PAINT SHOP SCHEDULING IN A BUS PLANT: RE-ENTRANT HYBRID FLOW SHOP MODEL WITH ZERO BUFFER AND NO-WAIT STATIONS

by

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ABSTRACT

PAINT SHOP SCHEDULING IN A BUS PLANT: RE-ENTRANT HYBRID FLOW SHOP MODEL WITH ZERO BUFFER AND NO-WAIT STATIONS

In this study, an optimization model is constructed to propose the scheduling plan for a paint shop of a bus production plant. In bus production plant, the paint shop is located between the body shop and assembly line. Both the release time and the delivery time of the buses are restricted and in takt time base. Although the buses follow single flow line in body shop and assembly line, due to painting variety options, buses re-visit some of the operations more than once depending on the color and decorations of the buses. There are identical parallel machines operate the same processes. The paint shop has limited space for buffers since the buses are large items and production area is valuable due to operating costs. Some stations can be used as buffer if the next station is blocked by another job. On the other hand, due to the nature of the operations some stations cannot be used after the process is finished, those are called no-wait stations.

There are many studies in literature that cover some of the issues mentioned above. However, no study is found that covers all those in their scope. We propose a linear mixed integer programming model that covers re-entrance, zero buffer, and no-wait stations in hybrid flow shop. The objective is to minimize the total tardiness with a stepwise cost function. The model is tested with combination of two different real case parameters, which are the changing re-entrant job ratios and the increasing daily production volume.

ÖZET

OTOBÜS FABRİKASINDA BOYAHANE ÇİZELGELEMESİ: SIFIR ARA DEPOLAMA VE BEKLEMESİZ İSTASYON; DÖNGÜLÜ HİBRİT AKIŞ TİPİ MODELİ

Bu çalışmada, otobüs üretim tesisindeki bir boyahanenin üretim planını önermek için bir en iyileme modeli oluşturulmuştur. Otobüs üretim tesisinde, boyahane, karoseri ve montaj hattı arasında yer almaktadır. Otobüslerin hem bırakma süresi hem de teslimat süresi takt zamanı bazında ve belirlidir. Otobüsler karoseri ve montaj hattında tek akış hattını takip etse de, boyama seçenekleri nedeniyle otobüslerin renk ve dekorasyonlarına bağlı olarak bazı istasyonları bir defadan fazla kez ziyaret ederler. Boyahanede aynı işlemleri yapan paralel istasyonlar bulunmaktadır. Otobüsler büyük parçalar olduğu ve üretim alanı işletme maliyetleri nedeniyle değerli olduğu için boyahane ara depolama alanları bakımından sınırlı alana sahiptir. Bir sonraki istasyonda işin başlaması başka bir iş tarafından engellendiğinde mevcut istasyonlar ara depolama olarak kullanılabilir. Öte yandan, işlemlerin niteliği nedeniyle, bazı istasyonlarda işlem tamamlandıktan sonra otobüsler bekleme yapamamaktadır, bunlar beklemesiz istasyonlar olarak adlandırılır ve otobüsler işlem sürelerinden daha uzun bir süre bu istasyonlarda tutulamazlar.

Literatürde yukarıda bahsedilen kapsamlardan bazılarını ele alan birçok çalışma vardır. Ancak, tüm bu kapsamları içeren bir çalışma bulunamamıştır. Bu çalışmayla, döngüleri, sıfır ara depolama alanları ve beklemesiz istasyonları kapsayan hibrit akış tipindeki boyahane üretim alanı için doğrusal karma tam sayılı bir programlama modeli öneriyoruz. Amaç, kademeli bir maliyet fonksiyonu ile toplam gecikmeyi en aza indirmektir. Model, üretim adetleri içinde araçların renk ve dekor tipine bağlı olarak değişen döngü oranlarını ve artan günlük üretim hacmini ele alan iki farklı gerçek durum parametresi ile test edilmiştir.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ACRONYMS/ABBREVIATIONS

B&B	Branch and Bound
FS	Flow Shop
GAMS	General Algebraic Modeling System
HFS	Hybrid flow shop
JS	Job Shop
LMIP	Linear mixed integer programming
MIP	Mixed integer programming
NP	Nondeterministic polynomial

1. INTRODUCTION

Scheduling is a method to determine operation start and completion times on resources to attain some objectives while taking customer demands and system limitations into account. Scheduling problems are hard to solve computationally due to the NP-complete nature of the problems. In most of the cases, it is not possible to find an optimal solution to the real life problems within a reasonable amount of time.

In this study, we aim to construct an optimization model to propose the scheduling plan for a paint shop of a bus production plant. In bus production plant, the paint shop is located between the body shop and assembly line. After bus body is constructed in body shop, the body is transported to the paint shop for painting operations and then the painted body and parts are moved to the assembly line for further operations.

Although there are limited options for the body design of a bus, there are high level of customer requests in painting and decoration design. That means, bus production company treats each order as a customized product, which also increase the complexity of the operations especially in paint shop, and the dependence to handcraft. In the paint shop, a large number of painting jobs are processed depending on the colors and decoration of the ordered products. Although the jobs flow throughout the production line in body shop and in assembly line, due to several painting operations in paint shop, jobs may re-enter some of the stations more than once. Unlike the body shop and assembly line, the buses do not follow a single production route in the paint shop. Therefore, their sequence may change frequently depending on the number of the painting and decoration operations. That means the entrance sequence of the buses to the paint shop may not be same with the exit sequence of the buses from the paint shop.

The delivery schedule of the buses to the customers depends on the customer orders. Therefore, the production plan is determined mainly based on the sales department’s data. Since the assembly line is the last part of the production, the sequence of the buses produced in the assembly should match the delivery schedule of the products. Thus, assembly line affects the sequence of the buses in the paint shop as well. As mentioned above, paint shop is located between body shop and assembly line. So, the earliest start time of the initial process in paint shop is determined by the production schedule in the body shop as well. Also the delivery time is constrained by the assembly line in which the main trigger is the customer orders.

Any deviation from the desired plan causes an extra cost to the bus production factory. The optimization model penalizes such undesired delivery schedule according to the magnitude of the late delivery in the takt time base. Takt time is a terminology which is used commonly in the production environment to synchronize pace of production with the pace of customer demand. Minimizing the late deliveries is handled via the objective function in the proposed model, which will be explained in Section 4.1 in detail.

Although the paint shop is designed as a flow shop, following properties increases the complexity of the paint shop scheduling.

- Hybrid: In the flow shop, there are multiple identical parallel stations, which operates the same process.
- Re-entrant: Depending on the colors and the decoration properties of the buses, several additional painting operations are needed to be performed. Therefore, buses may re-enter some of the stations more than once.
- Zero buffer: Since the buses are large volume products and the production facility has a space limit, there are limited buffer space. If we assume that the buffers are the stations, which have zero processing time, without loss of generality, we may assume that there are no buffers between the stations.
- No-wait: Due to the nature of the process, buses cannot be kept any longer than the predetermined processing times in some stations. At the end of that process, buses are needed to be removed from the stations immediately to the next station. Those stations are called no-wait stations.

With the above properties, the model for the paint shop is a re-entrant hybrid flow shop with zero buffer and no-wait stations.

In this thesis, we concentrate on the following issues:

- We propose an optimization model that will generate a feasible schedule of operations in the system. We hope that the proposed model will decrease the last minute chaos between related departments, which may be the case when the desired schedule is not realistic. In other words, the model will provide a proper due date communication between the departments and decrease the burden of production planning department.
- We assume that the body shop and assembly line schedules are fixed. Delivery from the body shop will determine the earliest start time of the initial process in the paint shop and it is thought as a hard constraint. Although the assembly schedule is also fixed, delivery sequence and dates are assumed as soft constraints and the objective function punishes late deliveries from the desired due dates, in other words deliveries after the assembly line request date, and not delivering a bus on the assigned possible due dates.
- Due to increasing export demands from the bus plant, plant is about to increase daily production volume with the existing investment. In addition, customers' requests on the paint variety is increasing. The proposed model will also guide the management team whether the additional investment to the paint shop is required. This is handled in the problem by changing the re-entrance ratio of the products and increase in the delivery frequency, in other words decrease in the takt time.

The rest of the thesis is organized as follows. Section 2 describes the problem in detail. Section 2.1 discuss the takt time definition used especially in the production environment. Section 3 gives the overview of the literature. Section 4 describes the assumptions and formulates the optimization model. Section 4.5 describes the initial state. Experimentation and results are reported in Section 5. Finally, Section 6 states the conclusion and final assessment.

2. PROBLEM DEFINITION

Depending on the market demand, 14 buses are produced per day currently in the bus production plant. However, due to high level of orders coming from abroad, production capacity is about to increase up to 17 buses per day. And when overall the demand decreases, the daily production volume is decreased down to 8 to 10 buses. There are 70 to up to 150 operations in paint shop to be performed, depending on the number of colors to be painted on the bus. The actual paint shop includes more than 40 stations. That means, on average 40 buses are processed simultaneously in the paint shop at any time. Due to operation variety and number of operations, paint shop is a complex environment. Moreover, increase in production amount is another challenge for the paint shop.

While the production planning department prepares the production plan, considering the vehicle type, type and the number of the painting operations, they allocate time for paint shop three days in general and up to nine days in special cases when the buses are needed extra operations due to their designs.

Most of the vehicles are opaque and single color, which means vehicles use the painting cabins and ovens only once. Some of the vehicles are metallic and single color. Metallic colored vehicles require extra work in operations that means those re-visit some of the stations more than once. In special orders, the vehicles can be ordered with multicolored and decorations, so depending on the colors the vehicles will have, vehicles may re-visit some operations three times or more. In our case, all re-entrances are deterministic and quality issues do not cause an unexpected re-entrance to the system.

As mentioned earlier, buses are large items and storage capacity is limited. So there are limited space as buffers. We treat the limited buffer spaces as stations with zero processing time. Therefore, we can model conveniently the paint shop as a zero buffer system.

Both zero buffer and re-entrant properties bring the possibility of deadlock occurrence "which may take place only in the systems with the closed structure at the resources belonging to the basic cycle." [1]. That means, buses block each other's entries to the stations where they form a loop with fully occupied stations. Deadlock implies that the buffer spaces between stages are limited or does not even exist. The jobs must wait in the previous stage, until the operations of the job on the next stage is completed and it is released.

Painting operations are complex and have some limitations due to the nature of the process itself. There are some stations, so that the buses cannot wait in the stations after their process is completed. Those stations are no-wait stations. In the paint shop, ovens are the no-wait stations. The buses cannot stay in the oven more than the allowed processing time since the heat in the oven cannot be decreased immediately after the operation is completed. Thus, it is impossible to use such stations as buffers when the next station is not available for entrance. If we assume that, there are no buffer spaces immediately after the no-wait stations, this may also result in deadlock occurrences in the paint shop.

Figure 2.1 represents the layout of the production plant. The simplified visualization model of the paint shop is shown in Figure 2.2. Rectangles in the Figure 2.2 show stations in the paint shop. Arrows indicate possible flow directions. There are some fixed operations and some operations have to be repeated depending on the number of colors the buses have, those follow different routes in the paint shop. After their operations in the paint shop are completed, they go through the assembly line.

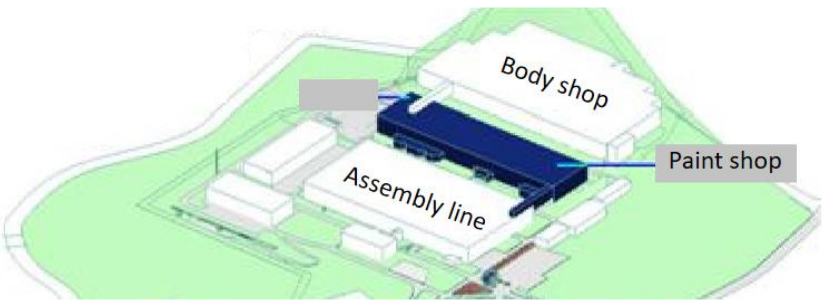

Figure 2.1. Layout of the production plant

In Figure 2.2, F and D stations represent buffer spaces, which are modeled as stations with zero processing time. Parallel stations (like M10 and M11) are identical stations, which do the same operations. Dark blue rectangles are the stations in which heating operations are performed. Due to the nature of the production operations, buses should be removed from those stations as their processes are finished. There are binding stations so that if a machine is selected than machine in the next stage is also determined due to the layout of the paint shop area. For example, if M20 is selected than the next stage will be processed in M30 or if M21 is selected than the next stage will be processed in M31.

As an example, routings of single-color and two-color buses are given below.

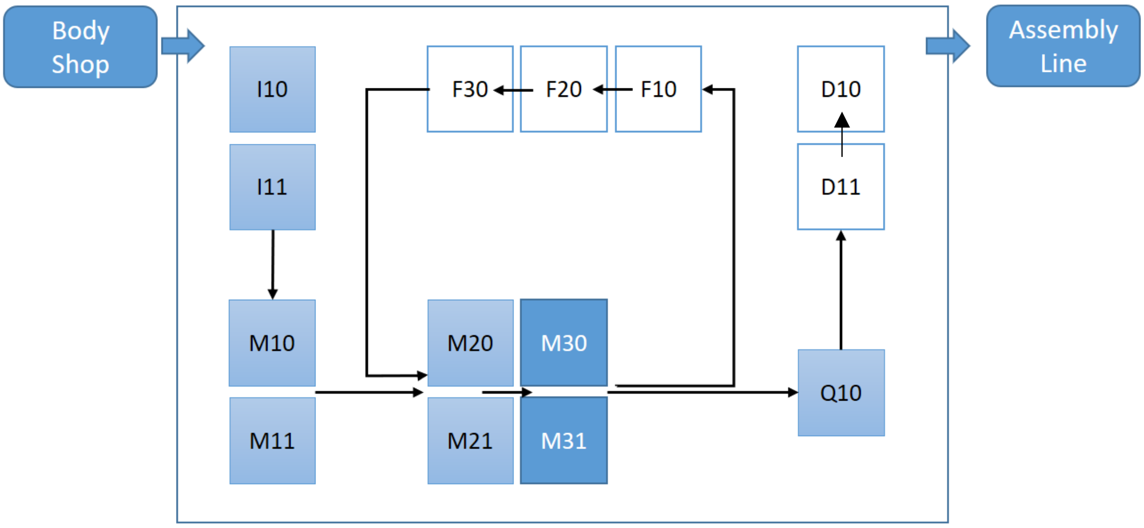

Figure 2.2. The simplified visualization of the paint shop

- Single-color route: (I10 or I11) → (M10 or M11) → ([M20 → M30] or [M21 → M31]) → Q10 → D10 → D11
- Two-color route: (I10 or I11) → (M10 or M11) → ([M20 → M30] or [M21 → M31]) → F10 → F20 → F30 → (M10 or M11) → ([M20 → M30] or [M21 → M31]) → Q10 → D10 → D11

In order to express the decision environment and borders of the system better, a context diagram is presented in Figure 2.3. It gives insight about the relationship and information flow between the stakeholders of the whole process. Production planning department submits the production orders to the paint shop. Ready times are given to the paint shop by the body shop. In our model, ready times are fixed, known, and not negotiable. The desired due dates are given to the paint shop by the assembly line. There might be some situations where creating a schedule with the given due dates is not feasible. The optimization model will find the solution by changing delivery order of some buses when necessary. The model will assign one bus to each takt time. Not delivering on the assigned due date is punished with a large penalty cost in order not to make the model infeasible. Any late delivery from the desired due dates will be penalized, so those will be minimized by the objective function. After the scheduling model runs, the paint shop will deliver the buses to the assembly line, which is responsible to deliver the final product to sales department.

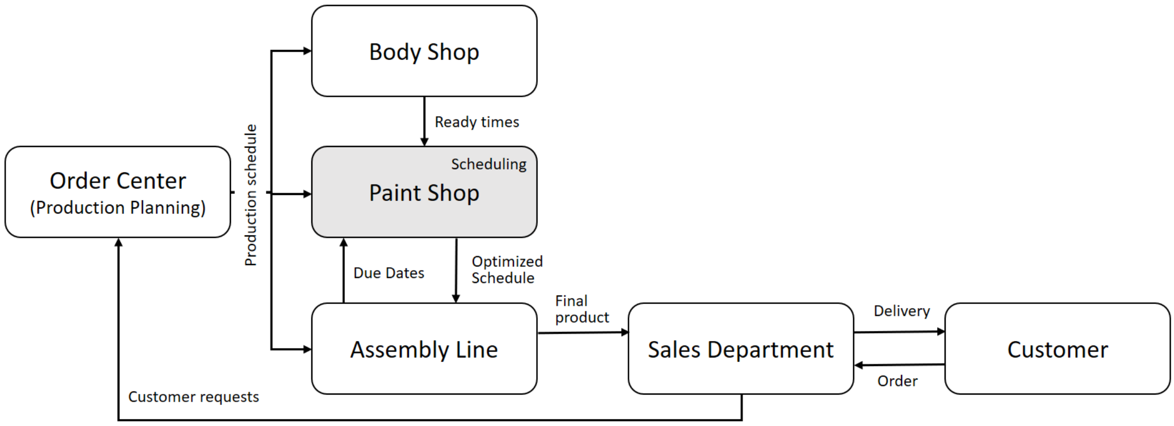

Figure 2.3. Context diagram

The model will optimize the paint shop scheduling via minimizing the cost of late deliveries to the customer. In the long run, the model will also be beneficial for the sales department, in order to manage delivery promises, if the optimization model is extended to model whole production stages.

2.1. Takt Time

Takt time relates customer demand to available production time and is used to pace the production [2]. In other words, takt time is calculated by available time divided by required output, which can be different from cycle time in the sense that cycle time is time to complete a process from start to finish on one unit. In ideal state, cycle time and takt time should match.

Takt time allows optimizing the capacity in the most appropriate way to meet the demand without keeping too much inventory. The concept of takt time is not suitable for all manufacturing types. It is usually used in discrete production such as automotive industry. To comply with the production jargon, we used takt time as the term to indicate the delivery frequency. In bus production plant, the flow shop is also organized based on the takt time concept.

Within the paint shop, jobs are not moved based on the takt time, although the arrival from the body shop and delivery to the assembly line are done based on takt time. Processing times may change depending on the painting area on the buses, and line balancing may not be possible, so application of takt time may reduce the productivity of the paint shop while increasing the complexity.

We do not have to move the buses according to takt time in every single operation within paint shop. There are some final stations with zero processing time at the end of the paint shop which are treated as buffer zones. It is enough to send one bus at every takt time to the assembly line.

3. LITERATURE REVIEW

Flow shop optimization problems are the search topics for many scholars since mid-twentieth century. The interesting aspect of flow shop problem is that inclusion of every real-life constraints into the model makes the problem more complex. Studies include different combinations of the real-life limitations. In the literature survey, we focused on the studies that cover tardiness minimization, no-wait scheduling, zero buffer scheduling, re-entrant hybrid flow shops considered. And a table is constructed in the end of this section to summarize the related works by taking the certain aspects.

The hybrid flow shop is a general form of the classical flow shop in which there are parallel machines to process some operations [3]. In this system, usually there are a set of jobs that must be processed in a series. [4] presents a wide literature review on exact, heuristic and metaheuristic methods that have been proposed for hybrid flow shop scheduling problems. [5] studies two stages problem with identical parallel machines in the second stage and uses a Branch and Bound method by the minimizing the total tardiness. Problem samples of up to 15 jobs are shown to be solvable in reasonable amount of time. Whereas, [6] studies the case where the stage one has parallel machines and stage two has only one machine. They also reported a B&B that is able to get good solutions in a reasonable amount of time. [7] studies two stage HFS with parallel machines in both stages with the minimization of makespan which is handled very effective with B&B which generates optimal solutions up to 1,000 jobs. But, their method cannot solve many medium sized samples optimally, and in some situations they observed relative gaps. [8] represent their HFS problem as a MIP and use a regular solver to obtain solution. [31] investigates the different formulations of MIP for no preemption jobs in hybrid flow shops.

Many researchers concentrated on minimizing the makespan. [4] reported that 60% of the reviewed literature are dealing with the makespan minimization, whereas only 1% is dealing with the earliness and tardiness minimization although it is so important for the real problems. [9] studies HFS problems by taking the tardiness related criteria as objectives. [10] minimizes tardiness and earliness in their HFS problem with sequence dependence setup times by using advanced metaheuristics. [11] minimizes the weighted earliness and tardiness from the due window by using local search methods. [12] presents two hybrid metaheuristic approaches for a single machine scheduling problem where tardiness cost of a job increases stepwise with various due dates and the objective is minimizing the total cost of tardiness.

3.1. No-wait Scheduling

No-wait can be defined as the successive operations have to be processed without any time lag between the operations. In other words, the start time of an operation must be equal to the completion time of its preceding operation.

[13] deals with the no-wait flow shop scheduling problem with due date constraints. The study takes the due date constraints as hard constraints and tries to minimize the makespan. They develop different mathematical models based on different decision variables, namely mixed integer programming model, quadratic mixed integer programming models and also an exact algorithm. They performed experiments to compare the performance of the developed models and algorithms. They revealed that the algorithm is significantly faster. [14] and [30] study the no-wait HFS problem as well. [15] also consider no-wait property where they study an m-machine no-wait flow shop scheduling problem to minimize the weighted sum of maximum completion time and maximum lateness. They propose a simulated annealing algorithm and a hybrid genetic algorithm to solve their problem.

3.2. Zero Buffer Scheduling

T.J.Sawik and his colleagues have put lots of effort and published many articles in scheduling using mixed integer programming. One of the studies, [16], converts scheduling parallel machine problem with finite in-process buffers into one with no buffers. So, buffers are viewed as special stations with zero processing times but with blocking. We used the same principle to the buffers and blocking property that we have at the last stage of the paint shop to deliver the jobs exactly on the possible due dates. Those buffers keep the early finished jobs until the next takt time, since assembly line works according to takt time principle and we cannot deliver buses earlier than the next takt time.

There are some other studies in the literature that covers zero buffer issue in the models. And [17] also models the flow shop scheduling problem as a mixed integer programming with zero buffer constraints and with blocking in-process by minimizing the total earliness and tardiness.

3.3. Re-entrant Flow Shop Scheduling

Re-entrance is another issue considered in flow shop scheduling. [18] models a flexible flow line with blocking and re-entry. Some parts are needed to revisit the complete line two times due their process needs.

[19] proposes wolf pack based algorithm as a global optimization method for their re-entrant scheduling models. [20] propose a farness particle swarm optimization algorithm (FPSO) to solve reentrant two-stage parallel machine flow shop scheduling problems in order to minimize earliness and tardiness. [21] uses multi-objective function namely the maximization of the utilization rate of the bottleneck and the minimization of the maximum completion time for their re-entrant hybrid flow shop problem. They use a form of genetic algorithm to solve the problem. [24] focuses on a hybrid re-entrant flow shop scheduling problem with identical parallel machines. They propose the list-scheduling method to solve the problem.

In the literature, the studies on flow shop scheduling cover only a subset of the above mentioned issues. In our study, we deal with a bus paint shop case which includes all of those properties. The optimization problem models the re-entrant flow shops with zero buffer and no-wait stations. The objective intends to minimize tardiness of the schedule by penalizing it in a linear step-wise function.

The Table 3.1 below shows the summary of the reviewed literature which has the common properties with our study. As it can be easily observed that there is found no study which covers the whole aspects of our problem.

Table 3.1. Summary of the some literature review studies based on our problem scope

Authors	**(FS) /(JS)**	**Hybrid**	**Re-entrant**	**Buffer**	**No-wait**
Ruiz, *et al.* [4]	FS	Yes			
Lee, *et al.* [5]	FS	Yes			
Unlu, *et al.* [31]	FS	Yes			
Sawik [18]	FS	Yes	Yes	Finite	
Samargandhi, *et al.* [13]	FS	Yes			Yes
Sawik [16]	FS	Yes	Yes	Finite	
Allahverdi, *et al.* [15]	FS	Yes			Yes
Ronconi, *et al.* [17]	FS	Yes		Finite	
Salvador [14]	FS	Yes			Yes
Han, *et al.* [19]	FS	Yes	Yes		
Huang, *et al.* [20]	FS	Yes	Yes		
Dugardin [21]	FS	Yes	Yes		

4. OPTIMIZATION MODEL

4.1. Objective Function

There are many different performance measures in scheduling. [22] classifies the performance measures used in the literature into three main categories. Those categories are; criteria based on completion time, criteria based on due dates and criteria based on inventory cost and utilization. Without any doubt, all categories are important. For our case, conforming due dates is the prior issue to satisfy customer satisfaction and the overall profitability of the company. Late or inaccurate deliveries can have a major negative impact on customers' future decisions, which is measured by the loss of market share in the end. In the short run, there are also negative effects such as compensation of customers' lose due to late delivery, since the products are used mainly for commercial purposes. And since most of the vehicles are exported, there are fixed shipment dates for the product batches which are negotiated beforehand. Therefore, lateness is penalized with unutilized capacity for shipment and additional cost of the unit for the next shipment.

Assembly line is also the customer of the paint shop. Their expectation is to receive the buses with the given order in each takt time. Any deviation from the desired due dates will cause to decrease the productivity of assembly line, too. Cost of not to deliver any job to assembly line is a major problem which cause to idle time in assembly line. That is also penalized with a huge amount of costs by using a slack variable to identify if there exists no delivery at any takt time.

The objective function of the model considers above issues with two elements and aims minimizing the tardiness accordingly. The first element is a cost of delivering a job later than the assembly line's desired due date which is constructed as a stepwise tardiness function in which up to a predefined upper limit, cost of being tardy for a job stays the same. Those limits can be defined based on the cost effects of late deliveries. On the other hand, the second element is a cost of not delivering a job on the assigned due dates. Each possible takt time from the takt time set is needed to match with a job from the job set in the model.

Deliveries later than those dates are penalized with a huge amount of cost. The slack variable relaxes the system to prevent infeasible solutions. Based on those assumptions, a linear cost function is constituted not to increase complexity of the proposed model.

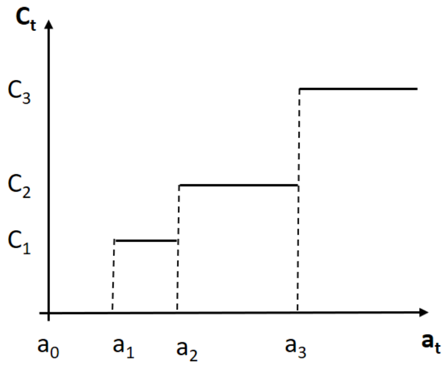

Figure 4.1. Illustrative step-wise tardiness cost function

Due to above reasons, tardiness cost of a job increases stepwise with the specified maximum tardiness limits. This kind of tardiness cost occurs in several real life cases particularly when there is transportation. This is also the case in the bus plant, since the buses are produced make-to-order based and transportation of the buses to the different locations are set a quite long time before the production is completed.

The study [23] also works with a stepwise function to minimize total tardiness cost. The step-wise tardiness cost function for a job can be illustrated in Figure 4.1 and can be explained as a function as follows:

$$f(W_j)=\begin{cases} 0\,, & m_{ij} - d_j \leq a_0 \\ C_1\,, & a_0 < m_{ij} - d_j < a_1 \\ C_2\,, & a_1 < m_{ij} - d_j < a_2 \\ \cdot & \\ \cdot & \\ \cdot & \\ C_t\,, & m_{ij} - d_j \geq a_t \end{cases} \qquad \text{where i is equal to the last stage of job j}$$

4.2. Notation: LMIP for Paint Shop Scheduling

4.2.1. Indices

i	stages
j	jobs
k	machines
w	due dates
n	slots
t	tardiness level

4.2.2. Sets

I_j	set of stages of job j
J	set of jobs
K_{ij}	set of machines that can process job j at its stage i
N	set of slots in each machine
W	set of due dates
T	set of tardiness levels
W	set of due dates

4.2.3. Parameters

d_j	desired due date of job j to be delivered to assembly line
rel_j	release date of job j from body shop (earliest processing start time in the paint shop)
p_{ij}	processing time of job j at its stage i

a_t upper bound on tardiness level t

C_t cost at tardiness level t

D_w takt time corresponds to due date w

M large number

E penalty for delivering jobs after the assigned due dates

z_{ij} $1 \text{ if stage i of job j is no} - \text{wait, else } 0$

b_{ij} $1 \text{ if stage i and i} + 1 \text{ belongs to the fixed route of job j, else } 0$

4.2.4. Decision Variables

c_{ij} completion time of job j at its stage i

m_{ij} move time of job j from its stage i

y_{ijkn} 1 if job j on its stage i is processed at machine k as the n^{th} $job, else$ 0

S_{jw} slack variable for movement of job j from its last stage at due date w

P_{jt} 1 if tardiness of job j is between a_{t-1} and a_t , $else$ 0

g_{jw} 1 if job j is assigned to due date w, $else$ 0

4.3. Model

We consider a linear objective function which tries to assign desired due dates to each bus by minimizing the stepwise tardiness cost and force them to be delivered on the assigned due date. The first part of the objective function tries to assign a cost to a job by minimizing the tardiness level where the job belongs to. The second part tries to minimize the slack variable of the job on the assigned due date.

The objective function is:

$$\min \sum_{j \in J} \sum_{t \in T} P_{jt}\, C_t + \sum_{w \in W} S_{jw}\, E$$

s.t.

4.3.1. Assignment of tardiness interval constraints

Each job should be assigned to only one tardiness interval based on the difference between the move time from the last stage of operations and desired due date.

$$\mathrm{m_{ij}} - \mathrm{d_j} \le \mathrm{a_t} + (1 - \mathrm{P_{jt}})\,\mathrm{M}\,; \qquad \forall \mathrm{j}, \mathrm{t} \text{ and i is the last stage of job j} \tag{4.1}$$

$$\sum_{t \in T} P_{jt} = 1\,; \qquad \forall j \tag{4.2}$$

4.3.2. Due date assignment constraints

Equation 4.3 satisfies that each job should be assigned exactly one due date from the possible due date set. And Equation 4.4 satisfies that each due date should be assigned to exactly one job.

$$\sum_{w \in W} g_{jw} = 1\,; \qquad \forall j \tag{4.3}$$

$$\sum_{j \in J} g_{jw} = 1\,; \qquad \forall w \tag{4.4}$$

Equation 4.5 tries to satisfy that each job should not move from the last stage of its operations before the assigned due date. The slack variable in the Equation 4.5 is penalized by a cost element in the objective function.

$$\mathrm{m_{ij}} - S_{jw} \le D_w + (1 - \mathrm{g_{jw}})\,\mathrm{M}\,; \qquad \forall \mathrm{j}, \mathrm{w} \text{ and i is the last stage of job j} \tag{4.5}$$

4.3.3. Machine assignment constraints

Each job at its every stage should be assigned to only one machine and only one slot on that machine. And every machine and its slots can process at most one job.

$$\sum_{k\in K_{ij}}\sum_{n\in N} y_{ijkn} = 1\,; \qquad \forall i,j \tag{4.6}$$

$$\sum_{j\in J}\sum_{i\in I_j} y_{ijkn} \leq 1\,; \qquad \forall k,n \tag{4.7}$$

4.3.4. Sequence constraints

From the jobs that are assigned to the same machine, only one of them can be processed before the other one. The constraint below relates the moving time of the jobs if they are processed on the same machine consecutively.

$$\mathrm{M}\big(2 - \mathrm{y_{ijkn}} - y_{lrk,n+1}\big) \geq \mathrm{m_{l-1,r}} - \mathrm{m_{ij}}\,; \quad \forall \mathrm{i,j,n} \quad \mathrm{j} \neq \mathrm{r},\ \ \mathrm{l} > 1,\ \ \mathrm{k} \in K_{ij} \tag{4.8}$$

$$\mathrm{M}\big(2 - \mathrm{y_{ijkn}} - y_{lrk,n+1}\big) \geq rel_r - \mathrm{m_{ij}}\,; \quad \forall \mathrm{i,j,n} \quad \mathrm{j} \neq \mathrm{r},\ \ \mathrm{l} = 1,\ \ \mathrm{k} \in K_{ij} \tag{4.9}$$

Equation 4.10 ensures the ordering between the slots of a machine so that the slots are used sequentially and move times are in increasing order.

$$\sum_{j\in J}\sum_{i\in I_j} y_{ijkn} \geq \sum_{j\in J}\sum_{i\in I_j} y_{ijk(n+1)}\,; \qquad \forall k,n \tag{4.10}$$

4.3.5. No pre-emption constraints

We assume that as soon as a job arrives a station, processing starts immediately and it is completed without any preemption. Hence, completion time should be greater than the move time of the job to the station plus the processing time on that station. For the initial operation in paint shop, move times are fixed and equal to the release time of the jobs from the body shop.

$$c_{1j} \geq rel_j + p_{ij}\ ; \qquad \forall j \tag{4.11}$$

4.3.6. Zero buffer constraints

After the completion of an operation on a station, a job may leave the station and move to the next station only if the next station is not occupied by another job. If the next station is occupied by another job, the job may wait on the station that is currently finished processing. So that, the move time of the job may differ from the completion time of the job.

$$c_{ij} \leq m_{ij} \qquad \forall i, j \tag{4.12}$$

As there are no space for buffer between the stations, each operation starts immediately after they move from the previous station.

$$m_{i-1,j} + p_{ij} = c_{ij}; \qquad \forall j, \qquad i > 1 \tag{4.13}$$

4.3.7. No-wait stations constraints

Due to processing requirements on some stations, every job processed on those stations has to depart from the station immediately after their processing is completed. The set of constraints below are active only if the station i of job j is no-wait. And the constraint forces completion time to be equal to move time to satisfy the inequality.

$$z_{ij}\left(c_{ij} - m_{ij}\right) \geq 0\ ; \qquad \forall i, j \tag{4.14}$$

4.3.8. Connected machines constraints

Due to the layout of the paint shop, there are connection between some stations. So that, if a station is selected in previous stage, the station for the upcoming stage is also determined, since there is no possibility to transfer the job to a parallel machine. If two stations are on the fixed route, then the machine assignment is guaranteed by the following inequality.

$$b_{ij}\left(y_{ijkn} - y_{i+1,j,sn}\right) \geq 0 ; \qquad \forall i, j, k, s, n \tag{4.15}$$

4.3.9. Non-negativity and integrality constraints

$$c_{ij} \geq 0 ; \qquad \forall i, j \tag{4.16}$$

$$m_{ij} \geq 0 ; \qquad \forall i, j \tag{4.17}$$

$$S_{jw} \geq 0 ; \qquad \forall j, w \tag{4.18}$$

$$y_{ijkn} \in \{0,1\} ; \qquad \forall i, j, k, n \tag{4.19}$$

$$g_{jw} \in \{0,1\} ; \qquad \forall j, w \tag{4.20}$$

$$P_{jt} \in \{0,1\} ; \qquad \forall j, t \tag{4.21}$$

4.4. Model Assumptions

Below assumptions are taken into account when the mathematical model is constructed. In real production environment, most of the assumptions below may occur in the time of the production simultaneously and those make the production environment even more complicated.

- Each job is an entity, even though the job is composed of distinct operations, no two operations of the same job may be processed simultaneously.
- The number of jobs is known and fixed.
- No job may be cancelled before completion.
- No job may be split or preempted.
- The release time of the jobs from the body shop is known and fixed.
- Delivery time of the jobs determined according to the takt time. Desired delivery time of jobs are also known.

- One job will be delivered to the assembly line at each takt time.
- The processing times of the jobs are known and fixed.
- There are identical parallel machines. In each stage, those machines can handle the same operations.
- Processing time in parallel machines in each stage is the same for a job j.
- Setup time and transportation time are considered as a part of the processing time and are independent of the job sequence. Therefore, no need to consider separate setup time and transportation time in the model.
- All machines are available at the beginning and never breakdown during the scheduling period.
- Materials required for the processes are ready before any operation is started.
- No machine may process more than one job at a time.
- Machine(s) may remain idle.
- Re-entrance is deterministic and known. And it only depends on the number of painting process of the job. There is no re-entrance due to quality issues.
- Only tardiness is considered as an undesired situation. Earliness of deliveries to the assembly line can be managed somehow.

4.5. Initialization

Production continues in the bus production plant. Hence, when one wants to optimize the schedule of a batch of new orders, the stations will be occupied with the buses whose operations are already started in the paint shop. So, that requires to define an initial state of the paint shop to obtain a realistic solution.

There are several initialization techniques used in literature. [25] analyzes the performance of the initialization methods in scheduling using three different methods namely random initialization, orthogonal initialization and chaotic initialization in their study. The result of their analysis shows that orthogonal initialization performs better for their case.

The performance of initialization methods can also be a search topic in another study. In this study, we are dealing with the effects of re-entrant job ratios on tardiness level and changes in the production volume.

One option and the simplest assumption is to place work-in progress buses in all stations assuming that all buses are one-color, which means none of the buses will revisit the same set of operations. Although this assumption simplifies the initialization of the paint shop, it is less likely to satisfy the real paint shop statement.

In our thesis, we use the random initialization such that we randomize the sequence of jobs and assign them to stations by adjusting their stage definitions and remaining processing times accordingly. Moreover, in order not to violate the feasibility of the shop at the initial state, we just fill a reasonable percentage of the stations to run our mathematical model. Assignment of jobs to the no-wait stations is critical in the initialization phase. If the succeeding stations are not available for the first move from the no-wait stations, then infeasibility is obtained at the initial stage. In order to prevent such infeasibility, initialization is made carefully in the test cases. As the number of station increases, setting an initial schedule also becomes complicated. One may need to generate an algorithm to set the initial stage. Since we are using a quite small model the run our test, we handled that situation without an algorithm.

5. EXPERIMENTATION

5.1. Experimentation Setup

In the experimentation part, we analyze the results of the model by changing two different environmental parameters:

(i) The percentage of the jobs that requires re-entrance. It affects directly the workload of the paint shop as well. The customers painting and decoration demands are changing. In real environment, based on the painting preferences, a bus can visit the same operation sets up to four times and those additional operations extends the total production time of the buses accordingly. Due to competition in the market, producers have to answer the customers' requests. In the experimentation part for the sake of the tests, we consider that re-entrance is possible only once.

(ii) Different takt times indicate the daily production volume of the production plant. Due to increasing and decreasing demand from the market, existing production plant is needed to adjust its production volume by changing the takt time accordingly. Due to increase in export volume, this is one of the hot topic of the management team of the plant as well. In the production plant, currently 14 buses are produced. Due to increasing order from abroad, the plant is about to increase its capacity to produce 17 buses per day. Decreasing the takt time means increasing the daily production volume. At the same time, it also means that increasing the ratio of the longest processing time with respect to the takt time.

For each environmental parameters, three different scenarios are created. Scenarios belongs to the parameters can be seen in the Table 5.1 and Table 5.2 below.

Table 5.1. Environmental parameter 1 with three different scenarios (re-entrant job ratio)

Percentage of jobs based on the number of re-entrance	Scenario1	Scenario2	Scenario3
No re-entrant	80%	66.7%	40%
Re-entrant only once	20%	33.3%	60%

Table 5.2. Environmental parameter 2 with three different scenarios (increase in production volume)

	Scenario 1	Scenario 2	Scenario 3
Ratio of processing time of heat treatment (longest processing time) to the takt time	1	1.5	2

Table 5.3 shows the combinations of scenarios, which are explained in the Table 5.1 and Table 5.2, and number of replications used for the test cases.

Table 5.3. Test cases

Test Case	Re-entrant job ratio	Ratio of longest processing time to takt time	Number of replications
1	20%	1	5
2	20%	1.5	5
3	20%	2	5
4	33.3%	1	5
5	33.3%	1.5	5
6	33.3%	2	5
7	60%	1	5
8	60%	1.5	5
9	60%	2	5

To see how the mathematical model reacts to those test cases, we constructed very small machine setup in which we can use all the assumptions and constraints that we have stated in the model which are explained in detail in the Section 4. In the experimentation, we used 5 machines, 4 different groups of operations, 15 jobs in total. There are 2 types of jobs namely jobs with single flow and jobs with re-entrance properties. So, the stage number for the single flow jobs is 3, whereas for the re-entrant jobs the stage number is 5. To keep the precedence relationships of the jobs on the same machine, we define 15 slots to each machine. Even in such a small setup, the problem has 70,984 rows, 5,927 columns, 268,330 non-zeroes and 5,587 discrete columns.

In order to create data sets for test cases based on the predefined scenarios, we used Spyder environment in Pyhton programming language. We started with creating the job sets based on the predefined re-entrant job ratios. For each replication, we randomize the order of the jobs within the job sets. Depending on the number of re-entrance of the job, namely whether the bus is single color or two-colored, the number of stages are associated with the jobs. So, the jobs have different routes. For each stage of a job, possible stations that process the identical operations are defined. No-wait stations are defined, given as binary parameters and related with the job stages. The processing times of the jobs at each stage are assigned. Possible due dates are defined so that the time between the two successive due dates is exactly the takt time. Each due date is assigned to a job. Those are the times that the assembly line requests the jobs to start its operations. Similarly, the release times are given. Those are the times so that the body shop delivers the jobs to the paint shop. The release times also indicate the earliest start time of the operations in the paint shop. It is assumed that they are fixed and cannot be negotiated. Similar to the due date definitions, in every takt time, exactly one job arrives to the paint shop from the body shop. To assess the effects of the increase in production volume, we adjust the relative time of the longest operation in the paint shop, which is the heating process, to the takt time with three scenarios which are defined in Table 5.2. Randomized job sets with different re-entrance ratios, which are defined in Table 5.1, are matched with different processing times accordingly.

For initial state of the paint shop at time zero, early jobs are assigned to the machines so that the initial stage does not violate the feasibility of the system. And for those jobs, stages are redefined based on their routes and remaining operations. The completed operations of the initial state jobs are no longer parameters in the model.

We defined the cost parameters of the objective function accordingly. Any late delivery of the jobs results in disturbance in the assembly line operations. Late deliveries from the desired due dates are penalized with a cost based on their tardiness level. Usually one takt time late delivery of the orders is accepted as reasonable by the management, and this does not disturb the operations in the assembly line significantly. However, late deliveries that exceed one takt time is assumed to have much greater effect than the one takt time late delivery. So, the tardiness intervals for the cost function are assigned as the follows. There is no cost for early and on time deliveries. There is a penalty for one takt time late delivery. And greater and a same cost coefficient is applied when the deliveries are greater than one takt time.

As well as deliveries on the desired due dates, for the assembly line, delivering a job at each takt time is also important for using the production capacity efficiently. Slack variable is used to handle such cases within the given period of scheduling time. And a fixed amount of penalty is applied for all jobs that have positive slack variable amount. It is kept as high as possible in order to prevent not delivering or late deliveries of the jobs from the assigned due dates. Using such a slack variable prevents our model to get infeasible solutions.

Based on the test cases that are stated in Table 5.3, 45 test instances are constructed. The results of the experimentation are discussed in the Section 5.2 in detail.

5.2. Experimentation Results

We used GAMS modeling environment to obtain solutions with ‘Cplex’ solver to our linear mixed integer programming model. We performed 45 test instances in total. Computational time for the each instance was unstable. Solutions were achieved in some instances with relative gap up to 10%. Also in some instances, the ‘Cplex’ solver was not able to find an integer solution.

The Figure 5.1 shows the average total costs of runs which is simplified by using greatest common divisor across all test cases. First of all, the results show in the Figure 5.1, as expected, cost increases as the production volume increases by keeping the re-entrant job ratio fixed. If we look at the test case 8 and 9 results, where the re-entrant job ratios are 60%, we can easily observe that there is a huge cost increase when we compare them with test case 7, in which the re-entrant job ratio is also 60%. And there is no significant difference between the cost amounts of test case 8 and 9. We can interpret that the paint shop capacity is already used when the processing time to takt time ratio is increased to 1.5. We use a stepwise tardiness function, so late deliveries more than one takt time have the same effect on the total cost in the setup. For the test cases 4, 5 and 6, where the re-entrant job ratios are 33.3%, there are slight increases in cost. And similarly for the test cases 1, 2 and 3, where the re-entrant job ratios are 20%, there are slight increases in cost. For those, we can interpret so that increasing the production volume can be manageable with the existing paint shop capacity when the re-entrant ratio is not so high.

Secondly, in the Figure 5.1, the results also show that, again as expected, tardiness cost increases, as the number of re-entrant jobs increases. In test cases 1,4 and 7, the cost increases slightly. But in test cases set consist of 2, 5 and 8, and the test cases set consist of 3,6 and 9, there is an obvious increase in cost when the re-entrant job ratio is increased to 60%. We can interpret that, if the plant wants to increase its production volume, the color options of the scheduled jobs should be decreased unless there is no investment to increase the capacity of the bottleneck stations. Re-entrant jobs require extra capacity usage of the paint shop and use the same set of operations after they leave them. The total time that they spend in the paint shop increases as well. Moreover, for every re-entrance, there will be competing jobs to use the same set of stations which have limited maximum capacity. In addition, those stations may not be used with full capacity, since there is no-wait property in the system. Some stations cannot be used even if they are empty unless the succeeding stations are eligible to start processing of the jobs immediately when the processes are finished at the no-wait stations.

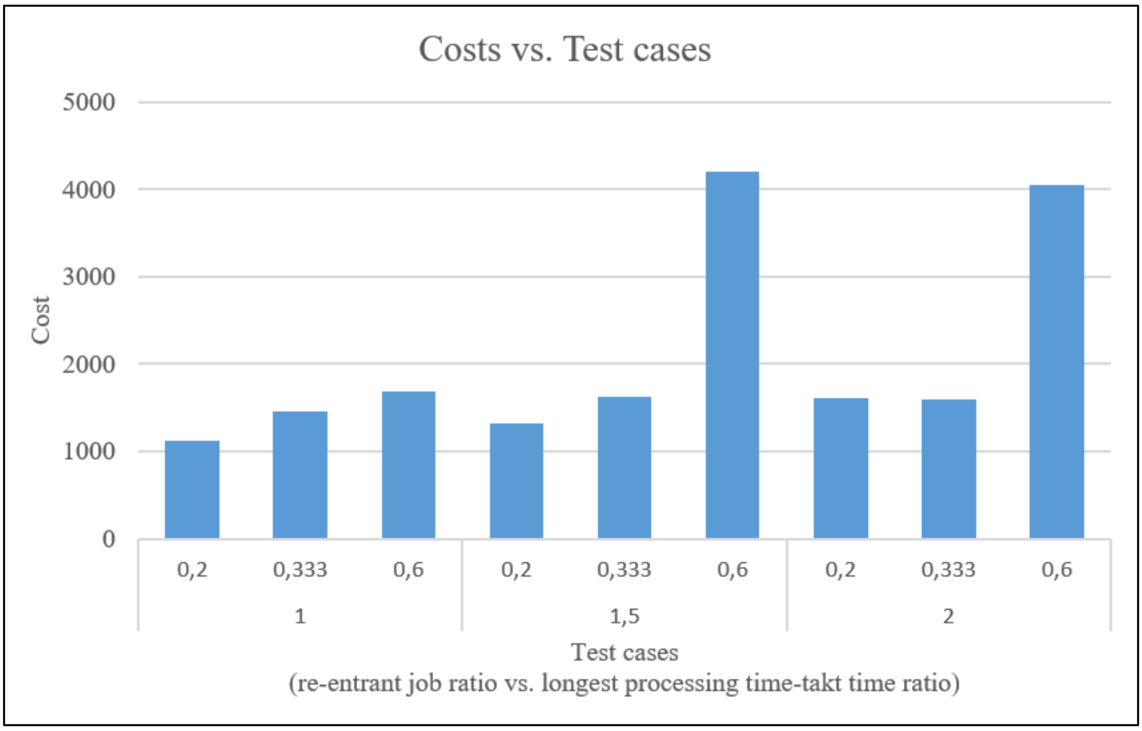

Figure 5.1. Average simplified total costs of the test cases

In the real production environment, the employees report that if multi re-entrant jobs are planned consecutively, the paint shop may not deliver jobs to the assembly line at every takt time. In real life, they say that such cases are handled via the regular meetings to review the production plan of the plant.

As we explained in the Section 5.1, we used a small setup to test the performance of the mathematical model. Results of one the test instances can be seen in Figure 5.2 via Gantt chart of the optimum scheduling solution. The time bar at the top of the Gantt chart is also show the release and delivery times of the jobs. Time at '0' shows the initial state. The jobs are assigned to the machines based on the routes of the jobs and desired due dates. Only 80% of the machines are filled with jobs in the initial state in order to prevent infeasibility at the first move of the jobs. M1 operates the initial stage of the jobs that enter to the paint shop from the body shop, whereas M5 is the last stage of the jobs before delivering the jobs to the assembly line. M2 and M3 stations are identical parallel stations that process exactly the same operations within the same processing time. These stations also represent the heating operations in the real paint shop. The jobs cannot stay more than the processing time in those stations due to the nature of the process. Those stations are also our no-wait stations. The jobs cannot be blocked when their operations are finished in no-wait stations. Equation 4.14 handles such cases by equalizing the completion time to the move time of the job. If the succeeding stations are not available at the move time, the jobs wait after their process are finished in the preceding stations. Finally, M4 represents the buffer station and it is used when the jobs are re-entrant.

machines \ time	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
M1	j4	j4	j5	j6	j7	j8	j9	j10	j11	j12	j13	j14	j15				
M2	j2	j2	j4	j5	j6		j8	j6	j10	j11	j12	j13	j14	j12			
M3					j3	j7		j9			j9			j15		j15	
M4	j3	j3	j3	j3		j6	j6		j9	j9		j12	j12		j15		
M5	j1	j1	j2	j4	j5	j3	j7	j8	j6	j10	j11	j9	j13	j14	j12		j15
delivery sequence		j1	j2	j4	j5	j3	j7	j8	j6	j10	j11	j9	j13	j14	j12		j15

Figure 5.2. Gantt chart of an instance from the test case 4.

The jobs are indicated with j index in the Figure 5.2. The re-entrant job ratio is the 33.3% in that test case. And all processing times are equal to each other and 1. Jobs j3, j6, j9, j12 and j15 are the re-entrant jobs, so they uses M4 in their route to re-visit the stations M2 or M3. We handled Equation 4.15 for connected machines in such a small test environment with a little change. We assign the re-entrant jobs in their second tour to the same parallel machine (M2 or M3) that they use in their first route.

It is important to see that the release order of the jobs is not same with the delivery orders, since the route that they follow and the total processing times are different. Delivery time of the jobs are assigned starting from 1 to 15 accordingly with the job index. Since the jobs are randomized in total processing times via the assignment of the re-entrance property, job 15 cannot be delivered to the assembly line on time. In that case, both the slack variable and stepwise tardiness function punish the late delivery.

As mentioned before in Section 4.1, the objective function has two elements. First element is the stepwise tardiness function which has used with only three intervals. The other analysis is done by using that tardiness interval assignment.

The Figure 5.3 shows the percentage of the jobs that are delivered within the defined tardiness intervals for each test case. As the percentage of the re-entrant jobs increases, the percentage of the late deliveries also increases, if we look the first three bar sets in the Figure 5.3. When only one fifth of the jobs are re-entrant, delivering on time or before the due date is 80%. Whereas the re-entrant job ratio is 60%, then on time delivery or early deliveries are decreased to 50%. On the other hand, as the production volume increases, the percentage of late deliveries increase. But there is no significant difference between the longest processing time to takt time ratio 1,5 and 2 in terms of the assigned intervals. This may indicate that we have already reached the upper limit of the system.

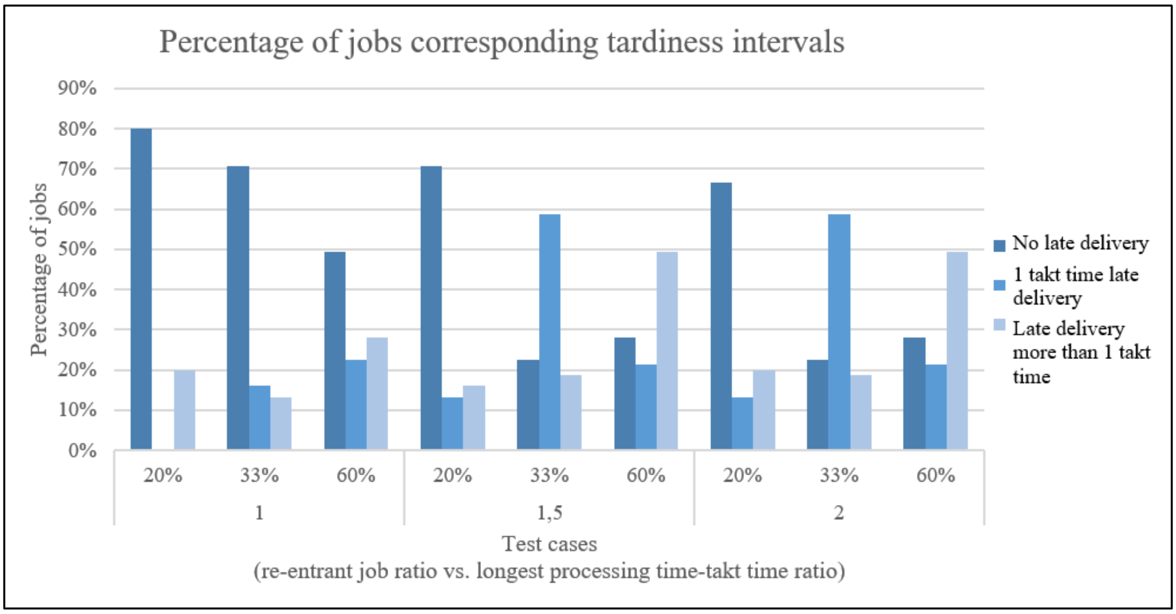

Figure 5.3. Percentage of jobs that are delivered within the defined tardiness intervals for each test case

The results show that the proposed mathematical model is able to reflect the expected outcomes for the small instances. We tried to test the mathematical model also with the larger setups. Adding just one parameter to the tested setup, increases the model size significantly. For example, the problem size increases dramatically, when we want to use the setup shown in the Figure 2.2 with 14 machines, 42 jobs, 17 stages (jobs with two re-entrance version is also placed), 42 machine slots, the number of rows in the model reaches above the 17,7 million and 69,5 million non-zeros. Even in smaller setup with 8 machines, 18 jobs, 8 stage and 18 machine slots, the number of the rows of the model 356 thousand rows and over 1,3 million non-zeros. For those setups, optimal solutions within the reasonable time are not found.

6. CONCLUSION

In this study, we have modelled a bus production plant which is a re-entrant hybrid flow shop with zero buffer and no-wait stations. A mixed integer linear programming (MILP) formulation is generated to find an optimal solution for a small sample of the complex scheduling problem. Our aim is to submit the buses from the body shop to the assembly line in line with the desired due dates.

We used GAMS modeling environment to obtain solutions with ‘Cplex’ solver. We obtained the solutions for the small instances quickly, which helped us to see lots of solutions in a limited time. Another important aspect is that we can see the move time of the buses from the stations which enable us to micromanage the shop as well. The results helped us to see the paint shop production capacity and how efficient the paint shop can operate.

We designed an experimental version of the paint shop to perform statistical tests to observe the results of two different aspects. Firstly, by changing the percentage of the re-entrant jobs, the effects on the late delivery ratios of jobs are observed. The results show that, as expected, as the percentage of the re-entrant jobs increases, deliveries to the assembly line after the desired due dates are also increasing. Secondly, the production plant adjusts its daily production volume within the limits of its production capacity in order not to produce for inventory. Since properties of the buses heavily depend on the customer special requests, most of the buses are treated as special order, especially in terms of painting variety. So, changing the daily production capacity is one of the top topic in the plant. Both cases are investigated by minimizing the cost of tardiness which consist of two elements, namely a stepwise cost function and a cost element assigned to a slack variable. The results of the experimentation show that proposed mathematical model projects the expected outcomes.

The real production environment in the paint shop is a bit more complicated than the modeled problem. Due to physical conditions of the production area, longer buses cannot be painted in some paint stations. So, in the real environment not all parallel machines operate identical processes. For simplicity reasons, such details are not included in the optimization model. Part painting is also neglected in the model. But in real world, parts of the buses are painted separately and they also consume the capacity of the paint shop. Moreover, violation of any assumptions such as availability of required materials on time, machine breakdowns, etc. would deteriorate the fitness of the model to the real world.

A shortcoming of this study is that it is difficult to solve the problem with the real data. A robust algorithm can be searched to find a reasonable solutions to the real environment instances.

The release times of the buses are assumed fixed. But in real life, there are regular meetings between the order center, body shop, paint shop and assembly line executives to review the production plan. So, the release times and orders of the buses can also be negotiable. Modeling such a system in which ready times are also soft constraints can be an interesting further search topic.

To sum up, we can say that the representative model of the paint shop gives satisfactory solutions for the complex problem. And it may also beneficial for the management team when making further investment decisions. For example, the effects of adding one machine to the paint shop can be assessed by the cost function. So, the model can give an insight for the return on investment, feasibility analyses.

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