THE ULTIMATE BEARING CAPACITY OF SOILS

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June 10 th ,1964

The auther wishes to express his gratitude to Prof. Vahit Kumbasar, his advisor, for his very valuable suggestions and great interest in the work of this thesis.

CONTENTS

INTRODUCTION

From the very early ages, ultimate bearing capacity of soils is a very important problem. The importance comes from the fact that unless build on a rock any kind of a structure (a bridge a fortication a dam a railroad any type of a pavement an airport harbors quays wharves docks etc...)must be founded on soil. The second reoson for the importance of the problem is the fact that usually any type of a failure of the upper structure can be tolerated or corrected whereas a foundation failire is never corrected.

Unlike other construction materials (steel , concrete etc...) properties of soil cannot be controlled within economical and succesfull limits and the civil engineer is forced to use the soil in its natural conditions.

Since we cannot avoid the use of soil as construction material we must know in our design (as in all engineering problems) what causes the failure, what tests should be fallowed to reveal the type of failure and what is the mathematical expressions connecting soil properties, test results and the failure load. The fallowing pages deal with methods of obtaining mathematical relations for the failure of the soil and a critical review of these theories.

SUMMARY OF THE METHODS WIDELY USED FOR THE PREDICTION OF BEARING CAPACITY

For any work in which the design of a foundation is necassary one needs the ultimate bearing capacity of the soil. This will be obtained from

1) Soil loading tests in place
2) Analytical methods of calculation
3) Laborotary testing of soils
4) Building codes official regulations and civil engineering handbooks.

The subject of my thesis will consist in general a thorough discussion of item 2 above, since it forms the basis of other calculations, at least it justifies their use.

Building codes on the other hand does not give ultimate bearing capacity directly but assuming a suitable factor of safety and multiplying the given value by that, an estimate of ultimate load is obtained. However in codes the condition of soil is discussed rather in general but in arbitrary terms.

Laborotory testing of soils are also of no great help because of the scale effect of the models cause large deviation of the values encountered in practice. Loading tests are rather empirical in nature but it is interesting to note Housel's method and indicate the defects.

<u>Housel's Bearing Capacity Method :</u>

The method is based upon Housels experimental work on clayey soils. He uses two models to determine the bearing capacity of the prototype. He assumes thet the bearing capacity of soil is given by a linear relationship.

$$A\sigma_0 = W = \sigma A + m l$$

According to this the bearing capacity of theprototype can be obtained from the model by a ~~sub~~ suitable correction of the bearing area.

$$\sigma_0 = \sigma + m \frac{L}{A} \qquad (1)$$

in that case L/A is the perimeter area ratio of the bearing area. Using two plates of different L/A ratio the unknown σ and m can be found out that in each case is the same to produce the same settlement. The simultaneous solution of the equation (1)

$$\begin{aligned} \sigma_{01} &= \sigma + m \frac{L_1}{A_1} \\ \sigma_{02} &= \sigma + m \frac{L_2}{A_2} \end{aligned} \qquad (1a)$$

gives values of σ and m and this is used to find bearing capacity of the prototype. However this is nothing but an extrapolation of the results of model tests.

Analytical Methods Of Calculation of Bearing Capacity :

These methods can be grouped into four divisions as follows:

1) Extreme methods

2) Theories of elasticity

3) Theories of plasticity

4) Limit analysis

A brief account of each theory will be given .

Extreme Methods : By this method the bearing capacity problem is solved by a single condition of equilibrium in connection with a maximum- minimum condition . A rupture surface, among possible rupture surfaces , is chosen and which the earth pressure resultant determined from the equation of equilibrium attains an extreme value i. e . passive and active pressures reaches their limiting values in the opposite sense.
Analytical solutions will give the same number of equetions as the number of gemetrical parameters required to describe the rupture line. With this rupture line the problem can be solved provided that the earth pressure center and direction of the resultant pressure is known.
Pauker, Rankine Coulomb , and Bell obtained the bearing capacity of soil by the above method. Except Coulomb - Rankine formula the other solutions have only historical values today and they are not used. There are also some graphical methods suggested however by these methods the ultimate bearing capacity can not be calculated directly.
Theories of Elasticity : A second approach to the problem is by

elastic analysis. The state of stress in the soil at failure is assumed and then the failure load is related to that state of stress by using methods of elasticity. Fröhlich's critical edge pressure theory, ~~Kötters Equation and~~ Schleicher's equation are derived by this method and will be discussed later.
In this method we should stress one point, that is at failure the failure of soil is plastic rather than elastic and because of that the validity of the above method is rather questioned.

Theories Of Plasticity : This method is widely applied for the solution of the bearing capacity problem. The first solution of the problem is obtained by Prandatl for weightless materials however later the solutions are elaborated to ~~kaga~~ to cover other cases which are frequently met in practice. The method is based on the assumption that the state of Failure is reached at any point in a certain zone or on a certain curve. In cases of plane strain (two dimensional strain) Coulomb -Mohr criterea is accepted as the condition of failure. It is given by

$$\tau = c + \sigma \tan \varphi \qquad (2)$$

τ shear stress along the point of aplication of σ which is the normal stress and c is the cohesion. Kötter using this and Mohr theory of stress obtained the differential equation and this equation is used to obtain the earth pressure along any surface of soil. However the results are correct only in cases where the equations are applied to correct failure surfa_ ces. Later Terzaghi and Meyerhoff making use of Prandatl theory Kötter's equation and with some simlifying assumtions evolved theories of bearing capacity which is widely used today and more complete than other theories with regard to extent of ap lication. However the assumptions are not fully justified and therefore they are not to be taken as exact solutions of bearing capacity problem.

Limit Analysis : Drucker and Prager developed a method of Limit analysis to be applied to earth pressure and bearing capacity problems. The solutions depend on fulfillment of some conditions on two admissable fields :

1) Statically admissable stress field defined by the equilibrium equations and inequalities, as well as statical boundary conditions.

2) Kinematically admissabla velocity field which satisfies the kinematical boundary conditions and the equation (3)

$$\varepsilon_1 + \varepsilon_2 = a_{mn} \sin\varphi \qquad (3)$$

at any point. In that ε_1 and ε_2 = rate of dilatation and a_{mn} rate of change of maximum shear. To solve the problem one must assume a certain stress distribution. If this stress field is in a statically admissable field than the solution is on the safe side. If that stress field is in a kinematically admissable field than the solution is on the unsafe side . Needless to say that if stress field is in both fields the exact solution is obtained, however up to today no such solution is available. With such a solution we will be able to find the interval in which the earth pressure lies and applying the same methods the interval in which the bearing capacity lies, will be solved.

FRÖHLICH'S CRITICAL EDGE PRESSURE THEORY

Fröhlich obtains his soulition by elastic analysis, assuming the principle stress conditions. Using the fig.(1)

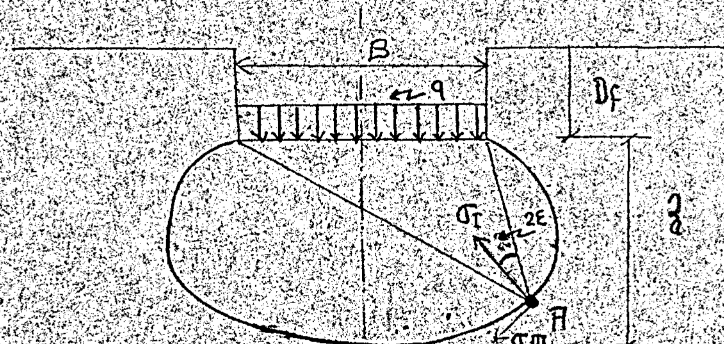

Fig-1

one obtains the compenent of principal stresses as follows, σ_I', σ_{III}' stresses due to applied net load (applied load - weight of soil excavated)

$$\sigma_I' = \frac{q}{\pi}(2\varepsilon + \sin 2\varepsilon) \qquad (4a)$$
$$\sigma_{III}' = \frac{q}{\pi}(2\varepsilon - \sin 2\varepsilon)$$

Using Terzaghi assumption the stress above foundation level is taken as a surcharge γD_f and the stresses σ_I'', σ_{III}'' are

$$\sigma_I'' = -\frac{\gamma D_f}{\pi}(2\varepsilon + \sin 2\varepsilon) \qquad (4b)$$
$$\sigma_{III}'' = -\frac{\gamma D_f}{\pi}(2\varepsilon - \sin 2\varepsilon)$$

than at any depth the stresses are

$$\sigma_I''' = \gamma(D_f + z) \qquad (4c)$$
$$\sigma_{III}''' = K_o\gamma(D_f + z)$$

Taking $K_o = \frac{\nu}{1-\nu}$ $\nu = 0.5$ then $K_o = 1.0$

The total stresses are

$$\sigma_I = \frac{q - \gamma D_f}{\pi}(2\varepsilon + \sin 2\varepsilon) + \gamma(D_f + z) \qquad (5)$$
$$\sigma_{III} = \frac{q - \gamma D_f}{\pi}(2\varepsilon - \sin 2\varepsilon) + \gamma(D_f + z)$$

After certain value of Q some points in the soil start to fail this is found from mohr stress theorry as

$$z = \frac{q-\gamma D_f}{\pi\gamma}\left(\frac{\sin 2\varepsilon}{\sin\phi} - 2\varepsilon\right) - D_f - \frac{c}{\gamma \, tg\,\phi} \qquad (6)$$

for any depth z such a relation exist. Then finding maximum value for z

$$\frac{dz}{d\varepsilon} = \frac{q-\gamma D_f}{\pi\gamma}\left(2\frac{\cos 2\varepsilon}{\sin\phi} - 2\right) = 0$$

$$\cos 2\varepsilon = \sin\phi$$

$$2\varepsilon = \frac{\pi}{2} - \phi$$

and

$$z_{max} = \frac{q-\gamma D_f}{\pi\gamma}\left[ctg\,\phi - \left(\frac{\pi}{2} - \phi\right)\right] - D_f - \frac{c}{ctg\,\phi} \qquad (7)$$

$$z_{max} = 0$$

$$q_d = \gamma D_f\left(\frac{\pi}{ctg\,\phi - (\frac{\pi}{2} - \phi)} + 1\right) + \frac{c}{tg\,\phi} \cdot \frac{\pi}{ctg\,\phi - (\frac{\pi}{2} - \phi)} \qquad (8)$$

and values of α is given for a changing value of ϕ

$$\alpha = \frac{\pi}{ctg\,\phi - (\frac{\pi}{2} - \phi)} + 1$$

$$q_d = \gamma D_f\,\alpha + \frac{c}{tg\,\phi}(\alpha - 1) \qquad (8a)$$

(Housel's Method or Schleichers equation do not give ultimate bearing capacity in a strict sense but they give the load such as p_1 on fig 5 for a settlement s_1

SCHLEICHER'S EQUATION

Schleicher integrated the stress caused by a uniform surface load under the assumption of perfectly elastic load on an elastic isotropic medium where Hookes law holds and E and m (Young's modulus and strain in the lateral direction respectively) remains constant. He got an expression for the elastic settlement of soil as

$$s = \omega \frac{\sigma_0 \sqrt{A}}{E} \frac{m^2 - 1}{m^2} \qquad (9)$$

another form of this equation taking into account the flexible slab settlements

$$s = \omega \frac{\sigma_0 \sqrt{A}}{C} \quad \text{or} \quad \sigma_0 = C \frac{s}{\omega \sqrt{A}} \qquad (10)$$

then from this last equation it can be calculated for
tolerable settlement s
the shape coefficient w
the size A of the loading area
soil properties defined by a constant C
This equation is in contradiction with Housels statement that for same settlement the bearing pressure under plates of two different sizes are the same because for this to happen
If

$$\frac{\omega\, \sigma_{01} \sqrt{A_1}}{\omega\, \sigma_{02} \sqrt{A_2}} = \frac{s_1}{s_2} = 1$$

$$\sigma_{02} = \sigma_{01} \sqrt{\frac{A_1}{A_2}}$$

which shows that contact pressure is in inverse ratio of the areas of the bearing plate. That is for Housel formula to hold the areas of plates should be the same and perimeters should be different. However above equations have no great value.

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KÖTTER'S EQUATİON

For the solution of the bearing capcity problem a rupture line is defined the one in which the shear stesses defined by eq (2) is not satisfied. We can take the limiting case which τ is equal to shear stresses along the rupture line as the starting point. Mohr circle can be used to define the state of stress at any point. From fig (1) it is seen that two different rupture lines can pass through the same point making angles 90 - o with each other. The lines of principal stress bisect the angles between the rupture lines.

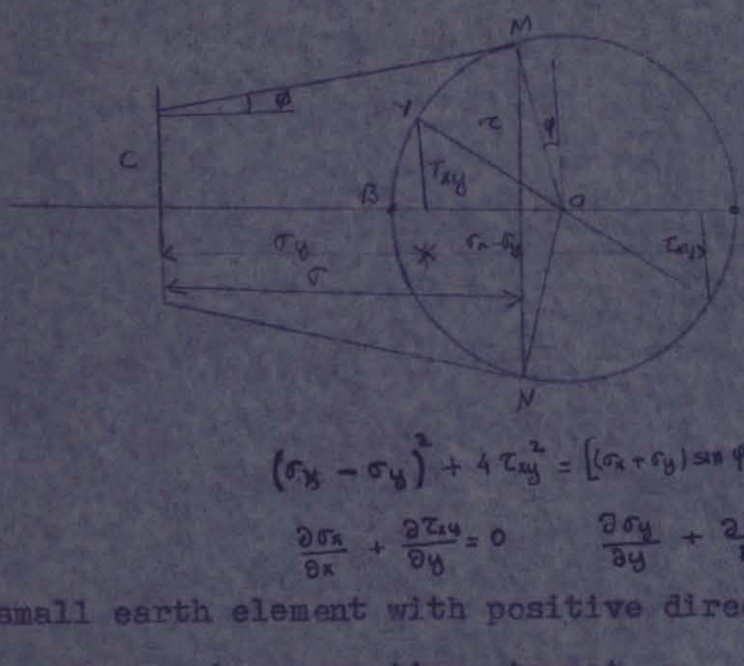


Fig-2

$$(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2 = \left[(\sigma_x + \sigma_y)\sin\varphi + 2\cos\varphi\right]^2 \qquad (11)$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \qquad \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = \gamma \qquad (12)$$

using small earth element with positive direction of y is down ward. We can use above equations to get σ_x, σ_y, τ_{xy}

$$\frac{\partial t}{\partial s} + 2(t\tan\varphi + c\sec\varphi)\frac{\partial v}{\partial s} + \gamma\sin(v+\varphi) = 0 \qquad (13)$$

in this equations s arc length of rupture line. For cohesion less soils the equation becomes

$$\frac{\partial t}{\partial s} + 2\tan\varphi\,\frac{\partial v}{\partial s} + \gamma\sin(v+\varphi) = 0 \qquad (14a)$$

for cohesive soils with angle of friction is zero (14b)

$$\frac{\partial t}{\partial s} + 2\frac{\partial v}{\partial s} + \gamma\sin(v) = 0$$

Kötters equation is used generally for two major cases

1) For weightless materials with no angle of internal friction and the rupture line is unknown

2) For cases where the rupture line is known

For the first case eq. 14 b) can be integrated and by suitable choice of boundary conditions the problem is solved. This method is called boundary methods.

For the second case with the help of the equation of the rupture line, using three conditions of equilibrium the problem is again solved.

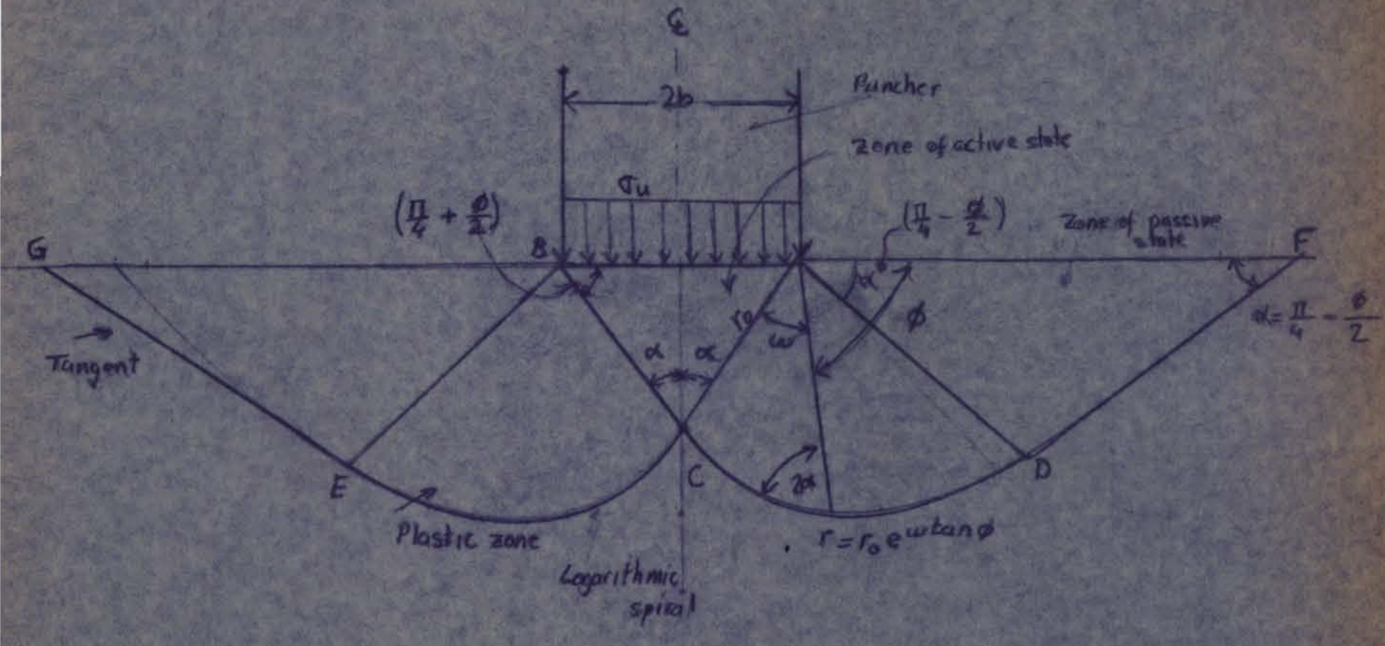


Fig-3 - Prandtl system of study

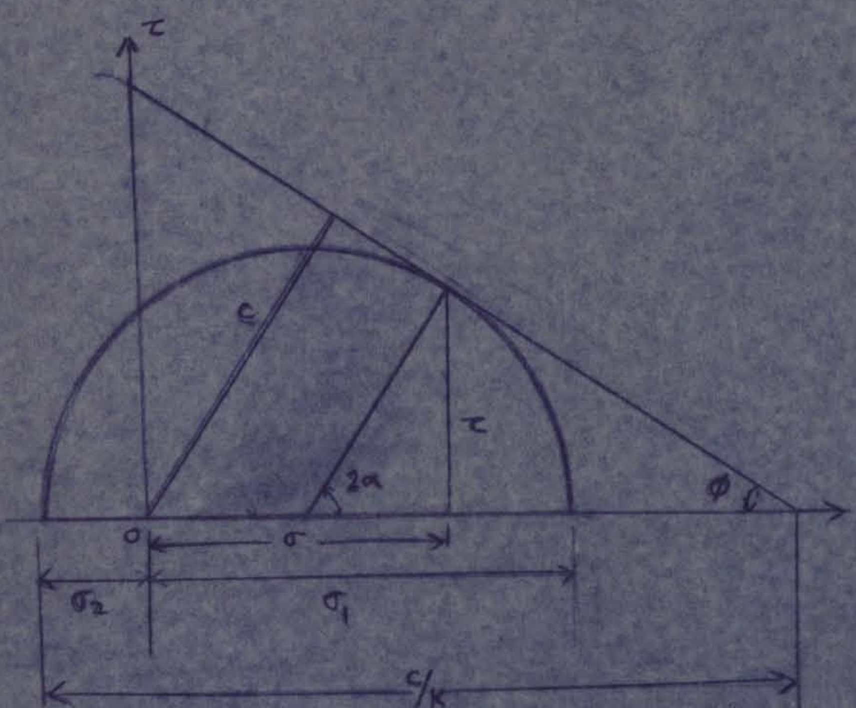


Fig 4 - Mohr stress diagram as used by Prandtl

PRANDTL'S THEORY

Prandatl studied the penetration of a hard body into a softer ideal material. He assmmed a puncher of width 2b applied on the surface of an infinitely extending body. He used the fallowing line of reasoning: since the elastic deformations of most materials are very small compared to plastic deformations they can safely be neglected and the parts of the body undergoing elastic deflection are taken as a rigid body. But it is also true in a metal the plastic strains are very small and therefore they cannot cause any change of shape. So under the assumption of no elastic deformation and zero volume change the failure in the softer material occurs by sliding. A study of the fig (3) will reveal the fallowing . When the wedge ABC is pushed into the material the triangles ADF and BEG would be translocated and and pushed out by sectors ACD and BCE. The pressures coming from triangles AOC and BOC are transferred to triangles ADF and BEG respectively and the stress conditons are given in fig (4) by Mohr circle. in Abc there is a uniform stress conditon. There exist vertically a puncher stress and horizontally, a lesser reactive stress. The weight of material in ABC (fig 3) is neglected. ABC acts as a rigid body and it moves downward with the puncher. This zone may be considered as a zone of active pressure. A uniform stress condition prevails in ADF and BEG . There the vertical stress is zero and only compressive stresses existh. ADF and BEG will slide along planes GE ard DF like a rigid body and can be considered as the zone of passive pressure.

Between those two triangles there are two sector like elements which are sections of a logorithmic spiral. The poles of the spiral are at the points A and B and the radii in both sectors are sliding or rupture surfaces. The plastic sector is divided into two by a surface emanating from the pole and having an angle of 2α with the spiral. These sectors deform plastically. Along any radius vector stress is constant but changes with different radius vector. The plastic zone is also called zone of radial shear.

Prandtl considers the plastic equilibrium of sectors assuming that the stress on AC is maximum and it is the same stress acting on AD also. Taking minor principal stresses equal to zero, he from Mohr theory of stress and Airy's stress function, obtained a differential equation of the second order which gives analytical expression for the ultimate stress .

$$\sigma_u = -\frac{c}{K}\left(\frac{1+K}{1-K}e^{\pi\tan\phi} - 1\right) \tag{15}$$

where $k = \sin\phi$ in these formulas c and ϕ are taken

$c = C\cos\phi$ as physical constants of the material.

the minus sign indicates the nature of stress (i. e . compressive

$$\sigma_u = \frac{c}{\tan\phi}\left(\frac{1+\sin\phi}{1-\sin\phi}e^{\pi\tan\phi} - 1\right)$$

or

$$\sigma_u = \frac{c}{\tan\phi}\left[\left(\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\right)e^{\pi\tan\phi} - 1\right] \tag{16}$$

this ultimate stress σ_u on AC and BC is the ultimate bearing capacity at the surface AB of the punched material. For above solutions σ_u is assumed to be of hydrostatic nature.

In these calculations one is also interested in the extent of failure surface that is the ratio of base b from the distance from the toe to the failure surface.

From fig 3

$$AF = 2\ AD \cos\left(\frac{\pi}{4} - \frac{\phi}{2}\right)$$
$$AD = AC\ e^{(\pi/2)(\tan\phi)}$$
$$AB = 2\ AC \sin\left(\frac{\pi}{4} - \frac{\phi}{2}\right)$$
$$\frac{AF}{AB} = \frac{\left(2AC\ e^{\frac{\pi}{2}\tan\phi}\right)\left(\cos\left(\frac{\pi}{4} - \frac{\phi}{2}\right)\right)}{2\ AC \sin\left(\frac{\pi}{4} - \frac{\phi}{2}\right)}$$
$$= \operatorname{ctg}\left(\frac{\pi}{4} - \frac{\phi}{2}\right) e^{\frac{\pi}{2}\tan\phi}$$
$$AC = r_0 = \frac{b}{\cos\left(\frac{\pi}{4} - \frac{\phi}{2}\right)}$$
$$AD = r_0\ e^{(\pi/2)\tan\phi}$$
$$AF = 2r \cos(\pi/4 - \phi/2) \qquad (17)$$

Prandatl obtained this solution for the case of metals but it can be extended to soils. In that case, the puncher is thought as an infinitely long strip footing; with a base width of 2b. The softer material is the soil and when the shearing strength of the the soil exceeds a certain value the failure of the soil occurs, with the expulsion of two sided sectors according to the mode given in the prandatl system.

Application of the theory to the case of soils:

The bearing pressure in the wedge is transmitted undiminished in all directions (assuming Pascals law holds), also the initial stress p_i distributes itself hydrostatically. Therefore the pressure intensity σ_u on the face AC is given

$$\sigma_a = \sigma_u + p_i \qquad p_i = \frac{c}{\tan\phi} \qquad (18)$$

Assuming the face is of unit length, the total compressive force on this face is given

$$P_a = \sigma_a A$$

and according to the stress theory

$$\frac{\sigma_3}{\sigma_1} = \frac{1 - \sin\phi}{1 + \sin\phi} = \tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right) = \frac{1}{K_p}$$
$$\sigma_1 = \sigma_3 K_p$$

$$\sigma_u + p_1 = \frac{p_1}{K_a} = p_1 K_p = \sigma_p \qquad (19)$$

The passive pressure an AD

$$DA = r_0 e^{\frac{\pi}{2}\tan\phi}$$

$$P_p = \sigma_p r = (\sigma_u + p_1)(r_0 e^{\pi/2 \tan\phi})$$

$$P_p = p_1 K_p e^{\frac{\pi}{2}\tan\phi}$$

$$r_0 = 1$$

$$P_a = P_p \quad \text{For force equilibrium}$$

$$(\sigma_u + p_1) = p_1 K_p e^{\frac{\pi}{2}\tan\phi}$$

For moment equilibrium

$$1/2(\sigma_u + p_1) = 1/2(p_1 K_p e^{\pi/2 \tan\phi})(e^{\frac{\pi}{2}\tan\phi})$$

solving this equation for we obtain prandatl's solution for the case af soils as

$$\sigma_u = \frac{c}{\tan\phi}\left[\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) e^{\pi\tan\phi} - 1\right] \qquad (20)$$

but a survey of the above equations show that for cases w here c : thebearing capacity diminishes. This fact together with the assumption of weightless material forms the weakness of the theory. Several people starting from this point and by addingempirical terms or making some simplifying assumptions tried to improve the above weaknsses of the problem. Terzaghi and Meyerhoff(whose theories to be discussed later) started from this point and by some assumptions arrived a fairly good solutions of the problem, how_ever we shold stress here that these solutions are not rigorous. It is of interest to note here the corrections introduced by Taylor however it is a regrettable fact that no account is given about the way which these corrections are obtained.

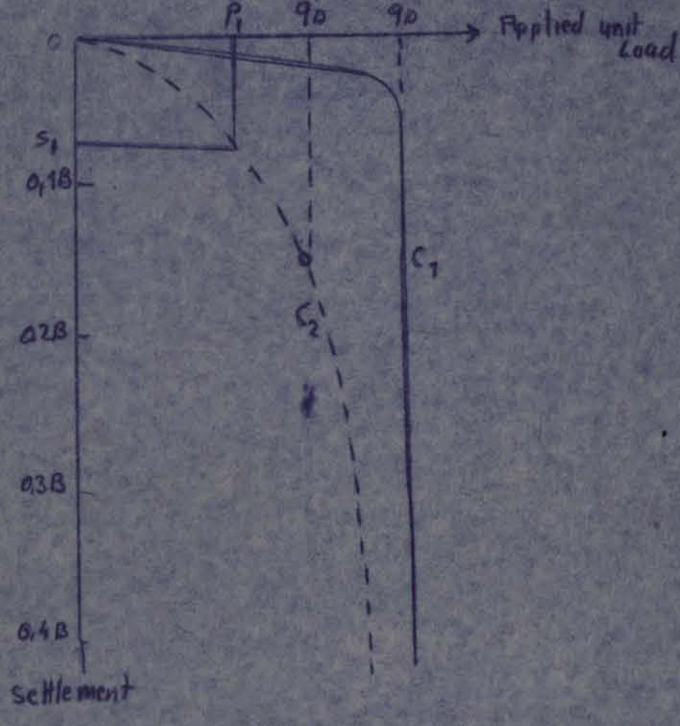


Fig 5- Relation between load and settlement

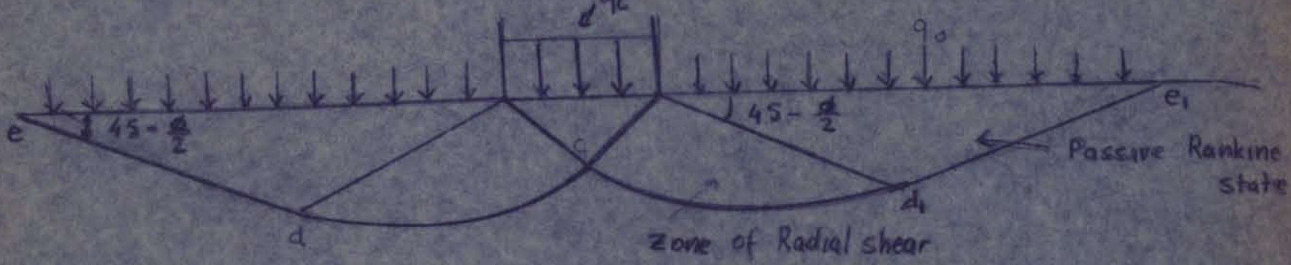
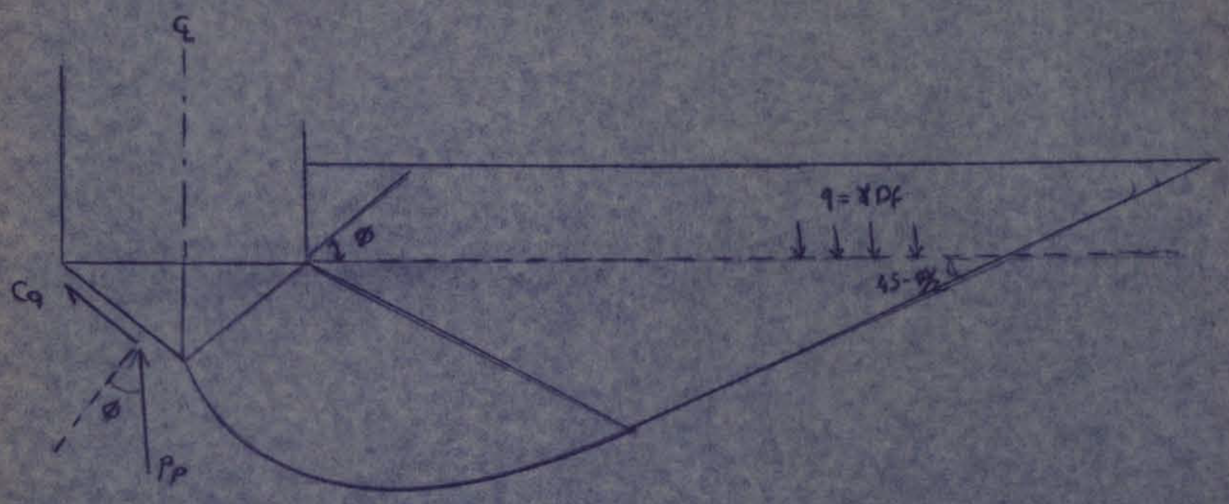
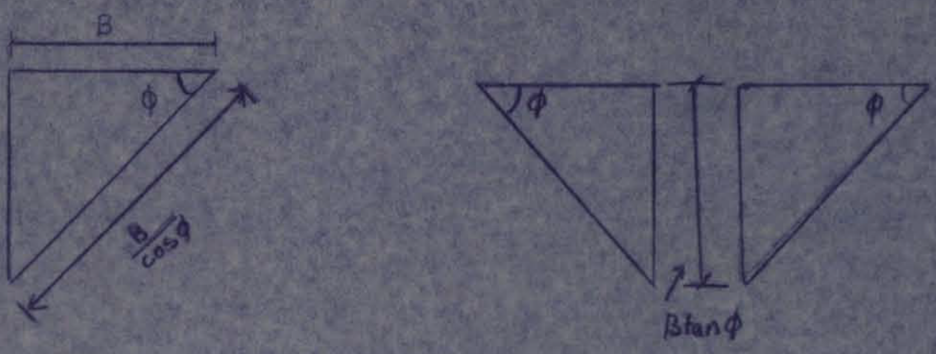


Fig-6 Plastic Flow in ideal soil


℄
q = γDf
Ø
45-Ø/2
Ca
Ø
Pp


B
ϕ
B/cosϕ
ϕ
ϕ
Btanϕ

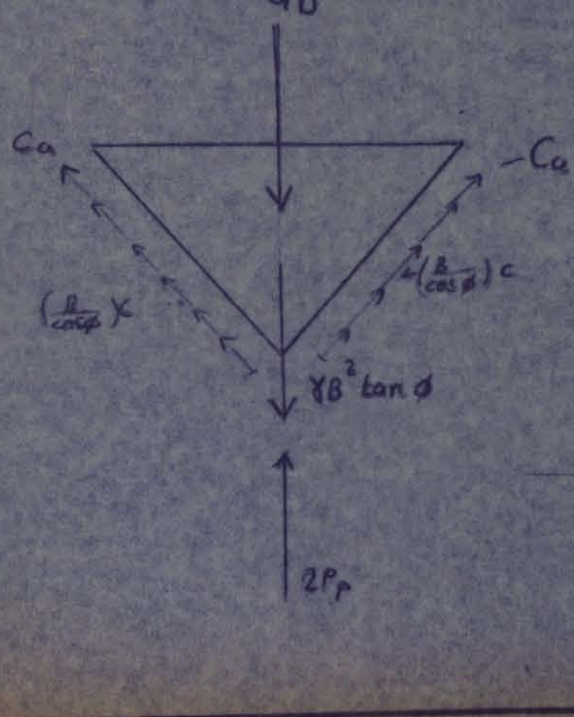

Q0
Ca
-Ca
(B/cosϕ)c
(B/cosϕ)c
γB² tan ϕ
2Pp

Fig - 9.

TERZAGHİ'S THEORY
OF BEARİNG CAPACİTY

General Definitions :

When a soil is loaded it settles, we can measure these setlement and we can plot them against the load causing this settlement. Such a plot is called the settlement curve. This curve usually lies between the two limiting curves of fig (5) . Curve C_1 shows the trend for a dense soil whereas curve C_2 shows the trend for loose soil. The area on which we applied the load is called the bearing area and the load which caused the failure of the soil support under this are is called the ultimate load or "ultimate bearing capacity of soil." Terzaghi assumes that mechanical properties of the soil, the size of the loded area the shape of the area location of the load with respect to the surface are the factors governing the bearing capacity of soil. In all his work, soil is assumed to be homogeneous from the surface to a depth far below the base of the footing. A continious footing is the one in which the length is very large compared to the width of the footing, whereas in spread footings the length to width ratio around one two,-two.

Types of failures :

As the load applied to the soil the state of elastic equilibrium of the soil changes and with the increase of load state of plastic equilibrium is established. In the transition phase from elastic to plastic state the soil reactions over the base of the footing also changes, resulting in the orientation of princi_ pal stress planes. These changes first begins at the base and at the outer edges. If in this state the plastic strains are small, that is the if the settlement curve is nearer to (C_1) the footing does not sink untill the condition of fig (6) occurs. The failure occurs by sliding as shown on fig (6). def is one of these sliding surfaces and it consists a cúrved part de and a plane ef . Such type of failure is called the general shear failure.

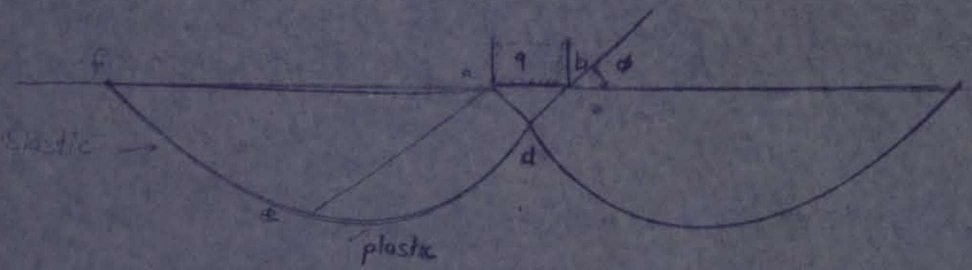


Fig - 7 - Zones of Plastic Flow
Real soil rough base

in the usual practicewe don't get plastic plastic equilibrium within the entire upper part of zone aef , and zones of failure are given for real soils in fig (7).

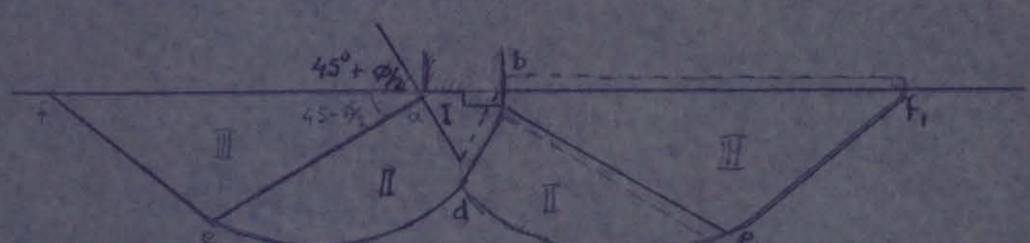


Fig 8a - Ideal soil smooth base

If plastic flow of the soil is preceded with very high strains, we can assume that failure occurs at a certain inclination of the tangent to the failure curve. In this case the load settlement curve is nearer to curve C_2 and such type of failures are called failure by local shear.

Shear failure of soil support of shallow continious fooings :

A shallow footing is defined as the one in which depth of footing below the surface of soil is equal of less than the width of the footing. Under these conditions the neglection of shear forces in soil above the foundation level is a good approximation and the error is an the safe side. When the shear forces are neglected the soil above can be replaced by a surcharge equal to the weight of soil above this level. If γ is the unit weight and depth is D_f the surcharge is given

$$q = \gamma D_f \qquad (21)$$

and if we further assume that the friction and adhesion at the base of the footing is zero we can get the zones shown on fig 8a In case of a smooth base due to lack of soil friction soil spreads laterally ~~and we get fig 8b .But~~ in case of a rough base the friction below the base restrains the movement of the soil and then soil acts as a unit and it moves together with the footing. fig (8b)

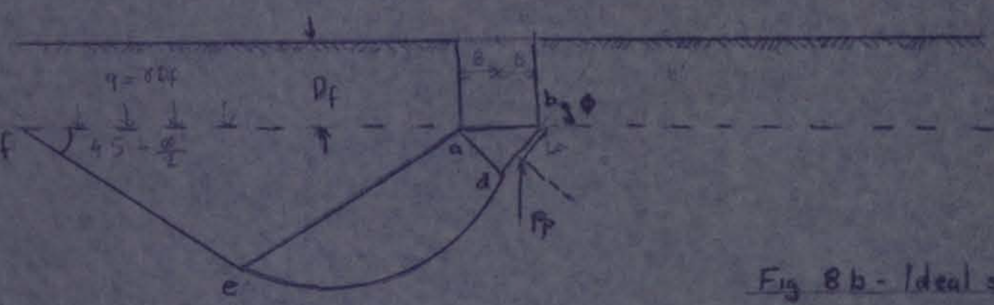


Fig 8 b - Ideal soil Rough base & surcharge

In these figures the zones of plastic equilibrium is represented by the area ff_1ede_1 can be sudivided into a wedge shaped zone (I)

As seen from the figure it is located beneath the loaded footing and pricipal stressesare vertical. Zoness of radial shear ade and bde_1 (II) and whose boundaries intersect the horizantal at angles of $45 - \phi / 2$ and $45 + \phi / 2$. Two passive Rankine zones (III).

In the real case of soils with rogh base the angle of slope of the boundaries are greater than $45 \pm \phi / 2$ but not as great as ϕ .

The ultimate bearing capacity for such footings can be calculated from the eqilibrium of the sliding surface and the passive pressures with the addition of adhesion acting on this surface. Due to previos reosning for shallow footings the soil above foundation level canbe replaced by a surcharge,

$$q = \gamma D_f$$

the shearing resistance is determined from

$$s = c + \sigma \tan \phi$$

which is known as the coulomb-Mohr criterea. At the instant of failure shearing stress on the wedge adb fig(9) is given

$$P_{pt} = c + P_{pn} \tan \phi \qquad (22)$$

where P_{pn} isthe normal component of the passive earth pressure per unit area of the contact face and P_{pt} is the ressistance encountred tangent to thefaces. Since the base of the footing is rough the faces of the soil wedge riseat an angle ϕ to the horizantal. The passivepressure acting on the faces have two components, one acting at an angle ($\phi = \delta$) to the normal and the other is the adhesion component.

$$C_a = \frac{B}{\cos \phi} c \qquad (23)$$

The value of F can be found by spiral or by friction circle the changes in β and ϕ . The values obtained by this method is in good agreement with the values obtained by other methods.(Fig 12a)

Calculation of bearing capacity of a strip foundation :

To calculate (to estimate rather) the bearing capacity of a strip foundation located at a depth D below the surface of the ground the parameters (β, p_o, s_o) must be compatible with the D in the formula below . The author gives an approximate formula ~~for~~ for depth assuming that the ground level passes through the intersection of the failure surface and the equivalent free surface.

$$D = \frac{\sin \beta \cos \phi \, e^{\theta \tan \phi} \, B}{2 \sin(45 - \phi/2) \cos(\eta + \phi)} \qquad (50)$$

$$P_s = \frac{\gamma D^2}{2 \cos \delta} K_s \qquad (51)$$

all the variables as defined before.

The free surface stresses can be calculated from the known value of adhesion force C_a and thrust P_s acting on the foundation shaft.

$$C_a = c_a D$$

$$P_s = p_s D$$

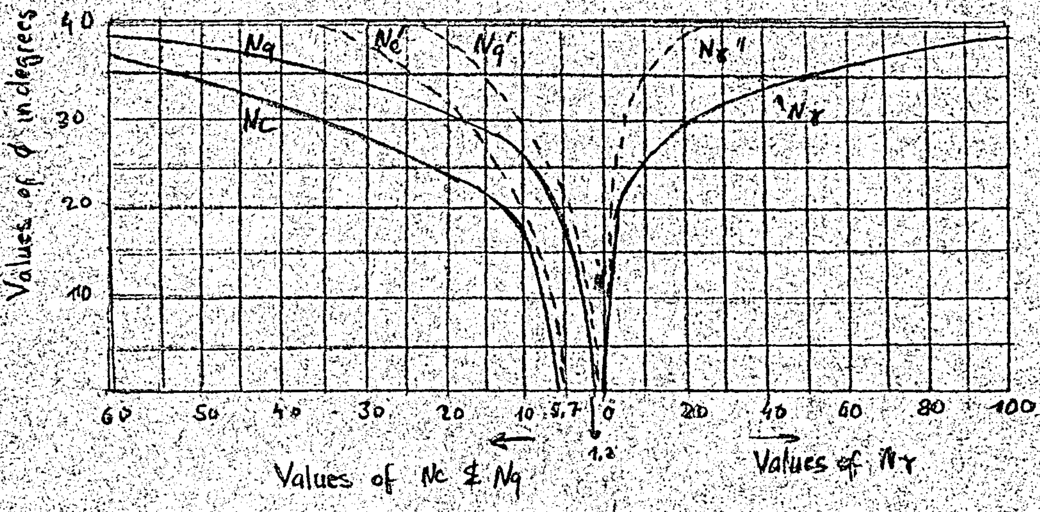
where

p_s = avarage unit earth pressure on shaft within failure (zone

K_s = coefficient of " " " " " " " " "

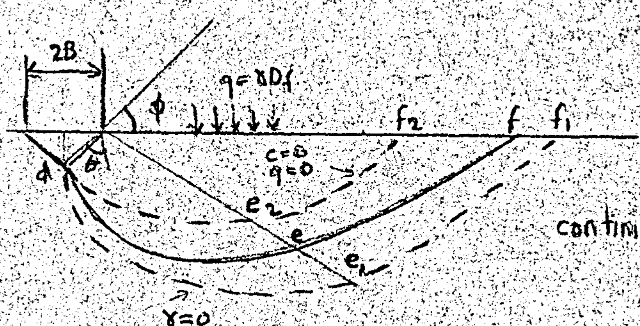
C_a = unit adhesion

it is to be noted that K_s depends on the mechanical properties of the soil and the physical properties of the foundation.

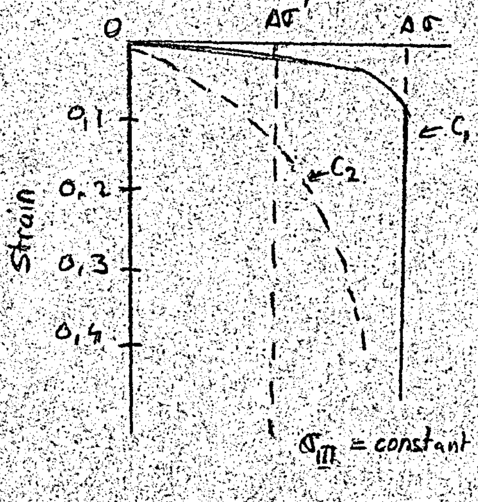


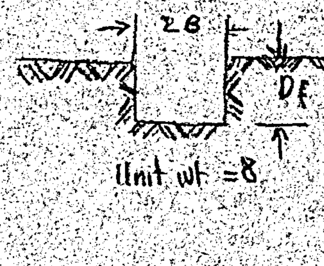
$\phi = 44^\circ \quad N_\gamma = 260$
$\phi = 48^\circ \quad N_\gamma = 780$

Fig - 10



continuous footing with rough base.





tioned before. The equation is given

$$P_{p_n} = \frac{H}{\sin\alpha}\left(cK_{p_c} + q\,K_{pq}\right) + \frac{1}{2}\,\gamma H^2\,\frac{K_{p\gamma}}{\sin\alpha} \qquad (27)$$

where P_{p_n} is the ~~athe~~ normal component of the passive earth pressure on a plane contact face with a height H. α is the slope angle of the same face and K_{p_c}, K_{pq}, and $K_{p\gamma}$ are coeffients whose values are independent of H and α. If

$$H = B\tan\phi \quad \alpha = 180 - \phi \quad \delta = \phi \quad c_a = c$$

$$P_p = \frac{P_{pn}}{\cos\delta} = \frac{P_{pn}}{\cos\phi}$$

$$Q_D = 2Bc\left(\frac{K_{pc}}{\cos^2\phi} + \tan\phi\right) + 2Bq\,\frac{K_{pq}}{\cos^2\phi} + \gamma B^2(\tan\phi)\left(\frac{K_{p\gamma}}{\cos^2\phi} - 1\right) \quad (28)$$

~~then combining~~. These equations are valid only for failures by general shear. Fig 10a shows a continious footing with a rough base. When $\phi = 0$ we get a circle with radius r_0, the logarithmic spiral turns into a circle. In this case the sliding surface is independent of cohesion and surcharge. In case $\gamma = 0$ the failure occurs along def_1 and de is a logarithmic spiral with center at b given by the equation

$$r = r_0\, e^{\theta \tan\phi}$$

for a weightless material the failure load for the surface def_1 is

$$Q_c + Q_q = 2Bc\left(\frac{K_{pc}}{\cos^2\phi} + \tan\phi\right) + 2Bq\,\frac{K_{pq}}{\cos^2\phi} \qquad (29)$$

$$= 2Bc\,(\;N_c\;) \qquad + 2Bq\,(N_q)$$

for this case the factors N_c and N_q are dependent on angle ϕ and given by coulomb equation.

For cases of a soil with weight while cohesion and surcharge is zero the curved part is not rigorously solved but it is fairly well known that de_2 is above de_1. The critical load which causes failure along def is given,

for cases where soil has cohesion surcharge and located below the surface of the soil ,the failure surface is given by the surface def which lies as seen in fig(10a)between def_1 and def_2. The results of many numerical computations show that the critical load Q is slightly greater than the sum of the loads Q_c and Q_q Q_γ which are obtained from different failure surfaces. Therefore we can assume with a good accuracy that

$$Q_p = Q_c + Q_q + Q_\gamma = 2Bc\,N_c + 2Bq\,N_q + 2B^2\gamma N_\gamma \qquad (30)$$

where 2B is the width of the footing,and substituting $q = \gamma D_f$

$$Q_p = Q_c + Q_q + Q_\gamma = 2B(cN_c + \gamma D_f N_q + \gamma B N_\gamma) \qquad (31)$$

than the coefficients N_c, N_q and N_γ are known as bearing capa city factors, and the values are given graphically. The value s are obtained from the fallowing formulas .

$$N_c = \cot\phi \left[\frac{a_\theta^2}{2\cos^2(45 + \phi/2)} - 1 \right] \qquad (31\,a)$$

$$N_q = \frac{a_\theta^2}{2\cos^2(45 + \phi/2)} \qquad (31\,b)$$

$$a_\theta = e^{(3/4\,\pi - \phi/2)\tan\phi} \qquad (31\,c)$$

these formulas are obtained rigorously from Airy's stress func tion by Prandatl and Reissner.

Case Of inclined loads

Terzaghi do not give analytical expressions for the evalua tion of bearing capacity including the effects of inclined loads. He states that for loads with small inclination the bearing ca pacity is not effected and can be treated as a symetrical case. For larger inclinations no account is given.

Conditions for local shear failure:

The stress conditon for a cohesive soil at failure is given by the equation,

$$\sigma_I = 2c\tan\left(45^\circ + \frac{\phi}{2}\right) + \sigma_{III}\tan^2\left(45 + \frac{\phi}{2}\right) \qquad (32)$$

where σ_I = major principal stress   c and ϕ as before

σ_{III} = minor principal stress

when the soil is in its loose state the lateral compression required to spread plastic equilibrium is greater than the lateral compression produced in sinking. In this case soil fails by local shear. In order to obtain the minimum value of bearing capacity for this case the settlement curve is replaced by an (Fig 5) ideal plastic curve Ocd .The curve represents a material with a smaller cohesion c and smaller angle of internal friction . From practical results it is safe to assume these reduced values as

$$c' = (2/3)\,c \qquad \tan\phi' = (2/3)\,\tan\phi$$

and using this fact in the previous eq (31) we get

$$Q_D = 2B\left(\tfrac{2}{3}\,c\,N_c' + \gamma D_f N_q' + \gamma B N_\gamma'\right) \qquad (33)$$

where N_c' N_q' and N_γ' are corresponding bearing capacity factors. For soils with settlement curves intermediate between C_1 and C_2 the bearing capacity lies between Q and Q .(Fig 5)

Bearing Capacity of Shallow or Circular Footings:

We can use the previous definitions of shallow footings and the assumptions concerning the shear forces above foundation level. In this case the failure occurs in radial directions unlike the case of continious footings in which the movement is in parellel directions. By the same line of reasoning the bearing capacity of a foundation with a radius R

$$Q_{D_r} = \pi R^2 \left(c\,n_c + \gamma D_f n_q + \gamma R n_\gamma\right) \qquad (34)$$

in this equations n_c, n_q and n_γ are pure numbers dependent on ϕ only. But due to mathematical diffuculties the factors are not evaluated rigorously. From the results of tests and fram experience the fallowing provisional eqation can be given

$$Q_D = \pi R^2 q_D = \pi R^2 (1.3 \, c \, N_c + \gamma D_f \, N_q + 0.6 \, \gamma R N_\gamma) \qquad (35)$$

where N N and N are the bearing capacity factors for continious footingsb with the same arguments and with the same deficiencies the bearing capacity for square footings are given

$$Q_D = 4 B^2 q_d = 4 B^2 (1.3 \, c \, N_c + \gamma D_f N_q + 0.8 \, \gamma B N_\gamma) \qquad (36)$$

In case of a loose soil the factors for local shear failure should be used. If q_D is the bearing capacity for general shear failure by local shear occurs at 1.3 q_D for circular footings on clay 0.6 q_D for footings on sand.

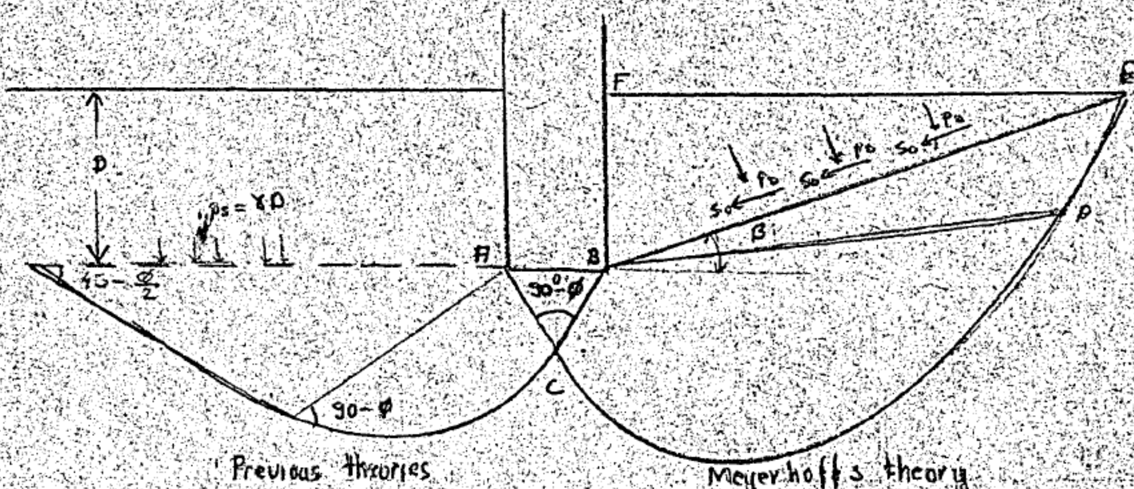

F
E
D
$p_s = \gamma D$
A
B
$45 - \frac{\phi}{2}$
$90°-\phi$
p_0
s_0
D
β
C
$90 - \phi$
Previous theories
Meyerhoff's theory

Fig - 10 -

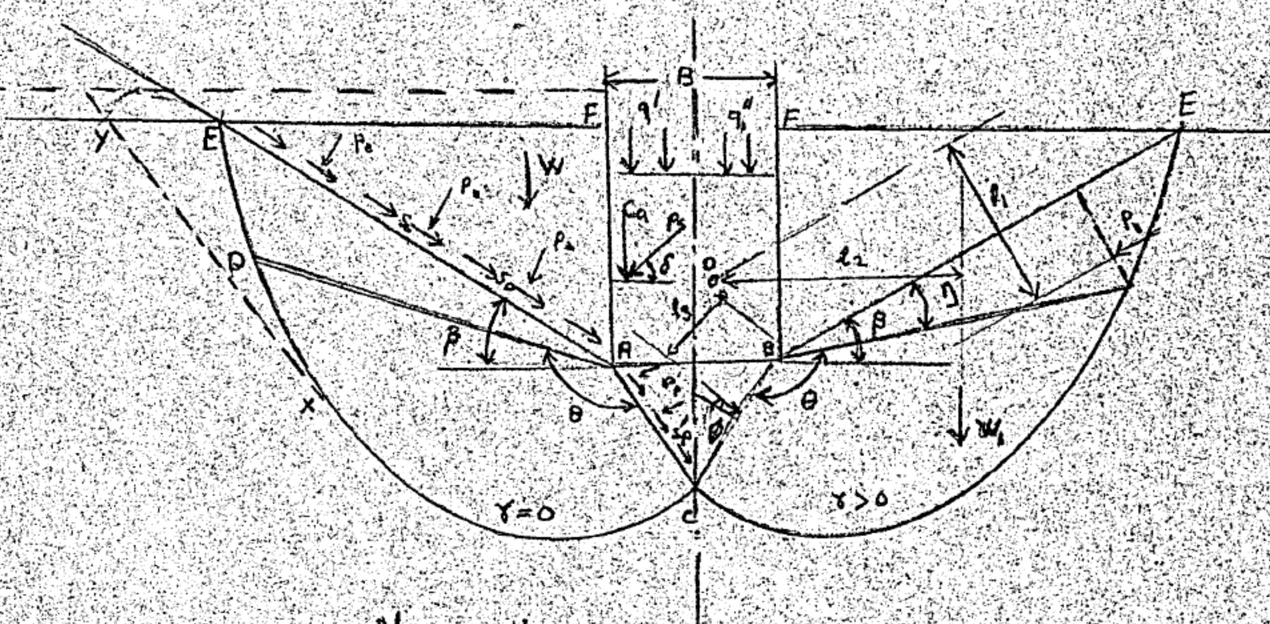

B
F
E
q'
q''
Y
W
C_a
P_p
δ
l_2
l_1
β
η
A
B
θ
$\gamma = 0$
$\gamma > 0$
C
N_c AND N_q
(b) N_γ

Fig - 11

MEYERHOF'S SOLUTION

Meyerhof takes Terzaghi's definitions of bearing capacity and applees to his work. Since this is previously mentioned, the reader is referred to Terzaghi's definitions. A theoretical method for estimation of this bearing capacity is outlined in the following paragraphs. The theory deals with soils with cohesive materials and friction, cohesive materials without friction, and cohesionless materials. Also cases of inclined and eccentric loads on foundations are investigated.

THEORY OF BEARING CAPACITY

Cohesive material with internal friction : The ultimate bearing capacity of a foundation usually depends on the following factors

I) Mechanical properties of the soil (such as density, shearing strength, deformation characteristics etc.)

II) Original stresses and water conditions on the ground

III) Physical properties of the foundation (size ,shape, depth roughness, etc.)

IV) Method of installation

However due to mathematical diffuculties not all of the above-mentioned factors are taken into account. Because of that soil is assumed to be a rigid material and in the case of deep foundation compressibility is empirically taken into account. Also the effect of factor IV above is an experimental evidence. The material is assumed to obey Coulomb - Mohr criteria for failure given by

$$S = c + p \tan \phi$$

where all variables are as defined earlier.

A section through the soil is shown on Fig (11) . It consists of three zones as follows:

I) Central zone ABC(This is the zona of elastic equilibrium and acts as a part of the foundation.)

II) Zone of radial shear ACD

III) Mixed zone ADEF (In this zones shear varies between the limits of radial shear and plane shear.)

According to the author the bearing capacity for a foundation increases with depth and it is maximum in the case of a deep foundation. The size of zones vary with a) roughness b) shape c)depth of foundation. For the same depth the size of zones vary with ~~xxxx~~ roughness and shape of and bearing capacity is maximum for circular smooth foundations.

The plastic equilibruim in the above-mentioned zones can be established from the boundary conditions starting at the foundation shaft. To take into accountthe surcharge above the foundation level a simplified method different than Terzaghi's "equivalent surcharge" is used. The forces on BF (Fig 11) and BEF are represented by p_o and s_o acting on the surface inclined β to the horizantal. The inclined surface is called the equivalent free surface and the stresses on it are called "Equivalent free surface stresses. The bearing capacity can be represented approximately by the equation

$$q = cN_c + pN_q + \gamma \frac{B}{2} N_\gamma \qquad (37)$$

Here N_c N_q N_γ are general bearing capacity factors dependent upon Depth, shape, roughnes of foundation andon angle of internal friction of soil. To get the first two terms of the above equation works of Prandtl and works of Reissner is used, for the case of a weightles materials the equation becomes:

$$q = cN + p_0 N_q \qquad (38)$$

the last term is obtained from the works of Ohde (1938)which takes the weight of soil into account and gives

$$q = \gamma \frac{B}{2} N_\gamma \qquad (39)$$

but in the case of rough foundations,

$$q_{rough} = (q_{smooth})(2) \qquad (40)$$

in the following work the general bearing capacity factors are derived in terms of p_o, β, s_o . It is also more convenient to express the bearing capacity

$$q = c\ N_{cq} + \gamma \frac{B}{2} N_{\gamma q} \qquad (41)$$

in this case the bearing capacity factors N_{cq} and $N_{\gamma q}$ depending upon N_c, N_q and N_γ, N_q respectively. In order to get the total bearing capacity of a foundation one must add the skin friction along the shaft to the above values obtained from the formulas.

General bearing capacity factors for strip foundations :

As seen from fig (12) the free surface AE makes an angle β with the horizontal and on that surface p_o and s_o act normally and tangentially respectively. In zone ADE ,the plastic equilibrium requires that s_1 and p_1 to be fully mobilized along AD and DE and given by,

$$s_1 = c + p_1 \tan\phi \quad - \text{Eq. s.}$$

If we call angle DEA = η from the use of Mohr diagram we can get

$$\cos(2\eta + \phi) = \frac{s_o \cos\phi}{c + p_1 \tan\phi}$$

$$= (c + p_1 \tan\phi)\ m \cos\phi \ / \ c + p_1 \tan\phi \qquad (42)$$

in these expressinns the subscrips refer to the surfaces on which these stresses act . m shows the degree of mobilization and its value $0 \leqslant m \leqslant 1$ .Following the calculation,

$$p_1 = \frac{c + p_1 \tan \phi}{\cos \phi} \left[\sin \left[2\eta + \phi \right] - \sin \phi \right] + p_o \qquad (43)$$

and in this equation one can find η and p_1 forgiven values of p_o s_o and ϕ

Angle θ in the radial shear zone (fig 12) ACD at A is given

$$\theta = 180 - (45 + \phi / 2) + \beta - \eta$$
$$\theta = \beta \mp 135 - \eta - (\phi / 2)$$

It was shown by Prandtl that ~~ad~~ CD is a logarithmic spiral and along the surface and along the radial sections the shearing ~~st~~ strength is fully mobilized for a weightless materials. Along AC

$$P_n = (s_\rho' - c) \cot \phi \qquad (44)$$
$$s_\rho' = (c + p_1 \tan \phi)\, e^{2\theta \tan \phi}$$

where P_n and s_ρ are normal and tangential components of the passive earth pressure. Therefore the bearing capacity is given by the expression-

$$q' = P_n' - s_\rho' \cot \left(45^\circ - \phi / 2 \right) \qquad (45)$$

then substituting the three equations above into this we get

$$q' = c \left\{ \cot \phi \left[\frac{(1 + \sin \phi)\, e^{2\theta \tan \phi}}{1 - \sin \phi \sin (2\eta + \phi)} - 1 \right] \right\} + p_o \left\{ \left[\frac{1 + \sin \phi\, e^{2\theta \tan \phi}}{1 - \sin \phi \sin 2\eta + \phi} \right] \right\} \qquad (46)$$

$q = c\, N_c - p_o N_q$ where N_c and N_q given by the brackets above

Values of N_c and N_q are calculated and given in the form of graphs an investigation of these values show that the variation of m between zero and one do not effect the results for practical pur poses. Therefore we can treat β and ϕ as variables. The values obtained for $\beta = 0$ and $m = 0$ which is the case of a surface foun dation is in close agreement with the values found by Reissner and Prandtl. (Fig 12a)

Calculation of N_γ:

The author choses to use the logorithmic spiral since it is the exact solution for weightless materials and also experimental evi dence shows that it is reosonably close to the observed mechanism of failure. As shown in fig(12) taking moments about any point O of the forces P_1 W_1 P and P_p'' where-

P_1 = soil resistance due to soil wedge DEG (can be found from Mohr diagram

W_1 = weight of the soil in segment BCDG

P_p'' = overturning resultant thrust acting at an ϕ to the normal on the faces BC and 1/3 BC above C then,

$$P_p'' = \frac{P l_1 - W_1 l_2}{l_3} \qquad (47)$$

the above equation is solved for different centers O untill the minimum value of P_p'' is reached, and this denotes the total pass sive earth pressure. Then

$$q'' = \frac{\gamma B}{2}\left[\frac{4\ P_p''\ \sin(45 - \phi/2)}{\gamma B^2} - \frac{1}{2}\tan\left(45 + \phi/2\right)\right] \qquad (48)$$

$$q'' = \gamma\frac{B}{2}\ N_\gamma \qquad (49)$$

N_γ is again given in a graphical form as in the previous case, the variation in m does not affect the results, but it is affected by

GENERAL BEARING CAPACITY FACTORS

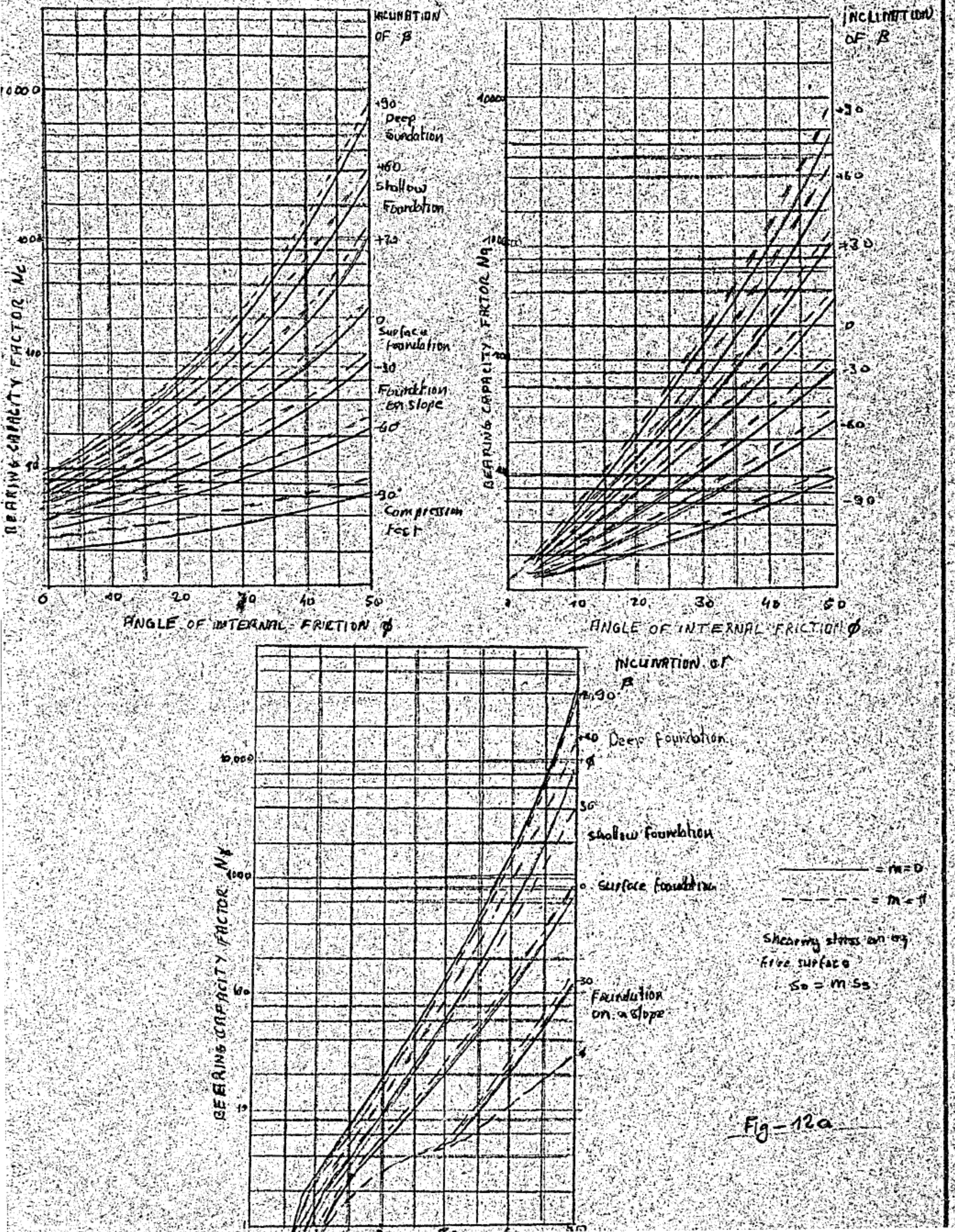


Fig-12a

the changes in β and ϕ. The values obtained by this method is in good agreement with the values obtained by other methods. (Fig 12a)

Calculation of bearing capacity of a strip foundation :

To calculate (to estimate rather) the bearing capacity of a strip foundation located at a depth D below the surface of the ground the parameters (β, p_o, s_o) must be compatible with the D in the formula below . The author gives an approximate formula ~~for~~ for depth assuming that the ground level passes through the intersection of the failure surface and the equivalent free surface.

$$D = \frac{\sin\beta \cos\phi \, e^{\theta \tan\phi} \, B}{2 \sin(45 - \phi/2) \cos(\eta + \phi)} \qquad (50)$$

$$P_s = \frac{\gamma D^2}{2\cos\delta} K_s \qquad (51)$$

all the variables as defined before.

The free surface stresses can be calculated from the known value of adhesion force C_a and thrust P_s acting on the foundation ~~x~~ shaft.

$$C_a = c_a D$$

$$P_s = p_s D$$

where

p_s = avarage unit earth pressure on shaft within failure (zone

K_s = coefficient of " " " " " " " " "

C_a = unit adhesion

it is to be noted that K_s depends on the mechanical properties of the soil and the physical properties of the foundation.

Referring to fig (11) we can find out that the weight of the soil wedge AEF is given by

$$W = \gamma \frac{D^2}{2} \cot \beta \qquad (52)$$

then by the values of c_a p_s, W we can calculate p_o, s_o from

$$p_o = \frac{p_o \sin \beta}{D} \qquad (53)$$

$$s_o = \frac{s_o \sin \beta}{D} \qquad (54)$$

The results obtained by the above mathod were derived by the author on the assumption that the free surface is extending boyond E (fig 11). Since in reality the ground surface is horizontal and the resistance is low D must be very high in order to have compatible bearing capacity factors N_c N_q N_γ for a calculated set of foundation parameters β, p_o, s_o .

An approximate method is suggested to ascertain whether a greater depth is required for the baering capacity factors to hold.

Referring to the fig (11) at any point X the major stress between EX and X is equal to maximum principal force S_1 and in the direction XY the resultant of that stress and the resistance of the sliding block EXY must be adequate, with this the curve such as EY is calculated, and difference between the foundation level and a horizontal tangent to this curve gives the depth, required for the material outside the failure surface to overcome the local resistance.

The usual computations give satisfactory results for ϕ equal or larger than 30° although the factors are based on different failure surfaces. But for other computations minimum foundation depth should be calculated by a method suggested above.

Bearing capacity of a Purely cohesive material:

In case of cohesive materials we have $\phi = 0$ and therfore s=c and this fact simplifies the solutions. As in the general theory we can divide the failure surface into tow zones 1) Radial shear zone and 2) palane shear zone. with the simplifying condition above the bearing capacity can be represented by the equation

$$q = c\ N_c + p_o \qquad (55)$$

since $N_q = 1$ and $N_\gamma = 0$ and further ,

$$N_c = 3 - \frac{\pi}{2} + 2B + 1 + \sqrt{1 - m^2} - \cos^{-1} m \qquad (56)$$

this may be obtained from the equations (46) and (42).
in this case N_c isindependent of the degree of adhesion and is directly proportional to the inclination of the equivalent free surface. This factor is again calculated for lower and upper limits of m and for a suitable variation of β . Using above formula one can see that for $m=0$ and m -1 the factor is greater than the lower one $\pi / 2 - 1 = 0.57$.
By a suitable choice of variables bearing capacity for a depth D is given by

$$q = c\ N_{cq} - K_s \gamma D \qquad (57)$$

in that Ks may be taken equal to unity and N_{cq} is a resultant bearing capacity factor dependent on N_c and N_q but latter to a smaller extent than the former. This factor varies between limits of 5.14 and 8.28 for shallow footings and the maximum value of the factor is reached at a depth twice the width of foundation. To calculate the bearing capacity one needs to add the skin fric

$$x'' = R\,(1 + a\cos(\beta+\eta)) \qquad (62)$$

$$a = \sqrt{2}\,(1 - x/R) \qquad (63)$$

where ψ denotes the angle at B between AB and point (r ,z) Fig 13
q appears in the following form after necassary oprations are carried out using equations (60),(61),(62) and integrating

$$c\left[\log_e \frac{1+a\cos\beta/\cos\eta}{1-a/\sqrt{2}} + \cot(\eta+\beta)\log_e\frac{1+a\cos\beta/\cos\eta}{1+a\cos(\eta-\beta)} + \frac{3\pi}{4}+\beta-\eta\right]$$
$$+c\left[-\frac{2}{\sqrt{1-a^2}}\left[\tan^{-1}\left(\sqrt{\frac{1+a}{1-a}}\cot\left(\frac{\eta-\beta}{2}\right)\right)\right]-\tan^{-1}\left\{\sqrt{\frac{1+a}{1-a}}\,(\sqrt{2}-1)\right\}\right] \qquad (64)$$

for a <1 in case a >1 the last term in the above equation isto be replaced by

$$c\left[-\frac{2}{\sqrt{a^2-1}}\coth^{-1}\left\{\sqrt{\frac{a+1}{a-1}}\cot\left(\frac{\eta+\beta}{2}\right)\right\}-\coth^{-1}\left\{\sqrt{\frac{a+1}{a-1}}\,(\sqrt{2}-1)\right\}\right]$$

Then we can obtain the bearing capacity from the following formulae

$$q_r = q + \frac{2}{R^2}\int_0^R \Delta q\, x\, dx \qquad (65)$$

$$q_r = c\, N_{cr} - p_o \qquad (66)$$

$$N_{cr} = N_c + \frac{2}{cR^2}\int_0^R \Delta q\, x\, dx \qquad (67)$$

for the value of Δq the integrations should be carried out numerically. The value for N_{cr} for smooth foundation is one half the value of a perfectly rough foundation so that rougnes affects the vaules of N_{cr} . The results of the analysis for N_{cr} is shown on fig 14 .One can note from the graph the linear variation between N_{cr} and β .These values are bigger than those for strip foundations because of the existance of hoop stresses. For maximum end for minimum values the factors are the same because of the neuralization of the hoop stress. The bearing capacity of a circular foundation is given by

$$q_r = c\, N_{cqr} - K_s\,\gamma\, D \qquad (68)$$

tion shown in fig 13 . The contact pressure remains constant for all depths. (Fig. 14 - gives N_{cq})

Circular foundation ⊙

In the case of a circular foundation the plastic flow occurs in horizontal and vertical directions. Hoop stresses act normal to to radial planes. They are equal to minor principal stresses and are in accordance with the coulomb -Mohr failure criterea. Work of Hencky showed that the plastic zones in radial planes and the composite failure surface is similar in shape but smaller in extent to that of under in strip footings. for an approximation the solution of the problem the two surfaces are assumed to be identical and again that plane is divided as before into radial and plane shear zones. The bearing capacity is given by

$$q_r = c\, N_{cr} + p_o \qquad (58)$$

in this case N_{cr} bearing capacity for a circular foundation. Hencky (1923) derived differential equations of stress in cylindirical coordinates (r , z) to be , where $r = x$

$$Q_x = q + c\left(\log_e \frac{x'}{x} - \int_{C'}^{E'} \frac{dz}{r}\right) \qquad (59)$$

$$= q - \Delta q$$

in this case q = the bearing capacity of a similar strip foundation and Δq contact pressures due to hoop stresses at failure. ~~Referring to fig~~ One can see that x and x' are circular radial coordinates of C' and E' of the slip line C'D'E' giving the contact pressure Q_x=

$$\int_{C'}^{E'} \frac{dz}{r} = a\int_{\pi/2}^{\pi+\beta-\eta} \frac{\cos\psi}{1-a\cos\psi}\, d\psi - \cot(\eta - \beta)\log_e \frac{x'}{x''} \qquad (60)$$

$$\bar{x}' = R(1 + a\cos\beta/\cos\eta) \qquad (61)$$

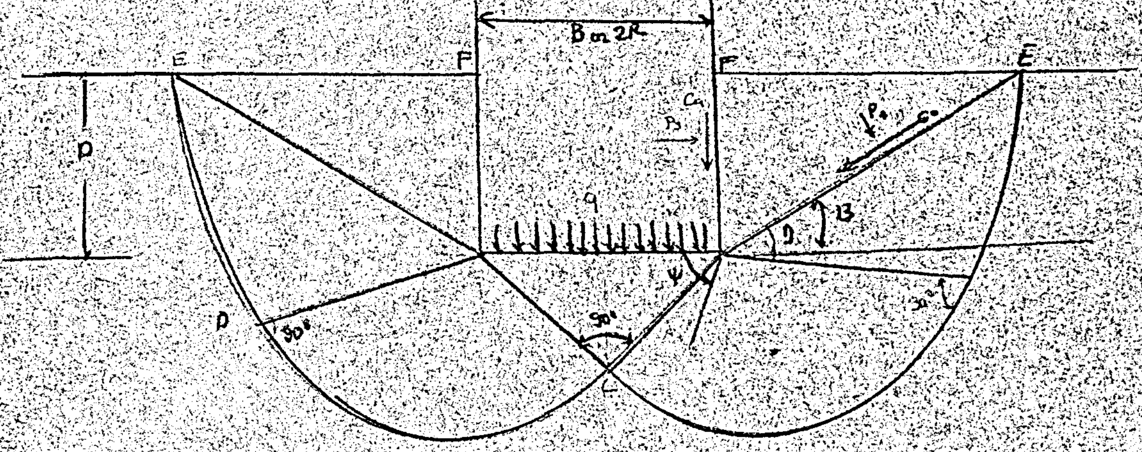


Fig 13- Determination of N_c, N_{cr}

Bearing Capacity Factors for strip and circular foundations on cohesive soils

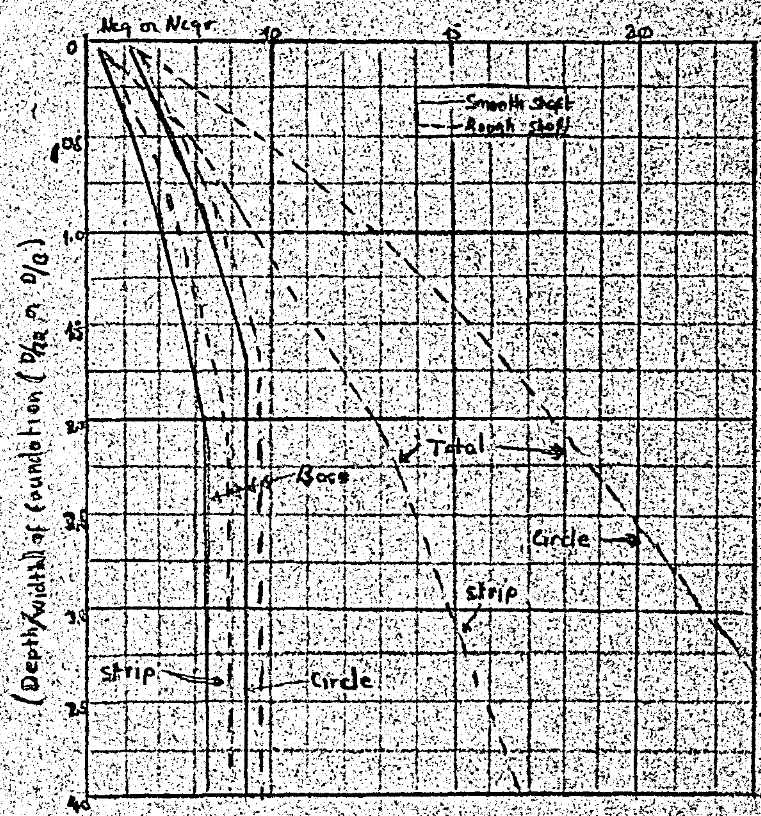


Fig- 14

in that case N_{cqr} the resultant bearing capacity factor for a circular footing and $K = 1$. As in the previous case to obtain the total bearing capacity the skin friction must be added. The increase of bearing capacity follows a similar trendd as in the case of a strip foundation and the factor is 20% greater than the latter. Under circular footings the contact pressure increases towars the center and for practical purposes it is trapzoidal. The deformation characteristics redüce the bearing capacity of the soil for larger depths.

Rectangular and Square foundations :

Depending on the previous assumptions that the failure surface of the strip foundation and circular foundation is the same, we can extend the analysis to the case of rectangular foundations of langth = L and width = B. In that case we assume a founda_ tion with circular ends $R = B / 2$, so that for the portion $L - B$ the stresses are the same for a similar strip foundation. In that case the longitidional stresses are identical to hoop stresses that arises in circular foundations. The bearing capa city for a rectangular foundation at a depth D is given by the following expression

$$q_l = c\ N_{cql} + K_s \gamma D \qquad (62)$$

for that case N_{cql} is the resultant bearing capacity factor for a rectangular footing, given by

$$N_{cql} = \left[1 + \frac{N_{cqr}}{N_{cq}} - 1\right]\left[\frac{\pi}{4} + 0.17\frac{B}{L}\right]\left[\frac{B}{L}\right] N_{cq} \qquad (63)$$

$$= \left[1 + \frac{N_{cqr}}{N_{cq}} - 1\right]\left[\frac{B}{L}\right] N_{cq} \text{ approximately}$$

~~odr~~ we can use a factor λ such as $N_{cql} = (N_{cq})(\lambda)$ (64)

as usual N_{cql} and N_{cq} shows the resultant bearing capacity factors and λ may be suitably termed as the shape factor which is defined by the equations (64). Since no solution is obtained for square foundations we can only assume that it will be approximately equal to the bearing capacity for a square foundation. So we can use the bearing capacity factors of a circular footing. By using a rectangular foundation with square ends we can obtain

$$N_{cql} = (1 - 0.15\, B / L)\, N_{cq} \qquad (65)$$

Bearing Capacity Of Cohesionless Materials :

For the case of cohesionless materials $C = 0$ and $s = p \tan \phi$ in the case of a strip foundation failure surface can be approximmated by the figs (11) & (12) on page . Then the bearing capacity is given by

$$Q = \gamma \frac{B}{2} N_\gamma + p_o N_q = \gamma \frac{B}{2} N_{\gamma q} \qquad (66)$$

for the foundations with smooth bases the first term should be divided by a factor of 2 so that first term reads $Q = \gamma \frac{B}{4} N_\gamma + p_o N_q$ The complete solution of the above equation is given by the fig (15) for a combined bearing capacity factor $N_{\gamma q}$ which depend on N_γ and N_q obtained from eq. (65). As seen from the graph the effect of N_γ on the combined factor is larger at smaller depths. We can also note that the assumption of no shearing strength of overburden holds at the surface only. Its use is very conservative except for cases of very shallow foundations. Above considerations also leads to the conclusion that the combined factor is directly proportional to depth and the earth pressure coefficient. The earth pressure coefficient mainly depends on the density, strength and deformation characteristics of the material, the stress strai history of the ground and the method of installation of the foundation. It has a value between the passive andacative earth pressure coefficients and cah only be determined from the field test In cases where the soil is submerged the eq. (66) is to be modifie and thus the eq. (66) becomes

$$q = \gamma' \frac{B}{} N_{\gamma q} + \gamma_w D \qquad (67)$$

where γ' = submerged density = $\gamma - \gamma_w$

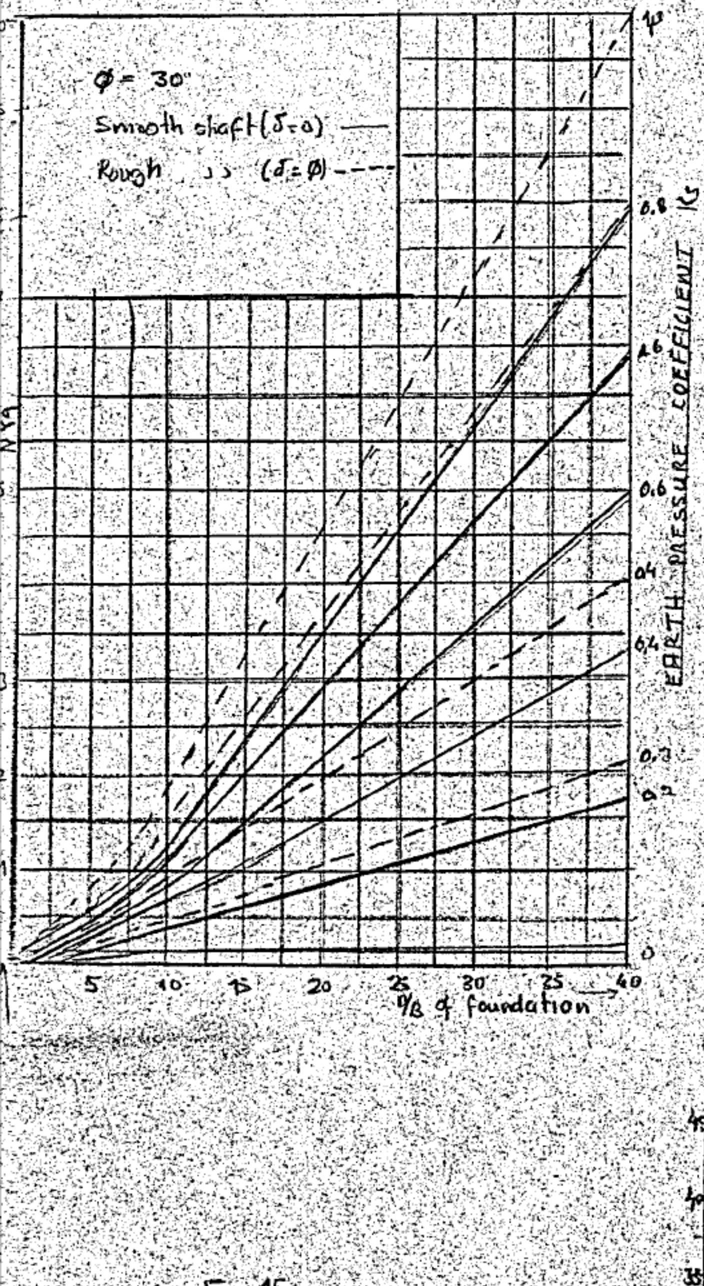
γ_w = density of water

the contact pressuure at the base is trapezoidal and increases to a maximum towards the center.

Circular Foundations :

For this case, only trial solutions are obtained, because the problem is very diffucult to solve mathematically. By carrying out slip circle analysis, which is only approximate, it is found out that the bearing capacity factor $N_{\gamma r}$ is around one half the factor for strip foundation. Meyerhoff obtained only a graphical solutions of the problem using radial sections on which the hoop stresses are equal. He found out that on these radial planes the worst failure surface is circular and cuts the ground surface at distance of 2R from the foundation where R is the radius of the foundation. The plastic zones are smaller in extent than those of strip foundation and varies with the angle of internal friction.

Since the approach used in that case is not very rigorous for other types of footings use of shape factors will be more appropriate.



BEARING CAPACITY FACTORS FOR
COHESIONLESS SOIL
(strip foundation)

φ = 40°

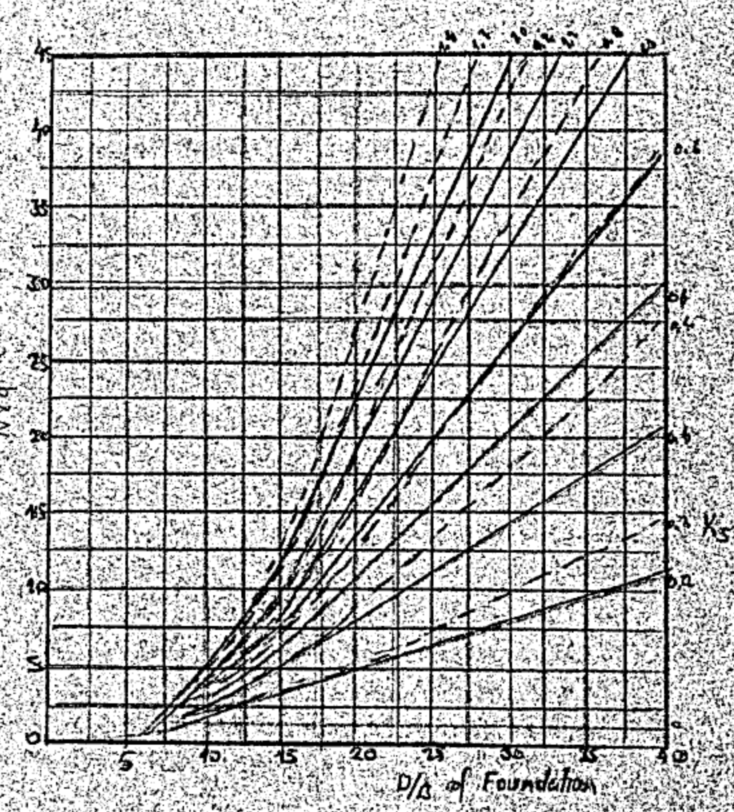


Fig-15.

THE BEARING CAPACITY OF FOUNDATIONS UNDER ECCENTRIC AND INCLINED LOADS

Theory:

A foundation when subjected to eccentric load i.e. a load plus a bending moment it tilts to in the direction of applied bending moment. Within the range of safe working loads we assume that the cotact pressures under the footing vary linearly from the maximum at the tilted edge to a minimum at the other edge. But for a footing loaded near to the failure load the pressure distribution is entirely different. In that respect Meyerhoff makes the assumption that same state of pressure prevails below an eccentrically loaded footing as in the case of a centrally loaded one. But for that case he assumes that the foundation width is reduc

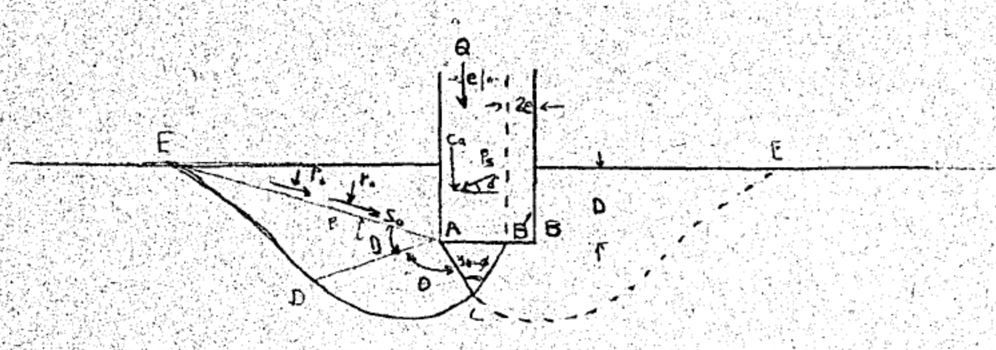


Fig-16

As shown in the fig. (16) above for a strip foundation carrying a load Q and located at a depth D, if the load has an eccentricity e than the reduced with of foundation is given by

$$B' = B - 2e \qquad (68)$$

and calculations showed that this assumption is on the safe side. In that case zones of shear failure on the eccentrically loaded sides are the same as the ones for the centrally loaded column, of the reduced size. Using γ=unit weight c=cohesion and ϕ angle of internal friction the bearing capacity can be represented by

$$Q = q' B \quad \text{or} \tag{69}$$

$$Q = q B' \tag{70}$$

in this case

$$q = c N_c + \gamma \frac{B'}{2} N_{\gamma q} \tag{71}$$

and again N_{cq} and $N_{\gamma q}$ depends upon depth ratio D / B' and angle ϕ of the soil. To obtain the total bearing capacity of a foundation one needs to add the skin fricion on the shaft. This is g ven by

$$F = C_a + P_s \cos \delta \tag{72}$$

The procedure outlined above can be extended to cases of individual footings with eccentricities on one or on two directions. The solution of that problem depends on finding maximum effecttive area of contact whose centroid coincides with the load then using the shape factor λ we can obtain the bearing capacity for that case as,

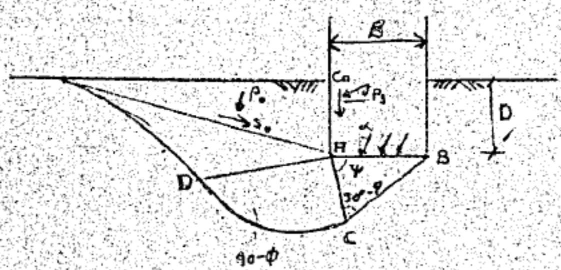
$$Q = \lambda q A' \tag{73}$$

where A' = maximum effective area under eccentric loadings. It is necassary to add that the shape factor depends on the avarage, (length / width) ratio.

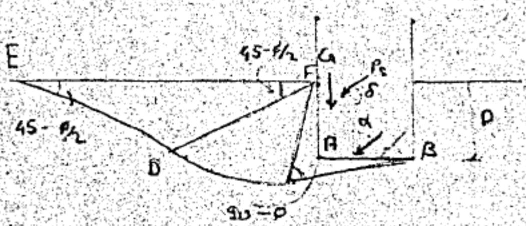
Inclined loads :

Under inclined loads the central zone is tilted and adjacent zones are effected from this change in the central zone.

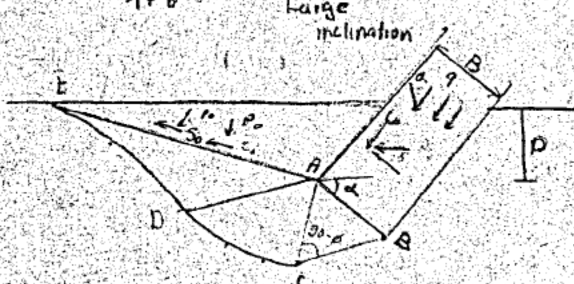
we can have two cases of inclined loads that is, 1) Loads normal to the base (footing is inclined) 2)Loads inclined to the base (footing is horizontal.) Figs. 17-a-b-c illustrates this condition.



17-a Small inclination of load



17 b Large inclination



17-c Inclined base

Horizontal Foundation :

With a load inclined to the vertical we have as in fig an elastic shear zone ABC a radial shear zone ADC and a mixed zone ADEF. As in other cases the equivalent surface stresses p and s can be found from the resultant forces acting on AF and adding to this the force on AE which is the load due to the weight of the soil wedge AEF. We should note that the plane AE makes an angle To the horizontal. we can represent the bearing capacity by Q and we can take its horizontal component as

$$q_v = q \cos\alpha \tag{74}$$

$$= c\,N_c + p_o N_q + \gamma \frac{B}{2} N_\gamma \tag{75}$$

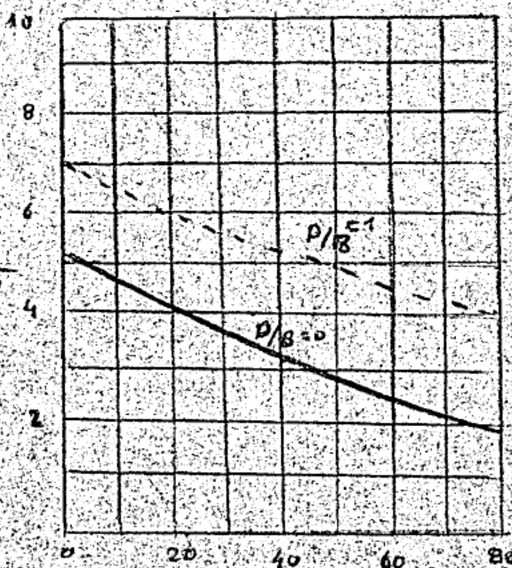
or

$$q_v = q'_v - q''_v \tag{76}$$

where

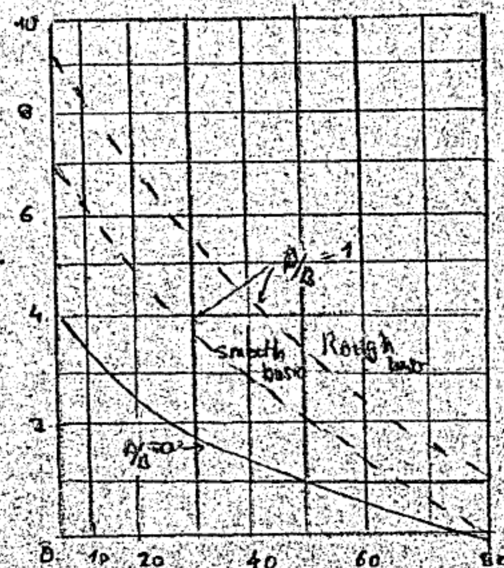
$$q'_v = c\,N_c - p_o N_q \tag{76a}$$

$$q''_v = \gamma \frac{B}{2} N_\gamma \tag{76b}$$



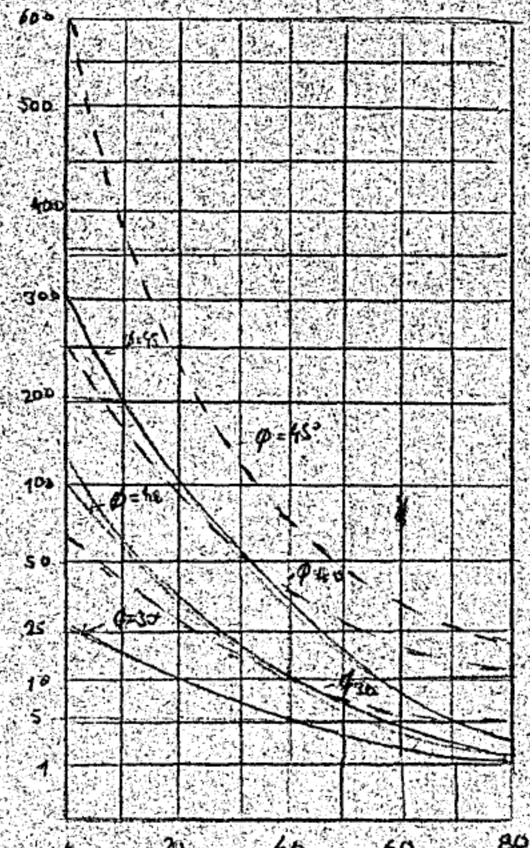
α
(Cohesive soil)

Fig – 18a



Cohesive soil —— P/B = 0
- - - P/B = 1

Fig– 19a



φ Cohesionless soil

Fig – 18 b

INCLINED FOUNDATION

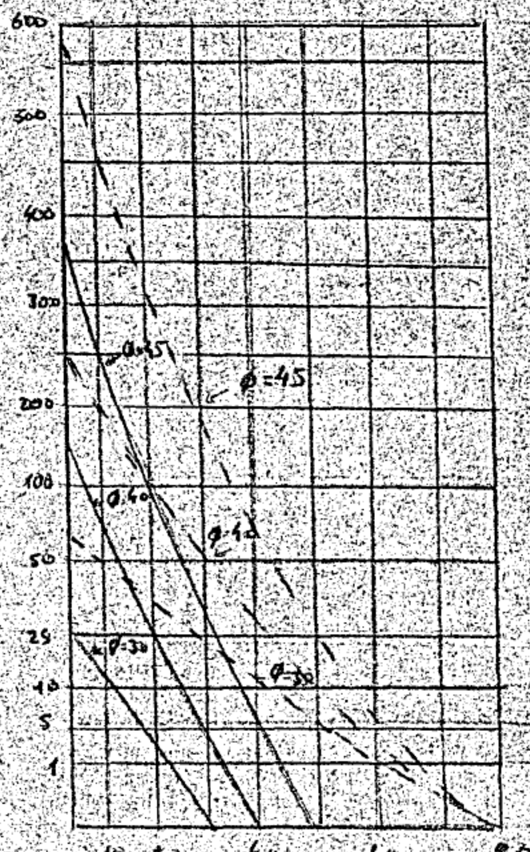


Fig – 19 b Cohesionless soil

HORIZONTAL FOUNDATION

N_c, N_q N_γ are the usual bearing capacity factors. These factors are given for cases a) Purely cohesive soils b) Cohesionless soils. And the eq. 72 gives the bearing capacity exclusive of the skin friction. The calculation of the above bearing capacity factors fallow the similar lines of procedure as carried in the previous cases and for other shapes of foundations suitable shape factors are used. In case tha angle of inclination is very large then we should have the following condition to be satisfied

$$q \tan\alpha \leq c_a + q_v' \tan\delta' \qquad (77)$$

where c_a = unit base adhesion

δ' = angle of base friction.

and in this case we should also take into account the horizontal component of the passive earth pressure on the front of the foundation.

Base normal to the Load :

For a shallow strip foundation for the cases of inclined loads normal to the base the zones are similar to those of a horizontal foundation. We can use the previous approach to arive the relation

$$q = c\,N_c + p_o N_q + \gamma \frac{\beta}{2} N_\gamma \qquad (78)$$

in that case N is obtained to be

$$N_\gamma = \frac{4P_p'' \sin\left(45+\frac{\emptyset}{2}\right)}{\gamma B^2} - \frac{1}{2}\tan\left(45+\frac{\emptyset}{2}\right)\cos\alpha \qquad (78a)$$

Where P_p'' is the minimum passive resistance obtained as indicated earlier. Then we can expressthe bearing capacity in terms of combined factors N_{cq}, $N_{\gamma q}$ as

$$Q = c\,N_{cq} + \gamma \frac{\beta}{2} N_{\gamma q} \qquad (79)$$

and these factors are given by figs 18a, 18b .

It is an interesting fact to note that the bearing capacity of a foundation with a base normal to the load has a larger bearing capacity than a foundation with a horizontal base. And this shows that practically it is more suitable to build inclined footings whenever it is possible.

Bearing capacity of a foundation with eccentric inclined load :

In that case one can always superimpose the above results to find the resultant bearing capacity. We can define three major types of problem and indicate the solutions.

1) Foundations with a horizontal force and a moment in the dire direction of the force.

2) Foundations (rectangular or square) loaded with a load and a moments in two directions.

3) Foundations with a horizontal force and a moment acting in the opposite directions to this force. (That case depends upon the effect of the force and the moment and the and foundation is analized in favor of the larger effect.

For case one we should use effective contact width B' in eq. (78) and bearing capacity is given by (79) . For case two the above pr procedure is followed with a proper shape factor λ . For case III (three) the foundation capacity is estimated for both effects and the lower value of the bearing capacity is usued. Bearing capacity factors are given Fig. 19

Experimental evidence :

Tests on clay : The test results show that the avarage bearing capacity decreases linearly, with increase in eccentricity, to zero for $(e_x/B) = 0.5$ also for a given eccentricity in one direction, increase in the eccentricity in the other direction, decrea ses the bearing capacity.

For sand : Bearing capacity in this case decreases parabolically for increase in eccentricities $(e_x / B) = 0.5$ but for a given eccentiricty in one direction an increase of the eccentricity in the other direction decreases the bearing capacity linearly. For large eccentricities the experimental values are somewhat greater than the predicted values due to the probable increase of angle of internal friction with the increase in pressure. The last important experimental evidence to note is that the middle third rule in case of failure is arbitrary and it is a better practice to design foundations with central loading.

ULTIMATE BEARING CAPACITY OF FOUNDATIONS ON SLOPES

For foundations on slopes the plastic zones are disturbed and they are greater on the side of the slope . For that case the stress distribution is shown on fig (29) .The stressed regions are again divided into zones of plane and radial shear. Then the load on AEF can be represented by a combination of equivalent free surface stresses p_o, s_o acting on plane AE making an angle α to the horizontal. We again assume that the coulomb-Mohr critera of failure holds true in this case also. Then as always the case the bearing capacity is given in terms of the bearing capacity factors as

$$q = c\,N_{cq} + \gamma \frac{B}{2} N_{\gamma q} \qquad (80)$$

in this case the value $N_{\gamma q}, N_{cq}$ are given for different (D / B) ratios

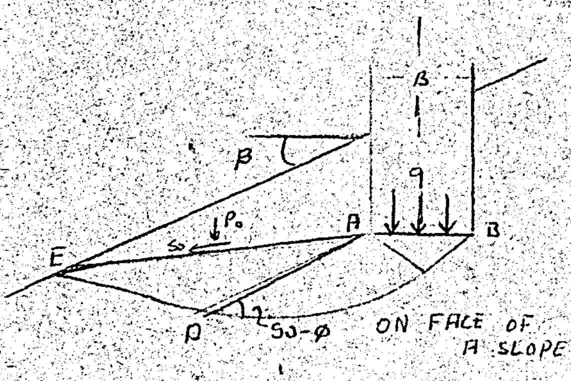
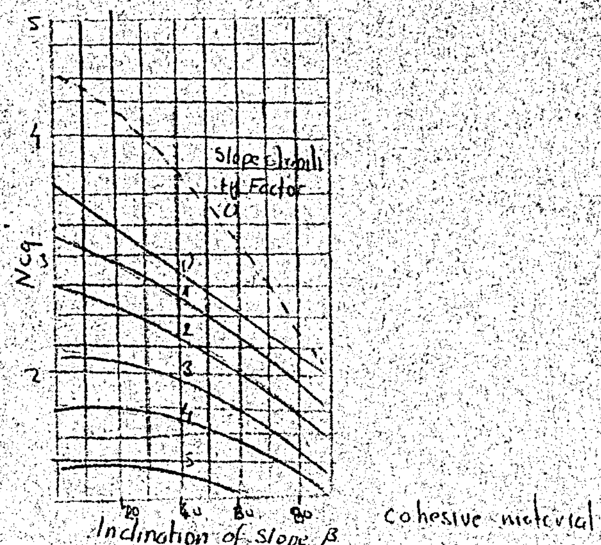
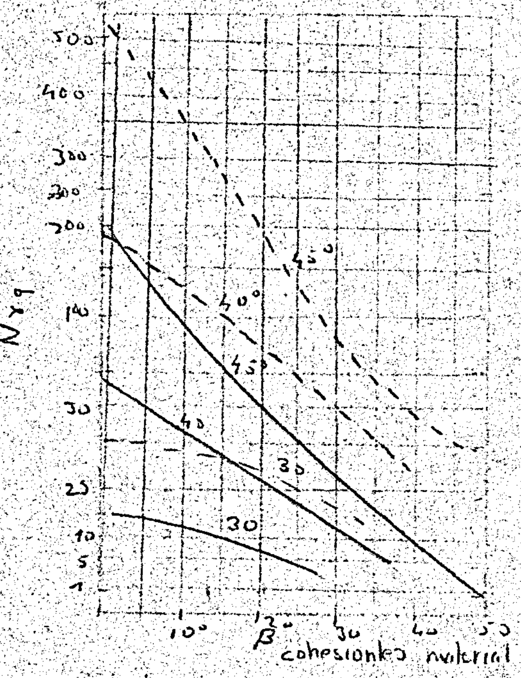


Fig-19





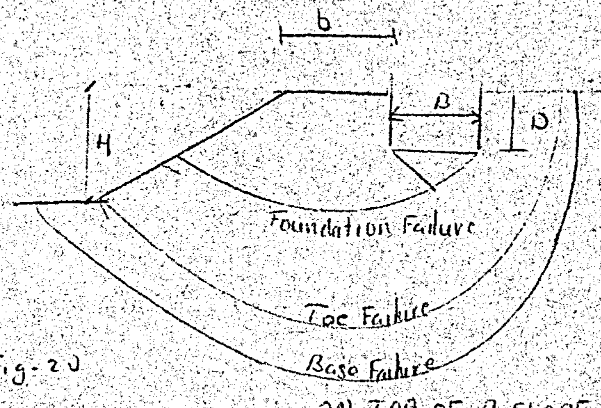
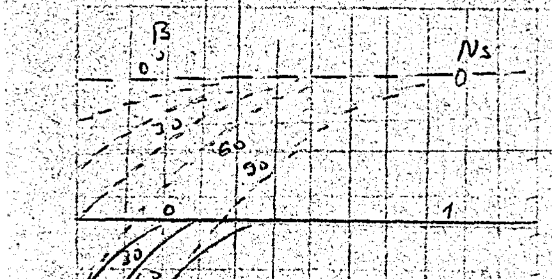


Fig-20



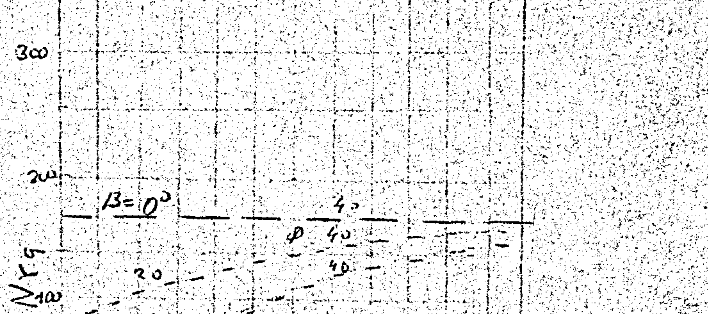


Fig-21

(where D is the foundation depth and B foundation width x 1/2) slope angle β and the angle of internal friction ϕ Figs 20a, 20b ~~they~~ cover the casas of cohesive and cohesionless materials. The examination of the values of N_{cq} shows that it decreases with greater inclination of the sldpe and becomes mininimum for $\beta = 90^\circ$ this mininmum in sandy soils is reached when $\beta = \phi$ .However this should be noted that at this angle cohesionless soil becomes unstable. For normal cases in which $\beta \leq 30^\circ$ bearing capacity of clays do not change appreciably but for sands and gravels this change is considerable, because in this case the bearing capa city decreases parabolically for such slopes.

For foundations below a stationary water table we can use $\gamma = \gamma'$ where γ'= submerged unit weight of soil. But in cases where the water percolates into the soil, flow net is to be drawn for better analysis. In cases of rapid drowdown it is suggested to use ϕ

$$\phi' = \tan^{-1}\left(\frac{\gamma}{\gamma'} \tan \phi\right) \tag{81}$$

For all above calculations when the foundation is not a strip footing, the convenient shape factor is to be taken into account. In cohesive materials the bearing capacity is in some cases limited by the stability of the slope. For usual slopes in purely cohesive soil, in great depth the base failure of an unloaded slope occurs along a critical midpoint circle., so that foundation below midpoint section increase the overall stability of the s slope. The upper limit is given by

$$q = c\, N_{cq} + \gamma D \tag{82}$$

and the lower limit is given for the case of a foundation located on the top of a slope.

Bearing Capacity For a Foundation on Top of a SLope :

Fig on page shows the zones of plastic flow for a foundatinn loaded to failure and which is located at the top of a slope. The above analysis may again be repeated for a foundation ~~m~~ referred above . The bearing capacity factors for this case is given on fig.(21) These factors decrease rapidly with the incli nation of the slope but increases with distnce b from the slope and as b =(2-6)B (depending on the angle of shearing resistance of the soil and (D / B) ratio)the bearing capacity becomes in dependent of the slope and can be calculated as a usual case of a foundatinn on long horizontal surface.

In case of wide foundations (in which $B \geq H$ slope height) Meyerhoff uses Janbu's work and in the equation of bearing capa city, the factor is assumed to depend on (N_{cq}) b , β ,and a stability factor defined by

$$N_s = \frac{\gamma H}{c} \qquad (83)$$

The examination of fig21shows that this stability factor incre ases from minimum, in slopes of smalll height, to a maximum in slopes of very great heights. That is the failure of founda tions on the latter types of slopes are due to slope failure rat her than the bearing capacity failure. In case of purely cohesive soils under the action of ground water and with a crack we should use a reduced cohesion factor given by

$$c' = \left(1 - \frac{0.8\beta z_c}{90^\circ H}\right) c \qquad (84)$$

where z_c depth of tension crack filled with water.

SKEMPTON'S WORK
ON BEARING CAPACITY OF CLAYS

When a foundation is constructed there are some settlements due to two main effects.

1) Immediate settlement due to deformation of soil taking place without change in water content,

2) Consolidation settlement due to a volume reduction caused by extrusion of some pore water from the soil. Because of fine particles of clay, for immediate loading the elastic settlements play the major role. But there is in all cases a small decrease in the moisture content of the soil beneath the foundation which will causes a small increase in strength. Neglection of this phenomena is conservative and leads to a great siplification of bearing capacity calculation. Most clays are found in saturated or nearly saturated conditon and they behave as purely cohesive and non-frictional materials, under applied loads. Therefore the angle of internal friction $\phi=0$.

The assumption of $\phi=0$ forms the basis of the theory and only in special cases o with prolonged loading or with very silty clays such an assumption is not valid and needs more elaborate analysis.

Ultimate bearing capacity of clays :

As a general case the bearing capacity can represented in terms of bearing capacity factors as

$$q' = \left[c N_c + p_o (N_q - 1) + (\gamma B N_\gamma) \tfrac{1}{2} \right] + p \qquad (85)$$

where N N and N genaral bearing capacity factors cohesion

P_o=effective overburden pressure at foundation level

p=total overburden pressure at foundation level

γ= density of soil beneath the foundation

using the condition of $\phi = 0$ the factors N_c and N_q reduces to unity and zero respectively and the bearing capacity is given for clays

$$q = c\ N_c + P \qquad (86)$$

therefore the bearing capacity of clay can be determined if c is measured and N_c can be calculated for different values of b L and D .

<u>Measurement of cohesion c :</u>

For the measurement of cohesion, undisturbed samples are required. These samples are obtained from the boreholes. The depth of these boreholes should go down to depths at which thes stresses caused by the proposed foundation becomes negligible.

Unconfined compression tests and/or triaxial tests should be made on the cores obtained fbom the holes. It is to be noted that tri axial tests should be performed as undrained tests. If on the spe cimen we have σ_I and σ_{III} as major and minor principal stresses respectively at failure we get for c

$$c = \frac{1}{2}(\sigma_I - \sigma_{III}) \qquad (87)$$

In some cases of very sensitive soils to exclude the worst effects of disturbances, in-situ vane tests are performed instead of bore-holes. Also for cases in which only shear strenth c is required this test is more economical.

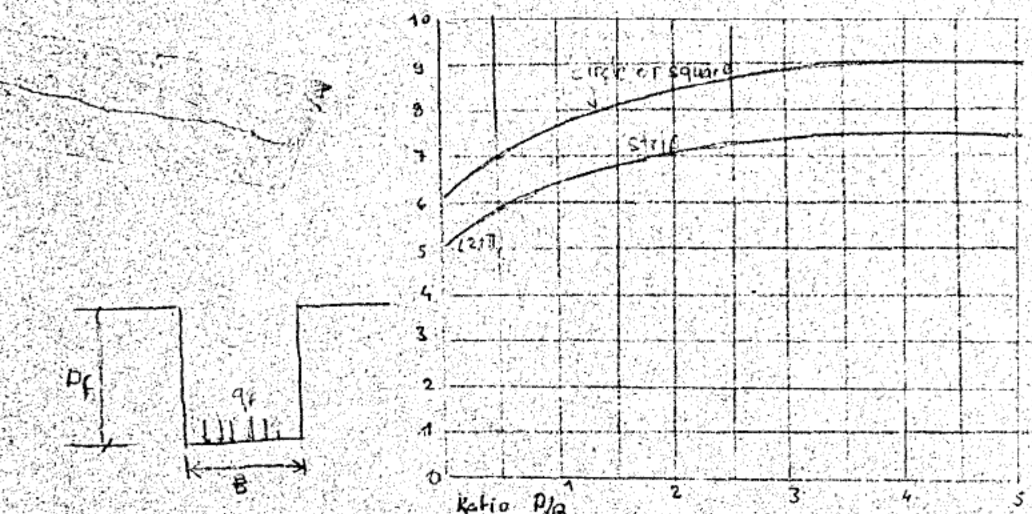
for cases where shear strenth of clay varies with depth the strength at 2/3 B depth should be taken. But this rule gives satisfactory results if the strength does not vary more than 50% within the ~~dpt~~ depth required for measurement of c .

Derivation of N_c :

In strict sense this is not a derivation but a comparison, but it is called "derivation" by the author (A. W. Skempton)
The analysis of Prandatl work gives N_c for a continious footing at the surface a value of 5.14 and Meyerhof using the same theory but with some modifications gets a vaue for N_c as 8.3. These values can be considered as limits of N_c. By the same argument for circular footings it wass found out that the upper and lower limits are 5.68 and 6.3 .
Gibson extended the large strain theory of Swainger assuming that the penetration of a footing at failure load is equivalent to to expanding a spherical hole in clay of diameter equal to the diameter of the footing. With these assumptions the bearing capacity factor is given as

$$N_c = \frac{4}{3}\left[\log_e \frac{E}{c} + 1\right] + 1 \qquad (88)$$

E being the Young's modulus for clay. Since the practical limits of (E/c) is 50 - 200 the above factor is around 7.6 - 9.4.
Another work along different lines by Gutlac - Wilson gave the vaue of $N_c = 8.0$. He takes in his work the failure load as the load which causes stresses high enough to merge the two plastic zones.
In the light of these works and with the help of experimental evidence a set of simple rules are given with a graphical figure.
Fig - 22



D/B	Nc (circle)	Nc (strip)
0	6.2	5.14
0.5	7.1	5.9
0.75	7.5	6.2
1.00	7.7	6.4
1.50	8.1	6.8
2.00	8.4	7.0
2.50	8.6	7.2
3.00	8.8	7.4
4.00	9.0	7.5
>4.00	5.0	7.5

Fig- 22

the above figures can be used interchangibly with the fallowing rules .

1) For surface loding $N_{co} = 5.0$ for strip footings

$N_{co} = 6.0$ for square or circular footings

2) For depths $D/B < 2.5$ $N_{cD} = (1 + 0.2\ D/B)\ N_{co}$

3) For depths $D/B > 2.5$ $N_{cD} = 1.5\ N_{co}$

4) AT any depth

$$N_c\ (\text{rectangular}) = \left[1 + 0.2\ B/L\right] N_c(\text{strip})$$

$$N_c\ (\text{rectangular}) = \left[0.84 + 0.16\ B/L\right] N_c(\text{square})$$

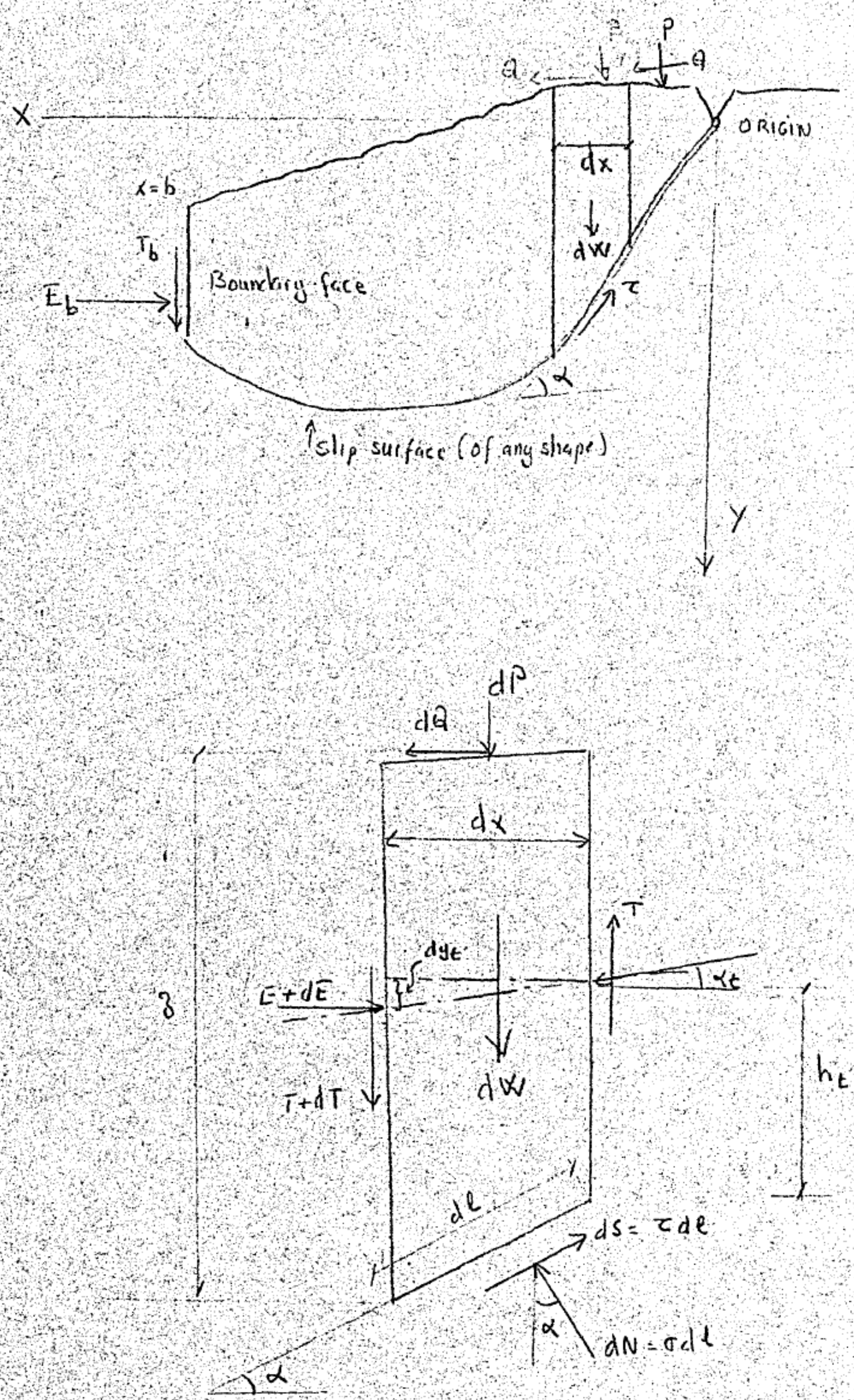

X
ORIGIN
dx
x = b
T_b
dW
E_b
Boundary face
τ
α
slip surface (of any shape)
Y
dP
dQ
dx
T
dyt
E+dE
αt
y
h_t
T+dT
dW
dl
ds = τdl
α
dN = σdl
α

Fig - 23

BEARING CAPACITY CALCULATIONS

BY GENERALIZED PROCEDURE OF SLICES

Equations of Equilibrium

The principal considerations upon which the formulas are based is given with reference to fig.(23) which shows the notation and conditions of equilibrium. Although the figure shows a vertical wall it is used only for illustration, the procedure can be applied to any desired shape of slip surface, regardless of the condition of the wall or the foundation.

Equations of	Vertical	$dW + dP + dT = dS \sin\alpha + dN \cos\alpha$	(89)
Equilibrium for	Horizontal	$dE - dQ = -dS \cos\alpha + dN \sin\alpha$	(90)
Each slice	Moment @M (Fig. 23)	$T dx + E\, dy_t + dQ z - dE h_t = 0$	(91)

Equilibrium of the whole sliding mass

$$\int_0^b (dS \sin\alpha + dN \cos\alpha) = \int_0^b (dW + dP) + T_b \qquad (92)$$

$$\text{i.e.} \int_0^b dT = T_b$$

$$\int_0^b (-dS \cos\alpha + dN \sin\alpha) = E_b - Q \qquad (93)$$

$$\text{i.e.} \int_0^b dE = E_b$$

and further

$$p = \frac{dW}{dx} + \frac{dP}{dx} = \gamma z + q \qquad t = \frac{dT}{dx}$$

Factor of safety : If the shearing resistance is given by

$$\tau = c + \sigma \tan\phi$$

than using $\tau_f = \frac{\tau}{F}$ $\tan\phi_e = \tan\phi/F$ $c_e = \frac{c}{F}$ we will get (94)

$$\tau_f = c_e + \sigma' \tan\phi_e$$

Normal and shearing stress:

$$dW + dP + dT = dS \sin\alpha + dN \cos\alpha \qquad (95)$$

Using $p = \frac{dW}{dx} + \frac{dP}{dx}$ $t = \frac{dT}{dx}$

and remembering the abbreviations

$$(p+t)\,dx = dS\sin\alpha + dN\cos\alpha \tag{96}$$

$$dx = dl\cos\alpha \qquad dS = \tau\, dl$$

$$p+t = \frac{dS}{dl}\tan\alpha + \frac{dN}{dl} \tag{97}$$

$$\sigma' = (p+t-u) - \tan\alpha$$

using - $\tau = c_e + \sigma'\tan\phi_e$ and $\sigma' = \frac{\tau - c_e}{\tan\phi_e}$

$$\frac{\tau - c_e}{\tan\phi_e}(p+t-u) - \tan\alpha$$

then

$$\tau = \frac{c_e + (p+t-u)\tan\phi_e}{1 + \tan\alpha\tan\phi_e} \tag{98}$$

internal forces and boundary conditons : If we eliminate dN between equations 89 and 90 we will get

$$dE = dQ + (p+t)\tan\alpha - \tau\cos^{2}\alpha\; dx \tag{99}$$

The horizantal force E at any point is obtained by integrating eq 99 from 0 to x. The corresponding shear force is expressed in terms of E and Q from 91 by dividing dx

$$T = -E\tan\alpha_t + h_t\frac{dE}{dx} + z\frac{dQ}{dx} \tag{100}$$

If we consider that this resultant force at the boundary equals E an T , the equilibrium of the free body requires that

$$\int_0^b dE = E_b \qquad \int_0^b dT = T_b \tag{101}$$

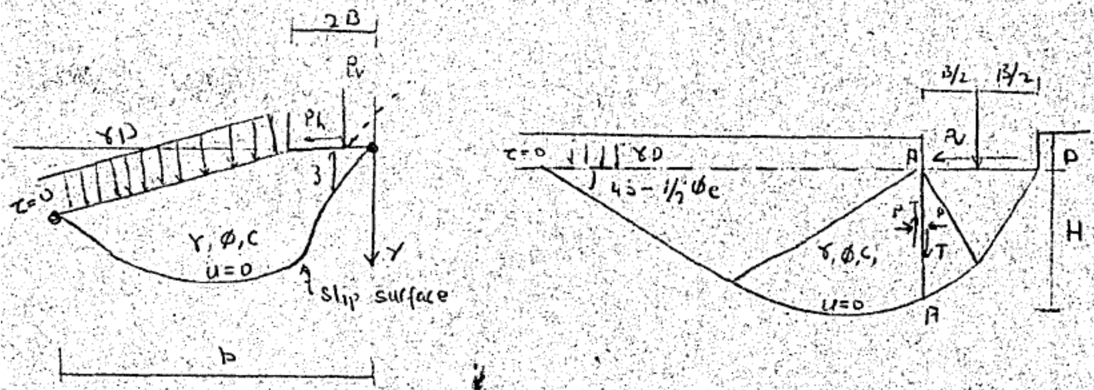
and moment equilibrium is given by (100) above.

Stability Critereon :

For the proposed procedure below it is found more canvenient to use equations 99 and 101 .

$$Q + \int_0^b (p+t)\tan\alpha\; dx - \int_0^b \tau\cos^{-2}\alpha\; dx = E_b \tag{102}$$

because of no simplified assumptions are made the equations above



$p = \gamma z + \frac{P_v}{B} \quad B \geq x \geq 0$

$p = \gamma (z + D) \quad b \geq x \geq B$

P $\quad T = 0$ except at A-A

$c = 0$

Horizontal equilibrium

$K_a (1/2\, \gamma H^2 + \frac{P_v}{B} H) + P_h = K_p (1/2\, \gamma H^2 + \gamma D H)$

Key sketches for bearing capacity calculations

~~le~~tting $\quad I = \int_0^B \tan(\alpha - \phi_e)$

and subtituting the value of p observing the discontinuity of the function we end up with an equation of the form

$$\frac{P_V + N_h P_h}{B} = \frac{1}{2} N_\gamma \gamma B + N_q \gamma D + N_c c_e \tag{108}$$

where $\quad P_h \leq P_V \tan\phi_e \quad$ no sliding on horizontal plane

and in that case we can represent

$$N_h = \frac{B}{I} \tag{109}$$

$$N_\gamma = -\frac{2}{BI}\int_0^b z \tan(\alpha - \phi_e)\, dx \tag{110}$$

$$N_q = -\frac{1}{2}\int_B^b \tan(\alpha - \phi_e)\, dx \tag{111}$$

$$N_c = \frac{\cot\phi_e}{I}\left[d - \int_0^b \tan(\alpha - \phi_e)\, dx\right] \tag{112}$$

Asumtion of T = 0 leads conservative values but by using eq. 100 we can find better values of T . The above formulas include a forth bearing capacity factor which is not usually encountered in the works of others. The values of the remaining factors are comporable to the value of the factors obtained by Terzaghi and Meyerhoff. The values are given in graphical form in Fig- 24 are available for any slip surface whether chosen or specified for any type of failure such as earth pressure.,slope stability or bearing capacity.

$$E_b - \int_0^b u\,dy = Q + \int_0^b \left[(p+t-u) c_e \cot\phi_e\right] \tan(\alpha-\phi_e)\,dx - \int_0^b c_e \cos\phi_e \,dy \qquad (103)$$

For a specified value of F we can get the above equation and for a workable formula

$$F = \frac{\sum \tau_f \cos^{-2}\alpha \,\Delta x}{Q - E_b + \sum (p+t)\tan\alpha \,\Delta x} \qquad (104)$$

where

$$\tau_f = \frac{c' + (p+t-u)\tan\phi'}{1 + \tan\alpha \tan\phi'/F} \qquad (105)$$

pocedure :

The value of t is indeterminate and we need to find out by succesive approximation using equations 99 and 100 . In this way we can get E and T . As a first aproximation $t = 0$ and with this value of t from eq 99 and 100 E_0 and T_0 is found out. than since $t = \frac{dT_0}{dx}$ t is obtained from the same equations then these steps are repeated for a desired number of times untill satisfac tory values of E_1, T_1 and t_1 is obtained.

If we can estimate the value of T at the beginning of our calcu lations we can use eq 100 to control the accuracy of the value of T . The positive directions are shown in fig 23 and they must be used to get correct results from the above formulas.

Application To Bearing Capacity Calculations :

Bearing capacity can be calculated from the eq 102 if the value p is substituted into

$$E_b - \int_0^b u\,dy = Q + \int_0^b \left[(p+t-u) + c_e \cot\phi_e\right] \tan(\alpha-\phi_e)\,dx - \int_0^b c_e \cot\phi_e \,dy \qquad (106)$$

Assuming no pore pressure and centric loads

$$E_b = Q + \int_0^b p \tan(\alpha-\phi_e)\,dx + \int_0^b t(\tan\alpha - \phi_e)\,dx + c_e \int \cot\phi_e \tan(\alpha-\phi_e) - c_e \int_0^b \cot\phi_e \,dy \qquad (107)$$

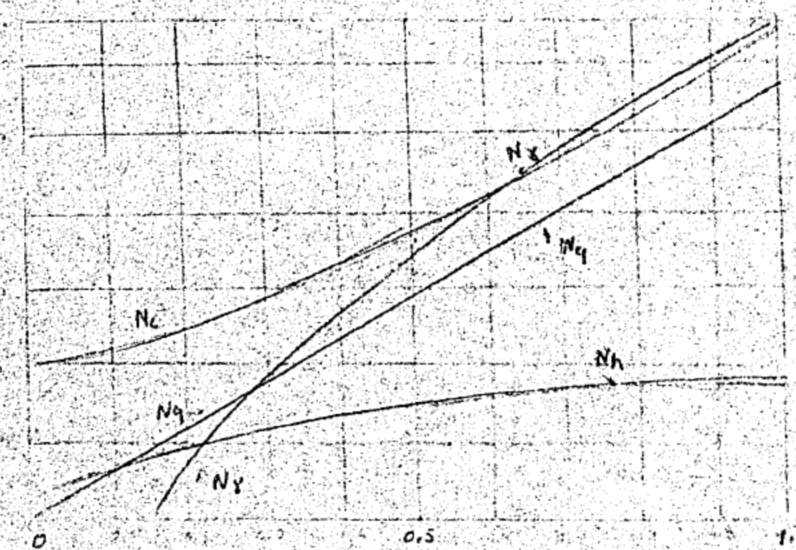


Fig-24

Shallow Footings

For the case of a horizantal terrain we can obtain the bearing c pacity factors from conditions of horizontal equilibrium where cohesion $c = 0$

$$\frac{P_v}{B} + \frac{P_h}{H K_a} = \frac{1}{2}\left(\frac{K_p}{K_a} - 1\right)\gamma H + \frac{K_p}{K_a}\gamma D$$

using values for K_a and K_p

$$N_h = \frac{B}{H K_a} = \sec\left(\frac{\pi}{4} + \frac{1}{2}\phi_e\right)\left(\frac{\pi}{4} - \frac{1}{2}\phi_e\right)\tan\phi_e$$

$$N_q = \frac{K_p}{K_a} = \tan^2\left(\frac{\pi}{4} + \frac{1}{2}\phi_e\right)\pi\tan\phi_e$$

$$N_\gamma = \left(\frac{K_p}{K_a} - 1\right) = \frac{1}{2}(N_h)(N_q - 1)$$

$$N_c = (N_q - 1) = (N_q - 1)\cot\phi_e$$

these values are in agreement with the prandatl's values for a smooth base and N_γ has a value close to the one given by Terzaghi.

LATEST SOLUTION OF BEARING CAPACITY

This is taken from   issue of the ASCE journal of Foundation Engineering division. The discussion on the article is closed   .

The problem deals with shallow foundations with vertical loads. As shown in the fig (25)

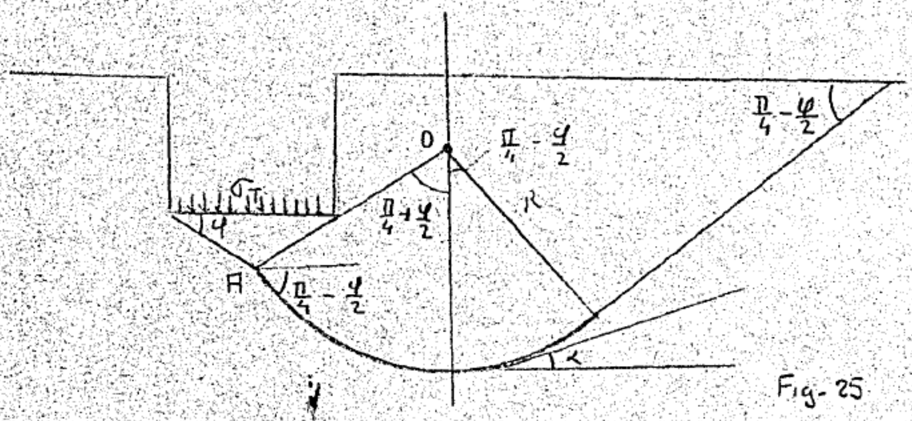


Fig- 25

Terzaghi assumption of elastic wedge below the foundation is also accepted here, and failure by general shear is treated. Unlike other methods the author (Arpad Bella) assumes that the failure surface consists a plane and a cylindirical segment. Al thogh it is claimed by the author that this is the only kinema tically possible sliding surface the discussions showed that such a surface is one of the possible surface. It is assumed also that the shear τ_{zy} at A and at the surface is zero, and for other depths it is given by

$$\tau_{zy} = \tau \, \frac{\cos(2\alpha + \phi)}{\cos\phi} \qquad (113)$$

$$\tau_{zy} = 0 \quad \phi \quad \alpha = \frac{\pi}{4} - \frac{\phi}{2} \qquad \alpha = -\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \qquad (114)$$

According to this

$$\alpha_A = -\left(\frac{\pi}{4} - \frac{\phi}{2}\right) \qquad \alpha_E = \left(\frac{\pi}{4} + \frac{\phi}{2}\right) \qquad (115)$$

Equilibrium Of The Infinitestimal Part :

Using Kötter equation for the sliding surface for the passive state of stress

From (13) $$\frac{\partial \tau}{\partial s} + \frac{\partial \alpha}{\partial s}(2\tan\phi)\tau - \gamma \sin\phi \sin(\alpha+\phi) = 0$$

and solving the equation for the cylindirical and plane parts the fallowing equations of stress is obtained.

$$\tau = \tau_2(-\alpha 2\tan\phi) - R\gamma \frac{\sin\phi}{1+4\tan^2\phi}\left[2\tan\phi \sin(\alpha+\phi) - \cos(\alpha+\phi)\right] \quad (117)$$

$$\tau = s\gamma \sin\phi \sin(\alpha+\phi) + \tau_1 \quad (118)$$

$$= z\gamma \sin\phi \frac{\sin(\alpha+\phi)}{\sin\alpha} + \tau_1 \quad (119)$$

$$\tau = z\gamma \sin\phi \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \quad (120)$$

Taking these state of stress for each part and accounting for the weight of the wedge a solution for equilibrium is reached and after some computation we get

$$q_f = b\gamma\rho\left[\rho F_4 + F_5 \tan\phi\right] + D\gamma\left[1+\rho F_5\right] + c\left[\tan\phi + \rho F_6\right] \quad (121)$$

Where F_1 F_2 ... show different functions of angle of internal fric tion ϕ.

ρ = coefficient to express the radius of the cylindirical sur face in terms of width b

$$R = \rho b \quad (122)$$

using above knowledge and conventional method of exprssing bearin capacity the eq

$$q_f = b\gamma N_\gamma + D\gamma N_q + cN_c \quad (123)$$

is obtained. For this case value of N_γ contains variables such as ρ and b thus it is more elaborate. For an actual calcula tion the magnitude of ρ is needed. This will be obtained from equations of equilibrium and the final form of the equation is

given by
$$\rho^3 F_{16} + \rho^2 \left[\frac{1}{2} F_4 \left[\frac{D}{b} + \tan\phi\right] F_{17} + \frac{c}{b\gamma} F_{18}\right] + \left[\frac{1}{2}\left(\frac{D}{b} + \tan\phi\right) F_5 + \left(\frac{D}{b} + \tan\phi\right)^2 F_{11} + \frac{c}{b\gamma} F_{19} + \left(\frac{D}{B} + \tan\phi\right) \frac{c}{b\gamma} F_{12} + \left[\left(\frac{D}{b} + \tan\phi\right)^3 F_{20} + \frac{c}{b\gamma} \frac{1}{2} \tan\phi\right] = 0$$

A grapical solution of ρ for different values of D/b ratios are given infig 26 so that ρ can be calculated easily.

The values obtained by this method are much closer to Muh's experimantal values than the values given by Terzaghi and Meyerhoff.

DISCUSSION

Classical and Modern methods used for the solution of bearing capacity problem, is presented in this thessis. Allmost all methods are based on plastic analysis since the failure of soil can be described much better than any other method. The elastic analysis methods described at the beginning of thi thesis does not give the solution of the bearing capacity problem. But if failure is assumed for a certain settlement to occur than by these methods we can find the load for this specified settlement.

The bearing capacity problem is very hard to solve. Soil is not Homogeneous ,isotropic and does not obey Hookes Law. The solution which will take into account all the above conditons will be very involved and the results (if obtained) will be very complex to apply in practice. Since no attempt is made for such a solution and since such a soluiton will be unpractical we must be satisfied with simpler solutions. A two dimensional problem on ideal soil (i.e. Homogeneous isotropic soil of infi nite extent) is satisfactory for our purposes. However even in this case the solutions require many simplifying assumptions to overcome mathematical diffuculties.

For the solution of the ultimate bearing capacity problem one needs to know the critical rupture line and stress distribution along all possible rupture surfacesb. Despite all the simplifica cations the exact shape of the rupture line is not known. So as a further simplification Prandtl assumed a weightless material

For this case he obtained the exact shape of the rupture surface as a logarithmic spiral. But such a solution is not directly a applicable to soil .

Kötter derived an equation for the stress distribution along a rupture line according to Coulomb -Mohr failure criterea. Coenen recently (1948) showed that if the equation is applied to an approximate rupture surface the resulting stresses are also approximate. Needles to say that the stresses are exact for correct rupture surfaces.

For plastic methods the correct rupture surface is a surface w which is statically and kinematically possible .Statical possibility implies that all the stresses defined by a failure criterea is maximum along the rupture surface which is in statical equilibrium. Kinematical possibility implies that movement of the material should be possible along the chosen rupture surface. Solutions of rupture surfaces satisfying both conditions are not available Terzaghi and Meyerhoff use logarithmic spirals as an approximate solution. The two solutions make use of Prandtl's work and Kötter's equation. Terzaghi assuming a surcharge and Meyerhoff assuming equivalent surface stresses inroduce the we weight of soil into the bearing capacity calculation. However in both cases the bearing capacity factors are calculated for different failure surfaces and at the end it is assumed that the bearing capacity calculated in this way will be close to the bearing capacity calculated by from bearing factors for one rupture surface.

The solution of bearing capacity problem by slice methods do not involve assumtions about ruptere surfaces and therefore they do not have the above weaknessess however the assumptions about the stresses on slices introduce errors of unknown magni tude.

In the works off Bella some promising improvements are made. The rupture surface formed by a cylinder and a plane is kinematically possible, and the bearing capacity factors are obtained from the same failure surface. However the theory is very recent so no practical use of it is made and as the Author suggests the range of application is very limited.

CONCLUSION

All the bearing capacity calculations except for the very simple cases suffer from simplifying assumtions to the degree that th e resulting solutions are only approximately correct. The shape of rupture surface is known definitely only $\gamma = 0$. The Rupture sur faces used by Meyerhoff and Terzaghi are not kinematically pos sible. Today even for the case of a two dimensional problem N_γ is not known and no attemt is made to solve for the bearing ca pacity of three dimensional problem which is frequently met in practice. But soils near failure exhibit properties similar to idel soil.

In the light of experiments and calculated values the use of T Terzaghi formula for local shear failures of all soils and general shear failure formula for clays give satisfactory results. For clays Skemtons' work and for sand Meyerhoffs work can be sug gested. Also Meyerhoffs theory is the only available theory for foundations on slopes or foundations under eccentric loads.

For stratified soils almost no theory is available for satisfac tory use.

The future work should be directed to

1) Semi - empirical solutions of the problem with practical value

2) Works on models to evelve a theory which can be applied to models and prototypes satisfactorily.

3) Solutions depending on iterative processes so that the prob lem can be handled by computers.

4) Investigation of the effects of particle size, soil structure, moisture content changes, comressibility and anisotropy of soil. on bearing capacity

A P P E N D I X

Application of different theories

Given: strip foundation

$2B = 8-0$

$D_f = 10-0$

Calculate q_o – for clay & sand given below

Sand $\gamma = 110\ lb/ft^3$, $\phi = 30°$, $c = 0$	Clay $\gamma = 110$, $\phi = 0$, $c = 1000\ lb/ft^2$
Terzaghi – Using fig. 10	Terzaghi – Using Fig. 10
$\frac{Q_o}{2B} = \gamma D_f N_q + \gamma B N_\gamma$	$q_o = cN_c + \gamma N_\gamma B$
$N_q = 21$	$N_q = 0$
$N_\gamma = 20$	$N_c = 5.7$
$q_o = (110)(10)(21) + (110)(4)(21)$	$N_\gamma = 1.0$
$q_o \simeq 32\,000\ lb/ft^2$	$q_o = (5.7)(1000) + (110)(1)(4)$
$q_d = 16\ t/sq.ft$	$q_o \cong 6000\ lb/sq.ft$
	$q_o = 3\ t/sq.ft$
	Skempton – Using fig. 22
	$q_o = cN_c + p \qquad p = \gamma D_f$
	$N_c = 6.2$
	$q_o = (6.2)(1000) + (110)(8)$
	$\cong 7000\ lb/sq.ft$
	$q_o = 3.5\ t/sq.ft$

Megerhoff - Using - fig 13 a

$B = 2B$

$q_p = \gamma \frac{B}{2} N_{\gamma q}$

$N_{\gamma q} = 0.09 \times 10^{-3}$

$K_s = 0$

$q_p = (110)(\frac{B}{2})\, 0.09$

$q_p \simeq 40\,000\ lb/ft^2$

$q_p = 20\ t/sq.ft$

Megerhoff - Using Fig 14

$q = N_{cq}\,(\text{total})\, c$

$N_{cq} = 10$

$q = (10)(10000)$

$q = 10000\ lb/sq\,ft$

$q = 5\ t/sq.ft$

From above it is seen that

For sand - the bearing capacity varies 16 - 20 t/sq ft

For clay the bearing capacity varies 3 - 5 t/sq ft
But closer to 3 ton/sq ft

For safe approximation take

$q_{p\,sand} = 18$

$q_{p\,clay} = 3.5$

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