										Common Terms	Units
1. Elevation	+12'	+8'	+4'	0	-4'	-8'	-12'	-16'	-20'		
2. Soil Press. p	165	297	429	578 1750	1910	2010 683	766	848	931		lb/ft^2
3. Eq. Conc for $\bar{p}$	627	1182	2591	1585 5410	11460	6050 2132	4595	5089	5565	$h/6$	lb/ft
4. Shear V_z		627	2409	5000	11995	23455	31637	36232	41321	$h/6$	lb/ft
5. Moment M_z	0	627	3036	8036	20031	43486	75123	111,355	152676	$h^2/6$	ft-lb/ft
6. Moment M_z	0	1.67	8.10	21.40	53.40	116.	200	297	407		ft-kip/ft
7. Angl. Change α	0	-1.67	-8.10	-21.40	-53.40	-116	-200	-297	-407	$1/EI$	1/ft
8. Conc. Ang. Ch. $\bar{\alpha}$		-0.0348	-0.1041	-0.2755	-0.6714	-1.413	-2.413	-3.577	-4.899	$\frac{1000h}{12EI}$	
9. Slope θ				(0)	-0.276	-0.947	-2.360	-4.773	-8.350	$\frac{1000h}{12EI}$	
10. Deflection y				0	-0.28	-1.22	-3.58	-8.36	-16.7	$\frac{1000h^2}{12EI}$	ft
For Free Earth Support Solution											
11. z V_z					16.0	93.8	210.	338	496		ft.kip/ft
12. z. A_p = M_{Ap} (Free Earth Support)				0	-119	-237	-356	-474	-593		ft.kip/ft
13. Net Moment = Line 6 + Line 12				21.4	-65.6	-121	-156	-177	$\boxed{-186}$		ft.kip/ft
For Fixed Earth Support Solution											
14. z. A_p = M_{Ap} (Fixed Earth Support)				0	-110	-220	-330	-440	-550		ft.kip/ft
15. Net Moment = Line 6 + Line 14				21.4	-56.6	-104	-130	-143	$\boxed{-143}$		ft.kip/ft
For Equivalent Beam Solution											
16. z. A_p = M_{Ap} (Equivalent Beam)				0	-104	-207	-311	-415	-519		ft.kip/ft
17. Net Moment = Line 6 + Line 16				21.4	-50.6	-91.0	-111	$\boxed{-118}$	-112		ft.kip, ft

TABLE 5-1 (cont'd)

												Common Terms	Units
1.	-24	-28	-32	-36	-40	-44	-48	-52	-56	-60	-64		
2.	1013	1096	429	-239	-907	-1574	-2242	-2909	-3577	-4244	-4912		lb/ft²
3.	6079	5826	1954 619	-1424	-5439	-9444	-13450	-17454	-21460	-25465	-29471	$\frac{h}{6}$	lb/ft
4.	46906	52985	58811	61382	59948	54509	45065	31615	14161	-7299	-32764	$\frac{h}{6}$	lb/ft
5	199582	252567	311377	372760	432708	487217	532282	563897	578058	570759	537995	$\frac{h^2}{6}$	ft-lb/ft
6.	532	674	830	994	1150	1300	1420	1504	1541	1522	1435		ft-kip/ft
7	-532	-674	-830	-994	-1150	-1300	-1420	-1504	-1541	-1522	-1435	$\frac{1}{EI}$	1/ft
8	-6.401	-8.102	-9.968	-11.920	-6.59	-7.22 -7.50	-8.06 -8.30	-8.71 -8.87	-9.13 -9.20	-9.24 -9.20	-9.00 -8.82	$\frac{1000h}{12EI}$	–
9.	-13.249	-19.650	-27.750	-37.720	-49.640	-63.430	-79.000	-96.010	-114.010	-132.440	-150.700	$\frac{1000h}{12EI}$	–
10.	-30.0	-49.6	-77.4	-115	-165	-228	-301	-403	-517	-650	-801	$\frac{1000h^2}{EI\,12}$	ft
	For Free Earth Support Solution												
11.	688	918	1176	1391	1579	1526	1382	1054	510	-282	-1354		ft-kip/ft
12	-712	-830	-949	-1067	-1186	-1305	-1423						"
13	-180	-156	-119	-73	-36	-5	+3						"
	For Fixed Earth Support Solution												
14.	-660	-770	-880	-990	-1100	-1210	-1320	-1430	-1540				"
15	-128	-96	-50	+4	+50	+90	+100	+74	+1				"
	For Equivalent Beam Solution												
16	-622	-726	-830										"
17	[illegible]	-52	0										"

$$\frac{\bar{M}_z z^2}{3\ EI} = \bar{\theta}_z z - \bar{y}_z \text{ .... } (5\text{-}7)$$

In the actual evaluation of Eq. 5-7, by substituting values from table 5-1, the EI-terms drop out and the expression reduces from

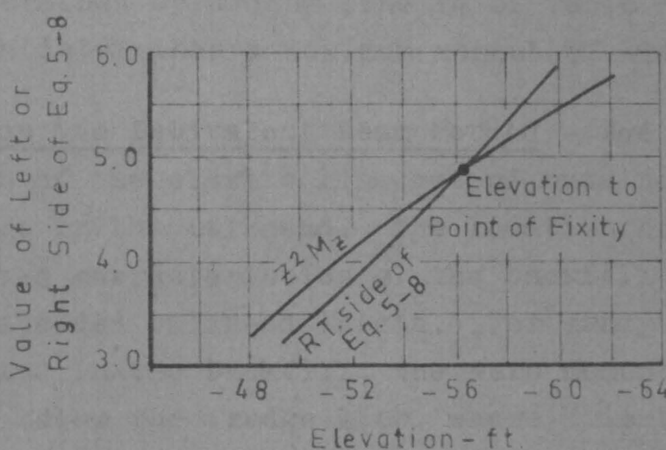


Fig. 5-6 Graphical Solution of Eq. 5-8

At a particular depth, z_1, the numerical values of $\bar{M}_{z1}$, $\bar{\theta}_{z1}$, and $\bar{y}_{z1}$, will satisfy Eq. 5-7, which establishes that

(a) the bending moment in the pile is zero at depth $\bar{z}_1$,

(b) the slope of the pile is zero at depth $\bar{z}_1$,

(c) the deflection of the pile is zero at depth z_1; and

(d) the anchor point deflection is zero.

$$\text{to} \quad \frac{\bar{M}_z z^2}{3\ E\ I} = (\text{No. for } \bar{\theta}_z)\ \frac{1000\ h\ z}{12\ E\ I} - (\text{No. for } \bar{y}_z)\ \frac{1000\ h^2}{12\ E\ I}$$

$$z^2 \bar{M}_z = (\text{No. for } \bar{\theta}_z) 250\ h\ z - (\text{No. for } \bar{y}_z)\ 250\ h^2 \text{ .... } (5\text{-}8)$$

which may be solved graphically as indicated on Fig. 5-6. The intermediate computations are given in Table 5-2. Note that for use in

Col. 3 of Table 5-2, it is necessary to evaluate the slop, $\bar{\theta}_z$, and the deflection at the same z depth.

The graphical solution of Eq. 5-8 on Fig. 5-6 determine the point of fixity at El. -56 ft, or D_1 = 28 ft. Then the total depth of embedment

$$D = 1.2\ D_1 = 33.6 \text{ ft.}$$

The anchor pull is A_p = M-56/56 = 27,5000 lb per ft. The net bending moment is determined by adding line 14 of Table 5-1 to line 6 to give line 15, which determines a maximum moment of -143 kips per ft.

Design Based on the Equivalent Beam Method - The equivalent beam is a simplification of the elastic line method obtained by considering an imaginery hinge in the bulkhead. The location of this zero moment poin depends upon the characteristics of the backfill and the material supporting the embedded bulkhead length. For sandy materials supporting the bulkhead and in the backfill, the zero moment point is approximately at 0.1 H below the dredge line, where H is the distance from the top of the pile to the dredge line as shown in Fig. 5-1(c). For the example considered herein, H = 40 ft, and the hinge is assumed to be located 0.1 H = 4 ft below the dredge line, or at El. -32 ft.

The moment given in line 6 of Table 5-1 is due to the soil loads alone acting on a cantilever pile. In order to produce a point of zero moment at El. -32 ft, an anchor pull must be applied which produces a moment equal and opposite to the soil moment at that elevation. For the equivalent beam the anchor pull must be

$$A_p = \frac{M_z\ (\text{at Elev. } -32\ ft)}{32\ ft} = \frac{830}{32} = 26.0^{K}$$

The distribution of anchor pull moment with depth is given in line 16, Table 5-1 and when this is added to line 6 the net moment is

TABLE 5-2 - EVALUATION OF QUANTITIES IN EQ. 5-8

z, in ft. (1)	$z^2 M_z/10^6$ (2)	$(\text{No for } \theta_z)\ 1000\ z / 10^6$ (3)	$(\text{No. for } y_z)\ 4000 / 10^6$ (4)	Col(3) - Col (4) $/ 10^6$ (5)
43	3.272	4.190	1.228	2.962
60	5.479	8.498	2.600	5.898

given in line 17. By the equivalent beam method the maximum bending moment in the bulkhead amounts to -118 kips per ft.

The concentrated reaction R_B at the hinge location (Fig. 5-1(c)) is obtained by subtracting the anchor pull from the total soil shear load given in line 4 of Table 1:

$$R_B = (58811 + 1954)\frac{h}{6} - A_p = 40500 - 26000 = 14500 \text{ lb/ft}$$

From Equivalent Beam Method, the expression for the total depth of embedment required is

$$D = 1.2 D_1 = 1.2 \left[\frac{3}{2} \frac{H K_A}{(K_p - K_A)} - \frac{x}{2} + \sqrt{\frac{6 R_B}{(K_p - K_A)\gamma}} \right] \qquad (5-10)$$

For this example, x = 0.1 H = 4 ft, H = 40 ft, K_P = 3.0, K_A = 0.33, and γ = 62.5 lb per ft. After substituting these values in Eq. 10, D was found to be 33.9 ft.

TABLE 5-3 SUMMARY OF DESIGN VALUES

METHOD	DEPTH D (in ft)	ANCHOR PULL A_p (in pounds/ft.)	MAX. BENDING MOM M (in pounds-foot/foot)
Free Earth Support	18.2	29700	186 000
Elastic Line	33.6	27500	143000
Equivalent Beam	33.9	26000	118000

CHAPTER VI

CONCLUSION

The general equations of the seven selected methods of analysis, namely the

1. Free Earth Support
2. Danish Rules
3. Fixed Earth Support
4. Equivalent Beam Support
5. Tschebotarioff Method
6. Rowe's Method
7. Numerical Method

including the Cantilever Sheet Pile Wall equations were derived.

Bulkhead design literature contains many theories and mathematical studies on the action and effect of earth pressure. The very number of these analysis support the fact that no single method has universal application, and that a wide range of error in the assessing of pressures can be expected.

The purpose here is not to advance a new procedure that will remove all uncertainties from bulkhead design. This paper contains suggestion for revisions on the basis of the experimental and observational data which have been secured during the last few decades.

The procurement of site and soil data is of paramount importance. A site survey that includes the history and general geology of the area is essential. Unless soil conditions are knownwith certainty, borings should be taken at representative locations throughout the site. Undisturbed samples of cohesive type of soils and disturbed samples of cohesionless materials should be tested for their physical properties. Active and passive earth pressures should be determined from these data.

The importance of the errors involved in the estimate of the soil constants appearing in the equations depends to a large extent on the complexity of the structure of the strata into which the sheet piles are driven, on the degree of uniformity of the material in the borrowpit area, and on the quality of the subsoil exploration. Therefore, it would be unwarranted to establish rigid rules for the factors of safety that should be used. One of the responsibilities of the designer is to evaluate the prevailing uncertainties and to choose the factors in accordance with his findings.

No bulkhead theory can possibly anticipate all the varieties of subsoil and hydraulic conditions that may be encountered in practice, and every case requires a certain amount of independent judgment. Hence, if the subsoil conditions do not conform to a standard pattern, designers who are not thoroughly familiar with the basic principles and techniques of soil mechanics are advised to assume free earth support and to use conservative factors of safety. When solving unusual problems, the designer should consult the observational data on which the design procedures are based.

Suggestions for Further Study - During the course of this investigation improvements and possibilities for further study have become apparent. The following list serves as tabulation of these items:

1. No precise recommendations can yet be made concerning the design of bulkheads where the natural soil and the backfill are composed of sand-clay mixtures. Until further research is performed all such design computations should include larger factor of safety.

2. In the case of bulkheads a computation based on a state of failure (for example, one in which a yield hinge is supposed to develop in the wall) has until recently not been possible because methods have not been available for determining the earth pressures corresponding to arbitrary movements of the wall. However, following the development and application of such methods of investigation to anchored bulkheads, nothing should prevent the engineer from designing such structures on the basis of a state of failure.

3. The relatively new science of soil mechanics is responsible for greatly improved means of evaluating earth pressures, based on the sampling and testing of materials. The most promising avenue to further progress seems to be in the field of prototype or field measurements for testing the validity of existing design methods.

APPENDIX. NOTATION

The following letter symbols conform essentially with ASCE Manual of Engineering Practice No. 22 ("Soil Mechanics Nomenclature") and American Standard Letter Symbols for Structural Analysis (ASA Z10.8-1949) prepared by a Committee of the American Standards Association, with ASCE participation, and approved by the Association in 1949:

A_p = anchor pull;

D = depth of sheet-pile penetration;

d = the ratio of deflection to the height of a wall;

E = the modulus of elasticity;

f = the allowable stress in bending of the piles;

f_y = the yield point;

G_s = the factor of safety;

H = the height of a lateral support, total length of sheet piles;

H_a = the vertical distance from the anchor to the dredge line;

H_f = the vertical distance between the dredge line and the surface of the backfill;

H_u = the vertical distance between the free water level and the water table in the backfill;

I = the moment of inertia of the cross section of a sheet pile;

i = the hydraulic gradient;

K_o = the pressure coefficient for earth at rest;

K_A = the coefficient of active earth pressure (the ratio between the normal component of the earth pressure on the lateral support and the corresponding fluid pressure);

K_P = the coefficient of passive earth pressure;

L_1, L_2, and L_3 = the vertical distances from centers of pressure;

M = the maximum bending moment in a sheet pile:

$M(max)$ = the maximum bending moment in a sheet pile computed on the assumption of free earth support;

M' = the allowable bending moment for a sheet pile with a given flexibility number;

M_a = the maximum bending moment in a sheet pile with flexibility number ;

M_1 = the maximum bending moment in a sheet pile with a given flexibility number;

m = the ratio between the hori-ontal distance from the wall to the height of the wall;

n = the ratio between the depth below the surface of the back-fill and the height of the wall;

P = the total normal pressure on a lateral support produced by a point load;

P_u = the unbalanced water pressure;

p = the unit pressure:

p' = the hori-ontal pressure produced by a line load per unit of wall length;

$\bar{p}$ = the effective unit pressure;

p_A = intensity of active earth pressure;

p_P = intensity of passive earth pressure;

p_W = pore water pressure;

p_1 = the hori-ontal unit pressure produced by a point load along the intersection between the inner face of a wall and a vertical section through the point load, perpendicular to the wall;

Q = a point load;

q = a uniformly distributed surcharge, per unit of area:

q' = a unit line load;

q_u = the unconfined compressive strength of cohesive soil;

γ = the unit weight of a soil including the weight of water contained in its voids:

γ' = the submerged unit weight;

$\bar{\gamma}$ = the effective unit weight of silt or clay (the saturated unit weight above the water table and the submerged unit weight below the water table);

$\Delta\gamma'$ = the reduction in the submerged unit weight resulting from seepage pressure exerted by rising ground water;

$\Delta\gamma''$ = the increase in the effective unit weight;

γ_w = the unit weight of water;

δ = the angle of wall friction;

ρ = the flexibility number;

ρ_a = the flexibility number corresponding to the point of intersection N

ρ_c = the critical flexibility number at which M becomes smaller than M(max);

σ = the normal stress;

σ_x = the hori-ontal unit pressure;

ϕ = the angle of internal friction;

ϕ' = the angle of partly mobilized international friction; and

θ = an angle

A SELECTED BIBLIOGRAPHY

1. Design of Sheet Piling Bulkheads
By R. A. Pennoyer
Civil Engineering (1933) November p. 615

2. Theoretical Soil Mechanics
By K. Ter-aghi
John Wilet and Sons (1943)

3. Anchored Sheet Pile Walls
By P. W. Rowe
Proceedings Inst. of C.E. London (1953) Vol. 1 Part 1 pp 27-70

4. Anchored Bulkheads
by K. Ter-aghi
Trans. ASCE (1954) Vol. 119 pp. 1243-1280

5. Numerical Procedure for Computing Deflections, Moments and Buckling Loads
by N. W. Newmark
Trans. ASCE (1943) Vol. 108 pp. 1161-1188

6. Analysis for Sheet Pile Retaining Walls
by F. E. Richart, Jr.
Trans. ASCE Vol. 122 pp. 1113-1132

7. Field Study of a Sheet Pile Bulkhead
by C. M. Duke
Trans. ASCE Vol. 118 (1953) p. 1131

8. The Design of Flexible Bulkheads
By J. R. Ayers and R. C. Stokes
Trans. ASCE Vol. 119 (1954) p. 373

9. Design Curves for Anchored Steel Sheet Piling
By H. M. Lummis and W. C. Boyer
Trans. ASCE Vol. 119 -1954) p. 639

10. Analysis of Sheet Pile Bulkheads
By Paul Baumann
Trans. ASCE Vol. 100 (1935) p. 707

11. The Design and Construction of Engineering Foundations
By F. D. Henry
McGraw Hill (1956) pp. 270-282

12. Deep Foundations and Sheet Piling
By Donovan H. Lee
Concrete Publications Limited London (1961) pp. 29-109

13. Substructure Design and Analysis
By Paul Anderson
The Ronald Press Comp. (1956) pp. 44-65

14. Soil Mechanics, Foundations and Earth Structures
By G. P. Tschebotarioff
McGraw Hill (1951) pp. 494-520

15. Earth Pressure Calculation
By J. Brinch Hansen
The Danish Technical Press
Civil Engineers, Copenhagen (1953) pp. 192-219

16. Zemin Mekaniği
by P. L. Capper, W. F. Cassie and V. Kumbasar, F. Kip
Istanbul Teknik Üniversitesi Kütüphanesi
Sayı: 496 (1962) pp. 159-164

17. Zemin Mekaniği
By H. Peynircioğlu
Istanbul Teknik Üniversitesi Kütüphanesi
Sayı: 573 (1964) pp. 120-165

18. Final Report - Large Scale Earth Pressure Tests with Model Flexible Bulkheads
by G. P. Tschebotarioff
Princeton University (1949)

19. Soil Mechanics in Engineering Practice
By K. Terzaghi and R. B. Peck
John Wiley and Sons Inc (1948) pp. 147-150

20. Pressure Distribution on Retaining Walls - Proceedings, First International Conference on Soil Mechanics and Foundation Engineering Discussion
By R. D. Mindlin, Vol. 3 (1938) pp. 155-156

21. Horizontal Pressures on Retaining Walls Due to Concentrated Surface Loads
By M. G. Spangler, Bulletin 140 No. 140 Iowa Eng. Experiment Station, Iowa City, Iowa

22. Anchored Bulkhead Design
by F. E. Richard
Journal of the Soil Mechanics and Foundations Division
Proc. of the ASCE Vol. 86, Part I, Feb. 1960 pp. 29-45