

DESIGN AND ANALYSIS OF COLLECTION SYSTEMS FOR
INCENTIVE-DEPENDENT PRODUCT RECOVERY

by

Ayşe Gönül Tanuğur Karaarslan

B.S., Industrial Engineering, Boğaziçi University, 2005

Submitted to the Institute for Graduate Studies in
Science and Engineering in partial fulfillment of
the requirements for the degree of
Master of Science

Graduate Program in Industrial Engineering
Boğaziçi University

2008

ACKNOWLEDGEMENTS

Firstly, I would like to express my sincere gratitude to Necati Aras and Deniz Aksen for their continuous support and guidance in the M.Sc.program. I have learned a lot from them which were the very basic and most important things to acquire on my way to academic career.

I would like to thank to Aybek Korugan for participating in my thesis committee.

Also, I am grateful to my friends Özgü Turgut, Özlem Çavuş, Hande Küçükaydın, Mehmet Gönen, Özgü Özsoy, Buket Avcı, Engin Durmaz, Güray Güler and Kamer Sözer for their friendship, courage and help.

Finally, and above all, I would like to give special thanks to my parents, my sister and my husband for being so patient and supportive to me at all times. I dedicate this thesis to them.

I am partly supported by the M.Sc. scholarship (2210) from TÜBİTAK during my masters program.

This research has been partly supported by Boğaziçi University Research Fund Grant No: 06HA303.

ABSTRACT

DESIGN AND ANALYSIS OF COLLECTION SYSTEMS FOR INCENTIVE-DEPENDENT PRODUCT RECOVERY

In this thesis, we propose three reverse logistics models in which we address the problem of locating collection centers of a profit seeking company that aims to collect used products (cores) from product holders via a pick-up strategy.

Firstly, we formulate a mixed-integer nonlinear facility location-allocation model to find both the optimal locations of collection centers and the optimal quality dependent incentive values to be paid to product holders for returning their cores.

Furthermore, we elaborate on two bilevel programming formulations to model the relationship between the government and the company engaged in core collection operations. Since the company seeks only economic profitability, the collected amounts may not be aligned with the target collection rate imposed by the government. In both models, the government pays a unit subsidy to the company for each core collected. The two models differ from each other by the attitude of the government towards the company as being supportive or legislative.

We propose heuristic methods to solve medium and large size instances. For the company's problem, the main loop of the method is based on tabu search performed in the space of collection center locations and Nelder-Mead simplex search is called to determine the best incentives and the corresponding net profit. For the government's problem, we propose a solution approach based on Brent's method, which is a root finding method. Our heuristics obtain good results in all models compared to the results of commercial solvers.

ÖZET

ÜRÜN GERİ ALIMLARININ TEŞVİK MİKTARINA BAĞLI OLDUĞU DURUMDA TOPLAMA SİSTEMLERİNİN TASARIM VE ANALİZİ

Bu çalışmada kar amacı güden bir şirketin müşterilerden kullanılmış ürünleri toplaması ve toplama merkezi yer seçimi problemini içeren üç tane tersine lojistik modeli öneriyoruz.

İlk modelde şirketin karını en iyileyecek şekilde en uygun toplama merkezi yerleri ve müşterilere ürünlerini idae etmeleri için ödenecek teşvik miktarlarını belirlemeye yönelik doğrusal olmayan bir karışık tamsayı modeli oluşturulmuştur.

Ayrıca hükümet ve şirketler arasındaki ürün geri toplama açısından ilişkiyi inceleyen iki seviyeli iki adet model kurulmuştur. Bu modellerde şirket sadece karlılık arttırmaya odaklandığından toplanılan ürün miktarı hükümetin hedeflediği toplama oranının gerisinde kalabilir. Hükümet şirketi bu anlamda desteklemek için her bir toplanan ürün başına belli bir miktar parasal destek vermektedir. Her iki model birbirinden hükümetin şirketin kullanılmış ürün toplama problemine destek verici veya kanuni olarak zorlayıcı şekilde yaklaşmasıyla ayrılır.

Orta ve büyük boyuttaki problemleri çözmek için sezgisel yöntemlerden yararlandık. Şirketin problemini çözerken tabu arama metodu ile toplama merkezlerinin yeri belirlenir. Daha sonrasında ise Nelder-Mead simplex arama yöntemi ile uygun teşvik miktarı bulunur. Hükümetin problemi ise Brent kök bulma metodu ile çözülür. Bütün modellerde ticari programların sonuçlarına göre iyi sonuçlar elde edilmiştir.

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LIST OF SYMBOLS/ABBREVIATIONS

| | |
|---------------|---|
| a_k | Maximum incentive level of product holder of type k |
| c_1 | Vehicle operating cost |
| c_2 | Cost per unit distance traveled |
| d_{ij} | Travel distance between customer zone j and candidate site i |
| f_i | Fixed cost of opening a CC at candidate site i |
| G | Unit subsidy offered for each collected used product |
| h_j | Number of product holders living in zone j |
| h_{jk} | Number of product holders living in zone j ($h_{jk} = \gamma_{jk}h_j$) |
| i | Index for potential collection center locations |
| j | Index for customer locations |
| k | Index for quality classes of used products |
| p | Number of collection centers to be opened |
| q | Vehicle capacity |
| R_k | Unit incentive offered (unit acquisition price) for a used product of type k |
| s_k | Unit cost savings from a used product of type k |
| V_{ij} | Number of vehicles required to transport returns from zone j to the CC at site i |
| Y_i | Binary variable indicating whether a CC is opened at site i |
| X_{ijk} | Fraction of potential returns of type k collected in zone j and transported to the CC at site i |
| γ_{jk} | Proportion of product holders of type k living in zone j |
| ρ | Target profitability ratio |
| τ | Target collection rate |
| BIP | Bilevel integer programming |
| BP | Bilevel programming |
| BPP | Bilevel programming problem |

| | |
|--------|--|
| CC | Collection center |
| CCLP | Collection center location problem |
| CP | Company's problem |
| ECM | Environmentally conscious manufacturing |
| ECP | Environmentally conscious production |
| GP | Government's problem |
| GSCSDP | Government-subsidized collection system design problem |
| MILP | Mixed-integer linear programming |
| MINLP | Mixed-integer nonlinear programming |
| MOP | Multi-objective programming |

1. INTRODUCTION

The development of distribution channels and systems for the recycling industry was first mentioned in Guiltinan and Nwokoye (1975). This was a time when the industry did not feel obligated to governmental regulations and customer perspective on environmental issues. After the first legislations of Environmentally Conscious Production and Manufacturing (ECP/ECM) were introduced, ECP/ECM began to draw the attention of production researchers and practitioners in early 1990s. It was mainly driven by the escalating deterioration of the environment, such as diminishing raw material resources, overflowing waste sites and increasing levels of pollution. Even in non-regulated markets, some manufacturers engaged in ECP/ECM to reduce production costs, enhance brand image and reputation, meet changing customer expectations, protect aftermarkets, and preempt pending legislation or regulations (Güngör and Gupta, 1999). In today's context of sustainable development, ECP/ECM has become a popular business strategy.

Motivated by both environmental and economical facts, in this thesis we develop three models which deal with reverse flow of goods taken from the product holders and carried to collection centers. The first part of this study is currently an article in press referred as Aras et al. (2007). In this study, we propose a facility location-allocation model to find the optimal locations of a predetermined number of collection centers (CCs) as well as the optimal incentives offered by the company to product holders depending on the condition of their used items. We consider a pick-up scenario in which the company collects used products from the premises of the product holders and all the collection related costs, i.e., cost of operating the vehicles and transportation cost of used products are incurred by the company. The willingness of product holders to return is assumed to be affected by the amount of the financial incentive offered. It is important to note that we are only modeling the collection operation of the company. Therefore, the decisions about the shipment of collected used products from collection centers to disassembly centers or to remanufacturing facilities are out of the scope of this study.

Based on this company's collection center location problem, in the second part of the thesis we develop a bilevel programming (BP) framework to model a company's relationship with the local, regional or national government which imposes a target collection rate on used products. We propose two interrelated formulations in which the follower's role is assigned to the company in the lower level problem, and the government takes on the role of the leader in the upper level problem. In the first formulation, the government's objective is to minimize the standard subsidy payable to the company for each used product collected from customers while meeting a minimum collection rate cumulative over all customer zones. The follower's problem is then to maximize its net profit from the collection operations for fixed value of subsidy from the leader (government). In the latter formulation the government's objective is still to minimize the unit subsidy it will pay to the company; however, the government now has to guarantee that the company achieves or exceeds a target profitability ratio in its product recovery efforts. This time the responsibility of meeting the minimum cumulative collection rate is transferred to the company. We believe that our work establishes the first BP methodology in the literature where government subsidization is merged into the design of a collection system for incentive-dependent product recovery.

The thesis is organized as follows. The second chapter includes further reviews of the relevant academic literature. Chapter 3 develops a single level and two improved bilevel model formulations on the research problem. Original tabu search based solution procedures are described in Chapter 4. Computational results on randomly generated problem instances are presented in Chapter 5 alongside with sensitivity analysis. Finally, Chapter 6 offers a conclusion, and suggests future research directions.

2. LITERATURE SURVEY

2.1. Reverse Logistics Concepts

A precise definition of reverse logistics is made by Rogers and Tibben-Lembke (1998) as:

“The process of planning, implementing, and controlling backward flows of raw material, in-process inventory, packaging, and finished goods from a manufacturing, distribution or reuse point, to a point of origin for the purpose of recapturing value or proper disposal.”

An increasing number of organizations in Asia, Europe and North America engage in voluntary or mandatory end-of-life product management. The most promising corporate endoflife strategies create both economic and environmental values (Geyer and Jackson, 2004). Dowlatshahi (2000), for example, has found that remanufacturing can reduce the unit cost of production by 40 to 60 per cent by reutilizing the product components. Toffel (2004) observes that concerns for the end of life products are motivated by legislation across Europe where the electrical/electronics industry has experienced some of the highest regulatory pressures. The WEEE Directive of the European Parliament and of the Council (Directive 2002/96/EC), establishes target component, material and substance reuse and recycling rates at 75 per cent by weight for large household appliances such as refrigerators, washing machines, and dishwashers. For desktop/notebook computers as well as printers the target rate is set at 65 per cent by weight (EUR-Lex, 2003).

From a logistics point of view, product recovery creates a reverse flow of goods that originates at the locations of product holders, also referred to as customer zones. After used products are consolidated at some collection facilities, they are shipped to disassembly centers where inspection, sorting, and disassembly operations are performed. The final destination of the returns is either remanufacturing facilities where product recovery actually takes place, or disposal sites.

Fleischmann et al. (2000) list the following activities found in product recovery which is considered an integral part of ECP/ECM:

- Collection of used products (returns) from product holders,
- Determining the condition of the returns by inspection and/or separation,
- Reprocessing the returns to capture their remaining value,
- Disposal of the returns which are found to be unrecoverable due to economic and/or technological reasons, and
- Redistribution of the recovered products.

The type of the product recovery is dependent on the condition of a return. The possibilities are repairing, refurbishing, remanufacturing, cannibalization, and recycling Thierry et al. (1995).

One of the key concerns of the companies involved in product recovery is used product acquisition or collection as mentioned by Guide et al. (2003). It is indeed the first activity of product recovery, and triggers the later activities of the recovery system. Güngör and Gupta (1999) argue that collection of retired products must be planned ahead in order to perform product recovery profitably and according to applicable laws and regulations. In the authors' view collection decisions involve:

- Location selection of collection centers (CCs) where used products are collected and stored prior to distribution to recycling or remanufacturing facilities,
- Layout design of CCs (including material handling and storage),
- Transportation (designing the transportation networks to bring used products from many origins to a single CC).

The biggest challenge in collection related problems is the level of uncertainty involved in the quality and quantity of the used products collected. Some manufacturers have been able to influence the quantity of returns by using buy-back (take-back) campaigns and offering financial incentives to product holders. A successful implementation in the power tools industry is mentioned in Klausner and Hendrickson (2000). Xerox

Europe obtained over \$80 million savings by implementing an end-of-life equipment take-back and reprocessing program in 1997 (Maslennikova and Foley, 2000). Clearly, the amount of incentive offered by the company (also called unit acquisition price) influences the quality level of collected returns. Accepting all end-of-use products in the waste stream is not viable for most companies since a high percentage of these will have a poor quality, hence will not be recoverable. As a consequence, adopting a proactive approach and offering the appropriate incentive depending on the quality status of a used product is crucial for a company engaged in product recovery.

2.2. Existing Models for Used Product Collection

Logistics literature is remarkably rich in papers that deal with the collection operations in the context of product recovery and recycling. With the list below, we would rather make the reader aware of the most pertinent published works that paved the road to our research.

2.2.1. Discrete Facility Location-Allocation Models with Deterministic Quantities of Collection

Jayaraman et al. (1999), Fleischmann et al. (2001), Salema et al. (2006), Lu and Bostel (2007) are some of the papers which consider not only the reverse flow of used products, but also the forward flow of new and remanufactured products to satisfy customer demand. However, the mixed-integer linear programs (MILP) developed do not take collection facilities into consideration, and assume that used products are shipped from customer zones directly to disassembly centers. Even more restrictive is the assumption that the quantity of returns is known for each customer zone. The number and locations of CCs alongside with refurbishing centers are considered in Jayaraman et al. (2003) who exclusively model the reverse logistics of hazardous products with a multi-level warehouse location model. Theirs is also an MILP model where the number of hazardous products to be returned at each originating site is known exactly. Another recent paper that explicitly deals with collection facilities is due to Min et al. (2006). They solve an MILP to determine the optimal number and locations of

collection points as well as centralized return centers. A solution method based on genetic algorithms is developed. Once again, the volume of products returned by consumers is a deterministic parameter. There exist also case studies in the literature which address reverse logistics network design problems in the framework of recycling. For example, Barros et al. (1998) focus on a sand recycling network and formulate a two-level capacitated facility location model. Louwers et al. (1999) present a nonlinear facility location-allocation model for the collection and preprocessing of carpet waste.

2.2.2. Incentive-Dependent Quantities of Collection

Wojanowski et al. (2007) develop a model for optimally designing a drop-off facility network and determining the sales price under deposit-refund requirements using a continuous modeling approach. The customer is informed that the sales price of the item includes a deposit which will be paid back when the used product is returned at a collection facility. Customers' purchasing and return decisions are incorporated by a stochastic utility choice model. Aras and Aksen (2008) analyze an uncapacitated CC location problem (CCLP) for incentive- and distance dependent returns. In the presented profit maximization model a drop-off policy is in effect, i.e., customers are asked to bring their used products to the centers by themselves. Their decision whether or not to participate in this product recovery campaign is affected by the distance to the nearest CC and by the financial incentive offered. The authors propose and solve two mixed-integer nonlinear programming (MINLP) models with a tabu search heuristic for the fixed-charge and p-median versions of the CCLP, respectively. Used products owned by the participating customers are sorted into a finite number of quality classes, and a different incentive is offered for each class.

2.3. Governmental Regulations on the Throughput of Collection Operations

The importance of environmental awareness for companies that anticipate tighter environmental regulations in the future is highlighted clearly by Rodrigue et al. (2001). These companies want to leverage "greenness" of their production as a competitive ad-

vantage, however, they fear diminishing profitability due to compulsory product recovery. All of the aforementioned works stem from a company standpoint, and formulate models to increase the company's welfare. Thus, the best solutions obtained may not always satisfy target collection rates imposed by the government. In other words, the profit maximization objective of the company does not match the objective of the government to reach a desired collection rate. To ensure that this rate is met, the government may choose to offer a subsidy for each collected item. Certainly, it tries to keep the level of this subsidy as low as possible. The interactions between government, company and product holders are illustrated in Figure 2.1, which has been adopted from Young et al. (1997), and further augmented with government subsidies.

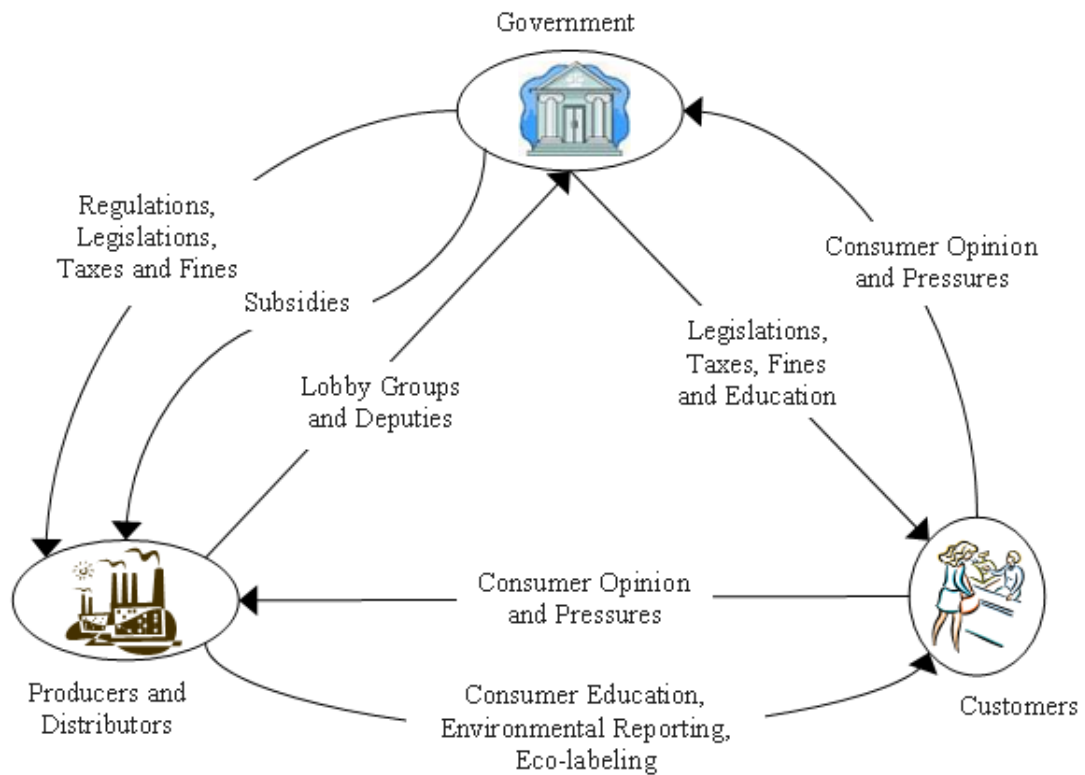


Figure 2.1. Interactions between government, users, producers and distributors

The role of government subsidies is analyzed in the literature in different contexts. For example, Kulshreshtha and Sarangi (2001) discuss a case where the government gives a subsidy to a firm involved in recycling of product packages. In another study, Sheu et al. (2005) indicate the difficulty of integrating logistics flows in a green-supply chain on the grounds that coordinating the activities of all chain members is a compli-

cated issue, and externalities such as end-customer behavior and governmental policies affect the performance. In their paper, the authors propose a linear optimization model to improve the performance of a green-supply chain, which involves both cross-functional product (logistics) and used product (reverse logistics) flows. The model Sheu et al. formulate has a composite multi-objective function together with corresponding operational constraints. The latter include also subsidies for used-product recovery, return ratio enforced by the government for environmental protection, and recycle fees charged to manufacturers. They find that used product return ratio and corresponding unit subsidy significantly influence the green-supply chain management performance.

A recent paper by Quariguasi et al. (2008) develops a framework for the design and evaluation of sustainable logistic networks, in which profitability and its environmental impact are balanced. The particular logistic network described in this paper has multiple agents; i.e., producers, consumers, third parties working on recycling, incineration and energy generation. The paper uses linear multi-objective programming (MOP), and also introduces a technique based on the commonalities between data envelopment analysis and MOP to calculate the efficiency of existing logistic networks. The authors then illustrate their findings as an efficient frontier for the European pulp and paper sector.

2.4. Bilevel Programming

Bialas (2002) presents an easy-to-understand introduction to multilevel mathematical programming explaining the underlying motivation in detail. He argues that many decision making problems require compromises among the objectives of several interacting individuals or entities, and they are often arranged within a hierarchical structure with independent and perhaps conflicting objectives. The solution to such problems is determined collectively by the choices of multiple distinct decision makers. A bilevel programming problem (BPP) is a special case with two parties, one of whom takes the leader's position, and the other one is the follower making his or her plan based on the leader's decision. The effect of each party's decision on the other party

is only indirect. In other words, there is not a single body that can make the trade-off between the independent or conflicting objectives.

2.5. BP Applications in Network Design Problems

BP especially fits when the objectives cannot be weighted and aggregated into a single objective in such a way that the resulting solution is accepted by both parties. This situation arises quite frequently in transportation and network design problems. Migdalas (1995), for instance, published an early review of BP in traffic planning where the leader and follower relationship is found between the public sector and passengers. Recent studies are due to Kara and Verter (2004) and Erkut and Gzara (2008) both of which tackle the network design for hazardous material transportation as a bilevel integer programming (BIP) problem. The government agency is in a leader position aiming to minimize risk while the carriers are followers who comply with the agency's regulation and aim to minimize cost at the same time. Kara and Verter transform the bilevel program into a single-level mixed integer program by replacing the lower-level problem by its Karush-Kuhn-Tucker conditions and by linearizing the complementary slackness constraints, whereas Erkut and Gzara develop a heuristic method that exploits the network flow structure to overcome the difficulty of the BIP model. Another interesting application of BIP is given in Scaparra and Church (2008). The researchers address a system planner (defender) making decisions about which facilities of an existing system to secure or fortify against a potential attacker (interdictor) who deliberately hits unprotected facilities in order to cause maximal reduction in system efficiency which is measured in terms of accessibility or service provision costs endured by the users of the system. In this inherently bilevel problem, the leader's and follower's roles are assigned to the defender and attacker, respectively. The newest comprehensive review on BP inclusive of literature survey, sample applications, and existing methods is due to Colson et al. (2007). Finally, two principal textbooks on the subject written by Bard (1999) and by Dempe (2002) particularly deserve the interested reader's attention.

The only work that synthesizes a BP approach with the collection operations in product recovery is due to de De Figueiredo et al. (2007). This is a very recent paper

addressing a minimum-cost recycling network design problem with incentive-dependent recyclable product collection and required quantity of recycled items per time period which is referred to as throughput. The specific problem setting in this study is similar to the properties of our problem. There is a recycler who wishes to determine the optimal number and location of receiving centers as well as the correct incentive (price per used/unrecoverable item to be recycled) it must offer to the collecting agents in order to stimulate collection from a number of geographical regions to a degree required for regulatory reasons. The authors model this problem as a large bilevel nonlinear mixed-integer program, and propose a three-stage heuristic algorithm due to its complexity. An illustrative case study in the recycling of unrecoverable tires in Southern Brazil is also presented.

Our research distinguishes from De Figueiredo et al. (2007) in several aspects. First of all, recyclable products are not classified into distinct quality levels; hence, collecting agents (analogous with the product holders in our problem) are offered a uniform incentive. Secondly, the government does not play the leader's role in their upper-level problem, and unlike in our case, it does not pay a unit subsidy to the recycler to mitigate his conformation to the specified throughput requirement. Thirdly, the recycler does not determine the number and loading scheme of vehicles used to haul collected items from customer regions to the receiving centers or from those centers to the processing centers. Instead, transportation cost is calculated by simply multiplying the amount of collected items with the respective costs per unit item and per unit distance. Finally, the authors represent the differences in individual collecting agents' decision processes on whether or not to cooperate with the recycler using a binomial logit function that accounts for factors such as the offered incentive, varying levels of environmental awareness, and transportation costs. In this thesis, we resort to a right triangular distribution function the only parameters of which are incentive payment and product holders' quality-dependent reservation price for used products. In fact, our problem combines all of the following merits:

- A nonlinear discrete facility location-allocation and pricing model for the collection operations of product recovery.

- A pick-up policy with vehicles of limited capacity dispatched from CCs to customer zones.
- Profit maximization through quality-dependent financial incentives to control the intrinsic willingness of customers to make a used product return.
- A minimum collection rate imposed by the government.
- A unit subsidy paid by the government for each used product collected.
- BP structure with the government and the company as the leader and its follower, respectively.

3. MODEL DEVELOPMENT

3.1. Base Collection System Model

3.1.1. Model Preliminaries

The company under consideration manufactures products for a wide customer base having different usage habits. Depending on the usage rate and the duration at each use, the deterioration of used products, hence their condition will vary giving rise to diverse quality states of the returns. The influence of quality-based collection incentives on the quality and quantity of product returns is studied by only a few papers. Guide et al. (2003) categorize returned cellular phones into quality grades each with a different remanufacturing cost, and find optimal acquisition prices for each grade to maximize the profit. In a similar fashion, Aras and Aksen (2008) divide used products owned by the customers (product holders) into K separate groups with respect to their quality conditions referred to as types. For each core type, a different level of incentive is offered by the company. This way, the company can pursue an endogenous quality differentiation in its product recovery efforts. In our study, we adopt the very same quality differentiation as applied in Aras and Aksen (2008). We denote the proportion of product holders in zone j having cores of type k ($k \in \mathbf{K}$) by γ_{jk} which is assumed to be known. The number of product holders in zone j who own type k cores is then given by $h_{jk} = \gamma_{jk}h_j$ where h_j is the total number of product holders in zone j . For convention, type 1 cores are assumed to have the highest quality and type K cores the lowest quality. The remaining value that can be captured from the collected cores can be regarded as revenue due to the savings in the production cost. The unit cost saving from a core of type k is denoted by s_k . It represents the positive difference between the production cost of a new product and the sum of the handling and recovery cost (remanufacturing or material recycling) of a core. When the company offers incentive R_k for a core of type k , the profit associated with that core becomes $(s_k - R_k)$. As a matter of fact, $R_k < s_k$ must hold true for each type k so that the collection operation is economically viable.

We model product holders' return decisions using the notion of consumer surplus. We assume that each product holder having used product of type k (referred to as product holder of type k) has a reservation incentive for returning his/her used product. In other words, each product holder would be willing to return if the company offered an incentive that is at least as large as the reservation incentive R_{0k} . Clearly, the product holders are heterogeneous in terms of their willingness. One of the most important factors in this respect is people's environmental consciousness. Like other authors who take product holder willingness into account in their models (Ray et al., 2005; Wojanowski et al., 2007; Aras and Aksen, 2008) we also use the uniform distribution to model the heterogeneity of the product holders because it not only provides analytical tractability in the formulation of the facility location-allocation model, but also helps to incorporate a large degree of variability among product holders. With this choice, R_{0k} takes on values in the interval $[0, a_k]$ where $a_k > 0$ represents the maximum reservation incentive level of product holder of type k . This means that if $R_k = a_k$, then every product holder of type k will return his/her used product. Therefore it is not viable for the company to offer an incentive $R_k > a_k$ for a return of type k , and $R_k \leq a_k$ must always be true. Since $R_{0k} \sim U(0, a_k)$, the probability density and cumulative distribution functions of R_{0k} are given by $f(R_{0k}) = 1/a_k$ and $F(R_{0k}) = R_{0k}/a_k$, respectively.

As mentioned before, the only factor affecting the decision of a product holder is the incentive offered by the company. When the company offers incentive R_k , the consumer surplus for a product holder of type k is

$$R_k - R_{0k}. \quad (3.1)$$

Using the notion of consumer surplus, the proportion P_{jk} of product holders of type k located in zone j that are willing to return a used product can be written as follows:

$$P_{jk} = \Pr(R_k - R_{0k} > 0) = \frac{R_k}{a_k}. \quad (3.2)$$

Since a_k is not dependent on where the product holders are located, we can set $P_k = P_{jk}$

for all j . This way, the return probability of a product holder of type k will be the same regardless of his/her location. On the other hand, consider two product holders having used products of the best quality class (product holder of Type 1) and the second best quality class (product holder of Type 2). When they are offered the same incentive (i.e., $R_1 = R_2$), the willingness of product holder of Type 1 to return will be obviously less than that of product holder of Type 2. This implies that the maximum reservation incentive level of product holder of Type 1 (a_1) should be higher than that of product holder of Type 2 (a_2). For all product holder types, the following inequalities must hold true: $a_1 > a_2 > \dots > a_K$.

As P_k is the proportion of product holders of type k willing to return their used products, the total number of potential returns of type k from zone j becomes $h_{jk}P_k = h_{jk}R_k/a_k$. Hence the maximum possible profit Π'_{jk} can be computed as $h_{jk}R_k(s_k - R_k)/a_k$ which is realized if *all* returns are collected. When $s_k > a_k$, we can write Π'_{jk} explicitly as follows:

$$\Pi'_{jk} = \begin{cases} 0 & R_k < 0 \\ \frac{h_{jk}}{a_k}(s_k R_k - R_k^2) & 0 \leq R_k < a_k \\ h_{jk}(s_k - R_k) & R_k \geq a_k \end{cases} \quad (3.3)$$

When $s_k \leq a_k$, then the total profit from returns of type k in zone j becomes

$$\Pi'_{jk} = \begin{cases} 0 & R_k < 0 \\ \frac{h_{jk}}{a_k}(s_k R_k - R_k^2) & 0 \leq R_k < s_k \\ 0 & R_k \geq s_k \end{cases} \quad (3.4)$$

At this point we want to emphasize that it may not be viable for the company to collect all the returns of type k in zone j because of the collection-related costs. As a result, the *net* profit, i.e., profit obtained from the returns less the collection cost thereof is a function of the number of returns collected. For that matter, we define a quantity $X_{i^*jk} \in [0, 1]$ to denote the fraction of potential returns of type k collected in zone j and carried to the nearest collection center at site i^* .

The used product collection cost consists of two parts. The first one is the operating cost of a vehicle. Depending on whether the firm prefers to have its own vehicle fleet or rent such a fleet for collection operations, the operating cost of a vehicle can either be its rent or its initial purchasing cost discounted on a periodic basis. This cost component may also include driver wages and other overhead costs such as insurance and tax. The volume of returns and the vehicle capacity determine together the number of vehicles to be operated by the company. If we assume that the vehicle fleet is homogeneous, i.e., all vehicles have the same capacity q , then the number of vehicles needed to carry the returns from zone j is given by $\left\lceil \frac{\sum_k X_{i^*jk} h_{jk} R_k / a_k}{q} \right\rceil$ where $\lceil z \rceil$ denotes the smallest integer greater than or equal to z for $z > 0$. Thus, for a unit operating cost of c_1 the total vehicle operating cost equals $c_1 \left\lceil \frac{\sum_k X_{i^*jk} h_{jk} R_k / a_k}{q} \right\rceil$. The second part of the collection cost is the traveling cost of the vehicles. If the nearest collection center to customer zone j is located at site i^* , each vehicle has to go from i^* to zone j and come back. Denoting the road distance between the two locations by d_{i^*j} , the traveling cost of a single vehicle becomes equal to $2c_2 d_{i^*j}$ where c_2 is the cost per unit distance. Here we assume that the traveling cost is linearly proportional to the total distance, and is independent of the load on the vehicle. To summarize, the collection cost can be expressed as $\left\lceil \frac{\sum_k X_{i^*jk} h_{jk} R_k / a_k}{q} \right\rceil (c_1 + 2c_2 d_{i^*j})$.

Actually, there is also another cost component that should be taken into account: the fixed cost of opening and operating collection centers. Under the assumption that this cost is site-independent, i.e., it is the same at all candidate sites, the total fixed cost of opening and operating collection centers becomes a constant since we are formulating a model with a predetermined number of collection centers which is also the case in the well-known p -median problem. Therefore, this cost component can be altogether eliminated from further consideration.

Consequently, the net profit Π_j from the collected returns in zone j equals

$$\begin{aligned} \Pi_j &= \sum_k X_{i^*jk} \Pi'_{jk} - \left\lceil \frac{\sum_k X_{i^*jk} h_{jk} R_k / a_k}{q} \right\rceil (c_1 + 2c_2 d_{i^*j}) \\ &= \sum_k \frac{X_{i^*jk} h_{jk} R_k (s_k - R_k)}{a_k} - \left\lceil \frac{\sum_k X_{i^*jk} h_{jk} R_k / a_k}{q} \right\rceil (c_1 + 2c_2 d_{i^*j}). \end{aligned} \quad (3.5)$$

It is obvious that if the collection cost equals or exceeds the profit from zone j , i.e., if $\Pi_j \leq 0$, then there is no motivation for the company to collect used products from that zone. The profitability of zone j is also affected by the locations of open collection centers since the collection cost is a function of the distance d_{i^*j} between zone j and the nearest collection center at site i^* . Another factor is the amount of the incentive R_k offered by the company as it has a direct impact on both the unit profit $(s_k - R_k)$ and the proportion R_k/a_k of product holders of type k who are willing to return their used item. It is possible to establish some guidelines which would help determine the number of vehicles dispatched from the nearest collection center at site i^* to zone j . We make use of these guidelines later on in our heuristic solution procedure.

1. If a fully loaded vehicle carrying type k returns is not profitable, then it is optimal not to operate any vehicles carrying only type k returns from zone j to collection center at site i . Mathematically speaking, this means if $q(s_k - R_k) \leq c_1 + 2c_2d_{ij}$, then a partially loaded vehicle with *only* type k returns will not be profitable. This is so because the collection cost $(c_1 + 2c_2d_{ij})$ associated with a single vehicle is independent of the number of returns transported by the vehicle, whereas its profit is increasing in the number of returns.

2. If a fully loaded vehicle carrying type k returns is profitable, then as many fully loaded vehicles as possible should be put into service to carry *only* type k returns. However, a partially loaded vehicle with exclusively type k returns is not guaranteed to be profitable.

3. When operating a partially loaded vehicle with only type k returns proves unprofitable, loading returns of lower quality classes $k+1, k+2, \dots$ to utilize the spare capacity of the vehicle may yield profit.

3.1.2. Model Formulation

In the base model, there are n customer zones ($j \in \mathbf{J}$) and m ($i \in \mathbf{I}$) candidate sites for collection centers to be opened. As is the case with the p -median problem

the number of collection centers to be opened is predefined as p . The objective is to determine the best p sites, the number and load composition of vehicles to be dispatched from centers to each customer zone, the quality dependent incentive values, and the amount of each used product type to be collected from the zones with the aim of maximizing the total net profit $\Pi = \sum_j \Pi_j$. The indices and parameters used in the model are defined as follows:

I : Set of candidate sites for opening a CC = $\{1, \dots, m\}$

J : Set of customer zones = $\{1, \dots, n\}$

K : Set of types (quality states) of used products = $\{1, \dots, K\}$

d_{ij} : travel distance between customer zone j and candidate site i

c_1 : vehicle operating cost

c_2 : cost per unit distance traveled

q : vehicle capacity

h_j : number of product holders living in zone j

γ_{jk} : proportion of product holders of type k living in zone j ($h_{ij} = \gamma_{jk} h_j$)

h_{jk} : number of product holders of type k living in zone j

s_k : unit cost savings from a used product of type k

a_k : the maximum reservation incentive level of product holder of type k

p : number of collection centers to be opened

The decision variables used in the model are as follows:

Y_i : binary variable indicating whether a CC is opened at location i

X_{ijk} : fraction of potential returns of type k collected in zone j and transported to the CC at site i

V_{ij} : number of vehicles required to transport returns from zone j to the CC at site i

R_k : unit incentive offered (unit acquisition price) for a used product of type k

The collection center location problem (CCLP) is formulated as:

$$\text{CCLP : } \max \Pi = \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} \sum_{k \in \mathbf{K}} X_{ijk} h_{jk} R_k (s_k - R_k) / a_k - \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} (c_1 + 2c_2 d_{ij}) V_{ij} \quad (3.6)$$

s.t.

$$\sum_{i \in \mathbf{I}} X_{ijk} \leq 1 \quad j \in \mathbf{J}; k \in \mathbf{K} \quad (3.7)$$

$$X_{ijk} \leq Y_i \quad i \in \mathbf{I}; j \in \mathbf{J}; k \in \mathbf{K} \quad (3.8)$$

$$\sum_{i \in \mathbf{I}} Y_i = p \quad (3.9)$$

$$V_{ij} \geq \frac{\sum_{k \in \mathbf{K}} X_{ijk} h_{jk} R_k / a_k}{q} \quad i \in \mathbf{I}; j \in \mathbf{J} \quad (3.10)$$

$$R_k \leq a_k \quad k \in \mathbf{K} \quad (3.11)$$

$$R_k \leq s_k \quad k \in \mathbf{K} \quad (3.12)$$

$$R_k \geq 0 \quad k \in \mathbf{K} \quad (3.13)$$

$$X_{ijk} \geq 0 \quad i \in \mathbf{I}; j \in \mathbf{J}; k \in \mathbf{K} \quad (3.14)$$

$$V_{ij} \geq 0 \text{ and integer} \quad i \in \mathbf{I}; j \in \mathbf{J} \quad (3.15)$$

$$Y_i \in \{0, 1\} \quad i \in \mathbf{I} \quad (3.16)$$

The objective function consists of two parts. The first part calculates the sum of profits from each customer zone j , which is obtained by multiplying the unit profit $(s_k - R_k)$ for each type k with the amount collected $(X_{ijk} h_{jk} R_k / a_k)$ and summing over all types. The second part of the objective function is the collection cost. Recall that since the fixed cost of opening and operating collection centers is site-independent and the number of collection centers is given as p , this cost component is not included in the objective function. Constraints (3.7) guarantee that either all used products of type k in zone j are collected and shipped to the nearest collection center ($\sum_i X_{ijk} = 1$), or only a fraction of them are collected ($0 < \sum_i X_{ijk} < 1$), or they are not collected at all ($\sum_i X_{ijk} = 0$). Constraints (3.8) ensure that used products of type k collected from zone j can only be shipped to an open collection center. Constraint (3.9) enforces that the number of open collection centers be equal to p . Constraints (3.10) determine the number of vehicles V_{ij} required to transport the returns from zone j to collection

center at site i . Since the sense of the objective is maximization, each V_{ij} is assigned the smallest integer value that is greater than or equal to the fractional term $\frac{\sum_k X_{ijk} h_{jk} R_k / a_k}{q}$. Constraints (3.11) and (3.12) are upper limits on the incentive R_k . Depending on the relative magnitudes of s_k and a_k one of these constraints will be redundant.

The formulation of the CCLP is a mixed-integer nonlinear programming (MINLP) model with $\{(n+2)K + (K+1)mn + 1\}$ constraints, $(mn+1)K$ nonnegative linear variables, mn nonnegative integer variables, and m binary variables. From the viewpoint of locational analysis, the resemblance of the CCLP to the well-known p -median problem, which is proven to be \mathcal{NP} -hard (Kariv and Hakimi, 1979), is evident.

It should be pointed out that there is no upper bound on the total number of items to be collected by the company either via an explicit upper bound or a purchasing budget limitation. This implies that the model allows the firm to acquire as many used products as would be profitable to remanufacture. In other words, it is an implicit assumption that there is sufficient demand for remanufactured products.

3.2. Government-Subsidized Bilevel Models

3.2.1. Problem Setup and Model Development

We formulate two BP models called GSCSDP1 and GSCSDP2 (Government-Subsidized Collection System Design Problem) for the used product collection operations of the company. The government is the leader and the company is the follower in both models. The models differ from each other in the government's action by either forcing or encouraging the company to collect more cores from the customers.

Model GSCSDP1 (supportive model):

The government announces that a unit subsidy G will be paid to the company for each core collected. If the amount of G is sufficiently high, the company will be better off by collecting more used products. The government's problem (GP1) is to

determine the least possible value of G so as to make the company reach the minimum collection rate τ . The company's problem (CP1), on the other hand, is to maximize its total net profit by determining the optimal number and locations of the CCs, the number and loads of vehicles to be dispatched from each CC to pick up the cores, the amount of the quality-dependent incentives to be paid to product holders for each core returned, and the amount of each core type to be collected. A bilevel framework to represent GSCSDP1 schematically is depicted in Figure 3.1.

Model GSCSDP2 (legislative model):

The company is obligated by governmental legislation to reach at a minimum target collection rate τ . This, however, can cause the company to suffer a net loss, i.e., a negative net profit from its collection operations. The government's problem (GP2) becomes minimizing the unit subsidy S paid to the company by guaranteeing that the company achieves a net profitability ratio at least as large as a target value ρ . The company's problem (CP2) is the same as CP1 with an additional constraint which fulfills the government's collection rate legislation. A graphic representation of the BP model GSCSDP2 is shown in Figure 3.2. In both problems, the government is certainly not interested in which type of cores are actually collected by the company. It is also not an issue, which customer zones participate in product recovery to what extent. The government's regulation will be satisfied once the overall cumulative collection ratio reaches the target.

For these bilvel models, we revised the company's problems based on the model depicted in Section 3.1 with two main differences. Firstly, instead of uniform distribution we assume that R_{0k} follows a right triangular distribution (RTD) given as in expression (3.17). By preferring the RTD we believe that we can better capture the customers' characteristic wisdom of "the more, the better" regarding the incentive offering of the company. In other words, the RTD implies that the change in the number of potential core returns occurs at an increasing rate per unit increase in R_k (see Figure 3.3). Moreover, this wisdom can be incorporated into the model formulation without sacrificing the analytical tractability that would be ensured otherwise by the use of

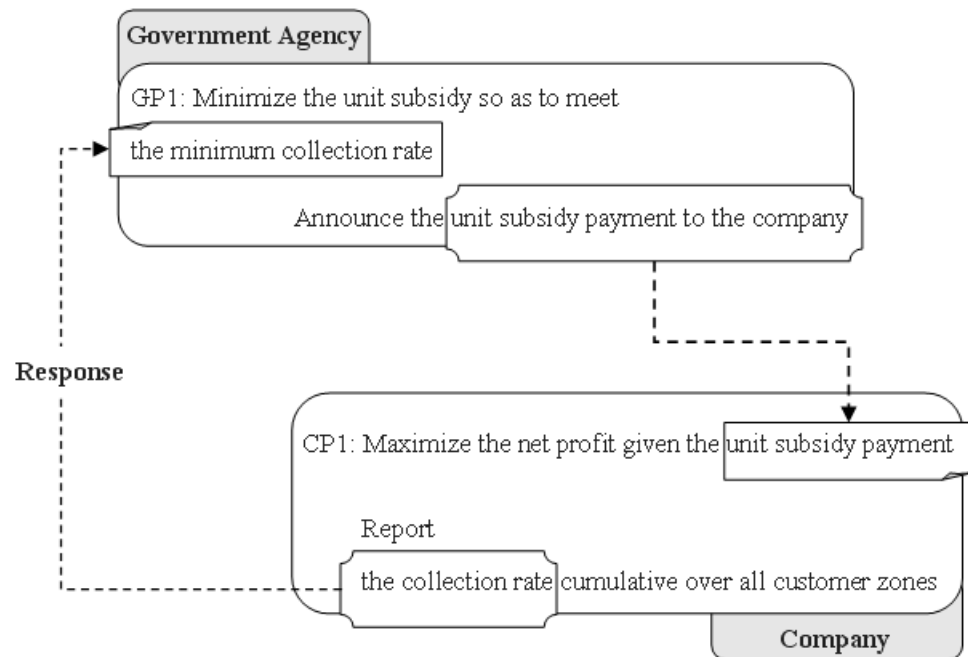


Figure 3.1. The bilevel framework for GSCSDP1

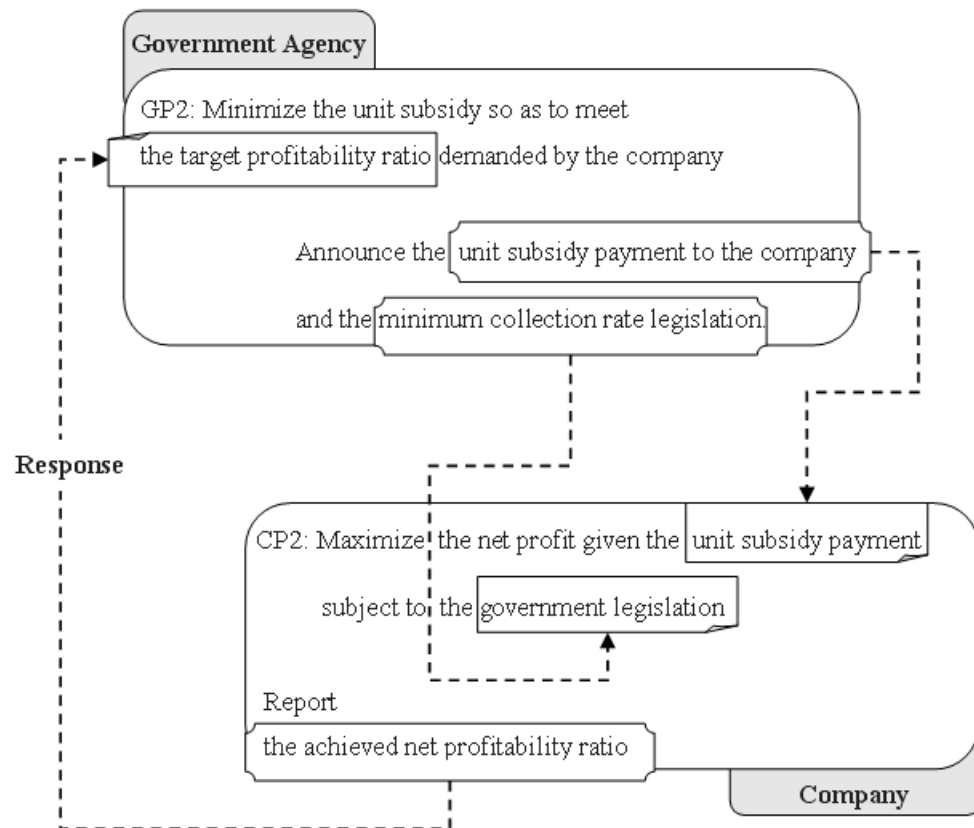


Figure 3.2. The bilevel framework for GSCSDP2

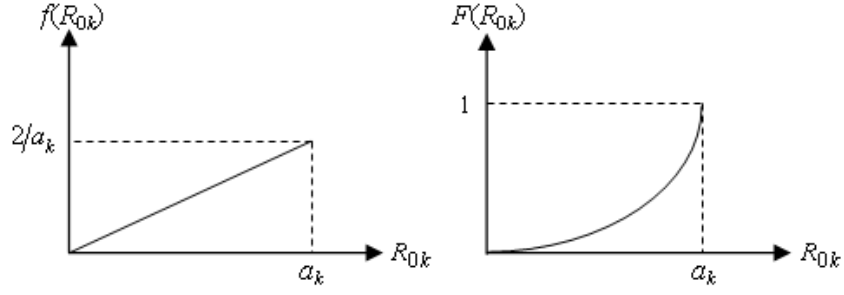


Figure 3.3. The RTD distribution of the reservation incentive

the uniform distribution. By implementing RTD, the proportion $P_k (= P_{jk})$ of product holders of type k in zone j who are willing to return their cores when the company offers incentive R_k per unit core can be written as

$$f(R_{0k}) = 2R_{0k}/a_k^2 \quad (3.17)$$

$$P_k = Pr(R_k \geq R_{0k}) = F(R_k) = R_k^2/a_k^2. \quad (3.18)$$

Note that R_{0k} again takes on values in the interval $[0, a_k]$ where $a_k > 0$ represents the maximum incentive level of product holder of type k . As P_k is the proportion of product holders of type k willing to return their used products, the total number of potential returns of type k from zone j becomes $h_{jk}P_k = h_{jk}R_k^2/a_k^2$. Also, for each core collected the company receives a unit subsidy G . Hence the maximum possible profit Π'_{jk} can be computed as $h_{jk}(G + s_k - R_k)R_k^2/a_k^2$ which is realized if all returns are collected. When $G + s_k > a_k$ we can write explicitly as follows

$$\Pi'_{jk} = \begin{cases} 0 & R_k < 0 \\ \frac{h_{jk}}{a_k^2} ((G + s_k)R_k^2 - R_k^3) & 0 \leq R_k < a_k \\ h_{jk}(G + s_k - R_k) & a_k \leq R_k < G + s_k \\ 0 & R_k > G + s_k \end{cases}. \quad (3.19)$$

When $G + s_k \leq a_k$, then the total profit from returns of type k in zone j becomes

$$\Pi'_{jk} = \begin{cases} 0 & R_k < 0 \\ \frac{h_{jk}}{a_k^2} ((G + s_k)R_k^2 - R_k^3) & 0 \leq R_k < G + s_k \\ 0 & R_k \geq G + s_k \end{cases} \quad (3.20)$$

Due to collection related costs like vehicle operating and facility opening, it may not be profitable to collect all the cores customers agree to give away. In order to reflect this situation in the model formulation we define a variable $X_{i^*jk} \in [0, 1]$ to denote the fraction of potentially returned cores of type k collected in zone j and carried to the nearest CC at site i^* . So, the total profit realized from zone j for $0 \leq R_k \leq \min\{a_k, G + s_k\}$ is then given by

$$\sum_k X_{i^*jk} h_{jk} (G + s_k - R_k) R_k^2 / a_k^2. \quad (3.21)$$

Furthermore, differing from CCLP we formulate company's problems in such a way that the number of collection centers to be opened is also a decision variable like in the well-known fixed charge facility location problem. In that case, the trade-off between the total fixed cost of opening and operating collection centers and the total collection cost can be incorporated by adding the term $f_i Y_{i^*}$ to the objective function where f_i denotes the fixed cost of opening and operating a collection center at site i . Moreover, constraint (3.9), i.e., $\sum_i Y_i = p$ will not be included the model. To summarize, the total collection cost can be expressed as

$$\left\lceil \frac{\sum_k X_{i^*jk} h_{jk} R_k^2 / a_k^2}{q} \right\rceil (c_1 + 2c_2 d_{i^*j}) + f_i Y_{i^*}. \quad (3.22)$$

Thus, the total net profit from zone j becomes

$$\sum_k X_{i^*jk} h_{jk} (G + s_k - R_k) R_k^2 / a_k^2 - \left\lceil \frac{\sum_k X_{i^*jk} h_{jk} R_k^2 / a_k^2}{q} \right\rceil (c_1 + 2c_2 d_{i^*j}) - f_i Y_{i^*}. \quad (3.23)$$

3.2.2. Model Formulations

We note that CP1 and CP2 are both variants of the uncapacitated discrete facility location-allocation problem in which there are n customer zones indexed by j and m candidate sites to open CCs indexed by i . Below we define additional parameters:

f_i : fixed cost of opening a CC at candidate site i

ρ : target profitability ratio

τ : target collection rate

The decision variables used in the models are as follows:

Y_i : binary variable indicating whether a CC is opened at location i

X_{ijk} : fraction of potential returns of type k collected in zone j and transported to the CC at site i

V_{ij} : number of vehicles required to transport returns from zone j to the CC at site i

R_k : unit incentive offered (unit acquisition price) for a used product of type k

G : unit subsidy offered for each collected used product

The GSCSDP1 is formulated as:

$$\text{GP1 : } \min_{G \geq 0} G \tag{3.24}$$

s.t.

$$\sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} \sum_{k \in \mathbf{K}} X_{ijk} h_{jk} \frac{R_k^2}{a_k^2} \geq \tau \sum_{j \in \mathbf{J}} h_j \tag{3.25}$$

where X_{ijk} , R_k solves:

$$\begin{aligned} \text{CP1 : } \max \Pi_{net} = & \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} \sum_{k \in \mathbf{K}} X_{ijk} h_{jk} \frac{R_k^2}{a_k^2} (G + s_k - R_k) - \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} (c_1 + 2c_2 d_{ij}) V_{ij} \\ & - \sum_{i \in \mathbf{I}} f_i Y_i \end{aligned} \quad (3.26)$$

s.t.

$$\sum_{i \in \mathbf{I}} X_{ijk} \leq 1 \quad j \in \mathbf{J}; k \in \mathbf{K} \quad (3.27)$$

$$X_{ijk} \leq Y_i \quad i \in \mathbf{I}; j \in \mathbf{J}; k \in \mathbf{K} \quad (3.28)$$

$$V_{ij} \geq \frac{\sum_{k \in \mathbf{K}} X_{ijk} h_{jk} R_k^2 / a_k^2}{q} \quad i \in \mathbf{I}; j \in \mathbf{J} \quad (3.29)$$

$$R_k \leq a_k \quad k \in \mathbf{K} \quad (3.30)$$

$$R_k \leq G + s_k \quad k \in \mathbf{K} \quad (3.31)$$

$$R_k \geq 0 \quad k \in \mathbf{K} \quad (3.32)$$

$$X_{ijk} \geq 0 \quad i \in \mathbf{I}; j \in \mathbf{J}; k \in \mathbf{K} \quad (3.33)$$

$$V_{ij} \geq 0 \text{ and integer} \quad i \in \mathbf{I}; j \in \mathbf{J} \quad (3.34)$$

$$Y_i \in \{0, 1\} \quad i \in \mathbf{I} \quad (3.35)$$

The model of GSCSDP2 is formulated as:

$$\text{GP2 : } \min_{G \geq 0} G \quad (3.36)$$

s.t.

$$\Pi_{net} \Big/ \left(\sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} \sum_{k \in \mathbf{K}} X_{ijk} h_{jk} \frac{R_k^3}{a_k^2} + \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} (c_1 + 2c_2 d_{ij}) V_{ij} + \sum_{i \in \mathbf{I}} f_i Y_i \right) \geq \rho \quad (3.37)$$

where X_{ijk} , R_k solves:

$$\begin{aligned} \text{CP2 : } \max \Pi_{net} = & \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} \sum_{k \in \mathbf{K}} X_{ijk} h_{jk} \frac{R_k^2}{a_k^2} (G + s_k - R_k) - \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} (c_1 + 2c_2 d_{ij}) V_{ij} \\ & - \sum_{i \in \mathbf{I}} f_i Y_i \end{aligned} \quad (3.38)$$

s.t.

$$\sum_{i \in \mathbf{I}} X_{ijk} \leq 1 \quad j \in \mathbf{J}; k \in \mathbf{K} \quad (3.39)$$

$$X_{ijk} \leq Y_i \quad i \in \mathbf{I}; j \in \mathbf{J}; k \in \mathbf{K} \quad (3.40)$$

$$V_{ij} \geq \frac{\sum_{k \in \mathbf{K}} X_{ijk} h_{jk} R_k^2 / a_k^2}{q} \quad i \in \mathbf{I}; j \in \mathbf{J} \quad (3.41)$$

$$\sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} \sum_{k \in \mathbf{K}} X_{ijk} h_{jk} \frac{R_k^2}{a_k^2} \geq \tau \sum_{j \in \mathbf{J}} h_j \quad (3.42)$$

$$R_k \leq a_k \quad k \in \mathbf{K} \quad (3.43)$$

$$R_k \geq 0 \quad k \in \mathbf{K} \quad (3.44)$$

$$X_{ijk} \geq 0 \quad i \in \mathbf{I}; j \in \mathbf{J}; k \in \mathbf{K} \quad (3.45)$$

$$V_{ij} \geq 0 \text{ and integer} \quad i \in \mathbf{I}; j \in \mathbf{J} \quad (3.46)$$

$$Y_i \in \{0, 1\} \quad i \in \mathbf{I} \quad (3.47)$$

The inner problems (company's problems) CP1 and CP2 are represented by constraints (3.27) – (3.35) and (3.39) – (3.47), respectively. The unit subsidy G in the outer problems (government's problems) GP1 and GP2 constitute an input parameter for the inner problems. GP1 and GP2 have both the same objective function which is to minimize the nonnegative value of the unit subsidy G in (3.24) and (3.36). The minimum collection rate constraint of GSCSDP1 for which the government is responsible in the outer problem is shown by (3.25), while the same constraint to be satisfied by the company in the inner problem of GSCSDP2 is shown by (3.42). The desired level of collection is obtained by multiplying the target collection rate τ by the total number of available cores. In both CP1 and CP2 we maximize the same objective function, namely the net profit as shown in (3.26) and (3.38), respectively, which is obtained by subtracting the total vehicle operating cost and the cost of opening CCs from the

total profit. The fraction of potential returns which are collected in reality should be less than or equal to 100 per cent as stated in (3.27) for GSCSDP1 and in (3.39) for GSCSDP2. Constraints (3.28) in GSCSDP1 (constraints (3.40) in GSCSDP2) ensure that this fraction is zero if there is no CC at site i . The number of vehicles V_{ij} required to transport returns from zone j to the CC at site i should be greater than or equal to the total amount of returns collected from zone j divided by the vehicle capacity q . Since vehicle related costs will be minimized in CP1 and CP2, V_{ij} is going to be set to the smallest integer value greater than $\sum_k X_{ijk}h_{jk}$. This is taken care of by the inequalities (3.29) as well as by the integrality constraints on V_{ij} in (3.34) for GSCSDP1 (inequalities (3.41) and integrality constraints (3.46) for GSCSDP2). For the incentive R_k , there are two upper bounds. One of them is a_k since all product holders would agree to make a return at or beyond this level of the incentive. The other upper bound equals $(G + s_k)$, because offering any incentive higher than $(G + s_k)$ will result in negative unit profit for CP1. In that case, the company would be better off if it did not engage in product recovery at all. However, the company of the legislative model may be obliged to declare an incentive higher than $(G + s_k)$ when $a_k > G + s_k$, because it has to fulfill the target collection rate requirement even though this triggers a decrease in the total net profit. From government's point of view, this implies at the cost of having negative profit in some core types due to R_k values above $(G + s_k)$, the company is responsible for reaching the target collection rate τ and by the contribution of the product recovery of other profitable core types the targeted overall profitability ratio ρ can eventually be attained. Constraints (3.30) and (3.31) (constraint (3.43)) serve as these upper bounds on R_k in GSCSDP1 (GSCSDP2). Finally, the constraints (3.32) – (3.35) and (3.44) – (3.47) are nonnegativity and integrality constraints for GSCSDP1 and GSCSDP2, respectively.

GP1 and GP2 are both continuous optimization problems in nonnegative variable G for given values of X_{ijk} and R_k . These problems are relatively easy to solve compared to CP1 and CP2. The latter models are MINLP problems with $mn(K + 1) + K(n + 2)$ and $mn(K + 1) + K(n + 1)1$ constraints, respectively.

4. SOLUTION PROCEDURE

4.1. Solution Methodology for Base Collection System

The CCLP is formulated as a MINLP model which is difficult to solve even for decent problem sizes. Hence heuristic methods can be used to obtain near-optimal solutions. In this context, we propose such a solution method which makes use of the following problem characteristic: given the locations of p collection centers, the CCLP reduces to a nonlinear problem in variables R_k for incentives and V_{ij} for vehicles. Thus it is possible to perform a search in a K -dimensional search space where each point corresponds to a feasible set of R_k and V_{ij} values. The optimal objective value of each point can be calculated by adding up the maximum net profits from customer zones. In order to establish such profits the correct number of vehicles allocated to each customer zone must be found based on the given vehicle capacity.

In the main loop of our heuristic we use a Tabu Search (TS) procedure to find good locations for the collection centers. The main motivation of using TS is the similarity of the CCLP to the well-known p -median problem. Very accurate solutions have been obtained for this problem by heuristic methods based on TS. Rolland et al. (1996) developed a TS procedure for the p -median problem which is found to be superior to the node interchange heuristic. Rosing et al. (1998) made a head to head comparison between the metaheuristics tabu search and heuristic concentration on a test bed of 21 challenging problems with 100 and 200 customers where p has been assigned values between 5 and 20. Recently, Mladenović et al. (2003) worked on both a tabu search and a variable neighborhood search method for solving the p -center problem rather than the classical p -median problem.

TS is a metaheuristic algorithm that guides the local search to prevent it from being trapped in premature local optima or in cycling (Glover and Laguna, 1997). This is achieved by prohibiting the moves that cause to return to previously visited solutions throughout a certain number of iterations. The basic TS algorithm starts

with an initial solution. At each iteration, a neighborhood of the current solution is created by one or more types of moves. The best solution from this neighborhood is selected as the new current solution if it is not classified as tabu. In the case this solution is restricted as tabu, it may still be admitted as the new current solution if it outperforms the *incumbent* (the overall best solution so far). This condition is called *aspiration criterion*. The incumbent is updated if the new current solution is both feasible and better than the incumbent. A *tabu list* which is updated at the end of each iteration keeps record of the tabu attributes of the accepted moves. The iterations are continued until one or more stopping criteria are satisfied. We propose a TS procedure that uses 1-, 2-, and 3-Swap moves in which, respectively, one, two, and three collection centers in the current solution are moved from their locations to candidate sites without a facility. In other words, opened collection centers are relocated.

A flowchart is provided in Figure 4.1 to describe the steps of our TS implementation referred to as TS-CCLP. First, we give the notation used in the flowchart.

| | |
|---------------------|---|
| num_iter | number of iterations performed so far |
| Max_Iter | maximum number of iterations |
| num_nonimp_iter | number of iterations throughout which the incumbent does not improve |
| Max_Nonimp_Iter | maximum number of iterations throughout which the incumbent does not improve |
| num_neigh | number of neighbors generated in the current iteration |
| $size_neigh_w$ | number of neighbors generated in the current iteration using the w^{th} move, $w = 1\text{-Swap}, 2\text{-Swap}, 3\text{-Swap}$ |
| Obj | objective value of a newly generated neighboring solution |
| Obj_Best_Neigh | objective value of the best neighboring solution |
| Obj^* | objective value of the incumbent |
| R_k | optimal incentive for type k for the newly generated neighboring solution |

| | |
|---------------------|--|
| R_k^{best} | optimal incentive for type k for the best neighboring solution |
| R_k^* | optimal incentive for type k for the incumbent |
| Max_Tabu_Tenure | maximum tabu tenure (i.e., maximum number of iterations during which a solution will be considered tabu-active.) |

Initial Solution

For each candidate site we calculate its cumulative distance summed over all customer zones. We obtain an initial solution by choosing the first p sites with the lowest cumulative distance value.

Neighborhood structure and tabu restrictions

The neighboring solutions are generated from the current solution by applying 1-Swap, 2-Swap, and 3-Swap moves. With this choice of the neighborhood structure, we prevent infeasible solutions—solutions in which the number of opened collection centers is not equal to p —from entering the search space. The neighborhood of a current solution is constructed by applying the three moves in such a way that an equal number of solutions are generated from each one and the best one is selected by comparing their net profit values.

As the current solution is updated throughout the iterations of TS-CCLP, we employ tabu restrictions so that solutions visited earlier are not selected repeatedly. Tabu restrictions are defined for the three moves as follows. In the 1-Swap move, if collection centers at sites i and j are swapped, i.e., one is added to and the other is dropped from the set of opened centers, then the 1-Swap move cannot be applied to them during the time they are tabu-active. In the case of the 2-Swap, suppose collection centers at sites i and j are about to be opened, i.e., to be added to the current configuration, and two centers opened previously at sites k and l are about to be closed, i.e., to be dropped from the current configuration. Suppose further that this 2-Swap move does not result in a better profit than the incumbent's profit, which

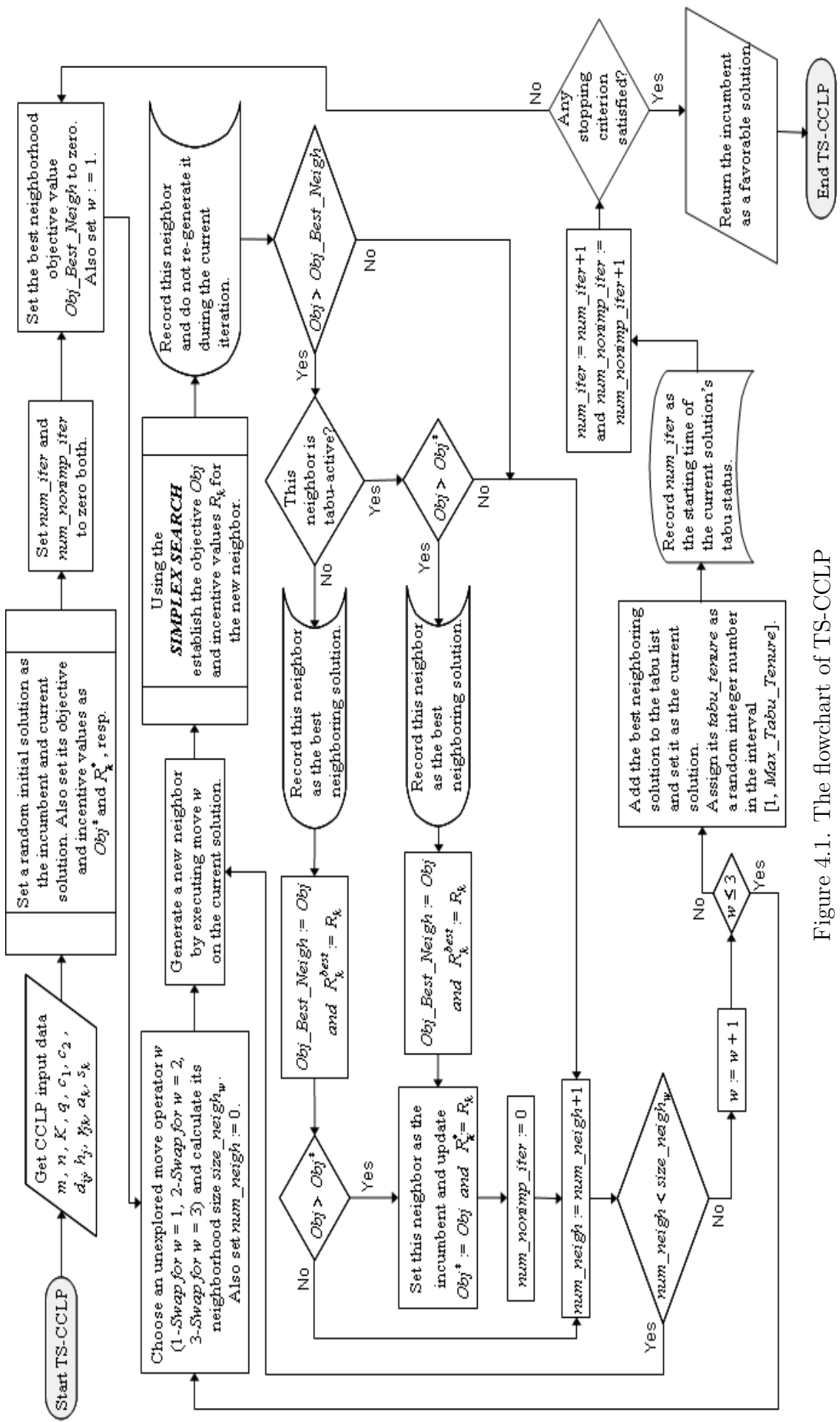


Figure 4.1. The flowchart of TS-CCLP

means that it does not satisfy the aspiration criterion. Thus, it cannot be executed if centers at sites i and j had been closed, and those at sites k and l had been opened during the previous ϵ iterations with ϵ denoting the then-assigned tabu tenure. This is, once a particular move has been performed upon some specific collection centers, it is declared as tabu-active for the next ϵ iterations, during which the outcome of the move should not be reverted. The tabu attributes of the 3-Swap move are defined in a similar way. According to the aspiration criterion, a move involving collection center sites that are tabu can be executed provided that it produces a solution with a higher net profit than the incumbent. The tabu tenure ϵ are assigned random integer values in the interval $[1, Max_Tabu_Tenure]$ where $Max_Tabu_Tenure = \lceil 1.5p \rceil$.

Termination Criterion

We use two termination criteria in TS-CCLP. The first one is the total number of iterations performed controlled by the parameter $Max_Iter = \max \{2m, 100\}$. The second criterion is the maximum permissible number of iterations during which the best solution (incumbent) does not improve. This criterion is controlled by the parameter $Max_Nonimp_Iter = \lceil Max_Iter/5 \rceil$. The values of these parameters are taken from Rolland et al. (1996).

As can be seen in Figure 4.1, for each alternative configuration of opened collection centers the TS-CCLP calls the simplex search method to determine the best values of the incentive variables $\mathbf{R} = \{R_k : k = 1, \dots, K\}$ and the corresponding net profit Π . Simplex search originally developed by Nelder and Mead (1965) for unconstrained non-linear optimization is a derivative-free direct search method for maximizing a function of multiple variables. In this method, the simplex is a polytope consisting of $z + 1$ vertices where z denotes the number of decision variables. Each vertex is represented by a z -dimensional vector. The worst vertex is rejected and replaced by a new vertex along the line joining this vertex and the centroid $\bar{\mathbf{R}}$ of the remaining vertices. The exact location of the new vertex is found by reflection, expansion, and contraction operations. These operations are repeated many times until the simplex approximately shrinks to

a single point which is declared as a local optimum. The $(K + 1)$ initial vertices in our problem are generated according to the suggestion in Bazaraa et al. (1993). The reflection (α), expansion (λ), contraction (β), and shrinkage (χ) coefficients are fixed at the following values as suggested by Nelder and Mead (1965): $\alpha = 1$, $\lambda = 2$, $\beta = 0.5$, and $\chi = 0.5$. If the new vertex is infeasible with respect to the bounds ($0 \leq R_k \leq s_k$), then the coefficient used in that step is adjusted so that each component R_k of the new vertex is within its lower and upper bounds. The steps of simplex search are provided in Appendix A.

To determine the best values for R_k the simplex search method needs to compute the profit for each vertex (i.e., a set of values assigned to R_k). As the total net profit Π can be calculated by summing up the individual net profits from customer zones, a procedure is required to find Π_j from a single customer zone j . Since simplex search is called inside the TS-CCLP with a given set of collection center locations, it is possible to assign each customer zone j to the nearest collection center at site i^* . In other words, the vehicles that will transport the returns from zone j will be dispatched from collection center at site i^* . This decision is optimal since the collection centers are assumed to have unlimited capacity. This would not be the case if we considered a finite capacity version of the CCLP. There are two interrelated issues that need to be resolved to determine the optimal profit in each zone. The first one is the amount of each type k return that will be collected with the maximum number being $h_{jk}R_k/a_k$. The other one is the number of vehicles that will be dispatched to carry the returns (of any type) from zone j . These decisions should be made simultaneously. By recalling the guidelines pointed out earlier, we develop an algorithm called vehicle loading procedure 1 (VLP-1) to compute Π_j at every zone j and hence $\Pi = \sum_j \Pi_j$ corresponding to a vertex represented by a set of R_k values.

First we sort the used product types in nonincreasing order with respect to their unit profits $s_k - R_k$. Let $s_{k[1]} - R_{k[1]} \geq s_{k[2]} - R_{k[2]} \geq \dots \geq s_{k[K]} - R_{k[K]}$ be this order. We start with item type $k[1]$ yielding the highest unit profit and calculate the net profit of a single vehicle if it is fully loaded with type $k[1]$ returns. This is the case when $h_{jk[1]}R_{k[1]}/a_{k[1]} \geq q$. The profit due to a fully loaded vehicle can be calculated

as $\pi = q(s_{k[1]} - R_{k[1]}) - (c_1 + 2c_2d_{i^*j})$. If $\pi > 0$, then it is always profitable to send as many vehicles as possible to zone j provided that all vehicles carry q returns of type $k[1]$. Thus, the number of vehicles turns out to be $V_j = \left\lfloor \frac{h_{jk[1]}R_{k[1]}/a_{k[1]}}{q} \right\rfloor$ where $\lfloor z \rfloor$ denotes the largest integer less than or equal to z for $z > 0$. The next question is whether it is also profitable to send the $(V_j + 1)^{\text{st}}$ vehicle that will be partially loaded with $(h_{jk[1]}R_{k[1]}/a_{k[1]} - V_jq)$ returns of type $k[1]$. We also have the situation of partial loading when type $k[1]$ returns are not sufficient to fully load a single vehicle (where $V_j = 0$), i.e., $h_{jk[1]}R_{k[1]}/a_{k[1]} < q$ and $(V_j + 1)^{\text{st}}$ vehicle has only $h_{jk[1]}R_{k[1]}/a_{k[1]}$ returns of type $k[1]$. Note that this includes the case of $h_{jk[1]}R_{k[1]}/a_{k[1]} = 0$ meaning that there exists no return of type $k[1]$.

Following the third guideline we load the $(V_j + 1)^{\text{st}}$ vehicle with returns of type $k[2]$, $k[3]$, \dots until it is full with q returns of various types or there are no returns remaining. If the net profit of this vehicle is positive, then we also dispatch this vehicle. Otherwise, the total number of vehicles sent to zone j is V_j . In case vehicle $(V_j + 1)$ is sent to zone j , vehicle $(V_j + 2)$ is loaded in a similar fashion starting with the returns yielding highest unit profit among the remaining ones that are still not loaded to any vehicle. The steps of VLP-1 to determine the optimal amount of collected returns of each type, the optimal number of vehicles to carry the returns from zone j to collection center at site i^* , and the optimal net profit Π_j in zone j are given in Figure 4.2.

Let q_l^{rem} and u_k^{rem} denote respectively the remaining capacity of vehicle l and the number of type k returns not collected. Also let π_l be the profit from vehicle l and V_j^* be the optimal number of vehicles. Note that the total number of vehicles necessary to collect all the possible returns in zone j is given as $\left\lceil \frac{\sum_k h_{jk}R_k/a_k}{q} \right\rceil$, which makes an upper bound on V_j^* .

Figure 4.2. Pseudo code for VLP-1

(To be run for each customer zone j , $j = 1, \dots, n$.)

1. (Initialization):

$$q_l^{rem} := q, \quad l = 1, \dots, \left\lceil \frac{\sum_k h_{jk} R_k / a_k}{q} \right\rceil.$$

$$u_k^{rem} := h_{jk} R_k / a_k, \quad k = 1, \dots, K.$$

$$\pi_l := 0, \quad l = 1, \dots, \left\lceil \frac{\sum_k h_{jk} R_k / a_k}{q} \right\rceil.$$

$$\Pi_j^* := 0.$$

$$V_j^* := 0.$$

2. Sort the item types in nonincreasing order with respect to unit profit $s_k - R_k$ and

let $s_{[1]} - R_{[1]} \geq s_{[2]} - R_{[2]} \geq \dots \geq s_{[K]} - R_{[K]}$ be this order. Also set,

$$l := 1, k := 1.$$

3. If $u_{[k]}^{rem} \geq q_l^{rem}$, then /* There are as much or more uncollected type k cores in zone j than the slack capacity of the current vehicle. */

$$\pi_l := \pi_l + q_l^{rem} (s_{[k]} - R_{[k]}) - (c_1 + 2c_2 d_{i^*j}).$$

If $\pi_l \leq 0$, then

Stop and go to 4. /* It is not profitable to collect any other returns from zone j . */

Else

$$\Pi_j^* := \Pi_j^* + \pi_l.$$

$$u_{[k]}^{rem} := u_{[k]}^{rem} - q_l^{rem}.$$

$$q_l^{rem} = 0.$$

$$V_j^* := V_j^* + 1. \quad /* \text{Acquire a new vehicle for customer zone } j. */$$

$$l := l + 1.$$

Go to 3.

EndIf

Else ($u_{[k]}^{rem} < q_l^{rem}$) /* The volume of uncollected returns of type k is less than the slack capacity of the vehicle. */

$$\pi_l := \pi_l + u_{[k]}^{rem} (s_{[k]} - R_{[k]}).$$

$$q_l^{rem} := q_l^{rem} - u_{[k]}^{rem}.$$

$$u_{[k]}^{rem} = 0.$$

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 $k := k + 1.$ 
If  $k \leq K$ , then
    Go to 3.          /* Proceed with the item type yielding the next highest
                        unit profit. */
Else
     $\pi_l := \pi_l - (c_1 + 2c_2d_{i^*j}).$  /* Compute the profit of the current vehicle. */
    If  $\pi_l > 0$ , then
         $\Pi_j^* := \Pi_j^* + \pi_l.$  /* Increase the total net profit by the current vehicle's
                                net profit. */
         $V_j^* := V_j^* + 1.$  /* Add the current vehicle to the set of vehicles allocated
                                for zone  $j$ . */
        Stop and go to 4.
    EndIf
EndIf
EndIf
4. Report  $\Pi_j^*$ ,  $V_j^*$ , and  $(h_{jk}R_k/a_k - u_k^{rem}).$ 

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4.2. Solution Methodology for Government-Subsidized Bilvel Models

Ben-Ayed and Blair (1990) proved that bilevel programs are \mathcal{NP} -hard; hence, efficient heuristic methods are needed to obtain good solutions for this type of problems. In both GSCSDP1 and GSCSDP2, the leader's problem GP is a continuous optimization problem with a single decision variable, namely the unit subsidy G . The follower's problem CP, however, is a MINLP model for a given value of G determined by the leader's problem. We propose a nested heuristic solution methodology, which mainly utilizes Brent's method (Brent, 1971) for solving outer problem GP, and a tabu search heuristic for solving inner problem CP. The nested heuristic for the supportive model GSCSDP1 is slightly different from that of the legislative model GSCSDP2. Therefore, we provide the details of our heuristic separately by starting with the supportive model.

4.2.1. A Nested Heuristic for the Supportive Model GSCSDP1

In GSCSDP1, the inner problem CP1 is to determine the maximum net profit that can be attained by the company for a value of the unit subsidy G given by the government. The government, on the other hand, wants to announce the smallest possible amount of the unit subsidy that ensures the minimum collection rate. Hence, it is possible to adopt an exhaustive search in the domain of G . We can start with the smallest possible value of G , which is zero, and solve CP1. If this solution satisfies the minimum collection rate constraint 3.25 of GP1, then we are done; otherwise we can update G by a small increment, and solve CP1 again. This iterative procedure continues until minimum collection rate constraint 3.25 is satisfied. Let us consider a sample problem with $n = 20$ customer zones and $K = 3$ core types for which we solve CP1 for different values of G in the interval $[0, 10]$ with increments of 0.5, and record the realized collection rate T which is computed as:

$$T = \left(\sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} \sum_{k \in \mathbf{K}} X_{ijk} h_{jk} \frac{R_k^2}{a_k^2} \right) / \sum_{j \in \mathbf{J}} h_j \quad (4.1)$$

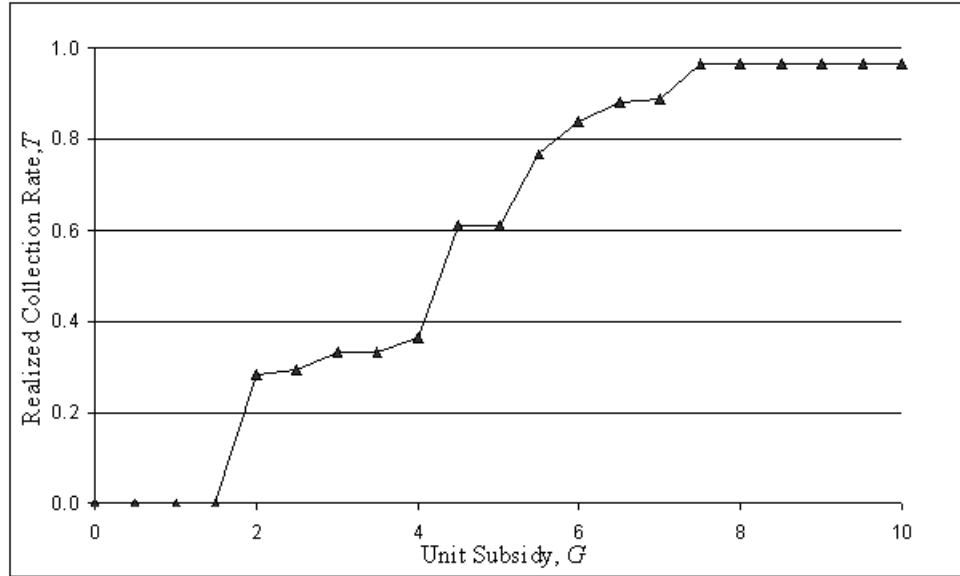


Figure 4.3. Realized collection rate T as a function of unit subsidy G

We plot T in Figure 4.3 as a function of the unit subsidy G . As can be seen, T is a monotonically increasing function of G implying that the company collects either

the same number of cores or more when the subsidy from the government increases. This observation is also valid for other problem instances we experimented with, which leads us to apply a more efficient method called Brent's method to solve GP1.

4.2.1.1. Solving the Outer Problem GP1 by Brent's Method. Brent's method is a root finding method that combines bisection method, secant method, and inverse quadratic interpolation. It can be employed when the function in question has a unique root. It has been shown that this method often converges superlinearly, and is never slower than the bisection method (Brent, 1971). In addition, it can be effectively applied also to discontinuous functions (Hedge and Kesera, 2005). In order to apply Brent's method we modify the objective function of GP1 by taking into account the infeasibility of the solution of inner problem CP1 with respect to constraint 3.25, which occurs if the realized collection rate T is less than the minimum collection rate τ . The new function $f_1(G)$, which is to be solved for its root by Brent's method, is defined as:

$$f_1(G) = \begin{cases} G & \text{if } T \geq \tau \\ -M(\tau - T) & \text{otherwise} \end{cases} \quad (4.2)$$

where M is a big positive number. The function $f_1(G)$ is a nondecreasing discontinuous function that takes on positive values equal to G for feasible solutions ($T \geq \tau$) and negative values equal to $-M$ for infeasible solutions ($T < \tau$). Moreover, it has a unique root at $T = \tau$. Brent's method can find this root, i.e., the minimum unit subsidy value G to be paid to the company by the government so that the realized collection rate T is at least as large as the minimum required rate τ .

4.2.1.2. Solving the Inner Problem CP1 by TS-1. Similar to solving CCLP, we implement a tabu search (TS) heuristic for the inner problem CP1. To this end, we modify the TS method developed in Aras and Aksen (2008) to solve an uncapacitated, fixed charge CC location problem so that it accommodates the government-subsidized core pick-up policy. Recent successful implementations of TS for finding optimal or near optimal solutions to the fixed charge facility location problem can be found in Sun

(2006), and Michel and Van Hentenryck (2004).

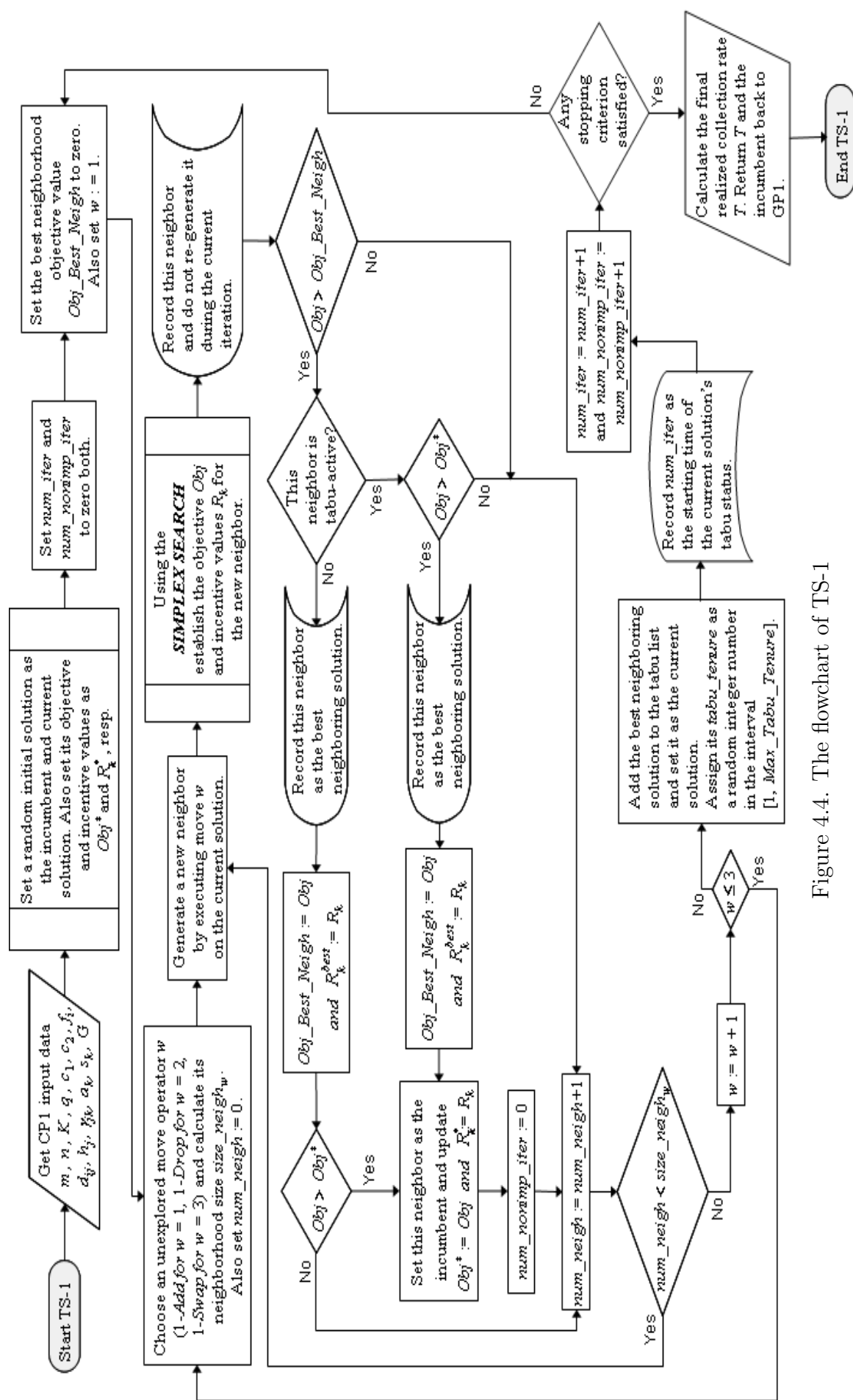
We name the TS procedure which tackles the inner problem CP1 in the supportive model as TS-1. Each neighborhood solution in this procedure corresponds to a particular CC location plan, and is obtained by the same moves as in the TS of Aras and Aksen (2008). Once a CC location plan is given, the optimal assignment of customer zones to the CCs opened in that plan is trivial. Since CCs are assumed to have unlimited capacity, collecting the returns in zone j with the vehicles departing from the nearest CC denoted by $i^*(j)$ will be optimal. A flowchart of TS-1 is provided in Figure 4.4. We note that TS-1 constitutes the middle shell of the nested heuristic for GSCSDP1. It runs at each iteration of Brent's method applied on GP1. TS-1 itself capitalizes on Nelder-Mead simplex search with embedded VLP-1 every time the objective value of a newly generated solution (a configuration of opened CCs) needs to be evaluated. General guidelines for TS-1 are given as follows. Same notation is used used with the TS-CCLP at Section 4.1.

Initial Solution

The initial solution is created by opening only one CC at a candidate site that is selected randomly.

Neighborhood structure and size

We generate neighboring solutions from the current solution by applying 1-Add, 1-Drop, and 1-Swap moves. This neighborhood structure proved efficient also in the TS implementation of Aras and Aksen (2008). We select the best of the neighboring solutions by comparing their objective (net profit) values, which we obtain from the simplex search embedded in TS-1. 1-Add move opens a CC at one of the candidate sites where no CC is available yet in the current solution. 1-Drop move removes an opened CC from the current solution. Finally, 1-Swap move removes one of the opened CCs in the current solution and at the same time opens a new one at a site that has none, i.e., it relocates an already opened CC. If we denote σ by the number of CCs



available in the current solution, then $size_neigh_w$ values, namely neighborhood sizes of the moves, will be as follows: $size_neigh_{1-Add} = (m - \sigma)$, $size_neigh_{1-Drop} = \sigma$, $size_neigh_{1-Swap} = \min\{3m, \sigma(m - \sigma)\}$. We ensure that no two neighboring solutions generated in a given iteration of TS-1 by a specific move are identical.

Tabu restrictions and aspiration criterion

In our TS-1 algorithm we resort to tabu restrictions in order not to be trapped in location plans that have been created earlier. Tabu restrictions are defined as follows. 1-Add move: If the CC at site i is added to the current location plan, then i is declared as tabu, and cannot be selected by the 1-Drop move during $tabu_tenure$ iterations. 1-Drop move: If the CC at site i is removed from the current location plan, then i is declared as tabu and cannot be selected by the 1-Add move while tabu-active. 1-Swap move: If the CCs at sites i and j are swapped, i.e., one is added to and the other is dropped from the current location plan, then 1-Swap move cannot be applied to them again during the time they are tabu-active. According to the so-called aspiration criterion, a move involving those sites which are declared as tabu restricted can be executed if it produces a solution with a higher profit than the incumbent's profit (current best profit). Like in the TS method of Aras and Aksen (2008), the $tabu_tenure$ is assigned random integer values in the interval $[1, Max_Tabu_Tenure]$ where Max_Tabu_Tenure equals 25.

Termination Criterion

We use the same termination criteria with TS-CCLP.

4.2.2. A Nested Heuristic for the Legislative Model GSCSDP2

In GSCSDP2 the minimum collection rate constraint has to be satisfied by the company in the inner problem CP2. This implies that it is mandatory for the company to reach a minimum collection rate τ by legislation. The government, on the other hand, ensures that the company achieves a net profitability ratio at least as large as a

target value ρ by declaring the smallest unit subsidy G . Then, it is possible to adopt an exhaustive search in the domain of G just as was the case with GSCSDP1. In other words, we can start with the smallest possible value of G , which is zero, and solve CP2. If this solution satisfies the minimum profitability constraint (3.37) of GP2, then we are done; otherwise we can increase G by a small amount, and solve CP2 again. This iterative procedure continues until the minimum profitability constraint (3.37) is satisfied. Let us consider a sample problem with $n = 50$ customer zones, $K = 2$ two core types, and minimum collection rate τ equal to 0.4. Using this setting, we solve CP2 for different values of G in the interval $[0, 10]$ with increments of 0.5, and record the realized profitability ratio P computed as:

$$P = \Pi_{net} \left/ \left(\sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} \sum_{k \in \mathbf{K}} X_{ijk} h_{jk} \frac{R_k^3}{a_k^2} + \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} (c_1 + 2c_2 d_{ij}) V_{ij} + \sum_{i \in \mathbf{I}} f_i Y_i \right) \right. \quad (4.3)$$

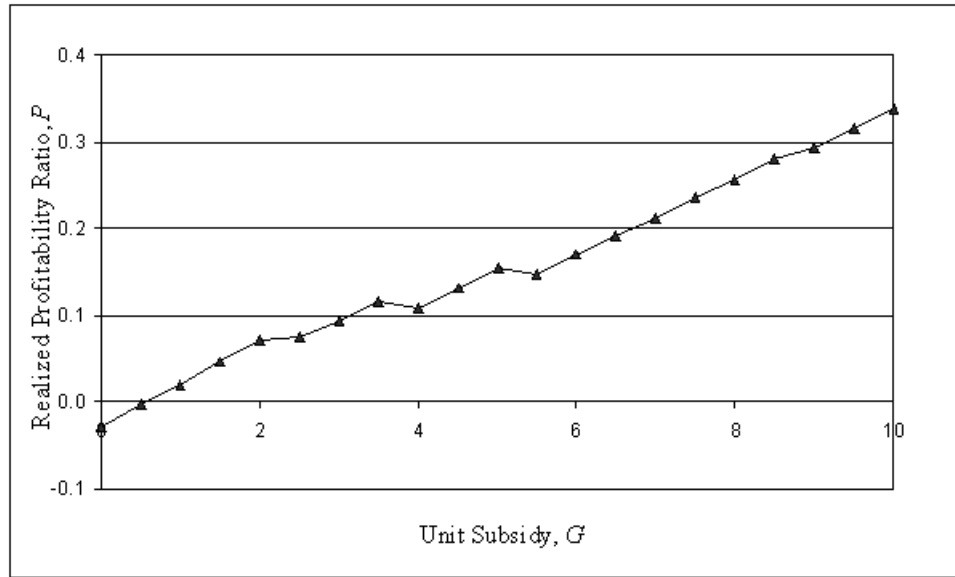


Figure 4.5. Realized profitability ratio P as a function of unit subsidy G

In Figure 4.5 we plot P as a function of the unit subsidy G . We observe that unlike the curve of the realized collection rate T in Figure 4.3, realized profitability ratio P is not a monotonically increasing function of G although it has an increasing trend in general with increasing values of G . This result is the outcome of the discrete nature of the inner problem CP2. The monotonicity of P in Figure 4.5 is destroyed when there occurs a drop in P as G increases. In fact, an increase in G gives rise to an

increase in both the net profit Π_{net} and total expenditures, but the amount of the latter surpasses that of the former leading to a reduction in P . This is most likely caused by the establishment of a new CC in the solution. Consequently, we apply Brent's method with the provision that the new function may have several roots.

4.2.2.1. Solving the Outer Problem GP2 by Brent's Method. First, we modify the objective function of GP2 by taking into account the infeasibility of the solution of inner problem CP2 with respect to constraint 3.37, which occurs if the realized profitability ratio P is less than the minimum profitability ratio ρ . The new function $f_2(G)$ is defined as:

$$f_2(G) = \begin{cases} G & \text{if } P \geq \rho \\ -M(\rho - P) & \text{otherwise} \end{cases} \quad (4.4)$$

where M is a large positive number. When Brent's method is applied to $f_2(G)$, it finds a unit subsidy value G at which $f_2(G)$ changes its sign, i.e., a root. However, because of the nonmonotonicity of $f_2(G)$ it may not be the root corresponding to the smallest value of G the company should receive from the government so that the realized profitability ratio P matches or exceeds the minimum required ratio ρ . Therefore, we adopt the following approach. After finding the best subsidy level (say G_1^*) at the first trial, we apply Brent's method anew by taking the initial interval as $[0, G_1^*]$ to find another possible root $G_2^* < G_1^*$ if there is any. If a second root G_2^* is found, Brent's method is started again with the new interval $[0, G_2^*]$. We remark that at each restart of Brent's method the initial interval gets smaller, and we stop when we find the minimum level of the unit subsidy G .

4.2.2.2. Solving the Inner Problem CP2 by TS-2. Just as the case with GSCSDP1, we apply a TS heuristic called TS-2 to deal with the inner problem CP2 in the legislative model GSCSDP2, which is run for each iteration of Brent's method employed on GP2. The structure of TS-2 is exactly the same as the one of TS-1, therefore we do not provide a separate flowchart for it (see Figure 4.4). TS-2 also depends on Nelder-Mead simplex search every time the objective value of a newly generated solution needs to

be evaluated. The only difference between TS-2 and TS-1 exists in the vehicle loading procedure called from simplex search for each vertex to compute the best profit Π_{net} . Recall that a vertex corresponds to a set of R_k values in simplex search. In the vehicle loading procedure VLP-1 of TS-1, the aim is to obtain for a given unit subsidy G the highest net profit, which is equal to the sum of the highest profits from each zone j , without paying attention to the amount of cores collected in customer zones. In TS-2, however, the collection rate constraint (3.42) must be satisfied in a solution to inner problem CP2. Therefore, a new component is added to VLP-1, which builds upon the solution provided by VLP-1 to restore the infeasibility with respect to the collection rate constraint. The new component of the procedure is referred to as VLP-2.

Note that if VLP-1 yields a solution in which the realized collection rate T is greater than or equal to the minimum collection rate τ , there is no need to call VLP-2. Otherwise, new cores have to be collected in addition to those determined by VLP-1, which definitely decreases the net profit Π_{net} . VLP-2 works by collecting new cores and operating new vehicles until the minimum collection rate is eventually reached at the expense of a reduction in Π_{net} . If τ is not attainable with the current R_k values due to the insufficient volume of potential returns, VLP-2 assigns a very large negative objective value to eliminate the corresponding vertex. The details of VLP-2 are described below.

Let L denote the minimum number of additional cores to be collected to reach the minimum collection rate τ . Also let v_{jk}^{rem} and z_j denote the number of uncollected cores of type k in zone j and the idle capacity of vehicles currently dispatched to zone j , respectively. At this point we use a result from Aras et al. (2007) where it is shown that only one of the vehicles dispatched to zone j from the nearest CC may have idle capacity. Unless $\sum_j \sum_k v_{jk}^{rem} < L$ holds, the next step is to determine (i) which zone j should be selected to collect the additional cores and (ii) how many cores from each type should be collected in the selected zone. These two issues need to be resolved so that the reduction in Π_{net} is kept at a minimum level. By taking into consideration the unit profits $s_{[k]} + G - R_{[k]}$ from different core types sorted in nondecreasing order, the number v_{jk}^{rem} of uncollected cores of type k in zone j , and the idle capacity z_j of

the vehicles dispatched to zone j , we compute the best possible batch of cores to be collected in zone j and the resulting profit loss Π_j^{loss} . Notice that the collection operation associated with additional cores is never profitable, because otherwise they would be identified by VLP-1. Zone j with the lowest profit loss, i.e., $j^* = \operatorname{argmin}_j \{\Pi_j^{loss}\}$ is selected to pick up new cores. After the values of $v_{j^*k}^{rem}$, z_{j^*} , and L are updated, the process is repeated until $L \leq 0$ at which point the minimum collection rate is attained. The pseudo code of VLP-2 is shown in Figure 4.6 followed by the corresponding notation.

Notation:

| | | |
|----------------|---|--|
| L | = | minimum number of additional cores to be collected to reach the minimum collection rate τ |
| v_{jk}^{rem} | = | number of uncollected cores of type k in zone j |
| z_j | = | idle capacity of vehicles dispatched to zone j |
| b_{jk} | = | number of type k cores selected from zone j temporarily |
| X_{i^*jk} | = | fraction of type k cores collected in zone j and transported to the nearest CC at i^* |
| V_j | = | best number of vehicles dispatched to zone j |
| Π_j^{loss} | = | profit loss from zone j after restoration iteration |
| Π_{net} | = | best net profit found in VLP-1 for given R_k values |
| δ | = | auxiliary variable used to determine the batch of cores to be collected at an iteration |
| M | = | a very large positive number |

Figure 4.6. Pseudo code for VLP-2

1. (Initialization)

$$L = \tau \sum_j h_j - \sum_i \sum_j \sum_k X_{i^*jk} h_{jk} R_k^2 / a_k^2$$

$$v_{jk}^{rem} = X_{i^*jk} h_{jk} R_k^2 / a_k^2 \text{ for each } k \in \mathbf{K} \text{ and } j \in \mathbf{J}$$

$$z_j = qV_{i^*j} - \sum_i \sum_j \sum_k X_{i^*jk} h_{jk} R_k^2 / a_k^2 \text{ for each } j \in \mathbf{J}$$

2. If $\sum_j \sum_k v_{jk}^{rem} < L$ /*If the number of cores not collected falls short of the τ */

then report $\Pi_{net} := -\infty$ and stop. /* To have simplex search mark the
current R_k values as infeasible */

3. While $L > 0$ Do /* While the minimum collection rate is not satisfied yet */

4. For each zone j Do

If $\sum_k v_{jk}^{rem} = 0$, then $\Pi_j^{loss} = M$ /* All cores in zone j have been already
collected */

Else /* There are some uncollected cores in zone j */

$$b_{jk} = 0 \text{ for } k \in \mathbf{K}.$$

$$\Pi_j^{loss} = 0.$$

If $z_j = 0$, then

$z_j = q$. /* Assign a new vehicle to zone j if idle capacity
is zero */

$\Pi_j^{loss} := \Pi_j^{loss} + (c_1 + 2c_2 d_{i^*j})$. /* Compute the cost of the new
vehicle */

EndIf

EndIf /* There are some uncollected cores in zone j */

5. $k = 1$

While $k \leq \mathbf{K}$ Do /* Collect cores starting from the most profitable
core type */

$$\delta = 0$$

If $v_{j[k]}^{rem} = 0$, then /* There are no uncollected cores of type k in
zone j */

$$k := k + 1.$$

Continued on Next Page

```

    Go to step 4
Else    /* There are uncollected cores of type  $k$  in zone  $j$  */
    If  $(s_{[k]} + G - R_{[k]}) \geq 0$ , then let  $\delta = \min(v_{j[k]}^{rem}, z_j)$ 
    Else let  $\delta = \min(v_{j[k]}^{rem}, z_j, L)$ 
    EndIf
EndIf

 $\Pi_j^{loss} := \Pi_j^{loss} - \delta(s_{[k]} + G - R_{[k]})$ .
 $b_{j[k]} := b_{j[k]} + \delta$ .
 $z_j = z_j - \delta$ .
If  $z_j = 0$  or  $(s_{[k]} + G - R_{[k]}) < 0$  then break WhileDo
 $k := k + 1$ .
EndDo    /* Consider the next customer zone */
6. Let  $j^* := \operatorname{argmin}_j \{\Pi_j^{loss}\}$ .    /* Select zone  $j^*$  with the lowest profit loss */
 $\Pi_{net} := \Pi_{net} - \Pi_{j^*}^{loss}$ .
 $v_{j^*k}^{rem} := v_{j^*k}^{rem} - b_{j^*[k]}$  for  $k \in \mathbf{K}$ .
 $L := L - \sum_k b_{j^*[k]}$ .
EndDo    /* End of the outermost while-do loop */
7. Report:

$$X_{i^*jk} = \frac{v_{jk}^{rem} a_k^2}{h_{jk} R_k^2} \text{ for each } j \in \mathbf{J} \text{ and } k \in \mathbf{K},$$


$$V_j = \left\lceil \frac{1}{q} \sum_k X_{i^*jk} h_{jk} \frac{R_k^2}{a_k^2} \right\rceil \text{ for each } j \in J, \Pi_{net}.$$


```

5. COMPUTATIONAL RESULTS

In this section, we first describe how we generate random problem instances. In all instances, the candidate sites for CCs coincide with the customer zones, implying $\mathbf{I} = \mathbf{J}$, and $|\mathbf{I}| = |\mathbf{J}| = m = n$. The x - and y -coordinates of customer zones are sampled independently from a discrete uniform distribution in the interval $[0, 100]$. The number of product holders in each zone is also generated from a discrete uniform distribution supported on $[1, 100]$. The travel distances between candidate sites and customer zones are calculated using the Euclidean distance. We assume that the proportion of product holders having cores of type k is the same across all customer zones, i.e., $\gamma_{jk} = \gamma_k$ for all j . Parameter values which differ with respect to core types, namely unit cost savings s_k , maximum reservation incentive a_k , and γ_k are given in Table 5.1.

Table 5.1. Parameter values of the base case scenario

| | $K = 2$ | | $K = 3$ | | |
|------------|---------|-----|---------|-----|-----|
| k | 1 | 2 | 1 | 2 | 3 |
| s_k | 25 | 15 | 25 | 20 | 15 |
| a_k | 15 | 5 | 15 | 10 | 5 |
| γ_k | 0.5 | 0.5 | 1/3 | 1/3 | 1/3 |

5.1. Results for Base Collection System Model

This section is divided into two subsections. In the first subsection we perform experiments to assess the performance of the TS-CCLP on randomly generated instances both in terms of the solution quality and computation time. In the second subsection we generate insights regarding the policy of the company to offer incentives depending on the quality of the returns. For this purpose, we examine the case where the company offers the same incentive to product holders *regardless* of the quality type of their returns. Our main objective is to compare the profits obtained under the two policies and to explore the situations in which offering quality-dependent incentives

brings about significant additional profit to the company.

5.1.1. Performance of TS-CCLP

By assigning five distinct values to n ($n = 10, 20, 50, 100, 200$), four distinct values to p ($p = 1, 2, 3, 4$) and two distinct values to K ($K = 2, 3$), we obtain $4 \times 5 \times 2 = 40$ instances where each instance is labeled as a triplet (n, p, K) . Also, other parameter values are taken as $c_1 = 90$, $c_2 = 1$, and $q = 10$.

TS-CCLP has been coded in C using Microsoft Visual C++ 6.0. All the experiments have been conducted on a desktop computer with 3.20 GHz Intel Pentium 4 HT processor and 2 GB RAM. In the implementation of TS-CCLP, the number of neighboring solutions (*num_neigh*) is taken as $p(m - p)$, which is the total number of possible 1-Swap moves. Since there are three types of moves, i.e., 1-Swap, 2-Swap, and 3-Swap, we generate $p(m - p)/3$ neighbors from each of them.

In order to evaluate the performance of our TS-CCLP heuristic, we employ two other methods to solve the test instances. One of them is exhaustive search and can only be used for small CCLP instances. In this approach, we try all possible $\binom{m}{p}$ combinations of p collection centers on $m = n$ candidate sites. For each combination we vary the incentives R_k in the range $[0, \min\{a_k, s_k\}]$ with increments of 0.01 to find the optimal incentives and the corresponding optimal net profit. The combination that provides the highest net profit and the associated incentive values becomes the optimal solution to the instance under consideration. In Tables 5.2 and 5.3 we present the results for two ($K = 2$) and three ($K = 3$) used product types, respectively. Those instances for which the number of facility location combinations is less than 200 are also solved by exhaustive search with grid size 0.01. Other instances (for which $\binom{m}{p} > 200$) cannot be solved within reasonable computation time by exhaustive search. Therefore, both the incentive values R_k and the net profits are missing for the latter as shown by $(-)$ sign. On the basis of these results we conclude that the objective values obtained by TS-CCLP are as good as the ones provided by the exhaustive search. The reason why TS-CCLP yields slightly better solutions than exhaustive search is that the latter

is limited by the resolution of the grid, which is 0.01.

We also compare the results of TS-CCLP with those obtained by solving the base collection model using commercial solvers. There are three solvers available for MINLP models within the optimization suite GAMS v22.0. These are DICOPT, OQNLP, and SBB (GAMS: The Solver Manuals, 2005). DICOPT is based on the outer approximation method in which a sequence of mixed-integer programs and nonlinear programs are solved. It is expected to perform better on models that have a significant and difficult combinatorial part. OQNLP is a multi-start heuristic algorithm designed to find global optima of constrained nonlinear programs that are smooth. By “multi-start” it is meant that the algorithm calls a nonlinear programming solver from multiple starting points which are determined by a scatter search application called OptQuest (Laguna and Marti, 2003). SBB works different from the former two solvers. It performs branch-and-bound where a nonlinear model is solved at each node of the branch-and-bound tree by making use of an existing nonlinear solver. SBB may give better results on models that are fairly nonconvex, or have more difficult nonlinearities. In order to make the decision on which solver to use we conducted a preliminary experiment. By putting a time limit of one hour, we tried to solve eight instances ($m = n = 10$, $p = 1, 2, 3, 4$, $K = 2, 3$) using the three solvers mentioned above with their default settings. The SBB solver has been employed two times by using nonlinear solvers MINOS 5.5 and CONOPT. The results were surprising in the sense that for none of the instances OQNLP could find a feasible solution at the end of the time limit, whereas SBB (with both MINOS 5.5 and CONOPT) and DICOPT produced feasible solutions for all of them. However, the profits corresponding to the feasible solutions obtained by DICOPT were significantly lower compared to the profit values of SBB. We decided to carry out the experiments with SBB coupled with MINOS 5.5 since it gave results as good as or better than those obtained by SBB with CONOPT.

Because the largest instance in our dataset is solved in 5967.328 seconds by the TS-CCLP (see the last row of Table 5.3), we set a time limit of two hours for each instance using the reslim option of GAMS. Among 40 instances, SBB can solve 15 instances within the imposed time limit. These are instances with 10 and 20 customer

Table 5.2. Results obtained for CCLP with $K = 2$ core types

| Instance (n, p, K) | Exhaustive Search | | | TS-CCLP | | | |
|---------------------------|-------------------|-------|------------|---------|-------|------------|----------|
| | R_1 | R_2 | Net Profit | R_1 | R_2 | Net Profit | CPU (s) |
| (10,1,2) | 6.73 | 2.91 | 686.52 | 6.742 | 2.907 | 686.641 | 0.328 |
| (20,1,2) | 6.28 | 3.06 | 753.62 | 6.269 | 3.065 | 753.680 | 0.953 |
| (50,1,2) | 5.44 | 2.44 | 1635.02 | 5.455 | 2.437 | 1635.109 | 4.593 |
| (100,1,2) | 5.61 | 1.88 | 3805.51 | 5.599 | 1.884 | 3805.515 | 36.39 |
| (200,1,2) | 5.28 | 1.77 | 5092.62 | 5.274 | 1.771 | 5092.646 | 272.641 |
| (10,2,2) | 6.74 | 2.91 | 975.47 | 6.742 | 2.907 | 975.671 | 1.281 |
| (20,2,2) | 6.37 | 3.03 | 1179.02 | 6.376 | 3.029 | 1179.189 | 4.375 |
| (50,2,2) | — | — | — | 6.061 | 2.205 | 2524.908 | 24.656 |
| (100,2,2) | — | — | — | 6.076 | 1.871 | 5518.736 | 175.547 |
| (200,2,2) | — | — | — | 6.161 | 2.113 | 8422.826 | 1259 |
| (10,3,2) | 6.94 | 2.84 | 1233.48 | 6.93 | 2.845 | 1233.651 | 2.937 |
| (20,3,2) | — | — | — | 6.742 | 2.907 | 1545.176 | 9.343 |
| (50,3,2) | — | — | — | 6.186 | 2.105 | 3166.757 | 54.953 |
| (100,3,2) | — | — | — | 6.241 | 2.001 | 6915.051 | 411.375 |
| (200,3,2) | — | — | — | 6.330 | 2.238 | 11124.785 | 3250.469 |
| (10,4,2) | — | — | — | 7.266 | 2.733 | 1400.434 | 3.406 |
| (20,4,2) | — | — | — | 7.420 | 3.145 | 1849.731 | 12.531 |
| (50,4,2) | — | — | — | 6.437 | 2.110 | 3664.761 | 80.015 |
| (100,4,2) | — | — | — | 6.412 | 1.944 | 7815.201 | 617.359 |
| (200,4,2) | — | — | — | 6.405 | 2.120 | 12613.231 | 4662.437 |

Table 5.3. Results obtained for CCLP with $K = 3$ core types

| Instance (n, p, K) | Exhaustive Search | | | | TS-CCLP | | | | |
|---------------------------|-------------------|-------|-------|------------|---------|-------|-------|------------|----------|
| | R_1 | R_2 | R_3 | Net Profit | R_1 | R_2 | R_3 | Net Profit | CPU (s) |
| (10,1,3) | 6.84 | 5.12 | 2.90 | 663.40 | 6.839 | 5.102 | 2.909 | 663.42 | 0.437 |
| (20,1,3) | 5.78 | 3.65 | 2.44 | 721.72 | 5.791 | 3.647 | 2.438 | 721.753 | 1.312 |
| (50,1,3) | 5.81 | 4.02 | 2.44 | 1601.98 | 5.806 | 4.019 | 2.444 | 1601.987 | 6.718 |
| (100,1,3) | 5.40 | 3.38 | 2.14 | 3606.93 | 5.395 | 3.384 | 2.140 | 3606.930 | 52.359 |
| (200,1,3) | 5.26 | 3.45 | 1.95 | 4861.28 | 5.259 | 3.450 | 1.949 | 4861.284 | 414.016 |
| (10,2,3) | 7.05 | 5.10 | 2.84 | 954.06 | 7.035 | 5.109 | 2.840 | 954.080 | 1.546 |
| (20,2,3) | 6.78 | 4.66 | 3.15 | 1145.40 | 6.788 | 4.668 | 3.143 | 1145.436 | 6.156 |
| (50,2,3) | — | — | — | — | 5.971 | 3.922 | 2.305 | 2477.887 | 31.516 |
| (100,2,3) | — | — | — | — | 5.880 | 3.670 | 2.055 | 5322.414 | 266.219 |
| (200,2,3) | — | — | — | — | 5.834 | 3.667 | 2.478 | 8094.920 | 1801.687 |
| (10,3,3) | 7.05 | 5.00 | 2.89 | 1224.85 | 7.038 | 5.011 | 2.888 | 1224.896 | 3.593 |
| (20,3,3) | — | — | — | — | 6.917 | 4.581 | 3.143 | 1515.056 | 12.063 |
| (50,3,3) | — | — | — | — | 6.095 | 3.988 | 2.319 | 3128.524 | 72.578 |
| (100,3,3) | — | — | — | — | 6.307 | 3.960 | 2.046 | 6771.334 | 526.984 |
| (200,3,3) | — | — | — | — | 6.089 | 3.843 | 2.305 | 10765.059 | 4355.453 |
| (10,4,3) | — | — | — | — | 7.339 | 4.951 | 2.818 | 1392.650 | 4.265 |
| (20,4,3) | — | — | — | — | 7.088 | 4.659 | 3.048 | 1840.563 | 16.296 |
| (50,4,3) | — | — | — | — | 6.507 | 4.100 | 2.170 | 3614.966 | 98.860 |
| (100,4,3) | — | — | — | — | 6.387 | 3.965 | 2.017 | 7709.316 | 785.813 |
| (200,4,3) | — | — | — | — | 6.294 | 3.931 | 2.326 | 12319.075 | 5967.328 |

zones (see Table 5.4), and only one of them is the same as that obtained by TS-CCLP while the remaining 14 solutions are inferior. The CCLP is a highly nonconvex MINLP, and commercial solvers have a great deal of difficulty in handling such models. As a result, heuristic methods are indeed necessary for solving moderate and large-size instances of the CCLP.

Table 5.4. Comparison of the results obtained for CCLP by SBB solver and TS-CCLP

| No. of customer zones | 10 | 20 | 50 | 100 | 200 |
|----------------------------------|----|----|----|-----|-----|
| No. of instances | 8 | 8 | 8 | 8 | 8 |
| No. of solutions returned | 8 | 7 | 0 | 0 | 0 |
| No. of solns. as good as TS-CCLP | 3 | 1 | - | - | - |
| No. of inferior solutions | 5 | 6 | - | - | - |

In order to explore the effect of some parameters on the results we perform now a sensitivity analysis by varying some parameters. We select the instances with $K = 3$ in which the used product types are equally distributed, i.e., $\gamma_1 = \gamma_2 = \gamma_3 = 1/3$ (there are 20 of them). First, we consider vehicle operating cost c_1 , and vehicle capacity q . Figures 5.1 and 5.2 display the net profit and the percentage of the potential returns collected as a function of c_1 and q , where c_1 takes on values in the interval $[70, 110]$ with increments of 5 while q assumes integer values in the interval $[6, 14]$. We remark that the values are the averages obtained over 20 instances. As one may expect, both the profit and the percentage of collected returns are decreasing in the vehicle operating cost whereas they are increasing in the vehicle capacity.

Now we turn our attention to unit cost savings (s_1, s_2, s_3) whose values are $(s_1, s_2, s_3) = (25, 20, 15)$ in the base case scenario. At this point we want to emphasize that the maximum incentive levels a_k at which all product holders of type k would be willing to return are kept at their original values $(a_1, a_2, a_3) = (15, 10, 5)$. Note that the effect of changing all s_i in the same way, i.e., increasing or decreasing them simultaneously, can easily be predicted. Namely, if they are increased (decreased), both the net profit and the percentage of collected returns increase (decrease) as well. Therefore, we investigate the effect of unit cost savings s_i by keeping $s_2 = 20$ and

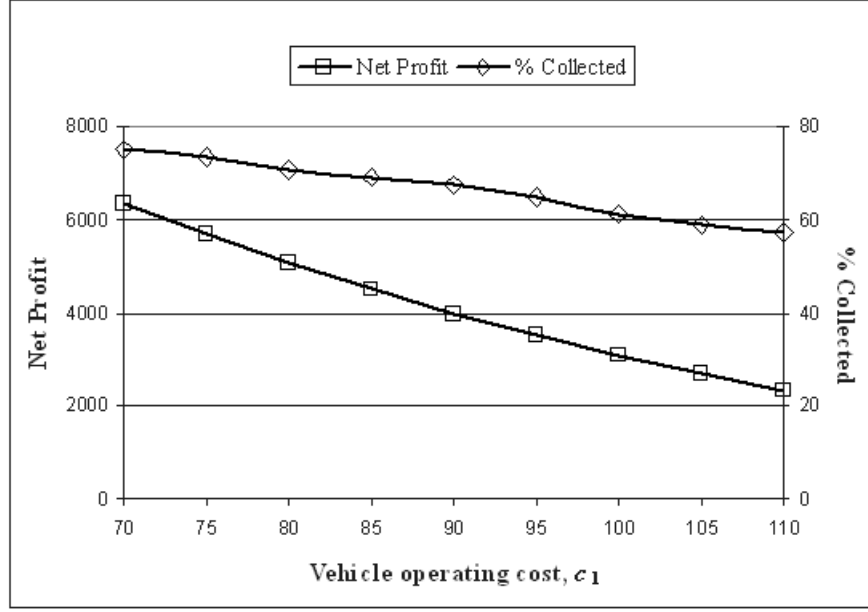


Figure 5.1. Effect of vehicle operating cost on the net profit ($K = 3$)

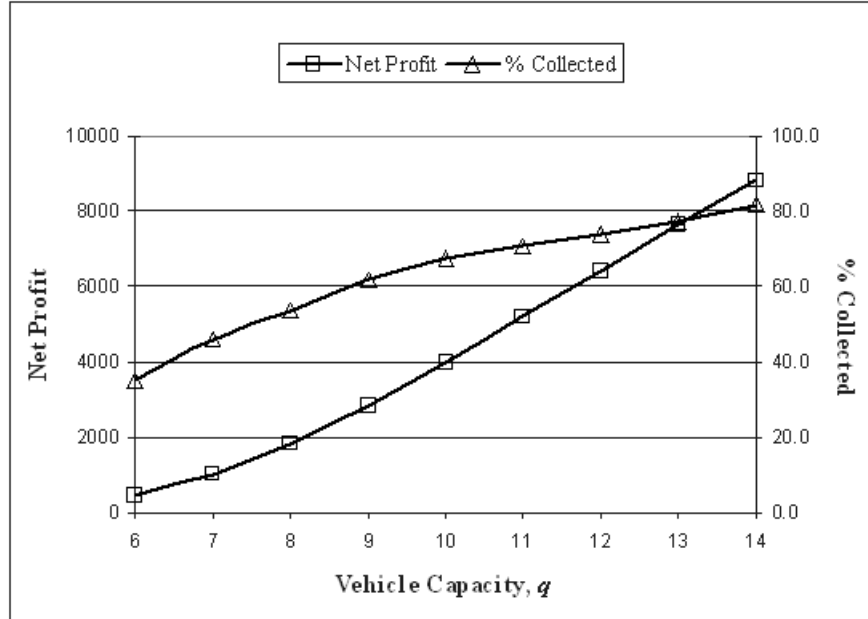


Figure 5.2. Effect of vehicle capacity on the net profit ($K = 3$)

vary the values of s_1 and s_3 concurrently either towards s_2 or away from s_2 . In other words, we experiment with different scenarios: $(s_1, s_2, s_3) = (20 + \Delta, 20, 20 - \Delta)$ where $\Delta = 1, 2, \dots, 9$. Effectively, the different scenarios correspond to different variability between used product types. While scenario $\Delta = 1$ represents the lowest variability among the types (unit cost savings are very close to each other), scenario $\Delta = 9$ represents the highest variability. The profits plotted in Figure 5.3 are averaged over 20

instances when $c_1 = 90$ and $q = 10$ (the values in the base case scenario). We also present in Table 5.5 the percentage of the collected returns for different quality types.

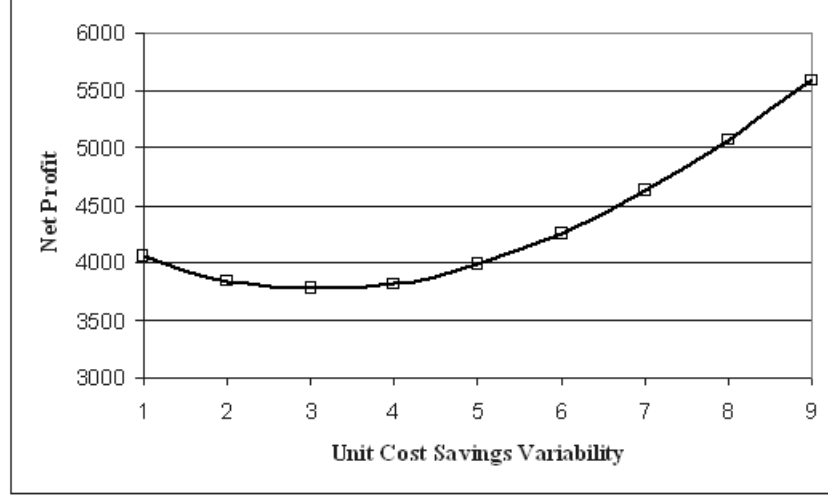


Figure 5.3. Effect of the cost savings variance on the net profit ($K = 3$)

The profit curve does not exhibit a monotonic behavior as was the case in Figures 5.1 and 5.2. When $\Delta = 1$, i.e., the variability among the quality types is lowest, the percentages of collected items become (23 per cent, 30 per cent, 47 per cent) for the three types, respectively. The number of collected returns of Type 3 is the largest. This happens because the reservation incentive of Type 3 product holders is, on average, lower than the other two reservation incentives ($a_3 < a_1$, $a_3 < a_2$) although the unit cost savings s_3 obtained from Type 3 returns is about the same as those obtained from other return types. This means it is possible for the company to offer a relatively low incentive R_3 and make a higher unit profit ($s_3 - R_3$) from Type 3 returns. As Δ increases, the difference between the unit cost savings of Type 1 and Type 3 returns becomes significant, and offering a higher incentive for Type 1 returns is justified. As a result, more Type 1 returns are collected. Besides, reduced unit cost savings s_3 of Type 3 returns cause a lower incentive R_3 leading to decreased collection amounts of that return type. The combined effect is an increase (decrease) in the percentage of collected Type 1 (Type 3) returns, which is evident in Table 5.5.

Table 5.5. Percentage of the returns collected with respect to quality types

| Δ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|----|----|----|----|----|----|----|----|----|
| Type 1 | 23 | 27 | 32 | 37 | 42 | 47 | 51 | 55 | 59 |
| Type 2 | 30 | 32 | 33 | 34 | 35 | 35 | 36 | 35 | 35 |
| Type 3 | 47 | 41 | 35 | 29 | 23 | 18 | 13 | 10 | 7 |

5.1.2. The Effect of the uniform incentive policy

Convinced with the performance of TS-CCLP, we now want to generate insights regarding the policy of the company to offer quality-dependent incentives. We examine the case in which the company offers the same incentive to product holders *regardless* of the quality of their returns. In other words, although the company recognizes the quality profile of the used products (distribution of the used products) in advance, it does not pursue incentive differentiation, but offers a uniform incentive R to all product holders. Obviously, such a policy will result in a reduced total net profit. Our aim is to investigate the magnitude of the profit reduction as a function of the quality profile. Presented in Table 5.6 is the percent loss in net profit averaged over 20 instances for various quality profiles when there are two and three used product types. Note that in the base case scenario we used quality profiles 5 and 1 for $K = 2$ and $K = 3$, respectively. To determine the optimal value of the uniform incentive (i.e., R^*), we have to use a derivative-free line search method for *one* variable only. Therefore, instead of the the simplex search we adopt the Fibonacci search method as it requires less function evaluations and therefore is more efficient than other search methods such as bisection search and golden section search (Bazaraa et al., 1993).

Table 5.6 suggests that the uniform incentive policy (UIP) results in a net profit loss for all quality profiles both when there are two and three quality types. Furthermore, the magnitude of this loss changes with respect to the quality profile. When $K = 2$, we observe that the benefit of the quality-dependent incentive policy (QDIP) decreases as the proportion γ_1 of higher quality items increases (or equivalently, as the proportion γ_2 of lower quality items decreases) except for γ_1 values below 0.3. This

observation can be explained with the help of optimal incentive values obtained under UIP and QDIP. It turns out that for all the instances we considered, the optimal uniform incentive R^* is between R_1^* and R_2^* of the QDIP regardless of the quality profiles. That is, $R_2^* < R^* < R_1^*$. When the firm offers a uniform incentive for all returns, R^* has to increase as the proportion γ_1 of higher quality items increases since this is the only way to collect this type of items whose unit profit is higher. Note that the unit profit of Type 1 returns is larger than the unit profit of Type 2 returns under the UIP, i.e., $s_1 - R^* > s_2 - R^*$ since $s_1 > s_2$. Therefore, R^* will get closer to R_1^* with increasing γ_1 . Consequently, the collected amounts of Type 1 returns will converge to each other under both policies. Although the volume of lower quality (Type 2) returns will be larger under the UIP, its effect is limited because of the small proportion of such items and their lower unit profits. In a similar fashion, R^* will be closer to R_2^* for relatively small values of γ_1 . This implies that most of the higher quality (Type 1) used products which were collected under the QDIP will not be returned by their holders when UIP is in effect. This gives rise to a relatively large discrepancy between the net profits of the two policies. However, when γ_1 is close to zero (e.g., $\gamma_1 = 0.1$), even under the QDIP the amount of collected Type 1 items will not be so high, and therefore the contribution to the net profit by those high quality items will be relatively limited. This is the reason why the value of percent net profit loss is smaller at $\gamma_1 = 0.1$ than at $\gamma_1 = 0.2$.

Table 5.6. Effect of the UIP with respect to quality profiles

| $(s_1=25, s_2=15)$ | | Quality Profile, $K = 2$ | | | | | | | | |
|----------------------------|--|--------------------------|-------|-------|-------|-------|-------|------|------|------|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| γ_1 | | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| γ_2 | | 0.9 | 0.8 | 0.7 | 0.6 | 0.5 | 0.4 | 0.3 | 0.2 | 0.1 |
| % Loss in Net Profit | | 15.94 | 25.52 | 26.58 | 21.23 | 17.73 | 13.37 | 9.70 | 5.73 | 2.41 |
| $(s_1=25, s_2=20, s_3=15)$ | | Quality Profile, $K = 3$ | | | | | | | | |
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | | |
| γ_1 | | 1/3 | 0.1 | 0.45 | 0.45 | 0.1 | 0.1 | 0.8 | | |
| γ_2 | | 1/3 | 0.45 | 0.1 | 0.45 | 0.1 | 0.8 | 0.1 | | |
| γ_3 | | 1/3 | 0.45 | 0.45 | 0.1 | 0.8 | 0.1 | 0.1 | | |
| % Loss in Net Profit | | 11.27 | 10.73 | 14.28 | 5.93 | 17.31 | 3.04 | 2.94 | | |

Also when $K = 3$, the same reasoning remains valid for the two extreme quality profiles (profiles 5 and 7). Similar to the case with profile 9 when $K = 2$, the benefit of incentive differentiation does not pay off for profile 7 where the proportion of Type 1 products dominates the others. On the other hand, the QDIP significantly outperforms the UIP for profile 5 in which the proportion of Type 3 products dominates the other types. For that profile, the percent loss in the net profit reaches nearly 17 per cent. The results for other profiles lie between these two extremes. With the limited number of sets of parameter values tested in this study, we can state that when the proportion of lowest quality items is significant with respect to the others, which is the case for profiles 1, 2, and 3, the percent loss ranges between 10-14 per cent. On the contrary, when the proportion of lowest quality items is considerably low (as in profiles 4, 6, and 7), the benefit of incentive differentiation drops below 6 per cent.

Obviously, the exact impact of offering a uniform incentive or equivalently the benefit of quality differentiation in offering incentives will depend on the values of the parameters such as s_k 's and a_k 's. For example, it is not surprising that as the difference between the savings of highest and lowest quality items ($s_1 - s_K$) increases, the percent loss in the net profit due to the UIP will also increase.

5.2. Results for Government-Subsidized Bilevel Models

As mentioned before, our nested heuristic consists of Brent's method to solve the outer problem GP1 (GP2), and TS heuristic TS-1 (TS-2) to solve the inner problem CP1 (CP2). Since the quality of the solutions generated by the nested heuristic would depend largely on the performance of TS-1 and TS-2, we first carry out experiments to assess the accuracy of the solutions found by these TS heuristics. While solving the problem instances some parameters are set to the following values: $c_1 = 75$, $c_2 = 1$, $q = 10$ and $f_i = 1000$.

Table 5.7. Comparison of the results obtained for CP1 by SBB solver and TS-1

| | | | |
|----------------------------------|----|----|----|
| No. of customer zones | 10 | 20 | 50 |
| No. of instances | 12 | 12 | 12 |
| No. of solutions returned by SBB | 12 | 12 | 11 |
| No. of solns. as good as TS-1 | 3 | 1 | - |
| No. of inferior solutions | 9 | 11 | 11 |

5.2.1. The Performance of TS-1 and TS-2

In order to test the performance of TS-1 and TS-2 we employ SBB which is one of the available solvers within the optimization suite GAMS v22.0 used for solving MINLP models. The reason why we choose this solver is based on the results presented in Section 5.1.1 where it is observed that SBB gives significantly better results than DICOPT and OQNLP.

We take the previously generated test instances with $n = \{10, 20, 50\}$ and $K = \{2, 3\}$ and we solve inner problem CP1 for six different values of $G = \{0, 2, 4, 6, 8, 10\}$ using both the SBB solver and TS-1. Using the same setting we also solve inner problem CP2 with values of the minimum collection rate τ from the set $\{0.2, 0.4, 0.6\}$ by the SBB solver and TS-2. This makes a total of 36 instances for CP1 and 108 instances for CP2. Recall that minimum collection rate requirement is not part of CP1, therefore parameter τ is not a parameter in CP1. To have a fair comparison we set a time limit for the SBB solver equal to two hours, which is more than the time required by our TS heuristics to solve the largest instance. TS-1 and TS-2 have been coded in C using Microsoft Visual C++ 6.0. All the experiments have been conducted on a desktop computer with 3.20 GHz Intel Pentium 4 HT processor and 3 GB RAM. The results are displayed in Table 5.7 and Table 5.8. SBB can solve 133 out of 144 instances in total. We observe that none of the results provided by SBB is better than those found by the TS heuristics. The quality of the solutions is the same for 55 instances while TS-1 and TS-2 outperform the SBB solver for the remaining 78 instances. The latter case is more prominent for large-size instances. Since both CP1 and CP2 are highly

nonconvex MINLPs, SBB has a great deal of difficulty in handling such models. Now by incorporating Brent's method in our nested heuristic, we turn our attention to the solution of the comprehensive problems GSCSDP1 and GSCSDP2.

Table 5.8. Comparison of the results obtained for CP2 by SBB solver and TS-2

| | | | |
|----------------------------------|----|----|----|
| No. of customer zones | 10 | 20 | 50 |
| No. of instances | 36 | 36 | 36 |
| No. of solutions returned by SBB | 36 | 35 | 27 |
| No. of solns. as good as TS-2 | 36 | 15 | - |
| No. of inferior solutions | - | 20 | 27 |

5.2.2. Results Obtained by the Nested Heuristic for GSCSDP1 and GSCSDP2

By choosing parameter values from the sets $n = \{10, 20, 50, 100\}$, $\tau = \{0.2, 0.4, 0.6\}$ and $K = 2, 3$, we generate $4 \times 3 \times 2 = 24$ test instances for GSCSDP1, and solve them using our nested heuristic. The results are shown in Table 5.9 and Table 5.10. When the number of customer zones is fixed, we observe that unit subsidy G , net profit Π_{net} , and realized profitability ratio P increase as the minimum collection rate τ becomes more stringent. This is an expected result, because the increase in the amount of collected cores, hence in the profit, is only affected by higher values of G decided by the government. Recall that the company in the supportive model GSCSDP1 never collects additional cores to help reach the minimum collection rate requirement if it is not profitable to do so.

Our next step is to solve test instances for the legislative model GSCSDP2, which take an additional parameter as input, namely the minimum profitability ratio ρ . Instead of assigning arbitrary values to ρ , we adopt the following approach that helps us also in the comparison of the results of the supportive model GSCSDP1 with those of GSCSDP2. We take the same 24 problem instances listed in Table 5.9 and Table 5.10, and solve them as GSCSDP2 instances with the minimum profitability ratio parameter ρ of each instance set to the value of realized profitability ratio P of the corresponding

GSCSDP1 instance. The results are provided in Table 5.11 and Table 5.12.

Table 5.9. Results obtained for GSCSDP1 with $K = 2$ core types

| Instance (n, τ) | No. CCs | | | | Net Profit, | Realized Coll. | Realized Prof. | CPU (s) |
|---------------------------|---------|-------|-------|------|-------------|----------------|----------------|------------|
| | opened | R_1 | R_2 | G | Π_{net} | Rate, $T(\%)$ | ratio, $P(\%)$ | |
| (10,0.2) | 1 | 12.09 | 5.00 | 1.97 | 0.08 | 38.1 | 0.0 | 98 |
| (10,0.4) | 1 | 11.95 | 5.00 | 2.79 | 187.82 | 41.3 | 3.5 | 101 |
| (10,0.6) | 1 | 11.80 | 5.00 | 6.14 | 1196.27 | 64.4 | 13.7 | 142 |
| (20,0.2) | 1 | 10.06 | 4.99 | 1.77 | 0.05 | 26.4 | 0.0 | 388 |
| (20,0.4) | 2 | 12.09 | 5.00 | 4.11 | 755.71 | 57.0 | 6.0 | 487 |
| (20,0.6) | 2 | 12.09 | 5.00 | 4.44 | 944.81 | 61.1 | 7.1 | 579 |
| (50,0.2) | 2 | 10.04 | 4.03 | 1.13 | 688.21 | 31.3 | 4.2 | 3113 |
| (50,0.4) | 2 | 10.81 | 4.92 | 1.70 | 1166.49 | 40.0 | 5.6 | 3122 |
| (50,0.6) | 4 | 12.04 | 5.00 | 3.63 | 3761.43 | 70.4 | 9.9 | 4795 |
| (100,0.2) | 3 | 9.58 | 3.91 | 0.00 | 2389.53 | 31.2 | 7.8 | 687 |
| (100,0.4) | 4 | 10.40 | 4.39 | 0.44 | 3214.12 | 42.1 | 7.9 | 24,919 |
| (100,0.6) | 5 | 11.15 | 4.97 | 1.46 | 6012.73 | 61.7 | 10.0 | 26,661 |

On the basis of these results, we can make an observation similar to the one made earlier for the GSCSDP1 instances. That is, for a given number of zones unit subsidy G and net profit Π_{net} increase as the minimum collection rate τ increases. More importantly, we can conclude that G is lower, and the company makes less profit in GSCSDP2 compared to GSCSDP1. In other words, by shifting from a supportive to a legislative role the government can reduce the subsidization budget allocated to collection operations. Actually, when the government does not guarantee a minimum profitability ratio ρ for the company in GSCSDP2, the company might even end up with a negative net profit as a result of the minimum collection rate requirement. This is never the case in the supportive model since the company would simply refrain from collecting any cores if the collection operations turn out to be unprofitable.

When we look at the CC location plans in GSCSDP2 versus in GSCSDP1, we see that the number of CCs opened in GSCSDP2 is either equal to or less than that in GSCSDP1. This observation can be explained as follows. Notice that for those instances in which GSCSDP2 ends up opening fewer CCs than GSCSDP1, there is a

Table 5.10. Results obtained for GSCSDP1 with $K = 3$ core types

| Instance (n, τ) | No. CCs opened | R_1 | R_2 | R_3 | G | Net Profit, Π_{net} | Realized Coll. Rate, $T(\%)$ | Realized Prof. ratio, $P(\%)$ | CPU (s) |
|---------------------------|-------------------|-------|-------|-------|------|----------------------------|---------------------------------|----------------------------------|------------|
| (10,0.2) | 1 | 12.30 | 8.97 | 5.00 | 2.05 | 0.10 | 38.1 | 0.0 | 214 |
| (10,0.4) | 1 | 11.88 | 9.22 | 5.00 | 2.92 | 200.52 | 41.5 | 3.6 | 218 |
| (10,0.6) | 1 | 11.74 | 8.92 | 5.00 | 6.14 | 1157.87 | 62.7 | 13.5 | 311 |
| (20,0.2) | 1 | 10.49 | 8.24 | 5.00 | 1.88 | 0.11 | 27.4 | 0.0 | 794 |
| (20,0.4) | 2 | 12.14 | 9.06 | 5.00 | 4.15 | 719.03 | 58.0 | 5.6 | 1086 |
| (20,0.6) | 2 | 12.20 | 9.03 | 5.00 | 4.44 | 883.76 | 61.1 | 6.5 | 1280 |
| (50,0.2) | 2 | 10.03 | 6.89 | 4.25 | 1.13 | 683.71 | 30.5 | 4.3 | 6516 |
| (50,0.4) | 3 | 11.19 | 8.17 | 5.00 | 2.25 | 1655.18 | 53.9 | 5.7 | 9262 |
| (50,0.6) | 3 | 11.31 | 8.17 | 5.00 | 3.87 | 4048.90 | 60.3 | 12.4 | 12,135 |
| (100,0.2) | 4 | 9.98 | 6.80 | 4.06 | 0.00 | 2352.37 | 35.4 | 6.8 | 1459 |
| (100,0.4) | 5 | 10.56 | 7.31 | 4.60 | 0.51 | 3294.49 | 48.8 | 6.8 | 50,942 |
| (100,0.6) | 5 | 11.40 | 8.22 | 5.00 | 1.51 | 5966.87 | 60.3 | 9.9 | 59,031 |

Table 5.11. Results obtained for GSCSDP2 with $K = 2$ core types

| Instance ($n, \tau, P\%$) | No. CCs | | | Net Profit, | | Realized Coll. | | Realized Prof. | | CPU (s) |
|--------------------------------|---------|-------|-------|-------------|-------------|----------------|----------------|----------------|--|------------|
| | opened | R_1 | R_2 | G | Π_{net} | Rate, $T(\%)$ | ratio, $P(\%)$ | | | |
| (10,0,2,0.0) | 1 | 12.09 | 5.00 | 1.97 | 0.08 | 38.1 | 0.0 | | | 170 |
| (10,0,4,3.5) | 1 | 11.95 | 5.00 | 2.78 | 185.68 | 41.3 | 3.5 | | | 229 |
| (10,0,6,13.7) | 1 | 11.80 | 5.00 | 5.97 | 1136.56 | 61.1 | 13.7 | | | 310 |
| (20,0,2,0.0) | 1 | 10.06 | 4.99 | 1.77 | 0.05 | 26.4 | 0.0 | | | 713 |
| (20,0,4,6.0) | 1 | 11.53 | 5.00 | 3.53 | 511.73 | 40.4 | 6.0 | | | 1161 |
| (20,0,6,7.1) | 2 | 12.09 | 5.00 | 4.41 | 924.70 | 60.0 | 7.1 | | | 1708 |
| (50,0,2,4.2) | 1 | 9.90 | 4.05 | 0.63 | 428.04 | 20.0 | 4.2 | | | 5798 |
| (50,0,4,5.6) | 2 | 10.81 | 4.93 | 1.69 | 1155.19 | 40.0 | 5.6 | | | 14,030 |
| (50,0,6,9.9) | 3 | 11.25 | 5.00 | 3.23 | 3130.50 | 60.1 | 9.9 | | | 19,825 |
| (100,0,2,7.8) | 3 | 9.58 | 3.91 | 0.00 | 2389.53 | 31.1 | 7.8 | | | 5196 |
| (100,0,4,7.9) | 4 | 10.40 | 4.39 | 0.44 | 3207.57 | 42.0 | 7.9 | | | 100,756 |
| (100,0,6,10.0) | 5 | 11.07 | 4.90 | 1.41 | 5834.76 | 60.1 | 10.0 | | | 103,906 |

Table 5.12. Results obtained for GSCSDP2 with $K = 3$ core types

| Instance | No. CCs | | | | | Net Profit, | | Realized Coll. | | Realized Prof. | | CPU |
|------------------|---------|-------|-------|-------|------|-------------|---------------|----------------|--|----------------|---------|-----|
| $(n, \tau, P\%)$ | opened | R_1 | R_2 | R_3 | G | Π_{net} | Rate, $T(\%)$ | ratio, $P(\%)$ | | | (s) | |
| (10,0.2,0.0) | 1 | 12.30 | 8.97 | 5.00 | 2.05 | 0.10 | 38.1 | 0.0 | | | 382 | |
| (10,0.4,3.6) | 1 | 11.88 | 9.22 | 5.00 | 2.90 | 195.68 | 41.5 | 3.6 | | | 497 | |
| (10,0.6,13.5) | 1 | 11.73 | 8.91 | 5.00 | 6.05 | 1125.43 | 61.0 | 13.5 | | | 695 | |
| (20,0.2,0.0) | 1 | 10.49 | 8.24 | 5.00 | 1.88 | 0.11 | 27.4 | 0.0 | | | 1592 | |
| (20,0.4,5.6) | 1 | 11.88 | 8.73 | 5.00 | 3.60 | 490.80 | 40.4 | 5.6 | | | 2487 | |
| (20,0.6,6.5) | 2 | 12.20 | 9.03 | 5.00 | 4.41 | 863.06 | 60.0 | 6.5 | | | 3970 | |
| (50,0.2,4.3) | 1 | 10.03 | 6.86 | 4.24 | 0.67 | 438.93 | 20.0 | 4.3 | | | 12,156 | |
| (50,0.4,5.7) | 2 | 10.49 | 7.45 | 4.67 | 1.82 | 1194.00 | 40.0 | 5.7 | | | 28,068 | |
| (50,0.6,12.4) | 3 | 11.26 | 8.20 | 5.00 | 3.83 | 3987.31 | 59.9 | 12.4 | | | 36,849 | |
| (100,0.2,6.8) | 4 | 9.98 | 6.80 | 4.06 | 0.00 | 2352.37 | 35.4 | 6.8 | | | 10,959 | |
| (100,0.4,6.8) | 4 | 10.02 | 6.74 | 4.00 | 0.29 | 2687.27 | 40.0 | 6.8 | | | 155,728 | |
| (100,0.6,9.9) | 5 | 11.40 | 8.22 | 5.00 | 1.50 | 5928.66 | 60.2 | 9.9 | | | 235,468 | |

wider gap between the unit subsidy values and realized collection rates. This implies that in order to realize the same minimum collection rate τ , unit subsidy $S_{GSCSDP1}$ has to be relatively higher than the value of $S_{GSCSDP2}$. However, since $\rho_{GSCSDP2}$ is set equal to $\rho_{GSCSDP1}$ for the same instance, despite lower subsidization the company still needs to be as much profitable in GSCSDP2 as it was in GSCSDP1. This can be only accomplished by collecting the same amount of cores with fewer CCs, thereby saving the heavy cost of opening a CC which has clearly a negative effect on profitability. Therefore, under the same minimum collection rate τ , the legislative model would open either an equal or lesser number of CCs in order to match the profitability of the equivalent supportive model. The proposed nested heuristic spends significantly more computational effort in solving GSCSDP2 instances as indicated by the CPU times in last columns of Tables 5.9–5.12. This is caused by restarting Brent’s method several times due to the nonmonotonicity of the function $f_2(G)$ as explained in Subsection 4.2.2.

Lastly, observe that CP1 and CP2 becomes MILP problems for given values of R_k ’s. MILPs can be solved to optimality with CPLEX solver. In order to test the performance of TS-1 and TS-2 again, we solved these problem instances with CPLEX by giving the realized R_k and G values we have found in Tables 5.9–5.12 as a parameter. 43 out of 48 problems gave the same results which are found by TS-1 and TS-2. However, with 5 instances CPLEX obtained better total profits. This indicates that these TS based procedures perform well but may not give optimal solutions since we are using heuristics.

6. CONCLUSIONS

In the first part of this thesis, we deal with CCLP of a company. We focus on a pick-up scenario in which a homogeneous fleet of vehicles with limited capacity is sent from CCs to customer zones in order to collect and bring the returns. The number of vehicles operated and the amount of each return type collected are also decision variables that are contingent on both the locations of collection centers and the incentives. We assume that the financial incentive offered by the company determines the willingness of product holders to return their used products. The amount of this incentive has two conflicting outcomes. Offering a lower incentive enhances on the one hand the company's unit profit from the returns; on the other hand, doing so may critically diminish product holders' willingness to return. For this problem we formulate an original MINLP model and develop a Tabu Search based heuristic called TS-CCLP which incorporates a simplex search procedure as a subroutine.

We generate a test bed consisting of 40 instances, and compare the accuracy of our heuristic with an exhaustive search on small instances, and with the SBB solver of the GAMS suite v22.0 on all instances. The experimental results reveal that for all the instances our heuristic exhibits a favorable performance both in terms of solution quality and running time. After being convinced with its performance, we use TS-CCLP to explore the effects of vehicle operating cost and capacity on the profitability and on the percentage of the collected items. We observe that while a larger vehicle capacity boosts both the profitability and the percentage of collected items, a higher unit vehicle operating cost has just the adverse effect. Another empirical finding of experimentation suggests that as the gap between the unit savings from returns of highest and lowest quality types increases with their respective reservation incentives and the average unit savings remaining unchanged, the collection percentage of higher quality returns as well as the total net profit show an upward trend.

We employ TS-CCLP also to scrutinize the effect of the policy to offer a uniform incentive for all quality types (UIP) opposed to the quality-dependent incentive policy

(QDIP). We experiment with different quality profiles when there are two and three quality types. We find that in terms of the total net profit the UIP is always inferior to the QDIP. Moreover, we explain why the UIP causes an even higher profit loss when the proportion of lowest quality products is relatively large. This insight is supported by the results of our scenario analysis under the UIP versus the QDIP.

Secondly, we develop two types of BP models. The first model (GSCSDP1) is a supportive model from the perspective of the government in the sense that it does not impose a minimum collection rate restriction on the company, but allows the company to be free in its profit maximization objective. The second model (GSCSDP2) is a legislative model in which the government issues a regulation requiring the company to reach or exceed a minimum core collection rate. This, of course, reduces the net profit of the company, and in order to not discourage the company from collecting cores, which is the triggering activity in reverse logistics and closed-loop supply chain management, the government guarantees that the profitability ratio of the company would not drop below a certain level. Both models have been formulated as bilevel programs, which are intrinsically difficult to solve.

For the solution of GSCSDP1 and GSCSDP2, we develop a nested heuristic solution methodology. The government's problem (outer problem) is solved with Brent's method, which is an effective root finding method for one-dimensional functions. A tabu search heuristic is used to solve the company's problem (inner problem), which performs tabu search in the solution space of CC location combinations and calls simplex search to find the best incentive values and the best net profit for a given set of CC locations.

We evaluate the performance of the proposed tabu search heuristics TS-1 and TS-2 in solving the inner problems CP1 and CP2 of GSCSDP1 and GSCSDP2, respectively, by comparing with the commercial solver SBB. This comparison proves the accuracy of TS-1 and TS-2. We carry out experiments on a set of randomly generated 24 test instances. The results demonstrate that the subsidy paid to the company by the government depends on the policy of the government regarding its support to reverse

logistics operations. In a supportive role, the amount of the subsidy will be higher, and the company makes a higher profit compared to the case of a legislative role of the government.

As an extension for future research, one may look into the location-routing version of this comprehensive problem in which vehicles visit more than one product holder address on a tour before returning to their base CCs. Vehicle tours will likely be constrained by capacity and maximum tour duration restrictions. From a methodological point of view, it is obvious that simultaneous determination of CC sites and associated vehicle routes complicates the solution of the government-subsidized collection system design problem by a great deal, and demands specifically tailored techniques. Also, value of information for knowing the number and condition of cores at each customer zone may be investigated. In addition, to be more realistic fixed cost of opening a CC may be zone dependent and the capacity related operating expenses of a CC may be considered as a cost component. Lastly, the demand for remanufactured items in the market can be included in the model since it will directly affect the profitability of the company.

APPENDIX A: Simplex search algorithm

1. Construction of the initial simplex: Choose points $\mathbf{R}^1, \mathbf{R}^2, \dots, \mathbf{R}^{K+1}$ to form a simplex in K . Choose a reflection coefficient $\alpha > 0$, an expansion coefficient $\lambda > 1$, a contraction coefficient $0 < \beta < 1$, and a shrinkage coefficient $\chi > 0$.
2. Initialization: Let $\mathbf{R}^{\max} = \arg \max_{1 \leq h \leq K+1} \Pi(\mathbf{R}^h)$, $\mathbf{R}^{\min} = \arg \min_{1 \leq h \leq K+1} \Pi(\mathbf{R}^h)$,

$$\bar{\mathbf{R}} = \frac{1}{K} \sum_{h=1| \mathbf{R}^h \neq \mathbf{R}^{\min}}^{K+1} \mathbf{R}^h.$$
3. Reflection: Let $\mathbf{R}^r = \bar{\mathbf{R}} + \alpha (\bar{\mathbf{R}} - \mathbf{R}^{\min})$.

 If $\Pi(\mathbf{R}^r) \geq \Pi(\mathbf{R}^{\max})$, go to Step 4.

 If $\Pi(\mathbf{R}^r) < \Pi(\mathbf{R}^{\max})$, but $\Pi(\mathbf{R}^r) \geq \min_{h| \mathbf{R}^h \neq \mathbf{R}^{\min}} \Pi(\mathbf{R}^h)$, then $\mathbf{R}^{\min} := \mathbf{R}^r$ to form a new set of $K+1$ points and go to Step 6.
4. Expansion: Let $\mathbf{R}^e = \bar{\mathbf{R}} + \lambda (\mathbf{R}^r - \bar{\mathbf{R}})$.

 $\mathbf{R}^{\min} := \mathbf{R}^r$ if $\Pi(\mathbf{R}^r) < \Pi(\mathbf{R}^e)$ and $\mathbf{R}^{\min} := \mathbf{R}^e$ if $\Pi(\mathbf{R}^r) \geq \Pi(\mathbf{R}^e)$ to yield a new set of $K+1$ points and go to Step 6.
5. Contraction: Let $\mathbf{R}^c = \bar{\mathbf{R}} + \beta (\hat{\mathbf{R}}^{\min} - \bar{\mathbf{R}})$ where $\hat{\mathbf{R}}^{\min} = \arg \max \{ \Pi(\mathbf{R}^{\min}), \Pi(\mathbf{R}^r) \}$.

 If $\Pi(\mathbf{R}^c) \geq \Pi(\hat{\mathbf{R}}^{\min})$, $\mathbf{R}^{\min} := \mathbf{R}^c$.

 If $\Pi(\mathbf{R}^c) < \Pi(\hat{\mathbf{R}}^{\min})$, $\mathbf{R}^h := \mathbf{R}^h + \chi (\mathbf{R}^{\max} - \mathbf{R}^h)$ for $h = 1, \dots, K+1$.
6. Termination: If $\left\{ \frac{1}{K+1} \sum_{h=1}^{K+1} [\Pi(\mathbf{R}^h) - \Pi(\bar{\mathbf{R}})]^2 \right\}^{1/2} < \varepsilon$, then stop and set $\mathbf{R}^{best} := \mathbf{R}^{\max}$, else go to Step 2.

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