PRIMORDIAL AND LATE-TIME INFLATION IN BRANS-DICKE COSMOLOGY

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ABSTRACT

PRIMORDIAL AND LATE-TIME INFLATION IN BRANS-DICKE COSMOLOGY

The basic motivation of this work is to attempt to explain the rapid primordial inflation and the observed slow late-time inflation by using the Brans-Dicke theory of gravity. We show that the ratio of primordial and late-time inflation parameters is proportional to the square root of the Brans-Dicke parameter ω ($\omega \gg 1$). We also calculate the Hubble parameter H and the time variation of the time dependent Newtonian gravitational constant G for both regimes. The variation of the Hubble parameter predicted by Brans-Dicke cosmology is shown to be consistent with recent measurements: The value of H in the late-time future is predicted as 0.86 times the present value of H_0. By using a linearized non-vacuum late time solution in Brans-Dicke cosmology we account for the seventy five percent dark energy contribution but not for approximately twenty-three percent dark matter contribution to the present day energy density of the universe. In the context of Brans-Dicke scalar tensor theory of gravitation, the cosmological Friedmann Equation which relates the expansion rate H of the universe to the various fractions of energy density is analyzed rigorously. And it is shown that Brans-Dicke scalar tensor theory of gravitation brings a negligible correction to the power of the scale size term in the matter density component of Friedmann equation. In addition to Ω_Λ and Ω_M in standard Einstein cosmology, another density parameter, Ω_Δ, is expected by the theory. This implies that if Ω_Δ is found to be nonzero, data will favor this model instead of the standard Einstein cosmological model with cosmological constant and will enable more accurate predictions for the rate of change of Newtonian gravitational constant in the future.

ÖZET

BRANS-DICKE KOZMOLOJİSİNDE, İLK-ZAMANA VE GEÇ-ZAMANA AİT ENFLASYON

Bu çalışmanın temel amacı, Brans-Dicke skaler-tensörel kütle çekim teorisi kullanılanılarak hızlı ilk zaman ve yavaş geç-zaman enflasyonlarına açıklık getirmektir. Gösterilmiştir ki, geç-zamana ve ilk-zamana ait enflasyon parametrelerinin oranı, Brans-Dicke teorisinin parametresi $\omega \gg 1$ için, ω nın kare kökü ile orantılıdır. Buna ek olarak, Hubble parametresi H nin ve zaman bağlı değişen Newton kütle çekim sabitinin değişim oran değerleri her iki rejim için de zamana bağlı olarak hesaplanmıstır. Brans-Dicke teorisi gereğince öngörülen Hubble parametresindeki değişimin en yeni gözlem sonuçlarıyla uyumlu olduğu gösterilmiştir: Geç zamanda H değerinin, bugün ölcülen H_0 değerinin yaklaşık 0.86 katı kadar olacağı öngörülmektedir. Brans-Dicke kozmolojisiyle elde edilen, lineerize edilmiş, kararsız ve vakum olmayan geç zaman çözümlerini kullanarak, evrenin bugünkü enerji yoğunluğuna yüzde yetmi beş oranında katkı veren karanlık enerjiye açıklık getirilebilirken; yaklaşık yüzde yirmi üç oranında katkı payına sahip karanlık maddeye açıklık getirilememiştir. Brans-Dicke skaler-tensörel kütle çekim teorisi çerçevesinde, bugünkü enerji yoğunluğunu, evrenin genileme hızına bağlayan, Friedmann denklemi daha da detaylı şekilde analiz edildi. Brans-Dicke skaler-tensörel kütle çekim teorisinin, Friedmann denkleminin madde bileşeninde ki evrenin skala olarak büyüklüğünü ifade eden terimin üstel derecesine $\omega \gg 1$ için, ihmal edilebilir bir düzeltme getirdiği gösterildi. Bunlara ek olarak, standard Einstein kozmolojisinde bilinen Ω_Λ ve Ω_M dan ayrı olarak yeni bir enerji yoğunluk parametrisi olan $\Omega_\vartriangle$ öngörülmektedir. Öyleki $\Omega_\vartriangle$ nın sıfıra eşit olmaması durumunda elde edilecek yeni kozmolojik verilerin standard Einstein teorisi yerine Brans-Dicke skaler-tensörel kütle çekim teorisine uygunluk göstereceği öngörülebilir ki bu da Newton kütle çekim sabitinin değişim oranı değerlerinin daha hassas ölçümünü sağlayabilecektir.

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LIST OF SYMBOLS/ABBREVIATIONS

a	Scale size of the universe
$\dot{a}$	Time rate of the scale size of the universe
a_*	Constant size of the vacuum universe in the Primordial regime
a_{obs}	Scale size of the universe measured by observer
a_{em}	Scale size of the universe when the light is emitted
a_0	Present value of the scale size of the universe
$b(t)$	Perturbative function to primordial vacuum universe
c	Speed of light
c_1, c_2, C, C_1, C_2	Dimensionless integration constants
dx^i	Spatial basis in coordinate frame
ds	Line element
d^4x	Four dimensional volume element
e^i	Basis one-form
$f(a)$	Linearized perturbative function to F
$F(a)$	Rate of change of Brans Dicke scalar field
F_p	Primordial value of the rate of change of BD scalar field
F_∞	Rate of change of BD scalar field value at the late-time
F_1, F_2, F_3	Perturbative constants of $F(a)$
$g_{\mu\nu}$	Metric tensor
$-g$	Determinant of the metric tensor $g_{\mu\nu}$
$\sqrt{-g}$	Metric tensor density
G_N	Newtonian gravitational constant
G	Dynamical Newtonian gravitational constant
G^μ_ν	Einstein tensor
$h(a)$	Linearized perturbative function to H
$H(a)$	Rate of change of the scale size of the universe
H_p	Primordial value of the Hubble parameter
H_∞	Hubble parameter value at the late-time
H_1, H_2, H_3	Perturbative constants of $H(a)$

H_0	Present Hubble parameter value
$\hbar$	Reduced Plank's constant
i^*	imaginary number i
i, j	Spatial indices
k	Curvature of space-time
l	Astrophysical length scale
L	Length dimension
L_J	Jordan Lagrangian
L_{BD}	Brans-Dicke Lagrangian
$L_M(\Psi, \Phi)$	Matter field Lagrangian
$L_M(\Psi)$	Matter field Lagrangian except BD scalar field
L_P	Planck length
m	Mass of the BD scalar field
p	Pressure
p_M	Pressure of everything except BD scalar field
$R^{\mu}_{\nu\sigma\rho}$	Riemann curvature tensor
R^{μ}_{ν}	Ricci tensor
R	Ricci scalar
$\vec{r}$	Physical distance vector
r	Radial parameter
S^3	Spatial slices of curved space
S	Action
t	Cosmological time
t_0	Present cosmological time
T^{μ}_{ν}	Energy momentum tensor
u^{μ}	4-velocity vector for fluid
v	Velocity of nearby galaxies
$\vec{x}$	Co-moving distance vector
z	red-shift
α	Dimensionless constant

$\eta^{\mu\nu}$	Minkowskian metric tensor
η^{00}	Time component of Minkowskian metric
ε	Perturbative factor
ϵ	Dimensionless constant
$\Phi(t, \vec{x})$	Jordan scalar field
$\varphi(t)$	Original Brans-Dicke scalar field
$\phi(t)$	Canonical formed Brans-Dicke scalar field
ϕ_0	Present value of the Brans-Dicke scalar field
Θ	General primordial scale size solution
δ	Variation operator
∇	Covariant derivative operator
∂	Ordinary differential operator
γ	Equation of state parameter
μ, ν	Both spatial and time indices
Λ	Cosmological constant
Ω	Density parameter
Ω_Λ	Density parameter due to cosmological constant
Ω_{DE}	Density parameter due to dark energy
Ω_M	Density parameter due to matter
Ω_{DM}	Density parameter due to dark matter
Ω_R	Density parameter due to curvature
Ω^μ_ν	Curvature two-form
$\Gamma^\sigma_{\mu\nu}$	Affine connection tensor
σ	Dimensionless constant
ρ	Energy density
ρ_{dust}	Energy density due to dust
ρ_{rad}	Energy density due to radiation
ρ_M	Energy density of everything except BD scalar field
ρ_c	Critical energy density
ρ_0	Present energy density
ω_J	Jordan coupling constant

ω	Brans-Dicke coupling constant
${\omega^i}_j$	Connection one-form
BD	Brans Dicke
CMBR	Cosmic microwave background radiation
DM	Dark Matter
FLRW	Freedmann Lemaitre Robertson Walker
GeV	Giga electron-volt
Mpc	Mega parsec

1. INTRODUCTION

The most incomprehensible thing about the world is that it is comprehensible. A. Einstein.

Concerning the recent observational evidences today, we may say that the field of cosmology is entering its 'golden age' which we are experiencing and living through. In the last twenty years, new observational data has allowed the field to advance as never before in its history. The most exciting innovations have strengthened the theory that the universe underwent a period of accelerated expansion at very early called primordial inflation and have also revealed the discovery that it is now experiencing another period of acceleration called late-time acceleration driven by some mysterious dark energy. Hence, this thesis is mainly motivated by making the physical explanation of these two accelerations. For this aim, we have chosen the underlying theory as Brans-Dicke (BD) scalar tensor theory of gravitation since it is the most serious alternative to general relativity. The leading idea to work with such scalar tensor theories is to study how the assumption of the underlying theory of gravitation affects the cosmological and astrophysical phenomena, and then to use that information for extracting testable predictions. The plan and some of the main results of the thesis are as follows: In the first chapter, the general review of standard cosmology will briefly be discussed so that some important cosmological results deduced from BD theory can be interpreted well compared to the results in standard cosmology. In the second chapter, primordial and late- time inflation will be studied under BD scalar tensor theory of gravitation, and the results coming from BD theory will be discussed from the point of standard cosmology. In the third chapter, by imposing linearized solution to the field equations of the theory, the issue of dark matter and dark energy will be examined and hence, it will be shown that BD theory well accounts for dark energy but not for dark matter. In the fourth chapter, the field equations of the theory will be examined rigorously so that the famous Friedmann Equation in standard Einstein cosmology will be modified in the power of the scale size term appearing in the matter component of Friedmann Equation. This correction is in the amount of $1/\omega$ and it is negligible when $\omega \gg 1$.

Also it is shown that as far as BD cosmology is concerned, the perturbative solutions of the theory predicts another density parameter $\Omega_{\vartriangle}$ which is different from the measured density parameters Ω_{Λ}, Ω_R, Ω_M and the measurement of its value in the future can be beneficial for more accurate measurement of the rate of change of G_N.

1.1. Friedmann-Lemaitre-Robertson-Walker Cosmology

1.1.1. The Metric

The cornerstone of modern cosmology is the belief that the place which we occupy in the universe is not a special one. In other words, there are no preferred positions in the universe [1, 2]. This statement that summarizes the basis of cosmology is known as the cosmological principle (or Copernican view) which underpins many models of the universe. The cosmological principle is widely accepted as a property of the global universe, since it breaks down when one looks at local phenomena. In this sense, this principle is not assumed to be an exact principle. But evidence that the universe becomes smooth on large scales supports the use of the cosmological principle. It is therefore believed that our large-scale universe possesses two important properties, homogeneity and isotropy. Homogeneity is the statement that the universe looks the same at each point, isotropy, on the other hand, is the statement that the universe looks the same in all directions. These do not automatically imply each other however if we require that a distribution is isotropic about every point, then that does enforce homogeneity as well according to a basic theorem of geometry. Astronomical observations of the Cosmic Microwave Background show that the Universe appears to be isotropic on large scales to 1 part in 10^5 [3]. Despite the existence of inhomogeneous structures such as stars and galaxies, the observable universe is remarkably homogeneous and isotropic [4] at scales larger than about $150h^{-1}$ Mpc, where 1 Mpc$\approx 3\times10^{24}$ cm is a convenient unit for extragalactic astronomy and $h = 0.72 \pm 0.07$ characterizes the current rate of expansion of the universe in dimensionless form. The mean distance between galaxies is about 1 Mpc while the size of the visible universe is about $3000h^{-1}$ Mpc. Structures on small scales, on the other hand, can be treated as perturbations to this homogeneous background. The conventional and highly successful approach to cosmology separates the study of large scale ($l \geq 150h^{-1}$ Mpc) dynamics of the universe from the issue of structure formation at smaller scales. The former is modelled by a homogeneous and isotropic distribution of energy density; the latter issue is addressed in terms of gravitational instability which will amplify the small perturbations in the energy density leading to the formation of structures like galaxies. This starting point in the large scale cosmology allows concepts of density and curvature to be idealized. The first approximation (working to 0th order in perturbations in the density and pressure) holds that the density of gas and the curvature of space is the same everywhere at a fixed cosmological time t. Starting from flat metric R^4 with a signature (+, +, +, +)

$$ds^2 = dw^2 + dx^2 + dy^2 + dz^2,$$

one can produce its curved form under the following transformations

$$w = r \cos \xi,$$

$$z = r \cos \xi \cos \theta,$$

$$y = r \sin \xi \sin \theta \sin \phi,$$

$$x = r \sin \xi \sin \theta \cos \phi.$$

Since spatial isotropy implies spherical symmetry about every point, using (ξ, θ, ϕ) is necessary in the metric and the form of R^4 metric in spherical polar coordinates is

$$ds^2 = dr^2 + r^2 \left[d\xi^2 + \sin^2 \xi (d\theta^2 + \sin^2 \theta d\phi^2) \right],$$

where r denotes the radius of S^3. As $r \to 1$, maximally symmetric S^3 metric is identified as

$$d\Omega_3^2 = d\xi^2 + \sin^2\xi(d\theta^2 + \sin^2\theta d\phi^2).$$

Hence, the most general metric called Friedmann-Lemaitre-Robertson-Walker (FLRW) metric must be the following form having S^3 describes homogeneous and isotropic spatial sections with radius $a(t)$. Considering the spacetime, FLRW metric is defined as

$$ds^2 = dt^2 - a^2(t)\left[d\xi^2 + f^2(\xi)(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{1.1}$$

so that the intrinsic spatial curvature is constant throughout the space. The three possibilities for $f(\xi)$ to get homogeneous and isotropic spatial sections are

$$f(\xi) = \{\sin\xi,\ \xi,\ \sinh\xi\}. \tag{1.2}$$

These alternatives of $f(\xi)$ are due to a purely geometric fact, independent of the details of general relativity.

In the rest of this thesis, we will be concerned with the behavior of the universe as a whole and will be assuming large-scale homogeneity and isotropy. In this thesis we will be working with the natural units in which $\hbar = c = 1$.The metric we use is the spatially conformal flat form of FLRW metric

$$ds^2 = dt^2 - a^2(t)\left[1 + \frac{k}{4}\vec{x}^2\right]^{-2} d\vec{x}^2, \tag{1.3}$$

where k geometrically describes the curvature of the spatial sections (slices at constant cosmic time), is the curvature constant; the term $\vec{x}^2$ denotes $\vec{x}^2 = x^2 + y^2 + z^2$; the time coordinate t, referred to as cosmic time, is called the proper time which is the time measured by a comoving observer who is at constant spatial coordinates. The function $a(t)$ is called as the scale size of the universe. In the FLRW metric the curvature k, governed by the amount of matter and energy inside the universe, does not change with the expansion of the universe. It can take three different values:

- $k = 1$ if $f(\xi) = \sin\xi$, corresponds to positively curved spatial sections, geometrically elliptical universe (locally isometric to 3-spheres),
- $k = 0$ if $f(\xi) = \xi$, corresponds to local flatness namely zero spatial curvature, a geometrically flat, Euclidean universe,
- $k = -1$ if $f(\xi) = \sinh\xi$, corresponds to negatively curved (locally hyperbolic).

1.1.2. Expansion and Red-Shift

The most dramatic piece of observational evidence in cosmology is that almost everything in the universe appears to be moving away from us [5]. And the further away something is, the more rapid its recession appears to be. These velocities are measured via the red-shift, which is basically the Doppler effect applied to light waves. Using the characteristic absorbtion and emission line spectra of the galaxies, whether a galaxy is receding or moving towards us can be identified. If the galaxy is receding, the characteristic lines move towards the red end of the spectrum and the effect is known as a red-shift z. In standard terminology, z, is defined by

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}}, \tag{1.4}$$

where λ_{obs}, λ_{em} are the wavelengths of light at the points of observation (us) and emission (galaxy) respectively. If an object is receding at a speed v, then its red-shift is

$$1 + z = \sqrt{\frac{1 + v/c}{1 - v/c}}, \tag{1.5}$$

for $v \ll c$

$$z = \frac{v}{c}, \tag{1.6}$$

where c is the velocity of light. Because the expansion is uniform, the relationship between real distance $\vec{dr}$ and the co-moving distance $\vec{dx}$ for nearby objects can be written

$$\vec{dr} = a(t)\vec{dx}, \tag{1.7}$$

where the homogeneity property ensures that a is a function of time only. The recession velocity of the nearby galaxies at fixed co-moving coordinates is $\vec{v} = d\vec{r}/dt$, and is proportional to their physical distance vector, $\vec{dr}$,

$$\vec{v} = H\vec{dr}. \tag{1.8}$$

This relation is known as Hubble's Law, where H is the constant proportionality, the Hubble parameter, at the cosmological time t. Hubble parameter, defined as $H = \dot{a}/a$, having the dimension $[LENGTH]^{-1}$, is constant over space but not over time; it determines the expansion rate of the universe, where $\dot{a} = da(t)/dt$. The red-shift of spectral lines that we used to justify the assumption of an expanding universe can also be related to the scale factor a in the form of

$$1 + z = \frac{\lambda_{\mathrm{obs}}}{\lambda_{\mathrm{em}}} = \frac{a_{\mathrm{obs}}}{a_{\mathrm{em}}}. \tag{1.9}$$

It tells us that as space expands, wavelength become longer in direct proportion. Namely, the wavelength difference in light is the same as the scale difference in the universe. One can think of the wavelength as being stretched by the expansion of the universe, and its change therefore tells us how much the universe has expanded since the light began its travel.

1.1.3. Dynamics: The Friedmann Equations

The FLRW metric is a purely kinematic consequence of requiring homogeneity and isotropy of the spatial sections. However, it tells us nothing about how the scale factor of universe $a(t)$ evolves in time. Concerning the dynamics, we must solve Einstein's field Equations [6, 7] in General Relativity:

$$G^{\mu}_{\ \nu} \equiv R^{\mu}_{\ \nu} - \frac{1}{2}R\delta^{\mu}_{\ \nu} = 8\pi G_N T^{\mu}_{\ \nu}, \tag{1.10}$$

where $R^{\mu}_{\ \nu}$ is the Ricci tensor, $R = R^{\mu}_{\ \mu}$ is the Ricci scalar, $T^{\mu}_{\ \nu}$ is the stress-energy tensor and G_N is the gravitational constant. Here we assume a perfect fluid where the stress-energy tensor is defined as

$$T^{\mu}_{\ \nu} = (p+\rho)u^{\mu}u_{\nu} - p\delta^{\mu}_{\ \nu}. \tag{1.11}$$

In a frame comoving with the fluid, the velocity $u^{\mu} = u_{\nu} = (1, 0, 0, 0)$. The individual elements are thus:

$$T^{0}_{\ 0} = \rho, \tag{1.12}$$

$$T^{1}_{\ 1} = T^{2}_{\ 2} = T^{3}_{\ 3} = -p, \tag{1.13}$$

$$T^{0}_{\ i} = T^{i}_{\ 0} = T^{i}_{\ j} = 0 \ (i \neq j). \tag{1.14}$$

Here ρ is the energy density and p is the pressure of the fluid. Diagonal form of the stress energy tensor is required by the diagonal metric $g_{\mu\nu}$. Solving Einstein's equations gives us two equations for the scale factor (see appendix B): $G^{0}_{\ 0}$ of Equation (1.10) yields

$$G^{0}_{\ 0} = 3(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}) = 8\pi G_N \rho,$$

so that the 'Friedmann equation' is

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \tag{1.15}$$

which relates expansion rate of the universe to the energy density ρ. The $G^i_{\ i}$ component of Equation (1.10) on the other hand yields

$$G^i_{\ i} = 2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = -8\pi G_N p. \tag{1.16}$$

Another equation which is called 'fluid equation' is deduced from the conservation of energy; namely from the fact that covariant derivative of energy momentum tensor must be zero, $\nabla_\mu T^\mu_\nu = 0$, (see appendix C)

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0. \tag{1.17}$$

This is the equation that maintains energy conservation. The first term in the brackets corresponds to the dilution in the density because the volume has increased as the universe expands, while the second corresponds to the loss of energy because the pressure of the material inside has done work as universe's volume increased. This energy has not disappeared entirely of course, since energy is always conserved. What really happens is that this energy lost from the fluid via the work done has gone into gravitational potential energy. Besides, since there are no pressure forces in a homogeneous universe, pressure gradient is zero, hence, pressure does not contribute a force helping the expansion along; its effect is solely through the work done as the universe expands. Lastly, if we subtract Equation (1.15) from Equation (1.16), we get precisely the Acceleration equation or sometimes called Evolution equation,

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}(\rho + 3p). \tag{1.18}$$

1.1.4. Evolution of Spacetime

In order to discover how the universe might evolve, we need idea of what is in it. In a cosmological context, this is done by specifying the relationship between the mass density ρ and the pressure p. This relationship is called as the 'equation of state' described as $p = p(\rho)$. The continuity equation (1.17) describes the evolution of the energy density $\rho = \rho(a)$ of the universe in terms of the expansion factor of the universe when coupled with the equation of state $p = p(\rho)$. In particular when $p = \gamma\rho$ is given, then using Equation (1.17) gives

$$\rho \sim a^{-3(1+\gamma)}, \tag{1.19}$$

where γ is a constant which is called as the equation of state parameter. Furthermore, if one assumes $k = 0$ or equivalently neglects k/a^2 which is strongly favored by the recent observations, and using Equation (1.19), it follows from Equation (1.15) that

$$a \sim t^{2/[3(1+\gamma)]}. \tag{1.20}$$

In standard cosmology, in addition to the assumption that the universe is flat, there is one more assumption that kinematics of the universe is essentially determined by one of the matter components or both. Namely, by pressureless matter, by radiation, or by the mixture of both. Now we will consider these cases briefly.

<u>1.1.4.1. Pressureless Matter (Dust; $\gamma = 0$).</u> This case illustrates the expansion of a universe filled with any type of material (particles) which exerts negligible pressure, $p = 0$. It is a good description of a collection of galaxies in the universe, as they have no interactions other than gravitational ones. Using Equation (1.17) and the equation of state $p = 0$, one can find

$$\rho_{\text{dust}} \sim \frac{1}{a^3}, \tag{1.21}$$

and this says that the dust density ρ_{dust} falls off in proportion to the volume of the universe. Throughout this thesis we will use the subscript '0' to indicate the present value of the quantities. Denoting the present density by ρ_0 fixes the proportionality constant

$$\rho_{\text{dust}} = \rho_0 \left(\frac{a_0}{a}\right)^3 . \tag{1.22}$$

By inserting Equation (1.22) into Equation (1.15) for a flat universe, we can also determine how $a(t)$ varies with time

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{2/3} . \tag{1.23}$$

In this solution, we see that despite the pull of gravity of the material in the universe, universe does not re-collapse but rather expands forever. However, the rate of expansion $H(t)$ decreases with time according to

$$H(t) \equiv \frac{\dot{a}}{a} = \frac{2}{3t}, \tag{1.24}$$

becoming infinitely slow as the universe becomes infinitely old.

1.1.4.2. Radiation ($\gamma = 1/3$). This case illustrates the expansion of the early universe during the Hot Bing Bang. The radiation obeys $p = \rho/3$ equation of state which is derived from the fact that Einstein-Maxwell energy- momentum tensor is traceless (see appendix C). The radiation energy density depends on a as

$$\rho_{\text{rad}} = \rho_0 \left(\frac{a_0}{a}\right)^4 , \tag{1.25}$$

the scale factor and the rate of expansion evolve as the following:

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{1/2} , \tag{1.26}$$

$$H(t) \equiv \frac{\dot{a}}{a} = \frac{1}{2t}. \tag{1.27}$$

Notice that the universe expands more slowly if radiation dominates than if dust dominates. The reason why this is so can be explained by the extra deceleration that the pressure supplies as it is displayed in Equation (1.18). In a flat radiation dominated universe the expansion will continue for all time but the Hubble parameter will go to zero as $t \to \infty$.

1.1.4.3. Mixture. Now let us investigate the case illustrates the more general situation that universe is composed of both dust and radiation. Then there are two separate fluid equations, one for each of the two components;

$$\rho_{\text{dust}} \sim \left(\frac{1}{a}\right)^3 \quad ; \quad \rho_{\text{rad}} \sim \left(\frac{1}{a}\right)^4. \tag{1.28}$$

Namely, the energy density $\rho = \rho_{\text{dust}} + \rho_{\text{rad}}$ is inserted in the Friedmann Equation (1.15) and it is going to be solved for a scale factor a in the case where the energy density ρ is much far away from being too large. Then we can study the regime where one of the components of the density is dominant. For example, suppose radiation is more important. Then, one would have

$$a(t) \sim t^{1/2} \quad ; \quad \rho_{\text{rad}} \sim \frac{1}{a^4} \sim \frac{1}{t^2} \quad ; \quad \rho_{\text{dust}} \sim \frac{1}{a^3} \sim \frac{1}{t^{3/2}}. \tag{1.29}$$

Here one can notice that the density in dust falls off more slowly than that in radiation. So the situation of radiation dominating can not last forever and eventually dust component might come to dominate. Hence, the domination of universe by radiation is an unstable situation. In the opposite situation where it is the dust which is dominant, one can obtain the solution

$$a(t) \sim t^{2/3} \quad ; \quad \rho_{\text{dust}} \sim \frac{1}{a^3} \sim \frac{1}{t^2} \quad ; \quad \rho_{\text{rad}} \sim \frac{1}{a^4} \sim \frac{1}{t^{8/3}}, \tag{1.30}$$

and this solution shows that dust domination is a stable solution; the dust becomes increasingly dominant over the radiation as time goes by. In other words, the evolution of a universe containing dust and radiation starts initially with radiation and eventually the dust comes to dominate and as it does so the expansion rate speeds up from $a(t) \sim t^{1/2}$ to the $a(t) \sim t^{2/3}$ law. Hence, it is very possible that this is the situation which applies in our universe.

1.1.5. Observational Parameters

Standard Big Bang model does not give a unique description of our present universe. However, it leaves quantities such as the present expansion rate, the present composition of the universe and the present value of cosmological constant to be fixed by observation.

1.1.5.1. Present Hubble Parameter H_0. The present value of Hubble constant $H_0 = 720 \pm 8$ km s^{-1} Mpc $^{-1}$ [8].

1.1.5.2. The Density Parameter Ω. It is convenient to measure the the energy densities of different components in terms of a critical energy density ρ_c, a special value of density, which would be required in order to make the geometry of the universe flat, $k = 0$. Using (1.15), it is given as

$$\rho_c(t) = \frac{3H_0^2}{8\pi G_N}. \tag{1.31}$$

Hence, instead of writing the matter density of the universe directly, it can be useful to write its value relative to the critical energy density. This dimensionless quantity called the density parameter is defined as

$$\Omega \equiv \frac{\rho_i}{\rho_c}, \tag{1.32}$$

and gives the fractional contribution of different components of the universe (i denotes Matter (Baryons+Dark matter), Radiation). The density parameter determines the geometry of the universe. With this new notation, one can rewrite the Friedmann Equation (1.15) as

$$\Omega - 1 = \frac{k}{a^2 H^2}, \tag{1.33}$$

and can state the possibilities for the geometries of the universe depending on the density parameter where $\Omega = \Omega_{\mathrm{M}} + \Omega_{\mathrm{R}}$.

- Open Universe: $0 < \Omega < 1 : k < 0 : \rho < \rho_c$
- Flat Universe: $\Omega = 1 : k = 0 : \rho = \rho_c$
- Closed Universe: $\Omega > 1 : k > 0 : \rho > \rho_c$.

1.1.5.3. The Cosmological Constant. When Einstein proposed general relativity, his field equation was in the form of Equation (1.10) where the left-hand side characterizes the geometry of spacetime and the right-hand side characterizes the energy sources. If the energy sources are a combination of matter and radiation, there are no solutions to Equation (1.10) describing a static, homogeneous universe as it is studied in Equation 1.1.4.3. Since astronomers at the time believed the universe was static, Einstein suggested modifying the left hand side of his equation to obtain

$$G_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}, \tag{1.34}$$

where Λ is a new free parameter, the cosmological constant. With this modifications, Friedmann-Lemaitre equation Equation (1.15) becomes

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2}. \tag{1.35}$$

Here Λ is dimensional and has dimension $[LENGTH]^{-2}$. Einstein's original idea was to balance positive curvature, Λ and ρ to get $H = 0$. This solution is called Einstein static universe. In fact this idea was rather misguided, since such balance proves to be unstable to small perturbations. Anyway, more commonly now the Λ term is considered in the context of universes with the flat Euclidean geometry, $k = 0$. In principle, Λ could be positive or negative, though the usual case is positive. Now let's analyze Einstein's static universe for the vacuum case and then for the case in which there is matter in the universe. Concerning the vacuum, Friedmann-Lemaitre Equation (1.15) gets the form of

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3} - \frac{k}{a^2}\,, \tag{1.36}$$

As Einstein's vacuum static universe is concerned, one can see that only possible solution for static universe is

$$a = \sqrt{\frac{3k}{\Lambda}}\,,$$

and the only plausible static universe is the closed one with $k = 1$. However if $k = 0$ then no static universe solution is yielded. Namely, there is an exponentially expanding or shrinking universe solution due to the sole effect of cosmological constant in vacuum as

$$a = a_0 e^{\lambda t}, \tag{1.37}$$

where $\lambda = \pm\sqrt{\frac{\Lambda}{3}}$. On the other hand, if the Einstein universe is assumed to be filled with matter, and it is static, Friedmann-Lemaitre Equation implies that

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3} - \frac{k}{a^2} + \frac{8\pi G}{3}\rho_{(dust)_0}\left(\frac{a_0}{a}\right)^3 = 0. \tag{1.38}$$

Since the considered universe should be big enough, we can ignore $\rho_{(dust)_0}\left(\frac{a_0}{a}\right)^3$ term compared to the curvature parameter term k/a^2. Hence this assumption yields

$$\frac{\Lambda}{3} = \frac{k}{a^2}. \tag{1.39}$$

At that point, we can argue that if $k = -1 < 0$, then this implies both $\Lambda < 0$ and the energy density $\rho = 2\Lambda/3 < 0$ is not physical. But if $k = 1 > 0$, then this implies both $\Lambda > 0$ and positive energy density with $\rho = 2\Lambda/3$ is plausible as far as FLRW cosmology is concerned. Hence, with positive spatial curvature and all the parameters ρ, p, and Λ are non-negative, instead of finding static universe solution as Einstein considered, one gets

$$a(t) = \lambda^{-1}\cosh(\lambda t), \tag{1.40}$$

expanding universe solution with matter inside. Although the need for a static universe has disappeared by improved astronomical observations, the cosmological constant somehow has kept its reputation alive until today. It is sometimes thought of as the energy density of 'empty' space; in particular in quantum physics. Its possible origin is as a type of 'zero-point energy' which remains even if no particles are present. It is useful to express the density as a fraction of the critical density, and it is convenient to normalize the cosmological constant by defining

$$\Omega_\Lambda = \frac{\Lambda}{3H^2}. \tag{1.41}$$

Although Λ is a constant, Ω_Λ is not since H varies with time. Now, we can state Friedman-Lemaitre Equation (1.35) as in the density fractional form of

$$\Omega_{\rm M} + \Omega_{\rm R} + \Omega_\Lambda - 1 = \frac{k}{a^2H^2}. \tag{1.42}$$

The condition to have a flat universe, $k = 0$ generalizes to

$$\Omega_{\rm tot} = \Omega_{\rm M} + \Omega_{\rm R} + \Omega_\Lambda = 1. \tag{1.43}$$

The equation of state of the universe with cosmological constant Λ, on the other hand, can be deduced from rewriting $G^0_{\ 0}$ and $G^i_{\ i}$ in the case of without matter energy but with cosmological constant. By treating $T_{\mu\nu} \sim g_{\mu\nu}$:

$$G^0_{\ 0} = (\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}) = \frac{\Lambda}{3} \ , \tag{1.44}$$

$$G^i_{\ i} = \frac{-1}{3}(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}) = -\frac{\Lambda}{3} \ ,$$

and these two equations are identical with $G^0_{\ 0}$ and $G^i_{\ i}$ with matter energy but without cosmological constant if we set

$$\frac{8\pi G}{3}\rho = \frac{\Lambda}{3} \ , \tag{1.45}$$

$$\frac{8\pi G}{3}p = -\frac{\Lambda}{3} \ , \tag{1.46}$$

and it is seen from the Equations 1.45 and 1.46 that the equation of state of the universe filled with cosmological constant is $\gamma = p/\rho = -1$.

1.1.6. Perplexing Observational Results

- Our universe has $0.98 \lesssim \Omega_{\mathrm{tot}} \lesssim 1.08$ [9]. The value of Ω_{tot} can be determined from the angular isotropy spectrum of the cosmic microwave background radiation (CMBR) and these observations now show that we live in a universe with critical density [9, 10].
- Observations of primordial deuterium produced in big bang nucleosynthesis (which took place when the universe is about 1 minute in age) as well as the CMBR observations [11] show that the total amount of baryons in the universe contributes about $\Omega_{\mathrm{B}} = (0.024 \pm 0.0012)\ h^{-2}$. Given the independent observations on the Hubble constant H_0 [8] which fix $h = 0.72 \pm 0.07$, it is concluded that

$\Omega_{\rm B} \cong 0.04 - 0.06$. These observations take into account all baryons which exist in the universe today irrespective of whether they are luminous or not. Combined with the first item we conclude that most of the universe is non-baryonic.

- Host of observations related to large scale structure and dynamics (rotation curves of galaxies, estimate of cluster masses, gravitational lensing, galaxy surveys ...) all suggest that the universe is populated by a non-luminous component of matter (dark matter; DM here thereafter) made of weakly interacting massive particles which does cluster at galactic scales. This component contributes about $\Omega_{\rm DM} \cong 0.20 - 0.35$.
- The universe also contains radiation contributing an energy density $\Omega_{\rm R} \cong 2.56 \times 10^{-5}$ today. Most of which is due to photons in the CMBR and is dynamically irrelevant today.
- Combining the last observation with the first, one can conclude that there must be (at least) one more component to energy density of the universe contributing about 70% of critical density. Early analysis of several observations [12, 13] indicated that this component is unclustered and has negative pressure. The observations suggest that the missing component has $\gamma = p/\rho \lesssim -0.78$ and contributes $\Omega_{\rm DE} \cong 0.60 - 0.75$.
- Briefly, all known observations are consistent with that our universe today has density compositions of $\Omega_{\rm DE} \cong 0.70$, $\Omega_{\rm DM} \cong 0.26$, $\Omega_{\rm B} \cong 0.04$, $\Omega_{\rm R} \cong 3 \times 10^{-5}$.

In the light of this introductory information about standard Einstein cosmology and observational results, one can see that how strange the present universe behaves cosmologically. Although the only component of the universe which we can understand theoretically well is the radiation (Cosmic Microwave Radiation), understanding the baryonic and dark matter density components is not trivial. Moreover, the issue of dark energy and slow-acceleration in present universe under the influence of this mysterious energy today is somehow perplexing and beyond the expectations of standard Einstein cosmology. Hence, to solve this puzzle, many alternative theories to Einstein's gravitational theory are widely being used in the literature. Scalar tensor theories are the most favorite ones of these theories. So, in this thesis, we have chosen Brans Dicke scalar tensor theory of gravity as an underlying theory to explain primordial inflation and late-time inflation essentially. Furthermore, whether Brans-Dicke scalar tensor theory can account for dark energy and dark matter or not will be studied. And finally, how Brans-Dicke theory modifies the famous Friedmann Equation in standard Einstein cosmology, and provides another density parameter $\Omega_{\vartriangle}$, different from the measured density parameters (Ω_Λ, Ω_R, Ω_M), will be shown rigorously.

2. PRIMORDIAL AND LATE-TIME INFLATION

The inflationary universe model whose key feature is a finite period of primordial rapid exponential expansion has been proposed to resolve a number of cosmological puzzles, including the horizon, flatness and monopole problems. In the original or 'old inflation model' [14], the universe super-cools into a false vacuum phase and its energy density acts as an effective cosmological constant which causes an epoch of de-Sitter (exponential) expansion. In this old inflation model, the de-Sitter expansion never ends and for a generic first order phase transition, there appears an energy barrier between the false vacuum and the true vacuum phases. This problem is known as the'graceful exit' problem. This problem was avoided with the invention of the new inflationary theory [15, 16]. In this theory, inflation may begin either in the false vacuum, or in an unstable state at the top of the effective potential. Then the inflaton field ϕ slowly rolls down to the minimum of its effective potential. The density perturbations produced during the slow-roll inflation are inversely proportional to $\dot{\phi}$ [17, 18, 19]. Thus the key difference between the new inflationary scenario and the old one is that the useful part of inflation in the new scenario, which is responsible for the homogeneity of our universe, does not occur in the false vacuum state, where $\dot{\phi} = 0$. Although this scenario was so popular in the beginning of the 80's, it had its own problems. One of them, for example, is that the inflaton field has an extremely small coupling constant in most versions of this scenario, so it could not be in thermal equilibrium with other matter fields. The theory of cosmological phase transitions, which was the basis of old and new inflation, did not work in this situation. Furthermore, inflation in this theory begins very late and during the preceding epoch, the universe can easily collapse or become so inhomogeneous that inflation may never happen again [20].

With the invention of the chaotic scenario, all problems of old and new inflation were resolved. According to this scenario, inflation may occur even in theories with simple potentials such as $V(\phi) \sim \phi^n$. Inflation may begin even if there was no thermal equilibrium in the early universe, and it may start even at the Planckian density, in which case the problem of initial conditions for inflation can be easily resolved [20]. The field in this scenario evolves slowly and at this stage the energy density of the scalar field, unlike the density of ordinary matter, remains almost constant, and expansion of the universe continues with a much greater speed than in the old cosmological theory. Inflation does not require supercooling and tunneling from the false vacuum [14], or rolling from an artificially flat top of the effective potential [15].

All models discussed so far have used field theories at very high energies to drive inflation. However, inflation may also be generated by changing the gravitational sector alone (R^2 inflation) [21, 22] or both the gravitational and the matter sectors (Extended inflation) [23, 24]. And the well known Jordan-Brans Dicke theories are widely used in this third type generation of inflation. Before going into the details of the primordial and late- time inflation in BD theory, it would be better to mention about the main differences in between Jordan and BD scalar theories as far as their Lagrangian definitions are concerned.

In the early 1910s, a “scalar” theory of gravity had been attempted by G. Nodström by promoting the Newtonian potential function to a Lorentzian scalar. However, owing to the lack of a geometrical nature, namely, it does not rely on equivalence principle which is one of the two pillars supporting the entire structure of general relativity, it was left outside the aim of the theory. After a decade of search for new concepts to make gravitational theory compatible with the spirit of special relativity, Einstein eventually arrived at a dynamical theory of space-time geometry. Thus, in Einstein’s hands, gravitation theory was transformed from a theory of forces into the first dynamical theory of geometry, the geometry of four dimensional curved space-time.

In spite of the widely recognized success of general relativity, it is now called the standard theory of gravitation, the theory has also feeded by many “alternative” theories for one reason or another. Among them, we particularly focus on the Jordan and Brans-Dicke scalar tensor theories of gravitation. The reason why these theories are also labeled with “tensor” is that this type of theories do not merely combine the two kinds of fields (scalar field and gravitational field), but also they are built on the solid foundation of general relativity and besides, since Einstein’s theory of relativity is a geometrical theory of space-time and the fundamental building block is a metric tensor field, $g_{\mu\nu}$, these kind of theories are named as "scalar-tensor" theories.

The scalar-tensor theory was studied originally by Jordan-Thiry [25, 26]. He showed that if a four dimensional curved manifold is embedded into a five dimensional flat space time, the constraint appeared in formulating the projective geometry can be four dimensional scalar field, which enables one to describe a space-time dependent Newtonian gravitational constant which is in accordance with Dirac's argument that gravitational constant should be time dependent [27]. Under these considerations, a general Jordan Lagrangian for the scalar field living in four-dimensional curved space-time:

$$L_J = \sqrt{-g}\left[\Phi^\sigma\left(R - \omega_J \frac{1}{\Phi^2} g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\right) + L_M(\Phi,\ \Psi)\right], \tag{2.1}$$

where $\Phi(t,\ \vec{x})$ is Jordan's scalar field, R is the Ricci scalar, $g^{\mu\nu}$ is the metric tensor, ω_J and σ are the Jordan's dimensionless coupling constants and Ψ is representing matter fields collectively. The term $\Phi^\sigma R$ has a specific name in this context called as a "nonminimal coupling term", and this is the term that marked the birth of the scalar tensor theory. Planck-length is defined in natural units as, $L_p^2 = 8\pi G_N$, where G_N is present Newton's gravitational constant. Thus, the dimension of the Jordan's scalar field, Φ, is chosen to be $[\Phi] = L^{-1}$ so that $G_{eff} \sim \Phi^{-2}$ has a dimension $[G_{eff}] = L^2$. In this unit system, we may also express the dimensions of the following physical entities such as $[L_J] = 1/L^4$, $[R] = 1/L^2$, $[g_{\mu\nu}] = [\omega_J] = [\sigma] = 1$, $[\partial_\mu] = 1/L$. Thus, dimensional analysis of the Lagrangian (2.1) yields that $\sigma = 2$, which shows that the term Φ^σ has the dimension L^{-2}, which is the same as that of G_{eff}^{-1} . Although this invariance need not to be satisfied if Φ enters the matter Lagrangian, the constants appear in the Lagrangian are dimensionless and this is true for any values of σ.

Jordan's this effort particularly had been re-studied by Brans and Dicke, but with a difference in defining the scalar field. They define their scalar field φ by

$$\varphi \equiv \Phi^\sigma, \tag{2.2}$$

which simplifies (2.1) in the sense that specific choice of the value σ is irrelevant. This process is actually a part of demanding that the matter part of the Lagrangian $(-g)^{1/2}L_M(\Phi,\, \Psi)$ be decoupled from $\Phi(t,\, \vec{x})$ as an implementation of their requirement that weak equivalence principle be respected, in contrast to Jordan's model. In BD model, Lagrangian is defined by as in the following form [28]

$$L_{BD} = \sqrt{-g}\left[\left(\varphi R - \omega\frac{1}{\varphi}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right) + L_M(\Psi)\right], \tag{2.3}$$

where the dimensionless ω is the only parameter of the theory, and L_M is the matter Lagrangian. According to our metric convention, (+ - - -), Equation (2.3) turns out to be

$$L_{BD} = \sqrt{-g}\left[\left(-\varphi R + \omega\frac{1}{\varphi}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right) + L_M(\Psi)\right]. \tag{2.4}$$

In particular, it is expected that $\varphi\ (t,\, \vec{x})$ is spatially uniform and evolves slowly only with cosmic time t such that $\varphi\ (t,\, \vec{x}) \to \varphi(t)$. As another point, the second term on the right hand side of (2.3) appears to be kinetic term of the scalar field but it is in an unlikely form, since the presence of the φ^{-1} which seems to indicate a singularity, and the presence of the coupling constant in multiplicative form is undesired. However, whole term can be transformed into the standard canonical form by re-defining the scalar field φ by introducing a new field ϕ, and a new constant ϵ in such a way that

$$\varphi = \frac{1}{8\omega}\phi^2. \tag{2.5}$$

In this new form, BD Lagrangian is redefined as

$$L_{BD} = \sqrt{-g}\left[-\frac{1}{8\omega}\phi^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + L_M(\Psi)\right], \tag{2.6}$$

where the signs of the non-minimal coupling term and the kinetic energy term are properly adopted to $(+\,-\,-\,-)$ metric signature in such a way that as $g^{\mu\upsilon} \sim \eta^{00}$, the kinetic term, $\frac{1}{2}\, g^{\mu\upsilon}\, \partial_\mu\phi\, \partial_\nu\phi$ becomes $\frac{1}{2}\, \dot{\phi}^2$.

In a brief manner, Jordan-Brans-Dicke scalar tensor theory of gravitation theories are a class of theories in which the effective gravitational coupling evolves with time, and asymptotically attains a value of G. The strength of the coupling is determined by a scalar field, ϕ, such that asymptotically, it tends to a value G^{-1}. The origins of Brans-Dicke theory is in Mach's principle according to which the property of inertia of material bodies arises because of their interactions with the matter distributed in the universe. In the modern context, the Brans-Dicke theory attempted to rescue the inflationary scenario from some of its problems. The theory is parameterized by a dimensionless constant ω, where as $\omega \rightarrow \infty$, Brans-Dicke theory approaches to the Einstein theory [29]. Present limits of the constant ω based on time-delay experiments [30, 31, 32] require $\omega > 10^4 \gg 1$.

In the conventional inflationary scenario, the universe underwent an exponential expansion for a brief period in its early phase. After the exponential phase is over the universe should transit to the normal cosmological phase. Within the framework of Einstein-Hilbert action, there is no satisfactory mechanism by which the universe transits to the normal phase. It was shown that within the framework of Brans-Dicke gravity, a constant energy density leads to a rapid power-law expansion instead of exponential. This is rapid enough to solve the problems in standard cosmology and at the same time slow enough to make the transition to normal state possible after the inflationary phase. This has come to be known as extended inflation [33, 34]. Extended inflation constrains ω to be less than 25. This bound comes from the fact that if it is more than 25, there will be much more anisotropy in the Cosmic Microwave Background Radiation [35] than what is observed today. This is, however, incompatible with the bound which constrains ω to be greater than 10^4. A large number of inflationary models were proposed in the framework of multi-scalar tensor gravity to solve the problem. For instance, the introduction of a potential for the scalar field ϕ and a scalar field dependent coupling constant $\omega(\phi)$ solved some problems [36, 37, 38, 39, 40].

In our work, we start out with a strong link between inflation and Brans-Dicke [28] theory of gravity. The proposed model in this thesis is simple in that no other phenomenon, such as the domination of the false vacuum over the scalar field energy density as in the extended inflation model is used. Since the recent progress in observational cosmology shifted attention towards experimental verification of various inflationary theories, the motivation of this work is accelerated with the recent measurements of the dependence of the Hubble parameter $H = \frac{\dot{a}}{a}$ on the scale size $a(t)$ of the universe. At this point it would be important to note that the classical Friedmann equation which is used for fitting the Hubble parameter, H, to the measured present density parameters ($\Omega_{0\Lambda}$, Ω_{0R}, Ω_{0M}) of the universe in such a way that $\Omega_{0\Lambda}+ \Omega_{0R}+ \Omega_{0M} = 1$ is given as

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{0\Lambda} + \Omega_{0R}\left(\frac{a_0}{a}\right)^2 + \Omega_{0M}\left(\frac{a_0}{a}\right)^3. \tag{2.7}$$

The present density parameter, $\Omega_{0i} \equiv \rho_{0i}/\rho_c$, is defined as the fractional ratio of the present energy density of any matter to the present critical energy density which is a special required density in order to make the geometry of the universe flat. i stands for Cosmological constant with Λ), matter with (M), and the curvature with R. In an other words, $\Omega_{0\Lambda}$, Ω_{0R}, Ω_{0M} are the fractions of contributions of the vacuum density, curvature density and the matter density to the present energy density of the universe respectively. H_0 is the present Hubble parameter [8]. According to recent data, their present measured values are $\Omega_\Lambda \cong 0.75$, $\Omega_{0R} \cong 0$ and $\Omega_M \cong 0.25$ [41]. In standard cosmology the Ω_Λ term would be induced by a cosmological constant. An immediate question which arises is the physical reason behind this cosmological constant. A universe, expanding under the sole influence of a cosmological constant $\Lambda = \lambda^2$ inflates as $a(t) \sim e^{\lambda t}$. For the present day expansion, $\lambda \simeq H_0$, whereas for the primordial expansion responsible for the present large size of the universe, λ is much bigger.

We will show a natural model where the large ratio of primordial inflation to present day inflation can be explained by Brans-Dicke theory which effectively replaces the Newtonian gravitational constant G_N in the Einstein-Hilbert action (A.30) by a power of the Brans-Dicke scalar field in such a way that

$$G^{-1} = \frac{2\pi}{\omega}\phi^2, \tag{2.8}$$

where G is the effective gravitational constant as long as the dynamical scalar field ϕ varies slowly. In units where $c = \hbar = 1$, we define Planck-length, L_p, in such a way that $L_P^2 \phi_0^2 = \omega/2\pi$ where ϕ_0 is the present value of the scalar field ϕ. Thus, the dimension of the scalar field is chosen to be L^{-1} so that G_{eff} has a dimension L^2.

The action is the following;

$$S = \int d^4x \sqrt{g} \left[-\frac{1}{8\omega} \phi^2 R + \frac{1}{2} g^{\mu v} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) + L_M \right], \tag{2.9}$$

where ϕ represents the Brans-Dicke scalar field and ω denotes the dimensionless Brans-Dicke parameter taken to be much larger than 1, $\omega > 10^4 \gg 1$ [32]. L_M, on the other hand, is the matter Lagrangian such that the scalar field ϕ does not couple with it. The nonminimal coupling term is $\phi^2 R$ and R is the Ricci scalar. The kinetic and potential terms of the scalar field behave effectively as time dependent cosmological constants. At this point, we have to point out three simple assumptions made in this work:

- The Brans-Dicke field ϕ does not couple to any other field except gravity.
- The Lagrangian of the field, in addition to the kinetic term of ϕ, contains the simplest chaotic inflation-style potential energy density $V(\phi) = \frac{1}{2}m^2\phi^2$ which is composed only of the scalar field mass term.
- In particular we may expect that ϕ is spatially uniform, but varies slowly with time. For simplicity we also restrict our analysis to the Robertson Walker metric 1.3 to emphasize that ϕ is necessarily spatially homogeneous.

After applying the variational procedure to the action (see appendix A) and assuming $\phi = \phi(t)$ and energy momentum tensor of matter and radiation excluding ϕ is in the perfect fluid form of $T^\mu_\nu = diag(\rho, -p, -p, -p)$ where ρ is the energy density and p is the pressure, and also noting that the right hand side of the ϕ equation below is set to be zero in accordance with our first assumption on L_M being independent of ϕ, the field equations reduce to

$$\frac{3}{4\omega}\,\phi^2\left(\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right)-\frac{1}{2}\,\dot{\phi}^2-\frac{1}{2}\,m^2\,\phi^2+\frac{3}{2\omega}\,\frac{\dot{a}}{a}\,\dot{\phi}\,\phi=\rho_M, \tag{2.10}$$

$$\frac{-1}{4\omega}\phi^2\left(2\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right)-\frac{1}{\omega}\,\frac{\dot{a}}{a}\,\dot{\phi}\,\phi-\frac{1}{2\omega}\,\ddot{\phi}\,\phi-\left(\frac{1}{2}+\frac{1}{2\omega}\right)\dot{\phi}^2+\frac{1}{2}\,m^2\,\phi^2=p_M, \tag{2.11}$$

$$\ddot{\phi}+3\,\frac{\dot{a}}{a}\,\dot{\phi}+\left[m^2-\frac{3}{2\omega}\left(\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right)\right]\,\phi=0, \tag{2.12}$$

where 'dot' denotes (d/dt). The right hand sides of the Equations (2.10, 2.11) are adopted to the matter energy density term ρ_M instead of ρ and p_M instead of p where the subscript M denotes everything except the ϕ field.

2.1. Primordial Inflation

In the primordial inflation analysis, we start to solve the field equations (2.10, 2.11, 2.12) for an empty-static universe by setting $\dot{a}=0$ and $p=\rho=0$ and get the following vacuum solutions;

$$\phi=\phi_o\,e^{F_p\,t}, \tag{2.13}$$

$$a=a_*=\text{const}, \tag{2.14}$$

$$k=1\ \text{(closed universe)}, \tag{2.15}$$

where

$$F_p^2 = m^2 \left(\frac{\omega}{2\omega + 3} \right), \tag{2.16}$$

$$a_*^2 = \frac{1}{m^2} \left(\frac{2\,\omega + 3}{3\,\omega + 3} \right) \left(\frac{3}{2\,\omega} \right). \tag{2.17}$$

From these solutions (2.13, 2.14, 2.15), we see that ϕ evolves exponentially with the expansion parameter F_p and, on the other hand, a_* is the constant size of this static universe. We regard that only the closed universe solution is possible as a positive aspect of this solution since homogeneity of the universe only makes sense if a closed universe undergoes big-bang. Let us also note that, since two variables $\phi\,(t)$ and $a\,(t)$ satisfy the field equations (2.10, 2.11, 2.12), these solutions (2.13, 2.14) are expected to be stable. To prove this stability we impose the size of the universe a and the field ϕ to be a function of time t in a perturbative manner;

$$a = a_* \left(1 + \varepsilon\, b\,(t)\right), \tag{2.18}$$

$$\phi = e^{F_p t} \left(1 + \varepsilon\, \psi\,(t)\right), \tag{2.19}$$

where ε is the perturbative factor ($\varepsilon \ll 1$) and $b\,(t)$, $\psi\,(t)$ are perturbative functions of $a\,(t)$ and $\phi\,(t)$ respectively. Using (2.18, 2.19) and (2.16, 2.17), it follows from the Equations (2.10, 2.11, 2.12) that

$$\dot{\psi}\,(t) - \frac{3}{2\,\omega}\,\dot{b}\,(t) + \frac{3}{2\,\omega\,F_p\,a_*^2}\,b\,(t) = 0, \tag{2.20}$$

$$\ddot{\psi}\,(t) + (4 + 2\omega)\; F_p\,\dot{\psi}\,(t) + \ddot{b}\,(t) + 2\,F_p\,\dot{b}\,(t) - \frac{1}{a_*^2} b\,(t) = 0, \tag{2.21}$$

$$\ddot{\psi}\,(t) + 2\,F_p\dot{\psi}\,(t) - \frac{3}{2\,\omega}\,\ddot{b}(t) + 3\,F_p\,\dot{b}\,(t) + \frac{3}{\omega\,a_*^2}\,b(t) = 0. \tag{2.22}$$

Solving Equations (2.20, 2.21, 2.22) simultaneously gives the only solution $\dot{b} = 0$ which directly implies $\ddot{b} = 0$, $\dot{\psi} = 0$ and $\ddot{\psi} = 0$. Namely, this closed universe vacuum solution where $a = a_* = cst$ and $\phi \sim e^{F_p t}$ is a stable solution.

Considering the radiation dominating early universe, we now investigate how the presence of radiation changes the behavior of the universe compared to this stable solution. Hence, as a first step, instead of solving the Equations (2.10, 2.11, 2.12) for the primordial equation of state $p = \frac{1}{3}\rho$, which is hard enough, we find that solving Equation (2.12) for $a\,(t)$ by keeping $\phi \sim e^{F_P t}$ is much more effective. As a second step, we plan to show that for possible solution for $a\,(t)$ and $\phi \sim e^{F_p t}$, equation of state $p = \frac{1}{3}\rho$ is satisfied automatically. By changing to the variable $a^2\,(t) = \Theta\,(t)$, the Equation (2.12), which turns out to be the following second order differential equation,

$$\frac{3}{4\omega}\,\ddot{\Theta} - \frac{3F_p}{2}\,\dot{\Theta} - \left(F_p^2 + m^2\right)\,\Theta = -\frac{3}{2\omega}, \tag{2.23}$$

has been solved for $\Theta(t)$, and we find,

$$\Theta\,(t) = a^2\,(t) = \left(\frac{3}{2\omega}\right)\left(\frac{1}{F_p^2 + m^2}\right) + c_1\,e^{-2F_p t} + c_2\,e^{2H_p t}, \tag{2.24}$$

where c_1, c_2 are integration constants and H_p is the primordial Hubble parameter. The relation between H_p and F_p is

$$H_p = (\omega + 1)F_p. \tag{2.25}$$

Using the Equations (2.16, 2.17), (2.24) reduces to

$$\Theta\,(t) = a^2\,(t) = a_*^2 + c_1\,e^{-2F_p t} + c_2\,e^{2H_p t}. \tag{2.26}$$

Considering $\omega \gg 1$ and using Equation 2.16, we may also display expansion parameter

F_p and primordial Hubble constants H_p in terms of the mass of the scalar field ϕ as

$$F_p \simeq 0.7m, \tag{2.27}$$

$$H_p \simeq 0.7\omega m. \tag{2.28}$$

Now, the Equation (2.26) can be expanded about $t = 0$ which becomes

$$\Theta(t) = a^2(t) \simeq a_*^2 \left(1 + c_1 + c_2 - 2c_1 F_p t + 2c_2 H_p t\right), \tag{2.29}$$

with the constraint $1 + c_1 + c_2 = 0$, due to the fact that as $t \to 0$, we want $a^2(t) \sim t$ as it is in standard cosmology (1.29). Thus, with the help of Equation (2.25), we end up with the general solution for the scale size of the universe in the primordial inflation regime with $\omega \gg 1$ as

$$a^2(t) \simeq a_*^2 \left[1 - (1 + c)\, e^{-2(H_p/\omega)t} + ce^{2H_p t}\right]. \tag{2.30}$$

Using Equation (2.28) provides the general solution (2.30) to be represented in terms of a scalar field mass of m as,

$$a^2(t) \simeq a_*^2 \left[1 - (1 + c)\, e^{-1.4mt} + ce^{1.4\omega mt}\right]. \tag{2.31}$$

This general solution is important for at least three reasons:

1. It is a natural solution in the sense that no equation of state for radiation is needed. It is solely deduced from the theory by using the stable-empty universe solution (2.13) in the Equation (2.12).
2. When we examine this inflationary solution concerning as $t \to 0$ and as $t \gtrsim 0$, we simultaneously see from Equation (2.29) and Equation (2.31) that, it is both consistent with $a(t) \sim \sqrt{t}$ as $t \to 0$ and the primordial rapid inflation described

by $a\,(t) \sim e^{H_p t} \sim e^{0.7m\omega t}$ for $\omega \gg 1$.

3. The equation of state parameter $\gamma \simeq 1/3$ is automatically satisfied under this solution. To show this substituting $\phi \sim e^{F_p t}$ and $a \sim \sqrt{t}$ into Equations (2.10, 2.11) yields

$$\phi^2 \left[\frac{1}{16\omega}\left(\frac{1}{t^2}\right) - \frac{1}{4\omega}\left(\frac{1}{t}\right) - \frac{1}{2\omega}\left(\frac{1}{t}\right) F_P - \left(\frac{1}{2} + \frac{1}{\omega}\right) F_P^2 + \frac{1}{2}\,m^2 \right] = p_M, \tag{2.32}$$

$$\phi^2 \left[\frac{3}{16\omega}\left(\frac{1}{t^2}\right) + \frac{3}{4\omega}\left(\frac{1}{t}\right) + \frac{3}{4\omega}\left(\frac{1}{t}\right) F_P - \frac{1}{2}\,F_P^2 - \frac{1}{2}\,m^2 \right] = \rho_M\,. \tag{2.33}$$

Using the relation $F_P^2 \simeq \frac{1}{2}\,m^2$ for $\omega \gg 1$ provides γ to be

$$\gamma \simeq \frac{\left[\frac{1}{16\omega}\left(\frac{1}{t^2}\right) - \frac{1}{4\omega}\left(\frac{1}{t}\right) - \frac{m}{2\sqrt{2}\omega}\left(\frac{1}{t}\right) - \frac{1}{4}m^2 \right]}{\left[\frac{3}{16\omega}\left(\frac{1}{t^2}\right) + \frac{3}{4\omega}\left(\frac{1}{t}\right) + \frac{3m}{4\sqrt{2}\omega}\left(\frac{1}{t}\right) - \frac{3m^2}{4} \right]}, \tag{2.34}$$

and as $t \to 0$, it becomes

$$\gamma \simeq \frac{\left[\frac{1}{16\omega}\left(\frac{1}{t^2}\right) - \frac{1}{4}\,m^2 \right]}{3\left[\frac{1}{16\omega}\left(\frac{1}{t^2}\right) - \frac{1}{4}\,m^2 \right]} = \frac{1}{3}. \tag{2.35}$$

As a result of this section, we can remark that as far as present big size of the universe is concerned, primordial inflation in BD universe satisfies the same results as the universe expanding exponentially under the sole influence of the cosmological constant in the standard cosmology. However, the main difference in between them is that although in standard cosmology, the expanding factor called cosmological constant is just a constant whereas in BD cosmology, the expansion factor is the scalar field ϕ and since it evolves only with time it can be assumed as a dynamical entity like dark energy.

2.2. Late-time Inflation

In this section, we aim to analyze how much today's universe is far from late-time inflation by considering the case of slowly expanding empty universe ($\rho = p = 0$) except the ϕ field in it. Since the considered universe should be big, we ignore the curvature parameter k/a^2 as $a(t)$ increases with the expansion of the universe. Under these considerations, in analogy with the previous section, we put $a = e^{H_\infty t}$ and $\phi = e^{F_\infty t}$ into the Equations (2.10, 2.11, 2.12) where H_∞, F_∞ are new constants to be determined and search for a stable solution. We get the following coupled equations for H_∞ and F_∞;

$$H_\infty^2 - \frac{2}{3}\,\omega F_\infty^2 + 2H_\infty\,F_\infty - \frac{2\omega}{3}\,m^2 = 0, \tag{2.36}$$

$$H_\infty^2 + \left(\frac{2}{3}\omega + \frac{4}{3}\right)\,F_\infty^2 + \frac{4}{3}\,H_\infty\,F_\infty - \frac{2\omega}{3}\,m^2 = 0, \tag{2.37}$$

$$H_\infty^2 - \frac{\omega}{3}\,F_\infty^2 - \omega\,H_\infty\,F_\infty - \frac{\omega}{3}\,m^2 = 0. \tag{2.38}$$

These equations have the solution;

$$H_\infty = 2\,(\omega + 1)\left(\frac{\omega}{6\omega^2 + 17\omega + 12}\right)^{1/2} m \approx 0.8\sqrt{\omega}\,m, \tag{2.39}$$

$$F_\infty = \left(\frac{\omega}{6\omega^2 + 17\omega + 12}\right)^{1/2} m \approx \frac{0.4}{\sqrt{\omega}}\,m, \tag{2.40}$$

where the approximations are again for $\omega \gg 1$. We see that although the primordial Hubble parameter H_p is $0.7\omega m$, the late-time Hubble parameter H_∞ is found to be $0.8\sqrt{\omega}\,m$. Namely, a factor $\sqrt{\omega}$ less than the primordial inflation parameter. This is a very important result in the sense that although there is an experimental lower bound on ω, there is no upper bound on it according to the observational results stating $\omega \gg 10^4$[32]. Hence in Brans-Dicke cosmology, the late-time inflation can be as small as one wishes compared to the primordial inflation. Now, we consider the case where the universe is closed ($k = 1$) and matter dominated $p \approx 0$. Since solving the field Equations (2.10, 2.11, 2.12) for $a(t)$ and $\phi(t)$ is hard enough under $p \approx 0$, we proceed to work by defining the fractional rate of change of ϕ as $F\,(a) = \dot{\phi}/\phi$ and the Hubble parameter as $H\,(a) = \dot{a}/a$ so that we rewrite the left hand-side of the field Equations (2.10, 2.11, 2.12) in terms of H, F and their derivatives with respect to a,

$$H^2 - \frac{2\omega}{3}\,F^2 + 2\,H\,F + \frac{1}{a^2} - \frac{2\omega}{3}\,m^2 = (\frac{4\omega}{3})\frac{\rho_M}{\phi^2}, \tag{2.41}$$

$$H^2 + \left(\frac{2\omega}{3} + \frac{4}{3}\right)\,F^2 + \frac{4}{3}\,H\,F + \frac{2a}{3}\,\left(H\,\acute{H} + H\,\acute{F}\right) + \frac{1}{3a^2} - \frac{2\omega}{3}\,m^2 = (\frac{-4\omega}{3})\,\frac{p_M}{\phi^2} \approx 0, \tag{2.42}$$

$$H^2 - \frac{\omega}{3}\,F^2 - \omega\,H\,F + a\left(\frac{H\,\acute{H}}{2} - \frac{\omega}{3}\,H\acute{F}\right) + \frac{1}{2a^2} - \frac{\omega}{3}\,m^2 = 0. \tag{2.43}$$

To check the stability of the vacuum solution $a \sim e^{H_\infty t}$ and $\phi \sim e^{F_\infty t}$ in late-time regime, we write them in perturbative manner such that $a = e^{H_\infty t}(1 + b\varepsilon)$ and $\phi = e^{F_\infty t}(1 + \psi\varepsilon)$ and put them into Equations (2.10, 2.11, 2.12). Using the relation $H_\infty = 2(\omega+1)F_\infty$ these equations reduces to

$$\begin{aligned}&\left(\frac{H_\infty^2(3\omega+4)(2\omega+3)}{8\omega(\omega+1)}\right) + \left(\frac{H_\infty}{4\omega(\omega+1)}\right)\dot{b} + \left(\frac{3}{2\omega}H_\infty\right)\dot{\psi} - \frac{1}{2}\,m^2\\ &+\left[\left(\frac{3H_\infty}{2\omega}\right)\dot{b} + \left(\frac{3H_\infty^2(2\omega+3)}{4\omega(\omega+1)}\right)b - \left(\frac{H_\infty}{2(\omega+1)}\right)\dot{\psi} + \left(\frac{H_\infty^2(3\omega+2)}{4\omega(\omega+1)^2}\right)\psi\right]\varepsilon = 0,\end{aligned} \tag{2.44}$$

$$\begin{aligned}&-\left(\frac{H_\infty^2(4\omega^2+13\omega+10)}{8\omega(\omega+1)^2}\right) - \left(\frac{H_\infty}{2\omega(\omega+1)}\right)\dot{b} - \left(\frac{H_\infty}{\omega}\right)\dot{\psi} + \frac{m^2}{2}\\ &-\left[\left(\frac{1}{4\omega}\right)\ddot{b} + \left(\frac{5H_\infty}{4\omega}\right)\dot{b} + \left(\frac{H_\infty^2(2\omega+3)}{2\omega(\omega+1)}\right)b\right]\varepsilon\\ &-\left[\left(\frac{1}{2\omega}\right)\ddot{\psi} + \left(\frac{H_\infty(5+2\omega)}{4\omega(\omega+1)}\right)\dot{\psi} + \left(\frac{H_\infty^2(3\omega+4)}{4\omega(\omega+1)^2}\right)\psi\right]\varepsilon = 0,\end{aligned} \tag{2.45}$$

$$
\begin{aligned}
-\left(\frac{H_\infty^2(12\omega^2+36\omega+23)}{8\omega(\omega+1)^2}\right)+\left(\frac{3H_\infty}{2(\omega+1)}\right)\dot{b}+3H_\infty\dot{\psi}+m^2 \\
-\left(\frac{3}{2\omega}\ddot{b}+\frac{15H_\infty}{2\omega}\dot{b}+H_\infty^2\frac{3(3\omega+4)}{2\omega(\omega+1)}b\right)\varepsilon \\
+\left(\frac{1}{2\omega}\ddot{\psi}+H_\infty\frac{3}{4\omega(\omega+1)}\dot{\psi}+H_\infty^2\frac{6\omega^2+6\omega+1}{4\omega(\omega+1)^2}\psi\right)\varepsilon=0.
\end{aligned}
\tag{2.46}
$$

Using the fact that as $\omega\to\infty$, $H_\infty^2\approx\omega m^2$ yields

$$
\begin{aligned}
\left(\frac{1}{2\sqrt{6}\omega^{3/2}}\right)\dot{b}+\left(\frac{3}{\sqrt{6}\omega^{1/2}}\right)\dot{\psi} \\
+\left(\frac{3}{\sqrt{6}\omega^{1/2}}\dot{b}+\omega b-\frac{1}{\sqrt{6}\omega^{1/2}}\dot{\psi}+\frac{1}{2\omega}\psi\right)\varepsilon=0,
\end{aligned}
\tag{2.47}
$$

$$
\begin{aligned}
\left(\frac{1}{\sqrt{6}\omega^{3/2}}\right)\dot{b}+\left(\frac{2}{\sqrt{6}\omega^{1/2}}\right)\dot{\psi} \\
+\left(\frac{1}{4\omega}\ddot{b}+\frac{5}{2\sqrt{6}\omega^{1/2}}\dot{b}+\frac{2}{3}b+\frac{1}{2\omega}\ddot{\psi}+\frac{1}{\sqrt{6}\omega^{1/2}}\dot{\psi}+\frac{1}{2\omega}\psi\right)\varepsilon=0,
\end{aligned}
\tag{2.48}
$$

$$
\begin{aligned}
\left(\frac{3}{\sqrt{6}\omega^{1/2}}\right)\dot{b}+\left(\sqrt{6}\omega^{1/2}\right)\dot{\psi} \\
+\left(-\frac{3}{2\omega}\ddot{b}-\frac{15}{\sqrt{6}\omega^{1/2}}\dot{b}-3b+\frac{1}{2\omega}\ddot{\psi}+\frac{3}{2\sqrt{6}\omega^{3/2}}\dot{\psi}+\psi\right)\varepsilon=0.
\end{aligned}
\tag{2.49}
$$

Namely,

$$
\left(\frac{\sqrt{6}}{12\omega^{3/2}}\right)\dot{b}+\left(\frac{\sqrt{6}}{2\omega^{1/2}}\right)\dot{\psi}=0,
\tag{2.50}
$$

$$
\left(\frac{\sqrt{6}}{6\omega^{3/2}}\right)\dot{b}+\left(\frac{\sqrt{6}}{3\omega^{1/2}}\right)\dot{\psi}=0,
\tag{2.51}
$$

$$\left(\frac{\sqrt{6}}{2\omega^{1/2}}\right)\dot{b}+\left(\sqrt{6}\omega^{1/2}\right)\dot{\psi}=0. \tag{2.52}$$

From these three Equations (2.50, 2.51, 2.52) it is seen that the only possible solution is $\dot{b}=0$ which directly implies $\ddot{b}=0$ and $\dot{\psi}=0$ which directly implies $\ddot{\psi}=0$. Namely, this flat universe vacuum solution where $a\sim e^{H_\infty t}$and $\phi\sim e^{F_\infty t}$ is a stable solution.

Expanding $F\left(a\right)$ and $H\left(a\right)$ in powers of $\left(\frac{a_o}{a}\right)$ up to third order, where a_0 is the present size of a universe,

$$H\left(a\right)=H_\infty+H_2\left(\frac{a_0}{a}\right)^2+H_3\left(\frac{a_0}{a}\right)^3, \tag{2.53}$$

$$F\left(a\right)=F_\infty+F_2\left(\frac{a_0}{a}\right)^2+F_3\left(\frac{a_0}{a}\right)^3, \tag{2.54}$$

and putting them into Equations (2.41, 2.42, 2.43), we get the perturbation constants for $(\omega\gg 1)$:

$$H_\infty\approx 0.8\sqrt{\omega}\,m, \tag{2.55}$$

$$F_\infty\approx\frac{0.4}{\sqrt{\omega}}m, \tag{2.56}$$

$$H_2\approx-\frac{1}{2a_o^2H_\infty}\approx-\frac{0.6}{\sqrt{\omega}a_o^2m}\approx 0, \tag{2.57}$$

$$F_2\approx\frac{3}{4\omega a_o^2H_\infty}\approx\frac{0.9}{\omega^{3/2}a_o^2m}\approx 0, \tag{2.58}$$

$$H_3 \approx \frac{2\omega}{3} F_3, \tag{2.59}$$

$$H_3 \approx -F_3. \tag{2.60}$$

Up to now, we note that all the constants required in our assumption for $H(a)$ and $F(a)$ in the late-time inflation regime are almost determined from the theory except H_3 and F_3. Indeed solving Equations (2.59) and (2.60) simultaneously gives us H_3 and F_3 to be zero. However, to explain the late-time universe, we may assume that $\gamma = p_M/\rho_M \ll 1$ rather than p_M being exactly zero. To overcome this problem, we use the relation (2.59) between H_3 and F_3 coming from the $\phi-$Equation (2.43) which is more exact comparing to the relation (2.60) coming from the $p-$Equation (2.42). We also use the classical Friedman formula (2.7) in such a way that using Equations (2.55, 2.57) and rearranging Equation (2.53) by leaving H_3 as a free parameter, it follows from the Equation (2.7) that

$$\Omega_{0\Lambda} \approx \frac{H_\infty^2}{H_0^2}, \tag{2.61}$$

$$\Omega_{0R} \approx \frac{2H_\infty H_2}{H_0^2}, \tag{2.62}$$

$$\Omega_{0M} \approx \frac{2H_\infty H_3}{H_0^2}. \tag{2.63}$$

By fitting all these above results to the present observational results $\Omega_{0M} \approx 0.25$, $\Omega_{0\Lambda} \approx 0.75$ and $\Omega_{0R} \approx 0$ [41], we determine H_3 and F_3 in terms of the parameters in the theory as,

$$H_3 \approx 0.13\sqrt{\omega}m \ \ (\text{for } \omega \gg 1), \tag{2.64}$$

$$F_3 \approx \frac{1.41}{\sqrt{\omega}} m \quad (\text{for } \omega \gg 1). \tag{2.65}$$

We may now write H and F as in the form in which all perturbative constants are appeared, namely;

$$H\left(a\right) \approx 0.8\sqrt{\omega}\, m + 0.13\sqrt{\omega} m \left(\frac{a_0}{a}\right)^3, \tag{2.66}$$

$$F\left(a\right) \approx \frac{0.4}{\sqrt{\omega}}\, m + \frac{1.4}{\sqrt{\omega}} m \left(\frac{a_0}{a}\right)^3. \tag{2.67}$$

After finding the perturbation constants explicitly for H and F, we also find it worthy to determine how the Hubble parameter $H\left(a\right) = \dot{a}/a$ and the time variation of G, where G is the time dependent value of the gravitational constant, change in the primordial and late-time regimes compared to their present values. To do so, we use the relation$G^{-1} = \frac{2\pi}{\omega}\phi^2$ (2.8) which entirely comes from the fact that the term $\frac{1}{8\omega}\phi^2$ in the action Equation (2.9) will be the same as that of the term $1/16\pi G$ in the Hilbert-Einstein since Brans-Dicke gravity becomes identical to Einstein gravity as ω approaches infinity. Then, taking time derivative of both sides of the Equation (2.8) gives,

$$\left(\frac{\dot{G}}{G}\right) = -2\left(\frac{\dot{\phi}}{\phi}\right) = -2F(a) \approx -\frac{0.8}{\sqrt{\omega}}\, m - \frac{2.8}{\sqrt{\omega}} m \left(\frac{a_0}{a}\right)^3. \tag{2.68}$$

Hence, if we use Equation (2.68) for the present scale size of the universe a_0, then the present rate of change of G is found to be

$$\left(\frac{\dot{G}}{G}\right)_0 \approx -\frac{3.6}{\sqrt{\omega}} m, \tag{2.69}$$

whereas in a similar manner, using Equation (2.56) in addition to Equation (2.68) expects the rate of change of G in asymptotic regime as

$$\left(\frac{\dot{G}}{G}\right)_{late-time} \approx \frac{-0.8}{\sqrt{\omega}} m, \tag{2.70}$$

and finally, using Equation (2.27) gives the rate of change G in primordial regime as,

$$\left(\frac{\dot{G}}{G}\right)_{primordial} \approx -1.4m. \tag{2.71}$$

Subsequently, we think that it would be beneficial to express parameter m, mass of the scalar field, in terms of the present value of Hubble parameter H_0. For this reason, we simply satisfy the Equation (2.66) for the present value of the Hubble constant H_0 and for the present value of the scale factor of the universe a_0 such that for $\omega \gg 1$, m is expected from the theory as,

$$m \approx \frac{1.08}{\sqrt{\omega}} H_0. \tag{2.72}$$

Hence, with Equation (2.72), we can also express the rate of change of G and Hubble parameters H in terms of the observationally measured quantity H_0:

$$\left(\frac{\dot{G}}{G}\right)_0 \approx -\frac{3.9}{\omega} H_o, \tag{2.73}$$

$$\left(\frac{\dot{G}}{G}\right)_{late-time} \approx \frac{-0.87}{\omega} H_0, \tag{2.74}$$

$$\left(\frac{\dot{G}}{G}\right)_{primordial} \approx -\frac{1.51}{\sqrt{\omega}} H_0. \tag{2.75}$$

$$(H)_{primordial} \approx 0.7m\omega \approx 0.75\sqrt{\omega} H_0, \tag{2.76}$$

$$(H)_{late-time} \approx 0.8\sqrt{\omega}m \approx 0.86H_0. \tag{2.77}$$

Finally, when we investigate the equation of state parameter for the late time regime where $\omega \to \infty$,

$$\gamma = \frac{p}{\rho} = \frac{\left[\frac{-(\omega+6)}{\omega(6\omega+6)} - \frac{60\omega+63}{50(3\omega^2+3\omega)}\right]}{\left[\frac{\omega+3}{\omega(2\omega+2)} + \frac{6}{25}\right]} \simeq 0 \tag{2.78}$$

we find that it is approaching a value of zero as expected.

3. CAN BRANS-DICKE SCALAR FIELD ACCOUNT FOR DARK-ENERGY AND DARK-MATTER?

Our universe seems, according to the present-day evidence, to be spatially flat and to possess a non vanishing cosmological constant [42, 43]. For a flat matter dominated universe, cosmological measurements [41] imply that the fraction Ω_Λ of the contribution of the cosmological constant Λ to present energy density of the universe is $\Omega_\Lambda \sim 0.75$. In standard cosmology Ω_Λ would be induced by a cosmological constant which is a dimensional parameter with units of $(\text{length})^{-2}$. From the point of view of classical general relativity, there is no preferred choice for what the length scale defined by Λ might be. Particle physics, however, gives a different point of view to the issue. The cosmological constant turns out to be a measure of the energy density of the vacuum and although we can not calculate the vacuum energy with any confidence, this allows us to consider the scales of various contributions to the cosmological constant. The energy scale of the constituent(s) of Λ which in Planck units is approximated to 10^{-123} is problematic since it is lower than the normal energy scale predicted by most particle physics models. To solve this problem, a dynamical Λ [44] in the form of scalar field with some self interacting potential [45] can be considered and its slowly varying energy density induces a cosmological constant. This idea called "*quintessence*" [44] is similar to the inflationary phase of the early universe with the difference that it evolves at a much lower energy density scale. The energy density of this field has to evolve in such a way that it becomes comparable with the mass density fraction Ω_M now. This type of specific evolution, better known as "*cosmic coincidence*" [46] problem, needs several constraints and fine tuning of parameters for the potential used to model quintessence with minimally coupled scalar field. To solve the cosmic coincidence problem, a new form of quintessence field called the "tracker field" [46] has been proposed. Such kind of quintessence field is mainly based on an equation of motion with a solution for such that for a wide range of initial conditions the equation of motion converge to the same solution. This type of solution is also called an 'attractor like' solution. There are a number of quintessence models proposed. Most of these involve minimally coupled scalar field with different potentials dominating over the kinetic energy of the field. Purely exponential [47] and inverse power law [45, 46] potentials have been extensively studied for quintessence fields to solve the cosmic coincidence problem. However the fact that the energy density is not enough to make up for the missing part of the cosmological constant or that the $p/\rho \equiv \gamma$ value found for the equation of state of quintessence is not in good agreement with the observed results makes such an explanation unlikely. The investigation of alternative models in which the equation of state parameter γ of the cosmological constant evolves with time has been proposed due to the conceptual difficulties associated with a cosmological constant [48, 49, 50, 51].

There have been quite a few attempts for treating this problem with non-minimal coupled scalar fields. Studies made by Bartolo *et al* [52], Bertolami *et al* [53], Ritis *et al* [54] have found tracking solutions in scalar tensor theories with different types of power law potential. In another work, Sen *et al* [55] have found the potential relevant to power law expansion in BD cosmology and Arık and Çalık [56] have shown that BD theory of gravity with the standard mass term potential $\frac{1}{2}m^2\phi^2$ is a natural model to explain the rapid primordial inflation and the observed slow late-time inflation.

In this chapter of thesis, we aim to show that a linearized non-vacuum solution about the stable cosmological vacuum solution with flat space-like section is capable of explaining how the Hubble parameter evolves with the scale size of the universe $a(t)$. In this framework, we also show that the standard Friedmann equation changes into a form in which the power of the scale size term with Ω_M is corrected by an amount $1/\omega$

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{0\Lambda} + \Omega_{0M}\left(\frac{a_0}{a}\right)^{3+\frac{1}{\omega}} . \tag{3.1}$$

Subsequently, under such a linearized solution, we point out that only a very small part of the dark matter can be accommodated into the contribution of the Brans-Dicke scalar field.

In the context of BD theory [28] with self interacting potential and matter field, the action we use in the canonical form is given by the Equation (2.9), and all the kinetic and potential term properties of the scalar field are the same as it is defined in the previous chapter. Excluding ϕ, as the matter field, we again consider a classical perfect fluid with the energy-momentum tensor $T^{\mu}_{\nu} = diag\left(\rho, -p, -p, -p\right)$ where ρ is the energy density, p is the pressure. Hence, the gravitational field equations derived from the variation of the action (2.9) with respect to Robertson- Walker metric are the same as the Equations (2.10, 2.11, 2.12). Since in the standard theory of gravitation, the total energy density ρ is assumed to be composed of $\rho = \rho_{\Lambda} + \rho_{M}$ where ρ_{Λ} is the energy density of the universe due to the cosmological constant which in modern terminology is called as "*dark energy*", re-organization of the right hand sides of the Equations (2.10, 2.11), adopted to the matter energy density term ρ_M instead of ρ and p_M instead of p where M denotes everything except the ϕ field, is being essential for this chapter in the sense that whether if the ϕ terms on the left-hand side of Equation (2.10) can accommodate a contribution to due to what is called dark matter.

In the light of encouraging result obtained by using the instability caused by the nonvacuum in the closed stable vacuum solution in explaining the rapid primordial inflation, we will show that a linearized non-vacuum solution about the flat stable vacuum solution can also be powerful in explaining the slow late time expansion. Since the universe becomes (approximately) flat in late times, we ignore the curvature parameter k/a^2 as $a\left(t\right)$ increases with the expansion of the universe. Under these considerations, in analogy with the assumption we use in explaining rapid primordial inflation, we first propose $a = e^{H_{\infty}t}$ and $\phi = e^{F_{\infty}t}$ and put into Equations (2.10, 2.11, 2.12) and search for a zeroth order stable vacuum (empty except the ϕ field) solution. H_{∞}, F_{∞} are the constants to be determined named as the late time Hubble parameter and the fractional rate of change of ϕ in the late time regime respectively. We have the same results for H_{∞} and F_{∞} as in the previous chapter. Thus, in our one hand, we have had an exact zeroth order stable-vacuum solution as,

$$a = e^{H_{\infty}t}, \tag{3.2}$$

$$\phi = \phi_{\infty} e^{F_{\infty}t}, \tag{3.3}$$

where $H_\infty \approx 0.8\sqrt{\omega}m$, $F_\infty \approx \frac{0.4}{\sqrt{\omega}}m$, ϕ_∞ is a constant. Then, after finding such a zeroth order exact stable solution, the question that stimulates us, similar to the primordial regime analysis, is that how the presence of matter affects this flat stable vacuum solution (3.2, 3.3). To understand such a perturbation phenomenon, we impose the following linearized first order non-vacuum solution for $H \equiv \dot{a}/a$ and $F \equiv \dot{\phi}/\phi$ which includes first order perturbation functions of $h(a)$ and $f(a)$ in addition to the constant terms H_∞ and F_∞ which appear in the flat stable vacuum solution (3.2, 3.3) respectively.

$$H = H_\infty + h(a), \tag{3.4}$$

$$F = F_\infty + f(a). \tag{3.5}$$

Since solving the field Equations (2.10, 2.11, 2.12) exactly for $a(t)$ and $\phi(t)$ under the condition $p = 0$ is hard enough, we put our imposed solution (3.4, 3.5) into the modified field Equations (2.41, 2.42, 2.43) for $p = 0$ and neglect higher terms in $h(a)$, $f(a)$ then we get $h(a)$ and $f(a)$ for all ω in the form of,

$$h(a) = C_1 H_0 \left(\frac{a_0}{a}\right)^{\frac{3\omega+4}{\omega+1}} - \left(\frac{1}{H_\infty a_0^2}\right) \frac{(\omega+1)(\omega+3)}{(\omega+2)(2\omega+3)} \left(\frac{a_o}{a}\right)^2, \tag{3.6}$$

$$f(a) = C_2 H_0 \left(\frac{a_0}{a}\right)^{\frac{3\omega+4}{\omega+1}} + \left(\frac{3}{2H_\infty a_0^2}\right) \frac{(\omega+1)}{(\omega+2)(2\omega+3)} \left(\frac{a_o}{a}\right)^2, \tag{3.7}$$

where C_1 and C_2 are dimensionless integration constants. Since letting $\omega \to \infty$ has a special meaning in the sense that the Brans-Dicke scalar tensor theory matches with standard Einstein theory under such limit, we display the linearized solution (3.4, 3.5) in the following form as $\omega \to \infty$,

$$H = H_\infty + C_1 H_0 \left(\frac{a_0}{a}\right)^{3+\frac{1}{\omega}} - \frac{1}{2}\left(\frac{1}{H_\infty a_0^2}\right)\left(\frac{a_o}{a}\right)^2, \tag{3.8}$$

$$F = F_\infty + C_2 H_0 \left(\frac{a_0}{a}\right)^{3+\frac{1}{\omega}}. \tag{3.9}$$

Hence, putting the solution (3.8) in the standard Friedmann equation (2.7), and using the present observational results on density parameters $\Omega_{0\Lambda} \simeq 0.75$, $\Omega_{0M} \simeq 0.25$, $\Omega_{0R} \simeq 0$ [41], we get $C_1 \simeq 0.15$ and $H_\infty = 0.86 H_0$ so that $F_\infty \approx H_\infty/2\omega \approx (0.43/\omega)\, H_0$ which provides $|C_2| \ll 0.43/\omega$. Namely, the first term in a linearized solution (3.9) is much greater than the second term. The curvature density parameter Ω_{0R}, on the other hand, is found to be in accordance with the recent measurements since the term $(1/H_0 a_0)^2 \approx \Omega_{0R} \simeq 0$ [41]. To compare Equation (2.41) with standard FLRW cosmology, we put the linearized solution (3.8, 3.9) into Equation (2.41) and transfer all terms except for $H^2 = (\dot{a}/a)^2$ to the right hand side. Neglecting the $1/a^2$ term, we end up with

$$H^2 = \frac{4\omega}{3\phi^2}(\rho_\Lambda + \rho_M + \rho_D). \tag{3.10}$$

Noting that in the late time regime, scalar field dependence on the scale size of the universe, $\phi \sim a^{1/2\omega}$, is approximately constant as a changes. Keeping this in mind, we identify the terms which do not explicitly depend on a with ρ_Λ and terms which depend on a as a^{-3} with the dark matter energy density ρ_D so that

$$\rho_D = (C_2 F_\infty H_0 \phi^2 - \frac{3C_2}{2\omega} H_\infty H_0 \phi^2 - \frac{3C_1}{2\omega} H_0 F_\infty \phi^2)(\frac{a_0}{a})^3, \tag{3.11}$$

$$\rho_\Lambda = \frac{1}{2} F_\infty^2 \phi^2 - \frac{3}{2\omega} H_\infty F_\infty \phi^2 + \frac{1}{2} m^2 \phi^2. \tag{3.12}$$

Using the recent observational results on density parameters of the universe ($\Omega_{0\Lambda} \equiv \rho_\Lambda/\rho_0 \simeq 0.75$, $\Omega_{0D} \equiv \rho_D/\rho_0 \simeq 0.23$) (1.1.6) where ρ_0 is the present measured energy density of the universe and the relations $H_\infty \approx 0.86 H_0$ and $F_\infty \approx H_\infty/2\omega \approx (0.43/\omega)\, H_0$ as $\omega \to \infty$, we fit Equations (3.11, 3.12) to the ratio $\Omega_\Lambda/\Omega_D \simeq 75/23$ and determine the $|C_2|$ integration constant to be $|C_2| \approx 0.20$ which is inconsistent with the requirement $|C_2| \ll 0.43/\omega$ imposed by the theory.

In conclusion, the first remarkable feature of this work that a linearized non-vacuum solution (3.8) about the stable cosmological vacuum solution (3.2) with flat space-like section is capable of explaining how the Hubble parameter $H \equiv \dot{a}/a$ evolves with the scale size of the universe $a(t)$. The second remarkable feature of this theory is that by fitting the linearized solutions (3.8, 3.9) of the theory to the recent observations [41], the late-time Hubble parameter $H_\infty = 0.86H_0$ and the fractional rate of change of ϕ in the late time regime $F_\infty = (0.43/\omega)\, H_0$ are successfully predicted in terms of today's observational measured value of Hubble parameter H_0. Another important prediction we note from this theory is that for a fixed H_0, since $F_\infty \approx (0.43/\omega)\, H_0$, F_∞ may not attain a large value because of its inverse dependence on ω which is measured to be, according to the recent observational data, as $\omega > 10^4 \gg 1$ [32]. This is the reason why F_∞ can not let the scalar field $\phi = \phi_\infty e^{F_\infty t}$ to blow up rapidly so that ρ_Λ (3.12), the energy density of the universe due to the cosmological constant, can grow slowly and reasonably. Hence we strictly agree on that this theory is successful in explaining the dark energy though the Brans-Dicke scalar field ϕ can not account for dark matter with its minor contribution to dark matter which is approximately less than 2 %. The last remarkable feature of this theory is that it enables us to estimate some dimensional parameters displayed in the theory. Using the relation $H_\infty \approx 0.86H_0 \approx 0.8\sqrt{\omega}m$ and the restriction on ω, we may estimate m for a fixed H_0 as

$$m \lessapprox 10^{-2} H_0, \tag{3.13}$$

where the present value of Hubble constant $H_0 = 720 \pm 8$ km s^{-1} Mpc $^{-1}$ [8]. Using appropriate conversion relation in relativistic units, present Hubble parameter is found to be

$$H_0 \approx 10^{-26}\mathrm{m}^{-1} \approx 2 \times 10^{-42}\mathrm{GeV}. \tag{3.14}$$

Furthermore, by using (3.13), one can re-estimate Hubble parameters and rate of change values in Newtonian gravitation constant for the primordial and the late time epochs considering linearization approach used in this chapter as

$$|\frac{\dot{G}}{G}|_P = 2F_P \approx (1.5/\sqrt{\omega})H_0, \tag{3.15}$$

$$|\frac{\dot{G}}{G}|_\infty = 2F_\infty \approx \frac{0.86}{\omega}\,H_0, \tag{3.16}$$

$$H_P \approx 0.7m\omega > 70\,H_0, \tag{3.17}$$

$$H_\infty \approx 0.8\sqrt{\omega}m \approx 0.86H_0. \tag{3.18}$$

The fractional rate of change of the scalar field ϕ, on the other hand, are given by

$$F_P \lessapprox 7 \times 10^{-3}\,H_0, \tag{3.19}$$

$$F_\infty < 43 \times 10^{-6} H_0. \tag{3.20}$$

4. FRIEDMANN EQUATION FOR BRANS DICKE COSMOLOGY

Recent observational data have strongly confirmed that we live in an accelerating universe [58] and have made it possible to determine the composition of the universe [9, 59]. According to these observations, nearly seventy five percent of the energy density in the universe is unclustered and has negative pressure by which it is driving an accelerated expansion [13, 60, 61, 62]. Furthermore, the energy density of the vacuum is much smaller than the estimated values so far. By itself, acceleration seems to be much more understandable in the context of general relativity (cosmological constant) [42, 43] and quantum field theory (quantum zero point energy); however, the very small and non-zero energy scale implied by the observations is not quite comprehensible. Because of these conceptual problems associated with the cosmological constant [48, 49, 50, 51], alternative treatments to the problem have been produced and they are being used widely in the literature nowadays [45, 46, 47, 52]. In such treatments, mostly, a scalar field ϕ is considered together with a suitably chosen $V(\phi)$ to make the vacuum energy vary with time. The reason for this is to get a model in which the current value of the cosmological constant can be expressed in a more natural way; without need of any fine tuning. In the literature, there exist number of studies on accelerated models in Brans Dicke theory [63, 64, 65, 66, 67, 68, 69, 70, 71]. In this regard, choosing the underlying theory as a BD scalar tensor theory of gravitation, we aim to calculate the corrections, in the context of BD cosmology, to the famous Friedmann Equation (2.7). According to recent observational results for the present universe, we have $\Omega_\Lambda \simeq 0.75$, $\Omega_M \simeq 0.25$, $\Omega_R \simeq 0$ [41]. In the light of these values, one can conclude that the universe is mostly filled with non-baryonic matter and it seems that this non baryonic matter is responsible for the expansion of the universe solely.

The gravitational field equations that we will use in this chapter are the ones modified in terms of $H(a)$, $F(a)$ and their derivatives with respect to the scale size of the universe a, namely, (2.41, 2.42, 2.43). From these three equations it can be shown that the continuity equation for the matter-energy excluding the BD scalar field is also satisfied (see appendix D) with the help of the ϕ Equation (2.43)

$$\dot{\rho_M} + 3\left(\frac{\dot{a}}{a}\right)(p_M + \rho_M) = 0, \tag{4.1}$$

and hence, instead of considering the p-Equation (2.42) solely as one of the dynamical equations to be satisfied, we choose continuity equation in addition to the density equation and the ϕ equation to be satisfied in any cosmological case we want to explain. That is because once the continuity equation is satisfied than p-Equation must already be satisfied automatically provided that $\dot{a}$ is nonzero. To eliminate the ϕ dependence in Equation (2.41), we take the time derivative of both sides of the ρ equation and after some rearrangements, we get Equation (2.41) purely in terms of $H(a)$, $F(a)$, $\rho(a)$ and their derivatives with respect to a.

$$\begin{aligned} H'(H^2 + HF) + F'(H^2 - \frac{2\omega}{3}HF) \\ = \frac{H^3}{2}\left(\frac{\rho^{'}}{\rho}\right) + \frac{2\omega}{3a}F^3 + H^2F\left[\left(\frac{\rho^{'}}{\rho}\right) - \frac{1}{a}\right] + F^2H\left[-\frac{2}{a} - \frac{\omega}{3}\left(\frac{\rho^{'}}{\rho}\right)\right] \\ + \frac{k}{a^2}\left[H\left(\left(\frac{\rho^{'}}{2\rho}\right) + \frac{1}{a}\right) - \frac{F}{a}\right] - \omega m^2\left[H\left(\frac{\rho^{'}}{3\rho}\right) - \frac{2F}{3a}\right]. \end{aligned} \tag{4.2}$$

After rewriting Equation (2.43) in the following form

$$3aHH^{'} - 2\omega HaF^{'} = -6H^2 + 2\omega F^2 + 6\omega HF - \frac{3k}{a^2} + 2\omega m^2 \tag{4.3}$$

we solve Equations (4.2, 4.3) for H', F' and get the general form of the solution in the sense that once the curvature constant k and energy density in terms of a is given than H and F can be solved from the following equations:

$$\begin{aligned} H' = \frac{[\omega a(\rho'/\rho) - 6]}{(2\omega+3)\,aH}H^2 - \frac{[4\omega^2 + 2\omega + 2a\omega^2(\rho'/3\rho]}{(2\omega+3)\,aH}F^2 + \frac{[8\omega + 2a\omega(\rho'/\rho)]}{(2\omega+3)\,aH}HF \\ -\frac{[2\omega^2 a(\rho'/3\rho) - 2\omega]}{(2\omega+3)\,aH}m^2 + k\frac{[2\omega + \omega a(\rho'/2) - 3]}{(2\omega+3)\,a^3H} \end{aligned} \tag{4.4}$$

$$F' = \frac{[3a(\rho'/2\rho)+6]}{(2\omega+3)\,aH}H^2 - \frac{[8\omega + a\omega(\rho'/\rho)+6]}{(2\omega+3)\,aH}F^2 - \frac{[6\omega - 3a(\rho'/\rho) - 3]}{(2\omega+3)\,aH}HF$$
$$-\frac{[\omega a(\rho'/\rho)+2\omega]}{(2\omega+3)\,aH}m^2 + k\frac{[6+3a(\rho'/2\rho)]}{(2\omega+3)\,a^3H}. \tag{4.5}$$

Hence, in the present epoch, to discover how the Hubble parameter H changes with the scale size of the universe a, we assume that the present universe is mostly flat and it necessarily obeys the $p_M = 0$ equation of state . Using Equation (4.1), we find that the energy density ρ evolves with a in the same manner as in standard Einstein cosmology when the universe is solely governed by matter,

$$\rho = \frac{C}{a^3}, \tag{4.6}$$

where C is an integration constant. Setting $k = 0$ and inserting this energy density into Equations (4.4, 4.5), we get the following form of the equations to be solved:

$$H' = \frac{-1}{H(2\omega+3)a}\left[3(2+\omega)H^2 + 2\omega(\omega+1)F^2 - 2\omega HF - 2\omega(\omega+1)m^2\right], \tag{4.7}$$

$$F' = \frac{1}{H(2\omega+3)a}\left[\frac{3}{2}H^2 - (5\omega+6)\,F^2 - 6(1+\omega)HF + \omega m^2\right]. \tag{4.8}$$

With the transformation $u = \left(\frac{a_0}{a}\right)^\alpha$, we rewrite Equations (4.7, 4.8) in terms of $H(u)$, $F(u)$ and their derivatives with respect to u

$$\frac{dH}{du} = \frac{1}{\alpha H(2\omega+3)u}\left[3\,(2+\omega)\,H^2 + 2\omega\,(\omega+1)\,F^2 - 2\omega HF - 2\omega(\omega+1)m^2\right], \tag{4.9}$$

$$\frac{dF}{du} = \frac{-1}{\alpha H(2\omega+3)u}\left[\frac{3}{2}H^2 - (5\omega+6)\,F^2 - 6(1+\omega)HF + \omega m^2\right]. \tag{4.10}$$

Since these coupled equations are hard enough to be solved analytically for H and F, our approach is to determine a perturbative solution in which both H and F vary about some constants H_∞ and F_∞ respectively:

$$H = H_\infty + H_1 u + H_2 u^2 + ... \tag{4.11}$$

$$F = F_\infty + F_1 u + F_2 u^2 + ... \tag{4.12}$$

where H_∞, F_∞, H_1, F_1, α, are all constants to be determined from the theory. Plugging this perturbative solution into Equations (4.9, 4.10) and keeping only the zeroth, first, second order terms of u and neglecting higher terms, we end up with two sets of solutions in the zeroth order

$$H_\infty = \frac{\sqrt{\omega}\,(2\omega+2)\,m}{\sqrt{(6\omega^2+17\omega+12)}}\,;\;\; F_\infty = \frac{H_\infty}{2(\omega+1)}, \tag{4.13}$$

and

$$H_\infty = \frac{2\sqrt{3\omega}\,m}{3\sqrt{3\omega+4}}\,;\;\; F_\infty = \frac{3}{2}H_\infty. \tag{4.14}$$

Comparing the first order terms of u, on the other hand, provides two linearly dependent equations for which the only possible solution is the trivial solution of $H_1 = 0$ and $F_1 = 0$,

$$\{[6(\omega+2) - \alpha(2\omega+3)]\,H_\infty - 2\omega F_\infty\}\,H_1 + [-2\omega H_\infty + 4\omega(\omega+1)F_\infty]\,F_1 = 0, \tag{4.15}$$

$$[-3H_\infty + 6(\omega+1)F_\infty]\,H_1 + \{[6\,(\omega+1) - \alpha(2\omega+3)]\,H_\infty + 2(5\omega+6)F_\infty\}\,F_1 = 0. \tag{4.16}$$

Since the solution in which H_1 and F_1 are nonzero is much more plausible for our aim, the coefficient matrix is properly constructed from Equations (4.15, 4.16) and its determinant is set to be zero to get the value of α for which H_1 and F_1 need not be zero simultaneously. We get two different α values

$$\alpha = 3 + \frac{1}{1+\omega}, \tag{4.17}$$

and

$$\alpha \sim \sqrt{\omega}, \tag{4.18}$$

corresponding to the solution sets (4.13) and (4.14) respectively. In this regard, we note two things here:

- Concerning the solution of H we seek for, the solution (4.17) is much more precious than the solution (4.18) which approaches to infinity as ω becomes infinitely large. On the other hand, in the same limit, Equation (4.17) gives $\alpha = 3$ which is the well known term in a matter dominated universe solution of standard Einstein cosmology.
- The correction factor $1/(1+\omega)$ in the solution (4.17) is solely coming from the exact solutions of the field equations of BD theory and it is not surprising that we get the same correction factor as we have had by a linearization method applied to the field equations in the previous chapter.

On the other hand, two linearly dependent equations are available when one compares the second order terms of u;

$$\begin{aligned} &\{[6(\omega+2) - 2\alpha(2\omega+3)]\, H_\infty - 2\omega F_\infty\}\, H_2 + [4\omega(\omega+1)F_\infty - 2\omega H_\infty]\, F_2 \\ &\qquad = [\alpha(2\omega+3) - 3(\omega+2)]\, H_1^2 - 2\omega(\omega+1)F_1^2 + 2\omega H_1 F_1, \end{aligned} \tag{4.19}$$

$$\begin{aligned} &[3H_\infty - 6(\omega+1)F_\infty]\, H_2 + \{[2\alpha(2\omega+3) - 6\,(\omega+1)]\, H_\infty - 2(5\omega+6)F_\infty\}\, F_2 \\ &\qquad = -\frac{3}{2}H_1^2 + F_1^2 + [6\,(\omega+1) - \alpha(2\omega+3)]\, H_1 F_1. \end{aligned} \tag{4.20}$$

Letting $\alpha = 3 + 1/(\omega+1)$ and $F_\infty = H_\infty/2(\omega+1)$ in Equations (4.19, 4.20) gives H_2 and F_2 only in terms of H_∞, H_1, F_1;

$$H_2 = \frac{1}{(3\omega+4)(2\omega+3)H_\infty}\left[-(3\omega^2+8\omega+6)H_1^2 + 2\omega(\omega+1)^2F_1^2 - 2\omega(\omega+1)H_1F_1\right], \tag{4.21}$$

$$F_2 = \frac{1}{(3\omega+4)(2\omega+3)H_\infty}\left[\frac{-(3\omega+3)}{2}H_1^2 + (\omega+1)F_1^2 - (5\omega+6)H_1F_1\right]. \tag{4.22}$$

Hence, with these perturbation constants found from theory, we can express H and F as

$$H = H_\infty + H_1\left(\frac{a_0}{a}\right)^{3+\frac{1}{\omega+1}} + H_2\left(\frac{a_0}{a}\right)^{6+\frac{2}{\omega+1}} + ... \tag{4.23}$$

$$F = F_\infty + F_1\left(\frac{a_0}{a}\right)^{3+\frac{1}{\omega+1}} + F_2\left(\frac{a_0}{a}\right)^{6+\frac{2}{\omega+1}} + ... \tag{4.24}$$

where Equation (4.13) gives

$$H_\infty = \left[2\left(\omega+1\right)\sqrt{\omega}m\right]/\sqrt{(6\omega^2+17\omega+12)}, \tag{4.25}$$

and

$$F_\infty = \left(\sqrt{\omega}m\right)/\sqrt{(6\omega^2+17\omega+12)}. \tag{4.26}$$

When the standard procedure of putting Equation (4.23) into the standard Friedmann Equation (2.7) and of fitting it to the present measured density parameters of universe is applied [41], we get

$$\left(\frac{H}{H_0}\right)^2 = \Omega_\Lambda + \Omega_M\left(\frac{a_0}{a}\right)^{3+\frac{1}{\omega+1}} + \Omega_\Delta\left(\frac{a_0}{a}\right)^{6+\frac{2}{\omega+1}}, \tag{4.27}$$

$$\Omega_\Lambda = \frac{H_\infty^2}{H_\Sigma^2} \simeq 0.75, \tag{4.28}$$

$$\Omega_M = \frac{2H_\infty H_1}{H_\Sigma^2} \simeq 0.25, \tag{4.29}$$

$$\Omega_\Delta = \frac{H_1^2 + 2H_\infty H_2}{H_\Sigma^2}, \tag{4.30}$$

where $H_\Sigma^2 = H_\infty^2 + 2H_\infty(H_1 + H_2) + H_1^2$. With the findings both from theory and observations, we can make the following statements:

1. Remarkable feature of this theory is that it enables us to estimate some dimensional parameters displayed in the theory. Using the ratio of Equation (4.28) to Equation (4.29), $H_1 \simeq 0.167H_\infty$ where $H_\infty \simeq 0.82\, m\, \omega^{1/2}$ as $\omega \to \infty$. If Equation (4.23) is satisfied for $H = H_0$, one can get $H_\infty \simeq 0.86\, H_0$ where H_0 is the present value of the Hubble parameter [8]. Using $H_\infty \simeq 0.86\, H_0 \simeq 0.82\, m\, \omega^{1/2}$ and ω restriction $\omega > 10^4 \gg 1$, we may estimate m for a fixed H_0 as $m \lessapprox 10^{-2} H_0$.
2. Investigating two cases for Ω_Δ can be meaningful for the sake of future measurements of the density parameter Ω:
   - If we set $\Omega_\Delta \simeq 0$ together with $H_2 \neq 0$ which is consistent with today's universe density compositions, by using Equation (4.21) and Equation (4.30) simultaneously, we get $F_1 \simeq 0.08H_\infty/\omega$, $F_2 \simeq -0.04H_\infty/\omega$ as $\omega \to \infty$. Remembering that $F_\infty \simeq H_\infty/2\omega$, we may say that F_∞ in Equation (4.24) is the dominating term in today's universe. This shows us that similar to the expansion rate of the universe H, the rate of change of the Newtonian gravitational constant has approached the asymptotic regime.
   - On the contrary, when we set $\Omega_\Delta \neq 0$ together with $H_2 \simeq 0$, we get $F_1 \simeq 0.2H_\infty/\sqrt{\omega}$, $F_2 \simeq -7 \times 10^{-3} H_\infty/\omega$ as $\omega \to \infty$. Since $F_\infty \simeq H_\infty/2\omega$, we may now say that F_1 is the dominating term in Equation (4.24). Namely, the rate of change of the Newtonian gravitational constant $\left(\dot{G}/G\right)$ has not approached to the asymptotic regime yet. However, theory predicts that

when the size of the universe exceeds $a \gg 0.6\omega^{1/6} a_0$, then the term F_∞ will become dominant so that asymptotic regime will be satisfied for $\left(\dot{G}/G\right)$. In addition to these, using Equations (4.28-4.30), we find that

$$\Omega_M = 2\sqrt{\Omega_\Lambda \Omega_\Delta}\ , \tag{4.31}$$

and with the constraint $\Omega_\Lambda + \Omega_M + \Omega_{{}_\Delta} = 1$, we express Equation (4.31) only in terms of Ω_Λ and Ω_M as

$$\Omega_M = 2\sqrt{\Omega_\Lambda}\left(1 - \sqrt{\Omega_\Lambda}\right). \tag{4.32}$$

If the ratio of today's universe density parameters $\Omega_\Lambda/\Omega_M = 3$ is still satisfied in this $\Omega_\Delta \neq 0$ case, we predict,

$$\Omega_\Lambda \simeq 0.73, \tag{4.33}$$

$$\Omega_M \simeq 0.24, \tag{4.34}$$

$$\Omega_{{}_\Delta} \simeq 0.03. \tag{4.35}$$

5. DISCUSSION AND CONCLUSION

In this work, we specifically have shown that primordial inflation and slow-rate late inflation can successfully be explained in the context of Jordan-Brans-Dicke scalar tensor theory of gravitation. Keeping in mind that the strength of the coupling to gravity is determined by a non-minimal term $\phi^2 R$ in these theories, we have just focused on the question what happens if we plug the simplest chaotic inflation-style potential energy density $V(\phi) = \frac{1}{2}m^2\phi^2$ composed only of the scalar field mass term in addition to the usual kinetic term $\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ in BD action. The reason for choosing such a simple potential energy term is to keep the originality of BD theory as possible as we can. For instance, the proposed model in this thesis is simple in that no other phenomenon, such as the domination of the false vacuum over the scalar field energy density as in the extended inflation model is used. Finding stable vacuum solutions and their perturbative solutions under the existence of matter is the common way of proceeding in this thesis. In this regard, in the first chapter, we have seen that such a solution technique is successful to explain primordial expansion. We have recognized that empty universe with a constant size a_* has a potential to expand if it is filled with radiation and scalar field $\phi \sim e^{F_p t}$. Its general equation for the scale size of the universe is appealing for at least three reasons: We get this solution without need any special equation of state for the matter as it is frequently required in the standard cosmology, using the stable-empty universe solution (2.13) in the Equation (2.12) has yielded this solution from the theory, and this inflationary solution is examined concerning as $t \to 0$ and as $t \gtrsim 0$. We recognized with the help of Equation(2.29) and Equation (2.31) that that this solution implies that $a(t) \sim \sqrt{t}$ as $t \to 0$ and the primordial rapid inflation $a(t) \sim e^{H_p t}$ for $\omega \gg 1$. When the equation of state $p = \frac{1}{3}\rho$ is studied for $\omega \gg 1$ by substituting $\phi \sim e^{F_p t}$ and $a \sim \sqrt{t}$ into Equations (2.10, 2.11), we have satisfied with the result of $\gamma = 1/3$ as expected in this regime.

Since the recent progress in observational cosmology shifted attention towards experimental verification of various inflationary theories, our motivation on this model is accelerated with the recent measurements of the dependence of the Hubble parameter

$H = \frac{\dot{a}}{a}$ on the scale size $a(t)$ of the universe. Hence we studied our model concerning the late-time regime. With perturbative solution approach we get the results of how the Hubble parameter $H\left(a\right) = \dot{a}/a$ and the time variation of G, where G is the time dependent value of the gravitational constant, change in the primordial and late-time regimes compared to their present values. We note that the recent measurement [41] of Ω_Λ and Ω_M has been used as input to derive these results. These are according to the expansion parameters of the scalar field ϕ found from the theory, $F_P \lessapprox 7 \times 10^{-3}\, H_0$, $F_\infty < 43 \times 10^{-6} H_0$, $|\frac{\dot{G}}{G}|_P = 2F_P \approx (1.5/\sqrt{\omega}) H_0$, $|\frac{\dot{G}}{G}|_0 \approx \frac{3.9}{\omega} H_o$, $|\frac{\dot{G}}{G}|_\infty = 2F_\infty \approx \frac{0.86}{\omega}\, H_0$. One interesting feature is that the predicted present day and late time values of $|\dot{G}/G|$ are comparable whereas the primordial value is much bigger. In any case a measurement of $\dot{G}/G$ will be crucial in determining the Brans-Dicke parameter ω. Hubble parameters found in both regimes, on the other hand, are $H_P \approx 0.7 m\omega > 70\, H_0$ and $H_\infty \approx 0.8\sqrt{\omega} m \approx 0.86 H_0$. Here it is noted that the ratio of the primordial and late-time inflation parameters is proportional to $\sqrt{\omega}$ is the most appealing feature of Brans-Dicke cosmology. This is a very important result in the sense that although there is an experimental lower bound on ω, there is no upper bound on it according to the recent observation $\omega \gg 10^4$[32], hence the late-time inflation in Brans-Dicke cosmology can be as small as one wishes compared to the primordial inflation. Besides this, the ratio $\gamma = \frac{p}{\rho}$ is found to be zero as universe approaches late-time inflation ($\omega \gg 1$). Thus, recent measurements, which imply that in today's universe $\Omega_\Lambda \neq 0$, require $1/\omega \neq 0$ and make this model attractive.

In the third chapter of this thesis, we have applied linearized solutions to the field equations of the theory. In this regard, it is checked that whether if there is dark matter contribution due to a scalar field ϕ to the energy density ρ_M or not. We have concluded that the probable contribution from the scalar field is approximately less than 2 % . Hence we strictly agree on that this theory is successful in explaining the dark energy though the Brans-Dicke scalar field ϕ can not account for dark matter with its minor contribution to dark matter. Another remarkable feature of this theory is that it enables us to estimate some dimensional parameters displayed in the theory. Using the relation $H_\infty \approx 0.86 H_0 \approx 0.8\sqrt{\omega} m$ and the restriction on ω, we may estimate m for a fixed H_0 as $m \lessapprox 10^{-2} H_0$ where the present value of Hubble constant $H_0 \approx$

$10^{-26}\text{m}^{-1} \approx 2 \times 10^{-42}\text{GeV}$ [8].

As a result, in the fourth chapter, it is shown that Brans-Dicke scalar tensor theory of gravitation brings a negligible correction to the matter density component of Friedmann equation by an amount $1/\omega$. Hence the standard Friedmann equation changes into a form (4.27). In the context of Brans-Dicke scalar tensor theory of gravitation, in addition to Ω_Λ and Ω_M, in standard Einstein cosmology, another density parameter, $\Omega_\vartriangle$, is expected by the theory. This implies that if $\Omega_\vartriangle$ is found to be nonzero, data will favor this model instead of the standard Einstein cosmological model with cosmological constant and will enable more accurate predictions for the rate of change of Newtonian gravitational constant G in the future.

APPENDIX A: Brans Dicke Action Variation

The action in Brans Dicke scalar tensor theory of gravitation is given

$$S = \int d^4x\, g^{1/2} \left[-\frac{1}{8\omega}\, \phi^2\, R + \frac{1}{2}\, g^{\mu\upsilon}\, \partial_\mu\phi\, \partial_\nu\phi - V\,(\phi) + L_M \right], \tag{A.1}$$

where $V\,(\phi)$ is solely composed of standard mass term $V\,(\phi) = \frac{1}{2}m^2\phi^2$. Here R is the Ricci scalar with its definition of

$$R = g^{\mu\nu} R_{\mu\nu}, \tag{A.2}$$

$R_{\mu\nu}$ is the Ricci tensor with its definition of

$$R_{\mu\nu} = R^a_{\mu a\nu}, \tag{A.3}$$

$R^a_{\mu a\nu}$ is the contracted Riemann tensor with its definition of

$$R^a_{bcd} = \partial_c\Gamma^a_{bd} + \Gamma^e_{bd}\Gamma^a_{ec} - \partial_d\Gamma^a_{bc} - \Gamma^e_{bc}\Gamma^a_{ed}, \tag{A.4}$$

and affine connection $\Gamma^\sigma_{\mu\nu}$ with its definition of

$$\Gamma^\sigma_{\mu\nu} = \frac{1}{2} g^{\sigma d}(\partial_\mu g_{\nu d} + \partial_\nu g_{\mu d} - \partial_d g_{\mu\nu}). \tag{A.5}$$

Besides, $g^{1/2}$ is defined as

$$g^{1/2} = (-\det g_{\mu\nu})^{1/2}, \tag{A.6}$$

and L_M is the matter Lagrangian except ϕ field in it. Variation of action yields,

$$\delta S = \int d^4x \, \left[-\frac{1}{8\omega}\delta \, \left(g^{1/2}\phi^2 \, R\right) + \frac{1}{2}\,\delta\left(g^{1/2}g^{\mu\upsilon}\,\partial_\mu\phi\,\partial_\nu\phi\right) - \delta\left(g^{1/2}\frac{1}{2}m^2\phi^2\right)\delta\left(g^{1/2}L_M\right)\right]. \tag{A.7}$$

Before going further, one can need the variation of the following geometrical objects defined in Einstein's General Relativity theory and the variation of the terms with respect to the scalar field ϕ,

$$\delta\, g^{1/2} = -\frac{1}{2}g^{1/2}g_{\mu\nu}\delta g^{\mu\nu}, \tag{A.8}$$

$$\delta\, R = \delta\left(g^{\mu\nu}R_{\mu\nu}\right) = R_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta R_{\mu\nu}, \tag{A.9}$$

$$\delta R^a_{bcd} = \nabla_c\delta\Gamma^a_{bd} - \nabla_d\delta\Gamma^a_{bc} \text{ (Palatini Equation)}, \tag{A.10}$$

$$\delta R^a_{\mu a\nu} = \delta R_{\mu\nu} = \nabla_a\delta\Gamma^a_{\mu\nu} - \nabla_\nu\delta\Gamma^a_{\mu a}, \tag{A.11}$$

$$\delta\Gamma^a_{\mu\nu} = \frac{1}{2}g^{ad}(\nabla_\mu\delta g_{\nu d} + \nabla_\nu\delta g_{\mu d} - \nabla_d\delta g_{\mu\nu}) \text{ (Palatini Equation)}, \tag{A.12}$$

$$\delta\partial_\mu\phi\,\partial_\nu\phi = 2\partial_\mu\phi\partial_\nu\delta\phi = 2\left(\partial_\mu\phi\delta\phi\right)_{,\,\nu} - 2\left(\partial_\mu\phi\right)_{,\,\nu}\delta\phi \tag{A.13}$$

$$\delta\,\phi^2 \, = 2\phi\delta\phi, \tag{A.14}$$

$$\delta\left(g^{1/2}L_M\right) \equiv -\frac{1}{2}g^{1/2}T_{\mu\nu}\delta g^{\mu\nu}. \tag{A.15}$$

Hence, variation of the action with respect to ϕ yields

$$\frac{\delta S}{\delta \phi} = \left[(\frac{1}{4\omega} R + m^2)\phi + \Box\ \phi \right] \sqrt{g}, \tag{A.16}$$

where $\Box\phi$ is defined as

$$\Box\phi = g^{-1/2}(g^{1/2} g^{\mu\nu} \partial_\mu \phi)_{,\ \nu}. \tag{A.17}$$

To get stationary action, Equation (A.16) is set to be zero

$$(\frac{1}{4\omega} R + m^2)\phi + \Box\phi = 0, \tag{A.18}$$

so that Equation (2.12) is deduced from Equation (A.18). Working with Cartan formalism under the Robertson Walker metric (1.3) yields Ricci scalar R as

$$R = -6(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}), \tag{A.19}$$

and each components of Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}$ as

$$G_{00} = 3(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}), \tag{A.20}$$

$$G_{ii} = 2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}, \quad (i = 1, 2, 3) \tag{A.21}$$

$$G_{ij} = 0\ (i \neq j). \tag{A.22}$$

Variation with respect to $g^{\mu\nu}$, on the other hand, yields

$$\frac{\delta S}{\delta g^{\mu\nu}} = \left\{ \begin{array}{c} \frac{-1}{8\omega} \left[G_{\mu\nu}\phi^2 + g^{-1/2}\nabla^\lambda \nabla_\lambda (g^{1/2} g_{\mu\nu}\phi^2) - g^{-1/2}\nabla_\mu \nabla_\nu (g^{1/2}\phi^2) \right] \\ + \frac{1}{2}\, \partial_\mu \phi\, \partial_\nu \phi - \frac{1}{4} g_{\mu\nu} g^{\alpha\beta}\, \partial_\alpha \phi\, \partial_\beta \phi + \frac{1}{4} m^2 g_{\mu\nu}\phi^2 - \frac{1}{2} T_{\mu\nu} \end{array} \right\} \sqrt{g}. \tag{A.23}$$

Here we assume a perfect fluid with the four-velocity $u^\mu = (1, 0, 0, 0)$, the stress-energy tensor is defined as

$$T^{\mu\nu} = (p+\rho)u^\mu u^\nu - p g^{\mu\nu}. \tag{A.24}$$

where ρ is the energy density, p is the pressure. Using Equation $T^\mu_\nu = g_{\nu\alpha}T^{\mu\alpha}$, one easily can show that

$$T^\mu_\nu = diag\,(\rho, -p, -p, -p)\,, \tag{A.25}$$

where the components of metric tensor are well defined from RW metric as

$$g_{00} = 1, \tag{A.26}$$

$$g_{ii} = -\frac{a^2(t)}{\left[1+\frac{k}{4}\vec{x}^2\right]^2}, \tag{A.27}$$

$$g_{ij} = 0 \ (i \neq j). \tag{A.28}$$

Hence, when Equation (A.23) is set to be zero for the sake of stationary action,

$$\frac{\delta S}{\delta g^{\mu\nu}} = \left\{ \begin{array}{c} \frac{-1}{8\omega}\left[G_{\mu\nu}\phi^2 + g^{-1/2}\nabla^\lambda\nabla_\lambda(g^{1/2}g_{\mu\nu}\phi^2) - g^{-1/2}\nabla_\mu\nabla_\nu(g^{1/2}\phi^2)\right] \\ +\frac{1}{2}\,\partial_\mu\phi\,\partial_\nu\phi - \frac{1}{4}g_{\mu\nu}g^{\alpha\beta}\,\partial_\alpha\phi\,\partial_\beta\phi + \frac{1}{4}m^2 g_{\mu\nu}\phi^2 - \frac{1}{2}T_{\mu\nu} \end{array} \right\} = 0, \tag{A.29}$$

other field Equations (2.10, 2.11) are obtained successfully.

The Einstein Hilbert action in the theory of gravitation, on the other hand, is given

$$S = \int d^4x\,\sqrt{g}\left[\frac{1}{16\pi G_N}\,R + L_M\right]. \tag{A.30}$$

Variation of action yields,

$$\delta S = \int d^4x \; \left[\frac{1}{16\pi G_N} \delta \; \left(g^{1/2} \, R \right) + \delta \; \left(g^{1/2} L_M \right) \right]. \tag{A.31}$$

Using Equations (A.8-A.15), it follows from Equation (A.31) that

$$\frac{\delta S}{\delta g^{\mu\nu}} = \left\{ \frac{1}{16\pi G_N} \left[(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}) + g^{-1/2} (\nabla^\lambda \nabla_\lambda g^{1/2} g_{\mu\nu} - \nabla_\mu \nabla_\nu g^{1/2}) \right] - \frac{1}{2} T_{\mu\nu} \right\} \sqrt{g}, \tag{A.32}$$

where the second term $g^{-1/2}(\nabla^\lambda \nabla_\lambda g^{1/2} g_{\mu\nu} - \nabla_\mu \nabla_\nu g^{1/2})$ in the brackets directly vanishes since $\nabla_\sigma g^{1/2} = 0$ and $\nabla_\sigma g_{\mu\nu} = 0$. Hence, to get the stationary action, Equation (A.32) is set to be zero

$$\frac{\delta S}{\delta g^{\mu\nu}} = \frac{1}{16\pi G_N} (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}) - \frac{1}{2} T_{\mu\nu} = 0, \tag{A.33}$$

so that Equation (A.33) reduces to the famous Einstein field equations

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}, \tag{A.34}$$

where G_N is the Newtonian gravitational constant.

APPENDIX B: Einstein's Field Equation

We will use basically Cartan's formalism in this calculation. The metric is the FLRW metric

$$ds^2 = dt^2 - a^2(t)\frac{d\vec{x}^2}{\left[1 + \frac{k}{4}\vec{x}^2\right]^2}. \tag{B.1}$$

and the basis one-forms deduced from the FLRW metric are as follows:

$$e^4 = dt, \tag{B.2}$$
$$e^i = i^* a(t)\frac{dx^i}{\left[1 + \frac{k}{4}\vec{x}^2\right]}, \quad (i^*)^2 = -1$$

where $\{i, j, k = 1, 2, 3\}$ are spatial indices, k is the curvature constant and i_* is chosen to be an imaginary number defined as $(i^*)^2 = -1$ lest to be confused with spatial index i. The metric tensor $g_{\mu\nu} \equiv \delta_{\mu\nu}$ since the basis are chosen to be orthonormal. Applying Cartan's first structure equation

$$de^\mu + \omega^\mu_{\ \nu} \wedge \omega^\nu = 0, \quad \{\mu,\ \nu,\ \sigma = 1, 2, 3, 4\} \tag{B.3}$$

provides

$$de^i = i^*\left[\frac{\dot{a}\left(dt \wedge dx^i\right)}{\left(1 + \frac{k}{4}x_j x^j\right)} - \frac{akx_j dx^j \wedge dx^i}{2\left(1 + \frac{k}{4}x_j x^j\right)^2}\right], \tag{B.4}$$

and the connection one forms $\omega^\mu_{\ \nu}$ as follows:

$$\omega^1_{\ 2} = \frac{i^* ky}{2a}e^1 - \frac{i^* kx}{2a}e^2, \tag{B.5}$$

$$\omega^1_{\ 3} = \frac{i^* kz}{2a}e^1 - \frac{i^* kx}{2a}e^3, \tag{B.6}$$

$$\omega^1{}_4 = \frac{\dot{a}}{a} e^1, \tag{B.7}$$

$$\omega^2{}_3 = \frac{i^* k y}{2a} e^1 - \frac{i^* k x}{2a} e^2, \tag{B.8}$$

$$\omega^2{}_4 = \frac{\dot{a}}{a} e^2, \tag{B.9}$$

$$\omega^3{}_4 = \frac{\dot{a}}{a} e^3. \tag{B.10}$$

Now we can apply Cartan's second structure equation

$$\Omega^i{}_j = d\omega^i{}_j + \omega^i{}_k \wedge \omega^k{}_j = 0, \tag{B.11}$$

where $\Omega^i{}_j$ is the curvature two-form. Applying (B.11) rigorously yields the non-zero curvature two-form terms as the following;

$$\Omega^1{}_2 = -\left(\frac{k}{a^2} + \frac{\dot{a}^2}{a^2}\right) \ e^1 \wedge e^2, \tag{B.12}$$

$$\Omega^1{}_3 = -\left(\frac{k}{a^2} + \frac{\dot{a}^2}{a^2}\right) \ e^1 \wedge e^3, \tag{B.13}$$

$$\Omega^1{}_4 = \left(-\frac{\ddot{a}}{a}\right) \ e^1 \wedge e^4, \tag{B.14}$$

$$\Omega^2{}_3 = -\left(\frac{k}{a^2} + \frac{\dot{a}^2}{a^2}\right) \ e^2 \wedge e^3, \tag{B.15}$$

$$\Omega^2{}_4 = -\left(\frac{\ddot{a}}{a}\right)\; e^2 \wedge e^4, \tag{B.16}$$

$$\Omega^3{}_4 = -\left(\frac{\ddot{a}}{a}\right)\; e^3 \wedge e^4, \tag{B.17}$$

and other curvature terms are zero. If we use the relation in between the curvature two form and the Riemann curvature tensor, namely,

$$\Omega^\mu{}_\nu = R^\mu{}_{\nu\sigma\rho}\; e^\sigma \wedge e^\rho, \tag{B.18}$$

we can find relevant Riemann tensor elements as

$$R^1{}_{212} = -\left(\frac{k}{a^2} + \frac{\dot{a}^2}{a^2}\right), \tag{B.19}$$

$$R^1{}_{313} = -\left(\frac{k}{a^2} + \frac{\dot{a}^2}{a^2}\right), \tag{B.20}$$

$$R^1{}_{414} = -\left(\frac{\ddot{a}}{a}\right), \tag{B.21}$$

$$R^2{}_{323} = -\left(\frac{k}{a^2} + \frac{\dot{a}^2}{a^2}\right), \tag{B.22}$$

$$R^2{}_{424} = -\left(\frac{\ddot{a}}{a}\right), \tag{B.23}$$

$$R^3{}_{434} = -\left(\frac{\ddot{a}}{a}\right), \tag{B.24}$$

and others are zero. Using the definition of Ricci tensor $R^{i}{}_{j} = R^{k}{}_{ikj}$ and the symmetry properties of Riemann tensor

$$R^{\mu}{}_{\nu\sigma\rho} = -R^{\mu}{}_{\nu\rho\sigma} = -R^{\nu}{}_{\mu\sigma\rho}, \tag{B.25}$$

we can determine also the relevant elements of Ricci tensor as

$$R^{1}{}_{1} = R^{1}{}_{111} + R^{2}{}_{112} + R^{3}{}_{131} + R^{4}{}_{141} = -2\frac{k}{a^2} - 2\frac{\dot{a}^2}{a^2} - \frac{\ddot{a}}{a}, \tag{B.26}$$

$$R^{2}{}_{2} = R^{1}{}_{212} + R^{2}{}_{222} + R^{3}{}_{232} + R^{4}{}_{242} = -2\frac{k}{a^2} - 2\frac{\dot{a}^2}{a^2} - \frac{\ddot{a}}{a}, \tag{B.27}$$

$$R^{3}{}_{3} = R^{1}{}_{313} + R^{2}{}_{323} + R^{3}{}_{333} + R^{4}{}_{343} = -2\frac{k}{a^2} - 2\frac{\dot{a}^2}{a^2} - \frac{\ddot{a}}{a}, \tag{B.28}$$

$$R^{4}{}_{4} = R^{1}{}_{414} + R^{2}{}_{424} + R^{3}{}_{434} + R^{4}{}_{444} = -3\frac{\ddot{a}}{a}, \tag{B.29}$$

where $R^{1}{}_{111} = R^{2}{}_{222} = R^{3}{}_{333} = R^{4}{}_{444} = 0$. On the other hand, Ricci scalar, R is defined as $R = R^{\mu}{}_{\mu}$ and this yields

$$R = R^{1}{}_{1} + R^{2}{}_{2} + R^{3}{}_{3} + R^{4}{}_{4} = -6\left(\frac{k}{a^2} + \frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right). \tag{B.30}$$

Hence since Einstein tensor is defined as $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}R$, we can identify G_{44};

$$G_{44} = R_{44} - \frac{1}{2}R = -3\frac{\ddot{a}}{a} + 3\frac{k}{a^2} + 3\frac{\dot{a}^2}{a^2} + 3\frac{\ddot{a}}{a}, \tag{B.31}$$

$$G_{44} = 3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right), \tag{B.32}$$

and G_{ii} as

$$G_{ii} = R_{ii} - \frac{1}{2}R, \tag{B.33}$$

in such a way that

$$G_{11} = G_{22} = G_{33} = -2\frac{k}{a^2} - 2\frac{\dot{a}^2}{a^2} - \frac{\ddot{a}}{a} + 3\frac{k}{a^2} + 3\frac{\dot{a}^2}{a^2} + 3\frac{\ddot{a}}{a}, \tag{B.34}$$

$$G_{11} = G_{22} = G_{33} = 2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}. \tag{B.35}$$

APPENDIX C: Energy conservation in standard cosmology

Energy conservation in general relativity requires that the covariant derivative of energy-momentum tensor is vanished. Starting from this requirement defined in general relativity, it follows

$$\nabla_\mu T^\mu_\nu = 0. \tag{C.1}$$

Using the definition of covariant derivative ∇ on (1, 1) type tensor yields

$$\nabla_\mu T^\mu_\nu = \partial_\mu T^\mu_\nu + \Gamma^\mu{}_{a\mu} T^a_\nu - \Gamma^a{}_{\nu\mu} T^\mu_a = 0. \tag{C.2}$$

For the $\nu = 0$ component, and remembering that T^μ_ν is diagonal (A.25), the relevant affine parameters are

$$\Gamma^0{}_{00} = 0 \qquad ; \qquad \Gamma^1{}_{01} = \Gamma^2{}_{02} = \Gamma^3{}_{03} = \frac{\dot{a}}{a}. \tag{C.3}$$

Substituting them in Equation (C.2), and keeping careful track of the summation over repeated indices, gives

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0. \tag{C.4}$$

APPENDIX D: Energy conservation in BD cosmology

We have checked the field equations (2.10, 2.11, 2.12) as far as energy conservation is satisfied. The easiest way of doing this is that taking the Equations (2.10, 2.11), and putting them directly into the energy conservation equation $\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0$, and checking whether the conservation equation is hold or not. For this aim, taking the time derivative of (2.10) gives,

$$\begin{aligned}\dot{\rho}_M =& \frac{3}{2\omega}\left(\frac{\ddot{a}}{a}\right)\left(\frac{\dot{a}}{a}\right)\phi^2 - \frac{3}{2\omega}\left(\frac{\dot{a}}{a}\right)^3\phi^2 + \frac{3}{2\omega}\left(\frac{\dot{a}}{a}\right)^2\dot{\phi}\phi - \frac{3k}{2\omega}\left(\frac{\dot{a}}{a^3}\right)\phi^2 + \frac{3k}{2\omega}\left(\frac{1}{a^2}\right)\dot{\phi}\phi \\ &-\ddot{\phi}\dot{\phi} - m^2\dot{\phi}\phi + \frac{3}{2\omega}\left(\frac{\ddot{a}}{a}\right)\dot{\phi}\phi - \frac{3}{2\omega}\left(\frac{\dot{a}}{a}\right)^2\dot{\phi}\phi + \frac{3}{2\omega}\left(\frac{\dot{a}}{a}\right)\ddot{\phi}\phi + \frac{3}{2\omega}\left(\frac{\dot{a}}{a}\right)\dot{\phi}^2. \qquad \text{(D.1)}\end{aligned}$$

The second term, on the right hand side of the energy conservation equation can be written as

$$\begin{aligned}&3(\rho + p)\frac{\dot{a}}{a} \\ &= \frac{9}{4\omega}\left(\frac{\dot{a}}{a}\right)^3\phi^2 + \frac{9k}{4\omega}\left(\frac{\dot{a}}{a^3}\right)\phi^2 - \frac{3}{2}\left(\frac{\dot{a}}{a}\right)\dot{\phi}^2 - \frac{3}{2}m^2\frac{\dot{a}}{a}\phi^2 + \frac{9}{2\omega}\left(\frac{\dot{a}}{a}\right)^2\dot{\phi}\phi \\ &-\frac{3}{2\omega}\left(\frac{\ddot{a}}{a}\right)\left(\frac{\dot{a}}{a}\right)\phi^2 - \frac{3}{4\omega}\left(\frac{\dot{a}}{a}\right)^3\phi^2 - \frac{3k}{4\omega}\left(\frac{\dot{a}}{a^3}\right)\phi^2 - \frac{3}{\omega}\left(\frac{\dot{a}}{a}\right)^2\dot{\phi}\phi - \frac{3}{2\omega}\left(\frac{\dot{a}}{a}\right)\ddot{\phi}\phi \\ &-\frac{3}{2}\left(\frac{\dot{a}}{a}\right)\dot{\phi}^2 - \frac{3}{2\omega}\left(\frac{\dot{a}}{a}\right)\dot{\phi}^2 + \frac{3}{2}m^2\frac{\dot{a}}{a}\phi^2. \qquad \text{(D.2)}\end{aligned}$$

Putting together (D.1) and (D.2), we end up with

$$\begin{aligned}&\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} \\ &= -\ddot{\phi}\dot{\phi} + \frac{3k}{2\omega}\left(\frac{1}{a^2}\right)\dot{\phi}\phi - m^2\dot{\phi}\phi + \frac{3}{2\omega}\left(\frac{\ddot{a}}{a}\right)\dot{\phi}\phi - 3\left(\frac{\dot{a}}{a}\right)\dot{\phi}^2 + \frac{3}{2\omega}\left(\frac{\dot{a}}{a}\right)^2\dot{\phi}\phi \qquad \text{(D.3)}\end{aligned}$$

in which there appears a second order time derivative of ϕ as a first term in the right hand side of the above equation. Now, at that point third field equation 2.12 comes into play and yields

$$-\ddot{\phi}\dot{\phi} = 3\left(\frac{\dot{a}}{a}\right)\dot{\phi}^2 + m^2\dot{\phi}\phi - \frac{3}{2\omega}\left(\frac{\ddot{a}}{a}\right)\dot{\phi}\phi - \frac{3}{2\omega}\left(\frac{\dot{a}}{a}\right)^2\dot{\phi}\phi - \frac{3k}{2\omega}\left(\frac{1}{a^2}\right)\dot{\phi}\phi. \qquad \text{(D.4)}$$

Combining (D.3) and (D.4) will give us that our field equations (2.10, 2.11, 2.12) satisfies energy conservation equation $\dot{\rho}_M + 3(\dot{a}/a)(\rho_M + p_M) = 0$ successfully.

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