QUANTITY COMPETITION FOR PERISHABLE PRODUCTS UNDER DEMAND SUBSTITUTION

by

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ABSTRACT

QUANTITY COMPETITION FOR PERISHABLE PRODUCTS UNDER DEMAND SUBSTITUTION

This paper provides optimal quantity ordering policies for perishable products under demand substitution in a competitive setting. An equilibrium point is found by using competitors' response functions. We analyse the equilibrium point behaviour under different scenarios and observe that the equilibrium point increases if competitors' main priority is not to get shortage of products. Similarly, the equilibrium point decreases if competitors' main priority is not to get outdated. We also see that the equilibrium point and total expected cost of competitors are inversely proportional with the amount of old products being brought into the new period. Numerical examples display that the demand substitution is beneficial for both players as it reduces the total cost of the system.

ÖZET

TALEP DEĞİŞİMİ ALTINDA BOZULABİLİR ÜRÜNLERİN ENVANTER REKABETİ

Bu tez talep değişimi altında bozulabilir ürünlerin envanter rekabetini araştırır. Oyuncuların cevap fonksiyonları kullanılarak bir denge noktası bulunmuştur. Bu denge noktasının farklı senaryolar altındaki davranışları incelenmiş ve oyuncuların darlık durumundan korktukları zaman denge noktasının yükseldiği gözlemlenmiştir. Ürünlerin elde kalıp bozulacakları senaryolarda ise denge noktası aşağı düşmüştür. Denge noktası ve toplam giderlerin mevcut bulunan eski ürünlerle ters orantılı olduğu görülmüştür. Sayısal örnekler talep değişiminin toplam giderleri düşürerek her iki oyuncuya da yarar sağladığını ortaya koymuştur.

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LIST OF SYMBOLS

D_i^j	Demand of player j in period i
$\tilde{D}_i^j$	Effective demand of player j in period i
F_{ij}	Cumulative distribution function of player j in period i
f_{ij}	Probability distribution function of player j in period i
L_i	Expected one-period cost function for player i
p	Shortage cost per unit
R_i	Response function for player i
x_i	One period old brought by player i
y_i	New products ordered by player i
Z^i	Random variable that gives outdated products of player i
α	Substitution multiplier
$\mu(ij)$	Mean value of demand function for player j in period i
Θ	Outdate cost per unit

LIST OF ACRONYMS/ABBREVIATIONS

cdf	Cumulative distribution function
Eq. Pt.	Equilibrium point
FIFO	First-in-first-out
pdf	Probability distribution function
pmf	Probability mass function

1. INTRODUCTION

The Pharmaceutical market has never been studied extensively from a mathematical point of view since the government of Turkey and most of other countries take away the price-setting option from the pharmacies and decide on fix prices by themselves. At first, this situation does not seem to be exciting as there are not many factors left to change the profitability of pharmacies. However, constant product price policy of the government actually makes inventory control management much more appealing for this sector, because pharmacies traditionally make little profit from the medicine. Wrong inventory management that results in shortage and outdates (since the medicine is a perishable product) can be devastating as pharmacies are already in a vulnerable economical position, because they do not decide the price of the product they sell. However, the pharmaceutical market has a vital advantage, which is the demand substitution. It is common to see multiple pharmacies located near each other. This case inspired us to develop a model that would yield optimal ordering quantities for perishable products under demand substitution in a competitive setting.

In this model, we have pharmacies as sellers and customers as buyers. Pharmacies sell medicine and customers visit pharmacies to buy medicine. Pharmacies hold inventory control management to replenish medicine that they sell to the customers. Medicine is a perishable product because medicine goes bad and cannot be sold after it surpasses its shelf life. It is assumed that customers have no preference for the medicine whether the medicine is brand new or it has a certain shelf life left before it reaches its expiration date. In case customers fail to find the medicine they seek in a pharmacy, demand substitution occurs and they visit another pharmacy close by with the hopes of finding the medicine.

After the model is prepared, the convexity of the expected cost functions is discussed. If both of the cost functions are proved to be convex, we expect to show that a Nash Equilibrium exists. We anticipate Nash Equilibrium to increase if shortage is the dominant cost in the market, as competitors hold more inventory. Likewise, we expect Nash Equilibrium to decrease if outdate is the dominant cost in the market, as competitors hold less inventory. We also think that initial inventory level and optimal ordering quantities are correlated. Competitors should order less if they have large initial inventory and vice versa.

The thesis is divided into five chapters. In chapter 2, related literature is reviewed. Chapter 3 introduces the model that calculates optimal ordering quantities. In chapter 4, an experimental setting is implemented to test the model and then we analyse how equilibrium point behaves under different scenarios including demand functions from uniform and poisson distribution. Chapter 5 concludes the thesis and provides a summary of the research.

2. LITERATURE REVIEW

The research in this thesis focuses on literature of two different areas: perishable inventory theory and demand substitution.

The most fundamental research about perishable inventories is proposed by Nahmias and Pierskalla [1]. At first, they compute optimal inventory ordering policy of a perishable product with two-period shelf life by using a generalized Newsvendor model with expected runout and outdates in a single period problem. Then, Nahmias [2], [3] extends the study into a dynamic model with finite and eventually infinite horizon, multi-period problem. An important finding of the study is that the decision of how much to order depends on the number of old products available. These studies coincide with this thesis because of the same context that has been used. The demand is independent of price and FIFO (first-in-first-out) policy is adapted to deliver the goods (so customer will be given the product, which is closest to its expiration date.) Pierskalla and Roach [4] actually show that FIFO issuing policy always gives optimal objective function value for perishable products when backlogging is allowed.

Nahmias and Pierskalla [5] work together to advance the study into two product perishable/nonperishable model, which can be applied to blood banks; because blood practically becomes a nonperishable product once it is frozen. Therefore, their model finds optimal ordering policies for frozen (nonperishable) and fresh (perishable) blood system that includes both products. Deuermeter [6] makes one of the later significant contributions on two product perishable inventory system with demand substitution in a single period model. In the model that Deuermeter proposes there exist two products, their respective demand functions and their initial inventory for each product. He computes optimal ordering policy for each product that minimizes total shortage and outdate costs of the system. If two products are rather defined as two players, the result can be interpreted as a centralized solution to the optimal inventory ordering policy for perishable product with two period shelf life under demand substitution and competition in a single period problem. This is actually the centralized solution to the model this thesis proposes, however there is a significant difference between our research. Deuermeter uses interdependent demand for products. He later explains this decision by a term called "economic substitution". It means that product A cannot be directly used to substitute for product B. He defines product demands as an aggregate function of demand classes. Therefore, even though there are no shortages, a small percentage of demand is always substituted to the other product. In case of the two player perishable inventory model (which is actually the same with two product perishable inventory model), this can be explained in an example. In Deuermeter's model, a small percentage of Pharmacy A customers will directly go to Pharmacy B, even though there are enough products to satisfy the demand in Pharmacy A, because how Deuermeter forms demand functions. On the contrary, in this thesis customers of Pharmacy A will never directly go to Pharmacy B if there is no shortage of product in Pharmacy A. This is the most major difference between our models and we believe the model in this thesis provides more a realistic context in case of a two player (namely pharmacy) perishable inventory model, and we analyse equilibrium behaviour.

Parlar [7] also works on optimal ordering policies for perishable and substitutable products of two period perish time in a single period model. He proves that the expected profit function for single period model is concave (his finding matches with Chazan and Gal's [8], because they also find that expected outdating is a convex function over the inventory level.) Parlar also provides numerical computations to show that substitution probability is a significant factor.

Sainathan [9] further extends perishable inventory model by modifying price with an utility function so that customers would prefer the product (old or new) that gives more utility. This change allows price to have impact on the demand. His model is constructed on a perishable product with two period shelf life over an infinite horizon. The most interesting finding is that in case of deterministic demand, selling old product is never optimal and it should be avoided at all cost. However, when the demand is stochastic, the retailer can gain more profit by selling both old and new product with correct pricing and ordering policy.

Recently, more and more researchers study perishable inventory models with demand substitution. Liu *et al.* [10] create model to make performance analysis on inventory levels of blood types in the blood bank system. This model consists of one way substitution, because demand of blood type of A can be substituted from inventory of type 0, however the reverse is not possible (for example satisfying the demand of blood type 0 with blood type A is not valid.) Yadavalli *et al.* [11] propose a similar model based on two substitutable perishable products. Their model also work with perishable products to review inventory levels on a continuous scale. However, even though both models are similar, none of them provides optimal inventory ordering policy as in the model of this thesis.

Demand substitution has been used widely in the literature. Deuermeter [6] and Parlar [7] are among the first researchers to bring the subject into perishable inventory theory. However, still new studies continue to hover its effect namely Sainathan [9]. Yet, the demand substitution model that we have been inspired is formed by Parlar [12]. He adds substitute demand into original demand to find effective demand for two Newsvendor competitors. Then, he derives the response function to characterize the Nash Equilibrium. Netessine and Rudi [13] also embrace the same concept of effective demand in their model. They extend the model into n players and show that a Nash Equilibrium exists for the decentralized model and they characterize the centralized solution.

The model that we propose is different than the ones mentioned above because of the following reasons. First of all, even though inventory ordering quantities are researched for perishable products by Nahmais [2], [3], and Nahmias and Pierskalla [1], [5], they never consider competition and therefore demand substitution. Despite Parlar [7] and Deuermeter [6] work on perishable inventory system with demand substitution, their studies also lack competition as demand substitution is used to substitute the product. Rather than substituting demand to a new player, Parlar and Deuermeyer study substitutable products, which results in a substituted product by the same player. Netessine and Rudi provide [13] demand substitution formulas that create effective demand, which are similar to this model as they substitute demand to another player.

However, their model completely ignore perishable products. Briefly, this model embraces perishable inventory models from Nahmais [2], [3], Deuermeter [6] and Nahmias and Pierskalla [1], [5] and then improves them to explain the demand substitution occurs in pharmaceutical market by using demand substitution and effective demand concepts found by Parlar [12], and Netessine and Rudi [13].

3. MODEL

The model consists of two players that are competing over a single perishable product. Each player has its own initial inventory as well as a decision to make on the amount of new product to order. Initial inventory levels and the amount of new product being ordered by the other player are known by both parties. Although the model is a single period problem, initial inventory and the demand of the previous period should be considered, because the product has a shelf life of two periods and backlogging between periods is allowed (This means new products can be used to satisfy the demand from previous period.) The timeline explaining the context from the perspective of player 1 can be seen in Figure 3.1. The goal of each player is to minimize total cost that occurs due to shortage and outdates. The model is inspired to reflect the pharmaceutical market as players represent two pharmacies that are close enough to each other to trigger demand substitution. If a customer fails to find the medicine in the first pharmacy, the customer will simply walk away to the next pharmacy to seek the medicine; which is the definition of demand substitution in this thesis. In Turkey and many other countries, the government controls the price of the medicine, so the price is fixed and pharmacies have no power to change it. Since the price is fixed in this model, players' only way to decrease their cost is to change the amount of new products being ordered. This is because the demand is independent of the price and the only way to increase demand is to make sure the product is available, when the customer walks in. This will cause the amount of new product being ordered to increase, which would also increases the chance of outdating and cost attached to it. This thesis investigates the interaction between shortage and outdates; and finds the optimum amount of new product to be ordered in a competitive (duopoly) setting based on different scenarios.

Following assumption and definitions are used in the model. x_1 and x_2 represent the amount of one period old product being brought into the new period by player 1 and player 2, respectively. y_1 and y_2 represent the amount of new product being ordered by player 1 and player 2, respectively (decision variable). All orders are placed

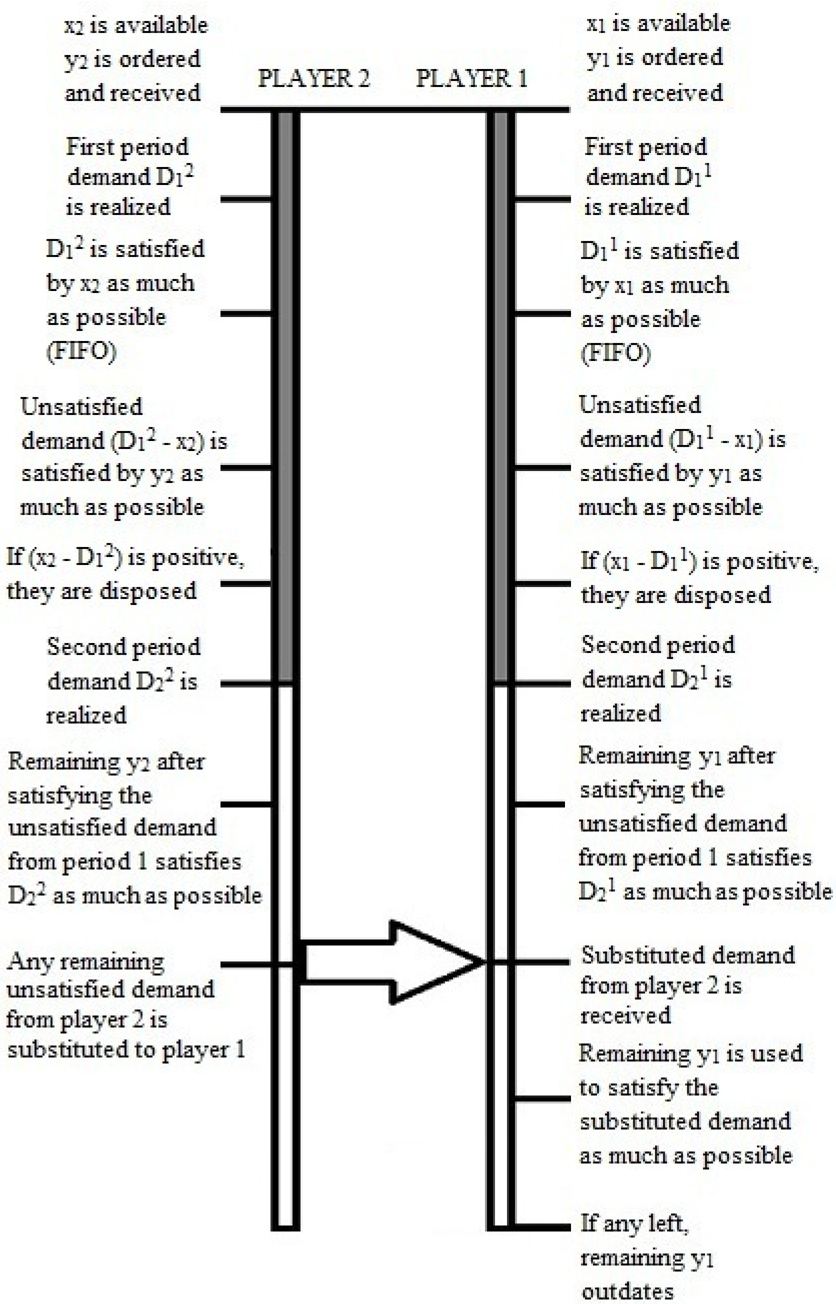

Figure 3.1. Order of Events from the Perspective of Player 1.

at the start of the period and they are received instantaneously. When orders are received, all of them are new and they have two periods of shelf life before outdating. p is the shortage cost per unit, penalty for ordering too little. Θ is the overage cost per unit, penalty for ordering too much. Shortage cost is applied in case of runouts, whereas overage cost is applied in case of outdates. D_1^1 and D_2^1 are independent random demands that occur in period 1 and period 2 for player 1, respectively. D_1^2 and D_2^2 are independent random demands that occur in period 1 and period 2 for player 2, respectively. All demands are independently distributed nonnegative random variables with their own distribution function F and density function f. $F_{11}(x_1)$, $F_{21}(y_1)$, $F_{12}(x_2)$ and $F_{22}(y_1)$ denote cdf; whereas f_{11}, f_{21}, f_{12} and f_{22} denote the pdf of D_1^1, D_2^1, D_1^2 and D_2^2, respectively. $\tilde{D}_2^1$ and $\tilde{D}_2^2$ are *effective demands* that occur in period 2 for player 1 and 2, respectively. (cf. eq. 3.5) FIFO policy is embraced to deplete inventory, because it is the most common policy used in perishable inventory theory. Furthermore, it is realistic as companies want to give away the products that are closer to expiration date first. Last but not least, it is easier to model.

Expected one period cost function for player 1 is;

$$L_1(x_1, x_2, y_1, y_2) = pE[Runouts] + \Theta E[Outdates] \tag{3.1}$$

where

$$E[Runouts] = E[D_2^1 + D_1^1 - (x_1 + y_1)] = \int_{x_1+y_1}^{\infty} [t - (x_1 + y_1)]g(t)dt \tag{3.2}$$

where g is the joint pdf of $D_1^1 + D_2^1$.

When calculating expected runouts, player is not punished for not satisfying the substitute demand. This assumption has also been used by Parlar [7]; because otherwise the player would be punished for competitor's lack of inventory, which is unfair. Also the system would punish players twice, which may exaggerate total shortage cost. Player 2 is already punished for not satisfying demand at its cost function

$L_2(x_1,x_2,y_1,y_2)$.

Normally, expected outdates is calculated by the following equation;

$$E[Outdates] = E[x_1 - D_1^1] = \int_0^{x_1} [x_1 - t] f_{11}(t)dt \tag{3.3}$$

However, in this method y_1 does not appear in excepted outdates. This indicates that new products, y_1 have no effect on amount of expected outdated products. It means single period model ignores the effects of outdating, therefore a new method has been introduced by Nahmias and Pierskalla [1]. This method calculates expected outdated cost one period into the future. Random variable Z^1 gives the total amount of outdated products, which is dependent on y_1 and y_2.

$$Z^1 = (y_1 - [\tilde{D}_2^1 + (D_1^1 - x_1)^+])^+ \tag{3.4}$$

The term $(D_1^1 - x_1)^+$, which will be referred as unsatisfied demand from period 1, represents the amount of demand from period 1 for player 1 that would be satisfied by new products, y_1. FIFO policy forces player 1 to supply demand with older products at first, which is x_1 in this case. Therefore, when D_1^1 is realized, player 1 tries to satisfy it by x_1 as much as possible. If $x_1 \geq D_1^1$, it means demand from period 1 for player 1 is fully satisfied and remaining $(x_1 - D_1^1)$ outdates. However, since we are calculating expected outdate cost one period into the future, the remaining x_1 is not added towards Z^1. If $x_1 < D_1^1$, remaining demand from $(D_1^1 - x_1)$ will be satisfied from new products, y_1 as much as possible. Henceforward, unsatisfied demand from period 1 is added to effective demand, $\tilde{D}_2^1$; and if $y_1 \geq \tilde{D}_2^1 + (D_1^1 - x_1)^+$, the total demand of period 2 for player 1 is satisfied by new products, y_1. Since, at the end of the period, the world ends and no more new demand is realized; remaining y_1 outdates and forms the random variable Z^1. If $y_1 < \tilde{D}_2^1 + (D_1^1 - x_1)^+$, it means that player 1 could not satisfy demand in period 2. Therefore, player 1 runs out of the product, which is already considered under $E[Runouts]$. This becomes part of the effective demand of player 2.

Effective demand considers demand substitution and is therefore a function of x_2 and y_2 as well. Effective demand of period 2 for player 1 is calculated by the following formula;

$$\tilde{D}_2^1 = D_2^1 + \alpha[D_2^2 + (D_1^2 - x_2)^+ - y_2]^+ \tag{3.5}$$

Where $(D_1^2 - x_2)^+$ represents the unsatisfied demand from period 1 for player 2. Like player 1's case, which is explained above, player 2 first tries to satisfy D_1^2 with older products, x_2. If $x_2 \geq D_1^2$, unsatisfied demand from period 1 is zero and the remaining x_2 outdates. However, if the demand is larger than the inventory ($x_2 < D_1^2$), the difference $(D_1^2 - x_2)$ will be unsatisfied demand from period 1. Now, player 2 tries to satisfy demand of period 2 for player 2 and unsatisfied demand from period 1 $[D_2^2 + (D_1^2 - x_2)^+]$ with new products, y_2 as much as possible. If $y_2 \geq D_2^2 + (D_1^2 - x_2)^+$, player 2 satisfies all of its own demand and no substitution would take place. However, if $y_2 < D_2^2 + (D_1^2 - x_2)^+$, player 2 will satisfy the demand as much as it can with y_2 and remaining demand $[D_2^2 + (D_1^2 - x_2)^+ - y_2]$, which is referred as the substituted demand, is substituted to player 1 with an α ($\alpha \in [0, 1]$) multiplier.

This allows us to write Z^1 by substituting $\tilde{D}_2^1$, under 8 different scenarios that cause y_1 to outdate. Figure 3.2 explains each scenario for player 1.

$$Z^1 = (y_1 - [D_2^1 + \alpha[D_2^2 + (D_1^2 - x_2)^+ - y_2]^+ + (D_1^1 - x_1)^+])^+ \tag{3.6}$$

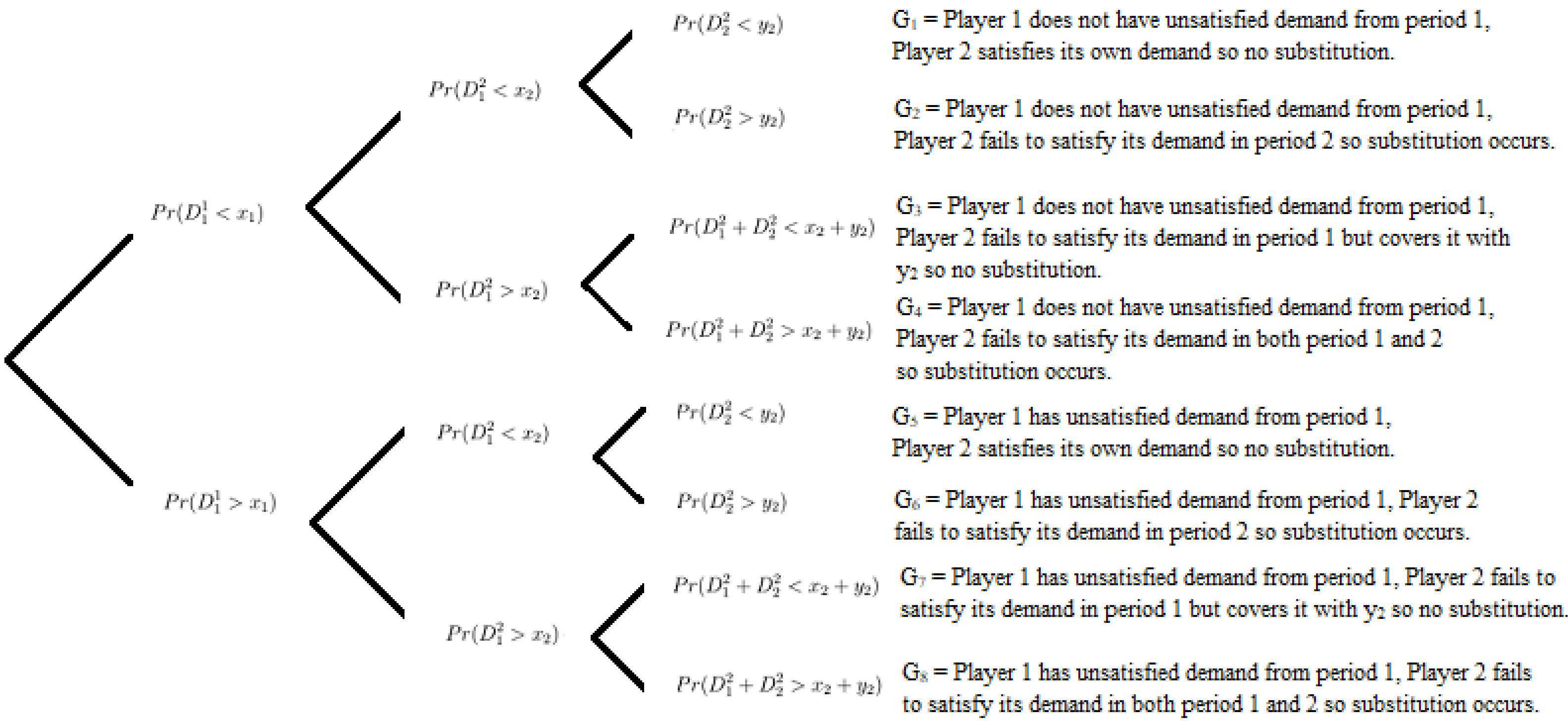

Figure 3.2. Explanation of 8 Different Scenarios that Cause y_1 to Outdate.

Following equations show how outdate probability is calculated for Z^1;

$$\begin{aligned}
Pr[Z^1] &= Pr((y_1 - [\tilde{D}_2^1 + (D_1^1 - x_1)^+])^+ \leq t) \\
&= Pr(y_1 - [\tilde{D}_2^1 + (D_1^1 - x_1)^+] \leq t) \text{ for } t \geq 0 \text{ and } t < y_1 \\
&= Pr(y_1 - [\tilde{D}_2^1 + (D_1^1 - x_1)^+] \leq t \text{ and } D_1^1 \leq x_1) \\
&+ Pr(y_1 - [\tilde{D}_2^1 + (D_1^1 - x_1)^+] \leq t \text{ and } D_1^1 > x_1) \text{ for } t \geq 0 \text{ and } t < y_1 \\
&= Pr(y_1 - (D_2^1 + \alpha[D_2^2 + (D_1^2 - x_2)^+ - y_2]^+) \leq t \text{ and } D_1^1 \leq x_1) \\
&+ Pr(y_1 - (D_2^1 + \alpha[D_2^2 + (D_1^2 - x_2)^+ - y_2]^+ + D_1^1 - x_1) \leq t \text{ and } D_1^1 > x_1) \\
&\text{for } t \geq 0 \text{ and } t < y_1
\end{aligned} \tag{3.7}$$

To compute (3.7), there are 8 probability functions inside $\Pr[Z^1]$ that correspond to the leaves of the tree in Figure 3.3, which illustrates the probabilities with associated functions for player 1. Since each probability function is independent of the other, they are calculated separately as G_i where $i = 1, 2, ..., 8$. For clarification the equations leading to the first scenario/probability function, G_1; where outdate probability, when $x_1 \geq D_1^1, y_2 \geq D_2^2$ and $x_2 \geq D_1^2$, as a function of y_1 is shown below;

$$\begin{aligned}
G_1 &= Pr(y_1 - [\tilde{D}_2^1 + (D_1^1 - x_1)^+] \leq t) \text{ for } t \geq 0 \text{ and } t < y_1 \\
&= Pr(y_1 - \tilde{D}_2^1 \leq t \text{ and } D_1^1 \leq x_1) \text{ for } t \geq 0 \text{ and } t < y_1 \\
&= Pr(y_1 - (D_2^1 + \alpha[D_2^2 + (D_1^2 - x_2)^+ - y_2]^+) \leq t \text{ and } D_1^1 \leq x_1) \text{ for } t \geq 0 \ \& \ t < y_1 \\
&= Pr(y_1 - D_2^1 \leq t \text{ and } D_1^1 \leq x_1, D_1^2 \leq x_2, D_2^2 \leq y_2) \text{ for } t \geq 0 \text{ and } t < y_1 \\
&= Pr(y_1 - D_2^1 \leq t).Pr(D_1^1 \leq x_1).Pr(D_1^2 \leq x_2).Pr(D_2^2 \leq y_2) \text{ for } t \geq 0 \text{ and } t < y_1 \\
&= F_{D_1^1}(x_1) F_{D_1^2}(x_2) F_{D_2^2}(y_2) \int_{y_1 - t}^{\infty} f_{21}(v) dv
\end{aligned} \tag{3.8}$$

We start the process from the equation 3.6. Since $D_1^1 \leq x_1$, $(D_1^1 - x_1)^+ = 0$. Similarly, $(D_1^2 - x_2)^+ = 0$ and $(D_2^2 - y_2)^+ = 0$, because $D_1^2 \leq x_2$ and $D_2^2 \leq y_2$. This only leaves probability of $(y_1 - D_2^1 \leq t)$ alongside with three cumulative probability functions. Note that $f_{21}(v)$ refers to density function of D_2^1.

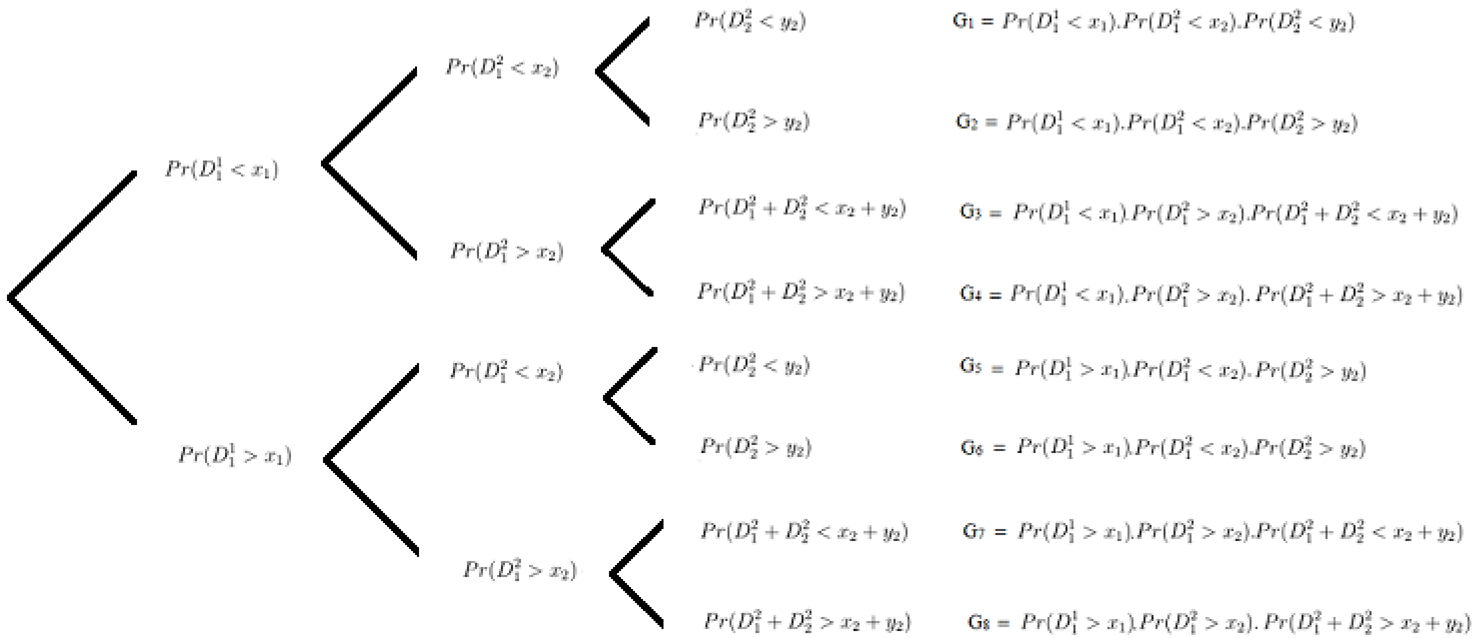

Figure 3.3. Probability Functions of 8 Different Scenarios that Cause y_1 to Outdate.

Remaining probability functions, G_2 to G_8 are defined analogously;

$$G_2 = F_{D_1^1}(x_1)F_{D_1^2}(x_2)[1 - F_{D_2^2}(y_2)] \int_0^\infty \int_{y_1-t-\alpha w+\alpha y_2}^\infty f_{22}(w)f_{21}(v)dvdw$$
$$G_3 = F_{D_1^1}(x_1)[1 - F_{D_1^2}(x_2)]F_{D_1^2+D_2^2}(x_2 + y_2) \int_{y_1-t}^\infty f_{21}(v)dv$$
$$G_4 = F_{D_1^1}(x_1)[1 - F_{D_1^2}(x_2)][1 - F_{D_1^2+D_2^2}(x_2 + y_2)] \int_0^\infty \int_0^\infty \int_{L(1)}^\infty f(y)dy$$
$$G_5 = [1 - F_{D_1^1}(x_1)]F_{D_1^2}(x_2)F_{D_2^2}(y_2) \int_0^\infty \int_{y_1-t-u+x_1}^\infty f_{11}(u)f_{21}(v)dvdu$$
$$G_6 = [1 - F_{D_1^1}(x_1)]F_{D_1^2}(x_2)[1 - F_{D_2^2}(y_2)] \int_0^\infty \int_0^\infty \int_{L(2)}^\infty f_{11}(u)f_{22}(w)f_{21}(v)dvdwdu$$
$$G_7 = [1 - F_{D_1^1}(x_1)][1 - F_{D_1^2}(x_2)]F_{D_1^2+D_2^2}(x_2 + y_2) \int_0^\infty \int_{y_1-t-u+x_1}^\infty f_{11}(u)f_{21}(v)dvdu$$
$$G_8 = [1 - F_{D_1^1}(x_1)][1 - F_{D_1^2}(x_2)][1 - F_{D_1^2+D_2^2}(x_2 + y_2)] \int_0^\infty \int_0^\infty \int_0^\infty \int_{L(3)}^\infty f(x)dx$$

where $L(1) = y_1 - t - \alpha w - \alpha z + \alpha x_2 + \alpha y_2$, $L(2) = y_1 - t - \alpha w + \alpha y_2 - u + x_1$, $L(3) = y_1-t-\alpha w-\alpha z+\alpha x_2+\alpha y_2-u+x_1$, $f(x)dx = f_{11}(u)f_{12}(z)f_{22}(w)f_{21}(v)dvdwdzdu$ and $f(y)dy = f_{12}(z)f_{22}(w)f_{21}(v)dvdwdz$.

Since each probability is independent, we can sum them up to find $G_{x_1,x_2,y_1,y_2}(t)$, which represents the distribution function of the random variable Z^1;

$$G_{x_1,x_2,y_1,y_2}(t) = G_1 + ... + G_8 \tag{3.9}$$

Expectation of distribution function $G_{x_1,x_2,y_1,y_2}(t)$ will give $\mathrm{E}[Z^1]$. Since Z^1 is a nonnegative random variable;

$$E[Z^1] = \int_0^\infty (1 - G_{x_1,x_2,y_1,y_2}(t))dt \tag{3.10}$$

Since $E[Z^1]$ is also $E[Outdates]$ and $E[Outdates]$ cannot be higher y_1 we have;

$$E[Outdates] = \int_0^{y_1} (1 - G_{x_1,x_2,y_1,y_2}(t))dt \tag{3.11}$$

This leads to the expected one-period cost function for player 1;

$$L_1(x_1, x_2, y_1, y_2) = p\int_{x_1+y_1}^{\infty} [t - (x_1 + y_1)]g(t)dt + \Theta\int_0^{y_1} (1 - G_{x_1,x_2,y_1,y_2}(t))dt \tag{3.12}$$

where p is the shortage cost and Θ is the outdate cost. It is also easy to show by numerical examples that $L_1(x_1, x_2, y_1, y_2)$ as well as $L_2(x_1, x_2, y_1, y_2)$ the expected one-period cost function for player 2 are convex. This was already proved by Parlar [7] as he showed that the expected profit function for single period model is concave. Following numerical values has been used to observe the convexity of $L_1(x_1, x_2, y_1, y_2)$ where $x_1 = 75$, $x_2 = 150$, $y_2 = 200$, $\alpha = 1$ and the demands are assumed to be distributed with continuous uniform distribution with the following values; $D_1^1 \sim U(50, 100)$, $D_2^1 \sim U(75, 125)$, $D_1^2 \sim U(125, 175)$, $D_2^2 \sim U(175, 225)$ as an illustration in Figure 3.4.

The response of player 1 is as follows:

$$R_1(x_1; x_2, y_2) = \arg\min_{y_1} L_1(x_1, x_2, y_1, y_2) \tag{3.13}$$

For player 2:

$$R_2(x_2; x_1, y_1) = \arg\min_{y_2} L_2(x_1, x_2, y_1, y_2) \tag{3.14}$$

Note that the responses are also functions of x_1 and x_2, one-year old inventory as well as the action of the competitor.

When both L_1 and L_2 are convex, if we can impose an arbitrary high bound y_1 and y_2 then using Debreu's theorem [14] we can argue that a Nash Equilibrium exists.

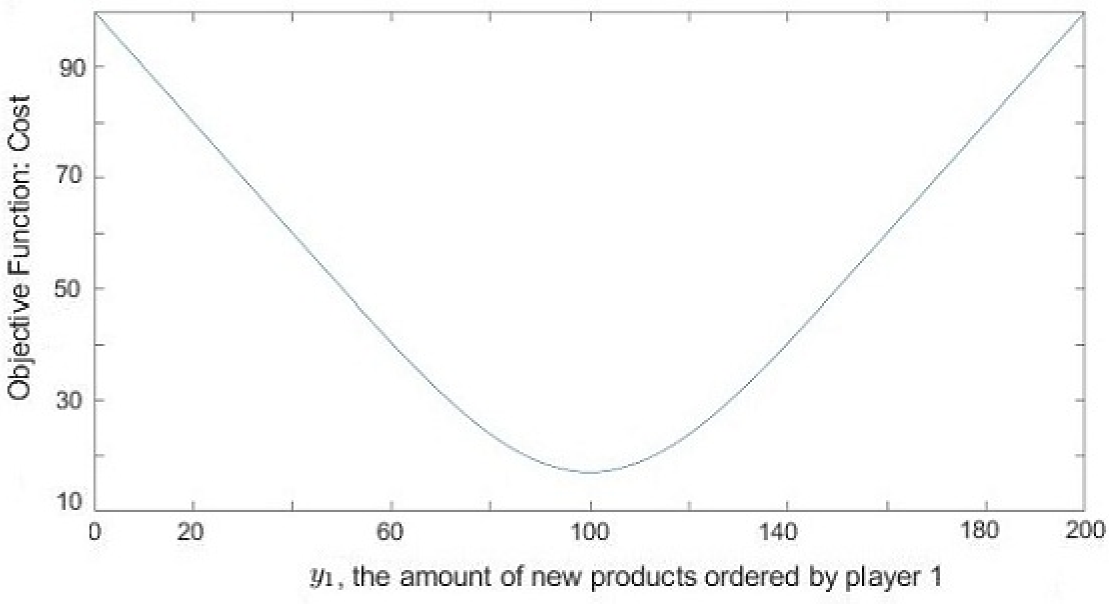

Figure 3.4. $L_1(x_1, x_2, y_1, y_2)$, Expected One-period Cost Function for Player 1 where $x_1 = 75$, $x_2 = 150$ and $y_2 = 200$.

4. COMPUTATIONAL RESULTS

In this chapter, we conduct a computational study to gain more insight into the problem. The computational study also strengthens the credibility of model by showing how the model works under different scenarios. The chapter consists of two sections to show the model can give reliable results with the demand functions that follows uniform and poisson distribution.

4.1. Uniform Distribution

First, in order to decrease the complexity of the problem and complete the calculations faster, the demands are assumed to be distributed with continuous uniform distribution with the following values; $D_1^1 \sim U(50,100)$, $D_2^1 \sim U(75,125)$, $D_1^2 \sim U(125,175)$, $D_2^2 \sim U(175,225)$. Uniform distribution is popular choice among newly introduced products since not enough data is available to work with. Uniform distribution is used in the literature to define these type of situation. In this case the product can be new medicine introduced to the pharmaceutical market. Furthermore, demand substitution multiplier α is assumed to be 1 to see effect of full demand substitution.

Table 4.1. The Experimental Setting for Uniform Distribution.

	x_1	x_2		p	Θ
Low	0	0	**Standard**	1	1
Medium	μ_{11}	μ_{12}	**Second**	10	1
High	$1.5\mu_{11}$	$1.5\mu_{12}$	**Third**	1	10

There are four sets of parameters that has an impact on the equilibrium result of the model. These are one period old products being brought into the new period, x_1 and x_2, as well as p, the shortage cost per unit and Θ, the overage cost per unit. An experimental setting is developed to test their influence over the equilibrium point.

x_1 and x_2 are given a value based on three settings; low, medium and high. Low level setting represents the case when both players are out of one period old products. ($x_1 = 0$ and $x_2 = 0$) Medium level setting represents the standard case when both players have one period old products with the mean value of their respective demand function. ($x_1 = \mu_{11}$ and $x_2 = \mu_{12}$) Lastly, high level setting represents when both players have excess amount of one period old products, which was expressed by one and half multiplier of the mean value of their respective demand function. ($x_1 = 1.5\mu_{11}$ and $x_2 = 1.5\mu_{12}$) Similarly, three settings are determined for the shortage and outdate costs. In the standard case, the shortage and outdate cost have the same weight. ($p = 1$ and $\Theta = 1$) In the second case, shortage is punished more with a 10 to 1 ratio between the shortage and outdate cost. ($p = 10$ and $\Theta = 1$) Finally, in the third case outdating is punished 10 times more than the shortage. ($p = 1$ and $\Theta = 10$) Table 4.1 illustrates the experimental setting.

Since there are 3 settings for both parameters x_1 and x_2; p and Θ, the experimental setting creates 9 unique scenarios using the combination of the parameters. Table 4.2 shows each unique scenario.

Table 4.2. 9 Unique Scenarios Based on the Experimental Setting for Uniform Distribution.

	x_1	x_2	p	Θ
Scenario 1.1	μ_{11}	μ_{12}	1	1
Scenario 1.2	μ_{11}	μ_{12}	10	1
Scenario 1.3	μ_{11}	μ_{12}	1	10
Scenario 2.1	0	0	1	1
Scenario 2.2	0	0	10	1
Scenario 2.3	0	0	1	10
Scenario 3.1	$1.5\mu_{11}$	$1.5\mu_{12}$	1	1
Scenario 3.2	$1.5\mu_{11}$	$1.5\mu_{12}$	10	1
Scenario 3.3	$1.5\mu_{11}$	$1.5\mu_{12}$	1	10

Starting from $y_2 = 0$, the response function of player 1 (cf. eq. 3.13) is calculated with an "increment of 5" on y_2 until the response function of player 1 stabilizes and do not respond significantly to the increase in y_2. Same procedure is applied to the response function of player 2. (cf. eq. 3.14) Following the response functions over y_1-y_2 plane reveals the equilibrium point of the system when the response functions intersect. Equilibrium point shows the amount of new products should be ordered by each player that minimizes total cost of the system. Figures 4.1 to 4.9 display the response functions and equilibrium points of Scenarios from 1.1 to 3.3. All calculations are performed by using Matlab R2018a.

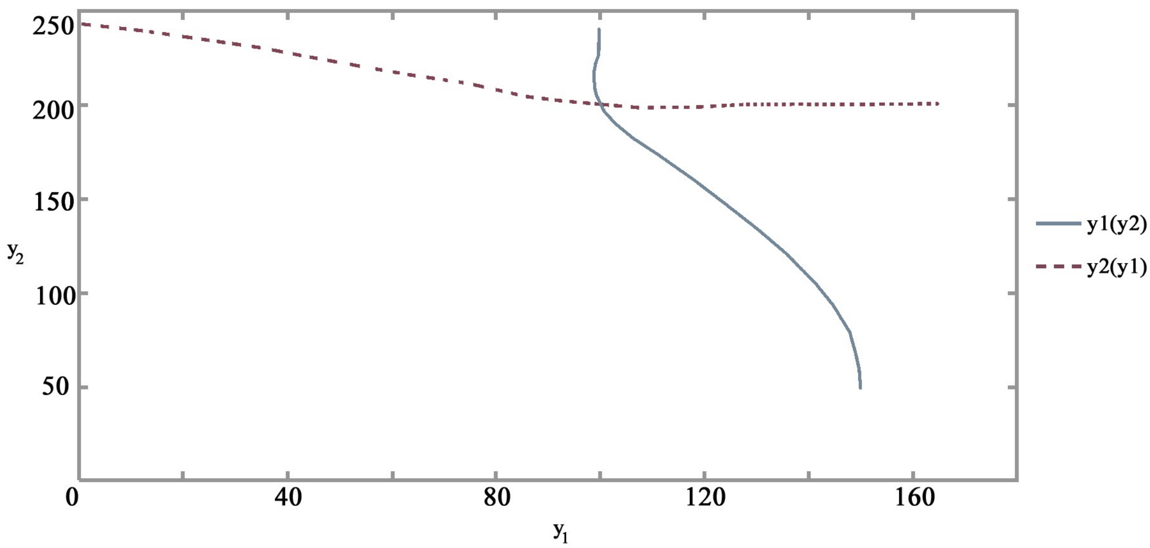

Figure 4.1. Response Functions of Scenario 1.1 ($x_1 = 75$, $x_2 = 150$, $p = 1$, $\Theta = 1$). The equilibrium order levels are $(y_1, y_2) = (100, 200)$.

Figure 4.1 shows the standard case that occurs in Scenario 1.1. In the standard case initial inventory levels of players are in the 'Medium' setting, which means the amount of one period old products being brought to the new period is equal to the mean value of their respective demand function. ($x_1 = \mu_{11} = 75$ and $x_2 = \mu_{12} = 150$) Moreover, the shortage cost is same with the outdate cost. ($p = \Theta = 1$) In Scenario 1.2, the shortage cost is now 10 times higher than the outdate cost. Therefore, both players react to this change by increasing the amount of new products they order. As it can clearly be seen from Figure 4.2, the equilibrium point shifts from $(y_1, y_2) = (100, 200)$

to $(y_1, y_2) = (128, 228)$; because players do not want to get short of products. Players are willing to take the risk of outdating by ordering more new products despite the number of old products available, x_1 and x_2 are constant in both Scenario 1.1 and 1.2. This behaviour arises because shortage is very costly. Table 4.3 also justifies the behaviour with numerical data as $E_1[Outdates]$ and $E_2[Outdates]$ increase from 8.68 to 29.70.

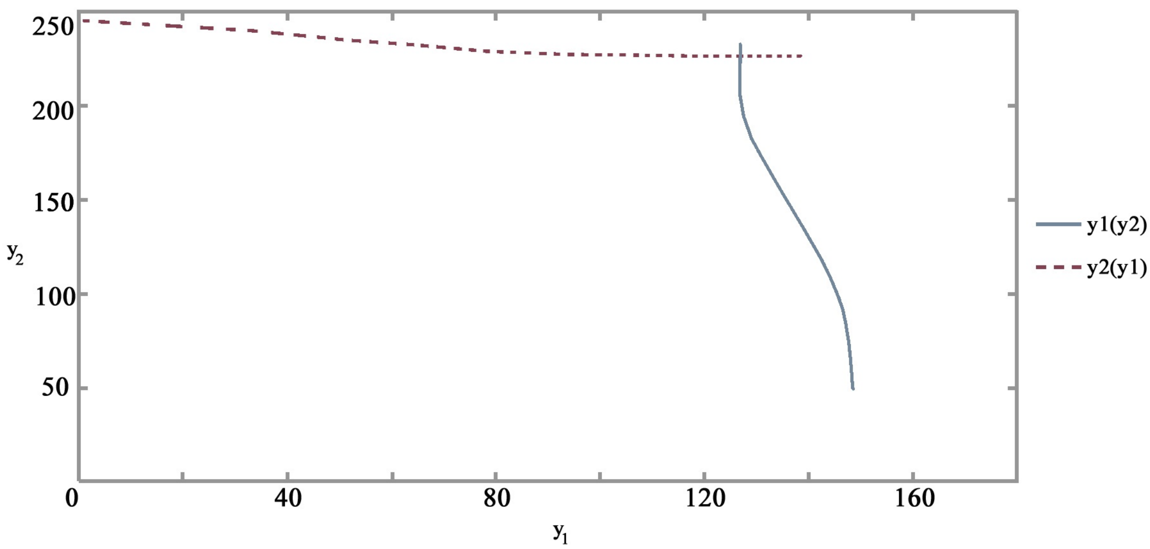

Figure 4.2. Response Functions of Scenario 1.2 ($x_1 = 75$, $x_2 = 150$, $p = 10$, $\Theta = 1$). The equilibrium order levels are $(y_1, y_2) = (128, 228)$.

In Scenario 1.3, the roles are now reversed and the outdate cost is 10 times higher than the shortage cost. Since outdating is very costly, both players decrease the amount of products they order. This way, players decrease the chance of outdating. Even though they will almost certain be short of products, it is a much better option than getting outdated. The equilibrium point reflects this behaviour as it shifts down to $(y_1, y_2) = (81.2, 181)$. This equilibrium point is lower than Scenario 1.1 and 1.2, because outdating is punished much more harshly in Scenario 1.3 than the first two. Table 4.3 demonstrates how much players fear of getting outdated as $E_1[Outdates]$ and $E_2[Outdates]$ decrease from 8.68 to 0.73 compared to standard case, Scenario 1.1. It corresponds to over 92% decrease in $E_1[Outdates]$ and $E_2[Outdates]$.

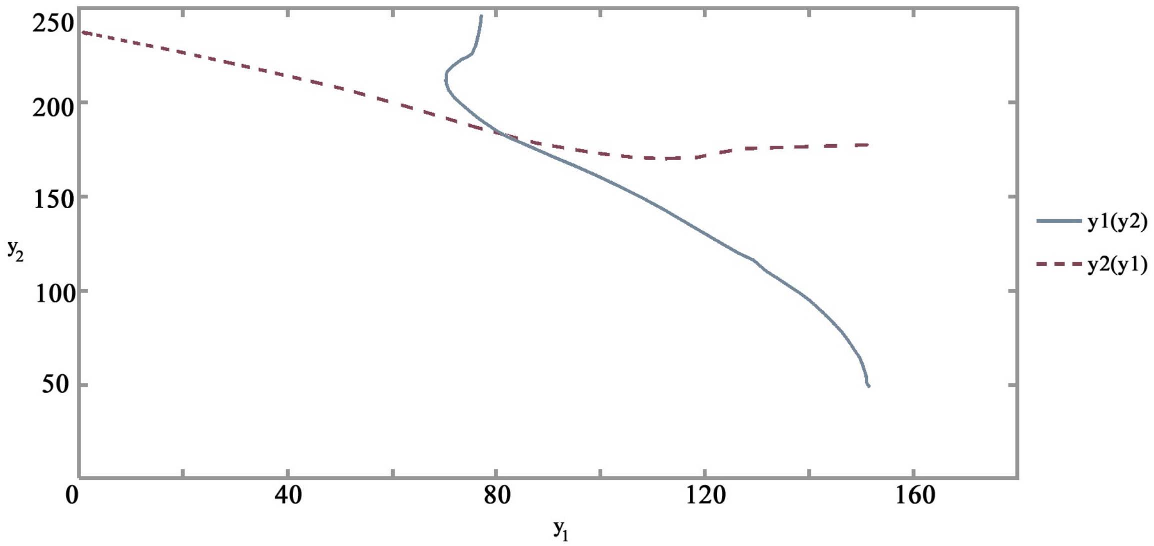

Figure 4.3. Response Functions of Scenario 1.3 ($x_1 = 75$, $x_2 = 150$, $p = 1$, $\Theta = 10$). The equilibrium order levels are $(y_1, y_2) = (81.2, 181)$.

In the second series of scenarios, the setting for one period old products is changed from 'Medium' to 'Low' and now we analyse the behaviour of players when they do not have any initial inventory to start with. ($x_1 = x_2 = 0$) In Scenario 2.1 players respond to this change by significantly increasing their ordering quantities compared to Scenario 1.1. As a result, the equilibrium point increases to $(y_1, y_2) = (175, 350)$, which can be observed in Figure 4.4. The reason for this change is that both players do not have any old products to cover their first period demand. Therefore, they increase the number of new products they order to match with the unsatisfied demand from period 1. Despite increasing the total amount of new products ordered by 75%, $E_1[Outdates]$ and $E_2[Outdates]$ are only increased by 13.2% and total expected cost by 7.7%.

In Scenario 2.2, shortage to outdate cost ratio is set to 10 whereas players still do not have any initial inventory. ($p = 10$, $\Theta = 1$) As it can be expected from the previous case, the equilibrium point significantly increases from $(y_1, y_2) = (128, 228)$ to $(y_1, y_2) = (204, 379)$ compared to Scenario 1.2, which shares the same setting for the shortage and outdate costs. The equilibrium point also increases compared to

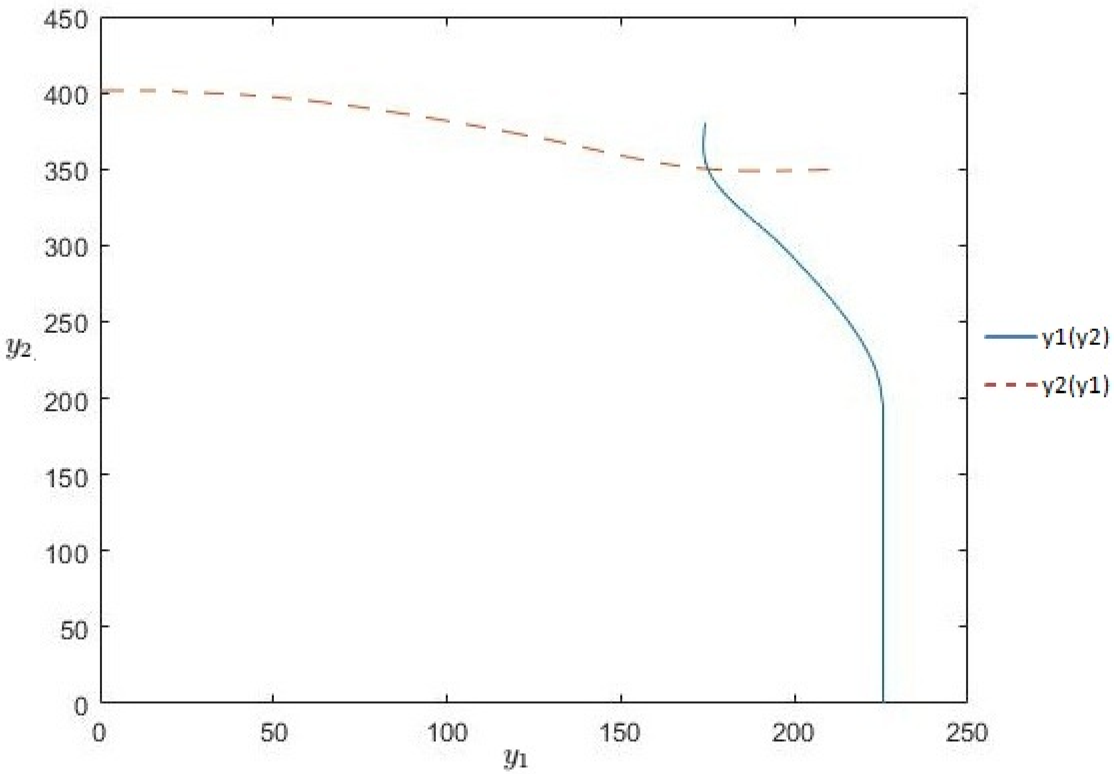

Figure 4.4. Response Functions of Scenario 2.1 ($x_1 = 0$, $x_2 = 0$, $p = 1$, $\Theta = 1$). The equilibrium order levels are $(y_1, y_2) = (175, 350)$.

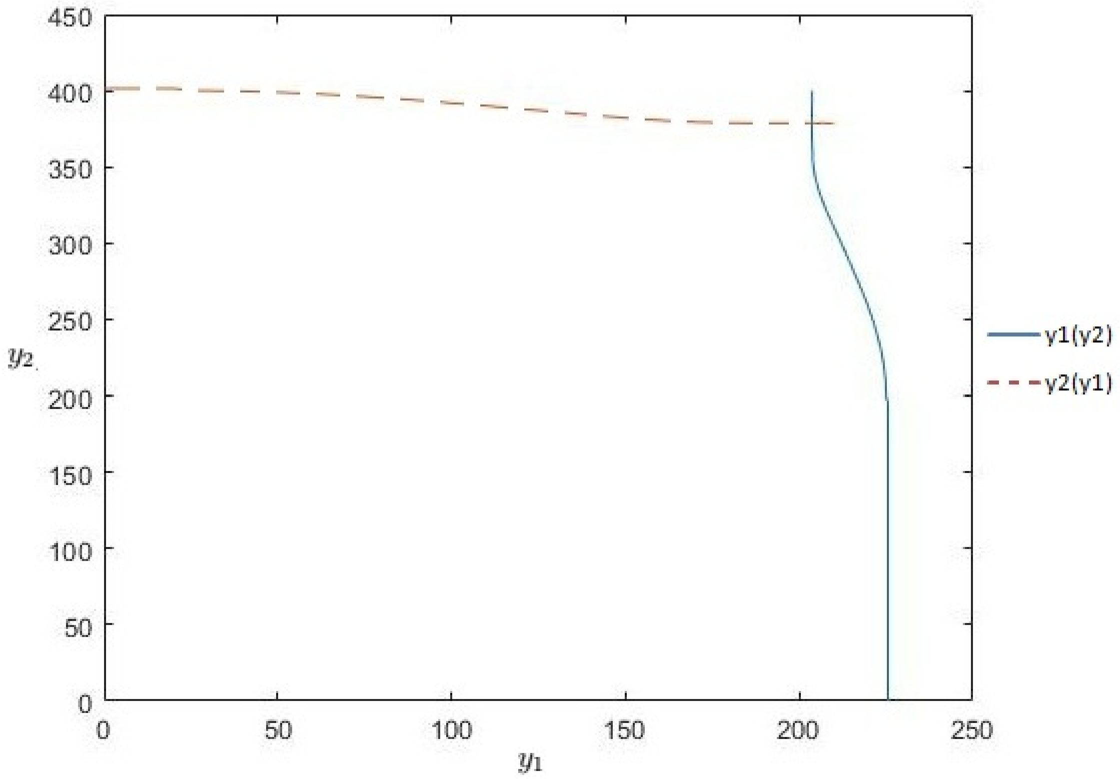

Figure 4.5. Response Functions of Scenario 2.2 ($x_1 = 0$, $x_2 = 0$, $p = 10$, $\Theta = 1$). The equilibrium order levels are $(y_1, y_2) = (204, 379)$.

Scenario 2.1 as players do not want to runout of products. Therefore, they order more inventory to be safe from high shortage costs. Figure 4.5 shows the response functions and equilibrium point for Scenario 2.2.

Scenario 2.3, which is displayed in Figure 4.6, shows expected behaviour from players. Just like any second series of scenarios, equilibrium point shifts up significantly compared to Scenario 1.3; where both scenarios have 10 times higher outdate cost than the shortage one. Compared to Scenarios 2.1 and 2.2, equilibrium decreases though, since players do not want to have outdated products at the end of the period because of high outdate cost.

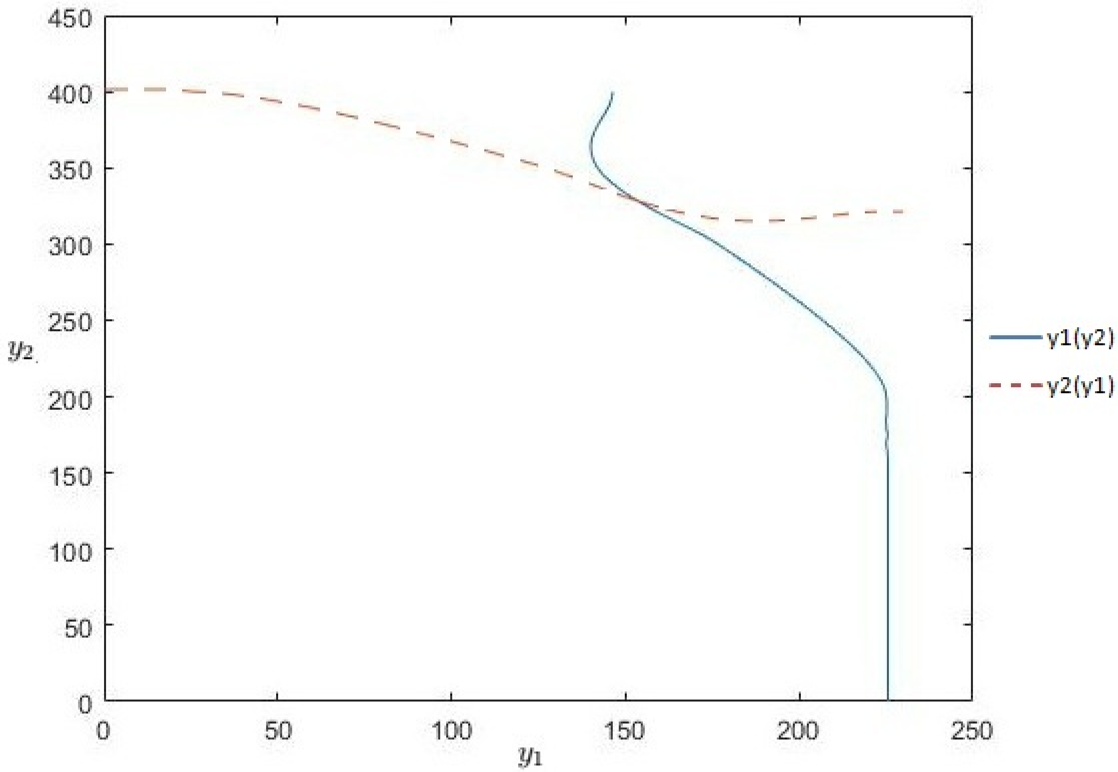

Figure 4.6. Response Functions of Scenario 2.3 ($x_1 = 0$, $x_2 = 0$, $p = 1$, $\Theta = 10$). The equilibrium order levels are $(y_1, y_2) = (153, 328)$.

One particular aspect about Scenario 2's that stand outs is how response functions are smoother than Scenario 1's. This is because initial inventory of both players is zero in Scenario 2's. That means they have unsatisfied demand from period 1 every single case. On the other hand, in Scenario 1's the initial inventory is equal to the mean of demand functions. This means that sometimes players have unsatisfied demand and sometimes not, if the demand is actually smaller than expected. Having an initial inventory creates extra complexity to the problem and add variance on the decision making even though x_1 and x_2 are only parameters, not decision variables. However, the picture is clearer in Scenario 2's because players know that unsatisfied demand from period 1 is coming without any uncertainty. At first glance, having initial inventory is perceived as a bonus, because it decreases the number of new products to order and therefore also reduces the cost which was also shown in Table 4.3 under L_1 and L_2. (Excepted cost functions are always higher in Scenario 2's compared to Scenario 1's.) However, it also makes the decision making harder for the player.

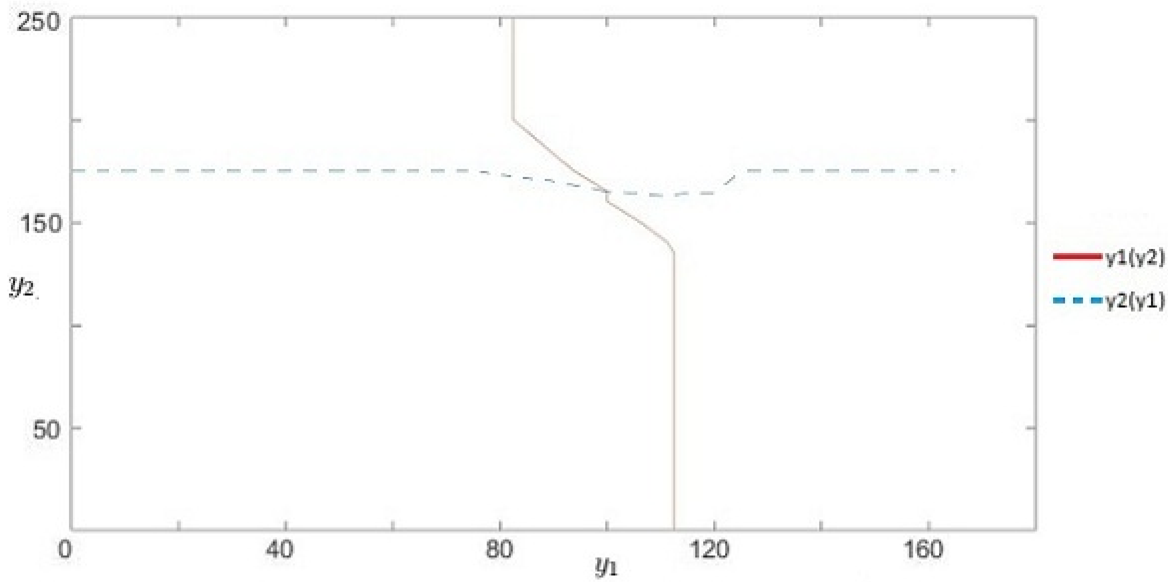

Figure 4.7. Response Functions of Scenario 3.1 ($x_1 = 112.5$, $x_2 = 225$, $p = 1$, $\Theta = 1$). The equilibrium order levels are $(y_1, y_2) = (100, 165)$.

In Scenario 3.1, the amount of old products that are brought into the new period is increased. ($x_1 = 112.5$, $x_2 = 225$) Compared to Scenario 1.1 and 2.1, equilibrium shifts down to ($y_1 = 100$, $y_2 = 165$) as players start with more initial inventory, therefore they afford to order less for the new period. Response functions and the equilibrium point is displayed in Figure 4.7.

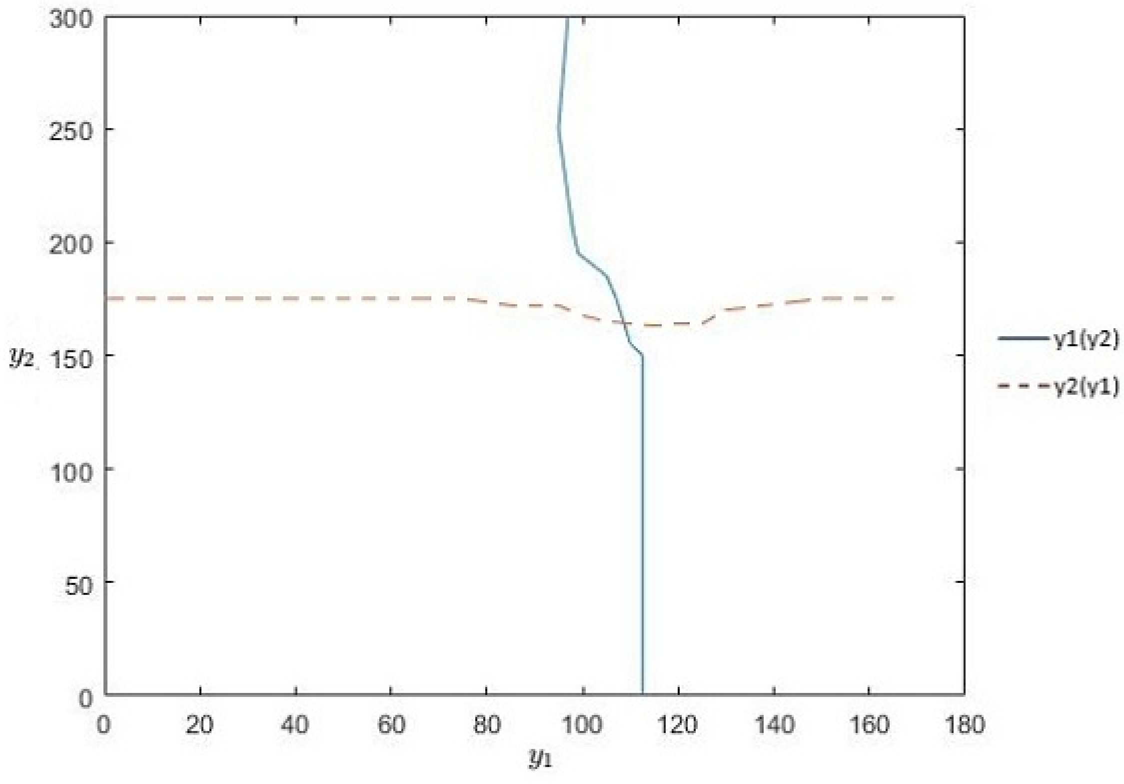

Figure 4.8. Response Functions of Scenario 3.2 ($x_1 = 112.5$, $x_2 = 225$, $p = 10$, $\Theta = 1$). The equilibrium order levels are $(y_1, y_2) = (109, 164)$.

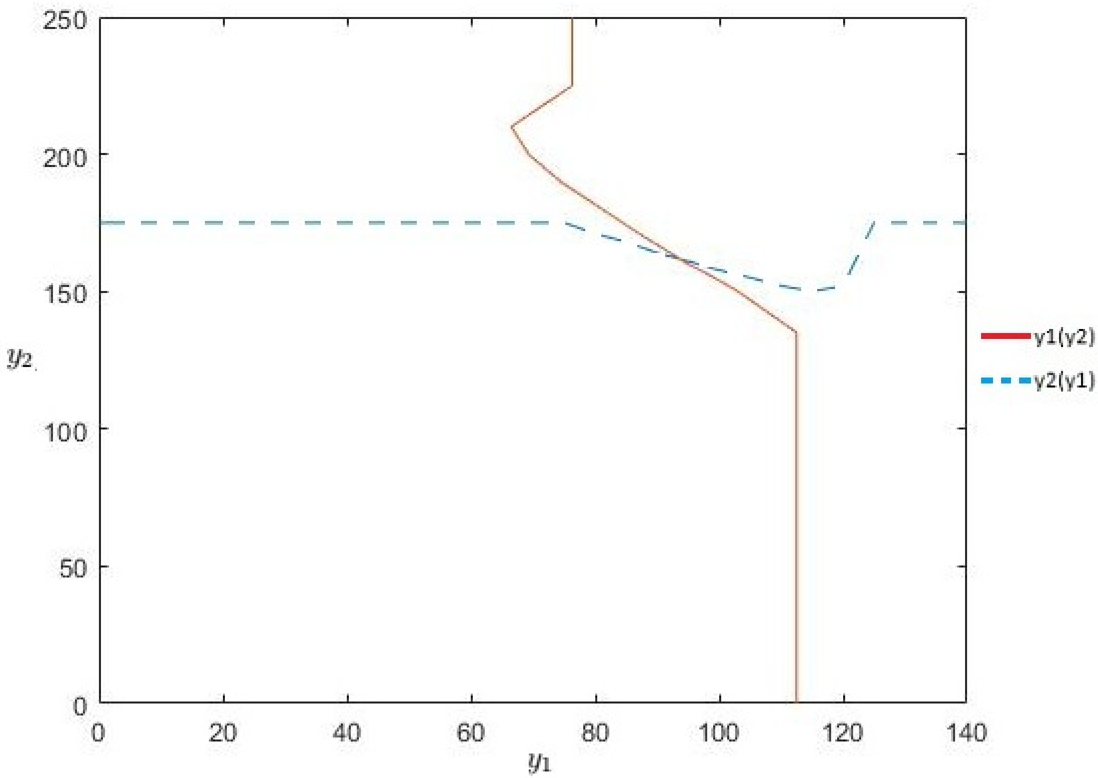

Figure 4.9. Response Functions of Scenario 3.3 ($x_1 = 112.5$, $x_2 = 225$, $p = 1$, $\Theta = 10$). The equilibrium order levels are $(y_1, y_2) = (93.7, 162)$.

Scenario 3.2 has an equilibrium point at ($y_1 = 109$, $y_2 = 164$). It is displayed in Figure 4.8. Equilibrium point is higher than Scenario 3.1 but lower than Scenario 2.2. Scenario 3.3 repeats the previous findings as the equilibrium point is the lowest among Scenario 3.1 and 3.2 since outdating is ten times costlier than shortage, so players order less. The equilibrium point of Scenario 3.3 is at $(y_1, y_2) = (93.7, 162)$, which can be observed from Figure 4.9.

Table 4.3 also exhibits that the expected cost is significantly lower if the shortage and outdate costs are the same at a level of one. When one cost becomes dominant and ten times more than the other, the expected costs increase as players balance their expenses by increasing either $E[Outdates]$ or $E[Runouts]$ depending on the scenario. One interesting comparison can be made between expected cost of Scenarios 1.2/2.2 and Scenarios 1.3/2.3. Excepted cost is higher in the equilibrium point when shortage cost is the dominant factor in the market. L_1 of Scenario 1.2 is 36.80 compared to 28.17 L_1 of Scenario 1.3. Demand substitution provides an explanation for this numerical difference. When the shortage cost is dominant, players buy more products not to end up in runout. Since they have more products than the expected demand, an environment for demand substitution does not develop and simply excess products outdate. However, when the outdate cost is dominant, players now buy less products. Because of it, some customers fail to find their products in the first store. Demand substitution occurs and they visit the next store. This way, second store now gains a second chance to sell its products, which would otherwise outdate. Therefore, we can argue that a market under demand substitution have a lower expected cost compared to the one with no demand substitution.

Figure 4.10 demonstrates summary of this chapter and shows how equilibrium points behave in different scenarios. Initial inventory level is inversely proportionate with the equilibrium. Scenario 2's gather around on the right top of the graph, because they have zero initial inventory. On the other hand, Scenario 3's assemble on the bottom of the graph as they have high level of initial inventory. Therefore, players' order quantities in the equilibrium decrease. Actually it was expected from Scenario 3's to gather around on the low bottom of the graph, however it does not happen,

Table 4.3. Equilibrium Points, Expected Outdates and Expected Costs of All 9 Scenarios for Uniform Distribution.

	y_1	y_2	$E_1[Outdates]$	$E_2[Outdates]$	L_1	L_2
Scenario 1.1	100	200	8.68	8.68	17.02	17.02
Scenario 1.2	128	228	29.70	29.70	36.80	36.80
Scenario 1.3	81.2	181	0.73	0.73	28.17	28.30
Scenario 2.1	175	350	10	10	18.33	18.33
Scenario 2.2	204	379	32.14	32.14	38.31	38.31
Scenario 2.3	153	328	0.85	0.85	31.95	31.95
Scenario 3.1	100	165	0.22	0.11	0.35	0.18
Scenario 3.2	109	164	0.81	0.25	0.84	1.15
Scenario 3.3	93.7	162	0.01	0.01	0.57	0.22

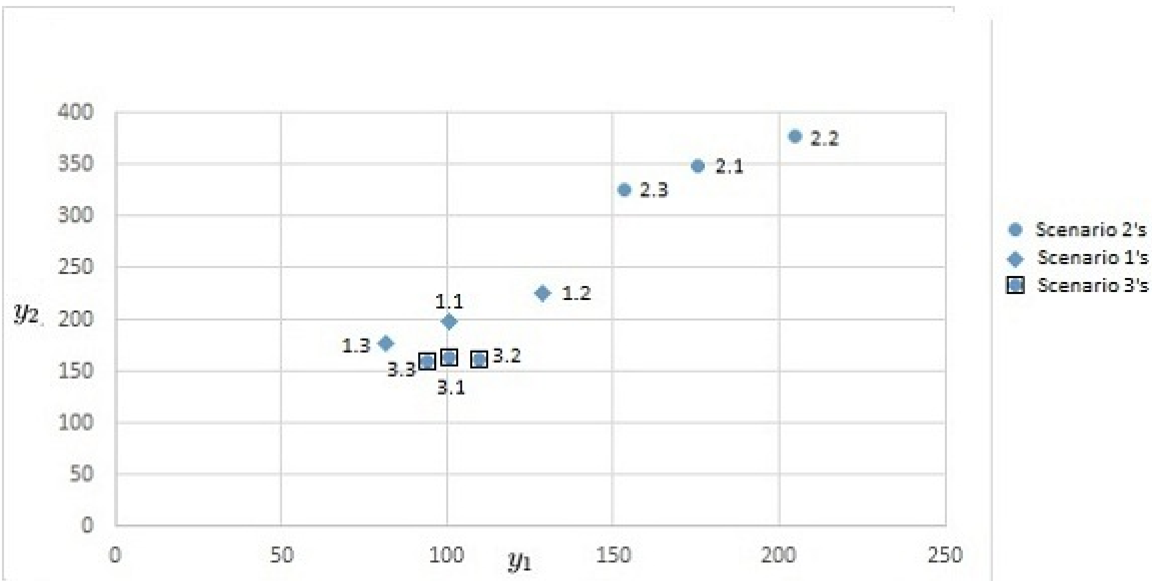

Figure 4.10. The Equilibrium Order Levels of All 9 Scenarios for Uniform Distribution.

because after a point the initial inventory level does not impact y_1 and y_2. It does not change if the initial inventory levels are very high, since after satisfying first period demand, they outdates and cannot be used to satisfy second period demand. Lastly, Scenario 1's stay in the middle as their initial inventory levels are between values of Scenario 2's and 3's.

Similar interpretation can be made from shortage and outdate cost scenarios. Scenarios with higher shortage cost setting are displayed on top right side of other scenarios that they share the initial inventory levels. In the higher outdate cost setting, equilibrium points stay on the left bottom side compared to other scenarios that have the same initial inventory level. When the shortage and outdate costs are the same and equal to one, equilibrium points stand between the dominant shortage and dominant outdate cost scenarios.

One question that arises after reviewing the results of scenarios is how can most of $E_1[Outdates]$ and L_1 be equal to $E_2[Outdates]$ and L_2. These results shows that even though the second player has more demand, it gets same expected outdates as well as same objective function value. In order to test this case, demand function of player 2 for period 2 is slightly adjusted. Rather than following $D_2^2 \sim U(175, 225)$, now it is changed to $D_2^2 \sim U(150, 250)$, which gives wider minimum-maximum range and higher variance since the demand is larger. The results are listed under Scenario 4.1 in Table 4.4.

Table 4.4. Comparison of Standard Case and High Variation Scenario.

	D_2^2	y_1	y_2	$E_1[Outdates]$	$E_2[Outdates]$	L_1	L_2
Scenario 1.1	175-225	100	200	8.68	8.68	17.02	17.02
Scenario 4.1	150-250	100	199.4	10.83	24.54	19.17	37.04

$E_1[Outdates]$ and L_1 slightly increased in Scenario 4.1 since the variation of second player's demand is larger. It has negative impact on player 1; because of the demand substitution and now player 1 gets more unexpected demand from player

2, which eventually increases its cost. However, the real difference can be observed in player 2's indicators. Since player 2's demand has larger minimum and maximum range, $E_2[Outdates]$ and L_2 increases more than 100% in Scenario 4.1 compared to Scenario 1.1. More importantly, $E_1[Outdates]$ and L_1 no longer match with $E_2[Outdates]$ and L_2. These two terms were same in the first case because actually pdf of player 1 and 2's second period demands are both 1/50, which makes function practically the same.

$$pdf of D_1^2 \text{ and } D_2^2 = \frac{1}{b-a} = \frac{1}{125-75} = \frac{1}{225-175} = \frac{1}{50}$$

$$pdf of new D_2^2 = \frac{1}{b-a} = \frac{1}{250-150} = \frac{1}{100}$$

Now pdf of player 2's second player demand has changed and it becomes 1/100. For this reason, $E_1[Outdates]$ and L_1 are not equal to $E_2[Outdates]$ and L_2; but smaller as expected.

4.2. Poisson Distribution

Poisson distribution is another way to define the demand functions, which can be implemented for very highly priced medicine like cancer drugs. Since these medicine are very expensive, pharmacies buy in small quantities just to satisfy the demand of their own customers. Calculations from uniform distribution have shown that running time for the model for a single product can extend up to 2-3 days. This means that the model cannot be used every single product yet, however it can be justified to run the model for very highly priced medicine because of their astronomic price and cost attached to it. Also, the running time for the model with poisson distribution decreases to roughly 1 hour, which makes viable to use the model. The demands are assumed to be distributed with poisson distribution with the respective λ values; $D_1^1 \sim Poisson(\lambda_1^1 = 3)$, $D_2^1 \sim Poisson(\lambda_2^1 = 4)$, $D_1^2 \sim Poisson(\lambda_1^2 = 6)$, $D_2^2 \sim P(\lambda_2^2 = 8)$.

Table 4.5. The Experimental Setting for Poisson Distribution.

	x_1	x_2		p	Θ
Low	0	0	**Standard**	1	1
Medium	μ_{11}	μ_{12}	**Second**	10	1
High	$2\mu_{11}$	$2\mu_{12}$	**Third**	1	10

Since poisson distribution is a discrete probability distribution, experimental setting is slightly adjusted. In the high level setting initial inventory levels are represented with two multiplier of the λ values instead of one and a half multiplier value. ($x_1 = 2\lambda_1^1$ and $x_2 = 2\lambda_1^2$) Remaining experimental setting stays the same and it can be reviewed from Table 4.5. Furthermore, demand substitution multiplier α is assumed to be 1.

Similarly to the uniform distribution case, since they are 3 settings for both paraments, the experimental setting creates 9 unique scenarios, which can be seen from Table 4.6.

Starting from $y_1 = 0$ and $y_2 = 0$, the objective function of player 1 (cf. eq. 3.12) is calculated with an "increment of 1" on y_1. Because of the convexity of L_1, optimal y_1 that minimizes the objective function for the given y_2 can be found by comparing the objective function values for adjacent y_1 values. ($y_1 - 1$ and $y_1 + 1$ for this case) If the objective function value of given y_1 and y_2 is better than objective function value of adjacent/neighbour points, then we can conclude that y_1 is the optimal ordering quantity for given y_2. Same procedure is applied again after y_2 is increased by 1 and continued until y_2 is a large number. The method is different than the method that we have used in uniform distribution since poisson distribution is discrete, therefore y_1 and y_2 should also be integers. This method is also applied for player 2 under its own objective function L_2 to find player 2's response function. We have now created

Table 4.6. 9 Unique Scenarios Based on the Experimental Setting for Poisson Distribution.

	x_1	x_2	p	Θ
Scenario 1	μ_{11}	μ_{12}	1	1
Scenario 2	μ_{11}	μ_{12}	10	1
Scenario 3	μ_{11}	μ_{12}	1	10
Scenario 4	0	0	1	1
Scenario 5	0	0	10	1
Scenario 6	0	0	1	10
Scenario 7	$2\mu_{11}$	$2\mu_{12}$	1	1
Scenario 8	$2\mu_{11}$	$2\mu_{12}$	10	1
Scenario 9	$2\mu_{11}$	$2\mu_{12}$	1	10

two response functions and following the response functions over y_1-y_2 plane reveals the equilibrium point of the system when the response functions intersect. Just like uniform distribution, equilibrium point shows the amount of new products should be ordered by each player that minimizes total cost of the system. However, for this case equilibrium points are always integer values. All calculations for the poisson distribution are also performed by using Matlab R2018a. Figures from 4.11 to 4.19 show scenarios from 1 to 9.

Just like Scenario 1.3 from Figure 4.3, similar response function behaviour can be observed in Scenario 3, which is demonstrated in Figure 4.13. After the equilibrium point, optimal y_1 goes even deeper and reduces to 1. However after y_2 increases, optimal y_1 stabilized at 2 by creating like a spoon movement.

One special case is observed in Figure 4.19, where response functions intersect in a line and therefore creating two equilibrium points at $(y_1, y_2) = (2, 4)$ and $(y_1, y_2) = (1, 5)$. Further analysis exposes that player 1 prefers the equilibrium point

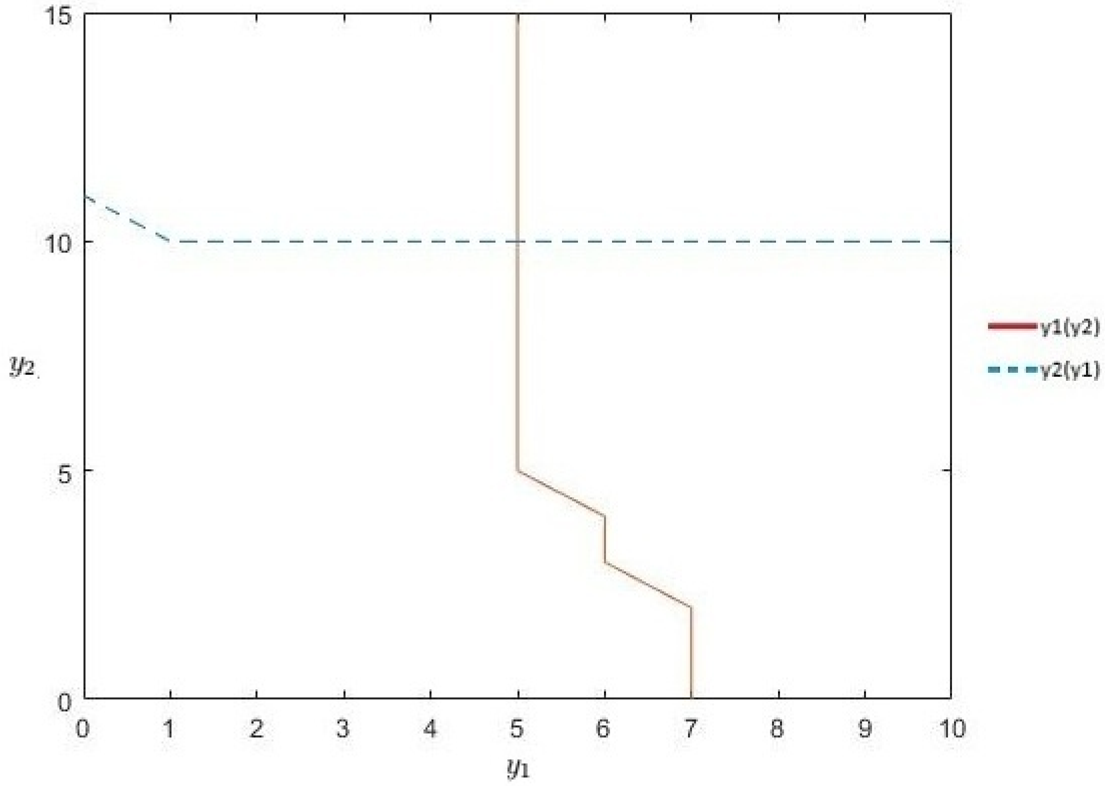

Figure 4.11. Response Functions of Scenario 1 ($x_1 = 3$, $x_2 = 6$, $p = 1$, $\Theta = 1$). The equilibrium order levels are $(y_1, y_2) = (5, 10)$.

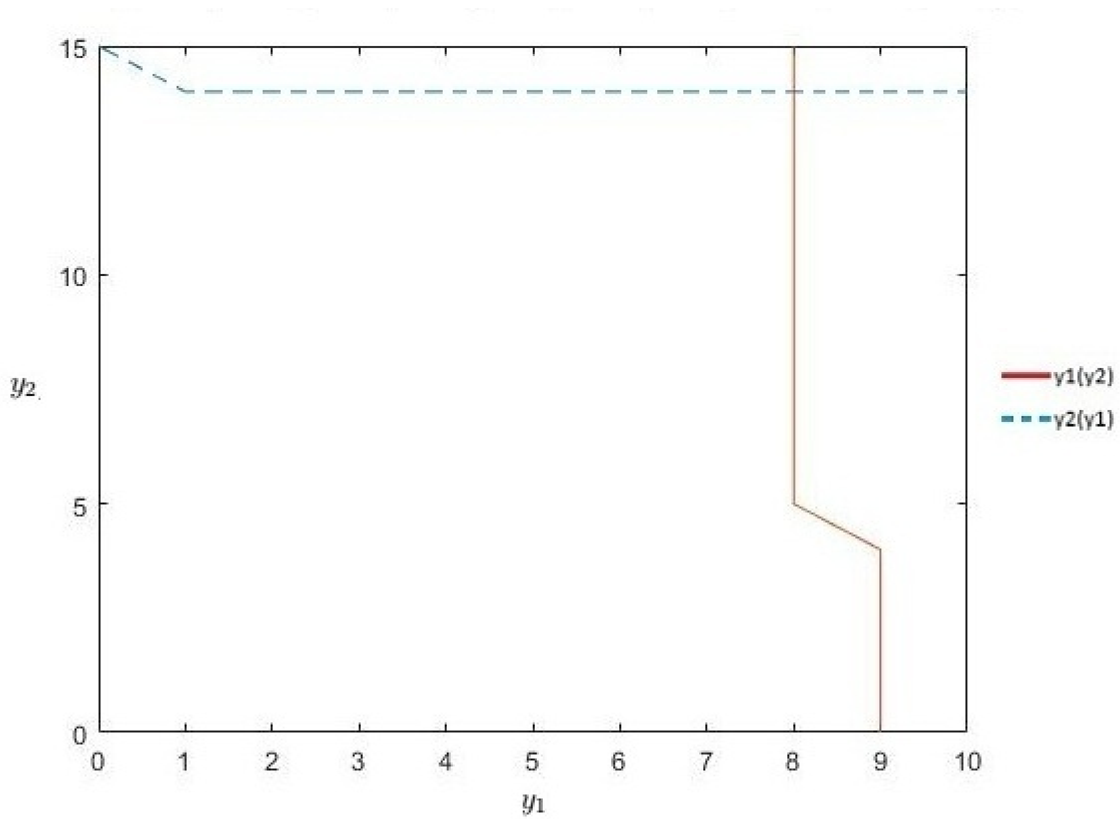

Figure 4.12. Response Functions of Scenario 2 ($x_1 = 3$, $x_2 = 6$, $p = 10$, $\Theta = 1$). The equilibrium order levels are $(y_1, y_2) = (8, 14)$.

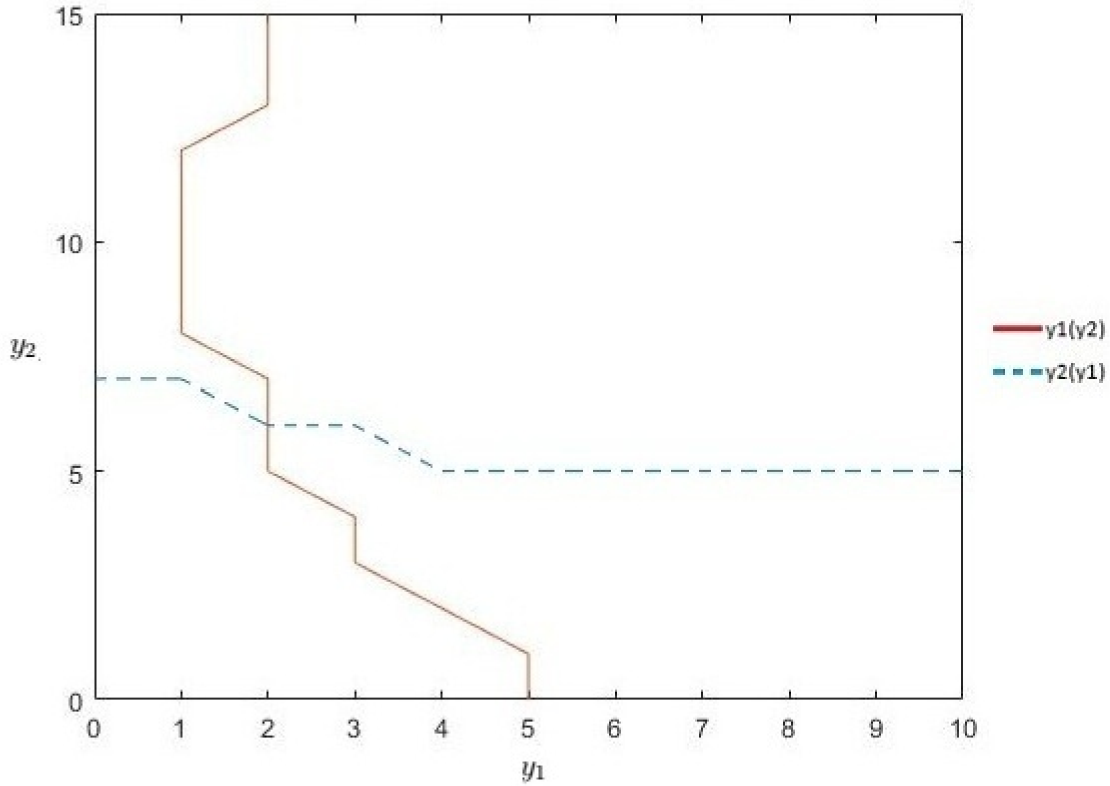

Figure 4.13. Response Functions of Scenario 3 ($x_1 = 3$, $x_2 = 6$, $p = 1$, $\Theta = 10$). The equilibrium order levels are $(y_1, y_2) = (2, 6)$.

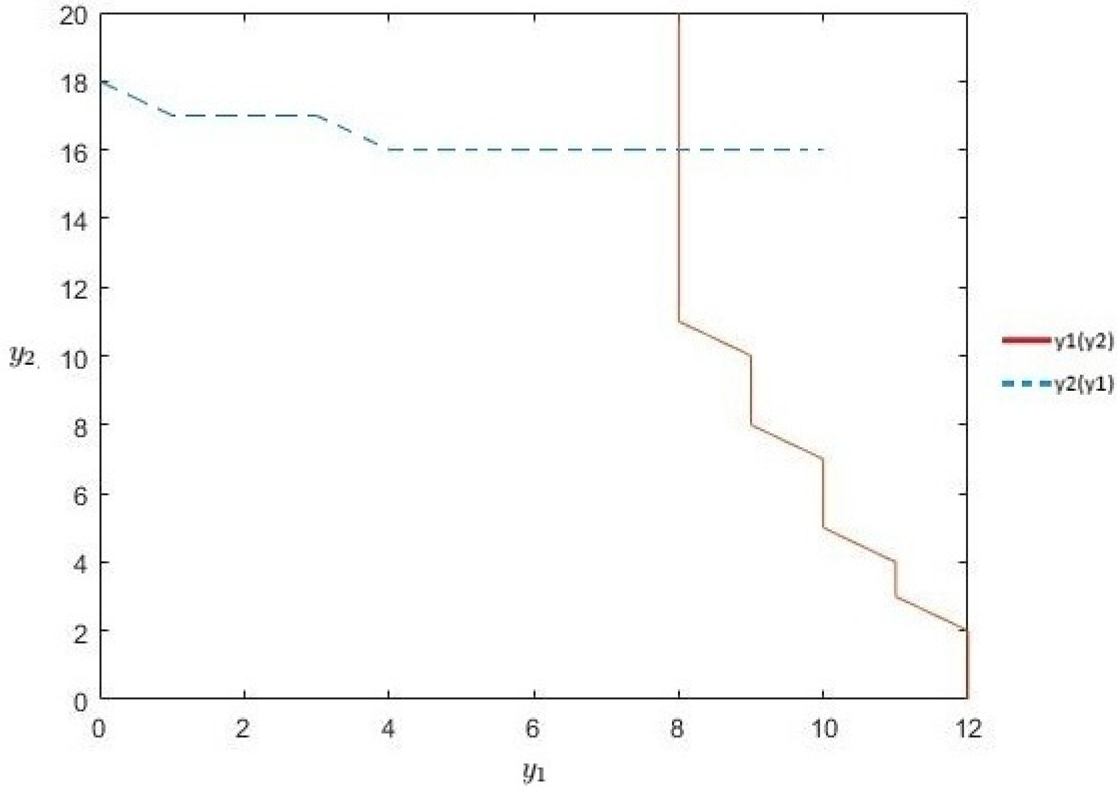

Figure 4.14. Response Functions of Scenario 4 ($x_1 = 0$, $x_2 = 0$, $p = 1$, $\Theta = 1$). The equilibrium order levels are $(y_1, y_2) = (8, 16)$.

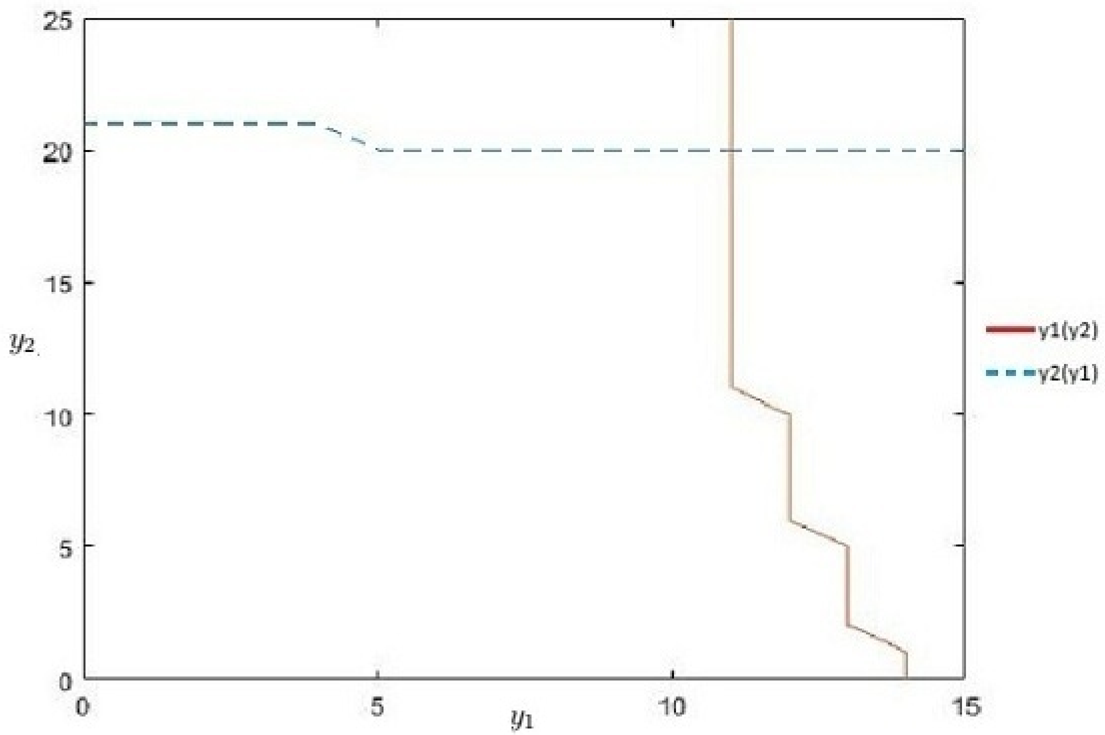

Figure 4.15. Response Functions of Scenario 5 ($x_1 = 0$, $x_2 = 0$, $p = 10$, $\Theta = 1$). The equilibrium order levels are $(y_1, y_2) = (11, 20)$.

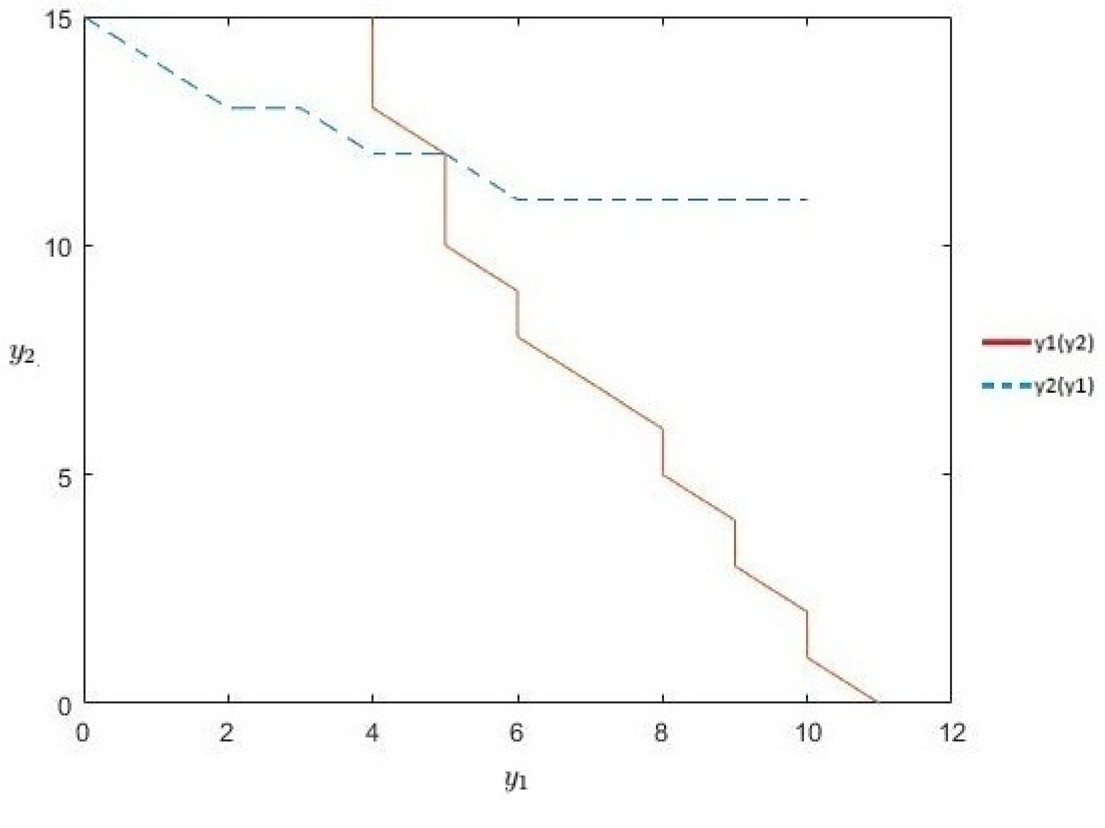

Figure 4.16. Response Functions of Scenario 6 ($x_1 = 0$, $x_2 = 0$, $p = 1$, $\Theta = 10$). The equilibrium order levels are $(y_1, y_2) = (5, 12)$.

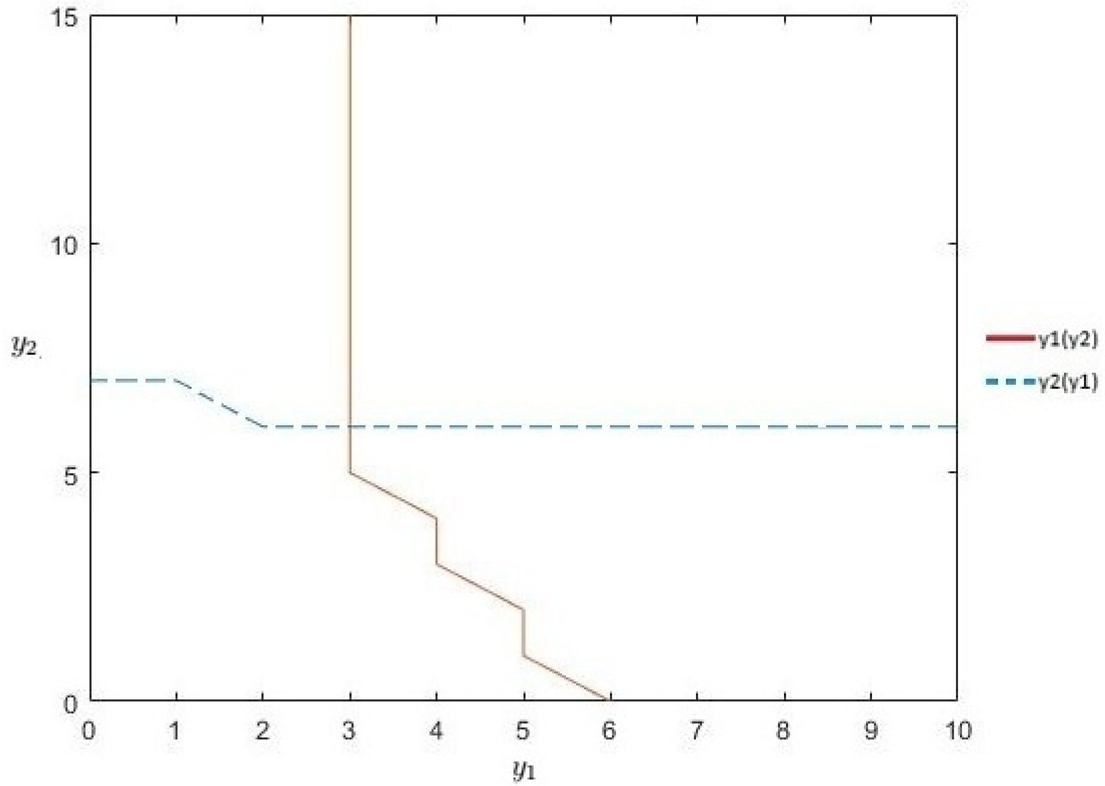

Figure 4.17. Response Functions of Scenario 7 ($x_1 = 6$, $x_2 = 12$, $p = 1$, $\Theta = 1$). The equilibrium order levels are $(y_1, y_2) = (3, 6)$.

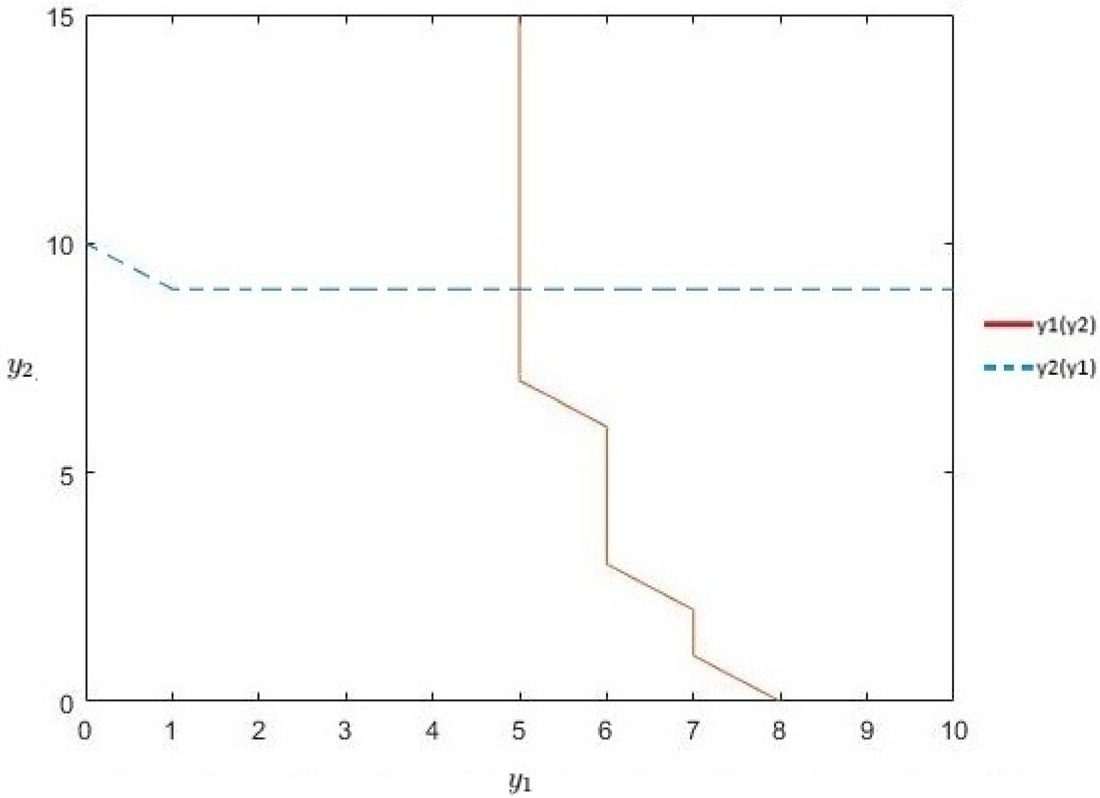

Figure 4.18. Response Functions of Scenario 8 ($x_1 = 6$, $x_2 = 12$, $p = 10$, $\Theta = 1$). The equilibrium order levels are $(y_1, y_2) = (5, 9)$.

$(2, 4)$ compared to player 2 that prefers the equilibrium point $(1, 5)$ because of lower total expected cost, L_2.

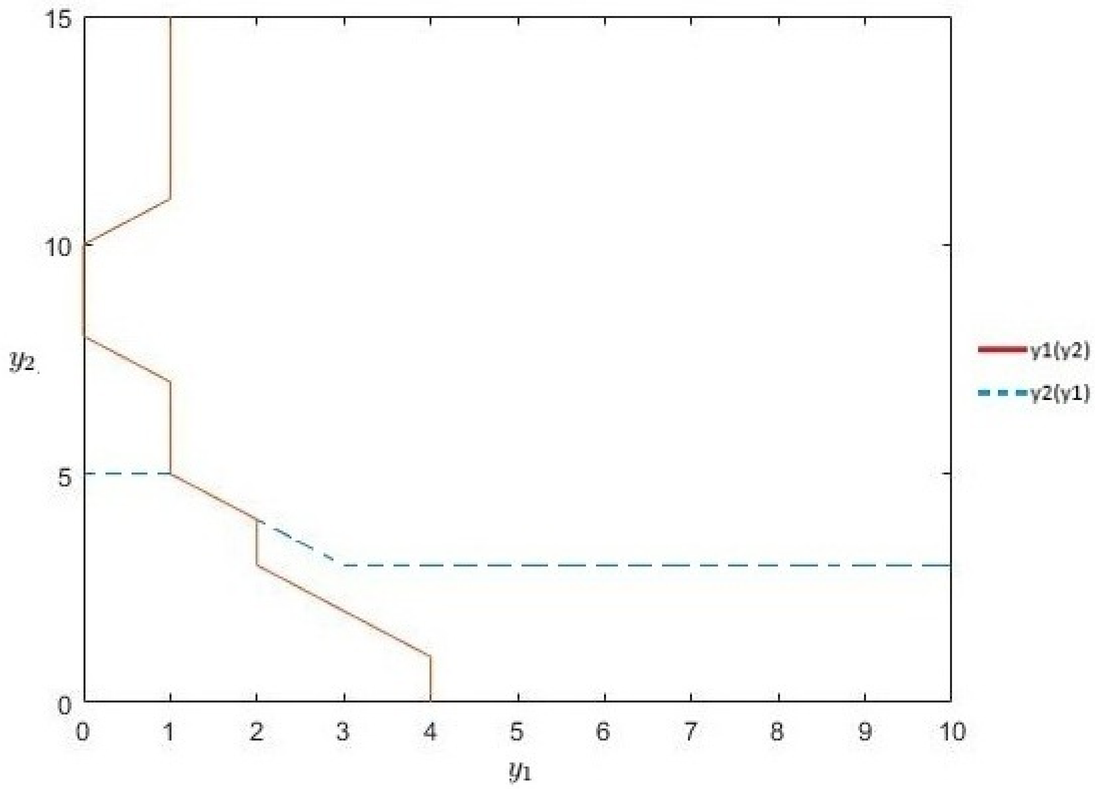

Figure 4.19. Response Functions of Scenario 9 ($x_1 = 6$, $x_2 = 12$, $p = 1$, $\Theta = 10$). The equilibrium order levels are $(y_1, y_2) = (2, 4)$ and $(y_1, y_2) = (1, 5)$.

Table 4.7 summarizes the results of the computational study and illustrates optimal ordering quantities y_1 and y_2 as well as expected cost and outdates for player 1 and 2 ($E_1[Outdates]$, $E_2[Outdates]$, L_1 and L_2) for all 9 scenarios concerning poisson distribution. Compared to the uniform distribution the values are not the same since they all have different pmf. Last row of the table is used for the second equilibrium point that belongs to Scenario 9.

Figure 4.20 shows how equilibrium points behave when they are categorized by initial inventory levels. Scenarios 4-6 hold the highest equilibrium order level with low initial inventory setting. ($(x_1, x_2) = (0, 0)$) Since initial inventory does not ex-

Table 4.7. Equilibrium Points, Expected Outdates and Expected Costs of All 9 Scenarios for Poisson Distribution.

	y_1	y_2	$E_1[Outdates]$	$E_2[Outdates]$	L_1	L_2
Scenario 1	5	10	2.4	3.52	3.48	5.11
Scenario 2	8	14	5.06	7.14	6.45	9.11
Scenario 3	2	6	0.29	0.49	8.13	12.76
Scenario 4	8	16	2.87	3.73	3.95	5.32
Scenario 5	11	20	5.38	7.27	6.77	9.24
Scenario 6	5	12	0.62	0.76	11.38	15.41
Scenario 7	3	6	0.56	0.59	1.14	1.18
Scenario 8	5	9	2.39	2.62	3.79	3.70
Scenario 9 Eq. Pt.1	2	4	0.1	0.1	1.99	2.54
Scenario 9 Eq. Pt.2	1	5	0.05	0.11	2.43	2.08

ist, players order more in quantities to compensate lack of initial inventory, which ultimately increases equilibrium points. When there is high inventory to start with ($(x_1, x_2) = (2\mu_{11}, 2\mu_{12})$), optimal strategy becomes to order less in quantities since they are already enough products to cover first period demand. Medium initial inventory settings ($(x_1, x_2) = (\mu_{11}, \mu_{12})$) lays between two. Another observation can be made about the distance between equilibrium points within same initial inventory setting. Both low and medium initial inventory setting create wide range of equilibrium points across the $y_1 - y_2$ plane. However, this range narrows in the high initial inventory level. After a point, having excess level of initial inventory does not bring any marginal advantage since they cannot be used for second period demand. That is why equilibrium points for Scenarios 7-9 clutch together at the bottom left corner of Figure 4.20, whereas others lay across the field.

Figure 4.21 again exhibits how equilibrium points behave, but now they are categorized by costing setting for p and Θ. When the shortage cost is ten times higher

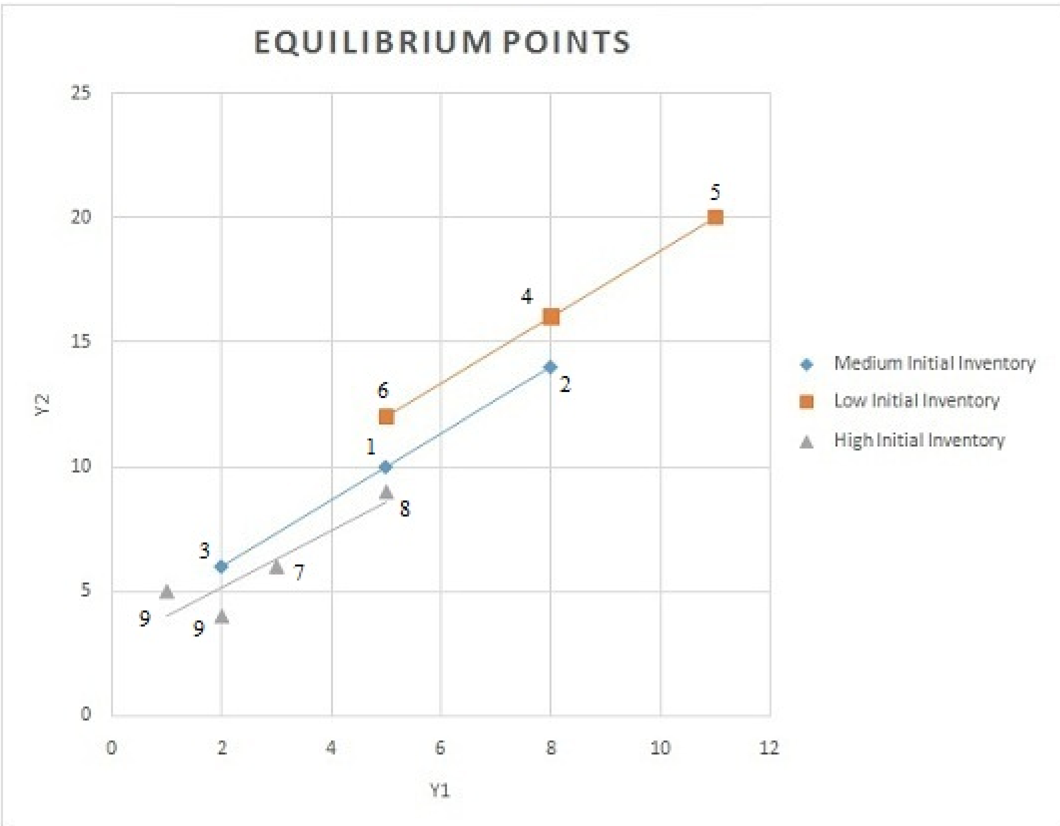

Figure 4.20. Equilibrium Points Categorized by Initial Inventory Level.

than outdate cost, equilibrium points increase as it can be seen from Scenarios 2, 5 and 8 on the right side of the figure. Players do not want to get short of products, which escalates the order quantities that leads an increase in equilibrium points. If outdate cost is ten times higher than shortage cost, players force to order less. At the end of the day, they have less leftover products, which would decrease total outdate cost. Figure 4.21 shows this behaviour as equilibrium points Scenarios 3, 6 and 9 are places at the left bottom.

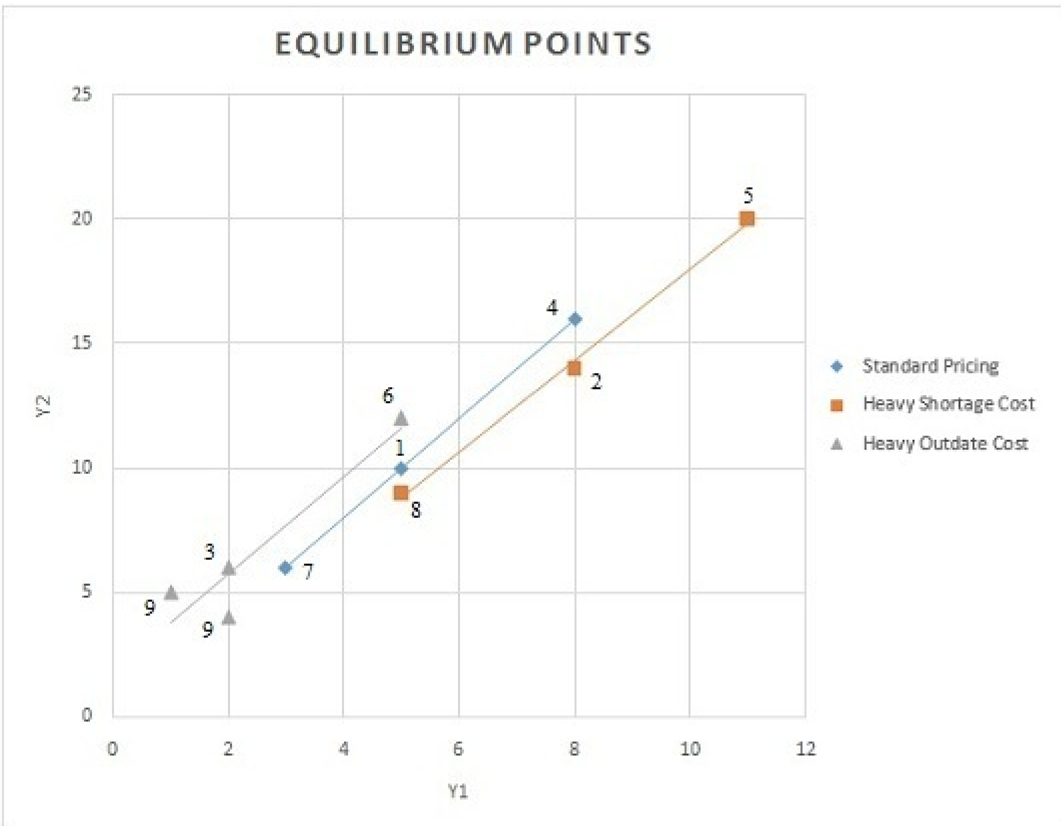

Figure 4.21. Equilibrium Points Categorized by Costing Settings.

Figure 4.22 demonstrates the relationship between expected outdates and initial inventory levels. Lowest expected outdates are achieved when players start with high initial inventory. Player do not need to worry about first period demand as high initial inventory would be sufficient enough to satisfy it. Since players only focus on second period demand, it decreases the variation of total demand, which ends up with more accurate results and less outdates. Low initial inventory setting creates highest expected outdates, however one thing to notice that player 2's expected outdates does change little between low and medium initial inventory setting. Scenario 1's $E_2[Outdates]$ is 3.52 compared to Scenario 4's 3.73. Scenario 2 and 5 also have little difference on $E_2[Outdates]$ with only 7.27-7.14=0.13 increase. These changes can also be spotted from Figure 4.22 as points 1-4 and 2-5 stay almost on the same horizontal line. Player 2 represents the big rival in this competition between two pharmacies. Since player 2's demand is higher, this results show that player 2 order in big quantities compared to player 1. So whether low or medium initial inventory does not change the outcome. However, player 1 orders in small quantities and therefore whether if 0 or 2 old products left from the previous period change expected outdates more drastically. This happens since they work in small quantities and an increase from ordering 1 product to 2 is more noticeable compared to an increase from ordering 11 products to 12.

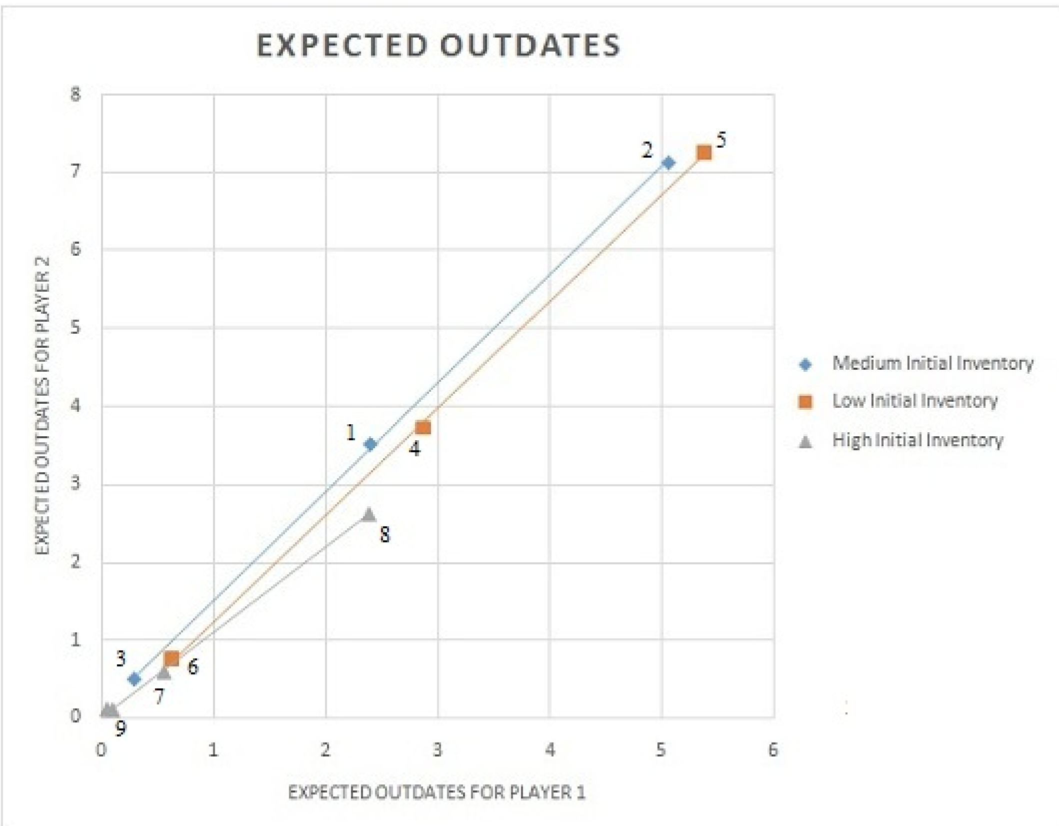

Figure 4.22. Expected Outdates Categorized by Initial Inventory Level.

The outcome is more clear in Figure 4.23, which refers to the relationship between expected outdates and costing setting. In case of heavy shortage cost, players order more; because they do not want to face high shortage cost. Since they order more, they get more expected outdates. On the hand, if the outdate cost is ten times higher than shortage cost, players order less to not get outdated products. Since they order less, expected outdates also decreases as it can be seen from Figure 4.23 in Scenario 3, 6, and 9 at the left bottom corner.

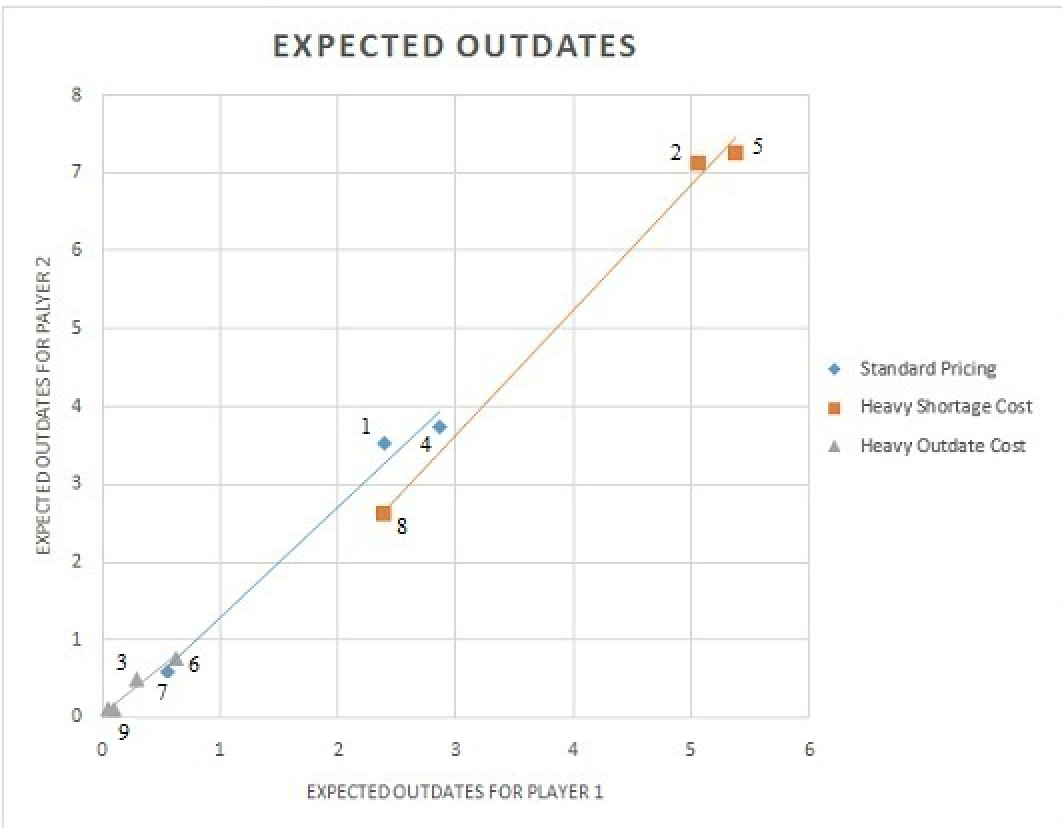

Figure 4.23. Expected Outdates Categorized by Costing Settings.

Figure 4.24 shows the interaction between expected cost of players and initial inventory levels. The figure is actually reflection of Figure 4.22, which was giving data about expected outdates. The results are basically same as high initial inventory decreases the costs. Low and medium initial inventory on the other hand increases expected costs.

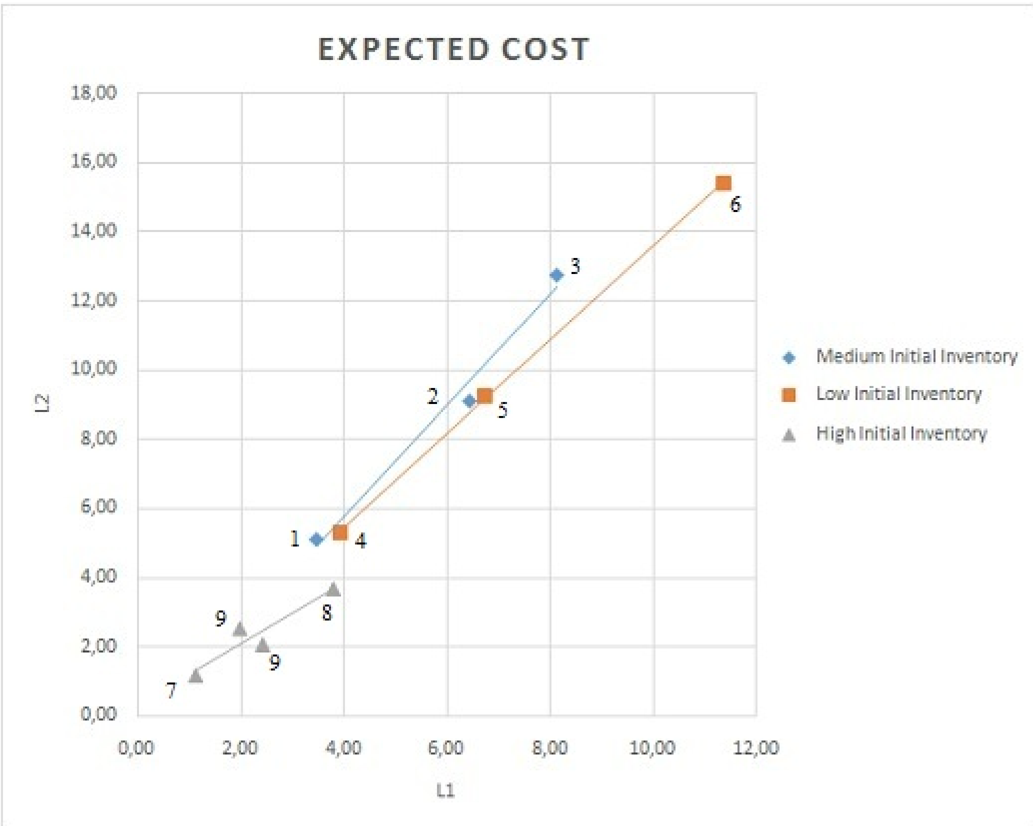

Figure 4.24. Expected Cost Categorized by Initial Inventory Level.

Figure 4.25 shows how expected cost occurs with different costing setting. This figure is by far the most complex on among all others. Standard pricing when outdate cost is equal to shortage cost gives the least expected cost. When shortage cost is ten times higher than outdate cost, expected costs increase. The real dilemma arise when outdate cost is tem times higher than shortage cost. In this setting, expected cost can be small just like in Scenario 9, but it can also increase significantly like in Scenarios 3 and 6.

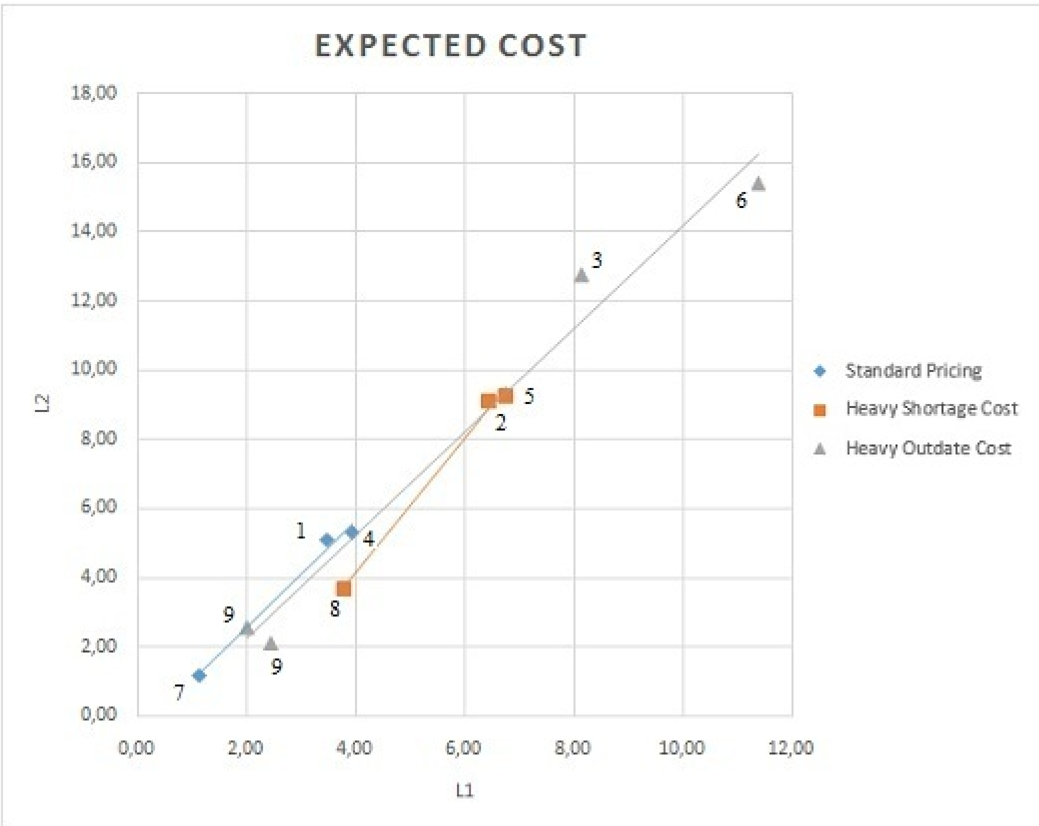

Figure 4.25. Expected Cost Categorized by Costing Settings.

If we compare Table 4.3 and 4.7, we can observe similar results. Comparisons are made against the standard scenarios, Scenario 1.1 for uniform distribution and Scenario 1 for poisson distribution. When shortage cost becomes ten times higher than outdate cost, both equilibrium points in uniform and poisson distribution increase. For example, equilibrium point increases from $(y_1, y_2) = (100, 200)$ to $(y_1, y_2) = (128, 228)$ in Scenario 1.2 for uniform distribution. Also equilibrium point increases from $(y_1, y_2) = (5, 10)$ to $(y_1, y_2) = (8, 14)$ in Scenario 2 for poisson distribution. Similarly, when outdate cost becomes ten times higher than shortage cost, both equilibrium points in uniform and poisson distribution decrease. For example, equilibrium point decreases from $(y_1, y_2) = (100, 200)$ to $(y_1, y_2) = (81.2, 181)$ in Scenario 1.3 for uniform distribution. Also equilibrium point decreases from $(y_1, y_2) = (5, 10)$ to $(y_1, y_2) = (2, 6)$ in Scenario 3 for poisson distribution. Equilibrium points react same way for initial inventory levels. If initial inventory decreases, both equilibrium points in uniform and poisson distribution increase. For example, equilibrium point increases from $(y_1, y_2) = (100, 200)$ to $(y_1, y_2) = (175, 350)$ in Scenario 2.1 for uniform distribution. Also equilibrium point increases from $(y_1, y_2) = (5, 10)$ to $(y_1, y_2) = (8, 16)$ in Scenario 4 for poisson distribution. The behaviour is similar since if the initial inventory increase, both equilibrium points in uniform and poisson distribution decrease. For example, equilibrium point decreases from $(y_1, y_2) = (100, 200)$ to $(y_1, y_2) = (100, 165)$ in Scenario 3.1 for uniform distribution. Also equilibrium point decreases from $(y_1, y_2) = (5, 10)$ to $(y_1, y_2) = (3, 6)$ in Scenario 7 for poisson distribution. Same trends for equilibirum points can be observed for $E_1[Outdates]$ and $E_2[Outdates]$.

However, one crucial difference can be seen for L_1 and L_2. Both uniform and poisson distribution has the lowest cost when shortage and outdate cost equal to 1. On the other hard, when heavy shortage cost is introduced ($p = 10$, $\Theta = 1$), uniform distribution reacts more as L_1 increases from 17.02 to 36.80 in Scenario 1.2 compared to heavy outdate cost setting ($p = 1$, $\Theta = 10$), where L_1 only increases to 28.17 in Scenario 1.3. The outcome is different in poisson distribution. Poisson distribution reacts more to heavy outdate cost setting ($p = 1$, $\Theta = 10$) as L_1 increases from 3.48 to 8.13 in Scenario 3 compared to heavy shortage cost setting ($p = 10$, $\Theta = 1$), where L_1 only increases to 6.45 in Scenario 2. This means poisson distribution is more vulnerable to outdate cost, whereas uniform distribution is more vulnerable to shortage cost. We think since uniform distribution deals with bigger product size, shortages are more crucial. For example, pharmacies are expected to hold on standard medicine like aspirin all the time. Therefore, when customers do not find it, it has more impact cost wise. Shortage cost is mainly consists of goodwill cost, so if a customer cannot even find a standard medicine as simple as aspirin, customers would not likely to come back and the implications are bigger. We use poisson distribution to define expensive medicine like cancer drugs. The results show that poisson distribution is more vulnerable to outdate cost. For example, it is common for a customer to visit multiple pharmacies before finding the expensive cancer drug. Therefore, it is not big deal to get shortage of expensive cancer drugs as it was also expected from customers. However, since these drugs are very expensive, the costs determiner in case of outdates since outdate cost is mostly sum of purchasing price, holding cost and disposal cost. (Pharmaceutical companies do not give any compensation for outdated medicine and it is pharmacies' duty to dispose the product.) Pharmacies pay significantly high prices to buy cancer drugs. If they fail to sell them and they outdate, they simply lost the money that they use to buy the cancer drugs. This was not big problem in uniform distribution since the prices are lower for common medicine. Therefore, pharmacies afford to lose some aspirins for example, after their shelf life ends because of their low purchasing cost.

5. CONCLUSION

In this research, we create a model that finds the optimal ordering quantities for perishable products under demand substitution in a competitive (duopoly) setting. By using a numerical example and prior knowledge from Parlar [7]; and Chazan and Gal [8], we show that the cost function of the model is convex. Since convexity holds for both players' cost functions, by using Debreu's theorem [14] we argue that a Nash equilibrium exists. Even though we did not prove it, we believe that Nash equilibrium is unique since a contrary case is not seen in Chapter 4 among all 19 examples. In the last example (Scenario 9) even through there exists two equilibrium points, response functions do only intersect once, making poisson distribution the real contributor for the second equilibrium point. Then, we create an experimental setting to test the model and observe the equilibrium point behaviour.

In the first case, demand functions are defined as uniform distribution to represent the case for newly introduced medicine to the market. Nine different scenarios are generated for this experimental setting including uniform distribution and an equilibrium point for each scenario is found. We repeatedly observe that if the shortage cost is higher than the outdate cost, players order more amount of products that results in higher equilibrium point. If the outdate cost is higher than the shortage cost, players order less amount of products that results in lower equilibrium. Expected cost functions in the equilibrium point show that the shortage costs are more dangerous to the market and they can sum up with an extra 20% to 30% of total cost compared to scenarios when the outdate cost is ten times higher than the shortage cost. When the initial inventory levels are decreased, players order more products to satisfy the difference between one period products and their respective demand. Even though, players may force to significantly increase the number of new products they order, the total cost is not linearly proportionate with the order quantity. In an extreme scenario when players increase y_1 and y_2 by 75%, the cost only increased by 7.7%. This actually shows that the system do not punish previous mistakes harshly and businesses can go back from bad decision without losing much profit. Indeed, the setting this model uses has a product with two-period shelf life, so it is not possible to generalize it for products with n-period shelf life. Furthermore, an argument about decision making is developed by comparing smoothness of response functions. We observe that every new parameter makes it harder to conclude a decision since it increases the variance.

We repeat the experimental setting by defining demand function with poisson distribution. Poisson distribution is suitable to represent very expensive medicine like cancer drugs, which has low demand; and therefore pharmacies stock very little. Most of the findings of poisson distribution match with uniform distribution. If the initial inventory is low, players order more. If the initial inventory is high, players order less. If outdate cost is high, players do not want to get outdate and therefore order less. If shortage cost is high, players do not want to get shortage and therefore order more. The most important finding about poisson distribution is that how the system protects the small player. Expected cost of big player, player 2 is roughly 35 to 50% more than small player 1. This difference between costs allow player 1 to compete with player 2 despite having less inventory. Only when players start with high inventory, cost of player 2 decreases and becomes similar to player 1. This case can also be justified because excess inventory occurs when the economy is bad and people cannot purchase the goods. In case of economic crisis, the small rival is affected more since it does not have cash flow to support during stagnation. On the other hand, big rival can simply hold on inventory and stay in the market on the short run. Moreover, the model with poisson distribution reflects pharmaceutical market as we observe pharmacies hold very little inventory like 1-2 medicine for very expensive drugs. Table 4.5 also confirms this action as the model favours small ordering quantities in general. Last but not least, we observe that poisson distribution is more costly in case outdate cost is higher compared to shortage cost. Since poisson distribution is used to define cancer drugs and cancer drugs have high purchasing cost for pharmacies, the implications are bigger in case of outdates. However, uniform distribution is affected more from heavy shortage cost scenarios since goodwill cost steps in. Uniform distribution is used to define more generalized medicine and if customers fail to find them in a pharmacies, they would likely not to comeback, which is reflected in total cost, L_1 and L_2.

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