DETERMINATION OF OPTICAL CONSTANTS OF THIN FILMS

by

Ali YILDIZ

B.S. in Physics, Boğaziçi University, 1996

Submitted to the Institute for Graduate Studies in
Science and Engineering in partial fulfillment of
the requirements for the degree of
Master of Science
in
Physics

Boğaziçi University

1998

ACKNOWLEDGMENTS

I am deeply indebted to my supervisor, Prof. Naci İNCİ for his constant support and his invaluable advice during my time as his research student. He has always been generous in sharing ideas and has been instrumental in instilling in me the tenacity and intellectual confidence which have sustained me throughout this research project.

Special thanks are also due to Prof. Dr. Haluk BEKER, who settled many a doubt in formula derivations in the theoretical sections.

My sincere gratitude is also due to Prof. Dr. Gülen AKTAŞ and Assoc. Dr. Alphan SENNAROĞLU, members of my advisory committee, for their constructive criticism and useful suggestions.

I would like to thank Mr. Gültekin GÜLŞEN, who was always ready to help whenever I needed it and provided lasting support, especially, in the experimental stage of this study. I would never have accomplished this thesis without his help. I would also like to express my gratitude also to Aslı UMUR for her ideas in the derivations of formula.

Special thanks are also due to my dear friends, particularly, to A. Serdar ARIKAN, Erdinç ATILGAN, Ertan YILDIZ and Özgür SADET, for their morale support and invaluable help during difficult times. I would also like to express my thanks also to Ekber KULİEV for the computer drawings.

Finally, I would like to thank to my family for all their love and unfailing support in all my endeavours.

ABSTRACT

In this thesis, a new approach is explored to determine the optical constants of thin dielectric layers. This study is based on the measurement of the reflection at various angles to determine the optical constants of a thin film. For this reason, this method allows using absorbing or non-transparent substrates. Study consists of not only theoretical derivations but also experimental measurements. A 650 nm laser diode was used as a light source and the reflected light from the surface of the film was measured by the BPX65 silicon photodiode. These reflection values were taken at various angles and numerical calculations were carried out. Results were consistent with previous studies. In addition, this method provided more information about the thin film, such as extinction coefficient, which was neglected in previous studies.

KISA ÖZET

Bu çalışmada ince filmlerin optik sabitlerinin belirlenmesinde çok kullanışlı olabilecek yeni bir yaklaşım teorik olarak ortaya konmuştur. Burada sadece teorik çıkarımlarla sınırlı kalınmamış aynı zamanda formüllerin doğruluğu deneysel sonuçlarla da desteklenmiştir. Deneyde, ışık kaynağı olarak 650 nm dalgaboyuna sahip bir lazer diyot kullanılmıştır. Işın ince filmin yüzeyine farklı açılarda gönderilmiş ve yansıyan ışın bu dalgaboyuna hassas bir fotodetektör ile ölçülmüştür. Bu ölçümler kullanılarak nümerik hesaplamalar yapılmış ve elde edilen sonuçların daha önceki çalışmalarla uyum içinde olduğu gözlenmiştir. Hatta, bu yeni metotta söndürme katsayısı ihmal edilmediği için, ince film hakkında önceki çalışmalarla elde edilen bilgilerden daha fazlasını elde etmek mümkün olmaktadır.

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d	Thickness of Film
g	Real Part of Complex Fresnel Reflection Coefficient
h	Imaginary Part of Complex Fresnel Reflection Coefficient
k_1	Extinction Coefficient of Film
k_2	Extinction Coefficient of Substrate
n_0	Refractive Index of Air
n_1	Refractive Index of Film
n_2	Refractive Index of Substrate
r	Fresnel Reflection Coefficient
$\boldsymbol{R}$	Reflection
R_p	Reflection of p Polarized Light
R_s	Reflection of s Polarized Light
t	Fresnel Transmission Coefficient
T	Transmission
δ	Phase Change
λ	Wavelength
σ	Conductivity
φ_0	Incidence Angle of Light

1. INTRODUCTION

The earliest recorded scientific studies on the optical behaviour of thin films were made in the seventeenth century. They appear, together with the results of a wide range of other optical experiments, in Sir Isaac Newton's early treatise on Opticks, written in a style which has a charm and fascination of its own. Although the mechanism of the propagation of light was in Newton's time shrouded in uncertainty, the observations made on the colors of thin films in relation to their thickness have proved useful for a long time.

With the development in recent years of methods of preparing thin films of materials, interest in the optical properties of films has been considerably stimulated. Although there remain many unsolved problems, the general features of the optical behaviour of thin films are now reasonably well understood. Developments of the techniques of producing and studying thin films have led to an understanding of their optical behaviour. There is growing interest in thin films for a wide range of applicaitons from antireflection coating to detector manufacture.

Many techniques have been used to determine optical properties of a thin dielectric layer so far. These techniques are based on the transmission spectrum [1]. Therefore, substrate must be transparent and have very little extinction cofficient. If substrate has large extinction coefficient to neglect or is non-transparent, optical constants cannot be calculated. Because only one reflection (film side) value can be obtained. But there are five unknowns d_1, n_1, k_1, n_2, and k_2, thickness, refractive index, extinction coefficient of the film, refractive index of substrate and extinction coefficient of substrate respectivelty. d_1 can be found from maxima and minima points of spectrum. (Even having known optical constants substrate is used, two unknown remain). This problem can be solved only by omitting k_1. But this is not desired always.

In this work, a new theoretical approach was proposed and studied. This study is based on determination of reflection of light launched with angle to an absorbing dielectric layer. Therefore, there is no need of transparent substrate and entire spectrum envelope. In addition, extinction coefficient can be calculated directly. Formula theoretically obtained were confirmed by experimental measurements. In experiment, light at oblique incidence used and only reflections were measured for different angles. Using these reflections values, optical constants were obtained by numerical calculation.

Absorpsion of the films minimizes the transmitted light. This limits the λ range where transmission measurements can be done. Our method can be used at all wavelengths.

The aim of the this work is to determine the optical constants of a thin dielectric layer from reflection at inclined incidence of radiation.

This study consists of five chapters. This first chapter provides an introduction to why our method is proposed and summarizes some advantages over previous works.

The second chapter gives the basic building blocks of the structure and detailed approach to theory of thin films. In the first part of this chapter, a detailed review of the electromagnetic radiation is given. In the second part, reflection coefficients from multilayer are determined at normal incidence for p and s polarized light[2].

In Chapter three, inclined incidence reflection is determined for a single absorbing thin dielectric layer coated on an absorbing substrate. All formulae are derived step by step in order to facilitate easy understanding of the theory.

In Chapter four, a detailed description of apparatus and experimental set-up are presented. In the second part of the same chapter experimental results are given and our results are compared to previous studies.

The last chapter focuses on the brief summary of this work, followed by recomendations for possible future directions.

2. REVIEW

2.1 Reflection and Transmission of Light at the Surface of a Transparent Medium

For an isotropic medium, the laws of electromagnetism are summarized by Maxwell's equations :

$$\nabla \boldsymbol{D} = \varepsilon \nabla \boldsymbol{E} = 4\pi\rho \tag{2.1}$$

$$\nabla \boldsymbol{B} = \mu \nabla \boldsymbol{H} = 0 \tag{2.2}$$

$$\nabla \times \boldsymbol{E} = -\frac{\mu}{c}\frac{\partial \boldsymbol{H}}{\partial t} \tag{2.3}$$

$$\nabla \times \boldsymbol{H} = \frac{4\pi\sigma \boldsymbol{E}}{c} + \frac{\varepsilon}{c}\frac{\partial \boldsymbol{E}}{\partial t} \tag{2.4}$$

where the symbols have their usual meanings. Electrical quantities are measured in electrostatic units and magnetic quantities in electromagnetic units. For a medium in which there is no space charge, these relations lead directly to the wave equations, representing the propagation of electromagnetic fields in the medium :

$$\frac{\varepsilon\mu}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} + \frac{4\pi\mu\sigma}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \nabla^2 \boldsymbol{E} \tag{2.5}$$

$$\frac{\varepsilon\mu}{c^2}\frac{\partial^2 \boldsymbol{H}}{\partial t^2} + \frac{4\pi\mu\sigma}{c^2}\frac{\partial \boldsymbol{H}}{\partial t} = \nabla^2 \boldsymbol{H} \tag{2.6}$$

For propagation in a non-conducting ($\sigma = 0$) medium, these reduce to

$$\frac{\varepsilon\mu}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \nabla^2 \boldsymbol{E} \tag{2.7}$$

$$\frac{\varepsilon\mu}{c^2}\frac{\partial^2 \boldsymbol{H}}{\partial t^2} = \nabla^2 \boldsymbol{H} \tag{2.8}$$

the well-known simple form of wave equation, which shows that disturbances are propagated with velocity $c/\sqrt{(\mu\varepsilon)}$. Since, the difference from unity is significant at optical frequencies, the velocity of propagation is $c/\sqrt{\varepsilon}$ where ε is the dielectric constant at the frequency of the light wave. From the definition of refractive index, we have the well-known result $n = \sqrt{\varepsilon}$.

The problem of determining the light reflected and transmitted at a boundary separating two media is dealt with by applying boundary conditions to the solutions of Maxwell's equations. These require that the tangential components of both electric and magnetic vectors be continuous at the boundary. Only sinusoidal solutions of the equations are considered. Solutions for other types of waves may in principle be dealt with by the aid of Fourier's theorem. Any variation of the propagation parameters with wavelength must be taken into account in effecting the summation.

A plane wave is considered as incident on the surface $z = 0$, the plane of incidence being the plane $x0z$, the angle of incidence φ_0 and the angle of refraction φ_1. It is assumed that the surface is infinite in extent so that it may accomodate the monochromatic plane wave of infinite extend.

The coordinate system is shown in figure 2.1. We denote the amplitudes of the electric vectors of the wave approaching the surface by E^+_{0p} and E^+_{0s} for the two components. Indices p and s are representing parallel and perpendicular components to the plane of incidence respectively. The reflected wave is denoted by E^-_{0p}, E^-_{0s} and the transmitted wave by E^+_{1p}, E^+_{1s}. The phase factors associated with the incident and reflected waves are of the form

$$\exp i(\omega t - \frac{2\pi n_0 x \sin\varphi_0}{\lambda} - \frac{2\pi n_0 z \cos\varphi_0}{\lambda}) \qquad \text{(incident)}$$

and $$\exp i(\omega t - \frac{2\pi n_0 x \sin\varphi_0}{\lambda} + \frac{2\pi n_0 z \cos\varphi_0}{\lambda}) \qquad \text{(reflected)}$$

while that for the transmitted wave is

$$\exp i(\omega t - \frac{2\pi n_1 x\sin\varphi_1}{\lambda} - \frac{2\pi n_1 z\cos\varphi_1}{\lambda})$$

where λ is the wavelength in vacuum.

At the boundary, which we take at $z = 0$, the point of incidence being the origin of coordinates, we have for the total components of the electric and magnetic vectors in the x- and y- directions

$$\left.\begin{aligned} E_{0x} &= (E_{0p}^{+} + E_{0p}^{-})\cos\varphi_0 \\ E_{0y} &= E_{0s}^{+} + E_{0s}^{-} \\ H_{0x} &= n_0(-E_{0s}^{+} + E_{0s}^{-})\cos\varphi_0 \\ H_{0y} &= n_0(E_{0p}^{+} - E_{0p}^{-}) \end{aligned}\right\} \tag{2.9}$$

for the first medium, and

$$\left.\begin{aligned} E_{1x} &= E_{1p}^{+}\cos\varphi_1 \\ E_{1y} &= E_{1s}^{+} \\ H_{1x} &= -n_1 E_{1s}^{+}\cos\varphi_1 \\ H_{1y} &= n_1 E_{1p}^{+} \end{aligned}\right\} \tag{2.10}$$

Applying the boundary conditions, we obtain equations which may be solved to give the amplitudes of the transmitted and reflected vectors in terms of those of the incident vectors. We obtain

$$\frac{E_{0p}^{-}}{E_{0p}^{+}} = \frac{n_0\cos\varphi_1 - n_1\cos\varphi_0}{n_0\cos\varphi_1 + n_1\cos\varphi_0} = r_{1p} \tag{2.11}$$

$$\frac{E_{1p}^{+}}{E_{0p}^{+}} = \frac{2n_0\cos\varphi_0}{n_0\cos\varphi_1 + n_1\cos\varphi_0} = t_{1p} \tag{2.12}$$

$$\frac{E_{0s}^{-}}{E_{0s}^{+}} = \frac{n_0\cos\varphi_0 - n_1\cos\varphi_1}{n_0\cos\varphi_0 + n_1\cos\varphi_1} = r_{1s} \tag{2.13}$$

$$\frac{E_{1s}^{+}}{E_{0s}^{+}} = \frac{2n_0\cos\varphi_0}{n_0\cos\varphi_0 + n_1\cos\varphi_1} = t_{1s} \qquad (2.14)$$

r_{1p}, r_{1s} are known as the Fresnel reflection coefficients and t_{1p}, t_{1s} the Fresnel transmission coefficients [2].

From eqs.(2.11) - (2.14) we see that $t_{1p} = 1 + r_{1p}$ and $t_{1s} = 1 + r_{1s}$ so that, for the case $n_0 > n_1$, the values of t_{1p} and t_{1s} exceed unity. This may at first sight appear strange since coefficients have been defined as the ratios of the amplitudes of the transmitted waves to those of the incident waves. Our fears lest the law of the conservation of energy has failed are allayed when we consider, with the aid of Poynting's theorem, the energy in each medium. The energy is represented by the Poynting vector **S**.

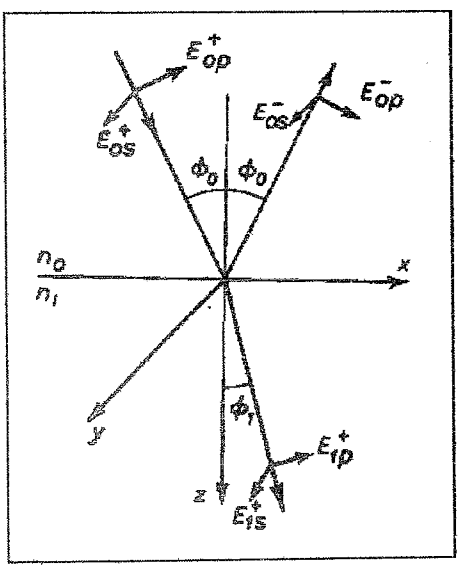

FIGURE 2.1. Schematic diagram of incident, reflected, and transmitted radiation

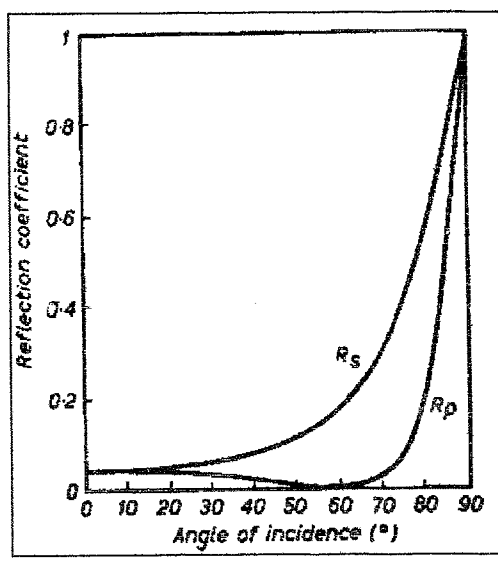

FIGURE 2.2. Variation of $\mathbf{R}_p$, $\mathbf{R}_s$ with angle of incidence

$$S = \frac{c}{4\pi}[E \times H] \qquad (2.15)$$

$$= \frac{c}{4\pi} n|E|^2 \qquad (2.16)$$

where we consider propagation in a non-absorbing medium of refractive index n.

The reflectances, defined as the ratios of reflected to incident energies, are simply given by

$$\boldsymbol{R}_p = \frac{\left(E_{0p}^{-}\right)^2}{\left(E_{0p}^{+}\right)^2} = r_{1p}^2 \qquad \text{and} \qquad \boldsymbol{R}_s = \frac{\left(E_{0s}^{-}\right)^2}{\left(E_{0s}^{+}\right)^2} = r_{1s}^2 \tag{2.17}$$

and the transmittances are given by

$$\boldsymbol{T}_p = \frac{n_1\left(E_{1p}^{+}\right)^2}{n_0\left(E_{0p}^{+}\right)} = \frac{n_1}{n_0}t_{1p}^2 \qquad \text{and} \qquad \boldsymbol{T}_s = \frac{n_1\left(E_{1s}^{+}\right)^2}{n_0\left(E_{0s}^{+}\right)} = \frac{n_1}{n_0}t_{1s}^2 \tag{2.18}$$

The variation of these coefficients with angle of incidence are shown in figure2.2 for the case $n_0 = 1$, $n_1 = 1.5$.

For normal incidence on an isotropic medium, the reflection and transmission coefficients, expressed in terms of refractive indices, become

$$\boldsymbol{R}_p = \boldsymbol{R}_s = \left(\frac{n_0 - n_1}{n_0 + n_1}\right)^2 \tag{2.19}$$

$$\boldsymbol{T}_p = \boldsymbol{T}_s = \frac{4n_0 n_1}{\left(n_0 + n\right)^2} \tag{2.20}$$

Suppose the angle of incidence, φ_1, is varied until the angle between the reflected and refracted beams is 90°. At this particular angle of incidence, experiments show that the reflected beam is completely polarized with its electric field vector parallel to the surface, while the refracted beam is partially polarized. The angle of incidence at which this occurs is called the *polarizing angle* (or *Brewster's angle*), φ_p.

2.2 Reflection at the Surface of an Absorbing Medium

The equations of propagation of light in a transparent medium may be taken over for the case of an absorbing medium by replacing the real refractive index by a complex term. The expressions for the fresnel coefficients, Eqs.(2.11)-(2.14), then also become complex and rather complicated. Replacing n_1 by $\boldsymbol{n}_1 = n_1 - ik_1$ we see that

$$\sin\varphi_1 = \frac{n_0 \sin\varphi_0}{n_1 - ik_1} \tag{2.21}$$

so that φ_1 is complex. For special case $\varphi_0 = \varphi_1 = 0$. For this case only, the Fresnel reflection coefficients, which are the same for both components of polarization, may be easily found.

$$r_{1p} = r_{1s} = \frac{n_0 - n_1 + ik_1}{n_0 + n_1 - ik_1} \tag{2.22}$$

which gives, for the reflectance of the surface

$$\boldsymbol{R}_p = \boldsymbol{R}_s = \frac{(n_0 - n_1)^2 + k_1^2}{(n_0 + n_1)^2 + k_1^2} \tag{2.23}$$

For other than normal incidence, exact expressions for the reflectance are cumbersome and approximations are used. For many absorbing materials, particularly metals in the visible region, $n^2 + k^2 >> 1$. To this approximation the reflectances reduce to

$$\boldsymbol{R}_p = \frac{(n^2 + k^2)\cos^2\varphi_0 - 2n\cos\varphi_0 + 1}{(n^2 + k^2)\cos^2\varphi_0 + 2n\cos\varphi_0 + 1} \tag{2.24}$$

$$\boldsymbol{R}_s = \frac{(n^2 + k^2) - 2n\cos\varphi_0 + \cos^2\varphi_0}{(n^2 + k^2) + 2n\cos\varphi_0 + \cos^2\varphi_0} \tag{2.25}$$

The Fresnel transmission coefficients have no direct significance for the wave entering the absorbing medium since the attenuation of the wave depends on the distance travelled in the medium [2].

2.3 Reflection and Transmission of Light by a Single Film

We may apply the results of Section 2.2 to the determination of the reflection and transmission coefficients for a single non-absorbing layer, bounded on either side by semi-infinite non-absorbing layers. We do this by considering a beam incident on the film which is divided into reflected and transmitted parts. Such division occurs each time the beam strikes an interface so that the transmitted and reflected beams are obtained by summing the multiply-reflected and muptiply-transmitted elements. For the case of the single layer and for this case only, the summation can be easily affected. The results are conveniently expressed in terms of Fresnel coefficients.

Suppose that a paralel beam of light of unit amplitude and of wavelength λ falls on a plane, parallel-sided, homogeneous, isotropic film of thickness d and refractive index n_1 supported on a substrate of index n_2 figure(2.3). The index of the first medium is n_0 and angle of incidence in this medium φ_0.

We may write down the amplitudes of the succesively reflected and transmitted beams in terms of the Fresnel coefficients, given by eqs.(2.11)-(2.14). From the definitions of these coefficients, it is clear that the values of r and t for a given boundary depend on the direction of propagation of light across the boundary. Thus for normal incidence on a boundary between media of refractive indices n_0 and n_1 the Fresnel coefficient for reflection for light incident from n_0 is given by $(n_0-n_1)/(n_0+n_1)$ while that for the reverse direction is $(n_1-n_0)/(n_1+n_0)$. The corresponding Fresnel coefficients for transmission are $2n_0/(n_0+n_1)$ for propagation from n_0 to n_1 and $2n_1/(n_0+n_1)$ for propagation from n_1 to n_0.

In treating the problem of the single layer, we shall denote the Fresnel coefficients for propagation from n_0 to n_1 by r_1 and t_1 as given by eqs.(2.11)-(2.14). The corresponding coefficients for propagation from n_1 to n_0 will be written r_1' and t_1'. The expressions given below will be valid for either direction of polarization provided that r and t are given the appropriate values from eqs.(2.11)-(2.14). The second suffix (p or s) will therefore be omitted. From the form of the expression for the fresnel reflection coefficient we see that r_1' is equal to r_1.

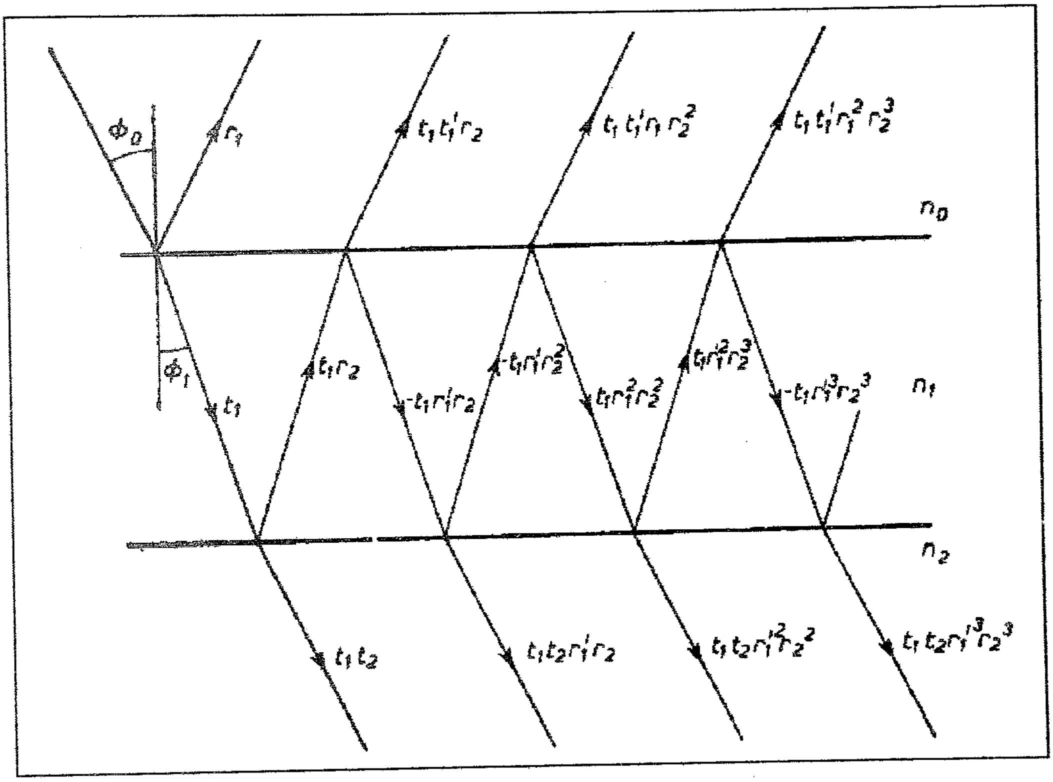

FIGURE 2.3. Reflection and transmission of light from a single film

The amplitudes of the successive beams reflected into medium n_0 are thus given by r_1, $t_1t_1'r_2$, $-t_1t_1'r_1r_2^2$, $t_1t_1'r_1^2r_2^3$, ... and the transmitted amplitudes by t_1t_2, $-t_1t_2r_1r_2$, $-t_1t_2r_1^2r_2^2$, ... Writing δ_1, for the change in phase of the beam on traversing the film, we have

$$\delta_1 = \frac{2\pi}{\lambda} n_1 d_1 \cos\varphi_1 \tag{2.26}$$

The reflected amplitude is thus given by

$$R = r_1 + t_1t_1'r_2e^{-2i\delta_1} - t_1t_1'r_1r_2^2e^{-4i\delta_1} + ...$$

$$= r_1 + \frac{t_1 t_1' r_2 e^{-2i\delta_1}}{1 + r_1 r_2 e^{-2i\delta_1}} \tag{2.27}$$

where the time-dependent factor is omitted. For non-absorbing media, this may be further simplified by writing the Fresnel transmission coefficients in terms of r_1, r_2. From conservation of energy we have

$$t_1 t_1' = 1 - r_1^2 \tag{2.28}$$

so that equation (2.27) becomes

$$R = \frac{r_1 + r_2 e^{-2i\delta_1}}{1 + r_1 r_2 e^{-2i\delta_1}} \tag{2.29}$$

The transmitted amplitude is given by

$$T = t_1 t_2 e^{-i\delta_1} - t_1 t_2 r_1 r_2 e^{-3i\delta_1} + t_1 t_2 r_1^2 r_2^2 e^{-5i\delta_1} - \ldots..$$

$$= \frac{t_1 t_2 e^{-i\delta_1}}{1 + r_1 r_2 e^{-2i\delta_1}} \tag{2.30}$$

If the film is absorbing, or if it is bouded by absorbing media, then the values of n_0, n_1, and n_2 are replaced by the corresponding complex quantities.

Since we have considered a wave of unit amplitude in the first medium, the reflectance and transmittance, defined as ratios of reflected and transmitted energy to the incident energy, are given by

$$R = \frac{r_1^2 + 2r_1 r_2 \cos 2\delta_1 + r_2^2}{1 + 2r_1 r_2 \cos 2\delta_1 + r_1^2 r_2^2} \tag{2.31}$$

$$T = \frac{n_2}{n_0} \frac{t_1^2 t_2^2}{\left(1 + 2r_1 r_2 \cos 2\delta_1 + r_1^2 r_2^2\right)} \tag{2.32}$$

The form of the expressions for the amplitudes of the reflected and transmitted beams for the case of normal incidence are reasonably compact even when expressed in terms of the refractive indices. From Eqs.(2.11)-(2.14) It can be seen that the Fresnel coefficients reduce to

$$r_1 = \frac{n_0 - n_1}{n_0 + n_1} \qquad t_1 = \frac{2n_0}{n_0 + n_1} \tag{2.33}$$

$$r_2 = \frac{n_1 - n_2}{n_1 + n_2} \qquad t_2 = \frac{2n_1}{n_1 + n_2} \tag{2.34}$$

2.4 The Extension to a System of Multiple Layers

Method by which the derivation of the reflection coefficient for a single layer may be extended to the case of any number of layers. There are two possible approaches. Since a single film bounded by two surfaces possesses an effective reflection coefficient and accompanying phase change, then such a film may be replaced by a single surface with these properties. Rouard starts with the film next to the supporting substrate and works step by step through the intervening layers to the top of the system. Vasicek starts with the top layer and moves downwards towards the substrate. The expression for the reflectivity of the system is slightly more tractable when Rouard's scheme as in figure(2.4) is used. Starting with two films with Fresnel coefficients r_1, r_2, r_3, we first compute the amplitude and phase of the light reflected by the lower film (d_2). Writing ρ_2 for the real amplitude and Δ_2 for the phase, we have

$$\rho_2 e^{i\Delta_2} = \frac{r_2 + r_3 e^{-2i\delta_1}}{1 + r_2 r_3 e^{-2i\delta_1}} \tag{2.35}$$

This is the effective Fresnel coefficient for the layer n_2 and is inserted into the corresponding expression for the Fresnel coefficient for the whole system which is now regarded as a film of thickness d_l lying on a surface whose Fresnel coefficient is $\rho_2 e^{i\Delta_2}$. We thus have

$$\rho_1 e^{i\Delta_1} = \frac{r_1 + \rho_2 e^{i\Delta_2} e^{-2i\delta_1}}{1 + r_1 \rho_2 e^{i\Delta_2} e^{-2i\delta_1}} \tag{2.36}$$

and on eliminating ρ_2, Δ_2 between eqs.(2.35) and (2.36), we obtain

$$\rho_1 e^{i\Delta_1} = \frac{r_1 + r_2 e^{-2i\delta_1} + r_3 e^{-2i(\delta_1+\delta_2)} + r_1 r_2 r_3 e^{-2i\delta_1}}{1 + r_1 r_2 e^{-2i\delta_1} + r_1 r_3 e^{-2i(\delta_1+\delta_2)} + r_2 r_3 e^{-2i\delta_2}} \tag{2.37}$$

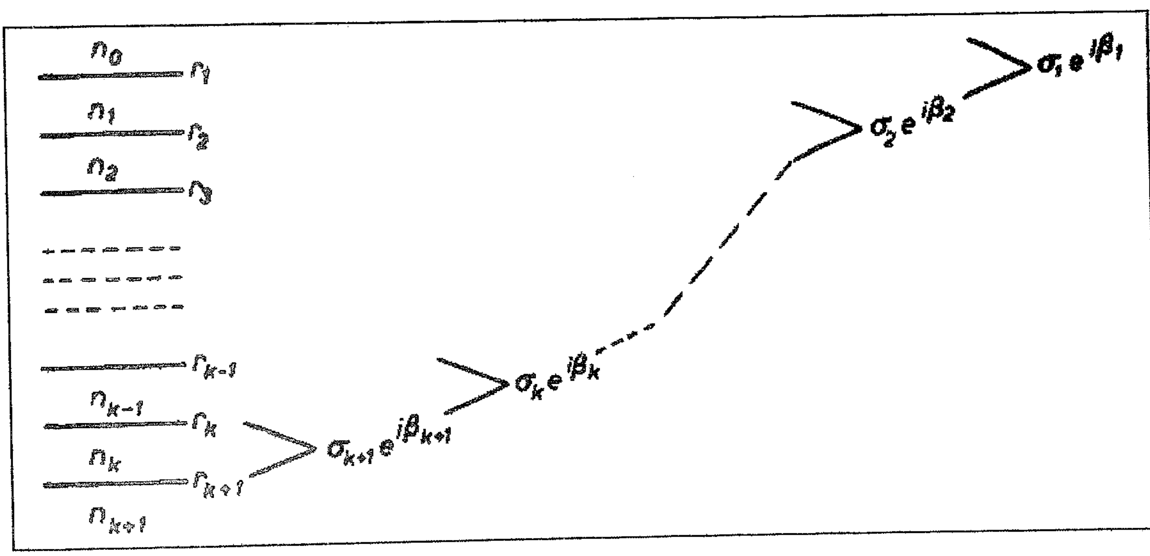

FIGURE 2.4. Demostration for dealing with multiple films

Consider now a sytem of k layers figure(2.4). The lowest layer (n_k) has an effective Fresnel coefficient given by

$$\rho_k e^{i\Delta_k} = \frac{r_k + r_{k+1} e^{-2i\delta_k}}{1 + r_k r_{k+1} e^{-2i\delta_k}} \tag{2.38}$$

On forming the product of this expression with its complex conjugate, we obtain

$$\rho_k^2 = \frac{r_k^2 + r_{k+1}^2 + 2r_k r_{k+1}\cos 2\delta_k}{1 + r_k^2 r_{k+1}^2 + 2r_k r_{k+1}\cos 2\delta_k} \tag{2.39}$$

Writing Δ_k in the form

$$\Delta_k = \eta_k - \xi_k \tag{2.40}$$

we see from eq.(2.38) that

$$\left.\begin{aligned} \tan\xi_k &= \frac{r_{k+1}\sin 2\delta_k}{r_k + r_{k+1}\cos 2\delta_k} \\ \tan\eta_k &= \frac{r_k r_{k+1}\sin 2\delta_k}{1 + r_k r_{k+1}\cos 2\delta_k} \end{aligned}\right\} \tag{2.41}$$

so that ξ_k, η_k and therefore Δ_k may be easily found from the Fresnel coefficients at each interface and from the optical thickness of the film.

The $(k-1)^{th}$ layer is then added giving an effective Fresnel coefficient

$$\rho_{k-1} e^{i\Delta_{k-1}} = \frac{r_{k-1} + \rho_k e^{i\Delta_k} e^{-2i\delta_{k-1}}}{1 + r_{k-1}\rho_k e^{i\Delta_k} e^{-2i\delta_{k-1}}} \tag{2.42}$$

ρ_{k-1} and Δ_{k-1} are calculated in the same way as for ρ_k and Δ_k and the process is repeated successively until the final coefficient ρ_1 is obtained, for the whole system. The reflection coefficient of the system is then simply given by ρ_1^2.

2.5 Analysis of Electric Vector

2.5.1 Electric Vector Parallel to Plane of Incidence

The tangential components are E_{kx} and H_{ky} where

$$\left.\begin{aligned} E_{kx} &= \left(E_{kp}^{+}e^{-i\mu_k z} + E_{kp}^{-}e^{+i\mu_k z}\right)\cos\varphi_k \\ H_{ky} &= \left(E_{kp}^{+}e^{-i\mu_k z} - E_{kp}^{-}e^{+i\mu_k z}\right)n_k \end{aligned}\right\} \tag{2.43}$$

We may choose the plane $z = 0$ on the left-hand side of the k^{th} layer and we then write down the values of E_{kx}, H_{ky} on this plane and on the plane $z = d_k$. On eliminating E_{kp}^{+} and E_{kp}^{-} from the resulting equations, we obtain the following relations between the tangential components. To simplify the notation, we write E_{k-1} for the tangential component of E on the plane separating $(k-1)$ from (k) (i.e. z = 0) and E_k for the plane $z = d_k$.

$$\left.\begin{aligned} E_{k-1} &= E_k\cos\mu_k d_k + \frac{i\cos\varphi_k}{n_k}H_k\sin\mu_k d_k \\ H_{k-1} &= \frac{in_k}{\cos\varphi_k}E_k\sin\mu_k d_k + H_k\cos\mu_k d_k \end{aligned}\right\} \tag{2.44}$$

2.5.2 Electric Vector Perpendicular to Plane of Incidence

For this case we have

$$\left.\begin{aligned} E_{ky} &= \left(E_{ks}^{+}e^{-i\mu_k z} + E_{ks}^{-}e^{+i\mu_k z}\right) \\ H_{kx} &= \left(-E_{ks}^{+}e^{-i\mu_k z} + E_{ks}^{-}e^{+i\mu_k z}\right)n_k\cos\varphi_k \end{aligned}\right\} \tag{2.45}$$

Following the procedure given above for the other plane of polarization, we obtain

$$\left.\begin{aligned} E_{k-1} &= E_k\cos\mu_k d_k - \frac{i}{\cos\varphi_k n_k} H_k \sin\mu_k d_k \\ H_{k-1} &= -in_k\cos\varphi_k E_k \sin\mu_k d_k + H_k\cos\mu_k d_k \end{aligned}\right\} \tag{2.46}$$

We may express eqs.(2.44) and (2.46) in a single form by introducing the characteristic admittance (η) of the medium, defined as the ratio of the tangential components of themagnetic to electric vector for the positive-going wave. For case (2.5.1) above, we have $\eta_{k\|} = n_k / \cos\varphi_k$ while for case (2.5.2) $\eta_{k|} = -n_k\cos\varphi_k$ (from equations (2.43) and (2.45), ignoring the E_{kp}^{-} and E_{ks}^{-} terms). Then, It may be written

$$\left.\begin{aligned} E_{k-1} &= E_k\cos\mu_k d_k + \frac{i}{\eta_k} H_k \sin\mu_k d_k \\ H_{k-1} &= i\eta_k E_k \sin\mu_k d_k + H_k\cos\mu_k d_k \end{aligned}\right\} \tag{2.47}$$

2.6 Matrix Method Using Fresnel Coefficients

We may notice that equations (2.47) representing the linear dependence of E_{k-1}, H_{k-1} on E_k, H_k, may be written in matrix form

$$\begin{pmatrix} E_{k-1} \\ H_{k-1} \end{pmatrix} = \begin{pmatrix} \cos\mu_k d_k & i\dfrac{\sin\mu_k d_k}{\eta_k} \\ i\eta_k \sin\mu_k d_k & \cos\mu_k d_k \end{pmatrix} \begin{pmatrix} E_k \\ H_k \end{pmatrix} \tag{2.48}$$

The components E_0, H_0 may thus be expressed in terms of E_k, H_k and hence the transmission coefficient of the system determined.

A form slightly more convenient for computation is obtained if the relation between the electric vectors in successive layers are expressed in terms of Fresnel coefficients,

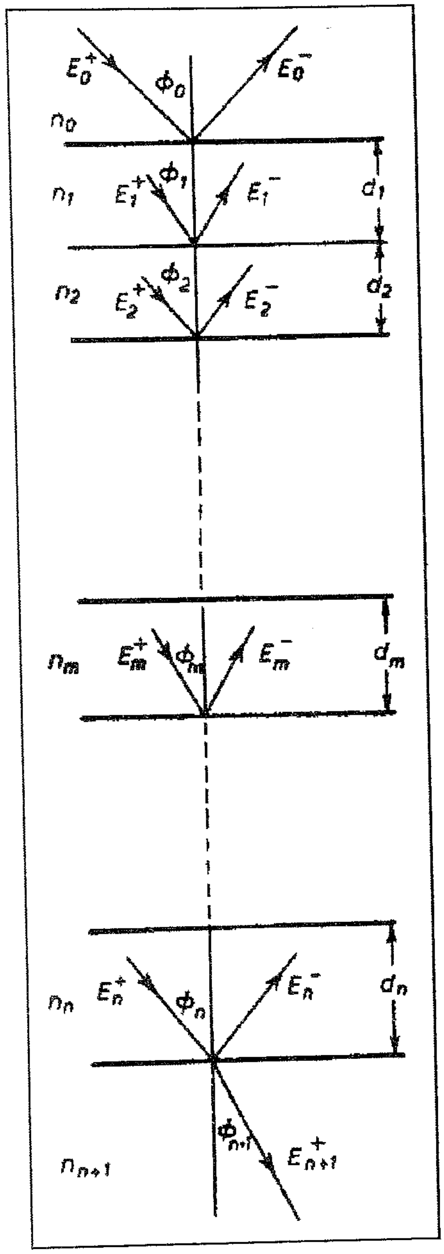

FIGURE 2.5. Reflection form each layer of multiple films

For the system of n layers shown in figure(2.5), we obtain for the x and y components of E and H in the m^{th} layer

$$\left.\begin{aligned} E_{mx} &= \left(E_{mp}^{+}e^{-i\mu_m z} + E_{mp}^{-}e^{+i\mu_m z}\right)\cos\varphi_m \\ E_{my} &= E_{ms}^{+}e^{-i\mu_m z} + E_{ms}^{-}e^{+i\mu_m z} \\ H_{mx} &= \left(-E_{ms}^{+}e^{-i\mu_m z} + E_{ms}^{-}e^{+i\mu_m z}\right)n_m\cos\varphi_m \\ H_{my} &= \left(E_{mp}^{+}e^{-i\mu_m z} - E_{mp}^{-}e^{+i\mu_m z}\right)n_m \end{aligned}\right\} \tag{2.49}$$

where $\mu_m = \dfrac{2\pi n_m \cos\varphi_k}{\lambda}$

Writing $c_m = \sum_{i=1}^{m-1} d_i$ we obtain, for the m^{th} surface, separating the $(m\text{-}1)^{th}$ and the m^{th} layers.

$$\left(E_{m-1,p}^{+}e^{-i\mu_{m-1}c_m} + E_{m-1,p}^{-}e^{+i\mu_{m-1}c_m}\right)\cos\varphi_{m-1} = \left(E_{mp}^{+}e^{-i\mu_m c_m} + E_{mp}^{-}e^{+i\mu_m c_m}\right)\cos\varphi_m \tag{2.50a}$$

$$n_{m-1}\left(E_{m-1,p}^{+}e^{-i\mu_{m-1}c_m} - E_{m-1,p}^{-}e^{+i\mu_{m-1}c_m}\right) = n_m\left(E_{mp}^{+}e^{-i\mu_m c_m} - E_{mp}^{-}e^{+i\mu_m c_m}\right) \tag{2.50b}$$

for the p component.

$$E_{m-1,s}^{+}e^{-i\mu_{m-1}c_m} + E_{m-1,s}^{-}e^{+i\mu_{m-1}c_m} = E_{ms}^{+}e^{-i\mu_m c_m} + E_{ms}^{-}e^{+i\mu_m c_m} \tag{2.51a}$$

$$\left(-E_{m-1,s}^{+}e^{-i\mu_{m-1}c_m} + E_{m-1,s}^{-}e^{+i\mu_{m-1}c_m}\right)n_{m-1}\cos\varphi_{m-1} = \left(-E_{ms}^{+}e^{-i\mu_m c_m} + E_{ms}^{-}e^{+i\mu_m c_m}\right)n_m\cos\varphi_m \tag{2.51b}$$

for the s component.

These equations may be written in terms of the Fresnel coefficients. The forms of the resulting equations for the p and s components are identical so that the suffix p and s from the Fresnel coefficients may be dropped. It must, however, be remembered that the values of the Fresnel coefficient depend on the plane of polarization considered and that the appropriate values, from eqs.(2.11) to (2.14) must be inserted.

It can be obtained, from eqs.(2.50) and (2.51) using the values of r, t as defined in section(2.1)

$$E_{m-1}^{+}e^{-i\mu_{m-1}c_m} = \left(E_m^{+}e^{-i\mu_m c_m} + r_m E_m^{-}e^{+i\mu_m c_m}\right)/\,t_m \tag{2.52a}$$

$$E_{m-1}^{-}e^{+i\mu_{m-1}c_m} = \left(r_m E_m^{+}e^{-i\mu_m c_m} + E_m^{-}e^{+i\mu_m c_m}\right)/\,t_m \tag{2.52b}$$

By writing $\delta_m = \mu_m c_m$ and a change of origin, eqs.(2.52) may be further simplified to

$$E_{m-1}^{+} = \left(e^{i\delta_{m-1}}E_m^{+} + r_m e^{i\delta_{m-1}}E_m^{-}\right) / t_m \tag{2.53a}$$

$$E_{m-1}^{-} = \left(r_m e^{-i\delta_{m-1}}E_m^{+} + e^{-i\delta_{m-1}}E_m^{-}\right) / t_m \tag{2.53b}$$

which recurrence relation may be written in matrix form

$$\begin{pmatrix} E_{m-1}^{+} \\ E_{m-1}^{-} \end{pmatrix} = \frac{1}{t_m}\begin{pmatrix} e^{i\delta_{m-1}} & r_m e^{i\delta_{m-1}} \\ r_m e^{-i\delta_{m-1}} & e^{-i\delta_{m-1}} \end{pmatrix}\begin{pmatrix} E_m^{+} \\ E_m^{-} \end{pmatrix} \tag{2.54}$$

For a system of n layers (figure(2.5)) we require to know the relation between E_{n+1}^{+} and E_0^{+} so that the transmission coefficient may be obtained; and also that between E_0^{-} and E_0^{+} for the reflection coefficient. From eq.(2.54) we obtain

$$\begin{pmatrix} E_0^{+} \\ E_0^{-} \end{pmatrix} = \frac{(C_1)(C_2) \,\ldots\ldots\, (C_{n+1})}{t_1 t_2 \,\ldots\ldots\, t_{n+1}}\begin{pmatrix} E_{n+1}^{+} \\ E_{n+1}^{-} \end{pmatrix} \tag{2.55}$$

where $$(C_m) = \begin{pmatrix} e^{i\delta_{m-1}} & r_m e^{i\delta_{m-1}} \\ r_m e^{-i\delta_{m-1}} & e^{-i\delta_{m-1}} \end{pmatrix} \tag{2.56}$$

We may therefore express E_0^{-} and E_{n+1}^{+} in terms of E_0^{+} and so obtain the reflection and transmission coefficients. We note that since there is no negative-going wave in the $(n+1)^{th}$ medium, we put $E_{n+1}^{-} = 0$.

2.6 Application of Matrix Method for Evaluating Reflectance and Transmittance

We first introduce a notation for the matrix elements which will enable expressions relating to any number of layers to be written in a form which is both succinct and convenient for numerical computation. It was shown above that the results are conveniently expressed in terms of Fresnel coefficients and these are, in general, complex. For the coefficients at the m^{th} interface we write $r_m = g_m + ih_m$ and $t_m = 1 + g_m + ih_m$. If both the $(m-1)^{th}$ and the m^{th} media are absorbing, with $\boldsymbol{n}_{m-1} = n_{m-1} - ik_{m-1}$ and $\boldsymbol{n}_m = n_m - ik_m$, then for the case of normal incidence [2]

$$\left.\begin{aligned} g_m &= \frac{n_{m-1}^2 + k_{m-1}^2 - n_m^2 - k_m^2}{\left(n_{m-1} + n_m\right)^2 + \left(k_{m-1} + k_m\right)^2} \\ h_m &= \frac{2\left(n_{m-1}k_m - n_m k_{m-1}\right)}{\left(n_{m-1} + n_m\right)^2 + \left(k_{m-1} + k_m\right)^2} \end{aligned}\right\} \tag{2.57}$$

If the thickness of the $(m-1)^{th}$ layer is d_{m-1} then the phase term $e^{i\delta_{m-1}}$ in the m^{th} matrix is written

$$\begin{aligned} \exp i\delta_{m-1} &= \exp\frac{i2\pi}{\lambda}\left(n_{m-1} - ik_{m-1}\right)d_{m-1} \\ &= \exp\alpha_{m-1}\exp i\gamma_{m-1} \end{aligned} \tag{2.58}$$

where $\alpha_{m-1} = \dfrac{2\pi}{\lambda}k_{m-1}d_{m-1}$ and $\gamma_{m-1} = \dfrac{2\pi}{\lambda}n_{m-1}d_{m-1}$

We note that all the elements of the matrices are complex and we write the m^{th} matrix as

$$\left(C_m\right) = \begin{pmatrix} p_m + iq_m & r_m + is_m \\ t_m + iu_m & v_m + iw_m \end{pmatrix}$$

A double suffix notation serves to denote the elements of product matrices. Thus the elements of $(C_1)(C_2)$ are written

$$\begin{pmatrix} p_{12}+iq_{12} & r_{12}+is_{12} \\ t_{12}+iu_{12} & v_{12}+iw_{12} \end{pmatrix}$$

The matrix elements are readily found from eqs.(2.56) and (2.57-58)

$$(C_m) = \begin{pmatrix} e^{i\delta_{m-1}} & r_m e^{i\delta_{m-1}} \\ r_m e^{-i\delta_{m-1}} & e^{-i\delta_{m-1}} \end{pmatrix} \equiv \begin{pmatrix} p_m+iq_m & r_m+is_m \\ t_m+iu_m & v_m+iw_m \end{pmatrix}$$

and

$$p_m = e^{\alpha_{m-1}}\cos\gamma_{m-1} \quad ; \quad q_m = e^{\alpha_{m-1}}\sin\gamma_{m-1}$$

$$r_m = e^{\alpha_{m-1}}\left(g_m\cos\gamma_{m-1} - h_m\sin\gamma_{m-1}\right) \quad ; \quad s_m = e^{\alpha_{m-1}}\left(h_m\cos\gamma_{m-1} + g_m\sin\gamma_{m-1}\right)$$

$$t_m = e^{-\alpha_{m-1}}\left(g_m\cos\gamma_{m-1} + h_m\sin\gamma_{m-1}\right) \quad ; \quad u_m = e^{-\alpha_{m-1}}\left(h_m\cos\gamma_{m-1} - g_m\sin\gamma_{m-1}\right)$$

$$v_m = e^{-\alpha_{m-1}}\cos\gamma_{m-1} \quad ; \quad w_m = -e^{-\alpha_{m-1}}\sin\gamma_{m-1}$$

3. THEORY AND EXPERIMENT

3.1 Previous Studies

In the previous chapter we derive a formula for reflection from a single layer thin film at normal incidence. But we must have at least three equations because the number of unknown optical constants are three (n, k, d; refractive index, extinction coefficient, and film thickness respectively). So, only one reflection value is not sufficient to solve these unknowns. Minkov [3] gets one transmission and two reflection (one from film side and other from substrate side) values to calculate optical canstants. But his method has some restrictions such as k, extinction coefficient, shall be very small with respect to n, refractive index, and substrate must be transparent. Panayotov [4] gets one transmission and one reflection spectrum from the film side. Swanepoel [5] uses only one tranmission spectrum but he neglects k and then from maxima and minima of spectrum, calculates n first, then, using this n value he calculates d; after this calculation, using this value of d, calculates corrected n value and with this corrected value calculates corrected d, but d has a large error [6].

3.2 Our Study

In our study, film side reflection is used at different angles. Extinction coefficient k is not neglected, entire spectrum is not needed. Also, optical constants of a film on a non-transparent substrate can be determined using this technique.

The geometry of the film on an absorbing substrate is shown in figure 3.1. In this figure, φ_1 and φ_2 are not real angles. They are complex. So, in reality, we cannot show them in figure. But to image the geometry, these angles are shown as if they are real.
The form $\mathbf{n} = n - ik$ was used throughout this study.

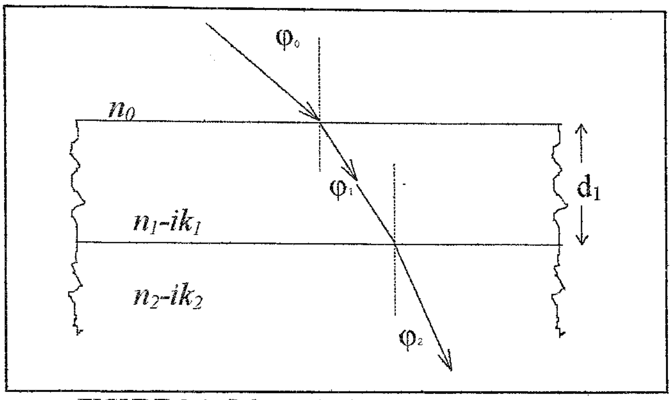

FIGURE 3.1. Schematic demostration of geometry

From Snell's law

$$n_0 \sin\varphi_0 = \boldsymbol{n}_1 \sin\varphi_1 \tag{3.1}$$

Hence,

$$\sin\varphi_1 = \frac{n_0 \sin\varphi_0}{\boldsymbol{n}_1} = \frac{n_0 \sin\varphi_0}{n_1 - ik_1}$$

where $\mathbf{n}_1 = n_1 - ik_1$. Although φ_1 is complex; φ_1 is not needed directly; instead, the following products are required:

1- $\cos\varphi_1 . \cos^*\varphi_1 = ?$

2- $\cos\varphi_1 - \cos^*\varphi_1 = ?$

3- $\cos\varphi_1 + \cos^*\varphi_1 = ?$

We can use $\cos^*\varphi$ instead of $\cos\varphi^*$ since $\cos^*\varphi = \cos\varphi^*$.

Proof:

$$[\cos(a - ib)]^* \overset{?}{=} \cos(a - ib)^*$$

$$[\cos a . \cos(ib) + \sin a . \sin(ib)]^* \overset{?}{=} \cos(a + ib)$$

$$[\cos a.\cosh b + \sin a. i\sinh b]^* \stackrel{?}{=} \cos a.\cos(ib) - \sin a.\sin(ib)$$

$$\cos a.\cosh b - i\sin a.\sinh b = \cos a.\cosh b - i\sin a.\sinh b \quad \checkmark$$

Proof is okey.

$$\sin\varphi_1 = \frac{n_0 \sin\varphi_0}{n_1 - ik_1} = \frac{n_0 \sin\varphi_0}{n_1 - ik_1}\,\frac{n_1 + ik_1}{n_1 + ik_1}$$

$$\sin\varphi_1 = \frac{n_0 \sin\varphi_0}{n_1^2 + k_1^2}\, n_1 + i\frac{n_0 \sin\varphi_0}{n_1^2 + k_1^2}\, k_1$$

let $\quad a \equiv \dfrac{n_0 \sin\varphi_0}{n_1^2 + k_1^2}$

So, we have

$$\sin\varphi_1 = a(n_1 + ik_1)$$

$$\begin{aligned}\sin^2\varphi_1 &= a^2(n_1 + ik_1)^2 = a^2(n_1^2 + 2in_1k_1 - k_1^2)\\ &= a^2(n_1^2 - k_1^2) + i2a^2n_1k_1\end{aligned}$$

Also we know from trigonometry;

$$\begin{aligned}\cos^2\varphi_1 &= 1 - \sin^2\varphi_1\\ \cos^2\varphi_1 &= 1 - \{a^2(n_1^2 - k_1^2) + i2a^2n_1k_1\}\\ &= \underbrace{1 - a^2(n_1^2 - k_1^2)}_{\equiv x} - i\underbrace{2a^2n_1k_1}_{\equiv y}\\ \cos^2\varphi_1 &= x - iy\end{aligned}$$

Therefore, we have

$$\cos\varphi_1 = \sqrt{x - iy} \qquad \Rightarrow \cos^*\varphi_1 = \sqrt{x + iy}$$

we firstly need;

1- $\cos\varphi_1 . \cos^*\varphi_1 = ?$

$$= \sqrt{x - iy}.\sqrt{x + iy} = \sqrt{x^2 + y^2} \equiv w$$

by De Moivre law [7];

if $z = r(\cos\theta + i\sin\theta)$ then $z^n = r^n(\cos n\theta + i\sin n\theta)$

where n is an integer or fractional number (positive or negative)

If $\cos\varphi_1$ is written in polar form ;

$$\cos\varphi_1 = \sqrt{x - iy} = (x^2 + y^2)^{1/4}\{\cos(\tfrac{1}{2}\tan^{-1}(y/x)) - i\sin(\tfrac{1}{2}\tan^{-1}(y/x))\}$$

$$\cos^*\varphi_1 = \sqrt{x + iy} = (x^2 + y^2)^{1/4}\{\cos(\tfrac{1}{2}\tan^{-1}(y/x)) + i\sin(\tfrac{1}{2}\tan^{-1}(y/x))\}$$

let $\beta \equiv \dfrac{1}{2}\tan^{-1}(y/x)$

we secondly need;

2- $\cos\varphi_1 - \cos^*\varphi_1 = ?$

$$\cos\varphi_1 - \cos^*\varphi_1 = \sqrt{x - iy} - \sqrt{x + iy} = (x^2 + y^2)^{1/4}\{-2i\sin\beta\}$$

$$\cos\varphi_1 - \cos^*\varphi_1 = -i(2\sqrt{w}\sin\beta)$$

Also we need

3- $\cos\varphi_1 + \cos^*\varphi_1 = ?$

$$= (x^2 + y^2)^{1/4}(2\cos\beta)$$

$$\cos\varphi_1 + \cos^*\varphi_1 = 2\sqrt{w}\cos\beta$$

At the second surface, our calculation is more diffucult than the first one because both medium, film and substrate, are absorbing. Therefore;

$$\boldsymbol{n}_1 \sin\varphi_1 = \boldsymbol{n}_2 \sin\varphi_2$$

$$\sin\varphi_2 = \frac{(n_1 - ik_1)\sin\varphi_1}{(n_2 - ik_2)}$$

we already know from the above calculations that

$$\sin\varphi_1 = a(n_1 + ik_1)$$

$$\sin\varphi_2 = \frac{(n_1 - ik_1).a(n_1 + ik_1)}{(n_2 - ik_2)}$$

$$= \frac{(n_1^2 + k_1^2).a}{(n_2 - ik_2)} = \frac{(n_1^2 + k_1^2)\,a\,(n_2 + ik_2)}{(n_2^2 + k_2^2)}$$

where $\quad a \equiv \dfrac{n_0 \sin\varphi_0}{n_1^2 + k_1^2}$

Hence, we have

$$\sin\varphi_2 = \frac{n_0 \sin\varphi_0}{(n_2^2 + k_2^2)}(n_2 + ik_2) \qquad \text{let} \qquad b \equiv \frac{n_0 \sin\varphi_0}{(n_2^2 + k_2^2)}$$

Therefore;

$$\sin\varphi_2 = b(n_2 + ik_2)$$

for second surface, we need

1- $\cos\varphi_2 . \cos^*\varphi_2 = ?$

2- $\cos\varphi_2 . \cos^*\varphi_1 - \cos\varphi_1 . \cos^*\varphi_2 = ?$

3- $\cos\varphi_2 . \cos^*\varphi_1 + \cos\varphi_1 . \cos^*\varphi_2 = ?$

$$\cos^2\varphi_2 = 1 - \sin^2\varphi_2$$
$$= 1 - b^2(n_2 + ik_2)^2$$
$$= \underbrace{1 - b^2(n_2^2 - k_2^2)}_{\equiv t} - \underbrace{i2b^2n_2k_2}_{\equiv z}$$
$$\cos^2\varphi_2 = t - iz$$

$$\cos\varphi_1 = \sqrt{t - iz} \qquad \Rightarrow \qquad \cos^*\varphi_1 = \sqrt{t + iz}$$

So, we can calculate

1- $\cos\varphi_2 . \cos^*\varphi_2 = ?$

$$= \sqrt{t - iz} . \sqrt{t + iz} = \sqrt{t^2 + z^2} \equiv s$$

2- $\cos\varphi_2 . \cos^*\varphi_1 - \cos\varphi_1 . \cos^*\varphi_2 = ?$

$$= \sqrt{t - iz} . \sqrt{x + iy} - \sqrt{x - iy} . \sqrt{t + iz}$$
$$= \sqrt{\underbrace{(xt + yz)}_{\equiv m} + i\underbrace{(yt - xz)}_{\equiv n}} - \sqrt{\underbrace{(xt + yz)}_{\equiv m} - i\underbrace{(yt - xz)}_{\equiv n}}$$
$$= \sqrt{m + in} - \sqrt{m - in}$$

$$\cos\varphi_2 . \cos^*\varphi_1 - \cos\varphi_1 . \cos^*\varphi_2 = i2\sqrt{p}\sin\xi$$

where $p \equiv \sqrt{m^2 + n^2}$ and $\xi \equiv \frac{1}{2}\tan^{-1}(n/m)$

therefore, with similar mathematical processes we easily get

3- $\cos\varphi_2 . \cos^*\varphi_1 + \cos\varphi_1 . \cos^*\varphi_2 = 2\sqrt{p}\cos\xi$

3.3 Derivation of Fresnel Coefficients

Above six equations were used to calculate the Fresnel coefficients (r_{1p},r_{1s},r_{2p}, and r_{2s} for a single film). Our aim is to write these coefficients as $r_m = g_m + ih_m$.

$r_{1p} = ?$

$$r_{1p} = \frac{n_0\cos\varphi_1 - n_1\cos\varphi_0}{n_0\cos\varphi_1 + n_1\cos\varphi_0} = \frac{n_0\cos\varphi_1 - (n_1 - ik_1)\cos\varphi_0}{n_0\cos\varphi_1 + (n_1 - ik_1)\cos\varphi_0} \tag{3.2}$$

$$= \frac{\{n_0\cos\varphi_1 - (n_1 - ik_1)\cos\varphi_0\} * \{n_0\cos^*\varphi_1 + (n_1 + ik_1)\cos\varphi_0\}}{\{n_0\cos\varphi_1 + (n_1 - ik_1)\cos\varphi_0\} * \{n_0\cos^*\varphi_1 + (n_1 + ik_1)\cos\varphi_0\}} = \frac{\mathrm{A}}{\mathrm{B}}$$

$$\mathrm{A} = n_0^2\cos\varphi_1\cos^*\varphi_1 + n_0(n_1 + ik_1)\cos\varphi_0\cos\varphi_1 - n_0(n_1 - ik_1)\cos\varphi_0\cos^*\varphi_1 - (n_1^2 + k_1^2)\cos^2\varphi_0$$

$$= n_0^2\underbrace{\cos\varphi_1\cos^*\varphi_1}_{=w} + n_0n_1\cos\varphi_0\underbrace{(\cos\varphi_1 - \cos^*\varphi_1)}_{=-i(2\sqrt{w}\sin\beta)}$$
$$+ in_0k_1\cos\varphi_0\underbrace{(\cos\varphi_1 + \cos^*\varphi_1)}_{=2\sqrt{w}\cos\beta} - (n_1^2 + k_1^2)\cos^2\varphi_0$$

$$= n_0^2w - 2in_0n_1\sqrt{w}\cos\varphi_0\sin\beta + 2in_0k_1\sqrt{w}\cos\varphi_0\cos\beta - (n_1^2 + k_1^2)\cos^2\varphi_0$$

$$= n_0^2w + 2in_0\sqrt{w}\cos\varphi_0(k_1\cos\beta - n_1\sin\beta) - (n_1^2 + k_1^2)\cos^2\varphi_0$$

$$\mathrm{B} = n_0^2\underbrace{\cos\varphi_1\cos^*\varphi_1}_{=w} + n_0n_1\cos\varphi_0\underbrace{(\cos\varphi_1 + \cos^*\varphi_1)}_{=2\sqrt{w}\cos\beta} + in_0k_1\cos\varphi_0\underbrace{(\cos\varphi_1 - \cos^*\varphi_1)}_{=-i(2\sqrt{w}\sin\beta)} + (n_1^2 + k_1^2)\cos^2\varphi_0$$

$$= n_0^2w + 2n_0\sqrt{w}\cos\varphi_0(n_1\cos\beta + k_1\sin\beta) + (n_1^2 + k_1^2)\cos^2\varphi_0$$

$$\boxed{g_{1p} = \frac{n_0^2w - (n_1^2 + \mathrm{k}_1^2)\cos^2\varphi_0}{n_0^2w + 2n_0\sqrt{w}\cos\varphi_0(n_1\cos\beta + k_1\sin\beta) + (n_1^2 + k_1^2)\cos^2\varphi_0}} \tag{3.3}$$

and

$$\boxed{h_{1p} = \frac{2n_0\sqrt{w}\cos\varphi_0(k_1\cos\beta - n_1\sin\beta)}{n_0^{\,2}w + 2n_0\sqrt{w}\cos\varphi_0(n_1\cos\beta + k_1\sin\beta) + (n_1^{\,2} + \mathrm{k}_1^{\,2})\cos^2\varphi_0}} \tag{3.4}$$

$r_{1s} = ?$

$$r_{1s} = \frac{n_0\cos\varphi_0 - n_1\cos\varphi_1}{n_0\cos\varphi_0 + n_1\cos\varphi_1} = \frac{n_0\cos\varphi_0 - (n_1 - ik_1)\cos\varphi_1}{n_0\cos\varphi_0 + (n_1 - ik_1)\cos\varphi_1} \tag{3.5}$$

$$= \frac{\{n_0\cos\varphi_0 - (n_1 - ik_1)\cos\varphi_1\} * \{n_0\cos\varphi_0 + (n_1 + ik_1)\cos^*\varphi_1\}}{\{n_0\cos\varphi_0 + (n_1 - ik_1)\cos\varphi_1\} * \{n_0\cos\varphi_0 + (n_1 + ik_1)\cos^*\varphi_1\}} = \frac{\mathrm{A}}{\mathrm{B}}$$

$$\mathrm{A} = n_0^{\,2}\cos^2\varphi_0 + n_0(n_1 + ik_1)\cos\varphi_0\cos^*\varphi_1 - n_0(n_1 - ik_1)\cos\varphi_0\cos\varphi_1 - (n_1^{\,2} + k_1^{\,2})\cos\varphi_1\cos^*\varphi_1$$

$$= n_0^{\,2}\cos^2\varphi_0 + n_0n_1\cos\varphi_0\underbrace{(\cos^*\varphi_1 - \cos\varphi_1)}_{=i(2\sqrt{w}\sin\beta)}$$

$$+ in_0k_1\cos\varphi_0\underbrace{(\cos^*\varphi_1 + \cos\varphi_1)}_{=2\sqrt{w}\cos\beta} - (n_1^{\,2} + k_1^{\,2})\underbrace{\cos\varphi_1\cos^*\varphi_1}_{=w}$$

$$= n_0^{\,2}\cos^2\varphi_0 + 2in_0\sqrt{w}\cos\varphi_0(k_1\cos\beta + n_1\sin\beta) - (n_1^{\,2} + k_1^{\,2})w$$

$$\mathrm{B} = n_0^{\,2}\cos^2\varphi_0 + n_0(n_1 + ik_1)\cos\varphi_0\cos^*\varphi_1 + n_0(n_1 - ik_1)\cos\varphi_0\cos\varphi_1 + (n_1^{\,2} + k_1^{\,2})\cos\varphi_1\cos^*\varphi_1$$

$$= n_0^{\,2}\cos^2\varphi_0 + n_0n_1\cos\varphi_0\underbrace{(\cos^*\varphi_1 + \cos\varphi_1)}_{=2\sqrt{w}\cos\beta}$$

$$+ in_0k_1\cos\varphi_0\underbrace{(\cos^*\varphi_1 - \cos\varphi_1)}_{=i(2\sqrt{w}\sin\beta)} + (n_1^{\,2} + k_1^{\,2})\underbrace{\cos\varphi_1\cos^*\varphi_1}_{=w}$$

$$= n_0^{\,2}\cos^2\varphi_0 + 2n_0\sqrt{w}\cos\varphi_0(n_1\cos\beta - k_1\sin\beta) + (n_1^{\,2} + k_1^{\,2})w$$

$$\boxed{g_{1s} = \frac{n_0^{\,2}\cos^2\varphi_0 - (n_1^{\,2} + k_1^{\,2})w}{n_0^{\,2}\cos^2\varphi_0 + 2n_0\sqrt{w}\cos\varphi_0(n_1\cos\beta - k_1\sin\beta) + (n_1^{\,2} + k_1^{\,2})w}} \tag{3.6}$$

and

$$\boxed{h_{1s} = \frac{2n_0\sqrt{w}\cos\varphi_0(k_1\cos\beta + n_1\sin\beta)}{n_0^{\,2}\cos^2\varphi_0 + 2n_0\sqrt{w}\cos\varphi_0(n_1\cos\beta - k_1\sin\beta) + (n_1^{\,2} + k_1^{\,2})w}} \tag{3.7}$$

$r_{2p} = ?$

$$r_{2p} = \frac{\boldsymbol{n}_1\cos\varphi_2 - \boldsymbol{n}_2\cos\varphi_1}{\boldsymbol{n}_1\cos\varphi_2 + \boldsymbol{n}_2\cos\varphi_1} = \frac{(n_1 - ik_1)\cos\varphi_2 - (n_2 - ik_2)\cos\varphi_1}{(n_1 - ik_1)\cos\varphi_2 + (n_2 - ik_2)\cos\varphi_1} \tag{3.8}$$

$$= \frac{\{(n_1 - ik_1)\cos\varphi_2 - (n_2 - ik_2)\cos\varphi_1\} * \{(n_1 + ik_1)\cos^*\varphi_2 + (n_2 + ik_2)\cos^*\varphi_1\}}{\{(n_1 - ik_1)\cos\varphi_2 + (n_2 - ik_2)\cos\varphi_1\} * \{(n_1 + ik_1)\cos^*\varphi_2 + (n_2 + ik_2)\cos^*\varphi_1\}} = \frac{\mathrm{A}}{\mathrm{B}}$$

$$\begin{aligned}\mathrm{A} = {} & (n_1^{\,2} + k_1^{\,2})\underbrace{\cos\varphi_2\cos^*\varphi_2}_{\equiv s} + (n_1 - ik_1)(n_2 + ik_2)\cos\varphi_2\cos^*\varphi_1 \\ & -(n_1 + ik_1)(n_2 - ik_2)\cos\varphi_1\cos^*\varphi_2 - (n_2^{\,2} + k_2^{\,2})\underbrace{\cos\varphi_1\cos^*\varphi_1}_{\equiv w}\end{aligned}$$

$$\begin{aligned}= {} & (n_1^{\,2} + k_1^{\,2}).s + (n_1n_2 + in_1k_2 - in_2k_1 + k_1k_2)\cos\varphi_2\cos^*\varphi_1 \\ & - (n_1n_2 - in_1k_2 + in_2k_1 + k_1k_2)\cos\varphi_1\cos^*\varphi_2 - (n_2^{\,2} + k_2^{\,2}).w\end{aligned}$$

$$= (n_1^{\,2} + k_1^{\,2}).s + (n_1n_2 + k_1k_2)(2i\sqrt{p}\sin\xi) + i(n_1k_2 - n_2k_1)(2\sqrt{p}\cos\xi) - (n_2^{\,2} + k_2^{\,2}).w$$

$$\begin{aligned}\mathrm{B} = {} & (n_1^{\,2} + k_1^{\,2})\cos\varphi_2\cos^*\varphi_2 + (n_1 - ik_1)(n_2 + ik_2)\cos\varphi_2\cos^*\varphi_1 \\ & +(n_1 + ik_1)(n_2 - ik_2)\cos\varphi_1\cos^*\varphi_2 + (n_2^{\,2} + k_2^{\,2})\cos\varphi_1\cos^*\varphi_1\end{aligned}$$

$$= (n_1^2 + k_1^2).s + (n_1 n_2 + k_1 k_2)\underbrace{(\cos\varphi_2 \cos^*\varphi_1 + \cos\varphi_1 \cos^*\varphi_2)}_{= 2\sqrt{p}\cos\xi}$$

$$+ i(n_1 k_2 - n_2 k_1)\underbrace{(\cos\varphi_2 \cos^*\varphi_1 - \cos\varphi_1 \cos^*\varphi_2)}_{= i(2\sqrt{p}\sin\xi)} + (n_2^2 + k_2^2).w$$

$$= (n_1^2 + k_1^2).s + (n_1 n_2 + k_1 k_2)(2\sqrt{p}\cos\xi) - (n_1 k_2 - n_2 k_1)(2\sqrt{p}\sin\xi) + (n_2^2 + k_2^2).w$$

$$\boxed{g_{2p} = \frac{(n_1^2 + k_1^2).s - (n_2^2 + k_2^2).w}{(n_1^2 + k_1^2).s + (n_1 n_2 + k_1 k_2)(2\sqrt{p}\cos\xi) - (n_1 k_2 - n_2 k_1)(2\sqrt{p}\sin\xi) + (n_2^2 + k_2^2).w}} \quad (3.9)$$

and

$$\boxed{h_{2p} = \frac{(n_1 n_2 + k_1 k_2)(2\sqrt{p}\sin\xi) + (n_1 k_2 - n_2 k_1)(2\sqrt{p}\cos\xi)}{(n_1^2 + k_1^2).s + (n_1 n_2 + k_1 k_2)(2\sqrt{p}\cos\xi) - (n_1 k_2 - n_2 k_1)(2\sqrt{p}\sin\xi) + (n_2^2 + k_2^2).w}} \quad (3.10)$$

$r_{2s} = ?$

$$r_{2s} = \frac{\boldsymbol{n}_1 \cos\varphi_1 - \boldsymbol{n}_2 \cos\varphi_2}{\boldsymbol{n}_1 \cos\varphi_1 + \boldsymbol{n}_2 \cos\varphi_2} = \frac{(n_1 - ik_1)\cos\varphi_1 - (n_2 - ik_2)\cos\varphi_2}{(n_1 - ik_1)\cos\varphi_1 + (n_2 - ik_2)\cos\varphi_2} \quad (3.11)$$

$$= \frac{\{(n_1 - ik_1)\cos\varphi_1 - (n_2 - ik_2)\cos\varphi_2\} * \{(n_1 + ik_1)\cos^*\varphi_1 + (n_2 + ik_2)\cos^*\varphi_2\}}{\{(n_1 - ik_1)\cos\varphi_1 + (n_2 - ik_2)\cos\varphi_2\} * \{(n_1 + ik_1)\cos^*\varphi_1 + (n_2 + ik_2)\cos^*\varphi_2\}} = \frac{\mathrm{A}}{\mathrm{B}}$$

$$\mathrm{A} = (n_1^2 + k_1^2)\cos\varphi_1 \cos^*\varphi_1 + (n_1 - ik_1)(n_2 + ik_2)\cos\varphi_1 \cos^*\varphi_2$$

$$- (n_1 + ik_1)(n_2 - ik_2)\cos\varphi_2 \cos^*\varphi_1 - (n_2^2 + k_2^2)\cos\varphi_2 \cos^*\varphi_2$$

$$= (n_1^2 + k_1^2)\cos\varphi_1 \cos^*\varphi_1 + n_1 n_2 (\cos\varphi_1 \cos^*\varphi_2 - \cos\varphi_2 \cos^*\varphi_1)$$

$$+ i n_1 k_2 (\cos\varphi_1 \cos^*\varphi_2 + \cos\varphi_2 \cos^*\varphi_1) - i n_2 k_1 (\cos\varphi_1 \cos^*\varphi_2 + \cos\varphi_2 \cos^*\varphi_1)$$

$$+ k_1 k_2 (\cos\varphi_1 \cos^*\varphi_2 - \cos\varphi_2 \cos^*\varphi_1) - (n_2^2 + k_2^2)\cos\varphi_2 \cos^*\varphi_2$$

$$= (n_1^2 + k_1^2).w + (n_1n_2 + k_1k_2)(-2i\sqrt{p}\sin\xi) + i(n_1k_2 - n_2k_1)(2\sqrt{p}\cos\xi) - (n_2^2 + k_2^2).s$$

$$\text{B} = (n_1^2 + k_1^2)\cos\varphi_1\cos^*\varphi_1 + (n_1 - ik_1)(n_2 + ik_2)\cos\varphi_1\cos^*\varphi_2$$
$$+ (n_1 + ik_1)(n_2 - ik_2)\cos\varphi_2\cos^*\varphi_1 + (n_2^2 + k_2^2)\cos\varphi_2\cos^*\varphi_2$$

$$= (n_1^2 + k_1^2).w + (n_1n_2 + k_1k_2)\underbrace{(\cos\varphi_1\cos^*\varphi_2 + \cos\varphi_2\cos^*\varphi_1)}_{=2\sqrt{p}\cos\xi}$$
$$+ i(n_1k_2 - n_2k_1)\underbrace{(\cos\varphi_1\cos^*\varphi_2 - \cos\varphi_2\cos^*\varphi_1)}_{=-i(2\sqrt{p}\sin\xi)} + (n_2^2 + k_2^2).s$$

$$= (n_1^2 + k_1^2).w + (n_1n_2 + k_1k_2)(2\sqrt{p}\cos\xi) + (n_1k_2 - n_2k_1)(2\sqrt{p}\sin\xi) + (n_2^2 + k_2^2).s$$

$$\boxed{g_{2s} = \frac{(n_1^2 + k_1^2).w - (n_2^2 + k_2^2).s}{(n_1^2 + k_1^2).w + (n_1n_2 + k_1k_2)(2\sqrt{p}\cos\xi) + (n_1k_2 - n_2k_1)(2\sqrt{p}\sin\xi) + (n_2^2 + k_2^2).s}} \quad (3.12)$$

and

$$\boxed{h_{2s} = \frac{(n_1k_2 - n_2k_1)(2\sqrt{p}\cos\xi) - (n_1n_2 + k_1k_2)(2\sqrt{p}\sin\xi)}{(n_1^2 + k_1^2).w + (n_1n_2 + k_1k_2)(2\sqrt{p}\cos\xi) + (n_1k_2 - n_2k_1)(2\sqrt{p}\sin\xi) + (n_2^2 + k_2^2).s}} \quad (3.13)$$

3.4 Derivation of Phase Change

For an angle of incidence, the *phase* δ_1 also changes:

$$
\begin{aligned}
\delta_1 &= \frac{2\pi}{\lambda} n_1 d_1 \cos\varphi_1 \\
&= \frac{2\pi}{\lambda}(n_1 - ik_1) d_1 \sqrt{x - iy} \\
&= \frac{2\pi}{\lambda} d_1 \{(n_1 - ik_1)\sqrt{w}(\cos\beta - i\sin\beta)\} \\
&= \frac{2\pi}{\lambda} d_1 \sqrt{w}\{n_1\cos\beta - in_1\sin\beta - ik_1\cos\beta - k_1\sin\beta\} \\
&= \frac{2\pi}{\lambda} d_1 \sqrt{w}(n_1\cos\beta - k_1\sin\beta) - i\frac{2\pi}{\lambda} d_1 \sqrt{w}(n_1\sin\beta + k_1\cos\beta)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
e^{i\delta_1} &= \mathrm{Exp}\{i\frac{2\pi}{\lambda} d_1 \sqrt{w}(n_1\cos\beta - k_1\sin\beta) + \frac{2\pi}{\lambda} d_1 \sqrt{w}(n_1\sin\beta + k_1\cos\beta)\} \\
&= \mathrm{Exp}\{\frac{2\pi}{\lambda} d_1 \sqrt{w}(n_1\sin\beta + k_1\cos\beta) + i\frac{2\pi}{\lambda} d_1 \sqrt{w}(n_1\cos\beta - k_1\sin\beta)\} \\
&= e^{\alpha_1} e^{i\gamma_1}
\end{aligned}
$$

where $\alpha_1 = \frac{2\pi}{\lambda} d_1 \sqrt{w}(n_1\sin\beta + k_1\cos\beta)$ and $\gamma_1 = \frac{2\pi}{\lambda} d_1 \sqrt{w}(n_1\cos\beta - k_1\sin\beta)$

The reflection at normal incidence is unique because Fresnel coefficients are the same for *p* and *s* polarizations for normal incidence. But our aim is to calculate reflection at inclined angle. So, Fresnel coefficients for *p* and *s* polarizations are different from each other.

3.5 Reflection of Polarized Light

3.5.1 Reflection of p Polarized Light

The reflection of p polarized light is derived by Heavens [2] as

$$R_p = \frac{t_{12p}^2 + u_{12p}^2}{p_{12p}^2 + q_{12p}^2} \tag{3.14}$$

where

$$\begin{aligned} p_{12p} &= p_{2p} + g_{1p}t_{2p} - h_{1p}u_{2p} \\ q_{12p} &= q_{2p} + h_{1p}t_{2p} + g_{1p}u_{2p} \\ t_{12p} &= t_{2p} + g_{1p}p_{2p} - h_{1p}q_{2p} \\ u_{12p} &= u_{2p} + h_{1p}p_{2p} + g_{1p}q_{2p} \end{aligned}$$

where

$$p_{2p} = e^{\alpha_1}\cos\gamma_1 \qquad q_{2p} = e^{\alpha_1}\sin\gamma_1$$

$$\begin{aligned} t_{2p} &= e^{-\alpha_1}(g_{2p}\cos\gamma_1 + h_{2p}\sin\gamma_1) \\ u_{2p} &= e^{-\alpha_1}(h_{2p}\cos\gamma_1 - g_{2p}\sin\gamma_1) \end{aligned}$$

3.5.2 Reflection of s Polarized Light

The reflection of s polarized light is given by

$$R_s = \frac{t_{12s}^2 + u_{12s}^2}{p_{12s}^2 + q_{12s}^2} \tag{3.15}$$

where

$$
\begin{aligned}
p_{12s} &= p_{2s} + g_{1s}t_{2s} - h_{1s}u_{2s} \\
q_{12s} &= q_{2s} + h_{1s}t_{2s} + g_{1s}u_{2s} \\
t_{12s} &= t_{2s} + g_{1s}p_{2s} - h_{1s}q_{2s} \\
u_{12s} &= u_{2s} + h_{1s}p_{2s} + g_{1s}q_{2s}
\end{aligned}
$$

where

$$
p_{2s} = e^{\alpha_1}\cos\gamma_1 \qquad q_{2s} = e^{\alpha_1}\sin\gamma_1
$$

$$
\begin{aligned}
t_{2s} &= e^{-\alpha_1}(g_{2s}\cos\gamma_1 + h_{2s}\sin\gamma_1) \\
u_{2s} &= e^{-\alpha_1}(h_{2s}\cos\gamma_1 - g_{2s}\sin\gamma_1)
\end{aligned}
$$

3.5.3 Reflection of Unpolarized Light

The reflection of unpolarized light is given by [8]

$$
R = \frac{R_p + R_s}{2} \tag{3.16}
$$

For all cases, we use numerical calculations to determine the optical constants since the equations are non-linear and have no analytical solutions.

Equation (3.16) was used to compute reflection R versus wavelength λ, from 500 nm to 1100 nm, for different inclined angles of incidence. Figure 3.3 shows R versus λ for $n_2 = 1.5$, $k_2 = 0$ and $d = 400$ nm. Changes of $n_1$and k_1 are shown in figure 3.2. It is seen that a considerable shift occurs in the reflection spectra when the incident angle is increased. We should notice that reflection at $\theta = 10^o$can be treated as normal, since the spectrum is almost unchanged when it is compared to $\theta = 0^o$'s spectrum.

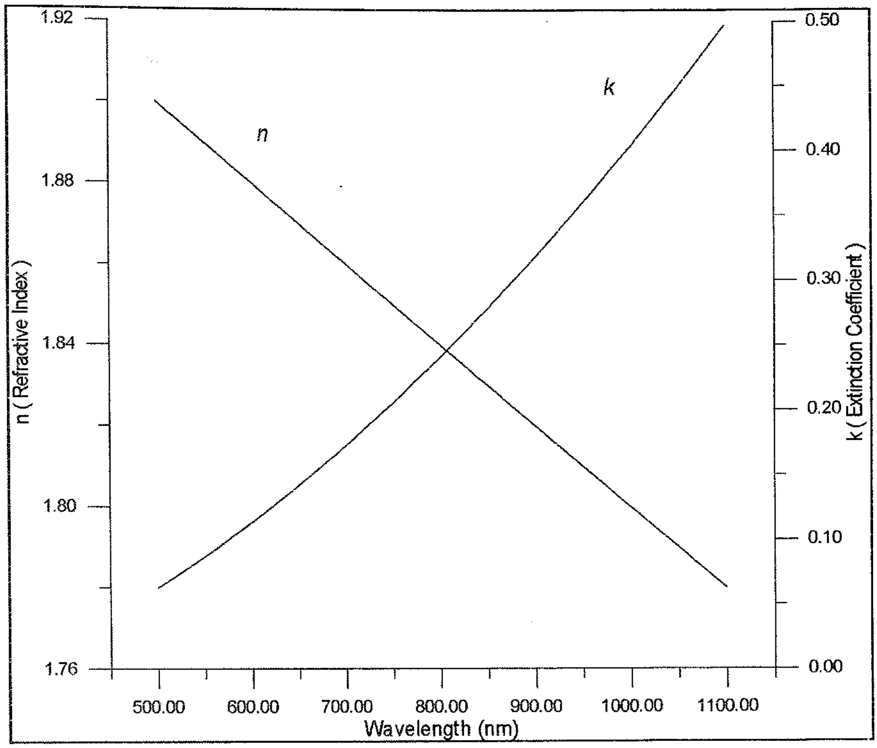

FIGURE 3.2. Typical changes of n and k of TiO_2 film with wavelength

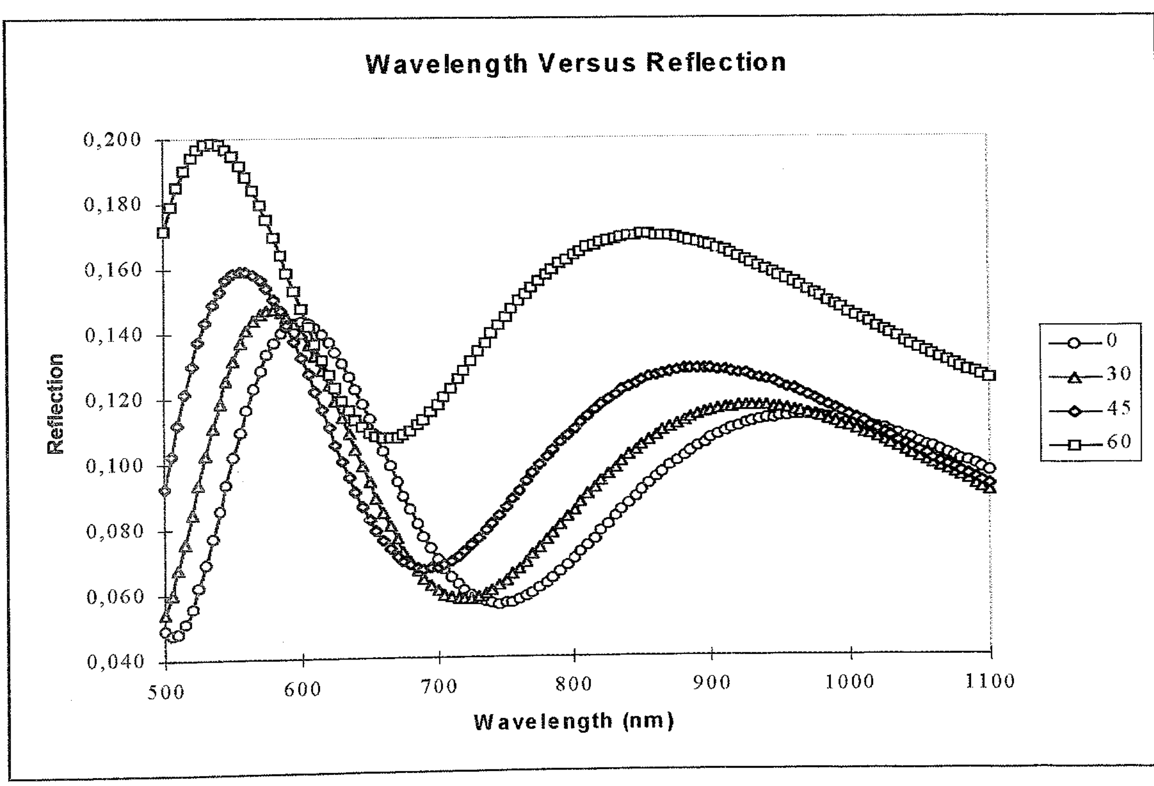

FIGURE 3.3. Reflection versus wavelength diagram obtained by computational analyses for n2=1.5, k2=0, d=400 nm, and changing n_1 and k_1 shown figure 3.2.

3.6 Experimental Work and Results

3.6.1 Description of Aparatus

3.6.1.1 Light Source

A 650 nm laser diode was used in our experiments as a light source. Such a source was used as a LED because laser light is coherent within the thickness of the substrate. So, one more reflection from other side of the subtsrate contributes the total reflection and data deviates from its true value.

This type of light source was chosen because of its important advantages such as its size, configuration compatibility to place anywhere suitable for setup and stable optical output [9-10]. It emits light at a specified wavelength which is easy to be detected by using silicon photodiodes sensitive at this wavelength. Besides; it is a cheap light source. Laser diose was operated using an electrical current which was below the threshold of lasing. Therefore, the laser emitted only spountaneous emission with *p* polarization at a wavelength of 650 nm.

3.6.1.2 Photodiode Detection

The photodiode used in this experiment was BPX65 silicon photodiode with 3kΩ feedback resistor. It is used to monitor the power reflected from the films. The photodiode produces a current proportional to the detected light intensity of reflected light [11]. A schematic diagram of the photodiode is shown in figure 3.4.

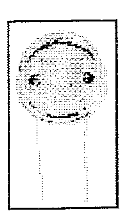

FIGURE 3.4. BPX65 silicon photodiode

3.7 Optical Arrangement

A schematic diagram of the experimetal setup is shown in Figure 3.4. Sample was mounted to a rotative circular platform by an attachment. Also photodiode detector was mounted to a second circular platform which was the same center with the first one to obtain the same path length. Laser diode was biased with a stable dc power supply. 650 nm wavelength light was launched to thin film surface at inclined incidence, and reflected light was detected by a photodiode which is sensitive to this wavelength.

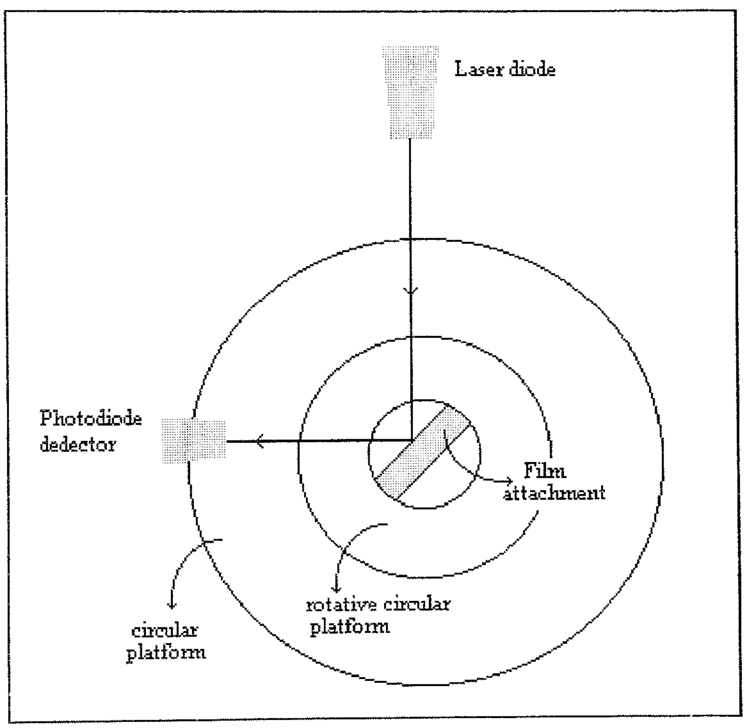

FIGURE 3.5. Experimental set-up

Before taking data, all laser light, thin film attachment, and detector were aligned horizontally. Then thin film was placed to the attachment. Platform and detector were rotated by angles $\theta_1$and $2\theta_1$ respectively. Detector was rotated $2\theta_1$ because reflected light rotates by 2θ if the film was rotated by θ. For this specific angle, reflected value was recorded and this procedure was repeated for angles desired. But large angles should not be preferred. Otherwise, measurement uncertainty may large due to inhomogeneoties in film thickness. For numerical calculation, the number of taken reflection values must be, at least, equal to those of unknowns parameters.

3.8 Results

3.8.1 TiO_2 Film Coated on a Transparent Substrate

Firstly, TiO_2 film coated on a transparent microscope slide was used for measurements. Data are listed in Table 3.1.

TABLE 3.1. Reflections of TiO_2 film at various angles

φ_0 ($^\circ$)	**Reflection**
20	0.0970
25	0.0926
30	0.0812
35	0.0542
40	0.0293

Reflection values are normalized by using a microscope slide which has a known refractive index and no extinction coefficient. For this reason, reflection values were calculated by using Fresnel's coefficients and these values were used to normalize the reflection of the sample.

By using the formula which were derived for inclined reflection in the previous chapter, numerical calculation was done according to the computational chart shown in figure3.3; and the results shown below were obtained. Our calculations were based on programming in Turbo Pascal. Since formula are non-linear, analytic solution could not be obtained.

TABLE 3.2. Experimental value of optical constants of TiO_2 film

d (nm)	**n**	**k**
400	1.870	0.03804
400	1.870	0.03805
400	1.870	0.03806
400	1.870	0.03807

Thickness of film d and refractive index n were calculated as 400 nm and 1.870 respectively. Four different values for k were obtained because reflection values are not precisely correct ones. They have 1.5 percent deviation, originating from detector misalignment, very little angle deviation and unsufficient precision of experimental apparatus. But three digits after decimal point are consistant with each other.

3.8.2 MgO Film Coated on a Non-transparent Substrate

Secondly, MgO film coated on a non-tranparent single crystal silicon was used for measurements. Data are listed in Table 3.3.

TABLE 3.3. Reflections of MgO film at various angles

φ_0 (°)	**Reflection**
20	0.0701
25	0.0699
30	0.06623
35	0.0570
40	0.0456

In this case, film thickness was known from production (50 nm). Again by using formula derived in the previous chapter, numerical calculation was done for optical constants of both the film and substrate. Below results were obtained. indices 1 and 2 represent film and substrate respectively.

TABLE 3.4. Experimental value of optical constants of MgO film and substrate single crystal Si

n_1	k_1	n_2	k_2
1.778	0.011	2.567	0.250
1.779	0.011	2.569	0.251
1.781	0.011	2.572	0.252
1.781	0.012	2.568	0.250

Refractive index of film n_1 and extinction coefficient k_1 were obtained as 1.7798 ± 0.0015 and 0.0113 ± 0.0005 respectively. Also refractive index of substrate n_2 and extinction coefficient k_2 were obtained as 2.569 ± 0.002 and 0.2508 ± 0.0010 respectively. Their correct values are listed in Table 3.5 [12].

TABLE 3.5. Optical constants of MgO and single crystal silicon (True values)

n_1	n_2	k_2
1.702	3.832	0.05

For MgO film and substrate, results are not precise. This is because of the silicon substrate. Silicon is a conductive material [13]. Therefore, at the MgO-Silicon interface, boundary condition $D_{2n}(t)$ - $D_{1n}(t) = 0$ is not satisfied. Boundary conditions are;

$$D_{2n}(t)\text{-}D_{1n}(t) = \sigma_f(t) \tag{3.17}$$

and

$$J_{2n} - J_{1n} = -\frac{\partial\sigma}{\partial t} \tag{3.18}$$

Since monochromatic radiation was used, for this reason, the surface charge density will vary as $e^{-i\omega t}$, then $\partial\sigma_f/\partial t = -i\omega t$. Thus Eqs.(3.17) and (3.18) become

$$\varepsilon_2 E_{2n} - \varepsilon_1 E_{1n} = \sigma_f \tag{3.19}$$

$$\sigma_{2c} E_{2n} - \sigma_{1c} E_{1n} = i\omega\sigma_f \tag{3.20}$$

For an arbitrary nonzero σ_f, Eqs.(3.19) and (3.20) can be combined by eliminating σ_f. Thus

$$\varepsilon_1 \left(1 + i\frac{\sigma_{c1}}{\varepsilon_1\omega} \right) E_{1n} = \varepsilon_{21} \left(1 + i\frac{\sigma_{c2}}{\varepsilon_2\omega} \right) E_{2n}$$

or

$$\left(1 + i\frac{\sigma_{c1}}{\varepsilon_1\omega} \right) D_{1n} = \left(1 + i\frac{\sigma_{c2}}{\varepsilon_2\omega} \right) D_{2n}$$

This result indicates that not only does the magnitude of the normal components of **E** and **D** change across the boundary but so do their phases [14]. Also we can say that the normal component of **D**, in general, will not be continuous because free charge will necessarily build up at the interface. $\sigma_1 = 0$ because first medium is MgO and it is an insulator [15].

3.9 Discussion

To use Swanepoel's method [16], entire transmission spectrum of film is needed. (see figure 3.6).

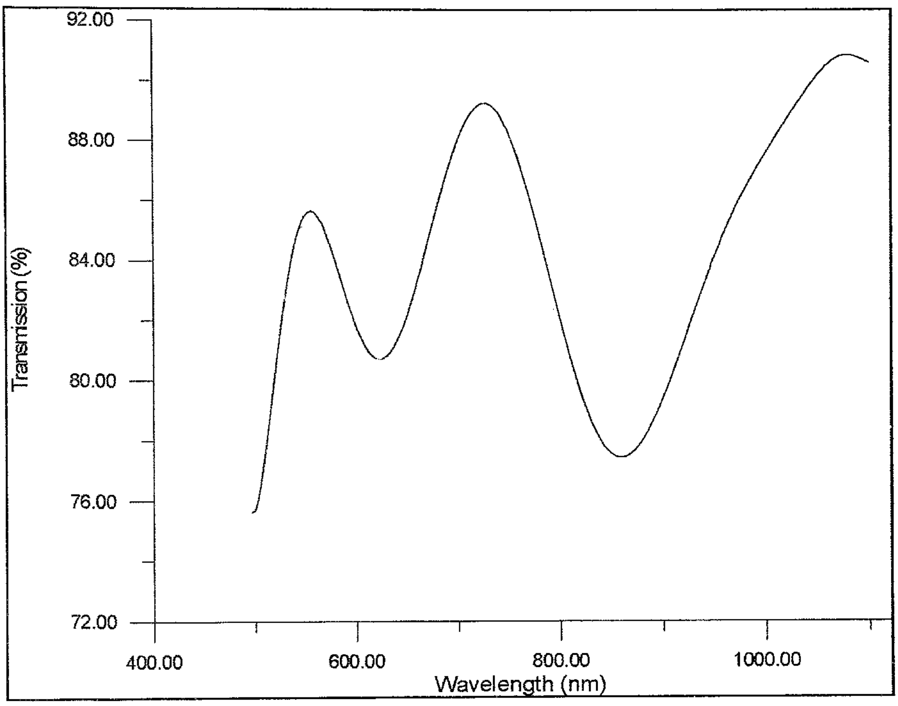

FIGURE 3.6. Transmission spectrum of TiO_2 film

Using maxima and minima points, results below were calculated for the TiO_2 film which we used.

TABLE 3.6. Optical constants of TiO_2 calculated by Swanepoel's method

d (nm)	n
511 ± 26	1.871

Our refractive index is consistent with the value calculated by Swanepoel's method while film thickness has 25 percent difference. But by Swanepoel's method, one can get an error as high as 100 percent in the film thickness [6]. Besides, in Swanepoel's method, extinction coefficient is omitted entirely. On the other hand, in our method there is no neglect for any optical constants, what its value is. Also, only one wavelength can be studied and no need to scan the entire spectrum.

For second sample, MgO film coated on a non-transparent silicon, Swanepoel's method would not work because substrate is non-transparent. So, transmission spectrum cannot be obtained. But, in our method, only reflections are needed and they can be obtained whatever substrate is. Only one exception, if substrate is conductive, results will deviate from its correct values. In our experiment, this deviation was approximately 5 percent for the MgO film. On the other hand, for substrate, this deviation was as high as 33 percent.

4. CONCLUSION

In this study, a new method have been developed to determine the optical constants of a thin dielectric layer. The most important feature of this method is to make possible using non-transparent substrate.

The experiments employed reflection measurements by launching the light output from a laser diode on a thin film surface at different incidence angles.

Thin film was attached on a rotating circular platform which is sensitive to 1/10 of a degree angle. Photodiode detector was mounted to a second circular platform which has the same center with the first one. If thin film was rotated by an angle θ, the photodiode detector must be rotated by an angle 2θ. A schematic diagram of experimental set-up is drawn in figure 3.5. It is easy to set up and to take data.

In this method, only film side reflection is used. For this reason, there is no limitation for substrate. It can be transparent, absorbing, or non-transparent medium. At the same time, extinction coefficient of thin film is not neglected. So, optical constants of a strongly absorbing film can be easily determined. On the other hand, this method has only one disadvantage. As the incidence angle gets larger, the spotsize of light on the thin film surface gets bigger. The thickness d may not be uniform in this large area. Therefore, small angles should be prefered.

In our method, non-transparent substrate can be used. Since our derivations was done for non-conducting media, both film and substrate should be chosen among the insulator materials to get good results. In the experiment, we used the conducting materials to check the results and to observe the deviation. Single crystal silicon which is a conductor, was used as a substrate for MgO film. In numerical calculations, conductivity of substrate was neglected. Results deviated 5 percent and 35 percent for MgO and Single crystal silicon respectively. Result deviated very little from correct value for the film. However, for substrate, deviation is larger and it is not reliable.

For conducting medium, boundary conditions change. This is future work to overcome. If this problem is solved, optical constants of thin film can be easily determined, for conducting as well as insulating substrates. In addition to this, variation of n with λ may be a very useful study.

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