THESIS

Grup
7

STUDY OF THE INFLUENCE OF UNBALANCED AND WAVE FORM OF THE APPLIED VOLTAGE SYSTEMS ON THE OUTPUT TORQUE AN ASYNCHRONOUS MACHINE

(WITH THE USE OF GENERALIZED MACHINE)

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STUDY OF POLYPHASE SYSTEMS

A system consisting of a number of winding groups displaced in space from each other, in which alternating emfs are produced of the same frequency but displaced in time from each other, is called a polyphase system.

The most important of the polyphase systems are the 2-, 3-, and 6-phase systems.

I - POLYPHASE BALANCED SYSTEMS

The windings of the types illustrated in Fig. 1 are called balanced polyphase windings, or, briefly, polyphase windings. They are characterized by the condition that the several component windings are symmetrically spaced.

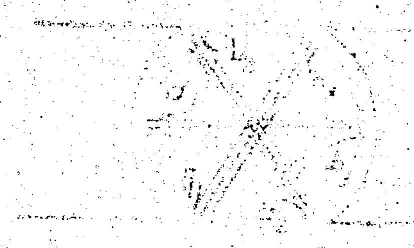


FIG. 1

A balanced polyphase winding having 9 separate windings is called a q-phase winding. The phase displacement between the emfs of adjacent windings is in general $2\pi/q$ rad. Thus, if the emf of one of the windings, arbitrarily designated as the first, is represented by

$$e_1 = \sqrt{2}\,E \cos wt \tag{1}$$

the emfs of the remaining (9-1) windings are successively displaced in phase by the angle $2\pi/q$, so that

$$e_2 = \sqrt{2}\,E \cos \left(wt - \frac{2\pi}{q}\right)$$

$$e_3 = \sqrt{2}\,E \cos \left(wt - 2\frac{2\pi}{q}\right)$$

$$e_9 = \sqrt{2}\,E \cos \left(wt - (q-1)\frac{2\pi}{q}\right) \tag{2}$$

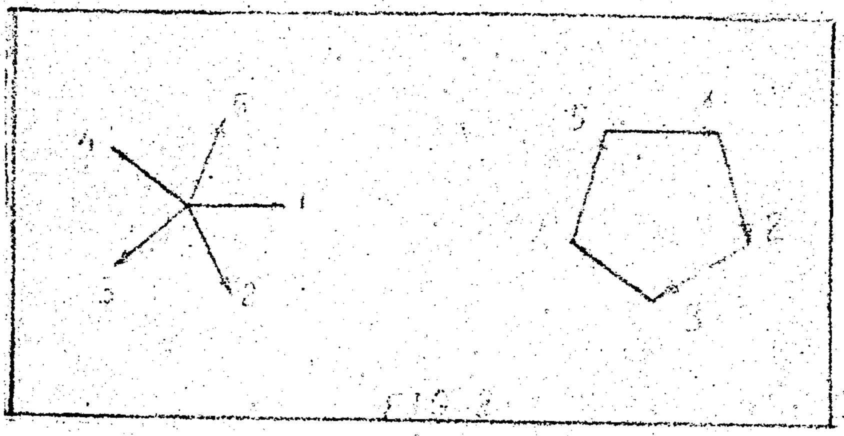
Such a system of emfs is called a poly phase system. In a balanced circuit a balanced voltage system produces a balanced current system. In a balanced poly phase circuit the conditions in all phases are the same, that is, the voltages upon all phases, the power factors for all phases are the same in magnitude.

The sum of all separate emfs of a balanced poly-phase system, at any given moment, is

$$e_1 + e_2 + e_3 + \cdots + e_q = 0 \tag{3}$$

The result does not hold for the special case where $q = 2$, in a two-phase winding.

The phasors of a polyphase balanced system form either a symmetrical star or an equal polygon.



If the vectors form a closed polygon, the algebraic sum of their individual projections on any axis of reference must be zero.

1. <u>The phase sequence of poly-phase system</u>

In Fig. 2 five equal and uniformly spaced vectors are intended to represent any general set of balanced phasors (say of voltage or current) comparising an q-phase system. This particular group is numbered consecutively in the negative direction (The clockwise direction) and the angle between consecutive phasors is $2\pi/q$ radians; such a phasor system is called <u>positive sequence system</u> of the first of order. (Fig. 3-(a)).

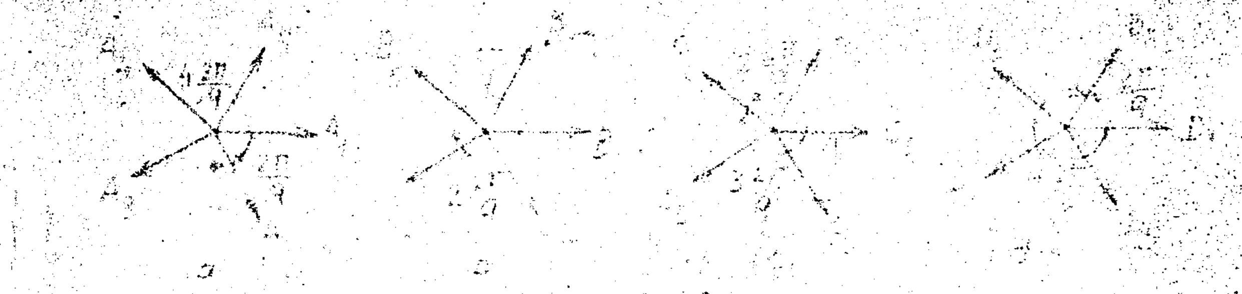


FIG. 3

Fig. 3(b) represents a wholly different set of balanced phasors for the same number of phases, the difference being due to the fact that consecutively numbered phasors differ in phase by $2\,(\frac{2\pi}{q})$ radians. This system is called positive sequence system of the second of order. In the same way, Figs 3c, 3d represent still other balanced groupings, the angles between consecutively numbered phasors being $3.(2\pi/q)$ radians in the first case and $4.(2\pi/q)$ radians in the second. These are positive sequence systems of the third and fourth of order. It will be seen that when $q = 5$, as in the diagram, no other balanced groupings are possible; in general when q has any value (except 2), the number of balanced groupings is $(q-1)$.

If a balanced phasor system is numbered consecutively in the positive direction, (counter clockwise direction) and the angle between consecutive phasors is $\frac{2\pi}{q}$ radians; such a phasor system is called <u>negative sequence system</u> of the first of order. It will also be seen that 5-phase systems in Fig. 3d, c, b, a are the negative sequence systems of the first, second, third, and fourth order consequently. Thus, $m(\frac{2\pi}{q})$ spacing results in a positive sequence of numbering is eqdivalent $(q-m)\,\frac{2}{q}$ spacing results in negative sequence system

Positive sequence components and negative sequence components of the same order are always conjugates.

The vector sum of any one of these groups of balanced vectors is zero;

If q is a prime number, it is possible to form positive sequence system $(q-1)$ of the 1., 2., ------ $(q-1)$ th of order.

It is also possible to form negative sequence system (φ-1)th of order; but they are equivalent to positive sequence system of the (φ-1)., (φ-2).,--------,2., 1th of order consequently.

If φ is not prime number, it is possible to form with prime numbers which are smaller than φ. The value of these prime numbers gives the positive sequence systems.

For instance, let's consider 12-phase balanced system; 1, 5, 7 and 11 are prime numbers with 12. Hence, it is possible to form

One 12-phase positive sequence system of the 1st order
" " " " " " " " 5th "
" " " " " " " " 7th "
" " " " " " " " 11th "

Each of these positive sequence systems may be taken as a negative sequence system of the (12-1) = 11., (12-5) = 7., (12-7) = 5., (12-11) = 1st order.

2. Connections of polyphase circuits

While it is evident that the separate windings shown in Fig. 1 may be connected to indpendent circuits, considerations of economy in the use of wire for the transmission and distribution lines call for interconnection of the phase windings so that the total number of wires shall be less than 2φ, and more specially, equal to φ, except in the spacial case where φ = 2.

Two different methods of interconnections may be used to produce a balanced system.

a) Star connection

The symbols b and e at the terminals of the several windings indicate, respectively, the "beginning" and "ending" points. In star connection, in Fig. 4 all the points marked b are connected together, and the points marked e are connecte to the line wires; it is equally possible to connect the all e points to a common junction, and the all b points to the line terminals. Such a system has φ phase line and 1 neutral line.

In case of poly-phase balanced currents the current flowing through the neutral line being always zero, this wire may be cancelled. The voltage between the neutral and the lead of any phase is called <u>star voltage</u> or <u>phase voltage</u>.

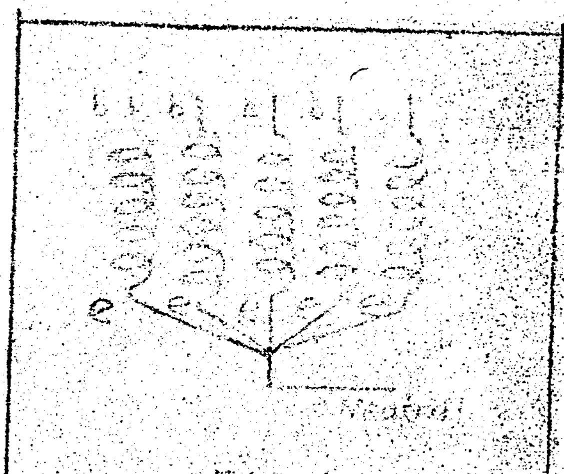


FIG. 4

The voltage between two consecutively numbered phases is called <u>line voltage</u>. This voltage is equal to the difference of consecutively numbered line voltages.

Consider the star voltages forming a positive sequence system of the first order. The grafical construction shows that the line voltages also form a positive sequence system.

$$U_{12} = V_1 - V_2$$
$$U_{23} = V_2 - V_3 \qquad (4)$$

From the geometry of Fig.5, it will be seen that the argument of line voltage,

$$\psi = \frac{1}{2}\left(\pi - \frac{2\pi}{q}\right) = (q-2)\,\frac{\pi}{2q} \qquad (5)$$

and the magnitude of the line voltage is given by the relation

$$U = 2\,V \sin\frac{\pi}{q} \qquad (6)$$

For three-phase systems ($q=3$) we obtain,

$$\psi = \frac{\pi}{6} = 30^{\circ} \qquad U = \sqrt{3}\,V$$

$$U_{12} = V_1 \sqrt{3}\,\underline{\big|\,\pi/6} \qquad (7)$$

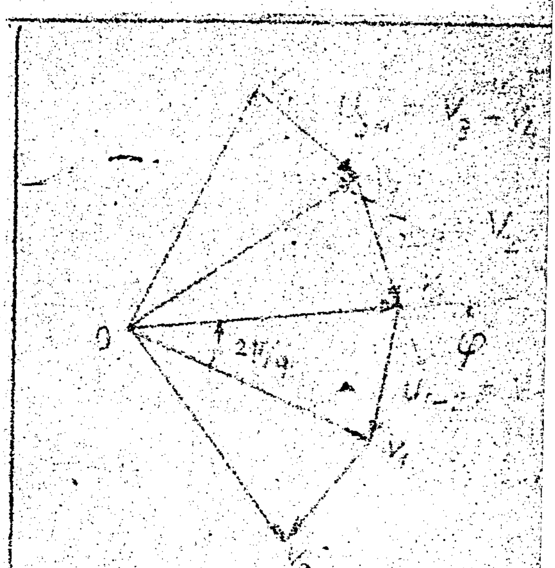


FIG. 5

Now, let's assume that the star voltages V form a negative sequence system; the line voltage system also form a negative sequence system. In a similar manner the argument and the magnitude of the line voltages are given by the following relations

$$\psi' = (q - 2)\frac{\pi}{2q} \quad , \quad U = 2\,V \sin\frac{\pi}{q} \tag{8}$$

In that case the line-voltage system lags in respect to the phase-voltage system.

For three-phase negative sequence system we obtain

$$\psi' = -\frac{\pi}{6} = -30^{o} \text{ and } U_{12} = V_1\sqrt{3}\,\underline{\big|-\frac{\pi}{6}} \tag{9}$$

b. Mesh connection

In the mesh connection, in Fig. 6, the "end" of each winding is connected to the "beginning" of the next phase winding, taken in cyclical order, and the q line wires are connected to q junction points. In the mesh connection the line voltage and phase voltage are same, but the line current at any junction is clearly equal according to the figure, to the difference of currents entering the junction from phases. To find the line current I_2 it is therefore necessary to take the phasor sum of I_{23} and $-I_{12}$ that is

$$I_2 = I_{23} - I_{12}$$

$$I_3 = I_{34} - I_{23} \tag{10}$$

From the above relations it follows that the magnitude of the line current in case of a q-phase balanced mesh connection is

$$I_1 = 2\,I_{12}\sin\frac{\pi}{q}, \tag{11}$$

and the phase sequence of line currents and mesh currents are same. It has been seen that if the currents form a positive sequence system, the line currents lags by $(q-2)\frac{\pi}{2q}$ to the

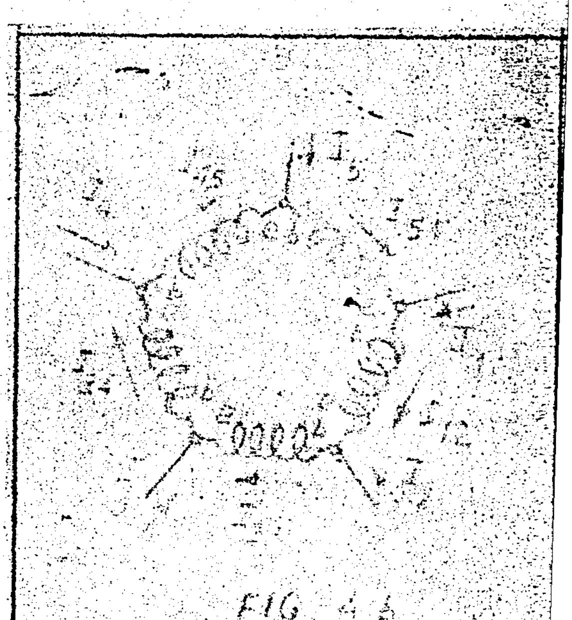
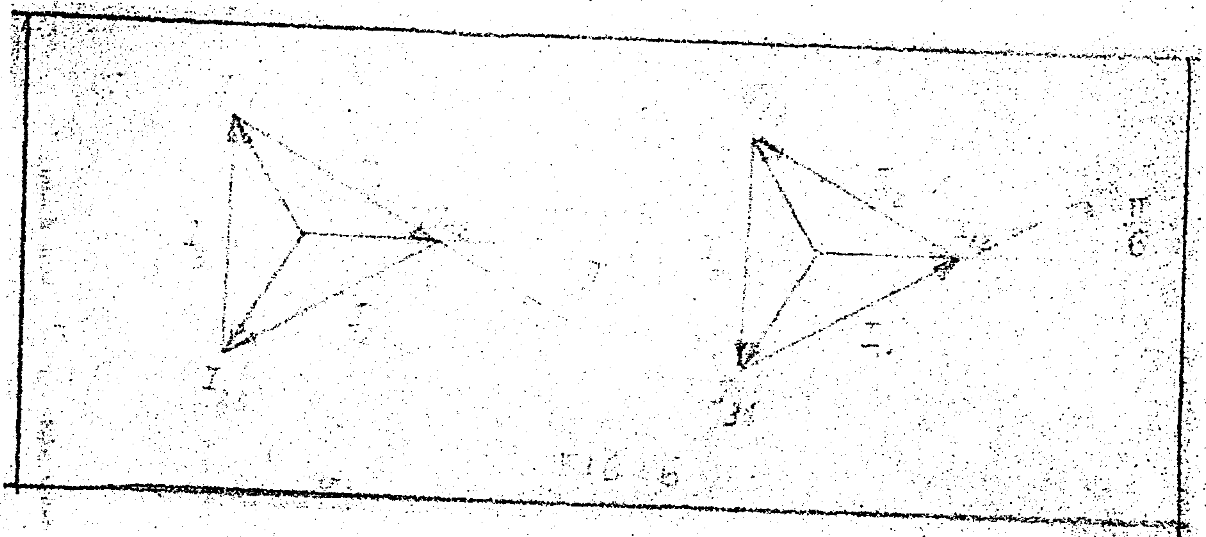


FIG. 6

phase currents, if they form a negative sequence system, line currents leads by $(q-2)\frac{\pi}{2q}$ to the phase currents Fig. 6.



For three-phase systems

At the positive sequence system $\qquad I_1 = \sqrt{3}\, I_{12} \angle -\frac{\pi}{6}$

At the negative sequence system $\qquad I_1 = \sqrt{3}\, I_{12} \angle +\frac{\pi}{6}$

or

At the positive sequence system $\qquad I_{12} = \frac{I_1}{\sqrt{3}} \angle +\frac{\pi}{6}$

at the negative sequence system $\qquad I_{12} = \frac{I_1}{\sqrt{3}} \angle -\frac{\pi}{6} \qquad (12)$

3. The Power in Balanced Polyphase Circuit

Consider a q-phase balanced circuit, let the successive emfs and currents in the q phase be

$$v_1 = \sqrt{2}\, V \cos wt$$

$$v_2 = \sqrt{2}\, V \cos \left(wt - \frac{2\pi}{q}\right)$$

$$v_3 = \sqrt{2}\, V \cos \left(wt - \frac{4\pi}{q}\right)$$

$$v_q = \sqrt{2}\, V \cos \left(wt - (q-1)\frac{2\pi}{q}\right)$$

and

$$\cancel{i_1 = 2\,V \cos\,(wt - (q-1)\,\frac{2}{q})}$$

$$i_1 = \sqrt{2}\,I \cos\,(wt - \varphi)$$

$$i_2 = \sqrt{2}\,I \cos\,(wt - \frac{2\pi}{q} - \varphi)$$

$$i_q = \sqrt{2}\,I \cos\,(wt - (q-1)\,\frac{2\pi}{q} - \varphi) \qquad (13)$$

The instantaneous power in the entire circuit is then

$$p = v_1\,i_1 + v_2\,i_2 + \text{----------} + v_q\,i_q \qquad (15)$$

and on multiplying the terms of Eq (14) in pairs and substituting the products in Eq (15), the result is in the form

$$p = 2\,V\,I \sum_{k=0}^{k=q} \cos\,(wt + k\frac{2\pi}{q})\,\cos\,(wt + k\frac{2\pi}{q} - \varphi) \qquad (16)$$

Since $\cos x \cos y = \frac{1}{2}\left[\cos(x-y) + \cos\,(x+y)\right]$, Eq. (16) can be transformed to

$$p = VI \sum_{k=0}^{k=q-1} \left[\cos\varphi - \cos\,(2wt + 2k\,\frac{2\pi}{q} - \varphi)\right] \qquad (17)$$

The summation of the q constant terms gives the total average power, or

$$p = q.V.I.\,\cos\varphi \qquad (18)$$

Where V and I are the effective values of the sinusoidal emfs and currents. As we see from the above expression the instanlenous power in the polyphase circuit is constant and the sum of the average powers of phases and the flustuant power, which alternates at double the frequency, is cancelled.

In the three-phase circuits, <u>active</u>, <u>reactive</u>, and <u>appar</u> powers are given by the following expressions:

$$p = 3\,VI \cos \qquad \Pi = 3\,VI \sin \qquad \mathcal{P} = 3VI \qquad (19)$$

or, since $U = \sqrt{3}\,V$

$$p = \sqrt{3}\,UI \cos \qquad = \sqrt{3}\,UI \sin \qquad \mathcal{P} = \sqrt{3}UI \qquad (20)$$

Where V is the phase voltage, U is the line voltage.

4. Representation of poly-phase systems by Complex Quantities.

Multiplication by $e^{j\frac{2\pi}{q}}$ rotates the radius vector in a positive sense (counterclocwise) by the angle $\frac{2\pi}{q}$. Euler's formula states that

$$\left(\cos\frac{2\pi}{q} + j\sin\frac{2\pi}{q}\right) = e^{j\frac{2\pi}{q}} \tag{21}$$

It is possible to abbreviated form if $a = e^{j\frac{2\pi}{q}}$

$$a = 1\underline{\left|\frac{2\pi}{q}\right.}$$

In a corresponding manner multiplication by $e^{-j\frac{2\pi}{q}}$ rotates the vector an angle $\frac{2\pi}{q}$ in a negative direction.

Referring to Fig. 3, the several sets of vectors may be represented as follws:

$$A_1,\quad e^{-j\frac{2\pi}{q}}A_1,\quad e^{-j2\frac{2\pi}{q}}A_1,\ \text{---------}\ e^{-j(q-1)\frac{2\pi}{q}}A_1$$

$$B_1,\quad e^{-j2\frac{2\pi}{q}}B_1,\quad e^{-j4\frac{2\pi}{q}}B_1,\ \text{---------}\ e^{-j(q-1)\frac{2\pi}{q}}B_1$$

$$C_1,\quad e^{-j3\frac{2\pi}{q}},\quad e^{-j6\frac{2\pi}{q}}C_1,\ \text{---------}\ e^{-j3(q-1)\frac{2\pi}{q}}C$$

$$D_1,\quad e^{-j4\frac{2\pi}{q}}D_1,\quad e^{-j8\frac{2\pi}{q}}D_1,\ \text{---------}\ e^{-j4(q-1)\frac{2\pi}{q}}D_1 \quad (2$$

or, in general by the expressions

$$A_1, \quad a^{-1} A_1, \quad a^{-2} A_1, \text{------------} \; a^{-(q-1)} A_1$$

$$B_1, \quad a^{-2} B_1, \quad a^{-4} B_1, \text{------------} \; a^{-2(q-1)} B_1$$

$$C_1, \quad a^{-3} C_1, \quad a^{-6} C_1, \text{------------} \; a^{-3(q-1)} C_1$$

$$L_1, \quad a^{-(q-1)} L_1, \quad a^{-2(q-1)} L_1, \text{--------} \; a^{-(q-1)^2} L_1 \qquad (24)$$

The phasor sum of any one of these groups of balanced phasors is zero; for example, the phasor sum of the first set is

$$1 + a^{-1} + a^{-2} + \text{------------} + a^{-(q-1)} = 0 \qquad (25)$$

Assuming that the ordinary laws of multiplication hold for this process, it follows that

$$a^{q} = 1 \qquad a^{q+1} = a \qquad a^{(q+2)} = a^2$$

$$a^{(q-1)} = a^{-1} \qquad a^{(q-2)} = a^{-2} \qquad a^{(q-3)} = a^{-3}$$

or in general

$$a^{(q+n)} = a^{n} \qquad (26)$$

Where n is any positive and negative Integer.
According to the above definitions, a positive sequence system of the first of order containing the vectors A_1, A_2, A_3, -------- may be written as follows:

$$(S)^{(1)}_{d}(A) = (A_1, \; a^{-1} A_1, \; a^{-2} A_1, \text{------} \; a^{-q} A_1); \qquad (27)$$

And the positive sequence systen of the second of order,

$$(S)^{(2)}_{d}(A) = (A_1, \; a^{-2} A_1, \; a^{-4} A_1, \text{--------} \; a^{2} A_1) \qquad (28)$$

As a result, the conjugate of positive sequence system of the -th of order gives a negative sequence system of the same order, that is

$$S_i^{(q)} = S_d^{(-m)} \qquad (29)$$

Three-phase systems

The three-phase system is the most important of the poly-phase systems. Now, consider 3-phase balanced systems. In case of 3-phase system the unite vector or operatör is

$$a = e^{j\frac{2\pi}{3}} \quad \text{or} \quad a = 1 \angle \frac{2\pi}{3}$$

$$\text{either} \quad a = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \tag{30}$$

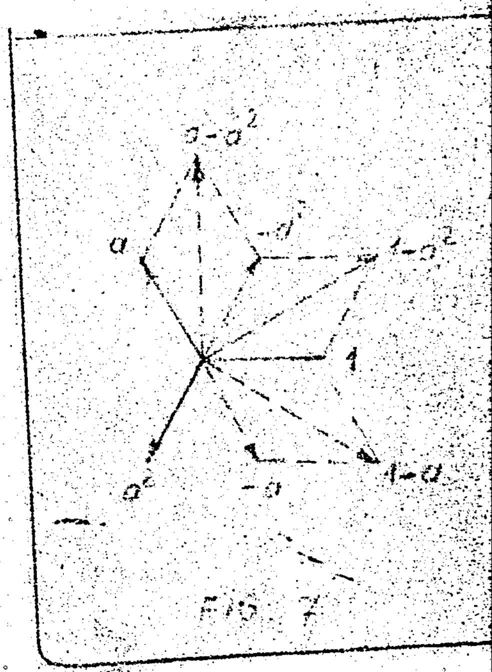
In the following table some properties of the unit vectors is given.

$$1 + a + a^2 = 0$$

$$a^3 = 1, \quad a^4 = a, \quad a^5 = a^2$$

$$a^{-1} = a^2, \quad a^{-2} = a$$

$$a^{3+n} = a^n \tag{31}$$



and

$$a = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = 1 \angle \frac{2\pi}{3}$$

$$a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} = 1 \angle -\frac{2\pi}{3}$$

$$1 + a = -a^2 = \frac{1}{2} \; j\frac{\sqrt{3}}{2} = 1 \angle \frac{\pi}{3}$$

$$1 + a^2 = -a = \frac{1}{2} - j\frac{\sqrt{3}}{2} = 1 \angle -\frac{\pi}{3}$$

$$a + a^2 = -1$$

$$1 - a = \frac{\sqrt{3}}{2}(\sqrt{3} - j) = \sqrt{3} \angle -\frac{\pi}{6}$$

$$1 - a^2 = \frac{\sqrt{3}}{2}(\sqrt{3} + j) = \sqrt{3} \angle \frac{\pi}{6}$$

$$a - a^2 = j\sqrt{3} \qquad = \sqrt{3} \angle \frac{\pi}{2}$$

$$\frac{1}{1-a} = \frac{1-a^2}{3} = \frac{1}{\sqrt{3}} \angle \frac{\pi}{6}$$

$$\frac{1}{1-a^2} = \frac{1-a}{3} = \frac{1}{\sqrt{3}} \angle -\frac{\pi}{6}$$

$$\frac{1}{a-a^2} = \frac{a^2-a}{3} = -\frac{1}{\sqrt{3}}\, j = \frac{1}{\sqrt{3}} \angle -\frac{\pi}{2} \qquad (32)$$

Note that, a and a^2 are conjugate complex quantities. And the positive sequence components and negative sequence components are always conjugates, that is

$$a^2 = a^* \quad \text{or} \quad a = a^{2*} \qquad (33)$$

Consider a 3-phase balanced system. Three-vectors may be two kind of systems.

1° positive-sequence system of the first of order Fig. 6a.

$$\mathcal{S}_d^{(1)}(A) = (A_1,\ a^2A_1,\ aA_1) \quad (33)$$

This also a negative sequence system of the second of order;

2° Negative-sequence system of the first of order

$$\mathcal{S}_i^{(1)}(A) = (A_1,\ aA_1,\ a^2A_1), \quad (34)$$

This also a positive sequence system of the second of order.

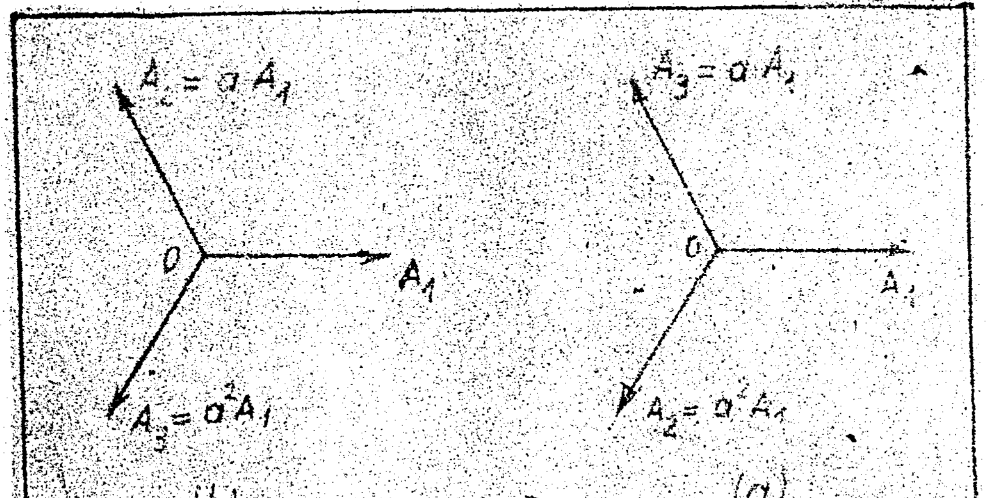


FIG 8

II. UNBALANCED POLYPHASE SYSTEMS

(Symmetrical Components Theory)

In the preceding articles we analysed that in the case of balanced polyphase systems, whether mesh or star connected, the circuit conditions in terms of the voltage, current, impedance, power, power factor, etc., concerning any one phase can be computed easily from the consideration of the equivalent single-phase circuit. Consequently, if an unbalanced system of voltages (or currents) could be resolved into component systems each of which is balanced, the effect of each of these components could be computed as for an equivalent single-phase circuit, and the separate results could then be recombined by the principle of super positions.

The analysis of an unbalanced system of polyphase phasors (voltages or current) due to C.L. Fortesque. This new analytical tool is called the method of symmetrical components.

1. The Symmetrical Components in three-phase system.

Consider an unbalanced set of three-phase phasors (voltages or currents) V_1, V_2, V_3, whose sum differs from zero, can be resolved into three groups of components, as follows:

a. A group consisting of three equal and cophasal vectors, each represents by V_0, where

$$V_o = \frac{1}{3}(V_1 + V_2 + V_3) \tag{35}$$

This group being called the uniphase or zero-sequence component.

b. A balanced three-phase group of positive sequence, represented by V_d, $a^{-1}V_d$, and $a^{-2}V_d$,

where

$$V_d = \frac{1}{3}\left[(V_1 - V_o) + a(V_2 - V_o) + a^2(V_3 - V_o)\right]$$

$$= \frac{1}{3}(V_1 + aV_2 + a^2V_3) - \frac{V_o}{3}(1 + a + a^2)$$

$$= \frac{1}{3}(V_1 + aV_2 + a^2V_3) \tag{36}$$

c. A balanced three-phase group of negative sequence, represented by V_i, aV_i and a^2V_i, where

$$V_i = \frac{1}{3}\left[(V_1 - V_o) + a^{-1}(V_2 - V_o) + a^{-2}(V_3 - V_o)\right]$$

$$= \frac{1}{3}(V_1 + a^{-1}V_2 + a^{-2}V_3) \text{ or since } a^{-1} = a^2,\ a^{-2} = a$$

$$= \frac{1}{3}(V_1 + a^2V_2 + aV_3) \qquad (37)$$

These three groups then posses the feature of complete symmetry.

2. Graphical Analysis of unbalanced three-phase phasors

The above mathematical analysis of an unbalanced three-phase system points the way to a graphical method for resolving a given set of vectors into symmetrical sets of components. The given phasors are V_1, V_2, V_3 ~~(Fig) (9a)~~. On adding them geometrically their resultant is $3V_o$, from which V_o is obtained by trisection, (Fig 9a).

Reference to Eq 36 shows that if V_2 is shows that if V_2 is rotated clockwise through 120 deg., and V_3 through 240 deg. in the same direction, the phasor sum of these new phasors, together with V_1, gives $3V_{d1}$, (Fig. 9b).

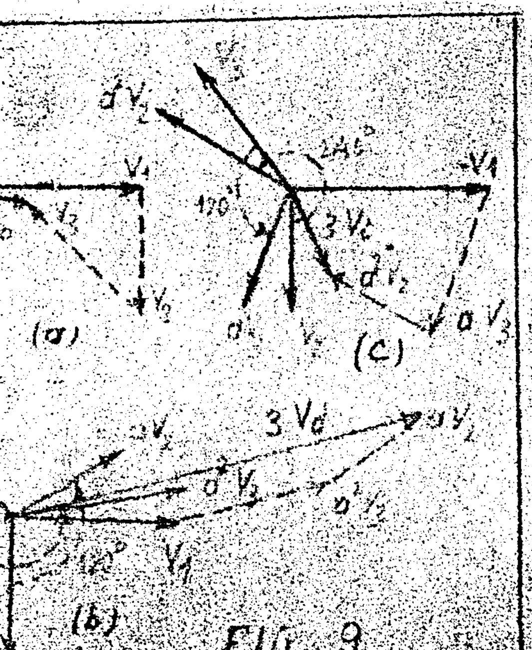


FIG. 9

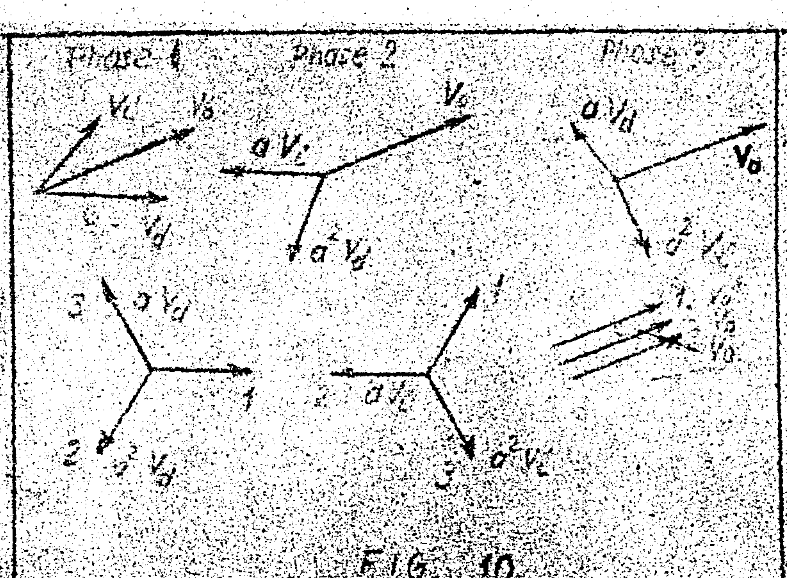


FIG. 10

Similarly, to find V_{i1}, phasors V_2 and V_3 are rotated counter clockwise through 120 and 240 deg., respectively, as in Fig 9c, and are then combined vectorially with V_1 to give $3\,V_{i1}$, in accordance with Eq(37). The remaining phasors, V_{d2} and V_{d3}, and V_{i2} and V_{i3} have been omitted From Fig. 9. The first pair is, of course, symmetrically placed with respect to V_{d1} insuch manner that any two of the three are 12o deg. apart and with the sequence of numbering in the positive direction similar relations hold for V_{i1}, V_{i2} and V_{i3} except that the sequence of numbering is in the negative direction.

3. Symmetrical Components in a q-phase system

It will be evident that if $V_1, V_2, V_3,$ ------- V_q represents any unbalanced group of q-phase phasors but subject to the condition that

$$V_1 + V_2 + V_3 + \text{----------} + V_q = 0 \qquad (35)$$

It is possible to write

$$V_1 = A_1 + B_1 + C_1 + \text{----------} + L_1$$

$$V_2 = a^{-1}A_1 + a^{-2}B_1 + a^{-3}C_1 + \text{-------} + a^{-(q-1)}L_1$$

$$V_3 = a^{-2}A_1 + a^{-4}B_1 + a^{-6}C_1 + \text{-------} + a^{-2(q-1)}L_1$$

$$V_q = a^{-(q-1)}A_1 + a^{-2(q-1)}B_1 + a^{-4(q-1)}C_1 + \text{------} + a^{-(q-1)^2}L_1$$

$$(36)$$

For an adding the separate equations in (36) the sum is identically zero. This means that any unbalanced group of q vectors whose phasor sum is zero, except when $q = 2$, may be resolved into $(q-1)$ balanced groups which have angular spacing respectively equal to $2\pi/q$, $a(2\pi/q)$, $3(2\pi/q)$, ----- $(q-1)(2\pi/q)$ radians.

The most general case that can occur is that of a group of q phasors, $V_1, V_2, V_3,$ ---------- V_q, whose phasor sum is not zero, in that case

$$V_1 + V_2 + V_3 + \text{---------} + V_q = qV_o \qquad (37)$$

which may be writteh in the form

$$(V_1 - V_o) + (V_2 - V_o) + (V_3 - V_o) + \text{-------} + (V_g - V_o) = 0$$

consequently if V_o, computed from

$$V_o = \frac{V_1 + V_2 + V_3 + \text{--------} + V_g}{g} \qquad (38)$$

is substracted from each of the given phasors the remainders satisfy the condition that their phasor sum shall equal zero, therefore it is possible that,

$$V_1 = V_o + A_1 + B_1 + C_1 + \text{-------------} + L_1$$

$$V_2 = V_o + a^{-1}A_1 + a^{-2}B_1 + a^{-3}C_1 + \text{------------} + a^{-(g-1)}L_1$$

$$V_3 = V_o + a^{-2}A_1 + a^{-4}B_1 + a^{-6}C_1 + \text{------------} + a^{-2(g-1)}L_1$$

$$V_g = V_o + a^{-(g-1)}A_1 + a^{-2(g-1)}B_1 + a^{-4(g-1)}C_1 + \text{------} + a^{-(g-1)^2}L_1 \qquad (39)$$

By adding the separate equations in (39) the sum is identically equal gV_o, in accordance with Eq. (37). This means that any unbalanced group of g phasors, whose phasor sum is not zero, may be resolved into $(g-1)$ balanced groups and one zero sequence component.

To find any one of the components, such as A_1, divide each equations of group (39) by the corresponding coefficient of A_1 and add the results. There is thus obtained.

$$V_o = \frac{1}{g}(V_1 + V_2 + V_3 + \text{-----------} + V_g)$$

$$A_1 = \frac{1}{g}(V_1 + aV_2 + a^2V_3 + \text{-----------} + a^{g-1}V_g)$$

$$B_1 = \frac{1}{g}(V_1 + a^2V_2 + a^4V_3 + \text{----------} + a^{2(g-1)}V_g)$$

$$L_1 = \frac{1}{g}(V_1 + a^{(g-1)}V_2 + a^{2(g-1)}V_3 + \text{---------} + a^{(g-1)^2}V_g) \qquad (40)$$

4. Symmetrical Components in a single-phase system

A single phase vector U may be assumed that it is identically equal the difference of two equal but opposite phasors of V_1 and V_2

$$V_1 = -V_2 = \frac{U}{2}$$

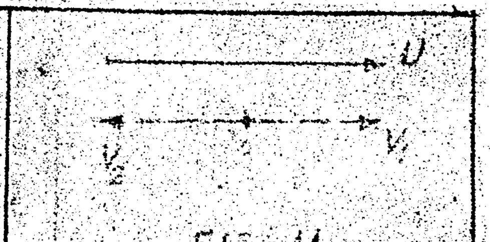


FIG. 11

Now, this two phasors also can be considered as an unbalanced three-phase system, in which the third one is zero. The symmetrical components of this three-phase system are:

$$V_o = \frac{1}{3}(V_1 + V_2 + V_3) = 0$$

$$V_d = \frac{1}{3}(V_1 + aV_2 + a^2V_3) = \frac{U}{2}\,\frac{1-a}{3} = \frac{U}{2\sqrt{3}}\,\underline{|-30^o}$$

$$V_i = \frac{1}{3}(V_1 + a^2V_2 + aV_3) = \frac{U}{2}\,\frac{1-a^2}{3} = \frac{U}{2\sqrt{3}}\,\underline{|30^o} \qquad (31)$$

This means that any single phase system may resolved into:

1° One positive sequence which have the magnitude $\frac{U}{2\sqrt{3}}$ and lays 30° to the reference phasor.

2° One negative sequence with same magnitude $\frac{U}{2\sqrt{3}}$ but leads 30° to the reference, Fig. 12.

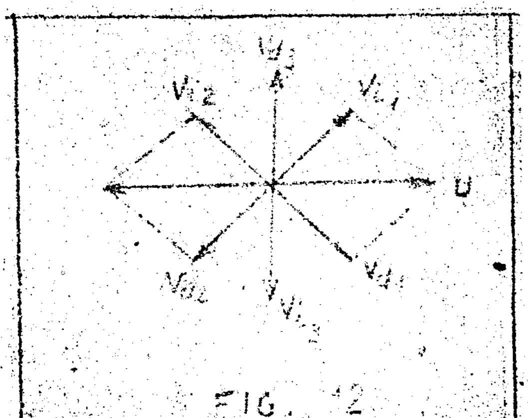


FIG. 12

5. Symmetrical Components in a Two-phase circuit

An unbalanced set of two phase phasors, say, of voltage or current, can always be resolved into one or the other of two combinations: (a) a single-phase system and one balanced two-phase system; or (b) two balanced two-phase systems of opposite phase sequence.

a. Let V_1 and V_2 in Fig. 13 be any pair of unbalanced two-phase vectors, and let V_o be the single-phase (zero sequence) component in each phase. It follows that $(V_1 - V_o)$ and $(V_2 - V_o)$ must represent two vectors of equal magnitude but which are in quadrature. Let it be assumed that

$$V_2 - V_o = E_2$$

is 90 deg. ahead of $V_1 - V_o = E_1$, in other words, that the sequence of the balanced two-phase vectors, E_1 and E_2 is positive. It follws than, that

$$\begin{aligned} V_1 - V_o &= E_1 \\ V_2 - V_o &= E_2 = jE_1 \end{aligned} \tag{32}$$

Taking first the sum, and then the difference, of Eqs. (32) it is found that

$$\begin{aligned} V_1 + V_2 &= 2V_o + E_1\ (1+j) \\ V_1 - V_2 &= E_1\ (1-j) \end{aligned} \tag{32}$$

when, by eliminating E_1,

$$V_o = \frac{V_1(1-j) + V_2(1+j)}{2} \tag{33}$$

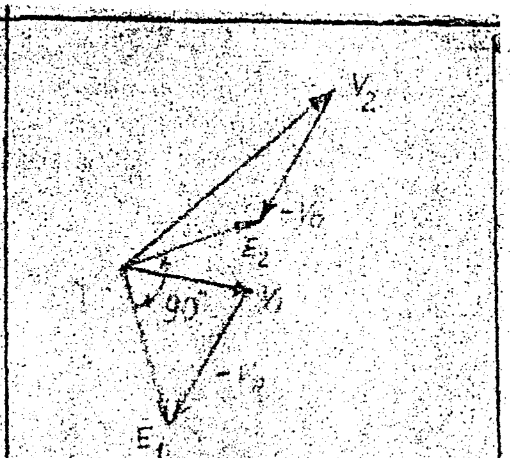


FIG. 13

If it is assumed that E_1 is 90 deg. ahead of E_2, correspon-ding to a negative sequence of vectors, it will be found that

$$V_o = \frac{V_1(1+j) + V_2(1-j)}{2} \tag{34}$$

It is seen that V_o is not half the sum of V_1 and V_2, as might have been expected by analogy with the three-phase and m-phase cases previously discussed.

b. If it is desired to resolve V_1, and V_2, and general pair of vectors, into two balanced two-phase groups of opposite sequence, Let A_1 and jA_1 in Fig. 14 be the pair having positive sequence, and B_1 and jB_1 be the pair having negative sequence.

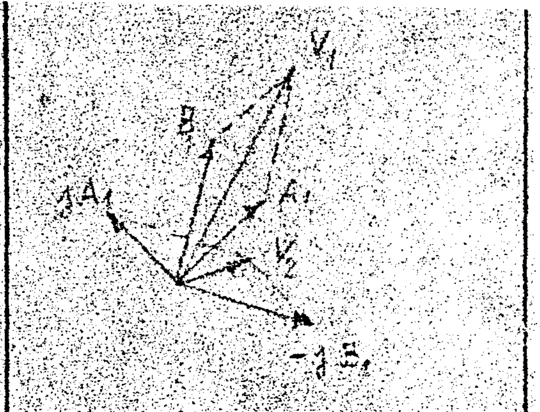


FIG 14

Then

$$V_1 = A_1 + B_1$$

$$V_2 = jA_1 - jB_1 \tag{35}$$

whence

$$A_1 = \frac{V_1 - jV_2}{2}$$

$$B_1 = \frac{V_1 + jV_2}{2} \tag{36}$$

Equations (58) to (62) lead/grafical methods for resolving V_1 and V_2 into either of the possible types of components included in case (a) and (b). The type of resolution represented by case (b) is ordinarily used in applying the method of symmetrical components to practical problems.

6. The Physical Meaning of Symmetrical Components

In symmetrical system the different sequences do not react each other; positive-sequence currents produce only positive-sequence voltages, negative sequence currents porduce only negative sequence voltages, and zero sequence currents produce only zero-sequence voltages. This method may be applied to both statio devices and rotating machines.

Static networks. Consider a static network shown in Fig 15 which may be represent a transmission or distribution line. If only balanced positive-sequence currents be made to flow through the line conductors, it follws that no current flows through the neutral. The positive-sequence currents produce only positive voltage drops. It can be similarly shown that negative sequence currents produce only negative-sequence voltage dropsl If only zero-sequence currents flow, equal currents flow in each line and the combined currents of the three line return through the neutral. In this case equal voltages will be induced in all three lines, and including the drop in the neutral impedance the drops will be the same in all three phases.

Rotating Machines. The positive-sequence durrents in the stator of a symmetrical machine produce a rotating field which rotates in the same direction as the rotor. It is apparent that under normal conditions with positive sequence voltages applied to the stator of rotating machines only positive-sequence currents are produced.

If negative-sequence voltages only are applied to the stator of induction machines a synchronously rotating field is produced which rotates in a direction opposite to the rotation of the rotor. This field induces currents in the rotor, which in turn produces a synchronously rotating field in a direction opposite to that of the rotor. Thus all the currents and voltages in the stator would be of negative-sequence. Because, the zero-sequence currents, which are in phase with each other in the three phases, produce no flux in the air gap. Hence these currents can produce only voltage drops of the zero sequence.

7. Sequence Impedances

It has been shown that in symmetrical networks the componemts of current of the different sequences do not react each other. When a voltage of a given sequence is applied to a piece of apparatus, a very definite current of the same sequence flows. The apparatus may be characterized as having a definite impedance to this sequence. Special names have been given to these impedances, namely: the impedance to positive-sequence currents, the impedance to negative-sequence currents, and the impedance to zero sequence currents or by means of simple expressions, negative-sequence impedance, positive sequence impedance, and zero-sequence impedance.

The impedance of symmetrical static networks are the same for the positive-and negative-sequences but may be different from the zero-sequence. For rotating machines the impedances will in general be different for all three sequences.

Now, let Z_1 and Z_2 and Z_3 in Fig. 37 represent three unequal Y connected impedances whose scalar magnitudes are independent if the current flowing through them; and let the three line currents be I_1, I_2 and I_3. The corresponding voltages drops in the three impedances are then

$$\begin{aligned} V_1 &= I_1 Z_1 \\ V_2 &= I_2 Z_2 \\ V_3 &= I_3 Z_3 \end{aligned} \qquad (37)$$

and the sume of these drops will in general differ from zero. They may therefore be resolved into components consisting of a zero sequence, a positive sequence and a negative sequence, given by

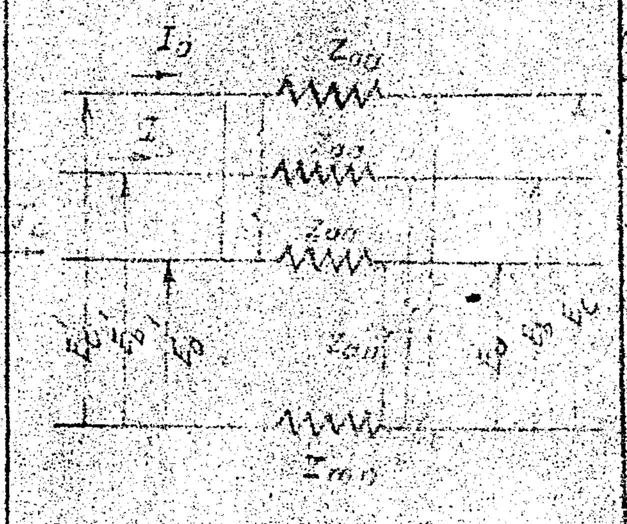


FIG. 15

$$V_o = \frac{1}{3}(V_1 + V_2 + V_3)$$
$$V_d = \frac{1}{3}(V_1 + aV_2 + a^2V_3)$$
$$V_i = \frac{1}{3}(V_1 + a^2V_2 + aV_3) \qquad (38)$$

In like manner, three unbalanced line currents may be resolved into similar symmetrical components such that

$$I_1 = I_o + I_d + I_i$$
$$I_2 = I_o + a^2I_d + aI_i$$
$$I_3 = I_o + aI_d + a^2I_i \qquad (39)$$

Substituting Eqs (37) and (39) in Eq (38), and collecting terms,

$$V_o = \frac{I_o}{3}(Z_1 + Z_2 + Z_3) + \frac{I_d}{3}(Z_1 + a^2Z_2 + aZ_3)$$
$$+ \frac{I_i}{3}(Z_1 + aZ_2 + a^2Z_3) \qquad (40)$$

From which it appears that the impedances are here arranged in groups similar to zero-, positive-, and negative-sequence groups of phasors. Writing

$$Z_o = \frac{1}{3}(Z_1 + Z_2 + Z_3)$$
$$Z_d = \frac{1}{3}(Z_1 + aZ_2 + a^2Z_3)$$
$$V_i = \frac{1}{3}(Z_1 + a^2Z_2 + aZ_3) \qquad (41)$$

it is found that

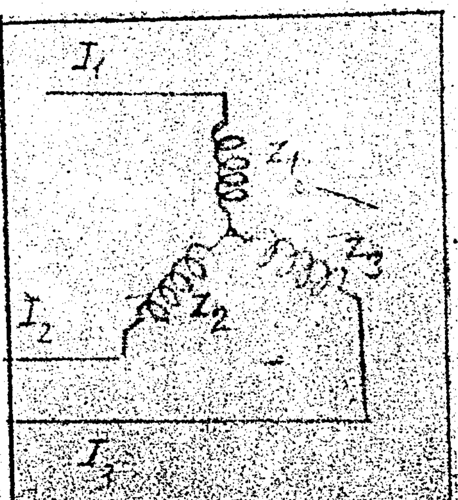


FIG. 16

$$V_o = I_oZ_o + I_dZ_i + I_iZ_d$$
$$V_p = I_dZ_o + I_iZ_i + I_oZ_d$$
$$V_n = I_iZ_o + I_oZ_d + I_dZ_d \qquad (42)$$

The three impedance Z_o, Z_d, Z_i are called, respectively the equivalent zero-, positive-, and negative- impedances, but they are not equivalent impedances in the sense ordinarily associated with that term. Thus, it is not true that the zero-sequence component of current I_o, multiplied by the zero-sequence impedance will give the zero-sequence component of voltage drop, as is plain from Eq. 42. The symmetric of the three equations in group (42) should be noted; thus, if the equivalent impedances are written in form,

$$\begin{matrix} Z_o & Z_i & Z_d \\ Z_o & Z_i & Z_d \\ Z_o & Z_i & Z_d \end{matrix}$$

The three component impedance drops V_o, V_d and V_i can be formed by inserting as multiplies the currents I_o, I_d, I_i taken in the cyclical order o-d-n, d-n-o, n-o-d.

In the case of a circuit that is physically balanced, the three branch impedances will be equal, and their separate magnitudes may be replaced by Z. It then follows from Eq. (41) that $Z_o = Z$, $Z_d = 0$, and $Z_i = 0$, and in that case, from Eq. (42) $V_o = I_oZ$, $V_d = I_dZ$, $V_i = I_iZ$. In order words, if the circuit is phsycally balanced, the component voltage drops are each directly related to the corresponding components of current in accordance with simple lows of ordinary circuits.

8. The Symmetrical Components of the resultaht voltages

Consider connected three-phase voltage system of V_1, V_2, V_3 with their symmetrical components V_o, V_i and V_d. The related resultabt or line voltages are

$$U_{12} = V_1 - V_2, \quad U_{23} = V_2 - V_3, \quad U_{31} = V_3 - V_1$$

Since the resultaht voltages form a closed triangle, it is determined that,

$$3U_o = U_{12} + U_{23} + U_{31} = 0$$

and the positive and negative sequences may be written as follows:

$$U_d = \frac{1}{3}(U_{12} + aU_{23} + a^2U_{31})$$

$$= \frac{1}{3}\left[U_{12}(1 - a^2) + U_{23}(a - a^2)\right] = \frac{1 - a^2}{3}(U_{12} - a^2U_{23})$$

$$U_i = \frac{1}{3}(U_{12} + a^2U_{23} + a\,U_{31})$$

$$= \frac{1}{3}\left[U_{12}(1-a) + U_{23}(a^2-a)\right] = \frac{1-a}{3}(U_{12} - aU_{23}) \qquad (43)$$

On the other hand let's express resultant voltage sequences U_d, U_i in terms of phase sequences V_d, V_i

$$U_{12} - a^2U_{23} = (V_1 - V_2) - a^2(V_2 - V_3)$$

$$= V_1 - V_2\,(1 + a^2) + a^2V_3 = 3V_d$$

$$U_{12} - aU_{23} = (V_1 - V_2) - a(V_2 - V_3)$$

$$= V_1 - V_2\,(1 + a) + aV_3 = 3V_i \qquad (44)$$

from this relations we obtain

$$U_d = (1 - a^2)\,V_d = \sqrt{3}\,V_d \underline{\left|\frac{\pi}{6}\right.}$$

$$U_i = (1 - a)\,V_i = \sqrt{3}\,V_i \underline{\left|-\frac{\pi}{6}\right.} \qquad (45)$$

Combining Eq. 44 with Eq. 45 it is possible to write

$$V_d = \frac{1}{3}(U_{12} - a^2\,U_{23})$$

$$V_i = \frac{1}{3}(U_{12} - a\,U_{23}) \qquad (46)$$

Similarly same relations may be written for the line currents I_1, I_2, I_3 and resultant currents that is $I_{12} = I_1 - I_2$, $I_{23} = I_2 - I_3$, $I_{31} = I_3 - I_1$

9. Determination of Positive-and negative-sequences of the resultant three-phase systems by the Graphical Method.

The positive and negative sequences of the phasors V_1, V_2, V_3 may be easily obtained from Eq. 46. To find V_d, E_{23} must be rotated 60 deg. in the counterclockwise direction and the resultant vector is equal to $3V_{d1}$. Similar construction is applicable to the negative sequence system. These construction are shown in Fig. 17.

10. Determination and Elimination of Zero-sequence Components

One method for determining the zero sequence component is shown in Fig. (10a). It is based on the fundamental equation (35) and consists in taking one-third of the vector sum of the three phase vectors.

Another method, shown, in Fig. 18 requires the determination of the neutral point of the system with zero-sequence components eliminated. The neutral point of this system is found at the point O, the intersection of two median lines. The vector connecting Oand O' gives directly the zero sequence voltage in its phase position with respect to the phase voltages V_1, V_2 and V_3.

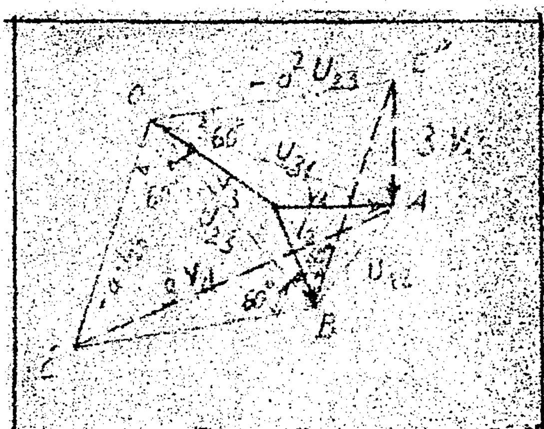


FIG. 17

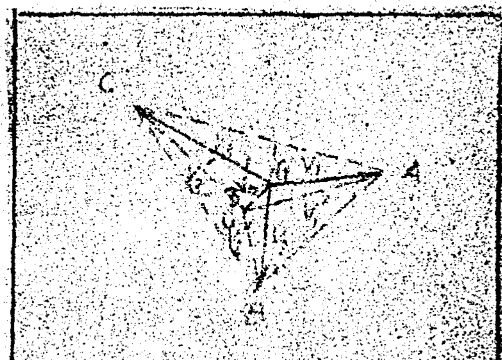


FIG. 18

NONSINUSOIDAL PERIODIC FUNCTION

1. Introduction

The usual theory and calculations of electrical circuits and machinery are based on the assumption of the sine wave. That is, it is assuming that the voltages and currents in the A.C. circuits are sine waves. While in most cases this is sufficiently the case, it is not always so, and especially they differ from the sine wave.

Such as distorted voltages and currents of a periodic nature can be resolved into a Fourier series consisting of a fundamental and higher harmonics. The result can always be expressed in the form of a series,

$$y = C_o + C_1 \sin(wt - \varphi_1) + C_2 \sin(2\,wt - \varphi_2) + \dots\dots\dots$$
$$+ C_n \sin(n\,wt - \varphi_n) + \cdots \qquad (47)$$

The wave $C_1 \sin(wt\ \varphi_1)$ of lowest frequency is the <u>fundamental</u>, its frequency determines the frequency of the irregular wave. The other component sine waves will have frequencies that are intergal multiples of the fundamental and are called <u>upper harmonics</u>. The wave having nwt is called n-th harmonic.

The above expression may also be written in the following form:

$$y = C_o + A_1 \sin wt + A_2 \sin 2wt + \dots\dots\dots + A_n \sin nwt +$$
$$+ B_1 \cos wt + B_2 \cos 2wt + \dots\dots\dots\dots + B_n \cos nwt \qquad (48)$$

The constants of the series is given by the following integration:

$$C_o = \frac{1}{T}\int_o^T y\,dt$$

$$A_n = \frac{2}{T}\int_o^T y \sin nwt\,dt$$

$$B_n = \frac{2}{T}\int_o^T y \cos nwt\,dt \qquad (49)$$

As we see from the above expression, C_o is the average value of the function for one period. The actual function therefore is equal the sume of C_o and the following series:

$$y = A_1 \sin wt + A_2 \sin 2wt + \dots\dots\dots + A_n \sin n\,wt + \cdots$$
$$+ B_1 \cos wt + B_2 \cos 2wt + \dots\dots\dots + A_n \cos n\,wt + \cdots \qquad (50)$$

2. The Relation Between the Shape of the Periodic Function and its Harmonics

1. In Fig. 19 shown one complete cycle of a distorted alternating wave form; if the loop B is moved back on the timeaxis to B', then A and B' are symmetrical with respect to the time axis. All alternating current and voltage waves produced by rotating machinery have this characteristic. Such a like curve satisfies that

$$f(wt+\pi) = -f(wt) \tag{51}$$

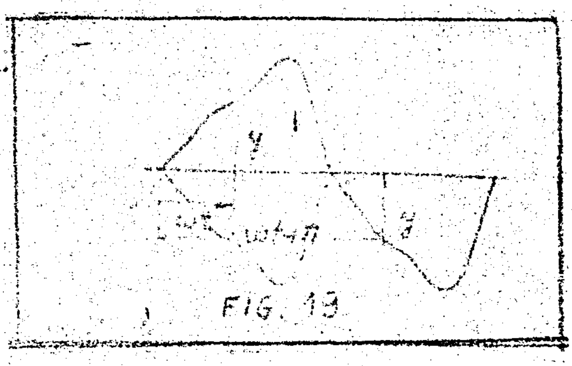


FIG. 19

When this function resolved into the Fourier Series, it is seen that, the series has only fundamentals and odd harmonics.

$$f(wt+\pi) = -A_1 \sin wt + A_2 \sin 2wt - A_3 \sin 3wt + \cdots$$
$$-B_1 \cos wt + B_2 \cos 2wt - \cdots$$

$$-f(wt) = -A_1 \sin wt - A_2 \sin 2wt - A_3 \sin 3wt - \cdots$$
$$-B_1 \cos wt - \cdots \tag{52}$$

By putting this two equations in Eq.(51) we obtain

$$y = A_1 \sin wt + A_3 \sin 3wt + \cdots + A_{(2n+1)} \sin(2n+1)wt + \cdots$$
$$+ B_1 \cos wt + B_3 \cos 3wt + \cdots + B_{(2n+1)} \cos(2n+1)wt + \cdots \tag{53}$$

For such like curves the integrals which are given the constants of harmonics may be taken only half period.

$$A_n = \frac{4}{T}\int_0^{\frac{T}{2}} y \sin n\,wt\, dt$$

$$B_n = \frac{4}{T}\int_0^{\frac{T}{2}} y \cos n\,wt\,dt \qquad (54)$$

2. In Fig. 20 is shown a flat-topped wave; if a line draw from the $\frac{\pi}{2}$ on the time axis, then A and B are symmetrical with respect to this line, that is

$$f(wt) = f(\pi - wt)$$

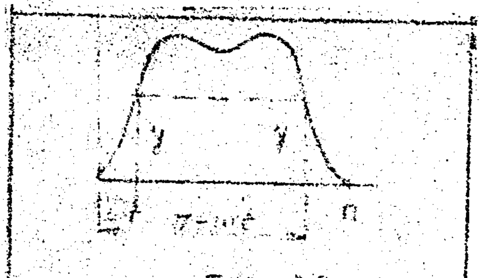


FIG 20

In that case all the cosine terms does not exist, and its enough to take integrals only for one-fourth period.

$$A = \frac{8}{T}\int_0^{\frac{T}{4}} y \sin n\,wt\,dt. \qquad (55)$$

3. At last, if the curve symmetric with respect to origin, that is

$$f(wt) = -f(-wt)$$

in that case all the cosine terms does not exist.

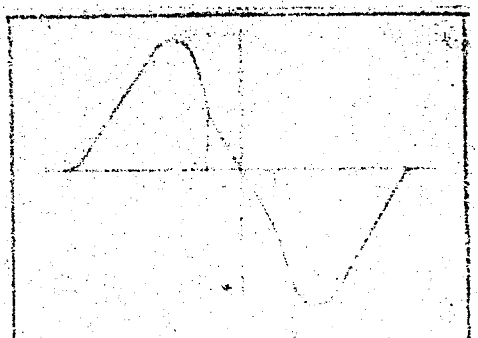


FIG 21

3. <u>The Determining of the coefficient of the Fourier Series</u>

When the function $y = f(x)$ is given in analytical form, Eqs(49) can be applied. If this is not the case, step-by-step integration can be applied. Referring to Eqs(49), these equations can be written in the form

$$C_o = \frac{1}{2\pi}\sum_{o}^{2} y\,\Delta x$$

$$A_n = \frac{1}{\pi}\sum_{o}^{2\pi} y \sin n\,x\,\Delta x$$

$$B_n = \frac{1}{\pi}\sum_{o}^{2\pi} y \cos n\,x\,\Delta x \qquad (56)$$

If the wave length, 2 radians, is divided in p equal parts, then $\Delta x = 2\pi/p.x$ is the distance to the mid-point of the interval and y the ordinate corresponding to x.

Effective Value of a Nonsinusoidal Current

The effective value of a variable current is defined as that continuous value which gives the same total i^2R loss. If I is the effective value of a periodic current i and T, the time of one cycle,

$$I^2RT = \int_o^T i^2 R\,dt$$

from which

$$I = \sqrt{\frac{1}{T}\int_o^T i^2\,dt} \qquad (57)$$

A nonsinusoidal current may be represent that,

$$i = I_o + \sqrt{2}\,I_1 \sin(wt - \varphi_1) + \sqrt{2}\,I_2 \sin(2wt - \varphi_2) + \cdots$$

The effective value of this current is

$$I^2 = \frac{1}{T}\int_o^T (I_o^2 + \sqrt{2}\,I_1 \sin(wt - \varphi_1) + \sqrt{2}\,I_2 \sin(2wt - \varphi_2) + \cdots)dt$$

$$= \frac{1}{T}\int_o^T I_o^2\,dt$$

$$+ \frac{1}{T}\int_o^T (\sqrt{2}\,I_1)^2 \sin^2(wt - \varphi_1)\,dt$$

$$+\frac{1}{T}\int_0^T (\sqrt{2}\, I_2)^2 \sin^2 (2wt - \varphi_2)\, dt$$

$$+\frac{1}{T}\int_0^T 2\sqrt{2}\, E_0 E_1 \sin(wt - \varphi)\, dt$$

$$+ \cdots$$

By solving the above integrals, we obtain

$$I^2 = I_0^2 + I_1^2 + I_2^2 + \cdots$$

or

$$I = \sqrt{I_0^2 + I_1^2 + I_2^2 + \cdots} \qquad (59)$$

Where I_0, I_1, I_2, are the effective value of fundamental and harmonics of the nonsinusoidal wave.

<u>Thus the effective value of a nonsinusoidal current is the square root of the sum of the squares of the effective values of all component currents.</u>

The effective cakue of a nonsinusoidal wave of voltage is similarly

$$E = \sqrt{E_0^2 + E_1^2 + E_2^2 + \cdots} \qquad (57)$$

Where E_0, E_1, E_2 etc. are the effective vadues of the fundamental and harmonic respectively.

4. Ohm's Law in a Circuit with Nonsinusoidal impressed Voltage

Consider a circuit containing resistance, inductance and capacitance in series, a nonsinusoidal voltage applied to this circuite. It is always possible to write that

$$e = Ri + L\frac{di}{dt} + \frac{1}{c}\int idt \qquad (58)$$

since $e = E_0 + \sqrt{2}\, E_1 \cos wt + \sqrt{2}\, E_2 \cos 2wt + \cdots$

It can be also write same relations for any components, that is

$$e_n = R\, i_n + L\frac{d\, i_n}{dt} + \frac{1}{c}\int i_n dt$$

Adding the equations for all harmonics we get

$$e = R\sum i_n + L\frac{d}{dt}\sum i_n + \frac{1}{c}\int \sum i_n dt \qquad (59)$$

Eq. 59 is the solving of Eq. 58. From 59 it is possible to write the following result: When a nonsinusoidal current applied to a circuit having R, L, C, the current is determined by superposition all component currents due to components of voltages.

If EMF is

$$e = E_0 + \sqrt{2}E_1 \cos wt + \sqrt{2}\, E_2 \cos 2wt + \cdots$$

the current will be

$$i = I_0 + \sqrt{2}\, I_1 (\cos wt - \varphi_1) + \sqrt{2}\, I_2 \cos (2wt - \varphi_2) + \cdots$$

where

$$I_0 = \frac{E_0}{R} \qquad I_n = \frac{E_n}{\sqrt{R^2 + (Lnw - \frac{1}{C\, nw})^2}} \qquad (60)$$

$$tg\, \varphi_n = \frac{Lnw - \frac{1}{Cnw}}{R}$$

(Note that, if the circuite contain C, $I_0 = 0$)

5. <u>Nonsinusoidal poly-phase systems.</u>

Let's consider an unbalanced nonsinusoidal 9-phase system of voltages with period T. If the voltages lag each other by $\frac{T}{g}$ respectively, that is

$$e_1 = f(t)\ , \quad e_2 = f(t - \frac{T}{g})\ , \quad e_3 = f(t - \frac{2T}{g})\ , \qquad (61)$$

They form a nonsinusoidal g-phase balanced system. Now it can to resolve all the voltages into the Fourier Series.

$$e_1 = A_1 \cos (wt - \varphi_1) + A_2 \cos (2wt - \varphi_2) + \cdots + A_n \cos(nwt - \varphi_n) +$$

$$e_2 = A_1 \cos\left[(wt - \frac{T}{9}) - \varphi\right] + A_2 \cos\left[2w(t - \frac{T}{9}) - \varphi_2\right] + \cdots$$

$$+ A_n \cos\left[nw(t - \frac{T}{9}) - \varphi_n\right] + \cdots$$

$$e_g = A_1 \cos\left[w(t - \frac{(g-1)T}{g}) - \varphi_1\right] + A_2 \cos\left[2w\ (t - \frac{(g-1)T}{g}) - \varphi_2\right] + \cdots$$

$$+ A_n \cos\left[nw\ (t - \frac{(g-1)\ T}{g}) - \varphi_n\right] + \cdots \qquad (62)$$

r

$${}_1 = A_1 \cos(wt - \varphi_1) + A_2 \cos(2wt - \varphi_2) + \cdots + A_n \cos(nwt - \varphi_n) + \cdots$$

$${}_2 = A_1 \cos\left[wt - \frac{2\pi}{g} - \varphi_1\right] + A_2 \cos\left[2(wt - \frac{2\pi}{g})\right] + \cdots$$

$$+ A_n \cos\left[n\,(wt - \frac{2\pi}{g}) - \varphi_n\right] + \cdots$$

$${}_g = A_1 \cos\left[(wt - \frac{(g-1)2\pi}{g}) - \varphi_1\right] + A_2 \cos\left[2\,(wt - \frac{(g-1)2\pi}{g} - \varphi_2\right] + \cdots$$

$$+ A_n \cos\left[n\,(wt - \frac{(g-1)2\pi}{g}) - \varphi_n\right] + \cdots \qquad (63)$$

If we consider only n-th terms of all phases, these also form a g-phase balanced system. Similarly, the terms of fundamentals of all phases also form a g-phase positive sequence system of the first order with the angular velocity of w, thus, though the n-th harmonics will have the phase angle with $-n\frac{2\pi}{g}$ respectively, these form a positive sequence system of the n-th order with the angular velocity of nw.

If the terms of fundamentals have the phase angle with $+\frac{2\pi}{g}$ and form a negative sequence system of the first order, then n-th harmonics also have the phase angle $n\frac{2\pi}{g}$ and form a negative-sequence system of the n-th order.

In general if the fundamentals form a positive-sequence system of the m-th order (or a negative-sequence system of the (g-m)th order), n-th harmonics form a positive-sequence system of the (n.m)-th order.

Now, consider the following 5-phase nonsinusoidal balanced system.

$$e_1 = A_1 \cos(wt - \varphi_1) + A_2 \cos(2wt - \varphi_2) + \cdots + A_n \cos(nwt - \varphi_n) + \cdots$$

$$e_2 = A_1 \cos\left[(wt - \frac{2\pi}{5}) - \varphi_1\right] + A_2 \cos\left\{2\,(wt - \frac{2\pi}{5}) - \varphi_2\right] + \cdots$$

$$+ A_n \cos\left[n(wt - \frac{2\pi}{5}) - \varphi_n\right] + \cdots$$

$$e_3 = A_1 \cos\left[(wt - \frac{2.2\pi}{5}) - \varphi_1\right] + A_2 \cos\left[2(wt - \frac{2.2\pi}{5}) - \varphi_2\right] + \cdots$$

$$+ A_n \cos\left[n(wt - \frac{2.2\pi}{5}) - \varphi_n\right] + \cdots$$

$$e_4 = A_1 \cos\left[(wt - \frac{3.2\pi}{5}) - \varphi_1\right] + A_2 \cos\left[2(wt - \frac{3.2\pi}{5}) - \varphi_2\right] + \cdots$$
$$+ A_n \cos\left[n(wt - \frac{3.2\pi}{5}) - \varphi_n\right] + \cdots$$

$$e_5 = A_1 \cos\left[(wt - \frac{4.2\pi}{5}) - \varphi_1\right] + A_2 \cos\left[2(wt - \frac{4.2\pi}{5}) - \varphi_2\right] + \cdots$$
$$+ A_n \cos\left[n\,(wt - \frac{4.2}{5}) - \varphi_n\right] + \cdots \qquad (64)$$

Fundamentals can be shown by the 5-phase balanced phasor system of the first order (Fig. 21(a))

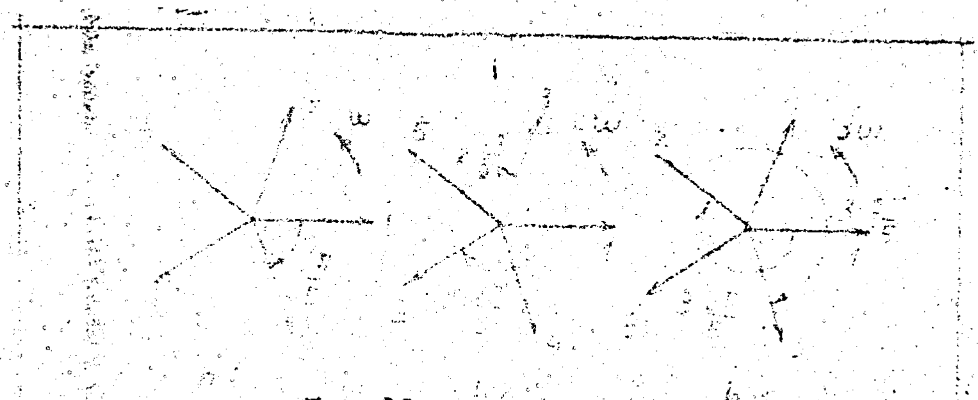


FIG. 22

Similarly, the terms of the second harmonics can also be shown by the positive-sequence system of the second step (Fig. 22(b)) and etc.

But in a q-phase system there is only q different systems, as the 0, 1, 2, ..., $(q-1)$th order.

The system of the 9-th order is equivalent to the zero-sequence system, and the system of the $(q+1)$th order is the system of the first order.

In general, if m is any integer and higher than q, then thus system is equal to the system of $(m-q)$ consequently, it may be expressed that:

1°- The terms of fundamentals in the nonsinusoidal q-phase balanced system form a q-phase system of the first order.

2°- $n = k\,q$-th harmonics form a zero sequence system; where n is any integer number.

3°- The harmonics having the steps $n = k\varphi - 1 = (k-1)\varphi + (\varphi - 1)$ form a φ-phase positive-sequence system $(\varphi-1)$th order, that is equal to φ-phase negative-sequence system.

As a sample, let's take three-phase condition. The following rule may be prepared by assuming the fundamental terms form a positive-sequence system.

The steps of harmonics				
1	4	7	10.....	The steps of the system which is formed positive-sequence system of the first order.
2	5	8	11.....	A pos.-seq. system of the second order or a neg.-seq. system of the first order
3	6	9	12.....	Zero-sequence system.

In case of four-phase $(\varphi = 4)$ system, by assuming also the fundamentals form a positive-sequence system of the first order, the following rule may be obtained.

The steps of harmonics				The steps of the system which is formed.
1	5	9	13.....	A pos.-seq. system of the first order.
2	6	10	15.....	A pos.-seq. system of the second order.
3	7	11	15.....	A pos.-seq. system of the third order or a neg.-seq. system of the first order.
4	8	12	16.....	Zero-sequence system.

6. <u>The Results</u>. Now, the following results may be written for the three-phase system.

1°. If the emf's of a three-phase system are produced into the delta connected winding group, then 3th, 6th, 9th ... etc. harmonics of the emf's add in algebraic and a circulating current exist into the delta connected winding group. The argument of this current depends to the zer-sequence impedance of the winding since the impedance especially in the rotating machinery is very small, high circulating currents exist due to this emf's.

2°. If the emf's is produced into the star connected winding, in that case 3th, 6th,.... harmonics doesn't exist into the line voltage.

7. Nonsinusodial unbalanced poly-phase systems

In a three-phase unbalanced system each harmonic having any steps forms a positive -, a negative- and zero sequence systems. Hence in a three-phase system the (3-6....etc) the harmonics are produced 3th, 6th.... harmonics into the line voltages.

GENERAL INSTRUCTIONS ABOUT THE ROTATING FIELD

1. The construction of the magnetic circuits of rotating machinery. The field distribution in space.

A rotating machine consists of two armatures with the same axis and separated by an air gap. Either both or one of them have coils in slots. This winding distributed in space so that they form the magnetic poles regularly.

The length of the arch τ between two poles axes is called pole pitch. If the number of pole pairs of armature is equal to p, the pole pitch is

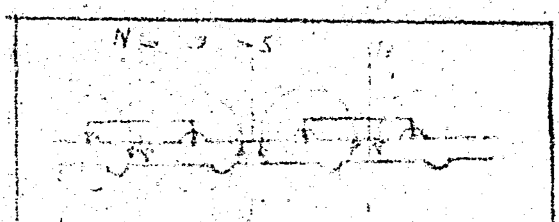


FIG 23

$$\tau = \frac{2\pi}{2p} = \frac{\pi}{p} \qquad (65)$$

The second armature may be a smooth phase or salient pole. This armature may have a distributed windings in slots or a concentrated winding in the salient pole.

The maximum value of the field is on the pole axes. Under the opposite pole axes it has opposite value and between two pole axes is zero. The field in space around the air-gap changes periodically; its period is equal to double of pole pitch; that is

$$\Theta = \frac{2\pi}{p} = 2\tau \qquad (66)$$

Such a periodic wave contains any number of space harmonics in addition to the fundamental of amplitude B, and let their amplitude be B' , B", ... etc. If the coordinate axis is taken through the center of the pole, $f(-p\theta) = f(p\theta)$ and only cosine terms are present. Further, since $f(p\theta+\pi) = -f(p\theta)$, only odd harmonics are present. It may be written that

$$\mathcal{B}(x) = B\cos\frac{2\pi}{2\tau}\theta + B'\cos 3\frac{2\pi}{2\tau}\theta + \cdots$$

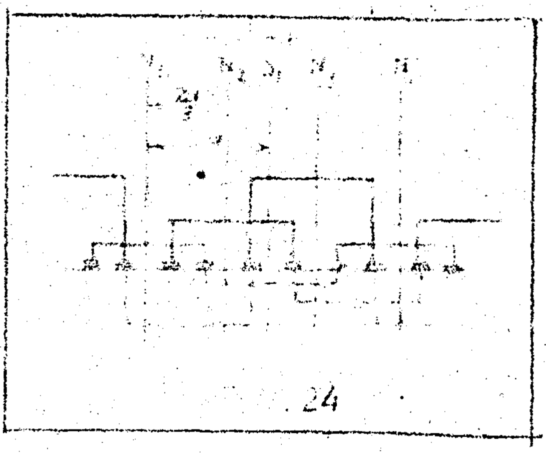
or

$$\mathcal{B}(x) = B\cos\frac{\pi}{\tau}\theta + B'\cos 3\frac{\pi}{\tau}\theta + \cdots \qquad (67)$$

2. Poly-phase armatures

A q-phase armature with 2p poles may be obtain by distributing q phase winding, which are separated each other by $\frac{2a}{q}$ and every one has full pole pitch.

Let's consider a three-phase armature: The pole axes of the first phase are N_1, S_1. The pole axes of the second phase are displaced by $\frac{2\tau}{3} = \frac{2\pi}{3p}$ from the pole axes of the first phase. And the pole axis of the third phase displaced $\frac{4\tau}{3} = \frac{4\pi}{3p}$ from the first phase. If the coordinate axis is taken through the axis of the first phase and the clockwise direction as a positive direction, then the field produced by the windings are,



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for the first phase, $\mathcal{B}_1(x) = B_1 \cos p\theta + B_1' \cos 3p\theta + \cdots$

for the second phase, $\mathcal{B}_2(x) = B_2 \cos (p\theta - \frac{2\pi}{3}) + B_2' \cos(3p\theta - 3\frac{2\pi}{3}) + \cdots$

for the third phase, $\mathcal{B}_3(x) = B_3 \cos (p\theta - \frac{4\pi}{3}) + B_3' \cos(3p\theta - 3\frac{4\pi}{3}) + \cdots$

or in general for k-th phase of a q-phase armature

$$\mathcal{B}_k(x) = \mathcal{B}_k \cos (p\theta - (k-1)\frac{2\pi}{q}) + B_k' \cos(3p\theta - 3(k-1)\frac{2\pi}{q}) + \cdots \quad (68)$$

3. Sinusoidal rotating fields. (a) Three-phase armature

Consider a three phase armature and take only fundamentals and neglect the harmonics.

If the second winding is displaced in the positive direction by $\frac{2\pi}{3}$ from the first winding, third winding is replaced also in the same direction by $\frac{4\pi}{3}$ from the first one, by taking the axis of the first

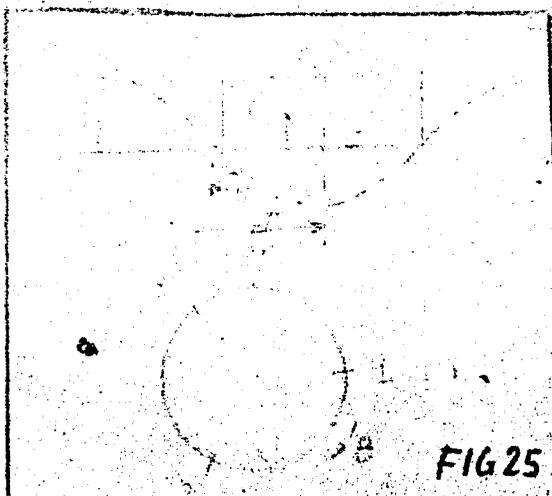


FIG 25

phase as ordinate, for given time three fields may be expressed as follows.

$$\mathcal{B}_1(x) = B_1 \cos p\theta$$

$$\mathcal{B}_2(x) = B_2 \cos (p\theta - \frac{2\pi}{3})$$

$$\mathcal{B}_3(x) = B_3 \cos (p\theta - \frac{4\pi}{3}) \tag{69}$$

These form a positive sequence system of the first order in the complex plane.

If the second winding is displaced by $\frac{2\pi}{3p}$ in the negative direction from the first winding's third winding also is displaced by $\frac{4\pi}{3p}$ in the same direction from the first one, similarly the three field are:

$$\mathcal{B}_1(x) = B_1 \cos p\theta$$
$$\mathcal{B}_2(x) = B_2 \cos (p\theta + \frac{2\pi}{3})$$
$$\mathcal{B}_3(x) = B_3 \cos (p\theta + \frac{4\pi}{3}) \tag{70}$$

Thus, these three fields form a negative-sequence system in the complex plan.

In general it may be written as:

$$\mathcal{B}_1(x) = B_1 \cos p\theta$$

$$\mathcal{B}_2(x) = B_2 \cos (p\theta - m\frac{2\pi}{3})$$

$$\mathcal{B}_3(x) = B_3 \cos (p\theta - 2m\frac{2\pi}{3}) \tag{71}$$

where, if the phase rotation is in the positive direction $m = 1$, if the phase rotation is in the negative direction $m = 2$.

Consider now three windings are excited by the sinusoidal three-phase current systems.

for first phase $\qquad i_1 = I_M \cos (wt - \varphi)$

" second " $\qquad i_2 = I_M \cos (wt - \varphi - m'\frac{2\pi}{3})$

" third " $\qquad i_3 = I_M \cos (wt - \varphi - 2m'\frac{2\pi}{3}) \qquad (72)$

If the currents form a positive sequence system, $m' = 1$ and if the currents form anegative sequence system, $m' = 2$. At last consider that the field produced by each winding is proportional to its currents. Since all the windings are identical, using same constand, it may be written as:

$$B_1 = A\, i_1 \;, \qquad B_2 = A\, i_2 \;, \qquad B_3 = A\, i_3 \qquad (73)$$

where constant A depends the number of windings, the length of the magnetic circuit and the kind of distribution of the windings, ... etc, then we shall determine its actual value.

Hence,

$$\mathcal{B}_1(x,t) = A\, I_M \cos p\,\theta \cos (wt - \varphi)$$

$$= \frac{A\, I_M}{2}\left[\cos (p\theta + wt - \varphi) + \cos (p\theta - wt + \varphi)\right]$$

$$\mathcal{B}_2(x,t) = A\, I_M \cos \left(p\theta - m\frac{2}{3}\right) \cos \left(wt - m'\frac{2}{3} - \varphi\right)$$

$$= \frac{A\, I_M}{2}\left\{\cos\left[p\theta + wt - \frac{2\pi}{3}(m + m') - \varphi\right]\right.$$

$$\left. + \cos\left[p\theta - wt - \frac{2\pi}{3}(m - m') + \varphi\right]\right\},$$

$$\mathcal{B}_3(x,t) = A\, I_M \cos \left(p\theta - 2m\frac{2\pi}{3}\right) \cos \left(wt - 2m'\frac{2\pi}{3} - \varphi\right)$$

$$= \frac{A\, I_M}{2}\left\{\cos\left[p\theta + wt - \frac{4\pi}{3}(m \;\; m') - \varphi\right]\right.$$

$$\left. + \cos\left[p\theta - wt - \frac{4\pi}{3}(m - m') + \varphi\right]\right\} \qquad (74)$$

The field at any point in space is equal to the sum of these three field, that is

$$\mathcal{B}_1 + \mathcal{B}_2 + \mathcal{B}_3$$

1°. if $m = m'$, that is, if the rotation of the windings in the air gap is the same the phase rotation of the currents in time

$$m + m' = 2$$

or

$$m + m' = 4$$

and

$$\cos(p\theta + wt - \varphi) + \cos\left[p\theta + wt - \varphi - (m+m')\frac{2\pi}{3}\right]$$

$$+ \cos\left[p\theta + wt - \varphi - (m+m')\frac{4\pi}{3}\right] = 0$$

$$\cos(p\theta - wt + \varphi) + \cos\left[p\theta - wt + \varphi - (m - m')\frac{2\pi}{3}\right]$$

$$+ \cos\left[p\theta - wt + \varphi - (m - m')\frac{4\pi}{3}\right]$$

$$= 3\cos(p\theta - wt + \varphi)$$

The resultant field therefore is:

$$\mathcal{B}_{(x,t)} = \frac{3\,A\,I_M}{2}\cos\left[p\theta - (wt - \varphi)\right] \tag{75}$$

This equation indicates a rotating phasor, it's amplitude is constant and rotates in the positive direction with angular velocity of $\frac{w}{p}$

2^{o}. If $m \neq m'$, $m = 1$ and $m' = 2$ or $m = 2$ and $m' = 1$ for both case

$$m + m' = 3 \qquad m - m' = 1$$

Hence

$$\cos(p\theta - wt - \varphi) + \cos\left[p\theta - wt + \varphi - (m - m')\frac{2\pi}{3}\right]$$

$$+ \cos\left[p\theta - wt + \varphi - (m - m')\frac{4\pi}{3}\right] = 0,$$

$$\cos(p\theta - wt - \varphi) + \cos\left[p\theta + wt - \varphi - (m + m')\frac{2\pi}{3}\right]$$

$$+ \cos\left[p\theta + wt - \varphi - (m + m')\frac{4\pi}{3}\right]$$

$$= 3\cos(p\theta + wt - \varphi)$$

and

$$\mathcal{B}_{(x,t)} = \frac{3\,A\,I_M}{2}\cos(p\theta + wt - \varphi) \tag{76}$$

This is the expression of a rotating field, which has amplitude $\frac{3A\ I_M}{2}$ and rotates in the negative direction with angular velocity of $-\frac{w}{p}$.

b. <u>The poly-phase armature</u>

In general, consider a poly-phase armature whichhas a q-phase symmetrical winding excited by a q phase balanced current system. Let m is the number of equal arch, $\frac{1}{p} \cdot \frac{2\pi}{q} = \frac{2\tau}{q}$, between the k-th and (k+1)-th winding.

The possible number of steps in the positive direction is (9-1), that is

$$m = 1,\ 2,\ \text{--------}\ (q-1). \tag{77}$$

On the other hand, let m' is the number of equal arch, $\frac{2\pi}{q}$, between kth - and (k+1)th phase currents on the complex plane. As we know, the possible number of steps is equal, that is

$$m' = 0,\ 1,\ 2\ldots\ldots (q-1) \tag{78}$$

Now, for any time the corresponding equations for the fields are:

$$\mathcal{B}_1(x) = B_1 \cos p\theta$$
$$\mathcal{B}_2(x) = B_1 \cos \left(p\theta - m\frac{2\pi}{9}\right)$$
$$\mathcal{B}_3(x) = B_3 \cos \left(p\theta - 2m\frac{2\pi}{q}\right)$$
$$\text{-------------------------}$$
$$\mathcal{B}_q(x) = B_q \cos \left[p\theta - (q-1)\, m\frac{2\pi}{q}\right] \tag{79}$$

and the currents through the phases are:

$$i_1 = I_M \cos (wt - \varphi)$$
$$i_2 = I_M \cos \left(wt - m'\frac{2\pi}{q} - \varphi\right)$$
$$i_3 = I_M \cos \left(wt - 2m'\frac{2\pi}{q} - \varphi\right)$$
$$\text{-------------------------}$$
$$i_q = I_M \cos \left[wt - (q-1)\, m'\frac{2\pi}{q} - \varphi\right] \tag{80}$$

Hence, the resulting field is:

$$= \mathcal{B}_1 + \mathcal{B}_2 + \cdots + \mathcal{B}_q$$

$$= \frac{A\ I_M}{2}\left[\cos\ (p\theta + wt - \varphi) + \cos\ (p\theta - wt + \varphi)\right]$$

$$+ \frac{A\ I_M}{2}\left\{\cos\left[p\theta + wt - \varphi - (m+m')\frac{2\pi}{q}\right] + \cos\left[p\theta - wt + \varphi - (m-m')\frac{2\pi}{q}\right]\right\}$$

$$+ \frac{A\ I_M}{2}\left\{\cos\left[p\theta + wt - \varphi - 2(m+m')\frac{2\pi}{q}\right] + \cos\left[p\theta - wt + \varphi - 2(m-m')\frac{2\pi}{q}\right]\right\}$$

$$+ - - - - - - - - - - - - -$$

$$+ \frac{A\ I_M}{2}\left\{\cos\left[p\theta + wt - \varphi - (q-1)(m+m')\frac{2\pi}{q}\right] + \cos\left[p\theta - wt + \varphi - (q-1)(m-m')\frac{2\pi}{q}\right]\right\} \quad (81)$$

1°. Where m and m' are integers between 1 and $(q-1)$, 0 and $(q-1)$ respectively. Being a any integer number (may be $a = 0$, $a = 1$), if $m - m' = aq$, $m + m' = 2m + aq$ is also any integer, since m is not zero.

Thus,

$$\sum_{n=0}^{n=q-1} \cos\left[p\theta + wt - \varphi - n(m+m')\frac{2\pi}{q}\right] = 0 \quad (82)$$

Because this algebraic series shows a closed regular polygon hence the sume of the projections of its sides on any axis is zero.

$$\sum_{n=0}^{n=q-1} \cos\left[p\theta - wt - \varphi - n(m-m')\frac{2\pi}{q}\right] = q\cos\ (p\theta - wt + \varphi)$$

from which

$$\mathcal{B}_{(x,t)} = \frac{q}{2} A I_M\ \cos\left[p\theta - (wt - \varphi)\right] \quad (83)$$

Therefore, if $m - m' = a\mathcal{g}$ and in private for $(a = 1)$, that is, if rotation of the windings on the armature and rotation of the phase currents are same, the resultant field is a rotating field which has constant magnitude of $\frac{\mathcal{g}}{2} AI_M$ and rotates in the positive rotation with angular velocity of $\frac{w}{p}$.

2°. If $m + m' = a\mathcal{g}$, $m - m' = a\mathcal{g} - 2m'$ is any integer differs from zero, then

$$\sum_{n=0}^{n=\mathcal{g}-1} \cos\left[p\theta - wt + \varphi - n(m - m')\frac{2\pi}{\mathcal{g}}\right] = 0$$

on the other hand

$$\sum_{n=0}^{n=\mathcal{g}-1} \cos\left(p\theta + wt - \varphi - n(m + m')\frac{2\pi}{\mathcal{g}}\right) = \mathcal{g}\cos(p\theta + wt - \varphi)$$

from which

$$\mathcal{B}(x,t) = \frac{\mathcal{g}}{2} AI_M \cos\left[p\theta + (wt - \varphi)\right] \qquad (84)$$

Therefore, if $m + m' = a\mathcal{g}$ an in private for $(a = 0)$, that is, if the steps of the windings on armature and the phase currents are equal, but their rotations are opposite each other, the total field gives a rotating field which has constant magnitude $\frac{\mathcal{g}}{2} AI_M$ and rotates as in the negative direction with angular velocity of $-\frac{w}{p}$.

3°. At the same time, if $m + m' \neq a\mathcal{g}$ and $m - m' \neq a\mathcal{g}$, then the two sum of the cosine terms are zero, hence the resulting field $(\mathcal{B} = 0)$ is zero.

4. Poly-phase symmetrical armature with unbalanced current system

As we know, a $\mathcal{g}$-phase unbalanced current system may be resolved into poly-phase $\mathcal{g}$ systems of the 0, 1, 2, ...($\mathcal{g}$-1)th order. When such an unbalanced current system is passed through the $\mathcal{g}$-phase winding of the symmetrical $\mathcal{g}$-phase armature of the first order $(m = 1)$, the current system of the first order $(m' = 1)$ produces a rotating field in the positive direction, because in that case $m - m' = 0$. The current system of the $m' = q$-1th order will produce a rotating field in the negative direction, because $m + m' = q$.

At last, each of the current systems of the $m' = 0$, 2, 3, -------(q-2)th order produces fields; but their sum always zero, because in that cases $m + m' \neq aq$ and $m - m' \neq aq$.

Now, consider a three-phase balanced armature windings in which unbalanced three-phase currents passed.

The positive-sequence components of the currents, that is the system of the first order, produce a rotating field in the positive direction.

The negative-sequence components, that is the system of the second order, produce a rotating field, which rotates in the negative direction.

The zero sequence components produce fields, their sum is always zero.

The circular and Elliptical rotating field

As we know, the field produced by a balanced poly-phase circuit is:

$$\mathcal{B}(x,t) = B_M \cos\left[\frac{x}{\tau} \mp (wt - \varphi)\right] \qquad (85)$$

This resultant field may be shown by a vector of constant magnitude B_M and argument $\mp(wt-\varphi)$. The tip of this phasor describes a circle.

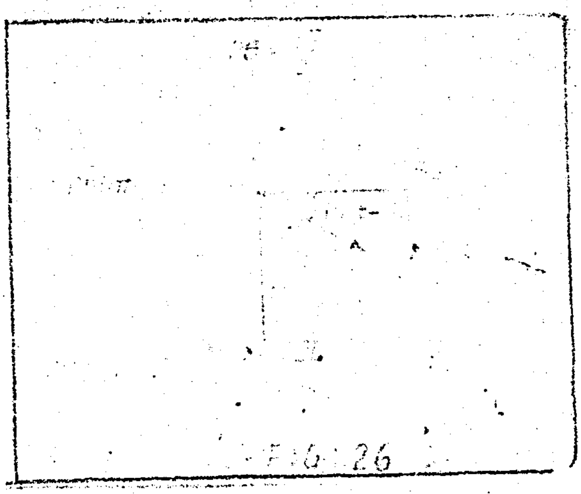


Fig. 26

Now, consider a field produced by an unbalanced current system; as we studied before, it can be divided into two components one of which may be indicated a rotating phasor in the positive direction,

$$\mathcal{B}_d(x,t) = B_{Md} \cos \quad p\theta - (wt - \varphi_d)$$

and its coordinates

$$x_d = B_{Md} \cos (wt - \varphi_d),$$
$$y_d = B_{Md} \sin (wt - \varphi_d) \qquad (86)$$

The other may be indicated another rotating phasor in the negative direction,

$$\mathcal{B}_i(x,t) = B_{Mi} \left[p\theta - (wt - \varphi_i) \right]$$

and its coordinates

$$x_i = B_{Mi} \cos (wt - \varphi_i)$$

$$y_i = B_{Mi} \cos (wt - \varphi_i) \quad (86)$$

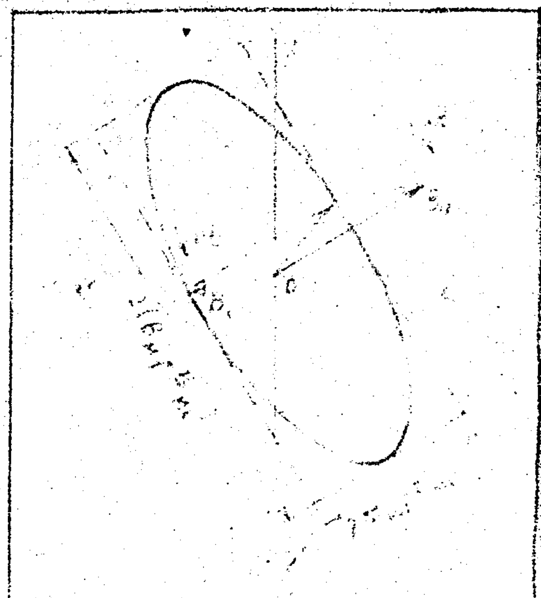


FIG. 27

Their resultant is an elliptically rotating field whose magnitude changes from point to point as the rotation continuous. This resultant field is

$$\mathcal{B} = \mathcal{B}_d + \mathcal{B}_i \quad (87)$$

its coordinates:

$$X = x_d + x_i = (B_{Md} \cos \varphi_d + B_{Mi} \cos \varphi_i) \cos wt + (B_{Md} B_{Mi} + B_{Mi} \sin \varphi_i) \sin wt,$$

$$Y = y_d + y_i = -(B_{Md} \cos \varphi_d - B_{Mi} \cos \varphi_i) \sin wt + (B_{Md} \sin \varphi_d - B_{Mi} \sin \varphi_i) \cos wt$$

or taking

$$\tan \beta_1 = \frac{B_{nd} \sin \varphi_d + B_{Mi} \sin \varphi_i}{B_{Md} \cos \varphi_d - B_{Mi} \cos \varphi_i}$$

$$\tan \beta_2 = \frac{B_{Mi} \cos \varphi_i - B_{Mi} \cos \varphi_i}{B_{Md} \sin \varphi_i - B_{Mi} \sin \varphi_i}$$

$$P = \sqrt{B_{Md}^2 + B_{Mi}^2 + 2 B_{Md} B_{Mi} \cos (\varphi_d - \varphi_i)}$$

$$Q = \sqrt{B_{Md}^2 + B_{Mi}^2 - 2 B_{Md} B_{Mi} \cos (\varphi_d - \varphi_i)} \quad (88)$$

its coordinates:

$$X = P \cos (wt - \beta_1)$$

$$Y = Q \cos (wt - \beta_2)$$

It may be seen that the tip of the phasor having coordinates X and Y desgribes an ellips. Infact Eqs. (88) can be written as follows:

$$\frac{X}{P} = \cos wt \cos \beta_1 \quad \sin wt \sin \beta_1,$$

$$\frac{Y}{Q} = \cos wt \cos \beta_2 + \sin wt \sin \beta_2 \qquad (89)$$

Multiplying first equation by $\sin \beta_2$ and second equation by $\sin \beta_1$ and than submiting one from the other, we obtain

$$\sin wt \sin (\beta_1 - \beta_2) = \frac{X}{P} \cos \beta_2 - \frac{Y}{Q} \cos \beta_1 \qquad (90)$$

Similarly, multiplying first equation by $\sin \beta_2$ and second by $\sin \beta_1$, and then subtracting one from the other, we also obtain.

$$-\cos w \sin (\beta_1 - \beta_2) = \frac{X}{P} \sin \beta_2 - \frac{Y}{Q} \sin \beta_1 \qquad (91)$$

Let's square Eq. 90 and 91 than add them.

$$\sin^2 (\beta_1 - \beta_2) = \frac{X^2}{P^2} + \frac{Y^2}{Q^2} - \frac{2XY}{PQ} \cos (\beta_1 - \beta_2) \qquad (92)$$

This is the equation of an ellips.

The axes of this ellips are $2(B_{Md} + B_{Mi})$ and $2(B_{Md} - B_{Mi})$, because the magnitude of vector B changes between $B_{Md} + B_{Mi}$, $B_{Md} - B_{Mi}$.

Single-phase Armature

A single-phase armature winding, carrying the current I, can be consider as a three-phase winding carrying the currents through the first phase $I_1 = I$, second phase $I_2 = -I$ and third phase $I_3 = 0$.

In the preceding chapter we saw that the positive and negative sequences of such a current system are:

$$I_d = \frac{I}{3}(1-a) = \frac{I}{\sqrt{3}} \angle -\frac{\pi}{6}$$

$$I_i = \frac{I}{3}(1-a^2) = \frac{I}{\sqrt{3}} \angle \frac{\pi}{6} \qquad (93)$$

The positive and negative sequences have same effective values.

Therefore, the single-phase armature produces two rotating fields, one of them rotates in the positive-, the other in the negative direction; the magnitudes of two fields are same. Let the instantaneous value of the current $i = I_M \cos wt$; the instantaneous values of two components are:

$$i_d = \frac{I_M}{\sqrt{3}} \cos\left(wt - \frac{\pi}{6}\right), \quad i_i = \frac{I_M}{\sqrt{3}} \cos\left(wt + \frac{\pi}{6}\right) \qquad (94)$$

and the expressions of two fields are:

$$\mathcal{B}_d = \frac{3}{2} \frac{AI_M}{\sqrt{3}} \cos\left[p\theta - \left(wt - \frac{\pi}{6}\right)\right]$$

$$\mathcal{B}_i = \frac{3}{2} \frac{AI_M}{\sqrt{3}} \cos\left[p\theta + \left(wt + \frac{\pi}{6}\right)\right] \qquad (95)$$

The ellips, which is drawn by the tip of the resultant field phasor limits to a straight line; because in that case the small axis $2(\mathcal{B}_{Md} - \mathcal{B}_{Mi})$ is equal to zero and the maximum magnitude of the resultant field are equal sum of the magnitudes of positive-. And negative sequences. $2\mathcal{B}_{nd} = \frac{3AI_M}{\sqrt{3}}$

<u>The poly-phase armature windings, which is distributed, sinusoidally in space, with a nonsinusoidal balanced poly-phase current system.</u>

Consider a q-phase armature winding of the m-th order, which is excited by a nonsinusoidal q-phase current system. As we saw before, if

$$m' n - m = aq \qquad (96)$$

then all the harmonics of the n-th order produce rotating fields, which rotate in the positive direction with the angular velocity of $\frac{nw}{p}$.

$$\text{If } m' n + m = aq \qquad (97)$$

in that case, all the harmonics of the n-th order also produce rotating fields, which rotate in the negative direction with the angular velocity of $-\frac{nw}{p}$.

All the other harmonics produce pulsaiting fields, their sum is always zero.

For instance, let's consider a three-phase winding of the first order and exict this winding with a three-phase current system of the first order.

Let's consider a three-phase winding and excite it by a three phase nonsinusoidal current system. In that case

$$n = 1 + 3a \qquad (a = 0, 1, 2, \ldots\ldots)$$

that is $n = 1, 4, 7, 10, 13, \ldots\ldots$

Therefore, these harmonics will produce rotating fields in the positive direction; and

$$n = 3a - 1$$

that is

$$n = 2, 5, 8, 11, \ldots.$$

these harmonics will produce rotating fields in the negative direction. At last, all the harmonics that are multiples of 3 will produce pulsaiting fields, their sum is always zero.

Nonsinusoidal Field Form

In general, let's consider the case, in which the fields produced by the windings, which are not distributed sinusoidally in space. In that case, each of the fields produced by every windings may be expressed as follows.

$$\mathcal{B}_1 = B_1 \cos p\theta + B_1' \cos 3p\theta + B_1'' \cos 5p\theta + \cdots$$

$$\mathcal{B}_2 = B_2 \cos\left(p\theta - \frac{2\pi}{q}\right) + B_2' \cos\left(3p\theta - 3\,\frac{2\pi}{q}\right) + B_2'' \cos\left(5p\theta - 5\,\frac{2\pi}{q}\right) + \cdots$$

$$\mathcal{B}_q = B_q \cos\left[p\theta - (q-1)\,\frac{2}{q}\right] + B_q' \cos\left[3p\theta - 3(p-1)\,\frac{2\pi}{q}\right] + B_q'' \cos\left[5p\theta - 5(q-1)\,\frac{2\pi}{q})\right] + \cdots \qquad (98)$$

a. The currents are sinusoidal

First of all let's assumed that all the windings are excited by the sinusoidal currents.

$$i_1 = I_M \cos(wt - \varphi), \quad i_2 = I_M \cos\left(wt - \frac{2\pi}{q} - \varphi\right),$$

$$i_q = I_M \cos\left[wt - (q-1)\frac{2\pi}{q} - \varphi\right]$$

Substituting the currents in Eqs. 73, for any components space harmonics of the field form, say the k-th, is then of the form

$$\mathcal{B}_1^{(k)} = AI_M \cos(wt - \varphi) \cos kp\theta$$

$$\mathcal{B}_2^{(k)} = AI_M \cos\left(wt - \frac{2\pi}{q} - \varphi\right) \cos\left(kp\theta - k\frac{2\pi}{q}\right)$$

$$\mathcal{B}_q^{(k)} = AI_M \cos\left[wt - (q-1)\frac{2\pi}{q} - \varphi\right] \cos\left[kp\theta - k(q-1)\frac{2\pi}{q}\right] \tag{99}$$

It will be noted that each term product of the form cos x cos y which can be written

$$\cos x \cos y = \frac{1}{2}\left[\cos(x - y) + \cos(x + y)\right]$$

Hence the above expressions are

$$\mathcal{B}_1^{(k)} = \frac{AI_M}{2}\left[\cos(kp\theta + wt - \varphi) + \cos(kp\theta - wt + \varphi)\right],$$

$$\mathcal{B}_2^{(k)} = \frac{AI_M}{2}\left[\cos\left(kp\theta + wt - \varphi - (k+1)\frac{2\pi}{q}\right),\right.$$

$$\left. + \cos\left(kp\theta - wt + \varphi - (k-1)\frac{2\pi}{q}\right)\right],$$

$$\mathcal{B}_q^{(k)} = \frac{AI_M}{2}\left[\cos\left(kp\theta + wt - \varphi - (k+1)(q-1)\frac{2\pi}{q}\right)\right.$$

$$\left. + \cos\left(kp\theta - wt + \varphi - (k+1)(q-1)\frac{2\pi}{q}\right)\right] \tag{100}$$

The resultant field of the k-th harmonics is

$$\mathcal{B}^{(k)} = \mathcal{B}_1^{(k)} \mathcal{B}_2^{(k)} + \text{---------} + \mathcal{B}_q^{(k)} \tag{101}$$

1°. If $k - 1 \neq a9$ and $k + 1 \neq a9$ (where a is any integer) In that case, as we indicated before this harmonics does not give any rotating field.

2°. If $k-1 = aq$ and $k+1 \neq aq$, the resulting field wave therefore is

$$\mathcal{B}^{(k)}(x,t) = \frac{\mathcal{G} AI_M}{2} \cos(kp\theta - wt + \varphi) \qquad (102)$$

That is the equation of a rotating field moving by $\frac{w}{kp}$ in the same direction of the main field (fundamental).

3°. If $k+1 = a\mathcal{G}$ and $k - 1 \neq a\mathcal{G}$, then the equation becomes

$$B^{(k)}(x,t) = \frac{\mathcal{G} AI_M}{2} \cos(kp\theta + wt - \varphi) \qquad (103)$$

this shows a rotating field moving by $\frac{w}{kp}$ in the opposite direction of the fundamental for instance, in case of three-phase armature 1, 7, 13, ... th odd harmonics produce rotating fields by the angular velocity of

$$\frac{w}{p}, \frac{w}{7p}, \frac{w}{13p}, \dots\dots\dots; \qquad (104)$$

in the same direction; 5, 11, 17 th harmonics produce rotating fields by the angular velocity of

$$\frac{w}{5p}, \frac{w}{11p}, \frac{w}{17p}, \dots\dots\dots; \qquad (105)$$

In the opposite direction. And all harmonics, that are multiplies of 3, don't give amy rotating fields.

While the fundamental wave produces the useful torque, the harmonics produce parasitic torques. Some of these torques end to drive the motor in the same direction as the fundamental and some in the opposite direction.

In addition to the parasitic torques, the harmonics may produce vibration in the core, causing undesirable noise, and also they may produce additional lossesin the iron and in the copper.

b. <u>Nonsinusoidal Current Form</u>

In general the currents, through the armature windings, may have harmonics. Now considering the harmonics of the field curve in space and the harmonics of the currents, it may be written as follow:

$$B \cos(\ell wt - \varphi) \cos kp\theta \qquad (106)$$

(Where γ is the order of the current harmonics)
By adding these fields in the different phases, we obtain rotating fields, which rotate in the positive and negative direction by velocity of $\frac{\gamma w}{kp}$.

$$\frac{\gamma B}{2} \cos \left[kp\theta \mp (\gamma wt - \varphi) \right] \qquad (107)$$

The terms having $\gamma = k$ give the rotating fields which rotate at the same speed as the fundamental.

THE POLY-PHASE INDUCTION MOTOR

The induction motor was invented by Nikolo Tesla in 1888. It differs from most other types of motors, in that no current is conducted to one of the stator or rotor.

The current in the stator or rotor results from an induced voltage and for that reason it is called an induction motor. Induction motors are sometimes called as asynchronous machines.

Induction motors may be either single-phase or poly-phase machines. There are two principle type of poly-phase induction motors: (a) the squirrel cage, (b) the wound-rotor machine. These two machines are very much the same in either principle of operation. Differences in construction exist in the secondary. In two type machines the stator always has a poly-phase winding. In the squirrel-cage type rotor conductors evenly spaced around the periphery of the rotor having their ends short-circuited by conductors in the form of end rings. The wound rotor is provided with poly-phase windings that are similar to these of the stator. The rotor must be wound for the same number of poles as the stator in which it is to operate.

In practice only three-phase motors are used. In that case the stator has a three-phase winding, rotor in case of squirrel-cage type has a squirrel cage, in case of wound rotor type has a three phase winding. Stator winding may be connected in star or delta.

In Fig. 28 the connection diagram of three-phase induction motor for both types are shown. The stator winding is connected to the three-phase line. The main purpose of this winding is to produce a rotating field.

If P is the number of stator pole pairs and f_1 the frequency of the line current, the speed of the rotating field or synchronous speed is given by

$$n_s = \frac{60 f_1}{P}$$

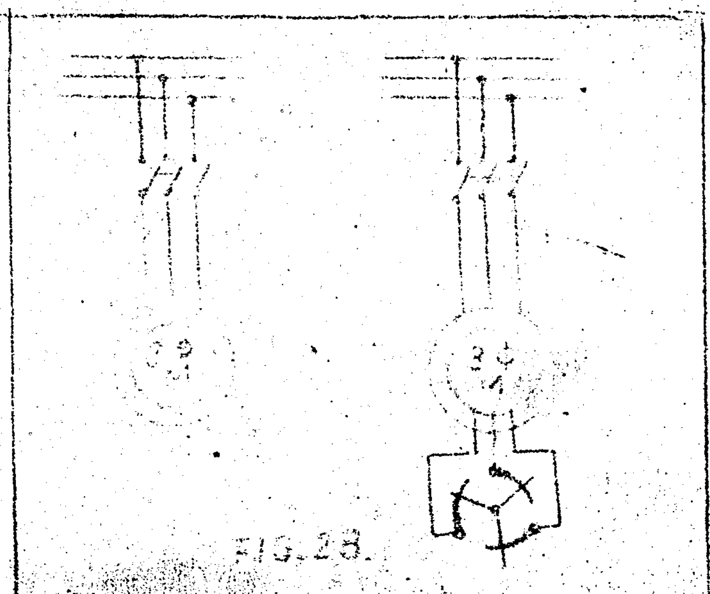


FIG. 28.

This rotating field produces the currents in the rotor and at the same time this field exerts a force on the current carrying conductors therefore a rotating torque on the rotor. The rotor travels and its speed increases, but it can not travel at exactly the same as the rotating field, for under this condition the rotor conductors would be stationary relative to the field and no voltage could be induced in them; consequently the rotor then would carry no current and no force would be exerted upon it thus the speed n of the rotor must be less than that of the rotating field (n_s). The difference $n_s - n$ is called slip; but it is always expressed in percentage of n_s, that is, the slip s of the rotor with respect to the rotating field is defined as

$$s = \frac{n_s - n}{n_s} \tag{109}$$

At no load the slip is almost 1%, at the full load 3÷5%.

The frequency of the rotor currents depends to $(n_s - n)$ and which is given by

$$f_2 = \frac{P}{60}(n_s - n) \tag{110}$$

Introducing Eqs. 108, 109 into Eq. 110, the result is

$$f_2 = sf_1 \tag{111}$$

E.M.F. and Current Relations

The EMF's induced in each phase of the rotor and stator due to rotating field will be given respectively by

$$E_1 = 4.44 \quad f_1 N_1 \quad k_{dp1} \phi_m 10^{-8} \quad \text{volts}$$

$$E_2 = 4.44 \quad f_2 N_2 \quad k_{dp2} \phi_m 10^{-8} \quad \text{"}$$

and

$$\frac{E_2}{E_1} = \frac{f_2}{f_1} \frac{N_2 \ k_{dp2}}{N_1 \ k_{dp1}} = s \frac{N_2 \ k_{dp2}}{N_1 \ k_{dp1}}$$

$$E_2 = s \frac{N_2 \ k_{dp2}}{N_1 \ k_{dp1}} E_1 \tag{112}$$

where

$k_{dpn} = \cancel{x}\ k_{pn} \times k_{dn}$

k_{dn} = distribution factor

k_{pn} = pitch factor

$N_1 N_2$ = turns in series per phase

Under standstill condition, since s = 1,

$$E_2 \rightarrow E_{20} = \frac{N_2\ k_{dp2}}{N_1\ k_{dp1}} E_1 = \text{const.} \tag{113}$$

when the slip is s, the e.m.f of rotor will be

$$E_2 = E_{20} \cdot S \tag{114}$$

If R_2 is the resistance and L_2 the coefficient of self-induction per phase due to the leakage flux of the rotor winding, then for any slip s the rotor current I_2 per phase is

$$I_2 = \frac{S\ E_{20}}{\sqrt{R_1{}^2 + (2\pi f_2 L_2)^2}}$$

If numerator and denominator divided by s,

$$I_2 = \frac{E_{20}}{\sqrt{(\frac{R_2}{S})^2 + (2\pi f_1 L_2)^2}} \tag{115}$$

in the last equation $2\pi f_1 L_2 = X_{20}$ is the leakage reactance of the rotor winding at standstill (S=1), when $f_2=f_1$. Since E_{20} is the emf induced in the rotor at standstill, it follows that the rotor current in a motor operating at a slip s is the same as that in a stationary rotor having a resistance equal to $\frac{R_2}{S}$ instead of R_2. This in order for the flux as wel as the current in the stationary induction moto to be the same in the machine operating at slip as resistance having the magnitude

$$R_e = \frac{R_2}{S} - R_2 = R_2 \frac{(1 - S)}{S} \tag{116}$$

must be connected in series with each phase of the rotor.

The induction motor when is running, therefore behaves as a transformer with a pure ohmic resistance. The load circuit of the induction motor may be considered as a transf er, contains only resistance and no reactance, since the pow of the rotating induction motor is a mechanical power and su power can be represented by a resistance and not by a reacta

It should be noted that the induction motor represents general kind of transformer; it does not only delivers elect power to the secondary (rotor), as ordinary transformer does but delivers mechanical power at the shaft at the same time; it thereby transformers the frequency and at the same time can transform the number of phases.

The Equivalent Circuit and Phasor Diagram

The equivalent circuit of a poly-phase induction motor can be developed on a per phase by considering the machine as a transformer. In Fig. 29 the stator winding per phase is shown as an ideal primary winding in series with a resistance R_1 and a constant leakage reactance X_1 with the impressed phase voltage V_1. It will be assumed that the primary and secondary windings are perfectly coupled and that the secondary resistance R_2 are external to the winding. The circuit has all the characteristics of a transformer except the secondary voltage E_2 and reactance X_2 are variable in nature (function of slip).

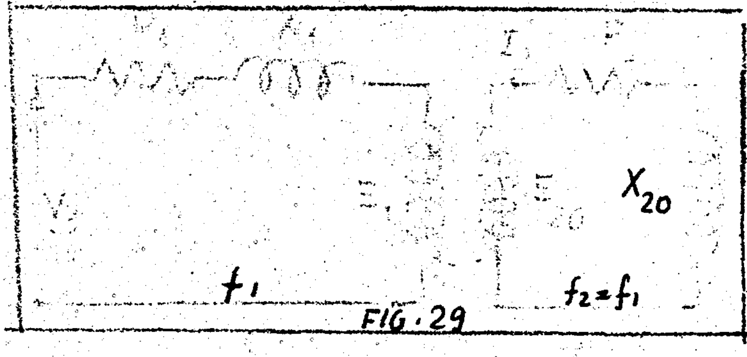


FIG. 29

In general the rotor will move and have a slip, the equivalent circuit can be form with the equivalent rotor circuit, which is shown in Fig. 30(a)

The term $\dfrac{R_2}{S}$ can be separate as the sum of $R_2 + R_2\left(\dfrac{1-S}{S}\right)$

As we saw from Fig. 30(b) the induction motor, when is running therefore behaves as a transformer loaded with a pure ohmic resistance

$$R_2 \frac{(1 - S)}{S}$$

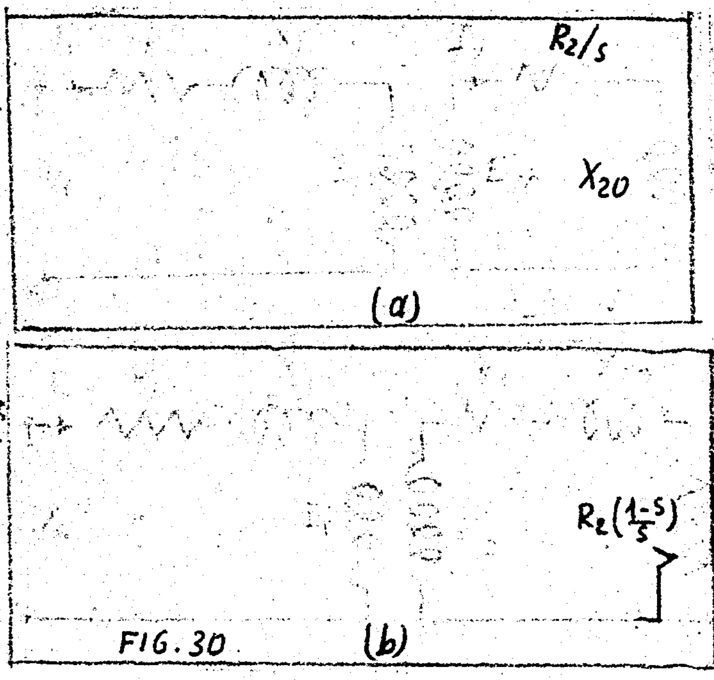
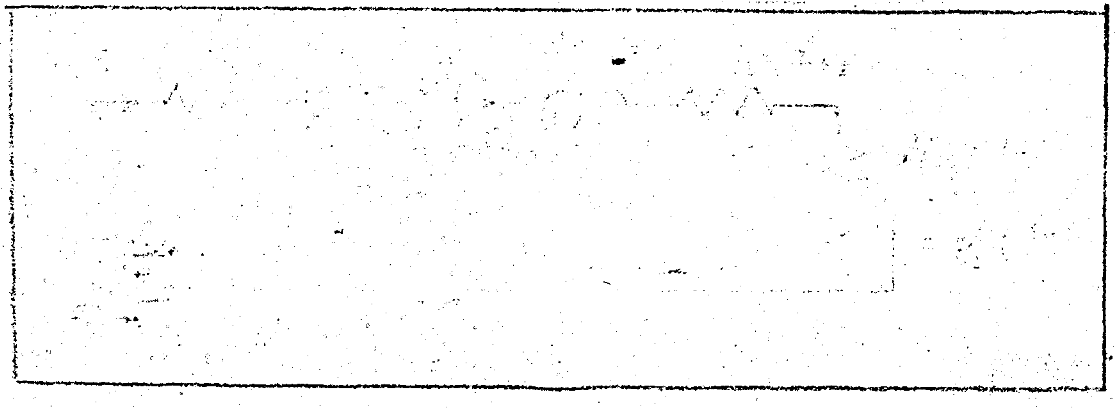


FIG. 30

As in the equivalent circuit of the transformer, all secondary values are referred to the stator. The reduction for the voltage is

$$a = \frac{m_1}{m_2} \quad \frac{N_1\, k_{dp1}}{N_2\, k_{dp2}} \qquad (117)$$

The rotor current must be reduced by the ratio of 1/a. Increasing rotor turns a ratio of a and the reduсing the current by 1/a will provide the same rotor M.M.F. The impedance required to limit the current to this new value must be such that the resistance is $a^2 \dfrac{R_2}{S}$ and the reactance a^2X_2. This equivalent circuit is shown in Fig.31.



The resistance $R_2' = a^2R_2$ is the rotor resistance referred to the primary circuit. Likewise $X_2' = a^2X_2$ and $I_2' = I_2/a$. The three circuits of Fig. 30 and Fig. 31 are equivalent as viewed from the primary side. Now, from knowledge developed for the transformer, the phasor diagram for fig 32 can be constructed as shown in Fig. 31.

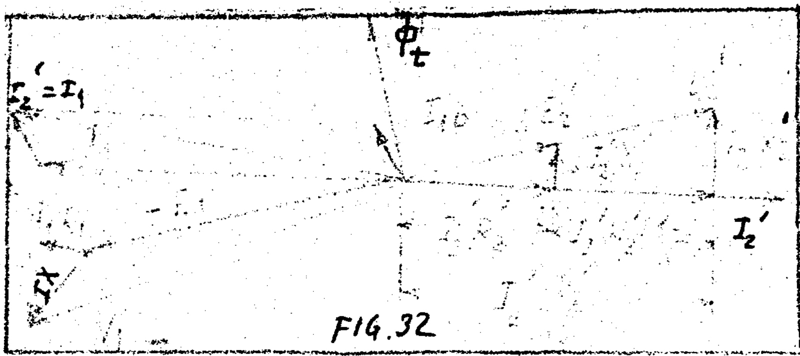


FIG. 32

Now, for any slip the losses, output, effi- ciency, torque, power factor, and primary can be calculate.

Power received by the rotor =

$m_2 I_2 E_2 \cos\theta_2 = m_2 I_2 (I_2 \frac{R_2}{S})$ watts.

Rotor losses $= m_2 I_2^2 R_2 = m_2 I_2'^2 R_2'$

The internal developed power $= m_2 I_2^2 R_2 \left(\frac{1 - S}{S}\right) = m_2 I_2'^2 R_2' \left(\frac{1 - S}{S}\right)$ watts.

$$\frac{\text{Rotor resistance loss}}{\text{Power received by rotor}} = \frac{m_2 I_2^2 R_2}{m_2 I_2^2 \left(\frac{R_2}{S}\right)} = \frac{m_2 I_2'^2 R_2'}{m_2 I_2'^2 \left(\frac{R_2'}{S}\right)} = S \qquad (118)$$

TORQUE:

The torque developed by the motor can be determined from the above relationships. Assume that the motor is operating at a slip s, its speed is N rpm, and developing a torque T foot-pounds. By definition, the developed horse power output is

$$HP = \frac{2\pi N T}{33\,000} = \frac{NT}{5250}$$

$$T = \frac{HP \times 33\,000}{2\pi N} = 5250 \frac{HP}{N} \quad \text{lb-ft} \qquad (119)$$

but the developed horse power is

$$HP = \frac{I_2^{\,2} R_2 \left(\frac{1-S}{S}\right)}{746} \quad \text{per phase}$$

$$T = \frac{I_2^{\,2} R_2 \left(\frac{1-S}{S}\right)}{746} \times \frac{33\ 000}{2\pi N} \quad \text{lb-ft} \qquad (120)$$

but $N = N_s\ (S - 1)$

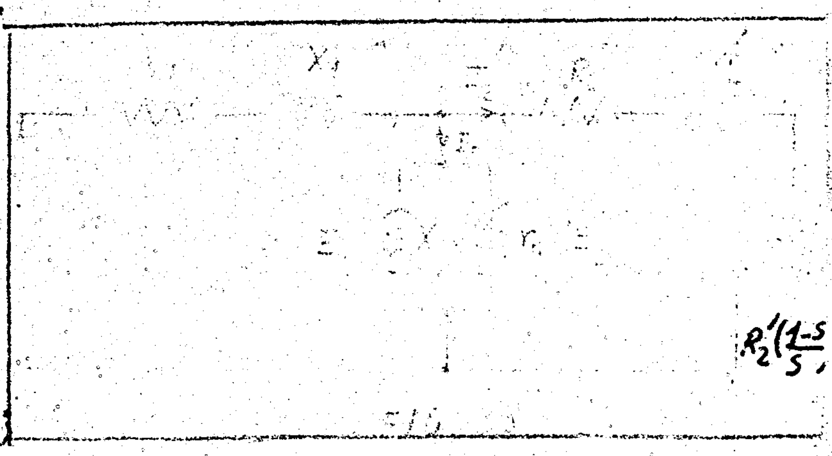
$$T = \frac{I_2^{\,2} R_2}{S} \times \frac{33\ 000}{746 \times 2\pi N_s} \quad \text{lb-ft}$$

$$T = \text{Power developed in watts} \times \frac{33\ 000}{746 \times 2\pi N_s} \quad \text{lb-ft}$$

$$= \frac{7.04 \text{ watts}}{RPM} \qquad (121)$$

The Approximate Equivalent Circuit

The two circuits in Fig 31 may be connect each other and it is also possible to treat the two windings as a shunt circuit carrying the no-load current.



Such circuit is shown in Fig.33. This circuit can be used to determine the characteristics of the machine. To make use of the circuit the following must be known: (a) the stator resistance and reactance, (b) the rotor resistance and reactance, (c) the magnetizing current. This can be determined from the input current at no load is used the resistance r_n. The input power is the friction, windage, and iron losses at no load. The resistance r_n is purely fictitious. It is only used to calculate the above losses For approximate calculations it is also to modify the circuite diagram of Fig.33 by assuming that the shunt path is connected directly across the voltage source shown in fig.34 this procedure tends to raise the voltage slightly on shunt circuit and would tend to indicate a slightly higher flux and core loss than actually exists in the machine. At the same time, the copper loss in the stator will be reduced because the non-load current no longer flows through the stator resistance.

The errors introducing in changing from the circuit of Fig. 33 to that of Fig. 34 tend to cancel. For this reason the approximate equivalent circuite of Fig. 34 is commonly used in the analysis of the machine. This circuit can be used to determine the characyeristics of the machine.

Other Torque Equations

From the approximate equivalent circuit with magnetizing branch moved to the machine terminals

(assuming a = 1)

$$V_1 = I_2 \left[(R_1 + jX_1) + \frac{R_2}{S} + j X_2 \right] \qquad (122)$$

Multiplying by s and eliminating the operator j,

$$SV \approx I_2 \sqrt{(R_1 S + R_2)^2 + S^2 (X_1 + X_2)^2}$$

since

$$I_2 = \frac{E_2}{\sqrt{(R_2/S)^2 + X_2^{\,2}}}$$

Then

$$E_2^{\,2} \approx V_1^{\,2} \; \frac{R_2^{\,2} + X_2^{\,2} \quad S^2}{(R_1 S + R_2)^2 + S^2 (X_1 + X_2)^2} \qquad (123)$$

Equation 121 gives the developed torque as

$$T = \frac{7.04}{N_s} \; \frac{m_1 I_2^{\,2} R_2}{S} \qquad \text{lb.ft}$$

Substituting the above value of I_2

$$T = \frac{7.04}{N_s} \; \frac{m_1 E_2^{\,2}}{(R_2/S)^2 + X_{20}^{\,2}} \times \frac{R_2}{S}$$

$$T = \frac{7.04}{N_s} \; \frac{m_1 E_2^{\,2} R_2}{R_2^{\,2} + S^2 X^2_{\,2}} \qquad (124)$$

Substituting E_2 from Eq. 123

$$T \approx \frac{7.04}{N_s} \quad \frac{m_1 V_1^{\,2} \quad R_2 \; S}{(R_1 \; S + R_2)^2 + S^2(X_1 + X_2)^2}$$

dividing by $(S)^2$ then

$$T = \frac{7.04}{N_s} \quad \frac{m_1 V_1^{\,2} \quad R_2/S}{(R_1 + R_2/S)^2 + (X_1 + X_2)^2} \tag{124}$$

The maximum torque may be found by differentiating Eq. 124 with respect to s. The slip at maximum torque or pull-out (S_T max) is thus found to be

$$S_T\text{max} = \frac{R_2}{R_1^{\,2} + (X_1 + X_2)^2} \tag{125}$$

The maximum (pull-out) torque which occurs at the above slip is

$$T\text{max} = \frac{7.04}{N_s} \quad \frac{m_1 V_1^{\,2}}{2\left[R_1 + \sqrt{R_1^{\,2} + (X_1 + X_2)^2}\right]} \tag{126}$$

The value of the maximum torque is independent of R_2 although the slip at which maximum occurs is a function of R_2. The starting torque (T_s) is obtained from Eq. 124 by setting s equal to 1.

$$T_s = \frac{7.04}{N_s} \quad \frac{m_1 V_1^{\,2} R_2}{(R_1 + R_2)^2 + (X_1 + X_2)^2} \tag{127}$$

Some useful relations may be derived from the above equations. At any given slip $T \sim V_1^{\,2}$ so that $T_s \sim V_1^{\,2}$ with a constant R_2 and V_1 $\quad T \sim I_2^{\,2} / S$

It is sometimes convenient to use a simplified form of the torque equation. It may be shown that, if the stator resistance is neglected ($R_1 = 0$) Eq. 125 may be written

$$\delta(T) = \frac{T}{T\text{max}} = \frac{2}{\dfrac{ST\text{max}}{S} + \dfrac{S}{ST\text{max}}} \tag{128}$$

This is the expression of the relative torque with respect tp S. Now, let us try to draw the curve of this function.

For $S/S_{T\text{max}} \ll 1$, $T/T\text{max} \approx 2S/S_{T\text{max}}$, this is the equation of a straight line, therefore within this range the torque changes as a straight line. At starting S/S_{Tmax} 1

neglecting the second term at the denominator we get T/T_{max} 2 S_{Tmax}/S ; within this range the curve describes a hyperbol. The straight line and Hyperbol cut each other at point of value $T/T_{max} = 2$ and $S/S_{Tmax} = 1$. But the actual totatifmaximum torque is equal 1 the above straight line and hyperbols are asemptots of the actual torque curve. These curves are shown in Fig. 35

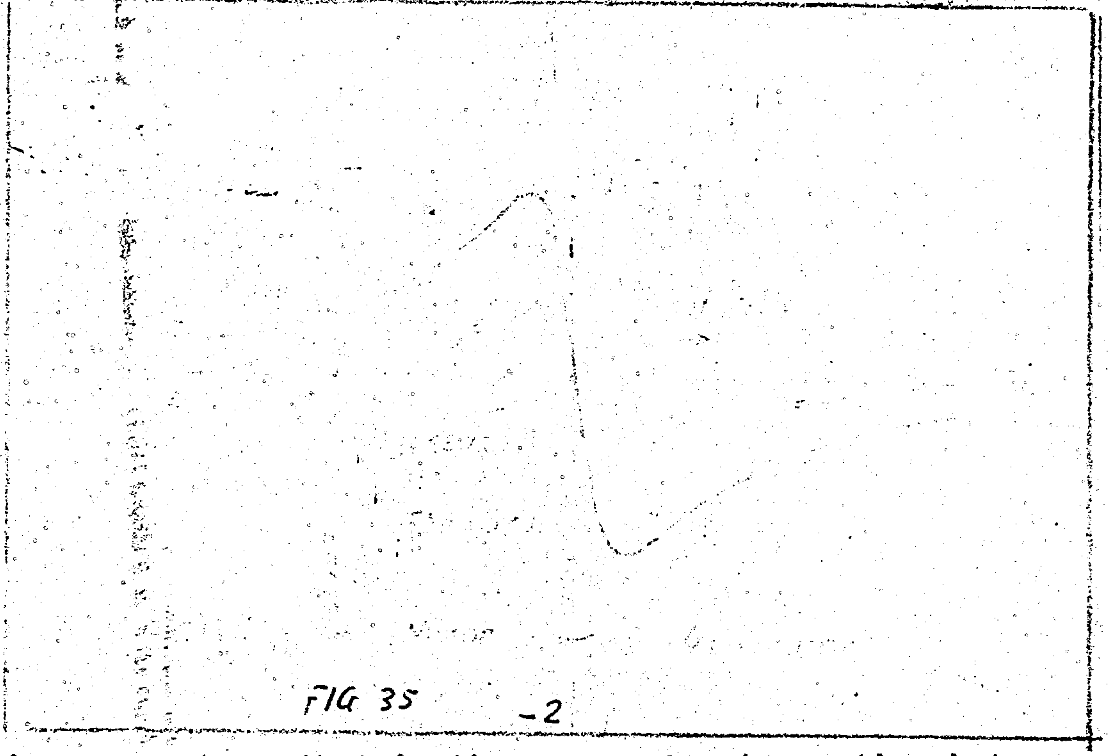


FIG 35

This curve shows that in the range at motor action between stand still and the speed at which maximum torque occurs, the torque increases more than proportionately to the speed. This means that within this range there is a condition of instability, in the sense that if the torque developed is greater then the resisting torque the speed will continue to rise until the point of maximum torque has been passed; beyond the point of maximum torque, and up to synchronism, the conditions become stable for here any increase of load torque causes the motor to slow down and therefore automatically to develop greater torque to balance the load requirement.

Operation of poly phase induction machine as generator and brake

The polyphase induction machine is a general machine, which has many kind of use. In the following table it is shown.

Operating point or range	n	s	f_2	The kind of use
ackward operating range	$n < 0$	$s > 1$	$f_2 > f_1$	Brake, Frequency increaser
tand still point	$n = 0$	$s = 1$	$f_2 = f_1$	Induction voltage regulator 1) Phase regulator 2) Voltage regulator
orward operating range	$0 < n < N_s$	$1 > s > 0$	$f_1 > f_2 > 0$	Induction motor Frequency decreaser
ynchronous operating point	$n = N_s$	$s = 0$	$f_2 = 0$ exciting by D.C.	Synchronous machine converter D-C machine
enerator operating range	$n > N_s$	$s < 0$	$f_2 < 0$	Asynchronous generator

If the prime mover drives the rotor at a speed is greater than syncronous speed N_s, the rotor emf is opposite in sign to the at a subsynchronous speed and the direction of the rotor current reversed, the machine operates as a generator and delivers current the lines. Thus an induction machine driven above synchronous ed operates as an asynchronous generator.

The prime mover can influence only active component of current not the reactive component. Consequently the induction generator t take its reactive current from the lines just the same as uction motor. Therefore an induction generator can operate only any synchronous generator are present in the line.

If the rotor is driven in a direction opposite to that of the ating field (n negative), s is positive and greater than unity. this region the machine operates as a brake. The machine eives electrical power from the supply line and also mechanical er through its shaft.

Typical speed-torque curves are plotted in Fig. 36. The two ves are identical; one being drawn for the forward direction rotation and the other for the reverse. The fact that the oring curve in the first quadrant extends into the fourth drant indicates that the positive torque is available even ugh the motor is rotating in the reverse direction. This same ve also extends into the second quadrant, indicating a negative que (braking) with speeds above synchronous.

Balanced Methods for Obtaining of the Torque

a) Impressed Voltages:

The torque at any value of slip varies as the square of the impressed voltage as indicated by Eq. 124. This relation may be used to complete the characteristics of a machine operating at any voltage, when the data are available for some one voltage. A set of speed-torque curves will be found in Fig. 37 which were computed by this method. The curves of Fig. 38 indicated that the slip at maximum torque is independent of the terminal voltage;

b) Rotor Resistance:

The addition of balanced resistor to the rotor circuit of a wound rotor motor will produce a change in the speed torque curve as indicated in Fig. 38. Fig. 38 shows that the value of pull-out torque is not affected by the magnitude of R_2, a fact verified by Eq. 126.

The family of curves for various values of R_2 can be computed quite accurately by noting that at a given value of torque the ratio R_2/S is a constant for all curves. Thus, if the resistance of the rotor circuit is known for any one curve, the characteristic for any other value of R_2 may be calculated directly from it.

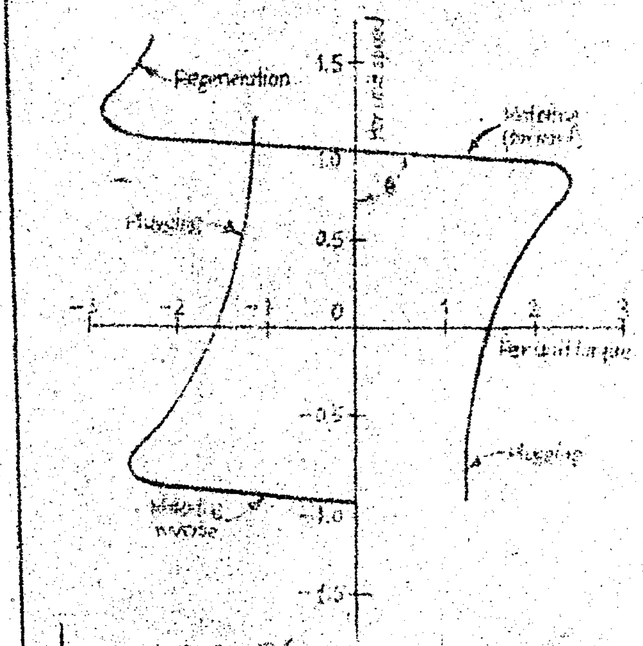


FIG. 36

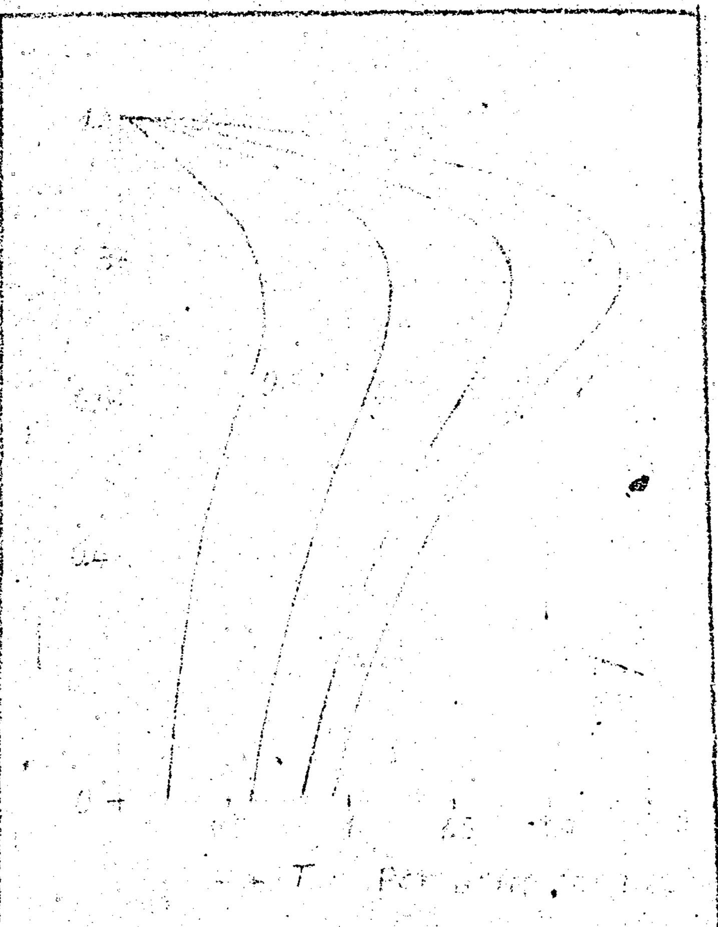
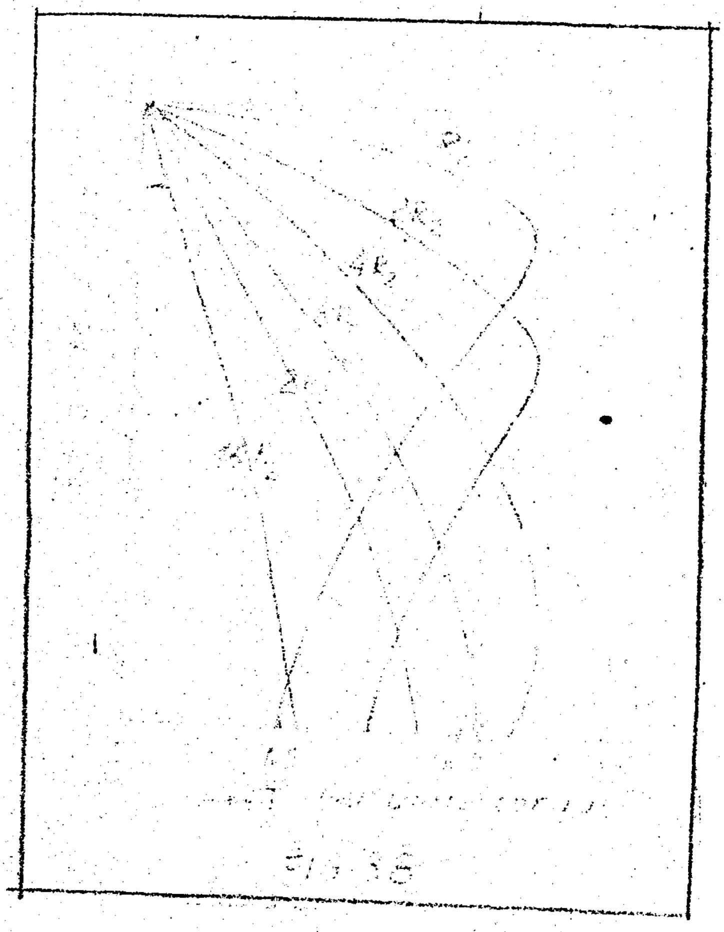


FIG 37

Unbalanced Methods for Obtaining the Torque Characteristics

The speed-torque characteristics of an induction motor may be determined considerably if the machine is used under conditions where the impressed voltages or the motor circuits are not balanced. The complete analysis of such unbalanced operation requires symmetrical components.



1a. Unbalanced impressed Voltage:

As we studies before an unbalanced system of three phase three-wire voltages may be resolved into its positive-and negative-sequence components, and each may be thought of as producing its own air-gap flux. The positive sequence flux thus rotates in the air gap at synchronous speed in the forward or positive direction, and the negative-sequence components rotates at the same speed in the opposite direction. This situation may be depicted graphically as shown in Fig. 39. A two pole machine is assumed, and w_{s1} and w_{s2} represents the velocities of the positive and negative sequence components of stator magneto mogtive force, respectively. The actual rotor velocity w_R is shown in the direction. There is a slip frequency in the rotor due to each of the two sequence voltages, and these result in the rotor magnetomotive voltages, and these result in the rotor magnetomotive force waves which move around the rotor surface. The positive and negative sequence components of these rotor waves are

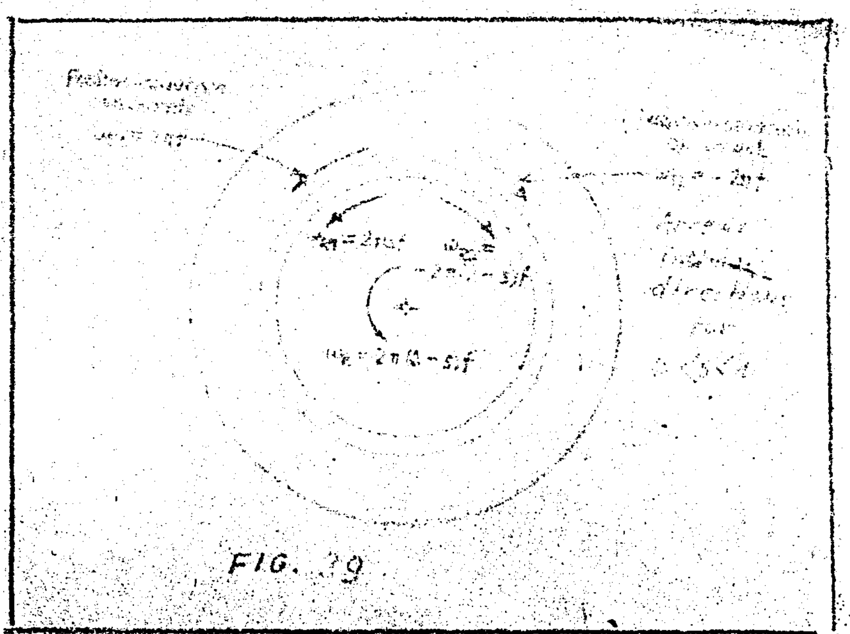


FIG. 39

designated w_{R1} and w_{R2}, respectively; these two angular velocities being measured with respect to the rotor. One may therefore write

$$
\begin{aligned}
\omega_R + \omega_{R1} &= \omega_{S1} \\
w_R + w_{R2} &= w_{S2}
\end{aligned}
\qquad (129)
$$

We shall adopt the view point that the production of torque arises as a result of induced currents in one winding interacting with the flux that produces them. The torque will then have a sign which is positive if the flux wave, viewed from the winding in which the current are induced, has a positive direction. Such a torque is a motor torque. A negative, or braking, torque arises when the flux wave has a negative velocity with respect to the winding in which the currents are induced. With this convention, it will be observed in Fig. 40 that the positive sequence torque is positive for all positive values of slip, and this torque component becomes negative only when the slip is negative. The negative sequence torque is seen to have a negative sign for all slips less than 2. At a slip of 2 the negative torque becomes zero, and at slips of greater than 2 this negative-sequence torque will have a positive sign. Calculation of the speed-torque curves for the case of unbalanced impressed voltages may be carried out in the following manner. The unbalanced voltage system is first resolved into its positive and negative sequence components by using Eqs. 124 V_d is positive sequence components of voltage, V_i is the negative-sequence component, V_1, V_2, V_3 are the unbalanced voltages. Having obtained the symmetrical components voltages, one may calculate the value of torque at any speed by combining the positive and negative-sequence torques. At any slip s the positive sequence Torque T_d is found from the relation

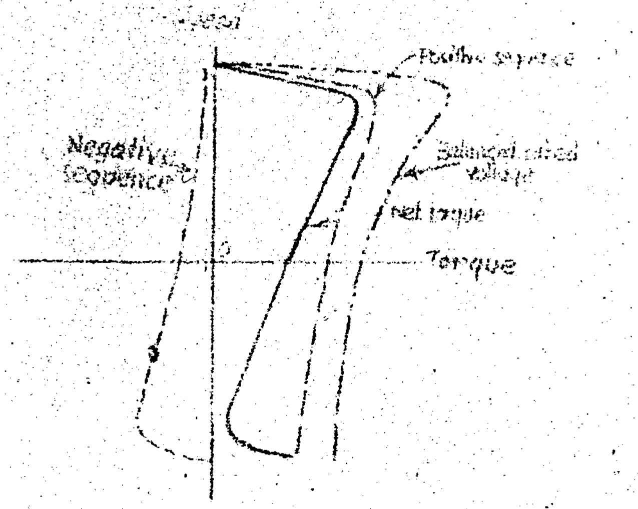


FIG. 40

$$
T_d = (T)_s \ (V_d/V)^2 \qquad (130)
$$

where $(T)_s$ is the torque for a balanced voltage V at the slip s. Likewise, the negative-sequence torque is found from the relation

$$T_i = (T)_{2-s} \quad (V_i/V)^2 \qquad (131)$$

where $(T)_{2-s}$ is the torque for a balanced voltage E at a slip 2-s. This is the slip for the negative-sequence voltage. The resultant torque is simply the difference

$$T_r = T_d - T_i \qquad (132)$$

The Circle Diagram of Induction Motor

A large number of graphical methods have been developed since 1894 for the analysis of induction motor characteristics. They differ greatly in their accuracy and simplicity of application. The circle diagram most frequently found in the text-books. It is based on the approximate equivalent network of Fig. 34. The method makes use of many approximations, but the errors introduced by these approximation are sufficiently small.

Derivation of the Circle Diagram

In the approximate equivalent circuit for the indu8tion motor, the no-load admittance branch is shifted to connect across the terminals of the motor circuit (Fig. 34). This means that the stator impedance drop caused by the no-load current is neglected. In most motors the error of this assumption is small.

If a constant voltage V is applied to the circuit, the load current flowing through the stator and rotor windings will be (assuming a = 1)

$$I_2 = \frac{V}{\sqrt{\left(R_1 + \frac{R_2}{S}\right)^2 + (X_1 + X_{20})^2}} \qquad (133)$$

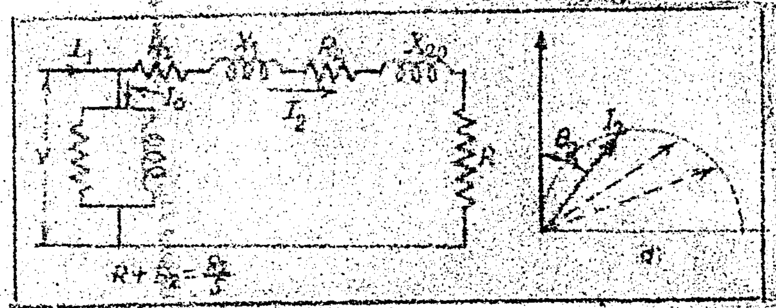
The current I_2 flowing this circuite is out of phase with the applied voltage by an angle θ2 such that

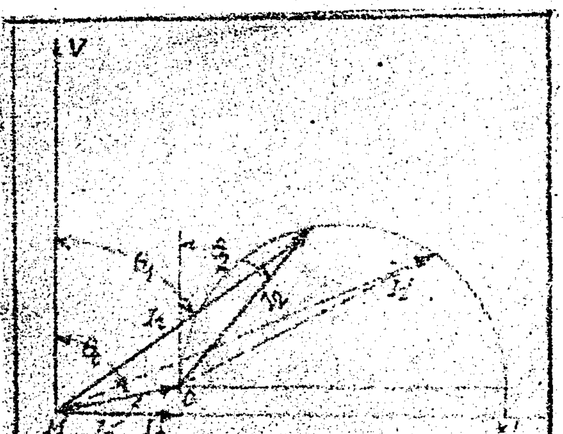
$$\sin \theta_2 = \frac{X_1 + X_{20}}{\sqrt{\left(R_1 + \frac{R_2}{S}\right)^2 + (X_1 + X_{20})^2}} \qquad (134)$$

By combining Eqs; 134 and 135 we get another expression for I_2 :

$$I_2 = \frac{V}{X_1 + X_{20}} \sin \theta_2 \qquad (135)$$

If the leakage reactances are assumed to remain constant regardless of the load, and the applied voltage is constant, then Eq. 135 is the polar equation of a circle having a diameter of $V/(X_1 + X_{20})$. By changing R_2/S (the load), the sin of θ_2 will vary and cause I_2 to change in magnitude and direction with respect to the voltage V. Such a locus is shown in Fig. 41 (a) the entire current taken by such an equivalent circuit is not only I_2 ; the second leg requires the constant current I_0. To conform to the physical facts, it is necessary that the change in load (or in R_2/S) must result in a variation in the current I_2 with no change in I_0. The circuit fulfills this requirement, and by adding the current vector I_0 as shown in Fig. 41 (b) the circle diagram is also in agreement. The total motor current per phase is then the vector sum of I_2 and I_0, or I_1. The diagram shows three pf. angles; the no load pf angle is θ_0, the secondary pf angle is θ_2, and the motor as a whole has the pf angle of θ_1.





(b)
FIG. 41

Construction of the Diagram

Two tests are necessary in order to predict the characteristics of an induction motor from laboratory data. These are the no-load and the rotor-blocked tests.

a) By operating the motor at no-load with full primary voltage, measuring the no-load current and power input by means of an ammeter and wattmeter, the point O (Fig. 42) is determined; under this condition the power input as measured by a wattmeter will be practically all due to core losses, friction losses and windings loss. Thus if at no load the primary voltage, current, and power input per phase are V , I_0 , P_0 respectively,

$$\cos \theta_0 = \frac{P_0}{V\, I_0} \tag{136}$$

Then use the applied voltage V as the reference vector, draw I_o behind V by θ_o degrees. The voltage and current scales can be chosen arbitrarily; the power and torque scales will be depend upon the first two.

Watts per inch = Vx Amperes per inch
Torque per inch = Vx Amperes per inch.

b) On blocking the rotor and impressing upon the primary just sufficient voltage to circulate full-load current in the primary, point a will be fixed. Under this condition the power input as measured by a wattmeter will be practically all due to the primary and secondary copper loss, since core loss will be negligibly small because of the small impressed voltage, and the friction and windage losses do not exist. If at short-circuit (standstill) the corresponding magnitudes are V_s, I_1, P_s, where I_1 is adjusted to approximately full-load value by suitable choise of V_s,

$$\text{Cos } \theta_{2s} = \frac{P_s}{V_s\, I_1}$$

and

$$I_s = \frac{V_1}{V_s} I_1 \qquad (137)$$

the points O and a must be lie on the locus circle. To draw the circle, determine its center by extending a horizontal line from o to K and erect a perpendicular bisector on the line connecting O and a. The intersection of this bisector with the horizontal extension from O is the center of the circle, shown as C.

Using C as the center, draw the semicircle OPK. Draw the vertical line ab and divide it by the point d so that add is to do as the rotor copper loss (in R_2) is to the stator "added" copper loss (in R_1), in terms of the resistances and the vectorial difference of current $(I_{b1} - I_o)$

$$\frac{(\ldots - I_o)^2 R_2}{(\ldots^2 - I_o^2)\, R_1} = \frac{ad}{do} \approx \frac{\text{Rotor effective resistance per phase in stator terms}}{\text{Stator effective resistance per phase}} \qquad (138)$$

If no resistance measurement can be obtained for the rotor, as is the case for squirrel cage or other short-circuited rotors, the point d can be determined as follows:

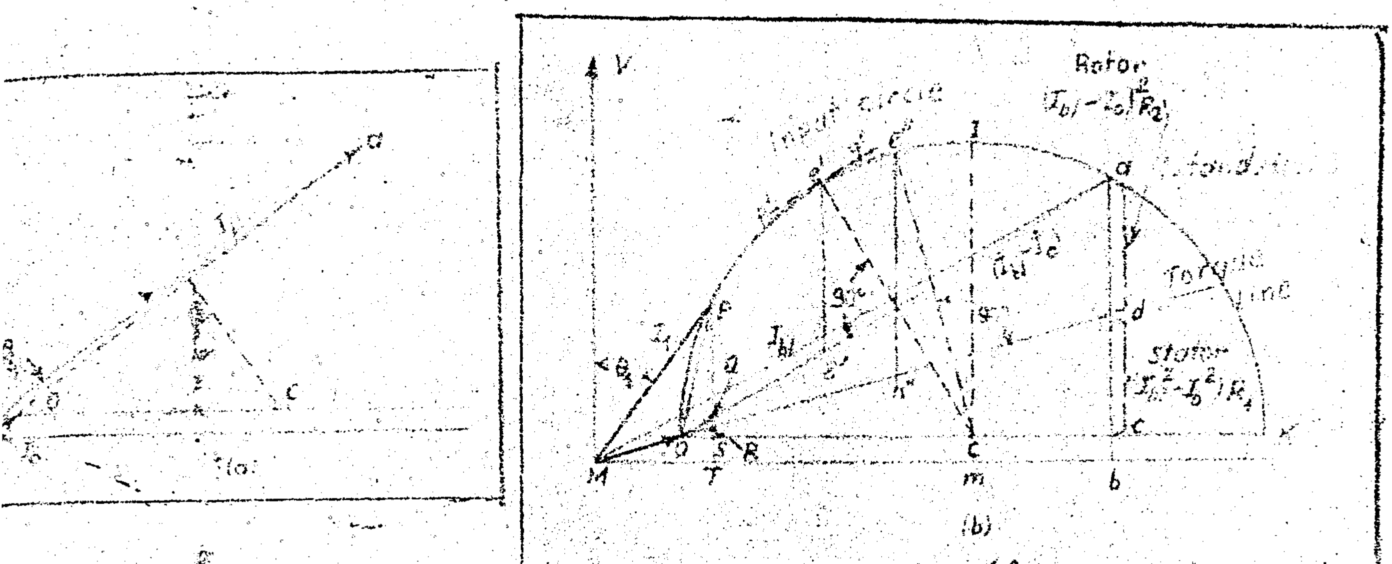


FIG. 42

Any vertical line drawn from the base to the circle represents an in phase component of current and hence is propertional to power. If the rotor current is I_{b1} and the stator resistance is R_1 ohms per phase, the stator copper loss on short circuit is $I_{b1}{}^2R_1$. The power component of current needed to supply the portion of this loss in excess of the no-load is

$$\frac{(I_{b1}{}^2 - I_o{}^2)\ R_1}{V \text{ per phase}} = I_{cd} \qquad (139)$$

Draw I_{cd} the current scale and thus determine the position of d. Draw od. The rated current of the motor per phase is I_1. Lay off MP equal to I_1, and draw the vertical line PT. With the correct scales the following values can now be determined.

No-load current = MO (amperes per phase)
Stator current = MP (amperes per phase)
Rotor current = OP (amperes per phase in the stator terms)
Power input to stator = TP (watts per phase)
Constant losses = TS (watts per phase)
Stator "added" copper loss = SR (watts per phase)
Rotor copper loss = RQ (watts per phase)
Power transferred across air gap = RP (watts per phase)
Useful output = QP (watts per phase)
Power factor = $\cos\theta_1$ = PT/MP
Slip = RQ/RP (as fraction)
Efficiency = QT/TP (as a fraction)

The torque is equal to RP in the watt scale. The torque scale in pound-feet equals the watt scale times

$$\frac{33{,}000}{746} \cdot \frac{1}{2\pi \text{ Synchronous rpm}} = \frac{7.04}{\text{Synchronous rpm}} \qquad (140)$$

By assuming different values of current I_1, the performance curves of the motor can be calculated over the entire load range.

Maxima.

The maxima pf at which the motor can operate will occur when the current vector I_1 is tangent to the circle. Inspection of the diagram shows that the maximum rotor pf occurs at no load. The maximum power which the motor can take from supply is indicated as mI. The maximum power output will occur when the length QP is a maximum. To determine this position it is necessary to draw the line p'q' tangent to the circle and parallel to Oa. A perpendicular bisector, erected on the line Oa is convenient means of locating this point of tangency. This bisector is shown as ce', and the maximum is then g'e' the current input for this condition is obviously MB.

Since the torque is proportional to RP, the maximum torque can be found by drawing a tangent to the circle parallel to the line Od. This maximum torque is indicated as h"e". If MP is the rated current, then RP represents the full-load torque , and the relative lengths of RP and h"e" give the ratio of full-load to pull-out torque.

The line Od is called torque line, since vertical distances from it to the circle represent the torque at the various slip. Then the starting torque will be proportional to the length, da, in as much as this represents unity slip condition. If MP represents full-load stator current, the full-load torque is RP and the ratio of starting torque to full-load torque is da/RP.

HIGHER HARMONICS IN INDUCTION MOTORS

As we indicated before, the usual theory and calculation of induction motors is based on the assumption of the sine wave. That is it is assumed that the voltage impressed upon the motor per phase, and therefore the magnetic flux and the current, are sine waves, and it is further assumed, that the distribution of the winding on the circumference of the armature is sinusoidal in space. While in most cases this is sufficiently the case, it is not always so, and especially or air-gap distribution of the magnetic flux may sufficiently differ from the sine shape, since harmonics may be present. These harmonics may be one of two varieties: (a) time or (b) space.

Time Harmonics:

If the voltage supplied to a polyphase induction motor contains harmonics, the air gap flux may have components that rotate at speeds other than that corresponding to the fundamental frequency. As an example, a fifth time harmonic would tend to rotate the motor at 5 times the speed of the fundamental. The time harmonic voltage is usually small in proportion to the fundamental voltage. The reactance of the motor to the time harmonics is high. Consequently, little harmonic current flows, and therefore the torque produced by this time harmonics are negligible.

Space Harmonics:

Space harmonics can be produced by sinusoidal currents received from the supply. They result when the stator windings do not posses perfect distribution. Fig. 43 a shows a stator carrying a current I, which is sinusoidal in time. The field wave B, produced by this sinusoidal current is rectangular in space (Fig. 43 b) but the amplitude of this rectangular wave varies sinusoidally with time. The space distribution is such that the field wave contains, in addition to its fundamental, an infinite number of space harmonics. Each of this harmonics has an amplitude that varies sinusoidally in time at the same frequency as the fundamental. As the current varies sinusoidally in time, as shown (Fig. 43 c), the amplitude of the rectangular wave will vary sinusoidally. The third space harmonic field wave B_3 is shown in Fig. 43 d. It should be noted that the amplitude of this component also varies sinusoidally in time with the current I. The fundamental B_1 of the space wave is also shown in Fig 43d. These effects could be produced by one phase of an induction motor if the windings of each phase are concentrated. It is evident from Fig. 43 d that the third-

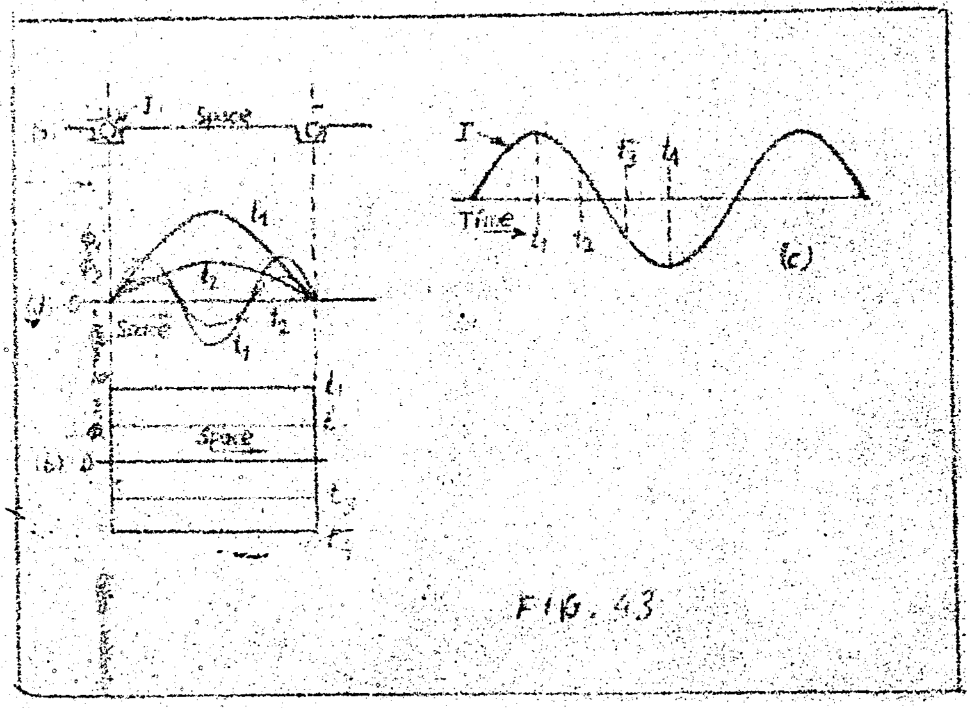


FIG. 43

harmonic flux produces 3 times as many magnetic poles as the fundamental component. Likewise, the fifth harmonic would produce 5 times the number of poles as the fundamental, etc. Therefore, if the space harmonics of the various phases do not cancel each other, they will produce rotating fields that travel around the air gap at a speed equal to the synchronous speed divided by the order of the harmonic. Harmonic fields may rotate in either direction, depending upon the number of phases and the order of the harmonic. Now let's study the influence of the time-and space harmonics to the output torque of induction motor. First of all let's consider two phase induction motor.

a) <u>Two-phase Induction Motor</u>

Let then:

$$E = E_1 \cos wt + E_3 \cos (3wt - \varphi_3) + E_5 (\cos 5wt - \varphi_5)$$
$$+ E_7 \cos (7wt - \varphi_7) + E_9 \cos (9\ wt - \varphi_9)$$

be the voltage impressed upon one phase of the induction motor. The voltage of the second motor phase, which lays 90^0 or $\frac{\pi}{2}$ behind the first motor phase, is:

$$e' = E_1 \cos\left(wt - \frac{\pi}{2}\right) + E_3 \cos\left(3wt - \frac{3\pi}{2} - \varphi_3\right) + E_5 \cos\left(5wt - \frac{5\pi}{2} - \varphi_5\right)$$

$$+ E_7 \cos\left(7wt - \frac{7\pi}{2} - \varphi_7\right) + E_9 \cos\left(9wt - \frac{9\pi}{2} - \varphi_9\right) + \cdots$$

$$= E_1 \cos\left(wt - \frac{\pi}{2}\right) + E_3 \cos\left(3wt - \varphi_3 + \frac{\pi}{2}\right) \quad E_5 \cos\left(5wt - \varphi_5 - \frac{\pi}{2}\right)$$

$$+ E_7 \cos\left(7wt - \varphi_7 + \frac{\pi}{2}\right) + E_9 \cos\left(9wt - \varphi_9 - \frac{\pi}{2}\right) + \cdots \qquad (141)$$

The magnetic flux produced by these two voltages thus consists of a series of component fluxes, corresponding to the successive components. The secondary currents induced by these component fluxes and the torque produced by the secondary currents, thus show the same components.

Thus the motor torque of the motor, given by the usual sine-wave theory of the induction motor, and due to the fundamental voltage wave;

$$\begin{aligned} &E_1 \cos wt \\ &E_1 \cos\left(wt - \frac{\pi}{2}\right) \end{aligned} \qquad (142)$$

is shown as T_1 in Fig. 44. As we know it is increasing from standstill, with increasing speed, up to maximum torque, and then decreasing again to zero at synchronism.

The thirs harmonics of the voltage waves are:

$$\begin{aligned} &E_3 \cos\left(3wt - \varphi_3\right) \\ &E_3 \cos\left(3wt - \varphi_3 + \frac{\pi}{2}\right) \end{aligned} \qquad (143)$$

As seen, these also constitute a two phase system of voltage but the second wave, which is ldgging in the fundamental, is 90° leading in the third harmonic, or in other words the third harmonic gives a backward rotation of the poles with triple frequency. It thus produces a torque in opposite direction to the fundamental, and would reach its synchronism, that is, zero torque, at one third of synchronism in negative direction, or at the speed $S = -1/3$, given infraction of synchronous speed. For back-ward rotation above one-third synchronism, this triple harmonic then gives an induction generator torque, and the complete torque curve given by the third harmonics thus is as shown by curve T_3 of Fig. 44

The fifth harmonics:

$$E_5 \cos (5wt - \varphi_5)$$
$$E_5 \cos (5wt - \varphi_5 - \tfrac{\pi}{2}) \qquad (144)$$

give again phase rotation in the same direction as the fundamental, that is, motor torque, and assist the fundamental. But synchronous is reached at one fifth of the fundamental, or at: $S = +1/5$, and above this speed, the fifth harmonic becomes induction generator, due to oversynchronous rotation, and retards. Its torque curve is shown as T_5 in Fig. 44

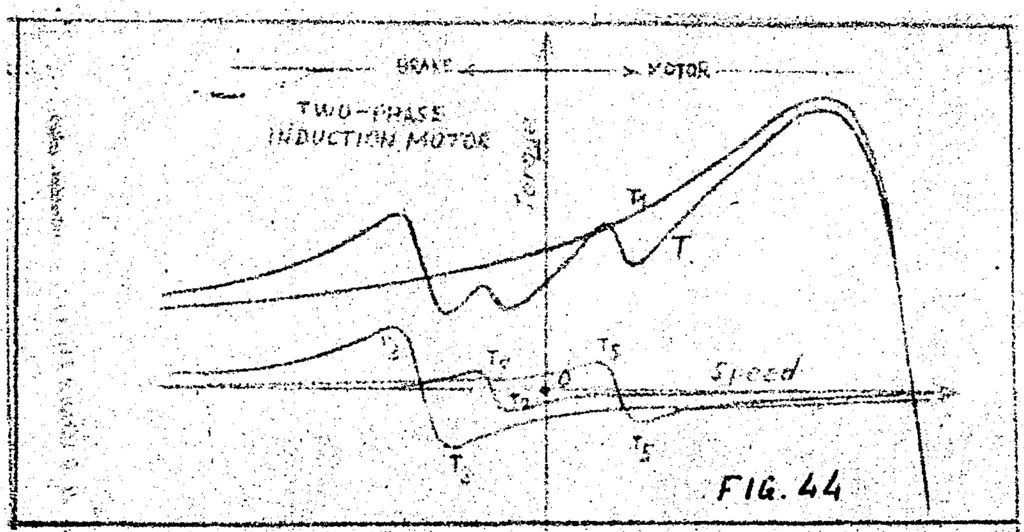


FIG. 44

The seventh harmonic again gives negative torque, due to backward phase rotation of the phases, and reaches synchronism as $S = -1/7$, that is, one-seventh speed in backward rotation shown by curve T_7 in Fig. 44.

The seventh harmonic again gives negative torque, due to back ward phase rotation of the phases, and reaches synchronism at $S = -1/7$, that is, one-seventh speed in back ward rotation, as shown by curve T_7 in Fig. 44.

The ninth harmonic again gives positive motor torque up to its synchronism, $S = +1/9$, and above this negative induction generator torque, etc.

We then have the effects of the various harmonics on the

Two-phase Induction Motor

Order of harmonics	1	3	5	7	9	11	13
Phase rotation	+	-	+	-	+	—	+
Synchronous speed: s =	+1	-1/3	+1/5	-1/7	+1/9	-1/11	+1/13
Torque positive up to: s = otherwise negative	+1	—	+1/5	—	+1/9	—	+1/13

Adding now the torque curves of the various voltages harmonics, T_3, T_5, T_7, to the fundamental torque curve, T_1, of the induction motor, gives the resultant torque curve, T.

As seen from Fig. 44, if the voltage harmonics considerable, the torque curve of the motor at lower speeds, forward and backward, that is, when used as brake, is rather irregular, showing depressing or "dead points".

b) <u>Three-phase Induction Motor</u>

Assume now, the general voltage wave (1) is one of the three-phase voltages, and is impressed upon one of the phases of a three-phase induction motor. The second and third phase then is lagging by $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$ respectively behind the first phase (1):

$$
\begin{aligned}
E' &= E_1 \cos\left(wt - \frac{2\pi}{3}\right) + E_3 \cos\left(3wt - \frac{6\pi}{3} - \varphi_3\right) \\
&+ E_5 \cos\left(5wt - \frac{10\pi}{3} - \varphi_5\right) + E_7 \cos\left(7wt - \frac{14\pi}{3} - \varphi_7\right) \\
&+ E_9 \cos\left(9wt - \frac{18\pi}{3} - \varphi_9\right) + \cdots \\
&= E_1 \cos\left(wt - \frac{2\pi}{3}\right) + E_3 \cos\left(3wt - \varphi_3\right) \\
&+ E_5 \cos\left(5wt - \varphi_5 + \frac{2\pi}{3}\right) + E_7 \cos\left(7wt - \varphi_7 - \frac{2\pi}{3}\right) \\
&+ E_9 \cos\left(9wt - \varphi_9\right) + \cdots
\end{aligned}
$$

$$
\begin{aligned}
E'' &= E_1 \cos\left(wt - \frac{4\pi}{3}\right) + E_3 \cos\left(3wt - \varphi_3\right) \\
&+ E_5 \cos\left(5wt - \varphi_5 + \frac{4\pi}{3}\right) + E_7 \cos\left(7wt - \varphi_5 - \frac{4\pi}{3}\right) \\
&+ E_9 \cos\left(9wt - \varphi_9\right) + \cdots
\end{aligned}
$$

Thus the voltage components of different frequency, impressed upon the three motor phases, are:

cos wt	$E_3 \cos (3wt - \varphi_3)$	$E_5 \cos (5wt - \varphi_5)$	$E_7 \cos (7wt - \varphi_7)$	$E_9 \cos (9wt - \varphi_9)$
cos (…t $- \frac{2\pi}{3}$)	$E_3 \cos (3wt - \varphi_3)$	$E_5 \cos (5wt - \varphi_5 + \frac{2\pi}{3})$	$E_7 \cos (7wt - \varphi_7 - \frac{2\pi}{3})$	$E_9 \cos (9wt - \varphi_9)$
cos (…t $- \frac{4\pi}{3}$)	$E_3 \cos (3wt - \varphi_3)$	$E_5 \cos (5wt - \varphi_5 + \frac{4\pi}{3})$	$E_7 \cos (7wt - \varphi_7 - \frac{4\pi}{3})$	$E_9 \cos (9wt - \varphi_9)$
…ndamental	3rd	5th	7th	9th

As seen, in this case of three-phase motor, the third harmonics have no phase rotation, but are in phase with each other, or single-phase voltages. The fifth harmonic gives backward phase rotation, and thus negative torque, while the seventh harmonic has the same phase rotation, as the fundamental, thus adds its torque up to its synchronous speed $S = +1/7$, and above this gives negative torque. The ninth harmonic again is single phase. Fig. 45 shows the fundamental torque, T_1, the higher harmonics of torque, T_5 and T_7, and the resultan torque, T.

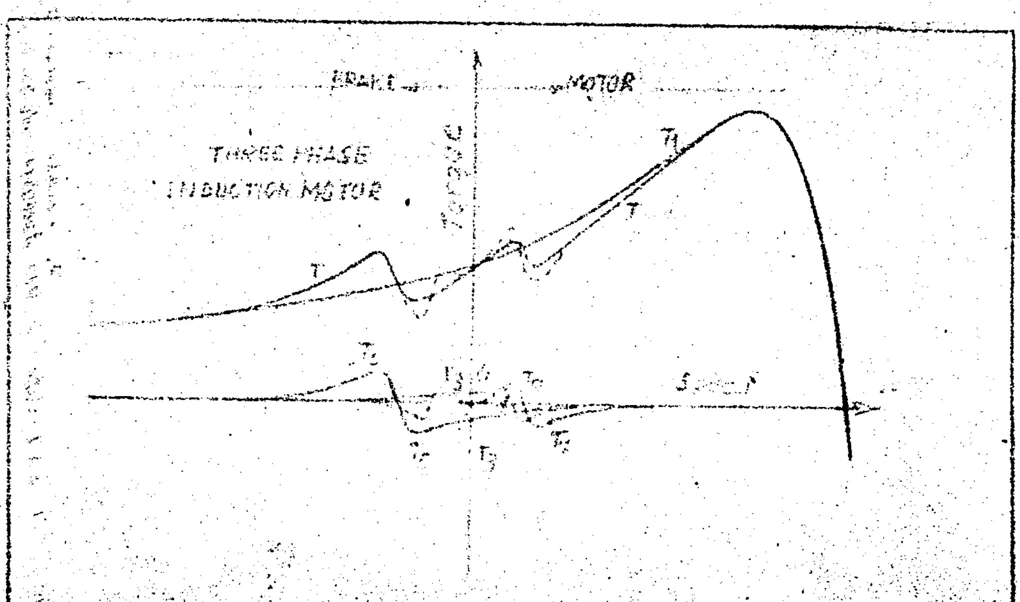


FIG. 45

As seen, the distortion of the torque curve is materially less, due to the absence, in Fig. 45, of the third harmonic torque.

However, while the third harmonic (and its multiplies) in the three-phase system of voltages are in phase, thus give no phase rotation, they may give torque, as single-phase induction motor has torque, at speed, though at stand still the torque is zero.

The fundamental motor torque, T_1, of Fig. 44 and 45, is given by a sine wave of voltage and thus of flux, if the winding of each phase is distributed around the circumference of the motor air gap in a sinusoidal manner.

This however, is never the case, but the winding is always distributed in a non-sinusoidal manner. The space distribution of magnetizing force and thus of flux of each phase, along the circumference of the motor air gap, thus can in the general case be represented by a trigono metric series, with θ as space angle, in electrical degrees, that is, counting a pair of poles as 2π or $360°$. It is then: The distribution of the flux of one phase, in the motor air gap:

$$B = B_o \left[\cos\theta + a_3 \cos 3\theta + a_5 \cos 5\theta + a_7 \cos 7\theta + a_9 \cos 9\theta + \cdots \right] \tag{146}$$

here the assumtion is made, that all the harmonics are in phase, that is, the magnetic distribution symmetrical. This is practically always the case, and if it were not, it would simply add phase angle, φ_m, to the harmonics, but would make no change in the result, as the component torque harmonics are independent of the phase relations between the harmonic and the fundamental.

In a two phase motor, the second phase is located $90°$ or $\theta = \frac{\pi}{2}$ displaced in space, from the first phase, and thus represented by the expression:

$$\begin{aligned} B' &= B_o \left[\cos\left(\theta - \frac{\pi}{2}\right) + a_3 \cos\left(3\theta - \frac{3\pi}{2}\right) + a_5 \cos\left(5\theta - \frac{5\pi}{2}\right) \right. \\ &\quad \left. + a_7 \cos\left(7\theta - \frac{7\pi}{2}\right) + a_9 \cos\left(9\theta - \frac{9\pi}{2}\right) + \cdots \right] \\ &= B_o \left[\cos\left(\theta - \frac{\pi}{2}\right) + a_3 \cos\left(3\theta + \frac{\pi}{2}\right) + a_5 \cos\left(5\theta - \frac{\pi}{2}\right) \right. \\ &\quad \left. + a_7 \cos\left(7\theta + \frac{\pi}{2}\right) + a_9 \cos\left(9\theta - \frac{\pi}{2}\right) + \cdots \right] \end{aligned} \tag{147}$$

Such a general or non sinusoidal space distribution of magnetic flux, as represented by B and B' can be considered as the super position of a series of sinusoidal magnetic fluxes:

$$\begin{array}{lll} \cos\theta & a_3 \cos 3\theta & a_5 \cos 5\theta \\ \cos(\theta-\frac{\pi}{2}) & a_3 \cos(3\theta+\frac{\pi}{2}) & a_5 \cos(5\theta-\frac{\pi}{2}) \\ a_7 \cos 7\theta & a_9 \cos 9\theta & \\ a_7 \cos(7\theta+\frac{\pi}{2}) & a_9 \cos(9\theta-\frac{\pi}{2}) & \end{array} \tag{148}$$

The first component:

$$\begin{array}{l} \cos\theta \\ \cos(\theta-\frac{\pi}{2}) \end{array} \tag{149}$$

gives the fundamental torque of the motor.

The second component of space distribution of field:

$$\begin{array}{l} a_3 \cos 3\theta \\ a_3 \cos(3\theta+\frac{\pi}{2}) \end{array} \tag{150}$$

gives a distribution, which makes three times as many cycles in the motor-gap circumference, than (10), that is, corresponds to a motor of three times as many poles. This component of space distribution of magnetizing force or field would thus, with the fundamental voltage and current wafe, give a torque curve reaching synchronism as one-third speed; with the third harmonic of the voltage. This torque curve would reach synchronism at one-ninth, with the fifth harmonic of the voltage wave at one-fifteenth of the normal synchronous speed.

In Eq. 150 the sign of the second term is reversed from that in Eq 149, that is, in Eq 150, the space rotation is backward from that of Eq. 149. In other words, Eq. 150 gives a synchronous speed $S = -\frac{1}{3}$ with the fundamental or full-frequency voltage wave.

The third component of space distrubution:

$$\begin{array}{l} a_5 \cos 5\theta \\ a_5 \cos(5\theta-\frac{\pi}{2}) \end{array} \tag{151}$$

gives a motor of fime times as many poles as Eq 149, but with

Order of harmonics	1	3	5	7	9	11	13	15	17
Two-phase motor:									
Phase rotation	+	–	+	–	+	—	+	—	+
Synchronous speed	+1	-1/3	+1/5	-1/7	+ 1/9	-1/11	+ 1/13	-1/15	+1/17
Time H — Frequency	f	3f	5f	7f	9f	11f	13f	15f	17f
Time H — No.of pole pairs	p	p	p	p	p	p	p	p	p
Space H — Frequency	f	f	f	f	f	f	f	f	f
Space H — No.of poles	p	3p	5p	7p	9p	11p	13p	15p	17p
Three-phase motor:									
Phase rotation	+	0	-	+	0	-	+	0	-
Synchronous speed	+ 1	(±1/9)	-1/5	+1/7	(±1/27)	-1/11	+ 1/13	(±1/45)	-1/17
Time H — Frequency	f	3f	5f	7f	9f	11f	13f	15f	17f
Time H — No.of pole pairs	p	(3p)	p	p	(3p)	p	p	(3p)	p
Space H — Frequency	f	f	f	f	f	f	f	f	f
Space H — No.pole pairs	p	0	5p	7p	0	11p	13p	0	17p

same space rotation as Eq 149, and this component thus would give a torque, reaching synchronism at $S = 1/5$. In the same manner, the seventh space harmonic gives $S = 1/7$, the ninth space harmonic $S = 1/9$ etc.

As seen the component torque curves of the harmonics of the space distribution of magnetizing force and magnetic flux in the motor air gap, have the same characteristics as the component torque due to the time harmonics of the impressed voltage wave, and thus are represented by the same torque diagrams:

Fig 44 for a two-phase motor,
Fig 45 for a three-phase motor.

Here again, we see that the three-phase motor is less liable to irregularities in the torque curve, caused by higher harmonics, than the two-phase motor is. Two classes of harmonics thus may occur in the induction motor, and give component torques of lower synchronous speed:

Time harmonics, that is, harmonics of the voltage wave, which are of higher frequency, but the same number of motor poles, and space harmonics, that is, harmonics in the air-gap distribution, which are of fundamental frequency, but of a higher number of motor poles.

Compound harmonics, that is, higher space harmonics of higher time harmonics, theoretically exist, but their torque necessarily is already so small that thay can be neglected, except where they are intentionally produced in the design.

We thus get the two classes of harmonics, and their characteristics:

Order of harmonics		1	3	5	7	9	11	13	15	17
Two-phase motor: Phase rotation		+	-	+	-	+	—	+	—	+
Synchronous speed		+1	-1/3	+1/5	-1/7	+ 1/9	-1/11	+ 1/13	-1/15	+1/17
Time H	Frequency	f	3f	5f	7f	9f	11f	13f	15f	17f
	No.of pole pairs	p	p	p	p	p	p	p	p	p
Space H	Frequency	f	f	f	f	f	f	f	f	f
	No.of poles	p	3p	5p	7p	9p	11p	13p	15p	17p
Three-phase motor: Phase rotation		+	0	-	+	0	-	+	0	-
Synchronous speed		+ 1	(±1/9)	-1/5	+1/7	(±1/27)	-1/11	+ 1/13	(±1/45)	-1/17
Time H	Frequency	f	3f	5f	7f	9f	11f	13f	15f	17f
	No.of pole pairs	p	(3p)	p	p	(3p)	p	p	(3p)	p
Space H	Frequency	f	f	f	f	f	f	f	f	f
	No.pole pairs	p	0	5p	7p	0	11p	13p	0	17p

As we indicated before, the space harmonics usually are more important then the time harmonics, as the space distribution of the winding in the motor usually materially differs from sinesoidal, while the deviation of the voltage wave from sine shape in modern electric power supply system is small, and the time harmonics thus usually negligible.

The space harmonics can easily be calculated from the distribution of the winding around the periphery of the motor air gap.