COMPETING VENDORS IN THE TELECOM VALUE CHAIN

by

Ünal Gök

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ABSTRACT

COMPETING VENDORS IN THE TELECOM VALUE CHAIN

We consider the competition between two vendors that serve a common telecommunication operator. The two vendors make independent investments on research and development of the same or substitutable innovative technology which strictly increases the demand of the operator if the innovation ever materializes. The operator buys the innovative technology from the vendor that achieves it first and then decides on the extra capacity to build in each period. In the first model, it is assumed that the operator chooses the cheaper vendor to buy the technology from. Whereas in the second model, the operator chooses the more profitable option if both vendors come up with the same technology.

For both models, the centralized solution is characterized in which a single decision maker oversees the whole system. This solution acts as a benchmark for the decentralized analysis. In the decentralized setting, the vendors play a simultaneous investment game first and then they offer the operator a contract. The operator's decisions are fully characterized for both models. For the first model we provide negative result via counter-examples for the non-existence of a Nash equilibrium. For the second model, we characterize the conditions under which a unique Nash equilibrium exists.

We provide a computational study to gain insight about how innovation plays a role in the vendors' game. Moreover, profit and revenue sharing coordinating contracts are proposed for the telecom value chain. The effect of the contract parameters on profit values is monitored by a numerical illustration.

ÖZET

TELEKOM DEĞER ZİNCİRİNDE REKABETÇİ TEKNOLOJİ SAĞLAYICILAR

Genel bir telekomünikasyon operatörüne hizmet veya ürün üreten ve birbirleriyle rekabet eden iki tedarikçi düşünülmüştür. Bu iki farklı tedarikçi operatörün talebini artırabilecek birbirinin yerine ikame edilebilir yeni bir teknoloji üzerinde çalışmaktadırlar. Teknoloji tedarikçileri, Ar&Ge çalışmalarına fon ayırarak yeni teknolojiyi keşfedecek ve eğer bu teknoloji kullanılabilecek hale gelirse operatöre satarak para kazanacaklardır. Müşteri talebinin artması için operatör yeni geliştirilmiş olan iletişim teknolojisine yatırım yapmakta ve dönemsel olarak kendi ağ kapasitesine karar vermektedir. İlk modelde, operatör yenilikçi teknolojiyi ilk keşfeden teknoloji sağlayıcısından alır. Ancak, ikinci modelde, her iki teknoloji sağlayıcısı da yeni teknolojiyi geliştirdiğinde operatör en karlı olan seçeneği tercih eder.

Her iki modelde de bir karar vericinin tüm sistemi kontrol ettiği merkezi çözüm üretilmiştir. Bu çözüm adem-i merkezi çözüm için bir karşılaştırma kriteridir. Merkezi olmayan sistemde, öncelikle teknoloji sağlayıcılar eş zamanlı olarak yatırım oyunu oynarlar ve sonrasında operatöre bir kontrat önerirler. Her iki model için de operatörün kararları tamamıyla tanımlanmıştır. İlk model için Nash dengesinin olmadığı olumsuz sonucuna zıt örnekler yardımıyla ulaşılmıştır. İkinci model için Nash dengesinin var olduğu koşullar tanımlanmıştır.

Teknoloji sağlayıcıların oyununda yenilikçiliğin nasıl rol oynadığını kavramak için sayısal çalışmalar yapılmıştır. Ayrıca, kar ve gelir paylaşımı kontratları telekom değer zinciri içinde koordinasyon için önerilmiştir. Sayısal analiz yöntemiyle kontrat değiştirgelerinin gelir değerlerini nasıl etkilediği gösterilmiştir.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS/ABBREVIATIONS

a_t	Revenue generated by operator per unit utilized capacity in period t
C_t	Capacity of operator's network in period t (Decision variable for the first model), $t = 1, 2$.
C_1	Capacity of operator's network in period 1 (Decision variable for the second model)
C_2^j	Capacity of operator's network in period 2 (Decision variable for the second model), $j = 1, 2$.
$D_t(I)$	Random demand in period t as a function of the technology investment
$F(x\|I)$	Probability distribution function of demand for a given I
$f(x\|I)$	Probability density function of demand for a given I
$G(x)$	Probability distribution function before technology adoption
$g(x)$	Probability density function before technology adoption
I_j	Technology investment of vendor j (Decision variable), $j = 1, 2$.
m	Manufacturing cost per unit capacity for the first period
m_j	Second period manufacturing cost of vendor j per unit capacity, $j = 1, 2$.
o_t	Operating cost of the operator per unit capacity in period t
v_t	Penalty cost of unsatisfied demand per unit capacity in period t
μ	Expected demand before technology adoption
π	Total expected profit of the supply chain
π_o	Total expected profit of the operator
π_1	Total expected profit of the first vendor
π_2	Total expected profit of the second vendor
R&D	Research and Development

1. INTRODUCTION

In today's fast changing highly competitive market environment, companies in most industries are turning to their suppliers to seek competitive advantage. Many firms have begun the reexamining of their supply management strategies to improve their financial performance and customer delivery service. As many business environments, the telecommunication supply chain has changed significantly during the past decades.

Formerly, telecom operators were more involved in innovation and research and development (R&D) activities, especially development activities. However, the picture of the sector has changed considerably. The vertical integration between operators and equipment manufacturers has shrunk. New outsourcing relationship, in which telecom operators have downsized their genuine long term R&D activities and have been focusing on short term service innovations and equipment producers have taken over long term R&D activities, has been formed between them [1, 2, 3, 4].

The recent practice in telecom value chain is that the vendors develop new technologies and manufacture equipments that are used in building a capacity. The operators buy equipment from the vendors to build capacity for serving their customers. At the same time, the operators can buy new technologies from the vendors to boost their demand. A recent example is the introduction of iPhone by Apple. The wireless network operator AT&T has been the exclusive provider of iPhones in the U.S. market with the hope that iPhones are going to increase the demand for AT&T. AT&T's wireless CEO states that "iPhone users generate lot of traffic for the network, about twice as much as customers using other phones" [5]. This is a typical example of a new technology developed by an OEM boosting up the revenues for the network operator. Technological innovation has been dubbed as a critical success factor in telecommunications [6].

This kind of technological improvement and the interaction between the innovators under competition motivate us to search the dynamics of innovation in the telecom sector. Critical questions are how much the vendors should invest for such an innovation activity, how they should respond to each other in a competitive business environment when the innovation capabilities of them get involved in competition, and finally, how the operator and the innovative vendors sustain new technologies in the telecommunication network. Briefly, our motivation originally stems from the R&D expenditures incurred by the innovators of the new technologies which attract the technology users and result in an increase in demand.

The main objective is to provide interpretation for R&D investment in the highly competitive environment of telecom supply chain. To light up on innovation competition, we consider the interactions of two rival vendors and one network operator. In two-stage telecom value chain models, the vendors determine irreversible, one-shot R&D expenditures at the beginning of the first period and they can not renege on this commitment during the game. If they innovate and materialize the new technology by the end of the first period they have an opportunity to sell it to the operator with a unit price (can be either a decision variable or an exogenous parameter) at the beginning of the second period. The operator installs a network capacity to satisfy the service demand of subscribers in each period. She uses this new technology to build extra network capacity for servicing her customers. Extra network capacity denotes a telecom network capacity that is built up by using new technology (hardware or software) on the current technology infrastructure of the operator. In addition, the telecom network capacity is assumed to be incremental. Demand structure of the telecom network is assumed to be effort dependent and stochastic and R&D investment to new technology is envisioned as an effort to boost up the demand of the network.

The first model in Chapter 3 is analyzed both from a centralized point of view, where a single decision maker controls the entire supply chain, and from decentralized point of view, agents (vendors and the operator) make their choices with respect to the objective of maximizing their own profits. In this model, the operator decides on her network capacity, besides, her two period network capacity decisions are contingent on likelihood of presence of the new technology. The vendors determine their R&D

investment levels and unit price of new technology. It is assumed that vendors are the leader of the game and start it either by announcing the price of new technology first and determining the R&D investment levels later (nested game) or by investing in innovation activities initially and announcing the price of it later (reverse nested game). After simultaneous movement (Nash) game between the vendors finishes, operator's capacity decisions follow the vendors' game (Stackelberg game). Eventually, the vendors run away from both nested games. According to our result, two degrees of freedom in decisions of the vendors (i.e. to set the investment level and the price of new technology), causes disruption in the game of the vendors. There is no Nash equilibrium for the nested game. However, in the reverse nested game, there can be Nash equilibrium in some situations for the identical vendors. This feature leads us to investigate the situation where the market exogenously determines the price of the new technology. In addition, there is always a Stackelberg equilibrium between the vendors and the operator in this game setting whenever Nash equilibrium exists.

The second model in Chapter 4 investigates particularly just R&D investment decisions of the vendors where the market determines the price of the new technology (exogenous parameter). Although it seems that the vendors lose their opportunity of pricing their innovation, it is obvious that naturally, the market can define the price of the new technology in some circumstances. In this model, contrary to the first one, the first period capacity decision of the operator is regarded as nominal capacity which is not associated with the new technology because the operator is assumed to be risk-free of the upcoming new technology and makes her upgrading network decision after the new technology appears. The second model is also evaluated both from centralized point of view and decentralized point of view. Nash equilibrium characteristics of the R&D investment game is stated clearly and some illustrative visual examples are given for it. Moreover, numerical examples are also provided to enhance the insights about the R&D investment dynamics. In addition, two coordinating contracts are designed, namely, profit sharing contract and revenue sharing contract, to coordinate the telecom value chain. For the profit sharing contract, we provide a numerical example showing the expected revenues gained by the vendors and the operator under different contract parameters, and for the revenue sharing contract, we show the acceptable range of contract parameters.

To summarize, R&D investments of two equipment suppliers and supplier-operator interactions are studied carefully. In this research, we look for R&D investment decisions of the vendors, capacity decisions of the operator and operational effects of these decisions for both sides as well as coordinating contracts for telecom value chain. We evaluate the telecom value chain via the eye of central decision maker and provide the maximum expected profit the telecom supply chain might gain. And, we disclose the conditions thoroughly where Nash equilibrium exists or not in R&D investment game between the competitive vendors and where Stackelberg equilibrium exists between the vendors and the operator. We also propose two different coordinating contracts to ensure that the vendors and the operator are allied to gain the centralized expected revenue.

The contribution of this thesis can be summarized as follows: (*i*) R&D investment decisions and its related issues such as whether Nash equilibrium exists or not in R&D investment game of competitive vendors, how the operational parameters of the vendors affect the equilibrium, and the amount of minimum investment level which can trigger the demand enough to make the operator is willing to use the new technology to upgrade her network, are enlightened in a service sector, (*ii*) the structure of operational decisions in terms of cost-revenue parameters of the agents and demand increase of the network for both equipment providers and the network provider is analyzed, and some managerial insights are provided for the real players of the telecom sector, (*iii*) two coordinating contracts, profit and revenue sharing contracts, under effort dependent stochastic demand are proposed for new usage area.

The rest of the thesis is organized as follows: in Chapter 2, the literature survey about telecom supply chain and R&D investment activities in technology development phase is given. In Chapter 3, the first model is described along with major assumptions and the analysis of the centralized solution in Section . A numerical study is also given to illustrate the centralized solution. Then, a section is devoted to the decentralized analysis. In Chapter 4, the second model's centralized and decentralized analysis are provided with both analytically and numerically, and also two coordinating contracts are defined. In Section 5 we conclude.

2. LITERATURE REVIEW

Even though the literature is extensive, the papers are selected basically regarding the new technology investments and telecom value chain papers.

First of all, the influence of R&D investments on innovation and firm performance has not received too much attention in the literature, especially in the telecommunication area. However, there exist some empirical studies and reports that try to explain the relationship among them. In fact, R&D expenditures in telecom sector have an increasing trend throughout the years [1, 2, 3, 7]. However, unlike most of the operators, most of the equipment manufacturers have a tendency to increase the R&D funds [1].

Furthermore, R&D investments are accepted as a parameter of innovation but it is obvious that R&D expenditures do not cover every aspect of innovation [1]. In addition, R&D outlay is on the rise, but the linkage between R&D investment and financial performance remains poor [7, 8].

After some empirical studies and reports akin to telecommunication area, Çanakoğlu and Bilgiç [9] model the two-stage telecommunication supply chain with technology dependent demand under a multiple period setting. R&D investment of equipment suppliers influences the innovation and that new opportunity always provides a better demand for operator. Their model consists of one operator which faces with stochastic market demand dependent upon the technology investment level, and decides on his capacity levels through the periods and one time R&D expenditure for the new technology. An algorithm which provides to find the centralized solution is given in the paper. Moreover, they enhance the results by proving the unique Nash equilibrium for the case in which the operator decides on her network capacity for each period and the vendor decides on his one time R&D investment at the beginning. After then, by considering the equipment manufacturer and the operator as different agents, they propose two different coordinating contracts, namely, a profit sharing contract where firms share both the revenue and operating costs generated throughout the periods along with initial technology investment and also a coordinating quantity discount contract where the discount on the price depends on the total installed capacity. However, the coordinating contracts are designed for the case in which the operator decides on both the capacity levels and the R&D investment level. This is a unique paper which focuses on the telecom value chain as service sector supply chain instead of the manufacturing supply chain.

Çınar and Bilgiç [10] extend the previous study by incorporating the probability of innovation period for manufacturer and implementation time of that innovation for operator. Moreover, they take into consideration the probabilities of both successful innovation and unsuccessful innovation and also the technology adoption delay to the existing network. Once the vendor gets innovation and materializes the new technology, the operator is able to improve her telecom network and starts to use it after some implementation time. As an extension to the paper of Çanakoğlu and Bilgiç [9], they, firstly constitute the solution of the centralized system contingent upon the probability of successful innovation. Moreover, they analyze the case in which the operator determines the capacities for each periods and, the vendor decides on the R&D expenditures, and they propose that the simultaneous movement game has a unique Nash equilibrium in terms of the supermodularity of the game. They also construct a profit sharing contract as coordinating contract in which the operator and the vendor are different, self-interested parties, but the operator gives the decisions of the system, though.

This work is a sequel to the efforts in characterizing the telecom value chain [9, 10]. In both of these earlier papers they have considered a two stage, serial chain comprised of one vendor and one operator. This thesis extends this line of work to two competing vendors and a single operator.

The major difference is that we incorporate the technology development race between the technology suppliers into the telecom value chain model. Furthermore, we also concentrate on the case in which the operator does business with one of the competing vendors, and her demand is effected by that individual R&D investment of the vendors, and the case in which the operator trades with either both or one of the vendors and her demand is jointly effected by R&D investment levels of the vendors at the same time.

Agrell *et al.* [2] consider a three stage telecom supply chain in a two-period investment-production game under stochastic demand and asymmetric information. In their paper, minimal agency model is used to compare the system currently used in telecommunication industry. They basically evaluate the supply chain performance via changing the magnitude of the bargaining power under asymmetric information. In contrast to them, we analyze two stage service sector rather than three stage manufacturing system. They are concentrated on the ongoing production system of telecommunication supply chain but not on the effect of the R&D investment activities of the upstream firms of the telecom operator. Moreover, the operator's revenue and the cost parameters are not involved to the analysis, she is almost inactive in the analysis.

Goyal and Netessine [11] study the effect of competition on a firm's technology selection (flexible and product dedicated) and capacity investment decisions. Two firms competing with each other in price dependent and uncertain demand markets are modeled. Firms make three decisions in the following sequence, selection of technology and capacity investment and production quantities. Technology and capacity decisions are made before demand curve is uncertain, and production quantities are decided after demand curve is revealed. Technology choice of the competing firms in the same market has examined and Nash equilibrium has been shown. Firstly, flexibility in manufacturing system requires initial investment but results various products to the market, likewise, new technology requires initial investment in service sector and results diverse service and better communication. Secondly, they modeled technology selection decisions of two rival firms while assuming that the technology is presence from the very beginning of the game. However, we model the interaction of the technology providers within themselves as well as with the technology implementer. Besides, they use price dependent demand under the assumption that the technology creates cost advantage, as a result, yield lower price, but we assume that new technology creates extra demand in telecom network, hence, we use effort dependent demand in our analysis.

There is a literature dealing with adoption time and diffusion of the new technology. Reinganum [12] studied the diffusion of new technology in her paper. Nash equilibria were obtained for a game where the early adoption of new technology yield better revenue with respect to rival, which uses existing technology, and lower operating cost [12]. Fudenberg and Tirole [13] extend the previous work and examine diverse game theoretic strategies for competing firms whose decisions are capacity and pricing over time in a technology driven market environment. Gaimon [14] formulates the two player dynamic game to explore the new technology acquisition decisions and the capacity choices of the firms in a competitive market where the new technology provides lower operating cost and more advantageous price under a price dependent demand structure as well as the solutions are obtained for open and closed loop strategies. Unlike the mentioned papers above, we try to model such a game that captures the interactions between the both sides of the technology game as a provider (vendors) and adopter (operator). We explore mutual actions which impact on the strategic decisions of the technology adopter (capacity choice) and the technology provider (R&D investment) simultaneously. Moreover, we deal with the technology selection which creates extra demand instead of the technology adoption which results lower operating cost and yields competitive advantage on market price. Finally, instead of exploring time of new technology adoption we handle the effort exerted by the technology providers to come up with a successful innovation. Justman and Mehrez [15] study a welfare analysis of innovation and provide numerical analysis of timing and diffusion of innovations in R&D markets where firms engage in R&D activities independently. In their model, the innovative firms accumulate their knowledge by exerting effort (R&D investment) throughout time, and eventually get successful new technology then sell it to the market, however, we also take into account of failure of innovation process. Moreover, in their paper, R&D investment treated as an effort affects the innovation probabilities of the firms positively, but in our model, innovation capabilities of our competitive firms are independent of how much they invest, and we regard R&D investment as the activator of demand of the new technology.

There is also a literature handling the irreversible investment decisions. According to Dixit and Pindyck [16], irreversible investments and the ability to delay such investments can have a strong impact on the decision to invest. A firm with an opportunity to invest is holding a real option like financial option in economics. Once a firm makes an irreversible investment, it activates, or loses, its option to invest. The lost option value is an opportunity cost that must be included as part of the cost of the investment. In our model, although the innovator firms make irreversible R&D investments and have an option of not to invest, they can not have opportunity of delaying their investments because we offer a periodical approach to telecom value chain to be more concentrated on the behaviors of both sides of technology pioneers rather than appearance time of new technology.

Cachon [17] in his review studies the supply chain coordination with contracts based on common newsvendor model. A number of contract types have been applied to this model: wholesale price contract, buyback contract, sales rebate contract, revenue sharing contract, quantity flexibility contract, quantity discount contract. Moreover, supply chain coordination is examined by altering the demand structure of the newsvendor model such as price dependent demand, effort dependent demand, and demand updating (signaling) opportunity. In the effort dependent demand structure, firms exert some effort to spur demand such as sale promotion, advertising and so on. In our model, we regard R&D investments of technology providers as the trigger of demand because naturally, tendency of the technology researches stimulates the choice of people and can create additional demand, or technology providers can allocate some portion of their R&D investments to create extra demand via various marketing strategies for their new technology.

Furthermore, coordination between the agents in telecom value chain under effort dependent demand is obtained by implementing a profit sharing contract [9, 10]. We are inspired such kind of profit sharing contract while trying to coordinate agents of supply chain. Cachon and Lariviere [18] examines the revenue sharing contract in a newsvendor setting. It is compared to several contracts that enhance the channel coordination such as buy back, quantity flexibility and sales rebate contracts. They advocate that revenue sharing contract is more capable than the others to coordinate a wide range of supply chains. Moreover, they also state that if demand is affected by the retailer's actions and sales effort costs are shared by two agents, revenue sharing contracts may not be attractive. However, although our game setting looks like this when R&D investment of vendors influences the demand of operator, and operator compensates the initial cost incurred by the vendors, we propose revenue sharing contract as a coordinating contract because it has gained attraction in practice in the telecom value chain.

Fudenberg and Tirole [19] systematically reveal the game theoretic approach towards the competitive models in economics and provide fundamental theories of it. Cachon and Netessine [20] reviews the game theoretic approach to supply chain management and particularly to the common newsvendor model. After a small historical review of game theory, non-cooperative static games where players choose their strategies simultaneously and thereafter committed to those strategies are analyzed. Techniques for demonstrating existence and uniqueness of Nash equilibrium are discussed and exemplified. After then, dynamic game settings are revealed such as sequential movements, Stackelberg equilibrium concept, simultaneous moves repeated over multiple periods, and differential games where decisions are made continuously. Not only non-cooperative game but also cooperative games are discussed in the review paper. They also analyze the signaling, screening and Bayesian games which results from asymmetric information.

3. INVESTMENT AND PRICING GAME OF VENDORS

We consider a two-stage value chain involving two competitive vendors and a single operator in which the vendors develop a competing substitutable technology and the operator can use this technology to boost up the demand.

The operator installs a network to satisfy the random demand from subscribers. We consider all capacity decisions are given with respect to the "peak demand" of the network operator. Usage of the network resources fills up the capacity of the network and revenue generated per unit capacity used is assumed to be constant.

The capacity of the network at a period t is defined as C_t for $t = 1, 2$. The equipment used to build up first period capacity is bought from the (spot) market at a cost of m per unit of capacity. There is no lead time consideration since the decision for the capacity was given long before the beginning of each period.

The operator incurs an operation cost per unit capacity in period t. Excess demand is lost and no backlogging is possible. The operator incurs a penalty cost as a result of lower customer satisfaction due to unsatisfied demand. It is unlikely to salvage unused capacity at the end of the planning period.

All cost parameters and the service price of the operator are stationary during the service periods. For the centralized system, the objective is to maximize the system-wide expected profit, whereas for the decentralized system, each firm optimizes its expected profits.

When the operator faces a new technology which causes an increase on demand she is willing to install that technology. The effect of technology on demand is modelled as follows: *total technology investment* made by both vendors increases the operator's demand. The news that both vendors have invested a certain amount on a new technology generates an expectation in the market. Although its demand structure is not the same as in this model, this situation is typical in the game console competition (between Sony, Nintendo and Microsoft). The news that all vendors are working on developing new technologies simultaneously creates a greater expectation in the market.

We consider a two-period interaction where the first period is used for technology development. Technology development process is uncertain but it does not take more than one period. At the beginning of the second period, one or both of the vendors have come up with a new technology or none of them could. If there is new technology available, the operator surely buys it. If both vendors came up with the same competing technology, the operator buys it from the cheaper vendor (the technology is perfectly substitutable).

The capacity installed in the first period can be used with the new technology in the second period. This is frequently observed in telecommunication networks where a new service based on a new technology usually requires a software configuration or update in the equipment that builds up the capacity.

We visualize the first model in Figure 3.1.

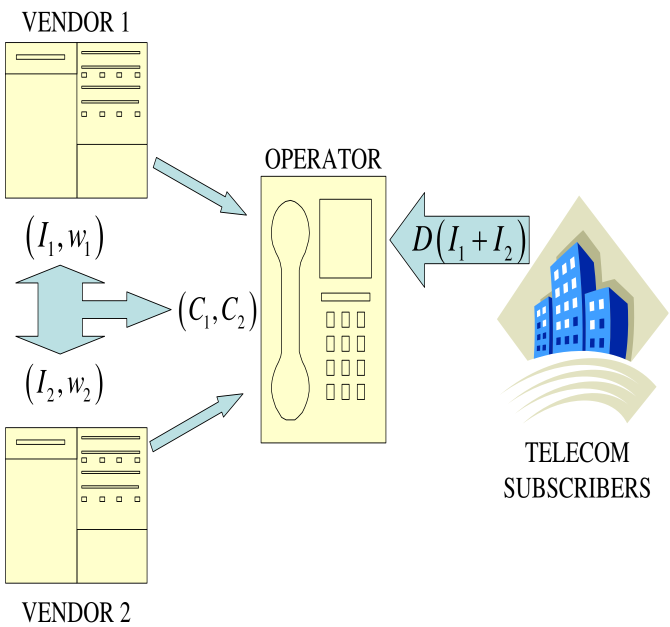


Figure 3.1. The description of the first model

Table 3.1 presents the notation used in the model.

Table 3.1. Model parameters and decision variables for the first model

$D_t(I)$:	Random demand in period t as a function of the technology investment
m :	Manufacturing cost per unit capacity for the first period
m_j :	Second period manufacturing cost of vendor j per unit capacity, $j = 1, 2$.
o_t :	Operating cost of the operator per unit capacity in period t
v_t :	Penalty cost of unsatisfied demand per unit capacity in period t
a_t :	Revenue generated by operator per unit utilized capacity in period t
C_t :	Capacity of operator's network in period t (Decision variable), $t = 1, 2$.
I_j :	Technology investment of vendor j (Decision variable), $j = 1, 2$.

Decision variables are endogenous to the models and all other parameters are exogenously determined.

3.1. Assumptions

$D_t(I)$ denotes the random demand during period t which depends on the technology investment I. We assume that demand is independent throughout the periods. Furthermore, demand is additive: $D_t(I) = \theta_t(I) + \varepsilon$, where $\theta_t(I)$ is a real-valued function and ε is a random variable such that $D_t(0) \geq 0$.

Let $G(x)$ denote the distribution and $g(x)$ denote the density function of demand without the effect of technology investment I. We assume G is twice differentiable, strictly increasing and $G(0) = 0$. Let $F(x|I)$ denote the distribution and $f(x|I)$ denote the density of demand after the successful technology is adopted. Note that, under the additive demand assumption:

$$F(x|I) = G(x - \theta(I))$$

The additive demand assumption is used extensively in the operations and economics literature [21]. Under this assumption, only the mean demand depends on I, and the uncertainty is captured by ε. The additive demand assumption also implies:

$$\frac{dE\left[D_t(I)\right]}{dI} = \frac{dD_t(I)}{dI} \qquad t = 1, 2$$

Each firm in the model is risk neutral, so each firm tries to maximizes its own expected profit. All firms have the same information at the start of the game, i.e., each firm knows all costs, parameters and rules and know that the other party knows this situation and so on.

Further assumptions of the model are given as follows. Expected demand is increasing in technology investment and it is always profitable to spend a non-zero amount on technology:

$$\frac{dE\left[D_t(I)\right]}{dI} > 0 \qquad t = 1, 2$$

This assumption is standard in marketing models. In our case, it means that technology investment, once innovation materializes and technology is adopted never decreases the demand.

Expected demand is diminishingly concave (i.e., it is "flattening out" as I approaches infinity) in technology spending:

$$\frac{d^2E\left[D_t(I)\right]}{dI^2} < 0 \qquad t = 1, 2$$

This assumption has empirical evidence in the marketing literature [22].

The demand function is strictly positive (i.e.,$F(x|I)$ is strictly increasing for fixed I). This is needed for the optimal set be a single point rather than a line segment [23].

In order for the value chain to earn a positive revenue:

$$\begin{aligned} a_1 &> m + o_1 \\ a_2 &> \max\{m_1, m_2\} + o_2 \end{aligned}$$

3.2. Analysis of the Centralized Model

We first assume that a central decision maker is going to determine the *total* technology investment, I that the company needs to make along with capacity decisions for the network. Note that $I = \lambda I_1 + (1-\lambda)I_2$ where $\lambda \in [0,1]$. Here λ roughly captures the "market share" or "brand value" of the first vendor. The first period is the development period for the innovation and this technology is adopted by the operator at the beginning of the next period only if it is materialized. Let $S(C_1)$ be the expected service delivered by the operator in the first period before the new technology is implemented. Furthermore, to isolate the effect of innovation we assume that the operator does not change her capacity in the second period if the innovation does not materialize.

$$\begin{aligned} S(C_1) &= E_{D_1}[\min(C_1, D_1(.))] \\ &= \int_0^{C_1} xG(x)dx + C_1(1 - G(C_1)) \\ &= C_1 - \int_0^{C_1} G(x)dx \end{aligned} \tag{3.1}$$

If successful innovation materializes and is adopted, the operator can buy *extra* capacity, C_2 with the expectation that the new technology (investment) is going to increase the demand. The expected service delivered in period 2 is given as:

$$
\begin{aligned}
S\left(C_1+C_2, I\right) &= \int_0^{C_1+C_2} xF\left(x\right| I) + \left(C_1+C_2\right)\bar{F}\left(C_1+C_2\right| I) \\
&= C_1+C_2-\int_0^{C_1+C_2} F(x|I)dx \qquad (3.2)
\end{aligned}
$$

where $\bar{F}(\cdot)$ denotes $1-F(\cdot)$.

If both vendors come up with the competing technology, we presume that the operator will buy it from the more effective (hence cheaper) vendor no matter how much technology investment he has made. In the centralized model this poses no problem as the whole value chain is owned by a single company.

Since the operator has an ability to switch to the cheaper technology provider, we create an asymmetry between two vendors such that one of them (the second vendor) is more likely to sell his technology.

The expected profit for the centralized system is given as:

$$
\begin{aligned}
\pi\left(C_1, C_2, I\right) &= \left(a_1+v_1\right) S(C_1) - v_1\mu_1 - o_1C_1 - mC_1 \\
&+ p_2\left\{\left(a_2+v_2\right) S(C_1+C_2, I) - v_2\mu_2\left(I\right) - o_2\left(C_1+C_2\right) - m_2C_2\right\} \\
&+ q_2p_1\left\{\left(a_2+v_2\right) S(C_1+C_2, I) - v_2\mu_2\left(I\right) - o_2\left(C_1+C_2\right) - m_1C_2\right\} \\
&+ q_2q_1\left\{\left(a_2+v_2\right) S(C_1) - v_2\mu_2 - o_2C_1\right\} - I \qquad (3.3)
\end{aligned}
$$

where the first line is the expected revenue of the first period. The rest of the equation is for the second period. The second line is the expected profit if the second vendor successfully comes up with the new technology. The third line is the expected second period profit if the second vendor fails and the first vendor succeeds. Finally, the last line is the expected second period profit when both of the vendors fail to develop the new technology. The first term of each line is the expected revenue generated by operating the network, the second term of each line is penalty cost for lost service, the third and the fourth terms have total acquisition and operating cost of the network. The final term is the total technology investment.

We assume that each vendor has an independent probability of successful innovation denoted by p_1 for vendor 1 and p_2 for vendor 2. The complement probabilities of failure are given as $q_1 = 1 - p_1$, and $q_2 = 1 - p_2$.

We first characterize the expected profit function.

Proposition 3.2.1 *The expected profit as given in (3.3) is jointly concave in* (C_1, C_2, I) *and the solution obtained from the first order conditions maximizes the total expected profit of the value chain.*

Proof: See Section A.1 in Appendix A.

The optimal capacity levels (C_1^*, C_2^*) and the optimal total investment I^* satisfy the first order conditions:

$$\frac{\partial \pi\left(C_1, C_2, I\right)}{\partial C_2} = (p_2 + q_2 p_1)[(a_2 + v_2)\bar{F}(C_1 + C_2|I) - o_2] - (p_2 m_2 + q_2 p_1 m_1) = 0$$

from which

$$\bar{F}(C_1 + C_2|I) = \left[\frac{p_2 m_2 + q_2 p_1 m_1}{(p_2 + q_2 p_1)} + o_2\right] / (a_2 + v_2) \tag{3.4}$$

$$\begin{aligned} \frac{\partial \pi\left(C_1, C_2, I\right)}{\partial C_1} &= (a_1 + v_1)\bar{G}(C_1) - (o_1 + m) + q_2 q_1[(a_2 + v_2)\bar{G}(C_1) - o_2] \\ &\quad +(p_2 + q_2 p_1)[(a_2 + v_2)\bar{F}(C_1 + C_2|I) - o_2] = 0 \end{aligned} \tag{3.5}$$

If we substitute (3.4) in (3.5) we derive C_1^* as:

$$C_1^* = G^{-1}\left[1 - \frac{(o_1 + m) + q_2 q_1 o_2 - (p_2 m_2 + q_2 p_1 m_1)}{[(a_1 + v_1) + q_2 q_1 (a_2 + v_2)]}\right] \tag{3.6}$$

$$\frac{\partial \pi\left(C_1, C_2, I\right)}{\partial I} = \frac{dE[D(I)]}{dI}(p_2 + q_2 p_1)\left[(a_2 + v_2)F(C_1 + C_2|I) - v_2\right] - 1 \tag{3.7}$$

If we substitute $F(C_1+C_2|I)$ from (3.4) in (3.7) we can derive optimal total investment, I^* from:

$$\left.\frac{dE[D(I)]}{dI}\right|_{I^*} = \frac{1}{(p_2 + q_2 p_1)(a_2 - o_2) - p_2 m_2 - q_2 p_1 m_1} \tag{3.8}$$

Note that the optimal investment level is independent of capacity decisions and the shortage penalty. As a managerial insight, the owner of the supply chain has an incentive to invest more as the marginal profit of selling unit new network capacity increases. Finally using the fact that $F(x|I) = G(x - \theta(I))$ we derive C_2^* as:

$$C_2^* = G^{-1}\left\{1 - \left[\frac{p_2 m_2 + q_2 p_1 m_1}{(p_2 + q_2 p_1)(a_2 + v_2)} + \frac{o_2}{a_2 + v_2}\right]\right\} + \theta(I^*) - C_1^* \tag{3.9}$$

In order for the capacity and investment decisions to make sense following two conditions have to be satisfied:

$$p_2 m_2 + q_2 p_1 m_1 < o_1 + m + q_2 q_1 o_2$$
$$(p_2 + q_2 p_1)(a_2 + v_2 - o_2) > p_2 m_2 + q_2 p_1 m_1$$

If the reverse of the first condition is true then there is no incentive to provide extra capacity in the second period (it is too expensive). The second condition states that if the expected revenue is larger than expected costs in the second period then it is worthwhile to invest in extra capacity.

3.2.1. Numerical Illustration

In this section, we illustrate the solution to the centralized system for two periods. We prepare a 2^3 factorial design to gain intuition about how the model parameters influence the decision variables and the expected profit. We determine the impacts of model parameters on network capacity and technology investment decisions statistically using factorial design. We determine three levels for all parameters for which the impact on system performance is not explicit. The random part of the demand, ε in both periods is taken as normally distributed with mean $\mu = 200$, and sigma $\sigma = 10$ or 20 or 40. The impact of investment I is reflected in the second period demand with $\theta(I) = \sqrt{I}$.

Unit sales price is fixed at $a = 100$ for both periods. First period manufacturing cost is $m = 30$ and manufacturing cost for both firms for new technology is also set $m_1 = m_2 = 30$. And, operating costs for both periods are $o_1 = o_2 = 20$. Penalty costs are fixed at $v_1 = v_2 = 25$ for both periods. Successful innovation probabilities, p_1 and p_2 of the firms have three levels, 0.2, 0.5, and 0.8. In our experimental design, we investigate the hidden effects of variance of demand and innovation probabilities of the vendors. Table 3.2 presents how the decision variables C_1, C_2, I change along with expected profit for different choices of model parameters. Note that probability of innovation is assumed to follow a geometric distribution and demand is stationary for the two periods if innovation does not materialize.

We analyze four different responses with respect to the variance of demand distribution, and innovation probability parameters by using analysis of variance (ANOVA) to gain some statistically supportable information.

First, we check the reaction of the first period network capacity, C_1, to the parameters. As it can be seen from Table B.1 in Appendix B, C_1 reacts to variance of demand distribution and innovation probabilities of the vendors, significantly. It is

Table 3.2. 2^3 Factorial design and responses

σ	p_1	p_2	C_1	C_2	I	Expected Revenue
10,00	0,80	0,50	209,08	27,47	506,25	26185
10,00	0,80	0,20	208,67	26,39	441,00	26055
10,00	0,20	0,20	206,63	16,42	81,00	25322
10,00	0,50	0,20	207,44	21,62	225,00	25618
10,00	0,80	0,80	209,57	28,48	576,00	26321
10,00	0,20	0,50	207,44	21,62	225,00	25618
10,00	0,50	0,50	208,13	24,67	351,56	25875
10,00	0,50	0,80	209,08	27,47	506,25	26185
10,00	0,20	0,80	208,67	26,39	441,00	26055
20,00	0,80	0,50	218,17	32,43	506,25	25283
20,00	0,20	0,50	214,87	28,23	225,00	24759
20,00	0,50	0,80	218,17	32,43	506,25	25283
20,00	0,80	0,20	217,33	31,77	441,00	25165
20,00	0,20	0,20	213,26	23,84	81,00	24476
20,00	0,50	0,50	216,27	30,58	351,56	25000
20,00	0,50	0,20	214,87	28,23	225,00	24759
20,00	0,20	0,80	217,33	31,77	441,00	25165
20,00	0,80	0,80	219,14	32,96	576,00	25404
40,00	0,80	0,80	238,29	41,91	576,00	23570
40,00	0,80	0,50	236,34	42,37	506,25	23478
40,00	0,50	0,20	229,74	41,46	225,00	23040
40,00	0,20	0,80	234,66	42,54	441,00	23385
40,00	0,50	0,50	232,54	42,42	351,56	23249
40,00	0,50	0,80	236,34	42,37	506,25	23478
40,00	0,80	0,20	234,66	42,54	441,00	23385
40,00	0,20	0,20	226,52	38,68	81,00	22785
40,00	0,20	0,50	229,74	41,46	225,00	23040

meaningful that first period network capacity of the operator is a new capacity and specifically depends on the first period demand characteristics. Moreover, first period network capacity is influenced by innovation probabilities of the vendors, hence first period capacity increases as innovation probabilities of the vendors increase.

Second, we check the reaction of the second period additional network capacity, C_2, to the parameters (Table B.2 in Appendix B). It is obvious that additional capacity is influenced by variance of demand distribution and innovation probabilities of the vendors significantly. Besides, additional network capacity C_2 has reaction to the joint effect of innovation probabilities (i.e., both vendors' innovation probabilities jointly affect the additional network capacity).

Third, we check the reaction of the total technology investment level, I, to the parameters (Table B.3 in Appendix B). As we foresee, investment levels are significantly influenced by innovation probabilities both independently and jointly when the other parameters are fixed.

Finally, we check the reaction of expected profit function of the centralized system, π, to the parameters (Table B.4 in Appendix B). It is clear that expected profit of the centralized system is influenced by the variance of the demand and the innovation probability of the vendors, both jointly and independently.

3.3. Decentralized Analysis

We now consider that each agent (two vendors and one operator) are independent self interested agents. In the decentralized setting, the game proceeds as follows: the vendors simultaneously announce their technology investment levels and unit prices. They offer the operator a contract; the operator accepts or rejects the contract; assuming the operator accepts the contract, she decides on the network capacity decisions for the upcoming two periods. Then the "nature moves" and two uncertainties are resolved: whether the innovation was successful or not is revealed and the demand is observed. The costs accrue and profits are collected at the end of the second period.

Hence a simultaneous (Nash) game is played between the vendors which is followed by a Stackelberg game between the vendors and the operator. In other words, the vendors announce their unit prices and investment levels of new technology, w_1, w_2, I_1, and I_2, then the operator follows by announcing her network capacities, C_1, and C_2, for the upcoming two periods. In what follows, we also assume that the game between the vendors is "nested" in the sense that the vendors first simultaneously announce their unit prices and then their technology investment levels in Section 3.3.3 and they announce their investment levels first followed by unit prices next in Section 3.3.5.

Moreover, the Stackelberg game between the vendors and the operator can only occur when technology is successfully adopted and the operator decides to *increase* her network capacity in the second period.

3.3.1. The Operator's Problem

On observing the vendors' decisions w_1, w_2, I_1 and I_2, the operator maximizes the following expected profit function:

$$\begin{aligned} \pi_o\left(C_1, C_2; w_1, w_2, I_1, I_2\right) &= \left(a_1 + v_1\right) S\left(C_1\right) - v_1 \mu - \left(o_1 + m\right) C_1 \\ &+ p_2\left(\left(a_2 + v_2\right) S\left(C_1 + C_2, I\right) - v_2 \mu\left(I\right) - o_2\left(C_1 + C_2\right) - w_2 C_2\right) \\ &+ q_2 p_1\left(\left(a_2 + v_2\right) S\left(C_1 + C_2, I\right) - v_2 \mu\left(I\right) - o_2\left(C_1 + C_2\right) - w_1 C_2\right) \\ &+ q_2 q_1\left(\left(a_2 + v_2\right) S\left(C_1\right) - v_2 \mu - o_2 C_1\right) \end{aligned} \tag{3.10}$$

where the interpretation is similar to (3.3) with manufacturing costs (m_1 and m_2) replaced by unit prices w_1 and w_2. Expected sales are influenced by the total technology investment of both vendors $I = \lambda I_1 + (1 - \lambda) I_2$.

Proposition 3.3.1 *The expected profit of the operator as given in (3.10) is jointly concave in (C_1, C_2) and the solution obtained from the first order conditions maximizes the total profit of the supply chain.*

Proof: See Section A.2 in Appendix A.

Since the operator's expected profit function is concave in (C_1, C_2), the optimal capacity levels (C_1, C_2) satisfy the following first order conditions:

$$\frac{\partial \pi_o\left(C_1, C_2\right)}{\partial C_2} = (p_2 + q_2 p_1)[(a_2 + v_2)\bar{F}(C_1 + C_2|I) - o_2] - (p_2 w_2 + q_2 p_1 w_1) = 0$$

from which

$$\bar{F}(C_1 + C_2|I) = \left[\frac{p_2 w_2 + q_2 p_1 w_1}{(p_2 + q_2 p_1)} + o_2\right] / (a_2 + v_2) \tag{3.11}$$

$$\begin{aligned} \frac{\partial \pi_o\left(C_1, C_2\right)}{\partial C_1} &= (a_1 + v_1)\bar{G}(C_1) - (o_1 + m) + q_2 q_1[(a_2 + v_2)\bar{G}(C_1) - o_2] \\ &\quad + (p_2 + q_2 p_1)[(a_2 + v_2)\bar{F}(C_1 + C_2|I) - o_2] = 0 \end{aligned} \tag{3.12}$$

If we substitute (3.11) in (3.12):

$$\begin{aligned} \frac{\partial \pi_o\left(C_1, C_2\right)}{\partial C_1} &= [(a_1 + v_1) + q_2 q_1(a_2 + v_2)]\bar{G}(C_1) - (o_1 + m) - q_2 q_1 o_2 \\ &\quad + [p_2 w_2 + q_2 p_1 w_1] = 0 \end{aligned} \tag{3.13}$$

from which we derive optimal capacity in period one as a function of unit capacity prices as:

$$C_1^*(w_1, w_2) = G^{-1}\left[1 - \frac{(o_1 + m) + q_2 q_1 o_2 - (p_2 w_2 + q_2 p_1 w_1)}{[(a_1 + v_1) + q_2 q_1(a_2 + v_2)]}\right] \tag{3.14}$$

Using the fact that $F(x|I) = G(x - \theta(I))$ we derive the second period capacity as a function of unit capacity prices and the total investment:

$$C_2^*(w_1, w_2, I) = G^{-1}\left\{1 - \left[\frac{p_2 w_2 + q_2 p_1 w_1}{(p_2 + q_2 p_1)(a_2 + v_2)} + \frac{o_2}{a_2 + v_2}\right]\right\} + \theta(I^*) - C_1^*(w_1, w_2) \tag{3.15}$$

Note that the first period capacity decision is independent of the technology investment as expected. However, as the expected cost of new technology $(p_2 w_2 + q_2 p_1 w_1)$ increases, C_1^* increases. The second period capacity increases with technology investment but its behavior with unit capacity prices is indeterminate.

There exists a minimum level of total investment which results nonnegative network capacity choice for the operator. The minimum level of investment to join the game is as follows:

$$\underline{I} = \theta^{-1}\left[C_1^*(w_1,w_2) - G^{-1}\left\{1 - \left[\frac{p_2w_2 + q_2p_1w_1}{(p_2+q_2p_1)(a_2+v_2)} + \frac{o_2}{a_2+v_2}\right]\right\}\right] \tag{3.16}$$

Note that if Nash equilibrium/equilibria exist/s in the vendors' game, the sum of their R&D investment levels have to be grater than the minimum total investment level in (3.16).

3.3.2. The Vendors' Problem

The vendors maximize their expected profit functions, π_1, and π_2 respectively as:

$$\begin{aligned} \max_{w_1,I_1} \pi_1\left(w_1, I_1; w_2, I_2\right) &= q_2p_1\left(w_1 - m_1\right)C_2(w_1,w_2,I) - I_1 \\ \text{s.t.} \qquad & w_1 \in [w_2, a_2 - o_2] \\ & I_1 \in (0,\infty) \end{aligned} \tag{3.17}$$

$$\begin{aligned} \max_{w_2,I_2} \pi_2\left(w_2, I_2; w_1, I_1\right) &= p_2\left(w_2 - m_2\right)C_2(w_1,w_2,I) - I_2 \\ \text{s.t.} \qquad & w_2 \in [m_2, w_1] \\ & I_2 \in (0,\infty) \end{aligned} \tag{3.18}$$

Both vendors make an initial investment in technology and then charge a unit price for unit capacity with the new technology. Both vendors aim to make the second period capacity of the operator, C_2 as high as possible.

Moreover, the lower bound of w_1 is determined by the asymmetric structure of the vendors as $w_1 \geq w_2$ (i.e., the first vendor cannot charge a unit price which is lower than the second vendor). The upper bound of w_1 is the profit obtained in the second period (since a higher price would make the operator to gain negative profits). Furthermore, it is obvious that the first vendor needs to make some positive R&D investment to enter the game. For the second vendor a natural lower bound for w_2 is m_2, the manufacturing price, and the upper bound is w_1.

We now characterize the expected profit of the vendors.

Proposition 3.3.2 *The expected profits of the first (second) vendor as given in (3.17) (as given in (3.18)) is concave in I_1, (I_2).*

Proof: See Section A.3 in Appendix A.

3.3.3. The Vendors' Nested Game

We analyze the case where the vendors play a nested game as follows: they first determine and announce their unit capacity prices for the new technology (w_1, w_2) simultaneously and then they decide on their levels of technology investments (I_1, I_2) and announce it simultaneously.

Note that the interaction of the vendors are via the second period capacity of the operator $(C_2(w_1, w_2, I))$.

The response function of the first vendor is given as follows:

$$\begin{aligned} I_1^*(w_1; w_2, I_2) &= \max_{I_1} \pi_1(w_1, I_1; w_2, I_2) \\ w_1^*(w_2, I_2) &= \max_{w_1} \pi_1(w_1, I_1^*(w_1; w_2, I_2); w_2, I_2) \end{aligned}$$

Analogously for the second vendor the response functions are given as:

$$
\begin{aligned}
I_2^*(w_2; w_1, I_1) &= \max_{I_2} \pi_2(w_2, I_2; w_1, I_1) \\
w_2^*(w_1, I_1) &= \max_{w_2} \pi_2(w_2, I_2^*(w_2; w_1, I_1); w_1, I_1)
\end{aligned}
$$

Proposition 3.3.2 guarantees that the responses are indeed functions (not correspondences). Since we know that the vendors' profit functions are concave in investments we first solve for $I_1^*(w_1; w_2, I_2)$ and $I_2^*(w_2; w_1, I_1)$ from:

$$
\left.\frac{\partial E[D(I)]}{\partial I_1}\right|_{I_1^*} = \frac{1}{q_2 p_1(w_1 - m_1)} \tag{3.19}
$$

$$
\left.\frac{\partial E[D(I)]}{\partial I_1}\right|_{I_2^*} = \frac{1}{p_2(w_2 - m_2)} \tag{3.20}
$$

Unfortunately these responses are not explicit but nevertheless computable. Once the investment responses are obtained, unit price responses can also be computed.

Hence the pair

$$
\{w_1^*(w_2^*, I_2^*), w_2^*(w_1^*, I_1^*)\}
$$

will be a Nash equilibrium by definition if it exists.

At this point, we are not able to characterize whether a (unique) Nash equilibrium exists unless we impose unnatural conditions that involve the derivative of the pdf of the demand.

Instead we impose more structure first to gain more insight about the nature of the problem in the next subsection.

3.3.4. A Counter Example under Uniform Demand

We provide a counter example showing that such a nested game under all assumptions of the game and demand distribution has no equilibrium. To construct such an example, demand distribution is determined as $D(I) = \sqrt{I} + \varepsilon$ where ε is distributed uniformly on a finite interval, [A,B]. Note that this demand distribution satisfies all the demand assumptions of the game.

First of all, in order for optimal capacity decisions of the operator, C_1 given in (3.14), and C_2 given in (3.15), to be legitimate capacity levels under uniform demand, we assume that $p_2 w_2 + q_2 p_1 w_1 < o_1 + m + q_2 q_1 o_2$ and $v_2 > 0$ hold simultaneously. If the reverse of the first condition is valid then uploading of the extra network is meaningless because it is too expensive.

The expected profit functions, $\pi_1(I_1; I_2(w_2), w_1)$, and $\pi_2(I_2; I_1(w_1), w_2)$ at the second stage are concave in I_1, and I_2 (see Proposition 3.3.2). Hence, we can find out the best technology investment by employing FOCs as follows:

$$I_1^*(w_1, I_2) = \frac{\left[\frac{1}{2}\lambda q_2 p_1 (w_1 - m_1)\right]^2 - (1-\lambda) I_2}{\lambda} \tag{3.21}$$

$$I_2^*(w_2, I_1) = \frac{\left[\frac{1}{2}(1-\lambda) p_2 (w_2 - m_2)\right]^2 - \lambda I_1}{(1-\lambda)} \tag{3.22}$$

At the first stage, the vendors play the game in wholesale prices, and technology investment levels now are the functions of wholesale prices such that

$$\pi_1(w_1; I(w_2)) = q_2 p_1 (w_1 - m_1) C_2 - I_1(w_1) \tag{3.23}$$

$$\pi_2(w_2; w_1(I_1)) = p_2 (w_2 - m_2) C_2 - I_2(w_2) \tag{3.24}$$

Proposition 3.3.3 *The expected profits of the first (second) vendor as given in (3.23) (as given in (3.24)) is concave in w_1 (w_2) for uniform demand distribution*

Proof: See Section A.4 in Appendix A.

Since the expected profit of first vendor (second vendor) at the first stage is concave, optimal w_1 (w_2) appears either at the FOCs or at the bounds of defined sets. See Section A.4 in Appendix A.

However, since Nash equilibrium is the best response to best response of the competitors if we substitute (3.21) in (3.22) and vice versa at the second stage:

$$\begin{aligned} I_1^*(w_1;w_2) &= \frac{\left[\frac{1}{2}\lambda q_2 p_1 (w_1 - m_1)\right]^2 - \left[\frac{1}{2}(1-\lambda) p_2 (w_2 - m_2)\right]^2}{2\lambda} \\ I_2^*(w_2;w_1) &= \frac{\left[\frac{1}{2}(1-\lambda) p_2 (w_2 - m_2)\right]^2 - \left[\frac{1}{2}\lambda q_2 p_1 (w_1 - m_1)\right]^2}{2(1-\lambda)} \end{aligned}$$

Since, a positive technology investment is required to start the game, i.e., $I_1^*(w_1;w_2) > 0$ and $I_2^*(w_2;w_1) > 0$, the following two inequalities must hold at the Nash equilibrium point(s).

$$\frac{1}{2}\lambda q_2 p_1 (w_1 - m_1) - \frac{1}{2}(1-\lambda) p_2 (w_2 - m_2) > 0 \tag{3.25}$$

$$\frac{1}{2}(1-\lambda) p_2 (w_2 - m_2) - \frac{1}{2}\lambda q_2 p_1 (w_1 - m_1) > 0 \tag{3.26}$$

Clearly inequalities (3.25) and (3.26) cannot hold simultaneously under the model restrictions and parameter assumptions. Therefore, the nested game of the vendors under the wholesale contract has no equilibrium. As a technical note, bounds of the uniform demand, [A,B], does not cause any problem in the analysis due to our additive demand assumption where uncertain demand is independent of R&D investment levels.

3.3.5. The Vendors' Reverse Nested Game

The vendors first determine and announce their levels of technology investments (I_1, I_2) simultaneously and then they decide on their unit capacity prices for the new technology (w_1, w_2) and announce it simultaneously. In addition, the interaction of the vendors are via the second period capacity of the operator $(C_2(w_1, w_2, I))$.

The response function of the first vendor is given as follows:

$$\begin{aligned} w_1^*(w_2, I_2) &= \max_{w_1} \pi_1(w_1, I_1; w_2, I_2) \\ I_1^*(w_1; w_2, I_2) &= \max_{I_1} \pi_1(I_1, w_1^*(I_1; w_2, I_2); w_2, I_2) \end{aligned}$$

Analogously for the second vendor the response functions are given as:

$$\begin{aligned} w_2^*(w_1, I_1) &= \max_{w_2} \pi_2(w_2, I_2; w_1, I_1) \\ I_2^*(w_2; w_1, I_1) &= \max_{I_2} \pi_2(w_2, I_2^*(w_2; w_1, I_1); w_1, I_1) \end{aligned}$$

Unfortunately, all the problems we have encountered in Section 3.3.3 are still valid in here. However, to light up some points and gain some perception we give an illustrative game example under uniform distribution, again.

3.3.6. Reverse Nested Game under Uniform Demand

This example is provided to show that such a reverse nested game has an equilibrium in a very limited game space, however, has no equilibrium rest of it. All the demand assumptions are the same in Section 3.3.4. Once again, the vendors play investment game at the first place and then they play unit capacity price game at the second stage of the game. Note that $\overline{w_1}$, and $\overline{w_2}$ stands for the maximum value that the unit price can take and $\underline{w_1}$, and $\underline{w_2}$ denote the minimum value of the unit price.

Proposition 3.3.4 *The expected pay-off functions of the vendors, $\pi_1(w_1; w_2, I)$ and $\pi_2(w_2; w_1, I)$, are concave under uniform demand distribution in w_1, and w_2, respectively. And, we can characterize the best unit price levels, w_1, and w_2, by employing FOCs as follows:*

1. *For the first vendor;*

$$w_1^* = \begin{cases} \overline{w_1} = a_2 - o_2 & if \qquad \left.\frac{\partial \pi_1}{\partial w_1}\right|_{\overline{w_1}} > 0 \\ w_1^{'} & if \quad \left.\frac{\partial \pi_1}{\partial w_1}\right|_{\underline{w_1}} > 0 \quad and \quad \left.\frac{\partial \pi_1}{\partial w_1}\right|_{\overline{w_1}} < 0 \\ \underline{w_1} = \max\{w_2, m_1\} & if \qquad \left.\frac{\partial \pi_1}{\partial w_1}\right|_{\underline{w_1}} < 0 \end{cases}$$

where $w_1^{'}$ can be obtained from $\left.\frac{\partial \pi_1}{\partial w_1}\right|_{w_1^{'}} = q_2 p_1 \left[C_2\left(w_1^{'}, w_2, I\right) - q_2 p_1 A\left(w_1^{'} - m_1\right)\right] = 0$ *such that* $A = \frac{1}{(p_2+q_2p_1)(a_2+v_2)g(\Omega)} + \frac{1}{[(a_1+v_1)+q_2q_1(a_2+v_2)]g\left(C_1^*\right)}$ *and*
$\Omega = G^{-1}\left\{1 - \left[\frac{p_2w_2+q_2p_1w_1}{(p_2+q_2p_1)(a_2+v_2)} + \frac{o_2}{a_2+v_2}\right]\right\}$

2. *For the second vendor;*

$$w_2^* = \begin{cases} \overline{w_2} = w_1 & if \qquad \left.\frac{\partial \pi_2}{\partial w_2}\right|_{\overline{w_2}} > 0 \\ w_2^{'} & if \quad \left.\frac{\partial \pi_2}{\partial w_2}\right|_{\underline{w_2}} > 0 \quad and \quad \left.\frac{\partial \pi_2}{\partial w_2}\right|_{\overline{w_2}} < 0 \\ \underline{w_2} = m_2 & if \qquad \left.\frac{\partial \pi_2}{\partial w_2}\right|_{\underline{w_2}} < 0 \end{cases}$$

where $w_2^{'}$ can be obtained from $\left.\frac{\partial \pi_2}{\partial w_2}\right|_{w_2^{'}} = p_2 \left[C_2\left(w_1, w_2^{'}, I\right) - p_2 A\left(w_2^{'} - m_2\right)\right] = 0.$

Proof: See Section A.5 in the Appendix.

When we start to solve investment game of the vendors, we encounter nine different cases, obtaining by cross matching of the cases for each vendor. Instead of case by case evaluation of the problem, we just illustrate sample cases where Nash equilibrium is possible to exist, and impossible to exist.

First of all, if it is optimal for one of vendors to set its unit price to its lowest value, $\underline{w_1}$, or $\underline{w_2}$, at the second stage of reverse nested game, it makes negative revenue by investing at the first stage. Recall that vendors only make money by selling extra network capacity to the operator with a unit price. (See the expected revenue function, (3.17) for the first vendor, and (3.18) for the second vendor) Therefore, there is no Nash equilibrium for the investment game of the vendors in the first stage of the reverse nested game when it is optimal to set the unit price of new technology to its lowest value.

Secondly, suppose that it is optimal to determine $w_1 = w_1^* = \overline{w_1} = a_2 - o_2$ for the first vendor at the second stage, and for the second vendor $w_2^* = \overline{w_2} = w_1 = a_2 - o_2$. Under this assumption, the pay-off functions, $\pi_1(I_1; w_1, w_2, I_2)$, and $\pi_2(I_2; w_1, w_2, I_1)$ are supermodular. Because cross partial derivatives of the pay-off functions given in (3.27) are nonnegative after employing a common trick in the literature such that $I_1 = -I_1^{'}$. Note that cross partial derivative of demand function is assumed to be negative due to the demand assumptions of the game. The pay-off functions are supermodular, hence, the investment game is supermodular. Therefore, supermodularity theorem holds for the game. It means the game has at least one Nash equilibrium [20]. However, due to our special game restrictions, Nash equilibrium exists above the minimum investment level such that $\lambda I_1^* + (1-\lambda) I_2^* \geq \underline{I}$.

$$\begin{aligned}\frac{\partial^2 \pi_1}{\partial I_1 \partial I_2} &= q_2 p_1 \left(a_2 - o_2 - m_1\right) \frac{\partial^2 ED\left[I\right]}{\partial I_1 \partial I_2} < 0 \\ \frac{\partial^2 \pi_2}{\partial I_2 \partial I_1} &= p_2 \left(a_2 - o_2 - m_2\right) \frac{\partial^2 ED\left[I\right]}{\partial I_2 \partial I_1} < 0\end{aligned} \tag{3.27}$$

Finally, suppose that the first vendor sets his unit price as a function of total investment, I, and unit price of his rival, w_2, such that $w_1^* = w_1^{'} = \tau_1\left(I, w_2\right)$, and the second vendor sets his unit price as a function of total investment, I, and unit price of his rival, w_1, such that $w_2^* = w_2^{'} = \tau_2\left(I, w_1\right)$. Unfortunately, we are unable to show w_1^*, and w_2^* as explicit functions derived from FOCs, nevertheless, we abbreviate them as $\tau_1\left(I, w_2\right)$, and $\tau_2\left(I, w_1\right)$, implicitly. Then, the vendors problem, π_1, and π_2, can be expressed as, respectively:

$$\begin{aligned}
\max_{I_1} \pi_1 \left(I_1; w_1^*(I, w_2), w_2^*(I, w_1), I_2\right) &= q_2 p_1 \left(w_1 - m_1\right) C_2(w_1, w_2, I) - I_1 \\
\text{s.t. } w_1 &= \tau_1 \left(I, w_2\right)
\end{aligned} \tag{3.28}$$

$$\begin{aligned}
\max_{I_2} \pi_2 \left(I_2; w_1^*(I, w_2), w_2^*(I, w_1), I_1\right) &= p_2 \left(w_2 - m_2\right) C_2(w_1, w_2, I) - I_2 \\
\text{s.t. } w_2 &= \tau_2 \left(I, w_1\right)
\end{aligned} \tag{3.29}$$

After substituting the w_1^*, and w_2^* values, obtained from FOCs (Proposition 3.3.4), into objective functions of the vendors, the pay-off functions turn out to be:

$$\begin{aligned}
\max_{I_1} \pi_1 \left(I_1; w_1^*(I, w_2), w_2^*(I, w_1), I_2\right) &= \frac{1}{A} \left[C_2(w_1^*, w_2^*, I)\right]^2 - I_1 \\
\max_{I_2} \pi_2 \left(I_2; w_1^*(I, w_2), w_2^*(I, w_1), I_1\right) &= \frac{1}{A} \left[C_2(w_1^*, w_2^*, I)\right]^2 - I_2
\end{aligned} \tag{3.30}$$

When we take the difference of the pay-off functions in (3.30), we end up with $\pi_1 - \pi_2 = -I_1 + I_2$. If we take the derivative of the difference of the pay-off functions with respect to decision variables, I_1, and I_2, we obtain the following result:

$$\begin{array}{l}
\frac{\partial(\pi_1 - \pi_2)}{\partial I_1} = -1 < 0 \\
\frac{\partial(\pi_1 - \pi_2)}{\partial I_2} = 1 > 0
\end{array} \tag{3.31}$$

The interpretation of (3.31) is as follows: the first vendor keeps losing money by investing one unit more, and makes his rival earn money, so does the second vendor. As long as the vendors are self-interested agents, none of the vendors has an incentive to invest under these conditions. This result resembles the preemption argument of Fudenberg and Tirole [19]. Therefore, there is no Nash equilibrium as long as the vendors are self-interested. The same argument used in this case is also valid for the rest of the cases where the vendors determine their unit price of technology as a function of their investment.

Briefly, there might be an equilibrium when unit prices, w_1, and w_2, are not functions of R&D investments, I_1, and I_2, but there is no equilibrium when the unit prices, w_1, and w_2, are functions of R&D investments. The interpretation of this situation can be that an innovator is more conservative about R&D investment as long as his rivals in the market are able to benefit from the demand which is manipulated by him. We have been contented with showing illustrative cases, because our main objective in this example is to reveal an equilibrium as an example, in contrary to Section 3.3.4.

4. SINGLE INVESTMENT GAME OF VENDORS

In this chapter we focus on how vendors in telecommunication value chain direct their research and development (R&D) investments. Plainly, we are concentrated on an industry composed of two competitive vendors and a single operator in a game theoretical framework. Vendors supply the operator's communication network either as hardware or software and race against each other to develop a new technology and sell it to the operator. In our game structure, each vendor makes a R&D investment at the beginning of development stage and sell their newly developed technology to the operator at the following period in case it is materialized. In particular, the operator is always willing to buy and adopt the new technology because it uses current best technology insofar as it's possible. What the new technology brings to the operator is it boosts up the operator's demand by providing better communication service.

We visualize the second model in Figure 4.1.

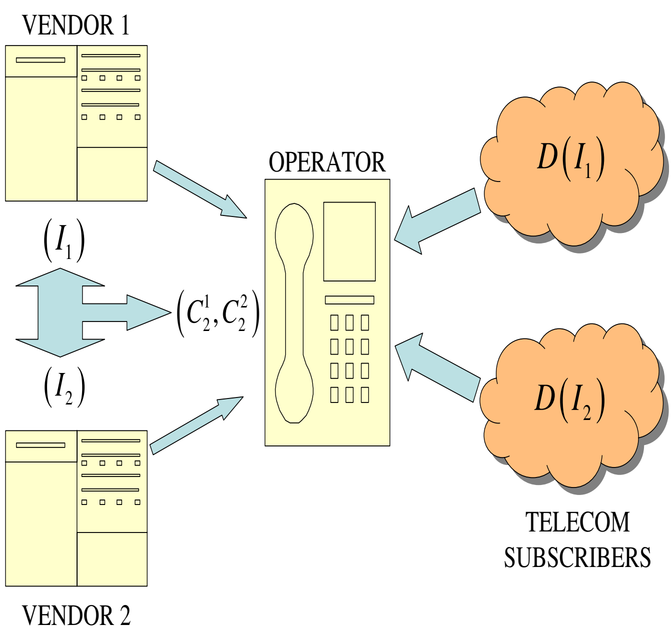


Figure 4.1. The description of the second model

When the new technology is materialized by either one of the vendors or both of them, they sell it to the operator at a unit price so that the operator expands its network capacity. The price of the new technology is determined by the market, which means the unit price is an exogenous parameter in our model. After that, the operator buys the new technology and uses it in its network to enhance the communication service to the customers. Again, it is assumed that the operator determines her network capacity with respect to the "peak demand" of the telecom network. All cost parameters and the telecom service price of the operator are constant during the periods. Moreover, the operator incurs penalty cost for unsatisfied network demand, and salvaging of unused capacity at the end of the planning period is not allowed.

Furthermore, vendors compete for the same, substitutable new technology in this game. It is assumed that each vendor either accomplishes the new technology with its own common innovation probability or does not with its complementary probability because in the real world, each firm has its own capability of innovation. Furthermore, the unit price of the new technology is assumed to be exogenous in the system due to the knowledge of free market, although there are cases where this is not true.

Another important assumption is that individual investment in new technology influences the demand of the operator positively though it can be true or not in different circumstances. However, the scope of this study is restricted to examine effort dependent demand, which means we limit ourselves to investigate the demand which can be raised by exerting some effort.

The vendors compete via unit price of new technology in the model in Chapter 3, but, in this model, they compete via the influence of new technology on service demand of the operator. The major difference in the demand structure is that *total* R&D *investment* made by the vendors was jointly triggering the demand of the operator in Chapter 3, but *individual* R&D *investment* affects the demand separately in this chapter. Another difference is that unit price of new technology, w was a decision variable, and endogenous to the model, but it is exogenous and determined by the market now. Final difference is that extra network capacity bought by the operator was totally substitutable in Chapter 3, however, the operator engages to business with only one of the vendors here whereas the new technology offered by the vendors is still entirely substitutable.

4.1. The Centralized Model

In the centralized model, we mainly investigate how the central decision maker who is totally able to control the telecommunication supply chain involving two vendors and an operator manages the system.

First of all, the operator is a start up company and has to determine adequate capacity for the first period initially. And, for the second period it has to decide on how much capacity it will add to its own network capacity by using new technology which is put up for sale by vendors. Moreover, it is assumed that the operator buys all extra network capacity from only one of the vendors who provides more profit to her. This assumption was structurally fixed in Chapter 3. At the beginning of the first period, the central decision maker determines the first period capacity with respect to that period's needs and the amount of R&D investments. In addition, at the beginning of the second period, it determines the extra necessary network capacity and also chooses the appropriate vendor.

Table 4.1 presents the notation used in the model.

Decision variables are endogenous to the models and all other parameters are exogenously determined.

In addition to demand assumptions in Section 3.1, we assume in order for the value chain to earn a positive revenue:

$$
\begin{aligned}
&a_1 > o_1 + m \\
&a_2 > o_2 + m_1 \qquad or \qquad a_2 > m_2 + o_2
\end{aligned}
$$

Table 4.1. Model parameters and decision variables for the second model

$D_t(I)$:	Random demand in period t as a function of the technology investment
m :	Manufacturing cost per unit capacity for the first period
m_j :	Second period manufacturing cost of vendor j per unit capacity, $j = 1, 2$.
o_t :	Operating cost of the operator per unit capacity in period t
v_t :	Penalty cost of unsatisfied demand per unit capacity in period t
a_t :	Revenue generated by operator per unit utilized capacity in period t
C_1 :	Capacity of operator's network in period 1 (Decision variable)
C_2^j :	Capacity of operator's network in period 2 (Decision variable), $j = 1, 2$.
I_j :	Technology investment of vendor j (Decision variable), $j = 1, 2$.

4.1.1. Analysis of the Centralized Model

The central decision maker has to decide on the amount of R&D investments for both of the vendors at the beginning of the first period (technology development period) and the amount of extra network capacity at the beginning of the second period (technology usage period). The first period is the development period of the new technology and its demand is not influenced by the likely upcoming technology at the next period. Let $S(C_1)$ be the expected service delivered by the operator in the first period before the new technology is adopted. Moreover, since we concentrate on the new technology adoption we do not let the operator cut its capacity down at the second period if the innovation does not appear. However, capacity reduction decision can easily be handled by creating an additional decision variable but it is left out of this study.

Expected service in the first period is given as exactly as in 3.1. If the successful new technology is produced by either one or both of the vendors at the beginning of the second period the central decision maker can load up extra capacity from only one of the vendors. In other words, the central decision maker can build up extra capacity either as C_2^1 or C_2^2 which denotes the second period extra network capacity bought from first vendor or second vendor, respectively. The expected service delivered in period 2 is given as:

$$
\begin{aligned}
S\left(C_1 + C_2^j, I_j\right) &= \int\limits_0^{C_1+C_2^j} xF\left(x|\, I_j\right) + \left(C_1 + C_2^j\right)\bar{F}\left(C_1 + C_2^j \middle|\, I_j\right) \\
&= C_1 + C_2^j - \int\limits_0^{C_1+C_2^j} F(x|I_j)dx \qquad j = 1, 2. \qquad (4.1)
\end{aligned}
$$

where $\bar{F}(\cdot)$ denotes $1 - F(\cdot)$.

The expected profit for the centralized system at the first period is given as:

$$\pi(C_1) = (a_1 + v_1)\, S(C_1) - v_1\mu_1 - (o_1 + m)\, C_1 \qquad (4.2)$$

Not only for necessity of telecommunication network existence before technology adoption but also for similarity to Chapter 3, we assume that there exists a telecommunication system before the innovation race starts. Apparently, first period network capacity, C_1 is a nominal capacity built with unit price m, which is not associated with possible imminent innovation. Simply, the expected revenue in the first period includes first period revenue and cost parameters.

Proposition 4.1.1 *The expected profit as given in (4.2) is strictly concave in* (C_1) *and the amount of capacity obtained from the first order condition maximizes the total expected profit of the value chain in the first period.*

Proof: See Section A.6 in Appendix A.

The optimal capacity level (C_1^*) satisfies the first order condition:

$$\frac{d\pi}{d\,(C_1)} = (a_1 + v_1)\, \overline{G}\,(C_1) - o_1 - m \qquad (4.3)$$

The optimal (C_1^*) can be extracted as shown below by equalizing (4.3) to zero:

$$C_1^* = G^{-1}\left(1 - \frac{o_1 + m}{a_1 + v_1}\right) \tag{4.4}$$

Unlike the best C_1^* given in (3.6), C_1^* given in (4.4) does not contain any cost related with manufacturing of new technology in the second period.

The expected profit for the centralized system at the second period, $\pi(I_1, I_2, C_2^1, C_2^2)$ is given as:

$$\begin{aligned}
= \quad & q_2 p_1 \left\{(a_2 + v_2)\, S(C_1 + C_2^1 \middle| I_1) - v_2 \mu_2 (I_1) - o_2 \left(C_1 + C_2^1\right) - m_1 C_2^1\right\} \\
+ \quad & q_1 p_2 \left\{(a_2 + v_2)\, S(C_1 + C_2^2 \middle| I_2) - v_2 \mu_2 (I_2) - o_2 \left(C_1 + C_2^2\right) - m_2 C_2^2\right\} \\
+ \quad & p_1 p_2 \left\{ \max \left(\begin{array}{c} (a_2 + v_2)\, S(C_1 + C_2^1 | I_1) - v_2 \mu_2 (I_1) - o_2 (C_1 + C_2^1) - m_1 C_2^1, \\ (a_2 + v_2)\, S(C_1 + C_2^2 | I_2) - v_2 \mu_2 (I_2) - o_2 (C_1 + C_2^2) - m_2 C_2^2 \end{array} \right) \right\} \\
+ \quad & q_1 q_2 \left\{(a_2 + v_2)\, S(C_1) - v_2 \mu_2 - o_2 C_1\right\} - I_1 - I_2
\end{aligned} \tag{4.5}$$

The first line is the expected gain if the first vendor comes up with a successful innovation and the second one fails. The second line reflects the exact opposite of the first line. The tricky part is the third line because it reflects that central decision maker decides on how much extra capacity to build up at the second period by comparing the revenues generated by choosing either first or the second vendor. To recall it again, we assumed that the operator establishes business relationship with just one of the vendors, logically with the most profitable one, when they both come up with a successful innovation. The final line represents the revenue gained by servicing with existing network capacity at the second period if none of the vendors succeed in innovation. The first term of each line is the expected revenue generated by operating the network, the second term of each line is penalty cost for lost service, the third and the fourth terms have total acquisition and operating cost of the network. Finally, the last two negative terms show the R&D investments by the decision maker at the beginning of the technology development stage.

Moreover, it is assumed that the successful innovation probabilities of the vendors are independent of each other and follow a geometric distribution. Obviously, p_j stands for the innovation probability for the j^{th} vendor and q_j is the complement of it, where $j = 1, 2$.

Through a series of transformations the expected profit of the centralized system in (4.5) can be written as in (4.9). We make two definitions:

$$\begin{aligned} \pi^0 &= (a_2 + v_2)\, S(C_1) - v_2\mu_2 - o_2 C_1 \\ \pi^j &= (a_2 + v_2)\, S(C_1 + C_2^j \big| \, I_j) - v_2\mu_2\,(I_j) - o_2\left(C_1 + C_2^j\right) - m_1 C_2^j, j = 1, 2 \end{aligned} \tag{4.6}$$

Lemma 4.1.1 *π^1, (π^2), as given in (4.6) is jointly concave and monotonously increasing in I_1, C_2^1, (I_2, C_2^2).*

Proof: See Section A.7 in Appendix A.

If we rewrite the expected revenue of the centralized system by substituting (4.6) into (4.5):

$$\pi = q_2 q_1 \pi^0 + q_2 p_1 \pi^1 + q_1 p_2 \pi^2 + p_1 p_2 \max\left(\pi^1, \pi^2\right) - I_1 - I_2$$

which can be written as:

$$\pi = \begin{cases} q_2 q_1 \pi^0 + p_2 \pi^2 + q_2 p_1 \pi^1 - I_1 - I_2 & if \quad \pi^1 \leq \pi^2 \\ q_2 q_1 \pi^0 + p_1 \pi^1 + q_1 p_2 \pi^2 - I_1 - I_2 & if \quad \pi^1 \geq \pi^2 \end{cases} \tag{4.7}$$

Lemma 4.1.2 *Without considering the equality conditions in (4.7) , there exist two pairs of (I_1, I_2) solutions for each branch of (4.7) such that ($\overline{I_1}, \overline{\overline{I_2}}$), and ($\overline{\overline{I_1}}, \overline{I_2}$) where*

$\overline{C_2^{j*}} = \theta\left(\overline{I_j}\right) + G^{-1}\left(1 - \frac{o_2+m_j}{a_2+v_2}\right) - C_1^*,\ j = 1, 2,\ and$

$$\begin{aligned}
\left.\frac{\partial ED(I_1)}{\partial(I_1)}\right|_{\overline{I_1}} &= \frac{1}{q_2 p_1 (a_2 - o_2 - m_1)} \\
\left.\frac{\partial ED(I_2)}{\partial(I_2)}\right|_{\overline{\overline{I_2}}} &= \frac{1}{p_2 (a_2 - o_2 - m_2)} \\
\left.\frac{\partial ED(I_1)}{\partial(I_1)}\right|_{\overline{\overline{I_1}}} &= \frac{1}{p_1 (a_2 - o_2 - m_1)} \\
\left.\frac{\partial ED(I_2)}{\partial(I_2)}\right|_{\overline{I_2}} &= \frac{1}{q_1 p_2 (a_2 - o_2 - m_2)}
\end{aligned}$$

Proof: See Section A.8 in Appendix A.

Lemma 4.1.2 implies that there are only two pairs of possible investment as the solution of this problem. Another observation is that the central chain owner tends to increase the R&D investment levels when the marginal profit of the new technology is getting higher.

Note that the central decision maker determines from which vendor it buys the new technology arbitrarily when both of the vendors materialize the successful innovation and cause same amount of profit for the operator since the operator's election system between the vendors depends on the profitability. Simply, both vendors are equally likely to sell their innovation.

Centralized system expected revenue function, (4.7), can be rewritten as follows using the fact that $q_j = 1 - p_j$, $j = 1, 2$:

$$\pi = q_2 q_1 \pi^0 + p_1 \pi^1 + p_2 \pi^2 - I_1 - I_2 - p_1 p_2 \begin{cases} \pi^1 & if \quad \pi^1 < \pi^2 \\ \{\pi^1, \pi^2\} & if \quad \pi^1 = \pi^2 \\ \pi^2 & if \quad \pi^1 > \pi^2 \end{cases} \tag{4.8}$$

Central decision maker tries to maximize his expected pay-off, therefore, his problem can be defined as follows equivalently to (4.8).

$$\max \pi = \max\left\{q_1 q_2 \pi^0 + p_1 \pi^1 + p_2 \pi^2 - I_1 - I_2\right\} - p_1 p_2 \min\left\{\max \pi^1, \max \pi^2\right\} \tag{4.9}$$

Lemma 4.1.3 *There are investment levels ($\widetilde{I}_1$ and $\widetilde{I}_2$) that maximize the first part of (4.9) such that*

$$\left.\frac{dED\left(I_1\right)}{d\left(I_1\right)}\right|_{\widetilde{I}_1} = \frac{1}{p_1\left(a_2 - o_2 - m_1\right)} \tag{4.10}$$

$$\left.\frac{dED\left(I_2\right)}{d\left(I_2\right)}\right|_{\widetilde{I}_2} = \frac{1}{p_2\left(a_2 - o_2 - m_2\right)} \tag{4.11}$$

Proof: See Section A.9 in Appendix A.

$\widetilde{I}_j$ is an upper bound in the sense that for any investment level $I_j > \widetilde{I}_j$, π is non-optimal. Then, extra network capacity levels associated with $\widetilde{I}_1$ and $\widetilde{I}_2$ in Lemma 4.1.3 can be characterized from the FOCs as follows (See Section A.9 in Appendix A):

$$\widetilde{C_2^1} = \theta\left(\widetilde{I}_1\right) + G^{-1}\left(1 - \frac{o_2 + m_1}{a_2 + v_2}\right) - C_1^* \tag{4.12}$$

$$\widetilde{C_2^2} = \theta\left(\widetilde{I}_2\right) + G^{-1}\left(1 - \frac{o_2 + m_2}{a_2 + v_2}\right) - C_1^* \tag{4.13}$$

where C_1^* is found from (4.4).

Lemma 4.1.4 *The expected revenue of the centralized system, (4.8) can be redefined as follows by using the upper level investments, ($\widetilde{I}_1$ and $\widetilde{I}_2$):*

$$\pi = \begin{cases} q_2 q_1 \pi^0 + p_1 \pi^1 + p_2 \pi^2 - p_1 p_2 \pi^1 - I_1 - I_2 & if \quad \pi^1\left(\widetilde{I}_1, \widetilde{C_2^1}\right) < \pi^2\left(\widetilde{I}_2, \widetilde{C_2^2}\right) \\ q_2 q_1 \pi^0 + p_1 \pi^1 + p_2 \pi^2 - p_1 p_2 \left\{\pi^1, \pi^2\right\} - I_1 - I_2 & if \quad \pi^1\left(\widetilde{I}_1, \widetilde{C_2^1}\right) = \pi^2\left(\widetilde{I}_2, \widetilde{C_2^2}\right) \\ q_2 q_1 \pi^0 + p_1 \pi^1 + p_2 \pi^2 - p_1 p_2 \pi^2 - I_1 - I_2 & if \quad \pi^1\left(\widetilde{I}_1, \widetilde{C_2^1}\right) > \pi^2\left(\widetilde{I}_2, \widetilde{C_2^2}\right) \end{cases} \tag{4.14}$$

Proof: See Section A.10 in Appendix A.

A special case for centralized solution in which manufacturing cost of new technology is equal for both vendors, $(m_1 = m_2)$ can be represented as follows:

$$\pi = \begin{cases} p_1\pi^1 + p_2\pi^2 - p_1p_2\pi^1 - I_1 - I_2 & if \quad p_1 < p_2 \\ p_1\pi^1 + p_2\pi^2 - p_1p_2\{\pi^1, \pi^2\} - I_1 - I_2 & if \quad p_1 = p_2 \\ p_1\pi^1 + p_2\pi^2 - p_1p_2\pi^2 - I_1 - I_2 & if \quad p_1 > p_2 \end{cases} \tag{4.15}$$

Finally, optimal capacity, (C_2^{1*}, and C_2^{2*}) and optimal investment levels, (I_1^*, and I_2^*) can be determined by using FOCs of the π function because each branch of π is mutually exclusive from each other and they are all strictly concave owing to the summation of strictly concave and linear functions.

Theorem 4.1.1 *Optimal decision variables that maximize (4.9) are characterized as follows:*

$$\left(I_1^*, I_2^*, C_2^{1*}, C_2^{2*}\right) = \begin{cases} \left(\overline{I_1}, \overline{\overline{I_2}}, \overline{C_2^1}, \overline{\overline{C_2^2}}\right) & if \quad \pi^1\left(\widetilde{I_1}, \widetilde{C_2^1}\right) < \pi^2\left(\widetilde{I_2}, \widetilde{C_2^2}\right) \\ \left(\widehat{I_1}, \widehat{I_2}, \widehat{C_2^1}, \widehat{C_2^2}\right) & if \quad \pi^1\left(\widetilde{I_1}, \widetilde{C_2^1}\right) = \pi^2\left(\widetilde{I_2}, \widetilde{C_2^2}\right) \\ \left(\overline{\overline{I_1}}, \overline{I_2}, \overline{\overline{C_2^1}}, \overline{C_2^2}\right) & if \quad \pi^1\left(\widetilde{I_1}, \widetilde{C_2^1}\right) > \pi^2\left(\widetilde{I_2}, \widetilde{C_2^2}\right) \end{cases}$$

where $\overline{C_2^j} = \theta\left(\overline{I_j}\right) + G^{-1}\left(1 - \frac{o_2+m_j}{a_2+v_2}\right) - C_1^* \qquad j = 1, 2.$, *and*

$$\begin{aligned} \left.\frac{\partial ED(I_1)}{\partial(I_1)}\right|_{\overline{I_1}} &= \frac{1}{q_2p_1(a_2-o_2-m_1)} \\ \left.\frac{\partial ED(I_2)}{\partial(I_2)}\right|_{\overline{\overline{I_2}}} &= \frac{1}{p_2(a_2-o_2-m_2)} \\ \left.\frac{\partial ED(I_1)}{\partial(I_1)}\right|_{\overline{\overline{I_1}}} &= \frac{1}{p_1(a_2-o_2-m_1)} \\ \left.\frac{\partial ED(I_2)}{\partial(I_2)}\right|_{\overline{I_2}} &= \frac{1}{q_1p_2(a_2-o_2-m_2)} \\ \left.\frac{\partial ED(I_1)}{\partial(I_1)}\right|_{\widehat{I_1}} &= \frac{1}{(q_2p_1+0.5p_1p_2)(a_2-o_2-m_1)} \\ \left.\frac{\partial ED(I_2)}{\partial(I_2)}\right|_{\widehat{I_2}} &= \frac{1}{(q_1p_2+0.5p_1p_2)(a_2-o_2-m_2)} \end{aligned}$$

Proof: See Section A.11 in Appendix A.

Note that $\overline{\overline{I_1}} = \widetilde{I}_1$, and $\overline{\overline{I_2}} = \widetilde{I}_2$. As it can be observed from Theorem 4.1.1, the alternative network capacity decisions of the central decision maker is dependent on only the different investment levels and the manufacturing cost of the new technology and independent of innovation probabilities. However, the investment level decisions of the central decision maker are related to the innovation probabilities of the vendors which captures the stochastic structure of the model.

The central decision maker allocates the whole investment with respect to innovation probability and manufacturing cost of each vendor, in other words with respect to effectiveness of the vendor, and determines the second period extra capacity by taking into account of demand increase and manufacturing cost of new technology. Finally, the expected pay-offs are associated with R&D investment share of each vendor and extra capacity upload of second period.

Centralized solution changes with respect to some parameters are summarized in a corollary.

Corollary 4.1.1 *1.* $\overline{I_1^*}$, $\widetilde{I}_1$, $\overline{\overline{I_1^*}}$, $\widetilde{C_2^1}$, $\overline{C_2^{1*}}$, *and* π *increases as* p_1 *increases.*

2. $\overline{I_2^*}$, $\widetilde{I}_2$, $\overline{\overline{I_2^*}}$, $\widetilde{C_2^2}$, $\overline{C_2^{2*}}$, *and* π *increases as* p_2 *increases.*

3. $\widetilde{C_2^1}$, C_2^{1*}, $(\widetilde{C_2^2}$, $C_2^{2*})$ *decreases as* m_1, (m_2) *increases.*

4. C_2^{1*}, (C_2^{2*}) *increases as penalty cost* v_2 *increases, and* $\pi \to -\infty$ *as* $v_2 \to \infty$.

Proof: See Section A.12 in Appendix A.

4.1.2. Numerical Analysis

In this section, we illustrate the solution to the centralized system for two periods. We try to gain intuition about how the model parameters influence the decision variables and the expected profit. The random part of the demand, ε in both periods is taken as normally distributed with mean $\mu = 200$, and sigma $\sigma = 10$, or $\sigma = 30$. The impact of investment I is reflected in the second period demand with $\theta(I) = \sqrt{I}$.

Unit sales price is fixed at $a = 100$ for both periods. First period manufacturing cost is $m = 30$. And, operating cost for the first period is $o_1 = 20$, and for the second period $o_2 = 20$. Penalty cost of first period is $v_1 = 25$, and either $v_2 = 25$ or $v_2 = 50$ for the second period. Successful innovation probabilities are either $p_1 = 0.25$, or $p_1 = 0.75$ for the first vendor, and either $p_2 = 0.25$, or $p_2 = 0.75$ for the second vendor. In addition, manufacturing costs are varied as $m_1 = 20$ or $m_1 = 40$ for the first vendor, and $m_2 = 20$ or $m_2 = 40$ for the second vendor to reveal the effect of manufacturing costs over decision variables.

The significant issue is that the central decision maker share his R&D investment budget between two under-controlled vendor, however, he performs business with one of them which firstly satisfies the natural rule that the one who gets innovation and materialize it, and secondly, who is the most efficient. In other words, extra telecom network capacity is uploaded by only and only one of the vendors. Therefore, C_2^1 and C_2^2 reflects the alternative additional capacities for the second period.

Table 4.2 presents how the decision variables C_1, I_1, I_2, C_2^1, and C_2^2 change along with expected profit for different choices of model parameters.Note that probability of innovation is assumed to follow a geometric distribution and demand is stationary for the two periods if innovation does not materialize.

As mentioned in Corollary 4.1.1, optimal I_1 decision of the central decision maker has tendency to increase, π increases as p_1 increases when the other parameters remain unchanged, like I_2 decision and π in p_2. Similar to C_2^2 decision, central decision maker attempts to decrease optimal C_2^1 decision when manufacturing cost, m_1, increases and others remain unchanged. Moreover, C_2^1 and C_2^2 tend to decrease as v_2 decreases, but expected profit has always tendency to decrease while this is happening. Finally, the effect of σ over the decision variables is not clear to observe, but it cause the profit loss almost every time.

One important thing to emphasize is that central decision maker reflects the optimal R&D decisions, (I_1, I_2), and the extra network capacity decisions, (C_2^1, C_2^2) in a mirror when p_1, and p_2 are not equal, and they interchange between themselves as long as the other parameters do not change. For example, if we change the values of p_1, and p_2 in any row of Table 4.2 in which $p_1 \neq p_2$, we end up with the transposition of the values of decision variables. I_1 value is transposed with I_2, same as C_2^1 and C_2^2. However, expected profit, π, remains the same as we expected. Furthermore, m_1 and m_2 transposition provide the same feature of the solution.

Finally, the decision maker has no obligation to add extra network capacity for the second period, however, he can chose the way of decreasing the telecom network at the second period, naturally. Since we are primarily interested in the cases of making R&D investments and uploading extra network rather than the circumstances under which not making investments and not building extra network up is optimal. In this sense, the central decision maker is bounded with the condition of $C_2^1 \geq 0$ and $C_2^2 \geq 0$. All of the intuitions can be gained from Table 4.2.

As a managerial insight, as marginal profit of selling capacity built by new technology increases the owner of the telecom value chain tends to increase R&D investment, naturally. However, competition of two identical firms can often cause profit loss for the centralized solution. In the long run, $\overline{\overline{I_j}}$ which is R&D investment level allocated to more probable vendor is used by the chain owner, and generally, $\overline{I_j}$ is wasted. The reason why competition between identical vendors cause profit loss is that one of the R&D investment, $\widehat{I_1}$, or $\widehat{I_2}$ is always waste, and $\widehat{I_j} > \overline{I_j}$, $j = 1, 2$. As it can be seen from Table 4.2, the expected centralized profit when the identical firms engage in R&D activities simultaneously is less than that when the vendors are non-identical.

Table 4.2. Numerical illustration of responses for the centralized model

σ	p_1	p_2	m_1	m_2	v_2	C_1	I_1	I_2	C_2^1	C_2^2	π
10	0.25	0.25	20	20	25	202.5	43.1	43.1	8.7	8.7	15683.8
10	0.25	0.25	20	20	50	202.5	43.1	43.1	10.3	10.3	15623.7
10	0.25	0.25	20	40	25	202.5	56.3	14.1	9.6	1.7	15667.9
10	0.25	0.25	40	20	50	202.5	14.1	56.3	3.8	11.2	15601.1
10	0.25	0.25	40	40	25	202.5	19.1	19.1	2.3	2.3	15635.8
10	0.25	0.25	40	40	50	202.5	19.1	19.1	4.4	4.4	15560.1
10	0.25	0.75	20	20	25	202.5	3.5	506.3	4.0	24.6	16111.4
10	0.25	0.75	20	20	50	202.5	3.5	506.3	5.6	26.2	16060.8
10	0.25	0.75	20	40	25	202.5	3.5	225.0	4.0	13.0	15829.9
10	0.25	0.75	40	20	50	202.5	1.6	506.3	1.3	26.2	16056.6
10	0.25	0.75	40	40	50	202.5	1.6	225.0	1.3	15.0	15748.4
10	0.75	0.75	20	20	25	202.5	197.8	197.8	16.2	16.2	15998.5
10	0.75	0.75	20	20	50	202.5	197.8	197.8	17.8	17.8	15951.1
10	0.75	0.75	20	40	25	202.5	506.3	14.1	24.6	1.7	16123.2
10	0.75	0.75	40	20	50	202.5	14.1	506.3	3.8	26.2	16069.1
10	0.75	0.75	40	40	25	202.5	87.9	87.9	7.3	7.3	15778.4
10	0.75	0.75	40	40	50	202.5	87.9	87.9	9.4	9.4	15697.6
30	0.25	0.25	20	20	25	207.6	43.1	43.1	13.0	13.0	14879.2
30	0.25	0.25	20	20	50	207.6	43.1	43.1	17.6	17.6	14698.8
30	0.25	0.25	40	20	50	207.6	14.1	56.3	3.8	18.6	14662.8
30	0.25	0.25	40	40	50	207.6	19.1	19.1	4.4	4.4	14603.8
30	0.25	0.75	20	20	25	207.6	3.5	506.3	8.3	28.9	15314.8
30	0.25	0.75	20	20	50	207.6	3.5	506.3	13.0	33.6	15162.9
30	0.25	0.75	20	40	25	207.6	3.5	225.0	8.3	8.9	15032.7
30	0.25	0.75	40	20	50	207.6	1.6	506.3	1.3	33.6	15154.2
30	0.25	0.75	40	40	50	207.6	1.6	225.0	1.3	15.0	14792.0
30	0.75	0.75	20	20	25	207.6	197.8	197.8	20.5	20.5	15204.5
30	0.75	0.75	20	20	50	207.6	197.8	197.8	25.1	25.1	15062.1
30	0.75	0.75	40	20	50	207.6	14.1	506.3	3.8	33.6	15166.7
30	0.75	0.75	40	40	25	207.6	87.9	87.9	3.3	3.3	14983.7
30	0.75	0.75	40	40	50	207.6	87.9	87.9	9.4	9.4	14741.3

4.2. The Decentralized Model

In the decentralized analysis, the agents (two vendors and the operator) are self-centred and trying to maximize their own pay-offs at the game. In the sequence of the decentralized game, two vendors concurrently announce their R&D investment levels. Then, they propose a "take it or leave it" contract to the operator. Although the operator has two different choices, our game goes on under the assumption of approval of the contract. After the acceptance of the contract, the operator decides on his required additional network capacity for the following period regarding outcome of the vendors' game and the market price of the new technology. After then, the mystery of whether the vendors come up with a successful innovation or not becomes clear at the beginning of the second period and all of the transfers between the players come true, they all gain the profits and incur corresponding costs at the end of the second period.

Initially, a simultaneous (Nash) game is played between the vendors regarding R&D investment levels. Then, a Stackelberg game follows it between the vendors and the operator regarding network capacity choice at the second period. To state the matter differently, the R&D investment levels, I_1 and I_2 are both announced by the vendors concurrently, and the announcement of the second period extra capacity by the operator, C_2^1 or C_2^2 pursues, afterwards. To make clear a point, since the operator is able to select one of the vendors considering the financial gain, he computes two separate second period network capacity, one of which is going to be uploaded to the actual network.

To emphasize a crucial aspect of the game, the Stackelberg game between the vendors and the operator can only occur when technology is successfully adopted and the operator decides to increase her network capacity in the second period. Otherwise, there is no game if no one invests to the new technology sufficiently to boost up the demand and attract the attention of the operator. Moreover, all information is common among the agents of the game.

4.2.1. The Operator's Capacity Determination Problem

At first place, the operator decides on his initial network capacity with respect to first period's costs and profits, because we assume that it is a new company in the market. Although the first period capacity decision has no effect in the investment game, we try to create resemblance with the model in Chapter 3.

The first period capacity decision of the operator is independent of the vendors. It is just a start-up nominal capacity level and constructed by any technology provider with a unit price m (exogenous). It means none of the vendors make money by supplying the first period network capacity of the operator. The vendors in our game structure run to collect money from new technology competition.

The operator has to decide on his first period network capacity, C_1 by maximizing the following pay-off function:

$$\pi\left(C_1\right) = \left(a_1 + v_1\right) S(C_1) - v_1\mu_1 - \left(o_1 + m\right) C_1 \tag{4.16}$$

where the interpretation is the same with (4.2), hence, the operator and the central decision maker can be regarded as the same corporate body without the investment game. As it can be seen from Proposition 4.1.1, the expected revenue function of the operator, (4.16), is concave in the network capacity decision, C_1, which can be derived as:

$$C_1^* = G^{-1}\left(1 - \frac{o_1 + m}{a_1 + v_1}\right) \tag{4.17}$$

Unlike the best C_1^* given in (3.14), C_1^* given in (4.17) does not contain any cost related with unit purchasing price of new technology in the second period. C_1^* given in (4.17) is nominal network capacity and independent of new technology decision of the operator, however, C_1^* given in (3.14) responds to maybe imminent new technology in the second period via possible unit price parameters, w_1, and w_2, of it. After observing the R&D investment levels of the vendors, the operator makes a decision about how much extra network capacity, C_2^1 or C_2^2, he adopts at the second period. Hence, he maximizes the following expected revenue function, $\pi_o(C_2^1, C_2^2; I_1, I_2)$:

$$\begin{aligned}
&= q_2 p_1 \left\{ (a_2 + v_2)\, S(C_1 + C_2^1 \middle| I_1) - v_2 \mu_2\,(I_1) - o_2 \left(C_1 + C_2^1\right) - wC_2^1 \right\} \\
&+ q_1 p_2 \left\{ (a_2 + v_2)\, S(C_1 + C_2^2 \middle| I_2) - v_2 \mu_2\,(I_2) - o_2 \left(C_1 + C_2^2\right) - wC_2^2 \right\} \\
&+ p_1 p_2 \left\{ \max \left(\begin{array}{c} (a_2 + v_2)\, S(C_1 + C_2^1 | I_1) - v_2 \mu_2\,(I_1) - o_2\,(C_1 + C_2^1) - wC_2^1, \\ (a_2 + v_2)\, S(C_1 + C_2^2 | I_2) - v_2 \mu_2\,(I_2) - o_2\,(C_1 + C_2^2) - wC_2^2 \end{array} \right) \right\} \\
&+ q_1 q_2 \left\{ (a_2 + v_2)\, S(C_1) - v_2 \mu_2 - o_2 C_1 \right\} \qquad (4.18)
\end{aligned}$$

The first line and the second line reflects the gain of the operator if one of the vendors, either second one or the first one, respectively, fails in the new technology development race and the other one sees the finish line with successfully materialized innovation. As one can refer to (4.5), but once again, the third line reveals that the operator transact with one of the vendors which literally cause more expected business profit for the operator. And, the last line shows the earnings of the operator if the innovation is not adopted flourishingly. Moreover, the first ingredient of each line is the expected revenue, the second one is the cost of lost service, and rest of them is the total purchasing and the operation cost of the telecommunication network.

In a similar way with the centralized model, we define the following equations:

$$\begin{aligned}
\pi_o^0 &= (a_2 + v_2)\, S(C_1) - v_2 \mu_2 - o_2 C_1 \\
\pi_o^j &= (a_2 + v_2)\, S(C_1 + C_2^j \middle| I_j) - v_2 \mu_2\,(I_j) - o_2 \left(C_1 + C_2^j\right) - wC_2^j \qquad (4.19) \\
j &= 1, 2.
\end{aligned}$$

If we rewrite the expected revenue of the operator by substituting (4.20) into

(4.18):

$$\pi = q_2 q_1 \pi_o^0 + q_2 p_1 \pi_o^1 + q_1 p_2 \pi_o^2 + p_1 p_2 \max\left(\pi_o^1, \pi_o^2\right)$$

Since one can easily observe that $\pi_o^1 = \pi_o^2$ when $I_1 = I_2$, three cases for the payoff function of the operator can be represented as below:

$$\pi = \begin{cases} q_2 q_1 \pi_o^0 + p_2 \pi_o^2 + q_2 p_1 \pi_o^1 & if \quad I_1 < I_2 \\ q_2 q_1 \pi_o^0 + q_2 p_1 \pi_o^1 + q_1 p_2 \pi_o^2 + 0.5 p_1 p_2 \left(\pi_o^1 + \pi_o^2\right) & if \quad I_1 = I_2 \\ q_2 q_1 \pi_o^0 + p_1 \pi_o^1 + q_1 p_2 \pi_o^2 & if \quad I_1 > I_2 \end{cases} \tag{4.20}$$

As the decision maker, the operator gives equal chances to the vendors when they exert effort equally. Obviously, without doing any favor to both vendors, they are equally likely to sell their innovation to the operator when they make the same R&D investments.

Proposition 4.2.1 *The expected profit of the operator as given in (4.20) is jointly concave in (C_2^1, C_2^2) and the solution obtained from the first order conditions maximizes the expected profit of the operator.*

Proof: See Section A.13 in Appendix A.

The alternative capacity decisions of the operator for the second period for all cases can be defined as follows: (See the FOCs from the proof (A.13) in Appendix A)

$$C_2^{1*} = \theta\left(I_1\right) + G^{-1}\left(1 - \frac{o_2 + w}{a_2 + v_2}\right) - C_1^* \tag{4.21}$$

$$C_2^{2*} = \theta\left(I_2\right) + G^{-1}\left(1 - \frac{o_2 + w}{a_2 + v_2}\right) - C_1^* \tag{4.22}$$

Obviously, the alternative network capacities are contingent only upon the R&D investments of the vendors. Moreover, the investment levels must yield nonnegative extra network capacity to ensure that the game exists between the vendor. Once again, as long as the operator chooses nonnegative extra network upload for the second period, there occurs a competition between the suppliers to satisfy this demand of the operator. As a result of this, there is a minimum level of investment which results in nonnegative network capacity choice for the operator. The minimum level of investment to join the game is as follows:

$$\underline{I_j} = \theta^{-1}\left[C_1^* - G^{-1}\left(1 - \frac{o_2 + w}{a_2 + v_2}\right)\right] \qquad for \qquad j = 1, 2. \tag{4.23}$$

Note that if Nash equilibrium/equilibria exist/s in the vendors' game R&D investment levels have to be grater than the minimum investment level revealed above.

4.2.2. The Vendors' Investment Problem

The vendors' aim in the simultaneous (Nash) game is to maximize their own expected revenue functions taking into consideration the decisions of the operator. The expected revenues of the vendors are well-adjusted to the operator's three cases.

Both vendors make an initial investment in technology and then charge a unit market price for unit capacity with the new technology. Both vendors' objective is to supply the operator's requested second period network capacity and make money via exerting sufficient effort to boost up the demand of the operator. The first vendor's problem can be expressed as follows:

$$\begin{aligned} \max_{I_1} \pi_1\left(I_1; I_2, C_2^1, C_2^2\right) &= \begin{cases} \pi_1^1\left(I_1\right) = q_2 p_1\left(w - m_1\right) C_2^1 - I_1 & if \quad I_1 < I_2 \\ \pi_1^2\left(I_1\right) = \left(q_2 p_1 + 0.5 p_1 p_2\right)\left(w - m_1\right) C_2^1 - I_1 & if \quad I_1 = I_2 \\ \pi_1^3\left(I_1\right) = p_1\left(w - m_1\right) C_2^1 - I_1 & if \quad I_1 > I_2 \end{cases} \\ \text{s.t.} \qquad & I_1 \in \left[\underline{I_1}, \infty\right) \end{aligned} \tag{4.24}$$

We now characterize the expected profit of the first vendor.

Proposition 4.2.2 *The expected profit of the first vendor as given in (4.24) is strictly concave in I_1 in left and right branches, π_1^1, and π_1^3, and not continuous when $I_1 = I_2$.*

Proof: See Section A.14 in Appendix A.

With the concavity of the expected pay-off of the first vendor, FOCs (Proposition 4.2.2) let us implicitly characterize the best investment levels for the first vendor for the left hand-side and the right hand-side of the function as follows:

$$\left.\frac{dED\left[I_1\right]}{d\left(I_1\right)}\right|_{\widehat{I_1}} = \frac{1}{q_2 p_1 \left(w - m_1\right)} \tag{4.25}$$

$$\left.\frac{dED\left[I_1\right]}{d\left(I_1\right)}\right|_{\widehat{\widehat{I_1}}} = \frac{1}{p_1 \left(w - m_1\right)} \tag{4.26}$$

From this point on, we abbreviate I_1 derived from (4.25) as $\widehat{I_1}$, and correspondingly I_1 derived from (4.26) as $\widehat{\widehat{I_1}}$. The investment level of the first vendor is dependent on the innovation probabilities, as it is expected. In addition, the effort dependent demand function is monotonously increasing as I_1 is raising and strictly concave by assumption, hence, $\widehat{\widehat{I_1}} > \widehat{I_1}$ when $p_2 \neq 0$.

The second vendor's problem can be expressed analogously:

$$\max_{I_2} \pi_2\left(I_2; I_1, C_2^1, C_2^2\right) = \begin{cases} \pi_2^1\left(I_1\right) = p_2\left(w - m_2\right) C_2^2 - I_2 & if \quad I_1 < I_2 \\ \pi_2^2\left(I_1\right) = \left(q_1 p_2 + 0.5 p_1 p_2\right)\left(w - m_2\right) C_2^2 - I_2 & if \quad I_1 = I_2 \\ \pi_2^3\left(I_1\right) = q_1 p_2 \left(w - m_2\right) C_2^2 - I_2 & if \quad I_1 > I_2 \end{cases}$$

$$\text{s.t.} \qquad I_2 \in \left[\underline{I_2}, \infty\right) \tag{4.27}$$

Proposition 4.2.3 *The first part and the third part of the expected profit of the second vendor as given in (4.27), π_2^1, and π_2^3, is strictly concave in I_2, and not continuous when $I_2 = I_1$.*

Proof: See Section A.15 in Appendix A.

With the concavity of the expected pay-off of the second vendor, FOCs (Proposition 4.2.3) let us implicitly characterize the best investment levels for the second vendor for the two different part of the function as follows:

$$\left.\frac{dED\left[I_2\right]}{d\left(I_2\right)}\right|_{\widehat{I_2}} = \frac{1}{q_1 p_2\left(w - m_2\right)} \tag{4.28}$$

$$\left.\frac{dED\left[I_2\right]}{d\left(I_2\right)}\right|_{\widehat{\widehat{I_2}}} = \frac{1}{p_2\left(w - m_2\right)} \tag{4.29}$$

From this point on, we abbreviate I_2 derived from (4.28) as $\widehat{I_2}$, and correspondingly I_2 derived from (4.29) as $\widehat{\widehat{I_2}}$. Similar to the first vendor, the investment level of the second vendor is dependent on the innovation probabilities, as it is expected. Moreover, $\widehat{\widehat{I_2}} > \widehat{I_2}$ when $p_1 \neq 0$ due to the same additive demand assumption.

Once more, the investment levels of the vendors are only dependent on the innovation probabilities, as it is expected. Each vendor gives reaction to rival's investment strategy over probabilities and its own manufacturing cost at the simultaneous investment game. The important interpretation is that best investment levels of the vendors are contingent on the marginal profit of the new technology. It means that the vendors have no incentive to allocate money to the new technology research unless selling it to the market is profitable enough.

4.3. Nash Equilibrium in Pure Strategies

The R&D investment problem can be modeled and solved in game theoretic framework when a competitor exists.

Definition 4.3.1 *The strategy space for the vendor j is $S_j = \left[\underline{I_j}, \infty\right)$, $j = 1, 2$. A* *pure strategy for vendor j is any scalar which ensures that the operator is willing to be involved in the new technology game.*

Definition 4.3.2 *The set of best responses for the vendor j to its rival is* $\Phi_j(I_k) = \{I_j \in S_j | \pi_j(I_j; I_k) \geq \pi_j(I_j^-; I_k), \forall I_j^- \in S_j\}$. *The mapping* $\Phi_j : S_k \Rightarrow S_j$ *is the j's best response correspondence.*

Definition 4.3.3 *A strategy pair* (I_1^N, I_2^N) *is a Nash equilibrium if*

- $I_j^N \in S_j$, $j = 1, 2$.
- $\pi_1(I_1^N; I_2^N) \geq \pi_1(I_1; I_2^N)$, $\forall I_1 \in S_1$; *and*
- $\pi_2(I_2^N; I_1^N) \geq \pi_2(I_2; I_1^N)$, $\forall I_2 \in S_2$.

In other words, the pair (I_1^N, I_2^N) *is a Nash equilibrium if* $I_1^N \in \Phi_1(I_2^N)$, *and* $I_2^N \in \Phi_2(I_1^N)$; *that is, each strategy is a best response to each other.*

Proposition 4.3.1 *The first vendor's best response mapping,* Φ_1, *can be expressed as:*
Case i) $\underline{I_1} < \widehat{I}_1 < \widehat{\widehat{I}}_1$;

$$\Phi_1(I_2) = \begin{cases} \widehat{\widehat{I}}_1 & if \quad I_2 \leq I_1^{'} \\ \widehat{I}_1 & if \quad I_2 > I_1^{'} \end{cases}$$

where $I_1^{'} = \widehat{I}_1 + (w - m_1)\left[p_1\theta(I_1^{'}) - q_2 p_1 \theta\left(\widehat{I}_1\right)\right] + p_1 p_2 (w - m_1)\left[G^{-1}\left(1 - \frac{o_2 + w}{a_2 + v_2}\right) - C_1\right]$
such that $\left.\frac{d\pi_1^3}{d(I_1)}\right|_{I_1^{'}} < 0$.

Case ii) $\widehat{I}_1 < \underline{I_1} < \widehat{\widehat{I}}_1$;

$$\Phi_1(I_2) = \begin{cases} \widehat{\widehat{I}}_1 & if \quad I_2 \leq I_1^{''} \\ no \quad response & otherwise \end{cases}$$

where $I_1^{''} = \underline{I_1} + (w - m_1)\left[p_1\theta(I_1^{''}) - q_2 p_1 \theta\left(\underline{I_1}\right)\right] + p_1 p_2 (w - m_1)\left[G^{-1}\left(1 - \frac{o_2 + w}{a_2 + v_2}\right) - C_1\right]$

such that $\left.\frac{d\pi_1^3}{d(I_1)}\right|_{I_1''} < 0$.

Case iii) There is no response when $\widehat{I_1} < \widehat{\widehat{I_1}} < \underline{I_1}$ *because* π_1 *is always negative for* $I_1 > 0$.

Proof: See Section A.16 in Appendix A.

Proposition 4.3.2 *The second vendor's best response mapping,* Φ_2*, can be expressed as:*

Case i) $\underline{I_2} < \widehat{I_2} < \widehat{\widehat{I_2}}$;

$$\Phi_2(I_1) = \begin{cases} \widehat{\widehat{I_2}} & if \quad I_1 \leq I_2' \\ \widehat{I_2} & if \quad I_1 > I_2' \end{cases}$$

where $I_2' = \widehat{I_2} + (w - m_2)\left[p_2\theta\left(I_2'\right) - q_1 p_2 \theta\left(\widehat{I_2}\right)\right] + p_1 p_2 (w - m_2)\left[G^{-1}\left(1 - \frac{o_2 + w}{a_2 + v_2}\right) - C_1\right]$
such that $\left.\frac{\partial\pi_2^1}{\partial(I_2)}\right|_{I_2'} < 0$.

Case ii) $\widehat{I_2} < \underline{I_2} < \widehat{\widehat{I_2}}$;

$$\Phi_2(I_1) = \begin{cases} \widehat{\widehat{I_2}} & if \quad I_1 \leq I_2'' \\ no \quad response & otherwise \end{cases}$$

where $I_2'' = \underline{I_2} + (w - m_2)\left[p_2\theta\left(I_2''\right) - q_1 p_2 \theta\left(\underline{I_2}\right)\right] + p_1 p_2 (w - m_2)\left[G^{-1}\left(1 - \frac{o_2 + w}{a_2 + v_2}\right) - C_1\right]$
such that $\left.\frac{\partial\pi_2^1}{\partial(I_2)}\right|_{I_2''} < 0$.

Case iii) There is no response when $\widehat{I_2} < \widehat{\widehat{I_2}} < \underline{I_2}$ *because* π_2 *is always negative for* $I_2 > 0$.

Proof: See Section A.17 in Appendix A.

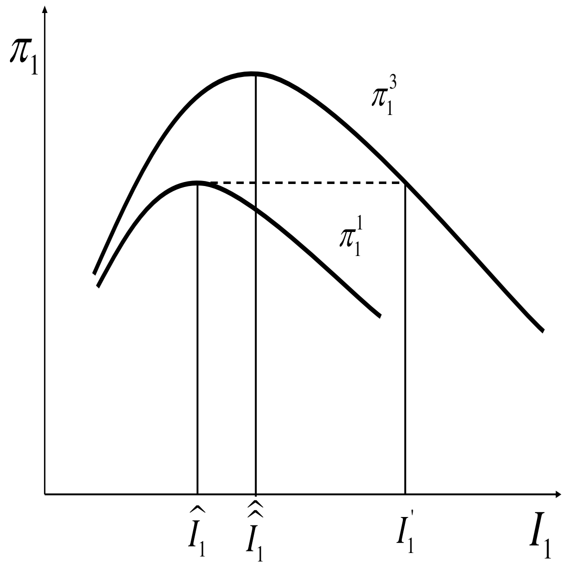


Figure 4.2. Visual interpretation of the expected profit of the first vendor

To make clear the response functions of the vendors we visualize the branches of the expected profit of the first vendor given in (4.24) in Figure 4.2. We can analogously illustrate the expected profit of the second vendor. Note that we disregard the minimum investment level, $\underline{I_1}$, in Figure 4.2.

The interpretation of $I_j^{'}$ or $I_j^{''}$ is that they are the upper bounds that one can increase the R&D investment level with the expectation of higher profit. The economic meaning is that they are the ultimate investment levels at which the expected profit of investing more with the hope that a vendor can make better money is equal to opportunity cost of investing less. Note that there are two investment levels (Proposition 4.3.1, and Proposition 4.3.2), and $\widehat{\widehat{I_j}} > \widehat{I_j}$, $j = 1, 2$.

We give some illustrative graphic examples to provide a visual interpretation for response functions of the vendors. For some game parameters, Nash equilibrium does not exist. For instance, when the vendors' responses are in case (iii) in Proposition 4.3.1, and case (iii) in Proposition 4.3.2. Note that the axes of Figures 4.3, and 4.4, are normalized at $\underline{I_1}$ and $\underline{I_2}$.

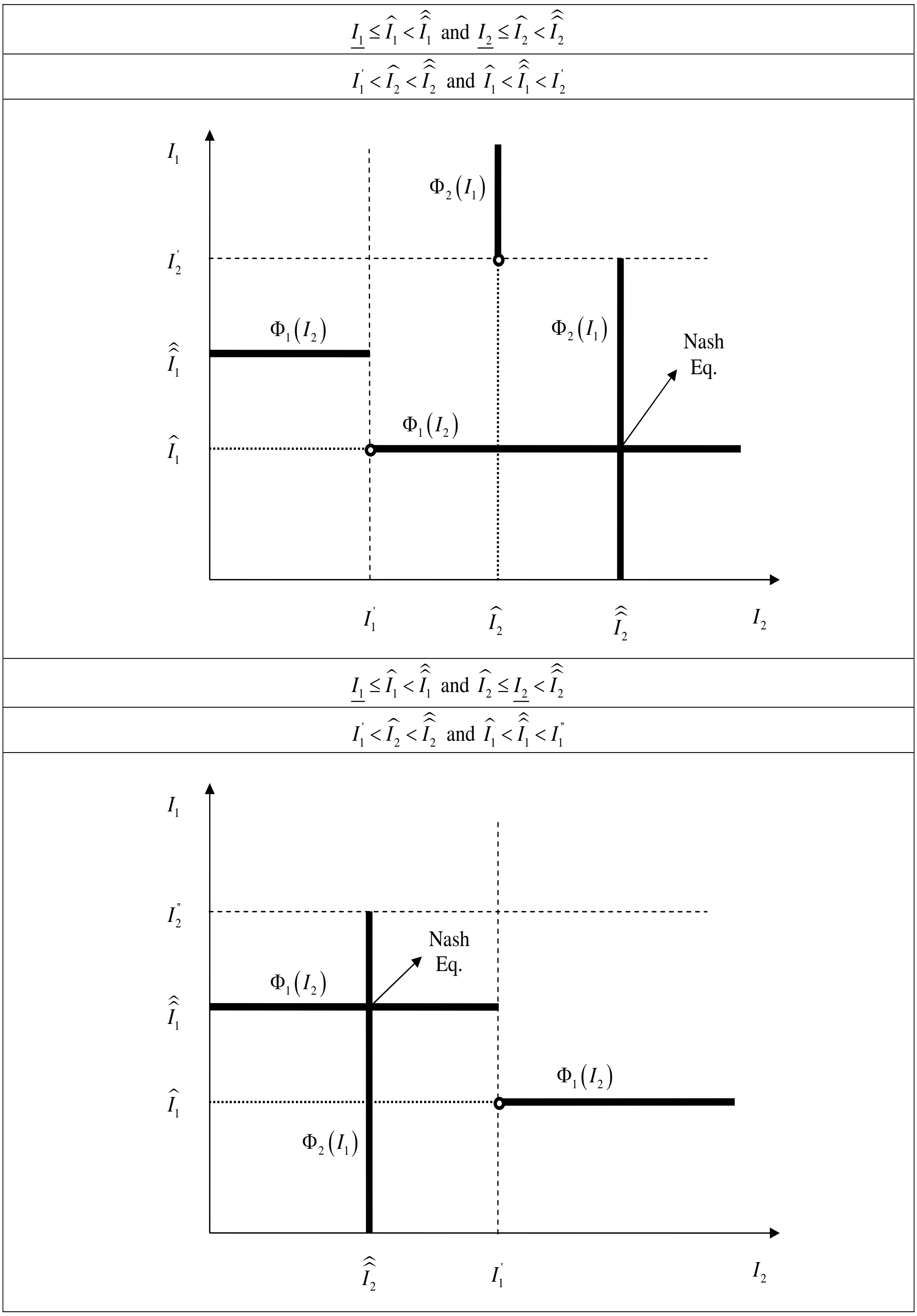
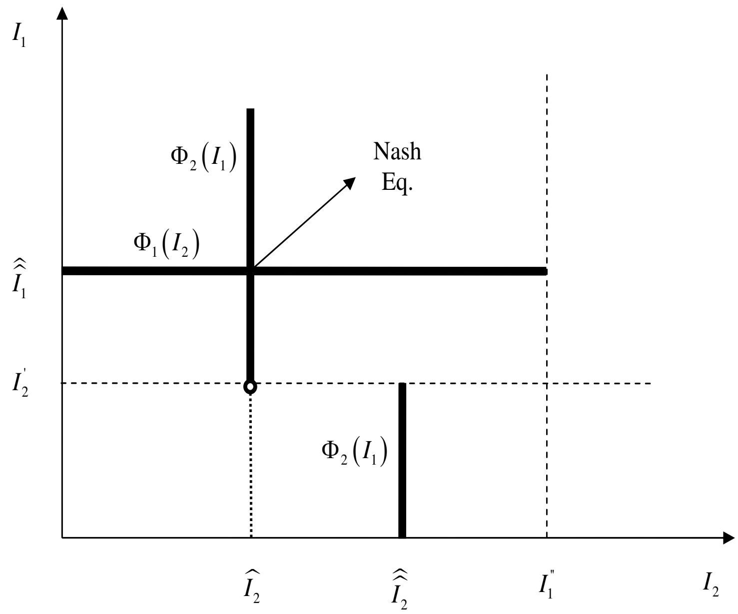


Figure 4.3. Response functions of the vendors-1

$\hat{I}_1 < \underline{I_1} \le \hat{\hat{I}}_1$ and $\underline{I_2} \le \hat{I}_2 < \hat{\hat{I}}_2$

$\hat{I}_2 < \hat{\hat{I}}_2 < I_1^{''}$ and $I_2^{'} < \hat{\hat{I}}_1$



$\hat{I}_1 < \underline{I_1} \le \hat{\hat{I}}_1$ and $\hat{I}_2 < \underline{I_2} \le \hat{\hat{I}}_2$

$\hat{\hat{I}}_2 < I_1^{''}$ and $\hat{\hat{I}}_1 < I_2^{''}$

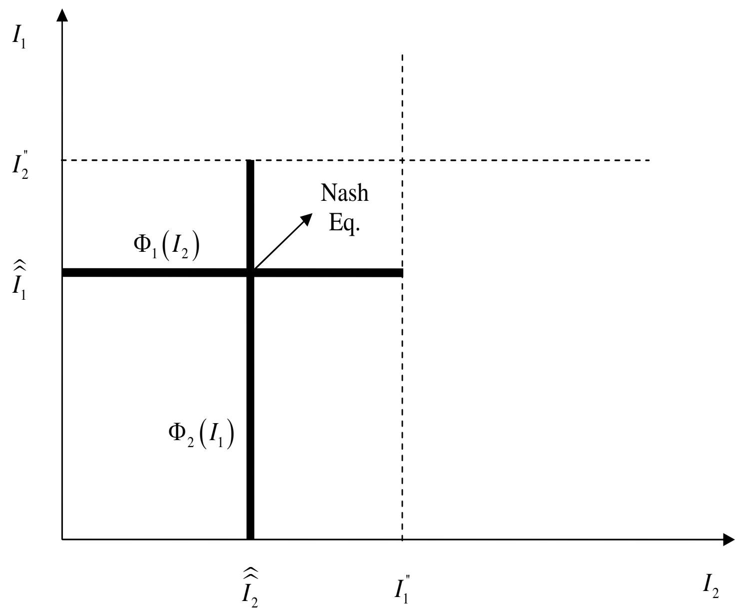


Figure 4.4. Response functions of the vendors-2

Theorem 4.3.1 *Case by case for all possible cases, Nash equilibrium can be expressed as follows:*

- $\underline{I_1} \leq \widehat{I}_1 < \widehat{\widehat{I}}_1$, *and* $\underline{I_2} \leq \widehat{I}_2 < \widehat{\widehat{I}}_2$;
    1. $\left(I_1^N, I_2^N\right) = \left(\widehat{I}_1, \widehat{\widehat{I}}_2\right)$ *for* $I_1' < \widehat{I}_2 < \widehat{\widehat{I}}_2$, *and* $\widehat{I}_1 < \widehat{\widehat{I}}_1 < I_2'$.
    2. $\left(I_1^N, I_2^N\right) = \left(\widehat{I}_1, \widehat{I}_2\right)$ *for* $\widehat{I}_2 \leq I_1' < \widehat{\widehat{I}}_2$, *and* $\widehat{I}_1 < \widehat{\widehat{I}}_1 < I_2'$.
    3. $\left(I_1^N, I_2^N\right) = \left(\widehat{\widehat{I}}_1, \widehat{\widehat{I}}_2\right)$ *for* $\widehat{I}_2 < \widehat{\widehat{I}}_2 \leq I_1'$, *and* $\widehat{I}_1 < \widehat{\widehat{I}}_1 \leq I_2'$.
    4. $\left(I_1^N, I_2^N\right) = \left(\widehat{\widehat{I}}_1, \widehat{I}_2\right)$ *for* $\widehat{I}_2 < \widehat{\widehat{I}}_2 < I_1'$, *and* $I_2' < \widehat{I}_1 < \widehat{\widehat{I}}_1$.
    5. $\left(I_1^N, I_2^N\right) = \left(\widehat{\widehat{I}}_1, \widehat{I}_2\right)$ *for* $\widehat{I}_2 < \widehat{\widehat{I}}_2 < I_1'$, *and* $\widehat{I}_1 \leq I_2' < \widehat{\widehat{I}}_1$.
- $\underline{I_1} \leq \widehat{I}_1 < \widehat{\widehat{I}}_1$, *and* $\widehat{I}_2 < \underline{I_2} \leq \widehat{\widehat{I}}_2$;
    1. $\left(I_1^N, I_2^N\right) = \left(\widehat{I}_1, \widehat{\widehat{I}}_2\right)$ *for* $I_1' < \widehat{\widehat{I}}_2$, *and* $\widehat{I}_1 < \widehat{\widehat{I}}_1 < I_2''$.
    2. $\left(I_1^N, I_2^N\right) = \left(\widehat{\widehat{I}}_1, \widehat{\widehat{I}}_2\right)$ *for* $\widehat{\widehat{I}}_2 \leq I_1'$, *and* $\widehat{I}_1 < \widehat{\widehat{I}}_1 \leq I_2''$.
    3. *No equilibrium for* $\widehat{\widehat{I}}_2 < I_1'$, *and* $I_2'' < \widehat{I}_1 < \widehat{\widehat{I}}_1$, *and for* $\widehat{\widehat{I}}_2 < I_1'$, *and* $\widehat{I}_1 \leq I_2'' < \widehat{\widehat{I}}_1$.
- $\widehat{I}_1 < \underline{I_1} \leq \widehat{\widehat{I}}_1$, *and* $\underline{I_2} \leq \widehat{I}_2 < \widehat{\widehat{I}}_2$;
    1. $\left(I_1^N, I_2^N\right) = \left(\widehat{\widehat{I}}_1, \widehat{I}_2\right)$ *for* $\widehat{I}_2 < \widehat{\widehat{I}}_2 < I_1''$, *and* $I_2' < \widehat{\widehat{I}}_1$.
    2. $\left(I_1^N, I_2^N\right) = \left(\widehat{\widehat{I}}_1, \widehat{\widehat{I}}_2\right)$ *for* $\widehat{I}_2 < \widehat{\widehat{I}}_2 \leq I_1''$, *and* $\widehat{\widehat{I}}_1 \leq I_2'$.
    3. *No equilibrium for* $I_1'' < \widehat{I}_2 < \widehat{\widehat{I}}_2$, *and* $\widehat{\widehat{I}}_1 < I_2'$, *and for* $\widehat{I}_2 \leq I_1'' < \widehat{\widehat{I}}_2$, *and* $\widehat{\widehat{I}}_1 < I_2'$.
- $\widehat{I}_1 < \underline{I_1} \leq \widehat{\widehat{I}}_1$, *and* $\widehat{I}_2 < \underline{I_2} \leq \widehat{\widehat{I}}_2$;
    1. $\left(I_1^N, I_2^N\right) = \left(\widehat{\widehat{I}}_1, \widehat{\widehat{I}}_2\right)$ *for* $\widehat{\widehat{I}}_2 \leq I_1''$, *and* $\widehat{\widehat{I}}_1 \leq I_2''$.
    2. *No equilibrium for* $I_1'' < \widehat{\widehat{I}}_2$, *and* $\widehat{\widehat{I}}_1 < I_2''$, *and for* $\widehat{\widehat{I}}_2 < I_1''$, *and* $I_2'' < \widehat{\widehat{I}}_1$.

where I_1', *and* I_1'' *are defined in Proposition 4.3.1, and* I_2', *and* I_2'' *are defined in Proposition 4.3.2.*

Proof: See Section A.18 in Appendix A.

Obviously, there is no Nash equilibrium under investment level of $\underline{I}_j$, $j = 1, 2$, investment level because the Stackelberg game occurs between vendors and the operator only above this investment level. Moreover, it is always unique whenever it exists.

An important observation is that Nash equilibrium does not appear because the marginal profit of the new technology is not attractive enough for the vendors to exert effort, and make investment. This observation explains why the firms engage innovation activities when the sale price of new technology is much higher than the cost of it, as it is emphasized from [16].

4.3.1. Numerical Analysis

In this section, we illustrate the solution to the decentralized system for two periods. We show the Nash equilibrium point numerically for some cases, and gain intuition about how the model parameters influence the decision variables and the expected profit. The random part of the demand, ε in both periods is taken as normally distributed with mean $\mu = 200$, and sigma $\sigma = 20$. The impact of investment I is reflected in the second period demand with $\theta(I) = \sqrt{I}$.

Unit sales price is fixed at $a = 100$ for both periods. First period manufacturing cost is $m = 30$. And, operating cost for the first period is $o_1 = 40$, and for the second period $o_2 = 20$. Penalty costs are fixed at $v_1 = v_2 = 25$ for both periods. Successful innovation probabilities, and manufacturing costs are varied to reveal some illustrative cases. Note that probability of innovation is assumed to follow a geometric distribution and demand is stationary for the two periods if innovation does not materialize.

The parameters in Figure 4.5 are $p_1 = 0.50$, $p_2 = 0.50$, $m_1 = 20$, and $m_2 = 20$. The vendors are identical, and simultaneous movement game equilibrium occurs at $(\widehat{\widehat{I}}_1, \widehat{\widehat{I}}_2)$=(100, 100) point. In addition, Stackelberg equilibrium between the vendors and the operator occurs at $(C_2^1(\widehat{\widehat{I}}_1),\ C_2^2(\widehat{\widehat{I}}_2)))$= (7.9, 7.9) point. However, centralized solution is $(I_1^*,\ I_2^*)$=(126.6, 126.6). The decentralized system composed of two vendors and the operator earns 14927, totally, but the centralized revenue is 15195. Apparently, the decentralized system makes money below the centralized setting.

In Figure 4.6, the vendors differ from each other with parameters $p_1 = 0.70$, $p_2 = 0.35$, $m_1 = 25$, $m_2 = 10$, and the first vendor is less efficient and more likely to get innovation. Nash equilibrium exists at $(\widehat{\widehat{I_1}}, \widehat{\widehat{I_2}})$=(150, 76.6) point. In addition, Stackelberg equilibrium between the vendors and the operator occurs at $(C_2^1(\widehat{\widehat{I_1}}),$ $C_2^2(\widehat{\widehat{I_2}})))$= (8.1, 4.6) point. However, centralized solution is (I_1^*, I_2^*)=(156.6, 150). The decentralized revenue is 15016, corresponding centralized revenue is 15342.

In Figure 4.7, the vendors are asymmetric, and the parameters are $p_1 = 0.55$, $p_2 = 0.40$, $m_1 = 20$, $m_2 = 30$. Nash equilibrium is $(\widehat{\widehat{I_1}}, \widehat{I_2})$=(121, 7.3). Moreover, Stackelberg game between the vendors and the operator occurs at $(C_2^1(\widehat{\widehat{I_1}}))$, $C_2^2(\widehat{I_2}))$=(8.9, 0.6) point. However, centralized solution is (I_1^*, I_2^*)=(272.3, 20.3). The decentralized revenue is 14960, corresponding centralized revenue is 15207.

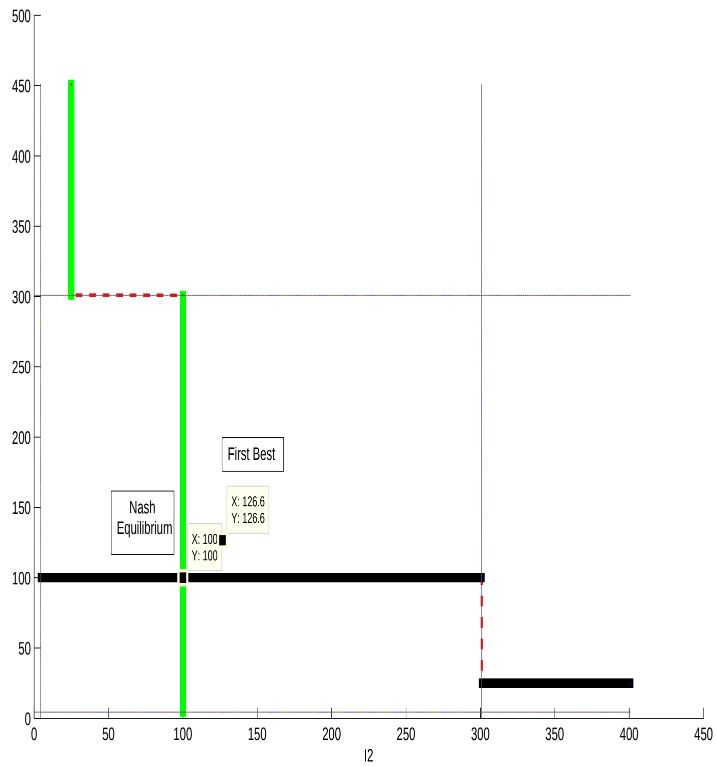


Figure 4.5. Nash equilibrium when the vendors are symmetric

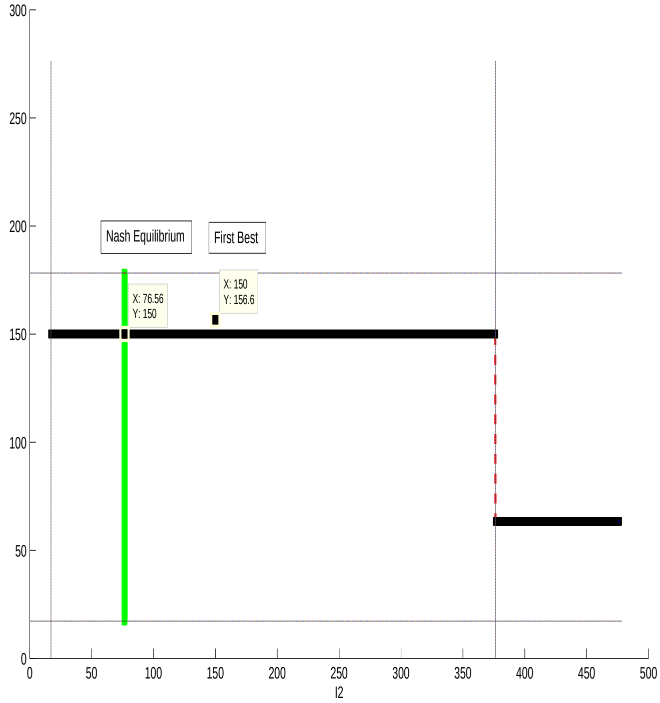


Figure 4.6. Nash equilibrium $(\widehat{\widehat{I}}_1, \widehat{\widehat{I}}_2)$ when the first vendor is more likely to succeed

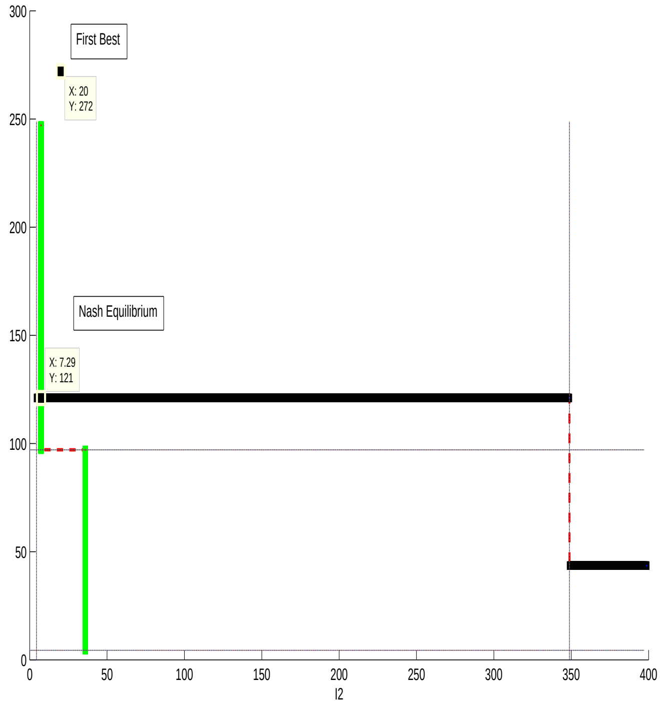


Figure 4.7. Nash equilibrium $(\widehat{\widehat{I}}_1, \widehat{I}_2)$ when the vendors are asymmetric

We are unable to show comparative statics results with respect to some important parameters of the model such as innovation probabilities, manufacturing costs, penalty costs and exogenous wholesale price. Because the response functions of the vendors are various forms which actually does not give us any chance to characterize the behavior of the Nash equilibrium point with respect to mentioned parameters. However, the equilibrium of Stakelberg game which happens between the vendors and the operator is plainer, since the operator chooses her decision without considering the innovation probabilities of the vendors. She announces two alternative extra network capacity, C_2^{1*}, and C_2^{2*}, with respect to I_1^*, and I_2^*, respectively. Therefore, the equilibrium of the Stakelberg game is only contingent on the equilibrium of the R&D investment game of the vendors.

4.4. Coordination

The coordinating contract is defined as the one that makes the independent agents decide on the same levels for the decision variables as the centralized solutions. In our case, the coordinating contract let the operator chooses the same amount of extra capacity for the second period and the vendors choose the same investment levels as the centralized solutions, and also simultaneous movement investment game must have an equilibrium for the vendors.

4.4.1. Profit Sharing Contract

Apparently, in wholesale price contract, there is not any coordination unless $w = m_i$ for $j = 1, 2$ which is unacceptable to the vendors in the decentralized setting. Therefore, a revenue, operating cost and investment sharing contract was proposed as a coordinating contract for the model we inspired [9]. We give a modified model of this contract here.

Initially, the vendors' technology investments are shared by the operator, after then the vendors participate both second period revenue and the operating cost of the operator in the profit sharing contract. Finally, the price of the technology is settled as regards the manufacturing cost of it.

Proposition 4.4.1 *A profit sharing contract with the transfer payments for both vendors given as*

$$T_1\left(C_2^1, I_1, w_1, \phi_1, \psi_1\right) = w_1 C_2^1 + (1-\phi_1)(a_2) S\left(C_1 + C_2^1 \middle| I_1\right) - (1-\lambda_1) o_2 C_2^1 + (1-\psi_1) I_1$$
$$T_2\left(C_2^2, I_2, w_2, \phi_2, \psi_2\right) = w_2 C_2^2 + (1-\phi_2)(a_2) S\left(C_1 + C_2^2 \middle| I_2\right) - (1-\lambda_2) o_2 C_2^2 + (1-\psi_2) I_2$$

where $0 \leq \phi_j \leq 1,\ \lambda_j \geq 0,\ 0 \leq \psi_j \leq 1,\ \phi_j a_2 + v_2 = \lambda_j (a_2 + v_2),\ w_j = \lambda_j m_j,$ *and* $\psi_j = \frac{(1-\lambda_j)(a_2+v_2-o_2-m_j)}{(a_2-o_2-m_j)}$ *for* $j = 1, 2.$

Proof: See Section A.19 in Appendix A.

To make clear the concept, we give an illustrative numeric example via using the parameters mentioned in section (4.1.2), and σ, p_1, p_2, m_1, m_2, v_2 parameters and C_1, I_1, I_2, C_2^1, C_2^2 decision variable in each row of 4.2 is the same with each row of Table 4.3. And, $\phi_1 = \phi_2 = 0.90$, or $\phi_1 = \phi_2 = 0.95$ which means the operator captures the 90 or 95 percent of the expected revenue gained from the sale. In this example, expected revenue shares of the vendors and the operator under profit sharing contract are given in Table (4.3) below.

Some of the basic intuitions, revealed in table (4.3), are that not surprisingly, the more likely the vendor innovates the more money he gains, and whenever the vendor decreases his manufacturing cost of the new technology he starts to collect more profit, and the operator can make more money if she pays less lost sale penalty cost. Moreover, the fluctuation of the demand cause profit loss for all parties of the game.

Although only two cases when $\phi_1 = \phi_2 = 0.90$, and $\phi_1 = \phi_2 = 0.95$ is revealed here, we can naturally state that the increase of ϕ_j, $j = 1, 2.$ is in favor of the operator because she makes her share of the sale profit larger. However, not all ϕ_j values can satisfy the restrictive conditions of the other parameters in Proposition 4.4.1. Even though the coordination among the different agents of the game is achieved by the profit sharing contract, it is unable to allocate the whole profit among the agents arbitrarily.

4.4.2. Revenue Sharing Contract

The coordination does not occur when the operator pays $w = m_i$, $j = 1, 2$, because the vendors are not willing to sell the technology to the operator due to double marginalization. Therefore, a revenue sharing and investment support contract is proposed as a coordinating contract for the model because such kind of contracts might have a chance in real practice.

At the beginning, the operator provides technology investment supports for the vendors, after then the vendors are involved in the second period revenue, and accept manufacturing cost support from the operator in the modified revenue sharing contract.

Proposition 4.4.2 *A revenue sharing contract with the transfer payments for both vendors given as*

$$T_1\left(C_2^1, I_1, \delta_1, \phi_1, \psi_1\right) = \delta_1 m_1 C_2^1 + \left(1 - \phi_1\right)\left(a_2\right) S\left(C_1 + C_2^1 \middle| I_1\right) + \left(1 - \psi_1\right) I_1$$
$$T_2\left(C_2^2, I_2, \delta_2, \phi_2, \psi_2\right) = \delta_2 m_2 C_2^2 + \left(1 - \phi_2\right)\left(a_2\right) S\left(C_1 + C_2^2 \middle| I_2\right) + \left(1 - \psi_2\right) I_2$$

where $0 \leq \delta_j < 1$, $\phi_j a_2 + v_2 = \lambda_j\left(a_2 + v_2\right)$, $,\lambda_j = \frac{o_2 + \delta_j m_j}{o_2 + m_j}$, *and* $\psi_j = \frac{(1-\varphi_j)a_2 - (1-\delta_j)m_j}{a_2 - o_2 - m_j}$ *for* $j = 1, 2$.

Proof: See Section A.20 in Appendix A.

To ensure that the revenue sharing contract perform the coordination properly, the contract parameters have to be chosen such that $\phi_j \geq \frac{o_2 + \delta_j m_j}{a_2}$, and $a_2 > o_2 + m_j + \frac{v_2 m_j}{o_2} - \frac{\delta_j m_j (a_2 + v_2 - o_2 - m_j)}{o_2}$. The logical interpretation of these might be that the operator is not willing to continue business under definite percentage of the sale's revenue, and firms of telecom sector keep on doing business if sale price per capacity is grater than operating and the manufacturing cost of it.

Table 4.3. Numerical illustration of revenues of agents under profit sharing contract

ϕ	π_1	π_2	π_o	ϕ	π_1	π_2	π_o	π
0.90	438.6	438.6	14806.6	0.95	219.3	219.3	15245.2	15683.8
0.90	439.0	439.0	14745.8	0.95	219.5	219.5	15184.7	15623.7
0.90	502.5	373.3	14792.2	0.95	251.2	186.7	15230.1	15667.9
0.90	373.2	502.8	14725.1	0.95	186.6	251.4	15163.1	15601.1
0.90	436.2	436.2	14763.4	0.95	218.1	218.1	15199.6	15635.8
0.90	436.1	436.1	14688.0	0.95	218.0	218.0	15124.1	15560.1
0.90	123.9	1563.6	14423.9	0.95	61.9	781.8	15267.7	16111.4
0.90	124.0	1564.8	14372.1	0.95	62.0	782.4	15216.4	16060.8
0.90	123.9	1535.5	14170.6	0.95	61.9	767.7	15000.2	15829.9
0.90	123.6	1564.8	14368.2	0.95	61.8	782.4	15212.4	16056.6
0.90	123.6	1534.9	14089.9	0.95	61.8	767.4	14919.1	15748.4
0.90	957.5	957.5	14083.5	0.95	478.7	478.7	15041.0	15998.5
0.90	958.2	958.2	14034.6	0.95	479.1	479.1	14992.8	15951.1
0.90	1563.6	373.3	14186.3	0.95	781.8	186.7	15154.8	16123.2
0.90	373.2	1564.8	14131.1	0.95	186.6	782.4	15100.1	16069.1
0.90	946.5	946.5	13885.4	0.95	473.2	473.2	14831.9	15778.4
0.90	946.1	946.1	13805.4	0.95	473.1	473.1	14751.5	15697.6
0.90	426.5	426.5	14026.2	0.95	213.3	213.3	14452.7	14879.2
0.90	427.5	427.5	13843.7	0.95	213.8	213.8	14271.3	14698.8
0.90	362.5	489.8	13810.5	0.95	181.2	244.9	14236.6	14662.8
0.90	423.6	423.6	13756.6	0.95	211.8	211.8	14180.2	14603.8
0.90	120.4	1522.2	13672.2	0.95	60.2	761.1	14493.5	15314.8
0.90	120.7	1525.6	13516.6	0.95	60.3	762.8	14339.7	15162.9
0.90	120.4	1494.0	13418.3	0.95	60.2	747.0	14225.5	15032.7
0.90	120.0	1525.6	13508.5	0.95	60.0	762.8	14331.4	15154.2
0.90	120.0	1492.1	13179.9	0.95	60.0	746.1	13986.0	14792.0
0.90	931.6	931.6	13341.4	0.95	465.8	465.8	14272.9	15204.5
0.90	933.8	933.8	13194.6	0.95	466.9	466.9	14128.4	15062.1
0.90	362.5	1525.6	13278.6	0.95	181.2	762.8	14222.6	15166.7
0.90	920.5	920.5	13142.6	0.95	460.3	460.3	14063.2	14983.7
0.90	919.4	919.4	12902.5	0.95	459.7	459.7	13821.9	14741.3

5. CONCLUSIONS

A two-stage supply chain with a technology dependent stochastic demand is considered comprehensively. Although the majority of the supply chain literature is interested in happenings between suppliers and retailers in a production sector we are are concentrated on a service sector by introducing a problem of a service operator. Besides, a literature about introduction and diffusion of innovative technology, which mainly pays attention to timing of new technology, and patent protection of it, has come into being, as well as a literature related to irreversible technology expenditures to enter a new market has also been constituted. However, to best of our knowledge, none of the papers has investigated the R&D investments in terms of operational parameters in a competitive environment.

In this paper, we have considered the interactions of two vendors competing to develop a new technology and then subsequently selling it to a common telecom network operator. We introduce a service operator who installs network capacity to provide a telecom service for its customers. This telecom network can also be used throughout the periods. A vendor supplies necessary equipment (hardware or software) to upgrade the network of the operator. The main objectives are to provide insights to conditions where such R&D investments take place, and to the operation of such a telecom value chain, as well as coordinating contracts.

If a single company owns the telecom value chain and manages it via a centralized decision maker, in order for the chain to achieve maximum expected profit each vendor invests in new technology and the operator increases the second period capacity under appropriate parameters for the new traffic to be created by the new technology for both of the models given in Chapter 3, and Chapter 4. A numerical study illustrates the behavior of the central solution for various parameter levels for both of the models. Note that we just focus on the situations where R&D investment boosts up the demand and causes uploading of extra new technology to telecom network, nevertheless, it might be optimal not to build up extra network for some cases where technology triggered demand is inadequate, but such cases are out of scope of this thesis.

In the model given in Chapter 3, the vendors and the operator interacts via unit price per unit of extra capacity of the new technology. As a result, we create an asymmetry between the unit price of the vendors so that one of the vendors sells his technology with a cheaper unit price with respect to its rival, and the operator always prefers him when he materializes his innovation. In this model, we take into account of two different nested game structures. In the first one, the vendors determine their unit price firstly, then they decide on the R&D investments afterwards and in the second one, all decisions are made in a reverse manner. If each player acts independently and maximizes its own profit, under a specific structure of the game (the unit price decisions are first and the investment decisions follow them) an equilibrium does not exist. None of the vendors invest in the new technology. However, under a reverse nested game structure (investment first, price next), the vendors might invest in a new technology when the unit price of the new technology does not depend on the R&D investment levels. But, the vendors have an incentive to distort the game when they determine the unit price of the new technology with respect to R&D investments they made at the first place.

First of all, R&D investment is defined as the act of incurring an immediate cost in the expectation of future pay-offs [24]. As a result, uncertainty makes firms pursue their rivals' strategies when they all serve to the same market and have the same opportunity to trigger the market demand. Besides, if the strategy of the vendors is to determine the unit price of the new technology he offers to market as a function of R&D expenditures he incurs during development period, the vendors endure the price risk as well as the demand risk. In addition, taking strategic interactions with their rivals in the market into account, companies need to deploy R&D budgets wisely, hence, they attempt to distort the game and they tend to be a follower in an innovative market. An important managerial insight is that innovative firms are more conservative when their rivals can benefit from the demand which is triggered by pioneers. Furthermore, this negative result (usually seen in public good investments in economics) can be due to several things such as:

- A different game structure at the vendor level (investment and price together) might still have an equilibrium.
- Under a revenue sharing contract, the game might have a different structure.

In the model given in Chapter 4, the vendors and the operator interacts via R&D investments directly as long as the operator chooses the vendor who can trigger the demand efficiently because the unit price of the new technology is exogenous, and determined by the market dynamics. When the vendors give the technology investment decisions simultaneously and the operator announces the extra network capacity afterwards, a unique Nash equilibrium appears under one-shot investment scheme of the game under particular conditions. Firstly, R&D expenditure per firm decreases as competition increases, however, the total R&D expenditure in the sector increases with competition. Secondly, the efficiency in some parameters such as firm's likelihood of innovation and production cost of new technology is an advantage in the competition in an innovative market, but it causes drawing the rivals back from investing in a new technology. The similar results are also stated by Justman and Mehrez [15], and their preceding papers. Finally, the R&D incentives of the firms when market exogenously determines the unit price of the new technology are modeled and all the operational cost/revenue parameters and their effects of this game are analyzed thoroughly. What makes sense is that the players of the innovation game can allocate their R&D investment when they have an information about how much money they can sell the end-product to the market (compared to the model in Chapter 3).

As a managerial insight, both the central owner of the value chain in the centralized setting and vendors individually in the decentralized setting increase the R&D funds as the marginal profit of the new technology increases. Moreover, innovative firms make R&D investment when the marginal profit of the new technology is attractive enough. Because of this fact, firms engage in innovation activities when the price of the new technology is much higher than its cost.

To summarize, we try to provide managerial insights about R&D investment levels of the firms in a competitive, innovative market environment. To fulfill such an objective, we focus on telecom sector as the most competitive and innovative market and we model the telecom value chain consisting of two vendors who supply the high technology equipment and the operator who buys and installs it to her network. R&D incentives of the firms via considering operational factors associated with R&D investment decisions are analyzed comprehensively and some insights are provided to light the phenomenon up.

As further research directions to this study, we propose two beneficial ways. The first one is that R&D investment decisions of the vendors can be examined under useful contracts which take place in the practice such as revenue sharing contract. The second one is that coordination either between the vendors or between the operator and the vendors can be examined. Because, according to the report of Lindmark *et al.* [3], collaborative innovation activities attract attention in telecom sector extensively.

APPENDIX A: PROOFS

A.1. Proof of Proposition 3.2.1

The concavity of the objective function w.r.t. C_1, C_2, and I can be shown by checking the Hessian matrix:

$$H = \begin{vmatrix} \frac{\partial^2\pi}{\partial C_1^2} & \frac{\partial^2\pi}{\partial C_1 \partial C_2} & \frac{\partial^2\pi}{\partial C_1 \partial I} \\ \frac{\partial^2\pi}{\partial C_1 \partial C_2} & \frac{\partial^2\pi}{\partial C_2^2} & \frac{\partial^2\pi}{\partial C_2 \partial I} \\ \frac{\partial^2\pi}{\partial C_1 \partial I} & \frac{\partial^2\pi}{\partial C_2 \partial I} & \frac{\partial^2\pi}{\partial I^2} \end{vmatrix}$$

The second order and cross partial derivatives are as follows:

$$\begin{aligned}
\frac{\partial^2 \pi (C_1, C_2, I)}{\partial C_1^2} &= -\left[(a_1 + v_1) + q_2 q_1 (a_2 + v_2)\right] g(C_1) \\
&\quad - (p_2 + q_2 p_1)(a_2 + v_2) f(C_1 + C_2 | I) \\
\frac{\partial^2 \pi (C_1, C_2, I)}{\partial C_2^2} &= -(p_2 + q_2 p_1)(a_2 + v_2) f(C_1 + C_2 | I) \\
\frac{\partial^2 \pi (C_1, C_2, I)}{\partial I^2} &= (p_2 + q_2 p_1)((a_2 + v_2) F(C_1 + C_2 | I) - v_2) \frac{\partial^2 E[D(I)]}{\partial I^2} \\
&\quad - (p_2 + q_2 p_1)(a_2 + v_2) f(C_1 + C_2 | I) \left[\frac{\partial E[D(I)]}{\partial I}\right]^2 \\
\frac{\partial^2 \pi (C_1, C_2, I)}{\partial C_1 \partial C_2} &= -(p_2 + q_2 p_1)(a_2 + v_2) f(C_1 + C_2 | I) \\
\frac{\partial^2 \pi (C_1, C_2, I)}{\partial C_1 \partial I} &= (p_2 + q_2 p_1)(a_2 + v_2) f(C_1 + C_2 | I) \frac{\partial E[D(I)]}{\partial I} \\
\frac{\partial^2 \pi (C_1, C_2, I)}{\partial C_2 \partial I} &= (p_2 + q_2 p_1)(a_2 + v_2) f(C_1 + C_2 | I) \frac{\partial E[D(I)]}{\partial I}
\end{aligned}$$

When we check the Hessian matrix we see that the determinant of the first principal submatrix is negative such that $H_1 = \frac{\partial^2\pi}{\partial C_1^2} < 0$, $\frac{\partial^2\pi}{\partial C_2^2} < 0$, and $\frac{\partial^2\pi}{\partial I^2} < 0$. And, the determinant of the second principal submatrix is positive such that

$$H_2 = \left[(a_1 + v_1) + q_2 q_1 (a_2 + v_2)\right](p_2 + q_2 p_1)(a_2 + v_2) g(C_1) f(C_1 + C_2 | I) > 0$$

And, the determinant of the third principal submatrix is negative such that

$$\begin{aligned} H_3 &= [(a_1+v_1)+q_2q_1(a_2+v_2)](p_2+q_2p_1)^2(a_2+v_2)\,g(C_1)\,f(C_1+C_2|\,I) \\ &\quad ((a_2+v_2)F(C_1+C_2|\,I)-v_2)\frac{\partial^2 E[D(I)]}{\partial I^2}<0 \end{aligned}$$

Hence, since hessian is negative definite, the centralized system revenue function is jointly concave in C_1, C_2, I.

A.2. Proof of Proposition 3.3.1

The concavity of the objective function w.r.t. C_1, and C_2 can be shown by checking the Hessian matrix:

$$H=\begin{vmatrix} \frac{\partial^2\pi_o}{\partial C_1^2} & \frac{\partial^2\pi_o}{\partial C_1\partial C_2} \\ \frac{\partial^2\pi_o}{\partial C_1\partial C_2} & \frac{\partial^2\pi_o}{\partial C_2^2} \end{vmatrix}$$

The second order and cross partial derivatives are as follows:

$$\begin{aligned} \frac{\partial^2\pi_o(C_1,C_2;w_1,I_1,w_2,I_2)}{\partial C_1^2} &= -[(a_1+v_1)+q_2q_1(a_2+v_2)]\,g(C_1) \\ &\quad -(p_2+q_2p_1)(a_2+v_2)\,f(C_1+C_2|\,I) \\ \frac{\partial^2\pi_o(C_1,C_2;w_1,I_1,w_2,I_2)}{\partial C_2^2} &= -(p_2+q_2q_1)(a_2+v_2)\,f(C_1+C_2|\,I) \\ \frac{\partial^2\pi_o(C_1,C_2;w_1,I_1,w_2,I_2)}{\partial C_1\partial C_2} &= -(p_2+q_2q_1)(a_2+v_2)\,f(C_1+C_2|\,I) \end{aligned}$$

Then, to check the hessian matrix whether it is negative semidefinite or not, we do the following analysis:

$$\begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{vmatrix} \frac{\partial^2\pi_o}{\partial C_1^2} & \frac{\partial^2\pi_o}{\partial C_1\partial C_2} \\ \frac{\partial^2\pi_o}{\partial C_1\partial C_2} & \frac{\partial^2\pi_o}{\partial C_2^2} \end{vmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\le 0$$

After some algebraic manipulation we end up with the following inequality:

$$\begin{aligned} - & \left[(a_1+v_1)+q_2q_1(a_2+v_2)\right] g(C_1)(x_1)^2 \\ - & (p_2+q_2p_1)(a_2+v_2) f(C_1+C_2|\, I)(x_1+x_2)^2 \leq 0 \end{aligned} \tag{A.1}$$

Since, the inequality (A.1) is nonpositive for all x_1, and x_2 variables, Hessian matrix of operator is negative semidefinite, hence, the expected revenue of the operator is concave in C_1, and C_2. Furthermore, if we do not allow our variables to take zero values, Hessian matrix of operator is negative definite, hence, it is strictly concave in C_1, and C_2.

A.3. Proof of Proposition 3.3.2

$$\frac{\partial^2 \pi_1(I_1; I_2(w_2), w_1)}{\partial I_1^2} = q_2p_1(w_1-m_1)\frac{d^2E[D(I)]}{dI_1^2} < 0$$

Since $\frac{d^2E[D(I)]}{dI_1^2} < 0$ by assumption, the expected profit function of the first vendor is concave in I_1 given w_1, so does the expected profit function of the second vendor in I_2 given w_2.

A.4. Proof of Proposition 3.3.3

The expected profit function of the first (second) vendor is concave in $I_1(I_2)$ as it is shown in Section A.3. Second order derivative of the expected profit function of the first (second) vendor at the first stage of game is as follows:

$$\begin{aligned} \frac{\partial^2 \pi_1(w_1; w_2(I_2))}{\partial w_1^2} &= -(q_2p_1)^2\left(\frac{2}{(p_2+q_2p_1)(a_2+v_2)f(C_1+C_2|I)} + \frac{2}{[(a_1+v_1)+q_2q_1(a_2+v_2)]g(C_1)} + \frac{\lambda}{2}\right) < 0 \\ \frac{\partial^2 \pi_2(w_2; w_1(I_1))}{\partial w_1^2} &= -2(p_2)^2\left(\frac{2}{(p_2+q_2p_1)(a_2+v_2)f(C_1+C_2|I)} + \frac{2}{[(a_1+v_1)+q_2q_1(a_2+v_2)]g(C_1)} + \frac{(1-\lambda)}{2}\right) < 0 \end{aligned}$$

Since the second order derivative of them are strictly negative, the expected profit functions of the vendors are strictly concave in w_1 and w_2 respectively. Then the first order conditions gives the global maximum if it is in the range of the wholesale prices. Otherwise, the global maximum point appears at the bounds of the wholesale prices. FOCs are as follows:

$$\frac{\partial \pi_1}{\partial w_1} = q_2 p_1 \left(C_2 - q_2 p_1 \left(w_1 - m_1 \right) \left(\frac{1}{(p_2+q_2q_1)(a_2+v_2)f(C_1+C_2|I)} + \frac{1}{[(a_1+v_1)+q_2q_1(a_2+v_2)]g(C_1)} + \frac{\lambda}{2} \right) \right) = 0$$
$$\frac{\partial \pi_2}{\partial w_1} = p_2 \left(C_2 - p_2 \left(w_2 - m_2 \right) \left(\frac{1}{(p_2+q_2q_1)(a_2+v_2)f(C_1+C_2|I)} + \frac{1}{[(a_1+v_1)+q_2q_1(a_2+v_2)]g(C_1)} + \frac{(1-\lambda)}{2} \right) \right) = 0$$

A.5. Proof of Proposition 3.3.4

FOCs of the pay-off functions, π_1, and π_2, with respect to w_1, and w_2, respectively, are given as:

$$\begin{aligned} \frac{\partial \pi_1}{\partial w_1} &= q_2 p_1 \left[C_2 \left(w_1, w_2, I \right) - q_2 p_1 A \left(w_1 - m_1 \right) \right] \\ \frac{\partial \pi_2}{\partial w_2} &= p_2 \left[C_2 \left(w_1, w_2, I \right) - p_2 A \left(w_2 - m_2 \right) \right] \end{aligned} \tag{A.2}$$

where $A = \frac{1}{(p_2+q_2p_1)(a_2+v_2)g(\Omega)} + \frac{1}{[(a_1+v_1)+q_2q_1(a_2+v_2)]g\left(C_1^*\right)}$ and $\Omega = G^{-1}\left\{1 - \left[\frac{p_2w_2+q_2p_1w_1}{(p_2+q_2p_1)(a_2+v_2)} + \frac{o_2}{a_2+v_2}\right]\right\}$. Note that A is a constant under uniform distribution. And, second order partial derivatives are given as follows, respectively:

$$\frac{\partial^2 \pi_1}{\partial (w_1)^2} = q_2 p_1 \left[-q_2 p_1 A - q_2 p_1 A \right] < 0$$
$$\frac{\partial^2 \pi_2}{\partial (w_2)^2} = p_2 \left[-p_2 A - p_2 A \right] < 0$$

Due to the fact that the second order partial derivative of the expected revenue function, π_1, (π_2) with respect to w_1, (w_2) is negative, π_1, (π_2) is strictly concave in w_1, (w_2). We characterize the best w_1^*, and w_2^* values via using FOCs because w_1^*, and also w_2^* are defined in its own interval. For Case(i), if $\left.\frac{\partial \pi_1}{\partial w_1}\right|_{\overline{w_1}} > 0$ then $w_1^* = \overline{w_1} = a_2 - o_2$ because π_1 still keeps increasing in w_1, hence, the best value is upper bound of $w_1 = \overline{w_1}$. For case (ii), π_1 gets its optimal value in intermediate value of w_1 if $\left.\frac{\partial \pi_1}{\partial w_1}\right|_{\underline{w_1}} > 0$ *and* $\left.\frac{\partial \pi_1}{\partial w_1}\right|_{\overline{w_1}} < 0$. Finally, π_1 keeps decreasing as w_1 increases if $\left.\frac{\partial \pi_1}{\partial w_1}\right|_{\underline{w_1}} < 0$, therefore the best value is lower bound of $w_1 = \underline{w_1}$. The same argument is also valid for the second vendor. If we obtain w_1^*, and w_2^* from FOCs, the following results are always satisfied:

$$\left.\frac{\partial \pi_1}{\partial w_1}\right|_{w_1^*} = 0 \Rightarrow C_2\left(w_1^*, w_2, I\right) = q_2 p_1 A\left(w_1^* - m_1\right)$$
$$\left.\frac{\partial \pi_1}{\partial w_2}\right|_{w_2^*} = 0 \Rightarrow C_2\left(w_1, w_2^*, I\right) = p_2 A\left(w_2^* - m_2\right)$$

A.6. Proof of Proposition 4.1.1

The expected revenue function in (4.2) is strictly concave in C_1 since the second derivative of it with respect to C_1 is strictly negative as shown below.

$$\frac{d^2\pi}{d\left(C_1\right)^2} = -\left(a_1 + v_1\right) f\left(C_1\right) < 0$$

A.7. Proof of Lemma 4.1.1

We first show that the separable parts of expected pay-off are concave in corresponding decision variables.

- π^1 is strictly concave in C_2^1 and I_1 since the Hessian matrix (shown below) is negative definite.

$$H = \begin{bmatrix} \frac{\partial^2 \pi^1}{\partial\left(C_2^1\right)^2} & \frac{\partial^2 \pi^1}{\partial\left(C_2^1\right)\partial(I_1)} \\ \frac{\partial^2 \pi^1}{\partial\left(C_2^1\right)\partial(I_1)} & \frac{\partial^2 \pi^1}{\partial(I_1)^2} \end{bmatrix}$$

$$\begin{aligned}
\frac{d^2\pi^1}{d\left(C_2^1\right)^2} &= -\left(a_2 + v_2\right) f\left(C_1 + C_2^1 \middle| I_1\right) \\
\frac{d^2\pi^1}{d\left(C_2^1\right) d\left(I_1\right)} &= \left(a_2 + v_2\right) f\left(C_1 + C_2^1 \middle| I_1\right) \frac{dED\left(I_1\right)}{dI_1} \\
\frac{d^2\pi^1}{d\left(I_1\right)^2} &= -\left(a_2 + v_2\right) f\left(C_1 + C_2^1 \middle| I_1\right) \left[\frac{dED\left(I_1\right)}{dI_1}\right]^2 \\
&\quad + \left(\left(a_2 + v_2\right) F(C_1 + C_2^1 | I_1) - v_2\right) \frac{d^2 ED\left(I_1\right)}{d\left(I_1\right)^2}
\end{aligned}$$

It turns out that the determiant of the fist principal submatrix of H, $|H_1| < 0$, and of the second principal submatrix H, $|H_2| > 0$. Hence, the hessian matrix is negative definite; therefore, π^1 is strictly concave in C_2^1 and I_1.

- π^2 is strictly concave in C_2^2 and I_2 and it can be easily shown just like the preceding proof of π^1.

First order conditions of the revenue functions are as follows:

$$\begin{aligned}
\tfrac{\partial \pi^1}{\partial (C_2^1)} &= \left[(a_2+v_2)\,\overline{F}(C_1+C_2^1|\,I_1)-o_2-m_1\right]=0\\
\tfrac{\partial \pi^1}{\partial (I_1)} &= \left[(a_2+v_2)\,F(C_1+C_2^1|\,I_1)-v_2\right]\tfrac{dED(I_1)}{dI_1}\\
\tfrac{\partial \pi^2}{\partial (C_2^2)} &= \left[(a_2+v_2)\,\overline{F}(C_1+C_2^2|\,I_2)-o_2-m_2\right]=0\\
\tfrac{\partial \pi^2}{\partial (I_2)} &= \left[(a_2+v_2)\,F(C_1+C_2^2|\,I_2)-v_2\right]\tfrac{dED(I_2)}{dI_2}
\end{aligned}$$

The optimal capacity levels (C_2^{1*} and C_2^{2*}) and the optimal investment levels (I_1^* and I_1^*) satisfy the first order conditions:

$$C_2^{j*} = \theta\,(I_j) + G^{-1}\left(1-\frac{o_2+m_j}{a_2+v_2}\right) - C_1 \qquad j=1,2. \tag{A.3}$$

If (A.3) is substituted into corresponding FOCs with respect to I_1 and I_2, we end up with the following:

$$\frac{d\pi^j}{d\,(I_j)} = \left[(a_2-o_2-m_j)\right]\frac{dE\left[D\,(I_j)\right]}{dI_j} \qquad j=1,2. \tag{A.4}$$

From (A.4), FOCs are strictly positive with respect to (I_1 and I_2), (since both $a_2 > o_2+m_j$ and $\frac{dE[D(I_j)]}{dI_j} > 0$ by model assumptions); hence, π_1, (π_2) is strictly increasing in I_1, (I_2). C_2^{1*}, (C_2^{2*}) as a function of I_1 is strictly increasing in I_1,(I_2). Therefore π_1, (π_2) is monotonously increasing in (I_1 and C_2^1), (I_2 and C_2^2).

A.8. Proof of Lemma 4.1.2

π^1, (π^2) is strictly concave function of I_1, and C_2^1, (I_2, and C_2^2) and from Lemma 4.1.1. And, π^0 is constant with respect to I_1, C_2^1, I_2, C_2^2. Each branch of (4.7) is summation of π^0 which is constant, and π^1, and π^2 which are concave functions, and linear investment functions, I_1, and I_2. Therefore, each branch of (4.7) is concave in I_1, C_2^1, I_2, C_2^2, and we can obtain optimal decision variables by employing FOCs given as:

$$\begin{aligned}
\tfrac{\partial \pi}{\partial (I_1)} &= (q_2 p_1)\left[(a_2+v_2)\,F(C_1+C_2^1 |\, I_1) - v_2\right] \tfrac{dED(I_1)}{dI_1} - 1 = 0\\
\tfrac{\partial \pi}{\partial (I_2)} &= (p_2)\left[(a_2+v_2)\,F(C_1+C_2^2 |\, I_2) - v_2\right] \tfrac{dED(I_2)}{dI_2} - 1 = 0\\
\tfrac{\partial \pi}{\partial (I_1)} &= (p_1)\left[(a_2+v_2)\,F(C_1+C_2^1 |\, I_1) - v_2\right] \tfrac{dED(I_1)}{dI_1} - 1 = 0\\
\tfrac{\partial \pi}{\partial (I_2)} &= (q_1 p_2)\left[(a_2+v_2)\,F(C_1+C_2^2 |\, I_2) - v_2\right] \tfrac{dED(I_2)}{dI_2} - 1 = 0
\end{aligned}$$

Note that $\overline{I_1}$, $\overline{\overline{I_2}}$, $\overline{\overline{I_1}}$, and $\overline{I_2}$ can be obtained in the order FOCs are given. C_2^{j*} can be obtained as given in Lemma 4.1.1. Furthermore, when we can substitute $(a_2 - o_2 - m_j)$ in place of $\left[(a_2+v_2)\,F(C_1+C_2^j \middle|\, I_j) - v_2\right]$ by reordering A.3.

A.9. Proof of Lemma 4.1.3

Initially, π^0 is constant in decision variables (I_1, I_2, C_2^1, C_2^2). Since π_1, (π_2) is monotonously increasing in (I_1 and C_2^{1*}), (I_2 and C_2^{2*}), maximum π^1, (π^2) goes to infinity as I_1, (I_2) goes to infinity. However, maximum π as given in (4.9) goes to negative infinity as I_1, (I_2) goes to infinity. Therefore, there must exist upper investment levels extracted from the following part of (4.9). Our problem turns out to be:

$$\max_{I_1, I_2, C_2^1, C_2^2} \pi = \max\left\{q_1 q_2 \pi^0 + p_1 \pi^1 + p_2 \pi^2 - I_1 - I_2\right\}$$

This function is a strictly concave function because its the summation of strictly concave and linear separable functions. Hence, the following FOCs gives us the upper bound investment levels:

$$\begin{aligned}
\frac{\partial \pi^1}{\partial \left(C_2^1\right)} &= p_1 \left[(a_2 + v_2)\, \overline{F}(C_1 + C_2^1 | \, I_1) - o_2 - m_1 \right] = 0 \\
\frac{\partial \pi^1}{\partial (I_1)} &= p_1 \left[(a_2 + v_2)\, F(C_1 + C_2^1 | \, I_1) - v_2 \right] \frac{dED(I_1)}{dI_1} - 1 = 0 \\
\frac{\partial \pi^2}{\partial \left(C_2^2\right)} &= p_2 \left[(a_2 + v_2)\, \overline{F}(C_1 + C_2^2 | \, I_2) - o_2 - m_2 \right] = 0 \\
\frac{\partial \pi^2}{\partial (I_2)} &= p_2 \left[(a_2 + v_2)\, F(C_1 + C_2^2 | \, I_2) - v_2 \right] \frac{dED(I_2)}{dI_2} - 1 = 0
\end{aligned}$$

Note that $F(C_1 + C_2^j | \, I_j) = 1 - \frac{o_2+m_j}{a_2+v_2}$ from the FOCs with respect to C_2^j, when we substitute this into the FOCs with respect to I_j, $\left[(a_2 + v_2)\, F(C_1 + C_2^j | \, I_j) - v_2\right]$ turns out to be $(a_2 - o_2 - m_j)$.

A.10. Proof of Lemma 4.1.4

Observe that (4.9) is strictly increasing function of π^1 and π^2, besides, it can be seen from (4.7). We also show that π^1, and π^2 reaches their maximum values when I_1, and I_2 goes to infinity, from Lemma 4.1.1. Second part of (4.9) is a negative term consisting of $max\pi^1$, and $max\pi^2$, and minimum of them has to be chosen. In Lemma 4.1.3, the existence of upper bound investment levels is revealed. The upper bound investment levels maximize the first part of (4.9). The interpretation of them is each technology supplier provides its best profit for the centralized system by investing its upper bound. If $\pi^1\left(\widetilde{I_1}, \widetilde{C_2^1}\right) < \pi^2\left(\widetilde{I_2}, \widetilde{C_2^2}\right)$, then π in (4.9) turns out to be $\pi = q_2 q_1 \pi^0 + p_1\pi^1 + p_2\pi^2 - p_1 p_2 \pi^1 - I_1 - I_2$ because the best value of π^2 is grater than π^1, and we know that there are two possible pairs of investment levels from Lemma 4.1.2. In other worlds, if there are two levels of investment for centralized decision maker, it is profitable to determine π^2 more. On the other hand, if $\pi^1\left(\widetilde{I_1}, \widetilde{C_2^1}\right) > \pi^2\left(\widetilde{I_2}, \widetilde{C_2^2}\right)$, then π in (4.9) turns out to be $\pi = q_2 q_1 \pi^0 + p_1\pi^1 + p_2\pi^2 - p_1 p_2 \pi^2 - I_1 - I_2$, because making π^1 bigger by investing more results more profit for the centralized system due to the fact that the centralized system determines two levels of investment (See Lemma 4.1.2). Finally, if $\pi^1\left(\widetilde{I_1}, \widetilde{C_2^1}\right) = \pi^2\left(\widetilde{I_2}, \widetilde{C_2^2}\right)$, then π in (4.9) turns out to be $\pi = q_2 q_1 \pi^0 + p_1\pi^1 + p_2\pi^2 - p_1 p_2 \left\{\pi^1, \pi^2\right\} - I_1 - I_2$ because π is defined by choosing π^1 or π^2 arbitrarily. The important point is to observe two different possible investment levels for each technology provider and assign which one results more profit for centralized system.

A.11. Proof of Theorem 4.1.1

The three branches of expected revenue function of the centralized system given in Lemma 4.1.4, can be separated in decision variables, (I_1 and C_2^1), (I_2 and C_2^2). π^1, (π^2) as a part of (4.14) is just function of I_1, and C_2^{1*}, (I_2 and C_2^{2*}). And, each branch of (4.14) is strictly concave due to the summation of constant, π^0, concave functions, π^1, and π^2, and linear functions, I_1, and I_2. (See the Section A.9 in Appendix A). Therefore, FOCs let us characterize the optimal solution. First order conditions of the expected revenue function in (4.14) are represented as follows:

$$\begin{aligned}
\tfrac{\partial\pi}{\partial(I_1)} &= (q_2p_1)\left[(a_2+v_2)\,F(C_1+C_2^1|\,I_1)-v_2\right]\tfrac{dED(I_1)}{dI_1}-1=0\\
\tfrac{\partial\pi}{\partial(I_2)} &= (p_2)\left[(a_2+v_2)\,F(C_1+C_2^2|\,I_2)-v_2\right]\tfrac{dED(I_2)}{dI_2}-1=0\\
\tfrac{\partial\pi}{\partial(I_1)} &= (p_1)\left[(a_2+v_2)\,F(C_1+C_2^1|\,I_1)-v_2\right]\tfrac{dED(I_1)}{dI_1}-1=0\\
\tfrac{\partial\pi}{\partial(I_2)} &= (q_1p_2)\left[(a_2+v_2)\,F(C_1+C_2^2|\,I_2)-v_2\right]\tfrac{dED(I_2)}{dI_2}-1=0\\
\tfrac{\partial\pi}{\partial(I_1)} &= (q_2p_1+0.5p_1p_2)\left[(a_2+v_2)\,F(C_1+C_2^1|\,I_1)-v_2\right]\tfrac{dED(I_1)}{dI_1}-1=0\\
\tfrac{\partial\pi}{\partial(I_2)} &= (q_1p_2+0.5p_1p_2)\left[(a_2+v_2)\,F(C_1+C_2^2|\,I_2)-v_2\right]\tfrac{dED(I_2)}{dI_2}-1=0
\end{aligned}$$

Note that $\overline{I_1}$, $\overline{\overline{I_2}}$, $\overline{\overline{I_1}}$, $\overline{I_2}$, $\widehat{I_1}$, and $\widehat{I_2}$ can be obtained in the order FOCs are given. The first pair of FOCs is the derivative of the first branch of (4.14) with respect to I_1, and I_2, the following pair is the derivative of the third branch, and the last pair is the derivative of the second branch. C_2^{j*} can be obtained as given in Lemma 4.1.1. Moreover, $\left[(a_2+v_2)\,F(C_1+C_2^j\middle|\,I_j)-v_2\right]$ can be substitutable with $(a_2-o_2-m_j)$ as expressed in Proof A.9. In Theorem 4.1.1, all decision variables are given with respect to these results.

A.12. Proof of Corollary 4.1.1

1. $\overline{I_1^*}$, $\widetilde{I}_1$, $\overline{\overline{I_1^*}}$ as given in Theorem 4.1.1 are increasing in p_1 due to the fact that derivative of the expected demand is monotonously decreasing as I_1 is increasing

because we assume that the expected demand is monotonously increasing and strictly concave in I_1. Besides, $\widetilde{C_2^1}$, $\overline{C_2^{1*}}$ as given in Theorem 4.1.1 are functions of I_1 and increasing as I_1 increases. Moreover, π is increasing as p_1 increases because π^1, a component of π, is monotonously increasing in I_1, and C_2^1 as argued in (A.7).

2. Analogous to case(1).
3. $\widetilde{C_2^1}$, $\overline{C_2^{1*}}$, ($\widetilde{C_2^2}$, $\overline{C_2^{2*}}$) as given in Theorem 4.1.1 are functions of m_1, (m_2) and decreasing in m_1, (m_2) because the inverse function of the demand G^{-1} is decreasing as m_1, (m_2) increases.
4. $\widetilde{C_2^1}$, $\overline{C_2^{1*}}$, ($\widetilde{C_2^2}$, $\overline{C_2^{2*}}$) as given in Theorem 4.1.1 are functions of v_2 and increasing in v_2, because the inverse function of the demand G^{-1} is increasing as v_2 increases. Besides, $\pi \to -\infty$ because $\pi^1 \to -\infty$, and $\pi^2 \to -\infty$ as $v_2 \to \infty$. $\pi^1 \to -\infty$ since $-v_2\mu_2(I_1)$ linear term more rapidly converges to negative infinity than $S(C_1 + C_2^1 | I_1)$ does, and the other terms also converge to negative infinity as $v_2 \to \infty$. Similar arguments can be used for π^2. Therefore, $\pi \to -\infty$ as $v_2 \to \infty$.

A.13. Proof of Proposition 4.2.1

Firstly, π_o^j is strictly concave in C_2^j, where $j = 1, 2$.

$$\frac{d^2\pi_o^j}{d\left(C_2^j\right)^2} = -\left(a_2 + v_2\right) f(C_1 + C_2^j \big| I_j) < 0 \qquad for \qquad j = 1, 2.$$

Since the extra network capacity decisions, C_2^1 and C_2^2, are contingent only upon R&D investments made by the vendors, by all means, π_o^1 and π_o^2 are just dependent on the investment levels of the vendors. Therefore, π_o^1 is equal to π_o^2 if and only if I_1 is equal to I_2, so that the operator expected revenue function breaks into pieces when $I_1 = I_2$.

Secondly, the operator's expected revenue function is differentiable at the break point of the function, $I_1 = I_2$, as shown below:

$$\lim_{I_1 \to I_2^-} \pi_o = p_2\pi_o^2 + q_2p_1\pi_o^2 = \left(p_2 + q_2p_1\right)\pi_o^2$$
$$\lim_{I_1 \to I_2^+} \pi_o = p_1\pi_o^2 + q_1p_2\pi_o^2 = \left(p_1 + q_1p_2\right)\pi_o^2$$

Since $\lim\limits_{I_1 \to I_2^-} \pi = \lim\limits_{I_1 \to I_2^+} \pi = \pi\left(I_1 = I_2\right)$, therefore, π_o is continuous at $I_1 = I_2$, and also differentiable because $\lim\limits_{I_1 \to I_2^-} \frac{\pi_o(I_1) - \pi_o\left(I_2^-\right)}{I_1 - I_2^-} = \lim\limits_{I_1 \to I_2^+} \frac{\pi_o(I_1) - \pi_o\left(I_2^+\right)}{I_1 - I_2^+} = 0$.

Finally, all the parts of the expected revenue function of the operator are the summation of separable, strictly concave functions, π_o^1 and π_o^2, and a constant, π_o^0, therefore, all branches of the function is strictly concave in itself.

To deal with the determination of the operator's network capacity decisions at the second period, C_2^1 and C_2^2, we simply equalize the first order conditions, shown below for all three branches of the expected revenue function of the operator. FOCs for $I_1 < I_2$;

$$\frac{\partial \pi_o}{\partial\left(C_2^1\right)} = q_2 p_1 \left[(a_2 + v_2)\,\overline{F}(C_1 + C_2^1|\,I_1) - (o_2 + w)\right] = 0$$
$$\frac{\partial \pi_o}{\partial\left(C_2^2\right)} = p_2 \left[(a_2 + v_2)\,\overline{F}(C_1 + C_2^2|\,I_2) - (o_2 + w)\right] = 0$$

FOCs for $I_1 > I_2$;

$$\frac{\partial \pi_o}{\partial\left(C_2^1\right)} = p_1 \left[(a_2 + v_2)\,\overline{F}(C_1 + C_2^1|\,I_1) - (o_2 + w)\right] = 0$$
$$\frac{\partial \pi_o}{\partial\left(C_2^2\right)} = q_1 p_2 \left[(a_2 + v_2)\,\overline{F}(C_1 + C_2^2|\,I_2) - (o_2 + w)\right] = 0$$

FOCs for $I_1 = I_2$;

$$\frac{\partial \pi_o}{\partial\left(C_2^1\right)} = (q_2 p_1 + 0.5 p_1 p_2) \left[(a_2 + v_2)\,\overline{F}(C_1 + C_2^1|\,I_1) - (o_2 + w)\right] = 0$$
$$\frac{\partial \pi_o}{\partial\left(C_2^2\right)} = (q_1 p_2 + 0.5 p_1 p_2) \left[(a_2 + v_2)\,\overline{F}(C_1 + C_2^2|\,I_2) - (o_2 + w)\right] = 0$$

As it can frankly be seen from the FOCs, in any cases, the operator selects the same extra network capacity for the second period regardless of vendors' innovation probabilities.

A.14. Proof of Proposition 4.2.2

Since the second derivative of the expected profit function's branches of the first vendor as shown below is negative, left and right branches of the pay-off function of the first vendor is strictly concave in I_1.

$$\frac{\partial^2 \pi_1^1}{\partial (I_1)^2} = q_2 p_1 (w - m_1) \frac{d^2 ED[I_1]}{d(I_1)^2} < 0$$
$$\frac{\partial^2 \pi_1^3}{\partial (I_1)^2} = p_1 (w - m_1) \frac{d^2 ED[I_1]}{d(I_1)^2} < 0$$

$$\lim_{I_1 \to I_2^-} \pi_1^1 (I_1) = q_2 p_1 (w - m_1) C_2^1 (I_2) - I_2$$
$$\lim_{I_1 \to I_2^+} \pi_1^3 (I_1) = p_1 (w - m_1) C_2^1 (I_2) - I_2$$

Since $\lim_{I_1 \to I_2^-} \pi_1^1 (I_1) \neq \lim_{I_1 \to I_2^+} \pi_1^3 (I_1)$ when $p_2 \neq 0$, π_1 is not continuous and not differentiable when $I_1 = I_2$. Moreover, $I_1 = I_2$ can not be a maximal point of the π_1 because $\lim_{I_1 \to I_2^+} \pi_1^3 (I_1) = p_1 (w - m_1) C_2^1 (I_2) - I_2 > \pi_1^2 (I_1 = I_2) = (q_2 p_1 + 0.5 p_1 p_2) (w - m_1) C_2^1 (I_2) - I_2$ when $p_2 \neq 0$. As it can easily be seen, the point which maximizes π_1, can be derived from the FOCs of either left hand-side or right hand-side of the pay-off function. FOCs are as follows:

$$\frac{\partial \pi_1^1}{\partial (I_1)} = q_2 p_1 (w - m_1) \frac{dED[I_1]}{d(I_1)} - 1 = 0$$
$$\frac{\partial \pi_1^3}{\partial (I_1)} = p_1 (w - m_1) \frac{dED[I_1]}{d(I_1)} - 1 = 0$$

A.15. Proof of Proposition 4.2.3

Since the second derivative of the left hand-side, π_2^3, and the right hand-side, π_2^1, of the expected profit function is negative, two branches of the pay-off function of the second vendor is strictly concave in I_2.

$$\begin{aligned}\frac{\partial^2 \pi_2^1}{\partial (I_2)^2} &= p_2 (w - m_2) \frac{d^2 ED[I_2]}{d(I_2)^2} < 0 \\ \frac{\partial^2 \pi_2^3}{\partial (I_2)^2} &= q_1 p_2 (w - m_2) \frac{d^2 ED[I_2]}{d(I_2)^2} < 0\end{aligned}$$

$$\begin{aligned}\lim_{I_2 \to I_1^-} \pi_2^3 (I_2) &= q_1 p_2 (w - m_2) C_2^2 (I_1) - I_1 \\ \lim_{I_2 \to I_1^+} \pi_2^1 (I_2) &= p_2 (w - m_2) C_2^2 (I_1) - I_1\end{aligned}$$

Since $\lim_{I_2 \to I_1^-} \pi_2^3 (I_2) \neq \lim_{I_2 \to I_1^+} \pi_2^1 (I_2)$ when $p_1 \neq 0$, π_2 is not continuous and not differentiable when $I_2 = I_1$. Moreover, $I_2 = I_1$ can not be a maximal point of the π_2 because $\lim_{I_2 \to I_1^+} \pi_2^1 (I_2) = p_2 (w - m_2) C_2^2 (I_1) - I_1 > \pi_2^2 (I_2 = I_1) = (q_1 p_2 + 0.5 p_1 p_2) (w - m_2) C_2^2 (I_1) - I_1$ when $p_1 \neq 0$. As it can easily be seen, the point which maximizes π_2, can be derived from the FOCs of either left hand-side or right hand-side of the pay-off function. FOCs are as follows:

$$\begin{aligned}\frac{\partial \pi_2^1}{\partial (I_2)} &= p_2 (w - m_2) \frac{dED[I_2]}{d(I_2)} - 1 = 0 \\ \frac{\partial \pi_2^3}{\partial (I_2)} &= q_1 p_2 (w - m_2) \frac{dED[I_2]}{d(I_2)} - 1 = 0\end{aligned}$$

A.16. Proof of Proposition 4.3.1

For the case $\underline{I_1} < \widehat{I}_1$;

- Firstly, $\pi_1^3 (I_1) > \pi_1^2 (I_1) > \pi_1^1 (I_1)$ for $\forall I_1$ when $p_2 \neq 0$.
- Secondly, $\widehat{\widehat{I}}_1 > \widehat{I}_1$ and $\pi_1^3 \left(\widehat{\widehat{I}}_1 \right) > \pi_1^1 \left(\widehat{I}_1 \right)$ because $\pi_1^3 \left(\widehat{\widehat{I}}_1 \right) > \pi_1^3 \left(\widehat{I}_1 \right) > \pi_1^1 \left(\widehat{I}_1 \right)$.
- Thirdly, there exists $I_1^{'} > \widehat{\widehat{I}}_1$ such that $\pi_1^3 \left(\widehat{\widehat{I}}_1 ; I_2 \right) > \pi_1^1 \left(\widehat{I}_1 ; I_2 \right)$ as $I_2 \leq I_1^{'}$, and $\pi_1^3 \left(\widehat{\widehat{I}}_1 ; I_2 \right) < \pi_1^1 \left(\widehat{I}_1 ; I_2 \right)$ as $I_2 > I_1^{'}$. Let $\psi \left(\widehat{I}_1 , I_1 \right) = \pi_1^3 (I_1) - \pi_1^1 \left(\widehat{I}_1 \right)$. $\psi \left(\widehat{I}_1 , \widehat{\widehat{I}}_1 \right) > 0$. $\psi \left(\widehat{I}_1 , M \right) < 0$, M refers to very large investment level. Since ψ monotonically decreases after $\widehat{\widehat{I}}_1$ as $\pi_1^3 (I_1)$ decreases, it follows by the intermediate

value theorem and the monotonicity of the ψ that there exists $I_1^{'} > \widehat{\widehat{I}}_1$.

- Finally, since $\pi_1^3\left(\widehat{\widehat{I}}_1\right) > \pi_1^1\left(\widehat{I}_1\right)$ roughly when $I_2 \leq \widehat{\widehat{I}}_1$, $I_1^{'}$ must be grater than $\widehat{\widehat{I}}_1$. To ensure this property, $\left.\frac{d\pi_1^3}{d(I_1)}\right|_{I_1^{'}} < 0$. Note that $I_1^{'}$ can be extracted when $\pi_1^3\left(I_1^{'}\right) = \pi_1^1\left(\widehat{I}_1\right)$.

For the case $\widehat{I}_1 < \underline{I_1} < \widehat{\widehat{I}}_1$;

Proof is similar to previous one but since $\widehat{I}_1 < \underline{I_1}$, the point which maximizes π_1^1 is $\underline{I_1}$ rather than $\widehat{I}_1$. π_1^1 starts to decreases after $\widehat{I}_1$. $\underline{I_1}$ is the minimum required investment level to join the race. Hence, the first vendor ends up with worst profit whatever he invests grater than $\underline{I_1}$. Furthermore, $\pi_1^3\left(\widehat{\widehat{I}}_1; I_2\right) > \pi_1^1\left(\underline{I_1}; I_2\right)$ as $I_2 \leq I_1^{''}$, and $\pi_1^3\left(\widehat{\widehat{I}}_1; I_2\right) < \pi_1^1\left(\underline{I_1}; I_2\right)$ as $I_2 \geq I_1^{''}$. Note that $I_1^{''}$ can be extracted when $\pi_1^3\left(I_1^{''}\right) = \pi_1^1\left(\underline{I_1}\right)$.

A.17. Proof of Proposition 4.3.2

The proof of Φ_2 is analogous to Φ_1, (A.16). For the case $\underline{I_2} < \widehat{I}_2$;

- Firstly, $\pi_2^1(I_2) > \pi_2^2(I_2) > \pi_2^3(I_2)$ for $\forall I_2$ when $p_1 \neq 0$.
- Secondly, $\widehat{\widehat{I}}_2 > \widehat{I}_2$ and $\pi_2^1\left(\widehat{\widehat{I}}_2\right) > \pi_2^3\left(\widehat{I}_2\right)$ because $\pi_2^1\left(\widehat{\widehat{I}}_2\right) > \pi_2^1\left(\widehat{I}_2\right) > \pi_2^3\left(\widehat{I}_2\right)$.
- Thirdly, there exists $I_2^{'} > \widehat{\widehat{I}}_2$ such that $\pi_2^1\left(\widehat{\widehat{I}}_2; I_1\right) > \pi_2^3\left(\widehat{I}_2; I_1\right)$ as $I_1 \leq I_2^{'}$, and $\pi_2^1\left(\widehat{\widehat{I}}_2; I_1\right) < \pi_2^3\left(\widehat{I}_2; I_1\right)$ as $I_1 > I_2^{'}$. Let $\psi\left(\widehat{I}_2, I_2\right) = \pi_2^1(I_2) - \pi_2^3\left(\widehat{I}_2\right)$. $\psi\left(\widehat{I}_2, \widehat{\widehat{I}}_2\right) > 0$. $\psi\left(\widehat{I}_2, M\right) < 0$, M refers to very large investment level. Since ψ monotonically decreases after $\widehat{\widehat{I}}_2$ as $\pi_2^1(I_2)$ decreases, it follows by the intermediate value theorem and the monotonicity of the ψ that there exists $I_2^{'} > \widehat{\widehat{I}}_2$.
- Finally, since $\pi_2^1\left(\widehat{\widehat{I}}_2\right) > \pi_2^3\left(\widehat{I}_2\right)$ roughly when $I_1 \leq \widehat{\widehat{I}}_2$, $I_2^{'}$ must be grater than $\widehat{\widehat{I}}_2$. To ensure this property, $\left.\frac{d\pi_2^1}{d(I_2)}\right|_{I_2^{'}} < 0$. Note that $I_2^{'}$ can be extracted when $\pi_2^1\left(I_2^{'}\right) = \pi_2^3\left(\widehat{I}_2\right)$.

For the case $\widehat{I_2} < \underline{I_2} < \widehat{\widehat{I_1}}$;

Proof is similar to previous one but since $\widehat{I_2} < \underline{I_2}$, the point which maximizes π_2^3 is $\underline{I_2}$ rather than $\widehat{I_2}$. π_2^3 starts to decreases after $\widehat{I_2}$. $\underline{I_2}$ is the minimum required investment level to join the race. Hence, the first vendor ends up with worst profit whatever he invests grater than $\underline{I_2}$. Furthermore, $\pi_2^1\left(\widehat{\widehat{I_2}}; I_1\right) > \pi_2^3\left(\underline{I_2}; I_1\right)$ as $I_1 \leq I_2^{''}$, and $\pi_2^1\left(\widehat{\widehat{I_2}}; I_1\right) < \pi_2^3\left(\underline{I_2}; I_1\right)$ as $I_1 \geq I_2^{''}$. Note that $I_2^{''}$ can be extracted when $\pi_2^1\left(I_2^{''}\right) = \pi_2^3\left(\underline{I_2}\right)$.

A.18. Proof of Theorem 4.3.1

Without taking into consideration to cases, $\Phi_1\left(I_2\right)$ is non-increasing and when represented by graph in (I_1, I_2) space, composed of either one or two horizontal half-lines which cover $\left[\underline{I_1}, \infty\right)$; $\Phi_2\left(I_2\right)$ is non-increasing and when graphed in (I_1, I_2) space, composed of either one or two vertical half-lines which cover $\left[\underline{I_2}, \infty\right)$, where $\underline{I_1} = \underline{I_2}$, see (4.23). When they are represented by graph case by case for all possible cases, they must intersect at least once for corresponding cases.

A.19. Proof of Proposition 4.4.1

For $\pi_1 \geq \pi_2$;

$$\begin{aligned}
\pi\left(I_1, I_2, C_2^1, C_2^2\right) &= p_1\left\{(a_2+v_2)\, S(C_1 + C_2^1 \middle| I_1) - v_2\mu_2\left(I_1\right) - o_2C_2^1 - m_1C_2^1\right\} \\
&+ q_1p_2\left\{(a_2+v_2)\, S(C_1 + C_2^2 \middle| I_2) - v_2\mu_2\left(I_2\right) - o_2C_2^2 - m_2C_2^2\right\} \\
&+ q_1q_2\left\{(a_2+v_2)\, S(C_1) - v_2\mu_2\right\} - o_2C_1 - I_1 - I_2 \qquad \text{(A.5)}
\end{aligned}$$

$$\begin{aligned}
\pi_o\left(C_2^1, C_2^2; I_1, I_2\right) &= p_1\left\{(a_2+v_2)\, S(C_1 + C_2^1 \middle| I_1) - v_2\mu_2\left(I_1\right) - o_2C_2^1 - T_1\left(.\right)\right\} \\
&+ q_1p_2\left\{(a_2+v_2)\, S(C_1 + C_2^2 \middle| I_2) - v_2\mu_2\left(I_2\right) - o_2C_2^2 - T_2\left(.\right)\right\} \\
&+ q_1q_2\left\{(a_2+v_2)\, S(C_1) - v_2\mu_2\right\} - o_2C_1 \qquad \text{(A.6)}
\end{aligned}$$

After we explicitly add the transfer payments as given in (4.4.1) to (A.7), it turns out to be, $\pi_o\left(C_2^1, C_2^2; I_1, I_2\right)$:

$$\begin{aligned} &= p_1\left\{\lambda_1\left(a_2+v_2\right)S(C_1+C_2^1\big| I_1)-v_2\mu_2\left(I_1\right)-\lambda_1 o_2 C_2^1-\lambda_1 m_1 C_2^1\right\} \\ &+ q_1 p_2\left\{\lambda_2\left(a_2+v_2\right)S(C_1+C_2^2\big| I_2)-v_2\mu_2\left(I_2\right)-\lambda_2 o_2 C_2^2-\lambda_2 m_2 C_2^2\right\} \\ &+ q_1 q_2\left\{\left(a_2+v_2\right)S(C_1)-v_2\mu_2\right\}-o_2 C_1-\left(1-\psi_1\right)I_1-\left(1-\psi_2\right)I_2 \qquad \text{(A.7)} \end{aligned}$$

When we check the FOCs with respect to C_2^j, $j=1,2$ we observe that $\frac{\partial \pi}{\partial\left(C_2^1\right)}=\lambda_1\frac{\partial \pi_o}{\partial\left(C_2^1\right)}$, and $\frac{\partial \pi}{\partial\left(C_2^2\right)}=\lambda_2\frac{\partial \pi_o}{\partial\left(C_2^2\right)}$. Therefore, the operator chooses the right extra network capacity with the central decision maker.

The first vendor's expected profit function can be represented as follows after adding transfer payment:

$$\pi_1=p_1\left\{\left(1-\lambda_1\right)\left(a_2+v_2\right)S\left(C_1+C_2^1\big| I_1\right)-\left(1-\lambda_1\right)\left(o_2+m_1\right)C_2^1\right\}-\psi_1 I_1 \quad \text{(A.8)}$$

When we check the FOCs of (A.8) with respect to I_1 we observe that

$$\frac{\partial \pi_1}{\partial\left(I_1\right)}=p_1\left[\left(1-\lambda_1\right)\left(a_2+v_2\right)-\left(1-\lambda_1\right)\left(o_2+m_1\right)\right]\frac{dED\left(I_1\right)}{dI_1}-\psi_1=0$$

Therefore, the optimal $\widehat{\widehat{I_1^*}}$ under profit sharing contract can be derived from

$$\left.\frac{dED\left(I_1\right)}{dI_1}\right|_{\widehat{\widehat{I_1^*}}}=\frac{\psi_1}{p_1\left(1-\lambda_1\right)\left(a_2+v_2-o_2-m_1\right)}$$

Finally, $\left.\frac{dED(I_1)}{dI_1}\right|_{\widehat{\widehat{I_1^*}}}=\left.\frac{dED(I_1)}{dI_1}\right|_{I_1^*}$ when $\psi_1=\frac{(1-\lambda_1)(a_2+v_2-o_2-m_1)}{(a_2-o_2-m_1)}$. Therefore, the first vendor determines the right investment level, $\widehat{\widehat{I_1^*}}=I_1^*$, with the central decision maker. Similar analysis can be made for the second vendor. He also decides on the right investment level, $\widehat{I_2^*}=I_2^*$, with the operator.

Similar analysis can be done for $\pi_1 \leq \pi_2$ case. Since our approach does not depend on different cases but it depends on the fact that we can define the operator's and also the vendors' expected profit functions λ away from the centralized expected revenue function, we can also get the same results with previous analysis. In conclusion, with a profit sharing contract, the independent agents of the technology game can be coordinated under the conditions of proposition (4.4.1).

The other important point is whether Nash equilibrium exists between the vendors under profit sharing coordinating contract. If the vendors and the central decision maker decide on the same amount of investment level, I_1, and I_2, the best response mapping of the vendors can be represented as, respectively:

$$\Phi_1 = \begin{cases} \widehat{\widehat{I}}{}_1^* & if \quad \pi_1 \geq \pi_2 \\ \widehat{I}_1^* & if \quad \pi_1 \leq \pi_2 \end{cases} \tag{A.9}$$

$$\Phi_2 = \begin{cases} \widehat{I}_2^* & if \quad \pi_1 \geq \pi_2 \\ \widehat{\widehat{I}}{}_2^* & if \quad \pi_1 \leq \pi_2 \end{cases} \tag{A.10}$$

(A.9) is non-increasing and when represented by graph in (I_1, I_2) space, composed of two horizontal half-lines which cover $\left[\underline{I_1}, \infty\right)$; (A.10) is non-increasing and when graphed in (I_1, I_2) space, composed of two vertical half-lines which cover $\left[\underline{I_2}, \infty\right)$. When they are represented by graph, they must intersect at least once for corresponding parameters. Therefore, Nash equilibrium exists between the vendors with profit sharing contract, and also the coordination is ensured among the agents in decentralized setting. Note that we presume the central decision maker is able to switch off the one of the vendors when they both provide equal expected profit.

A.20. Proof of Proposition 4.4.2

For $\pi_1 \geq \pi_2$; The centralized expected revenue function is given in (A.5). The operator's expected revenue function, $\pi_o\left(C_2^1, C_2^2; I_1, I_2\right)$, is given as:

$$\begin{aligned}
&= p_1\left\{\lambda_1\left(a_2+v_2\right) S(C_1+C_2^1 | \, I_1) - v_2\mu_2\left(I_1\right) - o_2C_2^1 - \delta_1 m_1 C_2^1\right\} \\
&+ q_1p_2\left\{\lambda_2\left(a_2+v_2\right) S(C_1+C_2^2 | \, I_2) - v_2\mu_2\left(I_2\right) - o_2C_2^2 - \delta_2 m_2 C_2^2\right\} \\
&+ q_1q_2\left\{\left(a_2+v_2\right) S(C_1) - v_2\mu_2\right\} - o_2C_1 - \left(1-\psi_1\right) I_1 - \left(1-\psi_2\right) I_2 \qquad \text{(A.11)}
\end{aligned}$$

Since the expected revenue of the operator, (A.11), is strictly concave in C_2^1, and C_2^2 because just the coefficients of the decision variables changed (See Section A.13 in the Appendix), FOCs provide the optimal network capacity levels for the second period. FOCs for $\pi_1 \geq \pi_2$;

$$\begin{aligned}
\frac{\partial \pi_o}{\partial (C_2^1)} &= p_1\left[\lambda_1(a_2+v_2)\,\overline{F}(C_1+C_2^1 | \, I_1) - (o_2+\delta_1 m_1)\right] = 0 \\
\frac{\partial \pi_o}{\partial (C_2^2)} &= q_1p_2\left[\lambda_2(a_2+v_2)\,\overline{F}(C_1+C_2^2 | \, I_2) - (o_2+\delta_2 m_2)\right] = 0
\end{aligned}$$

The optimal capacity levels (C_2^{1*} and C_2^{2*}) are given as follows:

$$C_2^{j*} = \theta\left(I_j\right) + G^{-1}\left(1 - \frac{o_2+\delta_j m_j}{\lambda_j\left(a_2+v_2\right)}\right) - C_1 \qquad j = 1, 2. \qquad \text{(A.12)}$$

In fact, if $\lambda_j = \frac{o_2+\delta_j}{o_2+m_j}$, then the operator determines the same extra network capacity with the central decision maker, whenever $C_2^j \geq 0$, $j = 1, 2$. The expected revenue of the vendors turns out to be:

$$\begin{aligned}
\pi_1 &= p_1\left\{\left(1-\lambda_1\right)\left(a_2+v_2\right) S\left(C_1+C_2^1 | \, I_1\right) - \left(1-\delta_1\right) m_1 C_2^1\right\} - \psi_1 I_1 \\
\pi_2 &= q_1p_2\left\{\left(1-\lambda_1\right)\left(a_2+v_2\right) S\left(C_1+C_2^2 | \, I_2\right) - \left(1-\delta_2\right) m_2 C_2^2\right\} - \psi_2 I_2 \qquad \text{(A.13)}
\end{aligned}$$

Since the expected revenue of the vendors are concave in I_1, and I_2 because $\frac{\partial^2 \pi_1}{\partial (I_1)^2} = p_1\left[\left(1-\lambda_1\right)\left(a_2+v_2\right) - m_1\right] \frac{d^2ED(I_1)}{d(I_1)^2} < 0$, and
$\frac{\partial^2 \pi_2}{\partial (I_2)^2} = q_1p_2\left[\left(1-\lambda_2\right)\left(a_2+v_2\right) - m_2\right] \frac{d^2ED(I_2)}{d(I_2)^2} < 0$, FOCs ensure optimal I_1, an I_2 as

follows:

$$\begin{aligned}\frac{dED(I_1)}{d(I_1)} &= \frac{\psi_1}{p_1[(1-\lambda_1)(a_2+v_2)-m_1]}\\ \frac{dED(I_2)}{d(I_2)} &= \frac{\psi_2}{q_1 p_2[(1-\lambda_2)(a_2+v_2)-m_2]}\end{aligned}$$

If $\psi_j = \frac{(1-\varphi_j)a_2-(1-\delta_j)m_j}{a_2-o_2-m_j}$, then right I_1, and I_2 are determined by the vendors in decentralized setting.

Similar analysis can be done for $\pi_1 \leq \pi_2$ case. In conclusion, under a revenue sharing contract, the independent agents, the operator and the vendors, of the technology game can be coordinated under the conditions of proposition (4.4.2). Note that an analogous proof with (A.19) can be expressed for the vendors about equilibrium of the simultaneous (Nash) game. In fact, the vendors operates at Nash equilibrium with revenue sharing contract. Once again, the central decision maker is in favor of one of the vendors when they yield the same amount of expected profit.

APPENDIX B: TABLES

B.1. ANOVA TABLES FOR C_1, C_2, I, AND CENTRALIZED EXPECTED REVENUE

Table B.1. ANOVA table for C_1

Analysis of variance table (ANOVA) Response C_1						
Source	Sum of Squares	df	Mean Square	F Value	p value Prob > F	
Model	3009.66	6	501.61	249.58	< 0.0001	significant
A-Sigma	2893.90	2	1446.95	719.93	< 0.0001	
B-p1	57.88	2	28.94	14.40	0.0001	
C-p2	57.88	2	28.94	14.40	0.0001	
Residual	40.20	20	2.01			
Cor Total	3049.86	26				

Table B.2. ANOVA table for C_2

Analysis of variance table (ANOVA) Response C_2						
Source	Sum of Squares	df	Mean Square	F Value	p value Prob > F	
Model	2415.35	10	241.54	111.51	< 0.0001	significant
A-Sigma	1388.61	2	694.30	320.55	< 0.0001	
B-p1	444.16	2	222.08	102.53	< 0.0001	
C-p2	444.16	2	222.08	102.53	< 0.0001	
BC	138.41	4	34.60	15.98	< 0.0001	
Residual	34.66	16	2.17			
Cor Total	2450.01	26				

Table B.3. ANOVA table for I

Analysis of variance table (ANOVA) Response I						
Source	Sum of Squares	df	Mean Square	F Value	p value Prob > F	
Model	6786634.43	8	848329.30	63660000.00	< 0.0001	significant
B-p1	3181890.06	2	1590945.03	63660000.00	< 0.0001	
C-p2	3181890.06	2	1590945.03	63660000.00	< 0.0001	
BC	422854.32	4	105713.58	63660000.00	< 0.0001	
Residual	0.00	18	0.00			
Cor Total	6786634.43	26				

Table B.4. ANOVA table for expected centralized profit

Analysis of variance table (ANOVA) Response Expected Profit						
Source	Sum of Squares	df	Mean Square	F Value	p value Prob > F	
Model	18.37	6	3.06	31.48	< 0.0001	significant
A-Sigma	2.08	2	1.04	10.69	0.0007	
B-p1	8.14	2	4.07	41.87	< 0.0001	
C-p2	8.14	2	4.07	41.87	< 0.0001	
Residual	1.94	20	0.10			
Cor Total	20.31	26				

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