QUANTITATIVE MODELS FOR DECISION MAKING IN REVERSE LOGISTICS NETWORK DESIGN

by

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ABSTRACT

QUANTITATIVE MODELS FOR DECISION MAKING IN REVERSE LOGISTICS NETWORK DESIGN

In this thesis, we focus on a problem in reverse logistics network design where the aim is locating distribution centers, inspection centers and remanufacturing facilities, determining the acquisition price as well as the amount of returned goods to be collected depending on the unit cost savings and competitor's acquisition price. The coordination of the forward and reverse flows in the network is also taken into account in order to minimize the transportation costs, fixed costs and used product acquisition costs.

A mixed-integer nonlinear programming problem has been formulated and exact algorithms have been suggested to solve it. When the acquisition price is set to a given value, the remaining problem becomes a mixed-integer programming problem which can be solved by Lagrangean relaxation, Benders Decomposition and Cross Decomposition algorithms. The best value of the acquisition price that minimizes the total cost is determined by the Golden Section search and computational results have been reported. Moreover, the effect of fixed cost, capacity as well as unit cost savings on the solution time have been analyzed.

ÖZET

TERSİNE LOJİSTİK AĞ TASARIMI KARARLARINDA SAYISAL MODELLER

Bu tezde, amacı dağıtım merkezleri, muayene veya kontrol merkezleri ve yeniden üretim tesisleri yer seçimi ile kullanılmış ürünlerin geri toplama fiyatının belirlenmesi olan bir tersine lojistik ağ tasarımı problemi uzerinde çalışılmıştır. İncelenen model aynı zamanda nakliye maliyetleri, sabit maliyetler ve geri toplama maliyetlerini enküçüklemek için ağdaki ileri ve geri akışları da koordine eder. Geri toplama fiyatı ve miktarı, geri toplamadan elde edilen fayda ve rakibin geri toplama fiyatına bağlıdır.

Bir karma tamsayılı doğrusal olmayan tersine lojistik ağ tasarım probleminin gösterimi verilmiş ve çözümü için kesin sonuç sağlayan algoritmalar önerilmiştir. Geri toplama fiyatı herhangi bir pozitif değere sabitlendiğinde geriye kalan problem bir tamsayı programlama problemidir. Bu problem Lagrange Gevşetmesi, Benders Ayrıştırması ve Çapraz Ayrıştırma algoritmaları ile çözülebilir. Toplam maliyeti enküçükleyen en iyi geri toplama fiyatı ise Altın Bölüm Arama tekniğiyle bulunmuş ve sayısal sonuçlar raporlanmıştır. Ayrıca, sabit maliyet, kapasite ve geri toplamadan elde edilen faydanın problemin çözüm zamanına olan etkisi analiz edilmiştir.

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a_i	Capacity of remanufacturing facility (RF) at site i
b	Unit cost savings
c_{ij}	Cost of sending one unit from plant i to distribution center (DC) at site j
cp_{ji}	Cost of sending one unit from inspection center (IC) at site j to RF at plant i
d_k	Demand at customer zone k
e_{jk}	Cost of sending one unit from DC at site j to customer zone k
ep_{kj}	Cost of reverse shipment and inspection from customer zone k to IC at site j
f_j	Fixed cost of opening a DC at site j
g_j	Fixed cost of opening an IC at site j
h_i	Fixed cost of opening an RF at plant i
H_i	Binary variable indicating whether an RF is opened at site i
i	Index for plants and RFs
j	Index for DCs and ICs
k	Index for customer zones
l	Acquisition price of the competitor
L	Used product acquisition price
n	Number of potential RFs
R_k	Collected amount of returns
s_i	Manufacturing capacity of plant i
T_j	Binary variable indicating whether an IC is opened at site j
U_{ij}	Amount shipped from plant i to DC at site j
V_{ji}	Amount shipped from IC at site j to RF at site i
W_{kj}	Amount shipped from customer zone k to IC at site j
X_{jk}	Amount shipped from DC at site j to customer zone k
Y_j	Binary variable indicating whether a DC is opened at site j

Z_0	Objective value of Benders relaxed master problem
z'_1	Objective value of Lagrangean subproblem1
z'_2	Objective value of Lagrangean subproblem2
Z_{dsp}	Objective of dual of Benders subproblem
α	Recovery ratio (fraction of returns found to be remanufacturable after inspection)
β_k	Dual variable corresponding to the related constraint in Benders subproblem
ϵ_i	Dual variable corresponding to the related constraint in Benders subproblem
γ_j	Dual variable corresponding to the related constraint in Benders subproblem
λ_k	Dual variable corresponding to the related constraint in Benders subproblem
μ_i	Dual variable corresponding to the related constraint in Benders subproblem
ω_i	Dual variable corresponding to the related constraint in Benders subproblem
ψ_{jk}	Dual variable corresponding to the related constraint in Benders subproblem
σ_{jk}	Dual variable corresponding to the related constraint in Benders subproblem
τ	Return ratio
θ_j	Dual variable corresponding to the related constraint in Benders subproblem

BB	Branch-and-bound
CD	Cross Decomposition
CDB	Cross Decomposition improvement by using Benders Decomposition
DC	Distribution centers
DSP_B	Dual of Benders subproblem
EU	European Union
GS	Golden Section

IC	Inspection centers
LB	Best lowerbound found by Benders Decomposition
LB_{LR}	Best lowerbound found by Lagrangean relaxation
LR	Lagrangean relaxation
MILP	Mixed-integer Linear Programming
MINLP	Mixed-integer Nonlinear Programming
MP	Master problem of Benders Decomposition
MD	Master problem of Lagrangean relaxation
OEM	Original Equipment Remanufacturers
P'_1	Lagrangean subproblem1
P'_2	Lagrangean subproblem2
RF	Remanufacturing facilities
SD_{η}	Dual subproblem (Lagrangean subproblem)
SP_B	Primal subproblem (Benders subproblem)
TC	Total cost
UB	Best upperbound found by Benders Decomposition
UB_{LR}	Best upperbound found by Lagrangean relaxation
UCS	Unit cost savings

1. INTRODUCTION

The importance of reverse logistics (RL) has increased in the past decade. The reuse opportunities lead to a flow of goods from the customers back to the manufacturers. The management of this reverse goods flow opposite to the conventional forward flow is the main concern of RL [1]. There are no worldwide estimates of the economic scope of reuse activities, but the number of firms engaged in this sector is growing rapidly in response to the opportunities for creating additional wealth and the growth in extended producer responsibility legislation in several countries. Take-back obligations, customer pressure, and economic motivation stimulate a number of companies to explore options for take-back and recovery of their products [2, 3]. Unfortunately, even with this significant development for the RL market in recent years, not enough analytical models exist which assist in RL strategic decisions.

European Union (EU) has two directives in effect to deal with the fast increasing waste stream of electrical and electronic equipment and complements: The first one is recycling of electrical and electronical home devices (2002/96/EC WEEE)[1] . The second one is about the limitation of the use of some hazardous materials (2002/95/EC RoHS). The WEEE directive covers a wide range of products such as large household appliances, small household appliances, IT and telecommunications equipment, consumer equipment, lighting equipment, electric tools, toys, sport and leisure equipment, medical devices, monitoring and control devices, and automated devices.

It is clear that as a candidate country targeting a full membership of EU, similar environmental directives prepared by The Ministry of Environment and Forests will also be effective in Turkey. The limitation of the use of some hazardous materials is ensured by the RoHS directive that has been prepared by The Ministry of Environment and Forests, and published in the Official Gazette on 30.05.2008 by number 26891. It will be effective as of 30 May 2009. Besides, the legislation about the control and management of used electrical and electronical home devices has been prepared as a draft

[1]WEEE, http://ec.europa.eu/environment/waste/weee/index_en.htm

version based on the WEEE of the EU. That directive includes the principle of the "Responsibility of the Manufacturer". This responsibility includes the remanufacturing of a predetermined percent of used products. At the same time, producers and importers have to carry out remanufacturing activities, compare with the marketed amount and verify this information. That directive is valid for all products and producers regardless of sales channels such as direct, remote, internet etc.

New planned legal regulations in Turkey for EU will enforce Turkish producers to recover and recycle at least a predetermined fraction of sold products. These activities involve collection of used products, inspection/separation to determine the condition of the return (i.e., whether it is recoverable or not), reprocessing the return (which may include reuse, recycling, remanufacturing or repair), disposal of returns which are found to be unrecoverable due to economic and/or technological reasons, and redistribution of recovered products [4].

The objective of this research is to determine the used product collection strategy in a reverse logistics framework. We focus on a problem which Turkish companies will face in near future and suggest a strategy for both used product acquisition and collection network design issues. We develop a mathematical programming model to answer questions such as "What should the pricing strategy of a firm be considering the reverse logistics issues?" and "How should a collection network be designed and applied?". Then, we suggest different solution techniques, and compare them on the basis of accuracy and efficiency.

The thesis is organized as follows. The second chapter summarizes the main characteristics of the reverse logistics networks and includes a review of the relevant literature. The proposed model is formulated in Chapter 3. The solution methodologies including Lagrangean relaxation, Benders Decomposition, and Cross Decomposition developed to solve the problem are explained in Chapter 4 along with the related literature on these techniques. Computational results obtained on randomly generated problem instances are presented in Chapter 5. Finally, Chapter 6 offers a conclusion and suggestions for future research.

2. REVERSE LOGISTICS NETWORK DESIGN

2.1. The Main Characteristics of Reverse Logistics Networks

There exist a variety of RL definitions in the literature. The first known definition of RL was published by The Council of Logistics Management [1] as "the term often used to refer to the role of logistics in recycling, waste disposal, and management of hazardous materials; a broader perspective includes all relating to logistics activities carried out in source reduction, recycling, substitution, reuse of materials and disposal". The European Working Group on Reverse Logistics [5] defines RL as follows: "The process of planning, implementing and controlling flows of raw materials, in process inventory, and finished goods from a manufacturing, distribution or use point to a point of recovery or point of proper disposal". According to Carter and Ellram [6], RL is the reverse distribution that includes resource reduction. Reverse distribution is the return, upstream movement of a good or material resulting from reuse, recycling, or disposal, and resource reduction is the minimization of waste which results in more efficient forward and reverse distribution processes. RL is different from Waste Management and Green Logistics, and it can be seen as part of sustainable development [7].

Fleischmann et al. [4] list the activities found in product recovery as follows:

- Collection of used products (returns) from product holders,
- Determining the condition of the returns by inspection and/or separation,
- Reprocessing the returns to capture their remaining value,
- Disposal of the returns which are found to be unrecoverable due to economic and/or technological reasons, and
- Redistribution of the recovered products.

The authors mention that supply uncertainty results in a more complex network structure. Güngör and Gupta [8] go over the driving forces behind companies and institutions to become active in RL and categorize the driving forces under three headings as

- Economics (direct and indirect),
- Legislation and take-back obligations, and
- Extended responsibility and customer expectations.

Brito and Dekker [7] analyze the topic from three main viewpoints: why, what and how.

- Why are things returned: They list the return reasons according to the usual supply chain hierarchy: manufacturing, distribution and customer returns. Manufacturing returns include:
  - Raw material surplus,
  - Quality-control returns,
  - Production leftovers.

  Distribution returns include:
  - Product recalls,
  - Commercial returns (e.g. unsold products, wrong/damaged deliveries),
  - Stock adjustments,
  - Functional returns.

  Customer returns include:
  - Reimbursement guarantees,
  - Warranty returns,
  - Service returns (repairs and spare-parts),
  - End-of-use,
  - End-of-life.
- What is returned: They describe the product characteristics (composition, use pattern and deterioration) which makes recovery attractive or compulsory and give examples based on real cases. Product composition is important from the point of view of ease of disassembly, homogeneity of constituting elements, presence of hazardous materials, and ease of transportation. The product use pattern shows the location of use and intensity, and duration of use.

- How RL works in practice: They list the actors and processes involved (how is value recovered from the products). Actors are the returners, the receivers and the collectors. Types of recovery are product recovery, component recovery, material recovery and energy recovery. There are four main reverse logistic processes: collection, the combined inspection / selection /sorting process, re-processing or direct recovery, and finally redistribution.

Figure 2.1 gives the reverse flows at various stages of the supply chain (Adapted from Thierry et al. [9]). Lund [10] and Jacobsson [11] describe three different types of companies that perform remanufacturing: Original Equipment Remanufacturers (OEM) which remanufacture their own products, Contracted Remanufacturers that are contracted to remanufacture products on behalf of other companies, and Independent Remanufacturers who remanufacture products with little contact with the OEM, and who need to buy or collect the used products (cores). Sometimes, these companies are paid by the last owner or distributor to pick up the discarded products [11].

Geyer and Jackson [12] examine the following constraints for the recycling and reuse of the products:

- Limited access to end-of-life products leaving the use phase,
- Limited feasibility of end-of-life product reprocessing, and
- Limited market demand for the secondary output from reprocessing.

Blackburn et al. [13] show that the time value of returned products varies widely across industries and product categories. Fisher [14] classifies the reverse supply chain strategies as efficient and responsive. Efficient strategy is chosen when delivering products at low cost is more important as in the case of functional products and responsive strategy is chosen when speed of the response is more important which occurs especially for innovative products. The positioning of the evaluation activity in the supply chain makes the main difference between the two strategies. Testing and evaluation are decentralized in order to determine the condition of the returned innovative product. When the cost efficiency is the objective, then a centralized evaluation activity

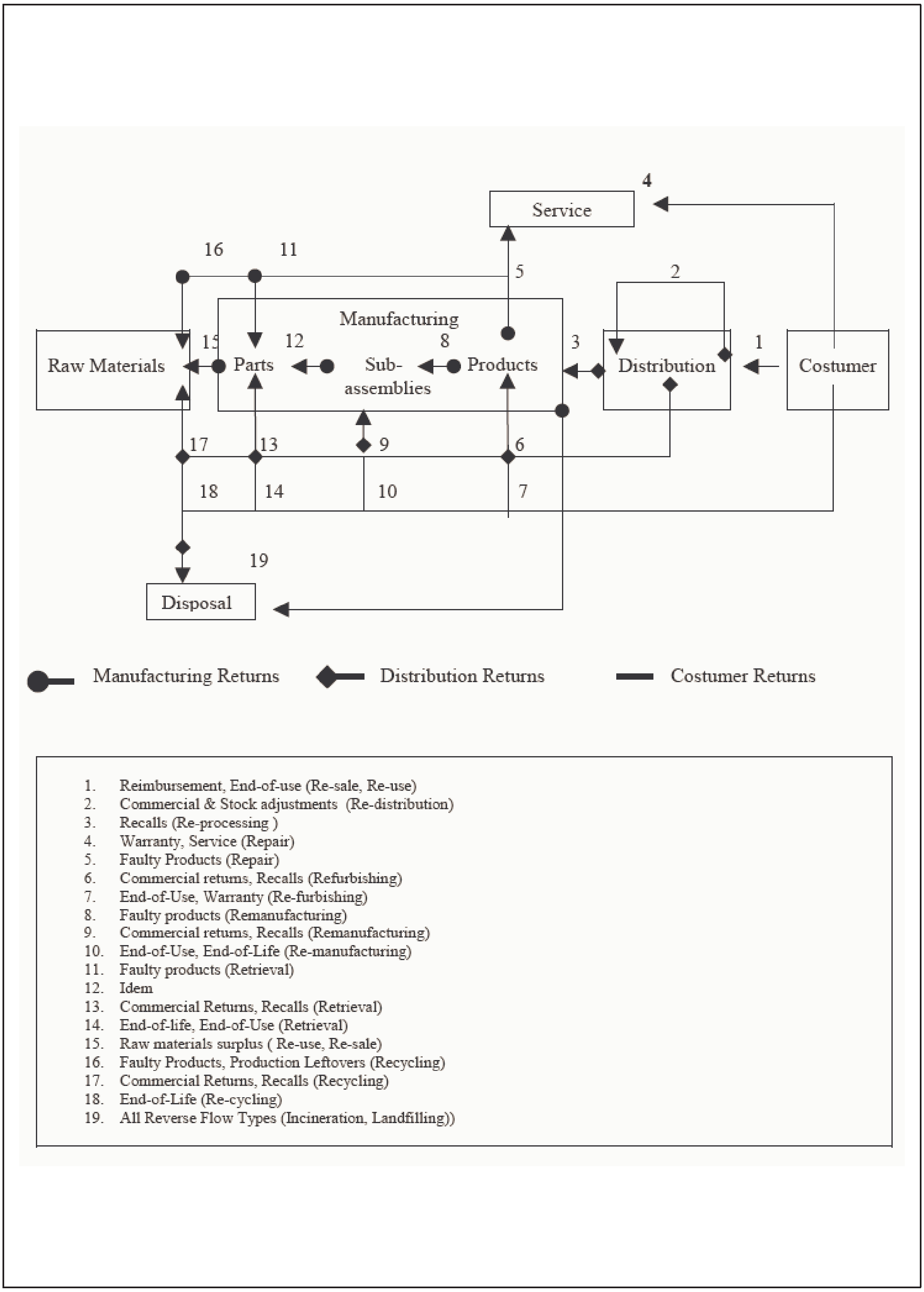

Figure 2.1. Reverse Flows in the Supply Chain

is needed. Achieving decentralization in evaluation is strongly related with technical capability of the resellers/retailers to evaluate and incentive alignments such as shared savings contracts to persuade them to the cooperation.

Sundin [15] explore how product and process design can contribute to successful remanufacturing by answering the following questions:

- Is product remanufacturing environmentally preferable in comparison to new product manufacturing and/or material recycling?
- What steps are to be included in a generic remanufacturing process?
- Which product properties are preferable for the remanufacturing steps?
- How can remanufacturing facilities become more efficient?
- How can design for remanufacturing aspects be integrated into manufacturing companies environmental management systems?

Brito and Dekker [7] develop a decision framework for reverse logistics including three levels: Strategic, tactical and operational. Recovery strategy, product design, network capacity and design, and strategic tools are included in the strategic level. At the tactical level integrating product returns with the overall organization is aimed. Then procurement, reverse distribution, coordination, production planning, inventory management, marketing, and information and technology issues are considered. Operational decision level consists of production scheduling and control, and information management.

Fleischmann et al. [16] divide the reverse logistics arena into three main areas, namely distribution planning, inventory control, and production planning. Distribution planning involves the physical transportation of used products from the end user to the manufacturer, and inventory control includes the transformation of the returned products into usable products again. Production planning consists of the scheduling of production activities related with product and material reuse taking into consideration disassembly level and high uncertainty with respect to timing, quantity and quality of the returns. They summarize the main characteristics of the RL networks as follows:

- Supply uncertainty,
- Complex network structure which includes quality inspection and combined transportation (collection and distribution),
- Large number of sources and low flow volumes.

The length of the product life cycle and the variability in the number of returns over time are very important while characterizing the RL network for outsourcing decisions according to Serrato et al. [17]. The amount of returns is affected by

- Where is a product located in its life cycle and whether it has a long or short life cycle
- Variability around expected value during each stage in the cycle
- Product's decreasing price in time
- Amount of money invested
- Cost of managing a return
- Financial incentives offered for the used products
- Amount of products managed by the firm
- Sales volume, life cycle etc. characteristics of the products
- Carry out effort for remanufacturing activity
- Inbound RL costs
- Required customer service
- Risk and control
- Importance of information reliability

2.2. LITERATURE SURVEY

Many researchers showed interest in retail collection network design from different points of view but there is lack of a complete understanding and obtaining framework of the matter. Fleischmann et al. [4] make a review of product recovery network design. Most of them include only the reverse flows and they categorize these case-based studies into three groups as bulk recycling networks, assembly product remanufacturing networks and reusable item networks. In the first group, Barros et al. [18] study a multi-level capacitated facility location problem as a mixed-integer linear program (MILP) when the volume and the locations of the demand are not previously known. They determine the optimal number, capacities and locations of the depots and cleaning facilities for recycling sand from construction waste. Louwers et al. [19] determine appropriate locations and capacities for the regional recovery centers by using investment, processing and transportation costs for carpet waste. They use volume dependent costs and develop a continuous nonlinear model and solve it by using standard software. Ammons et al. [20] use a multi-level capacitated facility location model again for carpet recycling and decide on the number and location of collection sites and processing plants and amount of carpet collected when delivery sites for recovered materials are known. Realff et al. [21] show that the volume is very important for the network design by using this model. Spengler et al. [22] propose a multi-level warehouse location model with piecewise linear cost functions for steel recycling. They decide on which recycling processes to install at which locations at what capacity level.

In the second group, Thierry [9] develops a capacitated linear programming model when the facility locations are fixed by combining forward and backward flows networks to determine the optimal flows for copy machines recovery. Berger and Debaillie [23] determine the location of disassembly centers for re-use to extend an existing distribution network. They propose multi-level capacitated MILPs for different versions of the problem to determine the location of disassembly centers and separated inspection and repair/disassembly centers. Jayaraman et al. [24] present a multi-product capacitated warehouse location model where the optimal number and locations of the remanufacturing facilities and the number of cores collected are determined considering investment, transportation, processing and storage costs for an electronic equipment remanufacturing company in USA. Krikke et al. [25] apply an MILP model in a copier manufacturer in The Netherlands for multi-echelon RL network design. Locations and goods flows for the recovery processes have been optimized and a choice has been made between two locations.

The third group includes the paper by Kroon and Vrijens [26] who focus on a deposit based system for rentable plastic containers. They model the uncapacitated warehouse location model as an MILP.

The design of RL networks requires consideration of many different issues in a multi-dimensional perspective. Ammons et al. [27] characterize forward production and distribution of the parts, the products, and reverse flows for reuse, recycling, and disposal of the used products and packaging for Electronic Assembly Reverse Production Systems. They develop an MILP model to support decision making for effective design and operation of the reverse production system and answered the following questions:

- Should sorting of used electronic products be centralized or decentralized ?
- Should they establish a single or multiple recycling centers ?
- What technology should be employed for carrying out the recycling tasks ?
- What should their expansion plan be to grow the network ?
- What are the most favorable end products and how do the locations of their recycling centers affect the profitability of the network ?
- What volumes of material are needed to justify capital intensive recycling tasks ?
- Should material be stored for future processing if existing capacity is exceeded ?

Krikke et al. [28] develop quantitative models to support decision-making concerning both the design structure of a product, i.e. modularity, repairability and recyclability, and the design structure of the logistic network. Environmental impacts are measured by linear-energy and waste functions. Economic costs are modeled as linear functions of volumes with a fixed set-up component for facilities. Then, they applied this model to a closed-loop supply chain design problem for refrigerators using real life research and development data of a Japanese consumer electronics company concerning its European operations. The model is run for different scenarios using different parameter settings such as centralized versus decentralized processing, alternative product designs, varying return quality and quantity, and potential environmental legislation based on producer responsibility.

Another issue which is taken into consideration while designing RL networks is the time value of commercial product returns [29]. Time (season), product category and global markets affect the return rates. Centralized structure (cost-efficient) vs. decentralized structure (responsive) decision also depends on the cost of the time delays and its effects on the asset recovery. Then the optimal level of the return handling capacity at the retailer and the inspection center, choice of the transportation modes with different levels of the responsiveness and the choice of the end-of-life or the return collection strategies should be decided. Souza et al. [29] study a simple queuing network model that includes the marginal value of time to identify the drivers of reverse supply chain design and examined how industry clockspeed generally affects the choice between an efficient and a responsive returns network.

Two main network design strategies in RL are drop-off and pick-up. Wojanowski et al. [30] focus on the use of a deposit-refund requirement by the government when the collection rate voluntarily achieved by the firms is deemed insufficient. They use a continuous modeling framework to design a drop-off facility network and to determine the sales price to maximize the firm's profit under a given deposit-refund. A discrete choice model with stochastic utilities is used and a parametric analysis is carried out to determine the net value that can be recovered from a returned product for the firm to voluntarily engage in collection. The authors conclude that the minimum deposit refund requirement did not achieve high collection rates for products with low return value and pointed out two complimentary policy tools that can be used by the government. They point out that the collection effectiveness depends on consumer's willingness to return a used product at the time of disposal and accessibility of collection facilities. Of course, rebate at the time of the return (incentives) will increase the willingness and pick up policies are (routing of collection vehicles) also very important. Government can use alternative policies such as taxes on the use of virgin materials, recycling subsidies, disposal fees, deposit refund requirements. Wojanowski et al. [31] also study incentive based collection strategies for product recovery by using a continuous model and stochastic utility choice model. They compare drop off and pick-up policies with respect to the acquisition of used products. They also examine the effects of variable collection cost parameters and amount of used products in collection strategy choice.

Listeş and Dekker [32] present a stochastic programming based approach by which a deterministic location model for product recovery network design may be extended to explicitly account for the uncertainties. They use GAMS as the modeling environment and CPLEX as the solver.

Listeş [33] studies a generic stochastic model for the design of systems in RL. He uses a decomposition method based on the branch-and-cut procedure known as the integer L-shaped method to solve the problem and concludes that volume is a powerful driver in integrated networks with remanufacturing options.

Ferrer and Swaminathan [34] analyze a multi-period model when remanufactured and new products are indistinguishable and conclude that if remanufacturing is very profitable, the firm tries to increase the available used products for remanufacturing.

Most of the existing models in the reverse logistics context put emphasis to the modeling aspect of the problem and use commercial software packages to solve the resulting MILPs mainly because of the increased complexity of the models. Verter and Aras [35] present a Lagrangean relaxation based solution method, evaluate its accuracy and running time-performance. By solving the same problem sequentially, i.e., optimizing first the forward flow and then the reverse flow, they identify conditions under which the sequential method provides good solutions in comparison with the integrated method.

Lu and Bostel [36] also use Lagrangean relaxation to solve a facility location model with reverse flows with three types of facilities. Although they use a similar model to ours with manufacturing, remanufacturing and intermediate centers, they do not consider acquisition prices for collecting used products. Furthermore, they have an uncapacitated model.

Marin and Pelegrin [37] analyze an MILP facility location model considering forward and backward flows and develop a heuristic solution by using Lagrangean decomposition. They assume that the number of returns is proportional to demand for each customer and the remanufacturing capacity of a plant is proportional to its manufacturing capacity.

Aras and Aksen [38] investigate a mixed-integer nonlinear facility location-allocation multi-type return model under drop-off strategy to determine both the optimal locations of the collection centers and the optimal incentive values for each return type. Customer motivation for return is the financial incentive offered by the company for the returned item and the distance to the nearest collection center. They propose a nested heuristic method based on a tabu search implementation for collection center selection and Fibonacci search to find the best incentives.

Aras et al. [39] develop a mixed-integer nonlinear facility location-allocation model to find both the optimal locations of a predetermined number of collection centers and the optimal incentive values for different return types under a pick-up strategy. Vehicles with limited capacities travel from the collection centers to customer zones to pick up used products. They propose NelderMead simplex search to obtain the best incentives and tabu search for collection center locations.

Salema et al. [40] propose an MILP formulation for the design of a reverse logistics network based on a warehouse location-allocation model where both forward and reverse flows are considered simultaneously. They first define a single product model with unlimited capacity and subsequently extend it to a multi-product capacitated recovery network model, where capacity limitations and a multi-product system is considered. They use commercial solver GAMS/Cplex.

In the work developed by Salema et al. [41], a multi-product model is proposed with capacity constraints, uncertain demand, and return rates. However, they solve their model with standard branch-and-bound (BB) techniques rather than using a decomposition method as in our study.

Salema et al. [42] develop a strategic location-allocation model for the simultaneous design of forward and reverse supply chains. Forward and reverse networks consist of two echelon structures, creating a link between factories and customers through warehouses or disassembly centers. Strategic decisions such as network design are accounted for together with tactical decisions, namely, production, storage and distribution planning. The integration between strategic and tactical decisions is achieved by developing a two-time scale, with a fully interconnected structure. This scale involves a macro time related to the strategic decisions, and a micro time related with the tactical decisions. An MILP formulation is obtained which is solved to optimality using standard BB techniques.

Min et al. [43], and Ko and Evans [44] use genetic algorithms for solving the reverse logistics models which include non-linear elements. Min et al. [43] solve only a reverse logistics network for product returns. Their model includes discounted transportation costs for large volumes. Returned products must be collected in reverse consolidation points in order to benefit from discounts. There is trade-off between inventory carrying costs of the consolidation points and freight rate discounts. Ko and Evans [44] solve also forward logistics network as in our problem. Their capacity of facilities may be expanded to different levels gradually, which makes the non-linear components of the model.

Üster et al. [45] use Benders Decomposition to solve a multi-product, single-source, closed-loop supply chain network design problem. They generate strong multiple Benders cuts by using the special decomposable structure of the single-source problem.

The following studies include a more comprehensive review. Rubio et al. [46] build up a database with the articles on reverse logistics published in the most relevant journals within the period 1995−2005. Demirel and Gökçen [47] review and classify the studies about RL network design problems for product recovery and analyze their main characteristics.

3. MODEL DEFINITION

We develop and analyze a mixed-integer nonlinear programming (MINLP) model which helps to simultaneously determine the number and locations of distribution centers (DCs), inspection centers (ICs) and remanufacturing facilities (RFs) in production/distribution systems so as to minimize the total cost. DCs receive the products from the plants and ship them to the customer zones, while inspection of the returned products is performed at ICs. Depending on the condition of the returns, they are either shipped back to the RFs or disposed of.

We consider a single product. The number and location of plants with limited capacity are given. Initially, all plants produce new products and we have to determine which of them should be equipped with remanufacturing capability. This decision incurs fixed costs. We want to locate DCs and ICs among potential sites. Opening DCs and ICs incurs also fixed costs. We note that they have unlimited capacity.

The number, locations and demand of customer zones are given. All demand should be satisfied by production or remanufacturing. At each customer zone, a fraction of the local demand is returned. The number of returns at customer zone k is a fraction of demand k. The collected amount is proportional to acquisition price and inversely proportional to competitor's acquisition price. There is only one competitor. The collected amount also depends on the unit cost savings b from a return. It can be defined as the difference between the manufacturing and remanufacturing cost per unit. Unit manufacturing and remanufacturing costs do not vary with plant location, and unit remanufacturing cost is lower than the unit manufacturing cost. Only some of the returns delivered to an IC are found to be remanufacturable after inspection.

Using the index set i for plants and RFs, j for DCs and ICs and k for customer zones, we define the parameters and variables given in List of Symbols/Abbreviations. We use the following binary variables:

$$Y_j = \begin{cases} 1 & \text{if a DC is located at site } j \\ 0 & \text{otherwise} \end{cases} \tag{3.1}$$

$$T_j = \begin{cases} 1 & \text{if an IC is located at site } j \\ 0 & \text{otherwise} \end{cases} \tag{3.2}$$

$$H_i = \begin{cases} 1 & \text{if an RF is located at site } i \\ 0 & \text{otherwise} \end{cases} \tag{3.3}$$

Flows of goods are shown in Figure 3.1:

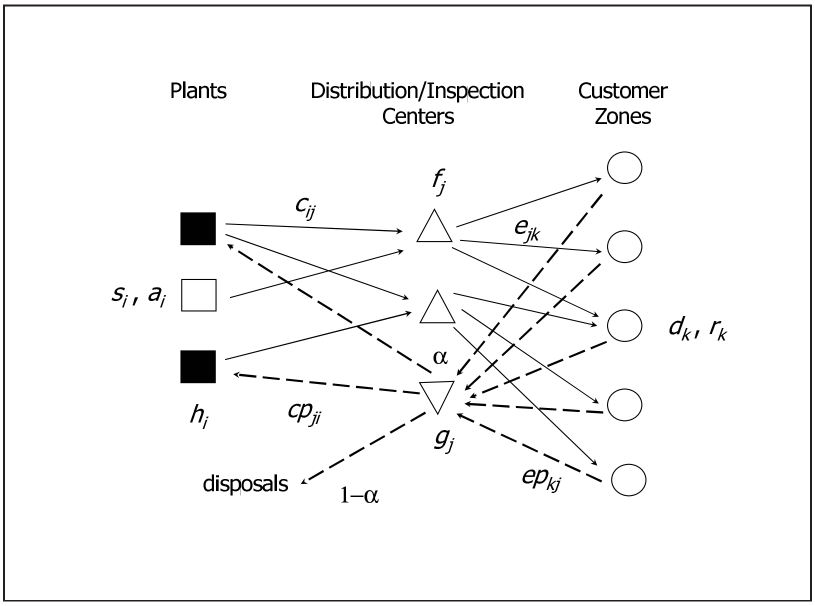

Figure 3.1. The Closed-loop Reverse Logistics Network

It is assumed that the total remanufacturing capacity of the plants is large enough to remanufacture the returns which are found to be remanufacturable after inspection and sent to RFs. Hence, the inequality $\sum_i V_{ji} \leq \sum_i a_i$ holds.

There are two opposite flows of goods in this formulation. The first one is the forward flow of finished products from the plants via DCs to customer zones, the other is the reverse flow of returned products from the customers via ICs back to the plants where facilities exist for remanufacturing.

When only the forward flow is considered, the problem is to find the number of DCs to open, to locate them at predetermined sites and assigning customers to the open DCs. This problem is called the forward problem. The reverse problem is concerned with the determination of the number and locations of ICs and RFs, and the amount of the acquisition price to be paid to collect returns. There is no capacity limitation for DCs and ICs whereas the plants are subject to capacity constraints with respect to both manufacturing and remanufacturing operations.

The unit manufacturing and remanufacturing costs are not explicitly considered in the model, because we assume that unit costs are the same at all plants. Besides this, we require that all the returns received by an RF have to be remanufactured and included in the shipment to the DCs. Therefore, total costs of manufacturing and remanufacturing are constant for any feasible solution to the problem and they are excluded in the model.

The product recovery network design problem can be formulated as the following MINLP.

$$\begin{aligned}\text{P:} \quad z \quad = \quad & \min \sum_j f_j Y_j + \sum_j g_j T_j + \sum_i \sum_j c_{ij} U_{ij} + \sum_j \sum_k e_{jk} X_{jk} + \sum_j \sum_i cp_{ji} V_{ji} \\ & + \sum_k \sum_j ep_{kj} W_{kj} + \sum_i h_i H_i + \sum_k (L-b) R_k\end{aligned}$$

s.t.

$$\sum_j X_{jk} = d_k \qquad \text{for } \forall k \tag{3.4}$$

$$\sum_j W_{kj} = R_k \qquad \text{for } \forall k \tag{3.5}$$

$$\sum_k X_{jk} = \sum_i U_{ij} \qquad \text{for } \forall j \tag{3.6}$$

$$\alpha \sum_k W_{kj} = \sum_i V_{ji} \qquad \text{for } \forall j \tag{3.7}$$

$$\sum_j U_{ij} - \sum_j V_{ji} \leq s_i \qquad \text{for } \forall i \tag{3.8}$$

$$\sum_j V_{ji} \leq \sum_j U_{ij} \qquad \text{for } \forall i \tag{3.9}$$

$$\sum_j V_{ji} \leq a_i H_i \qquad \text{for } \forall i \tag{3.10}$$

$$X_{jk} \leq d_k Y_j \qquad \text{for } \forall j,k \tag{3.11}$$

$$W_{kj} \leq \tau d_k T_j \qquad \text{for } \forall j,k \tag{3.12}$$

$$R_k = \tau d_k \frac{L}{L+l} \qquad \text{for } \forall k \tag{3.13}$$

$$X_{jk}, W_{kj}, U_{ij}, V_{ji}, L, R_k \geq 0 \qquad \text{for } \forall i,j,k \tag{3.14}$$

$$Y_j, T_j, H_i \in (0,1) \qquad \text{for } \forall i,j \tag{3.15}$$

The objective function of problem P includes the variable cost of the forward and reverse flow as well as the fixed cost of opening DCs, ICs and RFs. Constraints (3.4) show that the demand of each customer must be satisfied. Since this is not a single source model, it is possible that products are shipped from different DCs to the same customer to meet the demand. Constraints (3.5) ensure that the collected amount should be shipped to ICs. Constraints (3.6) and (3.7) are the flow conservation equations at DCs and ICs, respectively. Observe that the amount of returns being shipped from an IC to the plants is only a fraction of the returns arriving at the IC since some of the returns are not found to be remanufacturable after inspection. Constraints (3.8) ensure that the number of manufactured products is limited by the manufacturing capacity. Constraints (3.9) force the plants to remanufacture all the incoming returns received from ICs and to ship the remanufactured products along with the manufactured ones. Two important points need to be mentioned here. First, if these constraints are not included, then for any plant it is possible to receive returned products without making any shipment since satisfying the demand of DCs might be cheaper from another plant due to the lower unit transportation cost. Second, because remanufacturing is less costly than manufacturing, the remanufactured products are always included in the shipments from the plant. Constraints (3.10) ensure that remanufacturing capacity is not exceeded. Constraints (3.11) and (3.12) guarantee that forward and reverse flows are from open DCs to customer zones and from customer zones to the open ICs, respectively. Constraints (3.13) describe the collected amount of return to ICs as a function of acquisition price and competitor's acquisition price. Constraints (3.14) and (3.15) are the nonnegativity and integrality constraints, respectively.

These constraints are sufficient to define the model which includes both forward and reverse flows. The forward flow can be explained as the flow of finished products from the plants to the customer zones over DCs whereas the reverse flow is the flow of returned products from customers to the RFs over ICs. The forward and reverse problems are coupled in the above formulation by constraints (3.8) and (3.9). It is easy to observe that these constraints include both the variables U_{ij} of the forward problem and V_{ji} of the reverse problem.

4. SOLUTION METHODOLOGY

Now, we develop a solution method for the model P. It is based on the observation that when the acquisition price is fixed, the remaining problem becomes an MILP that can be solved by Lagrangean Relaxation, Benders Decomposition, and Cross Decomposition methods which are alternative solution techniques for the solution of MILPs. These methods are compared and Cross Decomposition is modified and improved. To find the best value of the acquisition price, Golden Section search (GS search) is applied recursively (see Winston[48] for GS search). The algorithm of recursive GS search is provided in Appendix A. We begin this chapter by discussing a Lagrangean relaxation of the problem which can be used to provide lower bounds on the optimal objective value.

4.1. Lagrangean Relaxation

When the acquisition price is fixed at a value L and constraints (3.4), (3.5), (3.6), (3.7), (3.8) and (3.9) of P are relaxed using Lagrange multipliers λ_k, β_k, γ_j, θ_j, μ_i, ω_i, the following subproblem P' also called the dual subproblem SD_η is obtained, where λ_k, β_k, γ_j, θ_j are unrestricted and μ_i, ω_i are positive variables. Also a redundant constraint, obtained by adding constraints (3.8) and (3.10), is added to tighten the lowerbound of the relaxed problem.

$$\sum_j U_{ij} \leq s_i + a_i H_i \qquad \text{for } \forall i \tag{4.1}$$

P′:

$$
\begin{aligned}
\min \quad & \sum_j f_j Y_j + \sum_j g_j T_j + \sum_i \sum_j c_{ij} U_{ij} + \sum_j \sum_k e_{jk} X_{jk} + \sum_j \sum_i cp_{ji} V_{ji} \\
+ \quad & \sum_k \sum_j ep_{kj} W_{kj} + \sum_i h_i H_i + \sum_k (L-b)\overline{R_k} + \sum_k \lambda_k \left(\sum_j X_{jk} - d_k \right) \\
+ \quad & \sum_k \beta_k \left(\sum_j W_{kj} - \overline{R_k} \right) + \sum_j \gamma_j \left(\sum_k X_{jk} - \sum_i U_{ij} \right) \\
+ \quad & \sum_j \theta_j \left(\alpha \sum_k W_{kj} - \sum_i V_{ji} \right) + \sum_i \mu_i \left(\sum_j U_{ij} - \sum_j V_{ji} - s_i \right) \\
+ \quad & \sum_i \omega_i \left(\sum_j V_{ji} - \sum_j U_{ij} \right)
\end{aligned}
$$

s.t.

$$
\begin{array}{rcll}
\sum_j V_{ji} & \leq & a_i H_i & \text{for } \forall i \\
X_{jk} & \leq & d_k Y_j & \text{for } \forall j,k \\
W_{kj} & \leq & \tau d_k T_j & \text{for } \forall j,k \\
\overline{R_k} & = & \tau d_k \frac{\overline{L}}{\overline{L}+l} & \text{for } \forall k \\
X_{jk}, W_{kj}, U_{ij}, V_{ji} & \geq & 0 & \text{for } \forall i,j,k \\
Y_j, T_j, H_i & \in & (0,1) & \text{for } \forall i,j \\
\sum_j U_{ij} & \leq & s_i + a_i H_i & \text{for } \forall i
\end{array}
$$

P′ can be decomposed into two subproblems P'_1 and P'_2 where

$P'_1 \; : \; z'_1 \; = \; \min \sum_i \sum_j c_{ij} U_{ij} + \sum_j \sum_i cp_{ji} V_{ji} + \sum_i h_i H_i - \sum_j \gamma_j \sum_i U_{ij} - \sum_j \theta_j \sum_i V_{ji} + \sum_i \mu_i \sum_j U_{ij} - \sum_i \mu_i \sum_j V_{ji} + \sum_i \omega_i \sum_j V_{ji} - \sum_i \omega_i \sum_j U_{ij}$

s.t.

$$\begin{aligned} \sum_j V_{ji} &\leq a_i H_i && \text{for } \forall i \\ \sum_j U_{ij} &\leq s_i + a_i H_i && \text{for } \forall i \\ U_{ij}, V_{ji} &\geq 0 && \text{for } \forall i, j \\ H_i &\in (0,1) && \text{for } \forall i \end{aligned}$$

and

$\mathrm{P}_2' : z_2' = \min \sum_j f_j Y_j + \sum_j g_j T_j + \sum_j \sum_k e_{jk} X_{jk} + \sum_k \sum_j ep_{kj} W_{kj} + \sum_k \lambda_k \sum_j X_{jk} + \sum_j \gamma_j \sum_k X_{jk} + \sum_j \theta_j \alpha \sum_k W_{kj} + \sum_k \beta_k \sum_j W_{kj}$

s.t.

$$\begin{aligned} X_{jk} &\leq d_k Y_j && \text{for } \forall j, k \\ W_{kj} &\leq \tau d_k T_j && \text{for } \forall j, k \\ X_{jk}, W_{kj} &\geq 0 && \text{for } \forall j, k \\ Y_j, T_j &\in (0,1) && \text{for } \forall i, j \end{aligned}$$

The solution of the following Lagrangean dual gives the best lower bound:

$$\max_{\lambda_k, \beta_k, \gamma_j, \theta_j, \mu_i, \omega_i} z_1' + z_2' + \sum_k \overline{R_k}(\overline{L} - b - \beta_k) - \sum_k \lambda_k d_k - \sum_i \mu_i s_i$$

Subproblem P_1' can be rewritten as

$$z_1' = \min \sum_i h_i H_i + \sum_i \sum_j U_{ij} \left(c_{ij} - \gamma_j + \mu_i - \omega_i\right) + \sum_j \sum_i V_{ji} \left(cp_{ji} - \theta_j - \mu_i + \omega_i\right)$$

and solved separately for each RF i. The solution is then given as a summation of the optimal solutions for all RFs.

For all i, the solution procedure can be summarized as shown in Figure 4.1:

1. Find $\min_j (c_{ij} - \gamma_j + \mu_i - \omega_i)$.
   If $\min_j (c_{ij} - \gamma_j + \mu_i - \omega_i) > 0$,
   then set $u_{ij} = 0$ for all j,
   else set $u_{ij} = s_i + a_i$ for that j.
2. Find $\min_j (cp_{ji} - \theta_j - \mu_i + \omega_i)$.
   If $\min_j (cp_{ji} - \theta_j - \mu_i + \omega_i) > 0$,
   then set $v_{ji} = 0$ for all j,
   else set $v_{ji} = a_i$ for that j.
3. If $h_i + \sum_j U_{ij} \min_j (c_{ij} - \gamma_j + \mu_i - \omega_i) + \sum_j V_{ji} \min_j (cp_{ji} - \theta_j - \mu_i + \omega_i) \leq 0$,
   then set $H_i = 1$,
   else set $H_i = 0$, $V_{ji} = 0$ for that j.
4. If $\min_j (c_{ij} - \gamma_j + \mu_i - \omega_i) < 0$,
   then set $U_{ij} = s_i$,
   else set $U_{ij} = 0$ for that j.

Figure 4.1. Solution Procedure for Subproblem P'_1

Subproblem P'_2 can be considered as the summation of two independent problems. To determine the locations of DCs and flows between DCs and customers, the following algorithm in Figure 4.2 has been applied for each j:

For each k,
If $e_{jk} + \gamma_j + \lambda_k < 0$,
then set $X_{jk} = d_k$,
else set $X_{jk} = 0$ for that j, k
If $\sum_k X_{jk} (e_{jk} + \gamma_j + \lambda_k) + f_j < 0$,
set $Y_j = 1$,
else set $Y_j = 0, X_{jk} = 0$.

Figure 4.2. Solution Procedure for Subproblem P'_2

Locations of ICs and flows between ICs and RFs can also be found like above by using the algorithm explained in Figure 4.3 for each j :

For each k,

If $ep_{kj} + \alpha\theta_j + \beta_k < 0$,

then set $W_{kj} = \tau d_k$,

else set $W_{kj} = 0$ for that j, k

If $\sum\limits_k W_{kj}\left(ep_{kj} + \alpha\theta_j + \beta_k\right) + g_j < 0$,

set $T_j = 1$,

else set $T_j = 0, W_{kj} = 0$.

Figure 4.3. Solution Procedure for Subproblem P'_3

Solution of P' provides a lower bound on the optimal value of P. We use subgradient optimization to update the Lagrange multipliers. The steps of subgradient optimization is given in Appendix B. For each GS iteration, a feasible solution providing an upperbound is calculated by fixing binary variables to the values found in subproblems P'_1 and P'_2 and solving the remaining LP.

4.2. Benders Decomposition

Benders Decomposition is successfully applied to a variety of mixed-integer programming applications in the literature [49]. This procedure is based on the principle that every mixed-integer program can be separated into two parts: Benders subproblem with continuous variables and Benders master problem with the complicating integer variables and one additional continuous variable. Solving these problems successively yields an efficient solution procedure for problems having appropriate structures.

4.2.1. Benders Subproblem

Fixing the binary location variables Y_j, T_j and H_i to feasible integer values $\overline{Y_j}$, $\overline{T_j}$ and $\overline{H_i}$, and L to $\overline{L}$ produces the following Benders subproblem SP_B:

$$SP_B : \min \sum_i \sum_j c_{ij} U_{ij} + \sum_j \sum_k e_{jk} X_{jk} + \sum_j \sum_i cp_{ji} V_{ji} + \sum_k \sum_j ep_{kj} W_{kj}$$

s.t.

$$\begin{aligned}
\sum_j X_{jk} &= d_k && \text{for } \forall k && (4.2)\\
\sum_j W_{kj} &= \tau d_k \frac{\overline{L}}{\overline{L}+l} && \text{for } \forall k && (4.3)\\
\sum_k X_{jk} &= \sum_i U_{ij} && \text{for } \forall j && (4.4)\\
\alpha \sum_k W_{kj} &= \sum_i V_{ji} && \text{for } \forall j && (4.5)\\
\sum_j U_{ij} - \sum_j V_{ji} &\leq s_i, && \text{for } \forall i && (4.6)\\
\sum_j V_{ji} &\leq \sum_j U_{ij} && \text{for } \forall i && (4.7)\\
\sum_j V_{ji} &\leq a_i \overline{H_i} && \text{for } \forall i && (4.8)\\
X_{jk} &\leq d_k \overline{Y_j} && \text{for } \forall j,k && (4.9)\\
W_{kj} &\leq \tau d_k \frac{\overline{L}}{\overline{L}+l} \overline{T_j} && \text{for } \forall j,k && (4.10)\\
X_{jk}, W_{kj}, U_{ij}, V_{ji} &\geq 0 && \text{for } \forall i,j,k && (4.11)
\end{aligned}$$

By using dual variables $\lambda_k, \beta_k, \gamma_j, \theta_j, \mu_i, \omega_i, \epsilon_i, \sigma_{jk}, and \psi_{jk}$ corresponding to the constraints (4.2), (4.3), (4.4), (4.5), (4.6), (4.7), (4.8), (4.9), and (4.10) respectively, the dual problem DSP$_B$ of SP$_B$ can be written as:

$$\begin{aligned}
DSP_B : \max Z_{dsp} &= \sum_k d_k \lambda_k - \sum_i \mu_i s_i - \sum_i \epsilon_i a_i \overline{H_i} - \sum_j \sum_k \sigma_{jk} d_k \overline{Y}_j \\
&\quad - \sum_j \sum_k \psi_{jk} \tau d_k \overline{T}_j + \sum_k \tau \beta_k d_k \frac{\overline{L}}{\overline{L}+l}
\end{aligned}$$

s.t.

$$
\begin{array}{rcll}
-\gamma_j - \mu_i + \omega_i & \leq & c_{ij} & \text{for } \forall i, j \\
\lambda_k + \gamma_j - \sigma_{jk} & \leq & e_{jk} & \text{for } \forall j, k \\
-\theta_j + \mu_i - \omega_i - \epsilon_i & \leq & cp_{ji} & \text{for } \forall i, j \\
\beta_k + \alpha\theta_j - \psi_{jk} & \leq & ep_{kj} & \text{for } \forall j, k \\
\mu_i, \omega_i, \epsilon_i, \sigma_{jk}, \psi_{jk} & \geq & 0, & \text{for } \forall i, j, k \\
\multicolumn{4}{c}{\lambda_k, \beta_k, \gamma_j, \theta_j \;\; \text{unrestricted in sign}}
\end{array}
$$

The solution of DSP_B is used to generate Benders' cuts which is added to the relaxed master problem as:

$$\min Z_0$$

s.t.

$$Z_0 \geq \sum_j f_j \overline{Y_j} + \sum_j g_j \overline{T_j} + \sum_i h_i \overline{H_i} + \sum_k (\overline{L} - b)\tau d_k \frac{\overline{L}}{\overline{L} + l} + Z_{dsp}$$

If the solution of DSP_B is unbounded, a ray is added to the relaxed master problem, which is found by solving the following problem:

$$\max dummy$$

s.t.

$$
\begin{array}{rcll}
\sum_k d_k \lambda_k - \sum_i \mu_i s_i - \sum_i \epsilon_i a_i \overline{H_i} - \sum_j \sum_k \sigma_{jk} d_k \overline{Y}_j & & & \\
- \sum_j \sum_k \psi_{jk} \tau d_k \overline{T}_j + \sum_k \tau \beta_k d_k \frac{\overline{L}}{\overline{L}+l} & = & 1 & \\
-\gamma_j - \mu_i + \omega_i & \leq & 0 & \text{for } \forall i, j \\
\lambda_k + \gamma_j - \sigma_{jk} & \leq & 0 & \text{for } \forall j, k \\
-\theta_j + \mu_i - \omega_i - \epsilon_i & \leq & 0 & \text{for } \forall i, j \\
\beta_k + \alpha\theta_j - \psi_{jk} & \leq & 0 & \text{for } \forall j, k
\end{array}
$$

If we summarize the procedure, Benders Decomposition algorithm [49] can be stated as in Figure 4.4:

1. initialization
2. fix binary variables to feasible values
3. set $LB := -\infty$
4. set $UB := \infty$
5. while $UB - LB > \epsilon$ do
   - solve DSP_B
   - if unbounded then
     get a ray
     add cut to master problem
   - else
     get extreme point
     add cut to master problem
     update UB if incumbent solution is better
   - end if
   - solve master problem
     $\min_{Y,T,H} \{z|cuts\}$
     update LB as $\overline{z}$
- end while

Figure 4.4. Benders Decomposition Algorithm

4.3. Cross Decomposition

Benders Decomposition is generally used for solving capacitated facility location problems [50] and certain classes of difficult problems such as stochastic programming problems and MINLP problems [49]. It has been used to reduce the computational difficulty of the moderate size problems by exploiting the special structure of the problem. Our problem is a large scale MINLP and it is computationally demanding to use conventional methodologies including Benders Decomposition. We use GS search to deal with the nonlinear part for each acquisition price value L set by a GS search iteration. Then, we solve the remaining MILP problem.

Benders Decomposition method exploits only the primal structure of the problem. However, our problem can be solved by using both primal and dual decomposition algorithms, so we use Cross Decomposition to solve the MILP part. It is one of the most powerful tools to solve large scale facility location problems [50]. Cross Decomposition was designed to exploit the primal and dual structure simultaneously which can be obtained by price directive decomposition (Lagrangean or Dantzig-Wolfe Decomposition) and resource directive decomposition (Benders Decomposition) respectively. It can be used to reduce the computational complexity by incorporating Benders Decomposition and Lagrangean relaxation techniques into a single framework of a standard decomposition scheme. When both techniques are put into one scheme, both primal and dual subproblems are easy to solve since the difficult constraints are in the master problems. Furthermore, Van Roy [51] show that primal and dual subproblems are relaxed master problems for each other.

In our problem, the primal subproblem SP_B can be obtained by fixing the primal variables Y_j, T_j and H_i to binary values in P. The dual subproblem SD_η can be obtained by fixing the dual variables λ_k, β_k, γ_j, θ_j, μ_i, and ω_i corresponding to the constraints (3.4), (3.5), (3.6), (3.7), (3.8) and (3.9) in P. The dual subproblem is also called Lagrangean subproblem or Lagrangean relaxation of P relative to the constraints (3.4), (3.5), (3.6), (3.7), (3.8) and (3.9).

Initializing the primal variables and ping-ponging between the following two steps produce upper and lower bounds on the optimal value of P in an iterative manner:

i) Set Y_j, T_j and H_i to one to provide an initial feasible solution.
ii) Solve the dual problem DSP_B of the primal subproblem SP_B to find dual variables λ_k, β_k, γ_j, θ_j, μ_i, ω_i.
iii) Set dual variables λ_k, β_k, γ_j, θ_j, μ_i, ω_i to current values and solve the dual subproblem SD_η for Y_j, T_j and H_i. Set Y_j, T_j and H_i to current values and go to step (ii).

These successive steps may produce tight bounds and if there is no duality gap, then equal values for lower and upper bound can be obtained, which shows the optimal objective value [52]. However convergence or monotonic improvement of bounds is not guaranteed. Van Roy incorporates these ping-ponging steps into a standard decomposition scheme to ensure progress toward an optimum as follows. Solutions of SP_B and SD_η have been used to generate new cuts for master problems of Benders Decomposition (MP) and Lagrangean relaxation (MD). We used subgradient optimization.

4.3.1. Primal (Benders) Decomposition

To get a primal subproblem, the binary variables Y_j, T_j and H_i should be fixed to zero or one in P. Then, for given feasible Y_j, T_j and H_i, the primal subproblem is:

$$SP_B : \min \quad \sum_i \sum_j c_{ij} U_{ij} + \sum_j \sum_k e_{jk} X_{jk} + \sum_j \sum_i cp_{ji} V_{ji} + \sum_k \sum_j ep_{kj} W_{kj}$$

s.t.

$$\begin{aligned}
\sum_j X_{jk} &= d_k && \text{for } \forall k \\
\sum_j W_{kj} &= \tau d_k \tfrac{\overline{L}}{\overline{L}+l} && \text{for } \forall k \\
\sum_k X_{jk} &= \sum_i U_{ij} && \text{for } \forall j \\
\alpha \sum_k W_{kj} &= \sum_i V_{ji} && \text{for } \forall j \\
\sum_j U_{ij} - \sum_j V_{ji} &\leq s_i && \text{for } \forall i \\
\sum_j V_{ji} &\leq \sum_j U_{ij} && \text{for } \forall i \\
\sum_j V_{ji} &\leq a_i \overline{H_i} && \text{for } \forall i \\
X_{jk} &\leq d_k \overline{Y_j} && \text{for } \forall j, k \\
W_{kj} &\leq \tau d_k \overline{T_j} && \text{for } \forall j, k \\
X_{jk}, W_{kj}, U_{ij}, V_{ji} &\geq 0 && \text{for } \forall i, j, k
\end{aligned}$$

The dual solution of SP_B provides an upper bound for the optimal value of P and Lagrangean (dual) multipliers corresponding to the related constraints. Lagrangean multipliers are used to generate Benders' cuts and Lagrangean (dual) subproblem.

4.3.2. Dual Decomposition (Lagrangean Relaxation)

Subproblem SD_η has been formulated and solved as explained in section Lagrangean Relaxation 4.1. Solution of SD_η provides a lower bound on the optimal value of P. Primal variables are used to generate Benders (primal) subproblem and subgradient optimization when needed.

4.3.3. Convergence Tests

Although the subproblems are master problems for each other, P cannot be solved just by iterating between them. Cycling may occur if a duality gap exists. So, convergence tests are used to prevent the algorithm from cycling between primal and dual subproblems. Figure 4.5 shows the Cross Decomposition method. A solution Y_j, T_j and H_i to SD_η is used for constructing the corresponding SP_B, while the dual solution λ_k, β_k, γ_j, θ_j, μ_i, ω_i to DSP_B is used for SD_η. Both subproblems are solved in an iterative manner until optimality is reached. If a primal convergence test fails in any iteration, Benders master problem is solved to obtain a new set of primal variables to generate the next primal subproblem. If a dual convergence test fails in any iteration, subgradient optimization is used to obtain a new set of dual variables to generate the next dual subproblem. For each GS iteration, a Lagrangean upperbound has been calculated by fixing binary variables to the values found in Cross Decomposition algorithm and solving the remaining LP.

The pseudocode of the Cross Decomposition algorithm has been explained in Figures 4.6, 4.7:

UB: Best upperbound found by Benders Decomposition.

LB: Best lowerbound found by Benders Decomposition.

UB_{LR}: Best upperbound found by Lagrangean relaxation.

LB_{LR}: Best lowerbound found by Lagrangean relaxation.

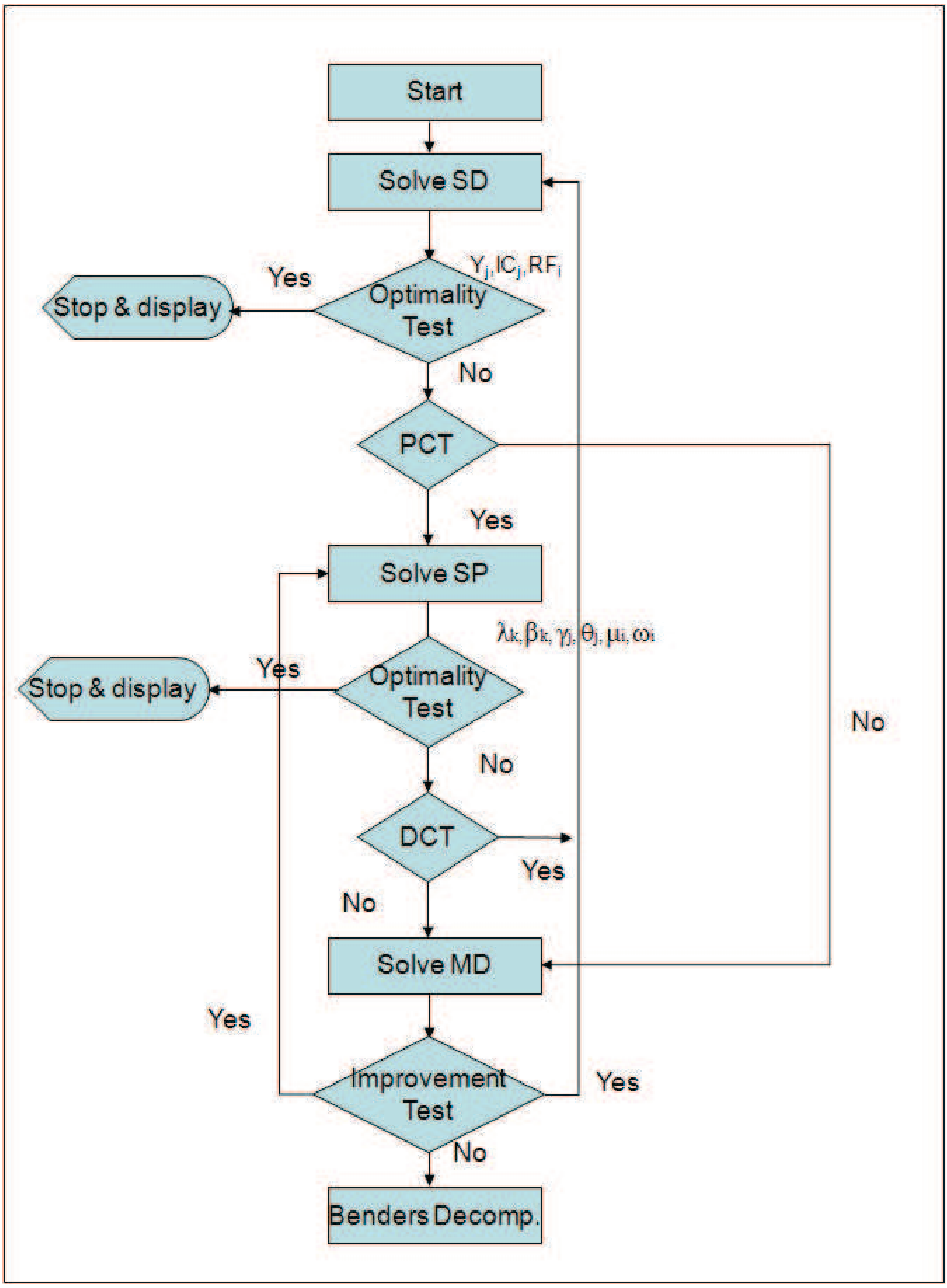

Figure 4.5. Cross Decomposition Flowchart

1. Set $UB = +\infty$ and $LB = -\infty$. Set $UB_{LR} = +\infty$ and $LB_{LR} = -\infty$.
2. Set gradients and dual variables to 0.
3. Set flag $continue = 1$, $\pi = 2$, $noimprovement = 0$.
4. While continue = 1 ,
   (a) Solve Lagrangean subproblem by inspection (Lagrangean relaxed problem),
      i. Set binary variables free (lower bounds to zero, upper bounds to 1),
      ii. Fix dual variables (Lagrange multipliers) at the last solution,
      iii. Solve Lagrangean subproblem by inspection to find a lower bound,
      iv. If binary variables ≤ 0.5 set them to 0, else set them to 1,
      v. If all binary variables are 0 for any type (Y_j,T_j,H_i), open the first facility (set binary variable to 1) for that type (i.e RF(1)=1),
      vi. If total capacity of open RFs are less than returned amount for remanufacturing, calculate the number of facilities needed to satisfy that difference and set as many RF as to 1.

   (b) Optimality test,
      i. if $lowerbound \geq LB_{LR}$ then set $LB_{LR} = lowerbound$,
      ii. if $UB - LB_{LR} \leq 0.01$, stop (set $continue = 0$),
      iii. if $UB_{LR} - LB_{LR} \leq 0.01$, stop (set $continue = 0$).

   (c) Convergence test: Calculate Benders' master problem's objective,
      i. if it is greater than UB, go to step 4g,
      ii. else go to step 4d.

   (d) Solve Benders Subproblem
      i. Fix binary variables to the last solution,
      ii. Set dual variables free (ie. set lower bounds to zero, upper bounds to 1 for positive dual variables),
      iii. Solve subproblem to find an upperbound z,
      iv. If it is unbounded, solve the modified subproblem,
      v. if $z \leq UB$, then set $UB = z$.

   (e) Optimality Test,
      i. If $UB - LB \leq 0.01$, stop(set continue=0),
      ii. If $UB - LB_{LR} \leq 0.01$, stop, set $continue = 0$.

Figure 4.6. Cross Decomposition Algorithm

(f) Convergence Test: Calculate Lagrangean relaxed problem (subproblem) objective,

i. If it is less than LB_{LR} go to step 4h,

ii. else go to 4a.

(g) Solve Benders Master Problem,

i. Set binary variables free (set lower bounds to zero, upper bounds to 1),

ii. Fix dual variables (Lagrange multipliers) to the last solution,

iii. Solve Benders master problem,

iv. If binary variables ≤ 0.5 set to 0, else to 1,

v. Go to 4d.

(h) Use subgradient optimization,

i. Fix binary variables to the last solution,

ii. Hold the last solution found for X_{jk},U_{ij},V_{ji},W_{kj},R_k,

iii. Solve original problem to find upperbound for Lagrangean relaxation,

iv. if $lowerbound \geq \mathrm{LB}_{LR}$, set $LB_{LR} = lowerbound$, $noimprovement = 0$,

v. if $upperbound \leq \mathrm{UB}_{LR}$, set $UB_{LR} = upperbound$, $noimprovement = 0$, else increment noimprovement by one unit,

vi. if $noimprovement \geq 30$, set $\pi = \pi/2$; $noimprovement = 0$, else increment noimprovement by one unit,

vii. calculate gradients,

viii. calculate step size,

ix. if $UB_{LR} - LB_{LR} \leq 0.01$, stop (set $continue = 0$),

x. update Lagrangean multipliers,

xi. Go to step 4.

Figure 4.7. Cross Decomposition Algorithm Continued

4.4. Cross Decomposition Improvement using Benders Decomposition

When the gap between the upper and lower bounds obtained from both subproblems remains persistent after a certain number of iterations, the branch and bound (B&B) phase is activated to close the gap. Van Roy also suggests to use Benders Decomposition alternatively in lieu of B&B. So we prefer to use Benders Decomposition when the gap between best lower bound obtained from Lagrangean subproblem and best upper bound obtained from Benders Decomposition reaches a predetermined percentage or half of the computation limit time is spent.

5. COMPUTATIONAL RESULTS

5.1. Preliminary Experiments

Prior to the main experiments, the following three methods have been compared to solve the problem:

(i) For each GS search iteration where the acquisition price value L is fixed, the remaining MILP is solved by Lagrangean relaxation (GS + LR),

(ii) For each GS search iteration where the acquisition price value L is fixed, the remaining MILP is solved by the commercial Cplex solver (GS + C),

(iii) For each integer L value fixed in the search interval, the remaining MILP is solved by the commercial Cplex solver (ES + C).

Two different data sets with two different unit cost savings and two fixed cost values are generated. Table 5.1 summarizes the instances used for the experiments. The search

Table 5.1. Scenarios for Preliminary Experiments

Instance	**Dataset**	**Unit Cost Savings**	**Fixed Cost**
1	1	20	5000-7500-10000
2	1	20	10000-15000-20000
3	1	40	5000-7500-10000
4	1	40	10000-15000-20000
5	2	20	5000-7500-10000
6	2	20	10000-15000-20000
7	2	40	5000-7500-10000
8	2	40	10000-15000-20000

interval has been set as $\left[\frac{l}{2}, \max\left\{1.5l, b\right\}\right]$ for L. In both data sets, the problem size is (x, y, z)=(5-10-20) where x shows the number of plants and potential RFs, y shows the number of potential DCs and/or ICs, and z shows the customer zones. We have selected the following parameters:

$$\begin{aligned} \tau &= 0.7 \\ \alpha &= 0.7 \\ l &= 10 \end{aligned}$$

The values assigned to the unit cost savings are 20 and 40. By assigning two distinct values to f_j (5000 and 10000), to g_j (7500 and 15000) and to h_i (10000 and 20000), we obtain two different fixed cost instances where each instance is shown as a triplet (f_j-g_j-h_i). These preliminary tests have been run on AMD Athlon (tm) 64X2 Dual Core Processor 4600+ 2.41 Ghz and 3.93 GB of RAM. All three methods have been coded within the GAMS v22.2 suite and solved by Cplex solver called from within GAMS.

In these experiments Lagrangean subproblems are solved by inspection as explained in Section 4.1. Then, we apply GS search to find the acquisition price L minimizing the total cost. We present the results in Table 5.2. Depending on the results, we conclude that the best method in terms of efficiency and accuracy is applying GS search with Cplex. The reason why GS search with Cplex yields slightly better solutions than Exhaustive Search is that the latter is limited by unit increments. The relaxation of six constraints degrade the quality of the lower bound in LR and there is no advantage of using LR in terms of both accuracy and solution time performance. So, we choose to apply GS search to find L and solve the remaining MILP with different algorithms.

Before assessing the performance of the different methods for larger problem instances, we perform additional tests using Exhaustive Search in the problem size (5-10-20) by enlarging the search interval of L to $(0, 100)$ and trying unit cost savings values in the interval $[0, 150]$ by increments of 10 to explore the nature of the total cost function. To ensure that a good solution is found, GS search is applied in an iterative manner. Our model is a highly nonconvex MINLP, and commercial solvers have a great

Table 5.2. Results for 5-10-20 Problem Instances

	Total Cost			*L*			**Time (sec)**		
Instances	**GS + C**	**ES + C**	**GS + LR**	**GS + C**	**ES + C**	**GS + LR**	**GS + C**	**ES + C**	**GS + LR**
1	28,610	28,610	28,688	7.07	7	6.95	4	45	1,000
2	61,110	61,110	61,188	7.07	7	6.95	5	44	977
3	19,068	19,070	19,218	12.18	12	12.74	4	47	839
4	51,568	51,570	51,718	12.18	12	12.74	4	45	1,395
5	28,532	28,533	19,163	6.99	7	4.25	6	55	1,986
6	61,032	61,033	62,119	6.99	7	4.18	6	57	2,996
7	18,589	18,589	39,086	12.09	12	13.09	6	55	2,551
8	51,089	51,089	91,543	12.09	12	12.09	6	52	2,706
Averages	39,950	39,950	46,590	10	10	9	5	50	1,806

deal of difficulty in handling such models. We have also tried other commercial solvers such as SBB, OQNLP, and DICOPT and the results were not satisfactory.

When the fixed cost triple is (5000-7500-10000), it is not cost effective to collect used products if their unit cost savings is below 80. When the fixed cost are twice as large, it is profitable to collect used products only if their unit cost savings is above 140. The full list of all graphs obtained are given in Appendix C.

We used the following capacity formula:

$$a_i = \left\lfloor \frac{\alpha\tau \sum_k d_k 1.5}{n} \right\rfloor$$

$$s_i = \left\lfloor \frac{1.8 \sum_k d_k - a_i n}{n} \right\rfloor$$

5.2. Main Results

As pointed out before, when the acquisition price is set to a value, the remaining problem becomes an MILP that can be solved by LR, Benders Decomposition (BD), and Cross Decomposition improvement using Benders Decomposition (CDB) methods. We decide not to use LR depending on the results in the previous section. In this section, we compare the performances of BD and CDB with that of Cplex solver in terms of the accuracy and efficiency. The methods are tested with randomly generated data sets of the following sizes: 10-50-100 and 10-50-200. In these experiments Benders' subproblem and master problem are solved by Cplex employed with the dual simplex method for LP, and Lagrangean subproblem in CD is solved by inspection. All tests were run on Intel(R) Xeon(R) CPU X5460@ 3.16 GHz, 27.9 GB of RAM. GS search has been applied iteratively to find the best acquisition price minimizing the total cost.

5.2.1. Selection of the Parameters

We try two different types of capacity: tight and normal. Tight capacity is the case where total demand cannot be satisfied without using remanufacturing facilities. They have been set according to the following formula:
Normal Capacity

$$\begin{aligned} a_i &= \left\lceil \frac{\alpha\tau \sum_k d_k}{n} \right\rceil \\ s_i &= \left\lceil \frac{\sum_k d_k}{n} \right\rceil \end{aligned}$$

Tight Capacity

$$\begin{aligned} a_i &= 5 \left\lceil \frac{\sum_k d_k - 0.8 \sum_k d_k}{n} \right\rceil \\ s_i &= 0.8 \left\lceil \frac{\sum_k d_k}{n} \right\rceil \end{aligned}$$

For tight capacity, lower bound of the search interval for L has been modified as

$$\frac{l\left(\sum_k d_k - s_i n\right)}{\alpha\tau \sum_k d_k \left(1 - \frac{sum(k,d(k)) - s_i n}{\alpha\tau \sum_k d_k}\right)}$$

to ensure feasibility.

By assigning two distinct values to fixed cost values (f_j, g_j, h_i) =(5000-7500-10000) and (f_j, g_j, h_i) = (10000-15000-20000), three distinct values to b (20, 80, 140) and two distinct values to capacity (normal, tight), we obtain $2 \times 3 \times 2 = 12$ instances for each problem size. The cost and demand parameters are generated as follows: First, all (x, y) coordinates are randomly generated in (0,1) with 5 digit accuracy for plants/potential sites of RFs, potential sites for DCs/ICs and customer zones. Then the cost parameters have been determined by calculating the Euclidean distances between these points. Demand parameters are randomly, uniformly generated in (0,100). Table 5.3 summarizes the instances used for the experiments.

Table 5.3. Problem Instances

Instance	**Fixed Cost**	**Capacity**	b
1	5000-7500-10000	Normal	20
2	5000-7500-10000	Normal	80
3	5000-7500-10000	Normal	140
4	5000-7500-10000	Tight	20
5	5000-7500-10000	Tight	80
6	5000-7500-10000	Tight	140
7	10000-15000-20000	Normal	20
8	10000-15000-20000	Normal	80
9	10000-15000-20000	Normal	140
10	10000-15000-20000	Tight	20
11	10000-15000-20000	Tight	80
12	10000-15000-20000	Tight	140

5.2.2. Results for 10-50-100 Problem Instances

We set a time limit of one hour for each GS search iteration and make comparisons between Cplex and proposed algorithms. The results are presented in Table 5.4. When b is 20 and regular production capacity is sufficient to satisfy the total demand (normal capacity case), used products are not collected. Cplex detects this situation by fixing the acquisition price value L to zero. BD and CDB algorithms cannot find this solution in one hour, when L is equal to zero. When b is 20 and regular production capacity is not sufficient to satisfy the total demand (tight capacity case), the minimum required amount is collected according to all three methods. When b increases, the proposed algorithms show better performance in terms of accuracy. If capacity is tight, we observe a slight improvement in performance. When b is 140, fixed cost is low, and capacity is normal BD and CDB have a per cent deviation of 0.2 from the result of Cplex. CDB finds the same results as BD, because CDB algorithm has been processed as pure BD after some conditions are satisfied as explained in Section 4.4. On the basis of these results we conclude that the objective values obtained by BD and CDB are close to the ones provided by Cplex, when the unit cost savings is high.

5.2.3. Results for 10-50-200 Problem Instances

We repeat the same experiments performed in the previous subsection for a 10-50-200 problem. The results are presented in Table 5.5. The proposed algorithms show a good performance in terms of accuracy, when the fixed costs and unit cost savings are high, and the capacity is tight. When b is 80, fixed cost is high, and the capacity is tight, BD shows the best performance by detecting an L value which minimizes the total cost by 6.87 per cent less than the best feasible solution found by Cplex. When fixed costs are high, the proposed algorithms either outperform Cplex or within one per cent interval of Cplex results, with a sole exception. (The only exception is when b value is set to 20.) CDB finds the same results as BD except one case, because CDB algorithm is implemented as pure BD after a certain time elapses. When b value is 20, the proposed algorithms cannot find satisfactory results compared to Cplex. On the basis of these results we conclude that the objective values obtained by BD and CDB are as good as the ones provided by Cplex, when the unit cost savings is high.

5.2.4. Results for Larger Problems

Since the time performance of the proposed algorithms is worse compared to Cplex's performance, we decide to enlarge the investigated problem sizes. The different algorithms to solve MILP for each GS search iteration are tested with 4 data sets of the following sizes (20-100-100, 20-200-200, 30-400-400, 30-800-800) by fixing L to a given value. We use the following capacity formula and parameters:

Table 5.4. Results for 10-50-100 Problem Instances

	CPLEX			BD				CDB			
Instance	L	**Total Cost**	**Time**	L	**Total Cost**	**Time**	%	L	**Total Cost**	**Time**	%
1	0.00	9,393	12,157	1.11	19,538	439,908	108.00	1.11	19,538	439,932	108.00
2	15.11	-90,908	26,145	19.54	-89,879	212,693	1.13	15.06	-89,879	212,702	1.13
3	23.39	-266,141	17,819	23.51	-265,607	219,952	0.20	23.51	-265,607	219,977	0.20
4	6.90	15,265	2,011	6.90	16,014	50,462	4.91	6.90	16,014	50,540	4.91
5	15.92	-122,343	4,539	15.92	-121,324	209,101	0.83	15.92	-121,324	209,160	0.83
6	28.58	-299,999	7,598	28.58	-299,284	140,594	0.24	28.58	-299,284	140,563	0.24
7	0.00	14,393	14,350	1.11	42,038	440,080	192.07	1.11	42,038	439,948	192.07
8	10.04	-21,460	36,094	10.04	-20,851	432,700	2.84	10.04	-20,851	432,580	2.84
9	23.51	-183,810	30,547	23.51	-183,107	295,912	0.38	23.51	-183,107	295,707	0.38
10	6.90	41,297	4,458	6.90	48,514	50,474	17.48	6.90	48,514	50,503	17.48
11	15.92	-79,843	9,118	15.92	-78,824	209,052	1.28	15.92	-78,824	209,180	1.28
12	28.58	-247,499	695	28.58	-246,784	140,619	0.29	28.58	-246,784	140,672	0.29
Averages		-102,638	13,794		-98,296	236,796	27.47		-98,296	236,789	27.47

Table 5.5. Results for 10-50-200 Problem Instances

	CPLEX			BD				CDB			
Instance	L	**Total Cost**	**Time**	L	**Total Cost**	**Time**	%	L	**Total Cost**	**Time**	%
1	0.00	12,960	62,647	2.49	14,669	428,891	13.19	2.49	14,669	429,220	13.19
2	19.78	-216,410	68,040	19.69	-215,306	144,245	0.51	19.69	-215,306	144,272	0.51
3	23.36	-541,002	33,201	23.36	-539,732	209,049	0.23	23.36	-539,732	209,340	0.23
4	6.85	1,860	10,078	6.85	2,442	50,466	31.26	6.85	2,442	50,490	31.26
5	15.76	-251,743	11,108	15.76	-249,970	194,755	0.70	15.76	-249,970	194,735	0.70
6	28.60	-557,928	50,035	28.54	-576,694	140,688	-3.36	28.54	-576,694	140,717	-3.36
7	0.00	17,960	44,088	1.11	39,870	429,096	122.00	1.11	39,870	429,232	122.00
8	10.00	-136,510	95,746	14.94	-144,982	281,402	-6.21	14.94	-144,982	281,587	-6.21
9	28.57	-444,820	25,973	23.24	-456,924	220,000	-2.72	23.24	-456,924	220,262	-2.72
10	6.85	34,360	9,639	6.85	35,710	104,601	3.93	6.85	35,710	104,644	3.93
11	19.85	-194,298	18,913	15.83	-207,651	209,211	-6.87	19.80	-193,177	209,215	0.58
12	28.60	-525,428	21,912	28.54	-524,194	140,674	0.23	28.54	-524,194	140,835	0.23
Averages		-233,417	37,615		-235,230	212,756	12.74		-234,024	212,879	13.36

$a_i = \left\lfloor \alpha\tau\frac{1.5}{n}\sum_k d_k \right\rfloor$
$s_i = \left\lfloor \frac{1.8\sum_k d_k - a_i n}{n} \right\rfloor$
$\tau = 0.5$, $\alpha = 0.5$, $l = 10$, $b = 20$, $f_j = 5000$, $g_j = 7500$, and $h_i = 10000$.

We design the experiments as follows. We fix the decision variable L to 14, which simplifies our problem from MINLP to MILP. We generated four different data sets for each problem size and we made the experiments with four different methods (Cplex, BD, CD, CDB). We observed that decomposition algorithms performed well compared to Cplex.

While solving MILP problem, different time limits have been set for each problem size, as can be seen in Table 5.6. Solution time limit for Cplex has been set different from that of the proposed algorithms in the larger problems because Cplex can not find any feasible solution in 7200 seconds. MILP problem's results for $L = 14$ have

Table 5.6. Solution Time Limits for each GS Iteration

Problem Size	**Time Limit(sec)**	
	Cplex	**Proposed Algorithms**
20-100-100	3600	3600
20-200-200	3600	3600
30-400-400	28800	7200
30-800-800	43200	7200

been presented in Appendix D. Average accuracy of the experiments for each problem size has been calculated and reported in Table 5.7.

In our experiments, we observed that when the fixed cost decreases, CDB shows better performance than BD in terms of accuracy and solution time. When the problem size increases, BD's accuracy closes to that of Cplex and outperforms for the largest problem in average as it can be examined in Table 5.7.

Table 5.7. Average Accuracy for Mixed-integer Problem

	Method		
Average Accuracy (%)	**BD**	**CD**	**CDB**
20-100-100	0.94	1.68	1.31
20-200-200	0.22	2.33	0.22
30-400-400	0.02	2.53	2.53
30-800-800	-1.63	0.60	0.60
Averages	-0.11	1.79	1.17

We also controlled the effect of fixed cost on the solution time performance. For fixed cost values (f_j, g_j, h_i) =(50-75-100), we compared solution quality of CD with Cplex in terms of accuracy for (30-400-400) problem. Except the last dataset, when the fixed cost is lower, solution quality decreases compared to the results in Table D.3 when fixed cost values are (f_j, g_j, h_i) =(5000-7500-10000). Because Cplex performance for low fixed cost is better than the performance for high fix cost. Results are available in Table 5.8.

Table 5.8. Results for 30-400-400 MILP with 50-75-100 Fixed Cost

Dataset1	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	92232.17					
CD	92	83292	92540	91717	92540	0.33
Dataset2	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	91483.69					
CD	87	83028	95783	90441	95613	4.51
Dataset3	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	92687.09					
CD	100	84023	96873	91777	96141	3.73
Dataset4	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	97431.73					
CD	80	88414	101435	96233	100209	2.85

6. CONCLUSIONS

In this thesis, we dealt with a mixed-integer nonlinear reverse logistics network design problem. New products and remanufactured products are sent from new product plants and remanufacturing facilities with limited capacity via distribution centers to the customer zones. Used products are collected at customer zones and brought to the inspection centers. We assumed that the financial incentives offered by the company and company's competitor determines the willingness of customers to return their used products. The aim of this study is to investigate and develop solution methods for a nonlinear reverse logistics model. The solution method is based on the observation that when the acquisition price is set to a given value, the remaining problem becomes an MILP. Lagrangean Relaxation, Benders Decomposition and Cross Decomposition methods are used as solution methods to solve the MILP. The acquisition price minimizing the total cost is found by using Golden Section search in an iterative manner.

The solution methods proposed have been tested with the randomly generated data sets of the following sizes (10-50-100 and 10-50-200) by assigning two distinct values to fixed cost values, three distinct values to b and two distinct values to capacity. Then the performances of the proposed algorithms have been compared to that of the Cplex.

We also conducted experiments for larger size problems (20-100-100, 20-200-200, 30-400-400, and 30-800-800). We dealt with the MILP part of the problem only by fixing acquisition price in order to compare the performances of the algorithms for larger problem instances. According to the experimental results, for all the instances our algorithms showed a good performance both in terms of solution quality and running time.

As an extension for future research, the multi-product, multi-period versions could be investigated. More than one competitor could be taken into consideration and the demand for remanufactured items in the market could be separated from the demand for new products.

APPENDIX A: GOLDEN SECTION SEARCH

1. lastL=infinite
2. CostofBestL=infinite
3. push the initial lowerbound and the initial upperbound of L at the end of boundlist
4. add the initial lowerbound and the initial upperbound to the list of local extreme points
5. while stack is not empty
   (a) pull a lowerbound and an upperbound from the end of boundlist
   (b) apply Golden section search with current bounds to find localL
   (c) if |localL-extremeL| $> \epsilon$ $\forall$ extremeL in the list of local extreme points

       add localL to the list of local extreme points

       if f(localL)<=CostOfBestL then

           bestL = lastL

           CostofBestL = f(lastL)

       end if
   (d) push current lowerbound and lastL as upperbound at the end of boundlist
   (e) push lastL as lowerbound and current upperbound at the end of boundlist.

APPENDIX B: SUBGRADIENT OPTIMIZATION

1. Let π be a user defined parameter satisfying $0 \leq \pi \leq 2$. Initialize upperbound Z_{UB} (from some heuristic). Decide upon an initial set (λ_i) of multipliers
2. Solve Lagrangean lowerbound with the current set of multipliers, to get a solution (X_j) of lowerbound Z_{LB}
3. Define subgradients G_i for the relaxed constraints such as: $G_k = \sum_j X_{jk} - d_k$
4. Define a step size T by $T = \pi(Z_{UB} - Z_{LB})/\sum_i G_i'2$
5. Update λ_i by $\lambda_i = \max(0, \lambda_i + TG_i)$
6. Go to step 2 until termination criteria (parameter π reaches 0.01 where π is halved if Z_{UB} has not improved in the last 30 subgradient iterations with the current value of π) is reached.

APPENDIX C: UNIT COST SAVINGS

As can be seen in Figures C.1, C.2 C.3, C.4, C.5, C.6, C.7, C.8, C.9, C.10, C.11, C.12, C.13, C.14, C.15, C.16, C.17, C.18, C.19, C.20, C.21, C.22, C.23, C.24, C.25, C.26, C.27, C.28, C.29, C.30, C.31, C.32, when the fixed cost triple is $(5000 - 7500 - 10000)$ it is not cost effective to collect used products if their unit cost savings is below 80. If the fixed cost is twice, it is profitable to collect used products whereas their unit cost savings is above 140.

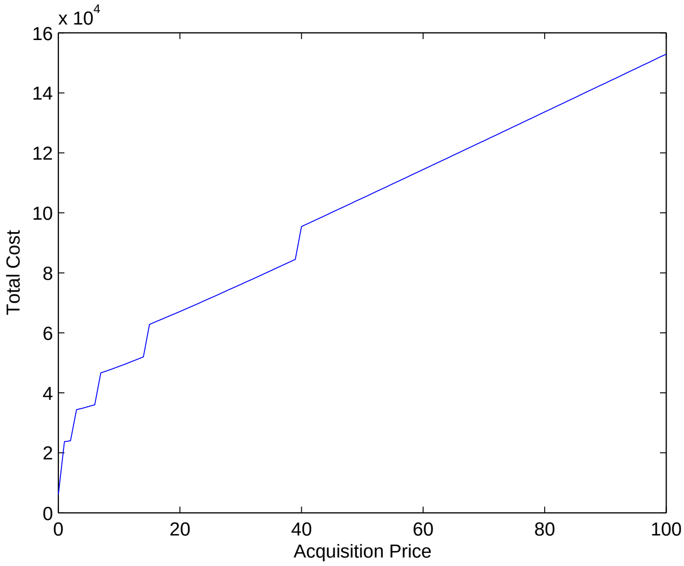

Figure C.1. Acquisition Price vs. Total Cost (b=0, (5000-7500-10000))

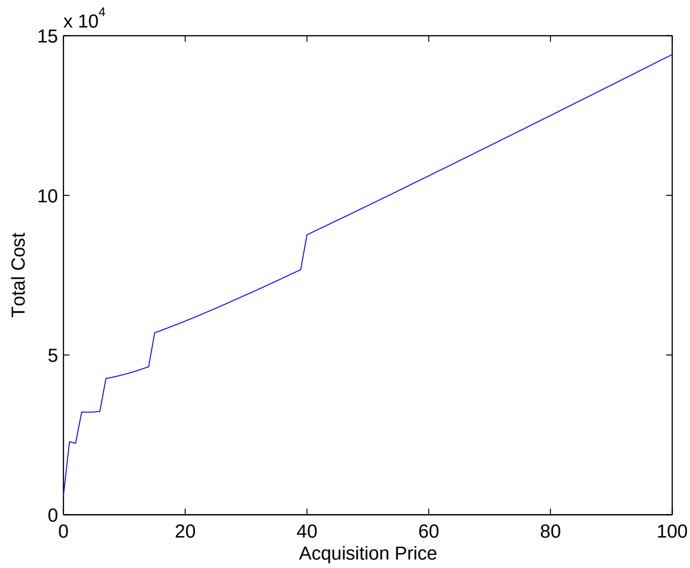

Figure C.2. Acquisition Price vs. Total Cost (b=10, (5000-7500-10000))

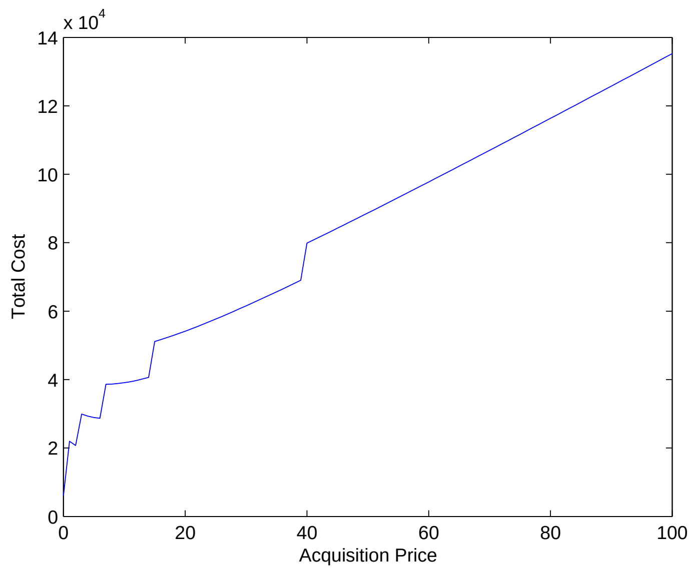

Figure C.3. Acquisition Price vs. Total Cost (b=20, (5000-7500-10000))

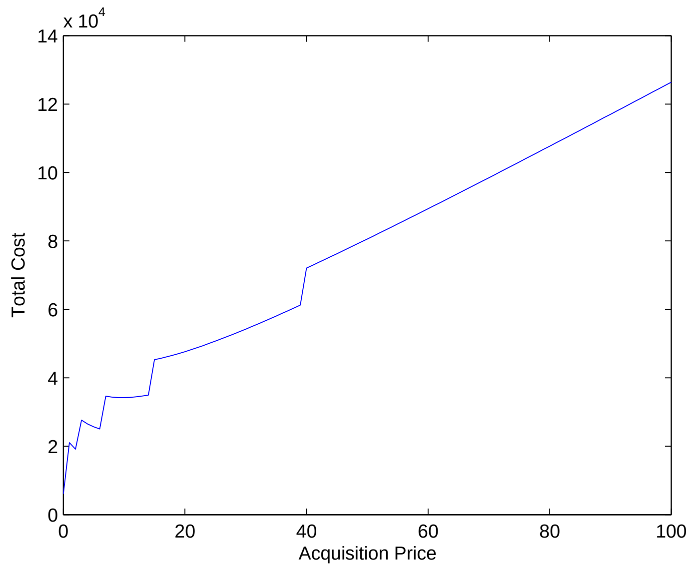

Figure C.4. Acquisition Price vs. Total Cost (b=30, (5000-7500-10000))

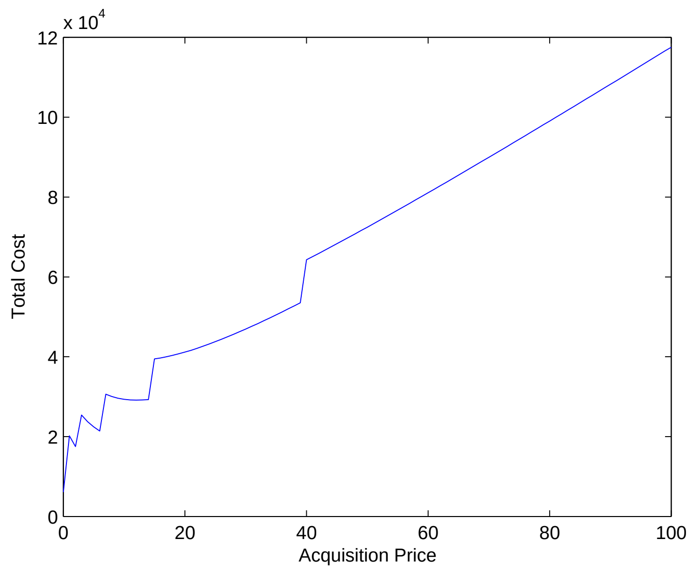

Figure C.5. Acquisition Price vs. Total Cost (b=40, (5000-7500-10000))

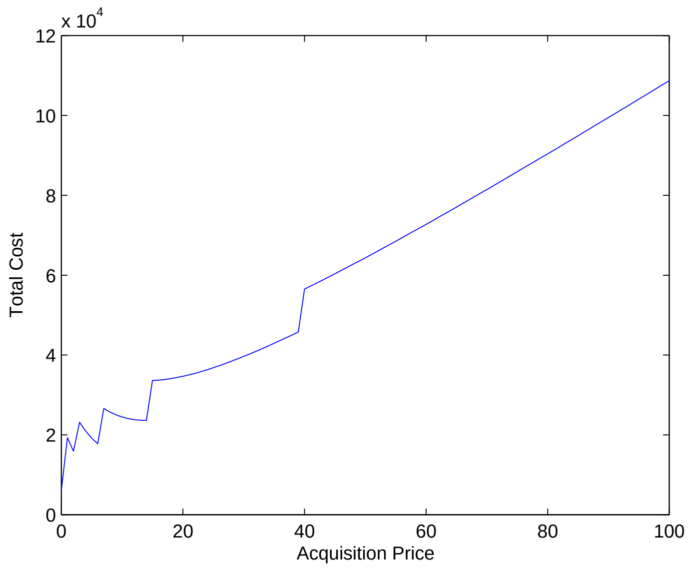

Figure C.6. Acquisition Price vs. Total Cost (b=50, (5000-7500-10000))

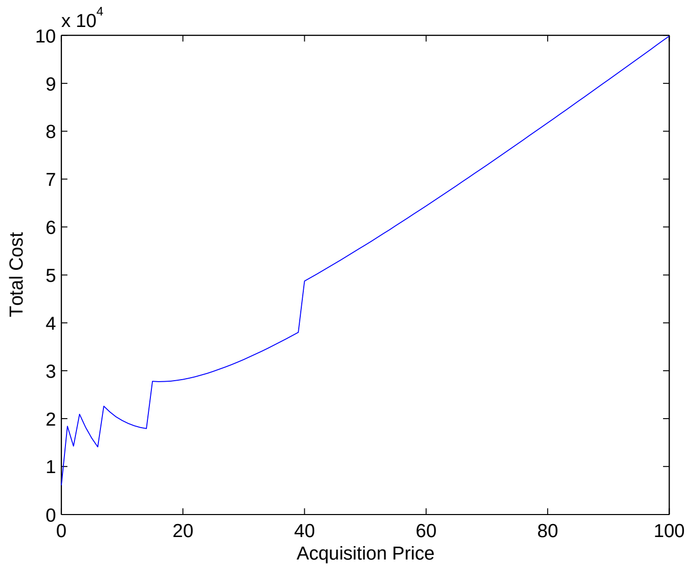

Figure C.7. Acquisition Price vs. Total Cost (b=60, (5000-7500-10000))

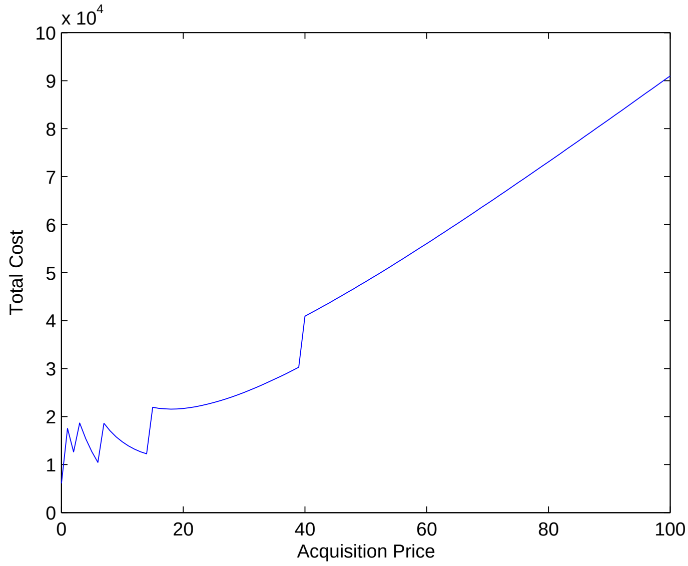

Figure C.8. Acquisition Price vs. Total Cost (b=70, (5000-7500-10000))

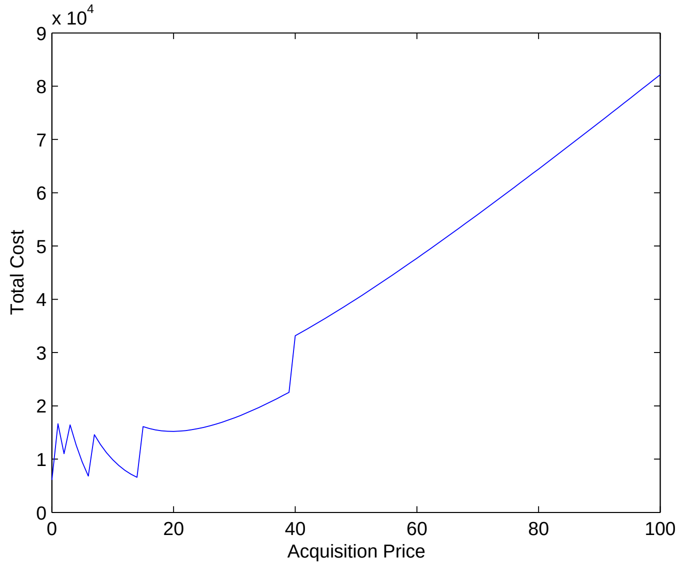

Figure C.9. Acquisition Price vs. Total Cost (b=80, (5000-7500-10000))

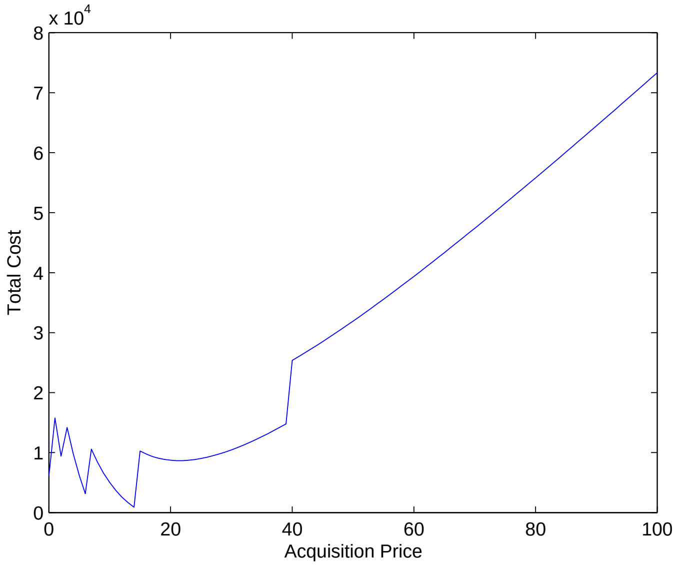

Figure C.10. Acquisition Price vs. Total Cost (b=90, (5000-7500-10000))

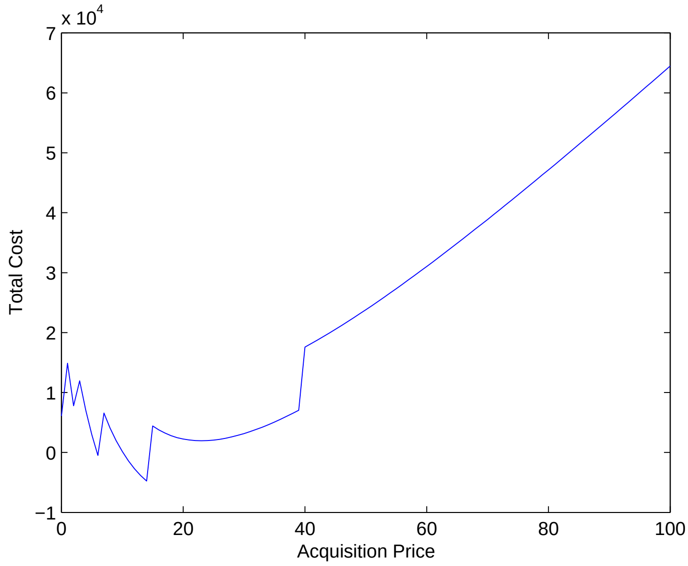

Figure C.11. Acquisition Price vs. Total Cost (b=100, (5000-7500-10000))

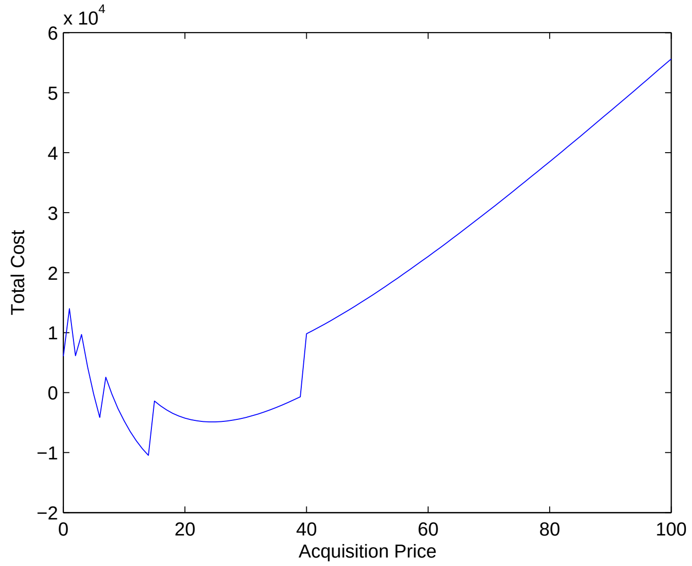

Figure C.12. Acquisition Price vs. Total Cost (b=110, (5000-7500-10000))

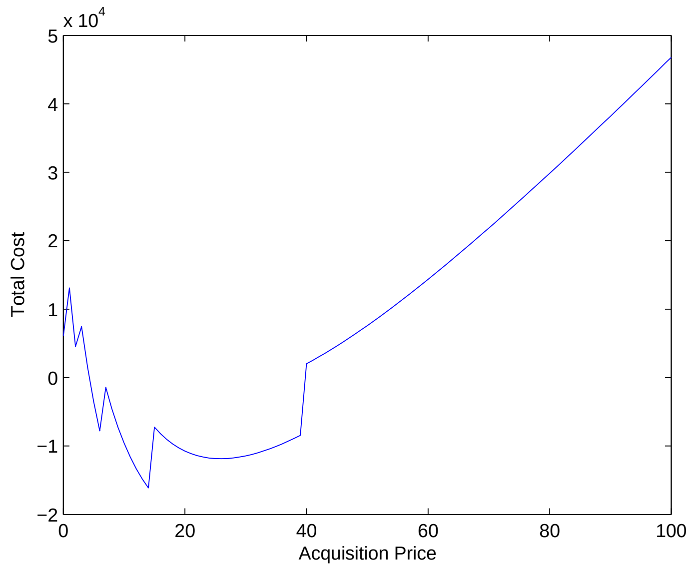

Figure C.13. Acquisition Price vs. Total Cost (b=120, (5000-7500-10000))

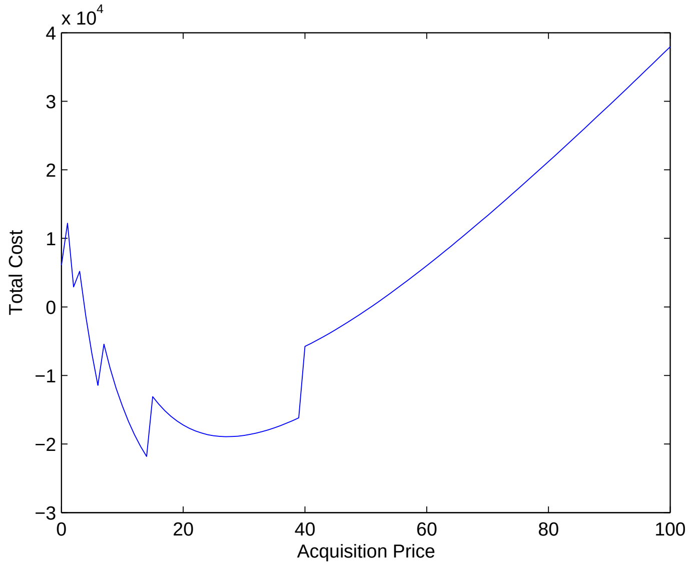

Figure C.14. Acquisition Price vs. Total Cost (b=130, (5000-7500-10000))

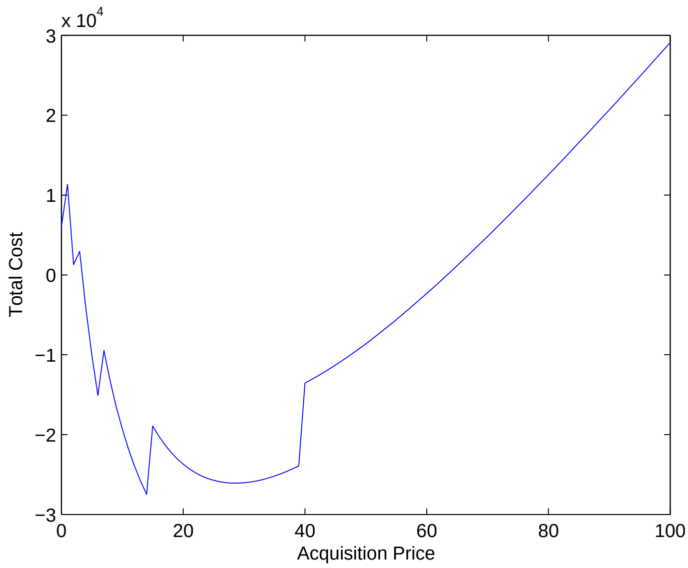

Figure C.15. Acquisition Price vs. Total Cost (b=140, (5000-7500-10000))

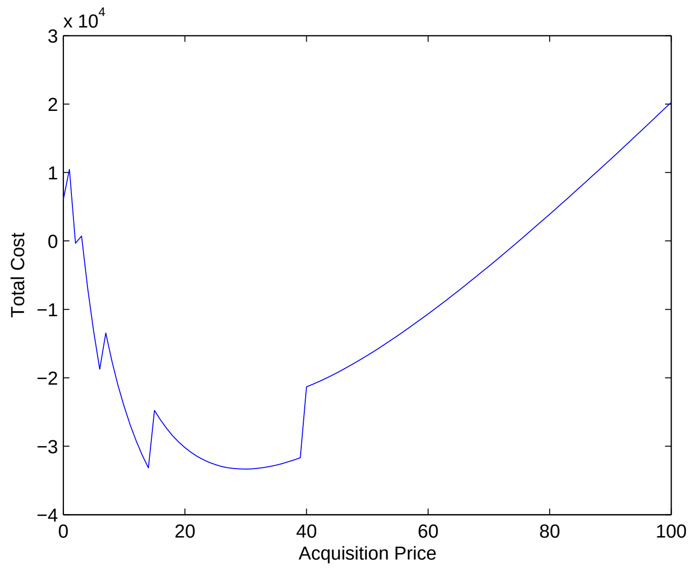

Figure C.16. Acquisition Price vs. Total Cost (b=150, (5000-7500-10000))

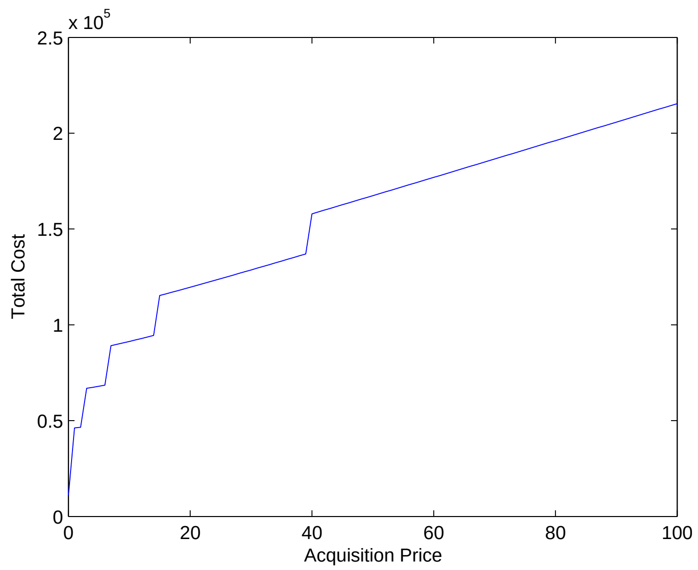

Figure C.17. Acquisition Price vs. Total Cost (b=0, (10000-15000-20000))

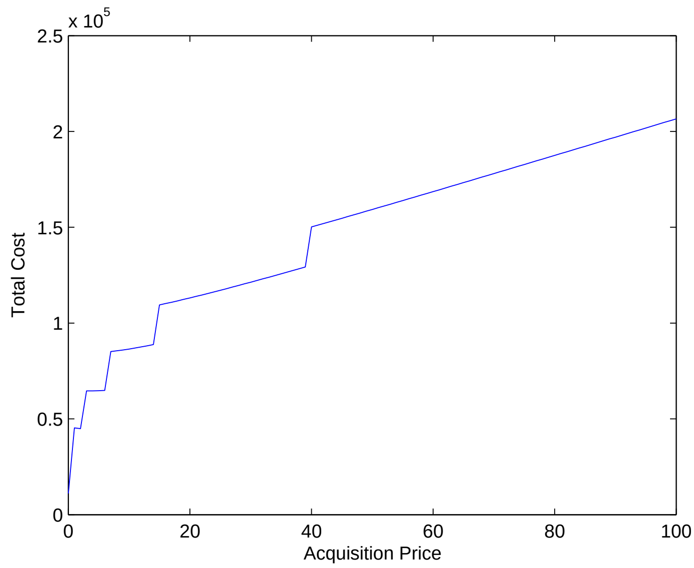

Figure C.18. Acquisition Price vs. Total Cost (b=10, (10000-15000-20000))

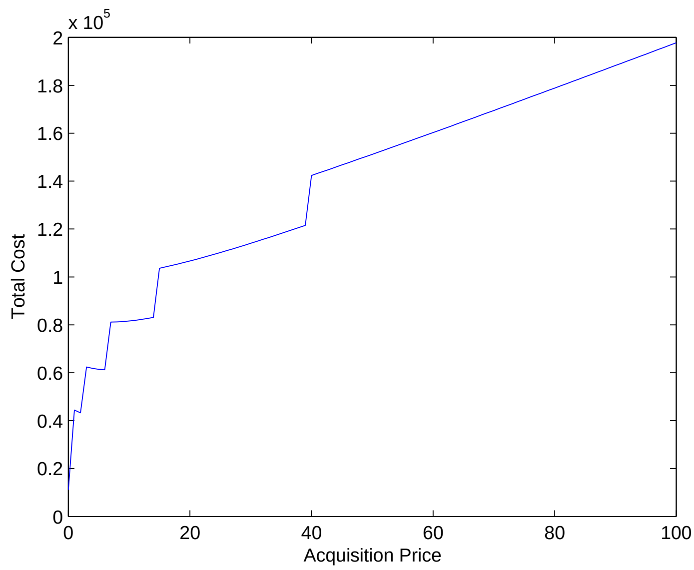

Figure C.19. Acquisition Price vs. Total Cost (b=20, (10000-15000-20000))

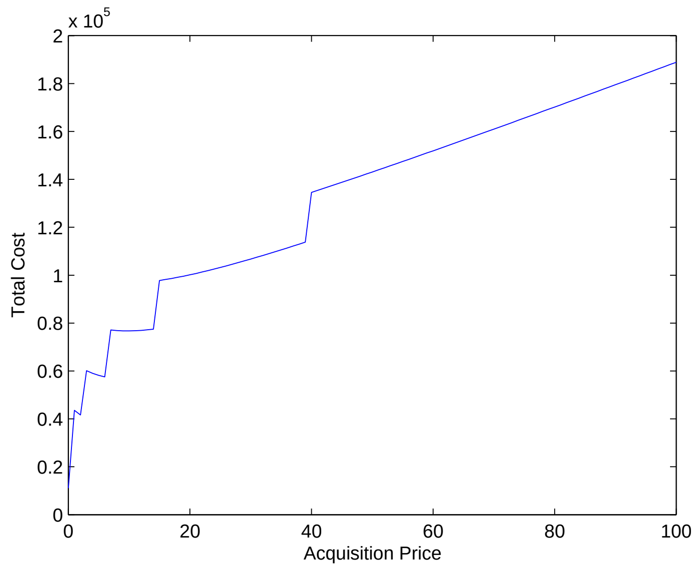

Figure C.20. Acquisition Price vs. Total Cost (b=30, (10000-15000-20000))

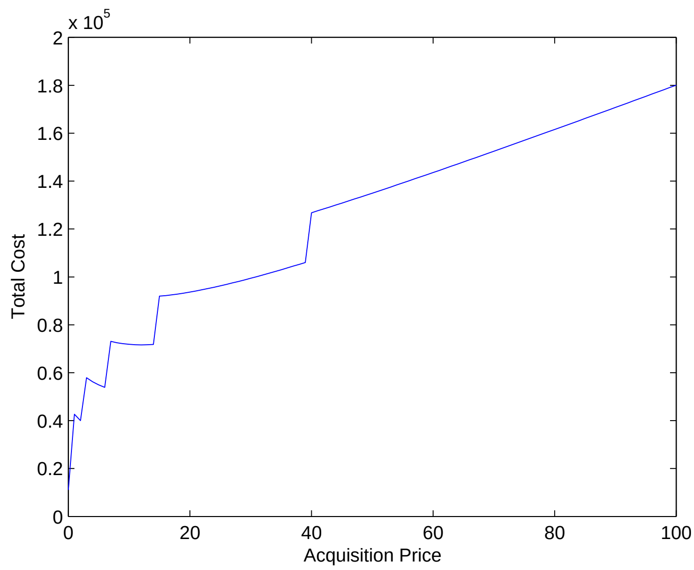

Figure C.21. Acquisition Price vs. Total Cost (b=40, (10000-15000-20000))

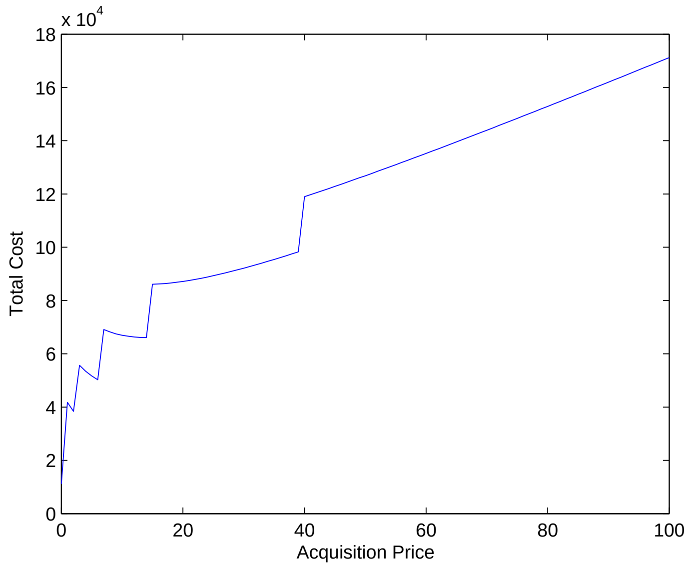

Figure C.22. Acquisition Price vs. Total Cost (b=50, (10000-15000-20000))

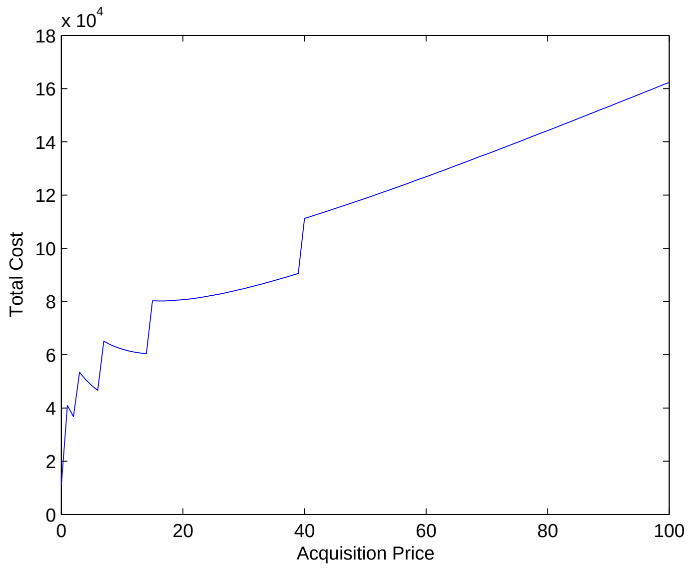

Figure C.23. Acquisition Price vs. Total Cost (b=60, (10000-15000-20000))

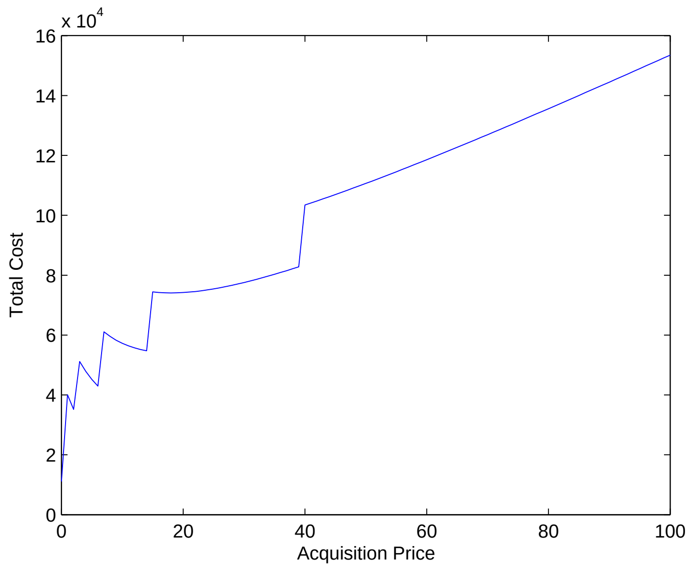

Figure C.24. Acquisition Price vs. Total Cost (b=70, (10000-15000-20000))

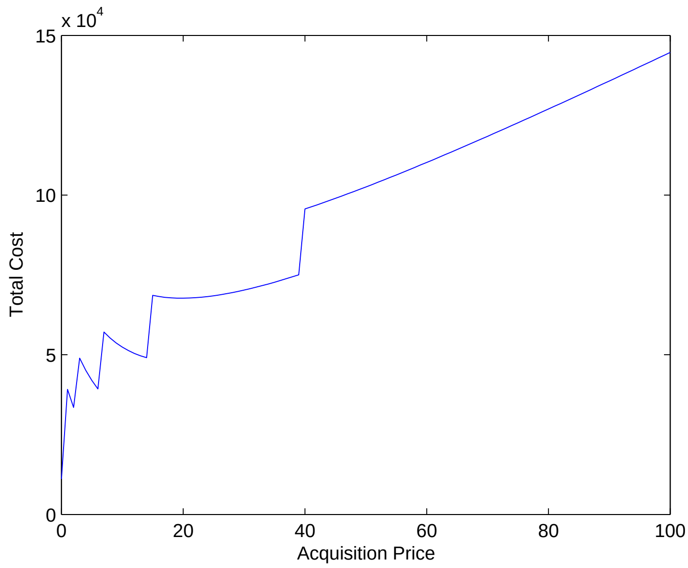

Figure C.25. Acquisition Price vs. Total Cost (b=80, (10000-15000-20000))

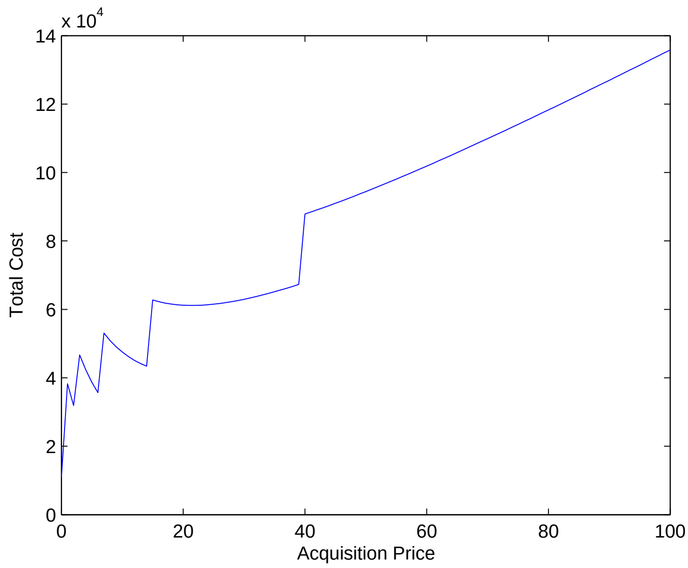

Figure C.26. Acquisition Price vs. Total Cost (b=90, (10000-15000-20000))

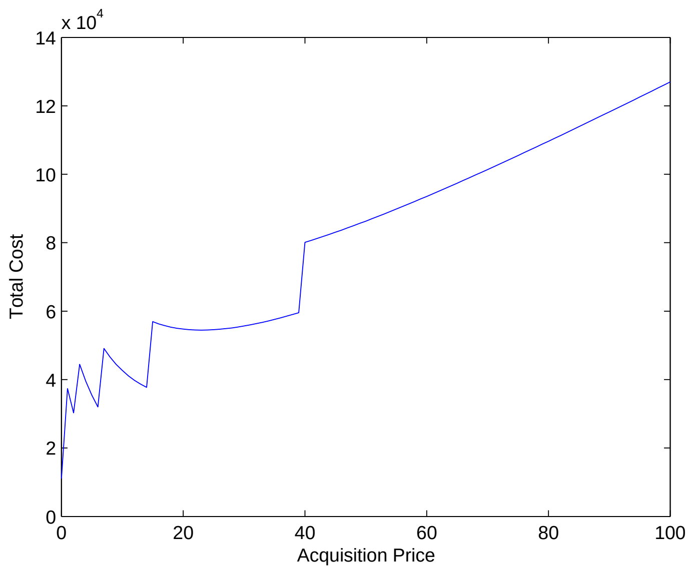

Figure C.27. Acquisition Price vs. Total Cost (b=100, (10000-15000-20000))

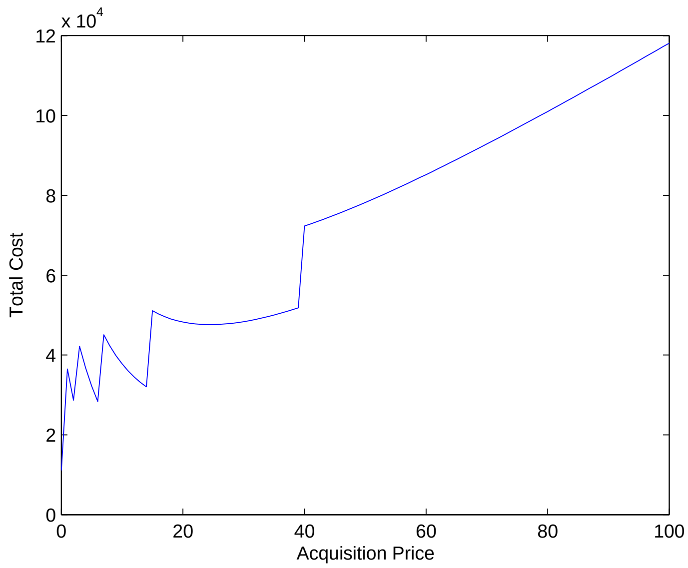

Figure C.28. Acquisition Price vs. Total Cost (b=110, (10000-15000-20000))

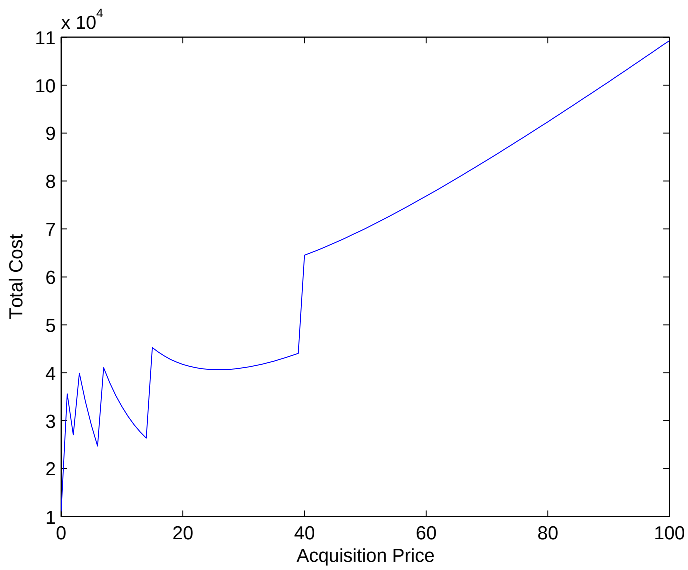

Figure C.29. Acquisition Price vs. Total Cost (b=120, (10000-15000-20000))

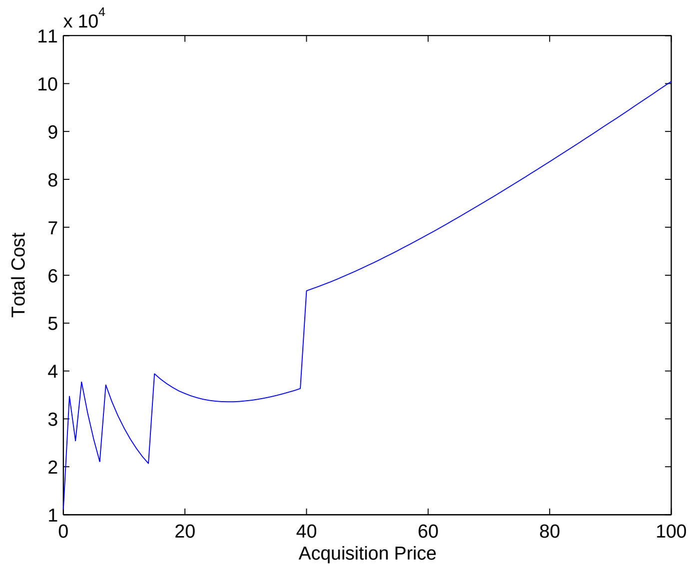

Figure C.30. Acquisition Price vs. Total Cost (b=130, (10000-15000-20000))

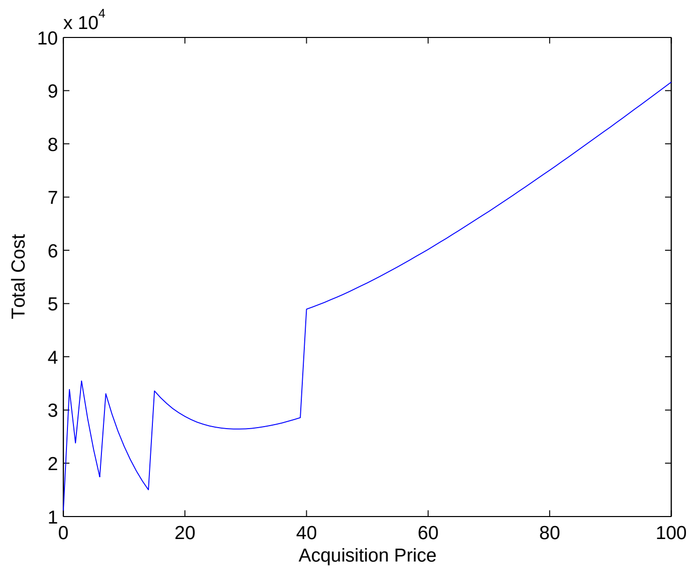

Figure C.31. Acquisition Price vs. Total Cost (b=140, (10000-15000-20000))

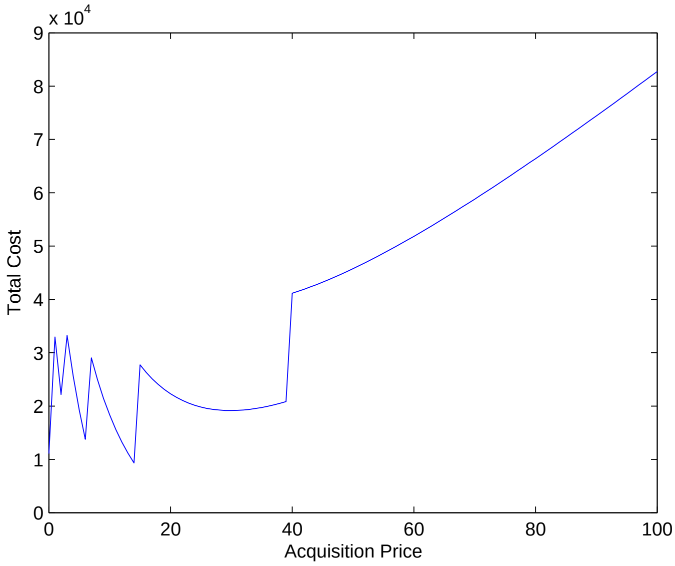

Figure C.32. Acquisition Price vs. Total Cost (b=150, (10000-15000-20000))

APPENDIX D: LARGER SIZE PROBLEMS

We have used following notation in the results tables of larger size problems:

z = Best integer solution found with Cplex

iter = Number of iterations in proposed algorithms

L-LB = Best lowerbound found in CD by Lagrangean subproblem

L-UB = Best upperbound found in CD by Convergence test

B-LB = Best lowerbound found in CD by Convergence test or best lowerbound found in BD

B-UB = Best upperbound found in CD by Benders subproblem or best upperbound found in BD

L = Acquisition price minimizing total cost found by GS search iterations

$Percentage = 100\frac{\text{Best integer}-\text{Cplex's best integer}}{\text{Cplex's best integer}}$

$Percentage - 24hr. = 100\frac{\text{Best integer}-\text{Cplex's best integer}}{\text{Cplex's best integer}}$

where each GS iteration is 24 hrs. for Cplex and 1 hr. for other algorithms

DC = Number of open DCs

IC = Number of open ICs

RF = Number of open RFs

Cplex = Cplex solver

$RelativeGap = 100\frac{\text{Best integer - Best estimate}}{\text{Best estimate}}$

Cplex-24 hr. = Each GS iteration is 24 hrs. for Cplex

Tables D.1, D.2, D.3, and D.4 show the results.

Table D.1. Results for 20-100-100 Mixed Integer Problem

Dataset1	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	123,420					
CD	576	25,419			124,136	0.58
CDB	832	25,419		32,552	124,136	0.58
BD	397			32,552	124,136	0.58
Relative Gap	1.39					
Dataset2	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	**125,299**					
CD	515	26,734			127,513	1.77
CDB	547	26,734		35,602	127,513	1.77
BD	321			35,602	127,513	1.77
Relative Gap	**Optimal**					
Dataset3	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	124,976					
CD	295	26,223		35,335	127,345	1.90
CDB	353	26,223		35,335	127,345	1.90
BD	344			35,335	125,681	0.56
Relative Gap	1.70					
Dataset4	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	**123,255**					
CD	486	24,745			126,299	2.47
CDB	515	24,745		34,368	124,494	1.01
BD	314			34,368	124,313	0.86
Relative Gap	**Optimal**					

Table D.2. Results for 20-200-200 Mixed Integer Problem

Dataset1	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	148,308					
CD	121	45,484			149,313	0.68
CDB	189	45,484		61,664	148,494	0.13
BD	98			61,664	148,494	0.13
Relative Gap	2.13					
Dataset2	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	149,976					
CD	115	46,766			153,393	2.28
CDB	161	46,766		61,927	150,568	0.39
BD	86			61,927	150,568	0.39
Relative Gap	1.98					
Dataset3	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	150,871					
CD	139	46,844			154,967	2.71
CDB	143	46,844		63,345	151,024	0.10
BD	116			63,345	151,024	0.10
Relative Gap	2.56					
Dataset4	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	149,517					
CD	139	45,917			154,976	3.65
CDB	150	45,917		63,436	149,889	0.25
BD	103			63,436	149,889	0.25
Relative Gap	2.13					

Table D.3. Results for 30-400-400 Mixed Integer Problem

Dataset1	z/iter	L-LB	L-UB	B-LB	B-UB	%
Cplex	232,281					
CD	126	81,381		106,502	232,835	0.24
CDB	141	81,381		106,604	232,835	0.24
BD	165			107,146	232,835	0.24
Relative Gap	1.06					
Dataset2	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	233,297					
CD	207	81,034			243,250	4.27
CDB	97	81,034		96,913	243,250	4.27
BD	126			106,725	233,787	0.21
Relative Gap	3.23					
Dataset3	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	234,255					
CD	20	81,912		103,332	235,901	0.70
CDB	14	81,912		104,207	235,901	0.70
BD	116			107,854	235,114	0.37
Relative Gap	2.73					
Dataset4	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	241,492					
CD	193	86,118			253,358	4.91
CDB	118	86,118		102,041	253,358	4.91
BD	144			113,475	239,760	-0.72
Relative Gap	3.70					

Table D.4. Results for 30-800-800 Mixed Integer Problem

Dataset1	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	no sol.					
CD	41	163,807			345,487	
CDB	96	163,807		-7,897,350	345,487	
BD	9			195,819	335,404	
Rel. Gap	no sol.					
Dataset2	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	344,493					
CD	41	168,634			352,255	2.25
CDB	90	168,634		-12,201,800	352,255	2.25
BD	9			200,007	339,626	-1.41
Rel. Gap	6.38					
Dataset3	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	no sol.					
CD	17	162,558		-16,660,900	350,776	
CDB	35	162,558		-16,660,900	350,776	
BD	8			194,276	333,671	
Rel. Gap	no sol.					
Dataset4	**z/iter**	**L-LB**	**L-UB**	**B-LB**	**B-UB**	**%**
Cplex	337,159					
CD	45	160,679			333,579	-1.06
CDB	96	160,679		-2,068,460	333,579	-1.06
BD	6			192,647	330,899	-1.86
Rel. Gap	5.34					

It is obvious that performance of CDB and BD in terms of efficiency and accuracy is satisfactory. When the problem size is increased (30-400-400 and 30-800-800), the proposed algorithms outperform Cplex. Cplex has not been able to find a feasible solution in 12 hrs for two data sets in (30-800-800) problem. CD, CDB and BD yield the same solution in first two datasets of (20-100-100) problem. CDB and BD found the same solution in (20-200-200) problem for all data sets. In (30-400-400) problem, CDB and BD found the same solution for only one dataset. In (30-800-800) problem, CD and CDB have found same solution whereas the best feasible solution of BD is different in all datasets.

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