ANALYSIS OF DISASSEMBLY SYSTEMS IN REMANUFACTURING USING KANBAN CONTROL

by

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Dedicated to
my parents

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ABSTRACT

ANALYSIS OF DISASSEMBLY SYSTEMS IN REMANUFACTURING USING KANBAN CONTROL

Disassembly is the most critical stage of remanufacturing activities. The condition of parts disassembled for reuse/remanufacturing display a high variance. Hence demand for different parts found in a core cannot always be satisfied by a single core. At this point the question is whether to partially or fully disassemble the second core. In this study, we concentrate on quantifying the potential benefits of this decision in a remanufacturing environment by using a queuing network model with kanban control. In our model, machine service times, demand and return arrival times to the remanufacturing facility are assumed to be independent and exponentially distributed random variables. The model is solved analytically by an approximate method to obtain the steady state performance measures of different types of disassembly systems. The accuracy of the approximation is validated by simulation experiments. Then steady state performance measures are aggregated to an expected total cost function to make a comparison based on the minimum total costs of the disassembly policies. To obtain the minimum of the total cost function, heuristic search procedures are proposed. Finally, some important results that can give managerial insights for the planners of disassembly systems are derived from experimentations and comments on the profitability of allowing partial disassembly are remarked.

ÖZET

YENİDEN İMALAT DEMONTAJ SİSTEMLERİNİN KANBAN KONTROLÜ KULLANILARAK İNCELENMESİ

Demontaj işlemi, yeniden imalat işlemlerinin en kritik aşamasını oluşturmaktadır. Yeniden kullanım ya da yeniden imalat için demonte edilen parçaların kalite düzeyleri yüksek değişkenlik göstermekte ve bu yüzden de parça taleplerinin tümü tek bir ürünün demontajı ile her zaman karşılanamayabilmektedir. Bu durumda ikinci ürünün demontajının kısmi mi yoksa bütüncül mü olacağı sorusu ortaya çıkmaktadır. Bu çalışmada söz konusu kararın yaratacağı olası yararların sayısallaştırılmasına odaklanılmış, bu amaçla kanban kontrolü ile beraber bir kuyruk ağı modelinden yararlanılmıştır. Modelde, makine servis sürelerinin, talep ve iade ürünlerin gelişler arası sürelerinin bağımsız ve üstel rassal değişkenler olduğu varsayılmaktadır. Farklı türlerdeki demontaj sistemlerinin uzun dönem performans göstergeleri, modelin analitik olarak çözülmesi ile elde edilmiş, çözüm yöntemi olarak da yaklaşık bir metot kullanılmıştır. Yaklaşık yöntemin doğruluğu benzetim deneyleri ile sınanmış ve geçerliliği gösterilmiştir. Ardından, uzun dönem performans göstergeleri bir beklenen toplam maliyet fonksiyonunda bir araya getirilerek, demontaj politikalarının karşılaştırılmasında bu fonksiyonun aldığı en küçük değerler kullanılmıştır. Toplam maliyet fonksiyonun en küçük değerlerinin bulunması için sezgisel arama yöntemleri öne sürülmüştür. Son olarak, sayısal deneyler aracılığıyla, demontaj sistemlerinin planlayıcıları için yönetimsel öngörü sağlayabilecek bazı önemli sonuçlar ortaya konmuş ve kısmi demontaja izin vermenin karlılığı üzerine yorumlar getirilmiştir.

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LIST OF SYMBOLS /ABBREVATIONS

B	Buffer capacity of returns
b_i	Backorder cost of a demand of part i
B_{ci}	Buffer capacity of disassembled component i
BL_i	Backorder limit of demands of part i
c_d	Operation cost of disassembly of a core
c_p	Acquisition (or transportation) cost of a core
c_{mi}	Manufacturing (or procurement) cost of a part i
c_{ri}	Operation cost of refurbishing a part i
c_M	Equal manufacturing costs of parts
c_R	Equal refurbishing costs of parts
$\mathrm{D_A}$	Queue of kanban A at J_1
$\mathrm{D_B}$	Queue of kanban B at J_1
d_i	Disposal cost of a part i
d_p	Disposal cost of a core
f_i	Allocation fraction of added value for part type i
$\mathrm{F_1}$	Queue of kanban 1 at J_1
G_i	Normalization constant associated with network i
h_p	Out of pocket inventory holding cost for the products
h_{di}	Out of pocket inventory holding cost for the dismantled parts of type i
h_{ri}	Out of pocket inventory holding cost for the refurbished parts of type i
H_p	Unit holding cost for the product per unit time
H_{di}	Unit holding cost for the dismantled component of type i per unit time
H_{ri}	Unit holding cost for the refurbished component of type i per unit time
J_0	Synchronization station between return flow and first stage
J_1	Synchronization station between first stage and downstream stages
J_A	Synchronization station between demand flow and stage A
J_B	Synchronization station between demand flow and stage B
K_{i}	Kanban size of stage i
n	Number of components core composed of
n_a	Current number of permits

n_1	Current number of stage 1 kanbans at J_1
n_A	Current number of stage A kanbans at J_1
n_B	Current number of stage B kanbans at J_1
n_{CA}	Current number of component A at J_1
n_{CB}	Current number of component B at J_1
n_{ci}	Current number of component i at J_1
n_{ij}	State variable that shows the distribution of kanban types through the stations of the single class network
N_a	Maximum number of permits
$\mathrm{P_A}$	Queue of disassembled component A at J_1
$\mathrm{P_B}$	Queue of disassembled component B at J_1
$P_{ij}(n_{ij})$	Marginal probability for the queue length of load dependent server j in single class network i
$\widetilde{P}_{ij}(n_{ij})$	Marginal probability for the number of class i customers, which are present in the isolated station
QB_A	Average queue length of backordered demands of part A
QB_B	Average queue length of backordered demands of part B
QC_A	Average queue length of disassembled part A
QC_B	Average queue length of disassembled part B
QFP_A	Average queue length of finished part A
QFP_B	Average queue length of finished part B
QR	Average queue length of returns
$QWIP_i$	Average work in process inventory of stage i
S_n	Index set of downstream stages when core is composed of n components
$T(n)$	Index set of customer classes viz. kanban types when the product is composed of n types of components
TC_{NA}	Total cost except holding and backorder costs for policy of not allowing partial disassembly
TC_A	Total cost except holding and backorder costs for policy of allowing partial disassembly
TH_1	Expected throughput of stage 1
TH_A	Expected throughput of stage A
TH_B	Expected throughput of stage B

$X(i)$	Index set of visited stations by customer class i
V_{ij}	Average visit ratio of station j in network i
Z	Total cost
Z_A	Total cost when partial disassembly is allowed
Z_{NA}	Total cost when partial disassembly is not allowed
Z_{opt}	Optimal (minimum) total cost
α	Inventory carrying charge
γ	Return arrival rate
λ_A	Demand arrival rate for part A
λ_B	Demand arrival rate for part B
λ_C	Demand arrival rate for part C
$\lambda_{ci}(n_{ci})$	Load dependent arrival rate of component i to J_1
$\lambda_a(n_a)$	Load dependent arrival rate of permits to J_1
$\lambda_a^k(n_a)$	Load dependent arrival rate of permits to J_1 when $n_1 = k$
$\lambda_{ij}(n_{ij})$	Load dependent arrival rate of class i customers to station j
$\tilde{v}_{ij}(n_{ij})$	Conditional throughput of station j in stage i in isolation
$\mu_{ij}(n_{ij})$	Load dependent service rate of station j in network i
CS	Cost saving of allowing partial disassembly
PDA-OD	Policy of allowing partial disassembly with overflow disposal
PDA-OP	Policy of allowing partial disassembly with holding overflow and overflow item priority
PDA-RP	Policy of allowing partial disassembly with holding overflow and regular item priority
PDNA	Policy of not allowing partial disassembly

1. INTRODUCTION

Changes in the environment create new challenges for the companies. Since the last decade, an important trend for the production companies is "remanufacturing". Remanufacturing can be defined as bringing a used product into "as good as new" condition through disassembly, refurbishing, rework and upgrading. Companies intending to perform these activities are faced with many problems. The changing structures of the supply chain and production activities force companies to change their traditional rules and policies. However, this is a big challenge, because there is a lack of theoretical research literature to establish and characterize the new rules in practical areas. So, researchers in production and operation management fields conduct new studies in this area to consider new problems that arise from remanufacturing activities.

The main economical motivation behind the remanufacturing decision is its higher profitability. A company decides to remanufacture used products not to face with high penalties as a consequence of recent environmental legislations. Especially in European Union countries, these penalties make the remanufacturing business an economically inevitable activity for the profitability of companies. The environmental legislations are the results of past economical activities, which are performed without considering the harmful environmental effects of manufacturing. Remanufacturing is an economical reaction to these negative environmental effects. As opposed to the traditional manufacturing process of raw materials, remanufacturing of used products consume less energy and release less amount of harmful emission. As governments introduce new regulations that force production companies to engage in collection operations of used products from the market, these companies are faced with two options: recycling and remanufacturing.

Existence of environmental legislations is an important factor on the marginal profitability calculations of remanufacturing operations and the cost structure of the companies. For instance, without these legislations, there will be no return flows to the company, so there will be no cost like the disposal cost associated with this flow. However, in countries with such legislations, there are return flows, even if the company is not engaged in remanufacturing. Here, the return flow generates an additional cost driver that will be taken into account when analyzing the models constructed to search for optimal production control rules and policies.

Another dimension of the remanufacturing decision is the cost and profit of the internal remanufacturing activities. In fact, even though there are environmental legislations that force the company to collect used products, the company does not intend to enter into the remanufacturing business if remanufacturing is less profitable than recycling. Actually, assessing the profitability depends on optimality of rules and policies governing the internal activity structure of the remanufacturing tasks. Hence analyzing the internal dynamics of remanufacturing activities in light of the external factors stated above is a critical issue.

As stated earlier, after collection of the used products, there are two options for the company: material recovery (recycling) or product recovery (remanufacturing). Since added value of the product is lost in material recovery, remanufacturing seems to be a better option. In remanufacturing, returned products are first disassembled to their subparts, then these parts are inspected, refurbished and reassembled. In hybrid production companies, remanufacturing and manufacturing activities are performed under the same roof. In these companies, subparts that will be assembled come from manufacturing of raw materials and disassembly process of returns simultaneously. Controlling these types of systems is much more difficult than other systems. In fact, the first major difficulty is the integration of the disassembly line with other production activities. Moreover, due to the quality uncertainty of components of the returns, there can be unbalanced supply streams for different components. Hence, there must be a selection rule for manufactured and remanufactured parts as there are two suppliers for the assembly line; disassembly and manufacturing. Also an optimal disposal policy must be determined for returns, by taking into account both disposal and holding costs of returns. When the stock level of returns reaches a predetermined level, newly returned products will have to be disposed pf.

After accepting a returned product, first critical stage is the disassembly process. Disassembly has a key role in remanufacturing operations. In disassembly, there is a diverging material flow (Kizilkaya and Gupta, 1998) on the contrary to the assembly setting. One core is disassembled to many parts having different characteristics. Also, in assembly, there is only one demand stream for the final assembled product, while in disassembly there are multiple demand streams, as many as the number of the types of disassembled components. Furthermore, because of the different characteristics of the components, these streams can be unbalanced, as components in the same core can have different quality levels and life cycles. As a result of these complexities, control rules that are established for the assembly line are not adequate for adapting in the disassembly setting. The main objective of this research is the characterization of new control policies for the disassembly operation, and assessing the profitability measures of these policies.

The rest of the thesis is organized as follows. In chapter 2, a survey about the existing literature on the control of remanufacturing and disassembly operations is given. In chapter 3, the objective of the research is stated in more detail. In chapter 4, problem definition is made, and the different models of the disassembly control systems are described. Also, performance evaluation and optimization models are explained in this chapter. In chapter 5, numerical results are illustrated, and, finally, in chapter 6, conclusions are drawn.

2. LITERATURE REVIEW

In this chapter, the related research on the control and performance evaluation tools of remanufacturing and disassembly operations is summarized briefly. In the remanufacturing literature, the main concern is the integration of the control of the return flow and the subsequent remanufacturing operations with the classical control policies of the manufacturing operations. In this chapter, we give some specific examples of related literature that contribute to this adaptation and integration effort by constructing strong theoretical foundations.

In the work of Korugan and Gupta (1998), a two echelon inventory system with return flow is modeled as an open queuing network with finite buffers. It is assumed that machine service times, demand and return inter-arrival times are independent and exponentially distributed random variables. The queuing network of the system is solved by the expansion methodology in order to get the steady state performance measures which are expected lost sales, transportation, remanufacturing, manufacturing and disposal rates with average inventory levels in the buffers. The total cost function is the weighted sum of these performance measures, and the weights are the unit costs. In that stochastic model, buffer limits are the main control points of the system. The authors optimize these buffer capacities with respect to the expected total cost of the system.

In the article of Aksoy and Gupta (2005), a more detailed model of the remanufacturing facility is analyzed. Each operation in the facility such as disassembly, inspection, disposition, etc. corresponds to a separate module in the model. Also, machines are unreliable and they are subject to operational break-downs. Repair times and inter-arrival times of the operational failures are assumed to be exponential random variables as well as the machine service times and inter-arrival times of the returns and demands. In fact, the analyzed remanufacturing facility is a combination of the Markovian queues and servers. For solving the resulting open queuing network model, the decomposition principle and the expansion methodology is used. Then by using a heuristic procedure, which gives near optimal solutions, the buffer limits of separate modules are optimized for a given total inventory level constraint by taking into account a total cost function that consists of different cost components which are similar to that of Korugan and Gupta (1998). In the previous study of the authors, in Aksoy and Gupta (2001), the same system is analyzed by using the same methodology that focuses on the effects of the variations of the reusable rates of returned products.

Another interesting work in this area is the study of Vorasayan and Ryan (2006a). They also use an open queuing network model for the analysis of a remanufacturing system. However, in this work, rather than analyzing only the cost of the remanufacturing operations, they focus on the profitability of all the operations in the closed-loop supply chain by taking into account the market cannibalization effect of the remanufactured products. For this purpose, they formulate an optimization problem and propose a mathematical model associated with the queuing network. They show how optimal price and quantity of refurbished products are affected by other factors in the system. In their model, it is assumed that service times of the servers follow exponential distributions. However, this assumption is relaxed in the subsequent paper of the authors (Vorasayan and Ryan, 2006b), where they analyze the same system by assigning general distributions for the service completion times. Another feature of this study is the capability of restricting the number of times a product can be refurbished rather than assuming it to be infinite. For the analysis, the parametric decomposition method is used to evaluate the performance of each server node in the network. Then a nonlinear optimization problem is solved by using the Karush-Kuhn-Tucker optimality conditions.

In the study of Korugan and Gupta (2001a), substitution policies in hybrid production companies are analyzed by using Markov Decision Process. In the analyzed system, there are two supply streams, viz. remanufacturing and manufacturing, but one demand class that can be satisfied by either of two different product types. In Bayindir *et al.* (2007), the profitability of the remanufacturing operation is analyzed under one-way substitution and the capacity constraint. In the analyzed system, there are two different demand classes for the remanufactured and manufactured products. It is assumed that demands for the remanufactured product can be satisfied by the manufactured product if the remanufactured product inventory is stock out. The authors investigate the optimal utilization ratio of the remanufacturing by deriving analytical conditions for the profitability of remanufacturing.

In Teunter and Vlachos (2002), the conditions that make the disposal option necessary for returned products that can be remanufactured are investigated by using simulation models for the hybrid production environment. For the analysis, it is assumed that there are more demands than returns and the remanufacturing is marginally profitable. In Guide *et al.* (2006), a queuing model is constructed to determine an optimal disposition decision for a product return with respect to its quality level. The authors have focused on the decreasing sales value of the return due to the delay in remanufacturing operations. Particularly for high-tech products having short life cycles, since low quality product returns cause more delay in remanufacturing operations, determining a threshold value for the processing time of the return is crucial to decide whether to remanufacture or to recycle the returned product. In the paper, this threshold value is determined by the help of queuing models.

Aras *et al.* (2004) investigate the possible cost reduction effect of the sorting activities of returns according to their quality levels before the remanufacturing operations by constructing continuous time Markov chains of the analyzed system. Also, in the study of Ferguson *et al.* (2006), by taking into account the different quality levels of the returns, the tactical production planning of the remanufacturing is considered. Effect of the quality levels of the returns to the cost of the remanufacturing operations is analyzed by constructing a linear programming formulation.

The effectiveness of using pull type control policies for hybrid production systems is first argued for inventory control. Van der Laan *et al.* (1999) showed that pull control strategy is more cost effective than the push control for inventory systems with return flows. Then in two subsequent papers, the pull control of hybrid systems is analyzed in the production control framework. First, in Korugan and Gupta (2001b), a pull controlled hybrid production system including two independent production subsystems, viz. remanufacturing and manufacturing, which are feeding the same stock, is modeled using continuous time Markov chains. In this research, different pull control strategies are compared based on the expected total cost function that consists of the backorder costs and linear holding costs of materials. These control strategies in the paper differ in the routing decision of the demand information. Then in their second study, Korugan and Gupta (2001c), propose an adaptive kanban control model for analyzing the same system.

Pull control of the disassembly process in remanufacturing is first analyzed by Kizilkaya and Gupta (1998). They propose traditional and flexible kanban models for coordinating different activities in the disassembly line. They show that, in the disassembly setting, flexible kanban control outperforms the traditional kanban control policy, with numerical results obtained by using simulation models. Gupta *et al.* (2004) propose a multi kanban model for the multi-product disassembly system with multiple demands. They show that the pull control of disassembly results in lower inventory levels than that of push control while achieving the same service level. For numerical experimentation, simulation models are used. A successful implementation of this new control mechanism in a real world application is given by Udomsawat and Gupta (2005) for an automobile disassembly facility.

In disassembly control literature, an important exception of using simulation models is the study of Ramakrishnan and Krishnamurthy (2005). They use an approximation technique, which is called parametric decomposition method, for the performance evaluation of a repair facility. Simulation models are just used to asses the accuracy of the approximation method proposed in the paper. The analyzed repair facility, which is modeled as a closed queuing network with fork/join stations, consists of disassembly, repair and assembly machines, respectively. Another interesting feature of the analyzed system is the use of CONWIP control policy for the coordination of production activities. However, assumptions of infinite supply of cores and saturated demand i.e. infinite demands make the study unrealistic from the remanufacturing point of view.

Quality of assemblies and the yield rate of the disassembly process are crucial factors that affect the performance of the disassembly systems. One of the important studies dealing with the quality aspect and the uncertainty of the yield of the disassembly operations is the work of Inderfurth and Langella (2006). In that study, a disassemble-to-order system with stochastic yields is analyzed by proposing some heuristics for solving the corresponding stochastic optimization model of the system. Deterministic demand rate and not allowing the partial disassembly of cores are the main assumptions of the analyzed disassembly system. In Langella (2007), new heuristics are proposed for the planning of the demand driven disassembly systems. The proposed heuristics are more powerful options compared to the alternative of using the integer programming (IP) approach for solving the optimization problem presented in the paper, due to the increasing computational complexity of the IP with the complex product structures and longer time horizons. In Kongar and Gupta (2006), a multi criteria optimization model is presented for a disassembly-to-order system under uncertainty. The main purpose of the study is to construct a model for determining the best combination of the number of each product type to be taken back from the last user and/or collectors. The authors propose a fuzzy goal programming technique to solve the presented problem.

In Teunter (2006), in order to determine an optimal disassembly and recovery strategy, a stochastic dynamic programming algorithm is constructed. The proposed algorithm is used for determining the optimal strategy with given disassembly trees, the process-dependent quality distributions of assemblies, and the quality-dependent recovery options and associated profits for the assemblies.

Finally, in a recent paper of Johar and Gupta (2007), a multi period inventory control problem of the disassembly process is analyzed to balance the inventory levels of different components while minimizing the total cost function. For this purpose, a linear programming formulation is used. Here core arrival and demand arrival rates are assumed to be deterministic in that mathematical model. However, randomness of these arrivals is one of the major characteristics of the remanufacturing environment.

In our study, we analyze different disassembly control strategies that differ in terms of allowing partial disassembly decision by using a queuing network model with kanban control mechanisms. For solving the model analytically, we use an approximation technique on the contrary to the previous studies that are using simulation models for evaluating the performance of the kanban controlled disassembly process

3. OBJECTIVES

Quality variations in the components of a core result in uncertainties in satisfying the demand. Sometimes a core is disassembled but a demanded component is out of order. Therefore, the demand has to be satisfied either by a new component or by a component from a second core. This characteristic of the disassembly line presents the decision of partial disassembly vs. complete disassembly. Either of these two policies can be performed depending on the demand arrival structure of different components. For instance, in a pull type control production environment, when there are unbalanced demands for different components, allowing partial disassembly, in other words allowing the disassembly of one core just for a more frequently demanded component, seems to be a better option than waiting for the demand occurrence of all components before disassembling a core. However, in this case, the residual item that is not demanded becomes an "overflow item" (Gupta *et al*., 2004) and we face the problem of storing these items. So, there is a strong need for quantifying the benefits of allowing or not allowing the partial disassembly. In this research, this quantification is made for a kanban controlled disassembly production system. In addition, for the partial disassembly case, we also investigate the effect of holding overflows rather than disposing them of.

The major reason of using kanban control in this study is the need for establishing a good counter for the overflow items. Also, as stated in Gupta *et al*. (2004), "pull" is better than "push" for controlling the disassembly operation. In fact, one of the simplest forms of the pull type control mechanisms is controlling the production stages by their fixed number of production authorization cards, viz. kanbans.

For the performance evaluation of kanban controlled production systems, using simulation models can be a good option, especially if we want to model the stochasticity of the machine service times and demand occurrence times. However, since simulation run times can be quite long for obtaining a tight confidence interval of the performance measures, their use for the optimization of the number of kanban cards is time consuming. On the other hand, an important alternative to the stochastic simulation is finding exact analytical solutions by using stochastic modeling tools such as continuous-time Markov chains. However, because of the large state space of the corresponding Markov chains of the analyzed systems, the effort of finding an exact solution results in significant computational complexity. So, using fast but approximate analytical techniques is a better option for the optimization if they are accurate enough.

In this research, an approximate analytical technique is utilized for the performance evaluation of the disassembly system. For this purpose, modifications are proposed on a former approximation technique for analyzing the disassembly process. Then, by an optimization procedure, optimal kanban numbers of each stage and optimal storage capacity of returns is determined by taking into account a total cost function composed of different costs such as backorder, holding and disposal costs. Based on the minimum total cost of the system, disassembly control policies that differ in allowing partial disassembly decision are compared numerically.

4. MODEL

4.1. Problem Description

4.1.1. Definition of the Remanufacturing and Disassembly Operations

The analyzed disassembly system is a subsystem of the remanufacturing operations. The interaction of the disassembly operation with other remanufacturing activities is shown in Figure 4.1. The first decision point in the system is the decision of the recycling/disposal or the disassembly of the end-of-life cores that are collected from customers. Mainly, in our system, this decision is made according to the state of the inventory level of cores. When the level of the stock of the cores reaches a predefined limit, new arrivals are disposed of directly. Furthermore, excess inventory occuring in the disassembly is also disposed of. In fact, a newly disassembled component is disposed of if the respective buffer of the component is full. In the disassembly operation, excess inventory in some components and starvation in other components can occur frequently because of the imbalance of the demand rates of the different components that is caused by the difference in the quality levels of the components. In the analyzed system, demands for components come from the assembly shop and low quality parts are not accepted for the assembly operation.

Orders coming from the assembly shop are backordered if the respective inventory of the disassembled component is out of stock. Since more backorders mean more waiting time for the order, we restrict the number of backorders by introducing a manufacturing/procurement option. So, when the number of backorders for a component reaches a pre-defined limit, new demands for that part are directed to the manufacturing operation or fulfilled by external procurement. By this policy, we prevent a bottleneck in the assembly operation, since the assembly operation cannot start before all types of components are available. Actually, in our system, there is a pull control between the assembly and disassembly operations, but we restrict our attention to the internal structures of the disassembly operation in order to be able to focus on the dynamics of the different disassembly control strategies.

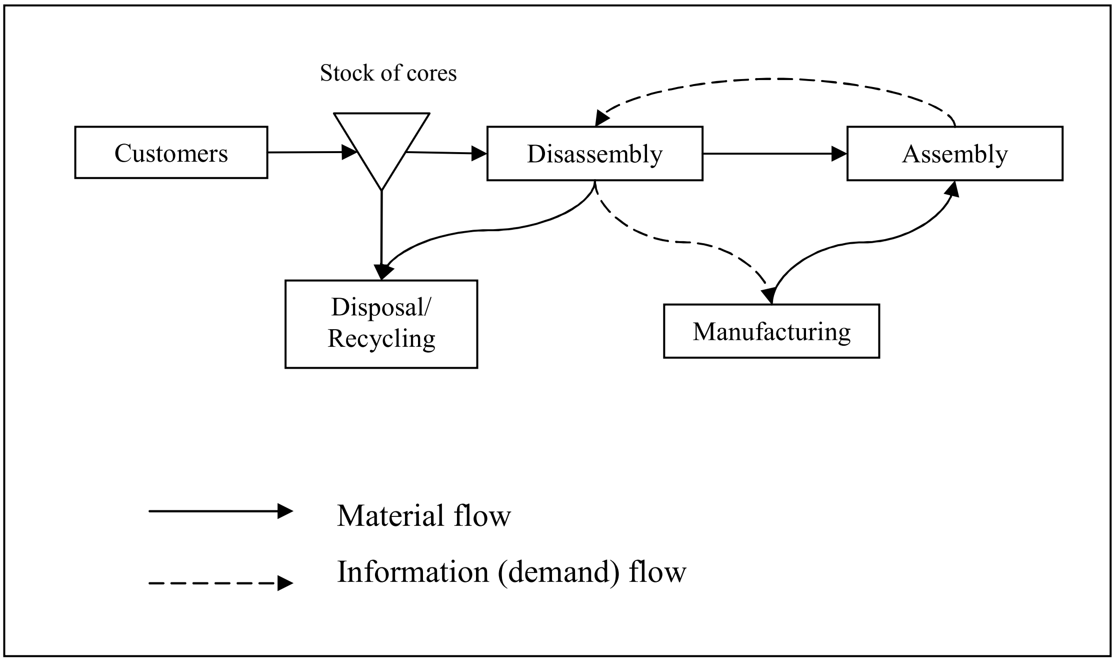

Figure 4.1. A simple sketch of the remanufacturing system

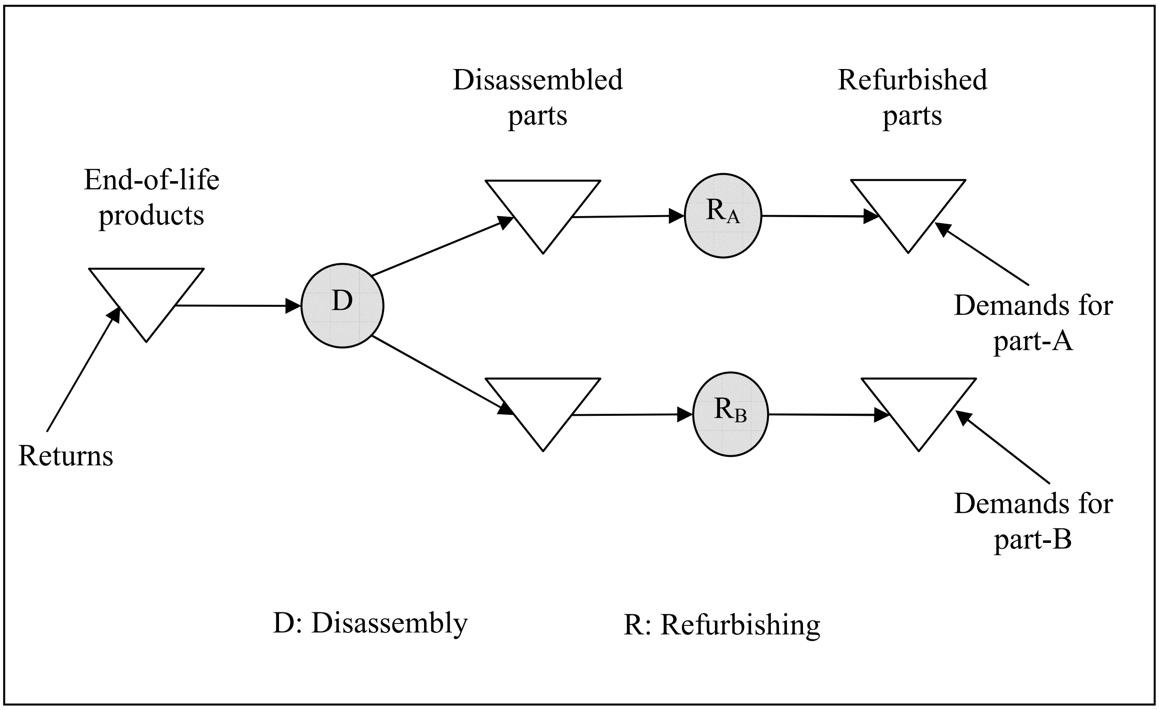

Figure 4.2. Disassembly system with two components

In Figure 4.2, a simple sketch of the disassembly system is given for the two-component case. As a first step, used products are dismantled to their subparts. Then, they are held for the next operations in different buffers that are assigned according to the part types. In Figure 4.2, there are two stocks for the newly dismantled parts since the product is composed of two components. After dismantling, parts are refurbished. Refurbished parts are held in the respective buffers, and demands are fulfilled from these buffers.

4.1.2. Definition of the Kanban Control Model

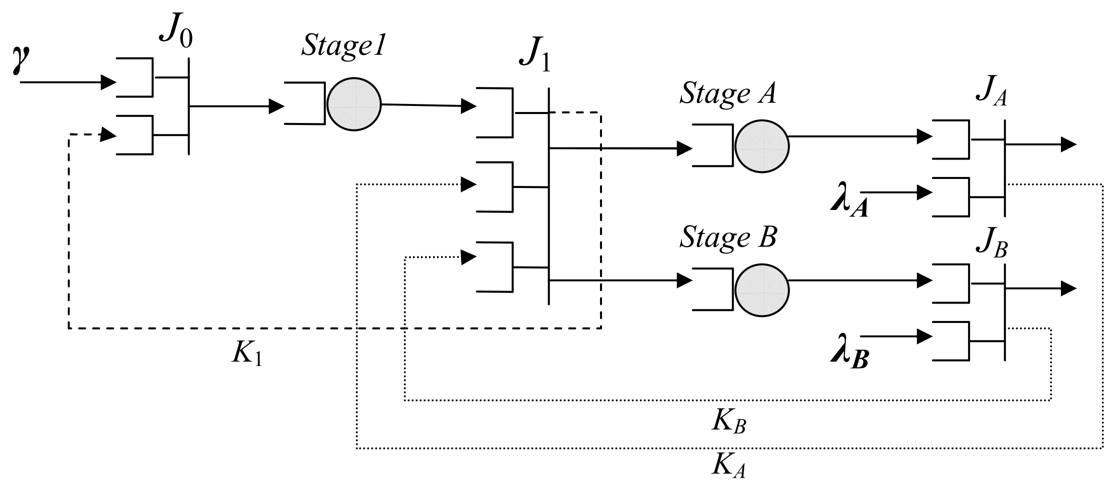

Figure 4.3. Kanban controlled disassembly system with n = 2 components

We consider a kanban controlled disassembly system where the product is composed of n components. Figure 4.3 describes the queuing model of such a system with n = 2. In this system, after returns are dismantled at Stage 1, the individual components get further processed through subsequent stages. Finally, they are synchronized with their respective demands.

There are n+1 stages and each stage has a constant number of kanbans. In Figure 4.3, K_1, K_A and K_B represent the kanban sizes of the respective stages. When a component demand arrives to the system, it is synchronized with the component if the finished part queue of that part is not empty. Otherwise, the demand is backordered and waits in the backorder queue in the respective synchronization station (J_A or J_B for n=2) until a new item arrives. On the other hand, if the finished part queue of a component is not empty, incoming demand causes a kanban release of that stage and the released kanban joins the kanban queue in synchronization station J_1.

In Figure 4.3, there are $n+1$ queues in synchronization J_1, n of them representing the kanban queues of downstream stages (Stage A and B for $n=2$) and the other one is the disassembled parts queue of Stage 1. Newly dismantled components in Stage 1 join that queue with their kanbans. In this queue, each Stage 1 kanban is attached to n types of newly dismantled components of the core. When a kanban of downstream stages synchronizes with a component, Stage 1 kanban is released and directed to the synchronization station J_0. If there is no core, Stage 1 kanban joins the kanban queue in J_0. Otherwise, it is attached with a core and directed to Stage 1. Also, when a return arrives to the system, it synchronizes with a Stage 1 kanban, or waits in J_0 until a kanban arrives.

We assume that there are independent arrival streams for returns and demands of different components to the system. We model these arrivals as Poisson processes with constant arrival rates. In Figure 4.3, γ, λ_A, λ_B are rates of arrivals of the returns, demands for component A and component B, respectively. Also, each stage consists of a single workstation and service times of these machines are independent and exponentially distributed random variables with a specific rate. We also assign limits for the return buffer and the backorder queues of finished parts. When the return queue reaches the capacity of buffer, newly arrived returns are disposed of. Likewise, demands arriving at the instant when numbers of waiting demands equal to the backorder limit are lost. Lost sales of the disassembly process are satisfied by the procurement or manufacturing of raw materials. So, we can say that the effective demand rate without lost sales is always smaller than or equal to the return rate.

There are two distinct control policies for controlling the disassembly process: i) allowing partial disassembly of cores and ii) not allowing the partial disassembly. Synchronization J_1 represents the main difference between the two different disassembly control systems. When partial disassembly is allowed, a kanban of a downstream stage can be synchronized with a respective component although one of the other kanban queues may be empty. In this case, a Stage 1 kanban is released, and the component which is not demanded becomes an overflow item and it is disposed of or it is kept in an extra buffer that is assigned for the overflow items instead of being disposed of. However, when partial disassembly is not allowed, a kanban of a downstream stage has to wait until there is at least one kanban in each of the kanban queues of other downstream stages to be synchronized with a disassembled core.

In partial disassembly, if we dispose the overflow item of, this disposal causes a reduction in the service level of that part. Hence, holding overflow items in a buffer can be beneficial particularly when the return arrival rate is smaller than demand arrival rates of the parts. So, in case of allowing partial disassembly, we are faced with the decision of choosing one of the two alternative sub policies: disposing overflows directly and holding them in additional buffers. In order to make a comparison between disposing and holding, we assign these buffers not only for overflow items but also for items with kanbans coming from the first stage.

In cases where partial disassembly is allowed and the overflow item is held, when parts come from the first stage, their kanbans are detached and detached kanbans and components join different queues. In Figure 4.4, P_A, P_B, D_A, D_B and F_1 represent the queues for components A and B and the waiting kanbans of Stage A, Stage B and Stage 1, respectively, for n = 2 component case. Here the buffers for components have respective capacity limits. If a component buffer reaches the limit, new arrivals to the queue are disposed. Kanbans coming from downstream stages synchronize with items waiting in these buffers. On the other hand, release of Stage 1 kanbans occurs differently depending on our priority decision. Giving priority to the overflow items and regular items are the two different sub policies of holding overflow policy. Here the number of overflow items is given by the difference between the number of components and the number of Stage 1 kanbans. If we give higher priority to the overflow items, then arrival of a downstream kanban does not cause a kanban release of Stage 1 kanbans when the respective component queue has a positive number of overflow items. However, if we give the priority to the regular items, every kanban arrival of downstream stages causes a kanban release of the Stage 1 without considering the overflow items. In fact, the number of waiting kanbans of Stage 1 represents the number of regular items.

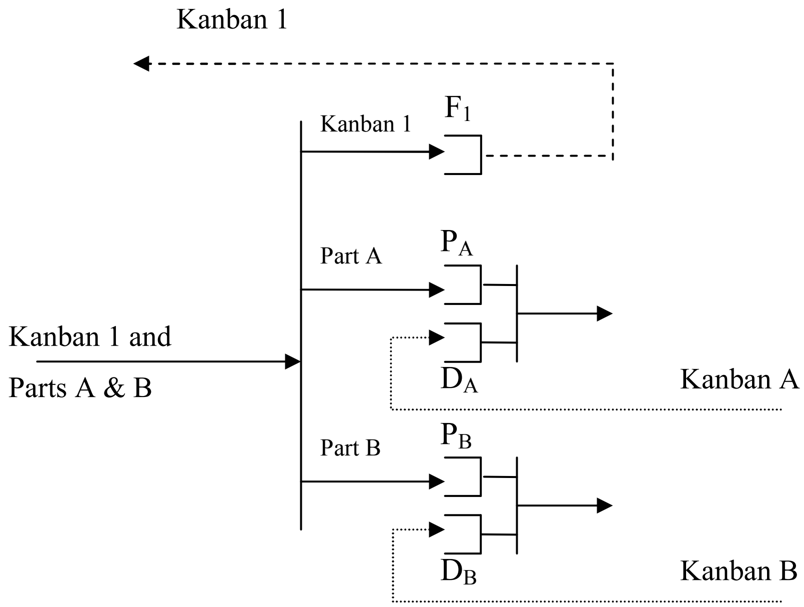

Figure 4.4. Synchronization station J_1 in case of holding overflows for n = 2 components

4.2. Performance Evaluation Model

4.2.1. Overview of the Method

In order to evaluate the performance of the system, we use an analytical method based on the product form approximation technique (Di Mascolo *et al.*, 1996). For the performance evaluation of multi stage kanban controlled production systems, Baynat's method (Baynat *et al.*, 2001), which is an improved version of the previously proposed technique in Di Mascolo *et al.* (1996), appears to be of special interest because of its accuracy and agility. In this method, the production system is modeled as a queuing network with synchronization mechanisms, and this network is solved approximately with the "multi class approximation technique" proposed by Baynat and Dallery (1996). In the algorithm, the whole system is approximated to a set of single class closed networks whose customers are kanbans for each stage. These small closed networks are analyzed by solving corresponding continuous time Markov chains for the isolated stations of closed networks to obtain the performance measures of the system such as the average number of work in process, the average number of finished goods, and the average number of backordered demands. This method is capable of analyzing complicated production systems such as kanban controlled assembly lines (Matta *et al*., 2005). In our research, Baynat's method is used for solving the queuing network model of the kanban controlled disassembly system.

In Korugan et *al*. (2006), a single stage kanban controlled disassembly system is analyzed by the exact solution of continuous time Markov chains. However, if the number of subparts in the core increases, the number of dimensions of the Markov chain increases as well causing a major increase in computational complexity for solving the Markov chains numerically. Hence, using Baynat's approximation technique seems to be a more appropriate alternative to solve the system exactly. However, even though the computational complexity caused by the large number of subparts is reduced significantly in Baynat's method, it does not disappear completely. For analyzing the isolated synchronization station (J_1 in Figure 4.3) between disassembly stage and its downstream stages, again the number of dimensions of the corresponding Markov chain increases when the number of subparts in the core increases. In these situations, the "class aggregation technique" proposed in Baynat and Dallery (1995) is used for analyzing the synchronization station. In this technique, instead of solving the complete Markov chain of the synchronization station, small Markov chains having only two dimensions are solved for each class of customers.

In cases where partial disassembly is not allowed, synchronization J_1 in Figure 4.3 is equivalent to an assembly kanban synchronization station. Therefore, the method proposed in Baynat and Dallery (1995) and the algorithm in Matta *et al*. (2005) can be utilized for solving that isolated station. However, when partial disassembly is allowed, class aggregation cannot be used directly. Thus, we need to modify this method to be able to analyze a partial disassembly system. In subsequent sections, we describe this modification for partial disassembly case. However, firstly, in Section 4.2.2, the multi class approximation technique of Baynat *et al*. (2001) is described.

4.2.2. Multi Class Approximation

The main approximation in the method is the decomposition of the multi-class network of the kanban controlled system to single class closed queuing networks having exponential servers with load dependent service rates. Each decomposed single-class network has a limited population, and kanbans are the customers of these closed networks. Load dependent servers in the single class queuing networks represent the machines and synchronization stations that are visited by the customers of the network viz. respective types of kanbans. The solution of separate networks is straightforward, since these single-class networks are Gordon-Newell networks (Baynat *et al*., 2001). In fact, product-form solution methods for these types of networks are already available in the literature (Bruell and Balbo, 1980).

However, by decomposition, we make the networks independent of each other. In the decomposed networks, there is no interaction between the different types of kanbans, while, in the multi class network, there is an interaction between the kanbans of different stages via the synchronization stations. This interaction effect is captured by the estimation of the load dependent service rates of the servers of the single class networks from the conditional throughputs of the corresponding stations of the multi class network. In order to find an estimate of the conditional throughputs of the stations, we isolate them from other stations. So, we analyze each station in isolated mode to approximate the service rates of the corresponding load dependent servers.

The method is iterative. To analyze stations in isolation mode, we use load dependent arrival rates obtained from the solution of single-class networks. Then by setting the load dependent service rates equal to the conditional throughputs of the isolated stations, we again solve the single class networks using these new estimates of load dependent service rates. These calculation steps are repeated until the convergence of the unknown parameters viz. load dependent service rates of the servers of the single class networks to stationary values.

Let $T(n)$ denote the index set of customer classes' viz. kanban types when the product is composed of n types of components. For our system, it is clear that $|T(n)| = n + 1$. For instance for a two component system $T(2) = \{1, A, B\}$. Also, let $X(i)$ denote the index set of visited stations by the customer class $i \in T(n)$ for $n \geq 2$. In our analyzed system, $|X(i)| = 3$ for all customer classes $i \in T(n)$. Finally, let K_i denote the kanban size of each stage for $i \in T(n)$.

To find the steady state probabilities of single-class networks, we use the following product-form solution formula

$$P(n_{i1}, n_{i2}, n_{i3}) = \frac{1}{G_i} \prod_{j \in X(i)} \left[\prod_{m=1}^{n_{ij}} \frac{V_{ij}}{\mu_{ij}(m)} \right] \qquad \text{for all } i \in T(n) \tag{4.1}$$

where n_{ij} is the state variable that shows the distribution of the kanban types $i \in T(n)$ through the stations of the single class network. V_{ij} is the average visit ratio of the station j in the network $i \in T(n)$. In our system, V_{ij} equals to one for all networks and stations. G_i is the normalization constant associated with the network i.

By using the steady state probabilities calculated by the above formula, we can obtain all performance measures such as throughput and average queue lengths. At this point, the estimation of the load dependent service rates $\mu_{ij}(n_{ij})$ becomes critical. As mentioned previously, we estimate these rates by analyzing the stations in isolation with load dependent arrival rates. Load dependent arrival rates to the isolated stations are calculated by the following relation (Baynat *et al.*, 2001):

$$\lambda_{ij}(n_{ij}) = \mu_{ij}(n_{ij}+1) \frac{P_{ij}(n_{ij}+1)}{P_{ij}(n_{ij})} \qquad \text{for } n_{ij} = 0,1,\ldots,K_i - 1 \tag{4.2}$$

The probabilities $P_{ij}(n_{ij})$ in the above formula are the marginal probabilities for the queue length of the load dependent server j in the single-class network i. These probabilities are obtained by formula (4.1).

Then by using load dependent arrival rates obtained from formula (4.2), we analyze the corresponding isolated station. Since the isolated stations are fed by state dependent Markovian process with rates $\lambda_{ij}(n_{ij})$, the analysis corresponds to the analytical solution of the corresponding continuous-time Markov chain of the respective isolated station. By the solution of Markovian model of the isolated station, we can obtain the marginal probabilities of the number of each customer class visiting the station. Then by using the following relation, we can find the conditional throughputs of the isolated stations (Baynat *et al*., 2001):

$$\widetilde{v}_{ij}(n_{ij}) = \lambda_{ij}(n_{ij}-1)\frac{\widetilde{P}_{ij}(n_{ij}-1)}{\widetilde{P}_{ij}(n_{ij})} \qquad \text{for } n_{ij} = 0,1,\ldots,K_i \text{ -1} \qquad (4.3)$$

The probabilities $\widetilde{P}_{ij}(n_{ij})$ in the above formula are the marginal probabilities for the number of class i customers, which are present in the isolated station. These probabilities are obtained by the solution of corresponding Markovian models of the isolated stations.

The main principle of the method is to set the service rates of the load dependent servers of the single-class networks to the conditional throughputs of the corresponding isolated stations, i.e.:

$$\mu_{ij}(n_{ij}) = \widetilde{v}_{ij}(n_{ij}) \quad \text{for } n_{ij} = 0,1,\ldots,K_i \text{ -1 and for all } i \in T(n) \text{ and } j \in X(i) \quad (4.4)$$

Then single class networks are solved again by using these new estimates of the load dependent service rates. These calculation steps are repeated until the values of the load dependent service rates converge to the stationary values. We summarize these calculation steps in the following algorithm. As reported in Baynat *et al*., (2001), the number of iterations to achieve convergence is usually reasonable (less than 10 in most examples).

Algorithm

Step 0. For $i \in T(n)$:

Initialize the unknown parameters $\mu_{ij}(n_{ij})$ to some initial values, for $j \in X(i)$ and n_{ij} $= 0,1,\ldots,K_i$

Step 1. For $i \in T(n)$:

Calculate the marginal probabilities $P_{ij}(n_{ij})$, for $j \in X(i)$ and $n_{ij} = 0,1,\ldots,K_i$, using formula (4.1)

Derive the state dependent arrival rates $\lambda_{ij}(n_{ij})$, for $j \in X(i)$ and $n_{ij} = 0,1,\ldots,K_i$-1, using relation (4.2)

Step 2. For $i \in T(n)$ and $j \in X(i)$:

Analyze the station j in isolation, for $j \in X(i)$, by solving the corresponding Markovian model

Calculate the marginal probabilities $\widetilde{P}_{ij}(n_{ij})$ for $j \in X(i)$ and $n_{ij} = 0,1,\ldots,K_i$

Calculate the conditional throughputs $\widetilde{\nu}_{ij}(n_{ij})$, for $j \in X(i)$ and $n_{ij} = 0,1,\ldots,K_i$, from relation (4.3)

Step 3. For $i \in T(n)$ and $j \in X(i)$:

Set the load-dependent service rates of station-j in the i-th single-class network to $\mu_{ij}(n_{ij}) = \widetilde{\nu}_{ij}(n_{ij})$ for $n_{ij} = 0,1,\ldots,K_i$ -1

Step 4. Go to Step 1 until the convergence of the parameters $\mu_{ij}(n_{ij})$ is achieved for a specified level of tolerance

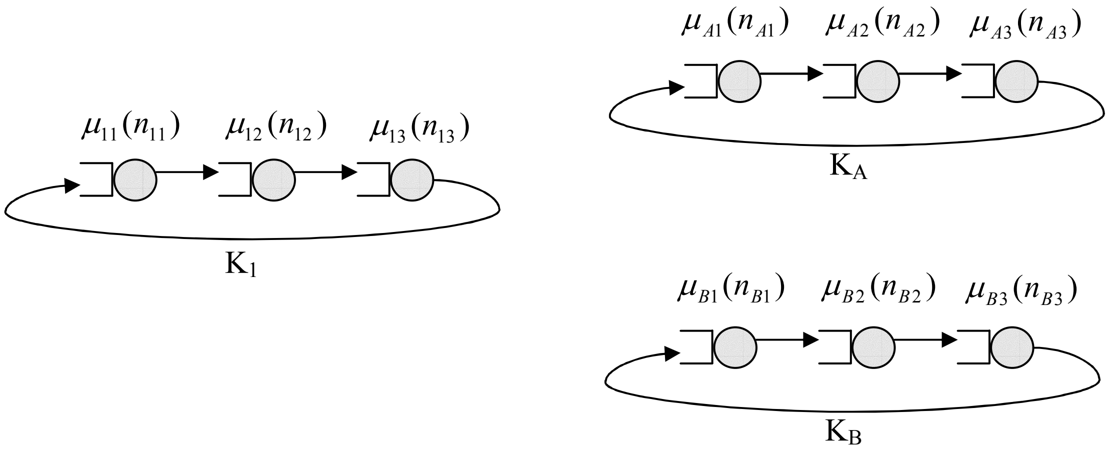

Figure 4.5. Decomposition of the multi-class network to the single-class closed queuing networks with load dependent servers

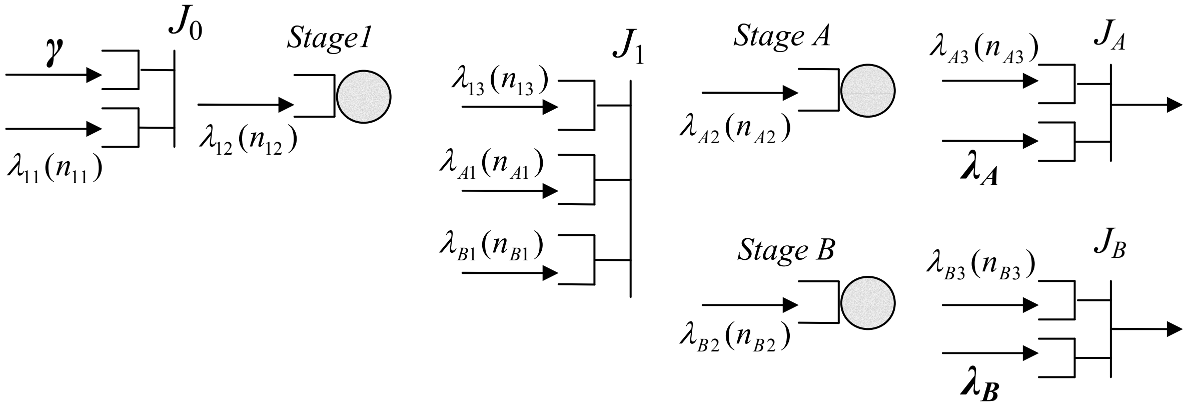

Figure 4.6. Decomposition of the network to the isolated stations with load dependent arrival rates

Since the most demanding calculation step of the algorithm in terms computational complexity is the step 2, in subsequent sections, we give detailed information about the calculations operations performed in this step.

4.2.3. Markovian Analysis of the Isolated Stations

In step 2 of the algorithm, continuous time Markov chains representing the behavior of the kanbans in the isolated stations are solved to obtain the stationary probabilities of the queue lengths which are used to estimate the conditional throughputs of these stations. In the isolation mode, there are two types of stations: synchronization stations and workstations. As pointed out in Dallery and Cao (1992), when the service time of the workstation is exponentially distributed, conditional throughput is simply equal to the load dependent service rate. So, the analysis procedure in step 2 can be skipped for these stations since in our analyzed system we assume that workstations have exponentially distributed service times. The only remaining part is solving the Markovian models of the isolated synchronization stations.

In our analyzed system, synchronization stations that are located at the input and output of the disassembly process are all made up of two queues. For instance, synchronization station J_0 in Figure 4.3 has two buffers: one for newly arrived returns and one for Stage 1 kanbans. Also, J_A and J_B have two queues which are the buffers of backordered demands and finished parts. So, the continuous time Markov chains of these isolated stations consist of only two dimensions and because of that reason, solution of them is straightforward. Here the major challenge is the solution of the Markovian model of the intermediate synchronization station J_1 that is located between the first stage and downstream stages. In fact, that synchronization station is composed of $n + 1$ queue when the product is composed of n types of components. This means that the number of dimensions of the analyzed continuous time Markov chain is also equal to the $n + 1$. When the product is composed of larger number of components than two types of components, solution of the corresponding Markov chain presents a higher computational complexity. Although numerical methods are available to solve the Markov chain numerically, an approximate solution is a more preferable alternative because of the high calculation speed. In our study, we analyze the isolated synchronization station J_1 by using the class aggregation technique, which is an approximation method giving accurate results.

4.2.4. Analysis of the Synchronization Station J_1 in Isolation Mode

The behavior of the synchronization station J_1 differs depending on the selected disassembly control policy. So, each control policy results in a different Markovian model of the synchronization station J_1 in isolation mode. Since when partial disassembly is not allowed this synchronization station is equivalent to the assembly kanban synchronization station, we do not give additional information for the Markovian analysis in isolation mode for this policy. Details of the analysis procedure that is used for the solution of the Markovian model of the synchronization J_1 when partial disassembly is not allowed can be found in Matta *et al*. (2005) and Baynat and Dallery (1995). In our study, we concentrate on finding new approximations for allowing partial disassembly policies, since they are not analyzed previously by the multi class approximation method. In subsequent sections, our proposed approximations for the analysis of the synchronization station J_1 are described for allowing partial disassembly policies.

4.2.4.1. Analysis of the Synchronization Station when Partial Disassembly is Allowed and the Overflow Item is Disposed. For the analysis, class aggregation technique of Baynat and Dallery (1995) is modified to solve the continuous time Markov chain of the synchronization station in isolation mode. The main idea of the class aggregation is to model each load dependent arrival process to the synchronization station individually with their resources approximated by other arrivals to the synchronization. We define these resources as permits. Here a two dimensional Markov chain that corresponds to a simple birth death process is obtained. By using this method for a disassembly system having n components and n + 1 stages, we solve n + 1 two dimensional Markov chains instead of solving one Markov chain having n + 1 dimensions to analyze the synchronization. In Figure 4.7, there is a representation of the method for the two component case.

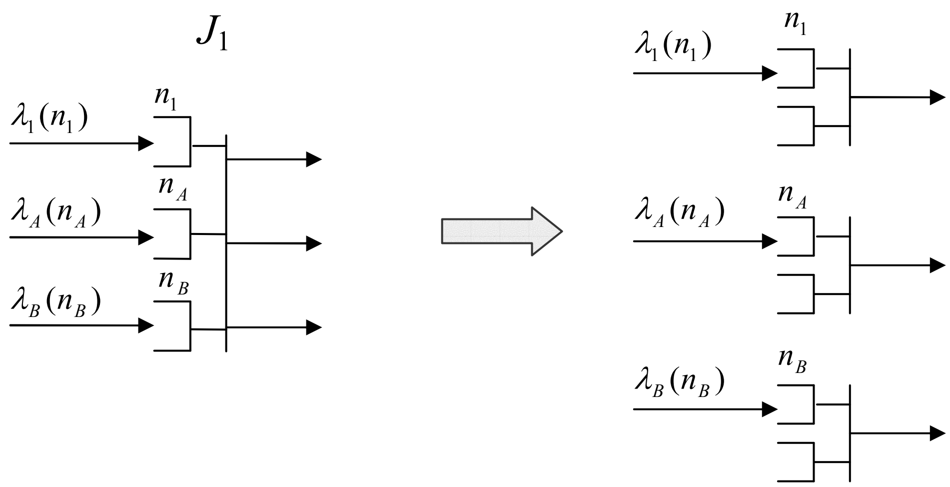

Figure 4.7. Class aggregation for n = 2 components

In this section and the subsequent sections, we only use the stage indexes of queue lengths and load dependent arrival rates by omitting the station indexes of them, for the sake of readability and simplicity. For instance, n_{13}, n_{A1} and n_{B1} are simply denoted by n_1, n_A and n_B, respectively.

In our model for $n = 2$ components, the number of permits can be defined as Max $\{n_A, n_B\}$ for Stage 1 kanbans and n_1 for both Stage A and B kanbans where n_A, n_B and n_1 are current numbers of waiting kanbans of Stage A,B and Stage 1 in the synchronization, respectively. Since, for both Stage A and B kanbans in the synchronization, the number of permits is simply equal to the number of first stage kanban in synchronization J_1, there is no need to aggregate the effect of other arrivals. Instead of aggregation, we embed these effects in the transition rates of their two dimensional Markov chains.

Here, for type A kanban in J_1, the state of the continuous time Markov chain is (n_A, n_{CA}), where n_A is the number of type A kanbans and n_{CA} is the number of components of type A, which denotes the number of components of other type and first stage kanbans simultaneously. When the state of the Markov chain is in the region of $n_A = 0$ and $n_{CA} > 0$, state transition rates must be adjusted to capture following effects of other system states of the synchronization J_1 that are not seen in the states of the two dimensional Markov chain. First of all, arrivals of other type kanbans can cause a reduction in n_{CA} because we dispose the overflow items. This effect is embedded to the Markov chain by adding arrival rates of other type kanbans ($\lambda_A(0) + \lambda_B(0)P_B(0)$ in Figure 4.8). This additional arrival rate is multiplied with the probability of having no kanban of that type in J_1, since $n_{CA} > 0$ means that $n_{ci} > 0$ for $i \neq A$ is true and so $n_i = 0$ must be true. On the other hand, if kanban queues of other types are not empty, arrival of components from Stage 1 does not cause an increase in n_{CA} since they can synchronize with existing kanbans by causing the disposal of newly arrived component A. We embed this effect to the Markov chain by weighting the rates of the arrival of components from Stage 1 with the probability of having no kanban of other types ($P_B(0)$ in Figure 4.8). The underlying Markov chain is shown in Figure 4.8, for the two-component case.

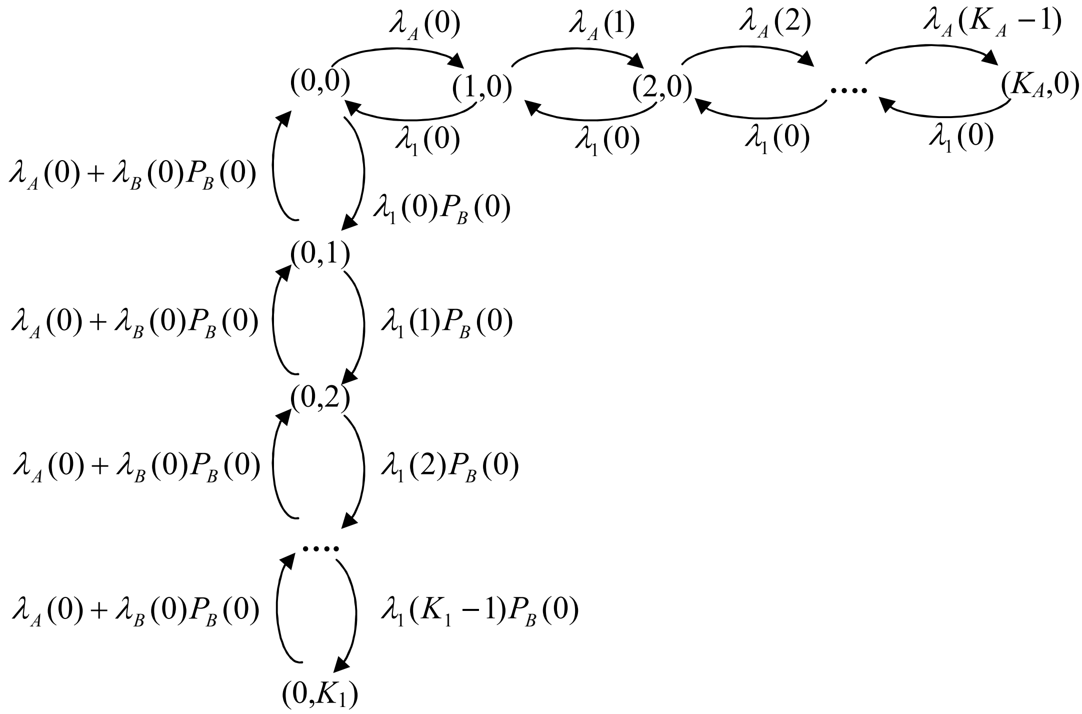

Figure 4.8. Markov chain representing the behavior of "Kanban type A" in synchronization station J_1 for the two-component case

The steady state probabilities $P(n_A, n_{CA})$ are the solutions of the following balance equations:

$$P(n_A, 0)\,\lambda_1(0) = P(n_A-1, 0)\,\lambda_A(n_A-1) \quad \text{for } n_A = 1,\ldots,K_A \tag{4.5}$$

$$P(0, n_{CA})\,[\lambda_A(0)+\lambda_B(0)P_B(0)] = P(0, n_{CA}-1)\,[\lambda_1(n_{CA}-1)P_B(0)] \quad \text{for } n_{CA}= 1,\ldots,K_1 \tag{4.6}$$

$$P(0,0)+\sum_{n_{CA}=1}^{K_1} P(0,n_{CA})+\sum_{n_A=1}^{K_A} P(n_A,0)=1 \tag{4.7}$$

where $P_B(0)$ is the probability of having no "kanban B" in J_I

Let S_n denote the index set of downstream stages when the product is composed of n components and let n_{ci} denote the current number of waiting type i components. For a system having $n \geq 2$ components and for $i, j \in S_n = \{A, B, ...\}$, above equations are restated as follows:

$$P(n_i, 0)\, \lambda_1(0) = P(n_i\text{-}1, 0)\, \lambda_i(n_i\text{-}1) \quad \text{for } n_i = 1,\ldots,K_i \tag{4.8}$$

$$P(0, n_{ci})\, [\lambda_i(0) + \sum_{j \neq i} \lambda_j(0) P_j(0)\,] = P(0, n_{ci\text{-}1})\, [\lambda_1(n_{ci} - 1) \prod_{j \neq i} P_j(0)\,] \quad \text{for } n_{ci} = 1,..,K_1 \tag{4.9}$$

$$P(0,0) + \sum_{n_{ci}=1}^{K_1} P(0, n_{ci}) + \sum_{n_i=1}^{K_i} P(n_i, 0) = 1 \tag{4.10}$$

By solving these equations, we can obtain marginal probabilities of kanbans of downstream stages that are used in the algorithm. However, for modeling the behavior of the Stage 1 kanbans in synchronization station J_1, we have to aggregate the arrivals of kanbans of downstream stages into single permit arrivals. Let the number of permits be defined as n_a and its state dependent arrival rate as $\lambda_a(n_a)$ for n_a=0,1,…,N_a-1 where n_a= Max$\{n_A, n_B\}$ and N_a= Max$\{K_A, K_B\}$ for n = 2 and $S_2 = \{A, B\}$. K_A and K_B are the assigned constant number of kanbans for stage A and stage B.

Aggregated permit arrival rate when the current number of permits is zero is equal to:

$$\lambda_a(0) = \lambda_A(0) + \lambda_B(0) \tag{4.11}$$

Otherwise:

$$\lambda_a(n_a) = \frac{[\lambda_A(n_a) P(n_A = n_a) P(n_B \leq n_a)] + [\lambda_B(n_a) P(n_B = n_a) P(n_B \leq n_a)]}{P(n_A \leq n_a) P(n_B \leq n_a) - P(n_A < n_a) P(n_B < n_a)},$$

$$\text{for } n_a = 1,\ldots,N_a\text{-}1 \tag{4.12}$$

Here, $\lambda_A(n_A)$, $\lambda_B(n_B)$ are state dependent arrival rates of Kanban type A and type B. The probability in the denominator in (4.12) is the estimated joint probability of Max$\{n_A, n_B\} = n_a$ by the product of marginal probabilities. The other probabilities in the numerator that are multiplied with the arrival rates of kanbans of downstream stages are the estimations of joint probabilities of the respective type of kanban queues in J_1 to have the maximum number of kanbans.

For a system having more than two components (n > 2), these formulas can be written as:

$$\lambda_a(0) = \sum_{i \in S_n} \lambda_i(0), \qquad \text{for } S_n = \{A, B, ...\} \tag{4.13}$$

$$\lambda_a(n_a) = \frac{\sum_{i \in S_n} \lambda_i(n_a) P(n_i = n_a) \prod_{j \neq i} P(n_j \leq n_a)}{\prod_{i \in S_n} P(n_i \leq n_a) - \prod_{i \in S_n} P(n_i < n_a)}, \text{ for } n_a = 1, \ldots, N_a\text{-}1 \text{ and } j \in S_n = \{A, B, \ldots\} \tag{4.14}$$

So, by solving the Markov chain having load dependent rates $\lambda_1(n_1)$ and $\lambda_a(n_a)$, we can also obtain marginal probabilities of first stage kanbans.

4.2.4.2. Analysis of the Synchronization Station when Partial Disassembly is Allowed and the Overflow Item is Held. In holding overflows policy, we are faced with two different sub policies: regular item priority and overflow item priority. Isolated synchronization stations in these sub policies have the same structure (Figure 4.9), but their behaviors are not the same.

In Figure 4.9, n_{CA}, n_{CB}, n_A, n_B and n_1 represent the number of waiting components A and B and the waiting kanbans of Stage A, Stage B and Stage 1, respectively, for $n = 2$ component case. Here n_1 represents the number of Stage 1 kanbans and the number of regular items which are present at the synchronization station J_1, simultaneously. Basically, $n_{ci} - n_1$ represents the number of overflow components of type i for $i \in S_n = \{A, B, ...\}$, while n_1 represents the number of regular components of each type. So, the following properties always hold.

For the regular item priority case, it is true that

$$n_1 \leq \underset{i \in S_n}{Min}[n_{ci}] \qquad \text{for } S_n = \{A, B, ...\} \tag{4.15}$$

However, for the overflow item priority case, the following relation must be true

$$n_1 = \underset{i \in S_n}{Min}[n_{ci}] \qquad \text{for } S_n = \{A, B, ...\} \tag{4.16}$$

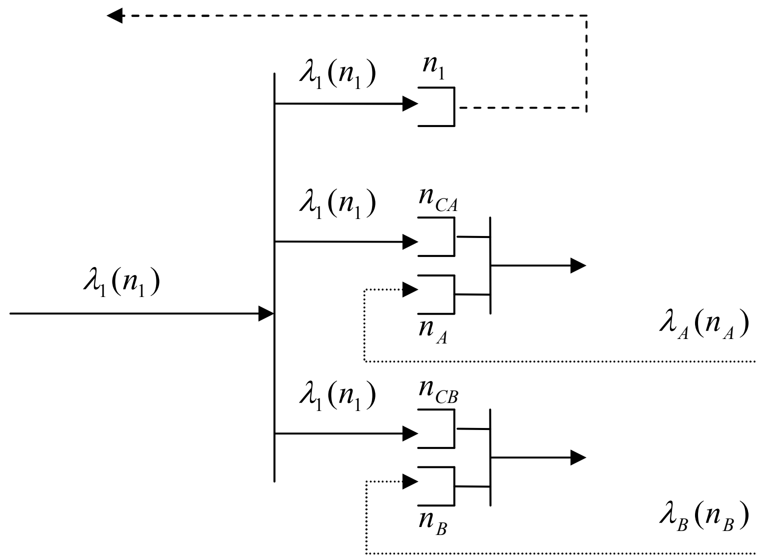

Figure 4.9. Synchronization station J_1 in case of holding overflows for n = 2 components in isolation mode

The difference in properties (4.15) and (4.16) reflects the difference of the behavior of the synchronization station according to the different priority policies. In case of overflow priority, a kanban of downstream stage i cannot be synchronized with a regular item if there is an overflow component of type i viz. if it is true that $n_{ci} - n_1 > 0$, until all overflows are consumed viz. until $n_{ci} - n_1 = 0$ becomes true. So, the minimum of the number of components of each type gives the number of Stage 1 kanbans. However, since in regular item priority case, Stage i kanban can be synchronized with a regular item even if there is an overflow item, the number of Stage 1 kanbans can be smaller than the minimum of the number of the different types of components.

We need properties (4.15) and (4.16) that relate the number of first stage kanbans and the number of components, because the load dependent arrivals feeding the component buffers depend on the number of Stage 1 kanbans present at the synchronization station J_1 rather than depending on the number of components (Figure 4.9).

Let B_{ci} denote the buffer capacity of type i components for $i \in S_n = \{A, B, ...\}$. For both priority policies, the following relation is true:

$$n_1 \le n_{ci} \le B_{ci} \qquad \text{for } i \in S_n = \{A, B, ...\} \tag{4.17}$$

Also, since the maximum number of components in a corresponding buffer cannot be smaller than the maximum number of Stage 1 kanbans, there is a constraint for selecting the buffer sizes of components and the kanban size of Stage 1. In fact, below inequality must be true for both priority policies

$$K_1 \le \underset{i \in S_n}{Min}[B_{ci}] \qquad \text{for } S_n = \{A, B, ...\} \tag{4.18}$$

To obtain the marginal probabilities of kanbans of downstream stages in J_1, instead of aggregation, we solve two dimensional Markov chains directly with load dependent arrival rates adjusted to reflect the above properties. For example for the Stage A kanban in J_1, the state of the continuous time Markov chain is (n_A, n_{CA}), where n_A is the number of kanbans and n_{CA} is the total number of components of type A including overflow and regular items. Let the load dependent arrival rates of component of type i for $i \in S_n = \{A, B, ...\}$ be denoted as $\lambda_{ci}(n_{ci})$ for $n_{ci} = 0,1,...,B_{ci}-1$. For the regular item priority case, this load dependent arrival rate can be calculated as a weighted average of the arrival rates $\lambda_1(n_1)$ by using the formula below.

$$\lambda_{ci}(n_{ci}) = \frac{\sum_{m=0}^{Min[n_{ci}, K_1-1]} \lambda_1(m) P(n_1 = m) \prod_{j \ne i} P(n_{cj} \ge m)}{\sum_{m=0}^{Min[n_{ci}, K_1-1]} P(n_1 = m) \prod_{j \ne i} P(n_{cj} \ge m)},$$

$$\text{for } n_{ci} = 0, 1, ..., B_{ci} - 1 \text{ and } i, j \in S_n = \{A, B, ...\} \tag{4.19}$$

On the other hand, for overflow item priority cases, the following formulas are used to estimate the load dependent arrival rates of components.

$$\lambda_{ci}(n_{ci}) = \sum_{m=0}^{n_{ci}-1} \lambda_1(m) \left[\prod_{j \ne i} P(n_{cj} \ge m) - \prod_{j \ne i} P(n_{cj} > m) \right] + \lambda_1(n_{ci}) \prod_{j \ne i} P(n_{cj} \ge n_{ci}),$$

$$\text{for } n_{ci} = 1, \ldots, K_1 - 1 \text{ and } i, j \in S_n = \{A, B, ...\} \tag{4.20}$$

$$\lambda_{ci}(K_1) = \sum_{m=0}^{K_1-1} \lambda_1(m) \left[\prod_{j \neq i} P(n_{cj} \geq m) - \prod_{j \neq i} P(n_{cj} > m) \right], \quad \text{for } n_{ci} = K_1 \text{ and } i, j \in S_n = \{A, B, ...\}$$

(4.21)

$$\lambda_{ci}(n_{ci}) = \frac{\sum_{m=0}^{K_1-1} \lambda_1(m) \left[\prod_{j \neq i} P(n_{cj} \geq m) - \prod_{j \neq i} P(n_{cj} > m) \right]}{1 - \prod_{j \neq i} P(n_{cj} > K_1)},$$

for $n_{ci} = K_1+1, \ldots, B_{ci}-1$ and $i, j \in S_n = \{A, B, ...\}$ (4.22)

The above formulas are the weighted sum of the arrival rates $\lambda_1(n_1)$ with weights differently estimated than that of the regular item priority case. The relationship of $n_1 \leq \text{Min}[n_{ci}]$ holds in the regular item priority case. However, in the overflow item priority case the equality of $n_1 = \text{Min}[n_{ci}]$ must be true. Therefore, our approximations are based on this equality for the overflow priority case. Moreover, in formula (4.22), we exclude the infeasible states by the corresponding probability in the denominator. In fact, all types of component queues can not exceed the kanban size of Stage 1 simultaneously in case of overflow priority, as a result of the property in (4.16).

Also, for both cases, following the relations in (4.15) and (4.16), it is apparent that $\lambda_{ci}(0) = \lambda_1(0)$ for $n_{ci} = 0$ and $i \in S_n = \{A, B, ...\}$. Since load dependent arrivals $\lambda_i(n_i)$ for $n_i = 0, 1, .., K_i-1$ are known and $\lambda_{ci}(n_{ci})$ for $n_{ci} = 0, 1, .., B_{ci}-1$, can be computed using the above formulas, the corresponding two dimensional Markov chains for stages $i \in S_n = \{A, B, ...\}$ can be analyzed. So, marginal probabilities of kanbans in the synchronization station J_1 that are used in the main algorithm can be obtained very easily. Also, the expected queue lengths in component buffers in the synchronization station J_1 can be calculated using the stationary probabilities obtained by the solution of Markov chain.

Class aggregation is also used to obtain the marginal probabilities of the Stage 1 kanbans in J_1. Let $\lambda_a^k(n_a)$ denote the load dependent arrival rates of permits when $n_1 = k$. For the regular item priority case, formulas (4.13) and (4.14) can be used for computation of these load dependent rates except for $\lambda_a^0(0)$ which is the arrival rate of permits when n_1

= 0 and $n_a = 0$. When $n_1 = 0$ and $n_a = 0$, the inequality $n_{ci} \geq 0$ must be true. Therefore, there can be as many as $n_{ci} - n_1$ overflow items for $i \in S_n = \{A, B, ...\}$. If $n_{ci} - n_1 > 0$ is true, the newly arrived "kanban *i*" does not have to wait since it can be synchronized with an overflow item. In this case, a new arrival of that kanban does not mean an arrival of a permit of the first stage kanbans. This condition is excluded from the load dependent arrival rate computation as follows:

$$\lambda_a^0(0) = \frac{\sum_{i \in S_n} \lambda_i(0) P(n_i = 0, n_{ci} = 0) \prod_{j \neq i} P(n_j = 0)}{\prod_{i \in S_n} P(n_i = 0) - \prod_{i \in S_n} P(n_{ci} > 0)}, \qquad \text{for } j \in S_n = \{A, B, ...\} \tag{4.23}$$

Probabilities in the formula can be obtained by the solution of Markov chains for stages $i \in S_n = \{A, B, ...\}$. For regular item priority case, using (4.13), (4.14) and (4.23) we can compute load dependent arrival rates of permits and solve the Markov chain to obtain marginal probabilities of the kanbans of Stage 1 in J_1.

For the overflow item priority case, computation of $\lambda_a^k(0)$ where $k>0$ must also be changed. Since we give the priority to the overflow items, synchronization and release of a first stage kanban can occur only when $n_{ci} = n_1$, viz. when there are no overflow items. Hence, we have to adjust load dependent arrival rates with the probability of having zero overflow items. For $k>0$, these rates are calculated using:

$$\lambda_a^k(0) = \frac{\sum_{i \in S_n} \lambda_i(0) P(n_{ci} = k) \prod_{j \neq i} P(n_{cj} \geq k)}{\prod_{i \in S_n} P(n_{ci} \geq k) - \prod_{i \in S_n} P(n_{ci} > k)}, \quad \text{for } k = 1, \ldots, K_1 \text{ and } j \in S_n = \{A, B, ...\} \tag{4.24}$$

Also for the overflow item priority case, by using load dependent rates computed by formulas (4.14), (4.23) and (4.24), we can solve the corresponding Markov chain to obtain marginal probabilities of the Stage 1 kanbans in J_1.

4.3. Accuracy of the Method

In this section, we give some numerical results showing the accuracy of the proposed approximations for the partial disassembly allowing policies. For this purpose, we have conducted several simulation experiments in order to show that approximation results which are obtained via analytical solution are accurate enough compared to the results obtained by stochastic simulation. The simulation results are obtained using Arena simulation software. In simulation experiments, number of replications is selected as 30 and each replication consists of 101,000 time units. The warm-up period is selected as 1,000 time units for each replication.

In the tables below, we report some examples from our experiments showing the accuracy of the proposed approximations by giving the expected values of the performance measures of the analyzed kanban controlled disassembly system in steady state. Mainly, these expected performance measures are the expected throughputs of different stages and the average queue lengths in different synchronization stations. In the tables below, TH and Q represent the expected throughput and the average queue lengths of the corresponding stages and the buffers, respectively. For instance, TH_1, TH_A and TH_B represent the expected throughputs of the Stage 1, Stage A and Stage B, respectively. Also, average queue lengths of return buffers (QR), average component queue lengths (QC_A and QC_B), average finished part queue lengths (QFP_A and QFP_B) and average backorder queue lengths (QB_A and QB_B) are reported. In fact, component queues denote the queues of disassembled components in the synchronization station J_1.

Since average queue lengths of workstations are more accurately estimated than that of synchronization stations in Baynat's multi-class algorithm, we only show the results of expected queue lengths in synchronization stations. Also, since the confidence intervals of the simulation results are very small and do not contribute any additional insights, they are omitted from the tables presenting the simulation results. Besides, since the accuracy of the approximation method that is used for analyzing the isolated synchronization station J1 when partial disassembly is not allowed is validated in Matta et al., (2005) and Baynat and Dallery (1995), we do not report additional simulation comparisons for this policy.

In Tables 4.1 to 4.6, PDNA represents the “partial disassembly not allowed” policy and PA-OD represents “partial disassembly allowed” policy with overflow disposal. Also, PDA-RP and PDA-OP represent the two different priority policies of the partial disassembly allowed with holding overflows policy: regular item priority and overflow item priority. Numerical results of the simulation and the approximation and the per cent relative error of the approximation method compared to the simulation results are reported for each policy. Approximation results are given under the title of “App” while the simulation results are reported under the title of “Sim”. Also, relative errors of the approximations are showed under the title of “RE”. Relative errors are calculated by using the formula below.

Relative Error = (Approximation - Simulation) / Simulation

We analyze two cases where the product is composed of two and three components respectively. Also, we consider three different arrival rate configurations. In the first configuration, the return arrival rate is smaller than the demand arrival rate of one component and larger than the demand arrival rate of other type of component (or types of components for the three components case). In the second arrival rate configuration, the return arrival rate is larger than the demand arrival rate of all components. Finally, in the third arrival rate configuration, the return arrival rate is smaller than the demand arrival rate of all components.

For the first configuration, we set the return arrival rate to 0.6 while the demand arrival rate of the more demanded part is 0.8 and that of the less demanded part(s) is 0.5. For the second configuration, we set the return arrival rate to 0.8 while the demand arrival rate of the more demanded part is 0.6 and that of the less demanded part(s) is 0.5. For the third configuration, we set the return arrival rate to 0.5 while the demand arrival rate of the more demanded part is 0.8 and that of the less demanded part(s) is 0.6. In all configurations, we select part-A as a more frequently demanded part than part-B (or part-B and part-C for the three component case). In the tables below, these configurations are represented by the arrival rates of returns, the demand arrival rate of the more demanded part and the demand arrival rate of the less demanded part respectively. For instance, 0.6 – 0.8 – 0.5 means that return arrival rate equals to 0,6 while demand arrival rate of the more demanded part equals to 0.8 and the demand arrival rate(s) of the less demanded part(s) equals to 0.5. In the three-component case, since the performance measures of the stages of the less demanded parts are the same due to the equality of the demand rates, their numerical results are shown in a single line in the tables.

In the experiments, average service rates for all machines are equal to one. Buffer capacities for returns and limits of backorder queues for each part are set to five. Results of these experiments are reported in the following tables with different kanban and component buffer sizes.

As reported in Tables 4.1, 4.2, 4.3 and 4.4, relative errors of approximation results compared to the simulation results are very small. Particularly, estimations of the expected throughputs are very accurate. In fact, most of the time, relative errors for the expected throughputs are smaller than one per cent. Even though the accuracy level in the estimation of average queue lengths is not as high as the accuracy level in the estimation of the expected throughputs, relative errors of them are also reasonably small. In fact, most of the time, relative errors for average queue lengths are smaller than 10 per cent.

However, since there is a two level approximation, we cannot distinguish the accuracy of our approximations from the estimation error of the global approximation algorithm. Here, the first level is the approximation of the multi class algorithm and the second level is the approximations that are proposed for the analysis of the synchronization station J_1 in isolation mode. So, to be able to assess the accuracy of our approximations separately, we conducted some numerical experiments where the continuous time Markov chain of the isolated synchronization station J_1 is solved exactly.

In Table 4.5 and 4.6, numerical results are reported for the policy of allowing partial disassembly with holding overflow items and overflow priority with different component buffer sizes, for a two-component case. Average service rates of machines, buffer capacity of returns, limits of backorder queues are the same as previous experiments. Kanban levels of each stage are set to five. In the tables, “Exact” and “App” represent the analysis type of the isolated synchronization J_1.

Table 4.1. Comparison of the simulation and the approximation results for the two components case with K_1=K_A=K_B=B_{CA}=B_{CB}=3

0.6 - 0.8 - 0.5	PDNA	PDA-OD			PDA-OP			PDA-RP		
	App	Sim	App	RE	Sim	App	RE	Sim	App	RE
TH_1	0.490	0.596	0.595	-0.2%	0.594	0.593	-0.2%	0.595	0.595	0.0%
TH_A	0.490	0.545	0.536	-1.6%	0.584	0.584	0.1%	0.581	0.581	0.0%
TH_B	0.490	0.464	0.459	-1.0%	0.484	0.488	0.8%	0.483	0.487	0.7%
QR	2.302	0.271	0.275	1.4%	0.346	0.356	2.8%	0.299	0.301	0.6%
QC_A	1.734	0.050	0.055	11.4%	0.279	0.264	-5.2%	0.260	0.255	-1.9%
QC_B	1.734	0.050	0.055	11.4%	1.085	1.073	-1.1%	1.064	1.052	-1.1%
QFP_A	0.073	0.096	0.108	12.6%	0.222	0.202	-9.0%	0.208	0.195	-6.2%
QFP_B	1.558	0.732	0.742	1.4%	1.471	1.455	-1.1%	1.458	1.438	-1.4%
QB_A	3.576	3.297	3.292	-0.1%	2.913	2.928	0.5%	2.954	2.954	0.0%
QB_B	0.502	1.342	1.387	3.4%	0.674	0.593	-12.1%	0.678	0.611	-9.9%
0.8 - 0.6 - 0.5	PDNA	PDA-OD			PDA-OP			PDA-RP		
	App	Sim	App	RE	Sim	App	RE	Sim	App	RE
TH_1	0.490	0.756	0.751	-0.7%	0.701	0.691	-1.5%	0.733	0.735	0.2%
TH_A	0.490	0.568	0.563	-1.0%	0.588	0.588	0.0%	0.586	0.586	0.0%
TH_B	0.490	0.489	0.488	-0.3%	0.498	0.498	0.0%	0.496	0.497	0.3%
QR	3.576	1.092	1.117	2.3%	1.978	1.940	-1.9%	1.492	1.365	-8.5%
QC_A	2.082	0.350	0.358	2.4%	1.532	1.478	-3.5%	1.443	1.376	-4.6%
QC_B	2.082	0.350	0.358	2.4%	1.888	1.848	-2.1%	1.826	1.768	-3.1%
QFP_A	0.409	0.931	0.910	-2.3%	1.538	1.490	-3.1%	1.500	1.433	-4.5%
QFP_B	1.558	1.332	1.324	-0.6%	1.970	1.941	-1.5%	1.957	1.908	-2.5%
QB_A	2.302	1.091	1.147	5.1%	0.513	0.521	1.5%	0.552	0.577	4.7%
QB_B	0.502	0.571	0.610	6.9%	0.200	0.199	-0.6%	0.210	0.219	4.0%
0.5 - 0.8 - 0.6	PDNA	PDA-OD			PDA-OP			PDA-RP		
	App	Sim	App	RE	Sim	App	RE	Sim	App	RE
TH_1	0.490	0.499	0.499	0.0%	0.498	0.499	0.1%	0.499	0.499	0.0%
TH_A	0.490	0.468	0.464	-0.8%	0.494	0.495	0.2%	0.494	0.494	0.0%
TH_B	0.490	0.454	0.450	-1.0%	0.481	0.486	0.9%	0.482	0.485	0.7%
QR	0.502	0.115	0.115	0.5%	0.124	0.125	0.7%	0.119	0.119	-0.6%
QC_A	0.582	0.013	0.014	14.7%	0.112	0.110	-2.0%	0.110	0.109	-0.1%
QC_B	0.582	0.013	0.014	14.7%	0.268	0.239	-10.7%	0.265	0.238	-10.3%
QFP_A	0.073	0.036	0.047	30.3%	0.082	0.077	-6.5%	0.080	0.076	-4.4%
QFP_B	0.409	0.187	0.207	10.6%	0.422	0.383	-9.3%	0.425	0.381	-10.4%
QB_A	3.576	3.757	3.726	-0.8%	3.553	3.549	-0.1%	3.554	3.552	-0.1%
QB_B	2.302	2.830	2.816	-0.5%	2.382	2.363	-0.8%	2.676	2.369	-11.4%

Table 4.2. Comparison of the simulation and the approximation results for the two components case K_1=K_A=K_B=B_{CA}=B_{CB}=5

0.6 - 0.8 - 0.5	PDNA	PDA-OD			PDA-OP			PDA-RP		
	App	Sim	App	RE	Sim	App	RE	Sim	App	RE
TH_1	0.497	0.598	0.598	0.0%	0.599	0.598	-0.1%	0.598	0.598	0.1%
TH_A	0.497	0.576	0.572	-0.7%	0.596	0.596	0.0%	0.595	0.595	0.0%
TH_B	0.497	0.477	0.478	0.1%	0.494	0.495	0.1%	0.491	0.495	0.6%
QR	2.122	0.094	0.094	-0.3%	0.125	0.126	0.7%	0.100	0.098	-1.7%
QC_A	3.200	0.030	0.032	8.4%	0.205	0.198	-3.3%	0.206	0.194	-5.7%
QC_B	3.200	0.030	0.032	8.4%	2.180	2.164	-0.7%	2.156	2.149	-0.3%
QFP_A	0.116	0.251	0.246	-1.9%	0.408	0.374	-8.4%	0.408	0.369	-9.5%
QFP_B	3.541	1.828	1.798	-1.6%	3.276	3.261	-0.5%	3.264	3.250	-0.4%
QB_A	3.519	2.967	3.001	1.2%	2.746	2.787	1.5%	2.780	2.798	0.7%
QB_B	0.161	0.786	0.829	5.4%	0.289	0.254	-11.9%	0.298	0.259	-13.2%
0.8 - 0.6 - 0.5	PDNA	PDA-OD			PDA-OP			PDA-RP		
	App	Sim	App	RE	Sim	App	RE	Sim	App	RE
TH_1	0.497	0.774	0.770	-0.4%	0.707	0.695	-1.6%	0.760	0.763	0.4%
TH_A	0.497	0.585	0.582	-0.4%	0.596	0.596	0.1%	0.594	0.595	0.3%
TH_B	0.497	0.497	0.496	-0.2%	0.500	0.500	0.0%	0.500	0.499	-0.1%
QR	3.519	0.689	0.705	2.3%	1.828	1.828	0.0%	0.977	0.843	-13.7%
QC_A	3.923	0.533	0.555	4.2%	3.041	2.962	-2.6%	2.837	2.736	-3.6%
QC_B	3.923	0.533	0.555	4.2%	3.670	3.615	-1.5%	3.522	3.470	-1.5%
QFP_A	0.818	2.291	2.189	-4.4%	3.378	3.332	-1.4%	3.324	3.241	-2.5%
QFP_B	3.541	2.993	2.949	-1.5%	3.957	3.936	-0.5%	3.930	3.904	-0.7%
QB_A	2.122	0.553	0.603	9.1%	0.177	0.173	-2.3%	0.202	0.206	2.4%
QB_B	0.161	0.216	0.234	8.2%	0.042	0.042	-1.1%	0.046	0.047	3.3%
0.5 - 0.8 - 0.6	PDNA	PDA-OD			PDA-OP			PDA-RP		
	App	Sim	App	RE	Sim	App	RE	Sim	App	RE
TH_1	0.497	0.499	0.500	0.2%	0.500	0.500	0.0%	0.500	0.500	0.0%
TH_A	0.497	0.490	0.490	-0.1%	0.499	0.499	0.0%	0.499	0.499	0.0%
TH_B	0.497	0.476	0.475	-0.2%	0.493	0.495	0.4%	0.493	0.494	0.4%
QR	0.161	0.025	0.028	12.1%	0.030	0.029	-0.3%	0.030	0.028	-6.4%
QC_A	0.483	0.004	0.004	4.6%	0.048	0.046	-5.2%	0.046	0.046	-1.9%
QC_B	0.483	0.004	0.004	4.6%	0.299	0.254	-14.9%	0.297	0.254	-14.5%
QFP_A	0.116	0.080	0.093	16.8%	0.128	0.119	-6.9%	0.127	0.119	-6.1%
QFP_B	0.818	0.453	0.476	5.3%	0.842	0.774	-8.1%	0.817	0.773	-5.4%
QB_A	3.519	3.597	3.570	-0.7%	3.503	3.505	0.1%	3.496	3.505	0.3%
QB_B	2.122	2.488	2.459	-1.2%	2.159	2.163	0.2%	2.183	2.164	-0.9%

Table 4.3. Comparison of the simulation and the approximation results for the two components case K_1=K_A=K_B=B_{CA}=B_{CB}=10

0.6 - 0.8 - 0.5	PDNA	PDA-OD			PDA-OP			PDA-RP		
	App	Sim	App	RE	Sim	App	RE	Sim	App	RE
TH_1	0.499	0.600	0.600	0.0%	0.600	0.600	0.0%	0.600	0.600	0.0%
TH_A	0.499	0.596	0.595	-0.1%	0.600	0.600	-0.1%	0.600	0.600	0.0%
TH_B	0.499	0.493	0.493	0.0%	0.499	0.499	0.1%	0.499	0.499	0.0%
QR	2.028	0.008	0.007	-6.4%	0.010	0.009	-5.0%	0.007	0.007	-5.2%
QC_A	7.441	0.006	0.007	25.7%	0.074	0.063	-13.8%	0.073	0.063	-12.7%
QC_B	7.441	0.006	0.007	25.7%	5.915	5.872	-0.7%	5.876	5.870	-0.1%
QFP_A	0.152	0.518	0.520	0.3%	0.655	0.618	-5.7%	0.656	0.617	-5.9%
QFP_B	8.628	5.374	5.375	0.0%	8.490	8.455	-0.4%	8.463	8.454	-0.1%
QB_A	3.499	2.771	2.769	-0.1%	2.701	2.719	0.7%	2.712	2.719	0.3%
QB_B	0.037	0.287	0.286	-0.2%	0.041	0.041	-1.3%	0.046	0.041	-10.9%
0.8 - 0.6 - 0.5	PDNA	PDA-OD			PDA-OP			PDA-RP		
	App	Sim	App	RE	Sim	App	RE	Sim	App	RE
TH_1	0.499	0.792	0.791	-0.1%	0.709	0.697	-1.7%	0.785	0.789	0.4%
TH_A	0.499	0.597	0.596	-0.1%	0.600	0.600	0.0%	0.600	0.600	0.0%
TH_B	0.499	0.499	0.500	0.1%	0.500	0.500	0.1%	0.500	0.500	0.1%
QR	3.499	0.225	0.236	4.9%	1.741	1.773	1.9%	0.352	0.276	-21.6%
QC_A	8.852	0.797	0.862	8.1%	7.533	7.450	-1.1%	7.127	7.115	-0.2%
QC_B	8.852	0.797	0.862	8.1%	8.541	8.485	-0.7%	8.332	8.318	-0.2%
QFP_A	1.558	6.580	6.371	-3.2%	8.411	8.402	-0.1%	8.386	8.361	-0.3%
QFP_B	8.628	7.792	7.703	-1.1%	8.992	8.990	0.0%	8.990	8.985	-0.1%
QB_A	2.028	0.125	0.135	8.5%	0.013	0.012	-6.1%	0.015	0.015	-4.3%
QB_B	0.037	0.020	0.022	10.6%	0.001	0.001	-15.0%	0.001	0.001	-10.8%
0.5 - 0.8 - 0.6	PDNA	PDA-OD			PDA-OP			PDA-RP		
	App	Sim	App	RE	Sim	App	RE	Sim	App	RE
TH_1	0.499	0.500	0.500	0.0%	0.500	0.500	0.0%	0.500	0.500	0.0%
TH_A	0.499	0.499	0.499	0.0%	0.500	0.500	0.0%	0.500	0.500	0.0%
TH_B	0.499	0.493	0.493	0.0%	0.499	0.499	0.0%	0.499	0.499	-0.1%
QR	0.037	0.001	0.001	-4.5%	0.001	0.001	-3.8%	0.001	0.001	5.5%
QC_A	0.371	0.0002	0.0002	-0.2%	0.005	0.004	-18.2%	0.004	0.004	-15.5%
QC_B	0.371	0.0002	0.0002	-0.2%	0.227	0.223	-1.8%	0.239	0.223	-6.6%
QFP_A	0.152	0.149	0.147	-1.5%	0.156	0.154	-1.5%	0.158	0.154	-2.7%
QFP_B	1.558	1.110	1.103	-0.7%	1.519	1.513	-0.4%	1.556	1.513	-2.7%
QB_A	3.499	3.493	3.498	0.1%	3.493	3.493	0.0%	3.484	3.493	0.2%
QB_B	2.028	2.145	2.157	0.5%	2.036	2.033	-0.1%	2.028	2.033	0.2%

Table 4.4. Comparison of the simulation and the approximation results for the three components case with K_1=K_A=K_B=K_C=B_{CA}=B_{CB}=B_{CC}=5

0.6 - 0.8 - 0.5	PDNA	PDA-OD			PDA-OP			PDA-RP		
	App	Sim	App	RE	Sim	App	RE	Sim	App	RE
TH_1	0.469	0.600	0.599	-0.2%	0.598	0.598	0.0%	0.599	0.598	-0.1%
TH_A	0.469	0.574	0.570	-0.7%	0.595	0.595	0.1%	0.595	0.595	0.0%
TH_{B-C}	0.469	0.477	0.477	0.0%	0.493	0.495	0.3%	0.493	0.495	0.4%
QR	2.511	0.091	0.091	0.2%	0.112	0.115	2.1%	0.102	0.098	-4.1%
QC_A	3.557	0.009	0.009	9.8%	0.207	0.196	-5.6%	0.214	0.194	-9.3%
QC_{B-C}	3.557	0.009	0.009	9.8%	2.161	2.156	-0.3%	2.148	2.149	0.1%
QFP_A	0.080	0.219	0.237	8.5%	0.410	0.371	-9.5%	0.416	0.369	-11.2%
QFP_{B-C}	2.108	1.781	1.773	-0.5%	3.271	3.254	-0.5%	3.269	3.250	-0.6%
QB_A	3.688	3.022	3.017	-0.2%	2.777	2.793	0.6%	2.780	2.798	0.7%
QB_{B-C}	0.979	0.808	0.839	3.8%	0.294	0.257	-12.7%	0.295	0.259	-12.3%
0.8 - 0.6 - 0.5	PDNA	PDA-OD			PDA-OP			PDA-RP		
	App	Sim	App	RE	Sim	App	RE	Sim	App	RE
TH_1	0.469	0.780	0.780	0.0%	0.743	0.732	-1.5%	0.759	0.763	0.5%
TH_A	0.469	0.581	0.578	-0.5%	0.596	0.596	0.0%	0.595	0.595	0.0%
TH_{B-C}	0.469	0.496	0.495	-0.2%	0.500	0.500	-0.1%	0.497	0.499	0.6%
QR	3.688	0.540	0.528	-2.1%	1.354	1.370	1.2%	0.986	0.849	-13.9%
QC_A	4.055	0.123	0.136	10.5%	2.889	2.820	-2.4%	2.815	2.737	-2.8%
QC_{B-C}	4.055	0.123	0.136	10.5%	3.566	3.523	-1.2%	3.530	3.471	-1.7%
QFP_A	0.560	2.006	1.906	-5.0%	3.332	3.278	-1.6%	3.321	3.242	-2.4%
QFP_{B-C}	2.108	2.762	2.691	-2.6%	3.939	3.917	-0.6%	3.943	3.904	-1.0%
QB_A	2.511	0.635	0.728	14.6%	0.195	0.193	-1.1%	0.198	0.206	4.3%
QB_{B-C}	0.979	0.261	0.293	12.4%	0.045	0.045	-0.9%	0.045	0.047	4.7%
0.5 - 0.8 - 0.6	PDNA	PDA-OD			PDA-OP			PDA-RP		
	App	Sim	App	RE	Sim	App	RE	Sim	App	RE
TH_1	0.491	0.499	0.500	0.2%	0.500	0.500	-0.2%	0.500	0.500	0.0%
TH_A	0.491	0.489	0.489	0.0%	0.500	0.499	-0.1%	0.500	0.499	-0.1%
TH_{B-C}	0.491	0.475	0.475	0.0%	0.493	0.494	0.3%	0.494	0.494	0.1%
QR	0.384	0.028	0.028	0.2%	0.029	0.028	-2.3%	0.025	0.028	11.4%
QC_A	0.988	0.001	0.001	-7.2%	0.049	0.046	-7.1%	0.048	0.046	-4.3%
QC_{B-C}	0.988	0.001	0.001	-7.2%	0.304	0.254	-16.4%	0.278	0.254	-8.8%
QFP_A	0.107	0.086	0.093	7.4%	0.133	0.119	-10.7%	0.138	0.119	-13.8%
QFP_{B-C}	0.755	0.451	0.474	5.2%	0.852	0.773	-9.3%	0.829	0.773	-6.8%
QB_A	3.559	3.584	3.572	-0.3%	3.489	3.505	0.5%	3.479	3.505	0.8%
QB_{B-C}	2.214	2.479	2.461	-0.7%	2.154	2.164	0.5%	2.165	2.164	0.0%

Table 4.5. Comparison of the simulation and the approximation results for the policy of PDA-OP with different component buffer sizes (unbalanced demand case)

0.6 - 0.8 - 0.5	$B_{CA} = B_{CB} = 10$					$B_{CA} = B_{CB} = 20$				
	Sim	Exact	RE	App	RE	Sim	Exact	RE	App	RE
TH_1	0.598	0.597	0.0%	0.597	0.0%	0.597	0.597	0.0%	0.597	0.0%
TH_A	0.598	0.597	0.0%	0.598	0.1%	0.597	0.597	0.0%	0.597	0.1%
TH_B	0.497	0.499	0.3%	0.499	0.3%	0.499	0.500	0.2%	0.500	0.2%
QR	0.154	0.139	-10.0%	0.141	-8.9%	0.149	0.140	-5.9%	0.140	-5.9%
QC_A	0.246	0.207	-15.8%	0.227	-7.4%	0.236	0.204	-13.7%	0.207	-12.2%
QC_B	5.986	5.990	0.1%	5.989	0.0%	15.282	15.219	-0.4%	15.150	-0.9%
QFP_A	0.433	0.380	-12.1%	0.385	-11.0%	0.427	0.380	-11.0%	0.380	-10.8%
QFP_B	3.744	3.762	0.5%	3.762	0.5%	3.990	3.996	0.1%	3.994	0.1%
QB_A	2.747	2.775	1.0%	2.767	0.7%	2.758	2.776	0.6%	2.774	0.6%
QB_B	0.132	0.093	-29.7%	0.093	-29.7%	0.043	0.035	-17.4%	0.036	-16.7%

0.8 - 0.6 - 0.5	$B_{CA} = B_{CB} = 10$					$B_{CA} = B_{CB} = 20$				
	Sim	Exact	RE	App	RE	Sim	Exact	RE	App	RE
TH_1	0.605	0.606	0.1%	0.612	1.1%	0.597	0.598	0.1%	0.599	0.4%
TH_A	0.597	0.597	0.1%	0.598	0.1%	0.597	0.597	0.1%	0.597	0.1%
TH_B	0.500	0.500	0.0%	0.500	0.0%	0.500	0.500	-0.1%	0.500	-0.1%
QR	2.779	2.710	-2.5%	2.654	-4.5%	2.795	2.775	-0.7%	2.758	-1.3%
QC_A	3.897	3.815	-2.1%	4.078	4.6%	3.521	3.441	-2.3%	3.532	0.3%
QC_B	6.886	6.782	-1.5%	7.235	5.1%	15.365	15.279	-0.6%	15.646	1.8%
QFP_A	3.484	3.453	-0.9%	3.461	-0.7%	3.466	3.437	-0.8%	3.438	-0.8%
QFP_B	4.022	4.011	-0.3%	4.017	-0.1%	4.027	4.029	0.1%	4.031	0.1%
QB_A	0.140	0.135	-3.4%	0.133	-4.9%	0.146	0.139	-4.6%	0.139	-4.8%
QB_B	0.030	0.031	1.8%	0.030	-0.7%	0.029	0.028	-2.8%	0.028	-3.7%

0.5 - 0.8 - 0.6	$B_{CA} = B_{CB} = 10$					$B_{CA} = B_{CB} = 20$				
	Sim	Exact	RE	App	RE	Sim	Exact	RE	App	RE
TH_1	0.500	0.500	0.0%	0.500	0.0%	0.499	0.500	0.1%	0.500	0.1%
TH_A	0.500	0.500	0.0%	0.500	0.0%	0.499	0.500	0.1%	0.500	0.1%
TH_B	0.497	0.499	0.3%	0.499	0.3%	0.499	0.500	0.1%	0.500	0.2%
QR	0.032	0.030	-5.3%	0.030	-4.6%	0.031	0.030	-2.3%	0.030	-1.4%
QC_A	0.057	0.050	-11.1%	0.053	-6.3%	0.056	0.051	-9.2%	0.054	-3.6%
QC_B	0.608	0.428	-29.5%	0.433	-28.7%	0.902	0.516	-42.8%	0.527	-41.6%
QFP_A	0.132	0.120	-8.8%	0.121	-8.4%	0.131	0.120	-7.9%	0.121	-7.5%
QFP_B	0.965	0.841	-12.8%	0.843	-12.6%	1.019	0.860	-15.7%	0.863	-15.4%
QB_A	3.495	3.502	0.2%	3.501	0.2%	3.501	3.502	0.1%	3.501	0.0%
QB_B	2.071	2.098	1.3%	2.096	1.2%	2.031	2.080	2.4%	2.077	2.3%

Table 4.6. Comparison of the simulation and the approximation results for the policy of PDA-OP with different component buffer sizes (balanced demand case)

$B_{CA} = B_{CB} =$ 10	$\gamma = 0.8.\ \lambda_A=\lambda_B=0.9$					$\gamma = 0.9.\ \lambda_A=\lambda_B=0.8$				
	Sim	Exact	RE	App	RE	Sim	Exact	RE	App	RE
TH_1	0.773	0.771	-0.3%	0.769	-0.4%	0.791	0.789	-0.2%	0.787	-0.5%
$TH_{A\text{-}B}$	0.766	0.763	-0.4%	0.767	0.1%	0.765	0.762	-0.4%	0.764	-0.1%
QR	0.701	0.696	-0.6%	0.718	2.4%	1.807	1.711	-5.3%	1.734	-4.0%
$QC_{A\text{-}B}$	1.463	1.421	-2.9%	1.623	10.9%	3.438	3.256	-5.3%	3.681	7.1%
$QFP_{A\text{-}B}$	0.929	0.853	-8.2%	0.885	-4.7%	2.113	1.988	-5.9%	2.038	-3.6%
$QB_{A\text{-}B}$	1.921	1.970	2.6%	1.931	0.5%	0.777	0.838	7.9%	0.804	3.5%
$B_{CA} = B_{CB} =$ 20	$\gamma = 0.8.\ \lambda_A=\lambda_B=0.9$					$\gamma = 0.9.\ \lambda_A=\lambda_B=0.8$				
	Sim	Exact	RE	App	RE	Sim	Exact	RE	App	RE
TH_1	0.771	0.769	-0.2%	0.767	-0.5%	0.777	0.775	-0.4%	0.775	-0.3%
$TH_{A\text{-}B}$	0.770	0.768	-0.3%	0.774	0.5%	0.768	0.766	-0.3%	0.769	0.1%
QR	0.735	0.723	-1.6%	0.754	2.6%	1.936	1.859	-3.9%	1.856	-4.1%
$QC_{A\text{-}B}$	1.950	1.935	-0.8%	2.451	25.7%	5.571	5.211	-6.5%	6.493	16.6%
$QFP_{A\text{-}B}$	0.976	0.896	-8.3%	0.950	-2.7%	2.175	2.076	-4.5%	2.155	-0.9%
$QB_{A\text{-}B}$	1.877	1.918	2.2%	1.853	-1.3%	0.727	0.777	6.9%	0.725	-0.3%
$B_{CA} = B_{CB} =$ 50	$\gamma = 0.8.\ \lambda_A=\lambda_B=0.9$					$\gamma = 0.9.\ \lambda_A=\lambda_B=0.8$				
	Sim	Exact	RE	App	RE	Sim	Exact	RE	App	RE
TH_1	0.771	0.769	-0.2%	0.767	-0.5%	0.770	0.768	-0.2%	0.769	-0.1%
$TH_{A\text{-}B}$	0.771	0.769	-0.2%	0.776	0.7%	0.768	0.767	-0.2%	0.769	0.0%
QR	0.729	0.726	-0.4%	0.761	4.3%	1.985	1.925	-3.1%	1.916	-3.5%
$QC_{A\text{-}B}$	2.127	2.086	-2.0%	2.821	32.6%	9.839	8.377	-14.9%	11.808	20.0%
$QFP_{A\text{-}B}$	0.984	0.902	-8.4%	0.964	-2.0%	2.210	2.115	-4.3%	2.222	0.6%
$QB_{A\text{-}B}$	1.871	1.911	2.1%	1.836	-1.8%	0.712	0.751	5.5%	0.681	-4.3%

It is observed that even though most of the estimations are reasonably accurate, there are some large differences between the simulation and analytical results in some of the performance measures of the unbalanced demand case. However, it can be seen that numerical results do not show significant difference with respect to the analysis type of the isolated synchronization station. In fact, exact and approximate analyses of the isolated synchronization station give approximately same results, although these results can be significantly different than the simulation results. So, it is observed that large errors are not due to our proposed approximations; rather these errors are caused by the approximation in the global algorithm.

4.4. Comparison of the Policies

In this section, we make an initial comparison of different disassembly policies with respect to the steady state performance measures reported in the previous section.

We see that by allowing partial disassembly with overflow disposal we obtain a higher service level in some of the cases. In Tables 4.1, 4.2, 4.3 and 4.4, this increase in service level can be observed as a decrease in the average level of backorder queues of the parts whose demand rate is lower than the return rate. Also, the average level of backorder queue of the part, whose demand rate is higher than that of the other part type, tends to decrease with allowing partial disassembly and disposing the overflows.

However, in some cases, the policy of allowing partial disassembly with disposal of the overflows can also cause a reduction in the service level. As observed in Tables 4.1, 4.2, 4.3 and 4.4, average level of backorder queues of the part, whose demand rate is lower than that of the other part type, tends to increase with allowing partial disassembly and disposing the overflows. This negative effect is due to the asynchronous behavior of the system that results in destruction of the limited supply by the disposal of the overflow items. In these situations, holding overflows becomes beneficial.

Another observation is that giving priority to different types of items (overflow vs. regular) in case of holding overflows does not change most of the performance indicators significantly. In case of giving priority to the overflow items, we are faced with the decrease of the first stage throughput and increase of the average level of return queue. This change occurs only when the return rate is higher than the demand rates of components. In fact, by giving priority to overflows, the imbalance between first and downstream stages is decreased. Since this gap represents the disposal of overflows, the disposal rate of overflows is also decreased. However, since the throughput of the first stage is decreased; the disposal rate of returns tends to increase in case of overflow priority.

Moreover, the negative effects of the larger number of components- such as the decrease of the production rate and the increase of the backorder level- are more apparent when partial disassembly is not allowed because of the necessity of the complete synchronization. In fact, when the number of components increases, the throughputs of all stages decrease when partial disassembly is not allowed. On the other hand, when partial disassembly is allowed, throughputs of all stages do not change significantly. Also, the gap between the first stage throughput and throughput of downstream stages remains unchanged. Actually, this gap between the first and downstream stages represents the disposal rates of overflow items.

4.5. Total Cost Function

The comparison in the previous section is only based on the steady state performance measures. However, in order to make a complete comparison, we have to aggregate each performance measure in a cost function with unit costs showing their relative importance. For instance, increasing buffer sizes results in an increase in the service level, but also causes an increase in the total inventory level. So, there is a trade off between the inventory holding cost and the total backorder cost. In this section, we develop a total cost function that can reflect such trade offs.

Our cost function mainly consists of two groups of cost components, which are calculated with expected throughputs and the average queue lengths respectively. In fact, costs that are calculated with average queue lengths are backorder costs and inventory holding costs. In order to evaluate the total backorder cost of a given system, we simply multiply the unit backorder cost with the average queue length of the backorder queue that is obtained by the analytical method. Also, to evaluate the total inventory holding cost, we multiply the unit holding cost with the average queue length in the corresponding buffer. The other group viz. cost components calculated with expected throughputs are the following: lost sales (manufacturing) cost, disassembly cost, refurbishing cost, acquisition cost, disposal cost.

Disassembly cost is the operation cost of the first stage of the analyzed disassembly system. So, this cost component is calculated as a multiplication of the first stage throughput with the unit operation cost of the disassembly machine. Likewise, refurbishing costs are calculated as a multiplication of the throughput of the corresponding downstream stage with the unit operation cost of the respective refurbishing machine.

Lost sales of the disassembly process are satisfied by the manufacturing or external procurement. Since lost sales rate of a part can be expressed as the difference between demand arrival rate of that part and the throughput of the corresponding downstream stage, lost sales cost can be calculated as a multiplication of this difference with the unit manufacturing cost of that part.

Acquisition cost includes all expenses for receiving an end of life product return such as transportation cost, collection cost etc. In our cost function, this cost calculated as the multiplication of the effective return arrival rate with the unit acquisition cost of a product return. In our model, the effective return arrival rate equals to the throughput of the first stage.

The difference between the return arrival rate and the throughput of the first stage represents the disposal rate of the product returns. In our cost function, product disposal cost is calculated as a multiplication of this difference with the unit disposal cost of the product return. So, the product disposal cost is the expense of rejecting a returned product. However, although disposition occurs only at the input port of the system when partial disassembly is not allowed, when partial disassembly is allowed, dismantled components can also be disposed whenever they become overflow items because of the excess inventory level. Since the disposal rate of a component equals to the difference between the throughputs of the first stage and the respective downstream stage, the disposal cost of a part is calculated as multiplication of this difference with the unit disposal cost of a part.

Actually, disposal cost can be positive or negative. In fact, if the end of life product has a salvage value, then it can be sold to a recycling facility. In this situation, disposal cost will be negative since it represents revenue. However, if it does not have a salvage value, then the operational expenses of the disposition process of the product return (or the dismantled part) results in a positive disposal cost for that item.

For the unit costs terms, following notation is used:

c_p : acquisition (or transportation) cost of a core.

c_d : operation cost of the disassembly of a core

c_{ri} : operation cost of the refurbishing a part *i*

d_p : disposal cost of a core

d_i : disposal cost of a part *i*

c_{mi} : manufacturing (or procurement) cost of a part *i*

b_i : backorder cost of a demand of part *i*

H_p : unit holding cost for the product per unit time

H_{di} : unit holding cost for the dismantled component of type *i* per unit time

H_{ri} : unit holding cost for the refurbished component of type *i* per unit time

When the product is composed of *n* types of components and for $i \in S_n = \{A, B, ...\}$, long run average cost terms can be expressed as follows.

Table 4.7. Components of the total cost function

Disassembly cost = $c_d\ TH_1$
Refurbishing cost = $\sum_i c_{ri} TH_i$
Lost sales cost = $\sum_i c_{mi}(\lambda_i - TH_i)$
Disposal cost = $d_p(\gamma - TH_1) + \sum_i d_i(TH_1 - TH_i)$
Acquisition cost = $c_p\ TH_1$
Backorder cost = $\sum_i b_i \cdot QB_i$
Holding cost = $H_p(QR + QWIP_1) + \sum_i \left[H_{di}(QC_i + QWIP_i) + H_{ri} \cdot QFP_i\right]$

Here $QWIP_i$ represents average work in process inventory of the stage *i*. Summation of the above cost terms gives the long run average total cost of the analyzed disassembly system.

4.5.1. Holding Cost Evaluation

In holding cost evaluation, determining the unit holding costs rates needs special attention. Because of the unique characteristics of the disassembly system, traditional rules that are established for determining the inventory holding cost rates for the traditional manufacturing systems are not suitable for the disassembly systems. There are two important studies focusing on the determination of the holding cost rates for the disassembly systems. One of them is the work of Teunter (2001). He uses net revenues associated with parts to evaluate respective unit holding cost rates of the corresponding parts. However, main danger in this approach is assigning more costs to the parts that give higher net-revenues. So, by using this approach, we can state wrong conclusions in comparison of different policies. In fact, this method can only be used when the disassembly strategy is fixed. The second work is the study of Akcali and Bayindir (2006). They are setting the unit holding cost rate of the core in a traditional way, but the unit holding cost rate of the parts are differently evaluated. Since the most appropriate method for the analyzed disassembly system is that of these authors, we use their holding cost setting rules that are proposed in Akcali and Bayindir (2006). In this section, we describe their methods and adaptation of these methods to the analyzed disassembly system.

Mainly, the unit inventory holding cost consists of two different parts: out-of-pocket cost and the opportunity cost (cost of capital). Out of pocket cost involves the expenses related to the physical storage and handling of the materials viz. product and parts. On the other hand, opportunity cost represents the amount of the capital tied up in the inventory. In fact, it is a function of the value of the product/part and inventory carrying charge α used by the facility.

Let h_p denote the out of pocket inventory holding cost for the products, and let h_{di} and h_{ri} denote out of pocket inventory holding cost for the dismantled parts and the refurbished parts of type i, respectively. For a product, we can calculate the total unit holding cost as $H_p=h_p+\alpha c_p$ under the traditional way of setting inventory holding cost rates, where c_p is the acquisition cost of the end of life product. However, for the disassembled parts, specifying the unit holding cost rate is not straightforward, because of the difficulty of determining the added value of the parts. In fact, cost of capital of the parts is represented by the unit operation cost of the disassembly machine. Here the question is how to allocate this cost to the each part.

Let f_i denote the allocation fraction of the added value for the part type i. Then the unit inventory holding cost for the dismantled part type i is calculated as $H_{di} = h_{di} + \alpha\ [(c_p + c_d)\ f_i]$ and, also, the unit inventory holding cost for the refurbished part type i is given by $H_{ri} = h_{ri} + \alpha\ [(c_p + c_d)\ f_i + c_{ri}]$.

There are three approaches for determining the added value allocation fraction: physical measure based, market value based and recovered value based approaches (Akcali and Bayindir, 2006). The approach that is most suitable for the analyzed system is the first one viz. physical measure based approach. In that approach, there are two alternatives for determining the allocation fractions. First option is to equally divide the added value to the parts. That is, when the product is composed of n types of components, $f_i = 1 / n$ for $i \in S_n = \{A, B, ...\}$. The other option is dividing the added value to the parts proportionally to their physical characteristics such as volume or weights. In some cases, the out of pocket inventory holding costs of the parts is proportional to the relative volume or weight of the components. In these cases, allocation fraction of the opportunity cost can be calculated depending on the ratio of out-of-pocket inventory holding costs of the parts. That is we can set $f_i = h_{di} / h_p$ for $i \in S_n = \{A, B, ...\}$.

4.6. Optimization of the Parameters

Although the total cost function gives a good comparison tool for the different disassembly policies, if the parameter values such as kanban levels are not optimized according to the selected policy then the comparison of the total costs of the different polices can yield wrong conclusions. So, the value of the each parameter of the system must be optimized based on the minimization of the total cost function. For this type of optimization problems, mostly used approach is the enumeration. For instance, in Duri *et al*. (2000) and in Koukoumialos and Liberopoulos (2005), an exhaustive enumeration technique is used for finding the optimal values of the kanban levels. However, in order to be able to perform a complete search, the number of different configuration alternatives of the parameter values must not be so large. In fact, in our system, we have to optimize kanban levels, buffer sizes and backorder limits. So, when the product is composed of n types of components, we have to optimize $2n + 2$ parameters if partial disassembly is not allowed or allowed with disposal of overflows. This number increases to $3n + 2$ when overflows are held, because of the necessity of the optimization of the buffer sizes of the disassembled components. So, in this study, we prefer to find local minimum of the total cost function instead of finding the global minimum. Comparison of different policies is made based on the local minimums of the cost of the different policies.

For finding a local minimum of the total cost, we propose a heuristic search procedure. In this procedure, we optimize each parameter individually by changing its value and observing the effect of this change on the total cost function while the other parameter values are held constant. When we find an optimal value of a parameter, we optimize the next parameter using this optimized value. Finally, we stop the algorithm when the total cost and the parameter values converge to local optimums. Lastly, we make a neighborhood search to verify that the obtained solution is a local minimum. Steps of our heuristic procedure are the following. (Step 3 is only for the policies of PDA-OP and PDA-RP)

Table 4.8. Steps of the heuristic search procedure

Step 1. Set initial values for the kanban levels of downstream stages and backorder limits
Step 2. Optimize return buffer and the kanban level of the first stage
Step 3. Optimize buffer sizes of each component
Step 4. Optimize kanban levels of each downstream stage
Step 5. Optimize backorder limits of each component
Step 6. Go to Step 2 until the convergence
Step 7. Make a neighborhood search for the optimality check

We explain these steps in more detail in the following search algorithms. We propose two search algorithms to find the optimal parameter configuration for different disassembly policies. For the policies of not allowing partial disassembly and allowing with overflow disposal (PDNA and PDA-OD), Algorithm 1 and for the other policies, viz.

allowing partial disassembly policies with holding overflows (PDA-OP and PDA-RP), Algorithm 2 is used for the optimization of the parameter values. The only difference between two algorithms is the need of optimizing the buffer capacities of the disassembled components that we are faced with when partial disassembly is allowed and overflows are held. These two algorithms are given below for the two-component case. For the cases of larger number of components than two, same algorithms are used with small modifications. For instance, for the three component case, the only modification needed is the addition of the loops for the optimization of the parameters related to the third component in corresponding steps of the procedure.

Here B and BL_i denote the buffer capacity of returns and backorder limit of demands of part i, respectively. Also, Z is the total cost for the current parameter configuration and Z_{opt} represents the optimal (minimum) total cost. Parameter values with asterisk i.e. K_1^* represent the optimized value of that parameter.

<u>Search Algorithm 1</u>

Set initial K_i and BL_i for $i \in S_2 = \{A, B\}$
Set Z_{opt} to a very large number
For e = 1 to 10
    For K_1 = 1 to 20
        For B = 0 to 20
            Calculate total cost Z by using analytical method
            Optimality check: If $Z < Z_{opt}$ then $Z_{opt} = Z$, $K_1^* = K_1$ and $B^* = B$
            Convexity check: Set $Z(B) = Z$ and If $Z(B) > Z(B\ -1)$ Exit For
        End For
        Convexity check: Set $Z(K_1) = Z(B\ -1)$ and If $Z(K_1) > Z(K_1\text{-}1)$ Exit For
    End For
    Set $K_1 = K_1^*$ and $B = B^*$
    For K_A = 1 to 20
        For K_B = 1 to 20
            Calculate new total cost Z
            Optimality check: If $Z < Z_{opt}$ then $Z_{opt} = Z$, $K_A^* = K_A$ and $K_B^* = K_B$
            Convexity check: Set $Z(K_B) = Z$ and If $Z(K_B) > Z(K_B\ -1)$ Exit For
        End For
        Convexity check: Set $Z(K_A) = Z(K_B\ -1)$ and If $Z(K_A) > Z(K_A\ -1)$ Exit For
    End For
    Set $K_i = K_i^*$ for $i \in S_2 = \{A, B\}$
    For BL_A = 0 to 20
        For BL_B = 0 to 20
            Calculate new total cost Z
            Optimality check: If $Z < Z_{opt}$ then $Z_{opt} = Z$, $BL_A^* = BL_A$ and $BL_B^* = BL_B$
            Convexity check: Set $Z(BL_B) = Z$ and If $Z(BL_B) > Z(BL_B\ -1)$ Exit For
        End For
        Convexity check: Set $Z(BL_A) = Z(BL_B\ -1)$ and If $Z(BL_A) > Z(BL_A\ -1)$ Exit For
    End For
    Set $BL_i = BL_i^*$ for $i \in S_2 = \{A, B\}$
    $Z(e) = Z_{opt}$
    If $Z(e) = Z(e\text{-}1)$ then Exit For
End For

Figure 4.10. Search algorithm for PDNA and PDA-OD

<u>Search Algorithm 2</u>

```
Set initial K_i, B_ci and BL_i for i ∈ S_2 = {A, B}
Set Z_opt to a very large number
For e = 1 to 10
        For K_1 = 1 to Min_{i∈S_n}[B_ci]
                For B = 0 to 20
                        Calculate total cost Z by using analytical method
                        Optimality check: If Z < Z_opt then Z_opt = Z, K_1* = K_1 and B* = B
                        Convexity check: Set Z(B) = Z and If Z(B) > Z(B -1) Exit For
                End For
        Convexity check: Set Z(K_1) = Z(B -1) and If Z(K_1) > Z(K_1-1) Exit For
        End For
        Set K_1 = K_1* and B = B*
        For B_CA = K_1 to 20
                For B_CB = K_1 to 20
                        Calculate new total cost Z
                        Optimality check: If Z < Z_opt then Z_opt = Z, B_CA* = B_CA and B_CB* = B_CB
                        Convexity check: Set Z(B_CB) = Z and If Z(B_CB) > Z(B_CB -1) Exit For
                End For
                Convexity check: Set Z(B_CA) = Z(B_CB -1) and If Z(B_CA) > Z(B_CA -1) Exit For
        End For
        Set B_CA = B_CA* and B_CB = B_CB*
        For K_A = 1 to 20
                For K_B = 1 to 20
                        Calculate new total cost Z
                        Optimality check: If Z < Z_opt then Z_opt = Z, K_A* = K_A and K_B* = K_B
                        Convexity check: Set Z(K_B) = Z and If Z(K_B) > Z(K_B -1) Exit For
                End For
                Convexity check: Set Z(K_A) = Z(K_B -1) and If Z(K_A) > Z(K_A -1) Exit For
        End For
        Set K_i = K_i* for i ∈ S_2 = {A, B}
        For BL_A = 0 to 20
                For BL_B = 0 to 20
                        Calculate new total cost Z
                        Optimality check: If Z < Z_opt then Z_opt = Z, BL_A* = BL_A and BL_B* = BL_B
                        Convexity check: Set Z(BL_B) = Z and If Z(BL_B) > Z(BL_B -1) Exit For
                End For
                Convexity check: Set Z(BL_A) = Z(BL_B -1) and If Z(BL_A)> Z(BL_A -1) Exit For
        End For
        Set BL_i = BL_i* for i ∈ S_2 = {A, B}
        Z(e) = Z_opt
        If Z(e) = Z(e-1) then Exit For
End For
```

Figure 4.11. Search algorithm for PDA-OP and PDA-RP

In the algorithms, in order to increase the speed of the procedure, we prefer to jointly optimize return buffer and the kanban level of the first stage. Moreover, we also jointly optimize the kanban levels of downstream stages and backorder limits. Since the algorithms terminate after a small number iteration, the maximum number of iterations for the global loop is selected as 10 in the algorithms.

After the iterations are stopped, we make a neighborhood search to verify the optimality of the total cost of the resulting configuration. In that step, we try all configuration alternatives that involve all neighbor values of the optimized parameter values by observing the total costs of these configurations. If we find a configuration that results in a lower total cost than the minimum cost obtained by the above search algorithms, we make a secondary neighborhood search for the newly founded configuration. However, in most of the experiments, we have found the optimal configuration by the search algorithms without a secondary neighborhood search.

In our search algorithms, we assume that the total cost function is convex respect to the parameter values. So, an individual parameter is optimized by incrementation starting from an initial value until the last increase does not cause a significant decrease on the total cost. We select initial values as one for the kanban levels and component buffers and zero for the return buffer and backorder limits. We increase the parameter values at most to 20, since additional increases after this value do not cause a significant change in the total cost for our experiment setting.

The convexity assumption of the total cost function with respect to the parameter values is a fairly reasonable assumption due to some clear reasons. For instance, an increase of the buffer size of returns (or kanban level of a stage or buffer size of a component) results in a decrease of the backorder cost, lost sales cost and disposal cost of the system while resulting in an increase of the holding cost, disassembly cost, refurbishing cost and acquisition cost. In other words, there are two opposite effects of the marginal increase of the parameter value on the total cost function. So, there is a trade-off point where the decreasing effect of the marginal increase of the parameter value on the total cost is dominated by the increasing effect of the marginal increase of that parameter value on the total cost. This convexity assumption is also valid for the backorder limits. In fact, an increase of the backorder limit results in a decrease of the lost sales cost and disposal cost while resulting in an increase of the backorder cost, disassembly cost, refurbishing cost and acquisition cost.

However, we have no possibility to assess the validity of the convexity assumption of the total cost function and to assess the sub optimality of the resulting minimum cost by comparing with the global minimum of the total cost, since it is not possible to find the global minimum of the total cost by performing an exhaustive search due to the extreme number of different configuration alternatives. For example, when the product is composed of only two types of components (n =2) and we try only 10 alternatives for each parameter, we are faced with $(2n+2)^{10} = 6^{10}$ or $(3n+2)^{10} = 8^{10}$ different configuration alternatives for performing a complete search to find the optimal one.

4.7. Marginal Profitability of the Disassembly Process

The marginal profitability of the disassembly process is an important measure for assessing the profitability of the different disassembly policies. Marginal profitability of the disassembly process is given as the below formula for $i \in S_n = \{A, B, ...\}$

$$\text{Marginal Profitability} = \frac{(d_p + \sum_i c_{mi}) - (c_p + c_d + \sum_i c_{ri})}{\sum_i c_{mi}} \tag{4.25}$$

The cost term in the numerator represents cost saving that can be achieved by preferring the disassembly operation for the returned products instead of disposing all of them and satisfying the demands by manufacturing. On the other hand, the term in the denominator represents the total cost that we will be faced with if we do not engage with the remanufacturing activities. In fact, this formula is designed with respect to the assumption that there will be no return flow to the facility if there is no remanufacturing activity. However, it may not be true in some cases if there are strong environmental legislations that forces companies to collect end of life products from the market. In this situation, the company is faced with a return flow even if the company is not engaged with the remanufacturing operations. So, in the case of existence of the environmental legislations, that formula can be restated as the following:

$$\text{Marginal Profitability} = \frac{(d_P + \sum_i c_{mi}) - (c_P + c_d + \sum_i c_{ri})}{(d_P + \sum_i c_{mi})} \tag{4.26}$$

In the next chapter, we analyze the effect of marginal profitability on the cost savings of different policies, and we select the first formula of the marginal profitability for our numerical analyses.

4.7.1. Profitability Condition of Allowing Partial Disassembly

As mentioned previously the total cost function comprises of two groups of cost components: costs that are calculated by using expected throughputs and the costs that are calculated by using average queue lengths. In this section, we concentrate on the first group of costs to find a threshold value of the marginal profitability for the profitability condition of the policy of allowing partial disassembly.

Let TC_{NA} and TC_A denote the total cost except the holding and backorder costs for the policies of not allowing partial disassembly and allowing partial disassembly, respectively. Here we assume that the product is composed of two components and each component has uniform characteristics, in other words, their manufacturing and refurbishing costs are equal. Let c_R and c_M denotes the equal refurbishing and manufacturing costs of the parts.

The total cost except the holding and backorder costs for each policy can be calculated as the summation of disassembly cost, refurbishing cost, lost sales cost, disposal cost and acquisition cost. For the policy of not allowing partial disassembly, we can write this summation as the following by keeping the same order.

$$TC_{NA} = c_d\, TH_1 + c_R\,(TH_A + TH_B) + c_M\,[(\lambda_A - TH_A) + (\lambda_B - TH_B)] + d_p\,(\gamma - TH_1) + c_p\, TH_1 \tag{4.27}$$

Since the throughputs of all stages are equal to each other when partial disassembly is not allowed, we can represent all of them as TH. That is $TH_1 = TH_A = TH_B = TH$. So, the above equation becomes

$$TC_{NA} = TH\,(c_p + c_d - d_p + 2c_R - 2c_M) + d_p\,\gamma + c_M\,(\lambda_A + \lambda_B) \tag{4.28}$$

On the other hand when partial disassembly is allowed, total cost except the holding and backorder costs becomes the following

$$\begin{aligned} TC_A = c_d\,TH_1 + c_R\,(TH_A + TH_B) + c_M\,[(\lambda_A - TH_A) + (\lambda_B - TH_B)] + d_p\,(\gamma - TH_1) \\ + d_A\,(TH_1 - TH_A) + d_B\,(TH_1 - TH_B) + c_p\,TH_1 \end{aligned} \tag{4.29}$$

If we assume that $d_A = d_B = d_p / 2$, then the above equation becomes

$$TC_A = (c_p + c_d)\,TH_1 + (c_R - c_M - d_p / 2)\,(TH_A + TH_B) + d_p\,\gamma + c_M\,(\lambda_A + \lambda_B) \tag{4.30}$$

Besides, the profitability condition of allowing partial disassembly can be stated as

$$TC_{NA} - TC_A > 0 \tag{4.31}$$

By using the equations (4.24) and (4.26), it becomes

$$(c_p + c_d)\,(TH - TH_1) - (c_M + d_p / 2 - c_R)\,(2TH - TH_A - TH_B) > 0 \tag{4.32}$$

By passing the first term to the right hand side of the inequality, we get

$$(c_M + d_p / 2 - c_R)\,(TH_A + TH_B - 2TH) > (c_p + c_d)\,(TH_1 - TH) \tag{4.33}$$

Further calculations take us to

$$(d_p + 2\,c_M) - (c_p + c_d + 2\,c_R) > (c_p + c_d)\left[\frac{2(TH_1 - TH)}{TH_A + TH_B - 2TH} - 1\right] \tag{4.34}$$

The left hand side of the above inequality represents the marginal profit, so when we divide both sides with the manufacturing cost, it becomes the marginal profitability.

$$\frac{(d_p + 2c_M) - (c_p + c_d + 2c_R)}{2c_M} > \frac{(c_p + c_d)}{2c_M} \left[\frac{2(TH_1 - TH)}{TH_A + TH_B - 2TH} - 1 \right] \tag{4.35}$$

Let assume that part A is more frequently demanded than part B. That is $\lambda_A > \lambda_B$. This means that if partial disassembly is not allowed $TH \approx TH_B$ becomes true. In other words, if partial disassembly is not allowed, throughputs of all stages will be equal to the throughput of the stage B when partial disassembly is allowed. Also, if partial disassembly is allowed, $TH_A > TH_B$ will be true. Besides, if we hold the overflow items and give the priority to these items, then the throughput of the first stage will be approximately equal to the throughput of the stage A. That is $TH_1 \approx TH_A$. Then the term depending on the throughputs in the above inequality will be equal to one. That is

$$X = \frac{2(TH_1 - TH)}{TH_A + TH_B - 2TH} - 1 = \frac{2TH_1 - TH_A - TH_B}{TH_A + TH_B - 2TH} \approx \frac{TH_A - TH_B}{TH_A - TH_B} = 1 \tag{4.36}$$

So if we hold overflows and give them the priority, then the profitability condition of allowing partial disassembly, for the uniform two parts case, becomes

$$\text{Marginal Profitability} > \frac{(c_p + c_d)}{2c_M} \tag{4.37}$$

This threshold value is valid if we only consider the costs that are calculated by the throughputs. Nevertheless, it can be beneficial by giving us a bound for the real threshold value. In the next chapter, we also analyze this condition: whether it is lower or upper bound for the real threshold value.

5. NUMERICAL RESULTS

In this chapter, the results of the numerical analysis, which can reveal managerial insights for the planners of the disassembly systems, are presented. To obtain these results, we used the analytical approximation method proposed in the previous chapter. The comparison of the different disassembly policies is made based on the minimum expected total cost of the policies that is obtained by the proposed heuristic search procedure. Mainly, we concentrate on the per cent cost saving that can be achieved by allowing partial disassembly compared to the not allowing. However, in some cases where the policy of allowing partial disassembly results in a cost increase, the cost saving becomes negative. So, in this chapter, we search for the conditions that make the cost saving of allowing partial disassembly non-negative. In fact, we focus on the conditions that differ with respect to the marginal profitability level of disassembly process, arrival rate configurations of returns and demands and the difference of the characteristics of the disassembled parts.

We calculate the per cent cost savings of allowing partial disassembly policies compared to the not allowing policy by using the below formula.

$$\text{Per cent cost saving} = \frac{Z_{NA} - Z_{A}}{Z_{NA}} \tag{5.1}$$

where Z_{NA} and Z_A represent the total costs that we are faced with when partial disassembly is not allowed and allowed, respectively.

In the experiments, it is assumed that the summation of the unit costs of the parts equals to the corresponding unit costs of the product. For instance, unit disposal cost of a product equals to the summation of unit disposal costs of parts. Likewise, unit manufacturing and holding costs of a product equals to the summation of unit manufacturing and holding costs of parts, respectively.

5.1. Effect of the Marginal Profitability of the Disassembly Process

In this section, we analyze the effect of the marginal profitability level of the disassembly process on per cent cost saving of allowing partial disassembly. In order to investigate the effect of the marginal profitability, one of the cost parameters is changed while others are held constant in the experiments. For this purpose, unit disposal cost, unit manufacturing cost and remanufacturing costs viz. unit acquisition, disassembly and refurbishing costs are changed and their affects are observed on the cost savings. In each of these cases, we consider two sub cases where the returned end-of-life product is composed of two and three components, respectively. In addition, we also analyze the effects of different arrival rate configurations of returns and part demands.

In the experiments, the uniform components case is taken into account. In other words, we assume that unit holding, refurbishing, manufacturing and disposal costs of the different components are equal to each other. The unit refurbishing and manufacturing costs of the parts are simply denoted by c_R and c_M respectively where $c_R = c_{ri}$ and $c_M = c_{mi}$ for $i \in S_n = \{A, B, ...\}$. Also, we assume that the summation of the unit disposal costs of components equals to the units disposal cost of the products. That is, when the product is composed of n types of components, $d_i = d_p / n$ for $i \in S_n = \{A, B, ...\}$. Also, it is assumed that out-of-pocket holding costs of different types of disassembled components are equal to each other and their summation equals to the out-of-pocket holding costs of the product. That is, when the product is composed of n types of components, $h_{di} = h_p / n$ for $i \in S_n = \{A, B, ...\}$. In holding cost evaluation, since the out-of-pocket holding costs of different types of disassembled components are assumed to be equal to each other, we have only one alternative for allocation fraction of the added value. That is $f_i = 1 / n$ for $i \in S_n = \{A, B, ...\}$.

Furthermore, we also assume that the out-of-pocket holding costs of disassembled and refurbished components are not different, since the refurbishing is not an operation that can change the physical characteristics of the parts significantly. So, their main physical characteristics (volume or weight) and out-of-pocket holding costs that are related to these characteristics remain the same after the refurbishing operations. That is $h_{di} = h_{ri} = h_p / n$ for $i \in S_n = \{A, B, ...\}$.

In the experiments, service rates of all machines are set equal to one. Out of pocket cost of a product is selected as one (h_p = 1), and inventory carrying charge is selected as α = 0.15. The unit backorder cost for part demands are assumed to be equal with respect to the part types and they are selected as b_i = 4 for $i \in S_n = \{A, B, ...\}$.

5.1.1. Change in Disposal Cost

One of the important parameters that can affect the level of the marginal profitability of the disassembly process is the unit disposal cost of returns. For instance, if we obtain high recycling revenue by disposing the product, then the disassembly operation may be unprofitable compared to the recycling of returns and manufacturing of parts from raw materials. For being able to observe this effect of the disposal cost, unit disposal cost of a core is changed from negative values (recycling revenue) to the positive values (landfill cost etc.) in the following experiment sets.

In Table 5.1, we present the unit costs set that are used for the analysis of the two component case. Here the unit disposal cost is changed while other unit costs are held constant. In Table 5.2, marginal remanufacturing cost (c_p + c_d + 2c_R), marginal manufacturing cost (d_p + 2c_M), marginal profit and marginal profitability levels are shown for each unit cost combination. In Table 5.3, minimum expected total cost of each policy and per cent cost savings of allowing partial disassembly policies compared to the not allowing policy are given for three different return and demand arrival rate configurations. These results are also shown in Figure 5.1, Figure 5.2 and Figure 5.3 for per cent cost savings.

Table 5.1. Experiment set 1 for two component case

Ex	c_p	c_d	c_R	d_p	c_M
1	10	10	10	-50	50
2	10	10	10	-40	50
3	10	10	10	-30	50
4	10	10	10	-20	50
5	10	10	10	-10	50
6	10	10	10	0	50
7	10	10	10	10	50
8	10	10	10	20	50
9	10	10	10	30	50
10	10	10	10	40	50

Table 5.2. Marginal profitability levels of experiment set 1

Ex	$c_p + c_d + 2c_R$	$d_p + 2c_M$	Marginal Profit	Marginal Profitability
1	40	50	10	10%
2	40	60	20	20%
3	40	70	30	30%
4	40	80	40	40%
5	40	90	50	50%
6	40	100	60	60%
7	40	110	70	70%
8	40	120	80	80%
9	40	130	90	90%
10	40	140	100	100%

Table 5.3. Total costs and per cent cost savings of experiment set 1

$\gamma = 0.6$ $\lambda_A = 0.8$ $\lambda_B = 0.5$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	38.55	39.59	-2.7%	39.33	-2.0%	39.59	-2.7%
2	42.56	43.40	-2.0%	42.70	-0.3%	42.97	-1.0%
3	45.87	46.50	-1.4%	45.64	0.5%	45.67	0.4%
4	48.94	49.22	-0.6%	48.01	1.9%	48.38	1.1%
5	51.72	51.81	-0.2%	49.92	3.5%	50.25	2.8%
6	53.82	53.70	0.2%	51.77	3.8%	52.03	3.3%
7	55.73	55.28	0.8%	53.53	4.0%	53.70	3.7%
8	57.64	56.67	1.7%	54.95	4.7%	55.04	4.5%
9	59.06	57.96	1.9%	56.00	5.2%	56.03	5.1%
10	60.56	59.17	2.3%	56.95	6.0%	56.96	5.9%
$\gamma = 0.8$ $\lambda_A = 0.6$ $\lambda_B = 0.5$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	19.05	21.40	-12.3%	20.78	-9.0%	20.83	-9.3%
2	25.38	26.83	-5.7%	25.29	0.3%	25.99	-2.4%
3	31.11	31.85	-2.4%	29.81	4.2%	30.42	2.2%
4	36.23	36.44	-0.6%	34.32	5.3%	34.79	4.0%
5	40.82	40.87	-0.1%	38.84	4.9%	39.16	4.1%
6	45.37	45.17	0.4%	42.63	6.0%	43.24	4.7%
7	49.30	49.42	-0.2%	46.21	6.3%	46.62	5.4%
8	53.14	52.90	0.5%	49.56	6.7%	49.92	6.1%
9	56.94	56.38	1.0%	52.98	7.0%	53.13	6.7%
10	60.68	59.79	1.5%	56.41	7.0%	56.27	7.3%
$\gamma = 0.5$ $\lambda_A = 0.8$ $\lambda_B = 0.6$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	48.14	49.92	-3.7%	48.61	-1.0%	48.73	-1.2%
2	50.98	52.05	-2.1%	51.16	-0.4%	51.30	-0.6%
3	53.43	53.97	-1.0%	53.21	0.4%	53.40	0.1%
4	55.58	55.78	-0.4%	54.40	2.1%	54.62	1.7%
5	56.77	56.94	-0.3%	55.65	2.0%	55.71	1.9%
6	57.88	57.74	0.2%	56.38	2.6%	56.68	2.1%
7	58.58	58.30	0.5%	56.97	2.7%	57.22	2.3%
8	58.98	58.78	0.3%	57.12	3.2%	57.38	2.7%
9	59.24	59.22	0.0%	57.29	3.3%	57.53	2.9%
10	59.49	59.62	-0.2%	57.44	3.5%	57.63	3.1%

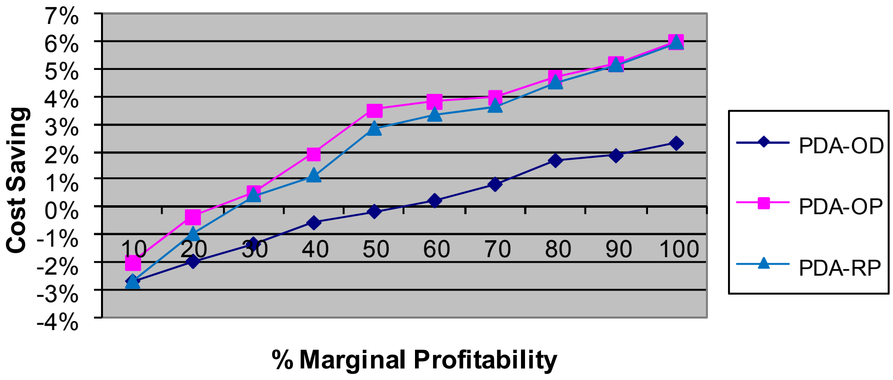

Figure 5.1. Cost saving versus marginal profitability for $\gamma = 0.6$, $\lambda_A = 0.8$ and $\lambda_B = 0.5$ with change in disposal cost for two-component case

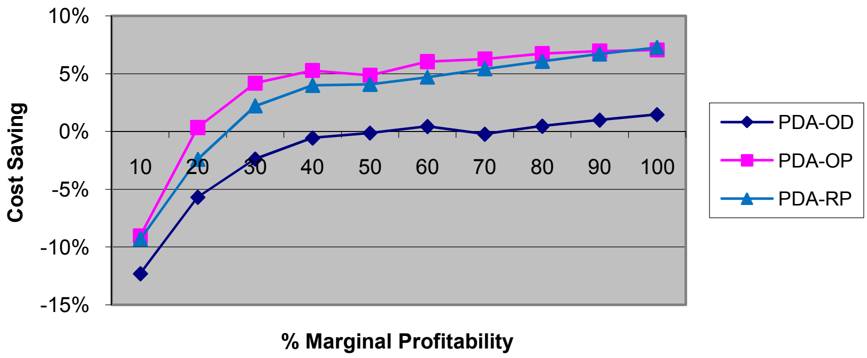

Figure 5.2. Cost saving versus marginal profitability for $\gamma = 0.8$, $\lambda_A = 0.6$ and $\lambda_B = 0.5$ with change in disposal cost for two-component case

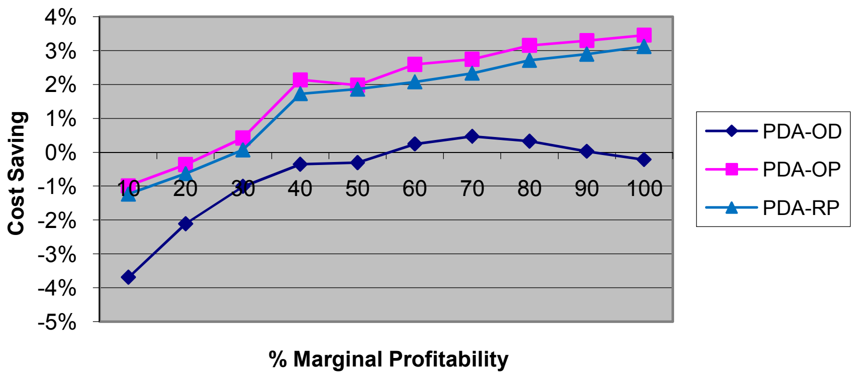

Figure 5.3. Cost saving versus marginal profitability for $\gamma = 0.5$, $\lambda_A = 0.8$ and $\lambda_B = 0.6$ with change in disposal cost for two-component case

In Table 5.4, we present the unit costs set that are used for the analysis of the three component case. Here, unit disposal cost is changed while other unit costs are held constant as in the previous case. In Table 5.5, remanufacturing cost, manufacturing cost, marginal profit and marginal profitability levels are shown for each unit cost combination. In Table 5.6, total cost of each policy and per cent cost savings of allowing partial disassembly policies compared to the not allowing policy are given for three different return and demand rate configurations. These results are also shown in Figure 5.4, Figure 5.5 and Figure 5.6 for per cent cost savings.

Table 5.4. Experiment set 2 for three component case

Ex	c_p	c_d	c_R	d_p	c_M
1	10	10	10	-40	33.33
2	10	10	10	-30	33.33
3	10	10	10	-20	33.33
4	10	10	10	-10	33.33
5	10	10	10	0	33.33
6	10	10	10	10	33.33
7	10	10	10	20	33.33
8	10	10	10	30	33.33
9	10	10	10	40	33.33
10	10	10	10	50	33.33

Table 5.5. Marginal profitability levels of experiment set 2

Ex	$c_p + c_d + 3c_R$	$d_p + 3c_M$	Marginal Profit	Marginal Profitability
1	50	60	10	10%
2	50	70	20	20%
3	50	80	30	30%
4	50	90	40	40%
5	50	100	50	50%
6	50	110	60	60%
7	50	120	70	70%
8	50	130	80	80%
9	50	140	90	90%
10	50	150	100	100%

Table 5.6. Total costs and per cent cost savings of experiment set 2

$\gamma = 0.6$ $\lambda_A = 0.8$ $\lambda_B = 0.7$ $\lambda_C = 0.5$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	46.56	48.16	-3.4%	48.24	-3.6%	47.95	-3.0%
2	50.99	51.94	-1.9%	51.68	-1.3%	51.40	-0.8%
3	54.99	55.21	-0.4%	54.93	0.1%	54.84	0.3%
4	58.24	58.02	0.4%	57.81	0.7%	57.84	0.7%
5	61.55	60.90	1.1%	59.86	2.7%	60.25	2.1%
6	64.36	63.53	1.3%	61.76	4.0%	61.77	4.0%
7	66.34	65.06	1.9%	63.66	4.0%	63.74	3.9%
8	68.29	66.50	2.6%	65.09	4.7%	65.13	4.6%
9	70.13	67.94	3.1%	66.47	5.2%	66.51	5.2%
10	71.96	69.25	3.8%	67.71	5.9%	67.74	5.9%
$\gamma = 0.6$ $\lambda_A = 0.8$ $\lambda_B = 0.5$ $\lambda_C = 0.4$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	36.90	38.77	-5.1%	39.11	-6.0%	38.80	-5.1%
2	41.82	42.83	-2.4%	42.71	-2.1%	42.39	-1.4%
3	46.01	46.53	-1.1%	46.30	-0.6%	45.99	0.0%
4	50.21	50.00	0.4%	49.53	1.4%	49.59	1.2%
5	53.71	53.47	0.4%	51.89	3.4%	51.89	3.4%
6	57.27	56.18	1.9%	54.20	5.4%	54.17	5.4%
7	60.83	59.21	2.7%	56.68	6.8%	56.77	6.7%
8	63.91	60.62	5.2%	58.65	8.2%	58.72	8.1%
9	67.12	62.72	6.6%	60.62	9.7%	61.01	9.1%
10	70.33	64.74	8.0%	63.00	10.4%	63.05	10.4%
$\gamma = 0.8$ $\lambda_A = 0.6$ $\lambda_B = 0.5$ $\lambda_C = 0.4$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	22.54	25.47	-13.0%	25.86	-14.7%	25.33	-12.4%
2	29.46	31.41	-6.6%	31.36	-6.5%	30.82	-4.6%
3	35.95	37.36	-3.9%	36.05	-0.3%	36.32	-1.0%
4	42.09	42.97	-2.1%	40.67	3.4%	40.69	3.3%
5	48.05	47.79	0.6%	45.30	5.7%	45.29	5.8%
6	53.35	52.17	2.2%	49.92	6.4%	49.89	6.5%
7	58.53	56.68	3.2%	54.27	7.3%	54.49	6.9%
8	63.49	61.19	3.6%	58.26	8.2%	58.36	8.1%
9	67.75	65.70	3.0%	62.08	8.4%	61.96	8.5%
10	72.41	69.64	3.8%	65.89	9.0%	65.72	9.2%

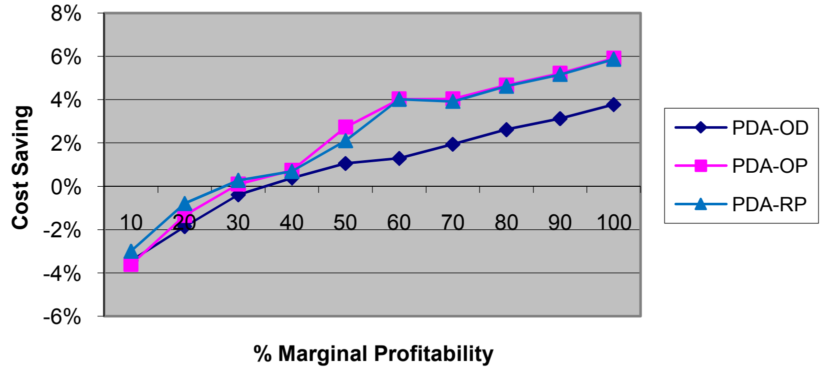

Figure 5.4. Cost saving versus marginal profitability for $\gamma = 0.6$, $\lambda_A = 0.8$, $\lambda_B = 0.7$ and $\lambda_C = 0.5$ with change in disposal cost for three-component case

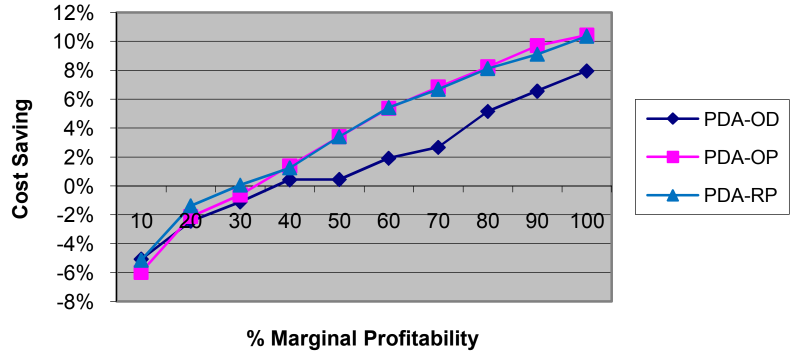

Figure 5.5. Cost saving versus marginal profitability for $\gamma = 0.6$, $\lambda_A = 0.8$, $\lambda_B = 0.5$ and $\lambda_C = 0.4$ with change in disposal cost for three-component case

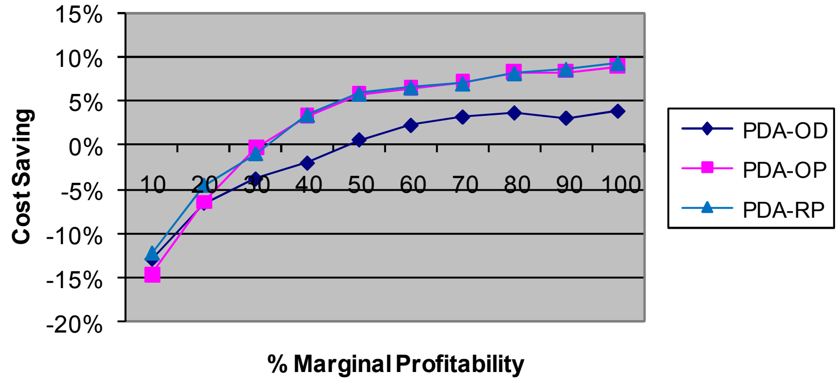

Figure 5.6. Cost saving versus marginal profitability for $\gamma = 0.8$, $\lambda_A = 0.6$, $\lambda_B = 0.5$ and $\lambda_C = 0.4$ with change in disposal cost for three-component case

As a result of these experiments, it is observed that allowing partial disassembly policies (PDA-OD, PDA-OP and PDA-RP) become more profitable compared to the not allowing policy when the marginal profitability level of the disassembly process becomes higher. Otherwise, viz. when the marginal profitability is low, allowing partial disassembly causes a significant increase in the total cost. These statements seem to be true for both the two and three component cases.

Moreover, when partial disassembly is allowed, holding overflows policies (PDA-OP and RP) outperforms the direct disposal policy (PDA-OD) in all marginal profitability levels and in all arrival rate configurations. Also, it is observed that the different priority choice in case of holding overflows does not cause a significant impact on the total cost. So, it can be stated that when the parameters of the system is optimized, priority decision is not an important factor in terms of the total cost.

In the configurations where return arrival rate is larger than the demand rates of components, the increase in the cost saving becomes smaller when the marginal profitability level becomes close to one hundred per cent. This converging behavior is due to the fact that the high marginal profitability means a high disposal cost, for this example, and this increase in disposal cost becomes a stronger effect, which is diminishing the cost reduction effect of the higher marginal profitability level of the disassembly process. Since the supply of returns is more than the demands in these arrival rate configurations, disposal cost of returns is a more effective cost component than that of the other configurations. So, this converging behavior is not seen in other configurations.

5.1.2. Change in Manufacturing Cost

Another reason that makes the disassembly less profitable is the low manufacturing costs of the parts. In fact, lost sales of the disassembly are satisfied by the manufacturing. If the manufacturing costs of the parts are low compared to the total disassembly cost, then manufacturing option becomes more cost effective than the disassembly of cores. So, the disassembly operation becomes less profitable. However, when the manufacturing costs of the parts increase, profitability level of the disassembly also increases with respect to the increase of unit manufacturing cost.

In the following experiment set that is shown in Table 5.7, we increase the manufacturing cost from a small value to large values to see the effect of the marginal profitability of the disassembly process on the cost savings of allowing partial disassembly policies. This experiment set is used for both of two and three component cases. In Table 5.8, marginal profitability levels of the disassembly process are given for two component case. In Table 5.9, total costs of each disassembly policies and per cent cost savings of allowing partial disassembly policies are reported for the two component case. Also, in the following graphics, cost savings of allowing partial disassembly policies are illustrated with respect to the marginal profitability level of the disassembly process for the two component case.

Table 5.7. Experiment set 3 for two and three component cases

Ex	c_p	c_d	c_R	d_p	c_M
1	10	10	10	20	11.11
2	10	10	10	20	12.50
3	10	10	10	20	14.29
4	10	10	10	20	16.67
5	10	10	10	20	20.00
6	10	10	10	20	25.00
7	10	10	10	20	33.33
8	10	10	10	20	50.00
9	10	10	10	20	100.00

Table 5.8. Marginal profitability levels of experiment set 3 for two component case

Ex	$c_p + c_d + 2c_R$	$d_p + 2c_M$	Marginal Profit	Marginal Profitability
1	40	42.22	2.22	10%
2	40	45.00	5.00	20%
3	40	48.57	8.57	30%
4	40	53.33	13.33	40%
5	40	60.00	20.00	50%
6	40	70.00	30.00	60%
7	40	86.67	46.67	70%
8	40	120.00	80.00	80%
9	40	220.00	180.00	90%

Table 5.9. Total costs and per cent cost savings of experiment set 3 for two component case

$\gamma = 0.6$ $\lambda_A = 0.8$ $\lambda_B = 0.5$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	31.25	33.73	-7.9%	33.00	-5.6%	33.00	-5.6%
2	32.61	34.86	-6.9%	34.09	-4.5%	34.10	-4.6%
3	34.35	36.32	-5.7%	35.47	-3.3%	35.52	-3.4%
4	36.66	38.26	-4.4%	37.41	-2.0%	37.41	-2.0%
5	39.56	40.60	-2.6%	39.80	-0.6%	39.99	-1.1%
6	43.37	44.00	-1.4%	43.22	0.3%	43.37	0.0%
7	49.13	49.29	-0.3%	48.11	2.1%	48.30	1.7%
8	57.64	56.67	1.7%	54.95	4.7%	55.04	4.5%
9	75.26	71.51	5.0%	68.31	9.2%	66.61	11.5%
$\gamma = 0.8$ $\lambda_A = 0.6$ $\lambda_B = 0.5$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	33.54	36.63	-9.2%	36.09	-7.6%	36.09	-7.6%
2	34.62	37.44	-8.2%	36.89	-6.6%	36.89	-6.6%
3	36.00	38.49	-6.9%	37.92	-5.3%	37.92	-5.3%
4	37.85	39.88	-5.4%	39.32	-3.9%	39.61	-4.7%
5	40.38	41.83	-3.6%	40.64	-0.7%	40.99	-1.5%
6	43.61	44.35	-1.7%	42.63	2.2%	42.99	1.4%
7	47.62	47.75	-0.3%	45.95	3.5%	46.17	3.0%
8	53.14	52.90	0.5%	49.56	6.7%	49.92	6.1%
9	62.03	59.17	4.6%	54.44	12.2%	53.57	13.6%
$\gamma = 0.5$ $\lambda_A = 0.8$ $\lambda_B = 0.6$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	29.96	32.42	-8.2%	31.18	-4.1%	31.18	-4.1%
2	31.45	33.73	-7.2%	32.45	-3.2%	32.45	-3.2%
3	33.37	35.32	-5.8%	34.08	-2.1%	34.08	-2.1%
4	35.81	37.32	-4.2%	36.21	-1.1%	36.26	-1.2%
5	38.98	40.05	-2.8%	39.18	-0.5%	39.30	-0.8%
6	43.44	43.97	-1.2%	43.37	0.1%	43.44	0.0%
7	49.71	50.06	-0.7%	49.06	1.3%	49.16	1.1%
8	58.98	58.78	0.3%	57.12	3.2%	57.38	2.7%
9	79.91	81.28	-1.7%	75.85	5.1%	76.08	4.8%

In Table 5.10, we give the marginal profitability levels of the disassembly process for the unit cost combinations in the experiment set 3 for three component case. In Table 5.11, total costs of each disassembly policy and per cent cost savings of allowing partial disassembly policies are reported for the three component case. Per cent cost savings are also shown in the following graphics.

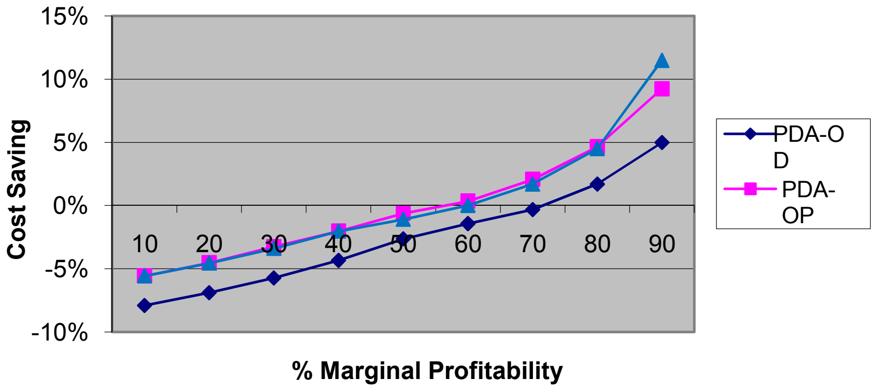

Figure 5.7. Cost saving versus marginal profitability for γ= 0.6, λ_A= 0.8 and λ_B= 0,5 with change in manufacturing cost for two-component case

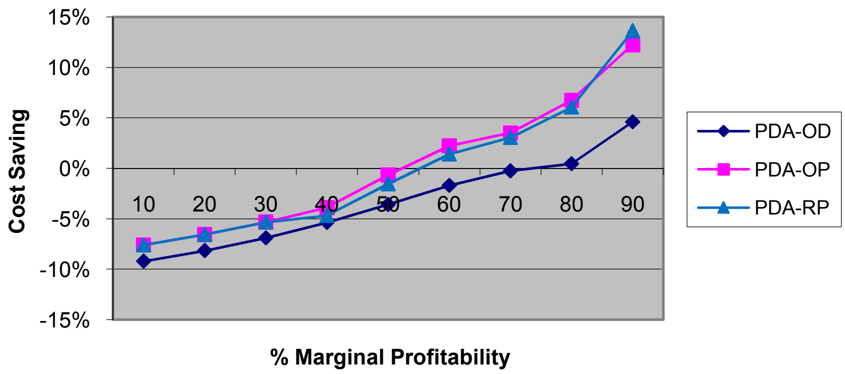

Figure 5.8. Cost saving versus marginal profitability for γ= 0.8, λ_A= 0.6 and λ_B= 0.5 with change in manufacturing cost for two-component case

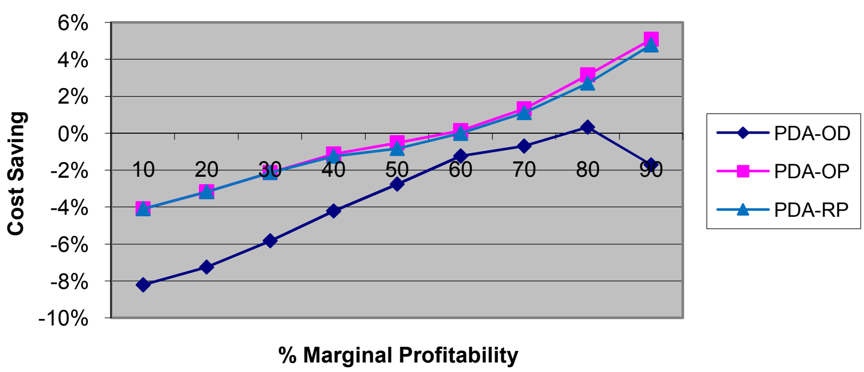

Figure 5.9. Cost saving versus marginal profitability for γ= 0.5, λ_A= 0.8 and λ_B= 0.6 with change in manufacturing cost for two-component case

Table 5.10. Marginal profitability levels of experiment set 3 for three component case

Ex	$c_p + c_d + 3c_R$	$d_p + 3c_M$	Marginal Profit	Marginal Profitability
1	50	53.33	3.33	10%
2	50	57.50	7.50	20%
3	50	62.85	12.86	30%
4	50	70.00	20.00	40%
5	50	80.00	30.00	50%
6	50	95.00	45.00	60%
7	50	120.00	70.00	70%
8	50	170.00	120.00	80%
9	50	320.00	270.00	90%

Table 5.11. Total costs and per cent cost savings of experiment set 3 for three component case

$\gamma = 0.6$ $\lambda_A = 0.8$ $\lambda_B = 0.7$ $\lambda_C = 0.5$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	38.91	41.08	-5.6%	41.50	-6.7%	41.21	-5.9%
2	41.19	43.00	-4.4%	43.21	-4.9%	42.93	-4.2%
3	44.12	45.48	-3.1%	45.41	-2.9%	45.13	-2.3%
4	47.66	48.61	-2.0%	48.35	-1.4%	48.06	-0.8%
5	52.32	52.54	-0.4%	52.27	0.1%	52.17	0.3%
6	58.23	57.79	0.7%	57.50	1.3%	57.52	1.2%
7	67.29	65.06	3.3%	63.69	5.4%	63.74	5.3%
8	78.29	74.79	4.5%	71.89	8.2%	72.94	6.8%
9	108.64	95.66	11.9%	91.06	16.2%	91.73	15.6%
$\gamma = 0.6$ $\lambda_A = 0.8$ $\lambda_B = 0.5$ $\lambda_C = 0.4$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	35.84	38.28	-6.8%	38.93	-8.6%	38.62	-7.8%
2	37.75	39.83	-5.5%	40.29	-6.7%	39.98	-5.9%
3	40.21	41.83	-4.0%	42.04	-4.6%	41.73	-3.8%
4	43.48	44.50	-2.3%	44.37	-2.0%	44.06	-1.3%
5	47.35	47.86	-1.1%	47.64	-0.6%	47.33	0.0%
6	53.02	52.57	0.8%	51.66	2.5%	52.22	1.5%
7	60.83	59.21	2.7%	56.68	6.8%	56.77	6.7%
8	70.85	66.61	6.0%	64.74	8.6%	64.73	8.6%
9	100.22	81.84	18.3%	78.04	22.1%	74.81	25.4%
$\gamma = 0.8$ $\lambda_A = 0.6$ $\lambda_B = 0.5$ $\lambda_C = 0.4$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	37.92	41.51	-9.5%	42.20	-11.3%	41.66	-9.9%
2	39.56	42.74	-8.0%	43.24	-9.3%	42.70	-8.0%
3	41.66	44.31	-6.4%	44.57	-7.0%	44.04	-5.7%
4	44.46	46.41	-4.4%	46.36	-4.3%	45.82	-3.1%
5	47.95	49.36	-2.9%	48.07	-0.2%	48.32	-0.8%
6	52.62	52.88	-0.5%	50.50	4.0%	50.55	3.9%
7	58.53	56.68	3.2%	54.27	7.3%	54.49	6.9%
8	66.23	62.24	6.0%	57.92	12.5%	58.25	12.1%
9	83.28	68.78	17.4%	62.96	24.4%	62.79	24.6%

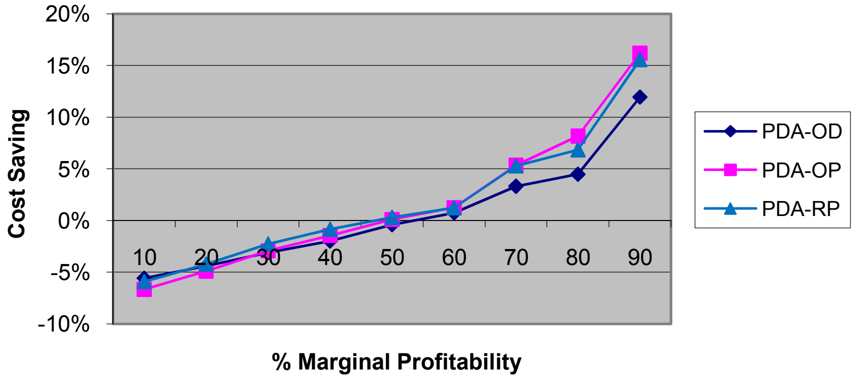

Figure 5.10. Cost saving versus marginal profitability for $\gamma = 0.6$, $\lambda_A = 0.8$, $\lambda_B = 0.7$ and $\lambda_C = 0.5$ with change in manufacturing cost for three-component case

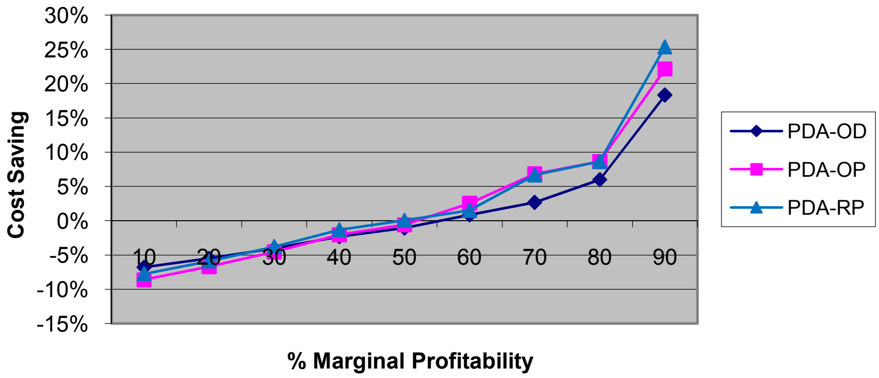

Figure 5.11. Cost saving versus marginal profitability for $\gamma = 0.6$, $\lambda_A = 0.8$, $\lambda_B = 0.5$ and $\lambda_C = 0.4$ with change in manufacturing cost for three-component case

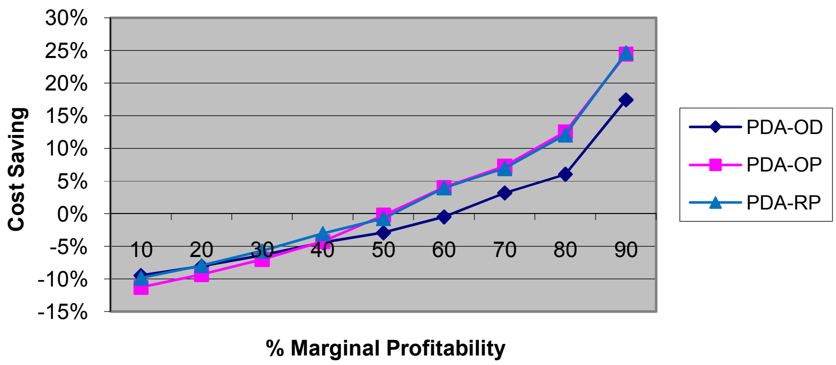

Figure 5.12. Cost saving versus marginal profitability for $\gamma = 0.8$, $\lambda_A = 0.6$, $\lambda_B = 0.5$ and $\lambda_C = 0.4$ with change in manufacturing cost for three-component case

As it is observed from the numerical results reported in the above tables and graphics, high marginal profitability level of the disassembly process makes allowing partial disassembly more profitable compared to not allowing. In fact, per cent cost saving achieved by allowing partial disassembly and holding overflows seems to be a monotonically increasing function of the marginal profitability level of the disassembly process in all demand and return rate configurations.

However, when the marginal profitability is low, allowing partial disassembly causes a significant increase in the total cost. The case where allowing partial disassembly policy is the most beneficial policy is the one where the return rate is larger than the demand rate of all components and the marginal profitability (or manufacturing cost) is very high.

Also, it is observed that when partial disassembly is allowed, holding overflows is more profitable than directly disposing them. Priority decision is not critical, since it does not affect the total cost significantly. Actually, these statements are the same as the ones we draw from the numerical results of the experiment sets where the unit disposal cost is changed while the other are held constant.

5.1.3. Change in Remanufacturing Costs

In this section, we observe the effect of marginal profitability of disassembly process by changing the remanufacturing costs while other unit costs are held constant. Remanufacturing costs involve the unit acquisition and disassembly of cores and the unit refurbishing costs of parts.

In Table 5.12, the unit cost set used for the analysis of the two component case is shown. In this experiment set, we change the unit acquisition, disassembly and refurbishing costs. However, since the unit holding cost rates are evaluated by using these unit costs, there is an additional effect of the change of the unit inventory holding costs on the total costs besides the effect of marginal profitability. So, in order to eliminate this additional effect, we change the inventory carrying charge so as to hold the unit holding cost rates constant. In Table 5.13, inventory carrying charges and unit holding cost rates are illustrated. In Table 5.14, marginal profitability levels are shown, and in Table 5.15, resulting total costs of different disassembly policies and per cent cost savings of allowing partial disassembly policies are reported for two component case.

Table 5.12. Experiment set 4 for two component case

Ex	c_p	c_d	c_R	d_p	c_M
1	50	50	50	20	100
2	45	45	45	20	100
3	40	40	40	20	100
4	35	35	35	20	100
5	30	30	30	20	100
6	25	25	25	20	100
7	20	20	20	20	100
8	15	15	15	20	100
9	10	10	10	20	100
10	5	5	5	20	100

Table 5.13. Inventory carrying charge and unit holding cost rates of experiment set 4

Ex	α	H_p	H_{di}	H_{ri}
1	0.030	2.5	2	3.5
2	0.033	2.5	2	3.5
3	0.038	2.5	2	3.5
4	0.043	2.5	2	3.5
5	0.050	2.5	2	3.5
6	0.060	2.5	2	3.5
7	0.075	2.5	2	3.5
8	0.100	2.5	2	3.5
9	0.150	2.5	2	3.5
10	0.300	2.5	2	3.5

Table 5.14. Marginal profit and marginal profitability levels of experiment set 4

Ex	$c_p + c_d + 2c_R$	$d_p + 2c_M$	Marginal Profit	Marginal Profitability
1	200	220	20	10%
2	180	220	40	20%
3	160	220	60	30%
4	140	220	80	40%
5	120	220	100	50%
6	100	220	120	60%
7	80	220	140	70%
8	60	220	160	80%
9	40	220	180	90%
10	20	220	200	100%

Table 5.15. Total costs and cost savings of experiment set 4

$\gamma = 0.6$ $\lambda_A = 0.8$ $\lambda_B = 0.5$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	143.56	147.22	-2.6%	143.24	0.2%	143.94	-0.3%
2	137.94	139.91	-1.4%	136.96	0.7%	136.94	0.7%
3	130.82	132.56	-1.3%	127.90	2.2%	129.95	0.7%
4	122.64	124.42	-1.5%	119.14	2.9%	119.28	2.7%
5	113.56	114.87	-1.2%	109.72	3.4%	110.18	3.0%
6	104.27	104.50	-0.2%	100.30	3.8%	99.08	5.0%
7	94.87	93.74	1.2%	89.54	5.6%	88.22	7.0%
8	85.08	82.69	2.8%	78.68	7.5%	78.44	7.8%
9	75.26	71.51	5.0%	68.31	9.2%	66.61	11.5%
10	65.41	60.29	7.8%	56.99	12.9%	55.08	15.8%
$\gamma = 0.8$ $\lambda_A = 0.6$ $\lambda_B = 0.5$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	128.38	135.92	-5.9%	127.96	0.3%	129.35	-0.8%
2	123.23	127.63	-3.6%	120.77	2.0%	121.49	1.4%
3	116.37	119.25	-2.5%	113.23	2.7%	113.63	2.4%
4	108.14	110.82	-2.5%	103.88	3.9%	105.41	2.5%
5	99.68	103.05	-3.4%	94.47	5.2%	95.20	4.5%
6	90.71	92.61	-2.1%	84.93	6.4%	85.11	6.2%
7	81.41	82.12	-0.9%	74.87	8.0%	74.80	8.1%
8	71.75	70.73	1.4%	64.61	9.9%	64.52	10.1%
9	62.03	59.17	4.6%	54.44	12.2%	53.57	13.6%
10	52.29	47.24	9.7%	43.62	16.6%	42.59	18.6%
$\gamma = 0.5$ $\lambda_A = 0.8$ $\lambda_B = 0.6$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	150.98	152.79	-1.2%	151.00	0.0%	151.34	-0.2%
2	145.59	146.14	-0.4%	143.63	1.3%	143.98	1.1%
3	137.88	139.02	-0.8%	134.98	2.1%	135.27	1.9%
4	128.97	130.19	-0.9%	126.40	2.0%	126.02	2.3%
5	119.49	120.55	-0.9%	116.57	2.4%	117.09	2.0%
6	109.71	110.75	-1.0%	106.60	2.8%	107.08	2.4%
7	99.83	100.93	-1.1%	96.54	3.3%	96.58	3.3%
8	89.88	91.10	-1.4%	86.13	4.2%	86.34	3.9%
9	79.91	81.28	-1.7%	75.85	5.1%	76.08	4.8%
10	69.92	71.43	-2.2%	65.56	6.2%	65.82	5.9%

In Table 5.16, we give the unit cost set used for the analysis of three component case. In Table 5.17, inventory carrying charges that are adjusted to make the unit holding rates constant are shown for the three component case. In Table 5.18, marginal profitability levels and in Table 5.19, resulting total costs with per cent cost savings are given.

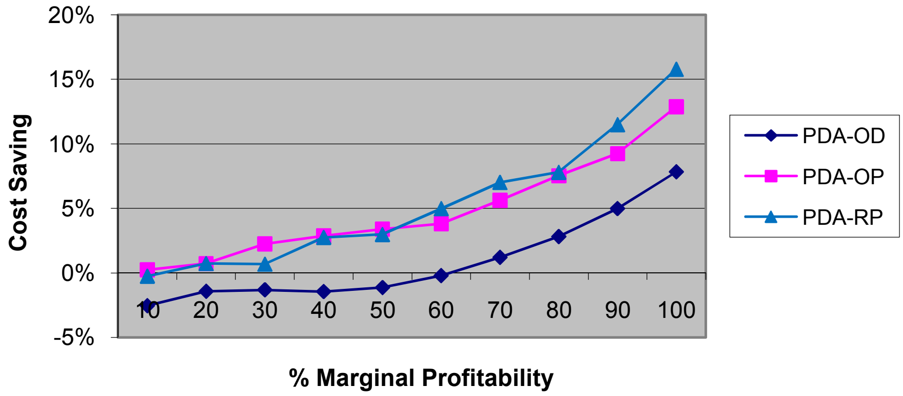

Figure 5.13. Cost saving versus marginal profitability for γ= 0.6, λ_A= 0.8 and λ_B= 0,5 with change in remanufacturing costs for two-component case

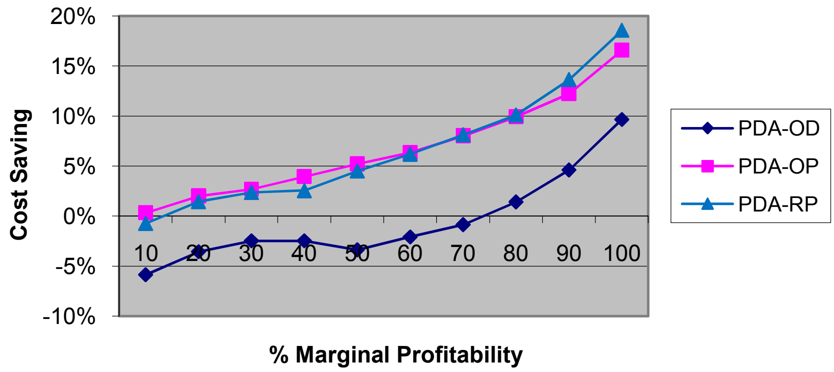

Figure 5.14. Cost saving versus marginal profitability for γ= 0.8, λ_A= 0.6 and λ_B= 0.5 with change in remanufacturing costs for two-component case

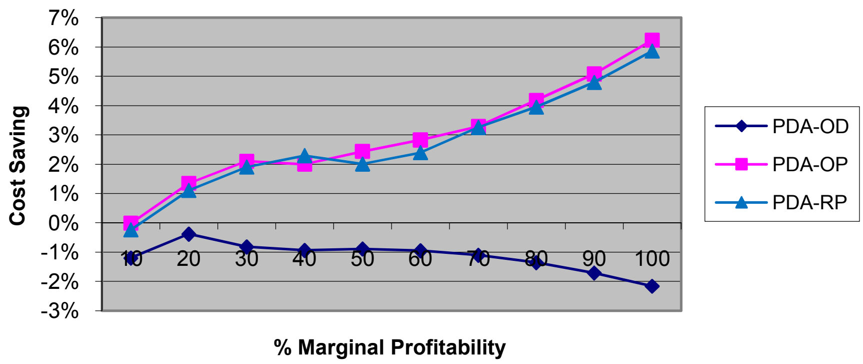

Figure 5.15. Cost saving versus marginal profitability for γ= 0.5, λ_A= 0.8 and λ_B= 0.6 with change in remanufacturing costs for two-component case

Table 5.16. Experiment set 5 for three component case

Ex	c_p	c_d	c_R	d_p	c_M
1	50	50	50	25	83.33
2	45	45	45	25	83.33
3	40	40	40	25	83.33
4	35	35	35	25	83.33
5	30	30	30	25	83.33
6	25	25	25	25	83.33
7	20	20	20	25	83.33
8	15	15	15	25	83.33
9	10	10	10	25	83.33
10	5	5	5	25	83.33

Table 5.17. Inventory carrying charges and unit holding cost rates of experiment set 5

Ex	α	H_p	H_{di}	H_{ri}
1	0.030	2.5	1.33	2.83
2	0.033	2.5	1.33	2.83
3	0.038	2.5	1.33	2.83
4	0.043	2.5	1.33	2.83
5	0.050	2.5	1.33	2.83
6	0.060	2.5	1.33	2.83
7	0.075	2.5	1.33	2.83
8	0.100	2.5	1.33	2.83
9	0.150	2.5	1.33	2.83
10	0.300	2.5	1.33	2.83

Table 5.18. Marginal profit and marginal profitability levels of experiment set 5

Ex	$c_p + c_d + 3c_R$	$d_p + 3c_M$	Marginal Profit	Marginal Profitability
1	250	275	25	10%
2	225	275	50	20%
3	200	275	75	30%
4	175	275	100	40%
5	150	275	125	50%
6	125	275	150	60%
7	100	275	175	70%
8	75	275	200	80%
9	50	275	225	90%
10	25	275	250	100%

Table 5.19. Total costs and per cent cost savings of experiment set 5

$\gamma = 0.6\ \ \lambda_A = 0.8\ \ \lambda_B = 0.7\ \ \lambda_C = 0.5$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	182.99	186.98	-2.2%	184.17	-0.6%	185.44	-1.3%
2	176.55	177.96	-0.8%	176.24	0.2%	176.53	0.0%
3	168.49	168.76	-0.2%	167.52	0.6%	167.69	0.5%
4	156.96	157.19	-0.1%	153.50	2.2%	153.73	2.1%
5	145.71	144.88	0.6%	139.92	4.0%	140.11	3.8%
6	134.43	131.86	1.9%	126.38	6.0%	126.30	6.0%
7	122.84	117.46	4.4%	113.67	7.5%	112.45	8.5%
8	111.29	103.57	6.9%	99.61	10.5%	100.10	10.1%
9	99.52	89.65	9.9%	85.46	14.1%	85.97	13.6%
10	87.57	75.72	13.5%	71.27	18.6%	71.79	18.0%
$\gamma = 0.6\ \ \lambda_A = 0.8\ \ \lambda_B = 0.5\ \ \lambda_C = 0.4$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	159.28	165.33	-3.8%	160.01	-0.5%	160.23	-0.6%
2	153.71	156.51	-1.8%	151.84	1.2%	151.84	1.2%
3	147.61	147.69	-0.1%	143.67	2.7%	143.45	2.8%
4	140.33	138.61	1.2%	134.73	4.0%	135.01	3.8%
5	128.77	127.98	0.6%	123.99	3.7%	124.12	3.6%
6	119.70	115.68	3.4%	112.72	5.8%	110.31	7.8%
7	110.56	103.24	6.6%	98.51	10.9%	98.40	11.0%
8	101.41	90.47	10.8%	85.95	15.2%	85.69	15.5%
9	92.07	77.55	15.8%	73.93	19.7%	72.84	20.9%
10	82.51	64.56	21.8%	61.57	25.4%	58.56	29.0%
$\gamma = 0.8\ \ \lambda_A = 0.6\ \ \lambda_B = 0.5\ \ \lambda_C = 0.4$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	147.92	158.67	-7.3%	150.22	-1.6%	150.93	-2.0%
2	143.05	149.03	-4.2%	140.98	1.5%	141.74	0.9%
3	136.01	138.75	-2.0%	132.05	2.9%	132.55	2.5%
4	127.41	128.77	-1.1%	121.87	4.3%	121.54	4.6%
5	118.43	118.79	-0.3%	110.17	7.0%	110.70	6.5%
6	109.45	108.72	0.7%	99.23	9.3%	98.95	9.6%
7	100.20	95.17	5.0%	87.45	12.7%	86.64	13.5%
8	90.95	82.62	9.2%	75.27	17.2%	74.36	18.2%
9	81.52	69.02	15.3%	62.83	22.9%	61.58	24.5%
10	70.03	55.42	20.9%	50.59	27.8%	50.36	28.1%

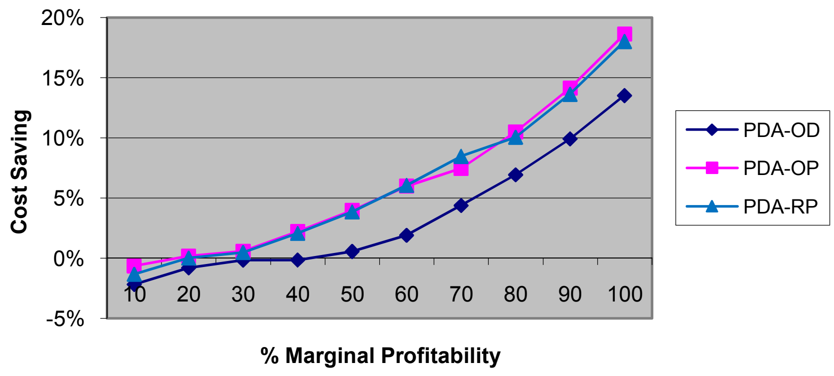

Figure 5.16. Cost saving versus marginal profitability for $\gamma = 0.6$, $\lambda_A = 0.8$, $\lambda_B = 0.7$ and $\lambda_C = 0.5$ with change in remanufacturing costs for three component case

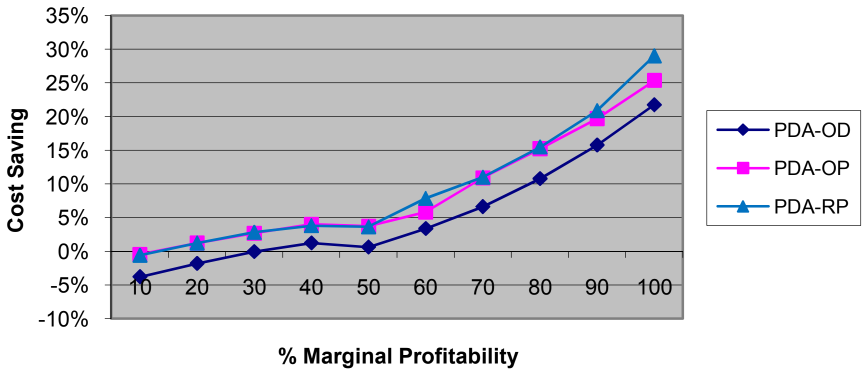

Figure 5.17. Cost saving versus marginal profitability for $\gamma = 0.6$, $\lambda_A = 0.8$, $\lambda_B = 0.5$ and $\lambda_C = 0.4$ with change in remanufacturing costs for three component case

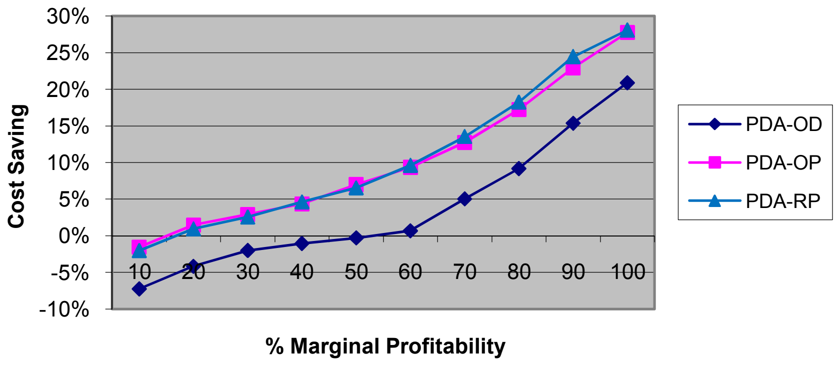

Figure 5.18. Cost saving versus marginal profitability for $\gamma = 0.8$, $\lambda_A = 0.6$, $\lambda_B = 0.5$ and $\lambda_C = 0.4$ with change in remanufacturing costs for three component case

When the marginal profitability of the disassembly is increased by decreasing remanufacturing costs, it is observed that allowing partial disassembly results in higher cost savings. Again, holding overflows is more profitable than disposing them for all cases. Also, the case where the largest cost saving is achieved by allowing partial disassembly is the one where the return rate is larger than the demand rates of all components and the marginal profitability of the disassembly process is high.

5.2. Holding vs. Disposing the Overflow Items

As observed in the experiments, when partial disassembly is allowed, policy of holding overflow items outperforms the direct disposal in all cases and in all configurations without exception. The main reason for this situation is not only the disposal cost of the components but also the reduction of service levels caused by the disposal of overflows. Especially, when the return rate is smaller than the demand rates, disposal policy becomes much more harmful, since the supply is more limited in this case and the limited supply is destructed by the disposal of overflow items. Also, the added values in these parts are lost by disposing them. The lost added value involves all expenses paid for these items before the disposal that are acquisition, disassembly and holding costs of these items. So, holding not demanded viz. overflow items until a certain limit is always more beneficial than the direct disposal.

5.3. Effect of the Return Arrival Rate

In this section we concentrate on the effect of the return arrival rate. In other words, we analyze the different behaviors of the disassembly systems with respect to the different arrival rate configurations of returns and demands. In fact, disassembly policies with a return rate that is smaller than the demand rates of components and with a return rate that is larger than the demand rates of components result in different expected total costs and different per cent cost savings. This situation can be easily observed from the results of the experiments in the previous section. Here, we gather these results with respect to the arrival rate configurations and draw some insights about the effect of the return arrival rate.

Since, in previous experiments, it is seen that holding overflows is more favorable than disposing them and also, priority preferences are indifferent with respect to the total cost, we focus on the holding policy with overflow priority (PDA-OP) for comparing different return and demand arrival rate configurations in this section.

Firstly, for the two-component case, let's consider following three cases

- Case 1: $\gamma = 0.5$, $\lambda_A = 0.8$, $\lambda_B = 0.6$
- Case 2: $\gamma = 0.6$, $\lambda_A = 0.8$, $\lambda_B = 0.5$
- Case 3: $\gamma = 0.8$, $\lambda_A = 0.6$, $\lambda_B = 0.5$

Per cent cost savings of PDA-OP policy for these arrival rate configurations are reported in Table 5.20 for experiment set 1, set 3 and set 4 where we observe the effect of marginal profitability by changing the unit disposal cost, manufacturing and remanufacturing costs, respectively, for two component case in previous section. These results are also shown in the following graphics with respect to the arrival rate configurations for each experiment set.

Table 5.20. Per cent cost savings of PDA-OP for experiment set 1, 3 and 4 of two component case

% Marginal Profitability	Set 1			Set 3			Set 4		
	Case-1	Case-2	Case-3	Case-1	Case-2	Case-3	Case-1	Case-2	Case-3
10	-1.0%	-2.0%	-9.0%	-4.1%	-5.6%	-7.6%	0.0%	0.2%	0.3%
20	-0.4%	-0.3%	0.3%	-3.2%	-4.5%	-6.6%	1.3%	0.7%	2.0%
30	0.4%	0.5%	4.2%	-2.1%	-3.3%	-5.3%	2.1%	2.2%	2.7%
40	2.1%	1.9%	5.3%	-1.1%	-2.0%	-3.9%	2.0%	2.9%	3.9%
50	2.0%	3.5%	4.9%	-0.5%	-0.6%	-0.7%	2.4%	3.4%	5.2%
60	2.6%	3.8%	6.0%	0.1%	0.3%	2.2%	2.8%	3.8%	6.4%
70	2.7%	4.4%	6.3%	1.3%	2.1%	3.5%	3.3%	5.6%	8.0%
80	3.2%	4.7%	6.7%	3.2%	4.7%	6.7%	4.2%	7.5%	9.9%
90	3.3%	5.2%	7.0%	5.1%	9.2%	12.2%	5.1%	9.2%	12.2%
100	3.5%	6.0%	7.0%	-	-	-	6.2%	12.9%	16.6%

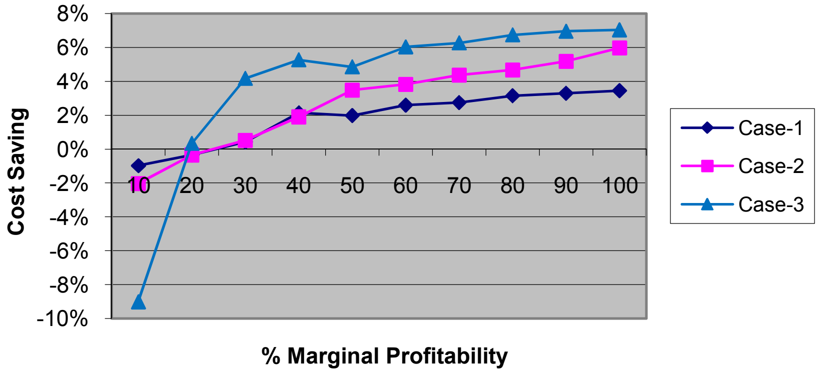

Figure 5.19. Cost saving vs. marginal profitability for experiment set 1 of two component case with different arrival rate configurations

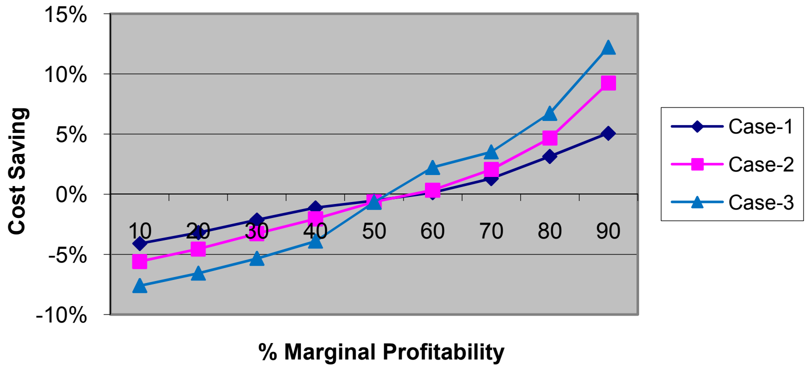

Figure 5.20. Cost saving vs. marginal profitability for experiment set 3 of two component case with different arrival rate configurations

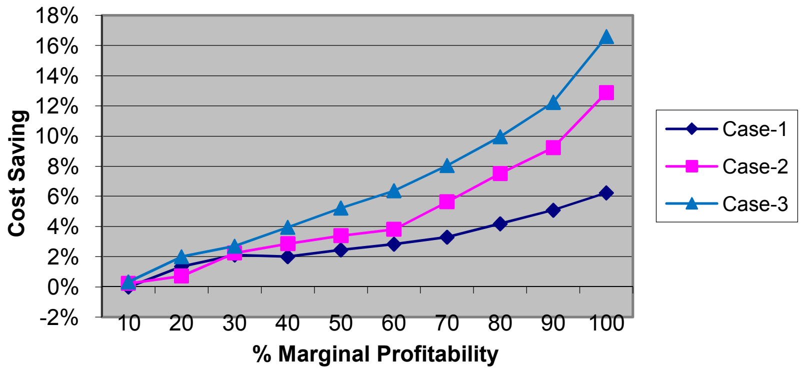

Figure 5.21. Cost saving vs. marginal profitability for experiment set 4 of two component case with different arrival rate configurations

Secondly, for the three-component case, let's consider following three cases

- Case 1: $\gamma = 0.6$ $\lambda_A = 0.8$ $\lambda_B = 0.7$ $\lambda_C = 0.5$
- Case 2: $\gamma = 0.6$ $\lambda_A = 0.8$ $\lambda_B = 0.5$ $\lambda_C = 0.4$
- Case 3: $\gamma = 0.8$ $\lambda_A = 0.6$ $\lambda_B = 0.5$ $\lambda_C = 0.4$

Per cent cost savings of PDA-OP policy for these arrival rate configurations are reported in Table 5.21 for experiment set 2, set 3 and set 5 where we observe the effect of marginal profitability by changing the unit disposal cost, manufacturing and remanufacturing costs, respectively, for three component case in previous section. These results are also shown in the following graphics with respect to the arrival rate configurations for each experiment set.

Table 5.21. Per cent cost savings of PDA-OP for experiment set 2,3 and 5 of three component case

% Marginal Profitability	Set 2			Set 3			Set 5		
	Case-1	Case-2	Case-3	Case-1	Case-2	Case-3	Case-1	Case-2	Case-3
10	-3.6%	-6.0%	-14.7%	-6.7%	-8.6%	-11.3%	-0.6%	-0.5%	-1.6%
20	-1.3%	-2.1%	-6.5%	-4.9%	-6.7%	-9.3%	0.2%	1.2%	1.5%
30	0.1%	-0.6%	-0.3%	-2.9%	-4.6%	-7.0%	0.6%	2.7%	2.9%
40	0.7%	1.4%	3.4%	-1.4%	-2.0%	-4.3%	2.2%	4.0%	4.3%
50	2.7%	3.4%	5.7%	0.1%	-0.6%	-0.2%	4.0%	3.7%	7.0%
60	4.0%	5.4%	6.4%	1.3%	2.5%	4.0%	6.0%	5.8%	9.3%
70	4.0%	6.8%	7.3%	5.4%	6.8%	7.3%	7.5%	10.9%	12.7%
80	4.7%	8.2%	8.2%	8.2%	8.6%	12.5%	10.5%	15.2%	17.2%
90	5.2%	9.7%	8.4%	16.2%	22.1%	24.4%	14.1%	19.7%	22.9%
100	5.9%	10.4%	9.0%	-	-	-	18.6%	25.4%	27.8%

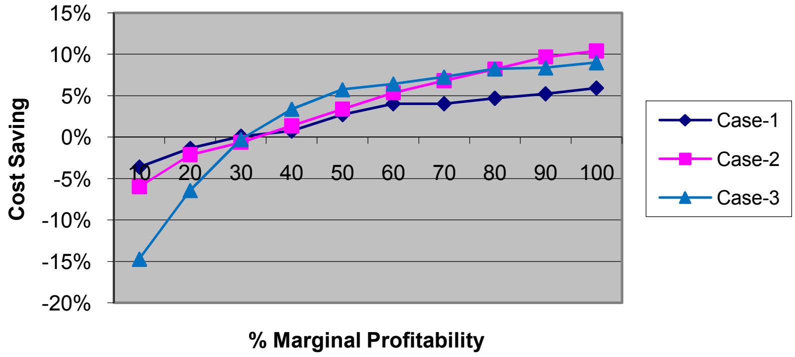

Figure 5.22. Cost saving vs. marginal profitability for experiment set 2 of three component case with different arrival rate configurations

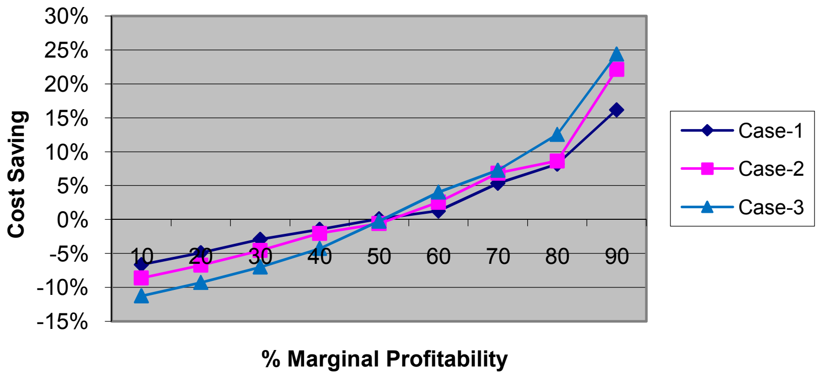

Figure 5.23. Cost saving vs. marginal profitability for experiment set 3 of three component case with different arrival rate configurations

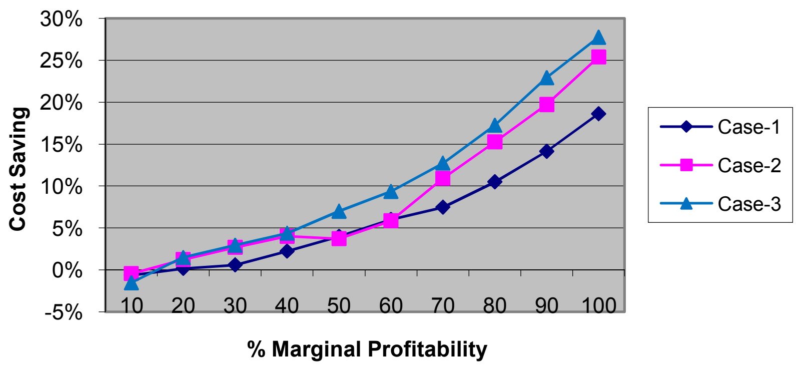

Figure 5.24. Cost saving vs. marginal profitability for experiment set 5 of three component case with different arrival rate configurations

It can be observed that return rate has different effects on cost savings in different marginal profitability levels. For instance, if the marginal profitability level of the disassembly process is low, then higher return rate causes less cost saving (or more cost increase). On the other hand, if the marginal profitability is high, higher return rate causes more cost savings.

Mainly, if partial disassembly is allowed and the marginal profitability level of disassembly process compared to the manufacturing is low, then higher return rate causes a higher increase in disassembly and disposal costs than the decrease in lost sales viz. manufacturing costs. So, higher return rate makes allowing partial disassembly less attractive in terms of the total cost. On the contrary, if marginal profitability is high, then higher return rate causes a smaller increase in disassembly and disposal costs than the decrease in the total lost sales viz. manufacturing costs. So, higher return rate makes allowing partial disassembly more advantageous in terms of the total cost.

We analyze three different cost factors affecting the marginal profitability level, viz. disposal costs, manufacturing costs and remanufacturing costs. However, effect of higher return rate when marginal profitability level is affected by the unit disposal cost is different than the effect of the higher return rate when marginal profitability level is affected by the unit manufacturing or remanufacturing cost. This fact can be observed from Figure 5.19 and 5.22. Since a higher return rate compared to the demand rates means a higher disposal rate, allowing partial disassembly policy becomes less beneficial in terms of the total cost because of the high disposal cost. So, it can be stated that a higher return rate with a higher unit disposal cost is not as advantageous as a higher return rate with a higher unit manufacturing cost (or lower remanufacturing costs) for allowing partial disassembly.

5.4. Allowing vs. Not Allowing Partial Disassembly

It is seen that when marginal profitability is low, allowing partial disassembly is a non profitable policy even if we hold the overflow items. This situation can be explained by the effect of allowing partial disassembly on the production speed of disassembly process. If the disassembly process is not a highly profitable process compared to the manufacturing, then the higher speed of the disassembly operation causes a higher increase in disassembly cost than the decrease in the lost sales viz. manufacturing costs. Thus, allowing partial disassembly becomes non-profitable.

So, here the main concern is the level of marginal profitability rather than the other effects such as the return arrival rate. In fact, there is a threshold value of marginal profitability for allowing partial disassembly. If the marginal profitability is lower than this value, then allowing partial disassembly results in a cost increase. Otherwise, a significant cost saving is obtained by allowing partial disassembly.

In Table 5.22, we show the estimated threshold values of marginal profitability levels of the experiment sets of two-component case that are experiment set 1, set 3 and set 4. Estimated threshold values are obtained by formula (4.37), which is $(c_p + c_d)/2c_M$. As pointed out in previous chapter, marginal profitability level is expected to be larger than this threshold value for obtaining a cost saving by allowing partial disassembly.

Table 5.22. Estimated threshold and marginal profitability levels of allowing partial disassembly

% Marginal Profitability	% Threshold		
	Set 1	Set 3	Set 4
10	20	90	50
20	**20**	80	45
30	20	70	40
40	20	60	**35**
50	20	**50**	30
60	20	40	25
70	20	30	20
80	20	20	15
90	20	10	10
100	20	-	5

As it can be observed in Figure 5.19 and Figure 5.20, estimated thresholds are close to the real threshold values. However, the estimated threshold value without considering holding and backorder costs seems to be an upper bound of the real threshold value for allowing partial disassembly. Because, there are some experimental results where allowing partial disassembly causes cost savings even if the marginal profitability is below of that estimated threshold value. For instance, as observed in Figure 5.21, allowing partial disassembly results in a cost saving even if the marginal profitability is below of the estimated threshold level. So, it can be stated that estimated threshold value of the marginal profitability level is an upper bound of the real threshold value for allowing partial disassembly. In other words, we can be sure about that allowing partial disassembly results in a cost saving if the marginal profitability is higher than the estimated threshold value.

5.5. Effect of the Non-Uniformity of the Parts

In previous experiments, we only take into account the uniform part case where the unit disposal, refurbishing, manufacturing and holding costs of different types of parts are equal to each other. In this section, we attempt to quantify the effect of the non uniformity of the parts in terms of their physical characteristics and manufacturing costs on the per cent cost savings of allowing partial disassembly. Here, we consider only two-component case to more precisely clarify the effect of the dominance of the one part type to the other part type in terms of the physical characteristics and manufacturing costs.

5.5.1. Effect of the Physical Characteristics of the Parts

Physical characteristics of the disassembled parts are important factors that can affect the unit out-of-pocket holding costs and unit disposal cost of the parts. In fact, the part type having more volume or weight than the other part type is expected to have larger unit out-of- pocket holding costs and larger unit disposal cost than that of the other part type. So, we assume that unit out-of-pocket holding and unit disposal costs of the parts are proportional to their volume or weight. We also assume that the summation of out-of-pocket holding and unit disposal costs of the parts are equal to the out-of-pocket holding and unit disposal costs of the core, respectively. We determine a ratio that we call physical ratio of a part type, and we compute the unit out-of-pocket holding and unit disposal costs of the parts by multiplying the unit out-of-pocket holding and unit disposal costs of the core with the physical ratio of corresponding part type, respectively. Summation of physical ratios of both part types are equal to one. So, by using this ratio that represents the respective volume or weight of components, we can observe the effect of the dominance of a part type to the other part type in terms of volume or weight on the cost saving of allowing partial disassembly.

In our experiments, service rates of all machines are set equal to one. The unit out-of-pocket holding cost of a product is selected as one (h_p = 1), and inventory carrying charge is selected as α = 0.15. In holding cost evaluation, we use two different allocation fractions: $f_i = h_{di} / h_p$ and $f_i = 1 / 2$ for $i \in S_2 = \{A, B\}$. Also, it is assumed that $h_{di} = h_{ri} = h_p / 2$ for $i \in S_2 = \{A, B\}$. The unit backorder cost for part demands are assumed to be equal with respect to the part types and they are selected as $b_i = 4$ for $i \in S_2 = \{A, B\}$.

The unit acquisition and disassembly costs of a core are selected as 10, that is $c_p = c_d = 10$. Unit disposal cost of a product (d_p) is set equal to 40. It is assumed that unit refurbishing and manufacturing costs of different types of parts are equal to each other. The unit refurbishing cost of a part is selected as $c_R = 10$. Also, the unit manufacturing cost of a part is selected as $c_M = 50$.

Table 5.23. Experiment set for non-uniformity of physical characteristics

	Physical ratios (%)		Unit out-of-pocket holding costs		Unit disposal costs	
Ex	Part A	Part B	Part A	Part B	Part A	Part B
1	10	90	0.1	0.9	4	36
2	20	80	0.2	0.8	8	32
3	30	70	0.3	0.7	12	28
4	40	60	0.4	0.6	16	24
5	50	50	0.5	0.5	20	20
6	60	40	0.6	0.4	24	16
7	70	30	0.7	0.3	28	12
8	80	20	0.8	0.2	32	8
9	90	10	0.9	0.1	36	4

Table 5.24. Total costs and per cent cost savings for allocation fraction $f_i = h_{di} / h_p$

$\gamma = 0.6$ $\lambda_A = 0.8$ $\lambda_B = 0.5$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	59.46	59.80	-0.6%	57.38	3.5%	56.93	4.3%
2	59.83	59.72	0.2%	57.61	3.7%	56.99	4.7%
3	60.18	59.61	1.0%	57.31	4.8%	57.82	3.9%
4	60.47	59.42	1.7%	57.15	5.5%	57.33	5.2%
5	60.56	59.17	2.3%	56.95	6.0%	56.96	5.9%
6	60.54	58.91	2.7%	57.01	5.8%	56.72	6.3%
7	60.38	58.62	2.9%	56.22	6.9%	56.80	5.9%
8	60.21	58.18	3.4%	55.92	7.1%	56.00	7.0%
9	59.94	57.72	3.7%	55.04	8.2%	55.02	8.2%
$\gamma = 0.8$ $\lambda_A = 0.6$ $\lambda_B = 0.5$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	59.10	59.84	-1.2%	56.21	4.9%	56.39	4.6%
2	59.55	60.27	-1.2%	56.13	5.7%	55.98	6.0%
3	59.95	59.95	0.0%	56.43	5.9%	56.29	6.1%
4	60.34	59.90	0.7%	56.50	6.4%	56.48	6.4%
5	60.68	59.79	1.5%	56.41	7.0%	56.27	7.3%
6	60.69	59.62	1.8%	56.31	7.2%	56.03	7.7%
7	60.61	59.58	1.7%	55.98	7.6%	55.79	7.9%
8	60.28	59.20	1.8%	55.63	7.7%	55.38	8.1%
9	60.32	58.81	2.5%	55.08	8.7%	54.77	9.2%
$\gamma = 0.5$ $\lambda_A = 0.8$ $\lambda_B = 0.6$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	58.75	59.59	-1.4%	57.30	2.5%	57.02	2.9%
2	59.05	59.79	-1.2%	57.37	2.8%	57.21	3.1%
3	59.35	59.86	-0.9%	57.47	3.2%	57.70	2.8%
4	59.43	59.77	-0.6%	57.49	3.3%	57.73	2.9%
5	59.49	59.62	-0.2%	57.44	3.5%	57.63	3.1%
6	59.42	59.34	0.1%	57.25	3.6%	57.36	3.5%
7	59.33	59.04	0.5%	57.01	3.9%	57.09	3.8%
8	59.07	58.74	0.6%	56.55	4.3%	56.68	4.0%
9	58.53	58.37	0.3%	55.92	4.5%	55.93	4.4%

In Table 5.23, cost parameters of the experiment set are shown. These parameters are unit out-of-pocket holding costs and disposal costs of parts that depend on the physical ratio of the parts. In Table 5.24 and Table 5.25, minimum expected total costs of the disassembly policies and per cent cost savings of allowing partial disassembly policies are reported for two different allocation fractions and three different arrival rate configurations. The results of per cent cost savings are also illustrated in the following graphics with respect to the physical ratio of part A.

Table 5.25. Total costs and per cent cost savings for allocation fraction $f_i = 1 / 2$

$\gamma = 0.6$ $\lambda_A = 0.8$ $\lambda_B = 0.5$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	60.47	60.71	-0.4%	57.85	4.3%	57.84	4.3%
2	60.50	60.38	0.2%	57.64	4.7%	57.71	4.6%
3	60.52	60.02	0.8%	57.75	4.6%	57.47	5.0%
4	60.54	59.59	1.6%	57.23	5.5%	57.21	5.5%
5	60.56	59.17	2.3%	56.95	6.0%	56.96	5.9%
6	60.58	58.74	3.0%	56.81	6.2%	56.70	6.4%
7	60.60	58.30	3.8%	56.37	7.0%	56.42	6.9%
8	60.58	57.85	4.5%	55.93	7.7%	56.08	7.4%
9	60.54	57.40	5.2%	55.75	7.9%	55.86	7.7%
$\gamma = 0.8$ $\lambda_A = 0.6$ $\lambda_B = 0.5$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	60.34	60.31	0.0%	56.44	6.5%	56.52	6.3%
2	60.43	60.29	0.2%	56.44	6.6%	56.48	6.5%
3	60.52	60.18	0.6%	56.44	6.7%	56.44	6.7%
4	60.60	60.01	1.0%	56.43	6.9%	56.36	7.0%
5	60.68	59.79	1.5%	56.41	7.0%	56.27	7.3%
6	60.73	59.56	1.9%	56.30	7.3%	56.12	7.6%
7	60.73	59.33	2.3%	56.12	7.6%	55.97	7.8%
8	60.71	59.45	2.1%	56.14	7.5%	55.86	8.0%
9	60.69	59.11	2.6%	55.84	8.0%	55.66	8.3%
$\gamma = 0.5$ $\lambda_A = 0.8$ $\lambda_B = 0.6$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	59.43	60.13	-1.2%	57.48	3.3%	57.84	2.7%
2	59.45	60.02	-1.0%	57.48	3.3%	57.80	2.8%
3	59.46	59.90	-0.7%	57.48	3.3%	57.76	2.9%
4	59.48	59.77	-0.5%	57.46	3.4%	57.71	3.0%
5	59.49	59.62	-0.2%	57.44	3.5%	57.63	3.1%
6	59.47	59.40	0.1%	57.41	3.5%	57.56	3.2%
7	59.45	59.17	0.5%	57.38	3.5%	57.47	3.3%
8	59.44	58.93	0.8%	57.35	3.5%	57.38	3.5%
9	59.42	58.70	1.2%	57.32	3.5%	57.30	3.6%

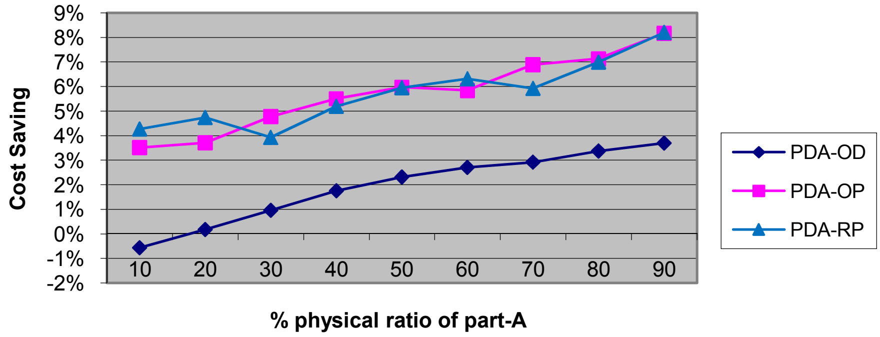

Figure 5.25. Cost saving versus marginal profitability for $\gamma = 0.6$, $\lambda_A = 0.8$ and $\lambda_B = 0.5$ with allocation fraction $f_i = h_{di} / h_p$

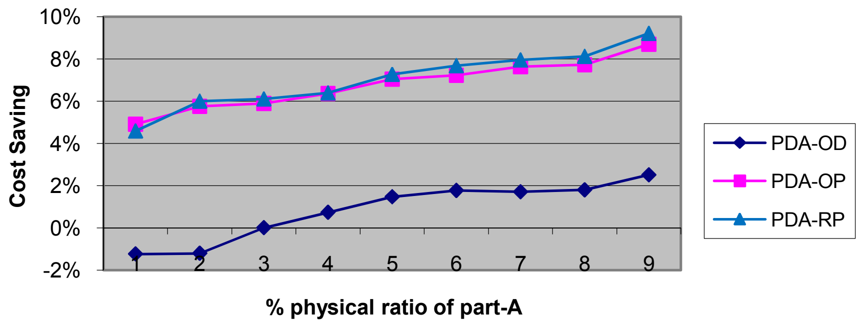

Figure 5.26. Cost saving versus marginal profitability for $\gamma = 0.8$, $\lambda_A = 0.6$ and $\lambda_B = 0.5$ with allocation fraction $f_i = h_{di} / h_p$

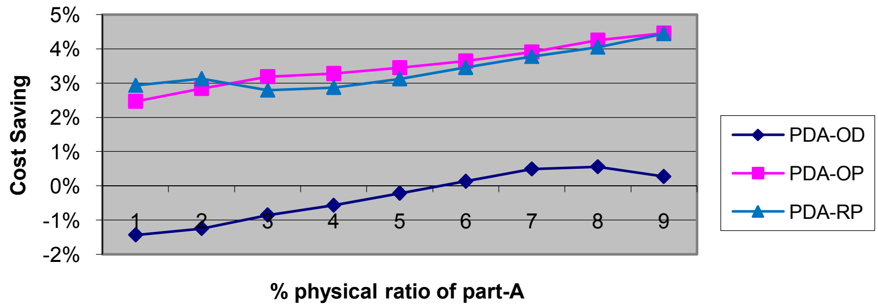

Figure 5.27. Cost saving versus marginal profitability for $\gamma = 0.5$, $\lambda_A = 0.8$ and $\lambda_B = 0.6$ with allocation fraction $f_i = h_{di} / h_p$

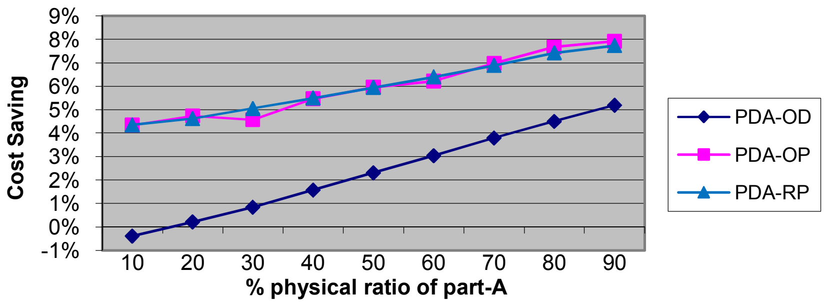

Figure 5.28. Cost saving versus physical ratio of part-A for $\gamma = 0.6$, $\lambda_A = 0.8$ and $\lambda_B = 0.5$ with allocation fraction $f_i = 1 / 2$

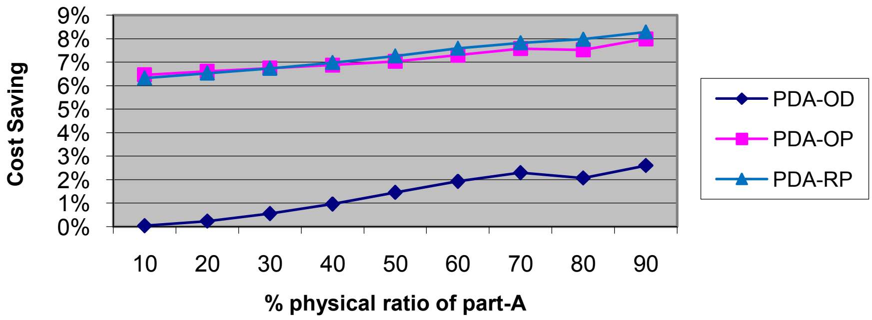

Figure 5.29. Cost saving versus physical ratio of part-A for $\gamma = 0.8$, $\lambda_A = 0.6$ and $\lambda_B = 0.5$ with allocation fraction $f_i = 1 / 2$

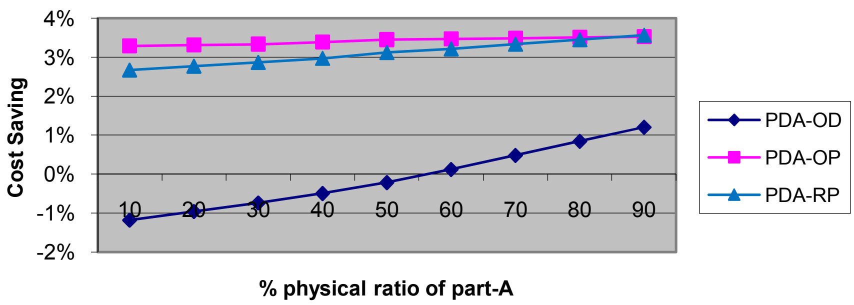

Figure 5.30. Cost saving versus physical ratio of part-A for $\gamma = 0.5$, $\lambda_A = 0.8$ and $\lambda_B = 0.6$ with allocation fraction $f_i = 1 / 2$

Since, in the experiments, the more demanded part is always part-A, cost saving of allowing partial disassembly becomes higher when the physical ratio of part-A becomes larger. The main reason to this situation is lower unit disposal and holding costs of the part-B. In fact, by allowing partial disassembly, we allow the system to disassemble the cores with the demand rate of more demanded part, and the not demanded parts viz. overflows are hold in finite buffers or they are directly disposed. Since the less demanded part is part-B in the experiments, when the unit disposal and holding costs of part-B is decreased, costs of disposing and holding of overflows caused by allowing partial disassembly is also decreased. So, allowing partial disassembly results in higher cost savings. Thus, it can be stated that allowing partial disassembly results in higher cost savings when more demanded part is dominant to the less demanded part in terms of the physical characteristics.

However, the change in percent cost savings of allowing partial disassembly with respect to the physical characteristics is very small. So, non uniformity of volume or weights of parts is not as critical as the marginal profitability of the disassembly process for the decision of allowing partial disassembly. In fact, when overflows are held, allowing partial disassembly can result in a cost saving even if the more demanded part has far smaller volume or weight than the less demanded part.

Also, it is observed that holding overflow items is always more profitable than disposing them. So, non uniformity of volume or weights of the parts does not have an effect on the superiority of holding policy compared to the direct disposal.

5.5.2. Effect of the Manufacturing Costs of the Parts

Different parts in a same core can have different manufacturing costs. For example, if a part is no longer actively being mass produced, then its production cost will be higher than that of the other parts that are still being mass produced. So, since lost sales of disassembly process are satisfied by new production, lost sales cost of that part will be higher than that of other parts. In this section, we analyze the effects of this non uniformity of the manufacturing costs of the parts on the cost saving of allowing partial disassembly.

In the experiments, we take the two-component case into account, and we assume that other unit costs except manufacturing and backorder costs are uniform with respect to the part types. In other words, unit holding, refurbishing and disposal costs of the different components are equal to each other.

Unit out-of-pocket holding costs of different components are assumed to be equal each other and so, used allocation fraction is $f_A = f_B = 1/2$. Also, it is assumed that $h_{di} = h_{ri} = h_p / 2$ for $i \in S_2 = \{A, B\}$.

In our experiments, service rates of all machines are set equal to one. The unit out-of-pocket holding cost of a product is selected as one (h_p = 1), and inventory carrying charge is selected as α = 0.15. The unit acquisition and disassembly costs of a core are selected as 10, that is $c_p = c_d$ = 10. Unit disposal cost of a product (d_p) is set equal to 40. It is assumed that unit refurbishing costs of different types of parts are equal to each other. The unit refurbishing cost of a part is selected as c_R = 10.

We determine a ratio that we call manufacturing cost ratio of a part type, and we compute the unit manufacturing and unit backorder costs of the parts proportionally to this ratio. Experiment set is shown in Table 5.26. Results of the experiments viz. total costs and per cent cost savings are shown in Table 5.27 for three different arrival rate configurations. Per cent cost savings of allowing partial disassembly policies are also illustrated in the following graphics.

Table 5.26. Experiment set for non-uniformity of manufacturing costs

	Manufacturing cost ratio (%)		Unit manufacturing costs		Unit backorder costs	
Ex	Part A	Part B	Part A	Part B	Part A	Part B
1	10	90	10	90	0.8	7.2
2	20	80	20	80	1.6	6.4
3	30	70	30	70	2.4	5.6
4	40	60	40	60	3.2	4.8
5	50	50	50	50	4	4
6	60	40	60	40	4.8	3.2
7	70	30	70	30	5.6	2.4
8	80	20	80	20	6.4	1.6
9	90	10	90	10	7.2	0.8

Table 5.27. Total costs and per cent cost savings of the experiment set that is for non uniformity of manufacturing costs of the parts

$\gamma = 0.6$ $\lambda_A = 0.8$ $\lambda_B = 0.5$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	48.22	49.76	-3.2%	46.96	2.6%	46.52	3.5%
2	51.86	52.71	-1.6%	49.90	3.8%	49.56	4.4%
3	54.88	55.23	-0.6%	52.66	4.0%	53.14	3.2%
4	57.76	57.42	0.6%	55.21	4.4%	55.38	4.1%
5	60.56	59.17	2.3%	56.95	6.0%	56.96	5.9%
6	63.12	60.75	3.7%	58.76	6.9%	58.82	6.8%
7	65.28	62.16	4.8%	60.72	7.0%	60.74	7.0%
8	67.13	62.98	6.2%	61.39	8.6%	61.58	8.3%
9	68.37	63.60	7.0%	62.33	8.8%	62.23	9.0%
$\gamma = 0.8$ $\lambda_A = 0.6$ $\lambda_B = 0.5$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	54.94	55.71	-1.4%	52.58	4.3%	52.52	4.4%
2	56.97	57.37	-0.7%	53.61	5.9%	53.79	5.6%
3	58.43	58.40	0.1%	54.64	6.5%	54.87	6.1%
4	59.72	59.13	1.0%	55.65	6.8%	55.52	7.0%
5	60.68	59.79	1.5%	56.41	7.0%	56.27	7.3%
6	61.24	60.45	1.3%	56.87	7.1%	56.96	7.0%
7	61.70	60.24	2.4%	57.14	7.4%	57.17	7.3%
8	61.83	60.04	2.9%	57.25	7.4%	57.30	7.3%
9	61.37	59.46	3.1%	57.16	6.8%	57.01	7.1%
$\gamma = 0.5$ $\lambda_A = 0.8$ $\lambda_B = 0.6$							
Ex	PDNA	PDA-OD	% CS	PDA-OP	% CS	PDA-RP	% CS
1	50.49	51.36	-1.7%	49.54	1.9%	49.27	2.4%
2	53.27	54.02	-1.4%	51.88	2.6%	51.98	2.4%
3	55.62	56.08	-0.8%	53.82	3.2%	54.01	2.9%
4	57.61	57.90	-0.5%	55.71	3.3%	55.82	3.1%
5	59.49	59.62	-0.2%	57.44	3.5%	57.63	3.1%
6	61.33	60.99	0.5%	59.02	3.8%	59.15	3.5%
7	62.81	62.20	1.0%	60.49	3.7%	60.36	3.9%
8	63.92	62.90	1.6%	61.77	3.4%	61.42	3.9%
9	64.38	63.28	1.7%	62.28	3.3%	62.07	3.6%

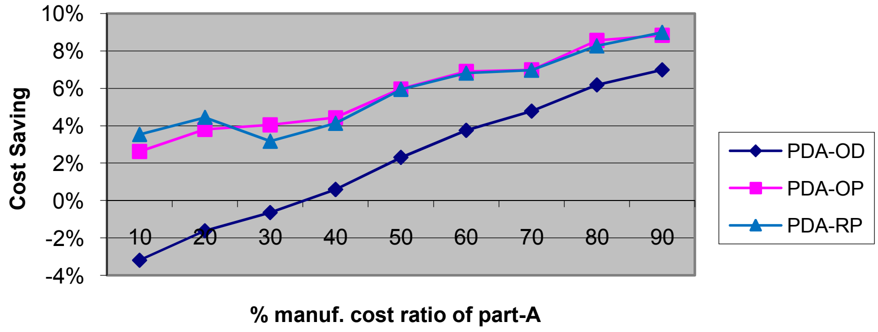

Figure 5.31. Cost saving versus manufacturing cost ratio of part-A for $\gamma = 0.6$, $\lambda_A = 0.8$ and $\lambda_B = 0.5$

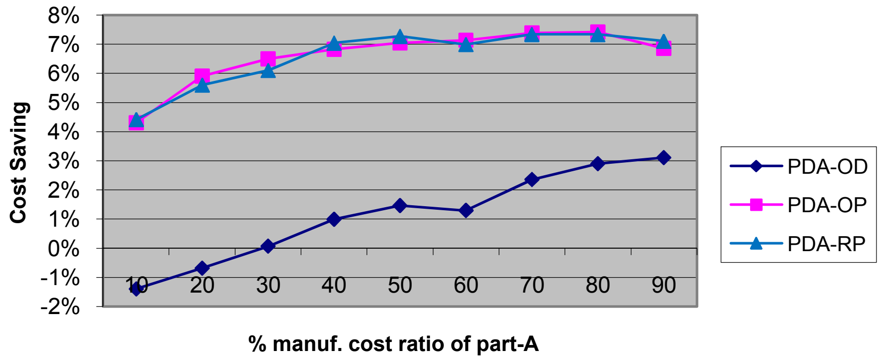

Figure 5.32. Cost saving versus manufacturing cost ratio of part-A for $\gamma = 0.8$, $\lambda_A = 0.6$ and $\lambda_B = 0.5$

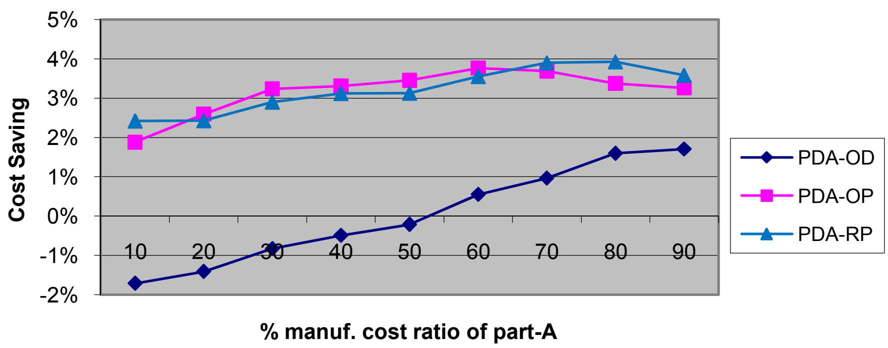

Figure 5.33. Cost saving versus manufacturing cost ratio of part-A for $\gamma = 0.5$, $\lambda_A = 0.8$ and $\lambda_B = 0.6$

It is observed that cost saving of allowing partial disassembly becomes higher when the manufacturing cost ratio of more demanded part viz. part-A becomes larger. Because, allowing partial disassembly results in a higher increase in the service level of more demanded part-A than that of the less demanded part-B. So, if manufacturing cost of the more demanded part is larger than that of less demanded part, then cost saving of allowing partial disassembly will be higher.

However, the change in percent cost savings of allowing partial disassembly with respect to the manufacturing cost ratio is very small. So, non uniformity of manufacturing costs of parts is not as critiacal as the marginal profitability of the disassembly process for the decision of allowing partial disassembly. In fact, when overflows are held, allowing partial disassembly can result in a cost saving even if the more demanded part has far smaller manufacturing cost than that of less demanded part.

Also, it is observed that holding overflow items is always more profitable than disposing them. So, non-uniformity of manufacturing costs of the parts does not have an effect on the superiority of holding policy compared to the direct disposal.

6. CONCLUSION

In this research, we analyzed different control policies of the disassembly process in remanufacturing. The analyzed policies differ with respect to the decision of allowing partial disassembly. The possible effects of this decision are quantified based on a total cost function that takes variable production, holding and disposal costs into account. We used a kanban control mechanism for coordination of production activities in disassembly process. The main reason for using kanban control, which is one of the simplest forms of the pull type control mechanisms, is the need of establishing a counter for overflow items that are consequences of allowing partial disassembly. Also, as reported in previous studies on disassembly control, pull control is more advantageous than the push control for disassembly process.

Allowing partial disassembly decision is more important in remanufacturing systems, because of the high variance of the quality levels of different parts found in the cores that are returned from customers. In fact, the quality levels of parts in a same core can be different, and so, demand that is unsatisfied because of the low quality can be satisfied by allowing partial disassembly. However, when partial disassembly is allowed for satisfying the demand of low quality part, the other disassembled parts become overflow items. So, these overflow items are disposed or held in extra buffers. Disposing and holding overflow items are two distinct policies that we are faced with when partial disassembly is allowed. Also, when overflows are held, we have two alternatives for satisfying the part demands: giving priority to the overflows items and giving priority to the regular items. In this study, we analyzed each of these policies by comparing the minimum total costs of the system with respect to the policies.

In order to obtain the results of the performance measures, the disassembly system is modeled as a multi class synchronized closed queuing network and a product form approximation technique, which is called “multi class approximation technique”, is used for solving this closed network. In the literature, there are applications of this technique for manufacturing and assembly systems, but not for disassembly processes. Here, for solving the queuing network model of the disassembly systems, we proposed new approximation based on the class aggregation technique. In our approximations, by adjusting the transition rates of continuous-time Markov chains, we derived new computation tools for the performance evaluation of the disassembly systems.

Once the performance measures are obtained by the approximation method, the calculation of the total cost is straightforward. However, evaluation of the unit holding cost rates needs special attention because of the unique characteristics of the disassembly system. In the disassembly setting, the material flow is diverging, therefore, the problem of how to allocate the added value to disassembled parts arises. In our study, for evaluating the unit holding cost rates of the parts, we used two different added value allocation rules that are based on the physical characteristics of the parts.

To obtain the minimum of the total cost function, parameters of the disassembly system viz. kanban levels, buffer sizes and backorder limits have to be optimized. For solving this optimization problem, we proposed heuristic search algorithms to find local optima of the objective function. We made our comparisons based on these local minimums of the total cost function.

In our numerical analysis, we searched for the conditions that make the allowing partial disassembly a profitable policy. Firstly, it is observed that cost saving caused by allowing partial disassembly depends on the marginal profitability level of the disassembly process. If the marginal profitability level is below a threshold value, allowing partial disassembly results in a cost increase. On the other hand, if the marginal profitability level is higher than this threshold, allowing partial disassembly results in a significant cost saving. To estimate this threshold, we made some calculations based on the cost components of the system that are obtained by only system throughputs. It is observed that our estimation gives an upper bound for the real threshold value.

Furthermore, it is seen that holding overflow policy outperforms the direct disposal in all experiments without exception. Also, when overflows are held, different priority policies did not affect the total costs significantly.

The effect of return arrival rate is also analyzed in the experiments. It is observed that return rate has a two sided effect on cost savings of allowing partial disassembly.

When the marginal profitability level is below the threshold value, a higher return rate makes allowing partial disassembly more costly compared to not allowing partial disassembly. On the other hand, when the marginal profitability level is higher than the threshold value, a higher return rate increases the cost saving of allowing partial disassembly.

Finally, we analyzed the effect of non-uniformity of parts in terms of physical characteristics and manufacturing costs. We observed that if the more demanded part is dominant in terms of physical characteristics or manufacturing costs, then allowing partial disassembly results in higher cost savings. However, it is also observed that the effect of non uniformity is not as strong as the effect of marginal profitability.

For further research, analysis of a disassembly system having a different topology can be of interest. For instance, a long disassembly line composed of a several number of tandem disassembly machines can be analyzed by the approximation technique proposed in this study. In that setting, allowing partial disassembly for some disassembly machines while not allowing for other machines can be analyzed. Also, analysis of a disassembly system with machines having service times that follow phase type distributions rather than exponential is a possible extension of our work. Furthermore, in order to assess the effect of the arrival process variability for returned products on the performance of the system with different disassembly policies, using phase type distributions having different squared coefficient of variations is another research point. In conclusion, our study establishes a framework that can give future research directions for the researchers in addition to giving important managerial insights for the planners of disassembly systems.

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