## MULTI AGENT INTERSECTION MANAGEMENT FOR AUTONOMOUS VEHICLES

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#### Abstract

\title{ MULTI AGENT INTERSECTION MANAGEMENT FOR } AUTONOMOUS VEHICLES


Traditional transportation systems cause traffic congestion especially at the intersections as the number of vehicles keeps increasing. This is also the main reason of air pollution and time wasted. Most of the people lose their time and money because of traffic congestion. Thanks to recent research on autonomous vehicles, intelligent transportation and wireless communication systems, efficient traffic management at the intersections with multi-agent scheduling methods will be possible.

The main objective of this thesis is intersection coordination for multi-agent systems by using time-based optimization and Model Predictive Control (MPC) methods while considering fuel economy at the intersections. Existing results show that these methods are efficient in comparison to the traditional methods when all the vehicles are autonomous. However, better trajectory planning can improve the total delay of the system. Besides, including fuel economy in the optimization function can also decrease fuel consumption which would be good for both humanity and nature. In this thesis, the effect of trajectory planning and different communication ranges on time-based optimization method is studied. It is shown that a wider communication range and better trajectory planning provide less time delay. Another contribution of this thesis is to propose centralized and decentralized MPC algorithms by including fuel consumption related costs in the objective function. As a result, fuel consumption is decreased at the expense of an increase in the time delay. In simulations, it is also observed that centralized MPC performs better than decentralized MPC.

## ÖZET

## OTONOM ARAÇLAR İÇİN ÇOK ETMENLİ KAVŞAK KONTROL MEKANİZMASI

Günümüzde yollarda araç sayısının artması ile birlikte geleneksel taşımacılık yöntemleri özellikle kavşaklarda trafik sıkışıklığına sebep olmaktadır. Bu da hava kirliliğine ve yolda boşa zaman geçirilmesinin temel nedenleri arasındadır. Son yıllarda otonom araçlar üzerine yapılan araştırmalar sayesinde yakın gelecekte kavşaklarda daha verimli bir şekilde trafik yönetimi mümkün olacaktır.

Bu tezin temel amacı, otonom araçlar için zaman temelli optimizasyon ve Model Öngörülü Kontrol (MÖK) yöntemleri ile yakıt tüketimini de ele alarak kavşak koordinasyonu sağlamaktır. Önceki araştırmalar bu yöntemlerin, otonom araçlar için geleneksel kavşak yönetim sistemlerine göre daha verimli sonuçlar elde ettiğini göstermiştir. Fakat, daha iyi yörünge planlama teknikleri kavşaklarda gecikme sürelerini azaltacaktır. Bunun yanında, optimizasyon problemi oluştururken yakıt tüketimi için de bir maliyet fonksiyonu eklemek araçların yakıt ekonomisi bakımından yararlı olacaktır. Bu tezde, verimli yörünge planlama tekniklerinin ve araçların haberleşmesi için kullanılan iletişim alanının genişliğinin zaman tabanlı optimizasyona etkileri incelenmektedir. Geniş iletişim alanının ve dinamik yörünge planlama tekniğinin daha az zaman kaybına yol açtığı gösterilmiştir. Bir diğer katkı ise, model öngörülü kontrol maliyet fonksiyonuna yakıt tüketimi maliyetini minimize edecek fonksiyonların da eklenmesi ve yakıt ekonomisine etkilerinin incelenmesidir. Ayrıca merkezcil model öngörülü kontrol sistemi merkezcil olmayan sisteme çevirilmiştir. Maliyet fonksiyonuna yakıt tüketimi denklemlerini eklemek her ne kadar yakıt tüketimini azaltsa da kavşak içinde gecikmeyi arttırdığ̆ı sonucuna varılmıştır. Ayrıca merkezcil MÖK'un merkezcil olmayana göre daha verimli sonuç elde ettiği de görülmüştür.

## TABLE OF CONTENTS

ACKNOWLEDGEMENTS ..... iii
ABSTRACT ..... iv
ÖZET ..... v
LIST OF FIGURES ..... ix
LIST OF TABLES ..... xi
LIST OF SYMBOLS ..... xii
LIST OF ACRONYMS/ABBREVIATIONS ..... xiv

1. INTRODUCTION ..... 1
1.1. Related Work ..... 2
1.1.1. Optimizing Traffic Flow With Traffic Lights ..... 2
1.1.2. Intersection Management Using V2I Communication ..... 3
1.1.3. Intersection Management Using V2V Communication ..... 4
1.1.4. Intersection Management Using Optimization Methods ..... 6
1.1.5. Intersection Management Using Model Predictive Control Frame- work ..... 8
1.2. Contributions of the Thesis ..... 8
1.3. Organization of the Thesis ..... 9
2. SYSTEM DESCRIPTION ..... 11
2.1. Problem Statement ..... 11
2.2. Assumptions ..... 11
2.3. Optimization Problem ..... 13
2.4. Simulation Environment ..... 14
2.5. Summary of the Chapter ..... 14
3. MULTI-AGENT INTERSECTION MANAGEMENT USING DEPARTURE TIME SCHEDULING APPROACH ..... 16
3.1. Optimization Problem ..... 16
3.2. Constraints ..... 18
3.2.1. Speed Limits and Maximum Deceleration ..... 18
3.2.2. Safety Time Gap along the Same Direction ..... 19
3.2.3. Overtaking ..... 19
3.2.4. Safety Time Gap along the Conflicting Direction ..... 19
3.3. Trajectory Planning ..... 20
3.3.1. Velocity Calculation For Acceleration ..... 20
3.3.2. Velocity Calculation For Deceleration ..... 21
3.3.3. Dynamic Trajectory Planning ..... 21
3.4. Communication Range ..... 22
3.5. Optimal Scheduling Controller Algorithm ..... 23
3.6. Numerical Analysis ..... 23
3.6.1. Simulation Setup ..... 25
3.6.2. Effects of Different Communication Circles ..... 25
3.6.3. Effects of Dynamic Trajectory Planning ..... 26
3.7. Summary of the Chapter ..... 27
4. A CENTRALIZED INTERSECTION MANAGEMENT USING MODEL PRE- DICTIVE CONTROL ..... 30
4.1. Vehicle Dynamics ..... 30
4.2. Cross Collision Point (CCP) and Risk Function ..... 31
4.3. Model Predictive Control Framework ..... 32
4.4. MPC Optimization Problem ..... 33
4.4.1. Traction Force ..... 34
4.5. Constraints ..... 35
4.5.1. Speed Limits and Maximum Acceleration/Deceleration ..... 35
4.5.2. Safety Distance Gap Along the Same Lane ..... 36
4.5.3. Safety Time Gap along the Conflicting Direction ..... 36
4.6. Controller Algorithm ..... 37
4.7. Numerical Analysis ..... 38
4.7.1. Simulation Setup ..... 38
4.8. Summary of the Chapter ..... 39
5. DECENTRALIZED INTERSECTION CONTROL USING MPC AND FUEL CONSUMPTION ..... 41
5.1. Hybrid Vehicle Model Dynamics ..... 41
5.2. Fuel Consumption Minimization ..... 42
5.3. MPC Optimization Problem ..... 43
5.4. Decentralized Conflict Resolution Problem ..... 44
5.4.1. Converting Centralized Problem into Decentralized One ..... 44
5.4.2. Policy for the Safety Constraints ..... 45
5.5. Controller Algorithm ..... 47
5.6. Numerical Analysis ..... 48
5.6.1. Simulation Setup ..... 48
5.7. Summary of the Chapter ..... 50
6. EXPERIMENTS AND RESULTS ..... 52
6.1. Effects of Vehicle Dynamics to Centralized MPC ..... 52
6.2. Performance Evaluation of CMPC and DMPC ..... 52
6.3. Summary of the Chapter ..... 54
7. CONCLUSION ..... 58
REFERENCES ..... 60

## LIST OF FIGURES

Figure 1.1. Example Scenario With No Conflict Point Occurs at the Intersec- tion [15] ..... 5
Figure 1.2. Example Scenario With Conflict Point Occurs at the Intersection [15] ..... 6
Figure 1.3. Platoon Based Intersection Management Architecture [17] ..... 7
Figure 2.1. Basic Intersection Definition ..... 12
Figure 2.2. Optimization Schema ..... 13
Figure 2.3. SUMO Crossroad intersection environment ..... 15
Figure 3.1. Illustration of VA approaching the isolated intersection [18] ..... 17
Figure 3.2. Actual and Reference Velocity Profile of a Vehicle ..... 22
Figure 3.3. Optimal Scheduling Controller Diagram ..... 24
Figure 3.4. Distance to Intersection Values for All Vehicle : 8 Vehicles Simula- tion Scenario ..... 26
Figure 3.5. Total Time Delay Under Different Communication Ranges ..... 27Figure 3.6. Reference and Actual Velocity Profile of a Vehicle with 100 metersand 250 meters Communication Ranges respectively28
Figure 3.7. Total Fuel Consumption Under Different Communication Ranges ..... 28
Figure 3.8. Total Delay Time Comparison Between Dynamic and Static Tra- jectory Planning ..... 29
Figure 3.9. Reference and Actual Velocity Profile of a Vehicle with Dynamic and Static Trajectory Planning Approaches Respectively ..... 29
Figure 4.1. Demonstration of the CCP: (a) safe situation, (b) unsafe situation [21] 31
Figure 4.2. Centralized Model Predictive Control Algorithm ..... 37
Figure 4.3. Distance to Intersection Values for All Vehicles ..... 39
Figure 5.1. Decentralized Model Predictive Control Algorithm ..... 47
Figure 5.2. Distance to Intersection Values for All Vehicles ..... 49
Figure 6.1. Detailed Travel Time Results for Decentralized MPC ..... 55
Figure 6.2. Detailed Travel Time Results for Centralized MPC ..... 56
Figure 6.3. Detailed Fuel Consumption Results for Decentralized MPC ..... 57
Figure 6.4. Detailed Fuel Consumption Results for Centralized MPC ..... 57

## LIST OF TABLES

Table 4.1. Detailed Travel Time and Fuel Consumption Results for All Vehicles With and Without Traction Force Cost ..... 40
Table 5.1. Results of Decentralized MPC ..... 50
Table 5.2. Detailed Travel Time and Fuel Consumption Results Using Hybrid Vehicle Dynamics With and Without Fuel Consumption Cost ..... 51
Table 6.1. Results of Centralized MPC ..... 53
Table 6.2. Travel Time Average Results ..... 54
Table 6.3. Fuel Consumption Average Results ..... 56

## LIST OF SYMBOLS

| $a_{\text {acc }}$ | Acceleration rate |
| :---: | :---: |
| $a_{\text {dec }}$ | Deceleration rate |
| $A_{F}$ | Frontal area of the vehicle |
| $C_{d}$ | Drag coefficient of the vehicle |
| $d_{i}$ | Current distance to the intersection departure point |
| $f_{r}$ | Rolling coefficient of the vehicle |
| $F_{i, j}(t)$ | Risk function |
| $F_{T}$ | Traction force |
| $g$ | gravity |
| H | Risk function weight |
| $\dot{m}_{\text {ifuel }}$ | Equivalent fuel consumption rate |
| M | Mass value of the vehicle |
| $n_{1}$ | Total number of vehicles travelling through west direction |
| $n_{2}$ | Total number of vehicles travelling through north direction |
| $n_{3}$ | Total number of vehicles travelling through east direction |
| $n_{4}$ | Total number of vehicles travelling through south direction |
| $p_{\phi_{i} \phi_{j}}$ | Distance to the CCP from end of lane |
| $P_{\text {icons }}$ | Effective power consumption |
| $r_{\text {min }}$ | Safe distance gap along the same lane |
| $R_{i, j}(t)$ | Risk indicator |
| $R_{\text {min }}$ | Safe distance gap conflicting lanes |
| $S_{i}$ | Travelled distance through prediction horizon |
| $t_{0}$ | Current time |
| $t_{\text {des }, i}$ | Desired departure time of vehicle i |
| $t_{i, w}^{D}$ | Departure time at the intersection through the west direction |
| $t_{i, w}^{E}$ | Entrance time at the intersection through the west direction |
| $t_{j, e}^{D}$ | Departure time at the intersection through the east direction |
| $t_{j, e}^{E}$ | Entrance time at the intersection through the east direction |


| $t_{k, n}^{D}$ | Departure time at the intersection through the north direction |
| :---: | :---: |
| $t_{k, n}^{E}$ | Entrance time at the intersection through the north direction |
| $t_{l, s}^{D}$ | Departure time at the intersection through the south direction |
| $t_{l, s}^{E}$ | Entrance time at the intersection through the south direction |
| T | step size of MPC |
| $u_{i}(t)$ | Acceleration/Deceleration of vehicle i at time step t |
| $u_{\text {max }}$ | Maximum acceleration rate |
| $u_{\text {min }}$ | Minimum acceleration rate |
| $v_{d}$ | Desired velocity |
| $v_{i}$ | Current velocity of the vehicle i |
| $v_{i}(t)$ | Velocity of vehicle i at time step t |
| $v_{\text {max }}$ | Maximum speed limit |
| $v_{\text {min }}$ | Minimum speed limit |
| $w_{u}$ | Weight of acceleration objective |
| $w_{v_{i}}$ | Weight of desired velocity objective |
| $x_{i}(t)$ | Distance to intersection of vehicle i at time step $t$ |
| $\alpha_{i}$ | Risk indicator constant for i |
| $\alpha_{j}$ | Risk indicator constant for j |
| $\Delta t$ | Remaining time to settled departure time |
| $\Delta T_{1}$ | Minimum necessary safe time gap between two consecutive vehicles |
| $\Delta T_{2}$ | Minimum necessary safe time gap between two conflicting vehicles |
| $\eta_{i e f f}$ | Propulsion efficiency |
| $\eta_{\text {irec }}$ | Recuperation efficiency |
| $\rho$ | Air density |
| $\sigma_{\phi_{i}, \phi_{j}}$ | Binary variable for collision function |
| $t_{s}$ | Time step interval |

## LIST OF ACRONYMS/ABBREVIATIONS

| AI | Artificial Intelligence |
| :--- | :--- |
| AV | Autonomous Vehicle |
| CC-IP | Concurrent Crossing-Intersection Protocol |
| CCP | Cross Collision Point |
| CDAI | Collision Detection Algorithm for Intersections |
| GPS | Global Positioning System |
| I2V | Intersection to Vehicle |
| ICU | Intersection Control Unit |
| IMA | Intersection Management Agent |
| IPM | Interior Point Method |
| ITS | Intelligent Transportation Systems |
| LVA | Leader Vehicle Agent |
| MPC | Model Predictive Control |
| MP-IP | Maximum Progression-Intersection Protocol |
| SUMO | Simulation of Urban Mobility |
| V2I | Vehicle to Intersection |
| V2V | Vehicle to Vehicle |
| VA | Vehicle Agent |

## 1. INTRODUCTION

Recent research in the autonomous vehicle industry pinpoints a future where intelligent transportation systems (ITS) will be a reality and vehicles will handle most of the driving tasks. Once autonomous vehicles become popular, the interaction of vehicles with the environment will be possible. To handle a driving task, autonomous vehicles (AVs) will have to interact with and understand the environment better, by communicating with not only roadside infrastructures (vehicle to infrastructure, V2I), but also with each other (vehicle to vehicle, V2V).

Traffic congestion is the main reason for excess fuel consumption and air pollution, especially at the intersections. Additionally, traffic congestion causes time delays which affect people directly. It also affects economic growth. For example, research studies show that American people lose about 4.8 billion hours and 3.9 billion gallons of gasoline every year because of traffic congestion. USA wastes about 115 billion dollars every year due to traffic congestion [1].

Traditional methods such as traffic lights, traffic signs, and police officers are insufficient in reducing the traffic congestion at the intersections as the number of human-driven vehicles are increasing day by day. Traffic lights and signs can cause unnecessary stops, even if the road is not so crowded. On the contrary, police officers may not be able to control the traffic flow if the road is very crowded. All these lead to idle time and air pollution in traffic.

As the development of V2I and V2V communication technologies are improving thanks to the recent increase in studies on autonomous vehicles industry, these technologies may lead to smart solutions for efficient intersection management. Cameras, road unit sensors, and other possible technologies can be placed as roadside units in order to collect traffic data. All these developments can help in analyzing and optimizing the traffic flow at the intersections. Thus, coordinating vehicles at the intersection without traffic lights will be possible. The primary purpose of the thesis is to find
an efficient solution for coordinating vehicles at the intersections without using traffic lights.

### 1.1. Related Work

Some studies show that traffic light optimization at intersections can be possible and more applicable in the near future since traffic lights are already planted to the intersections. However, intersection management using traffic lights will be a difficult and inefficient solution as the number of vehicles is increasing. Additionally, it is assumed that all vehicles will be autonomous and able to communicate with the new generation of infrastructures. Therefore most of the researchers have already started to work on intersection management using V2I communication. There are also several researchers who think that autonomous vehicles will able to communicate with each other and coordinate themselves for safe passing without manipulation of infrastructure at the intersection. In this section, some intersection management frameworks used in the literature are reviewed.

### 1.1.1. Optimizing Traffic Flow With Traffic Lights

Intelligent traffic light control can be considered as the first step for traffic management as autonomous and legacy vehicles will be on the road together in a few years and traffic lights are already planted. Optimizing traffic flow is a huge task and intelligent control of intersections via traffic lights will decrease time delays and fuel consumption efficiently.

There are several studies in the literature regarding traffic light optimization at the intersections. An application called Surtract System which is set up in Pittsburgh [2] utilizes an artificial intelligent (AI) algorithm to build a timing plan for each traffic light by collecting data from radar and camera sensors. It is shown that the pilot implementation of this system reduces travel time by 25 percent and idle time by over 40 percent. An adaptive traffic signal coordination approach based on V2I communication is proposed in [3] which improves travel time and reduces queuing at the intersection.

A dynamic predictive traffic signal control framework to minimize the intersection delays at a signalized intersection using platoon model and dynamic programming is proposed in [4]. Arel et al. use a reinforcement learning approach to optimize traffic signals efficiently in terms of total intersection delay [5]. A new algorithm for short term output maximization to avoid unnecessary stops at the intersection is given in [6]. A semi-centralized method with attribute base block-chain is proposed in [7].

### 1.1.2. Intersection Management Using V2I Communication

The traffic light approach is not efficient enough to minimize the delay time and fuel consumption where all the vehicles are autonomous because it causes queues and unnecessary stopping at the intersection. Therefore, an appropriate strategy without traffic lights should be used to coordinate vehicles and minimize intersection delays.

Some studies focus on reservation based V2I communication technology. A multiagent intersection management firstly introduced by Dresner and Stone is called Autonomous Intersection Management (AIM) [8]. Vehicles approaching an intersection send a reservation request to Intersection Management Agent (IMA) to pass the intersection without any collision. Since IMA can calculate future states of each vehicle, it can easily understand if there is a conflict point between vehicles or not. After IMA evaluates all reservation requests and if there is no potential collision between vehicles, it permits them in order to pass the intersection with a given velocity, acceleration, and time. Dresner and Stone also proposed a method where autonomous vehicles and human drivers met at the intersection [9].

Although non-conflicting vehicles are approved by IMA, in order to handle and give permission to the conflicting vehicles, some prioritizing rules should be defined. Dresner and Stone proposed the First Come First Serve (FCFS) method for AIM. If more than one vehicle have reservation request at the same time and there are some conflicts between the path of vehicles, the vehicle which enters the intersection communication area first, will be approved by IMA first. In this system, in order to detect conflict points, the intersection is divided into grids. In addition to the FCFS
method, some other prioritizing rules are examined in [10]. For example, emergency vehicles such as ambulance or fire truck can have the highest priority or if two conflicting vehicles have the same priority, attending priority on the lanes would be considered.

The FCFS policy coordinates vehicles according to their entrance times to the intersection communication area, but it may cause inefficiency when a vehicle can pass the intersection first but enter the intersection communication area last. For this reason, FCFS policy may have some disadvantages in terms of fuel consumption and time delay. Zhu et al. propose a look ahead intersection control policy [11] for intelligent vehicles to overcome the disadvantages of the FCFS policy. According to their policy, the first comer may not pass the intersection first if the time delay of this sequence is higher than other sequences. Their test results show that this method performs $25 \%$ better than FCFS policy.

Stone et al. propose a planning based motion controller algorithm in order to reduce the number of stops at intersections by using Little's queuing law [12] to schedule conflicting vehicles. The proposed technique relies on finding a velocity profile with the highest possible speed at the end of the intersection with minimum time delay [13].

### 1.1.3. Intersection Management Using V2V Communication

Although there are numerous intersection management strategies where intersection is controlled by an IMA, there are also many in which vehicles coordinate themselves by communicating with each other and pass the intersection without any collision. Two V2V based intersection protocols are proposed by Azimi et al. in order to detect a collision and schedule the conflicting vehicles [14]. In these works, the intersection is defined as a perfect square grid divided into small cells and every vehicle in the intersection communication area sends their cell list to be occupied during the intersection pass. Figure 1.1 shows an example scenario with no space conflict. In this scenario, the algorithm will detect no conflicting space. On the other hand, in a scenario, as shown in Figure 1.2, the algorithm will detect conflicting spaces. This collision detection algorithm, named as Concurrent Crossing-Intersection Protocol (CC-IP)


Figure 1.1. Example Scenario With No Conflict Point Occurs at the Intersection [15]
runs on all vehicles. If more than one vehicle has a conflicting point, the vehicle with the highest priority will pass the conflict point first, while other vehicles have to hold outside of the intersection area and wait until the prior vehicle passes the conflict point. Another intersection protocol called Maximum Progression Intersection Protocol (MPIP) which can solve the inefficiency of CC-IP is also introduced by Azimi et al. This approach, unlike the CC-IP, lets the prior vehicle pass the conflict point first, while letting other vehicles pass the intersection till the conflict point and wait until prior vehicle pass. The CC-IP and MP-IP have performance improvements over the traffic lights [15]. It is also concluded that MP-IP has $71 \%$ better performance than CC-IP. This study is also modified for traffic coordination through roundabouts [14]. Azimi et al. have also designed and developed a new intersection protocol with a realistic ground positioning system (GPS) model [16].

Jin et al. propose an intersection management protocol by considering the line of vehicles as a platoon and combines V2V and V2I communication to schedule traffic flow [17]. In this system, a platoon communicates with the intersection control unit through a leader vehicle agent (LVA) by sending their estimated arrival time to the intersection and estimated departure time from the intersection and receiving a reservation confirmation. LVA is also responsible for the trajectories of the follower vehicles. Figure 1.3 shows a detailed architecture of platoon based intersection management.
(a)

(b)


Figure 1.2. Example Scenario With Conflict Point Occurs at the Intersection [15]

### 1.1.4. Intersection Management Using Optimization Methods

Some researchers consider intersection management as an optimization problem. Jin et al. describe a V2I based improved multi-agent intersection management approach by optimizing departure times at the intersection instead of the first come first serve policy [18]. To minimize the total travel time, IMA takes entrance times in the controlled region into account and optimize departure time rather than arrival time to the intersection. To overcome connection lost and connection range limits at the communication area, a dynamic scheduling approach is proposed. In this approach, at each time step, the intersection management agent reschedules the departure time of the vehicle agents if there is a difference between the current and previous time step in terms of vehicle agents. To minimize the total travel time for all vehicle agents, the cost function is described as,

$$
\begin{equation*}
J=\sum_{i=1}^{n_{1}}\left(t_{i, w}^{D}-t_{i, w}^{E}\right)+\sum_{j=1}^{n_{2}}\left(t_{j, n}^{D}-t_{j, n}^{E}\right) \tag{1.1}
\end{equation*}
$$

where $n_{1}$ and $n_{2}$ are the total numbers of VAs traveling through west and north direction respectively; $t_{i, w}^{D}$ and $t_{i, w}^{E}$ are the departure and entrance times at the intersection, for the $i^{t h}$ vehicle agent through the west direction; $t_{j, n}^{D}$ and $t_{j, n}^{E}$ are the departure and entrance time at the intersection for the $j^{\text {th }}$ vehicle agent through the north direction. The objective is to minimize the total time delay controlling departure times of vehicle agents within the communication area under safety constraints by giving entrance


Figure 1.3. Platoon Based Intersection Management Architecture [17]
times of vehicles to the cost function as known variables. After all the vehicles within the communication area receive their proposed departure times from the optimal scheduler, they are responsible for planning their trajectory. In order to depart from the intersection at the scheduled time, the velocity profile is calculated based on [10].

Fayazi et al. propose a centralized mixed integer linear programming algorithm in order to minimize the total delay at the four-way intersection [19] as an extension of their previous work [20], subject to safety constraints. Like most of the previous studies, they assume that all vehicles are autonomous. In [20], the objective is to minimize the total travel time for each vehicle while preventing potential collisions by setting some constraints. Unlike the optimal scheduling approach, position and
velocity of each vehicle are taken into account to solve the linear optimization problem which is separated into two cost functions with weights. The objective of this work is to minimize the total travel time by calculating the arrival times of each vehicle instead of departure times under several safety constraints which are processed by a central server. After sending all the desired access times for all vehicles, they plan their trajectory. The proposed intersection model for the simulation is similar to [18].

### 1.1.5. Intersection Management Using Model Predictive Control Framework

Some studies are found in the literature where Model Predictive Control approach is used to coordinate vehicles at the intersection for collision-free travel. Kamal et al. study centralized Model Predictive Control framework as an intersection coordination method for smooth flows of vehicles without using any traffic light at the intersections [21] as an extension of their previous work [22]. In [21], a risk function is defined to estimate the potential collision risk of vehicles approaching the intersection. An Intersection Control Unit (ICU), which coordinates the intersection, receives state information from vehicle agents and sends them new control inputs after solving the optimization problem.

In [23], a fast and online V2V based decentralized optimal control schema is proposed by exploiting the low complexity of model-based heuristics. The results are presented in terms of efficiency, feasibility, and optimality.

### 1.2. Contributions of the Thesis

There exist different theories in the literature aiming to develop more efficient algorithms applied to intersection management with different control techniques using V2I or V2V communication topologies. Studies such as [10] have shown that safe intersection management is possible by using a centralized time optimization method, but trajectory planning of vehicles at the intersection is limited to acceleration and deceleration rates. In view of this fact, this thesis proposes a dynamic trajectory
planning algorithm in order to overcome the limitations of acceleration and deceleration rates.

A great deal of previous research into intersection management has also focused on centralized and decentralized model predictive control approaches. This thesis also aims to implement an efficient centralized MPC approach to coordinate vehicles at the intersection as it is known as providing smooth flows [21]. Additionally, decomposition of the centralized MPC into decentralized MPC based on [24] is studied.

It has been argued that minimizing the acceleration can also lead to minimum fuel consumption [21]. Along similar lines, investigating solutions for further decreasing fuel use by fuel consumption minimization is also in the scope of this thesis.

In view of the items discussed above, the main contributions of this thesis can be summarized as follows:

- Optimal scheduling approach calculating the desired departure time at the intersection is extended to four-way intersections. Dynamic trajectory planning to arrive at the intersection exit at a settled time is implemented.
- The minimum allowed communication range is formulated and the effects of different communication range at the intersection in terms of travel time are investigated.
- Centralized and decentralized model predictive approaches are studied considering fuel efficiency. To decrease fuel consumption, formulations of traction force, fuel efficiency, and hybrid vehicle dynamics are given.
- A simulation environment is constructed using SUMO simulation environment and all the numerical analysis are implemented using MATLAB environment.


### 1.3. Organization of the Thesis

The rest of the thesis is organized as follows. In Chapter 2, the intersection model and simulation environment that will be used throughout the thesis are introduced.

Communication topology between vehicle agents will be defined. Assumptions for designing the system are given. Optimization concept and MATLAB function that is used while solving the optimization problem are introduced.

In Chapter 3, intersection coordination using optimal scheduling approach and extension of the four way intersection are introduced. Vehicle coordination using departure time optimization is formulated. Formulation and topology of dynamic trajectory planning are given.

In Chapter 4, centralized intersection coordination using model predictive control (MPC) framework is explained. Vehicle models used for model predictive control are given. Necessary safety constraints for a safe intersection pass are defined. Risk function to estimate the risk of collision and traction force to decrease fuel consumption are formulated. To use in MPC, the objective function and its elements are introduced. Simulation results with and without traction force cost element are compared.

In Chapter 5, hybrid vehicle dynamics and fuel efficiency formulation to minimize fuel consumption are given. Conversion of a centralized optimization problem into a decentralized optimization problem via primal decomposition is defined. To solve the decentralized conflict resolution problem, assigning a consensus policy is explained. Comparing the results with and without fuel consumption cost, its contribution to minimize fuel consumption and travel time is examined.

In Chapter 6, the simulation results of the centralized and the decentralized MPC frameworks are compared in terms of travel time and fuel consumption. Finally, the thesis concludes with a short summary and future research directions in Chapter 7.

## 2. SYSTEM DESCRIPTION

In this chapter, an overview of intersection coordination for multi-agent systems is described. Based on the previous studies, some existing works on both centralized and decentralized strategies will be studied, and the effects of different factors on those will be examined. Some assumptions will be made to develop a common strategy and implementation. Once problem definition and assumptions are described, a simulation environment that is used in this thesis will be explained.

### 2.1. Problem Statement

In Figure 2.1, an overview of intersection collision concept is illustrated. If the vehicle agents are not updated, a potential collision at the intersection would be inevitable. This thesis seeks for an efficient way to coordinate vehicle agents in a safe way using optimization methods.

Figure 2.1 depicts a basic intersection definition being used in this thesis. The intersection consists of a communication zone and an intersection box. Outside of the communication zone, vehicles keep moving with their default velocity. Within the communication zone, vehicles start to communicate with either the intersection control unit (ICU) or each other based on V2V, V2I communication and broadcast their current states. In this zone, both vehicle scheduling and control tasks are handled. At each time step, once vehicles are given a new control input, they adjust their current states. After a vehicle passes the intersection box without any collision, it is not included in the optimization problem and adjusts its velocity profile subject to speed limits.

### 2.2. Assumptions

In order to develop a multi-agent control protocol, some conditions should be taken into account. In this work, the following assumptions are the basis for optimally scheduling the intersection.


Figure 2.1. Basic Intersection Definition

- Each vehicle is autonomous and equipped with the required networking system to communicate with each other and intersection control unit within a communication zone;
- Each vehicle has a global position system (GPS) in order to calculate their distances to the intersection;
- Robust and efficient communication between vehicles and ICU is assumed to be available. No latency in communication is considered;
- Only light-duty passenger vehicles are used in this work;
- All the vehicles have the same 2.5 meters length and 1000 kilograms weight;
- The maximum speed limit is $23 \mathrm{~m} / \mathrm{s}$ and the minimum speed limit is $3 \mathrm{~m} / \mathrm{s}$ (to avoid full stop);


Figure 2.2. Optimization Schema

- Vehicles are assumed to obey the traffic rules.
- The intersection has four arms (West, East, North, and South) with one lane in each direction;
- Weather conditions are not considered.


### 2.3. Optimization Problem

In this work, an optimization approach is utilized to centralized and decentralized coordination strategies. In general, the main objective is to coordinate vehicle agents by minimizing either travel time or fuel consumption or both. This goal is achieved by formulating an objective function and minimizing it by adjusting decision variables subject to some auxiliary conditions that need to be satisfied. The optimization schema is shown in Figure 2.2.

In this work, MATLAB optimization toolbox [25] is used to solve both linear and nonlinear constraint optimization problems. As a function fmincon is used with the formulation as

$$
\begin{array}{ll}
\underset{x}{\operatorname{minimize}} & f(x) \\
\text { subject to } & c(x) \leq 0, \\
& c_{e q}(x)=0, \\
& A \cdot x \leq b, \\
& A_{e q} \cdot x=b_{e q}, \\
& l_{b} \leq x \leq u_{b}
\end{array}
$$

where $b$ and $b_{e q}$ are vectors, $A$ and $A_{e q}$ are matrices. $c(x)$ and $c_{e q}(x)$ are functions that return vectors, and $f(x)$ is a function that returns a scalar. $f(x), c(x)$ and $c_{e q}(x)$ can be nonlinear functions. $x, l_{b}$ and $u_{b}$ can be considered as vectors or matrices. As an optimization method, interior point method (IPM) is used [26].

### 2.4. Simulation Environment

In this work, SUMO (Simulation of Urban Mobility) [27] is used to build the intersection model in order to evaluate the performance of the proposed strategies. SUMO is a microscopic traffic simulation environment which uses real-world vehicle model, car following, and lane changing algorithms to model the lateral and longitudinal movements of vehicles. Traffic control framework is implemented in SUMO over TraCI [28] which is MATLAB interface for SUMO.

### 2.5. Summary of the Chapter

In this chapter, an intersection coordination problem statement is defined along with the concepts of the experimental platform. In particular, first the intersection model and the assumed limitations (i.e. maximum speed and dimensions of the vehicles) on the model are detailed. These assumptions, although can be relaxed in the


Figure 2.3. SUMO Crossroad intersection environment
future, maintain the focus on the solution for the intersection coordination problem similar to the previous studies on the literature. Based on this problem definition, the optimization problem definition with its schema and constraints is formed. Finally, the simulation environment which is used to show the applicability of this method is given.

Overall, with the necessary implementation details and the problem given in this chapter, Chapter 3 will provide the departure time optimization based intersection coordination problem by considering different communication ranges and using better trajectory planning algorithm.

## 3. MULTI-AGENT INTERSECTION MANAGEMENT USING DEPARTURE TIME SCHEDULING APPROACH

Travel time optimization under safety constraints is one of the most popular methods for intersection management. In this chapter, the main objective is to calculate the departure time of each vehicle within communication range by obtaining entrance times subject to safety constraints. In [18], the two-way intersection is studied. To make this study more realistic, the optimal scheduling approach is extended to a four way intersection. Previous studies such as [18], [8] and [19] investigate different communication ranges. In this chapter, the minimum required communication range for efficient acceleration and deceleration to full stop is formulated. Additionally, a dynamic trajectory planning approach is introduced to overcome the limitations of acceleration and deceleration rates.

This chapter is organized as follows. Firstly, the time minimization problem arranging departure times subject to safety constraints in a four way intersection is defined. Then, the trajectory planning concept and minimum allowable communication range to control vehicles are formulated. Finally, the proposed controller algorithm and the simulation results are presented.

### 3.1. Optimization Problem

The main objective of this chapter is to minimize travel time of vehicles at the intersection. To calculate this, all vehicles in the communication range broadcast their entrance time and after obtaining these values, ICU decides the departure time of all vehicles. Finding efficient departure time also leads to minimize the delay time of the vehicles which is defined as the difference between departure time at maximum speed and the calculated departure time. Figure 3.1 illustrates the scenario where a vehicle agent is approaching the isolated intersection.


Figure 3.1. Illustration of VA approaching the isolated intersection [18]
To calculate departure times of all vehicles at the four way intersection, the optimization problem studied in [18] is extended as below,

$$
\begin{equation*}
\min \left(\sum_{i=1}^{n_{1}}\left(t_{i, w}^{D}-t_{i, w}^{E}\right)+\sum_{j=1}^{n_{2}}\left(t_{j, n}^{D}-t_{j, n}^{E}\right)+\sum_{l=1}^{n_{3}}\left(t_{l, e}^{D}-t_{l, e}^{E}\right)+\sum_{k=1}^{n_{4}}\left(t_{k, s}^{D}-t_{k, s}^{E}\right)\right) \tag{3.1}
\end{equation*}
$$

where
$n_{1}, n_{2}, n_{3}, n_{4}$ : the total number of vehicle agents travelling throughout west, north, east and south direction, respectively.
$t_{i, w}^{D}, t_{j, n}^{D}, t_{l, e}^{D}, t_{k, s}^{D}$ : Departure times at the intersection for the $i^{t h}, j^{t h}, l^{t h}$ and $k^{t h}$ vehicle agent along the Westbound, Northbound, Eastbound and Southbound respectively, which are defined as the decision variables for the optimization problem.
$t_{i, w}^{E}, t_{j, n}^{E}, t_{l, e}^{E}, t_{k, s}^{E}$ : Entrance time within the communication area for the $i^{t h}, j^{t h}, l^{t h}$ and $k^{t h}$ vehicle agent along the Westbound, Northbound, Eastbound and Southbound respectively which should be considered as known variables.

Note that the decision variables, $t_{i, w}^{D}, t_{j, n}^{D}, t_{l, e}^{D}, t_{k, s}^{D}$, appear linearly in the optimization problem (3.1).

### 3.2. Constraints

To schedule vehicle agents to pass the intersection without any collision, safetyrelated constraints such as

- Speed limits and maximum acceleration/deceleration of the vehicles,
- Safety time gap between vehicles in the same direction,
- Overtaking in the same lane,
- Safe time gap between two vehicles along the conflicting direction
should be defined. These constraints are explained in more detail below.


### 3.2.1. Speed Limits and Maximum Deceleration

It is assumed that each vehicle is travelling through the intersection at maximum speed. Since a vehicle at the intersection travelling with maximum speed departs from the intersection fastest, departure time at the intersection with maximum speed can be considered as the lower bound of decision variables. In other words, the departure time of each vehicle agent cannot be less than the departure time at maximum speed. This constraint can be formulated as

$$
\begin{equation*}
t_{i}^{D} \geq t_{i_{l, l}}^{D} \tag{3.2}
\end{equation*}
$$

where $t_{i_{l b}}^{D}$ is the departure time of the $i^{\text {th }}$ vehicle at the maximum speed. Note that this is a linear constraint.

### 3.2.2. Safety Time Gap along the Same Direction

There should be a safe time gap between two consecutive vehicle agents which are travelling in the same direction (e.g. northbound) so that rear and front collisions between two vehicles are prevented. The linear constraint can be formulated as

$$
\begin{equation*}
t_{i+1}^{D}-t_{i}^{D} \geq \Delta T_{1} \tag{3.3}
\end{equation*}
$$

where $\Delta T_{1}$ is the minimum necessary safe time gap between two consecutive vehicles that are travelling in the same direction. In this thesis, $\Delta T_{1}$ is calculated by dividing the minimum safety distance by the speed limit.

### 3.2.3. Overtaking

In this optimization problem, overtaking in the same lane is not allowed. In other words, if $t_{i}^{E} \leq t_{j}^{E}$, then $t_{i}^{D} \leq t_{j}^{D}$ where $t_{i}^{E}, t_{j}^{E}$ are entrance times within the intersection of $i^{t h}$ and $j^{\text {th }}$ vehicle agents, $t_{i}^{D}, t_{j}^{D}$ are departure time at the intersection of $i^{\text {th }}$ and $j^{t h}$ vehicle agents in the same direction.

### 3.2.4. Safety Time Gap along the Conflicting Direction

There should be a safe time gap between two vehicles on conflicting directions to prevent a collision. This time gap assures that a vehicle agent can only depart from the intersection after all other conflicting vehicles leave the intersection or vice versa. This safe time gap is strictly dependent on the intersection box width and vehicle speed limit. This nonlinear constraint can be formulated as

$$
\begin{equation*}
\left|t_{i, W}^{D}-t_{j, N}^{D}\right| \geq \Delta T_{2} \tag{3.4}
\end{equation*}
$$

where $\Delta T_{2}$ is the minimum safe time gap between two vehicles on different approaches which is calculated by dividing the length of the intersection by the speed limit.

Note that $\Delta T_{2}$ should always be higher than $\Delta T_{1}$ since conflicting vehicles approaching the intersection from different lanes should have a safer time gap than vehicles travelling along the same lane. This also ensures that the safe distance gap between two consecutive vehicles along the same direction cannot be more than the intersection box width to provide non-conflicting headway.

### 3.3. Trajectory Planning

After calculating the desired departure times for each vehicle agent, a trajectory planning algorithm should run on each vehicle to depart from the intersection at the desired time. This algorithm should run on each vehicle individually, and all the vehicles are responsible for their velocity profile.

A piece-wise linear function stated in [10] is used for planning the trajectory of each vehicle so that they can exit from the intersection at the desired time. In this approach, each vehicle travels with the maximum allowed speed outside of the communication range, and once a vehicle enters the intersection, the vehicle adjusts its velocity according to the settled departure time.

The main fact is that, if remaining travel of the vehicle agent time to exit point of the intersection at current speed is equal to the remaining time to its assigned departure time, the VA does not need to adjust its speed. This means that it is enough for this vehicle to maintain its current speed if $\Delta t=t_{d e s, i}-t_{0}=\frac{d_{i}}{v_{i}}$, where $d_{i}$ and $v_{i}$ are the current distance and velocity at current time $t_{0}$, and $t_{d e s, i}$ is the departure time assigned to the VA by ICU. However, in most cases, acceleration or deceleration is necessary to adjust vehicle speed as explained based on [19].

### 3.3.1. Velocity Calculation For Acceleration

To reach the intersection exit at the settled time, a vehicle agent may need to accelerate. In other words, if $t_{d e s, i}<t_{0}+\frac{d_{i}}{v_{i}}$, vehicle agent needs to be accelerated in order to exit from the intersection point at the settled time [19]. In this case, cruising
velocity can be calculated as

$$
\begin{equation*}
v_{\text {cruise }, i}=v_{i}+a_{a c c} \Delta t-\sqrt{2 a_{a c c}\left(\frac{1}{2} a_{a c c} \Delta t^{2}+v_{i} \Delta t-d_{i}\right)} \tag{3.5}
\end{equation*}
$$

where $a_{\text {acc }}$ is the acceleration value, $\Delta t$ is the remaining time to the settled departure time, $d_{i}$ is the remaining distance to the intersection exit point, and $v_{i}$ is the velocity of vehicle i at the current time. In this study, $a_{a c c}$ is set to $2.4 \mathrm{~m} / \mathrm{s}^{2}$.

### 3.3.2. Velocity Calculation For Deceleration

In this work, it is assumed that all vehicles are cruising at their maximum speed until they reach the communication area. This means that most of the vehicles should decelerate in order to reach the intersection exit at the settled delayed time. In other words, if $t_{\text {des }, i}>t_{0}+\frac{d_{i}}{v_{i}}$, the vehicle would reach the departure point earlier if it maintains its current velocity. Therefore, the vehicle needs to decelerate. In this case, cruising velocity can be calculated as

$$
\begin{equation*}
v_{\text {cruise }, i}=v_{i}+a_{\text {dec }} \Delta t+\sqrt{2 a_{\text {dec }}\left(\frac{1}{2} a_{\text {dec }} \Delta t^{2}+v_{i} \Delta t-d_{i}\right)} \tag{3.6}
\end{equation*}
$$

where $a_{\text {dec }}$ is the deceleration rate. It should be noted that in this work, $a_{\text {dec }}$ is set to $-2.4 \mathrm{~m} / \mathrm{s}^{2}$.

### 3.3.3. Dynamic Trajectory Planning

To handle the state errors and overcome the limitations of acceleration and deceleration capabilities of vehicle agents, in this dynamic trajectory approach, reference velocity is re-calculated in every simulation step to reach the intersection exit at the exact assigned departure time. To achieve velocity reference tracking, a PID controller is used.


Figure 3.2. Actual and Reference Velocity Profile of a Vehicle

As seen in Figure 3.2, the vehicle agent cannot follow the reference velocity profile correctly because of the limitations in acceleration and deceleration rates. Because of this, some of the vehicles cannot exit from the intersection at the desired time.To overcome this problem, the desired speed for the vehicles is calculated at each time step. Thanks to this approach, the velocity of the vehicles can be updated according to the state errors and vehicles can exit from the intersection at the desired time.

### 3.4. Communication Range

In the ideal case, all vehicles within the communication range send their state information to ICU. The maximum speed limit on urban roads can be considered as $80 \mathrm{~km} / \mathrm{h}$. Under these conditions, it is assumed that vehicles are obliged not to the exceed speed limit and all the vehicles approach to the intersection at the maximum speed limit. Deceleration distance to the intersection from $80 \mathrm{~km} / \mathrm{h}(22.2 \mathrm{~m} / \mathrm{s})$ to full stop should be as

$$
\begin{equation*}
x_{d e c}=\frac{v^{2}}{2 a_{d e c}}=\frac{22.2^{2}}{2 \times 2.4}=102.67 \mathrm{~m} \tag{3.7}
\end{equation*}
$$

The desired distance for a full stop is rounded to be 100 meters. Hence, the minimum communication and velocity profile adjustment range should be at least 100 meters. Otherwise, it can be said that vehicle agents cannot be accelerated or decelerated to their desired velocity and passing intersection will not be possible. The Maximum deceleration is chosen as $2.4 \mathrm{~m} / \mathrm{s}^{2}$, as stated in [29].

### 3.5. Optimal Scheduling Controller Algorithm

In order to coordinate vehicle agents at the intersection, entrance time of the vehicles to the communication are taken into account. In this pattern, all vehicle agents within the controlled region should keep communicating with ICU and update their states. In this study, it is assumed that there is no lack of communication between vehicle agents and ICU. In other words, all the vehicles and ICU are equipped with perfect V2I and I2V communication devices and ICU has enough hardware and software capability to calculate all the vehicles' departure times in an optimal way.

We illustrate the system architecture as in Figure 3.3. ICU is responsible for scheduling departure times of all the vehicles, and vehicle agents are responsible for adjusting their trajectory due to the departure time that is assigned for them.

Since it is not convenient to obtain entrance time and run optimization formula at once, a dynamic scheduling approach is presented. At each time step, ICU scans for the vehicles which have newly entered the communication area. Whenever a new vehicle agent which does not have any assigned departure time shows up, ICU runs the optimization problem and calculates departure time of that vehicle. There is no need to include the vehicles that have pre-assigned departure times in the optimization problem anymore.

### 3.6. Numerical Analysis

In this section, numerical analysis of the proposed method in this chapter is presented. SUMO is used as a simulation environment and a four-way intersection


Figure 3.3. Optimal Scheduling Controller Diagram
model is set up, which is mentioned in Chapter 2. The proposed algorithm is tested with different numbers of vehicles ( $8,16,24,32,40,60$ vehicles) at the intersection. Different communication ranges ( $50 \mathrm{~m}, 100 \mathrm{~m}, 150 \mathrm{~m}, 200 \mathrm{~m}$, and 250 m ) are examined and effects of the range of communication circle in terms of average travel time, total delay time and fuel consumption are discussed. The efficiency of the dynamic trajectory planning approach is also explained.

### 3.6.1. Simulation Setup

The simulation setup of this study is as follows:

- Each arm of the intersection is 250 meters long from the starting point of the road to the center of the intersection;
- SUMO vehicles are used for this simulation;
- The minimum safe distance between consecutive vehicles in the same lane is 2.5 meters;
- Vehicles are allowed to accelerate and decelerate between $-2.4 \mathrm{~m} / \mathrm{s}^{2}$ and $2.4 \mathrm{~m} / \mathrm{s}^{2}$;
- Simulation step is 0.1 seconds, and all the vehicles are processed every 0.1 seconds;
- $\Delta T_{1}$ is taken as 0.2 and $\Delta T_{2}$ is taken as 0.8 seconds;
- Sample time is chosen as 0.1 seconds.

Although the first three constraints are linear, because of the non-linearity of the fourth constraint in (3.2.4), the optimization problem becomes nonlinear. In this study, the nonlinear optimization problem is solved by using the interior point method [30] in the MATLAB environment.

Figure 3.4 illustrates simulation results in terms of position of the vehicles according to the intersection. All the vehicles travel with the same speed, and after a while, they start to accelerate or decelerate to avoid any collision and pass the intersection within different time slots.

### 3.6.2. Effects of Different Communication Circles

As mentioned in Section 3.5, the minimum communication circle range should be at least 100 meters. When we simulate the proposed algorithm with 50 meters communication range, it is seen that there is no possibility to have travel without collision at the intersection.


Figure 3.4. Distance to Intersection Values for All Vehicle : 8 Vehicles Simulation Scenario

As seen in Figure 3.5, total delay decreases with increasing communication range. The reason is that increasing the communication circle leads to decelerate the vehicles to a higher velocity. This means vehicle agents can follow the reference velocity profile with better settling time, as illustrated in Figure 3.6. It can be concluded that a higher communication range also leads to the higher average velocity. Higher average velocity also leads to less delay. It can be seen also that the higher communication range leads to more fuel-efficient travel, as depicted in Figure 3.7.

### 3.6.3. Effects of Dynamic Trajectory Planning

In this part, the benefits of dynamic trajectory planning in terms of total travel time are investigated. As discussed in Section 3.4.3, updating the desired velocity at each time step leads to a decrease in the total delay time of vehicle agents at the intersection. In Figure 3.8, the comparison of the efficiency of the dynamics and static trajectory planning is depicted. It can be seen that dynamic trajectory planning performs $13 \%$ better than static planning.


Figure 3.5. Total Time Delay Under Different Communication Ranges

Figure 3.9 illustrates the velocity profile of the two trajectory planning approaches. It can be seen that in the dynamic trajectory planning approach, the reference velocity is updated according to the state error so that the vehicles can exit from the intersection at the desired time.

### 3.7. Summary of the Chapter

In this chapter, the two-way approach of [18] is extended to four-way intersections by also including dynamic trajectory planning in the formulation. It is shown that the dynamic trajectory planning approach overcomes the limitations of deceleration capabilities. Minimum necessary communication range is formulated and numerical analysis confirms that fuel consumption and total delay decrease if communication range increases.

In Chapter 4, the intersection coordination problem is studied by using a centralized model predictive control approach in order to decrease fuel consumption of the vehicles at the intersection.


Figure 3.6. Reference and Actual Velocity Profile of a Vehicle with 100 meters and 250 meters Communication Ranges respectively


Figure 3.7. Total Fuel Consumption Under Different Communication Ranges


Figure 3.8. Total Delay Time Comparison Between Dynamic and Static Trajectory Planning


Figure 3.9. Reference and Actual Velocity Profile of a Vehicle with Dynamic and Static Trajectory Planning Approaches Respectively

## 4. A CENTRALIZED INTERSECTION MANAGEMENT USING MODEL PREDICTIVE CONTROL

Another intersection management method which is commonly used in the literature is model predictive control. It can be appraised as a quite powerful feature to have as it provides smooth flows and collision free travel at the intersections. An intersection coordination scheme by using a centralized MPC approach is proposed by Kamal et al. in [21] based on their previous work [22]. In these studies, it is pointed out that minimizing acceleration can also decrease the fuel consumption while providing smooth flows. In this chapter, the centralized MPC approach in [21] is extended by considering traction force minimization in the problem formulation.

The rest of the chapter is organized as follows. After introducing, the longitudinal vehicle dynamics, the concept of cross collision point and the risk function, the effect of traction force on total cost and fuel economy is discussed. Subsequently, the proposed control algorithm and simulation results are presented.

### 4.1. Vehicle Dynamics

To calculate the control input with MPC framework, the discrete time longitudinal mathematical model of a vehicle is formulated as

$$
\begin{array}{r}
x_{i}(t+1)=x_{i}(t)-v_{i}(t) t_{s}-\frac{1}{2} u_{i}(t) t_{s}^{2}  \tag{4.1}\\
v_{i}(t+1)=v_{i}(t)+u_{i}(t) t_{s}
\end{array}
$$

where $x_{i}(t)$ is the position (distance to the intersection), $v_{i}(t)$ is the velocity and $u_{i}(t)$ is the acceleration/deceleration value (control input) of the $i^{t h}$ vehicle at time step t; $t_{s}$ is the time step interval.


Figure 4.1. Demonstration of the CCP: (a) safe situation, (b) unsafe situation [21]

### 4.2. Cross Collision Point (CCP) and Risk Function

The cross collision point is demonstrated in Figure 4.1. Using position and velocity values of vehicles, their predicted trajectories are intersected at a common point. In other words, this common point is the point at which these two vehicles have potential collision risk if they travel with their current velocities. To avoid this, they should pass the collision point one after the other. The circle centering the collision point demonstrates the approximate safe area where only one vehicle is allowed to enter. For example, Figure 4.1 (a) shows that, while one vehicle is approaching the safety circle, the other vehicle has already left the zone, and this provides a safe pass for both vehicles. However, in Figure 4.1(b), both vehicles enter the safety circle at the same time interval and this leads to a collision. It can be deduced that a necessary condition for avoiding the collision of any pair of vehicles around their corresponding CCP is to prevent them from entering the safe zone at the same time. This way, the whole intersection can be safe.

Based on the CCP concept, a common risk indicator $R_{i, j}(t)$ which calculates the potential collision risk between the $i^{\text {th }}$ and $j^{\text {th }}$ vehicles at time t , can be described as

$$
\begin{equation*}
R_{i, j}(t)=e^{-\left(\alpha_{i}\left(x_{i}(t)+p_{\phi_{i} \phi_{j}}\right)^{2}+\alpha_{j}\left(x_{j}(t)+p_{\phi_{j} \phi_{i}}\right)^{2}\right)} \tag{4.2}
\end{equation*}
$$

where $\alpha_{i}$ and $\alpha_{j}$ are positive constants and $p_{\phi_{i} \phi_{j}}$ and $p_{\phi_{j} \phi_{i}}$ are the distances to the CCP from the end of each lane where the $i^{\text {th }}$ and $j^{\text {th }}$ vehicles travel. In a case as shown in Figure 4.1, $p_{\phi_{i} \phi_{j}}=\mathrm{b} / 2$ and $p_{\phi_{j} \phi_{i}}=3 \mathrm{~b} / 2 . \alpha_{i}$ and $\alpha_{j}$ are the safety circle constants, and by adjusting these values, the safety circle can be tuned according to vehicle lengths.

In order to use risk the indicator in the cost function, it should be generalized. Since not all the vehicles in the simulation has a CCP, risk indicator should be set to 0 for such collision free cases. To this end, a binary variable $\sigma_{\phi_{i}, \phi_{j}}$ is defined. If two vehicles have a CCP, $\sigma_{\phi_{i}, \phi_{j}}=1$, otherwise $\sigma_{\phi_{i}, \phi_{j}}=0$.

The risk function, $F_{i, j}(t)$, which can be used in the cost function, by using risk indicator (4.2) for vehicles i and j at time t can be formulated as,

$$
\begin{equation*}
F_{i, j}(t)=H \sigma_{\phi_{i}, \phi_{j}} e^{-\left(\alpha_{i}\left(x_{i}(t)+p_{\phi_{i} \phi_{j}}\right)^{2}+\alpha_{j}\left(x_{j}(t)+p_{\phi_{j} \phi_{i}}\right)^{2}\right)} \tag{4.3}
\end{equation*}
$$

where H is a positive high value that defines the weight of the risk function.

### 4.3. Model Predictive Control Framework

Model Predictive Control (MPC) is a control method that uses the system mathematical model to make a prediction for its future behaviour subject to some constraints so that the controller can better calculate the control input by considering future behaviour which is called the prediction horizon. MPC has several advantages which are summarized in [31]:

- Constraints for inputs and outputs can be considered in predictions.
- By estimating potential problems, it can offer accurate solutions for the model.
- Dynamic and static interactions between input and outputs are covered.

The main goals of an MPC framework have been well-described by Qin and Badgwell in [32] as follows:
(i) Control input can always be calculated by satisfying the input and output constraints.
(ii) Provides smooth control input.
(iii) Control as many process variables is possible even if there is no feedback mechanism from the system.
(iv) Maintains the outputs within specified constraints.

MPC framework is also widespread in decentralized control systems for formation control thanks to its flexibility [33]. Dunbar stated in [34] that the formation of vehicles could be maintained by using a local MPC controller that generates smooth outputs without too much deviation.

### 4.4. MPC Optimization Problem

In this work, the MPC framework is used for solving nonlinear constrained optimization problem with finite horizon to acquire optimal control strategy to coordinate vehicles. To design a centralized MPC objective function to provide safe and smooth travel through the intersection, the following objectives should be considered:

- Acceleration,
- Potential of a collision,
- Desired velocity tracking.

In the light of these information, the overall objective function can be formulated as

$$
\begin{equation*}
J=\sum_{t=0}^{T-1} \sum_{i=1}^{N} w_{v_{i}}\left(v_{i}(t+1)-v_{d}\right)^{2}+\sum_{t=0}^{T-1} \sum_{i=1}^{N} w_{u}\left(u_{i}(t)\right)^{2}+\sum_{t=0}^{T-1} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} F_{i, j}(t) \tag{4.4}
\end{equation*}
$$

where T (step size) is the MPC prediction horizon, N is the number of vehicles approaching the intersection, $v_{d}$ is the desired velocity, $w_{v_{i}}$ and $w_{u}$ are the weights.

In this cost function, three terms effect the performance of the whole system. The first one is related to velocity deviation from the desired velocity and the second term aims to minimize the acceleration of the vehicles. Minimizing these two terms mainly provide smooth velocity profile through the intersection. Setting $v_{d}$ at a higher speed, rapid intersection crossing would be available. The third term is related to the risk of a potential collision defined in (4.3). This term calculates and sums all the potential risks at the collision point through their prediction horizon and used to provide collision safe travel for all vehicles.

### 4.4.1. Traction Force

In (4.4), although the second term is related to fuel economy, minimizing the total traction force that is applied to the vehicles will also lead to minimize total fuel consumption. Traction force, including aerodynamic, rolling and gradient forces can be formulated based on [35] as

$$
\begin{equation*}
F_{T}(t)=M \frac{d v(t)}{d t}+M g f_{r} \cos (\alpha)+\frac{1}{2} \rho A_{f} C_{d} v^{2}(t)+M g \sin (\alpha) \tag{4.5}
\end{equation*}
$$

where $M$ is the mass of the vehicle, $f_{r}$ is the rolling coefficient, $A_{f}$ is the frontal area, $C_{d}$ is the drag coefficient of the vehicle, and $\rho$ is the air density. In (4.5), it can be seen that the required traction force for a vehicle agent to follow its speed profile is increased during acceleration. Although the derivative of the speed is zero at a constant speed, the counter force will be generated, when the vehicle agent intends to accelerate.

Since minimizing the total traction force is related to reducing both the acceleration and the velocity, adding this value to the cost function as an objective will contribute to reducing the total fuel consumption of the system. However it will also increase the delay time. In this scenario, agents can decide on whether fuel economy or time delay is significant.

By adding the traction force element to the cost function in (4.4), the final objective function is formulated as

$$
\begin{align*}
J= & \sum_{t=0}^{T-1} \sum_{i=1}^{N} w_{v_{i}}\left(v_{i}(t+1)-v_{d}\right)^{2}+\sum_{t=0}^{T-1} \sum_{i=1}^{N} w_{u}\left(u_{i}(t)\right)^{2}  \tag{4.6}\\
& +\sum_{t=0}^{T-1} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} F_{i, j}(t)+\sum_{t=0}^{T-1} \sum_{i=1}^{N} w_{f}\left(F_{T_{i}}(t)\right)
\end{align*}
$$

### 4.5. Constraints

In the MPC framework, a constrained nonlinear optimization problem is solved subject to linear and nonlinear constraints. To provide smooth velocity profiles and safe flows through the intersection, linear and nonlinear constraints are defined such as

- Speed limits and maximum acceleration/deceleration of the vehicles,
- A safe distance gap between the vehicles along the same lane,
- A safe distance gap along the conflicting direction.

These constraints are explained in more detail below.

### 4.5.1. Speed Limits and Maximum Acceleration/Deceleration

It is assumed that all the vehicles travel within speed limits and cannot exceed the acceleration-deceleration limits. Linear speed and acceleration/deceleration limits
can be formulated as

$$
\begin{gather*}
v_{\min } \leq v_{i} \leq v_{\max }  \tag{4.7}\\
u_{\min } \leq u_{i} \leq u_{\max }
\end{gather*}
$$

where $v_{i}$ and $u_{i}$ are the velocity and acceleration of $i^{\text {th }}$ vehicle; $v_{\text {max }}$ and $v_{\text {min }}$ are the maximum and minimum allowable speeds as constant values; and $u_{\max }$ and $u_{\text {min }}$ are the maximum and minimum permissible acceleration limits as constant values.

### 4.5.2. Safety Distance Gap Along the Same Lane

To avoid collision between two vehicles through the same direction, a safe distance between two consecutive vehicles should be defined, which can be formulated as

$$
\begin{equation*}
x_{i+1}-x_{i} \geq r_{\text {min }} \tag{4.8}
\end{equation*}
$$

where $x_{i+1}$ and $x_{i}$ are the two consecutive vehicles along the same lane, and $r_{\text {min }}$ is the minimum allowable gap, which can be formulated as in [36].

### 4.5.3. Safety Time Gap along the Conflicting Direction

To avoid any collision between vehicles along the conflicting lanes at the intersection, a safe distance gap should be defined. Since the vehicles may cross the intersection at different angles, a nonlinear inequality constraints is formulated as

$$
\begin{equation*}
\left(x_{i}(t)+p_{\phi_{i} \phi_{j}}\right)^{2}+\left(x_{j}(t)+p_{\phi_{j} \phi_{i}}\right)^{2} \geq R_{m i n}^{2} \tag{4.9}
\end{equation*}
$$

where $R_{\text {min }}$ is a constant value that provides a minimum safe distance gap between conflicting vehicles to avoid any collision at the intersection; $\phi_{i}$ and $\phi_{j}$ are the destination definition of vehicle $i$ and vehicle $j$.

Step 1: Collect position, velocity and destination information from all vehicles in the communication range.

Step 2: Calculate assumed states according to the prediction horizon using (4.1).
Step 3: Compute the control signals by minimizing the centralized optimization problem for all vehicles using (4.6).

Step 4: Compute speed value using vehicle dynamics in (4.1).
Step 5: Apply the speed value as an input to the vehicles in SUMO;
Step 6: Simulate one step and go to Step 1.
Figure 4.2. Centralized Model Predictive Control Algorithm

### 4.6. Controller Algorithm

The main advantage of the centralized MPC framework is that it controls the whole environment, and it is not constrained with any traffic policy. The only task of the controller is to minimize objective function. Therefore, vehicles can pass through the intersection more efficiently. However, increasing the number of vehicles within the intersection causes more computational cost for ICU.

The main algorithm for centralized MPC is given in Figure 4.2. In Step 1, ICU collects state information from all vehicles within communication range. In Step 2, assumed vehicle future dynamics are calculated through the prediction horizon T by (4.1). In Step 3, ICU runs the optimization problem and computes the control input by minimizing (4.4). In Step 4, ICU sends control input values to the corresponding vehicles and each vehicle calculates its velocity value by using (4.1). In Step 5, the reference velocities, which are calculated in the previous step are applied to the vehicles. After running simulation one step, new states are generated, and iteration starts over in Step 1.

### 4.7. Numerical Analysis

In this section, evaluation of the proposed framework is carried out in SUMO, and the simulation results are presented.

### 4.7.1. Simulation Setup

The simulation setup for the numerical analysis of this chapter is as follows:

- SUMO vehicle dynamics are used as a plant model;
- Eight vehicles are deployed to the simulation.
- Maximum and minimum allowable acceleration and deceleration capabilities are $5 \mathrm{~m} / s^{2}$ and $-6 \mathrm{~m} / \mathrm{s}^{2}$;
- Minimum safe distance between two consecutive vehicles is set to 2.5 meters;
- Minimum safe distance between two conflicting vehicles $\left(R_{\text {min }}\right)$ is set to 7 meters;
- Desired velocity $\left(v_{d}\right)$ is chosen as $16.67 \mathrm{~m} / \mathrm{s}$;
- Rolling coefficient $\left(f_{r}\right)$ is set to 0.015 , the frontal area $\left(A_{f}\right)$ is set to $2 \mathrm{~m}^{2}$, air density $(\rho)$ is set to $1.2 \mathrm{~kg} / \mathrm{m}^{3}$, drag coefficient $\left(C_{d}\right)$ is set to 0.3 ;
- Weights for the cost functions $w_{v_{i}}, w_{u}$ and $w_{f}$ are chosen as 2, 5, 0.001;
- Preceding horizon T is chosen as 20 ;
- Sample time $t_{s}$ is chosen as 0.2 seconds;
- In the risk function, H is chosen as 1000 and $\alpha$ is chosen as 0.05 .

In the simulations, we test our study with and without traction force cost element and inspect the efficiency of traction force minimization in terms of fuel economy and travel time. It can be stated that, at each time step, only the first two vehicles approaching the intersection at each lane are involved in the optimization problem. In Figure 4.3, the position of the vehicles according to the intersection, is illustrated. It is shown that all vehicles pass through the intersection without any collision by adjusting their velocities depending on the control input calculated by ICU.


Figure 4.3. Distance to Intersection Values for All Vehicles

Table 4.1 shows the detailed result of the simulation for all vehicles at the intersection. It can be seen that the traction force cost element decreases the average fuel consumption of the overall system while increasing the average travel time. However, fuel consumption of some vehicles (e.g., vehicle 8 ) increase with the use of the traction force cost. The reason of this is that, since the main objective of the centralized MPC framework is to optimize the whole system (all the vehicles), it has to sacrifice some vehicles in terms of fuel economy to minimize total fuel consumption.

### 4.8. Summary of the Chapter

In this chapter, the centralized model predictive control (CMPC) approach is discussed by introducing the risk function to estimate potential collisions and the traction force to minimize the fuel consumption. The details of the controller algorithm for ICU is explained. Simulation results support that traction force minimization has an advantage in terms of reducing the fuel consumption at the expense of increased travel time.

The next chapter moves on to discuss the decomposition of the centralized MPC approach into decentralized MPC. Fuel efficiency formulation of a combustion engine

Table 4.1. Detailed Travel Time and Fuel Consumption Results for All Vehicles With and Without Traction Force Cost

| Vehicles | Direction | With Traction Force Cost <br>  |  | Travel Time <br> $(\mathbf{s})$ | Fuel Consumption <br> $(\mathbf{m l})$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | 31 | Travel Time <br> $(\mathbf{s})$ | Fuel Consumption <br> $(\mathrm{ml})$ |  |
| vehicle 2 | Eastbound | 32.4 | 180.42 | 30.6 | 185.77 |
| vehicle 3 | Northbound | 33.6 | 168.44 | 31.8 | 169.09 |
| vehicle 4 | Northbound | 35.6 | 153.18 | 33 | 169.85 |
| vehicle 5 | Westbound | 30.6 | 187.81 | 14.8 | 157.25 |
| vehicle 6 | Westbound | 32.6 | 163.28 | 30.2 | 194.65 |
| vehicle 7 | Southbound | 34 | 163.53 | 32 | 169.09 |
| vehicle 8 | Southbound | 35.4 | 155.53 | 33.8 | 169.14 |
| Average |  | 33.15 | 167.19 | 33.8 | 150.66 |

will be included in the objective function to also provide more efficient travel in terms of fuel consumption.

## 5. DECENTRALIZED INTERSECTION CONTROL USING MPC AND FUEL CONSUMPTION

Traffic coordination at intersections is being frequently analyzed using decentralized MPC (DMPC) methods. The main motivation is due to the increase in V2V capabilities of vehicles, since they will be able to communicate with each other seamlessly, enabling decentralized approaches. With these approaches, minimizing delay time of the vehicles as well as fuel consumption under safety constraints is the main goal to coordinate vehicles at the intersection.

This chapter will present a DMPC method to coordinate the vehicles at intersections. In order to solve the consensus problem which is stated in [24], a prioritizing method which is commonly used in the literature will be defined. In addition to acceleration optimization which is stated in [21], fuel efficiency optimization will be explained to take fuel economy more into consideration.

### 5.1. Hybrid Vehicle Model Dynamics

In this MPC framework, fuel economy is also considered while providing smooth flows for the vehicles approaching the intersection. Therefore, basic longitudinal vehicle state dynamics are re-formulated by also taking traction force which is applied against the vehicle into consideration. To calculate MPC model future states, the longitudinal dynamics of a vehicle at intersection are modified based on [37], [38] as

$$
\begin{array}{r}
x_{i}(t+1)=x_{i}(t)-v_{i}(t) t_{s}-\frac{1}{2} u_{i}(t) t_{s}^{2} \\
v_{i}(t+1)=v_{i}(t)-\left(\frac{1}{2 M_{i}} C_{d} \rho A_{f_{i}} v_{i}(t)^{2}-f_{r} g+g \theta-u\right) t_{s} \tag{5.1}
\end{array}
$$

where $v_{i}, x_{i}, M_{i}$ and $A_{f_{i}}$ are velocity, distance to intersection, mass and frontal area of vehicle i respectively; $u_{i}$ is the control input of vehicle $\mathrm{i} ; t_{s}$ is the time step of the controller. In this system, acceleration is used as control variable. $M$ is mass, $f_{r}$ is
rolling coefficient, $A_{f}$ is frontal area, $C_{d}$ is drag coefficient of the vehicle, g is the gravity and $\rho$ is the air density. $\theta$ is the angle of the road slope which is assumed to be 0 in this thesis.

### 5.2. Fuel Consumption Minimization

One of the main goals of each vehicle, apart from crossing the intersection with minimum time delay without any collision, is to minimize fuel consumption by controlling vehicle traction and braking force per unit mass while satisfying other constraints. To minimize fuel consumption, the problem definition is given by [39],

$$
\begin{equation*}
\arg \min _{u_{i}(t)} \frac{1}{S_{i}(t)} \sum_{t=0}^{T} \dot{m}_{i f u e l}(t) t_{s} \tag{5.2}
\end{equation*}
$$

where $S_{i}$ is the total headway distance that vehicle i has through the preceding horizon $T$ (in other words travelled distance through prediction horizon). In (5.2), the equivalent fuel consumption rate $\dot{m}_{i f u e l}$ is formulated as

$$
\begin{equation*}
\dot{m}_{i f u e l}(t)=\frac{1}{\eta_{i e f f} H_{L H V}} P_{\text {icons }}(t) \tag{5.3}
\end{equation*}
$$

where $\eta_{\text {ieff }}$ is the propulsion efficiency of the vehicle i and $H_{L H V}$ is the lower heating value of gasoline. $P_{i c o n s}$ is the effective power consumption for vehicle i that is formulated as

$$
\begin{equation*}
\mathrm{P}_{i \text { cons }}(t)=\frac{1}{2} \rho C_{D} A_{f_{i}} v_{i}(t)^{3}+M_{i} g v_{i}(t)(f+\theta)+\beta\left[v_{i}(t) M_{i} u_{i}(t)\right]+(1-\beta)\left[-\eta_{\text {irec }}(t) M_{i} u_{i}(t) v_{i}(t)\right] \tag{5.4}
\end{equation*}
$$

where $\eta_{\text {irec }}$ is the recuperation efficiency of vehicle i; $M$ is the mass; $f_{r}$ is the rolling coefficient; $A_{f}$ is the frontal area and $C_{d}$ is the drag coefficient of the vehicle and $\rho$ is the air density. In general, $P_{i c o n s}$ should be always greater than zero. To ensure that,
a binary $\beta$ coefficient is defined as follows:

$$
\beta= \begin{cases}0 & \text { if } \quad u_{i} \leq 0  \tag{5.5}\\ 1 & \text { otherwise }\end{cases}
$$

### 5.3. MPC Optimization Problem

Once vehicles in the communication range broadcast their position and velocity profile to the network, each vehicle can calculate its optimal velocity profile with the MPC framework. To design a distributed MPC framework considering fuel economy, the following objectives should be considered:

- Fuel consumption for unit distance,
- Desired velocity tracking,
- Acceleration.

The overall minimization problem can be formulated as

$$
\begin{equation*}
\min _{u_{i}(t)}\left(\sum_{t=0}^{T}\left[w_{1} \frac{\dot{m}_{i f u e l} t_{s}}{x_{i}(T)-x_{i}(0)}+w_{2}\left(v_{i}(t+1)-v_{d}\right)^{2}+w_{3} u_{i}(t)^{2}\right]\right) \tag{5.6}
\end{equation*}
$$

where $w_{1}, w_{2}, w_{3}$ are the weight factors, $x_{i}(t)$ is the distance to the intersection at time $\mathrm{t}, v_{d}$ is the predefined common desired velocity which is the same value for all vehicles, T is the model predictive horizon, $v_{i}(t)$ is the velocity of vehicle i at time t and $u_{i}(t)$ is the control input which is the acceleration/deceleration value of vehicle i.

The first term of the objective function minimizes the total fuel consumption per unit distance through the model predictive horizon for vehicle i; the second term optimizes the deviation of the desired velocity which ensures that vehicle i will not stop during the optimization period; the last term focuses on the minimization of the acceleration/deceleration value. By obtaining current velocity and position of vehicle i within the communication area, the optimal velocity profile can be calculated.

The weights in the objective function are given by [39]

$$
\begin{array}{r}
w_{1}=10+100 e^{\left(0.05 v_{d}\right)} \\
w_{2}=10+500 e^{\left(-0.07 v_{d}\right)}  \tag{5.7}\\
w_{3}=2000+1000 e^{\left(-0.1 v_{d}\right)}
\end{array}
$$

where $v_{d}$ is the desired velocity for all vehicles. When the desired velocity decreases, velocity tracking and acceleration weight factors will increase while the weight factor of the fuel consumption term will decrease. In other words, if the desired velocity increases, fuel consumption will be more important for the cost function and vice versa.

In this work, interior point method [40] is used to solve the nonlinear model predictive control optimization problem, so that the optimal solution can be found over a given prediction horizon [41].

### 5.4. Decentralized Conflict Resolution Problem

In the previous chapter, the intersection coordination problem has been solved by a centralized model predictive control method. This section explains how a centralized coordination problem can be converted into a decentralized coordination problem by splitting the problem into smaller pieces so that each vehicle agent can solve its own coordination problem.

### 5.4.1. Converting Centralized Problem into Decentralized One

In order to solve the conflict resolution problem at the intersection in a decentralized manner, the centralized optimal MPC problem should be decomposed into local optimal MPC problems by applying the primal decomposition technique based on [24]. Similarly, the system dynamics, input and state constraints can be used independently. However, the safety constraints should be re-implemented especially for conflicting vehicles. To achieve this, the following issues should be handled:
(i) As formulated in (4.8) and (4.9), these safety constraints are the absolute value constraints which can be satisfied easily by the centralized solver, because the centralized solver control all the vehicle agents. For the decentralized problem, the constraints should be reformulated accordingly in order to be solved by vehicle agent instead of a central agent. In addition, this reformulation should provide efficient solution for the optimization problem.
(ii) A simple decomposition may not solve the conflicting resolution problem, because in a distributed scheme with non-convex coupled constraints, this approach may lead to convergence issues.

### 5.4.2. Policy for the Safety Constraints

Since (5.6) with coupled constraints has a non-convex nature, it is a challenging task to solve this kind of optimization problems. For decomposed objective functions and the coupled constraints, there are several methods like distributed sub-gradient methods [42], [43], proximal methods [44], or the alternating direction method of multipliers [45] to solve the decentralized convex optimization problem. These approaches generally utilize that Lagrangian multiplier of the coupled constraints can be handled as a consensus variable for the optimization problem. This consensus is obtained by satisfying constraints for two or more coupled subsystems. These convex optimization problems can be easily divided into local optimization problems and can be solved in a sequential manner, therefore fully parallelized computations are available. To consider the experimental simulations, parallelized and nested iterations may cause additional communication effort and because of the time consuming nature, solving these kind of optimization problems increase the overall computation time and may not be realistic when we consider the real time experimental simulations. Therefore, for the conflict resolution problems at the intersection, fully parallelized computations should be chosen. While aforementioned general methods are reasonable for convex decentralized problems, there is no only one way to solve non-convex problems.

In our decentralized non-convex optimization problem, without any consensus, safety constraints may cause to the dead-lock or collisions while solving the optimiza-
tion problem in a fully parallelized manner. If there is no common consensus between agents in simulation, each agent can change their intention according to cost of their minimization function. In other words, if there is no common traffic rule, each vehicle agent will intend to pass the intersection before others and without any collision. As a result of this, there will be no such a way for all the vehicles to pass the intersection without any collision, minimum time delay and fuel consumption.

To deal with the consensus problem, Katriniok et al. propose prioritized safety constraints instead of pairwise safety constraints in [24]. The basic idea is to introduce some priorities on agents such that only one of two conflicting vehicles can pass the conflicting point at one time. This idea is summarized in the following definition.

Definition 5.4.1. (Prioritized Safety Constraints) A prioritization function $\gamma: \mathcal{A} \rightarrow$ $\mathbb{N}_{+}$is defined which appoints a unique passing priority to vehicle i , where $i \in \mathcal{A}$ and the vehicle agent with higher priority will pass the conflict point always first. The vehicle with lower priority will always know that it has to pass the conflict point after the vehicle with higher priority. The formulation of this approach is as

$$
\begin{equation*}
\mathcal{A}_{c, \gamma}^{[i]}=\left\{l \in \mathcal{A}_{c}^{[i]} \mid l \neq i \wedge \gamma(l)<\gamma(i) \wedge s_{c, l}^{[i]} \neq \infty\right\} \tag{5.8}
\end{equation*}
$$

which requires that the vehicle agent 1 with higher priority than vehicle agent i , has to pass the conflict point before vehicle agent i.

In this thesis, as a prioritization policy, we use the first come first serve policy studied in [8]. Basically, the vehicle agent which is closer to the intersection will pass the conflict point first. We can formulate the safety constraint with the prioritization as

$$
\begin{array}{ll}
x_{l}(t)-x_{i}(t)>R_{\text {min }} & \text { if } \quad x_{i}(t) \geq x_{l}(t)  \tag{5.9}\\
x_{i}(t)-x_{l}(t)>R_{\text {min }} & \text { otherwise }
\end{array}
$$

where $x_{i}$ and $x_{l}$ are the positions of the $i^{\text {th }}$ and $l^{\text {th }}$ vehicles which have a common conflict point and $R_{\text {min }}$ is the minimum safe distance to pass the intersection.

Step 1: Collect position, velocity and destination information to all vehicles in the communication range.

Step 2: Calculate assumed states according to the prediction horizon using (5.1); for $i=1$ to $N_{\text {vehicles }}$ do

Step 3: Compute the control signals by minimizing the optimization problem for the vehicle i using (5.6);

Step 4: Compute speed value to apply using vehicle dynamics in (5.1);
Step 5: Apply the speed value as an input to the vehicle i in SUMO; end for

Step 6: Run the simulation for one step and go to Step 1.
Figure 5.1. Decentralized Model Predictive Control Algorithm

### 5.5. Controller Algorithm

The main advantage of the decentralized model predictive control (DMPC) is that it is capable of providing a solution for all vehicles independent from the number of vehicles at the intersection. However, increasing the number of vehicles at the intersection, computational and communication complexity will also be increased due to the vehicle to vehicle constraints.

The main algorithm for the DMPC is given in Figure 5.1. In Step 1, all vehicles broadcast their position and velocity information to other vehicles within the intersection and receive their state values. In Step 2, assumed vehicle dynamics are calculated through the prediction horizon T by (5.1). After all vehicles know each other's current and assumed position, velocity and destination information, the aforementioned nonlinear constraint optimization problem runs for each vehicle to calculate the control input signal in Step 3. Although the control signal for the entire horizon is calculated, the velocity input corresponding to the current time is calculated by using the control input at current time in Step 5. After one step simulation, new velocity, position, and destination values are generated and iteration starts over from Step 1.

### 5.6. Numerical Analysis

In this section, the evaluation of the algorithm in the SUMO simulation environment and simulation results are presented.

### 5.6.1. Simulation Setup

The simulation setup for the numerical analysis of this chapter is as follows:

- SUMO vehicle dynamics are used as a plant model;
- 8 vehicles are deployed to the simulation.
- Maximum and minimum allowable acceleration and deceleration capabilities are $5 \mathrm{~m} / s^{2}$ and $-6 \mathrm{~m} / \mathrm{s}^{2}$;
- Minimum safe distance between two consecutive vehicles is set to 2.5 meters;
- Minimum safe distance between two conflicting vehicles $\left(R_{\min }\right)$ is set to 7 meters;
- Desired velocity $\left(v_{d}\right)$ is chosen as $16.67 \mathrm{~m} / \mathrm{s}$;
- The propulsion ( $\mu_{i e f f}$ ) and recuperation ( $\mu_{\text {irec }}$ ) efficiency values are set to 0.2 and 0.8 , respectively;
- Rolling coefficient $\left(f_{r}\right)$ is set to 0.015 , the frontal area set to $\left(A_{f}\right)$ is set to $2 \mathrm{~m}^{2}$, air density $(\rho)$ is set to $1.2 \mathrm{~kg} / \mathrm{m}^{3}$, drag coefficient $\left(D_{d}\right)$ is set to 0.3 , the lower heating value of gasoline $\left(H_{L H V}\right)$ is set to 10 ;
- Preceding horizon T is chosen as 20 ;
- Sample time $t_{s}$ is chosen as 0.2 seconds.

In the simulation setup, we have tested our decentralized system with hybrid and basic vehicle (which is described in Chapter 4) longitudinal dynamics. We also have tested the algorithm with fuel consumption and without fuel consumption term in the objective function and inspected the effects of the fuel consumption elements to the decentralized model predictive control model.

In Figure 5.2, distance to intersection values of the vehicles through the simulation are depicted. It can be seen that, after 13 seconds, vehicles start to accelerate or


Figure 5.2. Distance to Intersection Values for All Vehicles
decelerate for safe travel. All the vehicles pass the intersection without any collision.

As seen in Table 5.1, simulation results show that using hybrid vehicle dynamics in (5.1) as MPC control model leads to travel with less energy loss compared to basic vehicle longitudinal dynamics in (4.1). Also, the fuel efficiency related cost in (5.6) provides fuel economic travel when we consider the overall system. However, the vehicles have the most delayed travel time with hybrid vehicle dynamics and the fuel efficiency related cost. These results confirm the association between fuel efficiency and travel time. In order to provide fuel efficient travel using fuel efficiency related costs, we should compromise the average travel time.

The detailed simulation results for all vehicles at the intersection are summarized in Table 5.2. Fuel consumption cost is beneficial for all vehicles in terms of fuel economy except Vehicle 1 which has to sacrifice itself in order to stick to the consensus policy and for the sake of fuel economy and travel time of all vehicles.

Table 5.1. Results of Decentralized MPC

|  | Fuel Consumption <br> $(\mathrm{ml})$ | Travel Time <br> $(\mathrm{s})$ |
| :---: | :---: | :---: |
| Basic Vehicle Dynamics <br> With Fuel Economy Related Cost | 178.32 | 35.9 |
| Basic Vehicle Dynamics <br> Without Fuel Economy Related Cost | 181.81 | 33.83 |
| Hybrid Vehicle Dynamics <br> With Fuel Economy Related Cost | 171.30 | 36.9 |
| Hybrid Vehicle Dynamics <br> Without Fuel Economy Related Cost | 179.74 | 34.57 |

### 5.7. Summary of the Chapter

In this chapter, converting the centralized MPC of Chapter 4 into a decentralized controller is examined. To provide collision-free passing through the intersection, FCFS policy is defined and included in the objective function.

In order to provide fuel efficient travel, basic longitudinal vehicle dynamics are reformulated to take traction force into consideration and called as hybrid model dynamics. By using hybrid model dynamics as MPC model and adding fuel efficiency formulation to the optimization problem, it is seen from numerical analysis that fuel consumption decreases comparing the basic vehicle dynamics and without fuel efficiency formulation. However, the fuel efficiency related costs increase the total travel time of the vehicles.

It should be also noted that some vehicles sacrificed themselves to obey the consensus policy and they are not able to decrease their fuel consumption in spite of including fuel efficiency formulation in the objective function.

Table 5.2. Detailed Travel Time and Fuel Consumption Values Using Hybrid Vehicle Dynamics for each Vehicles With and Without Fuel Consumption Cost

| Vehicles | Direction | Hybrid Vehicle Dynamics With <br> Fuel Consumption Cost |  | Hybrid Vehicle Dynamics Without <br> Fuel Consumption Cost |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | Fuel Consumption <br> (ml) | Travel Time <br> (s) | Fuel Consumption <br> (ml) |  |
| Vehicle 1 | Eastbound | 31.8 | 189.12 | 32 | 187.71 |
| Vehicle 2 | Eastbound | 41.4 | 135.51 | 36.8 | 161.70 |
| Vehicle 3 | Northbound | 40.6 | 170.19 | 37.8 | 180.50 |
| Vehicle 4 | Northbound | 46 | 170.6 | 39 | 177.28 |
| Vehicle 5 | Westbound | 31.8 | 183.26 | 32 | 184.79 |
| Vehicle 6 | Westbound | 41.8 | 138.80 | 37 | 162.21 |
| Vehicle 7 | Southbound | 30 | 209.88 | 30.4 | 209.10 |
| Vehicle 8 | Southbound | 31.7 | 173.05 | 31.6 | 174.68 |
| Average | 36.9 | 171.30 | 34.58 | 179.74 |  |

In Chapter 6, simulation results for both centralized and decentralized MPC approaches are compared in detail.

## 6. EXPERIMENTS AND RESULTS

In this chapter, the performance of the centralized MPC of Chapter 4 is compared with the decentralized MPC of Chapter 5 by considering both basic and hybrid longitudinal vehicle dynamics. Additionally, the efficiency of both approaches with and without fuel economy related cost is examined.

### 6.1. Effects of Vehicle Dynamics to Centralized MPC

In Chapter 4, centralized MPC with basic vehicle dynamics is explained. In this section, the efficiency of hybrid vehicle dynamics which is formulated in Chapter 5, to CMPC is investigated. All approaches are simulated with the same test setup and the results are compared in terms of fuel consumption and travel time.

Table 6.1 summarizes the average results for four conditions (hybrid vehicle dynamics with fuel economy related cost, hybrid vehicle dynamics without fuel economy related cost, basic vehicle dynamics with fuel economy related cost, basic vehicle dynamics without fuel economy related cost). Note that using hybrid vehicle dynamics and fuel efficiency related cost decrease the fuel consumption. However, fuel efficient travel increases time delay as in real life. Nevertheless, it can be acceptable when we consider the air pollution and the cost of the carbon-dioxide emission to nature.

### 6.2. Performance Evaluation of CMPC and DMPC

In this section, the performances of centralized and decentralized MPC studied in Chapter 4 and 5, are compared. While doing this, all the configurations are simulated with the same simulation setup and numerical values for both approaches.

Table 6.2 summarizes the simulation results of centralized and decentralized MPC studies in terms of travel time. The results, as shown in Table 6.2, indicate that centralized MPC performs better than decentralized MPC with all configurations. Detailed

Table 6.1. Results of Centralized MPC

|  | Fuel Consumption <br> $(\mathrm{ml})$ | Travel Time <br> (s) |
| :---: | :---: | :---: |
| Basic Vehicle Dynamics <br> With Fuel Economy Related Cost | 167.19 | 33.15 |
| Basic Vehicle Dynamics <br> Without Fuel Economy Related Cost | 169.43 | 32.62 |
| Hybrid Vehicle Dynamics <br> With Fuel Economy Related Cost | 166.26 | 33.32 |
| Hybrid Vehicle Dynamics <br> Without Fuel Economy Related Cost | 167.97 | 33.15 |

results of the centralized and the decentralized approaches for all vehicles in the simulation are given in Figures 6.1-6.2. It should be noted that in general, travel times decrease with the basic vehicle dynamics and without using fuel related cost optimization. In other words, considering fuel efficiency in the optimization problem increases the travel times of the vehicles. The second major finding is that some (e.g. veh7) vehicles are not be able to decrease their travel time unlike other vehicles. A possible explanation for this is that some vehicles have to compromise their travel times for the sake of the whole system.

The simulation results of the centralized and decentralized MPC studies in terms of fuel consumption are summarized in Table 6.2. Overall, these results indicate that CMPC performs better than DMPC in terms of fuel consumption. Results depicted in Figures 6.3-6.4 show that fuel consumption is decreased by including the hybrid longitudinal vehicle dynamics and fuel efficiency related cost in the optimization problem.

In general, the centralized MPC performs better than the decentralized MPC in terms of both travel time and fuel consumption with all configurations. This result

Table 6.2. Travel Time Average Results

|  | Travel Time (s) |  |
| :---: | :---: | :---: |
|  | Centralized MPC | Decentralized MPC |
| Basic Vehicle Dynamics <br> With Fuel Economy Related Cost | 33.15 | 35.92 |
| Basic Vehicle Dynamics <br> Without Fuel Economy Related Cost | 32.62 | 33.82 |
| Hybrid Vehicle Dynamics <br> With Fuel Economy Related Cost | 33.32 | 36.9 |
| Hybrid Vehicle Dynamics <br> Without Fuel Economy Related Cost | 33.15 | 34.575 |

may be explained by the fact that the centralized approach minimizes the whole system without any consensus and optimizes the trajectories of the whole system under safety constraints. On the other hand, the decentralized framework is constrained with the traffic policy.

The computational cost of the centralized framework is higher than the decentralized framework because, in the centralized framework, all the computations are done by one central intersection controller while in the decentralized framework, the computations are carried out by each agent individually. On the other hand, we can say that the centralized framework is better than the decentralized framework in terms of communication costs because the centralized framework only needs V2I communication while the decentralized framework needs both V2I and V2V communication which is another research area.

### 6.3. Summary of the Chapter

In this chapter, first, the effects of the hybrid vehicle dynamics on centralized MPC in terms of fuel economy and travel time are examined. In summary, the simu-


Figure 6.1. Detailed Travel Time Results for Decentralized MPC
lation results show that hybrid vehicle dynamics have the advantage to decrease fuel consumption. However, hybrid vehicle dynamics, as well as fuel efficiency related cost in the optimization problem, increase the travel time of vehicles.

Second, the comparison of centralized and the decentralized MPC is carried out. The centralized approach performs better than the decentralized approach in terms of travel time and fuel consumption. However, managing all the vehicles with only one agent causes computational and management costs. In other words, to handle centralized control task, ICU should be well equipped with communication and computation tools. On the other hand in the decentralized approach, the whole computation is decomposed into smaller agents. This decreases the amount of computation at the expense of increased communication.


Figure 6.2. Detailed Travel Time Results for Centralized MPC

Table 6.3. Fuel Consumption Average Results

|  | Fuel Consumption (ml) |  |
| :---: | :---: | :---: |
|  | Centralized MPC | Decentralized MPC |
| Basic Vehicle Dynamics <br> With Fuel Economy Related Cost | 167.19 | 178.31 |
| Basic Vehicle Dynamics <br> Without Fuel Economy Related Cost | 169.43 | 180.78 |
| Hybrid Vehicle Dynamics <br> With Fuel Economy Related Cost | 166.26 | 171.30 |
| Hybrid Vehicle Dynamics <br> Without Fuel Economy Related Cost | 167.98 | 179.74 |



Figure 6.3. Detailed Fuel Consumption Results for Decentralized MPC


Figure 6.4. Detailed Fuel Consumption Results for Centralized MPC

## 7. CONCLUSION

Multi-agent intersection management is a huge task including safe passing, time delays, and fuel economy. There are several studies in the literature which have used various methods for coordinating vehicles at the intersection. In this thesis, the objective of the overall system has been to develop a multi-agent intersection coordination scheme based on minimizing time delays and energy loss. Three optimization problems have been studied; namely (1) departure time-based optimization, (2) centralized optimization with MPC and (3) decentralized optimization with MPC. Finally, the efficiency of centralized and decentralized MPC to intersection management has been studied in Sumo with TraCi Matlab interface.

The first contribution of this thesis is an improvement in the V2I and I2V communication model. The optimal departure time scheduling approach which is already 50 \% better than the FCFS policy has been extended to the four-way intersection model. Vehicles communicate with ICU planted at the intersection and ICU optimizes the departure time of vehicles. Dynamic trajectory planning approach is defined for vehicles to depart from the intersection at the settled time. Simulation results confirm that the dynamic trajectory planning approach overcomes the limitations of acceleration and deceleration rates. The algorithm is also tested with different communication ranges and it has resulted that the travel time of the vehicles decreases as the communication range increases.

Another contribution of this thesis is to provide smooth flows at the intersections by using a centralized MPC approach. The main focus of this study has been to construct the CMPC algorithm in terms of nonlinear costs and constraints to solve the identified problem by also considering fuel efficiency. In addition to the nonlinear risk function, the traction force is formulated and included in the optimization problem. The simulation results confirm the association between traction force and fuel consumption. It is noted that minimizing traction force applied to the vehicles leads to a decreased energy loss of the vehicles.

Although V2I based centralized intersection management is a common study in literature, there are also various studies relying on V2V communication. The final contribution of this thesis is a V2V based decentralized intersection coordination scheme in order to coordinate vehicles to pass the intersection with minimum time delays and fuel consumption. To this end, the centralized MPC scheme of Chapter 4 is decomposed into decentralized MPC. To solve the consensus problem between vehicles, the common policy of FCFS is implemented as a constraint to the optimization problem. Additionally, hybrid vehicle dynamics and fuel efficiency cost are formulated in order to decrease fuel consumption. It is concluded that hybrid vehicle dynamics and fuel efficiency related cost are more efficient in terms of fuel economy. However, these terms also increase the average travel time of the vehicles.

The proposed CMPC and DMPC algorithms have been implemented and tested in the same simulation environment and conditions. Numerical analysis shows that the centralized MPC approach performs $8 \%$ better than the decentralized MPC in terms of travel time and fuel consumption. Centralized and decentralized control approaches have advantages and disadvantages in some cases. Doing a whole optimization problem in only one agent may cause computational costs which would be non-realistic when we consider applying this framework to real life. Additionally, if ICU is destructed, the whole communication will be lost and intersection management will not be available. On the other hand, decomposing the whole optimization problem into multi-agents induces communication costs which is another research area in telecommunications.

Finally, in light of the work done so far, these findings help us to shape our further improvements. In this thesis, all the vehicles travel with a constant speed before they reach to the communication area and all the vehicles have the same weight. As future work, a scenario where all the vehicles travel with different velocity and different vehicle types can be considered. Communication delays in V2I and V2V should also be taken into account. As human-driven legacy and autonomous vehicles will travel together in a very near future, a further study with more focus on coordination of legacy and autonomous vehicles at the intersections is therefore necessary.

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