STOCHASTIC PROGRAMMING MODELS FOR SUSTAINABLE ISSUES: EARTHQUAKE AND FOREST FIRES

by

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To

My Family

Halim, Müesser, Fatma Tülay

AND

Nursahver Turangil

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ABSTRACT

STOCHASTIC PROGRAMMING MODELS FOR SUSTAINABLE ISSUES: EARTHQUAKE AND FOREST FIRES

This study introduces a stochastic programming (SP) approach for modelling sustainable development issues under uncertainty. The aim is to point out how to apply SP for high risk with low probability events and how to form the best combination of mitigation alternatives by using SP logic. Two different problem areas are selected in this study, namely earthquake and forest fire hazards in Istanbul and its vicinity. For the earthquake risk mitigation problem, event based scenario approach is developed, and an SP model is proposed. The notion of utility theory with SP model is applied in the earthquake risk mitigation problem. First, we define mitigation alternatives, and the problem of choosing an alternative out of eight alternatives is described to minimize earthquake risk at sustainable level. Then, we develop an SP model including the cost of building damages, loss of lives, infrastructure damage, and the benefit of insurance return for each scenario. It is shown that "relocation" and "rebuild" options decrease the effect of big earthquakes at high marginal levels, and buying insurance is more useful especially in case of medium intensity levels of earthquake risk. In the second part, a different SP approach which is the time based scenario approach, is applied for the forest fire problem, where time periods separate stages. This model searches effective controlling for the forest-level under the risk of uncertain fire losses. After the problem definition, an SP model is developed to explain strategies for adaptation to stochastic fire loss. Harvest and enhancement are the decisions to be made before the fire whereas regeneration and rehabilitation are made after fire. The important results are that buffer stock area should be set in order to reduce fire loss effect on harvest quantity, and the application of mitigation techniques is effective to reduce fire loss. Furthermore, it is shown that the stochastic programming approach is a useful method for solving real life risk mitigation problems.

ÖZET

DOĞA OLAYLARI İÇİN STOKASTİK PROGRAMLAMA MODELLERİ: DEPREM VE ORMAN YANGINLARI

Bu çalışma, belirsizlik faktörünü içeren kıt kaynakların iyileştirilmesi konularını ve bu kaynaklarla ilgili çevresel problemleri modellemek amacıyla bir stokastik programlama yaklaşımını anlatmaktadır. Çalışmanın ana amacı, stokastik programlama mantığını yüksek risk ve düşük olasılık içeren çevresel olaylara nasıl uygulayabileceğimizi ortaya koymak ve stokastik programlama yöntemini kullanarak riski azaltıcı alternatiflerin en iyi kombinasyonlarını belirlemektir. Bu çalışmada, İstanbul ve çevresinde meydana gelebilecek deprem ve orman yangını risklerini modelleyen iki farklı problem uygulaması seçildi. Deprem riskini azaltma problemi için olay bazlı senaryo yaklaşımı geliştirildi, ve bu yaklaşımı uygulayan bir stokastik programlama modeli önerildi. Sonuçların analizinde kolaylık için fayda teorisi yaklaşımı stokastik programlama modeli içine dahil edilmiştir. Bu modelde, her bir senaryo için sigorta seçeneğinden elde edilen fayda dikkate alınıp deprem riskini azaltıcı sekiz alternatiften en iyi kombinasyon bulunduğunda; yaşam kaybı, alt yapı ve bina hasar maliyetlerini içeren bir stokastik programlama modeli geliştirildi. Bu model, yer değiştirme ve yeniden imar alternatiflerinin büyük depremlerin riskini ve kayıplarını yüksek marjinal seviyelerde düşürdüğünü ve özellikle orta şiddetli deprem riski için de sigorta yaptırmanın oldukça faydalı bir alternatif olduğunu ortaya koymaktadır. Araştırmanın ikinci bölümünde, orman yangını problemi için zaman bazlı senaryo yaklaşımını içeren farklı bir stokastik programlama konsepti uygulanmıştır. Bu model, belirsiz yangın risklerine sahip ormanlar için etkin bir kontrol yöntemini araştırmaktadır. Stokastik yangın kayıplarını azaltıcı yenileme ve rehabilitasyon seçenekleri modelde dikkate alınmıştır. Böylece, bu stratejilerin ağaç kesim miktarını olumsuz etkileyen yangın risklerini azalttığı görülmüştür. Genel olarak, bu çalışmada stokastik programlama kavramının gerçek hayattaki yüksek risk içeren problemlerin çözümü için etkin bir yöntem olduğu kanıtlanmıştır.

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LIST OF SYMBOLS/ABBREVIATIONS

A	First stage matrix
b	Right hand side vector in the first stage
c	First stage objective
E	Mathematical expectation operator
F	Feasibility Sets
h	Right hand vector side in the second stage
m	Number of constraints
n	Number of variables
p	Probability of a random element
P	Probability of events
q	Second stage objective vector
Q	Second stage value function
$\mathcal{Q}$	Second Stage expected value (recourse) function
T	Technology matrix
y	Second-stage decision vector
x	First stage decision vector
W	Recourse matrix
Z	Objective value
Ω	Set of all random events
γ	Real value
ω	Random event
ξ	Random vector
$\Im$	Random variable
EP	Exceedance Probability
EV	Expected Value
JICA	Japan International Cooperation Agency

LP	Linear Programming
MVP	Mean Value Problem
NSR	Not Sufficient Restocked
RP	Recourse Problem
SP	Stochastic Programming
TISP	Two stage Interval Stochastic Programming
WS	Wait and See

1. INTRODUCTION

In recent years, sustainable development issues including management of restricted natural resources, natural and men-made hazards, and ecological systems have become more important. Especially during the last two decades, both men made and natural disasters occurred more frequently and harmed the balance of environment and people's lives at the highest level. Actually, human lives and environment can be considered together because there is a vital loop and a direct relationship between them. Sustainable issues cover all the hazards or activities which directly or indirectly affect environment, but they also contain social factors. Thus, environmental protection and resource conservation are still challenges faced by scientists and authorities in the public and private sectors.

It is well-known that many studies of management, optimization, and problem solving through conventional linear programming techniques were undertaken in the real world. However, linear programming techniques assume that all of the problem or system requirements are known at deterministic level. That is in conflict with the real life practice because there is always uncertainty in the real world. As it is estimated, the value of information about future events is very high in order to overcome uncertainty. Moreover, it is very complex and costly to organize real data. Because of the complexity and uncertainty in the prescribed environment, there is always a demand for studies that incorporate various sustainable issues within a general framework and evaluate policy responses efficiently to address the analysis of those problems. To cope with uncertainty, one of the most effective techniques is the stochastic programming approach proposed in this study since it handles the problem without distorting its uncertain nature.

In this study, we focus on earthquake and forest fire hazards at sustainable level by mitigating their risks. The main reason for choosing those issues is that big earthquakes occurred in Turkey in recent years, and there is a high probability of a major earthquake. Furthermore, forest fires occur very frequently in Turkey, threaten human and wild life, and affect the level of timber supply. In addition, spatial and temporal variations exist in many components of those two problems. For example, in case the of earthquakes, the occurrence of earthquake, its time, magnitude, and impact level are not known before it happens. In the forest fire problem, fire loss is highly variable and has a destructive effect on trees. Thus, the main aim is to reduce or eliminate high risks in those events by taking some kinds of precaution before those natural disasters happen. The actions taken before an hazardous event are more effective than the actions taken during or after those events.

Two stochastic mathematical models are proposed based on the notion of risk mitigation for earthquake and forest fire problems by considering the basic concepts of stochastic programming approach. The main difference between those two models is that the earthquake risk mitigation model is event based whereas the forest fire risk mitigation model is time based. Moreover, the utility idea is included in the earthquake model.

This study is organized as follows: Firstly, it defines the main topics which are earthquake and forest fire problems, their features and basic properties of stochastic programming in Chapter 2. Then, a brief literature review is included about stochastic programming, sustainability, earthquake and forest fire separately in Chapter 3. Chapter 4 consists of discussions about the mathematical stochastic earthquake model, mitigation options, scenarios, specific assumptions, definition of variables and constraints. Moreover, the experimental design and computational results of earthquake risk mitigation model are given in this chapter. The development of the mathematical model, required data, experimental design and computational results of forest fire risk mitigation model are studied in Chapter 5. Finally, concluding remarks are given in Chapter 6.

2. PROBLEM DEFINITION

United Nations Brundtland Report defines sustainable development as "development that meets the needs of the present without compromising the ability of future generations to meet their own needs". The main underlying and simple idea of that definition is equity, common usage of world resources including natural resources within and between generations, and inter-generation equity. Hence a policy for sustainable development is typically defined as one that leaves future generations with the opportunity to attain similar or higher levels of well-being than the present one. In an economic sense, "opportunities" which are left to future generations depend on the total stock of wealth they inherit, including natural (resource and environmental) capital as well as man-made physical and human capital. "Social capital" that means social values and institutions could also importantly influence sustainable development.

A key issue is the degree of substitutability of the various forms of capital. To the extent they are substitutes, sustainability can be achieved by balancing a decline in natural capital by a proportionate increase in man-made capital. This is the "weak" concept of sustainability. The "strong" concept requires that some level of the stock of natural capital be preserved under any circumstance. In reality some forms of natural capital are critical for development and can be replaced with man-made capital to only a limited extent, while others are more fully substitutable. Most non-renewable resources are of the former category, as are renewable and environmental resources that are prone to very slow regeneration processes (fishery, forestry, biodiversity, ozone layer, etc.). As long as science is unable to provide reliable rules for sustainable depletion paths or viable alternatives for these forms of natural capital, depletion of non-renewable resources could present a high cost for future generations, because of irreversibility

By taking these issues into consideration globally, various countries signed Kyoto agreement, and tend to obligate its rules and organize their laws accordingly. The basic idea of Kyoto Agreement is about how prices could better reflect the social costs of environmental damage including through subsidy reduction, the role of better exploitation of knowledge, technology and innovation for resource productivity, and improved measurement of performance.

The main concern is to keep natural resources at a low level in order to guarantee the survival of lives for future. However, there are always hazards both natural and man-made that threaten environment and ecology. Such hazards are earthquakes, ozone depletion, pollution in air, contamination in water, energy management, radiation spread, floods, hurricanes, forest fires, global warming etc. Those hazards result in disaster that means any occurrence causing damage, ecological disruption, loss of human life, deterioration of health and health services on a scale sufficient to warrant an extraordinary response from outside the affected community or area.

Most of those hazards have high catastrophic risks, which are characterized by their less frequent but bulky damages. In order to cope with the risks of disasters, an effective disaster management which reduces the risks by mitigation should be built. The disaster management should cover all the layers of a public that include formal and informal organizations, individual responsibilities of people so on so forth. So it is the vital point that develops mitigation alternatives and policies in order to overcome and reduce the impacts of disasters in the presence of life and property. Policies aiming at cost-effectiveness are important to achieve sustainability as they allow faster wealth accumulation. There is often considerable uncertainty in the nature of disasters which affect sustainable development issues and policies. Thus, we propose to use the notion of stochastic programming in sustainable development issues.

Big earthquakes which occurred in various places all over the world in recent years caused very high damages. The scientists have been working on earthquake estimation, which is still very difficult to achieve. Thus, in spite of trying to know the certain occurrence of earthquake, it becomes more important to reduce the risks of earthquakes by mitigating. If the correct action is done at correct time, there is no more loss of lives and damage even in case of big earthquakes. Rebuilding based on earthquake proof design, retrofitting, relocation, taking earthquake insurance etc. are some of mitigation options which can be made before the earthquake. In this research, we focus on the evaluation of disaster mitigation activities by using stochastic programming idea instead of estimating occurrence of earthquakes. The intensity of an earthquakes depends on the some special geographic features of the affected area such as macro-seismic intensity, magnitude of earthquakes, the seismic activity, frequency of strong earthquakes, the scheme of lineaments, solidification and structure of land, and epicenter distance.

The second part of this study deals with forest fires. There are also unsystematic and systematic risks in forests. Unsystematic risks cause independent variations among stands such as genetic and site variations, whereas systematic risks cause correlated variation among stands which are fire flaps, insect epidemics, windthrow. It is necessary to investigate the effect of fire loss at the forest level because fire loss is highly variable and has the potential to destabilize the structure of forest which is related to timber supply.

In the literature, there are four different forest management models which are with and without fire. Those management models are stand level rotation with age control, forest-level regulation with area control, forest-level simulation with volume control, and forest-level optimization. The main focus is the forest level optimization model, in which the stands grow one age class to another class until they are harvested as the time period passes. If they are harvested, the area returns to the starting point which means that the area starts self regeneration. This loop continues unless there is a fire. If a fire occurs, the loop is broken, and the area returns to the starting point unexpectedly. Some treatment actions are taken to renew the burned areas such as regeneration and rehabilitation rather than self regeneration feature.

We can consider forest risk having three modules: fire risk which is likelihood of a fire starting, fire hazard which is the potential fire behavior, and the value of a forest at risk. Forest value depends on age, species, silviculture, productivity, location, and terrain. Then, the loss is calculated by subtracting salvage area after the fire from the economic value of forest.

Another striking difference is that our earthquake problem is modeled as stochastic programming with simple recourse whereas the forest fire model is considered as stochastic programming with complete recourse. In the earthquake model, our first stage decision is the implementation of mitigation alternatives, and the second stage decisions are building and infrastructure damages and loss of life in the presence of mitigation alternatives. In the forest model, harvest and enhancement is the first stage decisions, and in addition to rehabilitation and regeneration decisions, harvest and enhancement decisions are also considered as the second stage decisions.

2.1. Overview of Stochastic Programming

Linear and integer programming models have been very useful techniques to solve problems for many years. Those techniques are based on the assumption that their model parameters are known and the decisions should be made in deterministic environment. However, all of the real life problems have uncertainty feature. It causes that the decisions in real life are made in very complex and costly environment. To reduce the complexity of real life problems, they can be transformed to deterministic equivalent problems.

The concept of solving the problems which have uncertain parameters was born as "stochastic programming". Dantzig [1] and Beale [2] introduced stochastic programming with recourse as a optimization technique dealing with uncertain parameters and data. This technique possesses some characteristic of dynamic programming but more advanced with simulation. This technique brings meaningful and robust solutions to real life problems. However, mostly it causes the solution to be far away from optimal solutions. Sometimes, there can be infeasibility because many outcomes can occur and affect the objective function. Methods to overcome infeasibility are discussed in Birge and Louveaux [3].

In presence of uncertainty, many realizations of a given system are generally possible. In such cases, a question arises over the specification of the objective function when a deterministic optimization model is used to represent a stochastic system. Future uncertainties have usually been examined individually through deterministic (alternate) scenarios. However, such scenario analyses have a drawback: whenever two contrasted scenarios require us to behave widely different in the immediate future (i.e. prior to the resolution of uncertainty), this leaves us in a dilemma, since in real life only one set of actions may actually be selected. This is precisely what the SP paradigm attempts to clarify, by merging two or more alternate scenarios in a single model, and by recommending actions which are optimal in the presence of uncertainty. In addition, SP will always select a single course of action at all periods prior to the resolution of the uncertainty. It is therefore more realistic than the traditional scenario analysis.

SP has the scenario-based approach, which implies that it summarizes all possible results for an event within the scenario framework. Thus, the scenario-based approach attempts to represent a random parameter by forecasting all its possible future outcomes. The main drawback of this technique is that the number of scenarios increases exponentially with the number of uncertain parameters leading to an exponential increase in the problem size. SP is a mathematical (i.e. linear, integer, mixed-integer, nonlinear) programming technique but with a stochastic element present in the data.

SP is basically structured in the format of stages, such that stages and their decisions follow each other respectively. The simplest form of SP is a two-stage program, in which the set of decisions is divided into two parts:

- A number of decisions have to be made before the stochastic experiments. In other words, we know some certain parameters at first , and we make our decisions in accordance with those parameters. This stage is called as the "first stage". Decisions made in first stage are called first stage or design variables. ($x \in$RN1, where RN1 is the first stage matrix)
- A number of decisions can be taken after the experiment. Those are called the "second stage decisions, and the corresponding period is called "second stage". The decisions are also called the control or recourse variables. ($y \in$RN2, where RN2 is the second stage matrix)

Since infeasibility can occur frequently in those problems, in the second stage we need to use recourse to a further degree of flexibility. In this second stage, the decisions will be dependent on the particular realization of the stochastic elements observed.

2.1.1. Relationship between Deterministic and SP Models

An SP model can be considered as a Linear Programming (LP) model extended and refined by the introduction of uncertainty (see Figure 2.1). More precisely, the underlying LP optimization model is extended by taking into account the probability distribution of the LP coefficients which are random variables. Such distributions are provided by models of randomness (implemented in *scenario generators*), which are specific to the particular optimization problems under investigation.

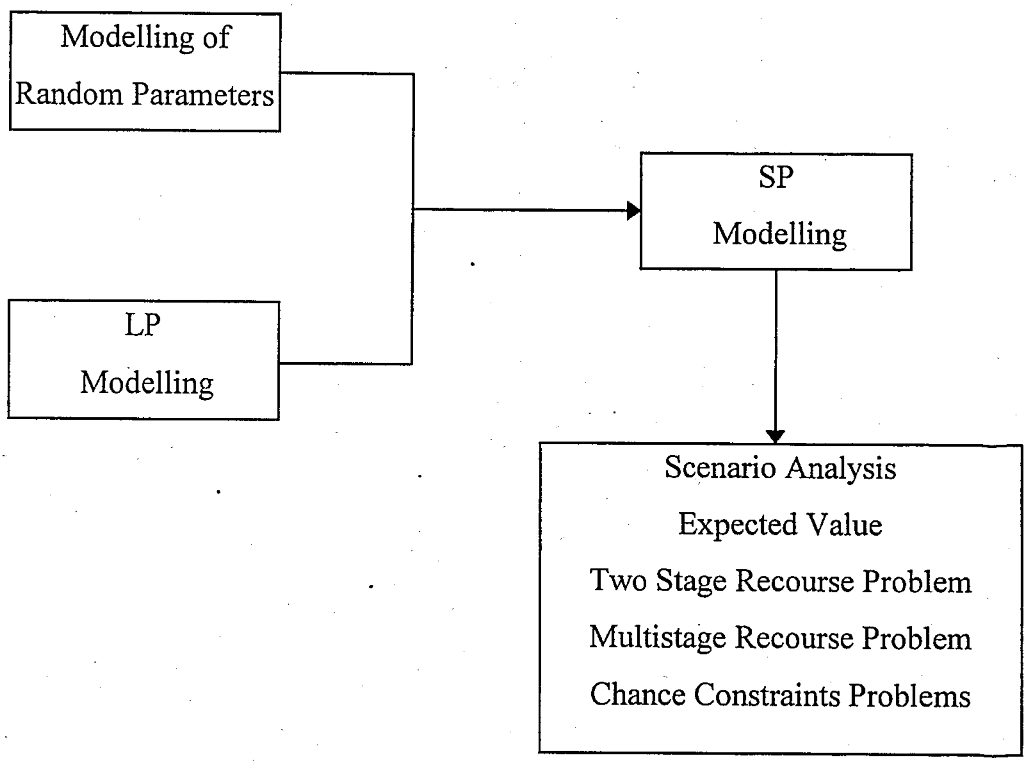

Figure 2.1. Overview of SP models

Therefore, it is always possible to identify an underlying deterministic model (also called the *core* model), which captures the logical structure of the problem as well as the dynamic relationships within decision variables, their bounds and the objective function. In a scenario-based recourse problem, for instance, the core represents the model associated with a particular sequence of realizations of the random parameters (scenario).

2.1.2. Underlying Deterministic Model

The core model of the underlying deterministic model could be linked to the model of randomness in two ways:

- Making variables, parameters and constraints explicitly parametric in the scenario index.
- Marking the appropriate coefficients as random parameters in such away that they can be treated implicitly

The first approach requires that a scenario dimension must be introduced a priori and precludes the possibility of describing models with continuous distributions; it also implies the replications of variables and constraints.

2.1.3. Definition of the Random Structure

Once the underlying deterministic problem has been implemented, it is necessary to merge it with the information related to the model of randomness which characterizes the problem. The items of information can be summarized as follows:

- *Scenario tree* represents the structure of the event tree for scenario-based problems.
- *Stages* show the time horizon of the underlying dynamic linear program can be partitioned into decisional stages.
- *Scenarios probability* shows the (discrete) probability distribution associated with the scenarios.
- *Scenario dimension* identifies a scenario index for scenario-based problems.
- *Time dimension* is the index used to describe the temporal horizon in the underlying model needs to be uniquely identified.
- *Random data* defines and marks the random parameters of the problem.

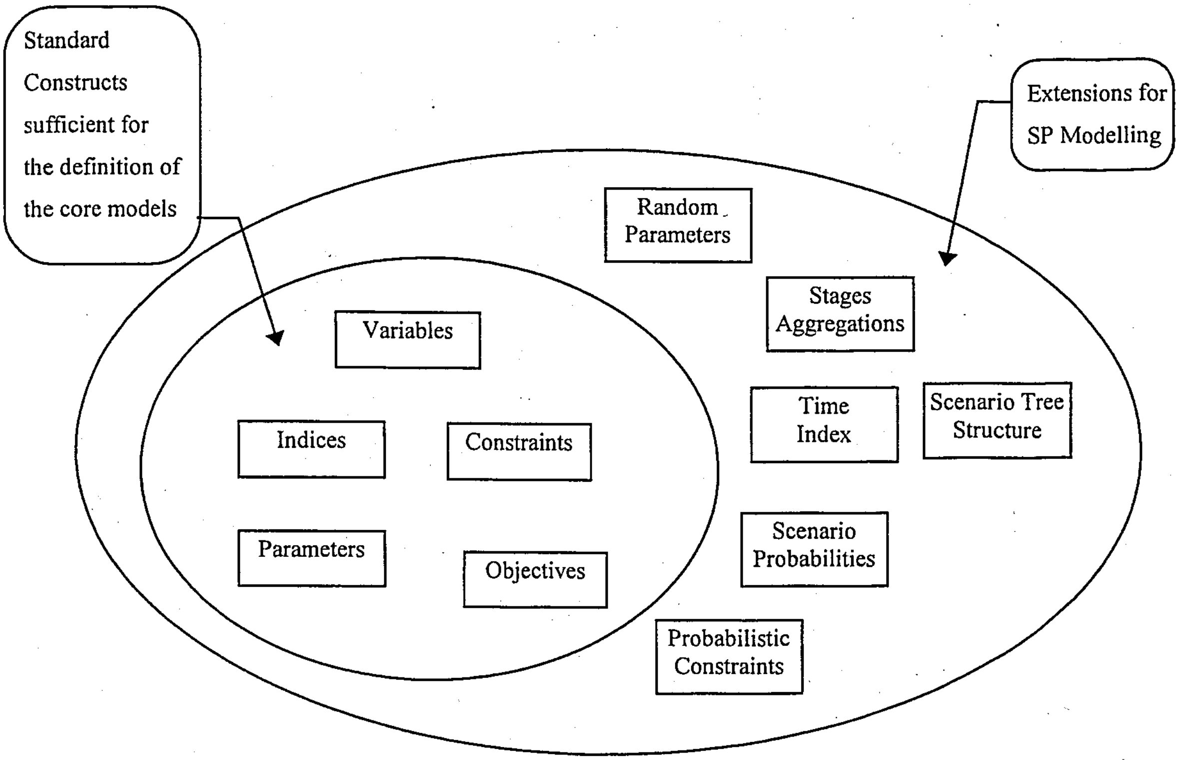

Figure 2.2. Construction scheme of SP models

2.1.4. Classification of SP Models

SP models can be classified as shown in Figure 2.3, and will be described briefly.

2.1.4.1. Distribution Problems: The optimization problems which provide the distribution of the objective function value for different realizations of the random parameters and also for the expected value of such parameters are broadly known as the distribution problems.

Expected Value Problem: The Expected Value (EV) model is constructed by replacing the random parameters by their expected values. Such an EV model is thus a linear program, as the uncertainty is dealt with before it is introduced into the underlying linear optimization model. It is common practice to formulate and solve the EV problem in order to gain some insight into the decision problem given by:

$$Z = \min cx$$

subject to

$$Ax = b$$
$$x \geq 0 \tag{2.1}$$

where $A \in R^{mxn}$; $c, x \in R^n$; $b \in R^n$

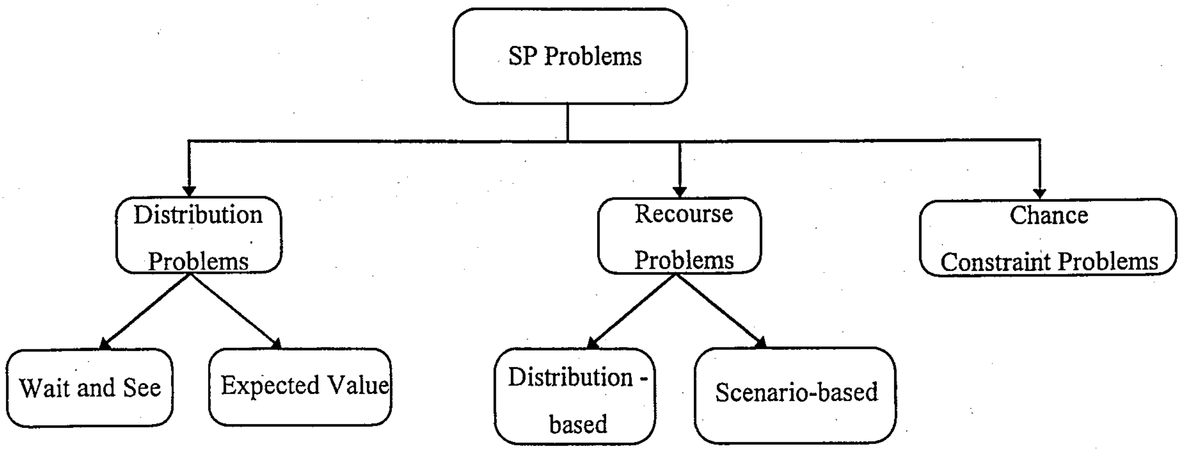

Figure 2.3. Taxonomy of SP problems

Let $(\Omega, \Im, P)$ denote a (discrete) probability space where $\xi(\omega)$, $\omega \in \Omega$ denote the realizations of the uncertain parameters. Let us denote the realizations of A, b, c for a given event ω as

$$\xi(\omega) \text{ or } \xi_\omega = (A, b, c)_\omega \tag{2.2}$$

The associated probabilities of these realizations are often denoted as $p(\xi(\omega))$ or $p_{\xi(\omega)}$ For notational convenience we denote these probabilities as $p(\omega)$. Let the feasible regions corresponding to the problem stated in (2.1) and (2.2) be defined as

$$F^{\omega} = \{x \mid Ax = b, x \geq 0\} \text{ for } (A,b,c)_\omega \text{ or } \xi(\omega) \tag{2.3}$$

We can reconsider (2.1) as an expected value or an average value problem where:

$$(\overline{A}, \overline{b}, \overline{c}) = \overline{\xi} = E[\xi(\omega)] = \sum_{\omega \in \Omega} p(\omega)\, \xi(\omega)$$

and the optimization problem is defined by

$$Z_{EV} = \min c\, x$$
$$\text{where } x \in \overline{F} \equiv \{x |\, \overline{A}x = \overline{b}\} \tag{2.4}$$

where x_{EV}* denotes an optimal solution to the above problem.

This solution can be evaluated for all possible realizations $\xi(\omega) \mid \omega \in \Omega$. We can thus determine the corresponding objective function values and compute the expectation of the expected value solution:

$$Z_{EEV} = \mathrm{E}[c(\omega)\, x_{EV}^*] \tag{2.5}$$

If for some $\omega \in \Omega$., $x_{EV}^* \notin F^{\omega}$, that is x_{EV}* is not feasible some realizations of $\xi(\omega)$ of the random parameters, we have

$$Z_{EEV} \to +\infty \tag{2.6}$$

Wait and See Problems: Wait and See (WS) problems assume that the decision-maker is somehow able to wait until uncertainty is resolved before implementing the optimal decisions. This approach therefore relies upon perfect information about the future. Because of its very assumptions, such a solution cannot be implemented and is known as the "passive approach". WS models are often used to analyze the probability distribution of the objective value, and consist of a family of LP models, each associated with an individual scenario. The corresponding problem is stated as:

$$Z(\omega) = \min c(\omega)\, x \tag{2.7}$$

subject to $x \in F^{\omega}$

The expected value of the WS solutions is defined as:

$$Z_{WS} = \mathrm{E}[Z(\omega)] = \textstyle\sum_{\omega \in \Omega} p(\omega)\, Z(\omega) \tag{2.8}$$

2.1.4.2. SP Problems with Recourse: SP problems with recourse are dynamic LP models characterized by uncertain future outcomes for some parameters. In general, SP problems can be formulated as follows:

$$\min z = c^T x + \mathrm{E}_\xi\,[\min q(\omega)^T\, y(\omega)]$$

Subject to:

$$\begin{aligned} A\,x &= b \\ T(\omega)\,x + W\,y(\omega) &= h(\omega) \\ x \geq 0,\ y(\omega) &\geq 0 \end{aligned} \tag{2.9}$$

x : first-stage decision vector

y : second-stage decision vector.

ω : random event

A : first stage matrix

b : first-stage right hand side

T : technology matrix

h: RHS in second stage

W: recourse matrix.

The first stage decisions are represented by the vector x. Corresponding to x are first stage vectors and matrices c , b, and A. In the second stage, a number of random events $\omega \in \Omega$ (universe) may realize. For a given realization w, the second stage problem data $q(\omega)$, $h(\omega)$, and $T(\omega)$ become known.

The second stage value function:

$$Q(x, \xi(\omega)) = \min_y \{\, q(\omega)^T y \mid W y = h(\omega) - T(\omega)x,\ y \geq 0 \,\} \tag{2.10}$$

The expected value of the second stage function:

$$Q(\omega) = E_\xi \, Q(x, \xi(\omega)) \qquad (2.11)$$

and the deterministic equivalent program is

$$\min z = c^T x + Q(x)$$

subject to:

$$Ax = b,$$

$$x \geq 0 \qquad (2.12)$$

Thus, $Q(x, \xi(\omega))$ is referred as recourse function . As the future unfolds in several sequential steps and subsequent recourse actions are taken, we deal with the generalization of the two-stage recourse problem known as the multistage SP problem with recourse. A decision made in stage t should take into account all future realizations of the random parameters and such decisions only affect the remaining decisions in stages t+1 … T. In SP this concept is known as non-anticipativity. This implies that if two different scenarios s and s' are identical up to time period T on the basis of information available about them at that time period, then the values of the variables which depend on these scenarios must also be identical up to time period T. The general formulation of a multistage recourse problem is given below:

$$Z = \min_{x_1} \left| c_1 x_1 + E_{\xi 2} \left(\min_{x_2} c_2 x_2 + E_{\xi 3/\xi 2} \left[\min_{x_3} c_3 x_3 + \ldots + E_{\xi\gamma/\xi\gamma-1/\ldots\xi 2} \min_{x_\gamma} c_\gamma x_\gamma \right] \right) \right| \qquad (2.13)$$

subject to

$$
\begin{array}{lllll}
A_{11}\, x_1 & & & & \\
A_{21}\, x_1 & + \; A_{22}\, x_2 & & & \\
A_{31}\, x_1 & + \; A_{32}\, x_2 & + \; A_{33}\, x_{33} & & \\
\vdots & & & \ddots & \\
A_{T1}\, x_1 & + \; A_{T2}\, x_2 & + \; A_{T3}\, x_{33} & + & \cdots
\end{array}
$$

$$l_t \leq x_t \leq u_t \tag{2.14}$$

where: $t = 1,..., T$ represents the planning horizon and the vectors

$$\xi_t = (b_t , c_t , A_{t1},...,A_{tT}) \qquad \forall t \in [2,...,T] \tag{2.15}$$

are random vectors on a probability space $(\Omega,\Im,P)$.

Scenario Based Recourse Problems: Let us reconsider the random parameter vector $\xi(\omega)$ introduced in (2.2) and used in the definition of the given class of models. In the discrete statement of the problem, the event parameter takes the range of values $\omega = 1,\ldots,|\Omega|$, and there are associated random vector realizations $\xi(\omega)$ and probabilities $p(\omega)$ such that:

$$\sum_{\omega\in\Omega} p(\omega) = 1 \text{ and } \Xi \cup_{\omega\in\Omega} \xi(\omega) \tag{2.16}$$

Here Ξ is the set of all random vectors and is often called the *set of scenarios*. For the multistage recourse problem (2.13)-(2.16), if the probability distribution of the random parameter vectors is discrete, the uncertainty defines a random structure in the form of an event tree, which represents the possible sequence of realizations (scenarios) over the time horizon . When the event tree is explicitly given, we refer to the model as a *scenario based recourse problem*. In general, individual scenarios are interpreted as leaf enumeration of the event tree. In the scenario based multistage problem, the event tree serves two purposes:

- to specify the model of randomness (the scenario generation) and
- to define the mathematical model structure including hierarchical (or recursive) non anticipativity restrictions.

Distribution Based Recourse Problems: An event tree can be also generated by defining the probability distributions of the random parameters, in which case the model is called *distribution based recourse problem (RP)*. Gassmann and Ireland [4] expand this concept in their work. This second class of problems, however, introduces various difficulties in the model specification using mathematical modelling languages and in terms of the solution process, in particular when some of the random parameters are continuously distributed. An approximation can be achieved by adopting appropriate sampling procedures, whereby the distributions may be replaced by an event tree.

2.1.4.3. Chance-Constrained Problems: In this case, some of the constraints or the objective are expressed in terms of probabilistic statements about first stage decisions. The description of the second stage or recourse action is thus avoided. This is especially useful when the cost and benefits of the second-stage decisions are difficult to assess.

3. LITERATURE SURVEY

This chapter is divided into two consecutive parts: stochastic programming studies and sustainable development issues. Sustainable issues are also specified in two parts: "earthquake" and "forest fire" risks which are our main research areas, focusing on risk mitigation issues related to stochastic programming. It is obvious that in the literature there are numerous studies, some of which are cited below without loss of generality.

3.1. Stochastic Programming

SP models aim to find out non-anticipative decisions before the occurrence of random variables to minimize the total expected recursive costs over the stages. It is important to note that decisions must be made under uncertainty, and recursive actions should be considered after the expected event realized.

The stochastic programming formulation and its concept are firstly released by Dantzig [1] who expressed them under the title of "Linear Programming under uncertainty". Beale [2] approaches that new concept from a different perspective by introducing n-dimensional newsboy problem which has become the example model for following researches.

Birge and Louveaux [3] organize the basic properties, solution procedures, application areas of stochastic programming. Their works become an introduction and a basis for stochastic programming idea for future studies. Mulvey [5] introduces the term of robust optimization to stochastic programming solutions which define the solution of stochastic programming which is near optimal and remains feasible even if the input data are changed. The robust optimizations include a penalty term to the objective function as variance or utility measures to keep the solution and model robust. Kanudia and Loulou [6] introduce this concept to energy planning, considering climate changes and economic growth. The analysis indicates significant savings of the overall system cost in using a hedging strategy over any of the perfect foresight ones. Dowman *et al.* [7] apply a two stage stochastic programming in agricultural area by considering a range of risk aversion factors in order to get more robust results compared to the corresponding deterministic model.

Escudero *et al.* [8] use linear programming and mixed integer techniques for the production and capacity planning problems with demand uncertainty. In the paper, scenarios are used to characterize the demand uncertainty, and different recourse actions such as multi period, multi product and single level production decision are defined in order to compose a non-anticipative policy for each scenario.

Powel and Frantzeskakis [9] introduce the idea that multistage SP problems can be formulated as networks with random arc capacities. They try to develop a good approximation for large scale problems by sampling a small number of scenarios to capture future uncertainties. Then, they point out the concept of hierarchical recourse used to synthesize and generalize earlier notions of nodal recourse and cyclic recourse. Glockner and NemHauser [10] develop a scheme called compath decomposition, which is derived from path decomposition for network flows with random arc capacities. Then, they use a polynomial algorithm to find the cheapest compath that can solve sub-problems, and it is extended to multi-commodity flow problems.

Stochastic programming models have multi-dimensional solutions depending on the realization of random variables. Monte Carlo approximation is a good technique to track that kind of stochastic programming models. Shapiro and Mello [11] present numerical results for two stage stochastic programming with recourse where the random data have continuos distribution by using Monte Carlo Simulation technique. Here, a statistical inference is developed, and used for estimation of the error, validation of optimality is calculated, and statistically based stopping criteria are adapted for an iterative algorithm.

Gupta and Maranas [12] apply two stage stochastic programming into supply chain planning problems. According to the paper, production decisions are made prior to the resolution of uncertainty whereas supply chain decision are thought as wait and see mode. They impose normality assumption for stochastic demands, and the evaluation of expected second stage costs was achieved by analytical integration of an equivalent convex mixed-integer nonlinear problem. The study shows that computational requirements for the proposed methodology are smaller than for Monte Carlo sampling.

Watkins *et al.* [13] use a primal simplex method and Bender decomposition techniques in their scenario based multistage stochastic programming model for water management. The model results can be improved by using a scenario generation technique based on sensitivity analysis of their model.

Maqsood and Huang [14] introduce a two stage interval stochastic programming (TISP) model for the planning of solid waste management systems under uncertainty. In the formulation, penalties are also included. The TISP model is converted into two deterministic sub-models, which corresponded to lower and upper bounds for desired objective value. Two special characteristics of that approach made it unique among the other optimization techniques that deal with uncertainties. First, TISP model provided a linkage to predefined policies determined by authorities that have to be respected when a modeling is undertaken; secondly, it furnishes the reflection of uncertainties presented as both probabilities and intervals, and the model reaches the stable interval solutions with different risk levels.

Wallace and Fleten [15] use stochastic programming technique for energy optimization models including electricity, gas, and oil with uncertainty of both demand and price. The distribution based recourse problem discussed in the previous chapter is developed by Gassmann and Ireland [4]. They use this concept to show an event tree consisting of probability distributions of random parameters.

The basic idea of L-shaped algorithm is approximate the nonlinear term in the objective of two stage stochastic programming problems. Van Slyke and Wets [16] add the feasibility and optimality cuts sequentially to L-shaped algorithm. The method uses outer linearization of random terms. The convergent cutting-plane and partial-sampling algorithm of Chen and Powel [17] is a sampling-based method lying between the stochastic decomposition and L-shaped method.

Ruszczynski [18] develops a decomposition method for multi-stage stochastic linear problems, in which the problem is represented in a tree-like form, and each node of the decision tree has a certain linear or quadratic subproblem. The sub-problems are solved in a backward manner to reach either an optimal solution or inconsistency after a number of finite iterations. The regularized decomposition method is also introduced by Ruszczynski [19], and extended by Ruszczynski and Swietanowski [20] to the new subproblem solution method underlying the primal simplex algorithm for linear programming.

Generally stochastic programming objects are non-convex. Berglann and Flam [21] develop an algorithm that combines the method of gradient projection with the heavy-ball method. Convergence is obtained under weak and natural conditions where an important condition is that marginal payoff, accumulated along the trajectory, yields a sum bounded above.

Kaut *et al.* [22] present a heuristic algorithm for generating scenario trees with specified first four moments and correlations. The algorithm generates a discrete distribution specified scenario tree by the first four marginal moments and correlations. The scenario tree is constructed by decomposing the multivariate problem into univariate ones, and using an iterative procedure that combines simulation to achieve correct correlation without changing the marginal moments. The heuristic generates large trees and solve more realistic problems.

3.2. Sustainability Issues

Sustainable issues cover a large area of ecological and environmental topics such as ozone depletion, greenhouse effects, climate changes effects, forest fires, floods, and earthquakes. Especially in recent years, the studies have been advanced in those areas because they are directly linked with the continuity of human lives in the future. Therefore, in this section, we try to discuss the main sustainable issues, catastrophic risks of those issues, and some stochastic programming researches out of numerous studies in the literature. Then, we will study the papers related to earthquake and forest fires respectively.

Patt [23] mentions about discussion between economists and ecologists on the issue of sustainable development. In his paper , the tools of cost benefit analysis and the decision sciences are examined to show members of the two disciplines often reach different results. First, economists and ecologists start from different perspectives about the point of reference against which policies should be judged. Second, economists and ecologists tend to apply different discount rates to future impacts of climate change. Third, economists and ecologists are likely to interpret differently the substantive findings and expressed uncertainties of the formal cost-benefit analysis.

Stripple [24] considers how to reduce the catastrophic risks of climate change, and distribution of financial losses. He analyzes securitization a new mechanism for spreading risks that is of interest to insurance companies to assure the supply of adequate financial capacity. Vourc'h and Jimenez [25], and van den Noord and Vourc'h [26] investigate sustainable growth in Finland and Norway respectively. Both emphasize that environmental issues need to be more closely integrated into sector policies, much of which conflicts with the country's environmental policy objectives.

Most of the sustainable issues related to natural hazards have generally catastrophic risks. Thus, in the literature, there are more researches about catastrophic risks and reduction of their effects. Bertens *et al.* [27] develop a conceptual approach towards risk assessment to synthesize physical and social components and to implement natural disaster management as a comprehensive and continuous activity. Ekenberg *et al.* [28] extend the risk evaluation process of catastrophic natural events by the integration of procedures for handling vague and numerically imprecise probabilities and utilities. Ermoliev *et al.* [29] briefly discuss the long-term effects of shocks and catastrophes on economic growth and the need for the co-existence of anticipative risk-averse (ex-ante) policies with adaptive risk- prone (ex-post) policies.

As it is pointed out above, all researches under sustainable development issues include the stochastic idea because of their unknown nature. Thus, SP idea is applied in catastrophic events generally and specifically. Ermolieva *et al.* [30] propose a general framework for the optimization capacity of an insurance industry in responding to catastrophic risks. Explicit geographical representation allows for sufficient differentiation of property values and insurance coverage in different parts of the region and for realistic modeling of catastrophes in space and time. They demonstrate the possibility of stochastic optimization techniques for optimal diversification of catastrophic exposure based on experiments. This is important for increasing the stability of insurers, their profits and for the financial protection of the population.

Lund [31] proposes a two stage formulation of flood control and demonstrates an explicit economic basis for developing integrated flood plain management plans. The approach minimizes the expected value of flood damages, costs, given a flow or stage frequency distribution. Nowak [32] formulates a stochastic model of damages caused by floods in order to allow the comparison of risk transfer instruments (such as catastrophe bonds and insurance) for various layers of the portfolio values.

3.2.1. Earthquake Risks and Mitigation

Since one of the main concerns of this study is to use stochastic programming model concept in earthquakes, we narrow our literature survey to especially mitigation activities, cost and risk evaluation, damage and impact estimation of earthquakes.

Grossi [33] studies the effects of residential earthquake insurance and structural mitigation techniques by using loss exceedance probability (EP) curves. She expands to include a sensitivity analysis of the HAZUS earthquake loss estimation methodology and the interaction of uncertainty with the effects of mitigation and insurance. The research promotes understanding of the uncertainty in earthquake risk and loss estimation as well as to advance the state-of-the-art in catastrophic risk modeling.

Tanimoto *et al.* [34] study risk allocation in a joint project in which all agents are exposed to the risk of allocated cost. They develop a stochastic cost allocation model by considering risk allocation itself. Yokomatsu and Kobayashi [35] scrutinize that catastrophe risks can be optimally, but not fully insured by insurance companies, state-contingently composed of mutual insurance to mitigate individual losses across individual households as well as contingent securities to hedge collective risks. They present a methodology to measure economic benefit of disaster mitigation investments, associated with the optimal allocation of disaster risks by insurance.

Grossi and Eeri [36] indicate that the views of structural engineers on the benefits of mitigation and contractors on the costs of mitigation are widely dispersed based on their survey results. They discuss these estimates in calculating economic losses from a significant earthquake event point of view.

Digas [37] develops a working tool for increasing capacity of insurance networks, which insure property against earthquakes including a generator of earthquake scenarios and losses. A "guaranteed" approach to finding coverage of the insurance companies is outlined. Sakakibara *et al.* [38] focus on safety degree and structural deterioration of old wooden houses based on the building codes. Mechler and Warner [39] phrase governmental precautions against earthquakes, floods, and storms, and their economic impacts .

Porter and Kiremidjian [40] propose a new approach to building damage estimation by the idea of assembly-based vulnerability which is different from empirical and heuristic methods. In this study, the building is treated as a collection of parts or *assemblies*, each of which is subjected to probabilistic demand that may be modeled using ground motion simulation and structural analysis. Each assembly is modeled as having a probabilistic capacity to resist damage. If demand exceeds capacity, an assembly fails and must be repaired or demolished and replaced. Miranda and Aslani [41] develop a loss estimation methodology by describing seismic performance quantitatively as real variables rather than discrete and subjective performance levels.

Bozorgnia and Bertero [42] examine two improved damage spectra. The improved damage spectra will be zero if the response remains elastic, and will be unity when the displacement capacity under monotonic deformation is reached. The proposed damage spectra are promising for various seismic vulnerability studies and post-earthquake applications Pricovic [43] studies multi-criteria decision making for natural hazard mitigation in the area affected by an earthquake. He develops a multi-criteria decision making procedure which consists of generating alternatives, establishing criteria, assessment of criteria weights, and application of the compromise ranking method.

Tamura *et al.* [44] develop a methodology of decision analysis of mitigating large earthquake risks arising with low probability for which expected utility theory is inadequate. They propose an alternative approach of decision analysis for such problems by using a value function under risk. They show that the value function under risk is a useful model for evaluating public risks of extreme events like large earthquakes with low probability.

McGuire [45] searches the efficiency of both probabilities and deterministic methods in seismic hazard and risk analyses for decision-making purposes. He proves that one method will have priority over the other, depending on how quantitative are the decisions to be made, depending on the seismic environment, and depending on the scope of the project (single site or a region). In many applications, a recursive analysis, where deterministic interpretations are triggered by probabilistic results and vice versa, will give the greatest insight and allow the most informed decisions to be made.

3.2.2. Forest Fire Risks and Mitigation

In this section, we review the research on forest fire hazard and its mitigation options. We first try to discuss some papers relating the hazard and effects of forest fires; and then try to illustrate the studies which apply stochastic programming to forest fires. In researches, fire is considered as stochastic variable, and it is tried to define its unexpected impact on harvest of timber management.

Wiering and Dorigo [46] describe a methodology for constructing an intelligent system which aims to support the human expert's decision making in fire control. The idea is based on first implementing a fire spread simulator and on searching for robust decision policies by reinforcement learning. Using reinforcement learning algorithms, they optimize the policies and interaction of agents that direct to learn cooperative strategies.

Ballart and Riba [47] examine the relation between government measures, volunteer participation, climate variables and forest fires. Taking a selection of fires with a certain size, a multiple regression analysis is used to find significant relations between policy instruments under the control of the government and the number of hectares burn in each case, controlling at the same time the effect of weather conditions and other context variables. Birot and Gollier [48] focus on the risk assessment options in forestry, emphasizing the need for a better integration of risks in forest management. The rationale of integration mentioned in the study lies in a rigorous assessment of all components of risk.

Rohner and Böswald [49] develop a simulation model called Forest Development and Carbon Budget Simulation Model (FORCABSIM), which provides both the development of timber stocks, sustainable levels of harvest, stocks and flow of carbon in forest at the same time by considering the economic effect of management practices on the value of forest and timber stocks. The combined study of these issues allows to assess scenarios with regard to the productive potential of forestry, the carbon cycle, and forest values.

Thompson *et al.* [50] describe a forest modelling system and examine several alternative operational interpretations of the accommodation and emulation of fire. A key element in the modelling system is a forest fire hazard model which estimates the potential for forest fire based upon forest attributes, forest utilization and topography.

Manley [51] builds an alternative approach to adjusting woodflows and cashflows to reflect wind and fire risk so that an additional risk premium has been included to the discount rate used for forest valuation purposes for high risk regions. The development of a model to estimate the value of plantations at risk from wildfire (and the potential salvage value) is described in the paper. Zhou [52] applies the stochastic optimization model to find a better selection of the planting method for the regeneration action of the next period by taking account of uncertainty in stocking levels of seedlings. Here, the sensitivity analysis of Zhou's stochastic model shows that decreasing the level of variation of mortality rate increases the expected net present value of forest.

McCarty *et al.* [53] emphasize optimal fire management strategies that incorporate trade-offs between biodiversity conservation and fuel reduction by using stochastic dynamic programming (a state-dependent decision-making tool). They consider three management strategies, namely lighting a prescribed fire, controlling the incidence of unplanned fire, and doing nothing.

The first forest level optimization formulations are pointed out by Johnson and Schuerman [54]. Then, Clutter *et al.* [55] describes the lineage of planning systems based on forest-level optimization models. Reed and Errico [56] introduce the notion of fire in their linear programming model of timber management, in which the expected burned area is subtracted from each age class in each time period, and added along with cutover area to the youngest age class in the following year

Gassman [57] formulates a smaller version of Reed and Enrico's problem as a multistage stochastic programming problem in which the proportion burned each period is stochastic. He tests different levels of discretization of the fraction burned, specifying them to provide bounds on the objective function. He first constrains the percent change in harvest quantity. The lower bound problem is highly restricted by the need for feasibility in the worst case scenario. Therefore, he allows violation of the harvest quantity constraints, but with an arbitrary penalty in the worst case scenario.

Montgomery *et al.* [58] show that computing fire damage based on the discounted value of lost timber overstated loss while the initial age class distribution and harvest flow policy are affected by the results. Martell [59] examines the effects of different regimes as well as single fires.

Forest management planning models are highly developed and used extensively, but only a few of them explicitly uses the effects of fire losses and uncertain losses which can be significant. Boychuk and Martell [60] develop forest-level timber management models with stochastic fire loss by using multistage stochastic programming based on a deterministic model. Then, they provide an insight into the impact of uncertainty on forest management planing.

4. EARTHQUAKE RISK MITIGATION PROBLEM

In this chapter, we try to propose an SP model for earthquake risk mitigation, which provides the problem description, assumptions, scenario tree and the mathematical model. Moreover, experimental design and computational results of the model indicate the stochastic programming framework that can be employed to solve selected sustainable development problem in general.

4.1. Stochastic Programming Model for Earthquake Risk Mitigation

As a realistic hypothetical setting, we show eight different alternatives that may be needed in optimal decision making for big earthquakes with low probability.

The aim is to propose some actions to mitigate the serious consequences of earthquakes in Turkey, especially in Istanbul. Earth-scientists have agreed upon the fact that the probability for the occurrence of a large earthquake in the Marmara Region (magnitude between 6.0 and 7.5) stands 65 per cent during the next 30 years. Thus, this situation makes our problem stochastic like most real life problems, and we develop a two-stage stochastic model within the framework of uncertainty.

For the sake of presenting the results in an integral manner, a pilot region is selected and the risk is modelled over the pilot area. Thus, the results can be interpreted more easily. The province of Zeytinburnu is selected to serve this purpose. The reasons for selecting Zeytinburnu will be discussed in the "Experimental Results" section.

The problem consists of two stages as the simplest version of SP. In the first stage, all of the mitigation alternatives are defined for the pilot area before an earthquake occurs so that mitigation alternatives can be compared in accordance with a reference point. The mathematical model evaluates the costs of mitigation alternatives separately and together. In addition, utility theory is included in the stochastic model to interpret the experimental results. Then, three different earthquake scenarios are defined at the second stage. It is assumed that earthquake scenarios and their impacts happen simultaneously and both are considered at the second stage because there is no such a big time gap between the occurrence of earthquake and its impacts. Moreover, three combinations of the earthquake (E/Q) impacts are considered together: building damage, loss of life and infrastructure damage as joint probability .function.

The objective function is to minimize the cost of preventive actions (mitigation), and the impact costs of those alternatives regarding the earthquake scenarios. Here, the insurance term is included into the objective function as a benefit term. Finally, we try to find out the best combination of actions to mitigate earthquake hazard risk.

4.1.1. Mitigation Alternatives

We actually consider four different mitigation options. Including the earthquake insurance factor, implementation of eight different alternatives is formed as the first stage decisions. In the model, it is assumed that all the alternatives occur at the same time.

Mitigation options are:

- Relocation: Detect E/Q resistant areas. Then, transfer the vulnerable buildings to those areas.
- Rebuild: Rebuild the building according to the new earthquake-proof design code.
- Retrofit / maintenance : Repair the building in a two or three-dimensional view point, perform diagonal reinforcement, exchange heavy roofing tiles by light ones and so on so forth
- Insurance: Buy earthquake insurance to transfer the risk of earthquake into third parties (insurance companies).

Eight alternatives are formed as follows:

A1: Relocation and take insurance

A2: Rebuild and take insurance

A3: Retrofit and take insurance

A4: Leave as it is and take insurance

A5: Relocation and do not take insurance

A6: Rebuild and do not take insurance

A7: Retrofit and do not take insurance

A8: Leave as it is and do not take insurance.

Then, three E/Q scenarios are defined by :

Scenario 1. An earthquake of magnitude 7 or bigger

Scenario 2 . An earthquake of magnitude between 5 and 7

Scenario 3. An earthquake of magnitude less than 5. (No damage cost even if an earthquake occurs.)

4.1.2. Assumptions

The following assumptions and definitions are made during the modelling process. In our formulation, parameters are defined in lower case, decision variables are defined in upper case.

- An unbiased probability distribution of earthquake scenarios is available.
- The idea of expected utility theory is applicable.
- Costs of mitigation options are known.
- Estimates of buildings damage are available for total, heavily and partial damage levels.
- Four different damage levels are assumed:
  - Total Damage: The building is completely collapsed after the earthquake
  - Heavy Damage: The building is damaged, but it can be repaired.
  - Partial Damage: The building is affected; but people can continue to live after some repair. Immediate occupancy is possible.
  - No Damage: There is no damage.

- Loss of life is assumed as a binary variable. If there is a death and injury, it is equal to "One". Otherwise, it is "Zero"
- Infrastructure damage variable is also a binary variable.
- The benefits of mitigation alternatives are estimated for each earthquake scenario s, and impact combination based on damage levels n for each alternative i. These also vary according to earthquake scenarios s and are also assumed to be additive. Moreover, these can be represented using piece-wise linearization even if the damage function is convex.

The unit effectiveness measures can be defined as the derivative of damage function given by

$$b_{i,s,n} = \frac{\partial D(s)}{\partial X(i)} \tag{4.1}$$

where $D(s)$ is the damage function for scenario s, and $X(i)$ is the mitigation decision.

4.1.3. Scenario Tree Representation

The structure of E/Q model is defined in Figure 4.2. Here, the stages are assumed to be event based, which means that the occurrence of earthquake determines the stages. The Figure 4.1 shows only a set of E/Q impacts (16 different combinations) for scenario "S2" of alternative A4. Scenario "S1" has also the same set of E/Q impacts. For scenario "S3", it is assumed that there is one combination of E/Q impact which is no damage, no loss of life, no infrastructure damage. Actually, the whole scenario tree has 264 combinations of E/Q impacts such that an alternative consists of 33 combinations of E/Q impacts and there are eight alternatives having the same combinations in scenario tree representation. Building damage abbreviations are defined as follows:

nd: no damage occurs
td: totally damaged buildings
hd: heavily damaged buildings

pd: partially damaged buildings

The following notation is employed in the scenario tree representation,

Loss of Life Value

Building Damage Level

N1 (0, 0, td)

Impact Number

Infrastructure Damage Value

Figure 4.1. The notation of scenario tree representation

$$\text{Loss of Life Value} = \begin{cases} 0, & \text{if there is no death and injury} \\ 1, & \text{otherwise} \end{cases}$$

$$\text{Infrastructure Damage Value} = \begin{cases} 0, & \text{if there is no infrastructure damage} \\ 1, & \text{otherwise} \end{cases}$$

The important point is that we should take into account the branches following "S3" for all the alternatives. There is one impact of "S3" comparing of "S1" and "S2" which have 16 different impacts as it is shown Figure 4.2. Thus, Figure 4.2 shows that "S3" has no building damage, loss of life infrastructure damage costs even if an earthquake occurs (magnitude is below 5). The simple logic of the tree ensures that any of the alternatives should be completed before the earthquake. Then, when anyone of the earthquake scenarios is realized, we are faced with the impact combinations of earthquake scenarios.

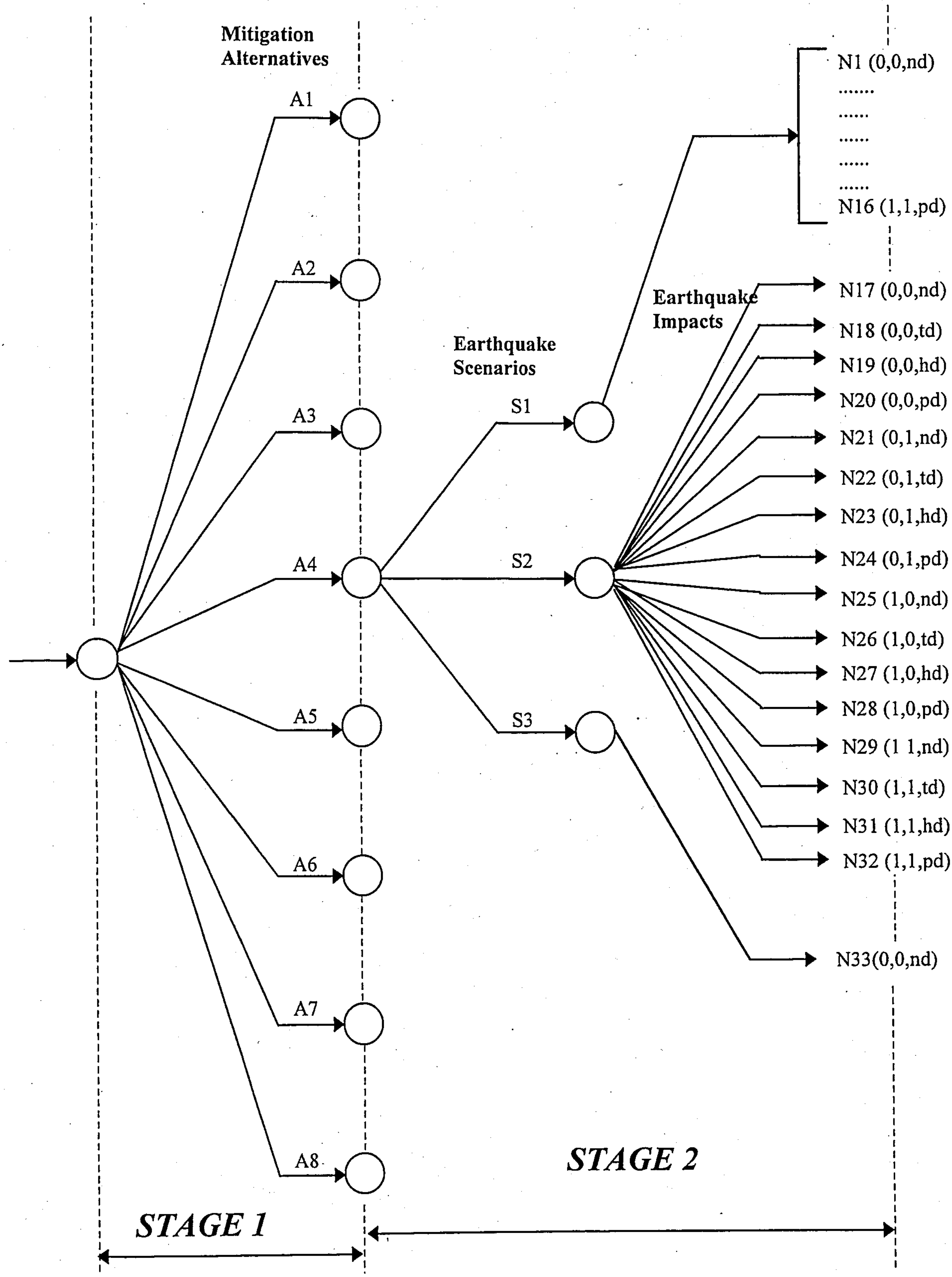

Figure 4.2. Scenario tree representation of the earthquake model

4.1.4. Development of the Mathematical Model for Earthquake Mitigation Problem

The mathematical model consists of several sets of constraints which are namely "implementation of mitigation alternatives constraint", "damage constraints", "insurance constraints", "cost equations", "utility equation" and "non negativity constraints". The objective function represents the investment costs of mitigation alternatives and damage costs simultaneously. The mitigation alternative costs are the sum of mitigation activity cost and damage cost minus the insurance return term.

Index Sets:

i : alternative sets; $i \in I = \{a1,a2,a3,a4,a5,a6,a7,a8\}$, $i \in I^{o} = \{a1,a2,a3,a4\}$ represents the set of alternatives with insurance and $i \in I^{u} = \{a5,a6,a7,a8\}$ represents the set of alternatives without insurance.

s : earthquake scenario subscript; $s \in S = \{s1,s2,s3\}$ for scenario 1 (magnitude over 7), scenario 2 (magnitude between 5 and 7) and scenario 3 (magnitude below 5) respectively.

j : damage levels subscript; $j \in J = \{d1,d2,d3\}$ for totally damaged buildings, heavily damaged buildings, partially damaged buildings respectively.

e : casualty subscript; $e \in E = \{e1=1\}$ means that there is death or injury.

v : infrastructure damage; $v \in V = \{v1=1\}$ means that there is infrastructure damage.

n : number of nodes for each scenario; $n \in N = \{n1,n2,.....,n33\}$

Variables:

Y_{isjn} : number of damaged buildings at level j in node n for earthquake scenarios s related to alternative i.

$$X_i : \begin{cases} 1, \text{ if mitigation alternative } i \text{ is taken} \\ 0, \text{ otherwise} \end{cases}$$

TY_{isj} : number of buildings damaged at level j for earthquake scenario s of alternative i.

M_{is} : percentage of dead people when earthquake scenario s is realized for alternative i.

$$AE_{nie} : \begin{cases} 1, \text{ if there is dead or injury for casualty state } e \text{ of alternative } i \text{ in node } n \\ 0, \text{ otherwise} \end{cases}$$

K_{is} : the percent of infrastructure damage when earthquake scenario s is realized for alternative i.

$$IDL_{niv} : \begin{cases} 1, \text{ if there is infrastructure damage for infrastructure damage level } v \text{ which} \\ \quad \text{occurs in node } n \text{ of alternative } i. \\ 0, \text{ otherwise} \end{cases}$$

IR_{is} : insurance benefit when earthquake scenario s is realized for alternative i.

U_{is} : scaled cost of building damages, loss of life and infrastructure value with scenario s for alternative i.

Cost Subscripts:

CTY_{isj} : cost of damaged buildings at level j for earthquake scenario s of alternative i.

CM_{is} : cost of life loss when earthquake scenario s is realized for alternative i.

CK_{is} : cost of infrastructure when earthquake scenario s is realized for alternative i.

$COST$: cost of objective function.

$ALTCOST_i$: actual cost (realized) for mitigation alternative i.

Parameters:

c_i : implementation cost of mitigation alternative i ($)

f_{sj} : building damage of level j in scenario s ($/number)

d_{ijn} : estimation of number of buildings damaged at level j for alternative i in node n.

iv : total infrastructure value of the area selected ($)

$vlife$: value of loss of life. That value is based on the amount of life insurance premium. ($/number)

np : number of people who live in the selected area before the earthquake

pb_{isn} : the probability of occurrence of node n belonging to scenario s for alternative i.

b_{ijn} : unit damage reduction benefits for alternative i under earthquake scenario s in node n.

p_s : probability of the occurrence of earthquake scenario s.

id_{is} : estimated infrastructure damage for alternative i under scenario s.

w_j : unit insurance benefit of damage level j.

$g1$, $g2$: scaling constants of mitigation and damage cost in the objective function respectively

$g3$, $g4$, $g5$, $g6$: scaling constants of building damage, loss of life, infrastructure costs and insurance benefit in utility cost term respectively.

Constraints:

Alternative Implementation Constraint : This constraint shows that every alternative occurs for the pilot region under the earthquake risk. .

$$X_i = 1 \text{ , } \forall\ i \tag{4.2}$$

Damage Constraints: After an earthquake; there will be three different categories of damage. The first is the structural damage, the second is the loss of life; and the last is the infrastructure damage.

- Building Damage Constraint: This constraint shows the effect of mitigation on damaged buildings after an earthquake. Thus, we can clearly observe the impacts of earthquake scenarios by considering alternatives separately.

$$b_{ijn}\ X_i + Y_{isjn} = d_{ijn} \text{ , } \forall\ i, s, j, n \tag{4.3}$$

$$TY_{isj} = \sum_{n \in N} pb_{isn}\ Y_{isjn} \text{ , } \forall\ i, s, j, n \tag{4.4}$$

- Loss of Life Constraint : This constraint provides the number of people affected by earthquake scenario s in alternative i.

$$M_{is} = \sum_{n \in N, e \in E} pb_{isn} \ AE_{nie} \tag{4.5}$$

- Infrastructure Damage Constraint : This is the total infrastructure damage for every scenario after an earthquake as a percent. Thus, it explains the total damage percentage for alternative i under scenario s.

$$K_{is} = \sum_{n \in N, v \in V} pb_{isn} \ IDL_{niv} \ id_{is} \tag{4.6}$$

Insurance Constraint : That constraint calculates the benefits of damaged buildings if they are insured by considering their damage level for scenarios s of every alternative *i* separately. This also shows the importance of risk transfer for especially big and unestimated natural hazards.

$$IR_{is} = \begin{cases} \sum_{n \in N, j \in J} pb_{isn} \ Y_{isjn} \ w_j \ , \ \text{for } i \in I^o \\ \\ 0 \ , \ \text{otherwise which means } i \in I^u \end{cases} \tag{4.7}$$

Cost Equations : The costs of building, loss of life and infrastructure damage are computed by

$$CTY_{is} = \sum_{j \in J} f_{js} * TY_{isj} \tag{4.8}$$

$$CM_{is} = vlife * M_{is} * np \tag{4.9}$$

$$CK_{is} = iv \ * \ K_{is} \tag{4.10}$$

Utility Equation : The cost is scaled to avoid the noise of biased data.

$$W\, U_{is} = g3 * CTY_{is} + g4 * CM_{is} + g5 * CK_{is} - g6 * IR_{is} \tag{4.11}$$

W is called recourse matrix in that formulation , which is fixed in here. The value of recourse matrix is equal to the value of identity matrix in this formulation. This allows us to characterize the feasibility region in a convenient manner of computation.

Non Negativity Constraint: All the variables in the model are positive variables. If any of them becomes negative; the problem becomes infeasible.

$$X, Y, M, K, TY, CTY, CM, CK, U, IR \geq 0 \tag{4.12}$$

Objective Function: The objective function aims at minimizing the implementation cost of all alternatives by using the idea of utility theory. It tries to balance the cost of alternatives and damage costs in one term. It minimizes the cost of impact of earthquake hazard under the assumption that all the alternatives occur in the pilot region. Moreover, we evaluate each of them one by one. Thus, we can point out which option is the best.

$$\min COST = g1 * \sum_{i \in I,} c_i X_i + g2 * \sum_{s \in S} p_s U_{is} \tag{4.13}$$

$$ALTCOST_i = g1 * c_i X_i + g2 * \sum_{s \in S} p_s U_{is} \tag{4.14}$$

We can reformulate the mathematical model by defining Expected Value (E.V.) function. The deterministic equivalent program (D.E.P.) of earthquake risk mitigation model is:

$$\min \text{COST} = g1 * \sum_{i \in I,} c_i X_i + g2 * Q(X) \tag{4.15}$$

s.t. Equations 4.2 - 4.12

where

$$Q(X) = \mathrm{E}[Q(X, s)] \tag{4.16}$$

equals to

$$Q(X) = \sum_{s \in S} p_s \, U_{is} \tag{4.17}$$

$Q(X)$ is called as the recourse or expected value function. It is computed by taking the expectation of first stage mitigation decisions.

4.2. Data Requirements of Earthquake Risk Mitigation Model

. The required data are provided by the final report of "Seismic Micro-Zonation, Disaster Mitigation Master Plan of Istanbul City prepared by JICA and sponsored by Istanbul Metropolitan Municipality in 2002". After 1999 August Earthquake, the authorities have become aware of the earthquake risk which threatens Istanbul. For that reason, various researches have been conducted to determine the possible combination of mitigation alternatives based on earthquake scenarios, and some master plans were developed. The aim is mainly to compare the effects and the costs of different earthquake scenarios.

The province of Zeytinburnu is selected as the pilot area because this region reflects the heterogeneous characteristics of Istanbul City very well. Moreover, this district has already been selected by the government as the pilot area to apply mitigation activities. The following assumptions are made to benefit from the available data:

- It is assumed the size of a typical flat is 100 m^2 in the pilot region.
- Insurance costs of buildings are deduced from the official web page of Turkish Catastrophic Insurance Pool (TCIP, www.dask.gov.tr).
- Objective function is based on expected utility theory.
- To balance the cost of mitigation and damage cost, scaling constants are used to make the model more robust. The purpose of using scaling constants is to overcome inconsistency between the damaging costs and investment costs of mitigation

alternatives in the objective function. Moreover, due to the distortion effect of loss of life, scaling constants are also used in cost calculation.

- The Infrastructure value consists of water line, natural gas line, electricity, and communication line. It is estimated according to Istanbul Metropolitan Municipality data
- Based on the TCIP web page, the insurance return value is calculated as \$21680. If the building is totally damaged, the insurer takes the whole amount. If it is damaged heavily, he takes 50 per cent of this. If it is damaged partially, he takes 10 per cent of it.
- For the synchronization of the used data; the data of model C and model A in the JICA report is used as scenario 1 and scenario 2 in the SP model. Model C is the worst case similar to scenario 1, and model A is the most probable one like scenario 2
- Number of damaged buildings, damage percents by its degrees, loss of life percents, and infrastructure damaged percents are calculated by based on JICA report data.
- By considering the commercial life insurance policies, there is a \$42000 insurance return if a person loses his life in any hazard.
- Mitigation benefit reduction ratios are determined empirically because of scarcity of data in that area, and the positive contribution of mitigation alternatives is calculated by a proposed estimation formula.
- All of the alternatives are implemented with the probability of 0.125.

Table 4.1. Cost of alternatives

Symbol	Alternative	Mitigation Cost (A)	Insurance Cost (B)	Total Alternative Cost (A)+(B)
a1	Relocation and Insurance	50000	364	50364
a2	Rebuild and Insurance	25000	364	25364
a3	Retrofit and Insurance	10000	364	10364
a4	Leave as it, and Insurance	2500	364	2864
a5	Relocation-No insurance	50000	0	50000
a6	Rebuild-no insurance	25000	0	25000
a7	Retrofit-no insurance	10000	0	10000
a8	Leave as it-no insurance	2500	0	2500

Table 4.1 shows the costs of alternatives that are used in objective function. As it can be seen, the total alternative cost is the summation of mitigation cost and insurance cost. Table 4.2 indicates the probability of realizations of scenarios. Table 4.3 shows population, the number of buildings, infrastructure value and loss of life cost based on life insurance return in the presence of death.

Table 4.2. Probability of earthquakes scenarios

Symbol	Scenario Name	Probability
s1	Magnitude 7 or higher	0.01
s2	Magnitude 5 and 7	0.10
s3	Magnitude less than 5-No damage	0.89

Table 4.3. Definition of parameters

Symbol	Definition	Value	Unit
NP	Population	239 927	people
NB	Number of buildings	15 995	quantity
IV	Infrastructure Value	218 000 000	$
LV	Loss of life Value	42000	($/person)

Table 4.4, Table 4.5 and Table 4.6 show insurance benefits, damage costs and damaged building estimates respectively. Those data are organized in accordance with the assumptions mentioned above.

Table 4.4. Insurance return benefits

Symbol	Definition	Unit ($)
W_1	Totally Damaged Buildings	21680
W_2	Heavily Damaged Buildings	10840
W_3	Partially Damaged Buildings	2170

Table 4.5. Damage costs

Scenario	Degree of Damage	Damage Cost ($)
1	Totally	50000
	Heavily	25000
	Partially	4500
2	Totally	30000
	Heavily	15000
	Partially	2500
3	No Damage	0
		0
		0

Table 4.6. Estimated number of buildings damaged in each scenario (for Zeytinburnu)

Scenario	Degree of Damage		
	Totally	Heavily	Partially
1	3036	5999	10184
2	2592	5296	9525
3	-	-	-

Table 4.7 indicates the probability of occurrence of damages for each alternative and earthquake scenario. Building damage percent is considered as the summation of totally, heavily and partially percents. The other striking point is that alternative 1 and 5, alternative 2 and 6, alternative 3 and 7 and alternative 4 and 8 are considered together because first four alternatives contain main mitigation choices with insurance factor, the remaining four alternatives contain the same mitigation choices without the insurance factor. Thus, this enables us to test the impact of the insurance factor for risk transfer. Moreover, the main mitigation alternatives (relocation, rebuilding, retrofitting) influence directly on damages (collapse of buildings, loss of life, infrastructure damage) whereas the insurance factor affects indirectly. In other words, it contributes by reducing the cost of damages in the objective function.

Table 4.7. Percentage of damage, loss of life, infrastructure damage in each scenario for each alternative

Alternative	Scenario	Building Damage Degree			Loss of life	Infrastructure
		Totally (%)	Heavily (%)	Partially (%)	(%)	(%)
1 and 5	1	2	2.5	3.7	1	5
	2	1	2	3	0.6	3.2
	3	0	0	0	0	0
2 and 6	1	5.5	4.1	6.4	1.7	13
	2	3.5	5	6	1.1	9
	3	0	0	0	0	0
3 and 7	1	10	10	13	2.3	20.5
	2	7	10	29	1.9	17.6
	3	0	0	0	0	0
4 and 8	1	19.5	10	26.9	3.1	25.7
	2	16.6	17.4	27.2	2.8	22.6
	3	0	0	0	0	0

Table A.1 shows the calculated node probabilities by using Table 4.7. In Table A.1, P_{isn} stands for the value at the end of each branch in the scenario tree representation of earthquake risk mitigation, and shows the realization when alternative i is implemented, scenarios s occurs, and impact n is realized. Furthermore, node probabilities are the joint function of building damage, loss of life and infrastructure values.

Effectiveness values in Table 4.8 are estimated logically such that relocation alternative causes the highest reduction in damage term close to hundred percent by regarding all scenarios because all of the constructions are removed from the vulnerable area to the safe area. Rebuilding alternative achieves around 70 or 80 percent damage reduction, and retrofitting is assumed to be 50 percent. The number of buildings which are protected from damage by applying mitigation alternatives are calculated by using the following estimation formula.

$$b(i, s) = k(i) \frac{\partial D(s)}{\partial X(i)} \tag{4.18}$$

Where $b(i,s)$ stands for benefit of the implementing alternative i in scenario s. $D(s)$ is the damage function for scenario s, $X(i)$ is decision vector of mitigation alternatives and $k(i)$ is constant of alternative i given by Table 4.8.

Table 4.8. Risk reduction percentages of mitigation alternatives (constant-k)

Alternative	Earthquake Scenarios		
	S1	S2	S3
1 and 5	90	95	99
2 and 6	75	80	90
3 and 7	55	60	75
4 and 8	0	0	0

Table 4.9 shows the number of undamaged buildings when Alternative i is implemented for scenario s. To make the objective function suitable for a two stage program, the cost and damage data and estimates are assumed to be linear or convex and piecewise linear.

4.3. Experimental Design of Earthquake Risk Mitigation Model

The experiment of E/Q risk mitigation model is designed in terms of using scaling constants in objective function (Equation 4.19) and damage cost function (Equation 4.20). There are three different experiments, all of which have three replications. The main aim of the experiment is to search the distortion effect of implementation cost of mitigation alternatives and damage cost. In other words, it tries to balance inconsistency between the damaging costs and investment costs of mitigation alternatives in the objective function.

Table 4.9. Number of non damaged buildings (b_{is})

Alternative	Scenario	Number of Protected Buildings		
		Totally Collapse	Heavily Collapse	Partially Collapse
1 and 5	1	2732	5399	9165
	2	2462	5031	9048
	3	0	0	0
2 and 6	1	2277	4499	7638
	2	2074	4237	7620
	3	0	0	0
3 and 4	1	1670	3300	5601
	2	1555	3178	5715
	3	0	0	0
4 and 8	1	0	0	0
	2	0	0	0
	3	0	0	0

In order to make the analysis easier, those two factors (alternative implementation and damage cost) are scaled to 1.0 by using empirical constants. Moreover, building damage, loss of life, infrastructure damage and insurance factors are also scaled to 1.0. Table 4.10 shows the definition of experimental design and constants.

$g1$: scaling constant of mitigation cost function in objective term

$g2$: scaling constant of total damage cost function (utility equation) in objective term

$$COST = g1 * \textstyle\sum_{i \in I,} c_i X_i + g2 * \textstyle\sum_{s \in S} p_s U_{is} \qquad (4.19)$$

$g3$: scaling constant of building damage cost in utility equation
$g4$: scaling constant of loss of life cost in utility equation
$g5$: scaling constant of infrastructure cost in utility equation
$g6$: scaling constant of insurance benefit in utility equation

$$U_{is} = g3 * CTY_{is} + g4 * CM_{is} + g5 * CK_{is} - g6 * IR_{is} \quad (4.20)$$

As it is seen in Equation 4.21 and 4.22, summation of those g values is equal to 1.

$$g1 + g2 = 1.0 \quad (4.21)$$

$$g3 + g4 + g5 + g6 = 1.0 \quad (4.22)$$

In addition to main aim of this experiment, the second aim of it is to determine the relationship between building damage cost and insurance benefit and effects of those two factors on damage cost function (Equation 4.20). Here, g4 and g5 keep their values at the same level for the replications of all experiments whereas g3 and g6 are changed as it is seen in Table 4.10. For the convenience of analyzing experiments, we introduce a coding system for them. In the coding system, first number represents experiment number and second one represents realization number. For example, "Design Code 1.1" stands for experiment 1 and realization 1. Then, the rest of the experiments are coded by this method.

Table 4.10. Experimental design parameters

Experiment Number	1	2	3
g1	0.1	0.2	0.5
g2	0.9	0.8	0.5
Replication Number	1	2	3
g3	0.5	0.55	0.60
g4	0.025	0.025	0.025
g5	0.05	0.025	0.025
g6	0.425	0.40	0.35

4.4. Computational Results of Earthquake Risk Mitigation Model

The model is coded by using General Algebraic Modelling System (GAMS) version 2.50 and it is solved by XA which is the mixed integer solver. The model has 13 block equations and 3417 single equations; 11 of block variables, 2625 of single variables; 6452 non zero elements; 3 derivative pool; 24 of discrete variables constraints. For the first run, the execution time is 0.530 seconds, resource usage is 0.380, and iteration count is 63.

The analysis of the results consists of four parts: execution time and resource usage of runs based on scaling constants, cost of alternatives based on utility function, insured and non insured alternatives, and value of stochastic programming.

Table 4.11. Execution time of experimental designs

Experiment 1		g1: 0.1 and g2 : 0.9			
g3	g4	g5	g6	Execution Time	Resource Usage
0.5	0.025	0.05	0.425	0.530	0.380
0.55	0.025	0.025	0.40	0.440	0.430
0.60	0.025	0.025	0.35	0.440	0.430
Experiment 2		g1: 0.2 and g2 : 0.8			
g3	g4	g5	g6	Execution Time	Resource Usage
0.5	0.025	0.05	0.425	0.520	0.440
0.55	0.025	0.025	0.40	0.430	0.550
0.60	0.025	0.025	0.35	0.430	0.550
Experiment 3		g1: 0.5 and g2 : 0.5			
g3	g4	g5	g6	Execution Time	Resource Usage
0.5	0.025	0.05	0.425	0.480	0.380
0.55	0.025	0.025	0.40	0.520	0.260
0.60	0.025	0.025	0.35	0.510	0.550

Table 4.11 shows that:

- The execution time of Experiment 1 decreases from 0.530 to 0.440 and execution time of Experiment 2 also decreases from 0.520 to 0.420 whereas in Experiment 3 that value increases from 0.480 to 0.510.
- Resource Usage of Experiment 1 increases from 0.380 to 0.430, and resource usage of Experiment 2 also increases whereas resource usage of Experiment 3 fluctuates.
- As it is easily seen, execution time and resource usage have negative correlation between each other.
- In general, when there are a little decrease in *g2* (from 0.9 to 0.8) and a huge gap between *g1* (0.1) and *g2* (0.9), execution time gives correlation among replications. However, when g1 (0.5) and g2 (0.5) are at the same level, deterioration occurs in execution time and resource usage. Because of the high implementation costs of alternatives, they distort the correlation between execution time and resource usage.

Table 4.12. Experimental design code and objective values

	Experiment 1			g1: 0.1 and g2 : 0.9		
Replication Nr	g3	g4	g5	g6	Design Code	Objective Value
1	0.5	0.025	0.05	0.425	1.1	6553.385
2	0.55	0.025	0.025	0.40	1.2	7069.502
3	0.60	0.025	0.025	0.35	1.3	7683.933
	Experiment 2			g1: 0.2 and g2 : 0.8		
Replication Nr	g3	g4	g5	g6	Design Code	Objective Value
1	0.5	0.025	0.05	0.425	2.1	8746.115
2	0.55	0.025	0.025	0.40	2.2	6303.608
3	0.60	0.025	0.025	0.35	2.3	6849.769
	Experiment 3			g1: 0.5 and g2 : 0.5		
Replication Nr	g3	g4	g5	g6	Design Code	Objective Value
1	0.5	0.025	0.05	0.425	3.1	5532.493
2	0.55	0.025	0.025	0.40	3.2	4005.926
3	0.60	0.025	0.025	0.35	3.3	4347.276

Table 4.12 indicates:

- In Experiment 1, when the insurance benefit factor ($g6$) decreases, objective value increases from 6553.385 to 7683.933. This is a reasonable result because the building damage factor increases and the insurance benefit decreases in damage function (Equation 4.20). Thus, insurance factor affects the SP model of E/Q positively.
- In Experiment 2, objective value decreases from 8746 to 6303 in replication 2, then increases to 6849 in replication 3 as the insurance benefit factor ($g6$) decreases. However, there is a distortion effect in the damage cost term (Equation 4.20) because in logic it should have been increased. Experiment 3 also has similar results to Experiment 2 whereas those are undesirable results which come from using utility equation as damage cost function.
- When we look at the relationship between experiments, as the damage value ($g2$) decreases, the objective function should also decrease logically. As it is also seen in Figure 4.3, this idea has been proved by the values of replication 2 and replication 3 for all experiments. Whereas, in replication 1 objective function fluctuates, which is an undesirable result.
- Finally, it is proved that distortion comes from mitigation activities. Because some of the activities have high implementation costs, sometimes those activities could not make enough contribution to objective value positively.

Table 4.13 shows that:

- All of the experimental runs verify that alternative 1 (relocation with insurance) and alternative 5 (relocation without insurance) are the most effective ways to reduce the impact of an earthquake. For example, Design 1.1 shows that Alternative 1 and Alternative 5 (172.628 and 178.478) have the least objective costs comparing with other alternatives.
- As it is known, alternative 1 to 4 include insurance factor, the rest of the alternatives do not include insurance factor. It can be easily observed that if people buy earthquake insurance, it also contributes to reduce the E/Q risk in addition to mitigation choices. For example, in design 1.1 objective function of Alternative 4

(Leave as it is and insurance) has 1452.611 whereas objective function of Alternative 8 (Leave as it is and no insurance) has 2527.677. Thus, it is proved that insurance factor gives positive contribution to the objective function.

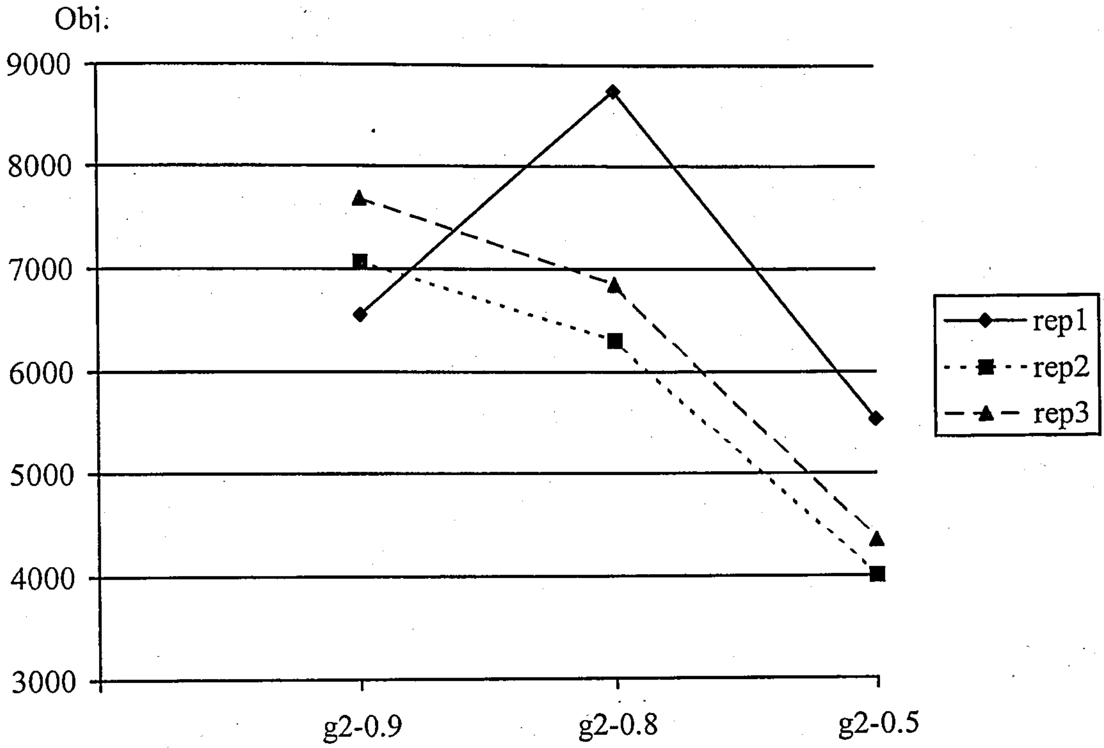

Figure 4.3. Representation of objective values for experiments

Table 4.13. Objective function value of alternatives

Design Code	Alternatives							
	Alt1	Alt2	Alt3	Alt4	Alt5	Alt6	Alt7	Alt8
1.1	172.628	331.937	644.507	1454.611	178.478	385.744	857.812	2527.667
1.2	173.840	343.299	690.505	1693.691	179.345	393.939	891.260	2703.624
1.3	175.606	358.903	752.146	2001.082	180.417	403.208	927.802	2884.769
2.1	303.161	553.275	1002.590	1915.882	308.321	601.063	1192.153	2869.669
2.2	160.121	307.973	614.934	1505.821	164.973	352.946	793.342	2403.499
2.3	161.690	321.843	669.726	1779.058	165.927	361.185	825.824	2564.517
3.1	208.362	355.308	630.505	1198.5	211.451	385.040	748.846	1794.481
3.2	118.962	201.994	388.220	942.212	121.858	229.966	499.589	1503.124
3.3	119.943	210.663	422.465	1112.985	122.454	235.116	519.890	1603.761

- Another interesting point in Table 4.13 is that positive contribution of insurance is relatively small for especially relocation and rebuild choices. When we look at "relocation" (Alt1 and Alt5) option, insurance factor reduces objective value from 178.468 to 172.628 (about 5.85 decrease in objective) in design 1.1. Whereas for "Leave as it is" option (Alt 4 and Alt 5) insurance factor reduces objective value from 2527.677 to 1452.611 (about an 1133.05 decrease in the objective value) in design 1.1. Therefore, especially for less effective mitigation options, buying insurance becomes a very important risk transfer tool. It is obvious that those interpretations are also true for all other experiments and replications in Table 4.13.

The value of the stochastic solution indicates the importance of knowing future activities and willingness to pay more to get information about future. It is also a kind of a reference point for the evaluation of SP solutions in the perspective of successful approximation. Table 4.14 shows the value of the stochastic solution (VSS).

Table 4.14. Expected value of stochastic solution of earthquake risk mitigation problem

Design Code	Stochastic Solution Recourse Problem (RP)	Expected (Mean) Value (EV)	Value of Stochastic Solution (RP-EV)	Per cent Improvement of Stochastic Solutions (RP-EV) / (RP)
1.1	6553,835	819,173	5734.662	88
1.2	7069.502	883.688	6185.814	87
1.3	7683.933	960.492	6723.441	87
2.1	8746.115	1093.264	7652.851	88
2.2	6303.608	787.951	5515.657	88
2.3	6849.769	856.221	5993.548	88
3.1	5532.493	691.562	4840.931	87
3.2	4005.926	500.741	3505.185	87
3.3	4347.276	543.410	3803.866	87

- Value of stochastic solution is a kind of SP model evaluation parameter because it is the difference between SP solution and EV solution. It shows the cost of ignoring uncertainty in the decision process. When we look at the Experiment 1 results, VSS changes from 5734 to 6723. The highest value of VSS is in Design 2.1 (Experiment

2- Trial 1) which is 7652. It starts decreasing considerably to 3505 in the third trial of Experiment 3.

- In Table 4.14, the per cent of cost of ignoring uncertainty is very high, which is 87. This value shows EV solution does not approximate SP solutions meaningfully. Thus, it could not be replaced by the solution of EV with solution of SP.

Table 4.15 shows the significant decrease of damage of buildings in alternative 1 for both scenario 1 and scenario 2. For example, in Alternative 1 (Relocation) there are only 6 building totally collapsed in scenario 1. However, there are 42 for Alternative 2 (Rebuild), 137 for Alternative 3 (Relocation), 593 totally collapsed buildings for alternative 4 in scenario 1. The similar building damage reduction patterns are realized for heavily and partially damage levels when Alternative 1 is selected instead of other alternatives. It is also proved that if people do any alternative, it results in positive effect on catastrophic risk much or less.

Table 4.15. Number of damaged buildings

Alternative	Damage Values		
	Totally	Heavy	Partial
a1.s1	6	15	38
a1.s2	1	5	14
a2.s1	42	61	163
a2.s2	18	53	114
a3.s1	137	270	598
a3.s2	72	212	457
a4.s1	593	1140	2732
a4.s2	430	921	2590
a5.s1	6	15	38
a5.s2	1	5	14
a6.s1	42	61	163
a6.s2	18	53	114
a7.s1	137	270	598
a7.s2	72	212	457
a8.s1	593	1140	2732
a8.s2	430	921	2590

Alternative 1 and 5, alternative 2 and 6, alternative 3 and 7, and alternative 4 and 8 give same results because alternative 4 to 8 are related not to take insurance which does not affect directly on building damages as we mentioned above.

Table 4.16. Percent of loss of life

Alternative	Scenario based percent of loss of life	
	S1	S2
a1	0.011	0.006
a2	0.017	0.011
a3	0.023	0.019
a4	0.031	0.028
a5	0.011	0.006
a6	0.017	0.011
a7	0.023	0.019
a8	0.031	0.028

In Table 4.16, it is observed that in Alternative 1 and Alternative 5 (relocation), the percentages of loss of life are relatively small for both Scenario 1 and Scenario 2. The casualty percent of Alternative 1 for Scenario 1 is 1.1 per cent and that of Alternative 1 for Scenario 2 is 0.6 per cent. Whereas, this percents go up to 3.1 per cent for Scenario 1 and 2.8 per cent for Scenario 2 in Alternative 4 and Alternative 8 respectively which are "Leave as it is" options. It is observed that implementation of one of the mitigation options instead of doing nothing leads to considerable decrease of casualties after the earthquake.

5. FOREST FIRE RISK MITIGATION PROBLEM

5.1. Stochastic Programming Model of Forest Fire Risk Mitigation

The second model related to the sustainable issues deals with the forest fire hazard. A forest is characterized by the area in various age classes. Normally, there are mainly two areas in a forest, which are called harvested and unharvested respectively. There is a cyclic relationship between harvested and unharvested areas. As time passes, the harvested area turns into the unharvested area by supplying the forest resources, which means that the harvested area makes a transition to the youngest age class according to this cyclic process unless it gets influenced by external effects such as insect epidemics and forest fires. When a fire happens in the forest, those transitions are cut off and all burned areas need to be renewed by rehabilitation or regeneration to complete the natural forest cycle.

In this study, "Belgrad Forest in Istanbul" is selected as the pilot area. We can develop the model and analyze results easily by using the pilot area data. The pilot forest is divided into five areas in the following manner: natural non-enhanced, basic non-enhanced, natural enhanced, basic enhanced and Not Sufficient Restocked (NSR) area.

There is a need for defining the terminology of basic, natural, enhancement and NSR area in order to understand the mathematical model clearly. In general, a forest has two types of trees based on their plantation or growing technique. First type is the trees which are naturally planted and grown up by themselves, those are called as "natural trees". The other type is the trees which are planted by people, those are called as "basic trees". "Enhancement" means the combination of some improvement techniques in order to increase the timber productivity and the number of trees in a selected area. Some enhancement techniques are thinning the stem of trees, trimming the branches of trees, and fertilizing the forest land, which increase the yields of forest and strengthen the forest for the next periods. In the "non-enhanced areas", those improvement techniques are not used, but non-enhanced area can still be harvested if the area is old enough. "Not Sufficiently

Restocked (NSR)" areas do not contain trees in the forest. In other words, those areas can be described as empty, unproductive and difficult to plant new trees.

The mathematical model of forest fire risk is a two-stage stochastic programming with deterministic equivalent solution. Uncertainty of the mathematical model stems from the occurrence of fire and its loss. Thus, some strategies are needed to reduce the effect of stochastic fire loss such as harvest quantity, regeneration intensity, NSR rehabilitation and stand enhancement. Once the first stage decisions - harvest and enhancement- are made, the second stage decisions - regeneration and rehabilitation actions- should be made after the fire to overcome its loss. The regeneration and rehabilitation actions can be thought as mitigation options for the stochastic model. New harvest and enhancement decisions should be made in the stage two after the fire.

The objective function is the minimization of costs for harvest, enhancement, mitigation activities, fire loss and unsuccessful mitigation activities by subtracting timber revenue obtained in those areas. The total cost can be minimized by reducing or distributing fire loss risk over the time periods.

5.1.1. Assumptions

The following assumptions are made in order to analyze the sustainable development of a forest in the selected area. Those assumptions help us to resemble the available data features into the stochastic programming logic.

- Two different fire loss scenarios are defined as "Low Risk" and "High Risk" ;
- For the sake of simplicity, only one harvest method is considered because the aim is not to compare different harvest methods or to select the best of them;
- The fire is assumed to occur in the middle of each time period. Enhancement and harvest decisions should be made before the fire starts (at the beginning of the period), and regeneration and rehabilitation decision should be made after the fire (at the end of the period);

- At the first, the model is developed as a two stage problem by considering two time periods, but it can be enlarged to a multi-stage stochastic programming;
- In the different perspective, the proportions of burned area can be distributed based on the age classes in a fire scenario. However, that makes the problem intractable. Thus, the portions are taken equally for all of the age classes;
- After the fire, burned area and NSR area can be regenerated by itself;
- There are regulated upper bounds on the harvest quantity;
- Time periods are scaled in terms of 20 years, which means period 1 covers 0 to 20, period 2 covers 21 to 40 years;
- It is assumed that two types of areas -"burned area" and "NSR area"- are treated after a fire;
- Two different types of regeneration and rehabilitation activities are attempted as "basic" and "natural". If the basic method is applied, it can result as "basic", "natural", or "NSR" area. Whereas if the natural method is implemented, it can result as "natural" or "NSR" areas.

5.1.2. Scenario Tree Representation

The scenario tree representation (Figure 5.1) shows that harvest and enhancement decisions should be made at the beginning of time period T1. Then, at the middle of time period T1, fire scenarios can be realized as low (L) or high (H) fire risk. Thus, some recursive actions should be taken according to rehabilitation and regeneration follows at the second stage. In addition to that, at the second stage, the harvest and enhancement decisions are considered again for the next period . At the right column of scenario tree representation, four different scenarios are shown, where the combinations of scenarios are: LL, LH, HH, HH. Another important point is that the scenario tree of the forest fire model is formed in the time based approach. The difference between the earthquake SP model and the forest fire SP model lies in the fact that the former is the event based approach whereas the latter is developed according to the time based approach..

After the fire, the burned area is treated to natural area, basic area or left to NSR area of the forest. In other words, regeneration of burned area can result as natural, basic or

NSR. Also, a portion of burned area can be regenerated by itself. Moreover, NSR area which is treated by rehabilitation can turn into natural, basic or NSR area after the fire. Hence, the treatment for turning burned and NSR area into natural or basic areas in the forest is the success criteria of our mitigation strategy whereas having NSR area after the treatments shows the unsuccessfulness of those mitigation actions.

5.1.3. Development of the Mathematical Model for Forest Fire Mitigation Problem

The mathematical model consists of two sets of constraints. The first one is "area balance" where area variables represent the state of the forest before activities occur in each period and the amount of area that is harvested, enhanced, treated successfully or not and the regenerated area in each period. The second one is "material balance" which represents the production of wood.

Index Sets:

c: forest cover type; $c \in C = \{N1, N2, B1, B2\}$ for non enhancement natural, enhanced natural, non enhanced basic, and enhanced basic respectively; $C^1=\{N1, B1\}$, $C^2=\{N2, B2\}$

a: age class; $a \in A = \{1,2,...,10\}$ for 0-20, 20-40, 40-60,...,180+yr. $A^H = (3,4,5,..,10)$ for harvestable age classes, $A^U = (1,2)$ for unharvestable age classes

t: time period; $t \in T^E = \{1,2,3\}$ for 0-20, 20-40, 60-80 yr ; $T = \{1, 2\}$, $t \in T^H = \{2, 3\}$

s: scenario set; $s \in S = \{1,2,3,4\}$ for {LL,LH, HL,HH} respectively.

r: realization of fraction burned; $r \in R = \{r1, r2\}$. $r1$ reflects the fraction of burned area under the low fire loss, whereas $r2$ is the fraction of burned area under the high fire loss.

ub : index of upper bound; $ub \in UB = \{ub1, ub2,.....,ub10\}$.

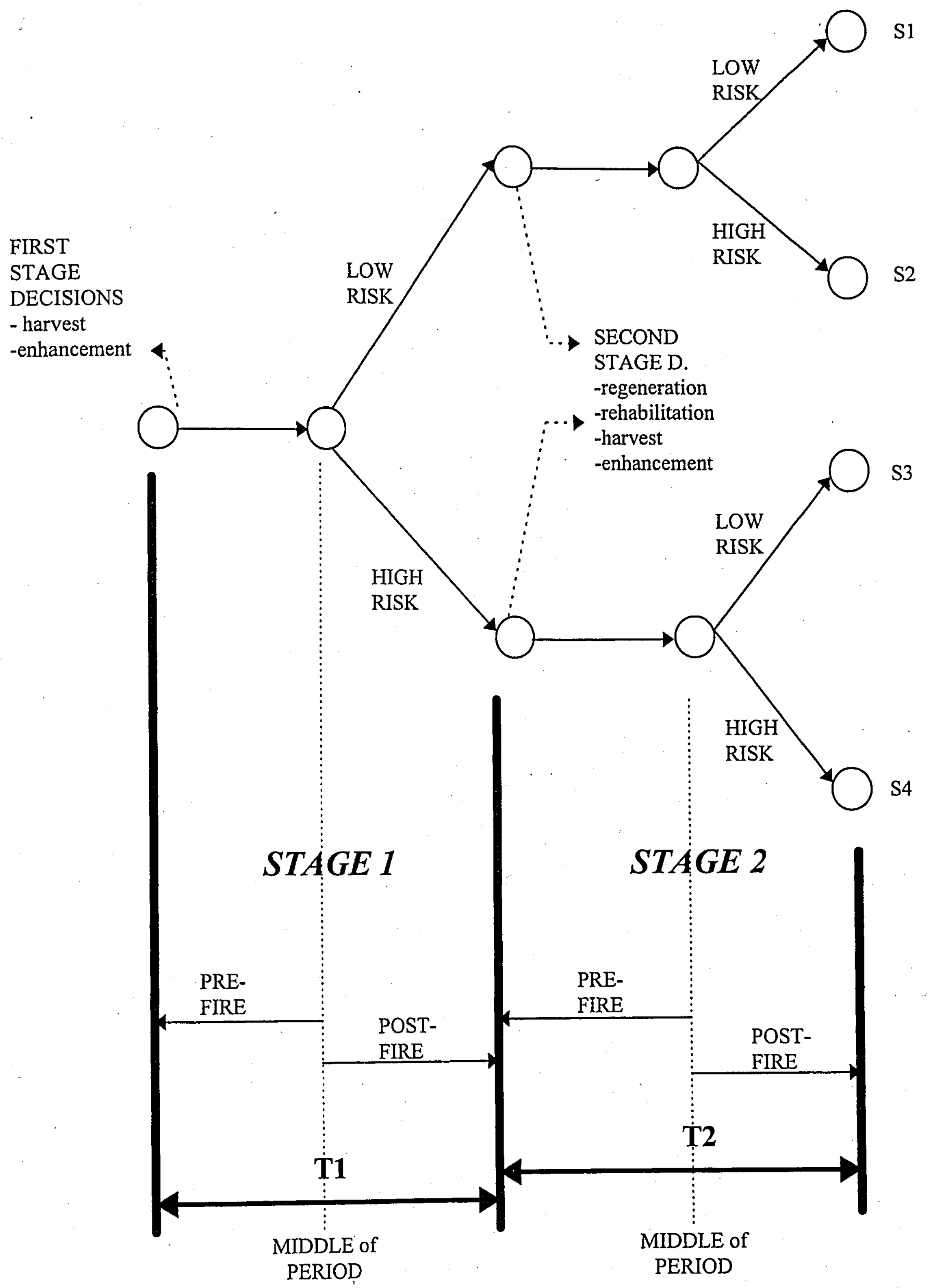

Figure 5.1. Scenario tree representation of the forest fire

Parameters:

P_s: probability of scenario s

Q_{st}: proportion of unburned area after a fire during time period t in scenario s

K_{st}: proportion of the forest that burns during time period t in scenario s. $Q_{st} = 1 - K_{st}$

S^F : the proportion of burned area that self-regenerates into natural cover type within the time period.

S^Y : the proportion of NSR area that self-regenerates into natural cover type within the time period..

$W^G{}_{ca}$: the net volumes of wood per unit area by age class a and cover type c when salvage methods G is used

PND1 : probability of node is occurred "before" fire in the scenario tree (Figure 5.1). Here, the node compromises the set of all decisions for scenario s at time t which are made in before the fire such as enhancement area, harvest quantity

PND2 : probability of node is occurred "after" fire in the scenario tree (Figure 5.1). Here, the node compromises the set of all decisions for scenario s at time t which are made in after the fire such as regeneration, rehabilitation, enhancement area, and harvest quantity

MV : market value of unsuccessful area (\$/m^3)

PC : market price of timber (\$/m^3)

The following costs have units \$/ha:

C^{HG} : cost of harvest method G

C^{EN} : cost of enhancement (improvement) of natural forest area

C^{EB} : cost of enhancement (improvement) of basic forest area

C^{FN} : cost of regeneration of burned area to natural forest area

C^{FB} : cost of regeneration of burned area to basic forest area

C^{YN} : cost of rehabilitation of NSR area to natural forest area

C^{YB} : cost of rehabilitation of NSR area to basic forest area

Basic forest area is grown by human whereas natural area is grown by itself. Not Sufficiently Restocked (NSR) area is the unproductive area of the forest. As it is mentioned, the initial forest area is composed of natural, basic and NSR areas before the fire. We apply mitigation actions which are regeneration and rehabilitation after the fire. The "basic forest" area can be regenerated as basic area and natural area, or be left as NSR area after the fire. Those are the realizations of this mitigation option. Whereas the "natural forest" area can be regenerated to natural area or be left as NSR area. Also, the "NSR basic area" can be rehabilitated to basic and natural area or left again as NSR area. However, "NSR natural area" can be rehabilitated to only natural forest area or be left as NSR area. The following parameters show the effectiveness ratios of risk mitigation actions :

E^{FBB}: effectiveness of burned basic area which is regenerated to basic area after a fire

E^{FBN}: effectiveness of burned basic area which is regenerated to natural area after a fire

E^{FNN}: effectiveness of burned natural area which is regenerated to natural area

E^{YBB}: effectiveness of NSR basic area which is rehabilitated to basic area after a fire

E^{YBN}: effectiveness of NSR basic area which is rehabilitated to natural area after fire

E^{YNN}: effectiveness of NSR natural area which is rehabilitated to natural area after a fire

ratio: proportion of enhanced (improved) forest area for the consecutive time period

X^1_{ca} : initial age class distribution for $c \in C$, $a \in A$

Y^1 : initial NSR area

$HQUB$: regulated upper bound for harvest quantity

$VUB(ub)$: regulated upper bound vector for harvest quantity

Variables:

All variables are non negative, Variables are defined on $c \in C$, $a \in A$, $t \in T$ unless otherwise indicated.

e_{scat} = newly enhanced area of cover type c, $c \in C^1$ and age class a in period t for scenario s.

f_{st} = burned area in time period t for scenario s.

f^b_{st} = non-self-regenerated burned area regenerated to basic in time period t for scenario s. (Non-self-regenerated refers to that portion of the burned area which does not spontaneously regenerate. Variable f^b_{st} refers to the area where money is spent to attempt regeneration to the basic cover type. Realizations of it will become BASIC, NATURAL, NSR).

f^n_{st} = non-self-regenerated burned area regenerated to natural area in time period t for scenario s.

f^y_{st} = non-self-regenerated burned area left to NSR for scenario s.

g_{scat} = harvest area from cover type c and age class a in time period t using harvest method G realized in scenario s.

u_{scat} = unharvested area from cover type c and age a in time period t occurred in scenario s.

x_{scat} = area of forest of cover type c and age class a in time t for scenario s.

y_{st}= NSR area in time t for scenario s.

y^{b}_{st} = non-self-regenerated NSR area rehabilitated to basic area in time period t for scenario s.

y^{n}_{st} = non-self regenerated NSR area rehabilitated to natural area in time t for scenario s.

y^{y}_{st} = non-self regenerated NSR area left to NSR area in time t for scenario s.

q_{st} = harvest quantity in time period t for scenario s.

$NSY_{s,t}$ = not successfully treated NSR area in time period t for scenario s.

$NBT_{s,t}$ = not successfully treated burned area in time period t for scenario s.

Area Balance Constraints:

Initial area: Initial forest area is composed of age distribution of trees belonging to the cover type c and the initial NSR area before fire.

$$x_{sca,1} = X^{1}_{ca}\,;\, c\in C,\, a\in A \tag{5.1}$$

$$y_{1,1} = Y^{1}\quad;\, s(1)\in S,\, t(1)\in T \tag{5.2}$$

Growth of harvestable enhanced area: Any amount of the harvestable area can be harvested by method G, the rest is left to grow.

$$x_{scat} = g_{scat} + u_{scat}\,;\, s\in S,\, c\in C^{2},\, a\in A^{H},\, t\in T \tag{5.3}$$

Growth of non-harvestable enhanced area: All of the non-harvestable area is left to grow. Non-harvestable area consists of trees which are not old enough to be harvested.

$$x_{scat} = u_{scat}\quad;\, s\in S\,,\, c\in C^{2},\, a\in (2,3),\, t\in \mathrm{T} \tag{5.4}$$

Enhancement of harvestable non-enhanced area: The harvestable non-enhanced area can be harvested, newly enhanced, or left to grow.

$$x_{scat} = g_{scat} + e_{scat} + u_{scat}\,;\, s\in S\,,\, c\in C^{1},\, a\in A^{H},\, t\in T, \tag{5.5}$$

Enhancement of non-harvestable non-enhanced area: The non-harvestable non-enhanced area can be enhanced, or left to grow.

$$\mathrm{x_{scat} = e_{scat} + u_{scat}\quad;\, s\in S\,,\, c\in C^{1},\, a\in A^{U},\, t\in T} \tag{5.6}$$

Enhanced area constraint: At the beginning of every time period, it is assumed 20 per cent of unharvestable area is decided to be enhanced by using fertilizers. By improving unharvestable areas,. the productivity of timber for the next period is increased.

$$e_{scat} = ratio * \ u_{scat}\qquad s\in S,\, c\in C^{1},\, a\in A,\, t\in T \tag{5.7}$$

Growth to oldest natural (basic) enhanced area: The oldest enhanced area natural (basic) area at the beginning of a time period is defined as the unburned oldest and second oldest unharvested previously enhanced natural (basic) area, plus the oldest and second oldest newly enhanced natural area (basic) area from the previous period.

$$x_{sN2,at} = Q_{s,t\text{-}1}\,(u_{sN2,a,t\text{-}1} + u_{sN2,a\text{-}1,t\text{-}1} + e_{sN1,a,t\text{-}1} + e_{sN1,a\text{-}1,t\text{-}1})\ ;\ s\in S,\ a\in(10),\ t\in T^H \tag{5.8}$$

$$x_{sB2,at} = Q_{s,t\text{-}1}\,(u_{s,B2,a,t\text{-}1} + u_{s,B2,a\text{-}1,t\text{-}1} + e_{s,B1,a,t\text{-}1} + e_{s,B1,a\text{-}1,t\text{-}1})\ ;\ s\in S,\ a\in(10),\ t\in T^H \tag{5.9}$$

Growth to intermediate age natural (basic) enhanced area: The intermediate age (age 3 to 9) enhanced natural (basic) area at the beginning of a time period is the immediately younger unburned unharvested previously enhanced natural(basic)area, plus the immediately younger newly enhanced natural(basic) area from the previous period.

$$x_{s,N2,at} = Q_{s,t\text{-}1}\,(u_{s,N2,a\text{-}1,t\text{-}1} + e_{s,N1,a\text{-}1,t\text{-}1})\ ;\ s\in S,\ a\in(3,4,....,9),\ t\in T^H \tag{5.10}$$

$$x_{s,B2,at} = Q_{s,t\text{-}1}\,(u_{s;B2,a\text{-}1,t\text{-}1} + e_{s,B1,a\text{-}1,t\text{-}1})\ ;\ s\in S,\ a\in(3,4,...,9),\ t\in T^H \tag{5.11}$$

Growth to age class 2 natural (basic) enhanced area: Age class 2 enhanced natural (basic) area at the beginning of a time period is the immediately younger unburned newly enhanced natural(basic) area from the previous period.

$$x_{s,N2,2,t} = Q_{s,t\text{-}1}\, e_{s,N1,1,t\text{-}1}\ ;\ s\in S, t\in T^H \tag{5.12}$$

$$x_{s,B2,2,t} = Q_{s,t\text{-}1}\ e_{s,B1,1,t\text{-}1}\ ;\ s\in S,\ t\in T^H \tag{5.13}$$

Growth to the oldest natural (basic) non-enhanced Area: The oldest non-enhanced natural (basic) area at the beginning of a time period is the oldest and second oldest unburned non-enhanced natural (basic) area from the previous period.

$$x_{scat} = Q_{s,t-1} (u_{s,c,a,t-1} + u_{s,c,a-1,t-1}) ; s \in S, c \in C^{J}, a \in (10), t \in T^{H} \quad (5.14)$$

Growth to intermediate age natural (Basic) non-enhanced area: The intermediate age (age 2 to 9) non-enhanced natural (basic) area at the beginning of a time period is the immediately younger unburned non enhanced natural (basic) area from previous period.

$$x_{scat} = Q_{s,t-1} (u_{s,c,a-1,t-1}) ; s \in S, c \in C^{J}, a \in (2,4,\ldots,9), t \in T^{H} \quad (5.15)$$

Burned area: The area of forest is burned after the fire. This is equivalent to K_{st} (proportion of the forest that burns during time period t in scenario s) times the total forest area.

$$K_{st} (\textstyle\sum_{c \in C2, a \in \{2,3,..10\}} u_{scat} + \sum_{c \in C1, a \in A} u_{scat} + \sum_{c \in C1, a \in A} e_{scat} + y_{st}) = f_{st} \quad (5.16)$$

Equation 5.16 can be rewritten as follows for the computational efficiency because the initial area is always equal to the total area at any time period t.

$$K_{st} (\textstyle\sum_{c \in C2, a \in A} X^{J}_{ca} + Y^{J}) = f_{st} \; ; s \in S, t \in T \quad (5.17)$$

Treatment of burned area: The non-self-regenerated burned area can be regenerated to natural or basic cover type, or left to NSR.

$$(1-S^{F}) f_{st} = f^{n}_{st} + f^{y}_{st} + f^{b}_{st} ; s \in S, t \in T \quad (5.18)$$

Treatment of NSR area: The non-self regenerated NSR area can be rehabilitated to natural or basic cover type, or left to remain NSR.

$$(1-S^{Y})\, y_{st} = y^{n}_{st} + y^{y}_{st} + y^{b}_{st}\,;\, s\in S,\, t\in T \tag{5.19}$$

Not successfully treated burned area : The area is not regenerated after the fire. It is equal to burned NSR area, left to NSR area from burned basic area and left to NSR area burned natural areas.

$$NBT_{s,t} = f^{y}_{s,t\text{-}1} + (1 - E^{FBN} - E^{FBB}) * f^{b}_{s,t} + (1 - E^{FNN}) * f^{n}_{s,t\text{-}1} \;\; ; s\in S,\; t\in T^{E} \tag{5.20}$$

Not successfully treated NSR area: The area is not rehabilitated after the fire. It is equal to burned NSR area, left to NSR area from burned basic area and left to NSR area burned natural areas.

$$NSY_{s,t} = y^{y}_{s,t\text{-}1} + (1 - E^{YBN} - E^{YBB}) * y^{b}_{s,t\text{-}1} + (1 - E^{YNN}) * y^{n}_{s,t\text{-}1} \;\; ; s\in S,\; t\in T^{E} \tag{5.21}$$

Transition to NSR: Non-self-regenerated and non rehabilitated NSR remains NSR. It is composed of unsuccessful rehabilitation of NSR and unsuccessful regeneration of burned areas.

$$y_{st} = f^{y}_{s,t\text{-}1} + y^{y}_{s,t\text{-}1} + (1\text{-}E^{FNN}) f^{n}_{s,t\text{-}1} + (1\text{-}E^{YNN})\, y^{n}_{s,t\text{-}1} + (1\text{-}E^{FBN}\text{-}E^{FBB}) f^{b}_{s,t\text{-}1} + (1\text{-}E^{YBN}\text{-}E^{YBB})\, y^{b}_{s,t\text{-}1}\,;\, s\in S,\; t\in T^{H} \tag{5.22}$$

Transition to the youngest basic cover type: The youngest basic cover type arises from the successful basic rehabilitation of NSR and basic regeneration.

$$x_{s,B1,1,t} = E^{FBB}\, f^{b}_{s,\,t\text{-}1} + E^{YBB}\, y^{b}_{s,t\text{-}1} \;\; ; \; s\in S,\; t\in T^{H} \tag{5.23}$$

Transition to the youngest natural cover type: The youngest natural cover type arises from unburned, self regenerated areas, successful natural regeneration and rehabilitation, and partially successful basic regeneration and rehabilitation.

$$x_{s,N1,1,t} = S^F f_{s,\,t\text{-}1} + S^Y y_{s,t\text{-}1} + E^{FNN} f^n_{s,t\text{-}1} + E^{YNN} y^n_{s,t\text{-}1} + E^{FBN} f^b_{s,t\text{-}1} + E^{YBN} y^b_{s,t\text{-}1} \;;\; s \in S,\ t \in T^H \tag{5.24}$$

Material Balance Constraints:

Harvest quantity : That refers the net merchantable harvest volume of wood per unit area by age class a and cover type c when harvested by method G in scenario s at time period t.

$$q_{st} = \sum_{c \in C2, a \in A^H} (W^G_{ca}\, g_{scat})\,;\ s \in S,\ t \in T^H \tag{5.25}$$

Upper Bound for Harvest Quantity: The upper bound is the allowable limit of harvest quantity in scenario s at time period t. It is used to test the effects of fire losses during the time periods. Here, 1.1 and 0.05 are the scaling constants used to search the effect of the stochastic fire loss and it is needed in the design of experiment.

$$q_{st} \leq HQUB \tag{5.26}$$

$$HQUB = VUB(ub) \tag{5.27}$$

$$VUB(ub) = 1.1 + 0.05\ VUB(ub\text{-}1\ ) \tag{5.28}$$

Objective Function:

The aim is to minimize the fire loss costs by applying mitigation actions. It is obvious that those recourse actions incur the total mitigation costs and if those actions are not successful, there should be penalty costs for them. In addition, there are also harvest costs to acquire timber over the time periods. Thus, the objective function is composed of the summation of all those terms. The terms used in the objective function are separately explained as follows:

ZTP (Forest Timber Profit) : This term refers to the benefit obtained by harvesting the forest area.

$$ZTP = PC\ q_{st} \tag{5.29}$$

ZHC (Harvest Method Cost) : This is the cost of using harvest method *G* to acquire timber.

$$ZHC = C^{HG}\ g_{scat} \tag{5.30}$$

ZESC (Enhancement Costs) : This is the cost of using enhancement (improvement) techniques to strengthen the trees in the forest and to increase timber productivity.

$$ZESC = C^{EN}\ e_{N1,at} + C^{EB}\ e_{B1,at} \tag{5.31}$$

ZMIT (Mitigation Investment Costs): This term refers to implementing cost of rehabilitation and regeneration

Regeneration cost : $C^{FB} f^{b}_{ts} + C^{FN} f^{n}_{ts}$

Rehabilitation cost: $C^{YB} y^{b}_{ts} + C^{YN} y^{n}_{ts}$

$$ZMIT = C^{FB} f^{b}_{ts} + C^{FN} f^{n}_{ts} + C^{YB} y^{b}_{ts} + C^{YN} y^{n}_{ts} \tag{5.32}$$

ZFC (Fire Loss Cost) : This term is obtained by multiplying penalty cost with the value of burned area.

$$ZFC = MV * f_{ts} \tag{5.33}$$

ZUMIT: This term refers to the unsuccessful mitigation costs.

$$ZUMIT = MV * y_{ts} \tag{5.34}$$

Finally; the objective term is;

$$\begin{aligned} \min z = {} & \textstyle\sum_{s\in S} \sum_{t\in T} PND1(s,t) * (C^{HG}\ g_{scat} + C^{EN}\ e_{sN1,at} + C^{EB}\ e_{sB1,at}) + \\ & \textstyle\sum_{s\in S} \sum_{t\in T} PND2(s,t) * (C^{FB}\ f^{b}_{ts} + C^{FN}\ f^{n}_{ts} + C^{YB}\ y^{b}_{ts} + C^{YN}\ y^{n}_{ts}) + \\ & \textstyle\sum_{s\in S} \sum_{t\in T} PND2(s,t)\ MV\ f_{ts} + \sum_{s\in S} \sum_{t\in T} PND2(s,t)\ MV\ y_{ts} - \\ & \textstyle\sum_{s\in S} \sum_{t\in T} PND1(s,t)\ PC\ q_t \end{aligned} \tag{5.35}$$

We can reformulate the objective function by defining recourse function (expected value) of this model. The forest fire model is complete recourse which means the first stage decisions (harvest and enhancement) are also taken into consideration as second stage decisions in addition to regeneration and rehabilitation decision. Thus, Equation 5.35 can be written as follows:

$$\begin{aligned} \min z = {} & \{(C^{HG}\ g_{c,a,1} + C^{EN}\ e_{N1,a,1} + C^{EB}\ e_{B1,a,1}) - (PC\ q_1)\} + \{[\textstyle\sum_{s\in S} PND1(s,2) * \\ & (C^{HG}\ g_{s,c,a,2} + C^{EN}\ e_{s,N1,a,2} + C^{EB}\ e_{sB1,a,2}) - \textstyle\sum_{s\in S} PND1(s,2)\ PC\ q_2] + [\textstyle\sum_{s\in S} \sum_{t\in T} \\ & PND2(s,t) * (C^{FB}\ f^{b}_{ts} + C^{FN}\ f^{n}_{ts} + C^{YB}\ y^{b}_{ts} + C^{YN}\ y^{n}_{ts}) + \textstyle\sum_{s\in S} \sum_{t\in T} PND2(s,t)\ MV\ f_{ts} \\ & + \textstyle\sum_{s\in S} \sum_{t\in T} PND2(s,t)\ MV\ y_{ts}]\} \end{aligned} \tag{5.36}$$

The second stage objective function $\varphi(FS,s)$ can be written as follows:

$$\begin{aligned} \varphi(FS,s) = \min \{ & (C^{HG}\ g_{s,c,a,2} + C^{EN}\ e_{s,N1,a,2} + C^{EB}\ e_{sB1,a,2}) - PC\ q_2] + [(C^{FB}\ f^{b}_{ts} + \\ & C^{FN}\ f^{n}_{ts} + C^{YB}\ y^{b}_{ts} + C^{YN}\ y^{n}_{ts}) + MV\ f_{ts} + MV\ y_{ts}]\} \end{aligned}$$

$$\text{s.t. : Equation 5.8 - 5.28} \tag{5.37}$$

Where t is equal to 2 and FS stands for the first stage decision sets (g-harvest method, e-enhancement and q-quantity).

The recourse function ($\Phi(FS)$) is calculated by $\Phi(FS) = \mathrm{E}_s\ \varphi(FS,s)$. Thus, recourse function ($\Phi(FS)$) is formed as follows:

$$\Phi(FS) = \{[\ \textstyle\sum_{s\in S} PND1(s,2) * (C^{HG}\ g_{s,c,a,2} + C^{EN}\ e_{s,N1,a,2} + C^{EB}\ e_{sB1,a,2}) - \sum_{s\in S} PND1(s,2)\ PC\ q_2\ ] + [\ \sum_{s\in S}\sum_{t\in T} PND2(s,t) * (C^{FB}\ f^{b}_{ts} + C^{FN}\ f^{n}_{ts} + C^{YB}\ y^{b}_{ts} + C^{YN}\ y^{n}_{ts}) + \sum_{s\in S}\sum_{t\in T} PND2(s,t)\ MV\ f_{ts} + \sum_{s\in S}\sum_{t\in T} PND2(s,t)\ MV\ y_{ts}\ ]\ \} \tag{5.38}$$

First stage objective (FSO) function is defined as follows:

$$FSO = \min z\{(C^{HG}\ g_{c,a,1} + C^{EN}\ e_{N1,a,1} + C^{EB}\ e_{B1,a,1}) - (PC\ q_1)\} \tag{5.39}$$

By considering definitions made above and Equation 5.38 and 5.39, the deterministic equivalent program (D.E.P.) of the forest fire mitigation model can be written as follows;

$$\min z = \{(C^{HG}\ g_{c,a,1} + C^{EN}\ e_{N1,a,1} + C^{EB}\ e_{B1,a,1}) - (PC\ q_1)\} + \Phi(FS)$$

$$\text{s.t. : Equation 5.1 - 5.7 and Equation 5.25-5.28} \tag{5.40}$$

Where t is equal to 1 and FS stands for first stage decision sets (g-harvest method, e-enhancement and q-quantity).

5.2. Data Requirement of Forest Fire Risk Mitigation Model

The data used in the forest problem are provided by "Directorate of Forest Institution in Istanbul Region ". The pilot area is selected as the total forest area in Istanbul and its vicinity since it is more difficult to collect data and to put them together within one data frame for the entire area of Istanbul. In this research, the area of "Belgrad Forests" is searched and modelled. Then, it is projected into the entire area. For the convenience of data structure; we consider the general probability of fire risk which is the same for all specific areas of forest in the Istanbul region. Some assumptions are needed to make the model be easily interpreted and analyzed.

General Definitions and Assumptions;

- The "Belgrad Forests" is modelled in this study. It is composed of two areas. One is "Bentler" and the other is "Kurtkemeri". Since data are available for them in separate forms, their data are combined together to reflect the total area.
- The probability and the risk of fire are calculated for the entire Istanbul region and it is assumed to be the same for all vicinity.
- The area unit is hectare (ha) which is equal to 10,000 square meters and costs are expressed in terms of US Dollar ($).
- The initial age class is distributed according to natural and basic non-enhanced types. "Scientific class type" is defined to be the basic class type and the remaining areas are defined as the natural class type.
- Because of intense risk reduction activities, there is a very low fire risk around Istanbul vicinity.
- Initially, natural and basic enhanced area for all age classes are assumed to be "zero", and not sufficiently restocked area is "125.3 hectare."

Table 5.1 shows the initial age class distributions in the selected area where it can be easily seen that the natural tree class is more dense than the basic class. Table 5.2 presents the net volume of timber harvested by the age classes. Enhanced volumes are 5 per cent and 3 per cent more than the non-enhanced volumes for natural and basic cover types, respectively.

Table 5.1. Initial age class distribution of Belgrad Forests

AGE CLASS	Actual Area (ha) NONENHANCED	
	natural X(n1,a)	basic X(b1,a)
I.	45.5	1
II	420.5	5
III	747	107.5
IV	423.5	7
V	356	170
VI	146.5	0
VII	288	0
VIII	864.5	49
IX	408	14.5
X	1474	0
TOTAL	5173.5	354

Table 5.2. Net merchantable volume by the harvest method and cover type (m^3 / ha)

	COVER TYPE			
AGE CLASS	NATURAL		BASIC	
	NONENHANCED	ENHANCED	NONENHANCED	ENHANCED
3	15000	15750	2000	2060
4	30000	31500	2800	2884
5	28000	29400	2800	2884
6	25000	26250	2650	2729.5
7	25000	26250	2500	2575
8	22000	23100	3200	3296
9	19000	19950	3080	3172.4
10	28000	29400	2900	2987

Table 5.3 represents the success factors of mitigation options so that we can compare them with each other. Then, Table 5.4 shows the self regeneration portions during each time period and the enhancement ratio for the following time periods.

Table 5.3. Effectiveness of natural and basic regeneration and rehabilitation of burned and NSR area

Symbol	Type of area	Type of regeneration / rehabilitation attempted	Type of regeneration / rehabilitation achieved	Proportion of achievement
E^{FBB}	Burned	basic	basic	0.85
E^{FBN}	Burned	basic	natural	0.15
	Burned	basic	NSR	0.00
E^{FNN}	Burned	natural	natural	0.95
	Burned	natural	NSR	0.05
E^{YBB}	NSR	basic	basic	0.75
E^{YBN}	NSR	basic	natural	0.20
	NSR	basic	NSR	0.05
E^{YNN}	NSR	natural	natural	0.83
	NSR	natural	NSR	0.17

Table 5.4. The self regeneration and annual enhancement ratios

Symbol	**Value**	**Definition**
S^F	0.71	Self regeneration ratio of burned area
S^Y	0.075	Self regeneration ratio of NSR area
RATIO(E)	0.157	annually enhancement ratio

The fires occurred between 2000 and 2003 are considered for determining the distribution of burnt areas of Istanbul. However, there are no major fires in that period. It is assumed that "LOW fire" is the case when the burned area is less than 1 hectare(ha) whereas "HIGH fire" is the case when the burned area is greater or equal to 1 hectare (ha). Thus, the scenario definitions are completed based on actual data, and weighted fire probabilities and burned area portions are calculated. The burned areas are partitioned to the total forest area of Istanbul which is 240960.1 hectare. The portions are calculated as 0.0002 for "Low fire" scenario, and 0.006 for "High fire" scenario by using Table 5.5. Moreover, Table 5.6 shows the fire statistics between the period of 2000 and 2003.

Table 5.5. Fire distributions between 2000 and 2003

	Year 2003			Year 2002		
Discrepancy	Number of Fire	Burned Area (ha)	Fire Definition	Number of Fire	Burned Area (ha)	Fire Definition
<1	167	21.66	LOW	91	8	LOW
SUB TOTAL	*167*	*21.66*		*91*	*8*	
1<....<5	29	48.6	HIGH	6	10	HIGH
5<.....<10	4	26.5	HIGH	1	6	HIGH
10+<..	4	107	HIGH	1	10	HIGH
SUB TOTAL	*37*	*182.1*		*8*	*26*	
TOTAL	204	203.8		99	34	
	Year 2001			**Year 2000**		
Discrepancy	Number of Fire	Burned Area (ha)	Fire Definition	Number of Fire	Burned Area (ha)	Fire Definition
<1	100	13	LOW	111	6	LOW
SUB TOTAL	*100*	*13*		*111*	*6*	
1<....<5	22	37.9	HIGH	15	25	HIGH
5<.....<10	3	22	HIGH	2	10	HIGH
10+<..	0	0	HIGH	6	1053.1	HIGH
SUB TOTAL	*25*	*59.9*		*23*	*1088.1*	
TOTAL	125	72.4		134	1094.1	

Table 5.6. Fire statistics

Years	LOW Fire Probability	HIGH Fire Probability
2003	0.82	0.18
2002	0.91	0.09
2001	0.80	0.20
2000	0.83	0.17
WEIGHTED	0.83	0.17

The costs in Table 5.7 are calculated based on actual forms obtained from Istanbul Forest Directorate, and the penalty cost is taken 30 percent of the market value of timber. The stochastic programming model has 3 time periods. Only the first two time periods have stochasticity of fire loss, and the remaining time period has expected fire loss.

Table 5.7. Cost symbols and their values

Symbol	Value	Unit	Definition
CHG	9.37	\$/ha	cost of harvest quantity
MV	0.9087	\$/m3	average market value of timber
PC*	0.2726	\$/m3	penalty cost
CEN	18.48	\$/ha	enhancement of natural forest area
CEB	60	\$/ha	enhancement of basic forest area
CYN	63.38	\$/ha	rehabilitation of NSR to natural forest area
CYB	86.27	\$/ha	rehabilitation of NSR to basic forest area
CFN	122.73	\$/ha	rehabilitation of burned area to natural forest area
CFB	245.51	\$/ha	rehabilitation of burned area to basic forest area

5.3. Experimental Design of Forest Fire Risk Problem

It is phrased that there are always systematic risks which cause high correlated variations between the trees in the forests. Fire is a kind of systematic risk and its loss effect is very high on timber management. In this study, in order to analyze the forest fire mitigation actions, we consider timber production in the presence of economic perspective. For that reason, we design our experiments based on the harvest quantity in the forest. It is known that when the fire happens in the forest, it firstly affects the harvest quantity and timber supply. Thus, in order to reduce the high risks of fire losses in the forest, the upper bound and regulated upper bound of harvest quantity are added to the mathematical model (Equation 5.26 and 5.28). In other words, upper bound is included to test the stochastic fire loss effect on the harvest quantity. It shows the allowable limits of harvest quantity during the decision process and is assumed that the initial value is 1.1. Thus, upper bound formulation is written as follows;

$$VUB(ub) = 1.1 + 0.05\ VUB(ub\text{-}1) \tag{5.41}$$

Based on Equation 5.41, 10 different experiments are designed. It is obvious that the number of upper bounds can be increased and some new factors can also be included into the experimental design such as enhancement quantity and age classes. As the model is developed as a two stage SP, the upper bound number is enough to interpret results because the problem size is not huge. The Table 5.8 shows the upper bound values and experiment numbers.

Table 5.8. Upper bound values and design codes

Upper Bound	ub1	ub2	ub3	ub4	ub5	ub6	ub7	ub8	ub9	ub10
Upper Bound Value	1.1	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50	1.55
Experiment Nr.	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10

5.4. Computational Results of the Forest Fire Risk Model

The model is coded using by General Algebraic Modelling System (GAMS) version 2.50 and it is solved by CONOPT2 solver. The stochastic programming model has 913 constraints, 1176 variables and 3156 non zero elements. Table 5.9 gives the solution time, the resource usage, and the optimum solution based on different upper bound values.

Table 5.9. Optimum solution of forest fire risk mitigation model by the upper bound limits

Experiment Nr	Upper Bound No	Solution Time (sn)	Resource Usage	Optimum Solution ($)
E1	ub1	0.600	0.660	1.8207
E2	ub2	0.280	0.333	1.7696
E3	ub3	0.160	0.220	1.7185
E4	ub4	0.220	0.220	1.6673
E5	ub5	0.170	0.221	1.6162
E6	ub6	0.21	0.21	1.5650
E7	ub7	0.280	0.280	1.5139
E8	ub8	0.220	0.220	1.4627
E9	ub9	0.221	0.221	1.4116
E10	ub10	1.260	1.310	1.3604

Table 5.9 is summarized as follows:

- When we look at the solution time, there is no specific pattern. It changes between 0.16 (for ub3) and 1.26 (for ub10). Additionally, resource usage has also no specific pattern. Whereas, the values of solution time and resource usage are near or equal to each other. For example, the solution time and the resource usage of ub10 are 1.26 and 1.31 respectively. Moreover, the solution time and resource usage of ub6 are equal to each other, the value of both is 0.21.

- The harvest quantity limit does not affect the solution time of the mathematical model as it is understood. If we extend the time period from t = 2 to t = 10, it is obvious that the solution time will be much longer because the size of the problem becomes large.
- The important remark is as the upper bound increases from 1.1 (ub1) to 1.55 (ub10), the optimal solution, which is the minimization of costs, starts decreasing from the value of 1.8207 to 1.3604. Increasing harvest limit affects objective value by decreasing the objective cost function. That means the increased harvest quantity reduces the effect of fire loss. In other words, if we enlarge the harvestable areas for the next periods, the impact of fire loss can be reduced.
- The other striking point in the table 5.9 is that the best optimum, value of which is 1.3604, is reached at ub10. Whereas, it takes much longer time (1.26) and uses more resources (1.31) compared to the previous upper bound solutions.
- Another considerable result is that the optimal solution decreases with a constant slope as the upper bounds increase. For example, the marginal decrease between ub1 and ub2 is equal to 0.0511 (1.8207-1.7696=0.0511). This marginal value is the same for the rest of the experiments, which means the increase of upper bound directly affect the objective function.

Table 5.10 presents the cost of every scenario, performance of rehabilitation and regeneration and expected cost of all scenarios. The results are as follows;

- Based on Figure 5.1, s1 is low (L) and low (L) fire loss, s2 is low (L) and high (H) fire loss , s3 is high (H) and low(L) fire loss, s4 is high(H) and high(H) fire loss scenarios. The abbreviations in Table 5.10 are;
    - ♦ Y: Not sufficient restocked area
    - ♦ NSY: Not successfully rehabilitated NSR area
    - ♦ FI: Burned area

- ♦ NBT: Not successfully regenerated burned area
- ♦ SY: Successfully rehabilitated NSR area. It is calculated by (Y - NSY)
- ♦ SBT: Successfully regenerated burned area. It is calculated by (FI - NBT).
- ♦ Succ. % : Success per cent of mitigation option.
- ♦ hq: Harvest quantity

- The most important point is that the low fire loss scenario (s1) has more timber cost than that of other scenarios. For example, in ub1 the timber cost of s1 is equal to 1.708 whereas it is equal to 0.098 for s2, 0.022 for s3, and 0.005 for s4. Since the low fire loss scenario occurs more often than that of other scenarios even the less area is burned in s1, it causes the highest timber cost. This result is also true for the rest of the experiments.
- When we look at the expected timber cost values, it goes down constantly by the increase of upper bounds. Its marginal decrease value is equal to 0.033 among the experiments. For instance, the expected timber cost (ETC) of ub1 is 1.194, and the ETC of ub2 is 1.161. The difference between those two values is equal to 0.033. This marginal value does not change for the rest of experiments.
- In ub1 the successful regenerated area percent is 100 per cent for the burned areas of s1 and s3 and 66.6 per cent for the burned areas of s2 and s4, which shows the less burned areas are completely treated comparing to the large burned areas. For rehabilitation option of NSR areas, the success percent is 0 per cent for both s1 and s3 whereas it is 9.5 per cent for s2 and 9.1 per cent for s4 in ub1, which shows rehabilitation is a more effective option for the high fire loss scenarios.
- For the rest of experiments; the success percent of "regeneration" option of s1 and s3 decreases from 100 per cent to 71 per cent whereas it keeps the value at 66.6 per cent for s2 and s4. Moreover, the success percent of "rehabilitation" of s1, s2 and s4 keeps their values as 0, 9.5 and 9.1 per cent respectively for all experiments whereas the success percent of s2 increases from 0 per cent to 4.8 per cent. Hence, it is easily seen that the rehabilitation and regeneration success percents are not affected by the increase of harvest quantity because they are the same for almost all experiments. Moreover, it is proved that regeneration alternative is more successful than the rehabilitation alternative.

Table 5.10. Scenario costs and performance of actions according to the upper bounds

ub1 - Expected timber cost: 1.194 hq: 1.1									
Scenario	Timber Cost	Y	NSY	SY (Y-NSY)	Succ. %	FI	NBT	SBT (FI-NBT)	Succ. %
s1	1.708	0.19	0.19	0.0	0	0.0002		0.0002	100
s2	0.098	0.21	0.19	0.02	9.5	0.006	0.002	0.004	66.6
s3	0.022	0.2	0.2	0.0	0	0.0002		0.0002	100
s4	0.005	0.22	0.2	0.02	9.1	0.006	0.002	0.004	66.6
ub2 - Expected timber cost: 1.161 hq: 1.15									
Scenario	Timber Cost	Y	NSY	SY (Y-NSY)	Succ. %	FI	NBT	SBT (FI-NBT)	Succ. %
s1	1.662	0.19	0.19	0	0	0.0002	0.000058	0.000142	71
s2	0.095	0.21	0.19	0.02	9.5	0.006	0.002	0.004	66.6
s3	0.021	0.21	0.2	0.01	4.8	0.0002	0.000058	0.000142	71
s4	0.004	0.22	0.2	0.02	9.1	0.006	0.002	0.004	66.6
ub3 - Expected timber cost: 1.128 hq: 1.2									
Scenario	Timber Cost	Y	NSY	SY (Y-NSY)	Succ. %	FI	NBT	SBT (FI-NBT)	Succ. %
s1	1.615	0.19	0.19	0	0	0.0002	0.000058	0.000142	71
s2	0.091	0.21	0.19	0.02	9.5	0.006	0.002	0.004	66.6
s3	0.020	0.21	0.2	0.01	4.8	0.0002	0.000058	0.000142	71
s4	0.004	0.22	0.2	0.02	9.1	0.006	0.002	0.004	66.6
ub4 - Expected timber cost: 1.096 hq: 1.25									
Scenario	Timber Cost	Y	NSY	SY (Y-NSY)	Succ. %	FI	NBT	SBT (FI-NBT)	Succ. %
s1	1.568	0.19	0.19	0	0	0.0002	0.000058	0.000142	71
s2	0.088	0.21	0.19	0.02	9.5	0.006	0.002	0.004	66.6
s3	0.019	0.21	0.2	0.01	4.8	0.0002	0.000058	0.000142	71
s4	0.004	0.22	0.2	0.02	9.1	0.006	0.002	0.004	66.6
ub5 - Expected timber cost: 1.063 hq: 1.3									
Scenario	Timber Cost	Y	NSY	SY (Y-NSY)	Succ. %	FI	NBT	SBT (FI-NBT)	Succ. %
s1	1.521	0.19	0.19	0	0	0.0002	0.000058	0.000142	71
s2	0.085	0.21	0.19	0.02	9.5	0.006	0.002	0.004	66.6
s3	0.018	0.21	0.2	0.01	4.8	0.0002	0.000058	0.000142	71
s4	0.004	0.22	0.2	0.02	9.1	0.006	0.002	0.004	66.6

Table 5.10. Scenario costs and performance of actions according the upper bounds(Cont')

ub6 - Expected timber cost: 1.030 hq: 1.35									
Scenario	Timber Cost	Y	NSY	SY (Y-NSY)	Succ. %	FI	NBT	SBT (FI-NBT)	Succ. %
s1	1.475	0.19	0.19	0	0	0.0002	0.000058	0.000142	71
s2	0.081	0.21	0.19	0.02	9.5	0.006	0.002	0.004	66.6
s3	0.018	0.21	0.2	0.01	4.8	0.0002	0.000058	0.000142	71
s4	0.004	0.22	0.2	0.02	9.1	0.006	0.002	0.004	66.6
ub7 - Expected timber cost: 0.997 hq: 1.40									
Scenario	Timber Cost	Y	NSY	SY (Y-NSY)	Succ. %	FI	NBT	SBT (FI-NBT)	Succ. %
s1	1.428	0.19	0.19	0	0	0.0002	0.000058	0.000142	71
s2	0.078	0.21	0.19	0.02	9.5	0.006	0.002	0.004	66.6
s3	0.017	0.21	0.2	0.01	4.8	0.0002	0.000058	0.000142	71
s4	0.003	0.22	0.2	0.02	9.1	0.006	0.002	0.004	66.6
ub8 - Expected timber cost: 0.964 hq: 1.45									
Scenario	Timber Cost	Y	NSY	SY (Y-NSY)	Succ. %	FI	NBT	SBT (FI-NBT)	Succ. %
s1	1.381	0.19	0.19	0	0	0.0002	0.000058	0.000142	71
s2	0.075	0.21	0.19	0.02	9.5	0.006	0.002	0.004	66.6
s3	0.016	0.21	0.2	0.01	4.8	0.0002	0.000058	0.000142	71
s4	0.003	0.22	0.2	0.02	9.1	0.006	0.002	0.004	66.6
ub9 - Expected timber cost: 0.932 hq: 1.50									
Scenario	Timber Cost	Y	NSY	SY (Y-NSY)	Succ. %	FI	NBT	SBT (FI-NBT)	Succ. %
s1	1.334	0.19	0.19	0	0	0.0002	0.000058	0.000142	71
s2	0.072	0.21	0.19	0.02	9.5	0.006	0.002	0.004	66.6
s3	0.015	0.21	0.2	0.01	4.8	0.0002	0.000058	0.000142	71
s4	0.003	0.22	0.2	0.02	9.1	0.006	0.002	0.004	66.6
ub10 - Expected timber cost: 0.899 hq: 1.55									
Scenario	Timber Cost	Y	NSY	SY (Y-NSY)	Succ. %	FI	NBT	SBT (FI-NBT)	Succ. %
s1	1.288	0.19	0.19	0	0	0.0002	0.000058	0.000142	71
s2	0.068	0.21	0.19	0.02	9.5	0.006	0.002	0.004	66.6
s3	0.014	0.21	0.2	0.01	4.8	0.0002	0.000058	0.000142	71
s4	0.003	0.22	0.2	0.02	9.1	0.006	0.002	0.004	66.6

Table 5.11. Expected value of stochastic solution of forest fire risk mitigation problem

Upper Bound (UB)	UB Values	Stochastic Solution of Recourse Problem (RP)	Expected (Mean) Value (EV)	Value of Stochastic Solution (RP-EV)	Per cent Improvement of Stochastic Solutions (RP-EV) / (RP)
ub1	1.10	1.8207	1.194	0.6267	34
ub2	1.15	1.7696	1.161	0.6086	34
ub3	1.20	1.7185	1.128	0.5905	34
ub4	1.25	1.6673	1.096	0.5713	34
ub5	1.30	1.6162	1.063	0.5532	34
ub6	1.35	1.5650	1.030	0.5350	34
ub7	1.40	1.5139	0.997	0.5169	34
ub8	1.45	1.4627	0.964	0.4987	34
ub9	1.50	1.4116	0.932	0.4796	34
ub10	1.55	1.3604	0.899	0.4614	34

The results of the stochastic programming problem (SPP) is compared with its corresponding mean value problem (MV) as shown in Table 5.11.

- Since replanning is inevitable, only the first period decisions are actually implemented. MVP is generally used for the approximation of stochastic programming problems. Thus, the difference between those two values shows how much we can ignore the cost of uncertainty in choosing a decision.
- Table 5.11 shows as the upper bound increases, the value of stochastic solution decreases from 0.6267 to 0.4614 constantly and its per cent is 34. Thus, we can say that MVP approximate 66 per cent of stochastic programming solution of forest fire risk mitigation model which is just above the average. In other words, people can ignore 34 per cent of cost of uncertainty when they make a decision

6. CONCLUSION AND FURTHER RESEARCH AREAS

The main target of this study is to establish SP models for destructive hazards at sustainable level. In general, it is analyzed how to mitigate and transfer catastrophic risks by implementing SP idea. Thus, two different types of problems are considered individually which are earthquake and forest fires respectively. In the following subsections, we discuss the models separately.

6.1. Discussion of the Earthquake Model

A mathematical model to evaluate alternatives by using stochastic programming idea for low probability high consequence events is formed. The proposed mathematical model integrates a variety of mitigation options economically and explicitly for coping with earthquake hazard when the probability of earthquake scenarios is given.

The proposed model seeks for optimum mitigation options mix for a given earthquake distribution. It is found that "relocation and purchase insurance" is the best combinations. If the "relocation" option is implemented, people do not need to prefer taking "insurance" because of less effect of "insurance" on reducing cost of risks. Whereas, "insurance" effect makes positive contribution in alternative3 (retrofit) and alternative4 (doing nothing). Moreover, it is shown that "relocation" and "rebuild" decrease the impact of big earthquakes at high marginal levels and "purchasing insurance" is more useful especially in medium intensity levels of earthquake risk.

It is found that if the "relocation" option with insurance factor is implemented, the number of buildings at all damage levels and the number of casualties decline markedly. It is also proved that high implementation costs of mitigation options cause a distortion in the objective value function. Thus, the mitigation costs should be kept at reasonable level.

The utility theory is included in the stochastic model which helps to explain the earthquake model. However, in some experiments it becomes inadequate and gives biased results. It is also attempted to embed the idea of value function instead of utility theory in stochastic model. Then, we compare both utility theory and value function solutions to derive national constraints. .

As it is mentioned before, the alternatives are just evaluated economically, although physical factors can be added to the model such as ground structure, distance from earthquake epicenter, solidification factors and others.

A benefit estimation formula is used in this study because of not having available real data to evaluate mitigation choices. In future work, a benefit reduction constraint can also be added to the model. .

In future researches, it is proposed that financing for mitigation alternatives can be added to the stochastic model of earthquake risk mitigation. People can finance some per cent of the total implementation cost of mitigation alternatives themselves while it requires some loans and credit such that government, municipalities or some non governmental organizations. The government can afford the total low interest rate credit amount for the "relocation" option whereas for rebuilding option it can give some subsidiary to people who suffer from earthquakes. Thus, the constraint for financing tools of mitigation options should be included in SP model from the point of public view.

We use more specific earthquake scenarios based on characteristic of the selected region and fault structure. That means scenarios should be developed a priori even considering a specific fault structure.

Moreover, we can introduce the emergency alternatives which should be undertaken during the earthquake. Secondary hazards such as fires and infectional epidemic can be added to the stochastic model for further analysis. It can be easily anticipated that those additional efforts make the proposed model more complex, but more realistic.

Numeric solutions indicate that a decision maker prefers applying "relocation" options in order to reduce the impact of earthquake significantly if he can afford the high cost of it. Moreover, "rebuilding" and "retrofitting" options which have lower cost than cost of relocation lead to considerable impact decrease for earthquake risk. The pilot area can be classified according to different risk levels from high to low, and mitigation options can be implemented by considering these risk levels such that for high risky areas "relocation" option, for medium risky areas "rebuilding" option and for low risky areas "retrofitting" option can be chosen. Furthermore, it is proved that the earthquake "insurance" should be purchased and implemented jointly with any mitigation option.

6.2. Discussion of the Forest Fire Model

As it is mentioned in previous sections, the forest fire model is developed to investigate the impact of fire losses at sustainable level and to reduce its impact on timber supply management. The stochastic model is later transformed to a deterministic equivalent problem. In order to avoid large fire losses, we can establish buffer stocks which would delay harvestable aged areas. That means, some portions of harvestable areas for the next time periods can be separated to avoid harvest quantity declines when a large fire loss occurs. However, the harvest decline in the solutions of stochastic forest fire risk model is not observed because there are mostly low fire risks in the pilot area. Actually, the main aim is to detect the success of mitigation options by taking recursive actions for the next time periods.

The empirical results of the forest fire risk model show that if the allowable limit of harvest quantity is increased, the impact of fire loss can be reduced on the timber management economically. The more frequent fires having low risk cause higher economic loss than the less frequent fires having high risk.

The "regeneration" alternative is more effective than the "rehabilitation" alternative such that most of the burned areas are successfully renewed for all scenarios by applying the regeneration alternative. On the other hand, the performance of "rehabilitation" remains at lower levels, although it leads to better results for fires having higher risks. Enhancement decision improves the volume of forest at a sustainable level that has a positive impact on harvest quantity.

In the forest fire risk model, the same fire loss proportion is allocated to every cover area type in a given time period since it has not been possible to obtain data needed to. The proportion could be varied by cover type including age class. In that case, each period's low or high loss would have effect for all cover types proportionally. However, this would make our stochastic programming approach computationally intractable.

The model is developed as a two stage stochastic programming for the sake of simplicity. In future studies, it can be developed to multi-stage stochastic programming by enlarging the time periods. The data pool can also be enlarged for the whole region or the country in order to get more meaningful and informative results. In further studies, the Markov Decision Process approach might also be applied if fire losses between stands are largely independent.

As possible extensions, prescribed fires which cause to renew forest could be included in the distribution of burned area proportion in the current model, and salvage could be incorporated into the current model with relatively little effort. Furthermore, the problem could be remodeled by considering the breakdown species of trees including age classes. However, it would also make the problem size bigger.

The model does not differentiate between areas which are enhanced at different ages and in different ways. This simplification was made to reduce model dimensionally while it is sufficient to indicate that stand improvement in general has a role in dealing with stochastic fire loss. In the expanded model, enhancement could be considered in greater detail.

The decision maker should establish some stock areas for the next periods to reduce the effect of fire loss. After the fire, he can apply the "regeneration" option as quick as possible for burned areas. Moreover, the "rehabilitation" option can be applied together with the "regeneration" option for especially the fires which lead to high economic loss

In conclusion, it is very difficult to collect data and represent the problems stochastically in the real world. Even if it is a simple problem, many factors can be unknown and many scenarios can be observed to cause model intractability. The stochastic models in this research are real life applications and they require the availability of many complex and unknown data while some mitigation actions need many years to be able to observe their positive contributions in real life. Thus, some data needed in these models are not available, and the number of experiments becomes inadequate to reach at specific certain results. However, this weakness shows the degree of difficulty for real life applications especially in case of sustainable issues.

In the extension of this study, the two stage SP models for earthquake and forest fires can be upgraded to multi-stages and multi-periods. Thus, more detailed results can be reached and the performance of the software used in this research can be tested for large size problems. Moreover, we should generally need to develop some estimation techniques for generating required data in future studies.

In this study, it is proved that stochastic programming idea merges a number of different or opposite scenarios within a single model and determines the optimal decisions in the presence of uncertainty; the best combinations turn out to be "relocation" and "rebuild" options for the earthquake risk mitigation model and "regeneration" option for reducing fire loss at the stand level of forest management. Hence stochastic programming model solutions provide pseudo-real solutions although if the stochastic programming models have high density of uncertainty causing infeasibility in general.

APPENDIX A: JOINT PROBABILITIES FOR EARTHQUAKE PROBLEM

Table A.1. Joint probabilities of building damage, infrastructure damage and loss of life

Alternative A1 and A5 - Scenario S1															
P_{111}	P_{112}	P_{113}	P_{114}	P_{115}	P_{116}	P_{117}	P_{118}	P_{119}	$P_{11(10)}$	$P_{11(11)}$	$P_{11(12)}$	$P_{11(13)}$	$P_{11(14)}$	$P_{11(15)}$	$P_{11(16)}$
86.32	1.87	2.35	3.47	4.53	0.1	0.12	0.18	0.94	0.01	0.02	0.04	0.05	0	0	0
Alternative A1 and A5 - Scenario S2															
$P_{12(17)}$	$P_{12(18)}$	$P_{12(19)}$	$P_{12(20)}$	$P_{12(21)}$	$P_{12(22)}$	$P_{12(23)}$	$P_{12(24)}$	$P_{12(25)}$	$P_{12(26)}$	$P_{12(27)}$	$P_{12(28)}$	$P_{12(29)}$	$P_{12(30)}$	$P_{12(31)}$	$P_{12(32)}$
90.45	0.96	1.92	2.89	2.99	0.03	0.05	0.1	0.54	0.01	0.01	0.01	0.02	0	0.01	0.01
Alternative A1 and A5 - Scenario S3															
$P_{13(33)}$	$P_{13(34)}$	$P_{13(35)}$	$P_{13(36)}$	$P_{13(37)}$	$P_{13(38)}$	$P_{13(39)}$	$P_{13(40)}$	$P_{13(41)}$	$P_{13(42)}$	$P_{13(43)}$	$P_{13(44)}$	$P_{13(45)}$	$P_{13(46)}$	$P_{13(47)}$	$P_{13(48)}$
100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Alternative A2 and A6 - Scenario S1															
P_{211}	P_{212}	P_{213}	P_{214}	P_{215}	P_{216}	P_{217}	P_{218}	P_{219}	$P_{21(10)}$	$P_{21(11)}$	$P_{21(12)}$	$P_{21(13)}$	$P_{21(14)}$	$P_{21(15)}$	$P_{21(16)}$
71.83	4.7	3.51	5.47	10.73	0.7	0.52	0.82	1.24	0.08	0.06	0.1	0.19	0.02	0.01	0.02
Alternative A2 and A6 - Scenario S2															
$P_{22(17)}$	$P_{22(18)}$	$P_{22(19)}$	$P_{22(20)}$	$P_{22(21)}$	$P_{22(22)}$	$P_{22(23)}$	$P_{22(24)}$	$P_{22(25)}$	$P_{22(26)}$	$P_{22(27)}$	$P_{22(28)}$	$P_{22(29)}$	$P_{22(30)}$	$P_{22(31)}$	$P_{22(32)}$
76.95	3.15	4.5	5.4	7.61	0.31	0.45	0.45	0.45	0.86	0.03	0.05	0.06	0.08	0	0.01
Alternative A2 and A6 - Scenario S3															
$P_{23(33)}$	$P_{23(34)}$	$P_{23(35)}$	$P_{23(36)}$	$P_{23(37)}$	$P_{23(38)}$	$P_{23(39)}$	$P_{23(40)}$	$P_{23(41)}$	$P_{23(42)}$	$P_{23(43)}$	$P_{23(44)}$	$P_{23(45)}$	$P_{23(46)}$	$P_{23(47)}$	$P_{23(48)}$
100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Alternative A3 and A7 - Scenario S1															
P_{311}	P_{312}	P_{313}	P_{314}	P_{315}	P_{316}	P_{317}	P_{318}	P_{319}	$P_{31(10)}$	$P_{31(11)}$	$P_{31(12)}$	$P_{31(13)}$	$P_{31(14)}$	$P_{31(15)}$	$P_{31(16)}$
52.04	7.77	7.77	10.1	13.42	2	2	2.6	1.23	0.18	0.18	0.24	0.31	0.05	0.05	0.06
Alternative A3 and A7 - Scenario S2															
$P_{32(17)}$	$P_{32(18)}$	$P_{32(19)}$	$P_{32(20)}$	$P_{32(21)}$	$P_{32(22)}$	$P_{32(23)}$	$P_{32(24)}$	$P_{32(25)}$	$P_{32(26)}$	$P_{32(27)}$	$P_{32(28)}$	$P_{32(29)}$	$P_{32(30)}$	$P_{32(31)}$	$P_{32(32)}$
57.39	5.65	8.08	9.7	12.26	1.21	1.73	2.07	1.11	0.11	0.16	0.19	0.25	0.02	0.03	0.04
Alternative A3 and A7 - Scenario S3															
$P_{33(33)}$	$P_{33(34)}$	$P_{33(35)}$	$P_{33(36)}$	$P_{33(37)}$	$P_{33(38)}$	$P_{33(39)}$	$P_{33(40)}$	$P_{33(41)}$	$P_{33(42)}$	$P_{33(43)}$	$P_{33(44)}$	$P_{33(45)}$	$P_{33(46)}$	$P_{33(47)}$	$P_{33(48)}$
100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Alternative A4 and A8 - Scenario S1															
P_{411}	P_{412}	P_{413}	P_{414}	P_{415}	P_{416}	P_{417}	P_{418}	P_{419}	$P_{41(10)}$	$P_{41(11)}$	$P_{41(12)}$	$P_{41(13)}$	$P_{41(14)}$	$P_{41(15)}$	$P_{41(16)}$
24.91	14.04	13.68	19.4	8.62	4.9	4.73	6.6	0.81	0.45	0.44	0.62	0.28	0.16	0.15	0.21
Alternative A4 and A8 - Scenario S2															
$P_{42(17)}$	$P_{42(18)}$	$P_{42(19)}$	$P_{42(20)}$	$P_{42(21)}$	$P_{42(22)}$	$P_{42(23)}$	$P_{42(24)}$	$P_{42(25)}$	$P_{42(26)}$	$P_{42(27)}$	$P_{42(28)}$	$P_{42(29)}$	$P_{42(30)}$	$P_{42(31)}$	$P_{42(32)}$
29.19	12.49	13.09	20.46	8.52	3.65	3.82	5.98	0.84	0.36	0.38	0.59	0.25	0.1	0.11	0.17
Alternative A4 and A8 - Scenario S3															
$P_{43(33)}$	$P_{43(34)}$	$P_{43(35)}$	$P_{43(36)}$	$P_{43(37)}$	$P_{43(38)}$	$P_{43(39)}$	$P_{43(40)}$	$P_{43(41)}$	$P_{43(42)}$	$P_{43(43)}$	$P_{43(44)}$	$P_{43(45)}$	$P_{43(46)}$	$P_{34(47)}$	$P_{43(48)}$
100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table A.1 shows the joint probabilities of building damage, infrastructure damage, and loss of life by using the data in Table 4.7. The probabilities in that table are realizations at the end of each node showed in Figure 4.2.

APPENDIX B: CODE OF THE STOCHASTIC PROGRAMMING MODEL OF EARTHQUKE PROBLEM

```
$title Stochastic Programming (SU,SEQ=187)
$ontext


$offtext
sets i alternative / a1*a8 /
    io(i) insured ones /a1*a4/
    iu(i) uninsured /a5*a8/
    j damage levels     / d-1,d-2,d-3 /
    s scenarios / s-1, s-2, s-3 /
    l live level /l1/
    v inf damage level /v1/
    n nodes      / n-1*n-33 /
;
Alias (s,sp), (i,ip), (n,nd)
Parameters c(i) present cost of alternative /a1 50.364, a2 25.364, a3 10.364, a4 2.864, a5
50.000, a6 25.000, a7 10.000, a8 2.500/
        PA(i)         / a1 0.125, a2 0.125, a3 0.125, a4 0.125, a5 0.125, a6 0.125, a7
0.125, a8 0.125/

Scalars
* scaling constants of objective function
        g1 /0.1/
        g2 /0.9/
* scaling constants of utility function
      g3 /0.55/
      g4 /0.025/
      g5 /0.025/
```

g6 /0.40/

table b(i,n,j) damage reduction benefit

	n-1.d-1	n-1.d-2	n-1.d-3	n-2.d-1	n-2.d-2	n-2.d-3	n-3.d-1	n-3.d-2	n-3.d-3	n-4.d-1	n-4.d-2	n-4.d-3
a1	0	0	0	2732	0	0	0	5399	0	0	0	9165
a2	0	0	0	2277	0	0	0	4499	0	0	0	7638
a3	0	0	0	1670	0	0	0	3300	0	0	0	5601
a4	0	0	0	0	0	0	0	0	0	0	0	0
a5	0	0	0	2732	0	0	0	5399	0	0	0	9165
a6	0	0	0	2277	0	0	0	4499	0	0	0	7638
a7	0	0	0	1670	0	0	0	3300	0	0	0	5601
a8	0	0	0	0	0	0	0	0	0	0	0	0

+	n-5.d-1	n-5.d-2	n-5.d-3	n-6.d-1	n-6.d-2	n-6.d-3	n-7.d-1	n-7.d-2	n-7.d-3	n-8.d-1	n-8.d-2	n-8.d-3
a1	0	0	0	2732	0	0	0	5399	0	0	0	9165
a2	0	0	0	2277	0	0	0	4499	0	0	0	7638
a3	0	0	0	1670	0	0	0	3300	0	0	0	5601
a4	0	0	0	0	0	0	0	0	0	0	0	0
a5	0	0	0	2732	0	0	0	5399	0	0	0	9165
a6	0	0	0	2277	0	0	0	4499	0	0	0	7638
a7	0	0	0	1670	0	0	0	3300	0	0	0	5601
a8	0	0	0	0	0	0	0	0	0	0	0	0

+	n-9.d-1	n-9.d-2	n-9.d-3	n-10.d-1	n-10.d-2	n-10.d-3	n-11.d-1	n-11.d-2	n-11.d-3	n-12.d-1	n-12.d-2	n-12.d-3
a1	0	0	0	2732	0	0	0	5399	0	0	0	9165
a2	0	0	0	2277	0	0	0	4499	0	0	0	7638
a3	0	0	0	1670	0	0	0	3300	0	0	0	5601
a4	0	0	0	0	0	0	0	0	0	0	0	0
a5	0	0	0	2732	0	0	0	5399	0	0	0	9165
a6	0	0	0	2277	0	0	0	4499	0	0	0	7638
a7	0	0	0	1670	0	0	0	3300	0	0	0	5601
a8	0	0	0	0	0	0	0	0	0	0	0	0

+	n-13.d-1	n-13.d-2	n-13.d-3	n-14.d-1	n-14.d-2	n-14.d-3	n-15.d-1	n-15.d-2	n-15.d-3	n-16.d-1	n-16.d-2	n-16.d-3
a1	0	0	0	2731	0	0	0	5399	0	0	0	9165
a2	0	0	0	2277	0	0	0	4499	0	0	0	7638
a3	0	0	0	1670	0	0	0	3300	0	0	0	5601
a4	0	0	0	0	0	0	0	0	0	0	0	0
a5	0	0	0	2731	0	0	0	5399	0	0	0	9165
a6	0	0	0	2277	0	0	0	4499	0	0	0	7638
a7	0	0	0	1670	0	0	0	3300	0	0	0	5601
a8	0	0	0	0	0	0	0	0	0	0	0	0

+	n-17.d-1	n-17.d-2	n-17.d-3	n-18.d-1	n-18.d-2	n-18.d-3	n-19.d-1	n-19.d-2	n-19.d-3	n-20.d-1	n-20.d-2	n-20.d-3
a1	0	0	0	2462	0	0	0	5031	0	0	0	9048
a2	0	0	0	2074	0	0	0	4237	0	0	0	7620
a3	0	0	0	1555	0	0	0	3178	0	0	0	5715

a4	0	0	0	0	0	0	0	0	0	0	0	0
a5	0	0	0	2462	0	0	0	5031	0	0	0	9048
a6	0	0	0	2074	0	0	0	4237	0	0	0	7620
a7	0	0	0	1555	0	0	0	3178	0	0	0	5715
a8	0	0	0	0	0	0	0	0	0	0	0	0

+	n-21.d-1	n-21.d-2	n-21.d-3	n-22.d-1	n-22.d-2	n-22.d-3	n-23.d-1	n-23.d-2	n-23.d-3	n-24.d-1	n-24.d-2	n-24.d-3
a1	0	0	0	2462	0	0	0	5031	0	0	0	9048
a2	0	0	0	2074	0	0	0	4237	0	0	0	7620
a3	0	0	0	1555	0	0	0	3178	0	0	0	5715
a4	0	0	0	0	0	0	0	0	0	0	0	0
a5	0	0	0	2462	0	0	0	5031	0	0	0	9048
a6	0	0	0	2074	0	0	0	4237	0	0	0	7620
a7	0	0	0	1555	0	0	0	3178	0	0	0	5715
a8	0	0	0	0	0	0	0	0	0	0	0	0

+	n-25.d-1	n-25.d-2	n-25.d-3	n-26.d-1	n-26.d-2	n-26.d-3	n-27.d-1	n-27.d-2	n-27.d-3	n-28.d-1	n-28.d-2	n-28.d-3
a1	0	0	0	2462	0	0	0	5031	0	0	0	9048
a2	0	0	0	2074	0	0	0	4237	0	0	0	7620
a3	0	0	0	1555	0	0	0	3178	0	0	0	5715
a4	0	0	0	0	0	0	0	0	0	0	0	0
a5	0	0	0	2462	0	0	0	5031	0	0	0	9048
a6	0	0	0	2074	0	0	0	4237	0	0	0	7620
a7	0	0	0	1555	0	0	0	3178	0	0	0	5715
a8	0	0	0	0	0	0	0	0	0	0	0	0

+	n-29.d-1	n-29.d-2	n-29.d-3	n-30.d-1	n-30.d-2	n-30.d-3	n-31.d-1	n-31.d-2	n-31.d-3	n-32.d-1	n-32.d-2	n-32.d-3
a1	0	0	0	2462	0	0	0	5031	0	0	0	9048
a2	0	0	0	2074	0	0	0	4237	0	0	0	7620
a3	0	0	0	1555	0	0	0	3178	0	0	0	5715
a4	0	0	0	0	0	0	0	0	0	0	0	0
a5	0	0	0	2462	0	0	0	5031	0	0	0	9048
a6	0	0	0	2074	0	0	0	4237	0	0	0	7620
a7	0	0	0	1555	0	0	0	3178	0	0	0	5715
a8	0	0	0	0	0	0	0	0	0	0	0	0

+	n-33.d-1	n-33.d-2	n-33.d-3
a1	0	0	0
a2	0	0	0
a3	0	0	0
a4	0	0	0
a5	0	0	0
a6	0	0	0
a7	0	0	0
a8	0	0	0

table f(j,s) cost of damage

	s-1	s-2	s-3
d-1	50	30	0
d-2	25	15	0
d-3	4.5	2.5	0

scalar NP number of population / 239927 /

NB number of buildings /15995/

LV loss of life value /42/

IV infra total value /218000/;

table stdat(s,*) scenario data

	prob
s-1	.01
s-2	.1
s-3	.89

table stadat(n,i,*) impact data

	a1.pr	a2.pr	a3.pr	a4.pr	a1.d-1	a1-d2	a1.d-3	a1.v1	a1.l1	a2.d-1	a2-d2	a2.d-3	a2.v1	a2.l1	a3.d-1	a3-d-2
n-1	.8632	.7183	.5201	.2491	0	0	0	0	0	0	0	0	0	0	0	0
n-2	.0187	.047	.077	.1404	3036	0	0	0	0	3036	0	0	0	0	3036	0
n-3	.0235	.0351	.0777	.1368	0	5999	0	0	0	0	5999	0	0	0	0	5999
n-4	.0347	.0547	.1010	.194	0	0	10184	0	0	0	0	10184	0	0	0	0
n-5	.0453	.1073	.1342	.0862	0	0	0	1	0	0	0	0	1	0	0	0
n-6	.001	.007	.02	.049	3036	0	0	1	0	3036	0	0	1	0	3036	0
n-7	.0012	.0052	.02	.0473	0	5999	0	1	0	0	5999	0	1	0	0	5999
n-8	.0018	.0082	.026	.066	0	0	10184	1	0	0	0	10184	1	0	0	0
n-9	.0094	.0124	.0123	.0081	0	0	0	0	1	0	0	0	0	1	0	0
n-10	.0001	.0008	.001	.0045	3036	0	0	0	1	3036	0	0	0	1	0	0
n-11	.0002	.0006	.0018	.0044	0	5999	0	0	1	0	5999	0	0	1	0	0
n-12	.0004	.0010	.0024	.0062	0	0	10184	0	1	0	0	10184	0	1	0	0
n-13	.0005	.0019	.0031	.0028	0	0	0	1	1	0	0	0	1	1	0	0
n-14	.0000	.0002	.0005	.0016	3036	0	0	1	1	3036	0	0	1	1	0	0
n-15	.0000	.0001	.0005	.0015	0	5999	0	1	1	0	5999	0	1	1	0	0
n-16	.0000	.0002	.0006	.0021	0	0	10184	1	1	0	0	10184	1	1	0	0
n-17	.9045	.7695	.5739	.2919	0	0	0	0	0	0	0	0	0	0	0	0
n-18	.0096	.0315	.0565	.1249	2592	0	0	0	0	2592	0	0	0	0	2592	0
n-19	.0192	.045	.0808	.1309	0	5296	0	0	0	0	5296	0	0	0	0	5296
n-20	.0289	.054	.097	.2046	0	0	9525	0	0	0	0	9525	0	0	0	0
n-21	.0299	.0761	.1226	.0852	0	0	0	1	0	0	0	0	1	0	0	0
n-22	.0003	.0031	.0121	.0365	2592	.0	0	1	0	2592	0	0	1	0	2592	0
n-23	.0005	.0045	.0173	.0382	0	5296	0	1	0	0	5296	0	1	0	0	5296
n-24	.001	.0053	.0207	.0598	0	0	9525	1	0	0	0	9525	1	0	0	0
n-25	.0054	.0086	.0111	.0084	0	0	0	0	1	0	0	0	0	1	0	0

n-26	.0001	.0003	.0011	.0036	2592	0	0	0	1	2592	0	0	0	1	2592	0
n-27	.0001	.0005	.0016	.0038	0	5296	0	0	1	0	5296	0	0	1	0	5296
n-28	.0001	..0006	.0019	.0059	0	0	9525	0	1	0	0	9525	0	1	0	0
n-29	.0002	.0008	.0025	.0025	0	0	0	1	1	0	0	0	1	1	0	0
n-30	.0	.0	.0002	.0010	2592	0	0	1	1	2592	0	0	1	1	2592	0
n-31	.0001	.0001	.0003	.0011	0	5296	0	1	1	0	5296	0	1	1	0	5296
n-32	.0001	.0001	.0004	.0017	0	0	9525	1	1	0	0	9525	1	1	0	0
n-33	1.0	1.0	1.0	1.0	0	0	0	0	0	0	0	0	0	0	0	0

+	a1.pr	a2.pr	a3.pr	a4.pr	a3.d-3	a3.v1	a3.l1	a4.d-1	a4-d2	a4.d-3	a4.v1	a4.l1
n-1	.8632	.7183	.5201	.2491	0	0	0	0	0	0	0	0
n-2	.0187	.047	.077	.1404	0	0	0	3036	0	0	0	0
n-3	.0235	.0351	.0777	.1368	0	0	0	0	5999	0	0	0
n-4	.0347	.0547	.1010	.194	10184	1	0	0	0	10184	1	0
n-5	.0453	.1073	.1342	.0862	0	1	0	0	0	0	1	0
n-6	.001	.007	.02	.049	0	1	0	3036	0	0	1	0
n-7	.0012	.0052	.02	.0473	0	1	0	0	5999	0	1	0
n-8	.0018	.0082	.026	.066	10184	1	0	0	0	10184	1	1
n-9	.0094	.0124	.0123	.0081	0	0	1	0	0	0	0	1
n-10	.0001	.0008	.001	.0045	0	0	1	3036	0	0	0	1
n-11	.0002	.0006	.0018	.0044	0	0	1	0	5999	0	0	1
n-12	.0004	.0010	.0024	.0062	10184	0	1	0	0	10184	0	1
n-13	.0005	.0019	.0031	.0028	0	1	1	0	0	0	1	1
n-14	.0000	.0002	.0005	.0016	0	1	1	3036	0	0	1	1
n-15	.0000	.0001	.0005	.0015	0	1	1	0	5999	0	1	1
n-16	.0000	.0002	.0006	.0021	10184	1	1	0	0	10184	1	1
n-17	.9045	.7695	.5739	.2919	0	0	0	0	0	0	0	0
n-18	.0096	.0315	.0565	.1249	0	0	0	2592	0	0	0	0
n-19	.0192	.045	.0808	.1309	0	0	0	0	5296	0	0	0
n-20	.0289	.054	.097	.2046	9525	0	0	0	0	9525	0	0
n-21	.0299	.0761	.1226	.0852	0	1	0	0	0	0	1	0
n-22	.0003	.0031	.0121	.0365	0	1	0	2592	0	0	1	0
n-23	.0005	.0045	.0173	.0382	0	1	0	0	5296	0	1	0
n-24	.001	.0053	.0207	.0598	9525	1	0	0	0	9525	1	0
n-25	.0054	.0086	.0111	.0084	0	0	1	0	0	0	0	1
n-26	.0001	.0003	.0011	.0036	0	0	1	2592	0	0	0	1
n-27	.0001	.0005	.0016	.0038	0	0	1	0	5296	0	0	1
n-28	.0001	..0006	.0019	.0059	9525	0	1	0	0	9525	0	1
n-29	.0002	.0008	.0025	.0025	0	1	1	0	0	0	1	1
n-30	.0	.0	.0002	.0010	0	1	1	2592	0	0	1	1
n-31	.0001	.0001	.0003	.0011	0	1	1	0	5296	0	1	1
n-32	.0001	.0001	.0004	.0017	9525	1	1	0	0	9525	1	1
n-33	1.0	1.0	1.0	1.0	0	0	0	0	0	0	0	0

+	a5.pr	a6.pr	a7.pr	a8.pr	a5.d-1	a5-d2	a5.d-3	a5.v1	a5.l1	a6.d-1	a6-d2	a6.d-3	a6.v1	a6.l1	a7.d-1	a7-d-2
n-1	.8632	.7183	.5201	.2491	0	0	0	0	0	0	0	0	0	0	0	0
n-2	.0187	.047	.077	.1404	3036	0	0	0	0	3036	0	0	0	0	3036	0
n-3	.0235	.0351	.0777	.1368	0	5999	0	0	0	0	5999	0	0	0	0	5999
n-4	.0347	.0547	.1010	.194	0	0	10184	0	0	0	0	10184	0	0	0	0
n-5	.0453	.1073	.1342	.0862	0	0	0	1	0	0	0	0	1	0	0	0
n-6	.001	.007	.02	.049	3036	0	0	1	0	3036	0	0	1	0	3036	0
n-7	.0012	.0052	.02	.0473	0	5999	0	1	0	0	5999	0	1	0	0	5999
n-8	.0018	.0082	.026	.066	0	0	10184	1	0	0	0	10184	1	0	0	0
n-9	.0094	.0124	.0123	.0081	0	0	0	0	1	0	0	0	0	1	0	0

n-10	.0001	.0008	.001	.0045	3036	0	0	0	1	3036	0	0	0	1	0	0
n-11	.0002	.0006	.0018	.0044	0	5999	0	0	1	0	5999	0	0	1	0	0
n-12	.0004	.0010	.0024	.0062	0	0	10184	0	1	0	0	10184	0	1	0	0
n-13	.0005	.0019	.0031	.0028	0	0	0	1	1	0	0	0	1	1	0	0
n-14	.0000	.0002	.0005	.0016	3036	0	0	1	1	3036	0	0	1	1	0	0
n-15	.0000	.0001	.0005	.0015	0	5999	0	1	1	0	5999	0	1	1	0	0
n-16	.0000	.0002	.0006	.0021	0	0	10184	1	1	0	0	10184	1	1	0	0
n-17	.9045	.7695	.5739	.2919	0	0	0	0	0	0	0	0	0	0	0	0
n-18	.0096	.0315	.0565	.1249	2592	0	0	0	0	2592	0	0	0	0	2592	0
n-19	.0192	.045	.0808	.1309	0	5296	0	0	0	0	5296	0	0	0	0	5296
n-20	.0289	.054	.097	.2046	0	0	9525	0	0	0	0	9525	0	0	0	0
n-21	.0299	.0761	.1226	.0852	0	0	0	1	0	0	0	0	1	0	0	0
n-22	.0003	.0031	.0121	.0365	2592	0	0	1	0	2592	0	0	1	0	2592	0
n-23	.0005	.0045	.0173	.0382	0	5296	0	1	0	0	5296	0	1	0	0	5296
n-24	.001	.0053	.0207	.0598	0	0	9525	1	0	0	0	9525	1	0	0	0
n-25	.0054	.0086	.0111	.0084	0	0	0	0	1	0	0	0	0	1	0	0
n-26	.0001	.0003	.0011	.0036	2592	0	0	0	1	2592	0	0	0	1	2592	0
n-27	.0001	.0005	.0016	.0038	0	5296	0	0	1	0	5296	0	0	1	0	5296
n-28	.0001	..0006	.0019	.0059	0	0	9525	0	1	0	0	9525	0	1	0	0
n-29	.0002	.0008	.0025	.0025	0	0	0	1	1	0	0	0	1	1	0	0
n-30	.0	.0	.0002	.0010	2592	0	0	1	1	2592	0	0	1	1	2592	0
n-31	.0001	.0001	.0003	.0011	0	5296	0	1	1	0	5296	0	1	1	0	5296
n-32	.0001	.0001	.0004	.0017	0	0	9525	1	1	0	0	9525	1	1	0	0
n-33	1.0	1.0	1.0	1.0	0	0	0	0	0	0	0	0	0	0	0	0

+	a5.pr	a6.pr	a7.pr	a8.pr	a7.d-3	a7.v1	a7.l1	a8.d-1	a8-d2	a8.d-3	a8.v1	a8.l1
n-1	.8632	.7183	.5201	.2491	0	0	0	0	0	0	0	0
n-2	.0187	.047	.077	.1404	0	0	0	3036	0	0	0	0
n-3	.0235	.0351	.0777	.1368	0	0	0	0	5999	0	0	0
n-4	.0347	.0547	.1010	.194	10184	1	0	0	0	10184	1	0
n-5	.0453	.1073	.1342	.0862	0	1	0	0	0	0	1	0
n-6	.001	.007	.02	.049	0	1	0	3036	0	0	1	0
n-7	.0012	.0052	.02	.0473	0	1	0	0	5999	0	1	0
n-8	.0018	.0082	.026	.066	10184	1	0	0	0	10184	1	1
n-9	.0094	.0124	.0123	.0081	0	0	1	0	0	0	0	1
n-10	.0001	.0008	.001	.0045	0	0	1	3036	0	0	0	1
n-11	.0002	.0006	.0018	.0044	0	0	1	0	5999	0	0	1
n-12	.0004	.0010	.0024	.0062	10184	0	1	0	0	10184	0	1
n-13	.0005	.0019	.0031	.0028	0	1	1	0	0	0	1	1
n-14	.0000	.0002	.0005	.0016	0	1	1	3036	0	0	1	1
n-15	.0000	.0001	.0005	.0015	0	1	1	0	5999	0	1	1
n-16	.0000	.0002	.0006	.0021	10184	1	1	0	0	10184	1	1
n-17	.9045	.7695	.5739	.2919	0	0	0	0	0	0	0	0
n-18	.0096	.0315	.0565	.1249	0	0	0	2592	0	0	0	0
n-19	.0192	.045	.0808	.1309	0	0	0	0	5296	0	0	0
n-20	.0289	.054	.097	.2046	9525	0	0	0	0	9525	0	0
n-21	.0299	.0761	.1226	.0852	0	1	0	0	0	0	1	0
n-22	.0003	.0031	.0121	.0365	0	1	0	2592	0	0	1	0
n-23	.0005	.0045	.0173	.0382	0	1	0	0	5296	0	1	0
n-24	.001	.0053	.0207	.0598	9525	1	0	0	0	9525	1	0
n-25	.0054	.0086	.0111	.0084	0	0	1	0	0	0	0	1
n-26	.0001	.0003	.0011	.0036	0	0	1	2592	0	0	0	1
n-27	.0001	.0005	.0016	.0038	0	0	1	0	5296	0	0	1
n-28	.0001	..0006	.0019	.0059	9525	0	1	0	0	9525	0	1

```
n-29  .0002  .0008  .0025  .0025   0     1   1   0     0     0     1   1
n-30  .0     .0     .0002  .0010   0     1   1   2592  0     0     1   1
n-31  .0001  .0001  .0003  .0011   0     1   1   0     5296  0     1   1
n-32  .0001  .0001  .0004  .0017   9525  1   1   0     0     9525  1   1
n-33  1.0    1.0    1.0    1.0     0     0   0   0     0     0     0   0

set sn(s,n)  scenario node  mapping / s-1.(n-1*n-16), s-2.(n-17*n-32), s-3.(n-33) /
   tree(s,n) / s-1.(n-1*n-16), s-2.(n-17*n-32), s-3.(n-33) /
parameter dam(n,i,j) stochastic damage
        AL(n,i,l)  being alive
        INFL(n,i,v) infrasructure damage level
        pr(n,i)  node probability;
dam(n,i,j) = stadat(n,i,j);
AL(n,i,l) = stadat(n,i,l);
INFL(n,i,v) = stadat(n,i,v);
pr(n,i) = sum(tree(s,n), stdat(s,'prob')*stadat(n,i,'pr'));
display pr;


table IDP(i,s) infrastructure damage percent
      s-1   s-2  s-3
a1    .02   .01  .0
a2    .03   .02  .0
a3    .05   .03  .02
a4    .06   .04  .03
a5    .02   .01  .0
a6    .03   .02  .0
a7    .05   .03  .02
a8    .06   .04  .03
parameters W(j) unit insurance benefit /d-1 21.680, d-2 10.840, d-3 2.170/;
variables x(i) alternative in time t-1
        y(i,j,s,n) number of buildings damaged
        ty(i,s,j) total buildings damaged by scenario
        m(i,s)  number of people died by percent
        k(i,s) percent of infrastr damage
```

```
        IR(i,s) insurance value
        cty(i,s)  cost of damaged buildings
        cm(i,s)   cost of lost lives
        ck(i,s)  cost of infrastructure damage
        U(i,s)    utility function
        cost

positive variables x,y,k,ty,cm,cty,ck,u

integer variable m;

equations
        objective
        bal(i)     decision of mitigation  alternative
        live(i,s)  being alive
        insur1(i,s)  taking earthquake insurance
        insur2(i,s)  not taking insurance
        infra(i,s)    infrastructure damage
        damex(i,s,j)  buildings damage because of  earthquake
        dambal(i,j,s,n) damage balance
        costdam(i,s)    cost of damage d buildings
        costlive(i,s)   cost of being alive
        costinf(i,s)    cost of infrastructure damage
        utility(i,s)    utility function

        lb(i,j,s,n);

bal(i).. x(i) =e= 1;

dambal(i,j,sn(s,n)).. b(i,n,j)*X(i) + y(i,j,sn) =e= dam(n,i,j);

live(i,s).. m(i,s)=e= sum((tree(s,n),l), stadat(n,i,'pr') * AL(n,i,l));
```

```
infra(i,s).. k(i,s) =e= sum((tree(s,n),v), stadat(n,i,'pr') * IDP(i,s)*INFL(n,i,v));

insur1(io,s).. IR(io,s)=e= sum( (tree(s,n),j), stadat(n,io,'pr') * y(io,j,s,n) * W(j));
insur2(iu,s).. IR(iu,s) =e= 0;

damex(i,s,j).. ty(i,s,j) =e= sum(n, stadat(n,i,'pr') *y(i,j,s,n));

costdam(i,s).. cty(i,s) =e= sum (j, f(j,s)*ty(i,s,j));
costlive(i,s).. cm(i,s) =e= LV* m(i,s)*NP;
costinf(i,s).. ck(i,s) =e= IV* k(i,s);

utility(i,s).. U(i,s) =e= g3*cty(i,s)+g4*cm(i,s)+g5*ck(i,s)-g6*IR(i,s);

lb(i,j,s,n).. y(i,j,s,n) =g= 0;

objective.. cost =e= g1*Sum(i, c(i)*x(i)) + g2*sum((i,s), stdat(s, 'pr') * U(i,s));

model ESP  Earthquake stochastic programming / all /;

* option lp = oslse;

PARAMETERS
ALTCOST(i)  ALTERNATIVE COST
EALTCOST    EXPECTED COST

solve su min cost us mip;
display x.l, y.l, m.l, k.l, IR.l, ty.l,U.l,cty.l, cm.l, ck.l, cost.l ;
ALTCOST(i) =  g1*(c(i)*x.l(i)) + g2*sum(s, stdat(s, 'pr') * U.l(i,s));
EALTCOST = SUM(i, ALTCOST(i)*PA(i));
DISPLAY ALTCOST, EALTCOST
```

APPENDIX C: CODE OF THE STOCHASTIC PROGRAMMING MODEL OF FOREST FIRE PROBLEM

```
$OFFUPPER
 $title Stochastic Programming Forest Fire (SU,SEQ=187)
$OFFSYMXREF OFFSYMLIST OFFUELLIST OFFUELXREF
$inlinecom /* */
FILE F1 /ffsp.prn/ PUT F1 ;F1.PC = 4 ;F1.ND = 3 ;F1.NZ = 1E-8
 PUT
  / 'The Effect of Fire '
  /
  / '<ch5b\rffsp> Version 1993.05.19 a'
  /
  / 'File:' SYSTEM.IFILE /
   'Date:' SYSTEM.DATE /
   'Time:' SYSTEM.TIME /
  / 'This run:' /
   ' - Regulated upper bound on harvest quantity' /
   ' - Stochastic programming problem' /
   ' - Economic criterion' /
   ' - Expected ACD' /
* Baseline model characteristics:
*
* - Multistage stochastic programming problem
* - Deterministic equivalent problem solution
* - Strategies for adaptation to stochastic fire loss:
*      - Harvest quantity
*      - Regeneration intensity
*      - NSR rehabilitation
*      - Harvest utilization
```

```
*       - Stand enhancement
* - Note regarding sequence of decisions and burn during period:
*       - Harvest and enhancement decided before burn
*       - Regeneration & rehabilitation decided after burn
* - Note that forest area is scaled to 1 (A) for numerical
*       stability purposes.  Model is linear, so that objective
*       function, harvest quantity, etc should be scaled up
*       accordingly.  Different forest areas have different fire
*       loss distributions, represented in F(r) and PrF(r).
*>>- No decline penalty
* - Non-selfregenerated cutover must be regenerated (cy=0)
*>>- Harvest quantity upper bounded for all s,t
*

Options Limrow=0, Limcol=0, Iterlim=400000, Reslim=400000, SOLPRINT = OFF,
SYSOUT = OFF


* Units:
*       Internal   Real
* Item    Unit     Unit
* ----   -----     ----
* Area     A        ha
* Volume   V        m**3
* Money    D        dolar
sets
     c forest cover type   / N1 natural initial, N2 natural enhanced,
                             B1 basic initial, B2 basic enhanced /
   c1(c) nonenhanced initial   /N1,B1/
   c2(c) enhanced     /N2,B2/
   a age class      / a1 0-20, a2 20-40, a3 40-60 ,
                      a4 60-80, a5 80-100, a6 100-120,
                      a7 120-140, a8 140-160, a9 160-180, a10 180+ /
   ah(a) harvestable age class /a3*a10/
```

```
    au(a) unharvestable age class /a1*a3/
    as(a) age23  /a2*a3/
    s scenarios  / s1*s4 /
    r Realization of fraction burned /r1*r2/
    ub index of upper bound /ub1*ub10/
    te extended time period /t1*t4/
    t(te) time horizon  / t1*t3 /
    ts(te)  time horizon with random loss /t1*t2/
    tr(t) time horizon of report / t1 0-20, t2 20-40, t3 40-60 /
;
Alias  (r,rp), (s,sp), (t,tp)

Parameters
       HQUB regulated upper bound   (V)
       VUB(ub)  regulated upper bound vector   (V) ;
VUB(ub) = 1.1+0.05* (ORD(ub)-1)

Scalars
     REDECOQ reduction of econ terms for quant obj /1.0000/

*-REDECOQ is the weight of economic terms in the objective for
*  the wood quantity criterion.  It is 1 for the economic
*  criterion.  A value of zero gives no weight, but small
*  weight is useful to identify the most profitable decisions
*  among multiple optima.  Search for a value small enough
*  to avoid competition with the wood quantity terms, but large
*  enough to avoid numerical problems (but, zero is fine).
*
*  0.01 worked; 0.0001 was "infeasible" for CDECR = 0.15
*
*  For reporting, note that F1.NZ & .ND affects conversion for
*  printing.
```

```
*
Abort $ (REDECOQ GT 1 OR REDECOQ LT 0) "REDECOQ > 1 OR < 0"

 PUT
  / 'No. of time periods (used & reported):' /
  CARD(t):0:0 CARD(tr):0:0 /
 LOOP(tr, PUT ORD(tr):0:0) PUT /
 PUT 'Time period cell labels (years):' /
 LOOP(tr, PUT tr.TE(tr)) PUT /
 PUT 'Age class distribution cell labels (years):' /
 LOOP(a, PUT a.TE(a)) PUT /

 PUT
  'No. of alternative upper bounds tested:' / CARD(ub):0:0 /
  'Upper bounds (V):' /

 LOOP(ub, PUT ORD(ub):0:0) PUT /
 LOOP(ub $ (ORD(ub) LE 10), PUT VUB(ub):0:2) PUT /
 LOOP(ub $ (ORD(ub) GT 10 AND ORD(ub) LE 20), PUT VUB(ub):0:2) PUT /
 LOOP(ub $ (ORD(ub) GT 20), PUT VUB(ub):0:2) PUT /

 PUT
  'Reduction of economic terms for quantity maximization' /
  '...criterion (1 for economic criterion):' /

;F1.NR = 0  /* Scientific notation */

 PUT REDECOQ
 PUT $ (REDECOQ EQ 1) '  (Economic criterion)' /
 PUT $ (REDECOQ LT 1) '  (Wood quantity criterion)' /

;F1.NR = 1  /* Normal notation */
```

Scalar

MV market value (D per V) /0.9087/

PC penalty cost /0.2726/

CHG cost of harvest method g (D per A) /9.37/

CEN cost of enhancement of natural (D per A) /18.48/

CEB cost of enhancement of basic (D per A) /60/

CFN cost of regeneration to natural (D per A) /122.73/

CFB cost of regeneration to basic (D per A) /245.51/

CYN cost of rehabilitation of NSR to natural (D per A) /63.38/

CYB cost of rehabilitation of NSR to basic (D per A) /86.27/

Scalar

EFBB efectiveness of burned area basic to basic / .85/

EFBN efectiveness of burned area basic to natural /.15/

EFNN efectiveness of burned area natural to natural /.95/

EYBB efectiveness of NSR area basic to basic /.75/

EYBN efectiveness of NSR area basic to natural /.20/

EYNN efectiveness of NSR area natural to natural /.83/

scalar

ININSR initial Not sufficenly restocked area (A) / 125.3 /

LAND total forest land area (A)

SF proportion of burned area that self regenerates during time period / 0.71/

SY proportion of NSR area that self regenerates during time period / 0.075/

RATIO proportion of enhanced area /0.157/

TABLE INIFOR(a,c1) Initial age class distribution (A)

	n1	b1	
a1	45.5	1	/* actual distribution */
a2	420.5	5	
a3	747	107.5	
a4	423.5	7	
a5	356	170	

```
   a6   146.5    0
   a7   288      0
   a8   864.5    49
   a9   408      14.5
   a10  1474     0

*    a1    46.7     0           /* reductive area */
*    a2    280.3    0
*    a3    379.6    0
*    a4    255.3    0
*    a5    224.8    0
*    a6    106.6    0
*    a7    211.5    0
*    a8    597.1    0
*    a9    313.9    0
*    a10   1017.2   0
;
LAND = SUM((c1,a), INIFOR(a,c1)) + ININSR
 DISPLAY 'LAND before scaling to 1:', LAND

*-Scale total land area to 1
;ININSR = ININSR / LAND
;INIFOR(a,c1) = INIFOR(a,c1) / LAND

;LAND = SUM((c1,a), INIFOR(a,c1)) + ININSR
 DISPLAY 'After scaling:', LAND, INIFOR, ININSR
TABLE YIELDG(ah,c) Net merchantable volume yield g  (V per A)
         n1      b1
   a3  15000    2000
   a4  30000    2800
   a5  28000    2800
   a6  25000    2650
```

```
   a7  25000     2500
   a8  22000     3200
   a9  19000     3080
  a10  28000     2900
;YIELDG(ah,'n2') = 1.05 * YIELDG(ah,'n1')
;YIELDG(ah,'b2') = 1.03 * YIELDG(ah,'b1')
DISPLAY YIELDG
 SCALARS
  TPERL    Time period length          (years)    / 20 /
  INT      Annual interest rate       (percent)  / 4.0 /

 PARAMETER
  ALPHA(te)  Discount factors

*-All activities assumed to occur in the middle of each period.
* If TPERL = 10, discount from start of years 10, 30, 50, ...
* If INT = 4.0, discount factors are 1.04**-5, 1.04**-15, ...

;ALPHA(t) = (1 + INT/100) ** (-(TPERL * ORD(t) - TPERL/2))
;ALPHA(te) $ (ORD(te) EQ CARD(te)) = INF
DISPLAY TPERL, INT, ALPHA
PARAMETERS
 F(r)    Fraction burned per period
        / r1  0.0002,
          r2  0.006 /
 PrF(r)  Probability that given fraction burns
        / r1  0.83,
          r2  0.17 /
 CPrF(r)  Cumulative probability that fraction burns
 EFBP     Expected fraction burned per period
 EFBA     Expected fraction burned per annum
 VFBP     Variance of fracation burned per period
```

```
 CVFBP    Coeff of var of fraction burned per period

;CPrF(r) = SUM(rp $ (ORD(rp) LE ORD(r)), PrF(rp))
;EFBP = SUM(r, PrF(r) * F(r))
;VFBP = SUM(r, PrF(r) * F(r) ** 2) - EFBP ** 2
;CVFBP = SQRT(VFBP) / EFBP
*-This is from the formula EFBP = 1 - (1 - EFBA) ** TPERL
;EFBA = 1 - (1 - EFBP) ** (1/TPERL)
 DISPLAY  F, PrF, CPrF, EFBP, EFBA, VFBP, CVFBP
* NS1(te) = (1 2)
* NS2(te) = (2 4)
*    <NS1>     <NS2>
*    PrND1     PrND2
*  x u e g h
*              P Q
*      (c)   cy cb cn
*      (f)   fy fb fn
*       y    yy yb yn
*       hq
*
*-Parameters $ CARD(te) = INF:
*
* ALPHA P Q PrND1 PrND2 ONE
*
PARAMETERS

 BRCHS      Number of branches at each node
 NS1(te)    Number of scenarios or nodes (before burn)
 NS2(te)    Number of scenarios or nodes (after burn)
 P(s,te)    Alternative proportions burned in branch
 Q(s,te)    '1 - P(s,te)'
```

```
  PrND1(s,te)  Probability of node -before- burn by s & t
  PrND2(s,te)  Probability of node -after- burn by s & t
  ONE1(te)     Check sum of PrND1
  ONE2(te)     Check sum of PrND2
;BRCHS = 2
;NS1(te) = POWER(BRCHS, CARD(ts))
;NS2(te) = NS1(te)
;NS1(ts) = POWER(BRCHS, ORD(ts) - 1)
;NS2(t) $ (ORD(t) LE CARD(ts)) = NS1(t+1)
 DISPLAY BRCHS, NS1, NS2
;P(s,t) $ (ORD(s) LE NS2(t)) = EFBP
;P(s,t) $ (ORD(t) LE CARD(ts) AND ORD(s) LE NS2(t)) =
      SUM(r $ (ORD(r) EQ MOD(ORD(s) - 1, BRCHS) + 1), F(r))
;Q(s,t) $ (ORD(s) LE NS2(t)) = 1 - P(s,t)
;P(s,te) $ (ORD(te) EQ CARD(te)) = INF
;Q(s,te) $ (ORD(te) EQ CARD(te)) = INF
 DISPLAY P, Q
;PrND1('s1','t1') = 1
 LOOP(t $ (ORD(t) GT 1 AND ORD(t) LE CARD(ts) + 1),
  LOOP(s $ (ORD(s) LE NS1(t)),
   PrND1(s,t) =
      SUM(r $ (ORD(r) EQ MOD(ORD(s) - 1, BRCHS) + 1), PrF(r))
    * SUM(sp $ (ORD(sp) EQ FLOOR(ORD(s) / BRCHS + 0.6)),
       PrND1(sp,t-1))
   )
  )

;PrND1(s,te) $ (ORD(te) GT CARD(ts) + 1) =
      SUM(tp $ (ORD(tp) EQ CARD(ts) + 1), PrND1(s,tp))

;PrND2(s,te) $ (ORD(te) LE CARD(t) AND ORD(s) LE NS2(te)) =
      PrND1(s,te+1)
```

```
;PrND1(s,te) $ (ORD(te) EQ CARD(te)) = INF
;PrND2(s,te) $ (ORD(te) EQ CARD(te)) = INF
;ONE1(te) = SUM(s $ (ORD(s) LE NS1(te)), PrND1(s,te))
;ONE2(te) = SUM(s $ (ORD(s) LE NS2(te)), PrND2(s,te))
PARAMETER  PrS(s)  Probability of scenario
;PrS(s) = SUM(t $ (ORD(t) EQ CARD(t)), PrND1(s,t))
 DISPLAY PrND1, ONE1, PrND2, ONE2, PrS

;PrND1(s,tr) $ (ORD(s) GT NS1(tr)) = -1
;PrND2(s,tr) $ (ORD(s) GT NS2(tr)) = -1

 PUT / PrND1.TS /
 LOOP(s,
  LOOP(tr, PUT PrND1(s,tr):0:5) PUT /
    )
 PUT / PrND2.TS /
 LOOP(s,
  LOOP(tr, PUT PrND2(s,tr):0:5) PUT /
    )
 PUTCLOSE F1  PUT F1 ;  F1.AP = 1
** PARAMETER TS1(s,te), TS2(s,te)
**;TS1(s,te) $ (ORD(s) LE NS1(te)) = 100 * ORD(s) + ORD(te)
**;TS2(s,te) $ (ORD(s) LE NS2(te)) = 100 * ORD(s) + ORD(te)
** DISPLAY TS1, TS2
positive variables

       fi(s,te)     Burned area                              (A)
       fy(s,te)    Non-self regen burn left to NSR           (A)
       fb(s,te)    Non-self regen burn regen to basic        (A)
       fn(s,te)    Non-self regen burn regen to natur        (A)
       NBT(s,te)    not successfully treated burned area     (A)
       NSY(s,te)     not rehablitated area                   (A)
```

y(s,te) Area of NSR (A)

yy(s,te) Non-self rehab NSR left to NSR (A)

yb(s,te) Non-self rehab NSR regen to basic (A)

yn(s,te) Non-self rehab NSR regen to natu (A)

x(s,c,a,te) Area of forest (A)

g(s,c,ah,te) Harvested area type g (A)

u(s,c,a,te) Unharvested area (A)

e(s,c1,a,te) Enhanced area (A)

hq(s,te) Harvest quantity in current period (V)

free variable z objective (D);

equations

harenharea(s,c2,ah,t) harvestable enhanced area (A)

NONHEA(s,c2,as,t) nonharvestable enhanced area (A)

HNONEA(s,c1,ah,t) harvestable nonenhanced area (A)

NONHNONEA(s,c1,au,t) nonharvestable nonenhanced area (A)

ENHANCED(s,c1,a,t) enhanced area (A)

OLDnatural(s,a,te) growth to oldest natural enhanced areaN2 (A)

OLDbasic(s,a,te) growth to oldest basic enhanced areaB2 (A)

INTnatural(s,a,te) growth to intermediate age 3to9 natural enhanced area (A)

INTbasic(s,a,te) growth to intermediate age 3 to9 basic enhanced area (A)

AGE2nat(s,te) growth to age class2 natural enhanced area (A)

AGE2bas(s,te) growth to age class2 basic enhanced area (A)

OLDNONNB(s,c1,a,te) growth to oldest natural(basic) nonenhanced area (A)

INTNONNB(s,c1,a,te) growth to intermediate age 2 to 9 to oldest natural(basic) nonenhanced area (A)

burn(s,te) burned area (A)

NTburn(s,te) not treated burned area (A)

NSNSR(s,te) not succfully rehabilitated aREA (A)

treatburn(s,te) treatment of burned area b n y (A)

treatNSR(s,te) treatment of NSR area b n y (A)

transNSR(s,te) transition to NSR (A)

```
        traYbas(s,te)  transition to youngest basic cover type (A)
        traYnat(s,te)  transition to youngest natural cover type (A)
        harvest(s,t)    harvest quantity  (V)
        HARVQUB(s,t)   harvest quantity upper bound (V)
        obj  objective function (D);

harenharea(s,c2,ah,t) $ (ORD(s) LE NS1(t))..
  x(s,c2,ah,t) =e= g(s,c2,ah,t) + u(s,c2,ah,t);

NONHEA(s,c2,as,t) $ (ORD(s) LE NS1(t))..
  x(s,c2,as,t) =e= u(s,c2,as,t) ;

HNONEA(s,c1,ah,t) $ (ORD(s) LE NS1(t))..
  x(s,c1,ah,t) =e= g(s,c1,ah,t) + u(s,c1,ah,t) + e(s,c1,ah,t);

NONHNONEA(s,c1,au,t) $ (ORD(s) LE NS1(t))..
  x(s,c1,au,t) =e= u(s,c1,au,t) + e(s,c1,au,t);

ENHANCED(s,c1,a,t) $ (ORD(s) LE NS1(t))..
  e(s,c1,a,t) =e= RATIO * u(s,c1,a,t) ;

OLDnatural(s,a,te-1) $ (ORD(s) LE NS1(te)
          AND ORD(a) EQ CARD(a))..
  x(s,'n2',a,te) =e= Q(s,te-1) * (
     SUM(sp $ ((ORD(te) LE CARD(ts) + 1
         AND ORD(sp) EQ FLOOR(ORD(s) / BRCHS + 0.6))
         OR (ORD(te) GT CARD(ts) + 1
         AND ORD(sp) EQ ORD(s))),
           u(sp,'n2',a,te-1) + u(sp,'n2',a-1,te-1)
         + e(sp,'n1',a,te-1) + e(sp,'n1',a-1,te-1)
     )
                    );
```

```
OLDbasic(s,a,te-1) $ (ORD(s) LE NS1(te)
            AND ORD(a) EQ CARD(a))..
  x(s,'b2',a,te) =e= Q(s,te-1) * (
      SUM(sp $ ((ORD(te) LE CARD(ts) + 1
          AND ORD(sp) EQ FLOOR(ORD(s) / BRCHS + 0.6))
          OR (ORD(te) GT CARD(ts) + 1
          AND ORD(sp) EQ ORD(s))),
            u(sp,'b2',a,te-1) + u(sp,'b2',a-1,te-1)
           + e(sp,'b1',a,te-1) + e(sp,'b1',a-1,te-1)
      )
                       );
INTnatural(s,a,te-1) $ (ORD(s) LE NS1(te)
            AND ORD(a) GE 3 AND ORD(a) LE 9)..
  x(s,'n2',a,te) =e= Q(s,te-1) * (
      SUM(sp $ ((ORD(te) LE CARD(ts) + 1
          AND ORD(sp) EQ FLOOR(ORD(s) / BRCHS + 0.6))
          OR (ORD(te) GT CARD(ts) + 1
          AND ORD(sp) EQ ORD(s))),
           u(sp,'n2',a-1,te-1) + e(sp,'n1',a-1,te-1)
      )
                       );
INTbasic(s,a,te-1) $ (ORD(s) LE NS1(te)
            AND ORD(a) GE 3 AND ORD(a) LE 9)..
  x(s,'b2',a,te) =e= Q(s,te-1) * (
      SUM(sp $ ((ORD(te) LE CARD(ts) + 1
          AND ORD(sp) EQ FLOOR(ORD(s) / BRCHS + 0.6))
          OR (ORD(te) GT CARD(ts) + 1
          AND ORD(sp) EQ ORD(s))),
           u(sp,'b2',a-1,te-1) + e(sp,'b1',a-1,te-1)
      )
                       );
AGE2nat(s,te-1) $ (ORD(s) LE NS1(te))..
```

```
      x(s,'n2','a2',te) =e= Q(s,te-1) *
         SUM(sp $ ((ORD(te) LE CARD(ts) + 1
            AND ORD(sp) EQ FLOOR(ORD(s) / BRCHS + 0.6))
            OR (ORD(te) GT CARD(ts) + 1
            AND ORD(sp) EQ ORD(s))),
             e(sp,'n1','a1',te-1)
         );
  AGE2bas(s,te-1) $ (ORD(s) LE NS1(te))..
     x(s,'b2','a2',te) =e= Q(s,te-1) *
        SUM(sp $ ((ORD(te) LE CARD(ts) + 1
           AND ORD(sp) EQ FLOOR(ORD(s) / BRCHS + 0.6))
           OR (ORD(te) GT CARD(ts) + 1
           AND ORD(sp) EQ ORD(s))),
            e(sp,'b1','a1',te-1)
        );
 OLDNONNB(s,c1,a,te-1) $ (ORD(s) LE NS1(te)
              AND ORD(a) EQ CARD(a))..
    x(s,c1,a,te) =e= Q(s,te-1) * (
       SUM(sp $ ((ORD(te) LE CARD(ts) + 1
          AND ORD(sp) EQ FLOOR(ORD(s) / BRCHS + 0.6))
          OR (ORD(te) GT CARD(ts) + 1
          AND ORD(sp) EQ ORD(s))),
             u(sp,c1,a,te-1) + u(sp,c1,a-1,te-1)
       )
                 );
INTNONNB(s,c1,a,te-1) $ (ORD(s) LE NS1(te)
             AND ORD(a) GE 2 AND ORD(a) LE 9)..
  x(s,c1,a,te) =e= Q(s,te-1) *
      SUM(sp $ ((ORD(te) LE CARD(ts) + 1
         AND ORD(sp) EQ FLOOR(ORD(s) / BRCHS + 0.6))
         OR (ORD(te) GT CARD(ts) + 1
         AND ORD(sp) EQ ORD(s))),
```

```
                u(sp,c1,a-1,te-1)
          );
 treatNSR(s,t) $ (ORD(s) LE NS2(t))..
    Q(s,t) *
        SUM(sp $ ((ORD(t) LE CARD(ts)
             AND ORD(sp) EQ FLOOR(ORD(s) / BRCHS + 0.6))
             OR (ORD(t) GT CARD(ts)
             AND ORD(sp) EQ ORD(s))),
          y(sp,t) * (1 - SY)
          )
            =e= yy(s,t) + yb(s,t) + yn(s,t) ;

 burn(s,t) $ (ORD(s) LE NS2(t))..
    fi(s,t) =e= LAND*P(s,t);
 treatburn(s,t) $ (ORD(s) LE NS2(t))..
   LAND * P(s,t) * (1 - SF) =e= fy(s,t) + fb(s,t) + fn(s,t)
 *  |- fi(s,te) -|
 ;

 transNSR(s,te-1) $ (ORD(s) LE NS1(te))..
   y(s,te) =e= fy(s,te-1) + yy(s,te-1)
         + (1 - EFBN - EFBB) * fb(s,te-1)
         + (1 - EYBN - EYBB) * yb(s,te-1)
         + (1 - EFNN) * fn(s,te-1)
         + (1 - EYNN) * yn(s,te-1)
 ;
 NTburn(s,te-1) $ (ORD(s) LE NS1(te))..
   NBT(s,te) =e= fy(s,te-1)
         + (1 - EFBN - EFBB) * fb(s,te-1)
         + (1 - EFNN) * fn(s,te-1)
         ;
```

```
NSNSR(s,te-1) $ (ORD(s) LE NS1(te))..
  NSY(s,te) =e= yy(s,te-1)
          + (1 - EYBN - EYBB) * yb(s,te-1)
          + (1 - EYNN) * yn(s,te-1)
;
traYnat(s,te-1) $ (ORD(s) LE NS1(te))..
  x(s,'n1','a1',te) =e=
  LAND * P(s,te-1) * SF
* |-- fi(s,te-1) -|
 + Q(s,te-1) *
     SUM(sp $ ((ORD(te) LE CARD(ts) + 1
         AND ORD(sp) EQ FLOOR(ORD(s) / BRCHS + 0.6))
         OR (ORD(te) GT CARD(ts) + 1
         AND ORD(sp) EQ ORD(s))),
        y(sp,te-1) * SY
     )
 + EFBN * fb(s,te-1) + EYBN * yb(s,te-1)
 + EFNN * fn(s,te-1) + EYNN * yn(s,te-1)
;
traYbas(s,te-1) $ (ORD(s) LE NS1(te))..
  x(s,'b1','a1',te) =e=
    EFBB * fb(s,te-1) + EYBB * yb(s,te-1)
;
harvest(s,t) $ (ORD(s) LE NS1(t))..
  hq(s,t) =e= SUM((c,ah), g(s,c,ah,t) * YIELDG(ah,c)) ;

HARVQUB(s,t) $ (ORD(s) LE NS1(t))..
  hq(s,t) =l= HQUB ;

obj..
 z =e=
 SUM(t,
```

```
   SUM(s $ (ORD(s) LE NS1(t)), PrND1(s,t) *  ALPHA(t) * (
    (
 (     CHG * SUM((c,ah), g(s,c,ah,t))
      + CEN * SUM(a, e(s,'n1',a,t))
      + CEB * SUM(a, e(s,'b1',a,t))     ) * REDECOQ
          )
                        )
    )
  + SUM(s $ (ORD(s) LE NS2(t)), PrND2(s,t) *
     ALPHA(t) * (
 (
     + CYB * yb(s,t)
     + CYN * yn(s,t)
     + CFB * fb(s,t)
     + CFN * fn(s,t)     ) * REDECOQ
         )
    )
+ SUM(s $ (ORD(s) LE NS2(t)), PrND2(s,t) *
    ALPHA(t) * (
(    + MV* FI(s,t)     ) * REDECOQ
         )
  )
+ SUM(s $ (ORD(s) LE NS2(t)), PrND2(s,t) *
    ALPHA(t) * (
(    + PC* Y(s,t)     ) * REDECOQ
         )
 )
- SUM(s $ (ORD(s) LE NS1(t)), PrND1(s,t) *
    ALPHA(t) * (
(     MV * hq(s,t)  ) * REDECOQ
         )
                      )   )
```

```
MODEL FFSP Forest Fire Stochastic Programming   / ALL /
;x.fx('s1',c2,a,'t1') = 0
;x.fx('s1',c1,a,'t1') = INIFOR(a,c1)
;y.fx('s1','t1') = ININSR
;
PARAMETERS

CKAR1(s,t)    Check of total area                 (A)
CKCS(s,t)     Check of harvested area             (A)
CKFS(s,t)     Check of f(t) substitution          (A)
CKFU(s,t)     Check of f(t) actually used         (A)
RESSTAT(ub)   Record of resources used
MODSTAT(ub)   Record of model status
SOLSTAT(ub)   Record of solver status
HQL(s,tr)     Harvest quantity by s & t (V)
EHQL(tr)      Expected harvest quantity by t (V)
ZREV          Objective value of timber revenue (D)
ZHTR          Objective value of harvest (D)
ZSIL          Objective value of silviculture (D)
ZMIT          objective value of mitigation costs (D)
ZFIRE         objective value of fire lost (D)
ZUMIT         objective value unsuccessful mitigation (D)
ZTIM          Sum of timber cost terms (D)
ZTIMS(s)      Timber COST by scenario (D)
EZTIMS        Expected timber cost done by scenario (D)
ZWOO          Total wood quantity over horizon (V)
ZWQC          Check objective for wood quantity criterion ()
ZCHK          Check objective for either criterion (D)()
TMP(s,tr)     Temporary denominator
TMPA(a)       Temporary denominator
XSPOBJ(ub)    Summary objective (D)
XSPEXH3(ub)   Summary expected harvest decline prop in period 3
```

```
 XSPWCH3(ub)   Summary worst case harvest decline prop in period 3
 LOOP(ub,
 HQUB = VUB(ub)
 PUT 'Model results for upper bound alternative no.:  *=#=#=#=#=#=*'
  / ORD(ub):0:0 /
* option lp = oslse;
SOLVE  FFSP USING LP MIN z ;
 RESSTAT(ub) = FFSP.RESUSD
;MODSTAT(ub) = FFSP.MODELSTAT
;SOLSTAT(ub) = FFSP.SOLVESTAT
** ABORT $ (FFSP.MODELSTAT NE 1) "Abort - not optimal"
** ABORT $ (FFSP.SOLVESTAT NE 1) "Abort - not normal completion"

** PUT $ (FFSP.MODELSTAT NE 1 OR REGSP.SOLVESTAT NE 1) / 'Aborted.'

*-Put some checks into the listing

;CKAR1(s,t) $ (ORD(s) LE NS1(t)) =
        SUM((c,a), x.l(s,c,a,t)) + y.l(s,t)

;CKCS(s,t) $ (ORD(s) LE NS1(t)) =
        SUM((c,ah), g.l(s,c,ah,t) )

;CKFS(s,t) $ (ORD(s) LE NS2(t)) =
        P(s,t) * (
      SUM(sp $ ((ORD(t)  LE CARD(ts) + 1
           AND ORD(sp) EQ FLOOR(ORD(s) / BRCHS + 0.6))
           OR (ORD(t)  GT CARD(ts) + 1
           AND ORD(sp) EQ ORD(s))),
       CKCS(sp,t) + y.l(sp,t)
      + SUM((c,a), u.l(sp,c,a,t))
      + SUM((c1,a), e.l(sp,c1,a,t))
```

```
        )
              )
 ;CKFU(s,t) $ (ORD(s) LE NS2(t)) = P(s,t) * LAND

 DISPLAY $ (ORD(ub) EQ 1)  LAND, CKAR1, CKFS, CKFU

 DISPLAY g.l, e.l, u.l, x.l, y.l, NSY.l, fi.l, NBT.l,hq.l

 *-Put results into a file

 *-Harvest quantity

 ; EHQL(tr) =
    SUM(s $ (ORD(s) LE NS1(tr)), hq.l(s,tr) * PrND1(s,tr))

 ;HQL(s,tr) =
   SUM(sp $ (ORD(sp) EQ
   FLOOR((ORD(s) - 1) / CARD(s) * NS1(tr) + 1.05)),
   hq.l(sp,tr))
 PUT / HQL.TS /
 LOOP(s,
  LOOP(tr, PUT HQL(s,tr)) PUT /
    )

 PUT / EHQL.TS /
 LOOP(tr, PUT EHQL(tr)) PUT /

*-Objective terms

;ZREV =
SUM(t,
  SUM(s $ (ORD(s) LE NS1(t)), PrND1(s,t) * (
```

```
      ALPHA(t) * (
        - PC * hq.l(s,t)
             )
                              )
      )
    )
   ;ZHTR =
    SUM(t,
      SUM(s $ (ORD(s) LE NS1(t)), PrND1(s,t) * (
        ALPHA(t) * (
        + CHG * SUM((c,ah), g.l(s,c,ah,t))
                 )
                              )
      )
    )
   ;ZSIL =
    SUM(t,
      SUM(s $ (ORD(s) LE NS1(t)), PrND1(s,t) * (
        ALPHA(t) * (
        + CEN * SUM(a, e.l(s,'n1',a,t))
        + CEB * SUM(a, e.l(s,'b1',a,t))
             )
                              )
      )
    )
   ;ZMIT =
    SUM(t,
      SUM(s $ (ORD(s) LE NS2(t)), PrND2(s,t) *
        ALPHA(t) * (
        + CYB * yb.l(s,t)
        + CYN * yn.l(s,t)
        + CFB * fb.l(s,t)
```

```
        + CFN * fn.l(s,t)
                )
            )
        )
    ;ZFIRE =
     SUM(t,
     SUM(s $ (ORD(s) LE NS2(t)), PrND2(s,t) *
         ALPHA(t) *
     (     + MV* FI.l(s,t)     )

            )
        )
    ;ZUMIT =
     SUM(t,
     SUM(s $ (ORD(s) LE NS2(t)), PrND2(s,t) *
         ALPHA(t) *
     (     + MV* Y.l(s,t)     )

            )
        )
    ;ZTIM = ZREV + ZHTR + ZSIL +ZMIT + ZUMIT + ZFIRE

    ;ZWOO =
     SUM(t,
       SUM(s $ (ORD(s) LE NS1(t)), PrND1(s,t) * (
         hq.l(s,t)
                                  )
          )
        )
    ;ZWQC = ZWOO + REDECOQ * ZTIM
    ;ZCHK = ZWOO $ (REDECOQ LT 1) + REDECOQ * ZTIM
```

```
PUT
/ 'Various objective function components (expected values):' /
'Computed objective:' /
z.l /
'-Revenue + Harvest + Silviculture+Mitigation+Unsuccessful+Fire = Timber Cost (D)' /
'(Equals computed objective for economic criterion):' /
ZREV ZHTR ZSIL ZTIM /
'Total wood quantity over planning horizon (V):' /
ZWOO /
'(Check computed objective for wood quantity criterion)' /
ZWQC /
'(Check computed objective for either criterion)' /
ZCHK /
*-Find cost by scenario
;ZTIMS(s) =
 SUM(t,
   SUM(sp $ (ORD(sp) EQ
   FLOOR((ORD(s) - 1) / CARD(s) * NS1(t) + 1.05)),
    PrND1(s,t) *  ALPHA(t) * (
     (
(     CHG * SUM((c,ah), g.l(s,c,ah,t))
     + CEN * SUM(a, e.l(s,'n1',a,t))
     + CEB * SUM(a, e.l(s,'b1',a,t))     )
          )
                              )
     )
 + SUM(sp $ (ORD(sp) EQ
  FLOOR((ORD(s) - 1) / CARD(s) * NS1(t) + 1.05)), PrND2(s,t) *
    ALPHA(t) * (
(
     + CYB * yb.l(s,t)
     + CYN * yn.l(s,t)
```

```
        + CFB * fb.l(s,t)
        + CFN * fn.l(s,t)     )
              )
        )
 + SUM(sp $ (ORD(sp) EQ
    FLOOR((ORD(s) - 1) / CARD(s) * NS1(t) + 1.05)), PrND2(s,t) *
      ALPHA(t) * (
 (     + PC* FI.l(s,t)     )
              )
   )
 + SUM(sp $ (ORD(sp) EQ
    FLOOR((ORD(s) - 1) / CARD(s) * NS1(t) + 1.05)), PrND2(s,t) *
      ALPHA(t) * (
 (     + MV* Y.l(s,t)     )
              )
  )
 - SUM(sp $ (ORD(sp) EQ
    FLOOR((ORD(s) - 1) / CARD(s) * NS1(t) + 1.05)), PrND1(s,t) *
      ALPHA(t) * (
 (       MV * hq.l(s,t)  )
              )
                              )  )
;EZTIMS = SUM(s, ZTIMS(s) * PrS(s))
 PUT / 'Distribution of cost by s (D)' /
 LOOP(s, PUT ZTIMS(s))
 PUT / 'Check expected cost (D):' /
  EZTIMS //
DISPLAY ZTIMS, EZTIMS
*-Store summary results
;XSPOBJ(ub) = z.l
;XSPEXH3(ub) = EHQL('t3') / EHQL('t1')
;XSPWCH3(ub) = SMIN(s, HQL(s,'t3')) / EHQL('t1')
```

```
*-Close loop ub
   )
 PUT 'Summary results:' /
  'Upper bound, Objective, Expected decline prop in period 3:' /
 LOOP(ub,
  PUT VUB(ub) XSPOBJ(ub) XSPEXH3(ub) /
   )
 PUT /  'Upper bound, Objective, Worst case decline prop in period 3:' /
 LOOP(ub,
  PUT VUB(ub) XSPOBJ(ub) XSPWCH3(ub) /
   )
 PUT / '==================' // 'Additional model data/features:' /
 PUT / 'Fire loss distribution:' /
  'EFBA EFBP VFBP CVFBP' /
  EFBA:0:4 EFBP:0:4 VFBP:0:4 CVFBP:0:4 /
  'PrF(r) F(r)' /
 LOOP(r, PUT PrF(r):0:4)
 LOOP(r, PUT F(r):0:4) PUT /
 PUT / 'Initial age class distribution:' /
  'ININSR INIFOR(a,n1)' /
  ININSR
 LOOP(a, PUT INIFOR(a,'n1')) PUT /
PUT / '==================' // 'Resources used:' /
 LOOP(ub $ (ORD(ub) LE 15), PUT RESSTAT(ub):0:1) PUT /
 LOOP(ub $ (ORD(ub) GT 15), PUT RESSTAT(ub):0:1) PUT /
 PUT 'Model & solver statuses (must all be 1):' /
 LOOP(ub, PUT MODSTAT(ub):0:0) PUT /
 LOOP(ub, PUT SOLSTAT(ub):0:0) PUT /
 PUT 'Errors in this output (must be 0):' / F1.ERRORS:0:0 /
 PUT '<End>'
*-End
```

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