COMPARISON OF MULTICARRIER CDMA WITH SUCCESSIVE INTERFERENCE CANCELLATION AND INTERFERENCE CANCELLATION WITH GIBBS SAMPLER

by

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ABSTRACT

COMPARISON OF MULTICARRIER CDMA WITH SUCCESSIVE INTERFERENCE CANCELLATION AND INTERFERENCE CANCELLATION WITH GIBBS SAMPLER

Recently Orthogonal Frequency Division Multiplexing (OFDM) has gained a lot of attention and is the technology for Fourth Generation (4G) wireless systems because it promises data rates up to 100 Mbps. A variation of OFDM is Multi-Carrier Code Division Multiple Access (MC-CDMA) which is a combined technique of Direct-Sequence (DS) CDMA and OFDM techniques, which is also an OFDM technique where the individual data symbols are spread using a spreading code in the frequency domain which results multi carriers. The spreading code associated with MC-CDMA provides multiple access technique as well as interference suppression.

In Code Division Multiple Access technologies, all other users different from the desired user in the system are also interference sources apart from noise. For this reason in CDMA systems the main resource is power. To suppress the interference level in the system and the interference caused by other users, because of the complexity of Multi User Detection (MUD) technique, much lower complexity than MUD technique which is called Interference Cancellation methods are used. Mainly there are two interference cancellation methods used which are Serial (Successive) Interference Cancellation (SIC) and Parallel Interference Cancellation (PIC). Gibbs sampler in interference cancellation gained a lot of attention which is an interference cancellation between serial and parallel type interference cancellation also introduce randomness in the interference cancellation scheme.

In this thesis, we apply these two interference cancellation techniques which are Successive Interference Cancellation and Interference Cancellation using Gibbs Sampler to the Multi Carrier CDMA system. We investigate the performance of these two systems with simulations and have observed better results from IC scheme with Gibbs sampler.

ÖZET

ÇOK TAŞIYICILI CDMA SİSTEMİNDE, BAŞARILI GİRİŞİM SİLME İLE GİBBS ÖRNEKLEMESİ İLE GİRİŞİM SİLMENİN KARŞILAŞTIRILMASI

Son zamanlarda Ortogonal Taşıyıcı Bölünmeli Çoklama (OFDM) tekniği dikkatleri topluyor ve 100 Mbps data hızlarına kadar destek sağlaması ile Dördüncü Jenerasyon (4G) için kullanılması düşünülen teknolojidir. Bir OFDM varyasyon tekniği olan Çok Taşıyıcılı Kod Bölünmeli Çoklu Erişim (MC-CDMA), Direk-Ardışık (DS) kod bölmeli çoklu erişim tekniği ile OFDM tekniğinin birleşim halidir. Çok Taşıyıcılı CDMA tekniği, bilginin frekans alanında yayılması ile oluşan bir OFDM tekniğidir ki, bu da çok taşıyıcılı bir sistem sonucu doğurur. Ayrıca, bilginin frekans alanına yayılması için çoklu taşıyıcılarda kullanılan ortogonal kodlar, ayrıca girişimin bastırılmasını sağlar.

Kod bölmeli çoklu erişim teknolojilerinde, çözülmek istenen kullanıcının haricindeki tüm kullanıcılar, çözülmek istenen kullanıcı için gürültünün haricinde girişim kaynağıdırlar. Dolayısıyla güç, CDMA teknolojilerinde en önemli kaynaktır. Sistemdeki girişim seviyesini ve diğer kullanıcılardan kaynaklanan girişimin engellenmesi için Çok Kullanıcıyı Yakalama (MUD) tekniğinin karmaşıklığı yüzünden daha az karmaşık bir çözüm olan Girişim Silme (IC) metotları kullanılmaktadır. Başlıca iki çeşit girişim silme metodu vardır. Bunlardan bir tanesi Seri (Başarılı) girişim silme, diğeri ise paralel girişim silme metotlarıdır. Gibbs örneklemeyi kullanan paralel ile seri arası girişim silme metodu auygulayan bir yöntemdir.

Bu tezde, Başarılı Girişim Silme ve Gibbs örnekleme kullanan Girişim Silme metodu gibi iki farklı girişim silme metodunu, Çok Taşıyıcılı CDMA sistemine uyguladık. Bu iki girişim silme metodunun başarımını benzetimlerle inceledik ve Gibbs örnekleme ile girişim silmede daha başarılı sonuçlar elde edildiği görüldü.

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LIST OF SYMBOLS / ABBREVIATIONS

a_1	Random variable
<i>a</i> ₂	Random variable
Α	Nominator coefficient of signal to interference ratio
b_k	Bit of the <i>k</i> th user
\hat{b}_k	Estimated bit of the <i>k</i> th user
В	Denominator coefficient of signal to interference ratio
В	Bit decisions in Gibbs Sampler
С	Orthogonal code
c^{I}	Inphase component of the orthogonal code
$c^{\mathcal{Q}}$	Quadrature component of the orthogonal code
$C_{k,m}$	Orthogonal code for user k and subcarrier m
d_1	Random variable
d_2	Random variable
d_k	Soft bit decision
D	Desired user signal
D_k	Desired user signal of user k
h	Channel model
h^{I}	Inphase component of the channel
$h^{\mathcal{Q}}$	Quadrature component of the channel
$h_{k,m}$	Channel model for user k and subcarrier m
Ι	Own carrier interference
I_k	Own carrier interference of user k
J	Other carrier interference
\boldsymbol{J}_k	Other carrier interference of user k
k	User index
l	Subcarrier index
m	Subcarrier index

n	Bit index
N_0	Noise power
P_e	Probability of error
Р	Power
P_k	Power of user k
q	Gain and phase adjustment factor
\hat{q}	Estimated gain and phase adjustment factor
$q_{k,m}$	Gain and phase adjustment factor for user k and subcarrier m
Q	Q function
Т	Bit duration time
<i>u_T</i>	Rectangular waveform
U	Decision signal
U_k	Decision signal for user k
x	Random error variable
у	Composite signal
$\mathcal{Y}_{k,m}$	Composite signal for user k and subcarrier m
Z	Regenerated signal
$Z_{k,m}$	Regenerated signal for user k and subcarrier m
β	Gain factor of the channel
$\hat{oldsymbol{eta}}$	Estimated gain factor of the channel
$oldsymbol{eta}_{k,m}$	Gain factor of the channel of user k and subcarrier m
ω	Carrier frequency
\mathcal{O}_m	Carrier frequency of subcarrier m
Ψ	Phase factor of the channel
Ŷ	Estimated phase factor of the channel
$oldsymbol{\psi}_{k,m}$	Phase factor of the channel of user k and subcarrier m
θ	Phase angle
$oldsymbol{ heta}_{k,m}$	Phase of user k and subcarrier m

ϕ	Received phase
$\hat{\phi}$	Estimated received phase
$\phi_{k,m}$	Receive phase of user k and subcarrier m
λ	Channel estimation error
λ'	Inphase component of channel estimation error
$\lambda^{\mathcal{Q}}$	Quadrature component of channel estimation error
ζ	Misalignment time
ζ_k	Misalignment time for user k
$\sigma_{\scriptscriptstyle{\lambda}}$	Variance of channel estimation factor
Ω	Other carrier interface phase
$\mathbf{\Omega}_{k,m,l}$	Other carrier interface phase of user k and subcarriers m and l
Γ	Signal to interference ratio
Γ_k	Signal to interference ratio of user k
η	Noise term
arY_b	Signal to noise ratio per bit
3G	Third Generation
4G	Fourth Generation
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CDMA	Code Division Multiple Access
DS-CDMA	Direct Spread CDMA
DSP	Digital Signal Processing
EGC	Equal Gain Combining
FFT	Fast Fourier Transform
IC	Interference Cancellation
ICI	Inter Carrier Interference
IFFT	Inverse Fast Fourier Transform
ISI	Inter Symbol Interference
MAI	Multiple Access Interference

MC-CDMA	Multi Carrier CDMA
MCMC	Markov Chain Monte Carlo
MLSE	Maximum Likelihood Sequence Estimation
MRC	Maximum Ratio Combining
MT-CDMA	Multitone CDMA
MUD	Multi User Detection
OFDM	Orthogonal Frequency Division Multiplexing
PC	Power Control
PIC	Parallel Interference Cancellation
S/P	Serial to Parallel
SIC	Successive Interference Cancellation
SINR	Signal to Interference and Noise Ratio
SIR	Signal to Interference Ratio

1. INTRODUCTION

1.1. Motivation

Third Generation (3G) mobile communication systems are already in deployment in several countries and this has enabled whole new ways to communicate, access information, conduct business and be entertained, liberating users from slow, cumbersome equipment and immovable points of access. In a way 3G has been the right bridge for mobile telephony and the internet. There are about two billion cellular subscribers worldwide, the majority of them using CDMA technology like 3G systems [1]. 3G services enable users to make video calls to the office and surf the internet simultaneously, or play interactive games wherever they may be. Second and 3G systems like EDGE, IS-95 and WCDMA can provide nominal data rates of about 50 - 384 Kbps. While 3G is just transforming itself into a reality from an engineer's dream, research efforts are already on to look into systems that can provide even higher data rates and seamless connectivity. Such systems are categorized under 4G and are predicted to provide packet data transmission rates of ten Mbps in outdoor macro-cellular environments and up to 100 Mbps in indoor and microcellular environments. While wide-band systems could be a natural choice to provide high data rates, service providers have to pay dearly for the spectrum necessary. Hence, spectrum efficiency is always a factor on the choice of any wireless technology. Very wide-band systems usually require complex receivers as the channel is frequency selective due to the presence of large number of resolvable multipaths.

Recently Orthogonal Frequency Division Multiplexing (OFDM) has gained a lot of attention and is a potential candidate for 4G systems. OFDM is very efficient in spectrum usage and is very effective in a frequency selective channel. Since recent improvements in Digital Signal Processing (DSP) and RF technologies, OFDM can provide higher data rates and is a very good choice for service providers to compete with wire-line carriers like DSL services. A variation of OFDM which allows multiple accesses is Multi-Carrier CDMA (MC-CDMA) which is essentially an OFDM technique where the individual data symbols are spread using a spreading code in the frequency domain [2].

Since the used frequency is same in CDMA systems all users are inherently interference source for the system. In other words, CDMA systems capacity is limited by multiple-access interference (MAI). So the power of the users are very important, when you lower or remove one of the user power from the system which also can be declared, removing from the received signal, results reduced interference effect.

Code division multiple access (CDMA) is a multiplexing technique where a number of users simultaneously and asynchronously access a channel by modulating and spread their information earing signals with preassigned signature sequences [3]. In CDMA systems all users within the same cell may transmit and receive at the same time and the users are distinguished by the unique codes addressed to them. Due to the multi path effect each users transmission becomes others interference. CDMA is interference limited system therefore tight power control is implemented the near far problem [2].

Instead of CDMA system disadvantages, because of nature of the multicarrier systems, they are weak to frequency offset and phase noise. Doppler shift between the transmitter and receiver and frequency jitter causes Inter Carrier Interference (ICI) which degrades the system performance unless appropriate compensation techniques are implemented.

To reduce the interference level in the system and to increase the capacity of CDMA systems, Multi User Detection (MUD) can be used but unfortunately industry has not yet adopted multiuser receivers to increase capacity [4, 5]. Despite of the complexity of MUD technique, much lower complexity solution approaches which also called Interference Cancellation is used [6]. There are two types of interference cancellation called Successive (Serial) Interference Cancellation and Parallel Interference Cancellation (PIC) for cancelling MAI.

SIC is regenerating users' signal after decoding and cancelling from received signal. Apart from traditional PIC systems also a new method introduced which use Gibbs Sampler for applying parallel interference cancellation. Basically Gibbs Sampler is estimating the user's data before decoding and cancelling from received signal to decode the desired user's data. Afterwards the decision is given with the help of a random number which is used with probability of decision for comparison.

In CDMA systems, the base station receives signals in the uplink from multiple users that interfere with each other since each user's code is orthogonal to the codes of the other users. The multiuser interference results in degradation in the received Signal to Interference Noise Ratio (SINR) and lowers the system capacity. If the receiver in the CDMA systems can perform interference cancellation, then the better results can be achieved in Bit Error Ratio (BER) in low Signal to Noise Ratios (SNR).

1.2. Interference Cancellation (IC)

Basically Interference cancellation is to remove multiuser interference from each user's received signal before giving decision for the users' data. There are two types of interference cancellation which are Successive Interference Cancellation (SIC) and Parallel Interference Cancellation (PIC). Also there are several ways to apply PIC to a system. In this thesis we deal with PIC which use Gibbs Sampler algorithm.

Successive Interference Cancellation is a simplest and a nonlinear type of Multiuser detection. In SIC, previous bit decisions are used to generate signal and cancel it from the received signal serially. First of all, all users are ordered according to their received signal power. Strongest user detected by conventional receiver. After detecting strongest user, its signal is regenerated using its chip sequence with the help of estimating of the channel. This regenerated signal is fed back to the detector of the second user and cancelled from the received signal. After cancellation of regenerated signal, second user decision is done according to new received signal. Whereas SIC helps cancelling interference caused by users in the system [7], also SIC is used to cancel other cell interference [8].

Afterwards, this new decision is used to regenerate second user spread signal and its' cancelled from the received signal which the first users' signal has already been cancelled before, for the third user decision. This algorithm is repeated until all the users' signals are decoded. The basic structure of SIC is shown in Figure 1.1. In this algorithm each user is effected different interference level. In this thesis we apply SIC to MC-CDMA system.



Figure 1.1. Successive interference canceller structure

PIC is different from SIC, to decode each users' data, remaining users' data are removed from the received signal simultaneously. In this process all users are effected same amount of interference [9]. The early researches that dealt with PIC recognized the desire to arrive at a structure that could be motivated by the maximum-likelihood approach. Also in different researches, a multistage iterative approach was suggested which at a given stage estimated a given user's bits in the same transmission interval needed to compute the multiuser interference could be replaced by estimates of these bits from the previous stage [10, 11, 12, 13, 14]. The basic PIC structure is shown in Figure 1.2. In traditional PIC algorithm, the bits which are used to cancellation in any stage are obtained from previous stage.



Figure 1.2. Basic L-stage PIC structure

Recently new statistical approached are examined in researches which is called Gibbs Sampler. Gibbs Sampler is used in different areas like economic approaches. In this thesis we used Gibbs Sampler for IC in MC-CDMA which is between SIC and PIC, and will be discussed in detail in the next sections. Shortly IC using Gibbs sampler, cancel all other users' interference from the desired user like PIC but apply this cancellation in order like SIC and there is an update of the bits during the interference cancellation in order. The Gibbs Sampler [15] is a Markov-chain Monte Carlo (MCMC) method that is often used in statistics work [16].

Interference cancellation schemes have advantages and disadvantages over themselves. Since SIC and kind of serial interference cancellation with Gibbs sampler is chosen in this thesis, we will focus on advantageous and disadvantageous of them. There are two major caveats to SIC which keeps it from becoming the method of choice for designing CDMA systems like MC-CDMA system. One of the caveats is that the decoding time increases approximately linearly with the number users in SIC. This caveat also exists in interference cancellation using Gibbs sampler but in addition, this time is increase also with the increase of the number of stage. Second one is in SIC, the estimation of the amplitude and phase of the users need to be accurate. However, in the interference cancellation using Gibbs Sample it does not need to be more accurate.

Despite of this caveats of SIC, there is also a major advantage of both interference cancellation schemes when performed at the receivers, a great deal of hardware is saved since there is only one decoder for all users in both SIC and interference cancellation with Gibbs sampler. Also there is a power save in both interference cancellation schemes. In the comparison of two interference cancellation scheme, while SIC needs more accurate amplitude and phase estimation, interference cancellation with Gibbs samples spend more time to decode all users. However, for both of them, there is only one decoder for all users and hardware save.

1.3. Outline of the Thesis

The rest of the Thesis is organized as follows: CDMA schemes are introduced in Section 2 which DS-CDMA, MC-CDMA, Multicarrier DS-CDMA and MT-CDMA. Section 3 introduced the MC-CDMA system model like transmitter, channel and main part receiver model which are analyzed in this thesis. Introduced MC-CDMA system, actually different receiver models depending on the interference cancellation schemes which are Successive Interference Cancellation and Interference Cancellation using Gibbs Sampler is analyzed in two different diversity techniques as Equal Gain Combining and Maximum Ratio Combining in Section 4. Section 5 present simulation results and simulation analysis of two models using different interference cancellation schemes BER performance. Section 6 concludes the thesis.

2. MULTI-CARRIER CDMA SCHEMES

It was in 1993, an epoch of Code Division Multiple Access application, that three types of new multiple access schemes based on a combination of code division and OFDM techniques were proposed, such as "Multi-Carrier CDMA", "Multicarrier Direct Spread (DS)-CDMA" and "Multitone CDMA (MT-CDMA)" [17]. These schemes were proposed, namely, MC-CDMA for example by N. Yee, J-P. Linnartz and G. Fettweis [18] and K. Fazel and L. Papke [19], Multicarrier DS-CDMA by V. Dasilva and E.S. Sousa [20] and MT-CDMA by L. Vandenporpe [21].

The Multi-Carrier CDMA schemes are mainly categorized into two groups. One spreads the original data stream using a given spreading code, and then modulates a different subcarriers with each chip (in a sense, the spreading operation in the frequency domain) [18], and other spreads the serial-to-parallel (S/P) converted data streams using a given spreading code, and then modulates a different subcarrier with each data stream (the spreading operation in the time domain) [20, 21], similar to a normal DS-CDMA scheme.

2.1. DS-CDMA Scheme

DS-CDMA transmitter spreads the original data stream using a given spreading code in the time domain. The capability of suppressing multi-user interference is determined by the cross-correlation characteristic of the spreading codes. The capability of distinguishing one component from other components in the composite received signal is determined by the auto-correlation characteristic of the spreading codes.

In Figure 2.1 the transmitter and power spectrum of transmitted signal of DS-CDMA scheme and receiver part are shown.



Figure 2.1. DS-CDMA scheme transmitter (a), power spectrum of transmitted signal (b) and rake receiver (c)

2.2. MC-CDMA Scheme

MC-CDMA is a combination of frequency domain spreading and multi-carrier modulation which transmitter spreads the original data stream over different subcarriers using a given spreading code in the frequency domain [18]. In a (synchronous) downlink mobile radio communication channel, we can use the Hadamard Walsh codes as an optimum orthogonal set, because we do not have to pay attention to the auto-correlation characteristic of the spreading code. In this thesis also we use Hadamard Walsh codes for the orthogonality.

In Figure 2.2 the transmitter and power spectrum of transmitted signal of MC-CDMA scheme and receiver part for Binary phase Shift Key (BPSK) scheme are shown. The number of subcarriers is four therefore the processing gain $G_{MC} = 4$. In this thesis we will apply interference cancellation techniques this scheme which is also called uncoded MC-CDMA scheme [7].



Figure 2.2. MC-CDMA scheme transmitter (a), power spectrum of transmitted signal (b) and receiver (c)

The transmitted signal of the j-th user in MC-CDMA can be written as:

$$S_{j}(t) = \sum_{i=-\infty}^{\infty} \sum_{m=1}^{G_{MC}} a_{j}[i] p_{s}(t-iT_{s}) \Re\{c_{j,m}e^{j(W_{m}t+\theta_{j,m})}\}$$
(2.1)

$$p_{s}(t) = \begin{cases} 1 & 0 \le t \le T_{s} \\ 0 & else \end{cases}$$
(2.2)

2.3. Multicarrier DS-CDMA Scheme

Multicarrier DS-CDMA is one of the combination of Time domain spreading and multicarrier modulation which its transmitter spreads the S/P converted data streams using

a given spreading code in the time domain so that the resulting spectrum of each subcarrier can satisfy the orthogonality condition with the minimum frequency separation [20]. This scheme can lower the data rate in each subcarrier so that a large chip time makes it easier to synchronize the spreading sequences. The Multicarrier DS-CDMA scheme is originally proposed for an up-link communication channel, because this characteristic is effective for establishment of a (quasi-) synchronous channel.

In Figure 2.3 the transmitter and power spectrum of transmitted signal of Multicarrier DS-CDMA scheme and receiver part for Binary phase Shift Key scheme are shown.



Figure 2.3. Multicarrier DS-CDMA scheme transmitter (a), power spectrum of transmitted signal (b) and receiver (c)

2.4. MT-CDMA Scheme

MT-CDMA is one of the combination of Time domain spreading and multicarrier modulation which its transmitter spreads the S/P-converted data streams using a given spreading code in the time domain so that the spectrum of each subcarrier before spreading operation can satisfy the orthogonality condition with the minimum frequency separation [21]. Therefore, the resulting spectrum of each subcarrier no longer satisfies the orthogonality condition. The MT-CDMA scheme uses longer spreading codes in proportion to the number of subcarriers, as compared with a normal (single carrier) DS-CDMA scheme, therefore, the system can accommodate more users than the DS-CDMA.

In Figure 2.4 the transmitter and power spectrum of transmitted signal of MT-CDMA scheme and receiver part for Binary phase Shift Key scheme are shown.



(c)

Figure 2.4. MT-CDMA scheme transmitter (a), power spectrum of transmitted signal (b) and receiver (c)

3. SYSTEM MODEL OF THE MC-CDMA

MC-CDMA system can be analyzed in two different models. One system is a more realistic and higher performance design. This system uses low-rate super orthogonal codes for spreading, with each coded symbol placed on a subcarrier rather than simple replicates of each bit. The decoding, and hence, subcarrier combining, is implemented using maximum-likelihood sequence estimation (MLSE) with a Viterbi decoder, rather than an integrator and threshold check. Further, a low-complexity inverse fast Fourier transform (IFFT) is used like OFDM for demodulation.

Second system model is also used in this thesis is uncoded system similar to that analyzed by J.G. Andrews and H. Y. Meng [7], X. Gui and T. S. Ng [22], and also E. A. Sonour and M. Makagawa [23] used in their works. The details of this system will be analyzed in the rest of this thesis report.

In order to attain analytical results for BER performance, and to compare meaningfully with previous work, the uncoded system is used for the analysis. In the BER analysis two interference cancellation schemes is used. In SIC scheme power control is used which results different Signal to Noise Ratio (SNR) and capacity increase. On the other hand in the system with interference cancellation using Gibbs Sampler all the user have same power and same SNR since a power control is used according to the near far problem but not because of the experienced interference.

In addition, in the system analyzed in this thesis, similar to J.G. Andrews and H. Y. Meng [7], differs to X. Gui and T. S. Ng [22] and T.Schmidl and A. Gatherer, X. Wang, R. Chen [16] imperfect channel estimation is assumed for both cases, SIC and interference cancellation using Gibbs sampler. For Interference cancellation with Gibbs Sampler used in CDMA, the performance of the system is analyzed with perfect channel estimation and synchronized. In the system analyzed in this thesis the system is not synchronized.

In general BPSK is used in MC-CDMA in researches as modulation technique in their researches. But also there exist researches which use different modulation technique and calculate the performance like MSK [24] and quasi-DPSK [25]. With the helping of some modulation techniques like DPSK, also good channel estimate is possible. In CDMA systems, to increase data speed, different kind of modulations are used, even in commercial CDMA systems like WCDMA. The binary phase shift keying (BPSK)-modulated signal is quadrature spread over the sine and cosine channels in order to allow better suppression of other-user interference [26] in both systems.

3.1. Transmitter

The transmitter for MC-CDMA is shown in Figure 2.5.



Figure 3.1. MC-CDMA transmitter

It can be easily obtained from Figure 3.1 the transmitted MC-CDMA signal where K is the number of users, M is the number of subcarrier and n denotes different bit sequence can be written as:

$$S_{k}(t) = \sqrt{P_{k}} \sum_{n=\infty}^{\infty} \sum_{m=1}^{M} b_{k}[n] u_{T}(t-nT) \Re\{c_{k,m}[n]e^{j(W_{m}t+\theta_{k,m})}\}$$
(3.1)

And we can rewrite transmitted signal:

$$S_{k}(t) = \sqrt{P_{k}} \sum_{n=\infty}^{\infty} \sum_{m=1}^{M} b_{k}[n]_{\mathcal{U}_{T}}(t-nT) \{ c_{k,m}^{T}[n] \cos(W_{m}t + \theta_{k,m}) - c_{k,m}^{Q}[n] \cos(W_{m}t + \theta_{k,m}) \}$$
(3.2)

In the transmitted signal equation, P_k is the transmit power of the kth user, $b_k[n]$ is the data bit of user k at time n, $c_{k,m} = c_{k,m}^I + c_{k,m}^Q$ is the complex *I* and *Q* spreading sequence for user k on subcarrier m in radians per second.

Since spreading sequences overlap both in the time and frequency domains, yet their crosscorrelation or inner product is zero [27]. It can be shown that the single user synchronization in a MC-CDMA system does not depend of the autocorrelation or other properties of spreading sequence [28], what is an advantage compared with conventional CDMA systems. However, the mutual interference between asynchronous users in the MC-CDMA systems does depend on the properties of spreading sequences, but the ordinary crosscorrelation function between the discrete sequences is not a proper measure of the mutual interference, as it is in the conventional DS-CDMA [29, 30].

Also $\theta_{k,m}$ is the phase for user k's *m*th subcarrier independent and is identically distributed (i.i.d) and uniformly distributed in $[0,2\pi)$, *T* is the symbol interval, or equivalently, the bit time, \Re {.} denotes the real part of the complex number, and $u_T(t)$ is defined as a rectangular waveform for the purpose of isolating successive symbols.

$$u_T(t) = \begin{cases} 1 & 0 \le t \le T_s \\ 0 & elsewhere \end{cases}$$
(3.3)

In this model, for the analysis, there are some assumptions. It is assumed that the symbol interval T is equal to the bit time as mentioned above, since it is assumed that the spreading factor is the same as the number of subcarriers. In general, this is not required, but this assumption made throughout the thesis in order to clarify the analysis. More bits per MC-CDMA symbol can be easily accommodated by simply extending the number of

subcarriers by an integer multiple of the spreading factor, as shown above and in [2] and [22]. This simplification does not change any of the analytical results.

Another assumption is that $\theta_{k,m}$ is i.i.d. uniform in $[0,2\pi)$ for all m is somewhat artificial. This assumption is also very common and made by [7], [2] and [22] to simplify the analysis and the simulation results shows that it is justified. Also in practice the spreading sequence is pseudorandom with a deterministic period. Because this period may be arbitrarily long in the proposed system, $c_{k,m}$ are assumed to be i.i.d. for all k,m, and for the I and Q channels.

3.2. Channel Model

In the study of communication systems the classical (ideal) additive white Gaussian noise (AWGN) channel, with statistically independent Gaussian noise samples corrupting data samples free of intersymbol interference (ISI), is the usual starting point for understanding basic performance relationships. The primary source of performance degradation is thermal noise generated in the receiver. Often, external interference received by the antenna is more significant than thermal noise. This external interference can sometimes be characterized as having a broadband spectrum and is quantified by a parameter called antenna temperature [31]. The thermal noise usually has a flat power spectral density over the signal band and a zero-mean Gaussian voltage probability density function (pdf).

For most practical channels, where signal propagation takes place in the atmosphere and near the ground, the free space propagation model is inadequate to describe the channel and predict system performance. In a wireless mobile communication system, a signal can travel from transmitter to receiver over multiple reflective paths; this phenomenon is referred to as multipath propagation. The effect can cause fluctuations in the received signal's amplitude, phase, and angle of arrival, giving rise to the terminology multipath fading. Another name, scintillation, which originated in radio astronomy, is used to describe the multipath fading caused by physical changes in the propagating medium, such as variations in the density of ions in the ionospheric layers that reflect high-frequency (HF) radio signals. Both of the names, fading and scintillation refer to a signal's random fluctuations or fading due to multipath propagation [32]. The main difference is that scintillation involves mechanisms (e.g., ions) that are much smaller than a wavelength. The end-to-end modeling and design of systems that mitigate the effects of fading are usually more challenging than those whose sole source of performance degradation is AWGN [33].

MC-CDMA systems are generally subject to frequency selective fading. By selecting a proper number of subcarriers, it can be guaranteed that each subcarrier experiences flat fading. Then, the channel gain for each subcarrier can be modeled by a zero-mean complex Gaussian random variable (RV), or equivalently, Rayleigh fading envelope and a random phase. Unfortunately, this also means that in practical systems adjacent subcarriers are subject to correlated fading because the frequency separation between them is less than the coherence bandwidth of the channel. The correlation coefficients depend on the frequency separation between subcarriers with respect to the coherence bandwidth of the channel [34, 35].

Depending on the surrounding environment, a transmitted radio signal usually, propagates through several different paths before it reaches the receiver. If there is no line-of-site between the transmitter and the receiver the attenuation coefficients corresponding to different paths are often assumed to be independent and identically distributed, in which path gain has a uniformly distributed phase and Rayleigh distributed magnitude [36].

In the thesis, for analysis, the channel is assumed to be a frequency selective Rayleigh fading channel. The main reason and motivation for using MC-CDMA is to allow a frequency selective fading channel to appear as flat fading on each subcarrier, assuming the number of subcarriers is sufficiently large. With this assumption, each subcarrier experiences a complex flat fading channel, which can be written as:

$$h_{k,m}(t) = h_{k,m}^{I}(t) + jh_{k,m}^{Q}(t) = \beta_{k,m}(t)e^{j\psi_{k,m}(t)}$$
(3.4)

which is a zero mean complex Gaussian random variable. Also it is assumed that $h_{k,m}(t)$ is uncorrelated and identically distributed for different k and m. This is a slight simplification over a real channel, which would be correlated in frequency, but typically the difference in performance for correlated modeling is small [22].

In communication systems, different types of channel estimation are suggested. The most popular method of performing channel estimation in wireless channels is to use pilot symbols. For MC-CDMA also there are some researches like [37] and [38]. However, channel estimation methods and estimation procedures are not under the concern of this thesis.

In this thesis, it is assumed that the channel estimation which $h_{k,m}(t)$ is not perfect. Since imperfect estimation is used the channel combining factor which will be introduced in system analysis section, $\hat{q}_{k,m}$ which also is based on $h_{k,m}(t)$, will not be exact and have deviation.

Second effect of imperfect channel estimation will be on the interference cancellation schemes. Since the channel, $h_{k,m}(t)$, estimated imperfectly, the users' bit will regenerated with a deviation which results error in cancellation of interference after regeneration. This fraction of cancellation error is denoted as ε_k which also means that a fraction ε_k of each received user's power will remain in the composite signal.

Since the interference cancellation is imperfect due to imperfect channel estimation, power control algorithm in SIC is effected which also will be explained in the system analysis section. This cancellation error, ε_k , normally is not known but because of this problem it must be considered in power control algorithm in SIC. In the system using interference cancellation with Gibbs Sampler, it will not be considered because of power control is not exist but it has effect on interference cancellation procedure.

For both the analytical and simulation results, errors are assumed (analytical) or induced (simulations) in the estimates of the received signal for user k, regenerated signal.

There are few assumptions for the estimation error like linear power and log power estimations and distributions [39]. The channel estimation error is assumed to follow a lognormal distribution with a mean of unity, and estimated channel with error can be formulized as:

$$\hat{h}_{k,m} = h_{k,m}^{I} \cdot \lambda_{k,m}^{I} + j h_{k,m}^{Q} \lambda_{k,m}^{Q}$$
(3.5)

and the lognormal distributed estimation error can be written as:

$$\lambda_{k,m}^{I,Q} = e^x, \qquad \mathbf{x} \sim N(0, \sigma_\lambda^2) \tag{3.6}$$

where with x, and hence, $\lambda_{k,m}^{I,Q}$ i.i.d. for k, m and the *I* and *Q* branches. Also, it is assumed that $\lambda_{k,m}^{I,Q}$ is unknown, although in some cases, the statistic σ_{λ}^2 may be known.

To sum up, $\lambda_{k,m}^{I,Q}$ is channel estimation error, $h_{k,m}(t)$ is the formulization of the real channel and is not known by the receiver and $\hat{h}_{k,m}$ is the formulization of the estimated channel with error. Only $\hat{h}_{k,m}$ is known by the receiver in this thesis which results error in interference cancellation.

3.3. Receiver Model

In MC-CDMA, the interference cancellation schemes haven't got any effect to the Transmitter and Channel model, but the receiver model will change according to the used interference cancellation scheme. In the transmitter part for the system SIC used, the power of the users changed but this is not effect any change to the transmitter model, only the transmitted powers changed. The mainly change is because of SIC is serial interference cancellation scheme and interference cancellation with Gibbs sampler is parallel interference cancellation scheme. That's why Receiver structure and model is different for two types of interference cancellation scheme.

Even the receiver structures are different; the received signal for both interference cancellation schemes is same. The transmitted signal which is shown in (3.2) and (3.3) will enter the channel which is described in section 3.2. Afterwards the received signal will be as shown in Equation (3.7), in the receiver.

$$r(t) = \sum_{n=-\infty}^{\infty} \sum_{k=1}^{K} \sqrt{P_{k}} \sum_{m=1}^{M} \beta_{k,m}(t) b_{k}[n] u_{T}(t - nT - \zeta_{k}) *$$

$$\Re\{c_{k,m}[n]e^{j(W_{m}^{t} + \phi_{k,m}(t))}\} + \eta(t)$$
(3.7)

In the received signal, ζ_{k} is the relative time misalignment of user k, and it is i.i.d. for different k and uniformly distributed over the interval [0,T) and the phase of the received signal can be written as (3.8),

$$\phi_{k,m}(t) = \psi_{k,m}(t) + \theta_{k,m} - \omega_m \zeta_k$$
(3.8)

 $\eta(t)$ is Additive White Gaussian Noise (AWGN) with two sided power spectral density (PSD) N₀/2.

For each user, all the multicarriers are used in the MC-CDMA system. Depending on the type of the interference cancellation scheme, one by one or all other users signal is cancelled from the composite signal.

3.3.1. Receiver Model for SIC

SIC approach is based on a simple and natural ides: if a decision has been made about an interfering user's bit, then that interfering signal can be recreated at the receiver and subtracted from the received waveform. This will cancel the interfering signal provided that the decision was correct; otherwise it will double the contribution of the interferer. Once the subtraction has taken place, the receiver takes the optimistic view that the resulting signal contains one fewer user and the process can be repeated with another interferer, until all but one user have been demodulated [27]. In the receiver model of the MC-CDMA with SIC, each users signal must be estimated and subtracted out from the composite signal before decoding next user. The receiver model for MC-CDMA that SIC used is shown in Figure 3.2.



Figure 3.2. MC-CDMA receiver structure with SIC

As shown in Figure 3.2 after giving a decision for one user's bit, that bit is used to regenerate the decoded user's signal and cancelled from the composite signal and update the composite signal. Afterward next user's decision will be given from new updated composite signal and this algorithm will go on till the last user's decision is given. The important thing all users decoding order will be done according to their power which means high power user will decoded first.

Since the user's signal regenerated and cancelled from the composite signal, first user will affected from multiple access interference (MAI) much more than the last user affected. First user will decoded as like no interference cancellation scheme is used but since power control scheme is used in successive interference cancellation scheme, first user will affected less MAI than the system which no interference cancellation scheme is used.

Since the detection is successive process, with each user decoded in turn. After decoding the kth user, the estimated bits $\hat{b}_k(t)$ are re-encoded to form an estimate of the received signal for that user. This process will go on till the last user encoded. The re-encoded signal after estimation can be written as (3.9) where $\hat{h}_{k,m}$ is the estimation of the channel is.

$$z_{k,m} = \sqrt{P_{k}} \Re\{\hat{h}_{k,m} c_{k,m} \hat{b}_{k} e^{j(W_{m}(t-\zeta_{k})+\theta_{k,m})}\}$$
(3.9)

This re-encoded signal $z_{k,m}$ subtracted from the current composite signal which can be seen in (3.10), Y_k , to form a cleaner signal that can be used to find the bits for user "k+1". In general for each user the composite signal used for detection in MC-CDMA is:

$$\mathbf{Y}_{k}(t) = \left[y_{k,1}(t), y_{k,2}(t), y_{k,3}(t), y_{k,4}(t), \dots, y_{k,M-1}(t), y_{k,M}(t) \right]^{T}$$
(3.10)

which shows all the components coming from all subcarriers assuming that there are M subcarriers. In the first user decoding process no detection and interference cancellation takes place so for the first user to be decoded, the composite signal is simply written as vector product, shown in (3.11).

$$\mathbf{Y}_{1}(t) = r(t)\mathbf{1}_{M} \tag{3.11}$$

where $\mathbf{1}_{M}$ is an M-vector of ones. It is also means that for the first user the received signals from all subcarriers are used as they are received. The interference cancellation procedure begins in the decoding process of the second users. In general, the composite signal used for detection for user *k* in MC-CDMA using SIC is described by its subcarriers as shown in (3.12)

$$y_{k,m}(t) = y_{k-1,m}(t) - z_{k-1,m}(t), \qquad k \ge 2, \forall m$$
(3.12)

where $z_{k,m}(t)$ is an estimate of the received signal for user k on subcarrier m which is estimated after the decoding user k is shown in (3.9). Hence, at each stage, the interference of the last decoded user is subtracted out of the signal, so that the next user experiences less total interference.

The received signal is processed successively, and the composite signal is used for giving decision. The (3.13) is used as decision statistic for user k.

$$U_{k} = \frac{1}{T} \int_{0}^{T} \sum_{m=1}^{M} y_{k,m} \Re\{\hat{q}_{k,m} c_{k,m}^{*} e^{-j(W_{m}(t+\zeta_{k})+\theta_{k,m})}\} dt$$
(3.13)

where $\hat{q}_{k,m}$ is a complex adjustment for phase in Equal Gain Combining (EGC) and for phase and amplitude in Maximum Ratio Combining (MRC). Adjustment for phase and amplitude is discussed in more detail in the analysis of the system section. Since $\hat{q}_{k,m}$ is dependent on imperfect channel estimates, it too is assumed to be imperfect and subject to the same error model as $\hat{h}_{k,m}$ which is discussed in the Channel model section.

Since we are using BPSK as modulation scheme, the sign of the decision statistic is used for the result in two interference cancellation schemes. This decision statistic result is used to re-encode the signal of the decoded user. Also this decision is used as a last result in the receiver for the users so only one stage is applied for one user in SIC different from the interference cancellation using Gibbs sampler scheme.

3.3.2. Receiver Model for Interference Cancellation with Gibbs Sampler

One well known method of interference cancellation at the receiver of the CDMA systems is parallel interference cancellation (PIC) with total or partial cancellation in each stage. In PIC with partial cancellation in each stage, tentative decisions are made for each of users, and then the interference is cancelled either by regenerating the interference at the

chip level or computing crosscorrelations at the symbol level and subtracting off the crosscorrelations.

Partial cancellation is performed in each stage, with the fraction of interference to be cancelled in each stage determined by exhaustive search over all the possible cancellation coefficients. Partial cancellation results in improved performance over full cancellation since the effect of biases in the decision can be reduced [40]. Interference cancellation with Gibbs Sampler is similar to parallel interference cancellation scheme with partial cancellation in each stage and its own properties which is between PIC and SIC.

A partial interference cancellation philosophy, in which the amount of interference canceled is related to the fidelity of the tentative decisions involved in forming the interference estimate, is in general superior to a brute force philosophy of entirely canceling the interference at each stage. After a number of stages, the final decision will be given.

In the receiver model of the MC-CDMA with interference cancellation with Gibbs Sampler, each users signal must be estimated and subtracted out from the composite signal parallel but in serial form with estimation of the other users signal and decoded users' signal before decoding next user. The receiver model for MC-CDMA that interference cancellation with Gibbs sampler used is shown in Figure 3.3.



Figure 3.3. MC-CDMA receiver structure with IC with Gibbs sampler

The Gibbs Sampler [15] is a Markov-chain Monte Carlo (MCMC) method that is often used in statistics. Bayesian inference of the unknown data is made from the noisy received signals. The idea is to generate ergodic random samples from the joint posterior distribution of all the unknowns which is the user bits and then to average the appropriate samples to obtain estimates of the unknown quantities.

As shown in Figure 3.3 after giving a decision for one user's bit, that bit is used to regenerate the decoded user's signal and update the composite signal which is subtracted from the received signal to decode the next user. Next user decision will be given afterwards. The composite signal which is used to cancel the interference from the other users is updated in a stage. Interference cancellation using Gibbs Sampler is different from conventional parallel interference cancellation scheme shown in Figure 1.2.

In the interference cancellation with Gibbs Sampler scheme, the all users' interferences are subtracted from the composite signal like conventional parallel
interference cancellation scheme, but the user's bits are decoded in sequence in one stage like serial interference cancellation scheme, which gives an advantage to use the decoded user's bit decision in the next user in the interference cancellation. In other word, the bit decisions which are used in interference cancellation are updated with the new bit decision in the same stage which is shown in Equation (3.18).



Figure 3.4. kth stage of total interference cancellation

As shown in Figure 3.4, the kth stage of the total PIC which parallel processing of multiuser interference simultaneously removes from each user the interference produced by the remaining users accessing the channel [40]. In conventional total parallel interference cancellation scheme, to cancel the interference in the re-generated signal, the decision bits which are received in the previous stage used. Also partial parallel interference cancellation scheme is discussed in [40]. However, in the interference cancellation using Gibbs Sampler algorithm is different which can be seen in Figure 3.5.



Figure 3.5. Block diagram of interference cancellation with Gibbs Sampler

As shown in Figure 3.5, the block diagram of the Gibbs sampler method of interference cancellation. Either the complex samples or the despread symbols are stored in memory. The receiver initializes the estimates of the received symbols by choosing them randomly different from SIC. In other word it is not needed to start from the user with the highest power. As it can be seen from Figure 3.3 and Figure 3.5, interference cancellation with Gibbs Sampler scheme is an interference cancellation scheme between serial interference cancellation and parallel interference cancellation.

From Figure 3.2, for each user k in stage n, the composite signal used for detection is:

$$\mathbf{Y}_{k}^{(n)}(t) = \left[y_{k,1}^{(n)}(t), y_{k,2}^{(n)}(t), y_{k,3}^{(n)}(t), y_{k,4}^{(n)}(t), \dots, y_{k,M-1}^{(n)}(t), y_{k,M}^{(n)}(t) \right]^{T}$$
(3.14)

For the first user to be decoded, the composite signal is simply vector product as shown in Equation (3.15):

$$\mathbf{Y}_{1}^{(n)}(t) = r(t)\mathbf{1}_{M} - z_{M}^{(0)}$$
(3.15)

where $\mathbf{1}_{M}$ is an M-vector of ones and $z_{M}^{(0)}$ is an M-vector of the sum of regenerated signal with the estimated bits for all users except first user. It is also means that in the first stage for the first user the interference cancellation is applied with the all estimated initial values. The interference cancellation procedure begins in the first user process different from SIC. In general, the composite signal used for detection for user *k* in MC-CDMA using interference cancellation with Gibbs sampler is described by its subcarriers as shown in (3.16)

$$y_{k,m}^{(n)}(t) = r(t) - \sum_{i=1}^{k-1} z_{i,m}^{(n)}(t) - \sum_{i=k+1}^{K} z_{i,m}^{(n-1)}(t), \qquad \forall k, \forall m$$
(3.16)

where $z_{k,m}^{(n)}(t)$ is an estimate of the received signal for user *k* on subcarrier m in stage n which is estimated after the decoding user *k* is shown in (3.17).

$$z_{k,m}^{(n)} = \sqrt{P_{k}} \Re\{\hat{h}_{k,m} c_{k,m} \hat{b}_{k}^{(n)} e^{j(W_{m}(t-\zeta_{k})+\theta_{k,m})}\}$$
(3.17)

With Equation (3.16), it can be easily seen that, the interference cancellation is applied to all users in all stages. If any of the user bit decision is given before decoding the interested user in the same stage, these new bit decisions are used in interference cancellation in the decoding process of the next user. There is always an update process in the re-generated interference signal used for interference cancellation in every stage. This will result that interference cancellation with Gibbs sampler scheme is an interference cancellation scheme between SIC and PIC.

Since all other users' signals are re-generated and cancelled form the composite signal during the decoding process of the desired user in one stage, all users experienced almost the same interference. Because of this, no power control is needed for different amount of experienced interference and all users will have the same signal to noise ratio. For the near far problem which is critical problem for CDMA systems, a power control is used to adjust the users power according to the channel. As a result, the received signal is processed as described, yielding the following decision statistic for user k in the n'th stage which is almost same with the SIC scheme:

$$U_{k}^{(n)} = \frac{1}{T} \int_{0}^{T} \sum_{m=1}^{M} y_{k,m}^{(n)} \Re\{\hat{q}_{k,m} C_{k,m}^{*} e^{-j(W_{m}^{(t+\zeta_{k})})+\theta_{k,m}}\} dt$$
(3.18)

where, as the same channel assumed also in this interference cancellation with Gibbs sampler scheme, $\hat{q}_{k,m}$ is a complex adjustment for phase in Equal Gain Combining (EGC) and for phase and amplitude in Maximum Ratio Combining (MRC). Adjustment for phase and amplitude is discussed in more detail in the analysis of the system section. Since $\hat{q}_{k,m}$ is dependent on imperfect channel estimates, it too is assumed to be imperfect and subject to the same error model as $\hat{h}_{k,m}$ which is discussed in Channel model section.

Since we are using BPSK as the modulation scheme also in the interference cancellation with Gibbs sampler scheme, the sign of the decision statistic is used for the result in two interference cancellation schemes. This decision statistic result is used to reencode the signal of the decoded user. Also this decision is used as an intermediate result in the receiver in the intermediate stages for the users in the interference cancellation using Gibbs sampler scheme. The last decision is given in the last decision stage which the decisions converges to the same decisions

4. THE MC-CDMA SYSTEM ANALYSIS

In the analysis of the system two different diversity techniques can be used since it is a multicarrier system. These two diversity techniques are Equal Gain Combining and Maximum Ratio Combining which are described below.

- Equal gain combining (EGC): The receiver corrects the phase rotation of the received signals caused by the fading channel and combines the received signals of different paths with equal weight. In MC-CDMA systems, the phase difference caused by the channel is estimated only. In this technique there is no deal with power changes of the received signal. The EGC strategy combines subcarriers at their received amplitudes, adjusting only the phase of the subcarriers.
- Maximum ratio combining (MRC): The optimum way (in the sense of the least BER) to use information from different paths to achieve decoding in an additive white Gaussian channel (AWGN). The receiver corrects the phase rotation caused by a fading channel and then combines the received signals of different paths proportionally to the strength of each path. Since each path undergoes different attenuations, combining them with different weights yield an optimum solution under an AWGN channel. In MC-CDMA systems, the phase and the power difference need to be estimated. In MRC, strong subcarriers are additionally amplified in proportion to the gain of the subcarrier. This technique maximizes the signal to noise ratio (SNR) at detection [41, 42]. However, this technique is not feasible and very hard to implement in the real systems.

In this thesis, two interference cancellation schemes are analyzed in both diversity technique, EGC and MRC. Since MRC technique is not feasible, most of the calculations are made in EGC mode but some calculations will be given also in MRC mode. As mentioned above the only difference between the two techniques is the estimation of the power of the subcarriers.

4.1. MC-CDMA System with SIC in EGC

Firstly the SIC will be considered and analyzed in EGC technique. As mentioned above in EGC technique only the phases of the subcarriers are estimated and combined in their received amplitudes. Since the system is analyzed in EGC technique, first of it is needed to mention the gain and the phase adjustment factor which is use in the receiver for the decision in SIC and is written in Equation (3.13). For EGC the gain and phase adjustment factor is:

$$q_{k,m} = e^{-j\psi_{k,m}}$$
(4.1)

where,

$$\Psi_{k,m} = \arctan(h_{k,m}^Q / h_{k,m}^I) \tag{4.2}$$

which means perfect channel estimation is used and the gain and phase adjustment factor is the same of the real channel effect. However, in the system analysis, it is assumed that the channel estimation is imperfect which implies that the imperfect subcarrier combining factor is:

$$\hat{q}_{k,m} = e^{-j\hat{\psi}_{k,m}}$$
 (4.3)

where,

$$\hat{\psi}_{k,m} = \arctan(\lambda_{k,m}^{Q} h_{k,m}^{Q} / \lambda_{k,m}^{I} h_{k,m}^{I})$$
(4.4)

is the imperfect channel estimation and phase estimate. Hence there is no added amplitude error for subcarrier combining in EGC, only phase error, since no amplitude adjustments are made. In SIC scheme, the power control (PC) algorithm, distribution of $\{P_k\}$ is explicitly designed to be robust to estimation error, and sets the signal to interference plus noise ratio (SINR) to be equivalent for all *K* users at decode time in presence of estimation error, as is mentioned and discussed more detailed in the next section. This causes the Bit Error Rate's (BER) to be approximately equivalent for all users, which fact allows the BER analysis to be carried out on the first user without loss of generality [43, 44]. BER equivalency of different users will be demonstrated in the simulation section.

The decision equation for user k in the receiver where SIC is used as interference cancellation scheme can also is written as:

$$U_k = D_k + \eta + I_k + J_k \tag{4.5}$$

and divided to the four different parts where D_k is the desired signal for user k, I_k and J_k are the same and other carrier interference experienced by user k, respectively, and η is AWGN term with variance $N_o M / 2T$ where T is the bit duration in the system. However, the analysis of the system for user k is a bit difficult, so for the analysis, we will use first user and the Equation (4.5) will rewritten for user one as:

$$U_1 = D_1 + \eta + I_1 + J_1 \tag{4.6}$$

where every part is same with the Equation (4.5) but for the first user. So with this division we can analyze each part independent from the other parts.

Before starting to analyze, it is assumed that bit duration is t = (0,T] and the channel is constant for this short duration. Since the bit duration is t = (0,T], the first user bit is assumed $b_1(0)$ in this period. So with this assumption and division to the parts we can write the decision equation for the first user as:

$$U_{1} = \int_{0}^{T} r(t) \sum_{l=1}^{M} \Re\{\hat{q}_{1,l} c_{1,l}^{*} e^{-j(\omega_{l}(t-\zeta_{1})+\theta_{1,l})}\} dt$$
(4.7)

where r(t) is the pure received signal since no interference cancellation is used yet and $\hat{q}_{1,l}$ is the imperfect subcarrier combining factor is changing according to the used diversity technique but this equation is valid for EGC and MRC. Other symbols are explained in the system model section. Equation (4.7) can be divided into the parts as shown in Equation (4.6).

$$U_{1} = \int_{0}^{T} \eta(t) \sum_{l=1}^{M} \Re\{\hat{q}_{1,l} c_{1,l}^{*} e^{-j(\omega_{l}(t-\zeta_{1})+\theta_{1,l})}\} dt + (D_{1} + J_{1} + I_{1})$$
(4.8)

Where the first term denoted by η is due to the Gaussian noise, D_1 is the contribution of the desired user, and I_1 and J_1 are the same and other carrier interference experienced from other K-1 users by first user, respectively. To identify every term in the equation, l is the subcarrier index for the M branches at the receiver. To solve the desired user signal from the decision signal, we can write the decision signal for the desired signal as:

$$D_{1} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sqrt{P_{1}} \sum_{m=1}^{M} \beta_{1,m}(t) b_{1}[n] u_{T}(t-nT-\zeta_{1}) \Re\{c_{1,m}[n]e^{j(W_{m}t+\phi_{1,m}(t))}\} \sum_{l=1}^{M} \Re\{\hat{q}_{1,l}c_{1,l}^{*}e^{-j(W_{l}(t+\zeta_{1})+\theta_{1,l})}\} dt$$

$$(4.9)$$

where *m* is the subcarrier index representing the M subcarriers in the received signal and we can write only first user's signal from the pure received signal to find out the desired user signal. The other K-1 users' signal will be analyzed in the interference term solution. Since we are solving the desired user signal we can combine the subcarrier indexes *m* and *l* the other multiplications of the different subcarriers are in the interference term of other carrier which is denoted as J_1 . Then we can rewrite Equation (4.9) with the combination of the subcarrier indexes as,

$$D_{1} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sqrt{P_{1}} \sum_{m=1}^{M} \beta_{1,m}(t) b_{1}[n] u_{T}(t-nT-\zeta_{1}) \Re\{c_{1,m}[n]e^{j(W_{m}t+\phi_{1,m}(t))}\} \Re\{\hat{q}_{1,m}c_{1,m}^{*}e^{-j(W_{1}(t+\zeta_{1})+\theta_{1,m})}\} dt$$

$$(4.10)$$

Till Equation (4.10), the equations are same for EGC and MRC mode. To continue solving the equation in EGC mode we can write the imperfect channel estimation factor $\hat{q}_{1,m}$ for EGC mode $|q_{1,m}| = |\hat{q}_{1,m}| = |q_{k,m}| = 1$ is valid and then the desired user signal will be,

$$D_{1} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sqrt{P_{k}} \sum_{m=1}^{M} \beta_{1,m}(t) b_{1}[n] u_{T}(t-nT-\zeta_{1}) \Re\{c_{1,m}[n]e^{j(W_{m}^{-1}+\phi_{1,m}(t))} \Re\{e^{-j\hat{\psi}_{1,m}}c_{1,m}^{*}e^{-j(W_{1}^{-1}+\zeta_{1})+\theta_{1,m}}\} dt$$

$$(4.11)$$

where, $\hat{\psi}_{k,m}$ is the imperfect channel estimation and phase estimate angle which is described above. We can combine all the angles with the channel estimate angle.

$$D_{l} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sqrt{P_{k}} \sum_{m=1}^{M} \beta_{l,m}(t) b_{l}[n] u_{T}(t-nT-\zeta_{1}) \Re\{c_{l,m}[n]e^{j(W_{m}^{-t} - \phi_{l,m}^{-(t)})} \Re\{c_{l,m}^{*}e^{-j(W_{l}^{-t} - \zeta_{1}) + \hat{\psi}_{l,m}^{*} + \theta_{l,m}}\} dt$$

$$(4.12)$$

And the angle part will be written with sinus and cosine.

$$D_{l} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sqrt{P_{k}} \sum_{m=1}^{M} \beta_{1,m}(t) b_{l}[n] u_{T}(t-nT-\zeta_{1}) \Big[C_{1,m}^{I}[n] \cos(\psi_{m}t + \phi_{1,m}(t)) - C_{1,m}^{Q}[n] \sin(\psi_{m}t + \phi_{1,m}(t)) \Big] \\ \Big[\sum_{l,m}^{*T} \cos(-(\psi_{l}(t+\zeta_{1}) + \hat{\psi}_{1,m} + \theta_{1,m})) - C_{1,m}^{*Q} \sin(-(\psi_{l}(t+\zeta_{1}) + \hat{\psi}_{1,m} + \theta_{1,m})) \Big] dt$$

$$(4.13)$$

Since cosine is even and sinus is odd function the equation will be,

$$D_{1} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sqrt{P_{k}} \sum_{m=1}^{M} \beta_{1,m}(t) b_{1}[n] u_{T}(t-nT-\zeta_{1}) \Big[C_{1,m}^{I}[n] \cos(\psi_{m}t + \phi_{1,m}(t)) - C_{1,m}^{Q}[n] \sin(\psi_{m}t + \phi_{1,m}(t)) \Big] \Big[C_{1,m}^{*I} \cos(\psi_{I}(t+\zeta_{1}) + \hat{\psi}_{1,m} + \theta_{1,m}) + C_{1,m}^{*Q} \sin(\psi_{I}(t+\zeta_{1}) + \hat{\psi}_{1,m} + \theta_{1,m}) \Big] dt$$

$$(4.14)$$

After multiplication,

$$D_{l} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sqrt{P_{k}} \sum_{m=1}^{M} \beta_{1,m}(t) b_{l}[n] u_{T}(t-nT-\zeta_{1}) \left\{ \int_{C_{1,m}}^{I} [n] \cos(\psi_{m}t + \phi_{1,m}(t)) C_{1,m}^{*T} \cos(\psi_{l}(t+\zeta_{1}) + \hat{\psi}_{1,m} + \theta_{1,m}) \right] \\ + \left[\int_{0}^{I} [n] \cos(\psi_{m}t + \phi_{1,m}(t)) C_{1,m}^{*Q} \sin(\psi_{l}(t+\zeta_{1}) + \hat{\psi}_{1,m} + \theta_{1,m}) \right] \\ - \left[\int_{0}^{Q} [n] \sin(\psi_{m}t + \phi_{1,m}(t)) C_{1,m}^{*T} \cos(\psi_{l}(t+\zeta_{1}) + \hat{\psi}_{1,m} + \theta_{1,m}) \right] \\ - \left[\int_{0}^{Q} [n] \sin(\psi_{m}t + \phi_{1,m}(t)) C_{1,m}^{*Q} \cos(\psi_{l}(t+\zeta_{1}) + \hat{\psi}_{1,m} + \theta_{1,m}) \right] \right] dt$$

$$(4.15)$$

to solve this equation in the system model section, the spreading sequence is pseudorandom in practice, with deterministic period. Because this period may be arbitrarily long in the proposed system, $c_{k,m}$ are assumed to identically independent distribution (i.i.d) for all *k* and *m*, and for I and Q channels. So the multiplications of spreading sequences are,

$$\begin{bmatrix} c_{1,m}^{I} c_{1,m}^{*Q} \\ c_{1,m}^{I} c_{1,m}^{*I} \end{bmatrix} = 0$$

$$\begin{bmatrix} c_{1,m}^{I} c_{1,m}^{*I} \\ c_{1,m}^{Q} c_{1,m}^{*Q} \end{bmatrix} = 1$$

$$\begin{bmatrix} c_{1,m}^{Q} c_{1,m}^{*Q} \\ c_{1,m}^{I} c_{1,m}^{I} \end{bmatrix} = 1$$
(4.16)

Then, using these properties of the spreading sequences, the desired user signal will be written as,

$$D_{1} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sqrt{P_{1}} \sum_{m=1}^{M} \beta_{1,m}(t) b_{1}[n] u_{T}(t-nT-\zeta_{1}) \{ \left[\cos(\psi_{m}t + \phi_{1,m}(t)) \cos(\psi_{l}(t+\zeta_{1}) + \hat{\psi}_{1,m} + \theta_{1,m}) \right] - \left[\sin(\psi_{m}t + \phi_{1,m}(t)) \sin(\psi_{l}(t+\zeta_{1}) + \hat{\psi}_{1,m} + \theta_{1,m}) \right] dt$$

$$(4.17)$$

and since the angle of the received signal, $\phi_{1,m}(t)$ is,

$$\boldsymbol{\phi}_{1,m}(t) = \boldsymbol{\psi}_{1,m}(t) + \boldsymbol{\theta}_{1,m} - \boldsymbol{\omega}_m \boldsymbol{\zeta}_1$$
(4.18)

and using the property of multiplication of the trigonometric equations, the desired user signal will be,

$$D_{l} = \frac{1}{2T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sqrt{P_{1k}} \sum_{m=1}^{M} \beta_{1,m}(t) b_{l}[n] u_{T}(t-nT-\zeta_{1}) \{\cos \varrho_{W_{m}}t + \psi_{1,m}(t) + \theta_{1,m} - w_{m}\zeta_{1} + \hat{\psi}_{1,m} + \theta_{1,m}) + \cos(w_{m}t + \psi_{1,m}(t) + \theta_{1,m} - w_{m}\zeta_{1} - w_{m}t - w_{m}\zeta_{1} - \hat{\psi}_{1,m} - \theta_{1,m}) - \cos(\varrho_{W_{m}}t + \psi_{1,m}(t) + \theta_{1,m} - w_{m}\zeta_{1} + w_{m}\zeta_{1} + \hat{\psi}_{1,m} + \theta_{1,m}) + \cos(w_{m}t + \psi_{1,m}(t) + \theta_{1,m} - w_{m}\zeta_{1} - w_{m}t - w_{m}\zeta_{1} - \hat{\psi}_{1,m} - \theta_{1,m}) \} dt$$

$$(4.19)$$

with summations and subtractions, the desired user signal will be,

$$D_{1} = \frac{1}{2T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sqrt{P_{1}} \sum_{m=1}^{M} \beta_{1,m}(t) b_{1}[n] u_{T}(t-nT-\zeta_{1}) \{2\cos(\psi_{1,m}(t)-\hat{\psi}_{1,m})\} dt$$
(4.20)

where only the angle of the channel and the imperfect channel estimate angle are remain. Before integration, taking out the multiplication of two,

$$D_{1} = \frac{1}{T} \sqrt{P_{1}} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sum_{m=1}^{M} \beta_{1,m}(t) b_{1}[n] u_{T}(t-nT-\zeta_{1}) \{\cos(\psi_{1,m}(t)-\hat{\psi}_{1,m})\} dt$$
(4.21)

then applying the integration,

$$D_{1} = \frac{1}{T} \sqrt{P_{1}} \sum_{m=1}^{M} \beta_{1,m} b_{1}[0] \cos(\psi_{1,m} - \hat{\psi}_{1,m}) * T$$
(4.22)

as shown in Equation (4.22), since the integration is applied in the time region (0,T], first user's first bit is decoded which is shown as $b_1[0]$ and for this short period all the components dependent on *t* are assumed constant, and as a result the desired user signal for the first user in EGC mode in MC-CDMA technology will be written as Equation (4.23).

$$D_{1} = \sqrt{P_{1}} b_{1} [0] \sum_{m=1}^{M} \beta_{1,m} \cos(\psi_{1,m} - \hat{\psi}_{1,m})$$
(4.23)

As shown in Equation (4.23), the desired user signal consists of desired user's bit and applied power of the desired user and the channel effect which are, $\beta_{1,m}$, power change effect and $\Psi_{1,m}$, angle change effect of the channel and the last term $\hat{\Psi}_{1,m}$ is the imperfect channel estimate angle. From Equation (4.23), it can be easily seen that if the channel estimate is perfect in EGC mode, the desired user signal power will only change according to the channel effect so desired signal energy reduces depending to the imperfect channel estimation by a factor of $\cos(\Psi_{1,m} - \hat{\Psi}_{1,m})$. However, desired user signal equation does not contain the interference terms caused by other users with the same or other carriers.

Apart from desired user signal and noise in the received signal, the superimposed interference signal can be defined as V_1 and shown as,

$$V_1 = I_1 + J_1 \tag{4.24}$$

and the superimposed interference term can be written as Equation (4.25) from the decision signal,

$$V_{1} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sum_{m=1}^{M} \sum_{k=2}^{K} \sqrt{P_{k}} \beta_{k,m}(t) b_{k} [n] \mu_{T}(t - nT - \zeta_{k}) \Re\{c_{k,m}[n] e^{j(\omega_{m}t + \phi_{k,m}(t))}\}.$$

$$\sum_{l=1}^{M} \Re\{\hat{q}_{1,l}c_{1,l}^{*}e^{-j(\omega_{l}(t - \zeta_{1}) + \theta_{1,l})}\} dt$$
(4.25)

where, as clarified before *l* is the subcarrier index for M branches at the receiver, while m is the subcarrier index representing the M subcarriers in the received signal. As Gui and Ng shows in their research [22], it is possible to change the order of summation by bringing the $\sum_{l=1}^{M}$ in front and carrying out the integration. Then the result of the superimposed interference can be written as,

$$V_1 = \sum_{l=1}^{M} I_{l,1} + \sum_{l=1}^{M} J_{l,1} = I_1 + J_1$$
(4.26)

where $I_{l,1}$ are terms in which m=l, and correspond to the interference on subcarrier l from the other K-1 users on subcarrier l. The other carrier interference term is $J_{l,1}$, which corresponds to the interference on subcarrier l from the other K-1 users' signals on $m \neq l$. The interference term correspond to the interference on the same subcarrier from other K-1 users can be found from the decision signal as,

$$I_{1} = \frac{1}{2T} \sum_{k=2}^{K} \sqrt{P_{k}} \sum_{m=1}^{M} \beta_{k,m} \left[(a_{1}^{T} \zeta_{k} + a_{2}^{T} (T - \zeta_{k})) \cos(\Delta \phi_{I}) + (a_{1}^{Q} \zeta_{k} + a_{2}^{Q} (T - \zeta_{k})) \cos(\Delta \phi_{I}) \right]$$

$$(4.27)$$

where the angles of the trigonometric functions,

$$\Delta \phi_I = \phi_{k,m}(0) - \hat{\phi}_{1,m}(0) \tag{4.28}$$

and the new random variable $\{a_i\}$'s are shown in Equation (4.29).

$$a_{1}^{I} = b_{k}(-1)c_{k,m}^{I}(-1)c_{1,m}^{I}(0)$$

$$a_{2}^{I} = b_{k}(0)c_{k,m}^{I}(0)c_{1,m}^{I}(0)$$

$$a_{1}^{\varrho} = b_{k}(-1)c_{k,m}^{\varrho}(-1)c_{1,m}^{\varrho}(0)$$

$$a_{2}^{\varrho} = b_{k}(0)c_{k,m}^{\varrho}(0)c_{1,m}^{\varrho}(0)$$
(4.29)

The result of interference term caused by the same subcarriers from the other K-1 users also can be called as own interference is shown in Appendix A. However, different from our solution Andrews and Meng in their research [7] found the own interference as shown in Equation (4.30).

$$I_{1} = \frac{1}{2T} \sum_{k=2}^{K} \sqrt{P_{k}} \sum_{m=1}^{M} \beta_{k,m} \left[(a_{1}^{\prime} \zeta_{k} + a_{2}^{\prime} (T - \zeta_{k})) \cos(\Delta \phi_{l}) + (a_{1}^{\varrho} \zeta_{k} + a_{2}^{\varrho} (T - \zeta_{k})) \sin(\Delta \phi_{l}) \right]$$

$$(4.30)$$

where all the terms are same with our solution but the sinus term is cosine in our solution. The other all terms like random variables $\{a_i\}$'s are same. On the other hand this difference doesn't affect any of calculation or simulation results. The calculations for the interference terms are equal to Andrews and Meng's research results. Since the expected values of the interference terms are zero, as shown in Appendix B, the variance of the first user's own interference which is conditioned on the channel and its estimated value can be found as,

$$\operatorname{var}\left[I_{1} \mid \lambda, h\right] = \frac{1}{6} \sum_{k=2}^{K} \sum_{m=1}^{M} P_{k} \beta_{k,m}^{2}$$
(4.31)

For the other carrier interference term of the first user has the same situation like the own interference term. Different result can be found from [7] if you applied same solution method which is shown in Appendix A for the own interference. So our result for the other carrier interference of the first user is,

$$J_{1} = \sum_{k=2}^{K} \sqrt{P_{k}} \sum_{m=1}^{M} \sum_{l=1 \atop l \neq m}^{M} \beta_{k,m} \frac{1}{2T} \{ \int_{0}^{\zeta_{k}} \left[(d_{1}^{T} \cos(\Omega_{k,l,m}(t)) + (d_{1}^{Q} \cos(\Omega_{k,l,m}(t))) \right] dt + \int_{\zeta_{k}}^{T} \left[(d_{2}^{T} \cos(\Omega_{k,l,m}(t)) + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t))) \right] dt + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t)) + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t))) \right] dt + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t)) + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t))) dt + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t)) + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t))) dt + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t)) + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t))) dt + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t)) + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t))) dt + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t)) dt + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t))) dt + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t)) + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t))) dt + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t)) dt + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t))) dt + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t))) dt + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t))) dt + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t))) dt + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t)) dt + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t))) dt + (d_{2}^{Q$$

where the angles of the trigonometric functions,

$$\Omega_{k,m,l}(t) = (\omega_m - \omega_l)t + \phi_{k,l}(0) - \hat{\phi}_{l,m}(0)$$
(4.33)

and the new random variable $\{d_i\}$'s are shown in Equation (4.34).

$$d_{1}^{I} = b_{k}(-1)c_{k,l}^{I}(-1)c_{1,m}^{I}(0)$$

$$d_{2}^{I} = b_{k}(0)c_{k,l}^{I}(0)c_{1,m}^{I}(0)$$

$$d_{1}^{\varrho} = b_{k}(-1)c_{k,l}^{\varrho}(-1)c_{1,m}^{\varrho}(0)$$

$$d_{2}^{\varrho} = b_{k}(0)c_{k,l}^{\varrho}(0)c_{1,m}^{\varrho}(0)$$
(4.34)

However, different from our solution Andrews and Meng in their research [7] found the other carrier interference as shown in Equation (4.35). Again the same situation

is valid in this equation also, all the terms and the random variable $\{d_i\}$'s are same but in the trigonometric expression, the second term is sinus in our solution but in this thesis, it is solved as cosine.

$$J_{1} = \sum_{k=2}^{K} \sqrt{P_{k}} \sum_{m=1}^{M} \sum_{l=1 \ l \neq m}^{M} \beta_{k,m} \frac{1}{2T} \{ \int_{0}^{\zeta_{k}} \left[(d_{1}^{l} \cos(\Omega_{k,l,m}(t)) + (d_{1}^{\varrho} \sin(\Omega_{k,l,m}(t))) \right] dt + \int_{\zeta_{k}}^{T} \left[(d_{2}^{l} \cos(\Omega_{k,l,m}(t)) + (d_{2}^{\varrho} \sin(\Omega_{k,l,m}(t))) \right] dt \}$$

$$(4.35)$$

Also for the other carrier interference solution, the difference does not affect any of the solution like expected value or variance. Since the expected value of the first user's other carrier interference conditioned on the channel and its estimated value is zero like the expected value in other word mean value of the own carrier interference conditioned on the channel and its estimated value, the variance of the first user's other carrier interference, J_1 , can be solved as Equation (4.36), which is also shown in Appendix B like own carrier interference of the first user.

$$\operatorname{var}[J_1 \mid \lambda, h] = \frac{1}{4\pi^2} \sum_{k=2}^{K} P_k \sum_{m=1}^{M} \sum_{l=1 \ l \neq m}^{M} \frac{\beta_{k,l}^2}{(m-l)^2}$$
(4.36)

It is already mentioned that for the interference terms, both own and other carrier interference, the expected value or it can be said that mean value is zero. It is already known that the noise term which is AWGN has zero mean. However, on the other hand in the decision signal there is one component which is desired user signal, and its expected value is not mentioned yet. To solve decision signal's expected value, it is enough to solve desired user signal's expected value.

To find the expected value of the decision signal which is actually desired user signal, let assume that desired user's bit which is transmitted at time zero is one. The mean conditioned on the channel and its estimated value of the decision signal, U_1 , can be found as Equation (4.37).

$$E[U_{1} | \lambda, h] = E[D_{1} | \lambda, h] = \sqrt{P_{1}} \sum_{m=1}^{M} \beta_{1,m} \cos(\psi_{1,m} - \hat{\psi}_{1,m})$$
(4.37)

In despite of the mean value of decision signal conditioned on the channel and its estimated value, the variance of the decision signal conditioned on the channel and its estimated value is the sum of the variances of the interference terms and noise term which are also conditioned on the channel and its estimated value. Then the variance of the decision signal can be found as Equation (4.38).

$$\operatorname{var}[U_1 \mid \lambda, h] = \frac{N_0 M}{2T} + \sum_{k=2}^{K} \sum_{m=1}^{M} P_k \left(\frac{\beta_{k,m}^2}{6} + \sum_{\substack{l=1\\l \neq m}}^{M} \frac{\beta_{k,l}^2}{4\pi^2 (m-l)^2} \right)$$
(4.38)

For the mean and variance expressions of the decision signal, λ and h have known distributions, and actual values for $E[U_1]$ and $var[U_1]$ can be yielded by removing the conditional expectation through Monte Carlo integration [7].

On the other hand, it is very interesting note that the imperfect channel estimate reduces the desired signal energy as mentioned above by a factor of $\cos(\Psi_{1,m} - \hat{\Psi}_{1,m})$, but does not increase the power of the interference terms, since the interference term phases are already assumed to be uncorrelated with the desired phases. And, if the channel estimate converge to the real value, the power of the desired user will converge to the its real value, since $\cos(\Psi_{1,m} - \hat{\Psi}_{1,m}) \approx 1$ if $\Psi_{1,m} \approx \hat{\Psi}_{1,m}$.

As it can be seen from the analysis, the interference cancellation process does not started yet for the first user so analysis of the first user in SIC is easy. For the next users, interference cancellation process is begin and since the bits which decisions are given are used, so the probability of the truth of the bits are not certain which results the analysis of the next users signal is not so easy. For each users the signal used for decision changes according to the interference cancellation used in the previous stage, decoding the previous user bit which is shown in Equation (3.12).

Since, no interference cancellation is used yet for the first user, first user in SIC experienced total interference and biggest interference from the other users in the system. So to make equal signal to interference ratio in SIC, a power control scheme is used. Because of the power control in SIC, the power of the users are different but signal to interference ratio for all users are same. The power control scheme is almost same for EGC and MRC scheme with minor differences according to the imperfect carrier combining factor.

In the next section, the power control algorithm in SIC will be explained and analyzed detailed.

4.1.1. Power Control for SIC in EGC

Power control (PC) is required for all realistic CDMA systems because of what is known as the near far problem which is users far from the base station experience far greater path loss than the users that are near the base station. Optimum power control is achieved when all users are decoded with the same signal to interference ratio (SIR) [45]. Otherwise, a user with a low SIR dominates the BER performance of the system, which is defined as the average BER over all users.

A wireless channel is quite dynamic and changes fairly rapidly as users enter, move within and leave the system. It is impractical for the receivers to update all users' powers constantly as the optimal power allocation varies. Also for the power control, channel estimation in the receiver is very important and according to the imperfect channel estimation, the fraction of the power cancelled from the next user changes.

In SIC, all users experienced different amount of interference, power control scheme is crucial scheme. Since first users experienced more interference, they need to have more power to compensate this high interference. A more complicated power control distribution is required to make full use of SIC, because the users must be received with differing powers, dependent on the order of decoding. In this thesis we will analyzed a general power control algorithm for SIC which is already analyzed in [43] and [44].

In order to maintain a similar SINR for all users in any SIC system, a power control algorithm is used. The IC is never perfect, and the PC distribution should achieve comparable BERs for all users regardless of their decoding order. The PC algorithm which is mentioned in [43, 44] is developed for MC-CDMA system in [7] for both EGC and MRC.

In a CDMA system, each user experience interference also caused by the other users in the system. So the SINR of one user will be written as,

$$SINR = \Gamma_k = \Gamma = \frac{P}{(K-1)P + N}$$
(4.39)

where $P_k = P$ is the received power of each user, and *K* is the number of users which is also defined in the previous section and *N* is the power of the background AWGN, which also includes other interference coming from other different reasons. Also this formula is valid when the channel and power estimation is perfect.

When SIC is used, the situation is significantly different. In this case, it is also desirable that each user experiences the same SIR at the time of decoding. However, interference is being subtracted out of the received signal after each user, so the first user to be decoded sees the most interference, the last user the least as mentioned above. If the successive cancellation scheme proceeds with no channel estimation error or bit errors, then finding the optimum power control scheme is straightforward as described in [46].

On the other hand, in the system that analyzed in this thesis the amplitude and phase estimation are not perfect. In the real systems also, it is never perfect and thus, it is desirable to know the optimum power solution in the presence of imperfect interference cancellation because of imperfect estimation. So because of the cancellation errors, *k*th user SINR will be written as,

$$SINR = \Gamma_k = \frac{P_k}{\sum_{k=k+1}^{K} P_k + \sum_{k=1}^{k-1} \varepsilon_k P_k + N}$$
(4.40)

where again *K* is again is the number of users and ε_k is the fraction of the *k*th user's power not cancelled, in other words, it is cancellation error. In this thesis, it is assumed that cancellation error is equal to the estimation error for the simulations.

If the successive cancellation scheme proceeds with channel estimation error, then the SINR values for the *K* users will be different. The following K equations describe the SINRs for each user in a normal CDMA system:

$$\Gamma_{1} = \frac{P_{1}}{\sum_{k=2}^{K} P_{k} + N}, \Gamma_{2} = \frac{P_{2}}{\sum_{k=3}^{K} P_{k} + \mathcal{E}_{1} P_{1} + N}, \dots$$

$$\Gamma_{k} = \frac{P_{k}}{\sum_{k=k+1}^{K} P_{k} + \sum_{k=1}^{k-1} \mathcal{E}_{k} P_{k} + N}, \dots$$

$$\Gamma_{K} = \frac{P_{K}}{\sum_{k=1}^{K-1} \mathcal{E}_{k} P_{k} + N}$$
(4.41)

For the MC-CDMA system that we analyzed in this thesis, these equations will be slightly different. Since the SINR value for the *k*th user can be found as,

$$\Gamma_{k} = \frac{(E[U_{k} \mid \lambda, h])^{2}}{\operatorname{var}[U_{k} \mid \lambda, h]}$$
(4.42)

so the equations given in equations (4.41) for all K users in MC-CDMA system by using the mean and the variance of the decision signal which are found in equations (4.37) and (4.38) will be,

$$\Gamma_{1} = \frac{A_{1}P_{1}}{B_{1}\sum_{k=2}^{K}P_{k} + N}, \Gamma_{2} = \frac{A_{2}P_{2}}{B_{2}(\sum_{k=3}^{K}P_{k} + \varepsilon_{1}P)_{1} + N}, \dots$$

$$\Gamma_{k} = \frac{A_{k}P_{k}}{B_{k}(\sum_{k=k+1}^{K}P_{k} + \sum_{k=1}^{k-1}\varepsilon_{k}P_{k}) + N}, \dots$$

$$\Gamma_{K} = \frac{A_{K}P_{K}}{B_{K}\sum_{k=1}^{K-1}\varepsilon_{k}P_{k} + N}$$
(4.43)

where N is noise power and the coefficients A_k and B_k for kth user which can be produced from mean and variance equations of the decision signal for EGC,

$$A_{k} = \left(\sum_{m=1}^{M} \boldsymbol{\beta}_{k,m} \cos(\boldsymbol{\psi}_{k,m} - \boldsymbol{\hat{\psi}}_{k,m})\right)^{2}$$

$$(4.44)$$

and,

$$B_{k} = \sum_{m=1}^{M} \left(\frac{\beta_{k,m}^{2}}{6} + \sum_{\substack{l=1\\l \neq m}}^{M} \frac{\beta_{k,l}^{2}}{4\pi^{2}(m-l)^{2}} \right)$$
(4.45)

where the statistics for A_k and B_k are independent of k. To simplify the SINR equations for PC derivation, the equations may be put in general form as written in Equation (4.46).

$$\Gamma_{k} = \frac{A_{k}}{B_{k}} \left(\frac{P_{k}}{\sum_{i < k} \varepsilon_{i} P_{i} + \sum_{i > k} P_{k} + N_{k}} \right)$$
(4.46)

In the equation, the noise term N'_k is $\frac{N}{B_k}$. The $\frac{A_k}{B_k}$ terms are simply part of the channel gain for user k, and may be neglected (in the same manner that path loss and shadowing terms are neglected without loss of generality) [7]. For PC, solving these equations to make equal SINR value for all users, the power of the all users can be found.

To solve the power values obtain from PC scheme, in the first equation it is needed to use $\Gamma_1 = \Gamma_2$ after neglecting the channel gain in MC-CDMA system and in the equation, the total power will be defined as $P_T = \sum_{k=1}^{K} P_k$ also the cancellation efficiency can be defined as $\eta_k = 1 - \varepsilon_k$ from the cancellation error [43]. Afterwards the first equation for the PC will be,

$$\frac{P_1}{P_T - P_1 + N} = \frac{P_2}{P_T - \eta_1 P_1 - P_2 + N}$$
(4.47)

If we solve the equation leaving alone P_2 , then the power of the second user can be found as a function of the power of the first user and the efficiency of the cancellation.

$$P_2 = P_1 - \frac{\eta_1 P_1^2}{P_T + N} \tag{4.48}$$

The other subsequent equations are straightforward. If we continue with the power of the third user, P_3 , and then, the general result for P_k can be found. The second equation which can be achieved by making equal the SINR values for second and third user,

$$\frac{P_2}{P_T - \eta_1 P_1 - P_2 + N} = \frac{P_3}{P_T - \eta_1 P_1 - \eta_2 P_2 - P_3 + N}$$
(4.49)

But to make a general form for the equations, we can introduce a convenient notation for the total remaining MAI to user k plus their own power,

$$V_k = P_T - \sum_{i=1}^{k-1} \eta_i P_i$$
(4.50)

and, if we use this MAI equation in the second equation which is given in equation (4.49), then we will get,

$$\frac{P_2}{V_2 - P_2 + N} = \frac{P_3}{V_2 - \eta_2 P_2 - P_3 + N}$$
(4.51)

and if we solve this equation as we did for Equation (4.47) by leaving the power of third user alone in one sight, then the power of the third user can be found as dependent on second user's power and cancellation efficiency of the second user as can be seen in Equation (4.52).

$$P_3 = P_2 - \frac{\eta_2 P_2^2}{V_2 + N} \tag{4.52}$$

Then the remaining equations can be solved as recursive solution, straightforward, the recursive solution for the *k*th user can be found as,

$$P_{k} = P_{k-1} - \frac{\eta_{k-1} P_{k-1}^{2}}{V_{k-1} + N}$$
(4.53)

and, if we replace cancellation efficiency with the cancellation error which is accepted equal to the channel estimation error, the general equation for the PC can be found as Equation (4.54).

$$P_{k} = P_{k-1} - \frac{(1 - \mathcal{E}_{k-1})P_{k-1}^{2}}{V_{k-1} + N}$$
(4.54)

This power values equalizes the SINR values at detection for all users and which result almost equal BER for all users. The power distributions of user decrease from the decoded first users to the decoded last users. Because of the PC, the total power will decrease and the capacity of the system will increase since the total power, in other word total interference will decrease by the help of PC.

On the other hand there are two important caveats that need to be addressed. First of all, the power distribution given in Equation (4.54) is in terms of the residual cancellation error ε_k , which is not typically known and so must be guessed in order to use

in the PC algorithm and in the simulations assumed equal to the estimation error. Because the cancellation error is because of the estimation error of the channel and the bit errors are made because of this estimation error. The vast majority of BER is because of channel estimation error. The second one is while BER is sensitive to the channel estimation error; on the other hand the PC is not which can be seen in the simulation section.

The recursive solution for the power distributions of the users derived from the PC algorithm gives us stability for any target SINR. Despite its apparent complexity, as shown above the iterative PC distribution is simple and no more complex then the used PC in CDMA systems [43, 44], even the cancellation error ε_k is not known.

4.2. MC-CDMA System using IC with Gibbs Sampler in EGC

As we analyzed SIC system in EGC mode, we also analyzed the IC using Gibbs sampler in EGC mode. Only the IC scheme is changes so the channel and channel estimations are all same. So the phase adjustment factor of EGC mode is given in equation (4.1) for the perfect estimation. Since perfect estimation is a utopia and in the system that we analyzed in this thesis, the channel estimate is imperfect, the subcarrier combining factor is given in Equation (4.3). As it can be seen from Equation (4.3), there is no added amplitude error for subcarrier combining in EGC, only phase error, since no amplitude adjustments are made.

The received signal is shown in Equation (3.7), but this received signal is not used purely, for giving decision. This criterion does not depend on which order the user is decoded. Since interference cancellation is applied to all users, other users' signals different from the desired user's signal is subtracted from the received signal. As explained below, the bit decisions are updated also in the same stage after every bit decision is given before decoded user.

Interference is either cancelled at the chip level or at the symbol level. In this thesis, the interference cancelled at chip level. At the chip level, the interference from the other users regenerated, which includes multiplying by the channel estimates, spreading and

scrambling. The regenerated interference is subtracted from the original received samples, which would be stored in the memory. After the interference is cancelled, the signal for the desired user can be descrambled, despread, and equal gain combined using the channel estimates for that user so that a soft decision may be made on the received symbol.

To understand the interference cancellation scheme with Gibbs sampler, first of all Gibbs sampler algorithm is needed to be explained. Let the vector of the unknown bits of the users be:

$$\mathbf{B} = [\mathbf{b}_1 \ \mathbf{b}_2 \ \mathbf{b}_3 \dots \mathbf{b}_K] \tag{4.55}$$

and let Y be the observed data. Suppose that we are interested in finding the a posteriori marginal distribution of parameter b_k conditioned on the observation Y, i.e., $p(b_k|Y)$. Direct evaluation involves integrating out the rest of the parameters from the joint a posteriori density, i.e.

$$p(b_k | Y) = \int \int \dots \int p(B | Y) db_1 \dots db_{k-1} db_{k+1} \dots db_K$$
(4.56)

And the all probability density functions of the all users' bits are:

$$p(b_1 | b_2, \dots, b_K); p(b_2 | b_1, b_3, \dots, b_K); \dots, p(b_K | b_1, b_2, \dots, b_{K-1})$$
(4.57)

In most cases such a direct evaluation is computationally infeasible especially when the parameter dimension K is large. The Gibbs Sampler is a Markov-chain Monte Carlo procedure for numerical evaluation of the multidimensional integral in Equation (4.56). The idea is to generate random samples from the joint posterior distributions using these samples. Given the values:

$$B^{(0)} = \left[b_1^0, b_2^0, b_3^0, \dots, b_K^0\right]$$
(4.58)

the algorithm iterates the loop described below. In the first iteration of the loop n=0, and then n is incremented each time the loop in completed. Every n is defines the number of stage. In one stage:

- Step 1: Draw sample $b_1^{(n+1)}$ from $p(b_1 | b_2^{(n)}, b_3^{(n)}, \dots, b_K^{(n)}, Y)$
- Step 2: Draw sample $b_2^{(n+1)}$ from $p(b_2 | b_1^{(n+1)}, b_3^{(n)}, \dots, b_K^{(n)}, Y)$
- Step 3: Draw sample $b_3^{(n+1)}$ from $p(b_3 | b_1^{(n+1)}, b_2^{(n+1)}, \dots, b_K^{(n)}, Y)$

. . .

•••

- Step K-1: Draw sample $b_{K-1}^{(n+1)}$ from $p(b_{K-1} | b_1^{(n+1)}, b_2^{(n+1)}, \dots, b_{K-2}^{(n+1)}, b_K^{(n)}, Y)$
- Step K: Draw sample $b_{K}^{(n+1)}$ from $p(b_{K} | b_{1}^{(n+1)}, b_{2}^{(n+1)}, \dots, b_{K-1}^{(n+1)}, Y)$ (4.59)

then, increment n and go back Step one, which also means to the next stage.

The convergence behavior of the Gibbs Sampler is investigated in [15], and general conditions are given for the following result. The distribution of $B^{(n)}$ converges geometrically to Equation (4.60).

 $p(B | Y) \qquad \text{as} \qquad n \to \infty \tag{4.60}$

Samples are drawn for many iterations, and there is an initial transient period before the Gibbs sampler converges. After the transient period, a histogram can be formed with the remaining samples, and estimated values for each variable can be computed by taking the mean of the histogram of each random variable [16, 47].

This transient period takes time if you compare with SIC. This transient period is taken a certain value in the analysis. However, the bit decision changes in the transient period are shown in Figure 4.1.



Figure 4.1. Fraction of bits that change from stage to stage

As shown in the Figure 4.1, the bit decision changes are stopping after a period, actually after transient period. In this thesis, for the simulation this transient time is taken as five stages which are enough to reach a stabile solution for all users in MC-CDMA system.

The composite signal used for the decision is given in Equation (3.16) for these iterations. It is very hard to analyze even the first user in the IC scheme using Gibbs

Sampler because the all users signal which are regenerated from the bit decisions from the same stage for the users decoded before desired user and bit decisions from the previous stage for the rest of the users, is cancelled from the received signal. The composite signal is shown in Equation (4.61) which is used in the *n*th stage

$$U_{k} = \frac{1}{T} \int_{0}^{T} \sum_{m=1}^{M} y_{k,m}^{(n)} \Re\{\hat{q}_{k,m} c^{*} e^{-j(W_{m}^{(t+\zeta_{k})} + \theta_{k,m})} dt .$$
(4.61)

where, the composite signal used for the decision is showed in Equation (3.16). Since the system analyzed in EGC mode the subcarrier combining factor $\hat{q}_{k,m}$ which is shown in Equation (3.16) is used.

The Gibbs sampler operates on one user at a time in a sequential manner. For the first user, the interference is cancelled using the method mentioned above which is chip level, and then a soft decision is made on that symbol. Using this soft decision and the estimated E_b/N_o , a conditional pdf of the data symbol can be estimated. A sample is chosen from this pdf and is used as the estimate of the received symbol for the particular user in the current iteration.

As an example of how this operation is performed, let the transmitted symbol be BPSK modulated as used in this thesis also for simulation so that it is either positive one or negative one. Assume that at the receiver the channel can be estimated perfectly and that the amplitude and phase are then removed from the received signal so that if there were no noise or interference, the received symbol would be positive one or negative one. Let the soft decision for user k be d_k and let the noise variance be denoted as *d*. The ratio of the conditional probabilities that b_k is positive one or x_k is negative one can be derived to be [16],

$$\frac{p[b_k = +1 \mid B_k, Y]}{p[b_k = -1 \mid B_k, Y]} = \exp\left\{\frac{2d_k}{\sigma^2}\right\}$$
(4.62)

where B_k the all bit decisions of all users except the bit decision of desired user bit b_k . Since these two conditional probabilities sum up to one, because only two probabilities exist, the conditional probability that b_k is negative one is given in Equation (4.63).

$$p[b_{k} = -1 | B_{k}, Y] = \frac{1}{1 + \exp\left\{\frac{2d_{k}}{\sigma^{2}}\right\}}$$
(4.63)

The probability drawing according to the different soft decisions given from composite signal in the IC scheme using Gibbs sampler can be seen below.



Figure 4.2. The probability of giving decision as negative one depending on the soft decision

In order to draw a sample in a stage from the conditional probability of b_k , a random variable "q" uniformly distributed from zero to one can be drawn. If "q" is less than the Equation (4.63) then the estimated symbol for user k is declared to be negative one, and otherwise it is declared to be positive one. As an example let $d_k=0.5$ and σ^2 be 0.5. The probability of the sample is drawn such that the symbol decision is negative one is about 12 per cent, and the probability that the symbol decision is positive one is about 88 per cent in this example [16]. Also this probability change according to the soft decision can be seen in Figure 4.2.

This IC scheme using Gibbs sampler is in contrast to the standard interference cancellation techniques which do not introduce any randomness into the cancellation process and would always produce a symbol decision of positive one when the soft decision is 0.5 which is given in the example above. Even there is randomness in IC scheme, after a transient period, the decision for all users not change so much. And this procedure goes on in every stage till the system stop the procedure and final decisions will be given in the last decision. Also this procedure is applied to all users in one stage which also means that IC scheme is applied to all users.

Since IC scheme using Gibbs sampler is applied to all users in the MC-CDMA system, all users are affected almost same amount of interference. Because of this reason, contrast of SIC; no power control is needed to adjust the SINR values of all users. However, a power control will be used to adjust the users' power because of near far problem in CDMA systems. There are different power control algorithms for CDMA systems can be applied to MC-CDMA which are not under the subject of this thesis [48, 49].

4.3. MC-CDMA System with SIC in MRC

The receiver corrects the phase rotation caused by a fading channel and then combines the received signals of different paths proportionally to the strength of each path. Since each path undergoes different attenuations, combining them with different weights yield an optimum solution under an AWGN channel. In maximum ratio combining (MRC), strong subcarriers are additionally amplified in proportion to the gain of the subcarrier. This technique maximizes the SNR at detection [18, 42], but may be harder to implement, and it is more sensitive to channel estimation error than EGC.

For MRC, the gain and phase adjustment factor is different from EGC as written Equation (4.64).

$$q_{k,m} = \beta_{k,m} e^{-j \Psi_{k,m}}$$

$$(4.64)$$

where, $\psi_{k,m}$ is shown in Equation (4.2) same as EGC mode, and

$$\beta_{k,m} = \sqrt{(h_{k,m}^{I})^{2} + (h_{k,m}^{Q})^{2}}$$
(4.65)

but as in EGC mode, the channel and phase estimation is not perfect is imperfect which implies that the imperfect subcarrier combining factor in MRC is:

$$\hat{q}_{k,m} = \hat{\beta}_{k,m} \, e^{-j\hat{\psi}_{k,m}} \tag{4.66}$$

where,

$$\hat{\beta}_{k,m} = \sqrt{(\hat{h}_{k,m}^{I})^{2} + (\hat{h}_{k,m}^{Q})^{2}}$$
(4.67)

is phase adjustment factor. With this altered gain factor, the expressions for the desired signal component, the same carrier interference and other carrier interference equations of the first user are changed as shown in the equations given below.

$$D_{1} = \sqrt{P_{1}} b_{1}[0] \sum_{m=1}^{M} \beta_{1,m} \hat{\beta}_{1,m} \cos(\psi_{1,m} - \hat{\psi}_{1,m})$$
(4.68)

$$I_{1} = \frac{1}{2T} \sum_{k=2}^{K} \sqrt{P_{k}} \sum_{m=1}^{M} \beta_{k,m} \hat{\beta}_{1,m} \left[(a_{1}^{T} \zeta_{k} + a_{2}^{T} (T - \zeta_{k})) \cos(\Delta \phi_{l}) + (a_{1}^{Q} \zeta_{k} + a_{2}^{Q} (T - \zeta_{k})) \cos(\Delta \phi_{l}) \right]$$

$$(4.69)$$

$$J_{1} = \sum_{k=2}^{K} \sqrt{P_{k}} \sum_{m=1}^{M} \sum_{l=1 \atop l \neq m}^{M} \beta_{k,m} \hat{\beta}_{1,m} \frac{1}{2T} \{ \int_{0}^{\zeta_{k}} \left[(d_{1}^{T} \cos(\Omega_{k,l,m}(t)) + (d_{1}^{Q} \cos(\Omega_{k,l,m}(t))) \right] dt + \int_{\zeta_{k}}^{T} \left[(d_{2}^{T} \cos(\Omega_{k,l,m}(t)) + (d_{2}^{Q} \cos(\Omega_{k,l,m}(t))) \right] dt \}$$

$$(4.70)$$

However, different from our solution Andrews and Meng in their research [7] found the own carrier interference and other carrier interference of the first user as shown in equations given below where all the parameters are same except last trigonometric value which is cosine in our solution but sinus in the Andrews and Meng's solution.

$$I_{1} = \frac{1}{2T} \sum_{k=2}^{K} \sqrt{P_{k}} \sum_{m=1}^{M} \beta_{k,m} \hat{\beta}_{1,m} \left[(a_{1}^{\prime} \zeta_{k} + a_{2}^{\prime} (T - \zeta_{k})) \cos(\Delta \phi_{l}) + (a_{1}^{\varrho} \zeta_{k} + a_{2}^{\varrho} (T - \zeta_{k})) \sin(\Delta \phi_{l}) \right]$$

$$(4.71)$$

$$J_{1} = \sum_{k=2}^{K} \sqrt{P_{k}} \sum_{m=1}^{M} \sum_{\substack{l=1\\l\neq m}}^{M} \beta_{k,m} \hat{\beta}_{1,m} \frac{1}{2T} \{ \int_{0}^{\zeta_{k}} \left[(d_{1}^{T} \cos(\Omega_{k,l,m}(t)) + (d_{1}^{\varrho} \sin(\Omega_{k,l,m}(t))) \right] dt + \int_{\zeta_{k}}^{T} \left[(d_{2}^{T} \cos(\Omega_{k,l,m}(t)) + (d_{2}^{\varrho} \sin(\Omega_{k,l,m}(t))) \right] dt \}$$

$$(4.72)$$

On the other as the own carrier interference and other carrier interference solutions of the first user, the expected value and variance of the first user decision signal is changed in MRC mode as in the equations given below.

$$E[U_{1} | \lambda, h] = E[D_{1} | \lambda, h] = \sqrt{P_{1}} \sum_{m=1}^{M} \beta_{1,m} \hat{\beta}_{1,m} \cos(\psi_{1,m} - \hat{\psi}_{1,m})$$
(4.73)

$$\operatorname{var}[U_{1} \mid \lambda, h] = \frac{N_{0}M}{2T} \sum_{m=1}^{M} \hat{\beta}_{1,m}^{2} + \sum_{k=2}^{K} P_{k} \sum_{m=1}^{M} \hat{\beta}_{1,m}^{2} \left(\frac{\beta_{k,m}^{2}}{6} + \sum_{l=1}^{M} \frac{\beta_{k,l}^{2}}{4\pi^{2}(m-l)^{2}} \right)$$
(4.74)

Although the imperfect phase estimate still does not increase the variance of the interference terms, the imperfect amplitude estimate does affect them in the case of MRC [7].

In the power control algorithm the channel gain components A_k and B_k are different in MRC but since the channel gain part can be neglected, the power control algorithm does not change. So the power control algorithm doesn't affect from the subcarrier combining type. But the parameters A_k and B_k can be written in equations given below.

$$A_{k} = \left(\sum_{m=1}^{M} \boldsymbol{\beta}_{k,m} \, \boldsymbol{\hat{\beta}}_{k,m} \cos(\boldsymbol{\psi}_{k,m} - \boldsymbol{\hat{\psi}}_{k,m})\right)^{2} \tag{4.75}$$

$$B_{k} = \sum_{m=1}^{M} \hat{\beta}_{1,m} \left(\frac{\beta_{k,m}^{2}}{6} + \sum_{\substack{l=1\\l \neq m}}^{M} \frac{\beta_{k,l}^{2}}{4\pi^{2}(m-l)^{2}} \right)$$
(4.76)

4.4. MC-CDMA System using IC with Gibbs Sampler in MRC

As we analyzed SIC system in MRC mode, only the gain and phase adjustment factor is change. Like SIC in the MRC, in IC using Gibbs sampler algorithm, the gain and phase adjustment factor is same given in Equation (4.64). And because of the imperfect channel estimate, the subcarrier combining factor in MRC is given in Equation (4.66). In MRC the combining is done also according to the received powers of the users.

Since only gain and phase adjustment factor is change in MRC and no change is happened on the algorithm of the Gibbs sampler for interference cancellation. So the algorithm is same as shown in the section which IC using Gibbs sampler in EGC is explained. However, no change happened in the algorithm of Gibbs sampler, since the BER performance of CDMA systems in MRC is better than EGC, the performance of IC using Gibbs sampler in MRC gives better result than EGC mode. But for the calculations and the probabilities, nothing has been changed. The soft decision used in the probability of the last decision will affected from subcarrier combining.

4.5. Bit Error Rate

In the interference cancellation schemes, SIC and IC using Gibbs sampler, it can be assumed that all interference terms are approximately Gaussian in both diversity techniques, EGC and MRC, because there are large number of interference terms. This expectation is also known as Central Limit Theorem [50]. So we can use Gaussian approach to find the BER of the two receivers with SIC and IC using Gibbs Sampler independent of diversity type.

If we start with the receiver model with SIC, for the first user and the later users, due to the residual interference for each user being uncorrelated, the Gaussian approximation is still reasonable. For Gaussian approach the bit error probability of the first can be written as Equation (4.77).

$$p_e = Q\left(\frac{E[U_1]}{\sqrt{\operatorname{var}[U_1]}}\right) \tag{4.77}$$

and by conditioning on the λ and h, the probability of bit error can be found as [7],

$$p_{e} = Q \left(\frac{E[E[U_{1} \mid \lambda, h]]}{\sqrt{E[\operatorname{var}[U_{1} \mid \lambda, h] + \operatorname{var}[E[U_{1} \mid \lambda, h]]]}} \right)$$
(4.78)

where Q(.) is the well known "Q-function" and the outer expectation is taken over both λ and h. The outer expectations were not found to analytically tractable, so were evaluated by Monte Carlo integration. This situation is valid for all users if you use IC using Gibbs sample because IC applied to all users. In the PC analysis which is presented in section 4.1.2, for simplification var $[E[U_1 | \lambda, h]]$ term is neglected when equalizing the user SINRs. For analyzing BER as a function of the noise in the system, the SNR per bit for user one which does not include MAI is,

$$\gamma_{b,1} = \frac{\left(E[D_1 \mid \lambda, h]\right)^2}{\frac{N_o M}{2T}} = \frac{2P_1 T}{N_0 M} A_1$$
(4.79)

where A_1 is given for both diversity types EGC and MRC which are given in Equation (4.44) and (4.75) respectively. The power of the first user in SIC is computed in PC algorithm for a fair system. For the later users, the randomness is exist therefore random terms are evaluated through Monte Carlo integration, so the formulations shown above are only for the first user.

On the other hand, for the receiver model with IC using Gibbs sampler is different from SIC because all users are processed in IC scheme so the received signal purely not used for decision for any of the user. Since randomness is in question in IC using Gibbs sampler, Monte Carlo integration is needed to evaluate these random terms. However, also for interference cancellation scheme, since there are a lot of interference terms, with the help of Central Limit Theorem, we can accept all of them as Gaussian, so the probability of error for Gaussian can be written as in Equation (4.80),

$$p_e = Q\left(\frac{E[U_1^0]}{\sqrt{\operatorname{var}[U_1^0]}}\right) \tag{4.80}$$

where U_1^0 is the signal used to decode first user in the first stage. So, as it can be understood from Equation (4.80), the probability of error is changed in different stages for the same user in the interference cancellation with Gibbs sampler scheme.

However, no power control is applied to the system with IC using Gibbs sampler so the system is fair in terms of SINR if the same power used for all users despite of the MC-CDMA system using SIC but of course there are always a power control exist in CDMA system because of near far problem of users. So it is very hard to evaluate a formulation of the bit error probability for the users in the system with IC using Gibbs sampler.

5. SIMULATIONS

Due to the system complexity because of interference cancellation which also results randomness in the signals, simulation plays a vital role in measuring the BER performance of the systems. While measurement and experimentation provide a means for exploring the "real world", simulation and analysis are restricted to exploring a constructed, abstracted model of the world. However, the role of simulation is very important because of the complexity of the subject area.

Analysis of the CDMA systems such as MC-CDMA which we analyzed in this thesis provides the possibility of exploring a performance of the system. The role of analysis of the system is fundamental because it brings with it greater understanding of the basic key factors in the system. On the other hand simulations are complementary to analysis, not only by providing a check on the correctness of the analysis, but by allowing exploration of complicated systems and the system with randomness which is very difficult to analyze.

In this thesis, we investigate BER performance and estimation error affect on the MC-CDMA system with SIC or IC with Gibbs Sampler. Also in the simulation of BER performance, the system with different number of users and two diversity techniques is used. Same notation is used for the variables with the used in the analysis section.

First of all, the system is analyzed with 40 user (K=40) and the total number of subcarriers is 32 (M=32). Also as diversity technique, EGC is used to simulate the system with 40 users and estimation error and cancellation error for SIC is 0.1 ($\varepsilon = \sigma_{\lambda} = 0,1$). So the estimation for both systems is imperfect and cancellation error in PC algorithm for SIC is same with the estimation error. The simulation results are obtained for different E_b /N_o but when referring to the E_b /N_o for SIC, the average E_b /N_o of the user with "average power" is being referred to, as different users have different average E_b /N_o , depending on the decoding order in MC-CDMA system with SIC because of the power of the earlier users are higher that the later users in SIC depending on the PC algorithm. On the other hand, since in the MC-CDMA system using IC with Gibbs sampler, all other users' signal
is cancelled from the received signal for decoding the desired user, the E_b / N_o of the users are almost same and does not depend on the decoding order. However, the SINR values for users in both systems are equal for the users. The SINR values held equal in SIC because of PC in IC with Gibbs sampler because of canceling all interference for all users.

Figure 5.1 shows BER performance of the first and last user according to the decoding order in the MC-CDMA system with SIC which have totally 40 users in the system. In SIC, the analytical results are nearly identical for first and last user, which shows that equal performance can be attained for all users, regardless of decoding order and even with imperfect channel estimation. IC with Gibbs sampler gives almost the same result in low E_b/N_o with SIC in the high E_b/N_o values.



Figure 5.1. BER performance for SIC in EGC for total 40 users.

On the other hand, Figure 5.2. shows the BER performance of the first and last user in MC-CDMA system using IC with Gibbs sampler which have totally 40 users in the system. The results are taken after five stages (n=5) in the Gibbs sampler. Like SIC also in the IC with Gibbs sampler, the BER performance of the first and last user is almost equal better than the BER performance of users in SIC.



Figure 5.2. BER performance for IC with Gibbs sampler in EGC for total 40 users.

As can be seen from the simulation results in the figures given in Figure 5.2 and in Figure 5.3, the agreement is very close between the simulation results and analytical expressions for both MC-CDMA systems with SIC or IC with Gibbs sampler. Also it shows that the assumptions do not make a big difference in the results.

If we decrease the number of users in the system, the BER performance of SIC in different E_b /N_o values for EGC and MRC can be seen in Figure 5.3 and Figure 5.4

respectively. When the number of users decrease, the BER performance of the users in the same E_b / N_o is getting better. Also in the MRC diversity technique, the BER performance of the system with SIC is better than EGC as expected.



Figure 5.3. BER performance for SIC in EGC for total 25 users.

Also for the system with 25 users, number of subcarrier is 32 and the estimation error is 0.1. The difference in the BER performance between MRC and EGC is in the higher E_b / N_o values, so in the low E_b / N_o values the BER performance is almost the same for both diversity technique, EGC and MRC.



Figure 5.4. BER performance for SIC in MRC for total 25 users.

Besides, IC with Gibbs sampler also gives better BER performance with the number of user decrease in the system. This simulation results can be seen in Figure 5.5 shows BER performance of IC with Gibbs sampler with 25 users in EGC and Figure 5.6 shows the same system with MRC. Again the number of stages Gibbs sampler applied is five. Comparison with SIC, the BER performance difference between EGC and MRC is not bigger but off course there is an improvement in the BER performance in MRC diversity technique.



Figure 5.5. BER performance for IC with Gibbs sampler in EGC for total 25 users.

The dependency on the diversity technique is lower if the enough number of stages is applied in IC with Gibbs sampler. This is also good property because as hardware perspective it is very hard to implement MRC diversity technique according to EGC. Also the BER performance difference with decrease of the number of user is not as big as SIC in IC with Gibbs sampler. So the number of user affects the system performance in IC with Gibbs sampler less than the SIC. However, overall system performance of IC with Gibbs sampler is better than SIC in the same estimation error.



Figure 5.6. BER performance for IC with Gibbs sampler in MRC for total 25 users.

The MC-CDMA system with SIC and IC with Gibbs sampler is simulated in the same estimation error which is 0.1 in the figures given Figure 5.6. Also for SIC the cancellation error is used equal to the estimation error in the simulations also in the analysis. To see the performance of the systems with different channel estimation errors, the two systems are simulated with the estimation error 0.3 with the MRC diversity technique. In Figure 5.7, the BER performance of the SIC for the first decoded user in the system in the two different channel estimation error, 0.1 and 0.3, in MRC is shown. Also the PC algorithm is applied with the cancellation error but it can be easily shown that there is not such a big difference in the BER performance with the perfect PC or imperfect PC with the cancellation error in SIC.



Figure 5.7. BER comparison according to estimation error in SIC.

As it can be seen in Figure 5.7, the BER performance of the SIC with the higher estimation error is decreasing. This difference is significant in the higher E_b /N_o values so for low E_b /N_o values the BER performance almost same with the change of estimation error. Actually this is estimation error statistics and every user has the same channel estimation error statistics.

For the second system which is IC with Gibbs sampler, the affect of the estimation error changes on the BER performance according to the different E_b /N_o values is simulated and the result is shown in Figure 5.8. Again the system simulated in MRC diversity technique with 25 users and 32 subcarriers and the results shown Figure 5.8 for the first user, like SIC results.



Figure 5.8. BER comparison according to estimation error in IC with Gibbs sampler.

As it can be seen in Figure 5.8, The MC-CDMA system with IC with Gibbs sampler is affecting from estimation error changes. With the high estimation errors, the BER is increasing but this affect like SIC is significant in higher E_b / N_o values. But for the same estimation errors, the MC-CDMA system using IC with Gibbs sampler gives better results than the MC-CDMA system with SIC in the same E_b / N_o values.

However, the channel estimation affect on the systems are different in some manner. For both of them, the BER performance is decreasing with increasing in the estimation error so both of them are affected. But the degrees of affect of the channel estimation on the systems are not the same. The BER performance of MC-CDMA system with SIC is decreasing more in the high estimation errors than the MC-CDMA system with IC with Gibbs sampler. According to the simulation results given in the Figure 5.1 and Figure 5.2, the IC cancellation with Gibbs sampler gives better results under the same conditions than the SIC. When the number of users in the system is increased, the BER performance of the SIC is decreasing more than the IC with Gibbs sampler. In CDMA systems, every user who enters the system behaves also as an interference source for all users in the system that is why MUD techniques are used. With IC techniques this affect is reduced and IC with Gibbs sampler is better results than the SIC. Also for all systems the estimation of the channel will be never perfect so it is needed to minimize the channel estimation error of the system and BER performance. The simulation results show us that the channel estimation error is not affecting the system with IC with Gibbs sampler as much as affecting the system with SIC.

Both systems are simulated with imperfect channel estimation and an imperfect PC algorithm is used in MC-CDMA system with SIC. Also for both systems the imperfect synchronization is assumed and misalignment is applied to the systems as considered in the system analyze. To sum up the overall simulation results, the MC-CDMA system using IC with Gibbs sampler gives better results in the simulations over MC-CDMA system with SIC.

6. CONCLUSION

In this thesis, we studied on MC-CDMA system model which is introduced in Section 3 analyzed in Section 4 with two different IC algorithms which are SIC and IC with Gibbs sampler for a comparison between two IC schemes

The MC-CDMA system is simulated much closer to the real system by using imperfect channel estimation and synchronization different from the system analyzed in the most of the previous research. Channel estimation success and its affects is a popular research subject which is not considered in this thesis. The Gibbs sampler algorithm is MCMC method that is often used in statistics and first Gibbs sampler is applied to CDMA system for IC, but IC with Gibbs sampler has been never applied to a MC-CDMA system before by any researcher.

The two IC schemes analyzed in this thesis have similar properties such as all users are decoded in a sequence manner in the both IC schemes. However, the amount of interference cancelled from the desired is user, in the SIC, previous users' interference is cancelled from the desired user. Besides, in the IC with Gibbs sampler, all users' interference is cancelled from the desired user, off course with an assumption of the users' signal in the first stage for the earlier users and this cancelled signal is updated after every decoded user. These properties of the both systems caused higher BER's when the previously decoded users are decoded wrongly. In this manner, two IC schemes have the same disadvantages.

Nevertheless, CDMA systems are affected from the total number of users enter the system so in the simulations it can be seen that both system gives almost same BER performance for the decoded first user and last user. SIC provide this with PC so decoding order is very important in SIC. This concludes that both IC schemes are working as fair systems for all users.

However, if the number of users in the system is changes, the system with SIC's BER performance is affected more than the system using IC with Gibbs sampler. In other

words, the system using IC with Gibbs sampler is working more stable than the system with SIC with the increase of the number of users in the system and BER performance of the users are not degrading much than the system with SIC when the systems are working in the same E_b / N_o values.

In the diversity techniques changes, the BER of the both systems are changes and both of them have better BER performance in MRC technique. Nevertheless, in EGC technique, IC with Gibbs sampler has better BER performance than SIC scheme again. But the BER performance increase in the change of diversity technique to MRC is less in IC with Gibbs sampler than IC. This can be also interpreted well in hardware sense because it is hard to implement MRC than EGC scheme.

Since both systems are very sensitive to the right decoding of the users because of the well interference cancellation and interference caused by other users are regenerated by estimating the channel affect. So the channel estimation is affecting the overall system BER performance. In the both systems, as the channel estimation error increase, the BER performance of the system is decreasing. Also in SIC, the cancellation error in the PC algorithm is increasing with the increase in the channel estimation error but previous researches shows us that this is not affect the system performance. Nevertheless, even the system using IC with Gibbs sampler's BER performance is decreasing with the increasing of the channel estimation error but the ratio of BER performance decrease in the IC with Gibbs sampler is less than the ratio of the BER performance decrease of SIC. IC with Gibbs sampler is resistive to the increase in the channel estimation error and working more stable than the SIC.

On the other hand, since the users are decoded serially in the both systems, with the increase in the number of total user in the system, the decoding time increase approximately linearly. For the stable results in IC with Gibbs sampler, a few number of stages need to be applied and in the simulation five stages is used. This results, increase in the time of decoding all of the users more than the decoding time of SIC. However, it is not presented in this thesis and also in the few number of stage, IC with Gibbs sampler also gives good results according to the SIC but still the decoding time is higher than the SIC scheme. But since the processor speed is increasing exponentially, this decoding time

increase complexity can be considered palatable. Also further, as data rate demands increase in the future, the number of users in the system will remain constant or possibly decrease.

As mentioned above, all users are decoded serially in the both systems, so both of them have the same advantageous when performed in the receiver, a great deal of hardware and power is saved by this scheme since there is only one decoder for all of users in the system. So it is very easy to implement in the receivers both IC schemes because of hardware save.

To sum up, IC with Gibbs sampler gives better results in the same E_b /N_o values and in the same number of users than SIC. Also SIC is more sensitive to channel estimation error and increase of the total number of users than the IC with Gibbs sampler. In addition, it is not too much needed to implement MRC diversity technique for better results in IC with Gibbs sampler since it also gives better results in EGC. However, the decoding time of IC with Gibbs sampler is higher than the SIC scheme. And in sense of hardware, both of the systems can be implemented similarly and very easily because of one decoder need.

The changes in BER performance of the IC with Gibbs sampler in the changes of the used number of stages used is not discussed in this thesis. The IC with Gibbs sampler can be compared with SIC in the capacity manner. Also both systems are compared with the same E_b / N_o values and different SINR values, so they are not compared with the same SINR values. These open issues should be studied in the future work.

APPENDIX A: OWN INTERFERENCE CALCULATION IN SIC FOR THE FIRST USER

As described in the thesis, in MC-CDMA with SIC, the pure received signal is used to give a decision for the first user. The received signal is given in Equation (3.7). And the decision integral for the first user is given in Equation (4.7). This decision signal is separated to the four parts as shown in Equation (4.6) where I_1 is own interference that first user is affected. So own interference term is,

$$I_{1} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \int_{k=2}^{M} \langle P_{k} \rangle \langle P$$

for own interference the subcarrier indexes are same, m=l.

$$I_{1} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sum_{k=2}^{M} \sum_{k=2}^{K} \sqrt{P_{k}} \beta_{k,m}(t) b_{k}[n] u_{T}(t-nT-\zeta_{k}) \Re\{c_{k,m}[n]e^{j(W_{m}t+\phi_{k,m}(t))}\} \Re\{\hat{q}_{1,m}c_{1,m}^{*}e^{-j(W_{m}(t-\zeta_{m})+\theta_{1,m})}\} dt, (A.2)$$

If we solve the own interference in EGC mode, so the subcarrier combining factor is only phase adjustment and the formula is,

$$I_{1} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sum_{k=2}^{M} \sqrt{P_{k}} \beta_{k,m}(t) b_{k}[n] u_{T}(t-nT-\zeta_{k}) \Re\{c_{k,m}[n]e^{j(W_{m}^{t+}\phi_{k,m}(t))}\} \Re\{e^{-j\hat{\psi}_{1,m}}c_{1,m}^{*}e^{-j(W_{m}^{t-}\zeta_{m}^{*})+\theta_{1,m}}\} dt, (A.3)$$

and, total angle will be,

$$I_{1} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sum_{k=2}^{M} \sum_{k=2}^{K} \sqrt{P_{k}} \beta_{k,m}(t) b_{k}[n] u_{T}(t-nT-\zeta_{k}) \Re\{c_{k,m}[n]e^{j(W_{m}^{t+}\phi_{k,m}(t))}\} \Re\{c_{1,m}^{*}e^{-j(W_{m}^{(t-}\zeta_{m})+\hat{\psi}_{1,m}^{*}+\theta_{1,m})}dt, (A,4)$$

if we extract real part,

$$I_{1} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty m=1}^{\infty} \sum_{k=2}^{M} \sqrt{P_{k}} \beta_{k,m}(t) b_{k}[n] u_{T}(t-nT-\zeta_{k}) \Big[C_{k,m}^{I}[n] \cos(\psi_{m}t + \phi_{k,m}(t)) - C_{k,m}^{Q}[n] \sin(\psi_{m}t + \phi_{k,m}(t)) \Big], \quad (A.5)$$

$$\left[c_{1,m}^{*I} \cos((\psi_{m}(t-\zeta_{m}) + \hat{\psi}_{1,m} + \theta_{1,m})) - c_{1,m}^{*Q} \sin((\psi_{m}(t-\zeta_{m}) + \hat{\psi}_{1,m} + \theta_{1,m})) \right] dt$$

after the odd and even function properties of trigonometric expressions,

$$I_{1} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sum_{k=2}^{M} \sqrt{P_{k}} \beta_{k,m}(t) b_{k}[n] u_{T}(t-nT-\zeta_{k}) \Big[\int_{C_{k,m}}^{T} [n] \cos(\psi_{m}t + \phi_{k,m}(t)) - \int_{C_{k,m}}^{Q} [n] \sin(\psi_{m}t + \phi_{k,m}(t)) \Big], \quad (A.6)$$

$$\left[\int_{1,m}^{*T} \cos(\psi_{m}(t-\zeta_{m}) + \hat{\psi}_{1,m} + \theta_{1,m}) + \int_{1,m}^{*Q} \sin(\psi_{m}(t-\zeta_{m}) + \hat{\psi}_{1,m} + \theta_{1,m}) \Big] dt$$

after multiplications,

$$I_{1} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty,m=1}^{\infty} \sum_{k=2}^{M} \sqrt{P_{k}} \beta_{k,m}(t) b_{k}[n] u_{T}(t-nT-\zeta_{k}) \left\{ \int_{C_{k,m}}^{T} [n] \cos(\psi_{m}t + \phi_{k,m}(t)) C_{1,m}^{*T} \cos(\psi_{l}(t-\zeta_{1}) + \hat{\psi}_{1,m} + \theta_{1,m}) \right] \\ + \left[\int_{C_{k,m}}^{T} [n] \cos(\psi_{m}t + \phi_{k,m}(t)) C_{1,m}^{*Q} \sin(\psi_{l}(t-\zeta_{1}) + \hat{\psi}_{1,m} + \theta_{1,m}) \right] \\ - \left[\int_{C_{k,m}}^{Q} [n] \sin(\psi_{m}t + \phi_{k,m}(t)) C_{1,m}^{*T} \cos(\psi_{l}(t-\zeta_{1}) + \hat{\psi}_{1,m} + \theta_{1,m}) \right] , \qquad (A.7) \\ - \left[\int_{C_{k,m}}^{Q} [n] \sin(\psi_{m}t + \phi_{k,m}(t)) C_{1,m}^{*Q} \sin(\psi_{l}(t-\zeta_{1}) + \hat{\psi}_{1,m} + \theta_{1,m}) \right] dt$$

where, the orthogonal function in Equation (A.7) have these properties.

$$\begin{vmatrix} c_{1,m}^{T} c_{1,m}^{*Q} \\ c_{1,m} c_{1,m} \end{vmatrix} = 0 \\ \begin{bmatrix} c_{1,m}^{T} c_{1,m} \\ c_{1,m} c_{1,m} \end{vmatrix} = 1 \\ \begin{bmatrix} c_{1,m}^{Q} c_{1,m}^{*Q} \\ c_{1,m} c_{1,m} \end{vmatrix} = 1$$
(A.8)

Also, the angle expressions in Equation (A.7) are,

$$\boldsymbol{\phi}_{k,m}(t) = \boldsymbol{\psi}_{k,m}(t) + \boldsymbol{\theta}_{k,m} - \boldsymbol{\omega}_{m}\boldsymbol{\zeta}_{k} \qquad \boldsymbol{\phi}_{1,m}(t) = \boldsymbol{\psi}_{1,m}(t) + \boldsymbol{\theta}_{1,m} - \boldsymbol{\omega}_{m}\boldsymbol{\zeta}_{1} \quad (A.9)$$

So, with these properties, the own interference can be written as,

$$I_{1} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty m=1}^{\infty} \sum_{k=2}^{M} \sqrt{P_{k}} \beta_{k,m}(t) b_{k}[n] u_{T}(t-nT-\zeta_{k}) \left\{ C_{k,m}^{I}[n] \cos(\psi_{m}t + \phi_{k,m}(t)) C_{1,m}^{*I} \cos(\psi_{l}(t-\zeta_{1}) + \hat{\psi}_{1,m} + \theta_{1,m}) \right] - \left[C_{k,m}^{Q}[n] \sin(\psi_{m}t + \phi_{k,m}(t)) C_{1,m}^{*Q} \sin(\psi_{l}(t-\zeta_{1}) + \hat{\psi}_{1,m} + \theta_{1,m}) \right] dt, \qquad (A.10)$$

with orthogonal functions arrangement,

$$I_{1} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sum_{k=2}^{M} \sqrt{P_{k}} \beta_{k,m}(t) b_{k}[n] u_{T}(t-nT-\zeta_{k}) \left\{ \int_{C_{k,m}}^{T} [n]_{C_{1,m}}^{*T} \cos(\psi_{m}t + \phi_{k,m}(t)) \cos(\psi_{m}(t-\zeta_{m}) + \hat{\psi}_{1,m} + \theta_{1,m}) \right], (A.11) \\ - \left[\int_{C_{k,m}}^{Q} [n]_{C_{1,m}}^{*Q} \sin(\psi_{m}t + \phi_{k,m}(t)) \sin(\psi_{m}(t-\zeta_{m}) + \hat{\psi}_{1,m} + \theta_{1,m}) \right] dt$$

and since $\phi_{k,m}(t)$ is given in Equation (A.9), then,

$$I_{1} = \frac{1}{T} \int_{0}^{T} \sum_{n=-\infty,m=1}^{\infty} \sum_{k=2}^{M} \sqrt{P_{k}} \beta_{k,m}(t) b_{k}[n] u_{T}(t-nT-\zeta_{k}) \left\{ C_{k,m}^{I}[n] C_{1,m}^{*I} \cos(\psi_{m}t + \phi_{k,m}(t)) \cos(\psi_{m}t + \hat{\phi}_{1,m}(t)) \right\} - \left[C_{k,m}^{Q}[n] C_{1,m}^{*Q} \sin(\psi_{m}t + \phi_{k,m}(t)) \sin(\psi_{m}t + \hat{\phi}_{1,m}(t)) \right] dt$$
(A.12)

And with arrangements, the formula of own interference is,

$$I_{1} = \frac{1}{2T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sum_{k=2}^{M} \sqrt{P_{k}} \beta_{k,m}(t) b_{k}[n] u_{T}(t-nT-\zeta_{k}) \\ \left\{ \begin{bmatrix} I \\ C_{k,m}[n] \\ C_{1,m}^{*I}[0] (\cos(w_{m}t + \phi_{k,m}(t) + w_{m}t + \hat{\phi}_{1,m}(t)) + \cos(w_{m}t + \phi_{k,m}(t) - w_{m}t - \hat{\phi}_{1,m}(t))) \end{bmatrix} , (A.13) \\ - \begin{bmatrix} C_{k,m}^{\varrho}[n] \\ C_{1,m}^{*\varrho}[0] (\cos(w_{m}t + \phi_{k,m}(t) + w_{m}t + \hat{\phi}_{1,m}(t)) - \cos(w_{m}t + \phi_{k,m}(t) - w_{m}t - \hat{\phi}_{1,m}(t))) \end{bmatrix} \right\} dt$$

with summations, the own interference will be as shown in Equation (A.14).

$$I_{1} = \frac{1}{2T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sum_{k=2}^{M} \sum_{k=2}^{K} \sqrt{P_{k}} \beta_{k,m}(t) b_{k}[n] u_{T}(t-nT-\zeta_{k})$$

$$\left\{ \begin{bmatrix} C_{k,m}^{I}[n] C_{1,m}^{*I}[0] (\cos(2w_{m}t+\phi_{k,m}(t)+\hat{\phi}_{1,m}(t))+\cos(\phi_{k,m}(t)-\hat{\phi}_{1,m}(t))) \end{bmatrix} - \begin{bmatrix} C_{k,m}^{Q}[n] C_{1,m}^{*Q}[0] (\cos(2w_{m}t+\phi_{k,m}(t)+\hat{\phi}_{1,m}(t))-\cos(\phi_{k,m}(t)-\hat{\phi}_{1,m}(t))) \end{bmatrix} \right\} dt$$
(A.14)

Since,

$$\int_{0}^{T} \cos(2w_{m}t + \phi_{k,m}(t) + \hat{\phi}_{1,m}(t))dt = 0$$
(A.15)

then using Equation (A.15),

$$I_{1} = \frac{1}{2T} \int_{0}^{T} \sum_{n=-\infty}^{\infty} \sum_{k=2}^{M} \sum_{k=2}^{K} \sqrt{P_{k}} \hat{\beta}_{k,m}(t) \hat{b}_{k}[n] u_{T}(t-nT-\zeta_{k})$$

$$\{ \left[C_{k,m}^{I}[n] C_{1,m}^{*I}[0] \cos(\phi_{k,m}(t) - \hat{\phi}_{1,m}(t)) \right] + \left[C_{k,m}^{Q}[n] C_{1,m}^{*Q}[0] \cos(\phi_{k,m}(t) - \hat{\phi}_{1,m}(t)) \right] \} dt$$
(A.16)

and, if we use separate integration because of misalignment, so there will be two integration with different two borders,

$$I_{1} = \frac{1}{2T} \int_{0}^{\zeta} \sum_{n=-\infty}^{\infty} \sum_{k=2}^{M} \sum_{k=2}^{K} \sqrt{P_{k}} \beta_{k,m}(t) b_{k}[n] u_{T}(t-nT-\zeta_{k})$$

$$\left\{ \begin{bmatrix} c_{k,m}^{I}[n] c_{1,m}^{*I}[0] \cos(\phi_{k,m}(t) - \hat{\phi}_{1,m}(t)) \end{bmatrix} + \begin{bmatrix} c_{k,m}^{\varrho}[n] c_{1,m}^{*\varrho}[0] \cos(\phi_{k,m}(t) - \hat{\phi}_{1,m}(t)) \end{bmatrix} \right\} dt \qquad (A.17)$$

$$+ \frac{1}{2T} \int_{\zeta}^{T} \sum_{n=-\infty}^{\infty} \sum_{k=2}^{M} \sum_{k=2}^{K} \sqrt{P_{k}} \beta_{k,m}(t) b_{k}[n] u_{T}(t-nT-\zeta_{k})$$

$$\left\{ \begin{bmatrix} c_{k,m}^{I}[n] c_{1,m}^{*I}[0] \cos(\phi_{k,m}(t) - \hat{\phi}_{1,m}(t)) \end{bmatrix} + \begin{bmatrix} c_{k,m}^{\varrho}[n] c_{1,m}^{*\varrho}[0] \cos(\phi_{k,m}(t) - \hat{\phi}_{1,m}(t)) \end{bmatrix} \right\} dt$$

with summations, the own interference will be,

$$I_{1} = \frac{1}{2T} \sum_{m=1}^{M} \sum_{k=2}^{K} \sqrt{P_{k}} \beta_{k,m} b_{k} [-1] \left\{ \begin{bmatrix} I \\ C_{k,m} [-1] C_{1,m} [0] \cos \phi_{k,m} (0) - \hat{\phi}_{1,m} (0) \end{bmatrix} + \begin{bmatrix} Q \\ C_{k,m} [0] \cos \phi_{k,m} (0) - \hat{\phi}_{1,m} (0) \end{bmatrix} \right\} \zeta_{k}, \quad (A.18)$$

$$\frac{1}{2T} \sum_{m=1}^{M} \sum_{k=2}^{K} \sqrt{P_{k}} \beta_{k,m} b_{k} [0] \left\{ \begin{bmatrix} I \\ C_{k,m} [0] \cos \phi_{k,m} (0) - \hat{\phi}_{1,m} (0) \end{bmatrix} + \begin{bmatrix} Q \\ C_{k,m} [0] \cos \phi_{k,m} (0) - \hat{\phi}_{1,m} (0) \end{bmatrix} \right\} \left\{ T - \zeta_{k} \right\}$$

and if we define angle as shown Equation (A.19) to shorten the Equation (A.18),

$$\nabla \phi = \phi_{k,m}(0) - \hat{\phi}_{1,m}(0)$$
(A.19)

the own interference will be written Equation (A.20) with this angles.

$$I_{1} = \frac{1}{2T} \sum_{m=1}^{K} \sum_{k=2}^{K} \sqrt{P_{k}} \beta_{k,m} b_{k} [-1] \{ \begin{bmatrix} I \\ C_{k,m} \end{bmatrix} [-1] C_{1,m}^{*I} [0] \cos(\nabla \phi)] + \begin{bmatrix} Q \\ C_{k,m} \end{bmatrix} [-1] C_{1,m}^{*Q} [0] \cos(\nabla \phi)] \} \zeta_{k}$$

$$\frac{1}{2T} \sum_{m=1}^{K} \sum_{k=2}^{K} \sqrt{P_{k}} \beta_{k,m} b_{k} [0] \{ \begin{bmatrix} I \\ C_{k,m} \end{bmatrix} [0] \cos(\nabla \phi)] + \begin{bmatrix} Q \\ C_{k,m} \end{bmatrix} [0] \cos(\nabla \phi)] + \begin{bmatrix} Q \\ C_{k,m} \end{bmatrix} [0] \cos(\nabla \phi)] \} (T - \zeta_{k})$$
(A.20)

For conclusion, if we define new variables to express own interference shortly, the own interference will be written as,

$$I_{1} = \frac{1}{2T} \sum_{k=2}^{K} \sqrt{P_{k}} \sum_{m=1}^{M} \beta_{k,m} \left[(a_{1}^{I} \zeta_{k} + a_{2}^{I} (T - \zeta_{k})) \cos(\Delta \phi_{I}) + (a_{1}^{\varrho} \zeta_{k} + a_{2}^{\varrho} (T - \zeta_{k})) \cos(\Delta \phi_{I}) \right], (A.21)$$

and, the variables in Equation (A.21) will be as shown Equation (A.22).

$$a_{1}^{I} = b_{k}(-1)c_{k,m}^{I}(-1)c_{1,m}^{I}(0)$$

$$a_{2}^{I} = b_{k}(0)c_{k,m}^{I}(0)c_{1,m}^{I}(0)$$

$$a_{1}^{\varrho} = b_{k}(-1)c_{k,m}^{\varrho}(-1)c_{1,m}^{\varrho}(0)$$

$$a_{2}^{\varrho} = b_{k}(0)c_{k,m}^{\varrho}(0)c_{1,m}^{\varrho}(0)$$
(A.22)

APPENDIX B: DERIVATION OF VARIANCE TERMS IN SIC FOR FIRST USER

As Appendix A, we will derive the variance terms in EGC mode. As we found the own interference term in Appendix A, the variance term of own interference can be derived which is given in Equation (4.31)

First of all, it is assumed that the $1/2\cos(2\omega_m t + \phi_{k,m})$ terms are driven to zero by an appropriate filter, and that all cross terms are uncorrelated due to the random phase of $\hat{\phi}_{k,m}$. Because I_1 is zero mean, the variance term is,

$$\operatorname{var}[I_1 \mid \lambda, h] = E[I_1 \mid \lambda, h]^2 \tag{B.1}$$

then the variance term will written as,

$$I_{1} = \frac{1}{4T^{2}} \sum_{k=2}^{K} E \left[\sqrt{P_{k}} \beta_{k,m}^{2} \left[(a_{1}^{T} \zeta_{k} + a_{2}^{T} (T - \zeta_{k})) \cos(\Delta \phi_{l}) + (a_{1}^{Q} \zeta_{k} + a_{2}^{Q} (T - \zeta_{k})) \cos(\Delta \phi_{l}) \right] \lambda, h \right]^{2}, \quad (B.2)$$

and we consider trigonometric channels are independent, since the integration of a symbol interval is long relative to period of the argument, thus,

$$E\left[\left(a_{1}^{\prime}\zeta_{k}+a_{2}^{\prime}(T-\zeta_{k})\right)\cos(\Delta\phi_{l})+\left(a_{1}^{\varrho}\zeta_{k}+a_{2}^{\varrho}(T-\zeta_{k})\right)\cos(\Delta\phi_{l})|\lambda,h\right]^{2}$$
(B.3)

And with the summation property of expectation value,

$$2E[(a_{1}^{I}\zeta_{k}+a_{2}^{I}(T-\zeta_{k}))\cos(\Delta\phi_{I})|\lambda,h]^{2}=E[a_{1}^{I}\zeta_{k}+a_{2}^{I}(T-\zeta_{k})]^{2}$$
(B.4)

And the random variables a_1 and a_2 have these properties given Equation (B.5),

$$E[a_1^I a_2^I] = 0$$
 and $E[a_1^I]^2 = E[a_2^I]^2 = 1$. (B.5)

Further, since ζ_k is uniformly distributed in [0,T),

$$E[\zeta^2] = \frac{T^2}{3} \tag{B.6}$$

then, the result using Equation (B.5) and Equation (B.6) is:

$$E\left[a_{1}^{\prime}\zeta_{k}+a_{2}^{\prime}(T-\zeta_{k})\right]^{2}=\frac{T^{2}}{3}$$
(B.7)

Using this result and along with the fact that $\{\beta_{k,m}\}$ and $\{P_k\}$ are constant when conditioned on $\{\lambda, h\}$, the variance term of own interference for first user can be written as shown Equation (B.8).

$$\operatorname{var}\left[I_{1} \mid \lambda, h\right] = \frac{1}{6} \sum_{k=2}^{K} \sum_{m=1}^{M} P_{k} \beta_{k,m}^{2}$$
(B.8)

Secondly to calculate the variance of other carrier interference term for the first user in MC-CDMA system with SIC, recognized that as own interference term also mean value of other carrier interference is 0. So the variance term is,

$$\operatorname{var}[J_1 \mid \lambda, h] = E[J_1 \mid \lambda, h]^2 \tag{B.9}$$

and, secondly,

$$\cos\left(\Omega_{k,m,l}\right) = \Re\left\{e^{2\pi j/T(m-l)t}e^{j\Delta\phi_J}\right\}$$
(B.10)

and integrating,

$$\int_{0}^{\zeta} \Re\left\{e^{j\Omega_{k,m,l}(t)}\right\} dt = \Re\left\{\int_{0}^{\zeta} e^{j\Omega_{k,m,l}(t)} dt\right\} = \frac{T}{2\pi(m-l)} \left[\sin(\Delta\omega\zeta_{k} + \Delta\phi_{j}) - \sin(\Delta\phi_{j})\right]$$
(B.11)

where,

$$\Delta \phi_J = \phi_{k,l}(0) - \hat{\phi}_{l,m}(0) \text{ and } \Delta \omega = \omega_m - \omega_l . \tag{B.12}$$

Also, we need to note and consider that all the terms in the sum are uncorrelated, and have zero crosscorrelation. This is due to the fact that the random phase component $\theta_{k,m}$ is i.i.d. for all *k* and *m*. Another fact is that users are uncorrelated, as ζ_k is i.i.d. Using these observations, the variance of the other carrier interference is,

$$E[J_{1} \mid \lambda, h]^{2} = \sum_{k=2}^{K} P_{k} \sum_{m=1}^{M} \sum_{l=1 \atop l \neq m}^{M} \frac{\beta_{k,l}^{m}}{16\pi^{2}(m-l)^{2}} \times E[(d_{1}^{I}(\sin(\Delta\omega\zeta_{k} + \Delta\phi_{j}) + \sin(\Delta\phi_{j}) + d_{1}^{Q}(\sin(\Delta\omega\zeta_{k} + \Delta\phi_{j}) + \sin(\Delta\phi_{j}))^{2}) + (d_{2}^{I}(\sin(\Delta\omega\zeta_{k} + \Delta\phi_{j}) + \sin(\Delta\omega\zeta_{k} + \Delta\phi_{j}) + d_{2}^{Q}(\sin(\Delta\omega\zeta_{k} + \Delta\phi_{j}) + \sin(\Delta\omega\zeta_{k} + \Delta\phi_{j}))^{2} \mid \lambda, h],$$
(B.13)

On the other hand, note that the cosine and sine cross terms have zero correlation due to the ζ_k term if they have different arguments and for the same arguments, they are orthogonal. Then the result is,

$$E[J_1 | \lambda, h]^2 = \sum_{k=2}^{K} P_k \sum_{m=1}^{M} \sum_{\substack{l=1\\l\neq m}}^{M} \frac{\beta_{k,l}^m}{16\pi^2 (m-l)^2} \left\{ E \left| d_1^I \right|^2 + E \left| d_2^Q \right|^2 + E \left| d_2^Q \right|^2 + E \left| d_2^Q \right|^2 \right\}$$
(B.14)

and, also the random variable d_1 and d_2 have same properties as shown in Appendix A for random variables a_1 and a_2 . Then the variance of the other carrier interference in MC-CDMA with SIC for the first user is written Equation (B.15).

$$E[J_1 \mid \lambda, h]^2 = \frac{1}{4\pi^2} \sum_{k=2}^{K} P_k \sum_{m=1}^{M} \sum_{\substack{l=1\\l\neq m}}^{M} \frac{\beta_{k,l}^m}{(m-l)^2}$$
(B.15)

APPENDIX C: SIMULATION CODE FOR SIC

% MC-CDMA with successive interference cancellation % It is appllied MC-CDMA for EGC

```
clear all
warning off MATLAB:divideByZero
No=3.9811e-011;
user=25; %max number of users
multicarrier=32; %number of multicarriers
bitsayisi=5; %number of bits to send
carrierdiff=200e3; %carrier differences between multicarriers
turn=10;
EbNo=0;
EbNos=10^(EbNo/10);
p(1:user)=No*EbNos; %initial power values
esterror=0.1; % Estimation error of channel model
cancelerror=esterror;
L=1:multicarrier;
[M,L] = meshqrid(L,L);
MLfark2=(M-L).^2;
carriers=2.1e9:carrierdiff:2.1e9+(multicarrier-1)*carrierdiff; %carriers for
multicarriers
d=zeros(user,bitsayisi); %initialize receiver part
b=sign(2*rand(user,bitsayisi)-1);% generate bit sets for the users to send
Wn=walsh(user*multicarrier,'+-'); %generate orthogonal walsh codes with the
format -1 and +1
Wn=flipud(Wn);
LWn=size(Wn,1); % calculate the length of code
spread=bitsayisi*LWn; %The max length of baseband signal
bb=imresize(b,[user spread]);
bbb=repmat(bb,multicarrier,1);
clear bb;
WW=repmat(Wn,1,bitsayisi);
baseband=bbb.*WW(1:size(bbb,1),:);
clear bbb WW;
baseband=reshape(baseband',spread,user,multicarrier);
baseband=permute(baseband,[2 1 3]); %The basaband signal which is multiplied
with orthogonal code
Wn=reshape(Wn(1:multicarrier*user,:)',LWn,user,multicarrier);
Wn=permute(Wn,[2 1 3]); %Orthogonal code sequence for receiver part
teta=2*pi*rand(user,multicarrier); %generate the phase angle of multicarriers in
sender part for each user
t=0:1/carriers(1)/20:LWn*bitsayisi/carriers(1)-1/carriers(1)/20; %create time
domain
```

lt=length(t); % Total time to send max number of bits misal=(LWn-20)*20*rand(user,1); % adding misalignment angle to the pase of the receiver part rt=zeros(1,lt/bitsayisi); ut=zeros(1,lt); % initiate decoded signal matrix rtt=zeros(1,lt); % initiate received signal matrix fierr(25,LWn*bitsayisi,32)=0; % estimated channel model phase betaerr(25,LWn*bitsayisi,32)=0; % estimated channel model absolute value dummy=zeros(user,lt/bitsayisi);

%SENDER PART

```
for rutin=1:turn:
for bi=1:bitsayisi;
  for ui=1:user;
     for mi=1:multicarrier;
      h=sqrt(0.5)*randn(1,lt/bitsayisi/20)+i*sqrt(0.5)*randn(1,lt/bitsayisi/20);
%generate channel model
                    beta=abs(imresize(h,[1 lt/bitsayisi])); %find channel absolute
value
                    fi=angle(imresize(h,[1 lt/bitsayisi]))+teta(ui,mi); %find
channel phase
        noise=No*sqrt(0.1)*randn(1,lt/bitsayisi); % noise at the receiver part
        r=sqrt(p(ui)).*beta.*imresize(baseband(ui,(bi-1)*LWn+1:bi*LWn,mi),[1
lt/bitsayisi])...
        .*real(exp(i*(2*pi*carriers(mi)*t((bi-1)*LWn*20+1:bi*LWn*20)+fi-
(carriers(mi)*misal(ui))))+noise;
                    %generate received signal in each multicarrier
        rt=rt+r;
        herri=exp(sqrt(esterror)*randn(1,lt/bitsayisi/20));
                    herrq=exp(sqrt(esterror)*randn(1,lt/bitsayisi/20)); %generate
channel model estimate error
        herr=(herri.*real(h))+(i*(herrq.*imag(h))); %estimate channel model
with error
                    fierr(ui,(bi-
1)*lt/bitsayisi/20+1:bi*lt/20/bitsayisi,mi)=angle(herr);%generate estimated
channel model phase
                    betaerr(ui,(bi-
1)*lt/bitsayisi/20+1:bi*lt/20/bitsayisi,mi)=abs(herr);%generate estimated channel
model absolute value
        Beta2(ui,mi)=(mean(beta)).^2;
        Aeta(ui,mi)=mean(beta)*cos(mean(fi)-mean(fierr(ui,:,mi)));
     end:
     rtt((bi-1)*lt/bitsayisi+1:bi*lt/bitsayisi)=rtt((bi-
1)*lt/bitsayisi+1:bi*lt/bitsayisi)+rt;
     %total received power for each user
```

rt=zeros(1,lt/bitsayisi); % pass to the next user end;

% RECEIVER PART

```
for ui=1:user;
     for mi=1:multicarrier;
        if ui \sim =1;
        z=(d(ui-1,bi)*sqrt(p(ui-1))).*imresize(Wn(ui-1,:,mi),[1 lt/bitsayisi])...
            .*imresize(betaerr(ui-1,(bi-1)*lt/bitsayisi/20+1:bi*lt/20/bitsayisi,mi),[1
lt/bitsayisi])...
            .*real(exp(i*(2*pi*carriers(mi)*t((bi-1)*LWn*20+1:bi*LWn*20)...
            +imresize(fierr(ui-1,(bi-1)*lt/bitsayisi/20+1:bi*lt/20/bitsayisi,mi),[1
lt/bitsayisi])...
            +teta(ui-1,mi)-(carriers(mi)*misal(ui)))));
        rtt((bi-1)*lt/bitsayisi+1:bi*lt/bitsayisi)=rtt((bi-
1)*lt/bitsayisi+1:bi*lt/bitsayisi)-z;
        end
        u((bi-1)*lt/bitsayisi+1:bi*lt/bitsayisi)=rtt((bi-
1)*lt/bitsayisi+1:bi*lt/bitsayisi)...
          .*imresize(betaerr(ui,(bi-1)*lt/bitsayisi/20+1:bi*lt/20/bitsayisi,mi),[1
%
lt/bitsayisi])...
%
          For MRC, the amplitude estimation is also used.
         .*imresize(Wn(ui,:,mi),[1 lt/bitsayisi]).*real(exp(-
i*(2*pi*carriers(mi)*t((bi-1)*LWn*20+1:bi*LWn*20)...
         +imresize(fierr(ui,(bi-1)*lt/bitsayisi/20+1:bi*lt/20/bitsayisi,mi),[1
lt/bitsayisi])...
         +teta(ui,mi)-(carriers(mi)*misal(ui)))));
         %decode signal in each multicarrier
        ut((bi-1)*lt/bitsayisi+1:bi*lt/bitsayisi)...
         =ut((bi-1)*lt/bitsayisi+1:bi*lt/bitsayisi)+u((bi-
1)*lt/bitsayisi+1:bi*lt/bitsayisi);
         %sum of the signal for user from all multicarriers
      end
      if ui = =1;
      d(ui,bi)=sign((sum(ut((bi-1)*lt/bitsayisi+1:(bi*lt/bitsayisi))))...
         *(t(2)-t(1))/max(t)/bitsayisi);
      else
         d(ui,bi)=sign((sum(dummy(ui-1,end-floor(misal(ui-1)):end)...
         +(sum(ut((bi-1)*lt/bitsayisi+1:(bi*lt/bitsayisi-floor(misal(ui-1)))))))...
         *(t(2)-t(1))/max(t)/bitsayisi);
      end
```

% give decision from the the received signal

```
if bi~=1;
dummy(ui,:)=ut((bi-1)*lt/bitsayisi+1:bi*lt/bitsayisi);
end
ut((bi-1)*lt/bitsayisi+1:bi*lt/bitsayisi)=0;
end
```

% POWER CONTROL PART

```
Beta2rs=imresize(Beta2,[multicarrier*user,multicarrier]);
MLfark2rp=repmat(MLfark2,user,1);
tria=Beta2rs./MLfark2rp/4/pi^2;
tria(tria==inf)=0;
triasum1=sum(tria,2);
triasum2=reshape(triasum1,multicarrier,user)';
Bk=sum(Beta2/6+triasum2,2);
Ak=sum(Aeta,2).^2;
Nou=No./Bk;
v=sum(p)-(1-cancelerror)*cumsum([0 p(1:user-1)]);
p(1)=No*EbNos/Ak(1);
p(2:user)=p(1:user-1)-(((1-cancelerror)*p(1:user-1).^2)./(v(1:user-1)+Nou(2:user)'));
```

end

% BER calculation part

```
errfirst=b(1,:)-d(1,:);
errlast=b(user,:)-d(user,:);
errfirst(errfirst==-2)=2;
errlast(errlast==-2)=2;
errfirsts(rutin)=sum(errfirst)/2;
errlasts(rutin)=sum(errlast)/2;
end
firstusererror=sum(errfirsts)/(bitsayisi*turn)*100
lastusererror=sum(errlasts)/(bitsayisi*turn)*100
```

% Walsh Code Script

function Wn=walsh(N,option); %WALSH Returns orthogonal walsh codes of length N which is used in program as number of multicarrier*number of user M = ceil(log(N)/log(2)); %find the power of 2 to match N, e.g. M=5 for N=32 if (nargin ~= 2),

```
option = '++';
                            %Set default to ones and zeros
end
if (option=='+-'),
     if 2^M = 1,
           Wn = [1a];
     elseif 2^M == 2,
           Wn = [1 1; 1 -1];
     else
           for k = 1:M-2,
                Wn = [Wn Wn; Wn (-Wn)];
           end
     end
else
     if 2^M == 1,
           Wn = [1a];
     elseif 2^M == 2,
           Wn = [1 1; 1 0];
     else
           Wn = [1 1 1 1; 1 0 1 0; 1 1 0 0; 1 0 0 1];
           for k = 1:M-2,
                Wn = [Wn Wn; Wn \sim Wn];
           end
     end
end
```

APPENDIX D: SIMULATION CODE FOR IC WITH GIBBS SAMPLER

The sender and other initial code for the IC with Gibbs sampler is same with SIC code. Also since no power control is needed in IC with Gibbs sampler, but there is a code for PC because of update procedure according to the channel changes. Here the code of receiver part of the IC with Gibbs sampler.

% RECEIVER PART FOR IC WITH GIBBS SAMPLER

```
for bii=1:bitsayisi;
```

for ui=1:user;

for mi=1:multicarrier;

```
zy=filter(coeffs,1,(d(ui,bii)*sqrt(p(ui))*imresize(Wn(ui,:,mi),[1 lt/bitsayisi])...
```

```
.*imresize(betaerr(ui,(bii-1)*lt/bitsayisi/20+1:bii*lt/20/bitsayisi,mi),[1
```

lt/bitsayisi]) ...

```
.*real(exp(i*(2*pi*carriers(mi)*t((bii-1)*LWn*20+1:bii*LWn*20)...
```

```
+imresize(fierr(ui,(bii-1)*lt/bitsayisi/20+1:bii*lt/20/bitsayisi,mi),[1 lt/bitsayisi])...
```

```
+teta(ui,mi)-(carriers(mi)*misal(ui))))));
```

```
z(ui,(bii-1)*lt/bitsayisi/10+1:bii*lt/bitsayisi/10,mi)=zy(10:10:end);
```

clear zy;

end

end

end

```
zs=sum(z,3);
```

```
for gibbs=1:gibbst;
```

for ui=1:user;

```
for mi=1:multicarrier;
```

```
u((bi-1)*lt/bitsayisi+1:bi*lt/bitsayisi)=(rtt((bi-1)*lt/bitsayisi+1:bi*lt/bitsayisi)...
-imresize(sum(sum(z(1:ui-1,(bi-1)*lt/bitsayisi/10+1:bi*lt/bitsayisi/10,:)),3),[1
```

```
lt/bitsayisi]) ...
```

-imresize(sum(sum(z(ui+1:user,(bi-1)*lt/bitsayisi/10+1:bi*lt/bitsayisi/10,:)),3),[1 lt/bitsayisi]))...

.*imresize(Wn(ui,:,mi),[1 lt/bitsayisi]).*real(exp(-i*(2*pi*carriers(mi)*t((bi-1)*LWn*20+1:bi*LWn*20)...

+imresize(fierr(ui,(bi-1)*lt/bitsayisi/20+1:bi*lt/20/bitsayisi,mi),[1 lt/bitsayisi])... +teta(ui,mi)-(carriers(mi)*misal(ui)))));

%decode signal in each multicarrier

ut((bi-1)*lt/bitsayisi+1:bi*lt/bitsayisi)...

=ut((bi-1)*lt/bitsayisi+1:bi*lt/bitsayisi)+u((bi-1)*lt/bitsayisi+1:bi*lt/bitsayisi);

%sum of the signal for user from all multicarriers

end

dik=(sum(dummy(end-floor(misal(ui)):end)...

```
+(sum(ut((bi-1)*lt/bitsayisi+1:(bi*lt/bitsayisi-floor(misal(ui))))))...
```

```
*(t(2)-t(1))/max(t)/bitsayisi);
```

```
pdk=1/(1+10^(2*dik/No));
```

qrand=rand(1);

```
if qrand<=pdk;
```

```
d(ui,bi)=-1;
```

end

```
if d(ui,bi)==-1;
```

```
for mi=1:multicarrier;
```

```
zy=filter(coeffs,1,(d(ui,bi)*sqrt(p(ui))*imresize(Wn(ui,:,mi),[1 lt/bitsayisi])...
```

```
.*imresize(betaerr(ui,(bi-1)*lt/bitsayisi/20+1:bi*lt/20/bitsayisi,mi),[1
```

lt/bitsayisi]) ...

```
.*real(exp(i*(2*pi*carriers(mi)*t((bi-1)*LWn*20+1:bi*LWn*20)...
```

+imresize(fierr(ui,(bi-1)*lt/bitsayisi/20+1:bi*lt/20/bitsayisi,mi),[1 lt/bitsayisi])...

```
+teta(ui,mi)-(carriers(mi)*misal(ui))))));
```

```
z(ui,(bi-1)*lt/bitsayisi/10+1:bi*lt/bitsayisi/10,mi)=zy(10:10:end);
```

clear zy;

end end

```
% give decision from the the received signal
```

```
dummy(ui,:)=ut((bi-1)*lt/bitsayisi+1:bi*lt/bitsayisi);
```

ut((bi-1)*lt/bitsayisi+1:bi*lt/bitsayisi)=0;

```
end
```

end

```
% POWER CONTROL PART
```

Beta2rs=imresize(Beta2,[multicarrier*user,multicarrier]);

```
MLfark2rp=repmat(MLfark2,user,1);
```

```
tria=Beta2rs./MLfark2rp/4/pi^2;
```

tria(tria==inf)=0;

```
triasum1=sum(tria,2);
```

triasum2=reshape(triasum1,multicarrier,user)';

```
Bk=sum(Beta2/6+triasum2,2);
```

```
Ak=sum(Aeta,2).^2;
```

Nou=No./Bk;

end

```
errfirst=b(1,:)-d(1,:);
```

```
errfirst(errfirst==-2)=2;
```

```
errlast=b(user,:)-d(user,:);
```

```
errlast(errlast==-2)=2;
```

```
errfirsts(rutin)=sum(errfirst)/2;
```

```
errlasts(rutin)=sum(errlast)/2;
```

end

```
firstusererror=sum(errfirsts)/(bitsayisi*turn)*100
```

```
lastusererror=sum(errlasts)/(bitsayisi*turn)*100
```

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