

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE

FOR REFERENCE

NOT TO BE TAKEN FROM THIS ROOM

A STUDY OF BEHAVIOR OF DIELECTRICS SUBJECT TO WEAK ELECTRIC FIELDS

by:

EROL GÖKTUNA

Advisor:

Prof. Dr. NECMİ TANYOLAÇ



Department of Electrical Engineering

School of Engineering

Robert College

Bebek, İstanbul, Turkey

December, 1969

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE i

ACKNOWLEDGEMENT

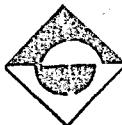
This report has been prepared for the partial fulfillment of the requirements of Robert College School of Engineering for the degree of Master of Science in Electrical Engineering.

I would like to express my sincere gratitude to my thesis supervisor, Prof. Dr. Necmi Tanyolaç, for his very kind interest, generous help and guidance.

I also wish to express my thanks to Cevat Gül for his help with the experimental apparatus, and Miss Mine Atalay, for her typing the manuscript.

Erol Göktuna

December 1969, Istanbul



124141

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE 11

ABSTRACT

The aim of this report is to investigate dielectric behavior at weak electric field. The behavior of permittivity, a defining property of dielectric materials, at weak fields is mostly discussed.

The theoretical approach implies that the permittivity is not influenced by the field intensity at relatively weak electric fields. However, the external disturbances such as frequency, temperature, and humidity, play an important role in determining the permittivity. The temperature and frequency dependence of dielectric materials are also considered in connection with the electric field.

The behavior of six different dielectric samples at relatively weak electric fields are investigated. These samples are being used in Electro-Odöcell, an objective odor measuring instrument, on which a research work is going on at Robert College Research Center, under the supervision of Prof. Dr. Necmi Tanyolaç.

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE

CONTENTS

	<u>Page</u>
ACKNOWLEDGEMENT	i
ABSTRACT	ii
INTRODUCTION	1
CHAPTER I Fundamental Theory of Dielectrics	3
CHAPTER II Electric Fields and Dipoles	8
A. Electric Fields	8
B. Dipoles	14
CHAPTER III Macroscopic Theory of Dielectrics	20
A. The Electric Polarization, P	20
B. Complex Permittivity - Time Dependent Fields	26
C. Anisotropy and Nonlinearity	38
Conclusions	42
CHAPTER IV Microscopic Theory of Dielectrics	44
A. Internal Fields	44
B. Electronic Polarization	52
C. Ionic Polarization	57
D. Orientational Polarization	59
E. Interfacial Polarization	66
F. Relaxation Time	67
Conclusions	70

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE

	<u>Page</u>
CHAPTER V Dielectric Behavior in Weak Fields, Measurements, and Applications	73
A. Dielectric Behavior in Weak Fields	73
B. Permittivity Measuring Methods	79
C. Static Permittivity	80
D. Discussion of the Experimental Results	84
E. Dielectric Applications	90
CHAPTER VI Conclusion	94
APPENDICES	97

INTRODUCTION

All materials, whether they are conductors, semi-conductors or insulators, conduct electricity to a greater or lesser extent. Every activity of electrical engineering involves some need of such materials that do not conduct electricity. The materials that possess this property are known as insulators or more correctly, dielectrics. According to ASA definition, a dielectric " is a medium having the property that the energy required to establish an electric field is recoverable, in whole or in part, as electric energy". Insulating materials are only imperfect dielectric materials. The non-conducting and energy-storage properties place the dielectrics among the most important practical engineering tools.

Recently, due to their important properties, the behaviors of dielectric materials have been studied with an increased interest. The results of these studies, of course, have established various classifications of dielectric materials, such as solid, liquid, gaseous dielectrics, or organic, inorganic dielectrics, or polar, non-polar dielectrics, etc. Still today, there are many research works going on for the development of the dielectric behavior and properties.

In this report our aim is to investigate the effects of weak electric fields on the dielectric materials. In order to realize this aim, first a model must be proposed. Beginning with this model, the behavior of dielectric materials at weak electric field intensities, can be examined from microscopic and macroscopic points of view. In this report we have tried to follow this procedure.

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE 2

In short, the first two chapters provide a brief discussion of fundamentals associated with dielectrics and fields. The third chapter explains the dielectric behavior from the macroscopic point of view. The fourth chapter relates molecular structure to two specific dielectric parameters. Finally, the fifth chapter deals with the discussion of the dielectric behavior in weak fields, evaluation of the results obtained from measurements, and applications of dielectrics. Chapter six summarizes the main conclusions of the report.

Sum dielectric data is also provided in appendix D.

The main abbreviations used in the reports are as follows:

1. The bar of the top of a letter represents a vector, \bar{A} .
2. The dot between two vector quantities represents dot product.
3. \oint represents a closed surface integral.
4. Σ stands for summation.

At the end of the report a selected bibliography is given for further references.

CHAPTER I

FUNDAMENTAL THEORY OF DIELECTRICS

In order to appreciate the microscopic and the macroscopic aspects of dielectric behavior in electric fields, a theoretical model based on the atomic and molecular structure is needed to provide a knowledge of the fundamental mechanisms involved in dielectric materials. In the days of Michael Faraday (1791-1867), who developed the concept of dielectric polarization, such a theoretical model was in a wide use in the continuum theory. The solid, in this classical model, was assumed to be composed of large number of pairs of equal but opposite point charges forming "electric dipoles" of infinitesimal dimensions.

This theoretical model can further be modified by visualizing that all matter is built up from electrically charged particles, positive nuclei surrounded by clouds of negatively charged particles, electrons. These nuclei and electrons form in many cases neutral atoms and molecules.

Thus formed atoms do not exhibit any dipolar characteristics when they are in stationary states and no external force acting on them. Combination of atoms of different types, sometimes, form molecules that do not normally share their electrons symmetrically. As a result of this phenomenon the electron clouds are shifted eccentrically toward the stronger binding atoms hence constituting electric dipoles. Such molecules, possessing permanent and appreciable dipole moments, are termed as "polar" molecules like H_2O . In cases where the constituent atoms of molecules are similar, the dipole moment must vanish for reasons of symmetry, an example being H_2 in contrast

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE 4

to H_2O molecules in which there is a displacement of charges between the bonding of H and the O atoms.

The movements of constituent charges characterize dielectrics in which, ideally, no net charge appears. The elementary charges that make up the dielectrics cannot move freely through the medium as it is the case in the conductors, but they can be displaced. In dielectrics there exist two different types of interaction forces. First, forces due to chemical bonding, van der Waal's attraction and atomic forces of repulsion, are effective between neighbouring atoms. Second, forces due to interaction between electrostatic dipoles which compared with dipoles, are of very long-range.¹

The basic model outlined in the previous paragraphs is fairly sufficient to describe qualitatively what happens when a dielectric is placed in a uniform external field, \vec{E}_0 . The application of a uniform external field onto a dielectric slab, by means of a pair of large-area parallel conducting plates carrying a fixed charge, q , will bring forth small displacements in the charges of the dielectric.

In the case of dielectrics possessing non-polar molecules, as the electric field is applied, the forces acting upon the charges result in small displacement of the electrons relative to the nuclei. The field causes all the positive charges to move along the direction of the field, and all the negative charges to move in the opposite direction, thus inducing electric dipoles into being. Since the forces bind the electrons to the nuclei, these displacements cannot extend beyond an atomic diameter for normal field intensities, and reactive forces arise that are proportional to the displacements. Figure 1 illustrates in a schematic way, the transition of

particles from pure neutral to dipolar condition.

In the case of dielectrics possessing polar molecules, however, the applied field tends to direct the permanent dipoles to align with the field itself. Because the molecules are in constant thermal agitation, the degree of alignment is not complete, but varies as the applied electric field or the temperature is varied. The permanent dipole behavior before and after the application of an electric field is illustrated in Figure 2.

In general, whether or not the molecules have permanent electric dipole moments, they acquire them by induction when placed in an electric field, and the induced dipoles are present only when the electric field is present. Therefore, the overall effect of alignment and induction is the polarization of the dielectric slab although it, as a whole, remains electrically neutral.

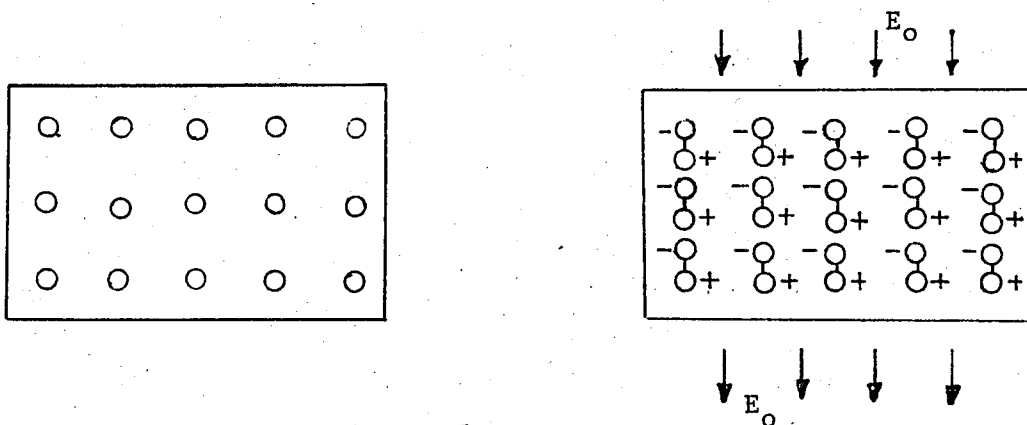


Figure 1

During the formation of polarization, a certain amount of charge is transported through every plane element in the dielectric slab. This transportation is called the displacement current.² Every volume element of the dielectric also acquires an induced dipole moment so that the whole slab behaves like an assembly of electric dipoles parallel to one another and very close together when the equilibrium state is reached.

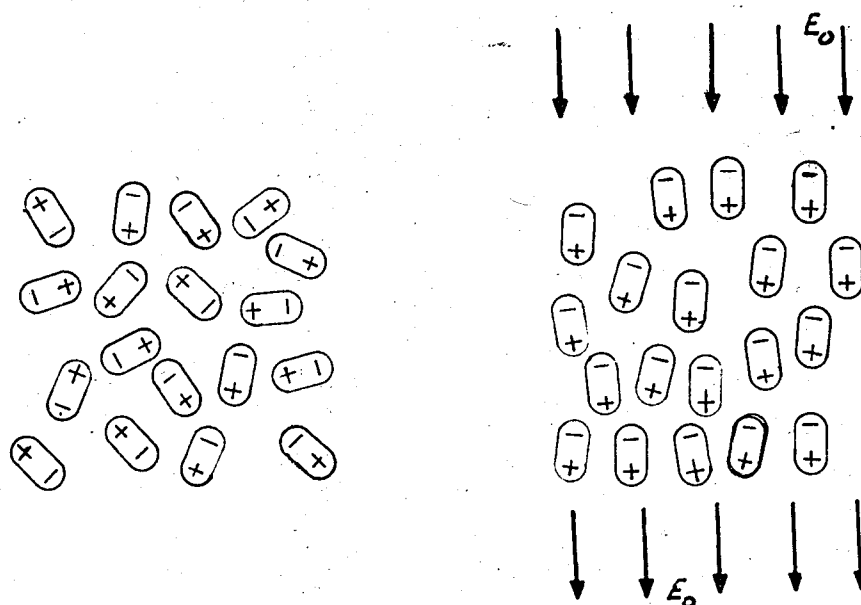


Figure 2

With the applied field present, within the slab of dielectric the dipoles' electric fields cancel, since no excess charge appears in any given volume element as stated earlier. At the plate surfaces this is not possible; the uppermost and the lowest layers of charges, positive charges on one layer and negative charges on the other, do not cancel. The magnitudes of these induced layers must be equal since the slab remains electrically neutral.

At the boundary surfaces of the dielectric and the conductor plates, the charges thus formed by polarization on the two extreme layers of the slab, neutralize the charges of equal value but of opposite sign of the conductor plates. The initial charges on the conductor plates are usually denoted as "true" charges, where the charges that are compensated by the polarization of the dielectric are termed as "bound" charges.³

Now a decrease in the applied field within the dielectric is expected since the true charges, which are responsible for the applied field, \bar{E}_0 , are decreased by an amount equal to the induced surface charges, bound

charges of the dielectric. In fact this is the case. A more simple approach to this phenomenon would be the consideration of the interaction between the electric fields set up by the true charges and by the induced

surface charges of the dielectric. The electric field, \bar{E}' , created by the induced charges, appears in such a way that it opposes the external field associated with the true charges on the conductor plates as illustrated in figure 3. The resultant field, \bar{E} , which is in the same direction of the applied field but smaller in magnitude is the vector sum of \bar{E}_0 and \bar{E}' . Therefore it can be easily stated that if a dielectric is placed in an electric field, induced

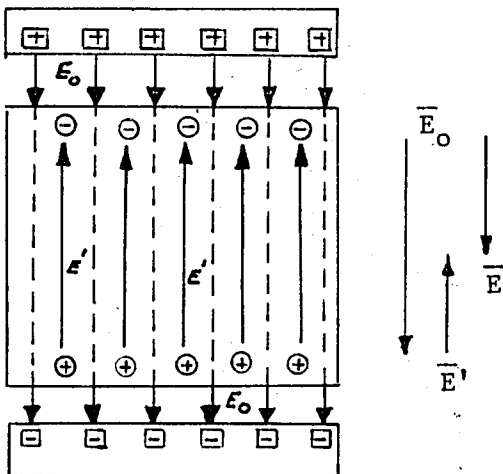


Figure 3

surface charges appear in such a way that they can tend to weaken the original field within the dielectric.

With the aid of this fundamental theory of dielectrics, behavior of the dielectrics in electric fields will be examined to a greater extent in the following chapters.

REFERENCES

1. H. Fröhlich, Theory of Dielectrics, Oxford University Press, 1958.
2. C.J.F. Böttcher, Theory of Electric Polarization, Elsevier Publishing Company, Amsterdam, 1952, p.37.
3. ibid, p.39.

CHAPTER II

ELECTRIC FIELDS AND DIPOLES

A. Electric Fields

Electric fields play an important role when dielectric behavior in electric fields is the subject of the discussion. It will be, therefore, convenient to choose the basic concepts of electric fields as the starting point of the discussion on its effects to the dielectrics.

Experiments conducted on electrically charged bodies show that there exists force between the charges. It is well known that like charges repel but opposite charges attract each other. This electrical force is found to depend on the medium in which the charges are embedded. The other interesting results obtained from experiments can be classified as, first, the force between the charged bodies is inversely proportional to the square of the distance between them, second, the force acts along the axis of the charges. All these facts can be summed up into a single expression. The statement, expressed in vector notation is

$$\vec{F} = K \frac{q_1 q_2}{\epsilon r^2} \vec{r}_0 \quad 2 - 1$$

This above expression is the well known Coulomb's law. $\vec{r}_0 = (\vec{r}/r)$ is the unit vector from one charge directed away from the other, and r is the distance between the charges. ϵ is a property of the medium and it is called the permittivity. The permittivity of a medium is related to the permittivity of the free space, ϵ_0 , through the relation $\epsilon = \epsilon_r \epsilon_0$, where

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE 9

ϵ_r is the relative permittivity. Relative permittivity is an unitless constant of proportionality. K is the constant of proportionality and in the rationalized MKSA¹ units it has a value of $1/4\pi$. The permittivity of the vacuum, ϵ_0 , has the value of $1/(36\pi)10^{-9}$ Farad/m.

It is seen from equation (2-1) that an electric charge experiences a net force from the neighbouring charge, as well it exerts force on the other. When more than two charges are involved, the force on any charge is the vector sum of the forces from each of the other charges. The region of influence of a charge is termed as its electric field. The strength of the force due to a charge at a point is defined as the force experienced by a unit test charge placed at that point. This strength is the electric field intensity, \vec{E} . Therefore, by definition, the electric field intensity, in general, can be expressed as

$$\vec{E} = \lim_{q \rightarrow 0} \frac{\vec{F}}{q} = \frac{\vec{F}}{q_t} \quad 2-2$$

One important aspect concerned with the field intensity is its dependence on the medium in which it exists. The field intensity arising from a point charge reveals this fact. The field intensity due to a charge, combining Eq. (2-1) and (2-2) is

$$\vec{E} = \frac{q}{4\pi\epsilon r^2} \vec{r}_o \quad 2-3$$

Hence, the field is a function of the medium, ϵ . Although it appears in the last relation that the field is a function of the distance, it is not always the case. The field intensity of a sheet of charge defined as ²

$E = Q_s/2\epsilon$, is not a function of the separation, where Q_s is the surface

charge density.

At this point it is convenient to introduce another concept in relation to the field intensity. It is the electric flux density, or as it is sometimes called, the electric displacement, \bar{D} . It is defined as

$$\bar{D} = \epsilon \bar{E} \quad 2-4$$

The flux density of a point charge, making use of Eq. (2-3) and Eq. (2-4), is given by,

$$\bar{D} = \frac{q}{4\pi r^2} \bar{r}_o \quad 2-5$$

which is not a function of the medium as it stands. In the MKSA system the unit associated with the flux density is coulombs per meter squared.

For an electric field, the number of lines of force that cut a surface is defined as the electric flux, ψ . Considering a point charge

surrounded by an arbitrary close surface, S , the flux density at any point on the surface is given by equation (2-5). The amount of the flux passing through an elemental area dS on S is determined by the component of the electric flux density normal to the area in consideration. This situation is illustrated in figure 4. The normal component, D_n , lies along the $d\bar{S}$, elementary area represented

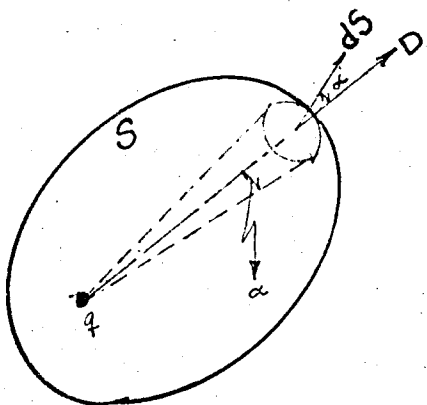


Figure 4

vectorially. Since \bar{D} is the flux density, flux per area, multiplying it

with the area yields the flux. Then total flux, $d\psi$, crossing $d\vec{S}$ is given by,

$$d\psi = \vec{D} \cdot d\vec{S} = D_n \cdot dS = D \cos\alpha \, dS \quad 2-6$$

Going back to Eq. (2-5), which simply gives the electric flux density due to a charge, and combining it with (2-6) yields,

$$d\psi = \vec{D} \cdot d\vec{S} = \frac{q \cos\alpha}{4\pi r^2} dS \quad 2-7$$

Hence, integrating over the closed surface, S , the total flux turns out to be

$$\psi = \oint_S \vec{D} \cdot d\vec{S} \quad 2-8a$$

or

$$\psi = \frac{q}{4\pi} \oint_S \frac{\cos\alpha}{r^2} dS \quad 2-8b$$

But on the other hand, the term inside the integral sign in the last equation is the solid angle, $d\omega$, subtended by a cone with a base area of dS tilted at an angle α . The flux crossing the base is $q/(4\pi) d\omega$. The total flux over the whole surface is determined by the integral of the solid angle over the whole surface. The solid angle integral is simply 4π spherical radians. As a result, the total flux is given by $\left[q/(4\pi) \right] 4\pi$ or simply q . Therefore, Eq. (2-8a) can be rewritten as :

$$q = \oint_S \vec{D} \cdot d\vec{S} \quad 2-9$$

The mathematical relation just derived is known as Gauss' Law which shortly states that the electric flux passing any closed surface is equal to the total charge enclosed.

The enclosed charge might be several point charges, in which case $q = \sum_n q_n$, or surface charge $q = \int_S \rho_S dS$, or free volume charge distribution of finite size $q = \int_V \rho_V dv$. For the last case Gauss' Law can be written as

$$\oint \vec{D} \cdot d\vec{S} = \int_V \rho_V dv \quad 2-10$$

The electrostatic potential difference, V , is defined as the work required per unit charge to move it from one point to another in the presence of an electric field. The potential difference increases when the work is done on the displaced charge. These definitions can be summarized in a single statement. That is,

$$V \approx -W/q = -\int \frac{\vec{F} \cdot d\vec{L}}{q} = -\int \vec{E} \cdot d\vec{L} \quad 2-11$$

With the use of vector relations, the converse relation of the last equation can be stated as

$$\vec{E} = -\vec{\nabla}V \quad 2-12$$

The divergence theorem can be stated in mathematical form as:

$$\int_V \text{div } \vec{E} \cdot dv = \oint_S \vec{E} \cdot d\vec{S} \quad 2-13$$

Combination of Eq. (2-13) with Gauss' Law yields,

$$\int_V \text{div } \vec{D} \cdot dv = q \quad 2-14a$$

or

$$\int_V \text{div } \vec{D} \cdot dv = \int_{V_01} \rho_V dv \quad 2-14b$$

From the preceding relations, it is obvious that

$$\vec{\nabla} \cdot \vec{D} = \text{div } \vec{D} = \rho_V \quad 2-15$$

where this last equation is known as one of the four Maxwell's relation. One point is worth of noting in this argument. It is the fact that D can be replaced by E , if the permittivity of the medium is not a function of the space in consideration. If the permittivity is not space dependent, then Eq. (2-15) can be rewritten as, $\nabla \cdot \bar{E} = \rho_v / \epsilon$.

On the other hand, assuming permittivity does not depend on the space, combination of Eq. (2-12) and the Maxwell's relation leads us to a new relation between the potential difference and the charge. This relation

$$\nabla \cdot \nabla v = \nabla^2 v = -\rho / \epsilon \quad 2-16$$

is the Poisson's equation.

For a region containing no net charge, Poisson's relation reduces to Laplace's equation of

$$\nabla^2 v = 0 \quad 2-17$$

where ρ_v is equal to zero justifying the condition.

The electrostatic energy present in a system of point charges or surface charge density or distributed volume charge density is given by,³

$$W_E = \frac{1}{2} \int_{vol} \bar{D} \cdot \bar{E} \, dv = \frac{1}{2} \int \epsilon E^2 \, dv \quad 2-18$$

This relation makes clear that the energy density, \mathcal{E}_v , or energy per unit volume associated with an electric field intensity is,

$$\mathcal{E}_v = \frac{dW_E}{dv} = \frac{1}{2} \bar{D} \cdot \bar{E} = \frac{\epsilon E^2}{2} \quad 2-19$$

It is worth of noting that energy as well as the energy density depends on the field intensity and on the medium in which it exists.

B. Dipoles

The dipoles, and the fields associated with them are quite important in that they form the basis for the behavior of dielectric materials in electric fields, as it was mentioned in the first chapter.

The electric moment of a point charge, q , with respect to a fixed point is defined as $q\bar{r}$, where \bar{r} is the vector representation of the distance from the fixed point to the charge. Eventually for a system containing a number of charges, the electric moment becomes,

$$\bar{m} = \sum q_i \bar{r}_i \quad 2-20$$

where \bar{r}_i is the vector from the origin to the (i)th charge.

If the net charge of the system is equal to zero, then the electric moment, \bar{m} , turns out to be independent of the choice of the origin. No net charge means there is equal number of positive and negative charges involved in the system. Owing to this fact, the relation in (2-20) can be resolved into two components each concerning one type of charges. This can be done by introducing the concept of electric centers of gravity of the positive and the negative charges, hence

$$\sum_{\text{positive}} q_i \bar{r}_i = \bar{r}_1 \sum q_i = \bar{r}_1 q \quad 2-21a$$

and

$$\sum_{\text{negative}} q_i \bar{r}_i = \bar{r}_2 \sum q_i = \bar{r}_2 q \quad 2-21b$$

where the radius vectors from the origin to these centers of gravity are denoted by \bar{r}_1 and \bar{r}_2 . The total positive charge is denoted by q . Therefore the electric moment of the system can be represented by,

$$\vec{m} = q(\vec{r}_1 - \vec{r}_2) \quad 2-22$$

The difference between the radius vectors is equal to the vectorial distance between the electric centers of gravity of both types of charges. Representing this difference vector by \vec{d} , extending from negative to positive center, the resulting electric moment of the system can be given by,

$$\vec{m} = q\vec{d} \quad 2-23$$

The last relation generally represents the electric dipole moment of a system with no net charge in the system. A simple case involved in electric moment would be a system containing only two opposite-polarity point charges separated by a distance (d). Such a system is called an electric dipole, where according to Eq. (2-23) its moment is equal to $q\vec{d}$. This case is illustrated in figure 5.

An ideal dipole, which is used extensively for simple derivation of

relationships for actual cases, is defined as the distance (d) between two point charges of opposite polarity is replaced by (d/n) and charge (q) by (qn) so that the limit approached as the number (n) tends to infinity represents the ideal dipole.⁴

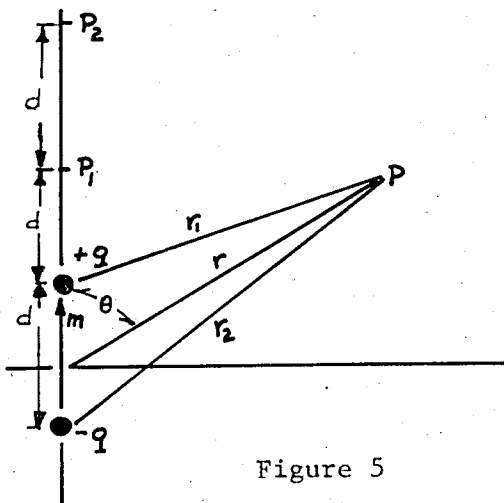


Figure 5

The potential difference due to a

dipole at a point, P, specified by the

quantities \vec{r} and θ , can be derived by making use of the Eq. (2-1) and Eq. (2-11).

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE 16

In vacuum, therefore, the potential difference is,

$$V = \sum_n V_n = V_1 + V_2 = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_1} - \frac{q}{r_2} \right) \quad 2-24$$

For the case where $r_1 \gg d$ some approximations can be made.⁵ If such is the case then, referring to figure 5,

$$r_1^2 = r^2 + \left(\frac{d}{2}\right)^2 - dr \cos\theta$$

dividing by r^2 yields

$$\left(\frac{r_1}{r}\right)^2 = 1 + \left(\frac{d}{2r}\right)^2 - \frac{d}{r} \cos\theta$$

solving for r/r_1 by binomial theorem gives

$$\frac{r}{r_1} = \left\{ 1 + \left(\frac{d}{2r}\right)^2 - \frac{d}{r} \cos\theta \right\}^{-1/2} = 1 - \frac{1}{2} \left(\frac{d^2}{4r^2} - \frac{d}{r} \cos\theta \right) + \frac{3}{8} \left(\frac{d^2}{4r^2} - \frac{d}{r} \cos\theta \right)^2$$

The terms smaller than $(d/r)^2$ can be neglected so that r/r_1 becomes approximately equal to

$$\frac{r}{r_1} \approx 1 + \frac{d}{2r} \cos\theta + \frac{d^2}{4r^2} \left(\frac{3\cos^2\theta - 1}{2} \right)$$

Similarly the ratio r/r_2 can be found. Hence

$$\frac{r}{r_2} \approx 1 - \frac{d}{2r} \cos\theta + \frac{d^2}{4r^2} \left(\frac{3\cos^2\theta - 1}{2} \right)$$

Substituting the ratios r/r_1 and r/r_2 into Eq. (2-24), the potential difference, then, is

$$V = \frac{q}{4\pi\epsilon_0} \cdot \frac{d\cos\theta}{r^2} = \frac{m\cos\theta}{4\pi\epsilon_0 r^2} = \frac{m \cdot r_0}{4\pi\epsilon_0 r^2} \quad 2-25$$

This last expression reflects the fact that it takes no work to bring a charge

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE 17

in along the line perpendicular to the dipole axis, since $\theta = 90^\circ$. The potential due to a dipole also decreases more with increasing distance from the dipole axis center than the potential due to one of the constituty charges.

Usually in molecular structures we assume ideal dipole case. For actual cases, non-ideal dipoles, deviation between ideal and non-ideal dipole is estimated by assuming ideal dipole case at smaller distances from the dipole.

As an extreme case, a point, P_1 , is considered on the dipole axis at a distance (d) from the positive charge. This case is illustrated in figure 5. The potential at P_1 , if contribution of each charge is taken seperately, is

$$V_{P_1} = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{d} - \frac{q}{2d} \right) = \frac{0.50}{4\pi\epsilon_0} \frac{q}{d}$$

whereas the relation in Eq. (2-25) would give

$$V_{P_1} = \frac{qd}{4\pi\epsilon_0 (3/2d)^2} = \frac{0.44}{4\pi\epsilon_0} \frac{q}{d}$$

At this small distance the error is about 12% when the formula for the potential of an ideal dipole is used in the case of non-ideal dipole. For a point P_2 on the dipole axis at a distance (2d) from the positive charge of the dipole, the potential difference, each charge being treated seperately, is

$$V_{P_2} = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{2d} - \frac{1}{3d} \right) = \frac{0.167}{4\pi\epsilon_0} \frac{q}{d}$$

On the other hand approximation of Eq. (2-25) gives

$$V_{P_2} = \frac{qd}{4\pi\epsilon_0 (5/2d)^2} = \frac{.160}{4\pi\epsilon_0} \frac{q}{d}$$

Hence at this distance the inaccuracy is already reduced to about 4% .⁶

The field intensity caused at a point by an ideal dipole moment can be

calculated by several methods depending on the choice of the coordinate system.

But in general it is easy to take $\vec{E} = -\nabla v$ as the starting point.

The field intensity of a dipole as a function of the electric moment and the distance can be determined through the use of Eq. (2-12) and Eq. (2-25).

Upon appropriate substitutions, the electric field intensity turns out to be

$$\vec{E} = - \frac{\nabla \vec{m} \cdot \vec{r}}{4\pi\epsilon_0 r^3} = \frac{1}{4\pi\epsilon_0} \left(-\vec{m} \cdot \vec{r} \nabla \frac{1}{r^3} - \frac{1}{r^3} \nabla \vec{m} \cdot \vec{r} \right) \quad 2-26$$

On the other hand due to the identity^X $\nabla(\vec{A} \cdot \vec{r}) = \vec{A}$, $\nabla(\vec{m} \cdot \vec{r}) = \vec{m}$, and

$\nabla(1/r^3) = (3/r^4)\nabla r = 3\vec{r}/r^5$, Eq. (2-26) can be rewritten as :

$$\vec{E} = \left(\frac{3\vec{m} \cdot \vec{r}}{r^5} \vec{r} - \frac{\vec{m}}{r^3} \right) \frac{1}{4\pi\epsilon_0} \quad 2-27$$

The electric field intensity thus resolved into two components, one along the radius vector and the other parallel to the dipole axis but in opposite direction of \vec{m} .

Another approach would be to find the field intensity at a point by using gradient relationships in spherical coordinates, then

$$\vec{E} = -\nabla v = - \left(\frac{\partial v}{\partial r} \vec{a}_r + \frac{1}{r} \frac{\partial v}{\partial \theta} \vec{a}_\theta + \frac{1}{r \sin \theta} \frac{\partial v}{\partial \phi} \vec{a}_\phi \right)$$

Substitution of $V = (m \cos \theta) / (4\pi\epsilon_0 r^2)$ into the previous expression results in

$$\vec{E} = \frac{m}{4\pi\epsilon_0 r^3} (2\cos\theta \vec{a}_r + \sin\theta \vec{a}_\theta)$$

It is interesting to note that the electric field intensity decreases now as the inverse cube of the distance from the dipole. Therefore it falls faster than the corresponding field of a point charge.

^X

Refer to Appendix A for the proof of the identity.

REFERENCES

1. H. Gürmen, " Units used in Electrical Engineering ", Robert College Research Journal, 1969, Şirketi Mürettibiye, İstanbul, p.41 .
2. W. H. Hayt, Engineering Electromagnetics, McGraw-Hill Book Company, Newyork, 1967 p.44 .
3. İbid, p.109 .
4. C. J. Böttcher, Theory of Electric Polarization, Elsevier Publishing Company, Amsterdam, 1952, p.2 .
5. A. Nussbaum, Electromagnetic Theory for Engineers and Scientists, Prentice-Hall, Inc., New Jersey, 1965, p.123 .
6. Examples are from C. J. Böttcher, Theory of Electric Polarization, Elsevier Publishing Company, Amsterdam, 1952, p.7 .

CHAPTER III

MACROSCOPIC THEORY OF DIELECTRICS

A. The Electric Polarization, P

It was mentioned in the first chapter that dielectric materials are considered to contain usually molecules that have permanent dipoles. On the other hand, some dielectrics can possess non-polar molecules which do not have dipolar arrangements until a field is applied. Whether dielectric material is composed of polar or non-polar molecules, permanent or induced dipoles may be described by their dipole moments. The dipole moment, in general, was developed in the preceding chapter as:

$$\bar{m} = q\bar{d} \quad 3-1$$

It was also mentioned in the second chapter that the moment of a set of charges with respect to an origin is

$$\bar{m} = \sum_i q_i \bar{r}_i \quad 3-2$$

Consequently, if there are (n) dipoles per unit volume in a dielectric with absence of net charge, then the total dipole moment in a volume element (v) around a fixed point R can be given by,

$$\bar{m}_t = \sum_i N_i q_i \bar{d}_i v = \bar{P}v \quad 3-3$$

where (N_i) is the average number of charges (q_i) with displacements (d_i) present in the volume element (v), and hence $n = \sum N_i$. The volume (v) is taken to be incremental but large enough to contain a large number of dipoles. Also displacement (d) is considered to be average in time to avoid the influence of the fluctuations due to molecular movements.

The vector \bar{P} introduced in relation (3-3) is generally called the electric polarization¹ and it is the "density" of electric polarization since it represents the dipole moment per unit volume.

If all the dipoles were identical, then the dipole moment would become

$$\bar{m} = \sum_i N_i q \bar{d} v = n q \bar{d} v \quad 3-4$$

and

$$\bar{P} = n q \bar{d} \quad 3-5$$

Polarization can also be explained by means of the charge transport through a plane element during the built-up of the polarization. In the dielectric material an incremental surface can be assumed. All the charges with displacements (d_i) crossing this small plane were present in the volume element $\bar{d}_i \cdot \Delta \bar{S}$ before the polarizing field was applied. $\Delta \bar{S}$ is the vectorial representation of the plane element. Consequently the number of charges in this volume element is $N_i \bar{d}_i \cdot \Delta \bar{S}$. Therefore, the net total charge which crosses the elemental surface in the direction of the surface is

$$\Delta q_b = \sum_i q_i N_i \bar{d}_i \cdot \Delta \bar{S} = \bar{P} \cdot \Delta \bar{S} \quad 3-6$$

where the subscript (b) associated with the charge denotes a bound charge but not a free charge.

If ΔS is an element of a closed surface and directed outward, then the net increase in the bound charge within the closed surface can be found by integrating the total bound charge passing the surface element, over the whole surface, that is

$$q_b = -\int_s \bar{P} \cdot d\bar{S} \quad 3-7$$

This charge can be considered to be distributed with a density of ρ_b or

$$q_b = \int_{vol} \rho_b \cdot dv \quad 3-8$$

Therefore

$$\int_{vol} \rho_b \cdot dv = - \int_s \bar{P} \cdot d\bar{S} \quad 3-9$$

Applying the divergence theorem, Eq.(2-13), to the right side of the previous relation, (3-9), it becomes

$$\int_{vol} \rho_b \cdot dv = - \int_{vol} \bar{\nabla} \cdot \bar{P} \cdot dv$$

and from this it follows that

$$\bar{\nabla} \cdot \bar{P} = - \rho_b \quad 3-10$$

On the other hand by Gauss' Law it was found that $\bar{\nabla} \cdot \bar{D} = \rho_v$. This shows that there must exist a close relation between the polarization and the flux density \bar{D} . This is indeed the case.

At a boundary surface of a dielectric and a conductor, the total charge density is obtained by taking the algebraic sum of the true charge

density arising from the conduction electrons of the conductor and the bound charge density arising from the polarization of the dielectric material. Therefore,

$$\rho_{\text{TOTAL}} = \rho_{\text{TRUE}} + \rho_{\text{BOUND}} \quad 3-11$$

(free)

If the permittivity of the dielectric material is not a function of the space, direction-independent, then Gauss' Law may be stated in terms of the electric field in free space as:

$$\nabla \cdot \vec{E} = \rho / \epsilon_0 \quad 3-12$$

The charge density ρ is the true charge density merely representing the total charge. This means that one can consider all the charges to reside in free space even with the material present. The use of ϵ_0 in Eq.(3-12) depends on this consideration. With the aid of Eq. (3-11), relation (3-12) can be rewritten as:

$$\nabla \cdot \epsilon_0 \vec{E} = \rho_f + \rho_b \quad 3-13$$

and using Eq.(3-10), the last equation becomes

$$\nabla \cdot \epsilon_0 \vec{E} = \rho_f - \nabla \cdot \vec{P} \quad 3-14$$

Rearranging the terms in (3-14), it follows that

$$\rho_f = \nabla \cdot \epsilon_0 \vec{E} + \nabla \cdot \vec{P} \quad 3-15a$$

It yields from Eq. (2-15) that

$$\rho_f = \nabla \cdot \vec{D} \quad 3-15b$$

hence the relation between the polarization, field intensity and the electric flux density is given by

$$\bar{D} = \epsilon_0 \bar{E} + \bar{P} \quad 3-16$$

This last equation shows that the electric polarization, introduced as the dipole moment per unit volume, turns out to be related in a simple way to the vectors \bar{D} and \bar{E} . It is worth of noting that the vector \bar{D} associated with the free space is a function of the total charge density, including the bound charges as well the true charges, Eq. (3-12). On the other hand, if consideration is given for \bar{D} , associated with permittivity other than that of the free space, then it is only a function of the free charges, Eq. (3-15b).

It is clear from Eq. (3-16) that there must be a relation existing between the electric field intensity, and the polarization. This relation should be a function of the type of the dielectric material. Usually, in actual crystalline dielectrics, the polarization vector is a linear function of the electric field strength, for fields of practical value. However, it is not necessary that this relation be a scalar one. A constant tensor χ_e termed the electric susceptibility of the dielectric material, is used to define this relationship. It is the ratio of bound charge density to the free charge density, or

$$\bar{P} = \epsilon_0 \chi_e \bar{E} \quad 3-17$$

In isotropic or polycrystalline materials χ_e is simply a scalar. But in anistropic, direction-dependent, crystals this is not the case.

Another relation between vector \vec{P} and \vec{E} can be derived by the aid of Eq. (3-16). If $\epsilon\vec{E}$ is substituted for \vec{D} in that equation, then

$$\epsilon\vec{E} = \epsilon_0\vec{E} + \vec{P}$$

and remembering $\epsilon = \epsilon_r \epsilon_0$, the relation becomes

$$\vec{P} = (\epsilon_r - 1)\epsilon_0\vec{E} \quad 3-18$$

hence it is clear that

$$\chi_e = \epsilon_r - 1 \quad 3-19$$

B. Complex Permittivity - Time Dependent Fields

The foregoing discussions of this chapter show that the electric field in a dielectric material can be easily described macroscopically by two field quantities. They are the electric field intensity and the electric flux density. As for the polarization, it can be obtained by making use of the Eq. (3-16). The relationship existing between the field intensity and the flux density is $\bar{D} = \epsilon \bar{E}$. The permittivity ϵ is the constant of proportionality and describes the dielectric property of the materials. At this point it should be emphasized that, ϵ may not be the same everywhere, and the proportionality between \bar{E} and \bar{D} may not hold at certain cases. For cases where \bar{E} gets too large, \bar{D} may not be proportional to it. This deviation from the proportionality may be experienced even at relatively small electric field intensities for certain materials. The permittivity may also be related to the time dependence of the field intensity. Therefore, the concept $\bar{D} = \epsilon \bar{E}$, is only an approximation for most real materials. On the other hand, it is perfectly satisfactory for cases where consideration is taken in vacuum.

The discussions carried so far on the macroscopic theory of dielectrics have dealt with the static fields. Next step will be then to examine what happens when time dependent fields are associated with the dielectric materials.

Consider first that a dielectric slab is placed between two parallel conducting plates, as it was mentioned in the first chapter. Such an arrangement is usually called a condenser. If a non-time-dependent electric field intensity is applied to this arrangement suddenly,

the dielectric requires some time to become totally polarized. The reason for this behavior is due to the inertia shown by the displacement of charges. Such a case is illustrated in figure 6. A similar case must be experienced for the time dependent field intensities.

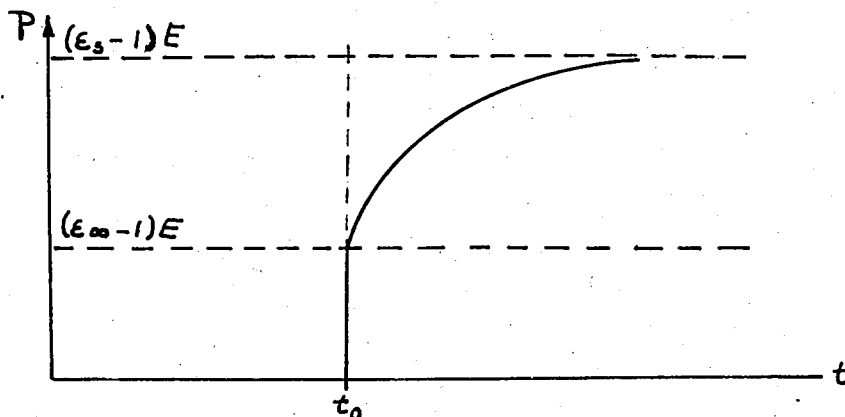


Figure 6

The potential difference for a parallel plate condenser in vacuum, is equal to $v = (dq)/(\epsilon_0 A)$ for static fields; where q is the charge on one plate, (A) is the area of one plate and (d) is the separation of the plates. It is clear from the preceding expression that the voltage is proportional to the charge if ϵ_0 is a constant quantity. ϵ_0 is always assumed to be constant. The constant of proportionality for this case is $(d)/(\epsilon_0 A)$. The inverse of this quantity is known as the capacitance (C) of the system. Capacitance is a dependent factor of the dimensions of the condenser as well as the permittivity of the dielectric. The permittivity of the vacuum in the previous expression can be replaced by the permittivity of the dielectric in consideration as long as ϵ does not depend on the field strength or space so that it remains constant in a

given condition. If the space between the plates is filled by a different medium than the vacuum, the proportionality constant, then, would become $C = (\epsilon_0 \epsilon_r A)/(d)$. Neglecting the fringing effects at the sides, the ratio of the capacitances of vacuum and the medium becomes

$$\frac{C}{C_0} = \epsilon_r \quad 3-20$$

where

$$C_0 = \frac{\epsilon_0 A}{d}.$$

If instead of static field, an alternating field, or voltage is applied across the plates of the condenser, the system experiences an alternating charge accumulation, hence current. The charges on the condenser plates vary with the voltage. For the case of vacuum, constant permittivity, the relationship between charge and voltage is

$$q = C_0 v \quad 3-21$$

It follows that

$$\frac{\partial q}{\partial t} = C_0 \frac{\partial v}{\partial t} \quad \text{or} \quad I = C_0 \frac{\partial v}{\partial t} \quad 3-22$$

Since voltage is now considered to be a periodic quantity, the expression for current becomes^x

$$i = i_C = j\omega C_0 v \quad 3-23$$

This current is usually termed as the charging current. The preceding expression shows that there is a 90° phase shift between the current

^x To take partial derivative of time dependent periodic functions with respect to time is equivalent to multiplying the corresponding phasor by ωj

and the voltage. This is true only for the case of vacuum or for perfect, ideal dielectric materials. However, this is not the case with the actual dielectric materials. In addition to the charging current, there is an in-phase component of current which flows between the conducting plates that make up the condenser. This component of current is termed as the loss current. This means that the total current flowing between the plates is

$$i = i_L + i_C = Gv + j\omega Cv = i' + ji'' \quad 3-24$$

This expression can have a meaning if the relative permittivity has two components; a real part and an imaginary part, or $\epsilon_r = \epsilon_r' - j\epsilon_r''$. Making use of this expression, the current, in the condenser becomes:

$$i = j\omega Cv = j\omega C_0 \epsilon_r v = j\omega C_0 v (\epsilon_r' - j\epsilon_r'')$$

$$i = \omega \epsilon_r'' C_0 v + j\omega \epsilon_r' C_0 v \quad 3-25$$

which simply states that, the current has a real part which corresponds to a resistive current flow, and a 90° phase shift component. The magnitude of ϵ_r'' is defined by the in-phase or "loss" component. It is also apparent that the loss component of current is a property of the medium. As the dielectric material approaches to a perfect one, i_L must vanish, hence ϵ_r'' , since dielectric becomes consequently loss free.

As a result of introducing loss component of current into the picture, the total current flowing in the condenser should incline by a power factor angle relative to the applied voltage. (Figure 7) In

another words, the total current makes an angle δ with the charging current. Tangent of this latter angle δ is called the loss tangent of the dielectric.

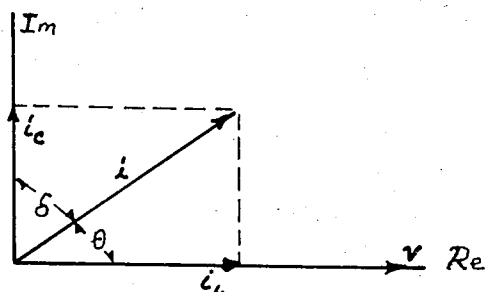


Figure 7

Loss tangent δ is the dissipation factor, that is, it is the ratio of loss current to the charging current. From Eq. (3-25), it follows that

$$\tan \delta = \frac{\epsilon_r''}{\epsilon_r'} \quad 3-26$$

On the other hand, since the field strength between the plates is $(v)/(d)$ the current density (J) becomes

$$i = \omega \epsilon_r'' C_0 E d + j \omega \epsilon_r' C_0 E d = \omega \epsilon_r'' \epsilon_0 A E + j \omega \epsilon_r' \epsilon_0 A E$$

$$J = \frac{i}{A} = \omega \epsilon_0 \epsilon_r'' E + j \omega \epsilon_0 \epsilon_r' E \quad 3-27$$

The dielectric conductivity, representing the sum of all mechanisms involved in the material, is defined as the real part of the ratio of current density to the electric field intensity, or

$$\sigma_d = \frac{J}{E} = \omega \epsilon_r'' \epsilon_0 = \frac{f(\epsilon_r'/\epsilon_0) \tan \delta}{1.8 \times 10^{12}} \quad \text{mho-cm} \quad 3-28$$

where $1/(1.8 \times 10^{12})$ is derived from $2\pi\epsilon_0$.

Besides, if it is assumed that there is no dielectric loss when static fields are associated, then the complex permittivity in the time-dependent fields should be identical with that of the static permittivity, as frequency approaches to zero. Permittivities having different values in static and time-dependent electric field intensities mean that ϵ_r'' and ϵ_r' are frequency dependent quantities. And it also means that ϵ_r'' should approach to zero with decreasing frequency as well as ϵ_r' to ϵ_s . Therefore,

$$\epsilon_r(\omega) = \epsilon_r'(\omega) - j\epsilon_r''(\omega) \quad 3-29$$

On the other hand, the variations of the permittivities ϵ_r' and ϵ_r'' with frequency can be such that the relation between the flux density and the field intensity be a linear one, $\bar{D} = \epsilon \bar{E}$. This means that the components of permittivities are not entirely independent. To explain this relation of dependency one can use the concept of linearity between the two field quantities, \bar{E} and \bar{D} , and the principle of superposition.

First let us assume that an electric field intensity $E(u)$ is applied to a dielectric at time (u) , and it does not exist after the time $(u + \partial u)$. The flux density arising from the application of the electric field intensity, should exist inside the small time interval ∂u . Besides, due to the inertia of the displacement of charges, the electric flux density persists at times $t > u + \partial u$. Since the field intensity is cut off outside the time interval, it follows that D must vanish after a period of time. Therefore it should be a decaying function of $(t - u)$. Considering the region where $t > u + \partial u$, this time function of flux density can be expressed as²

$$D(t - u) = E(u) \propto (t - u)\partial u \quad 3-30$$

where $\alpha(t - u)$ is the decay function which represents the vanishing of D . Inside the time interval, that is $u < t < u + \partial u$, however, one can imagine that the flux density is divided into two components, one of which immediately follows the applied electric field intensity. Hence, in this interval, the time function of the flux density can be written as

$$D(t - u) = \epsilon_{\infty} E(u) + E(u) \alpha_0 \partial u \quad 3-31$$

where the decaying function is considered to have a constant value, α_0 , in ∂u . ϵ_{∞} can be defined as the value corresponding to $\epsilon'(\omega)$, at highest frequencies, that is when (ω) approaches to infinity. Actually in practice these frequencies fall into the infra-red region. $E(u)\epsilon_{\infty}$ is the term which corresponds to the immediate following of the applied field.

Second, let us assume that after a period of time, at time u' , another field, $E(u')$, is applied onto the dielectric. This field persists between the time interval (u') and $(u' + \partial u')$. Similarly the flux density is a function of $(t - u')$ and decays at $t > u' + \partial u'$.

Now we can assume that the flux density as a function of $(t - u')$ is superposed linearly to the previous D , which was a function of $(t - u)$. This, in fact, is the principle of superposition. It follows from this fact that, the principle of superposition can be used for a continuous time dependent electric field intensities. If this is done, the flux density at any time, t , can be expressed as

$$D(t) = \epsilon_{\infty} E(t) + \int_0^{\infty} E(u) \alpha(t - u) \partial u \quad 3-32$$

Since it is clear that D vanishes at a certain time, t_0 , after the cut off of

the field intensity, the value of the integral does not change if it is integrated from zero to infinity. If a substitution $x = t - u$ is made into the equation (3-32), this equation becomes

$$D(t) = \epsilon_{\infty} E(t) + \int_0^{\infty} E(t-x) \alpha(x) \partial x \quad 3-33$$

Now if we consider a periodic field intensity of the form $E = E_0 e^{j\omega t}$ is applied, Eq. (3-33) consequently becomes,

$$D(t) - E_0 e^{j\omega t} \epsilon_{\infty} = E_0 \int_0^{\infty} e^{j(\omega t - x)} \alpha(x) \partial x$$

and

$$D(t) - \epsilon_{\infty} E_0 e^{j\omega t} = E_0 a^{j\omega t} \int_0^{\infty} e^{-j\omega x} \alpha(x) \partial x \quad 3-34$$

Remembering $D = \epsilon E$ and $\epsilon = \epsilon'(\omega) - j\epsilon''(\omega)$, the last expression takes the form of

$$\epsilon'(\omega) - j\epsilon''(\omega) - \epsilon_{\infty} = \int_0^{\infty} e^{-j\omega x} \alpha(x) \partial x \quad 3-35$$

and

$$\epsilon'(\omega) - \epsilon_{\infty} - j\epsilon''(\omega) = \int_0^{\infty} \cos \omega x \alpha(x) \partial x - j \int_0^{\infty} \sin \omega x \alpha(x) \partial x$$

It follows from this last relation that

$$\epsilon'(\omega) - \epsilon_{\infty} = \int_0^{\infty} \cos \omega x \alpha(x) \partial x \quad 3-36$$

and

$$\epsilon''(\omega) = \int_0^{\infty} \sin \omega x \alpha(x) \partial x \quad 3-37$$

Since both $\epsilon'(\omega)$ and $\epsilon''(\omega)$ can be derived from the same function $\alpha(x)$, they cannot be independent.

One can make use of the Fourier Transformations Theorem to simplify the preceding relations. From equation (3-36)

$$\alpha(x) = \frac{2}{\pi} \int_0^{\infty} \epsilon'(\ell) - \epsilon_{\infty} \cos \ell x \, \partial \ell \quad 3-38$$

and from equation (3-37)

$$\alpha(x) = \frac{2}{\pi} \int_0^{\infty} \epsilon''(\ell) \sin \ell x \, \partial \ell \quad 3-39$$

Introducing Eq. (3-39) into Eq. (3-36) yields,

$$\epsilon'(\omega) - \epsilon_{\infty} = \int_0^{\infty} \frac{2}{\pi} \, \partial x \, \cos \omega x \int_0^{\infty} \epsilon''(\ell) \sin \ell x \, \partial \ell$$

$$\epsilon'(\omega) - \epsilon_{\infty} = \frac{2}{\pi} \lim_{R \rightarrow \infty} \int_0^{\infty} \partial \ell \, \epsilon''(\ell) \int_0^R \cos \omega x \sin \ell x \, \partial x$$

$$\epsilon'(\omega) - \epsilon_{\infty} = \frac{2}{\pi} \lim_{R \rightarrow \infty} \int_0^{\infty} \epsilon''(\ell) \frac{1}{2} \frac{1 - \cos(\ell + \omega)R}{\ell + \omega} + \frac{1 - \cos(\ell - \omega)R}{\ell - \omega} \, \partial \ell$$

$$\epsilon'(\omega) - \epsilon_{\infty} = \frac{2}{\pi} \int_0^{\infty} \epsilon''(h) \frac{h}{h^2 - \omega^2} \, \partial h$$

where ℓ is a variable of integration. By a similar method, using Eq. (3-37) and (3-39),

$$\epsilon''(\omega) = \frac{2}{\pi} \int_0^{\infty} \epsilon'(\ell) - \epsilon_{\infty} \frac{\omega}{\omega^2 - \ell^2} \, \partial \ell$$

Returning back to Eq. (3-33), if we assume that the attainment of equilibrium is exponential, then the decay function can be represented by;

$$\alpha(t) = \alpha_0 e^{-t/\tau} \quad 3-40$$

where τ is called the relaxation time. It is infact independent of time.

This kind of solution was first proposed by Debye.³

If Eq. (3-40) is introduced into the Eq. (3-32) then it follows that

$$D(t) = \epsilon_{\infty} E(t) + \int_0^{\infty} E(u) \alpha_0 e^{-t/\tau} du \quad 3-41$$

and differentiating Eq. (3-41) with respect to time yields

$$\frac{\partial D(t)}{\partial t} = \epsilon_{\infty} \frac{\partial E(t)}{\partial t} + \frac{\partial}{\partial t} \int_0^{\infty} E(u) \alpha_0 e^{-t/\tau} du \quad 3-42$$

Now by using Leibnitz's formula⁴

$$\frac{\partial}{\partial t} \int_0^{\infty(t)} E(u) \alpha_0 e^{-(t-u)/\tau} du = \alpha_0 E(t) - \frac{1}{\tau} \int_0^{\infty(t)} E(u) \alpha(t-u) du$$

Therefore Eq. (3-42) becomes

$$\frac{\partial D(t)}{\partial t} = \epsilon_{\infty} \frac{\partial E(t)}{\partial t} + \alpha_0 E(t) - \frac{1}{\tau} \int_0^{\infty} E(u) \alpha(t-u) du \quad 3-43$$

Multiplying Eq. (3-43) by (τ) and then addition with Eq. (3-32) yields

$$\tau \frac{\partial}{\partial t} (D - \epsilon_{\infty} E(t)) + (D - \epsilon_{\infty} E(t)) = \tau \alpha_0 E(t) \quad 3-44$$

when the electric field intensity is time independent

$$\frac{\partial}{\partial t} (D - \epsilon_{\infty} E) = 0 \quad \text{and} \quad D = \epsilon_s E \quad 3-45$$

Therefore for constant field intensities, substituting Eq. (3-45) into Eq. (3-43) results in

$$\tau \alpha_0 = \epsilon_s - \epsilon_{\infty}$$

where ϵ_s represents the static permittivity. As a result, Eq. (3-44) turns out to be

$$\tau \frac{\partial}{\partial t} (D - \epsilon_{\infty} E(t)) + (D - \epsilon_{\infty} E(t)) = (\epsilon_s - \epsilon_{\infty}) E(t) \quad 3-46$$

If a sinusoidal voltage is considered, remembering $\tau \frac{\partial D}{\partial t} = \tau j\omega \epsilon E(t)$ and $\frac{\partial(E(t))}{\partial t} = j\omega E(t)$, Eq. (3-46) becomes

$$\tau j\omega \epsilon - \epsilon_{\infty} \tau j\omega + \epsilon - \epsilon_{\infty} = \epsilon_s - \epsilon_{\infty}$$

and hence

$$\epsilon(\omega) = \epsilon_{\infty} + \frac{\epsilon_s - \epsilon_{\infty}}{1 + j\tau\omega} \quad 3-47$$

But since $\epsilon(\omega) = \epsilon'(\omega) - j\epsilon''(\omega)$, upon appropriate separation of real and imaginary parts, $\epsilon'(\omega)$ turns out to be

$$\epsilon'(\omega) = \epsilon_{\infty} + \frac{\epsilon_s - \epsilon_{\infty}}{1 + \omega^2 \tau^2} \quad 3-48$$

and $\epsilon''(\omega)$ becomes

$$\epsilon''(\omega) = \frac{(\epsilon_s - \epsilon_{\infty}) \omega \tau}{1 + \omega^2 \tau^2} \quad 3-49$$

and

$$\tan \delta = \frac{\epsilon''(\omega)}{\epsilon'(\omega)} = \frac{(\epsilon_s - \epsilon_{\infty}) \omega \tau}{\epsilon_s + \epsilon_{\infty} \omega^2 \tau^2} \quad 3-50$$

It is clear from the last equations that as frequency, ω , approaches to infinity, ϵ' approaches to ϵ_{∞} , and ϵ'' approaches to the static case, that is to zero. ϵ' approaches to ϵ_s , and ϵ'' becomes zero, for $\omega \rightarrow 0$. Therefore for ϵ'' , there must be a value of frequency for which ϵ'' is maximum. This value, equating the time derivative of ϵ'' to zero, is

$$\omega = \omega_m = \frac{1}{\tau} \quad 3-51$$

For this value of ω_m

$$\epsilon''_{\max} = \frac{\epsilon_s - \epsilon_{\infty}}{2} \quad \text{and} \quad \epsilon'_m = \frac{\epsilon_s + \epsilon_{\infty}}{2}$$

The alternating field volume conductivity was given by

$$\epsilon''_r = \sigma = \frac{1}{\rho} = \frac{f(\epsilon'/\epsilon_0)\tan \delta}{1.8 \times 10^{12}} \text{ mho-cm}$$

If the values of ϵ' and $\tan \delta$ are substituted into the last relation it follows that

$$\sigma = \frac{1}{\rho} = \frac{f(\epsilon_s - \epsilon_\infty)\omega\tau}{\epsilon_0(1 + \omega^2\tau^2)} \quad 3-52$$

A diagram showing conductivity-resistivity as a function of ϵ''/ϵ' and f is given in the appendix. Also some examples are given in the appendix, showing the variations of ϵ' and ϵ'' with frequency.

On the other hand the relaxation time can be experimentally observable quantity for a macroscopic process. Dividing Eq. (3-49) by Eq. (3-48) yields

$$\frac{\epsilon''}{\epsilon' - \epsilon_\infty} = \omega\tau \quad 3-53$$

When $\frac{\epsilon''}{\epsilon' - \epsilon_\infty}$ is plotted against the frequency, the graph is a line passing through the origin with τ being the slope. Hence τ can be obtained from such an graph.

C. Anistropy and Nonlinearity

So far we have discussed the isotropic dielectric materials. In such systems it was assumed that space dependence was not existing for the field relations. In general, any dielectric crystal of non-cubic structure exhibits anistropic, space-dependence, properties. In anistropic materials the space dependence of fields plays an important role for the behavior of dielectric materials in electric fields. This importance is experienced mostly at optical frequencies.

Generally in a dielectric material, assuming linear relationships, the expression between the dipole moment density, P , and the electric field intensity is given by:

$$\overline{P} = \epsilon_0 \chi_e \overline{E} \quad 3-54$$

For isotropic materials, the χ_e tensor is simply a scalar. This implies that polarization points parallel to the electric field intensity vector. However for anistropic dielectrics, this is not completely parallel to the electric field intensity. Therefore the previous relation can be rewritten in a generalized form in its Cartesian components as:

$$P_i = \epsilon_0 \sum_k^3 \chi_{ik} E_k \quad 3-55$$

where $i = x, y, z$, and $k = 1$ stands for the (x) component, $k = 2$ for the (y) component and $k = 3$ for the (z) component, or in expended form

$$P_x = \epsilon_0 (\chi_{11} E_x + \chi_{12} E_y + \chi_{13} E_z)$$

$$P_y = \epsilon_0 (\chi_{21} E_x + \chi_{22} E_y + \chi_{23} E_z)$$

$$P_z = \epsilon_0 (\chi_{31} E_x + \chi_{32} E_y + \chi_{33} E_z)$$

The coefficients χ_{ik} are the linear susceptibilities. These equations are involved with six different susceptibilities since $\chi_{ik} = \chi_{ki}$.

The relation between the susceptibility and the relative permittivity was defined previously as:

$$\epsilon_r = 1 + \chi$$

which means that the value of the relative permittivity depends on the choice of the direction for anisotropic materials. In cubic dielectrics $\chi_{12} = \chi_{13} = \chi_{23} = 0$.

One important aspect associated with these susceptibilities is their dependence on the frequency variations. This phenomenon can be obtained when equations of motion are considered. The differential equations of motion of a harmonic oscillator system of particles due to a wave propagating in one direction is given by,

$$\frac{\partial^2 x}{\partial t^2} + \beta \frac{\partial x}{\partial t} + \omega_0^2 x = - \frac{e \bar{E}_x(t)}{m} \quad 3-56$$

This equation of motion has the solution of

$$x(t) = - \frac{e E_x}{m (\omega_0^2 - \omega^2 - j\beta\omega/m)}$$

and hence the dipole moment of the particle is

$$\bar{p}_x = \frac{e^2 \bar{E}_x}{m (\omega_0^2 - \omega^2 - j\omega\beta/m)}$$

Therefore from Eq.(3-55) the susceptibility χ_{11} is in the form of,

$$\chi_{11} = \frac{\chi_{11}(0) \omega_0^2}{\omega_0^2 - \omega^2 - j\omega\beta/m}$$

which proves the frequency dependence of the susceptibilities. Similarly the other coefficients can be calculated in terms of frequency. According to this result the permittivities along the directions can be written as:

$$\epsilon_{rik}(\omega) = 1 + \chi_{ik}(\omega)$$

On the other hand at higher optical frequencies the components of relative permittivity along the various directions of the dielectric slightly depend on E. This means that ϵ_{rik} is a weakly function of the electric field intensity at these frequencies.

Even at microwave frequencies this non-linearity effect can be observed⁵. But this condition is experienced at high field intensities. Dielectric anisotropy and non-linearity are dependent on the structure of the material. Further consideration on this subject can be obtained from the books edited by Beam, and Van Ziel^{6,7}.

Most of the dielectric materials are not anisotropic and non-linear. When an electric field is applied upon a dielectric material, the displacement of charges from their equilibrium positions results in change of material's physical dimensions. This case is often experienced for atomic and electronic polarizabilities which will be discussed in the next Chapter. This change of dimensions is referred as electrostriction. On the other hand if a dielectric material is mechanically strained electric

charges are induced on the surface of the dielectric giving rise to a voltage. Such dielectric materials are termed as piezoelectric dielectrics. One important aspect associated with piezoelectrics is that this phenomenon can not occur in materials having a center of symmetry, such as $\text{Zn}(\text{BrO}_3)_2 \cdot 6\text{H}_2\text{O}$ or CH_4 . In other words this effect is available only for anisotropic dielectric materials.

The most important dielectric materials that exhibit piezoelectricity are Quartz, Tourmaline, Rochelle Salt ($\text{NaKC}_4 \cdot 4\text{H}_2\text{O}$), Potassium Dihydrogen Phosphate (KH_2PO_4), or Ammonium Dihydrogen Phosphate ($\text{NH}_4\text{H}_2\text{PO}_4$).

Besides piezoelectricity, some of the dielectric materials have relative permittivities which depend on the field strength. These materials are ferroelectric materials. The Ceramics and Barium-Strontium Titanate are such materials. The spontaneous polarization, or spontaneous alignment of electric dipoles define the ferroelectric materials. The spontaneous polarization is defined by Curie temperature which will be mentioned in the next chapter. Ferroelectric materials also possess ferroelectric hysteresis loop upon appropriate heat treatment.

CONCLUSIONS:

1. The relation between polarization, field intensity and the electric flux density is given simply by:

$$\bar{D} = \epsilon_0 \bar{E} + \bar{P}$$

This relationship is a function of the type of the material.

2. For isotropic dielectric materials \bar{E} and \bar{P} are linearly related through

$$\bar{P} = \chi_e \epsilon_0 \bar{E} = (\epsilon_r - 1) \epsilon_0 \bar{E}$$

In ferroelectric materials this relationship is not linear and they show hysteresis effects.

3. \bar{D} is connected with the free charge only.
 \bar{P} is connected with the bound charge only.
 \bar{E} is connected with all charges that are actually present.
4. For time dependent fields, relative permittivity contains two components each being a function of the frequency of the field, or,

$$\epsilon_r(\omega) = \epsilon_r'(\omega) - j\epsilon_r''(\omega)$$

This shows that ϵ_r is a function of frequency.

5. At time-dependent fields actually dielectric materials have loss components represented by

$$\tan \delta = \frac{\epsilon_r''(\omega)}{\epsilon_r'(\omega)}$$

6. Dielectric conductivity is defined as

$$\sigma_d = \frac{J}{E} = \omega \epsilon_r'' \epsilon_0 = \frac{f(\epsilon_r'/\epsilon_0) \tan \delta}{1.8 \times 10^{-12}} \text{ mho-cm.}$$

7. The dielectric materials require sometime to be elapsed before becoming totally polarized upon application of a time-dependent electric field. This delay can be represented by a decaying function of the form $\alpha(t) = \alpha_0 e^{-t/\tau}$ where τ is called the relaxation time. In terms of the relaxation time the frequency dependent components can be written as

$$\epsilon''(\omega) = \frac{(\epsilon_s - \epsilon_\infty)\omega\tau}{1 + \omega^2\tau^2} \quad \text{and} \quad \epsilon'(\omega) = \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{1 + \omega^2\tau^2}$$

It follows from above relations that

$$\tan\delta = \frac{(\epsilon_s - \epsilon_\infty)\omega\tau}{\epsilon_s + \epsilon_\infty\omega^2\tau^2} \quad \text{and} \quad \sigma = \frac{1}{\rho} = \frac{f(\epsilon_s - \epsilon_\infty)\omega\tau}{\epsilon_0(1 + \omega^2\tau^2)}$$

where ϵ_s static permittivity and ϵ_∞ is the permittivity at infinite frequency.

REFERENCES

1. C.J.F. Böttcher, Theory of Electric Polarization, Elsevier Publishing Company, Amsterdam, 1952, p.37.
2. H. Fröhlich, Theory of Dielectrics, Oxford University Press, 1958, p.6.
3. Smyth, C., Dielectric Behavior and Structure, McGraw-Hill Book Co. Inc. New York, 1955, p. 10.
4. Bronwell, Arthur, Advanced Mathematics in Physics and Engineering, McGraw-Hill Book Co. Inc., 1953, p.111.
5. Von Hippel, A., Dielectric Materials and Applications, The MIT Press, Massachusetts, 1961, p.41.
6. Beam, W.R., Electronics of Solids, McGraw-Hill Book Co. Inc., New York, 1965.
7. Von der Ziel, Solid State Physical Electronics, Prentice Hall, Inc. New Jersey, 1968.

CHAPTER IV

MICROSCOPIC PROPERTIES OF DIELECTRICS

A. Internal Fields

The surface density of the bound charges as represented by

$$\bar{P} = (\epsilon_r - 1) \epsilon_0 \bar{E} \quad 4-1$$

provides a means to enter into the microscopic world from the macroscopic scale. When a polarisable body is subjected to an uniform electric field, polarisation, \bar{P} , appears in response to the macroscopic field. The dipole moment per unit volume may be thought as resulting from the additive action of (n) elementary dipoles per unit volume each having an average moment $\bar{\mu}_{av}$. Therefore the total polarization becomes

$$\bar{P} = n \bar{\mu}_{av} \quad 4-2$$

which is in agreement with the previously defined polarization. In the last expression $\bar{\mu}_{av}$ is used for the dipole moment of an atomic or especially molecular dipole instead of \bar{m} which represents dipole moment in general. In addition to this fact, $\bar{\mu}_{av}$ is generally related to a local field strength \bar{E} existing inside the dielectric through the relation

$$\bar{\mu}_{av} = \alpha \bar{E} \quad 4-3$$

where (α) is termed as the polarizability of the body which measures the flexibility of the particle or in other words the average dipole moment per unit field strength. This α is not the same as the decay function mentioned in

the previous chapter although the symbols are similar.

The combination of Eq. (4-1), Eq. (4-2), and Eq. (4-3) leads us to a new interpretation of the polarization, thus

$$\bar{P} = (\epsilon_r - 1)\epsilon_0 \bar{E} = \alpha n \bar{E}' \quad 4-4$$

linking the macroscopically measured polarization to three molecular parameters; the number of particles per unit volume, n , their total polarizability, α , and the internal electric field, \bar{E}' , acting locally. Since α is defined in terms of dipole moment per field strength, its magnitude will clearly be a measure of the extent to which electric dipoles are formed from the atoms or molecules.

Electric dipoles may arise through a variety of mechanisms, any or all of which can contribute to the value of polarizability. There are basically three different mechanisms of polarization. They are¹:

- 1) Orientation (rotation) of molecules. The field tends to direct permanent dipoles to align in the field direction.
- 2) The displacement of charged atoms or groups of atoms with respect to each other; atomic polarization.
- 3) Slight shift of the electrons with respect to the positive charges; electronic polarization.

These three mechanisms of polarization are due to charges that are locally bound in atoms, molecules or in the structure of solids and liquids. However, in addition, some charges or charge carriers exist in actual dielectrics, either because they become trapped in the material or on interfaces, or because they can not be freely discharged or replaced at the

electrodes.² These contribute a fourth method of polarization. This fourth mechanism is termed as the interfacial polarization.

The total polarizability can be expressed as the sum of the individual polarizabilities, each acting independently and arising from one particular mechanism. Hence total polarizability of a dielectric material can be written as:

$$\alpha = \alpha_e + \alpha_a + \alpha_o + \alpha_s \quad 4-5$$

where α_e , α_a , α_o , and α_s represent electronic, atomic, orientational, and interfacial polarizabilities respectively.

It was mentioned in the previous chapters that the electric field intensity at a point due to the surrounding dipole moments of particles is :

$$\bar{E}_a = \sum \frac{3 (\bar{m}_i \cdot \bar{r}_i) \bar{r}_i - r_i^2 \bar{m}_i}{4\pi\epsilon_0 r_i^5} \quad 4-6$$

Since the summation in the last equation is to be considered over the whole dielectric material, it will be somewhat very difficult to evaluate the field intensity. One way to solve this difficulty is to find such steps that will help to the evaluation of the summation.

For such a solution a dielectric material is considered to be placed in electric field intensity applied externally (\bar{E}_0). A dielectric specimen subjected to an external field is illustrated in figure 9.

A microscopic spherical body with a radius (a) is taken out of this specimen. It is important that the radius of this spherical body must be very small when compared with the physical dimensions of the specimen, but must be very large when compared with the atomic dimensions.

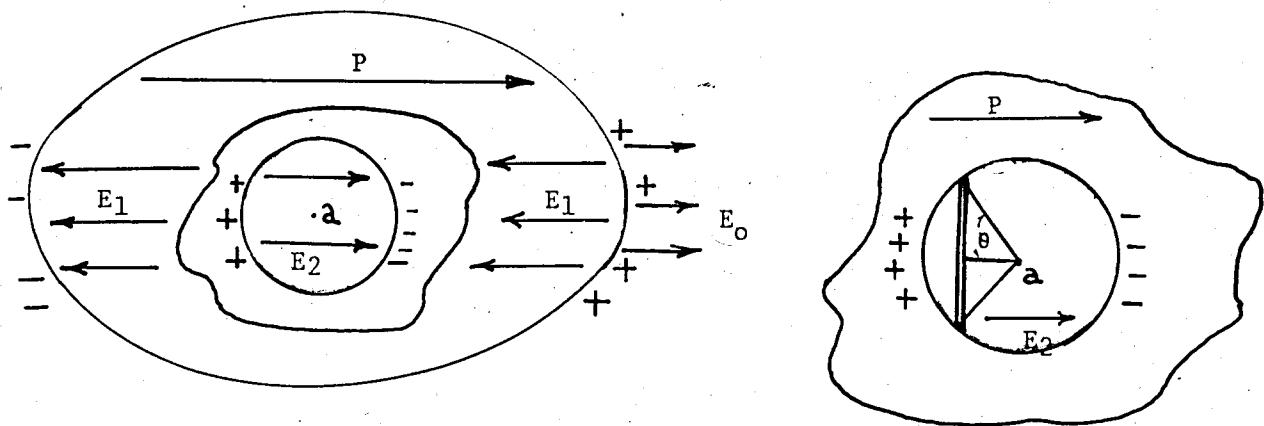


Figure 9

Now, the dipole electric field intensity at the point where an atom is located can be separated into three components :

- 1) \bar{E}_1 , which is due to the polarization charges on the outer surface of the specimen, induced by the application of the external field \bar{E}_0 .
- 2) \bar{E}_2 , which is due to the charges on the surface of the spherical cavity.
- 3) \bar{E}_3 , which is due to the contributions of the dipoles existing inside the spherical body.

The superposition of these fields must result in the actual dipole field acting on the (a)th atom, or

$$\bar{E}_1 + \bar{E}_2 + \bar{E}_3 = \bar{E}_a = \sum_i \frac{3 (\bar{m}_i \cdot \bar{r}_i) \bar{r}_i - r_i^2 \bar{m}_i}{4\pi\epsilon_0 r_i^5} \quad 4-7$$

If the external field \bar{E}_0 is taken into consideration, the total field acting on this atom can be given by

$$\bar{E}_{loc} = \bar{E}_0 + \bar{E}_1 + \bar{E}_2 + \bar{E}_3 = \bar{E}'$$

where \bar{E}_{loc} is known as the local electric field intensity (\bar{E}').

At this point it should be emphasized that \bar{E}' (\bar{E}_{loc}) is not the same with the actual macroscopic field, $\bar{E} = \bar{E}_0 + \bar{E}_1$, for latter was defined as the space-average electric field intensity over the whole dielectric slab.

The field \bar{E}_1 , which is due to the polarization charges, is usually termed as the depolarization field because it tends to depolarize the charges since it is in the opposite direction of the polarization \bar{P} . For a homogeneous ellipsoidal specimen with an uniform polarization, it can be related to the polarization through relation

$$\bar{E}_1 = -N \bar{P} \quad 4-9$$

And if polarization is separated into components along the cartesian axes, then the representation in Eq.(4-9) can be rewritten as :

$$\bar{E}_1 = -N_x \bar{P}_x - N_y \bar{P}_y - N_z \bar{P}_z \quad 4-10$$

where N_x , N_y , N_z , or N is called the depolarization factor. It is found that as long as the shape of the specimen is ellipsoid or some limiting case of ellipsoid, the polarization is uniform when placed in electric field³.

Furthermore the values of (N) depend on the dimensions of the specimen which means that \bar{E}_1 depends on the dimensions.^x The value of the depolarization factor for a sphere can be calculated by taking a circular ring on the sphere which is illustrated in figure 9. The charge on the ring is :

$$\partial q = P \cos\theta (2\pi r \sin\theta) r \partial\theta \quad 4-11$$

where $P \cos\theta$ is the surface charge density on the surface of the spherical slab.

^x Values of (N) for several shapes are given in appendix

The component of the electric field intensity along the z-axis at the center of the sphere can be given by

$$\partial \bar{E}_z = \frac{\partial q}{4\pi\epsilon_0 r^2} \quad \text{or} \quad E_z = \int_0^{\pi} \frac{P \cos\theta (2\pi r \sin\theta) r \partial\theta \cos\theta}{4\pi\epsilon_0 r^2} \quad 4-12$$

Therefore $\bar{E}_z = (\bar{P}) / (3\epsilon_0)$ by evaluation of the integral. On the other hand due to the symmetry $\bar{E}_z = \bar{E}_x = \bar{E}_y = (\bar{P}) / (3\epsilon_0)$, hence $N_z = N_y = N_x = 1 / (3\epsilon_0)$. It is interesting to note that the sum of the values (N) along the three principle axes should sum up to a constant value, that is

$$N_x = N_y = N_z = \frac{1}{3\epsilon_0} \quad 4-13$$

Due to this fact we can calculate the values of N_x , N_y , and N_z as long as x, y, z are the principle axes of the specimen.

The field \bar{E}_2 , which is due to the bound charges on the walls of the cavity can be calculated by a similar method used for the calculation of \bar{E}_z in the previous example for spherical slab. This field intensity, usually referred as the Lorentz field, has the value

$$\bar{E}_2 = \frac{P}{3\epsilon_0} \quad 4-14$$

The third field \bar{E}_3 in question is due to the dipoles existing inside the cavity. \bar{E}_3 should then depend on the structure of the material, since it results from the individual action of the particles in the dielectric.

The simplest structure is one with a cubic arrangement. In this kind of structure all atoms are considered to be replaced by point dipoles parallel to each other. Considering the axis of the dipoles are taken in

the direction of z-axis, then

$$E_{3z} = \sum_i \frac{3m_i z_i^2 - m_i r_i^2}{4\pi\epsilon_0 r_i^5} \quad \text{where} \quad \overline{m_i r_i} = m_i z_i \quad 4-15$$

On the other hand by the virtue of symmetry of the lattice and the cavity:

$$\sum_i z_i^2 = \sum_i x_i^2 = \sum_i y_i^2 \quad \text{and} \quad \sum_i z_i^2 + y_i^2 + x_i^2 = 3\sum_i z_i^2 = \sum_i r_i^2 \quad 4-16$$

Substitution of this result into Eq.(4-15) will result in \overline{E}_{3z} to become identically equal to zero, and hence $\overline{E}_3 = 0$. This last result is applicable for cases of cubic symmetry, and parallel dipole arrangements. Actually this result is a highly idealized one but it is a reasonable approximation to the other cases where the particles are arranged in complete disorder or the dielectric materials having similar cubic structures possessing highly symmetrical arrays. It is also applicable for cases when the particles do not possess permanent dipole moments and they are isotropically distributed.

It was stated previously that the field existing inside the dielectric slab is the vectorial sum of the externally applied field and the depolarization field, so that

$$\overline{E} = \overline{E}_0 + \overline{E}_1 \quad 4-17$$

This \overline{E} field is the same one that was used in Eq.(3-16). In terms of \overline{E} , the field strength acting locally upon a single atom (dipole) inside the dielectric slab having cubic structure can be expressed by

$$\overline{E}' = \overline{E} + \overline{E}_3 + \overline{E}_2 = \overline{E} + \frac{\overline{P}}{3\epsilon_0} \quad 4-18$$

The relation between the polarization and the local field \bar{E}' was given by Eq.(4-4). Substituting this relation into Eq.(4-18) will result in

$$\bar{P} = \frac{3\epsilon_0 n \alpha}{3\epsilon_0 - n \alpha} \bar{E} \quad 4-19$$

On the other hand substitution of the expression $P = (\epsilon_r - 1)\epsilon_0 \bar{E}$ into this last relation in Eq.(4-19) and rearranging the terms yields

$$\frac{n\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \quad 4-20$$

It should be noticed that Eq.(4-20) applies only to materials in which particles are spatially well organized, or in complete disorder. This means that the equation can be applied to the gases very easily. The value of ϵ_r has a value approximately 1 in gases at low temperature and pressures. The term $\epsilon_r + 2$ then is approximately equal to 3. Since this is the case Eq.(4-20) simplifies to

$$\frac{n\alpha}{\epsilon_0} \approx \epsilon_r - 1 = \chi_{\text{gas}} \quad 4-21$$

On the other hand since (n) is the number of molecules or particles present per unit volume, it can be replaced by a quantity $(N_0 \rho)/M$, where N_0 is the Avagadro's number, ρ is the density of the material and M is the molecular weight of the substance. From this substitution it follows that

$$\frac{N_0 \alpha}{3 \epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \frac{M}{\rho} \quad 4-22$$

where this last expression is known as the "Clasissus-Mosotti" relation.

B. Electronic Polarization

The mechanism that provides the strength of the induced moment μ_e for an atom to be proportional to the local field acting on the atom is known as the electronic polarization. Electronic polarization is most pronounced, undisturbed by other effects, in monatomic gases. In monatomic gases the particles are free to travel or to orient. Therefore the simplest model on which the calculations must be based is the one that can be easily applied to monatomic gases. In monatomic gases the electron clouds have spherical symmetry if the atoms are free of interacting with each other. These electron clouds can be imagined to be confined uniformly to spheres that have radii (r_0) for a given material, gas. When an atom of such a gas is placed in an electric field, the electrons as well as the nucleus experience a force that is proportional to the applied field, or in other words

$$\vec{F}_e = -Ze\vec{E}$$

4-23

where (Z) is the number of negatively charged particles in the atom. Due to this force there is a slight net shift of the centers of charge gravities by a distance (d) from the original equilibrium point. This produces an induced dipole moment. For small electric field intensities, the amount of displacement and hence the dipole moment is proportional to the applied field.

After the displacement occurs, a small portion of the negative charge distribution within a sphere of radius (d) can be thought as concentrated around the new center of gravity of the negatively charged electrons. The part of the electron cloud concentrated in the sphere with radius (d) can be

expressed as :

$$q = -Ze \frac{\frac{4\pi d^3}{3}}{\frac{4\pi r_o^2}{3}} = -Ze \frac{d^3}{r_o} \quad 4-24$$

The Coulombic force between the nucleus and the electron cloud should be equal in magnitude to the force due to the applied force, or

$$\overline{F}_C + \overline{F}_e = 0 \quad \text{and} \quad \frac{(Ze)^2 d^3}{\epsilon_o 4\pi r_o^3 d^3} \overline{d} = Ze \overline{E} \quad 4-25$$

But on the other hand for $Z = 1$ the field has induced a dipole moment in the atom which can be represented by,

$$\overline{\mu}_e = \alpha_e \overline{E} \quad 4-26$$

Therefore,

$$\alpha_{es} = \epsilon_o 4\pi r_o^3 \quad 4-27$$

where (s) denotes the static field condition.

So far we have assumed uniformly distributed concentration for the spherical charge cloud. This assumption is somewhat an approximation for actual cases, for quantum picture of atom leads to a more extended electron cloud concentration. According to this view the electron distribution decreases exponentially as the radius increases. Due to this fact the electrons that are far away from the nucleus are weakly attracted, hence contributing more to the polarization. For this reason there are some deviations in the values of the electronic polarizabilities that are obtained by calculations and by experiments.

The force equation under static field condition can be modified for the case of sinusoidally varying electric fields. If the electrons are considered to be quasi-elastically bound to equilibrium positions and react like linear harmonic oscillators, then the mathematics associated with such a system can be used to analyze the situation. The equation of motion in the presence of time-dependent electric field can be written, in one dimension, as :

$$m \frac{\partial^2 x}{\partial t^2} + \beta \frac{\partial x}{\partial t} + \gamma x = e \bar{E}_{loc} \quad 4-28$$

It is worth of noting that the loss mechanism, $\beta \frac{\partial x}{\partial t}$, is usually insignificant for the conditions where the gas or the liquid state is involved. However it is an important factor for the solids. Neglecting this term, which is just an idealization of the condition, the solution of the last equation must satisfy the behavior of the electronic polarizability under time-dependent electric fields.

In general the system shows some response to the field. The system has a natural frequency (ω_0), which is equal to $\sqrt{\gamma/m}$ where γ is the force constant and (m) is the mass of the electron.

The Coulombic force which is responsible for the restoring force in Eq. (4-28) is

$$\frac{e^2 d}{4\pi\epsilon_0 r_0^3} = \gamma d \quad 4-29$$

The natural frequency is therefore,

$$\omega_0 = \frac{\sqrt{\gamma}}{\sqrt{m}} = \frac{\sqrt{e^2}}{\sqrt{4\pi\epsilon_0 r_c^3 m}} \quad 4-30$$

It is interesting to note that

$$\omega_0^2 = \frac{e^2}{\alpha_{es} m} \quad 4-31$$

where α_{es} is the value for static electronic polarizability (Eq. 4-27).

The steady state solution of Eq. 4-28 for a time dependent field of the form $E_{loc} = \bar{E}_0 e^{j\omega t}$ turns out to be

$$\bar{x} = \frac{e/m\bar{E}}{\omega_0^2 - \omega^2 + j\omega\beta/m} \quad 4-32$$

so that the dipole moment has the value

$$\bar{p} = \alpha_e \bar{E} = e\bar{x} \quad 4-33$$

Therefore,

$$\alpha_{et} = \frac{e^2/m}{\omega_0^2 - \omega^2 + j\omega\beta/m} \quad \text{or} \quad \alpha_{et} = \alpha'_e - j\alpha''_e \quad 4-34$$

$$\text{where } \alpha'_e = \frac{e^2(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + (\omega\beta/m)^2} \quad \text{and} \quad \alpha''_e = \frac{e^2\omega\beta/m^2}{(\omega_0^2 - \omega^2)^2 + (\omega\beta/m)^2}$$

If the static case, $\omega = 0$, is considered then $\alpha'_e = \frac{e^2}{m\omega_0^2}$ and $\alpha''_e = 0$ which are in accordance with the Eq. (4-27).

So far the discussion dealt with one dipole. If there are (n) number of dipoles per unit volume, one can expect the solution will be somewhat similar but not the same that of the single dipole system. The Eq. (4-28) can be rewritten for the polarization, \bar{P} , if $\bar{p}_{av} = e\bar{x}$ so that $\bar{P} = n e\bar{x}$. By making use of the relation $E' = E + \frac{P}{3\epsilon_0}$, the new differential equation of motion is

$$\frac{\partial^2 \bar{P}}{\partial t^2} + \beta \frac{\partial \bar{P}}{\partial t} + (\omega_0^2 - \frac{ne^2}{3m\epsilon_0}) \bar{P} = \frac{ne^2}{m} \bar{E} \quad 4-35$$

Therefore the effect of the polarization of the other atoms is to lower the natural frequency squared of the individual oscillator by an amount $\frac{ne^2}{3m\epsilon_0}$. Considering this, the steady state solution of Eq. 4-35 turns out to be

$$\bar{P} = \frac{ne^2/m}{\omega_{op}^2 - \omega^2 + j\omega\epsilon/m} \bar{E} \quad 4-36$$

where

$$\omega_{op}^2 = \omega_0^2 - \frac{ne^2}{3m\epsilon_0}$$

The current density associated with this kind of system can be represented by,

$$\bar{J} = - \Sigma e \frac{\partial x}{\partial t} = \bar{E} n (\omega \alpha_e'' + j\omega \alpha_e') \quad 4-37$$

which is very similar in form to the current density mentioned in Chapter III.

The relative permittivity given by the Eq. (4-4) can be restated as,

$$\epsilon_r = 1 + \frac{P}{\epsilon_0 E} \quad 4-38$$

which becomes a complex quantity when α_{et} is substituted into this relation.

Therefore, the complex permittivity in molecular terms is,

$$\epsilon_r = 1 + \frac{ne^2/m\epsilon_0}{\omega_0^2 - \omega^2 + j\omega\beta/m} \quad 4-39$$

So far the assumptions were based on one type of oscillator. In general for a case of (k) oscillators, the last equation can be given by,

$$\epsilon_r = 1 + \Sigma_k \frac{n_k e^2 / \epsilon_0 m_k}{\omega_{ok}^2 - \omega^2 + j\omega\beta_k / m_k} \quad 4-40$$

C. Ionic Polarization

In the preceding paragraphs the structure of the electronic cloud was the main concern. But additional difficulties arise when the discussion is extended to molecules. When the molecules are considered, the symmetry experienced in monatomic gases or other monatomic cases does not hold. The distance between nuclei and their mutual orientations become important.

Ionic displacements are characterized by the movements of the heavy ions. In this mechanism the system can be again considered to execute harmonic oscillations with spring like restoring forces between neighbouring ions. It is important that the frequency response of this system is somewhat altered by these spring constants. When a system with a linear-chain model of two types of ion having different masses, and opposite but equal charges is considered, the applied field causes a displacement of each ion by a distance x . These displacements produce forces that are proportional to each spring's constant. Since each spring is deformed a distance $2x$, the force on the ion should be equal to $4\gamma x$. But this force on the ion is compensated by the force due to the applied field, hence

$$q\bar{E} = 4\gamma x \quad 4-41$$

The polarization produced by each ion is (qx) , therefore, the ionic polarizability at static field cases, is

$$\alpha_{is} = \frac{q^2}{4\gamma} = \frac{q^2}{\omega_{oi}^2 M} \quad 4-42$$

where $\omega_{oi} = \frac{4\gamma}{M}$, and $M = \frac{m_1 m_2}{m_1 + m_2}$

The response of this system to time dependent fields can be obtained through the force equation, similar in form to Eq. (4-28). The solution of such an equation gives 4

$$\alpha_{it} = \frac{q^2/M}{\omega_{oi}^2 - \omega^2 + j\omega\beta/M} \quad 4-43$$

which verifies Eq. (4-42) for static field cases, $\omega = 0$.

D. Orientational Polarization

It was mentioned previously that molecules possessing permanent dipole moments tend themselves to align with the applied field. Thermal agitation, however, tends to maintain a random distribution. In gases and liquids polar molecules are somewhat free to rotate in the applied field direction. In the absence of an electric field, individual dipoles are oriented in random directions, so that net moment per unit volume is zero. The application of electric field tends to orient the individual dipoles to produce net moment per unit volume. Although the polarization is not complete, there appears some amount of polarization.

An elementary dipole with dipole moment μ_d is considered to be placed in an electric field of intensity E . The potential energy of the dipole would be

$$U = - \vec{\mu}_d \cdot \vec{E} = -\mu_d E \cos\theta \quad 4-44$$

where θ is the angle between the axis of the polar molecule (dipole) and the field's direction. If there are (n) number of molecules distributed with their axes within a solid angle $\partial\Omega$, then according to Boltzmann distribution⁵

$$n = A \left(\exp \frac{-U}{kt} \right) \partial\Omega \quad 4-45$$

where (A) is a proportionality constant and depends on the total number of molecules in consideration. Since $\partial\Omega = 2\pi \sin\theta \partial\theta$ then,

$$n = A \left(\exp \frac{\mu_d E \cos\theta}{kT} \right) 2\pi \sin\theta \partial\theta \quad 4-46$$

The integration of the last equation results in the total number of polar molecules. But the total dipole moments in the direction of the field is:

$$2\pi \int_0^\pi A \left(\exp \frac{\mu_d E \cos \theta}{kT} \right) \mu \cos \theta \sin \theta d\theta \quad 4-47$$

The average moment per dipole in the direction of the field is, therefore,

$$\mu_{av} = \frac{A 2\pi \int \left(\exp \frac{\mu_d E \cos \theta}{kT} \right) \mu \cos \theta \sin \theta d\theta}{A 2\pi \int \left(\exp \frac{\mu_d E \cos \theta}{kT} \right) \sin \theta d\theta} \quad 4-48$$

Setting $X = \frac{\mu_d E}{kT}$ and $\xi = \cos \theta$

$$\frac{\mu_{av}}{\mu_d} = \frac{\int_{-1}^1 \exp(X\xi) \xi d\xi}{\int_{-1}^1 \exp(X\xi) d\xi} \quad 4-49$$

Evaluation of this last equation yields

$$\frac{\mu_{av}}{\mu_d} = \frac{\exp(X) + \exp(-X)}{\exp(X) - \exp(-X)} - \frac{1}{X} = \coth X - \frac{1}{X} \quad 4-50$$

where $X = \frac{\mu_d E}{kT}$ is known as the Langevin function, hence $\frac{\mu_a}{\mu_d} = L(X)$.

In weak fields where $\frac{\mu_d E}{kT} \ll 1$, $L(X)$ can be expended into Taylor series

which yields

$$\coth X = \frac{1}{X} + \frac{X}{3} - \frac{X^3}{45} \dots\dots\dots$$

so that $L(X) = \frac{X}{3} = \frac{\mu_d E}{3kT}$. Therefore

$$\frac{\mu_{av}}{\mu_d} = \frac{\mu_d E}{3kT} \quad 4-51$$

where μ_d is the dipole moment of the polar molecule.

The orientational polarizability is given, therefore by,

$$\alpha_{os} = \frac{\mu_d^2}{3kT} \quad 4-52$$

The total polarizability, including the contributions of electronic and ionic polarizabilities, may be written as

$$\alpha = \alpha_e + \alpha_i + \frac{\mu_d^2}{3kT} \quad 4-53$$

So far it was assumed that the molecules are free to rotate which is true for the case of gases. For the case of gases, by making use of Eq. (4-18) and Eq. (4-4), we get

$$\bar{P} = \frac{n\alpha\bar{E}}{1 - \frac{n\alpha}{3\epsilon_0}} \quad 4-54$$

On the other hand, since $\alpha_s = \alpha_{es} + \alpha_{is} + \frac{\mu_d^2}{3kT}$ there must be a critical temperature for which $\frac{n\alpha}{3\epsilon_0} = 1$.

Upon appropriate manipulations this critical temperature is found to be,

$$T_c = \frac{n\mu_d^2}{9\epsilon_0 K} \cdot \frac{1}{1 - \frac{n(\alpha_{es} + \alpha_{is})}{3\epsilon_0}} \quad 4-55$$

This means that the dielectric will become spontaneously polarized below this critical "Curie" temperature. If $\alpha_{es} + \alpha_{is}$ is much less than α_{os} then,

$$\alpha_{os} = \frac{\mu_d^2}{3kT} \quad \text{and} \quad T_c = \frac{n\mu_d^2\alpha}{9k\epsilon_0} \quad 4-56$$

But Eq. (4-4) and Eq. (4-54) show that

$$\chi_e = \epsilon_r - 1 = \frac{n\alpha_s/\epsilon_0}{1 - \frac{n\alpha_{os}}{3\epsilon_0}} \quad 4-57$$

Therefore, it follows from Eq. (4-57) that

$$\chi_e = \frac{3T_c}{T - T_c} \quad 4-58$$

The calculation of orientational polarization of solids is not as simple as that for the gaseous state. In solids molecules are not free to orient in any arbitrary direction. This is due to fact that the interactions between the neighbouring molecules are limited, that is there are limited number of stable orientations. These orientation states depend on the symmetry of the molecules as well as the bonding.

It is assumed by Fröhlich⁶ that particles in solids have different equilibrium positions but not the freedom of rotation. The simplest case is the one with two equilibrium positions. These positions (A) and (B) are separated by a distance (b). A particle with a charge (e) has the same amount of energy in both of the equilibrium positions if the electric field is absent. But due to thermal agitations this particle may have sufficient energy to travel between these positions. However, on a time average, it will spend as much time in (A) as it will spend in (B). Due to this fact the probability of finding the particles in (A) must be the same of finding it in (B).

On the other hand, application of an electric field alters the equilibrium positions as well as the probability. The potential energies (V_A) and (V_B) will be different in this situation. This difference can be expressed by,

$$V_A - V_B = Eeb\cos\theta = e\bar{b}.\bar{E} \quad 4-59$$

where θ is the angle between the direction of the field and \vec{b} . This shows that the probability of finding the particle in (B) is greater than finding it near (A). According to Boltzmann distribution law the probabilities are

$$p_A = \frac{\text{Exp}(-\frac{V_A}{kT})}{\text{Exp}(-\frac{V_A}{kT}) + \text{Exp}(-\frac{V_B}{kT})}$$

and

$$p_B = \frac{\text{Exp}(-\frac{V_B}{kT})}{\text{Exp}(-\frac{V_A}{kT}) + \text{Exp}(-\frac{V_B}{kT})}$$

4-60

such that $p_A + p_B = 1$

If now there are (n) particles in the system, the probable number of particles in the state (A) is $n_A = np_A$ and in the state (B) is $n_B = np_B$. Therefore,

$$\frac{n_B}{n_A} = \text{Exp}(\frac{Ebc\cos\theta}{kT}) \quad 4-61$$

On the other hand the average moment induced by the field is ⁷

$$\bar{\mu}_T = \frac{1}{2} eb(p_B - p_A) \quad 4-62$$

Therefore, the induced moments in the direction of the field is,

$$\bar{\mu} = \frac{1}{2} ebc\cos\theta \frac{n_B - n_A}{n_B + n_A}$$

and

4-63

$$\bar{\mu} = \frac{eb}{2} \cos\theta \tanh(\frac{eE\cos\theta}{2kT})$$

In weak field cases $\frac{eE}{kT} \ll 1$ so that the orientational polarizability for solid dielectrics turns out to be,

$$\alpha_{os} = \frac{(eb\cos\theta)^2}{2kT} \quad 4-64$$

The ability of a molecule in solids to orient itself with the field depends on its shape and on the strength of the interactions with the neighbouring molecules. It is a mere fact from experiments that a molecule can rotate more freely and quickly with the changing field if its shape is near to sphere, an example being solid methane (CH_4). Molecules having less symmetry such as HCl or H_2O , possess several equilibrium positions so that their response is relatively slow.⁸

The behavior of orientational polarization in time dependent fields can be obtained by a similar method used for the electronic and ionic polarizabilities. If $\bar{\mu}_{av}$ is the moment due to the permanent dipoles in a constant electric field and $\bar{\mu}_t$ is the moment introduced by the application of time dependent field, then the change of $\bar{\mu}_t$ with time must be equal to the difference between $\bar{\mu}_t$ and $\bar{\mu}_{av}$ divided by the relaxation time τ , that is

$$\frac{\partial \bar{\mu}_t}{\partial t} = \frac{1}{\tau} (\bar{\mu}_{av} - \bar{\mu}_t) \quad 4-65$$

but $\bar{\mu}_{av} = \frac{\mu_d^2 E}{3kT}$ due to the Eq. (4-51) hence

$$\tau \frac{\partial \bar{\mu}_t}{\partial t} = \frac{\mu_d^2 E}{3kT} - \bar{\mu}_t \quad 4-66$$

On the other hand, since $\alpha_{os} = \frac{\mu_d}{3kT}$, given by Eq. (4-52) then

$$\tau \frac{\partial \bar{\mu}_t}{\partial t} = \alpha_{os} E - \bar{\mu}_t \quad 4-67$$

If E sinusoidally varying field, then the steady state solution of Eq. (4-67) results

$$\bar{\mu}_t = \frac{\alpha_{os} E}{1 - j\omega\tau} \quad 4-68$$

yielding the complex orientational polarizability, α_{ot} . Then

$$\alpha_{ot} = \frac{\alpha_{os}}{1 + j\omega\tau}$$

or

4-69

$$\alpha_{ot} = \alpha'_{ot} - j\alpha''_{ot}$$

where $\alpha'_{ot} = \frac{\alpha_{os}}{1 + \omega^2\tau^2}$ and $\alpha''_{ot} = \frac{\omega\tau\alpha_{os}}{1 + \omega^2\tau^2}$

E. Interfacial Polarization

Interfacial polarizability, as it was mentioned previously, is due to the lattice imperfections in the crystals of dielectric materials. If there exist imperfections inside the crystal, then the charges accumulate in these regions. These defects likely give rise to a conduction in the materials and they contribute to the polarizability of the materials.

It is believed that under static fields, there is no conductivity in ideal dielectric materials. Actually due to the lattice defects, the conductivity in such materials is extremely small but not zero.

In dielectric materials' structures there exist an appreciable number of interstices that are partly occupied by the particles that make up the material. Conduction, consequently, can take part by the movements of the particles from one site to an unoccupied site..

The most important defects existing inside the materials are Frenkel defects and Schottky defects. Frenkel defects result from the displacement of a particle from one site into one interstitial leaving behind a vacant site. On the other hand, when a pair of ions create a positive and a negative vacancy pair, the imperfection thus formed is called the Schottky defect. It can be easily deduced that upon application of an electric field these vacancies move through the lattice. The increase in the number of vacancies depends on the temperature increase.

Anderson⁹ proposes a two-layer condenser model to calculate the interfacial polarization by the use of equivalent circuits.

This mechanism is mostly pronounced in materials that show semi-conducting properties, or in other words, in semi-conductors.

F. Relaxation Time

In the view of the discussions carried so far in this section, the relaxation time given in Chapter 3, (Eq. 3-53) can be obtained consequently. The particles in the dielectric material may be assumed to oscillate with a natural frequency, $f_0 = \omega_0/2\pi$, due to thermal agitation in the absence of an applied electric field. Since in a two equilibrium position system, the probability of finding a particle in one position is the same as finding in the other, the transition probabilities must be the same. These transition probabilities are proportional to the energies of the positions. However, when an electric field is applied, the potential of one position is increased with respect to the other. This increase is given by Eq. (4-59).

In the absence of the electric field, the probability of particles' movement from (A) to (B), or (B) to (A) in per unit time is $(1/(2\pi\omega_0))$. But in the presence of an electric field the transition probabilities may be expressed as

$$T_{AB} = \frac{\text{TRANSITION FROM A TO B}}{\text{Sec}} = \frac{\omega_0}{2\pi} p_B$$

$$T_{BA} = \frac{\text{TRANSITION FROM B TO A}}{\text{Sec}} = \frac{\omega_0}{2\pi} p_A$$

4-70

where (p_A) and (p_B) are the probabilities of finding the particles in (A) and (B) respectively, and given by Eq. (4-60). Therefore

$$T_{AB} = \frac{\omega_0}{2\pi} \frac{e^{-V_B/kT}}{K} \quad \text{and} \quad T_{BA} = \frac{\omega_0}{2\pi} \frac{e^{-V_A/kT}}{K}$$

4-71

$$\text{where } K = \text{Exp}\left(-\frac{V_A}{kT}\right) + \text{Exp}\left(-\frac{V_B}{kT}\right)$$

Now, making use of the Eq. (4-59), we can obtain

$$T_{AB} = \frac{\omega_o}{2\pi A} \exp\left(-\frac{V_A}{kT}\right) \exp\left(\frac{ebE\cos\theta}{kT}\right) \quad 4-72$$

and

$$T_{AB} = T_{BA} \exp\left(\frac{ebE\cos\theta}{kT}\right) \quad 4-73$$

Since $ebE \ll kT$ is assumed, then

$$T_{AB} = T_{BA} \left(1 + \frac{ebE}{kT} \cos\theta\right) \quad 4-74$$

At an instant (t), where the number of particles in position (A) and position (B) is (n_A) and (n_B) respectively, there is a transition from one position to the other. The number of particles that makes transition from (A) to (B) is $n_A T_{AB}$ and the number of particles that make transitions from (B) to (A) is $n_B T_{BA}$ per second. Therefore

$$\frac{\partial n_A}{\partial t} = -n_A T_{AB} + n_B T_{BA}$$

and

$$\frac{\partial n_B}{\partial t} = -n_B T_{BA} + n_A T_{AB}$$

hence

$$\frac{\partial}{\partial t} (n_B - n_A) = -(T_{AB} + T_{BA})(n_B - n_A) + (T_{AB} - T_{BA})(n_B + n_A) \quad 4-76$$

Using the Eq. (4-74) and sticking to the assumption made about $ebE \ll kT$ it follows

$$T_{AB} + T_{BA} = 2T_{BA} \left(1 + \frac{ebE}{kT} \cos\theta\right) = 2T_{BA} \quad 4-77$$

$$T_{AB} - T_{BA} = \left(\frac{ebE}{kT} \cos\theta\right) T_{BA}$$

so that

$$\frac{d}{dt} (n_B - n_A) = -2T_{BA}(n_B - n_A) + 2T_{BA} \frac{ebE}{2kT} n \cos\theta \quad 4-78$$

where $n = n_A + n_B$.

Before the application of the field, at $t = 0$, $n_A = n_B = \frac{n}{2}$ which yields

$$n_B - n_A = \frac{n}{2} \frac{ebE}{kT} (1 - \text{Exp}(-2T_{BA} t) \cos\theta) \quad 4-79$$

The above discussion shows that, since polarization induced is proportional to the number of particles, polarization must depend on the time involved. The term $\text{Exp}(-2T_{BA} t)$ represents the time dependence of the polarization. Remembering Debye's decay function $\alpha(t) = \omega_0 \text{Exp}(-\frac{t}{\tau})$, this term should be equivalent to $\text{Exp}(-\frac{t}{\tau})$, or

$$\tau = \frac{1}{2T_{BA}} = \frac{\pi}{\omega_0} \text{Exp}\left(\frac{V_A}{kT}\right) \quad 4-80$$

where (V_A) is the potential of the particles in position (A) before the application of the field. π/ω_0 is the time for a single oscillation, or it is the time spent between two successive collisions of the particles.

CONCLUSIONS:

1. The surface density of the bound charges can be related to the molecular parameters through relation

$$\bar{P} = (\epsilon_r - 1)\epsilon_0 \bar{E} = \alpha n E'$$

where n is the number of particles per unit volume, α is their total polarizability and E' is the internal electric field acting locally.

2. Four mechanisms of polarization are:

- a. Orientation of molecules; orientational polarization
- b. The displacement of charged atoms or groups of atoms with respect to each other; atomic polarization
- c. Shift of electrons with respect to nuclei; electronic polarization
- d. Trapping of charge carriers; interfacial polarization.

3. Total polarizability is $\alpha = \alpha_e + \alpha_i + \alpha_o + \alpha_s$

Internal field acting locally is found to be

$$E' = \bar{E} + \bar{P}/3\epsilon_0$$

which easily applies to materials in which particles are spatially well organized or in complete disorder.

4. The relation between polarizability and relative permittivity is given by

$$\frac{N_0 \alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \frac{M}{\rho}$$

which is known as "Clausius-Mossotti" relation. N_0 is the Avagadro's number, ρ is the density of the material and M is the molecular weight of the substance.

5. For electrostatic fields, electronic polarizability is found to be

$$\alpha_{e_s} = 4\pi\epsilon_0 r_o^3$$

where r_o is the radius of the orbit of the electron. For sinusoidally varying fields, it is

$$\alpha_{e_t} = \frac{e^2/m}{\omega_o^2 - \omega^2 + j\omega\beta/m}$$

These show that α_e is a constant quantity at constant field but varies with frequency at time dependent fields.

6. For ionic polarizability for constant fields it is

$$\alpha_{i_s} = q^2/\omega_{oi}^2 M$$

which is a constant quantity, and for time dependent fields

$$\alpha_{i_t} = \frac{q^2/M}{\omega_{oi}^2 - \omega^2 + j\beta\omega/m}$$

7. For orientational polarizability:

At static fields $\alpha_{o_s} = \mu d^2/(3kT)$ temperature dependent ;

at time varying fields $\alpha_{o_t} = \alpha_{o_s}/(1 + j\omega\tau)$ both temperature and frequency dependent.

8. Relaxation time, defined in chapter III, is equal to

$$\tau = \frac{\pi}{\omega_o} \text{Exp}(V_A/kT)$$

where V_A is the potential of the particles in one position before the application of field.

9. Interfacial polarizability is due to Lattice Imperfections of which Frenkel and Schottky defects are the most important parts.

REFERENCES

1. J. C. Anderson, Dielectrics, Reinhold Publishing Company, New York, 1964 Chapter 4 .
2. A. Von Hippel, Dielectric Materials and Applications, The M.I.T. Press, Massachusetts, 1961 .
3. J. A. Stratton, Electromagnetic Theory, McGraw-Hill Book Company, Inc. New York, 1941, p.213 .
4. S. Wang, Solid State Electronics, McGraw-Hill Book Company, Inc., New York 1966, p. 398 .
5. C. P. Smyth, Dielectric Behavior and Stucture, McGraw-Hill Book Company, Inc., 1955, p. 10 .
6. H. Fröhlich, Theory of Dielectrics, Oxford Press, London, 1958, Chapter 2 .
7. Ibid, p.19 .
8. C. Kittel, Introduction to Solid State Physics, 3rd edition, John Wiley and Sons, Inc., New York, 1966
9. J. C. Anderson, Dielectrics, Reinhold Publishing Company, New York, 1964 p. 92 .

CHAPTER V

DIELECTRIC BEHAVIOR IN WEAK FIELDS, MEASUREMENTS, AND APPLICATIONS

A. Dielectric Behavior in Weak Fields

The theory of dielectrics discussed up to this point shows us that when a dielectric material is placed in electric fields, surface charges are induced on the dielectric. These induced surface charges appear in such a way that they weaken the originally applied field within the material. In general, the density of dipole moments introduced by the induced surface charges is related to the electric field intensity inside the material through relation (3-17), $\bar{P} = \chi_e \epsilon_0 \bar{E}$.

At relatively weak electrostatic fields, the relations $\bar{P} = \chi_e \epsilon_0 \bar{E}$ and $\bar{P} = (\epsilon_s - 1) \epsilon_0 \bar{E}$ suggest a linear proportionality between electric field intensity and polarization. This linear proportionality can be realized provided that the term $(\epsilon_s - 1) \epsilon_0$ or $\chi_e \epsilon_0$ is a constant quantity. By "constant quantity" we mean that the proportionality constant should not be altered by external effects such as temperature, relative humidity, as well as electric field intensity.

From the theoretical point of view, developed in the preceding chapters, we can investigate the dependence of ϵ_s to the external disturbances. Naturally this theoretical approach must be verified by experiments which will be

^x ϵ_s is used instead of ϵ_r which is general. ϵ_s is the relative permittivity which corresponds to static field cases.

discussed in the following sections of this chapter.

Referring to Eq. (4-22), which assumes a relation between the relative permittivity and polarizability, we can be able to direct our discussions into microscopic concepts. Although it is not valid for all cases of dielectrics, for the time being we will base our discussions on this "Clausius-Mossotti" relation,

$$\frac{N_0 \alpha}{3 \epsilon_0} = \frac{(\epsilon_s - 1)M}{(\epsilon_s + 2)\rho}$$

The quantities N_0 , M , and ρ are constant quantities for a given material, and we assumed ϵ_0 also be a constant value. This means we are only left with total polarizability which has to be investigated.

We know from Eq. (4-5) that total polarizability is the sum of the electronic, ionic, orientational, and interfacial polarizabilities. This suggests that we can treat each polarizability individually and then combine them into Clausius equation.

For electrostatic fields the electronic polarizability derived in Eq. (4-29) is simply $\alpha_{es} = 4\pi\epsilon_0 r_0^2$. Although in derivation constant r_0 is assumed, an exact quantum-mechanical calculation resembles a quantity very close to the above value but differs by a number.¹ The electronic polarizability appears to be independent of external disturbances since for a given material r_0 is a constant quantity.

Referring to Eq. (4-43), the ionic polarizability for electrostatic fields is given by $\alpha_{is} = q^2/\omega_{oi}^2 M$ which indicates that it is also unaffected by

the external disturbances.

The orientational polarizability in weak electrostatic fields, $E \ll kT/\mu_d$, was derived in Chapter IV and it was found to be $\alpha_{os} = \frac{\mu_d^2}{3 kT}$, where (μ_d) is the permanent dipole moment of the particles that make up the dielectric material. Generally, permanent dipole moments of non-symmetrical molecules are between 0.5 and 5 Debye.² Debye is the unit associated with the electric dipole moment and 1 Debye is equal to 10^{-18} ESU or $3.336 \cdot 10^{-28}$ Coulomb-meter. The assumption $E \ll kT/\mu_d$ turns out to be $E \ll 248 \cdot 10^5$ Volts/Meter for .5 Debye and $E \ll 24.8 \cdot 10^5$ Volts/Meter for 5 Debye at room temperatures.

We recognize from the expression for orientational polarizability that it is a function of the absolute temperature in weak electrostatic fields.

Now if we combine all the polarizabilities and substitute into Clausius-Mossotti relation, then it follows that

$$\epsilon_s = 1 + \frac{3n (\alpha_{es} + \alpha_{is} + \alpha_{os} + \alpha_{ss})}{3\epsilon_0 - (\alpha_{is} + \alpha_{es} + \alpha_{os} + \alpha_{ss}) n}$$

This expression, therefore, for dielectrics possessing no polar molecules is a constant quantity at all conditions. However for a given dielectric, containing polar molecules, the static permittivity is affected by the temperature variations and it is a constant quantity only for a given temperature.

The Langevin function is the dominating factor in orientational polarizability. From Eq. (4-50) we see that when $kT = E\mu_d$, the orientational polarizability becomes a constant quantity. And also from this expression we see that orientational polarizability would be a function of the electric field intensity if $X^3/45$ were not neglected. But since we deal with weak

fields, the assumption made by neglecting this term is an appropriate one.

So far in this section we have used the equations which can be applied easily to gaseous state. For solid dielectrics the orientational polarizability is given by Eq.(4-64) as $\alpha_{os} = \frac{(eb \cos\theta)^2}{2kT}$ which also shows that it is a function of the temperature at weak electrostatic fields. For polar liquids and solids, Clausius-Mossotti equation usually does not hold. For such dielectrics Onsager³ has established the relation,

$$\frac{(\epsilon_s - 1)M}{(\epsilon_s + 2)\rho} - \frac{(\epsilon_\infty - 1)M}{(\epsilon_\infty + 2)\rho} = \frac{3\epsilon_s(\epsilon_\infty + 2)}{2(\epsilon_s + \epsilon_\infty)(\epsilon_s + 2)} \frac{4\pi\mu_d^2 N_0}{9kT}$$

This relation is more generalized by Kirkwood and Fröchlich^{4,5}, which can be expressed by,

$$\frac{(\epsilon_s - 1)M}{(\epsilon_s + 2)\rho} - \frac{(\epsilon_\infty - 1)M}{(\epsilon_\infty + 2)\rho} = \frac{3\epsilon_s(\epsilon_\infty + 2)}{2(\epsilon_s + \epsilon_\infty)(\epsilon_s + 2)} \frac{4\pi\mu_d^2 N_0}{9kT} \quad g.$$

As far as the external effects are concerned these equations are similar in form to Clausius equation.

The behavior of dielectric materials at weak alternating fields can be investigated from the previously derived relationships.

Eq.(4-34) indicates that the electronic polarizability is a function of frequency only. Using relation, the relative permittivity given by Eq.(4-4) can be written as:

$$\epsilon_r = 1 + \frac{ne^2/m\epsilon_0}{\omega_0^2 - \omega^2 + j\omega\beta/m}$$

For ionic polarizability, it also varies with frequency and given by Eq.(4-43) as:

$$\alpha_{it} = \frac{q^2/M}{\omega_{oi}^2 - \omega^2 + j\omega\beta/M}$$

Also for time-dependent fields, the orientational polarizability is,

$$\alpha_{ot} = \frac{\alpha_{os}}{1 + j\omega\tau}$$

The quantity (τ), relaxation time, is a function of the temperature as derived in Eq.(4-80). This means that orientational polarizability is a function of the frequency as well as the temperature. The complex dielectric permittivity of whose components are given by Eq.(3-48) and Eq.(3-49) in terms of Debye relaxation time, therefore depends on the temperature and frequency. However, theoretically if linear relation between \bar{E} and \bar{D} is assumed, the components of complex relative permittivity should not be independent of each other since they are derived from the same function. This proposes that relative permittivity is almost a constant quantity under frequency variations. On the other hand polarizabilities arising from different mechanisms are dependent factors of frequency, yielding the fact that relative permittivity varies with frequency.

We can deduce from the frequency variations of the polarizabilities that dielectric materials can be classified according polarizabilities. These are 1) dielectric materials that exhibit electronic polarizability, 2) dielectric materials that have both electronic and ionic polarizabilities, and

3) dielectric materials that exhibit all the polarizabilities.

The frequency dependence of polarizabilities is illustrated in figure 10 in a schematic way. At the microwave region the curve resembles that of the spectrum of complex permittivity defined by the relaxation time and given by Eq.(3-48) and Eq.(3-49).

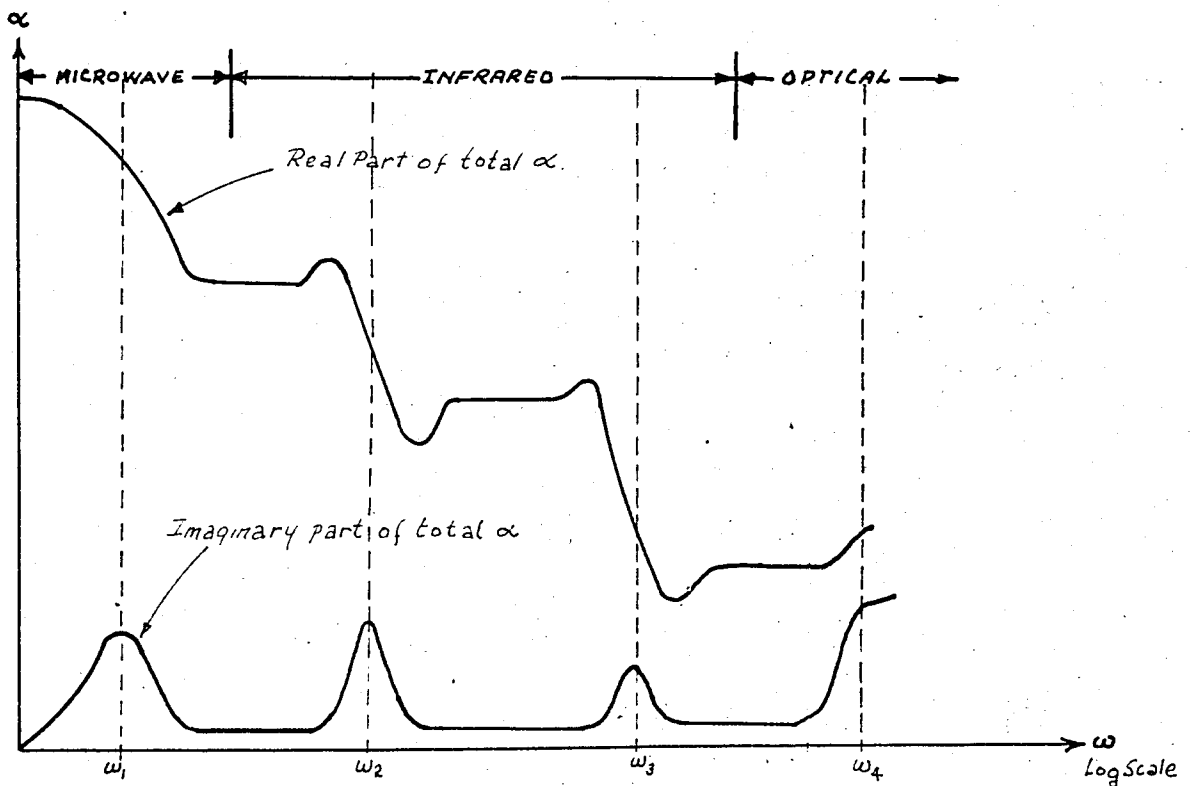


Fig. 10 - Variation of polarizability with frequency.

Therefore we can conclude that for weak time-dependent electric fields the relative permittivity is a function of both temperature and frequency. For weak electrostatic fields relative permittivity is a function of the temperature for dielectric materials that posses polar molecules.

B. Permittivity Measuring Methods^{6,7}

Permittivity is one of the important aspects associated with the dielectrics. There are variety of methods to determine the permittivities. Since the components of permittivity vary according to conditions, the methods of measurements vary with the conditions in which the measurements are to be performed.

Static Permittivity can be obtained by a time-constant technique which will be discussed in the following section. On the other hand for the permittivities at different frequencies, it is necessary to use different bridge or resonant circuits properly adjusted to the conditions. For frequencies in the range of 10^2 to 10^7 Hertz a typical bridge, Schering bridge, may be used. This type of bridge provides an accuracy of $\pm(2\% + 0.04 \mu\text{f})$ for the capacitance if the system is well-shielded and a standard precision capacitor is used. However, for frequencies of 10 MHz to 100 MHz, a.c. bridges fail to give accurate results due to increased stray capacitance effects. It usually happens that resonant circuits are used instead of a.c. bridges at this frequency range. One deficiency of this type of circuit is that it is limited to relatively low-loss materials with $\tan\delta \approx 0.05$. For the dielectrics having large loss tangents, a double resonance method is usually used. There are five different methods associated with these resonant circuits. They are (1) resistance variation, (2) conductance-variation, (3) susceptance-variation, (4) frequency variation and (5) resonance rise methods.

In very high frequency range, 100 - 1000 MHz, the tuned circuits cannot be used due to the unavailability of such circuits. Instead, distributed

circuits are used. Transmission - line and wave guides are common forms of these distributed circuits.

C. Static Permittivity Measurements

The value of the static permittivity for a given dielectric material at weak electrostatic fields can be obtained through the measurements described in the following paragraphs. This method is used to find the static permittivities of six different dielectric materials which are to be used in Electro-Odocell measurements. Electro-Odocell will be mentioned in the "applications" section. The graphs and results of the experiments in relation with this paper are given in Appendix B.

The basic concept associated with time-constant measuring technique for determining the static permittivity is to measure the elapsed time for a condenser system to discharge from an initial to a latter voltage. The diagram in figure (11) illustrates the circuit used in determining the d.c. relative permittivity.

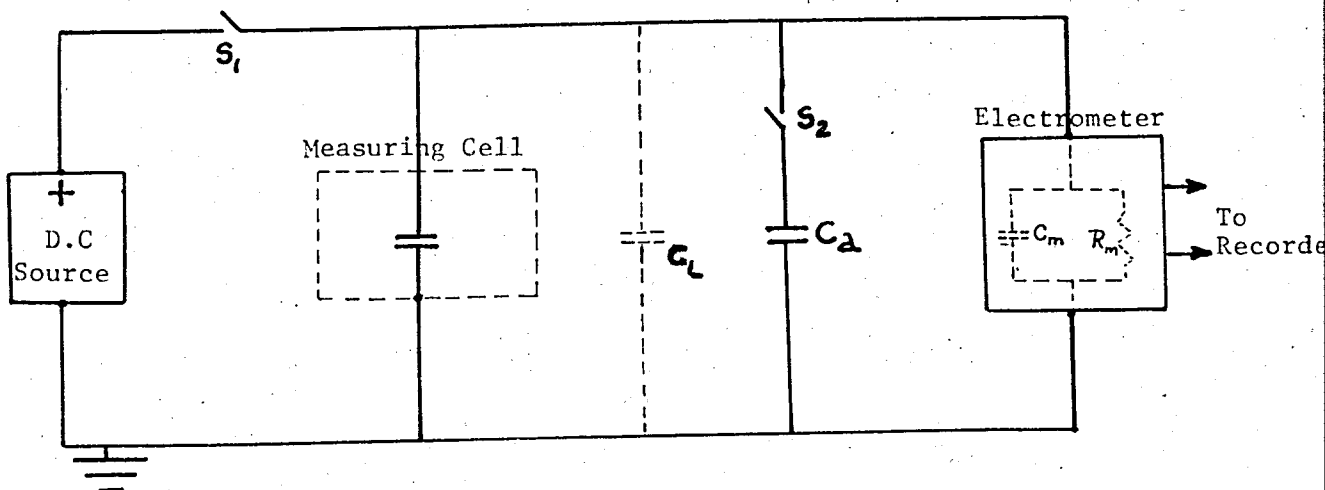


Fig. 11 - Measuring circuit used in experiments.

The measuring cell designed for this experiment consists of a micrometer electrode system. Micrometer system electrode is used in order to adjust the distance between the electrodes so that dielectric samples can be easily inserted or removed. Also it provides the measurement of capacitance when there is no dielectric specimen between the electrodes. This arrangement is placed in a grounded shield to eliminate induced voltages due to outside sources. This arrangement is shown in figure (12).

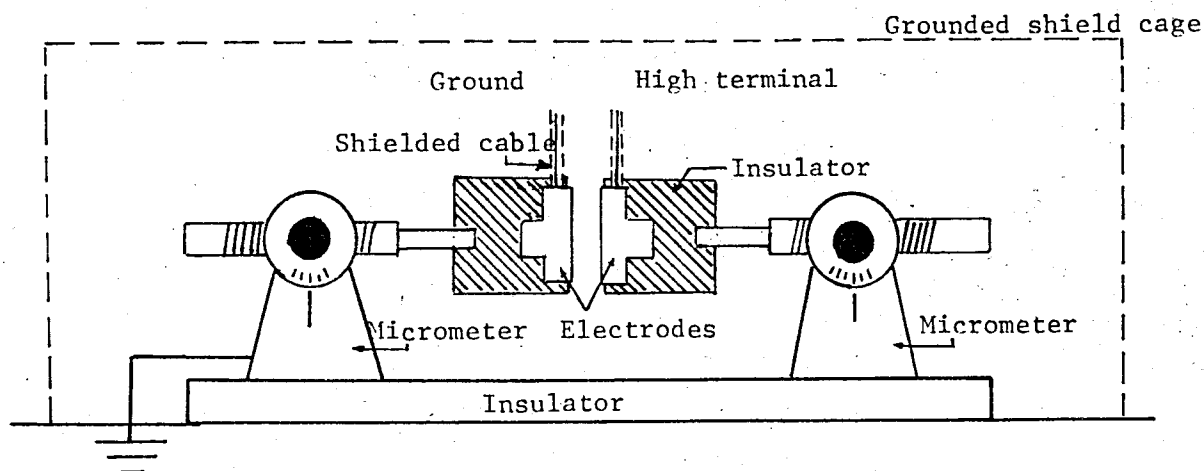


Fig. 12 - Measuring Cell.

Initially, without a dielectric specimen between electrodes, this two parallel, circular, plate condenser is charged to a voltage V_0 from a battery. The spacing between the electrodes is adjusted to the value which corresponds to the thickness of the dielectric under investigation. Charging of the condenser is performed by closing the switch (S_1). After the condenser is charged, the switch (S_1) is opened at the time (t_0). This operation leads the condenser to discharge through the high-valued resistance of the electrometer.^x During discharge, the time elapsed from t_0 and the

^x Electrometer: Model 610 B, .001 volt full scale to 100 volts in eleven 1x and 3x ranges, 10^{14} ohms shunted by 22 picofarads as the input impedance, input resistance may be selected in decade steps from 10 to 10^{11} ohms, Keithley instruments.

decreasing values of the voltage to any value (V) are obtained from the recorder^x connected to the electrometer. This recorder records the discharging curves, and provides a direct reading of the time for a voltage (V).

If the value of the voltage on the condenser at any time (t) is V(t), then

$$V(t) = V_0 \text{Exp}\left(-\frac{t}{RC'_0}\right)$$

where R is the resistance of the electrometer (R_m), and C'_0 is the capacitance of the system for figure 11, that is it is the sum of the C_L , C_m , and C_0 which is the true capacitance of air-filled electrodes.

Selecting $t_0 = 0$, if at time $t = \tau_0$, $V(t) = V_1$ is observed, then the capacitance of the system can be expressed by,

$$C'_0 = \frac{\tau_0}{R_m \ln(V_0/V_1)}$$

Secondly, the dielectric specimen under investigation is inserted between the electrodes. This arrangement is also charged to V_0 by means of closing the switch (S_1). The system is then allowed to discharge as in the previous case. During the discharge the time τ is found for the voltage to fall to a convenient value (V_2). According to exponential decay

$$\ln \frac{V_0}{V_2} = \frac{\tau}{C'R_p}$$

where C' is the capacitance of this last arrangement and R_p is the parallel combination of (R_m) with the losses of the dielectric sample represented by R_L . We see from the last relation that it contains two unknowns, R_p and C' . To

^x

Recorder: Varian recorder division, series F-80A, x-y recorder, x-time recorder, 0.2, 1, 2, 5, 10, 20, 50, 100, 200, 500 mV/cm; 1, 10, 20 V/cm range

eliminate one of them, another equation is needed. Such a relation can be obtained by connecting a low-loss air filled capacitor (C_a) in parallel to the arrangement^x. This is performed by closing the switch (S_2). The system is first charged to V_0 and then discharged. The time (τ') is measured for the initial voltage to drop to V_2 . Voltage (V_2) is the same value obtained for the previous case. Then from exponential decay,

$$\ln \frac{V_0}{V_2} = \frac{\tau'}{(C' + C) R_p} = \frac{\tau}{C' R_p} \quad \text{or} \quad C' = \frac{C_a}{(\tau'/\tau - 1)}$$

Now, since $C_0 = \epsilon_0 A/d$ and $C = \epsilon_0 \epsilon_r A/d$, it follows that

$$\epsilon_r = \frac{C}{C_0} = \epsilon_s$$

Now, we must point out that the values of C'_0 and C' obtained from discharge curves are not the same with C_0 and C which appear in $\epsilon_r = C/C_0$. C'_0 and C' contain the values of electrometer capacitance (C_m) and the capacitance due to electrodes and insulations (C_L), combined in parallel^{xx}. Therefore,

$$C = C' - (C_m + C_L) \quad \text{and} \quad C_0 = C'_0 - (C_m + C_L)$$

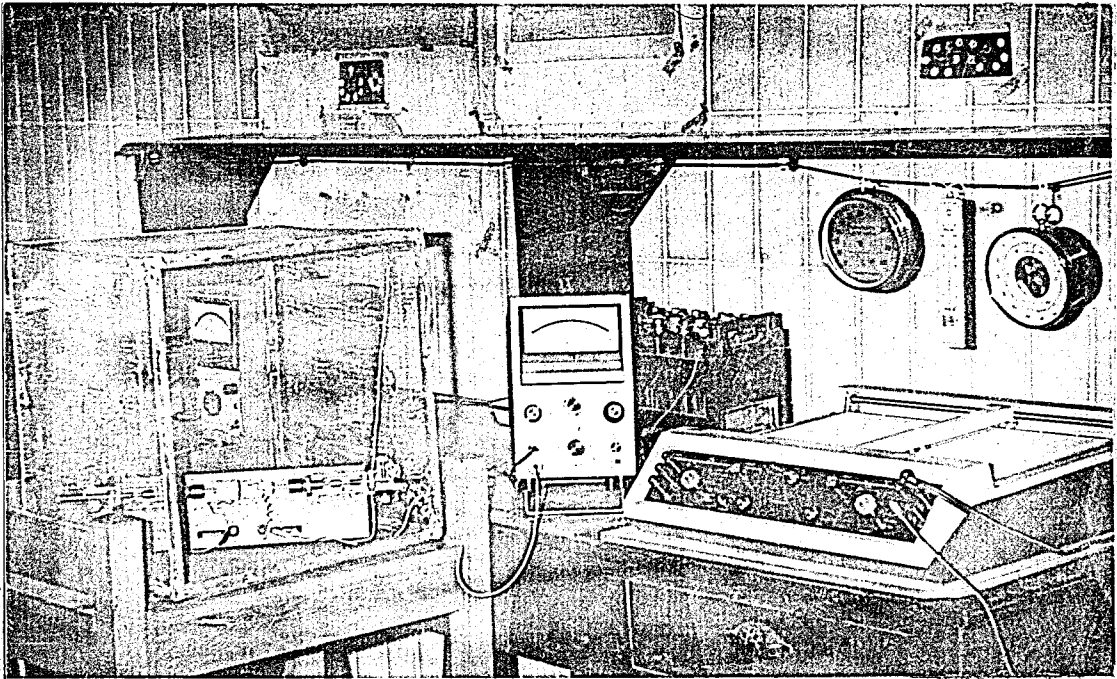
The experiments with 6 dielectric samples were conducted under constant temperature (25°C) and relative humidity (49-51%) conditions in order to avoid interference of these factors to the experiments. Five initial charging voltage is used in the experiments. These are: $V_0 = 2.7, 4.1, 5.4, 32, 45$ Volts.

^x

C_a : Since a variable low-loss air-filled capacitor was available, its capacitance was recorded after each experiment to avoid any error, (25-250pF).

^{xx}

C_L : It was measured by Impedance bridge, type 1650A, General radio Company, and its capacitance was found to be equal to 10 pF.



D. Discussion of the Experimental Results

The dielectric samples with which the experiments were performed are given in Appendix B.1. These samples have the following characteristics:

Sample 1 - Wrapping paper, having fiber content, semi-transparent,

thickness = $5/50 \text{ mm} = 1 \cdot 10^{-4} \text{ Meter}$.

Sample 2 - Plastic, semi-transparent, has small amount of electrostatic

surface charges, thickness = $15/50 \text{ mm} = 3 \cdot 10^{-4} \text{ Meter}$.

Sample 3 - Special paper used in airplanes as hand towels, large pores,

non-transparent, thickness = $18/50 \text{ mm} = 3.6 \cdot 10^{-4} \text{ Meter}$.

Sample 4 - Special paper used in airplane service trays, has electrostatic

surface charges, non-transparent, thickness = $11/50 \text{ mm}$

$= 2.2 \cdot 10^{-4} \text{ Meter}$.

Sample 5 - Cellophane paper, transperant, thickness = $3/50 \text{ mm} = .6 \cdot 10^{-4} \text{ M}$.

Sample 6 - Wrapping paper, transperant, thickness = $3/50 \text{ mm} = .6 \cdot 10^{-4} \text{ M}$.

The experiments are performed under constant temperature (25°C) and relative humidity (49-51%) conditions inorder to eliminate the effects that might arise from these effects. The results obtained from the experiments are tabulated in Table 5-1.

A series of bridge measurements was also conducted to compare the results^x. The results obtained from such measurements at 6 Volts and 1KHz are given in table 5-2.

The values of ϵ_r in Table 5-2 is calculated from the relation $\epsilon_r = \frac{C \cdot t}{\epsilon_0 A}$ where (C) is the capacitance measured by bridge, (t) is the thickness of the sample, and (A) is the area of one of the electrodes. In this approach, the application of the formula is based on the assumption that the plate dimensions are much larger than the thickness of the sample. This means that the condenser can be considered as having infinite plate area so that approximately the linear relation holds. A more careful approach would give better results but not appreciably different than the above results. For our purposes the assumption is applicable.

The results of Table 5-1 imply that, except sample No. 1, dielectric samples considered have almost constant relative permittivities at initial voltages of $V_0 = 2.7, 4.1, 5.4$ Volts, hence in weak initially applied external fields since $E_0 = V_0/t$. The differences in the values of ϵ_r of a specific

^x

Impedance bridge: Type 1650-A Impedance Bridge, General Radio Company
1 pf - 1000 μ f, series or parallel, 7 ranges for capacitance measurements, 1 KHz supplied internally.

dielectric sample can be attributed to the experimental errors, especially to the difficulty of reading time values from the graphs. As it is shown in Appendix B.2, a small difference, of the order of .5 mm, on the graph paper results in an appreciable change in the value of the capacitances. Although we tried to read approximately the correct values from the graphs, a more sensitive recording device with a scale less than 0.2 sec/cm would yield more reliable results.

Comparison of Tables 5-1 and 5-2 shows that $(\epsilon_r)_L$ for sample 1, $(\epsilon_r)_H$ for sample 2, $(\epsilon_r)_H$ for sample 3, $(\epsilon_r)_H$ for sample 4, $(\epsilon_r)_L$ for sample 5, and $(\epsilon_r)_L$ for sample 6 give reasonable values. Now let us investigate each sample individually.

Sample 1 - Table 5-2 implies that it has a relative permittivity of 6.66. In Table 5-1 there is no repeating value for ϵ_r . When $(\epsilon_r)_L$ is taken into account, there appears some values close to the value 6.66. The deviations in the values can be due to the experimental errors. This judgement is based on the fact that relative permittivity does not follow a regular pattern with increasing initial voltages. Although we are not definitely sure, we can state that sample 1 has a relative permittivity around 6.70, and does not depend on the field strength at weak fields.

Sample 2 - This sample has a relative permittivity of 4.65 from bridge measurements and 4.67 from $(\epsilon_r)_H$, which are very close. The only variation from these values is experienced at $V_0 = 32$ Volts. As a result we can easily state that ϵ_r for this dielectric sample is around 4.65 and it stays constant with increasing externally applied initial fields (E_0).

TABLE 5-1
(Values obtained from experiments)

Initial Voltage V_o Volts	SAMPLE 1			SAMPLE 2			SAMPLE 3		
	E_o V/mm	$(\epsilon_r)_L$	$(\epsilon_r)_H$	E_o V/mm	$(\epsilon_r)_L$	$(\epsilon_r)_H$	E_o V/mm	$(\epsilon_r)_L$	$(\epsilon_r)_H$
2.7	27	7.23	7.80	9	4.43	4.67	7.5	5.34	6.00
4.1	41	6.82	7.35	13.7	4.43	4.67	11.4	5.34	6.00
5.4	54	8.07	8.73	18.0	4.43	4.67	15.0	5.34	6.00
32.0	320	6.95	7.50	106.7	4.64	4.89	89.0	4.73	5.32
45.0	450	6.75	7.28	150.0	4.43	4.67	125.0	4.95	5.57
Initial Voltage V_o Volts	SAMPLE 4			SAMPLE 5			SAMPLE 6		
	E_o V/mm	$(\epsilon_r)_L$	$(\epsilon_r)_H$	E_o V/mm	$(\epsilon_r)_L$	$(\epsilon_r)_H$	E_o V/mm	$(\epsilon_r)_L$	$(\epsilon_r)_H$
2.7	12.3	3.19	3.21	45.0	5.62	5.77	45.0	4.00	4.12
4.1	19.1	3.50	3.58	68.5	5.62	5.77	68.5	4.07	4.18
5.4	24.6	3.19	3.21	90.0	5.60	5.75	90.0	4.03	4.14
32.0	145.0	3.17	3.20	533.5	6.60	6.75	533.5	4.07	4.18
45.0	205.0	3.27	3.29	750.0	-	-	750.0	4.36	4.50

TABLE 5-2
(Values obtained from bridge measurements.)

	SAMPLE 1	SAMPLE 2	SAMPLE 3	SAMPLE 4	SAMPLE 5	SAMPLE 6
C pF	185	43	47	41	252	180
ϵ_r	6.66	4.65	6.10	3.23	5.45	3.86

Sample 3 - Comparison of Tables 5-1 and 5-2 implies that sample 3 has a relative permittivity approximately equal to 6.00. The disbehaviors observed for ϵ_r at $V_0 = 32$ Volts and 45 Volts might be due to the experimental errors, since the variation order is irregular. Therefore we can say that relative permittivity does not depend on the fields of relatively weak intensities.

Sample 4 - For this sample, it is obtained 3.23 for the relative permittivity from bridge measurements and 3.20 from time-constant measuring results. Referring to Table 5-1 we can say that ϵ_r stays constant for increasing externally applied initial fields (E_0).

Sample 5 - This sample has almost a constant relative permittivity for initial voltages of $V_0 = 2.7, 4.1, \text{ and } 5.4$ Volts. However an irregular behavior is observed for higher voltages. This might be due to the experimental errors otherwise theoretical explanation would fail. Although this behavior is experienced at relatively high field intensities, it can not be attributed to dielectric breakedown since if it were the case then the decaying response would not be obtained. As a last alternative, there might be space charges in the dielectric sample which can contribute to the variation of ϵ_r at increased voltages. We can lastly say that for external fields up to about 100 v/mm, relative permittivity has a constant value around 5.50 .

Sample 6 - Only one irregularity is observed at $E_0 = 450$ Volts/mm . But for our purposes, at weak fields, ϵ_r has a constant value approximately equal to 3.86 from bridge measurements and around 4.00 from time-constant measurements.

It is also interesting to note that for most cases the values of (C) obtained from graph measurements somewhat resemble the values that are tabulated in Table 5-2

As a result we can say that, except experimental errors, the relative permittivities of experimented dielectric samples have constant values for externally applied weak electric fields.

E. Dielectric Applications

The basic properties that outline the dielectric materials are their non-conducting properties and energy storage capacities. Due to these properties dielectric materials are widely used. All kinds of electrical equipments, from power and distribution equipments to electronic devices, make use of dielectric materials.

Naturally, dielectrics, due to their non-conducting properties, are used when insulation problems are involved. They are used, for instance, in cables, in transformers, in switch gears as insulators.

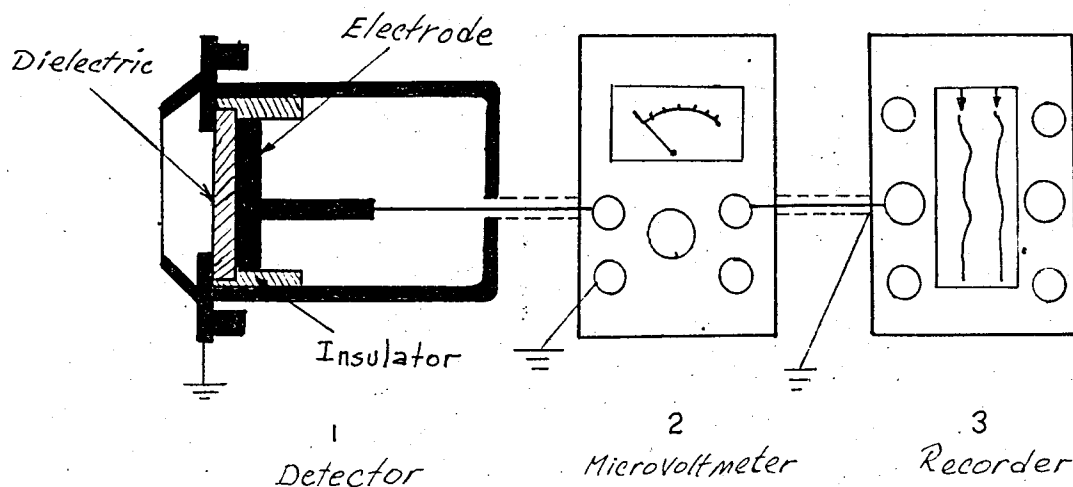
Besides the insulating properties, charge-storage capacity makes the dielectric materials to be widely used in electronic equipments. Capacitors are the most important devices that make use of this property.

Capacitors having large capacitances and small sizes are made of ferroelectric materials since these materials have high permittivities. However, the temperature dependence of these dielectric materials limits their applications. For cases where precision and constancy of the capacitance is required, they can not be used. For other dielectric materials, high capacitance can be obtained by reducing the thickness of dielectric slabs. This leads to thin-film capacitors to be used for high capacitance values. But reduction of the thickness results in an increase in the electric field intensity for a given voltage so that dielectric breakdown occurs at low potentials. This limits the application of thin-film capacitors as valves in circuits where there are high potentials associated with the system.

The capacitive property of dielectric materials finds another application in the Electro-Odocell, an objective odor measuring instrument which has been developed in the Research Center of Robert College.

Prof. N. N. Tanyolaç, who first developed Odocell with Prof. Eaton at Purdue University in 1948, describes the principle of operation of the Electro-Odocell as:^{8,9}

" The electro-Odocell detects and measures the change in potential or current on a dielectric when one surface is contaminated by the molecules given off by odorants. The Electro-Odocell consists of an odor sensitive transducer open to odor molecules, a microvoltmeter, and an automatic recorder."



"... The sensitive surface of the detector is open to the odor molecules and is made of dielectric materials like mica, glass, plastics, paper, etc."

It can be easily observed from above definition that the detector in a way acts like a capacitor. Introduction of odorant molecules to the dielectric

surface results in formation of a layer of charges which acts as the second electrode. Basically the voltage built up across the detector is dominated by the amount of odorant molecules collected at the dielectric surface as well as their orientation. Besides, the experiments carried in the Research Center of Robert College reveal the fact that relative humidity is also an important factor in odor measurements.

We see from the foregoing discussions that the voltage across the dielectric is somewhat related to the charges collected at the surface of the dielectric material of the detector. In order to investigate this relationship, we need the capacitance, hence relative permittivity, to remain constant at the field intensities introduced by the odor molecules. Also we need to find dielectric materials such that they give the best and the desired results.

For illustration purposes, curves obtained for Menthone by using samples 2 and 3, is given in Appendix C. These curves are obtained from an Electro-Odocell which has been used for further investigation in odor detection. It is interesting to observe that the shapes of these curves somewhat resemble that of the charging of capacitors. Also we see from these curves that the dielectric with $\epsilon_r = 4.65$ (sample 2) is more sensitive for menthone than the dielectric with $\epsilon_r = 6.00$ (sample 3). Presently this behavior is under investigation.

Ferroelectric materials are used as dielectric amplifiers since they possess hysteresis loops. If very thin ferroelectric capacitor is used, then any change in the biasing voltage would result in a large change in the incremental permittivity which is determined by the point on the hysteresis loop.

Besides these applications, piezoelectric properties make dielectric materials to be used as transducers, converting mechanical energy into electrical energy or vice versa.

REFERENCES:

1. R. Feynman, R. Leghton, M. Sands, Lectures on Physics, Addison-Wesley Publishing Co., Inc. Massachusetts, 1965, p. 11-3.
2. C.J.F. Böttcher, Theory of Electric Polarization, Elsevier Publishing Company, Amsterdam, 1952, p.3.
3. C. Smyth, Dielectric Behavior and Structure, McGraw-Hill Book, Co., Inc., New York, 1955, p.23.
4. Ibid, p. 28.
5. H. Fröhlich, Theory of Dielectrics, Oxford University Press, London 1958, Ch.2.
6. R.F. Field, "Lumped Circuits" in Dielectric Materials and Applications, The M.I.T. Press, Massachusetts, 1961.
7. W.B. Westphal, "Distributed Circuits", in Dielectric Materials and Applications, by A. Von Hippel, The M.I.T. Press, Massachusetts, 1961.
8. N.N. Tanyolaç, "The Electro-Odocell for Inspection and Grading of Meat and Fish" in Olfaction and Taste 2 by T. Hayashi, Wenner-Gren Center International Symposium Series, Volume 8, Pergamon Press, 1965.
9. N.N. Tanyolaç, "The Electro-Odocell for Odor Measurement and Surface Effects", in Surface Effects in Detection by J.I. Brennan and A. Dravnieks, Spartan, 1965.
10. N.N. Tanyolaç, "Electro-Odocell and Theories of Odor" in Theories of Odors and Odor Measurement by Necmi Tanyolaç, Technivision, London, 1968.

CHAPTER VI

CONCLUSION

In the foregoing discussions of this report we have dealt the dielectric behavior in relatively weak electric fields. We have began the discussion with a detailed description of macroscopic and microscopic properties of dielectric materials in electric fields. In the latter sections of this report, the dependence of relative permittivity to the external effects has been investigated. Some of the results obtained from the theory were tried to be verified by experiments.

The fundamental theory discussed in Chapter I implies that when a dielectric material is placed in an electric field, induced surface charges appear in such a way that they tend to weaken the original field, within the dielectric.

In Chapter II, we considered the fundamental theories associated with electric fields. We defined the relation $\vec{D} = \epsilon \vec{E}$, which makes use of the property of the medium. The second part of Chapter II dealt with the other important aspect of dielectrics; dipoles. The dipoles and the fields associated with them were discussed in this part. By making use of field theories, the electric field intensity due to a dipole, was found to be

$$\vec{E} = \frac{3 (\vec{m} \cdot \vec{r}) \vec{r} - r^2 \vec{m}}{4\pi\epsilon_0 r^5}$$

In Chapter III, the behavior of dielectric materials in electric fields was discussed from the macroscopic point of view. The important conclusions obtained from this chapter can be summarized as :

1. For isotropic dielectric materials, the relation between induced charge density, P , and electric field intensity, E , within dielectric is,

$$\bar{P} = \chi_e \epsilon_0 \bar{E} = (\epsilon_r - 1) \epsilon_0 \bar{E}$$

This linear relationship is not valid for anisotropic and ferroelectric materials.

2. For time-dependent fields, relative permittivity is composed of two components; $\epsilon_r(\omega) = \epsilon'_r(\omega) - j\epsilon''_r(\omega)$. In terms of relaxation time, which was derived and found to be a function of temperature in Chapter IV, the components of permittivity can be given by,

$$\epsilon'_r(\omega) = \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{1 + (\omega\tau)^2} \quad \text{and} \quad \epsilon''_r(\omega) = \frac{(\epsilon_s - \epsilon_\infty) \omega\tau}{1 + (\omega\tau)^2}$$

At very low frequencies and at constant τ conditions, that is $(\omega\tau)^2 \ll 1$, we see that ϵ_r is a constant quantity, ϵ_s . However this is not the case for large values of ω .

Chapter IV has dealt with the microscopic aspects of dielectrics under electric fields. We see in this chapter that macroscopic polarization is related to molecular parameters through relation

$$\bar{P} = \alpha n \bar{E}' = (\epsilon_r - 1) \bar{E}$$

Following this relation we reach the "Clausius-Mossotti" relation which links relative permittivity to microscopic polarizabilities. Each component of total polarizability has been examined. We have concluded that for electrostatic fields electronic and ionic polarizabilities have constant values where orientational polarizability is only constant for a given temperature at weak field intensities. However, for time-dependent fields, we have obtained the fact that all

polarizabilities are functions of frequency, and they vary with varying fields. Although for solid dielectric materials possessing polar molecules, the orientational polarizability seems to be affected by the field intensity, the term which is responsible for this behavior is negligible in weak field, so that it does not contribute to permittivity.

In the first parts of Chapter V, we have considered the behavior of dielectric materials at weak fields with the help of previously derived relationships, and concluded that for electrostatic fields, the relative permittivity is a constant quantity for a given temperature. On the other hand, we have also found that dielectric properties are affected by frequency as well as temperature at time-dependent fields.

Finally, the rest of Chapter V has been devoted to experimental verification of the theoretical observations, and applications of dielectric materials. Experiments have been conducted on 6 different dielectric samples in connection with the Electro-Odocell which has been under investigation in the Research Center of Robert College. Experiments have showed that out of 6 samples, 4 of them have almost constant relative permittivity values at different externally applied initial fields. However irregularities observed for sample 1 can be attributed to experimental errors. This suggests a more sensitive approach is needed for the experiments in order to eliminate the errors. Once this is done, we can obtain more reliable verification of the theory.

APPENDICES

APPENDIX A

A.1 Vector identities

$$\nabla \cdot (\nabla \bar{A}) = \bar{A} \cdot \nabla \nabla + \nabla \nabla \cdot \bar{A}$$

$$\nabla (\nabla \bar{W}) = \nabla \nabla \bar{W} + \nabla \nabla \bar{W}$$

$$\nabla \cdot (\bar{A} + \bar{B}) = \nabla \cdot \bar{A} + \nabla \cdot \bar{B}$$

$$\nabla \cdot (\nabla + \nabla) = \nabla \nabla + \nabla \nabla$$

$$\nabla \times (\nabla \bar{A}) = \nabla \nabla \times \bar{A} + \nabla \nabla \times \bar{A}$$

$$\nabla \cdot \nabla \times \bar{A} = 0$$

$$\nabla \cdot \nabla \nabla = \nabla^2 \nabla$$

A.2 Proof of $\nabla (\bar{A} \cdot \bar{r}) = \bar{A}$

$$\text{Let } \bar{r} = x \bar{i} + y \bar{j} + z \bar{k} \quad \text{and} \quad \bar{A} = A_x \bar{i} + A_y \bar{j} + A_z \bar{k}$$

$$\text{Then } \bar{A} \cdot \bar{r} = A_x x + A_y y + A_z z$$

$$\text{Since } \nabla (\nabla \bar{W}) = \nabla \nabla \bar{W} + \nabla \nabla \bar{W}, \quad \text{if } A_x, A_y, A_z, \text{ hence } \bar{A} \text{ is a constant}$$

$$\text{vector, then } \nabla (A_x x + A_y y + A_z z) = A_x \nabla x + A_y \nabla y + A_z \nabla z$$

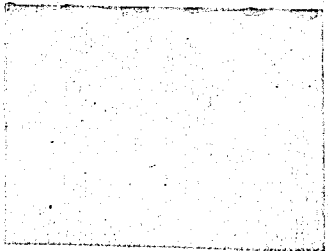
$$\text{But } \nabla x = \frac{\partial x}{\partial x} \bar{i} = \bar{i} \quad \nabla y = \frac{\partial y}{\partial y} \bar{j} = \bar{j} \quad \nabla z = \frac{\partial z}{\partial z} \bar{k} = \bar{k}$$

$$\text{Hence } \nabla (A_x x + A_y y + A_z z) = A_x \bar{i} + A_y \bar{j} + A_z \bar{k} = \bar{A}$$

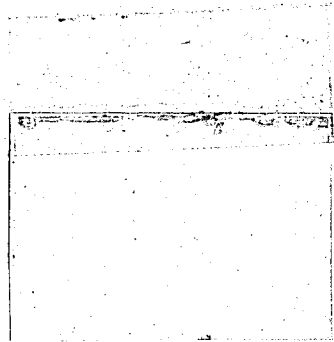
$$\text{Therefore } \nabla (\bar{A} \cdot \bar{r}) = \bar{A}$$

APPENDIX B

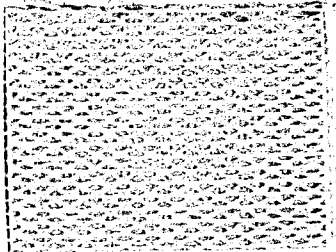
B.1 Dielectric samples used in experiments



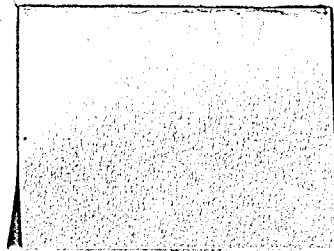
Sample 1



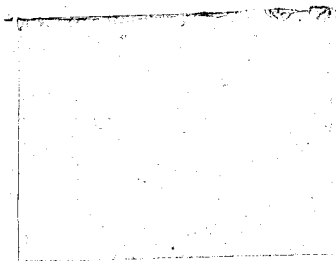
Sample 2



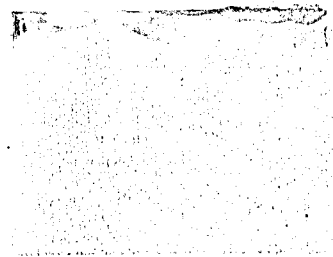
Sample 3



Sample 4



Sample 5



Sample 6

B.2 Calculations

i) Formulas

$$A = \pi r^2$$

$$C' = \frac{C_a}{(\tau'/\tau - 1)} \quad ; \quad C = C' - (C_m + C_L)$$

$$C'_o = \frac{\tau_o}{R_m \ln(V_o/V_1)} \quad ; \quad C_o = C'_o - (C_m + C_L)$$

$$(\epsilon_r)_L = \text{for highest value of } C_o = \frac{C}{C_{oh}}$$

$$(\epsilon_r)_H = \text{for lowest value of } C_o = \frac{C}{C_{ol}}$$

$$\epsilon_r \text{ from bridge measurements} = \frac{tC}{A\epsilon_o}$$

$$E_o = \frac{V_o}{t}$$

ii) Constant values

$$r = 10 \text{ mm} \quad R_m = 10^{11} \text{ Ohms} \quad C_m = 22 \text{ pf} \quad C_L = 10 \text{ pf}$$

$$\ln(5.4/3.9) = .326$$

$$\ln(4.1/3.1) = .278$$

$$\ln(2.7/2.2) = .207$$

$$\text{Temperature } 25^\circ\text{C}$$

$$\text{Relative humidity: } 49-51\%$$

iii) Calculations, results, and graphs

SAMPLE 1

$$V_O = 2.7 \text{ Volts}$$

$$\text{At } V_2 = 1.45 \text{ Volts}$$

τ', τ, τ_0 are obtained from graphs

$$\tau' = (15 \text{ mm})(.02 \text{ Sec/mm}) = .30 \text{ Sec.}$$

$$\tau = (13.5 \text{ mm})(.02 \text{ Sec/mm}) = .27 \text{ Sec.}$$

$$\frac{\tau'}{\tau} = 1.11$$

$$C' = \frac{25 \text{ pF}}{1.11 - 1} = 227 \text{ pF}$$

$$C = C' - (C_m + C_L) = 227 - 32 = 195 \text{ pF}$$

$$V_O = 4.1 \text{ Volts}$$

$$\text{At } V_2 = 3.1 \text{ Volts}$$

$$\tau' = (9 \text{ mm})(.02 \text{ Sec/mm}) = .18 \text{ Sec.}$$

$$\tau = (8 \text{ mm})(.02 \text{ Sec/mm}) = .16 \text{ Sec.}$$

$$\frac{\tau'}{\tau} = 1.125$$

$$C' = \frac{27 \text{ pF}}{1.125 - 1} = 216 \text{ pF}$$

$$C = 216 - 32 = 184 \text{ pF}$$

$$V_O = 5.4 \text{ Volts}$$

$$\text{At } V_2 = 3.9 \text{ Volts}$$

$$\tau' = (11 \text{ mm})(.02 \text{ Sec/mm}) = .22$$

$$\tau = (10 \text{ mm})(.02 \text{ Sec/mm}) = .20$$

$$\frac{\tau'}{\tau} = 1.1$$

$$C' = \frac{25 \text{ pF}}{1.1 - 1} = 250 \text{ pF}$$

$$C = 250 - 32 = 218 \text{ pF}$$

Similarly values of (C) are obtained for other initial voltages. They are tabulated in tables in this section.

$$V_o = 5.4 \text{ Volts} \quad \text{At } V_1 = 3.9 \text{ Volts}$$

$$\tau_o = (96 \text{ mm})(.02 \text{ Sec/mm}) = 1.92 \text{ Sec}$$

$$C'_o = \frac{1.92 \cdot 10^{-11}}{.326} = 59 \text{ pF}$$

$$C_o = 59 - 32 = 27 \text{ pF}$$

$$V_o = 2.7 \text{ Volts} \quad \text{At } V_1 = 2.2 \text{ Volts}$$

$$\tau_o = (60 \text{ mm})(.02 \text{ Sec/mm}) = 1.20 \text{ Sec.}$$

$$C'_o = \frac{1.20}{.207} 10^{-11} = 58 \text{ pF}$$

$$C_o = 58 - 32 = 26 \text{ pF}$$

$$V_o = 5.4 \text{ Volts}$$

$$(\epsilon_r)_H = \frac{218}{26} = 8.39$$

$$(\epsilon_r)_L = \frac{218}{27} = 8.07$$

$$V_o = 4.1 \text{ Volts}$$

$$(\epsilon_r)_H = \frac{184}{26} = 7.09$$

$$(\epsilon_r)_L = \frac{184}{27} = 6.82$$

$$V_o = 2.7 \text{ Volts}$$

$$(\epsilon_r)_H = \frac{195}{26} = 7.50$$

$$(\epsilon_r)_L = \frac{195}{27} = 7.23$$

xx If at $V_o = 2.7 \text{ Volts}$ and $V_2 = 1.45$, $\tau' = (15.5 \text{ mm})(.02 \text{ Sec/mm}) = .31 \text{ Sec.}$
then $\frac{\tau'}{\tau} = 1.148$

$C' = \frac{25 \text{ pF}}{1.148 - 1} = 169 \text{ pF}$, $C = 169 - 32 = 137 \text{ pF}$ which is appreciably different than the value calculated previously.

Similarly the values for permittivities for all samples are calculated.
They are given in the following Tables.

SAMPLE 1

V_o Volts	V_2 Volts	τ Sec.	τ' Sec.	C_a pF	C' pF	C pF
2.7	1.45	.27	.30	25	227	195
4.1	3.1	.18	.16	27	216	184
5.4	3.9	.20	.22	25	250	218
32	22	.30	.40	73	219.5	187.5
45	35	.17	.23	75	214	182
V_o Volts	V_1 Volts	τ_o Sec.	C_o' pF	C_o pF	$(\epsilon_r)_H$	$(\epsilon_r)_L$
2.7	2.2	1.20	58	26	7.50	7.23
4.1	3.1	1.64	59	27	7.09	6.82
5.4	3.9	1.90	59	27	8.39	8.07
32	-	-	-	-	7.22	6.95
45	-	-	-	-	7.00	6.75

SAMPLE 2

V_o Volts	V_2 Volts	τ Sec.	τ' Sec.	C_a pF	C' pF	C pF
2.7	2.2	1.50	2.40	45	75	43
4.1	3.1	2.25	3.60	45	75	43
5.4	3.9	2.35	3.75	45	75	43
32	27	0.72	1.14	45	75	45
45	40	0.45	0.72	45	75	43
V_o Volts	V_1 Volts	τ_o Sec.	C_o' pF	C_o pF	$(\epsilon_r)_H$	$(\epsilon_r)_L$
2.7	2.2	0.86	41.5	9.5	4.67	4.43
4.1	3.1	1.16	41.7	9.7	4.67	4.43
5.4	3.9	1.34	41.2	9.2	4.67	4.43
32	-	-	-	-	4.89	4.64
45	-	-	-	-	4.67	4.43

SAMPLE 3

V_o Volts	V_2 Volts	τ Sec.	τ' Sec.	C_a pF	C' pF	C pF
2.7	2.2	.18	.27	40	80	48
4.1	3.1	.22	.33	40	80	48
5.4	3.9	.22	.33	40	80	48
32	22	.40	.62	41	74.5	42.5
45	35	.26	.40	41	76	44.5
V_o Volts	V_1 Volts	τ_o Sec.	C_o' pF	C_o pF	$(\epsilon_r)_H$	$(\epsilon_r)_L$
2.7	2.2	.84	40.5	8.5	6.00	5.34
4.1	3.1	1.14	41.0	9.0	6.00	5.34
5.4	3.9	1.30	40.0	8.0	6.00	5.34
32	-	-	-	-	5.32	4.73
45	-	-	-	-	5.57	4.95

SAMPLE 4

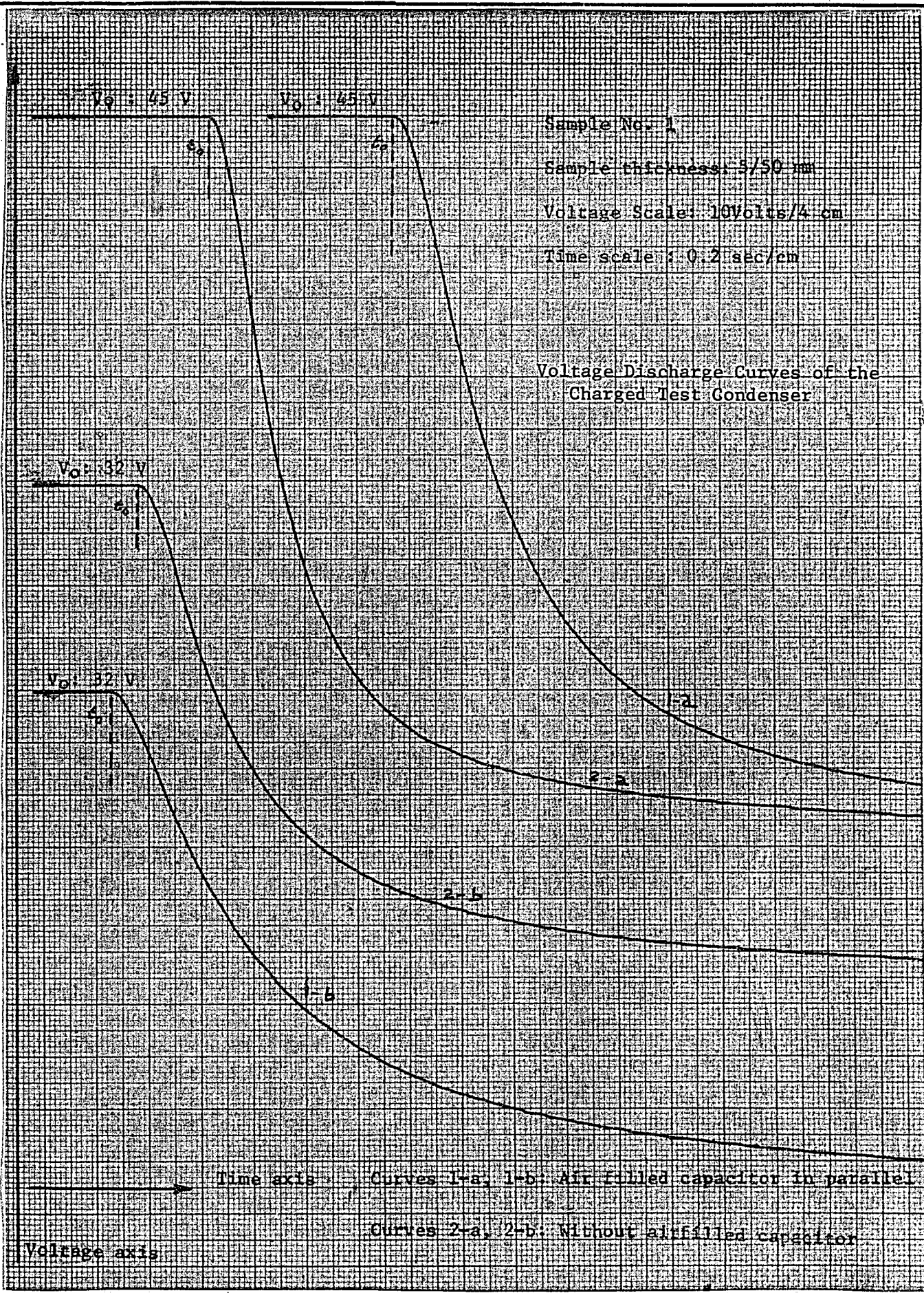
V_o Volts	V_2 Volts	τ Sec.	τ' Sec.	C_a pF	C' pF	C pF
2.7	2.2	1.45	2.35	45	72.5	40.5
4.1	3.1	2.08	3.25	43	76.4	44.5
5.4	3.9	2.10	3.40	45	72.5	40.5
32	27	.42	.67	43	72.3	40.3
45	40	.29	.46	43	73.5	41.5
V_o Volts	V_1 Volts	τ_o Sec.	C_o' pF	C_o pF	$(\epsilon_r)_H$	$(\epsilon_r)_L$
2.7	2.2	0.94	45.4	13.4	3.21	3.19
4.1	3.1	1.24	44.6	12.6	3.58	3.50
5.4	3.9	1.46	44.7	12.7	3.21	3.19
32	-	-	-	-	3.20	3.17
45	-	-	-	-	3.29	3.27

SAMPLE 5

V_o Volts	V_2 Volts	τ Sec.	τ' Sec.	C_a pF	C' pF	C pF
2.7	2.2	.19	.22	45	283	251
4.1	3.1	.19	.22	45	283	251
5.4	3.9	.21	.23	26	282	250
32	27	.13	.16	75	326	294
45	40	.16	.16	75	∞	∞
V_o Volts	V_1 Volts	τ_o Sec.	C'_o pF	C_o pF	$(\epsilon_r)_H$	$(\epsilon_r)_L$
2.7	2.2	1.56	75.5	43.5	5.77	5.62
4.1	3.1	2.10	75.5	43.5	5.77	5.62
5.4	3.9	2.50	76.6	44.6	5.75	5.60
32	-	-	-	-	6.75	6.60
45	-	-	-	-	-	-

SAMPLE 6

V_o Volts	V_2 Volts	τ Sec.	τ' Sec.	C_a pF	C' pF	C pF
2.7	2.2	.46	.66	92	211	179
4.1	3.1	.59	.84	91	214	182
5.4	3.9	.60	.86	92	212	180
32	27	.26	.37	90	214	182
45	35	.33	.46	90	227	195
V_o Volts	V_1 Volts	τ_o Sec.	C'_o pF	C_o pF	$(\epsilon_r)_H$	$(\epsilon_r)_L$
2.7	2.2	1.56	75.5	43.5	4.12	4.00
4.1	3.1	2.10	75.5	43.5	4.18	4.07
5.4	3.9	2.50	76.6	44.6	4.14	4.03
32	-	-	-	-	4.18	4.07
45	-	-	-	-	4.50	4.36



Sample No. 1

Sample thickness: 3/50 mm

Voltage Scale: 10Volts/4 cm

Time scale : 0.2 sec/cm

Voltage Discharge Curves of the
Charged Test Condenser

$V_0 = 45 \text{ V}$

$V_0 = 45 \text{ V}$

$V_0 = 32 \text{ V}$

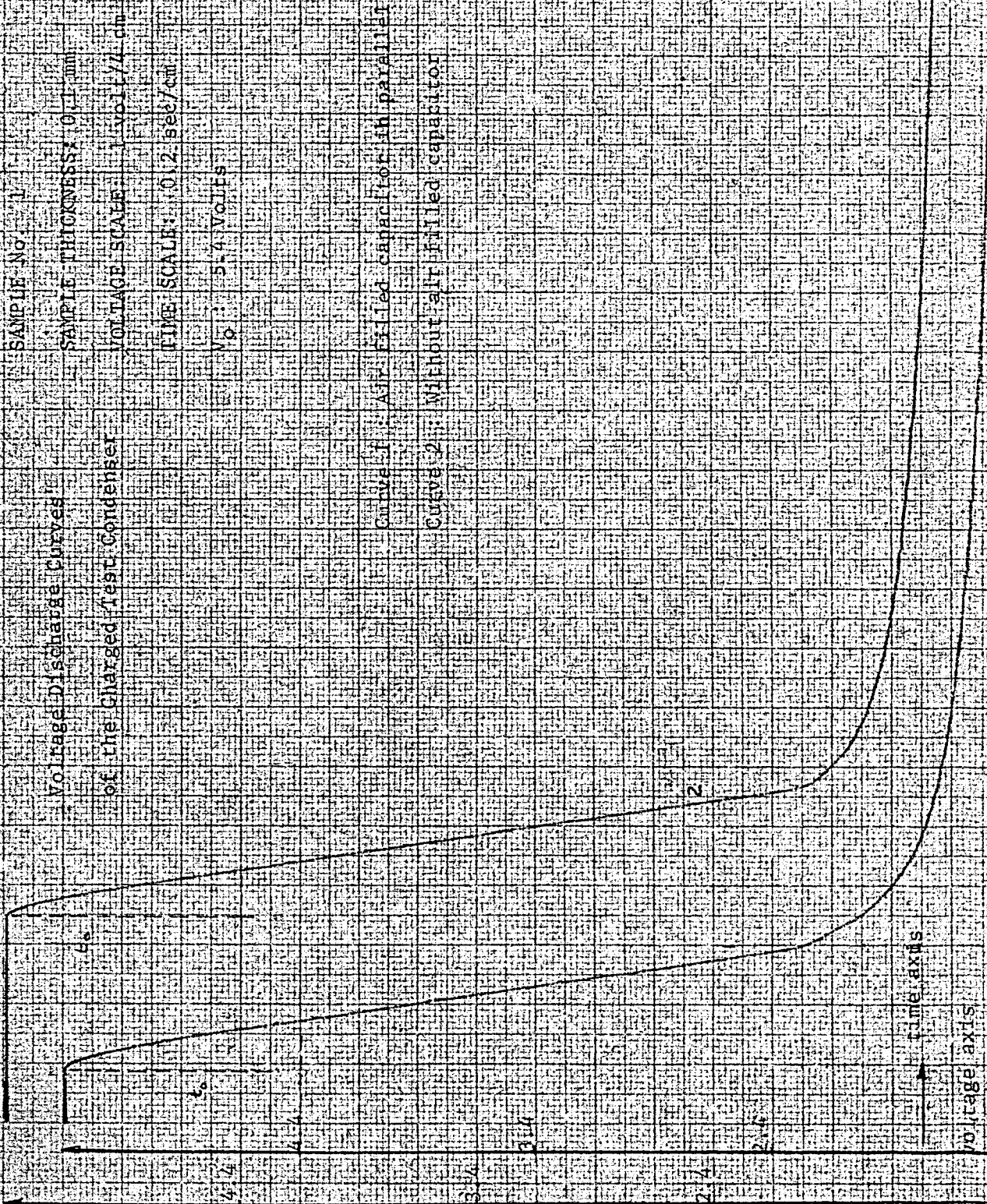
$V_0 = 32 \text{ V}$

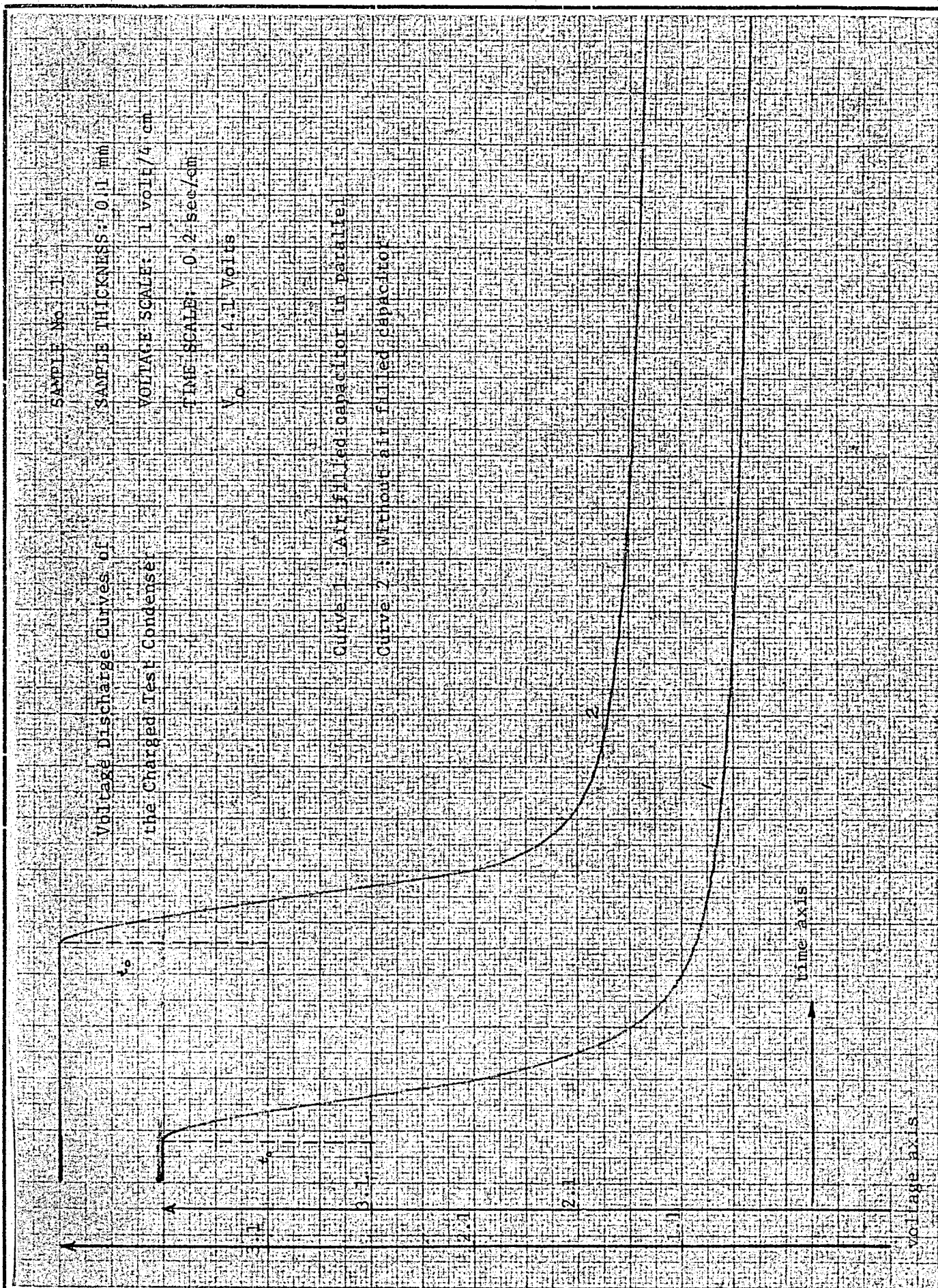
Time axis

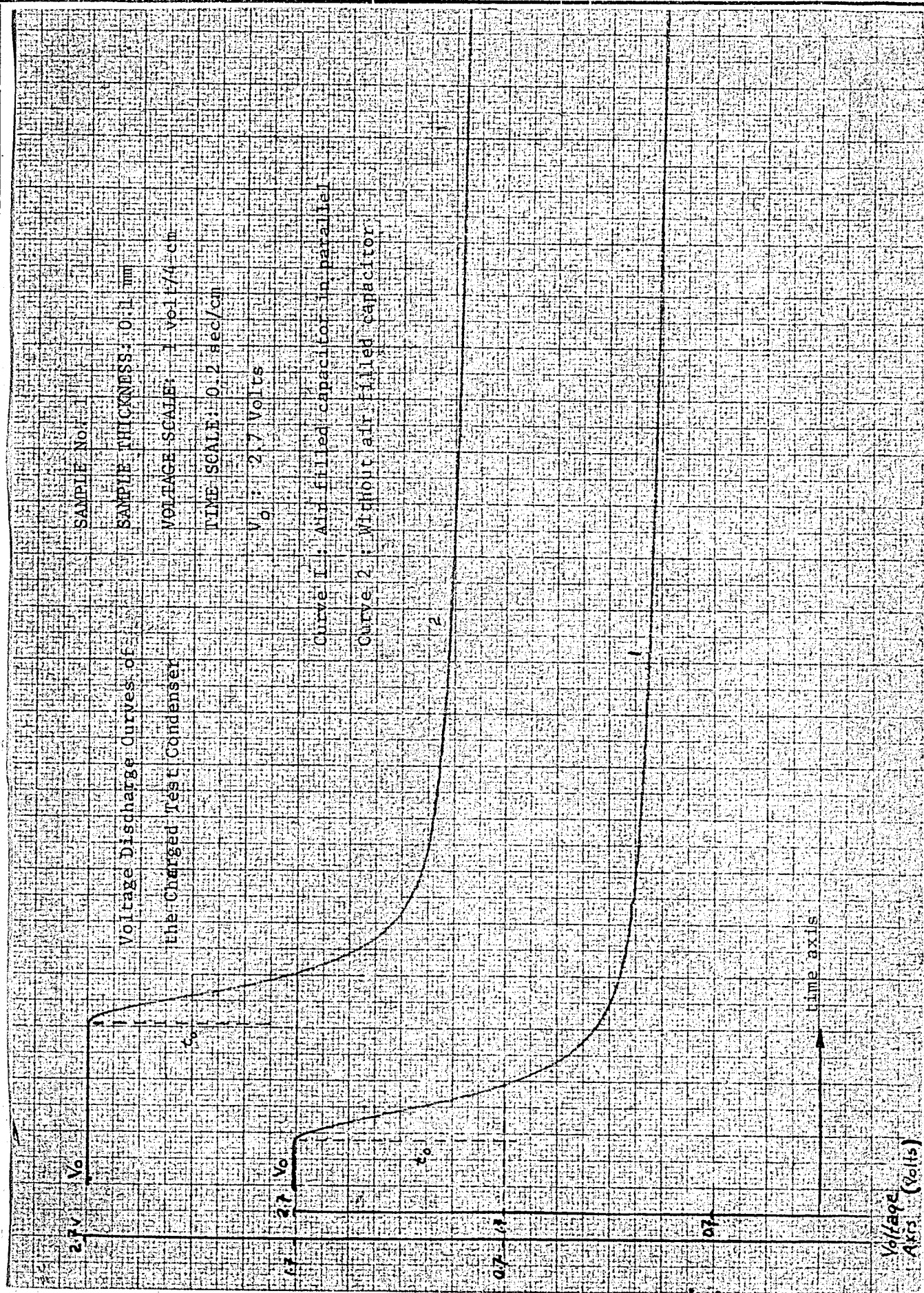
Curves 1-a, 1-b: Air filled capacitor in parallel

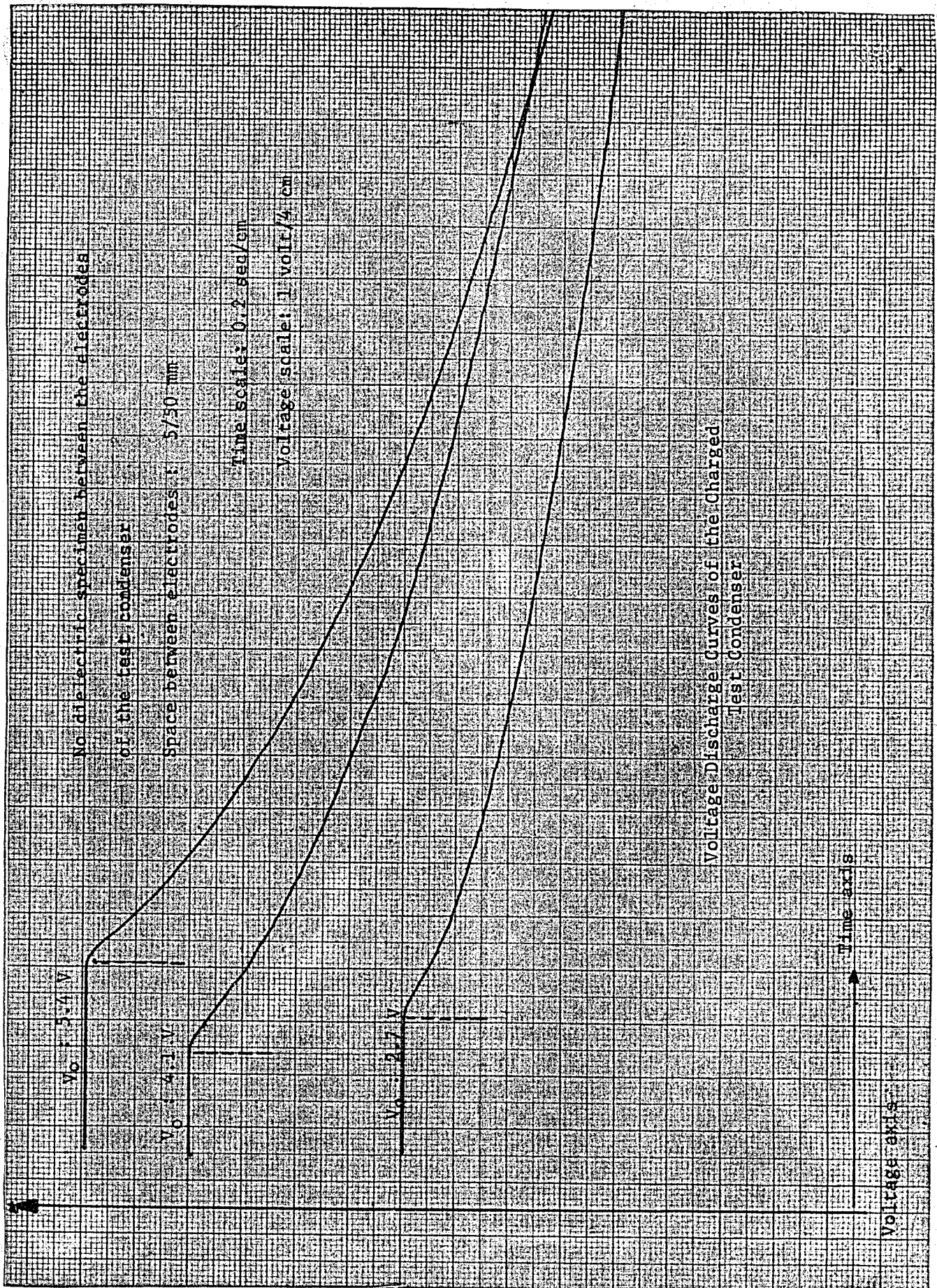
Curves 2-a, 2-b: Without airfilled capacitor

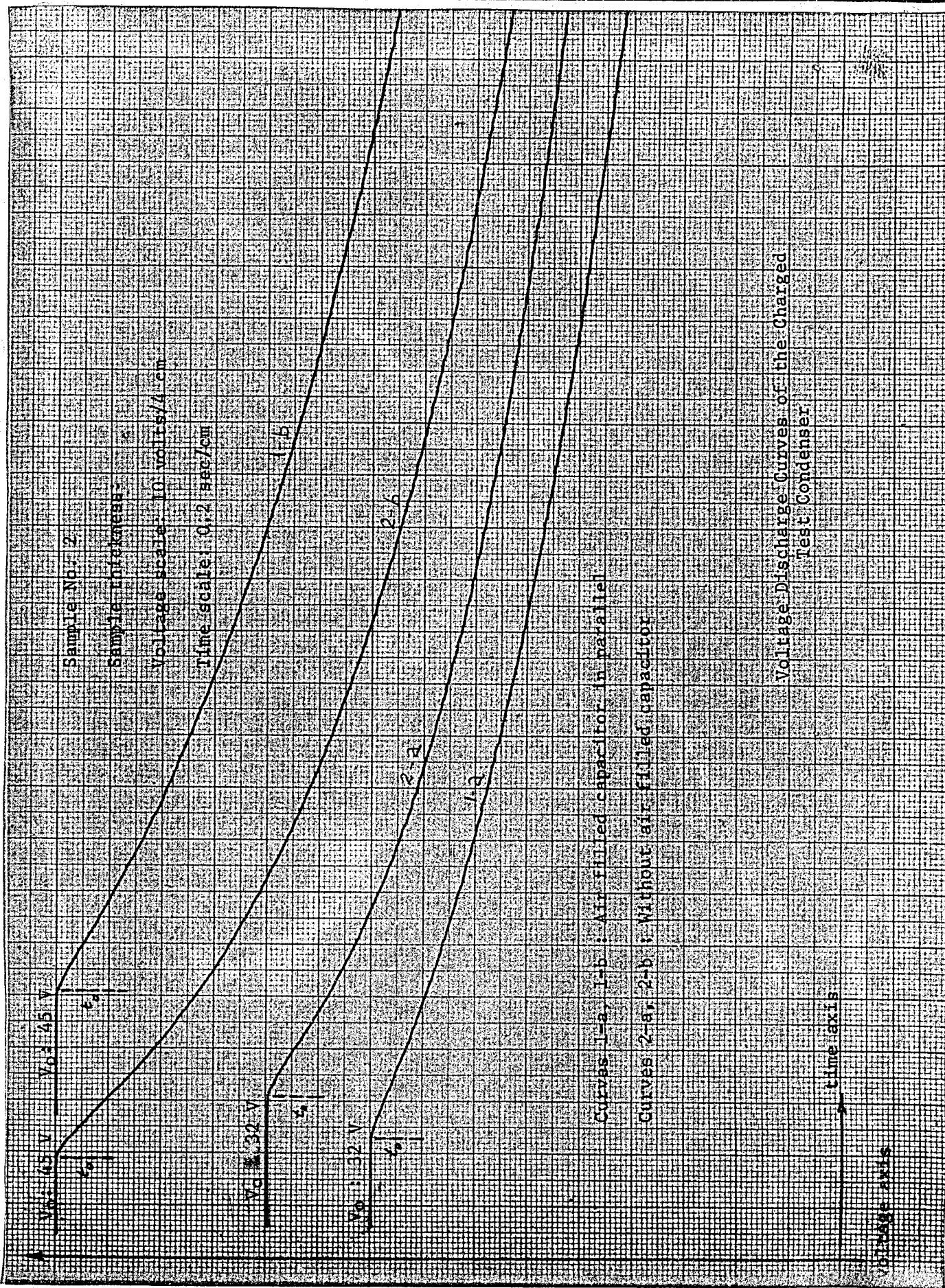
Voltage axis

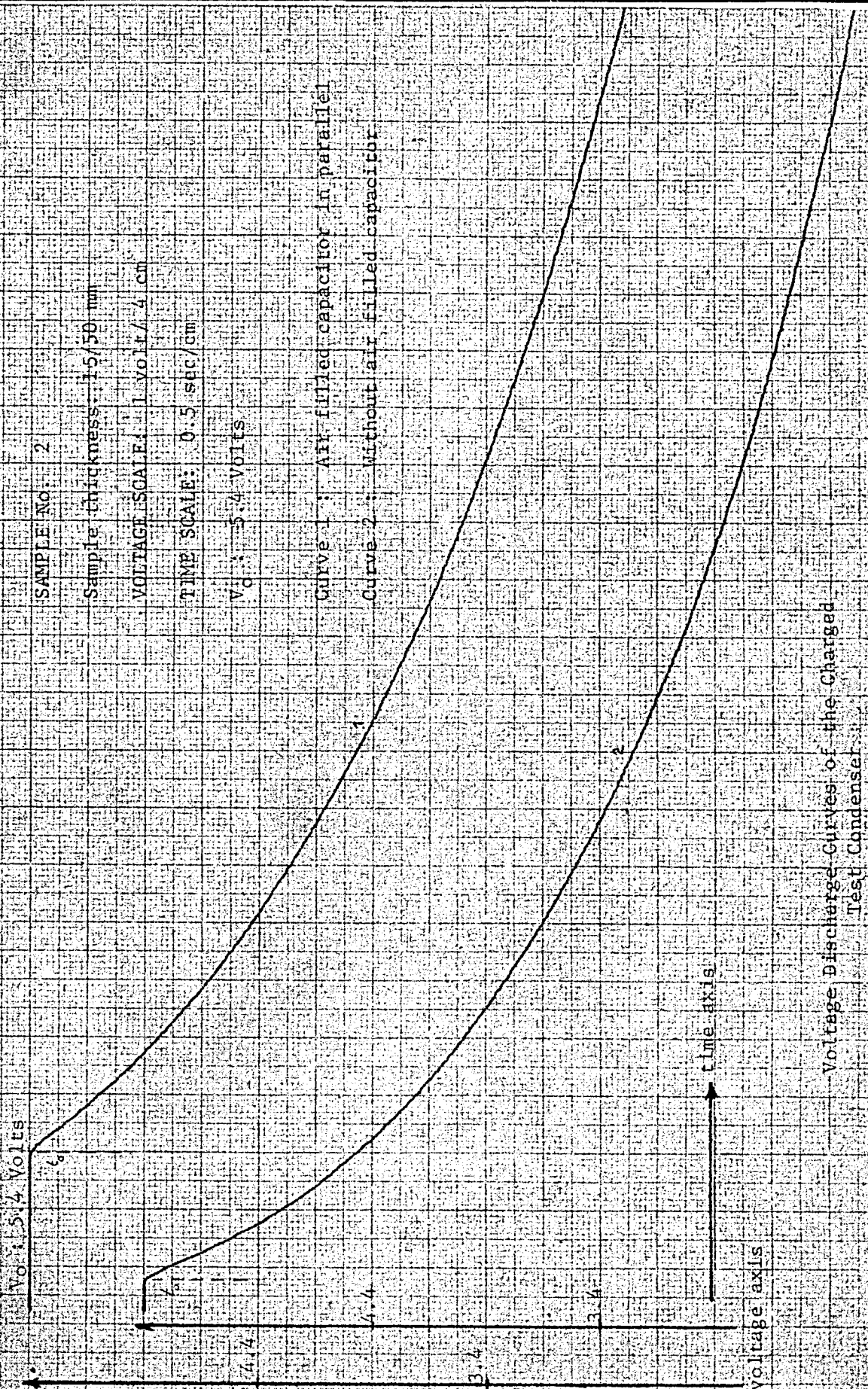












SAMPLE No. 2
SAMPLE THICKNESS: 15/50 mm
VOLTAGE SCALE: 1 volt/4 cm

TIME SCALE: 0.5 sec/cm

$V_0 = 4.1$ Volts

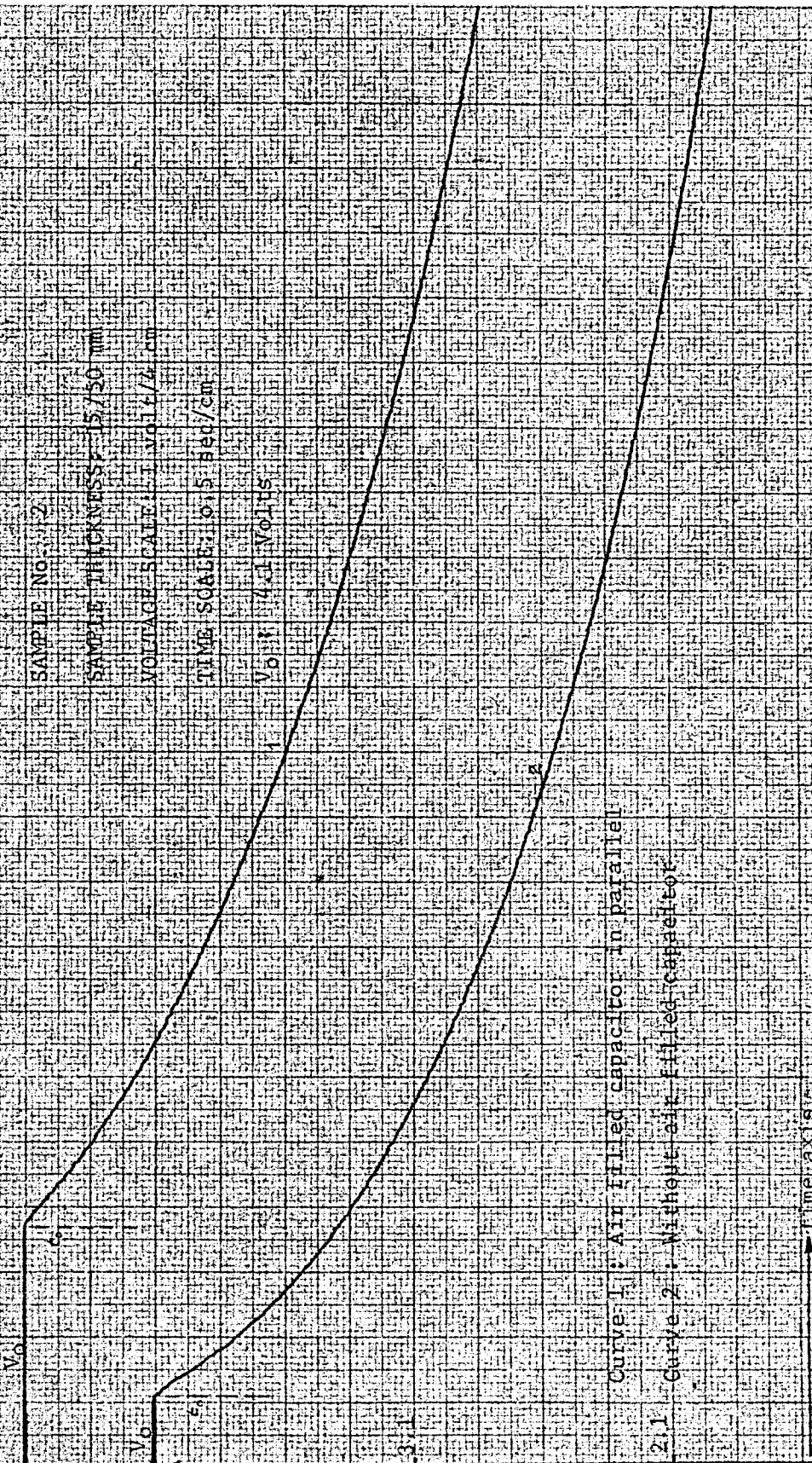
Curve 1: Air filled capacitor in parallel

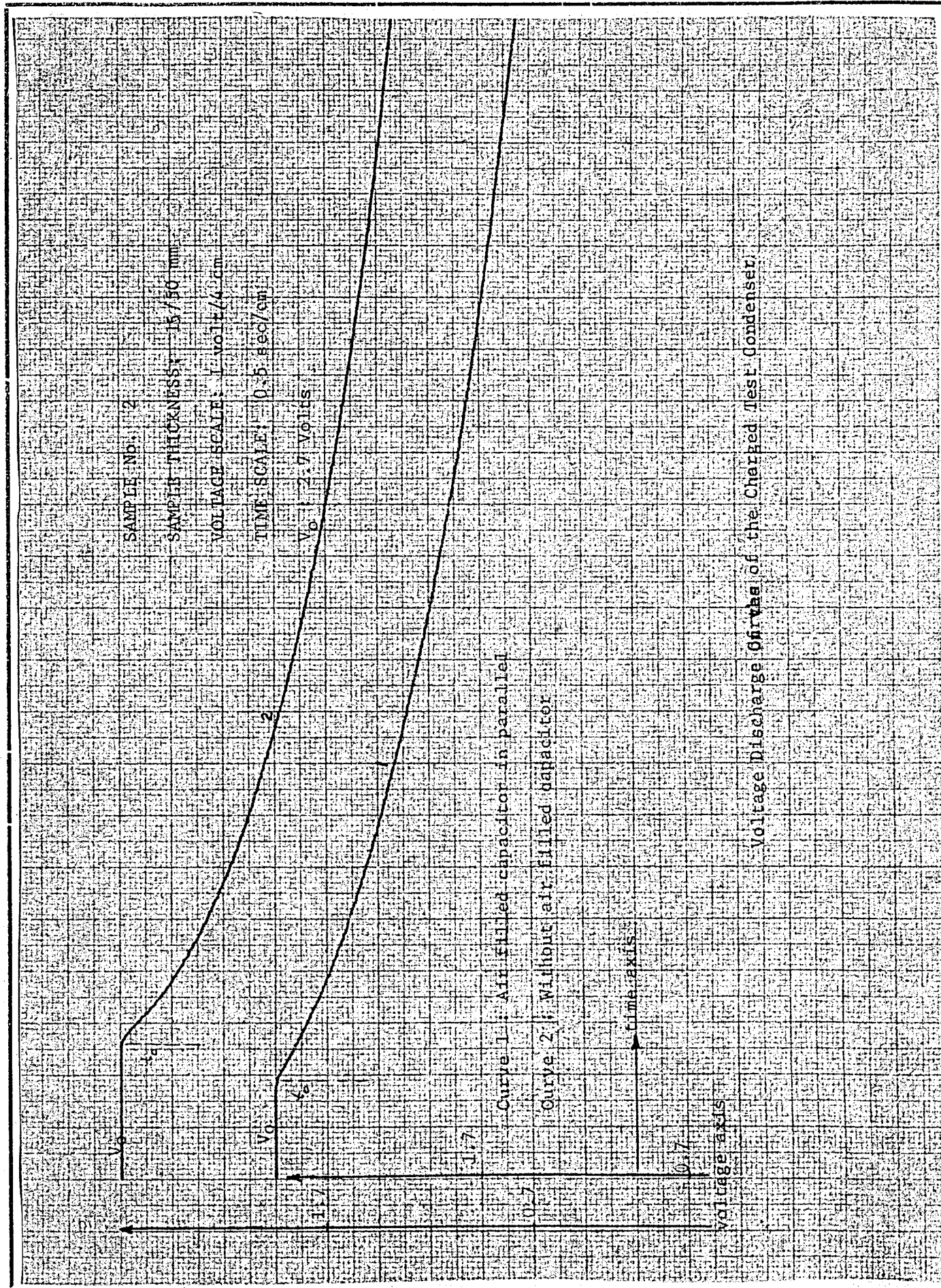
Curve 2: Without air filled capacitor

time axis

Voltage Discharge Curves of the Charged Test Condenser

voltage axis





Voltage Discharge Curves of the Charged Test Condenser

No dielectric specimen between the electrodes
of the test condenser

Space between electrodes: 15/50 mm

Time scale: 0.2 sec/cm

Voltage scale: 1 volt/4 cm

$V_0 = 5.4 \text{ V}$

t_0

$V_0 = 4.1 \text{ V}$

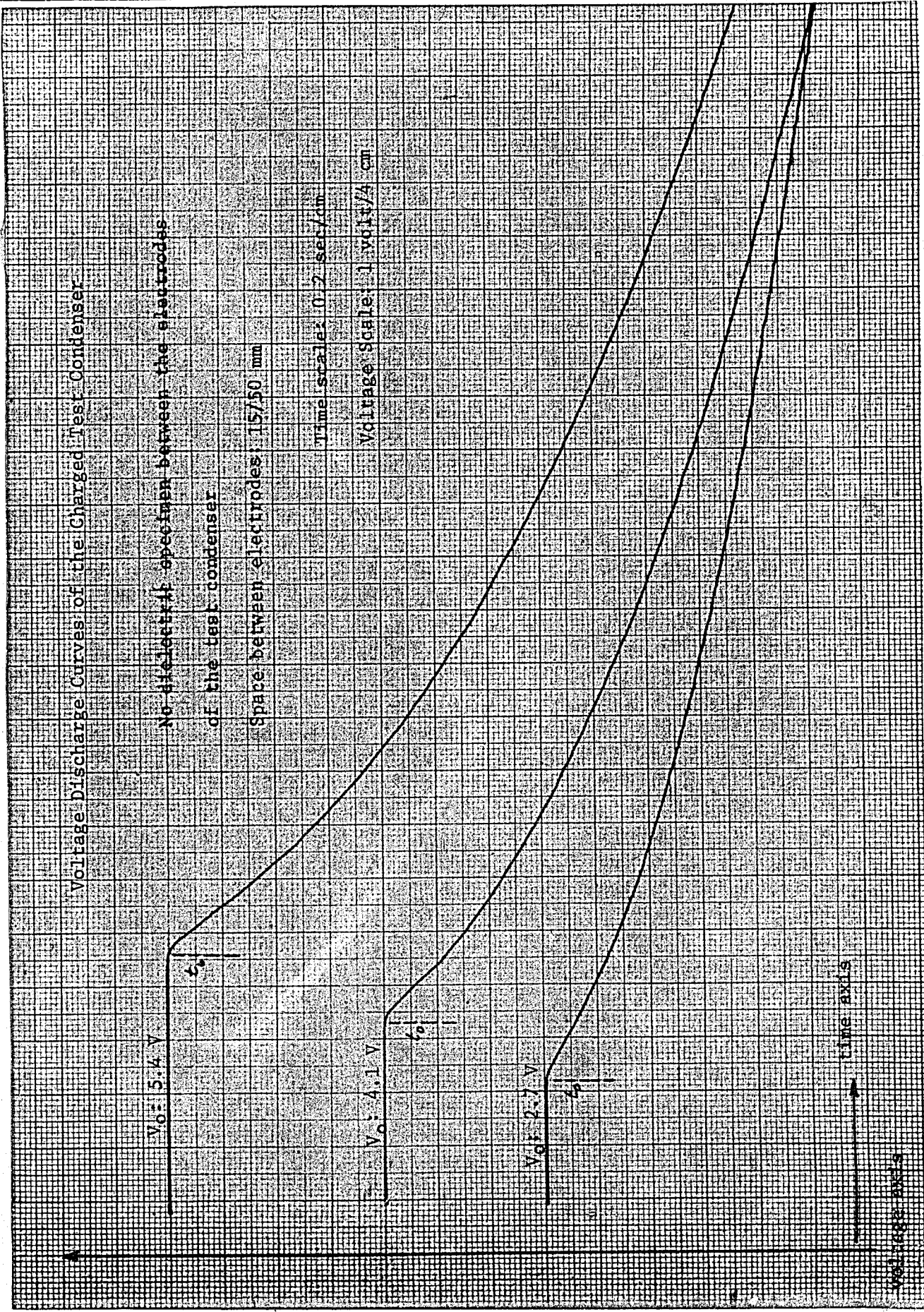
t_0

$V_0 = 2.7 \text{ V}$

t_0

time axis

voltage axis



THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE B-1

Voltage Discharge Curves of the Charged Test Condenser

$V_0 = 45 \text{ V}$

$V_0 = 45 \text{ V}$

Sample No. 3

Thickness: 18/50 mm

Voltage scale: 10 volts/cm

Time scale: 0.2 sec/cm

$V_0 = 32 \text{ V}$

$V_0 = 32 \text{ V}$

Curves 1-a, 1-b : Air filled capacitor in parallel

Curves 2-a, 2-b : Without air filled capacitor

Time axis

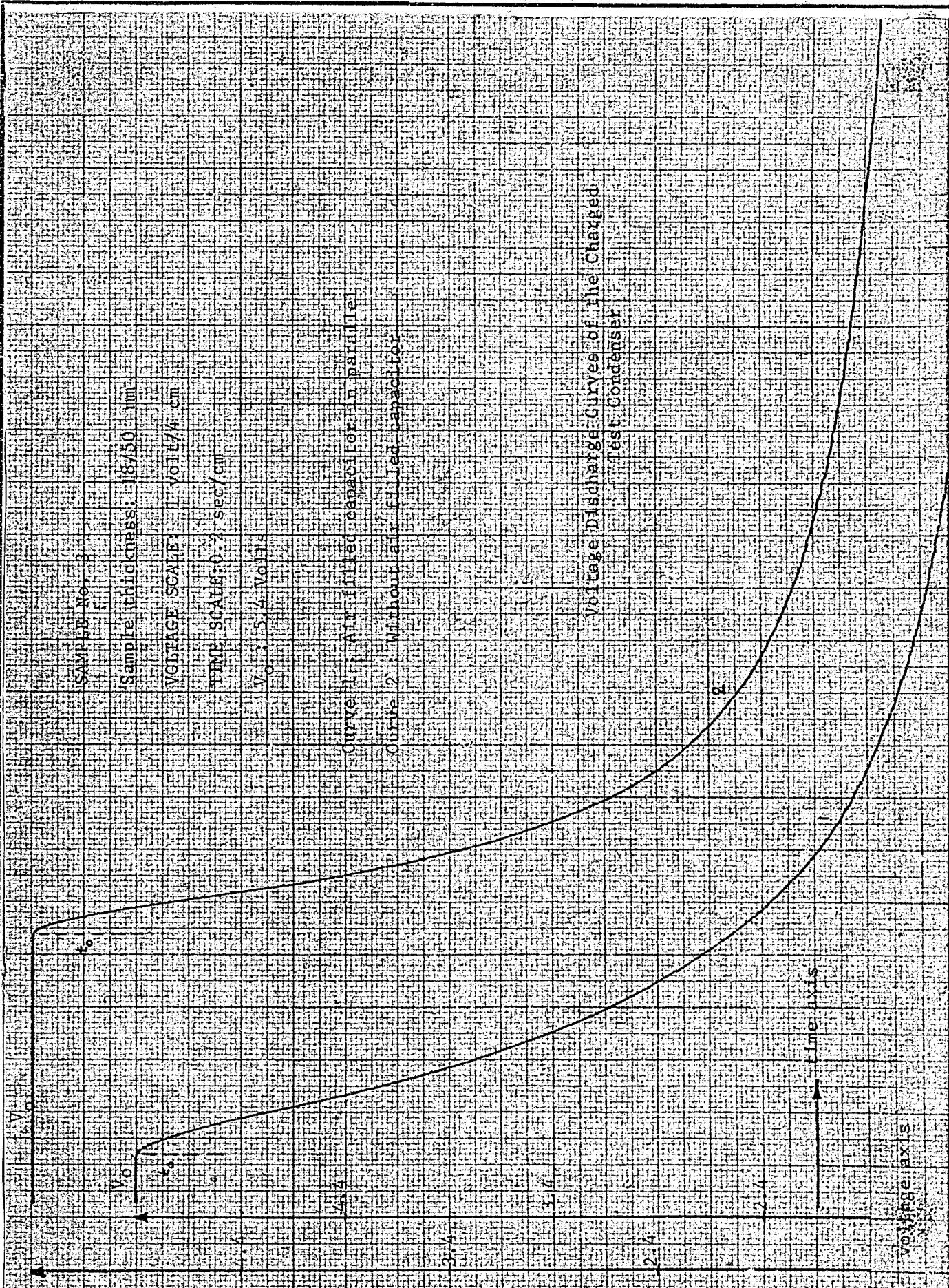
Voltage axis

THESIS

ROBERT COLLEGE GRADUATE SCHOOL

BEBEK, ISTANBUL

PAGE B-



THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE B-2

SAMPLE No. 3

Sample thickness: 18/50 mm

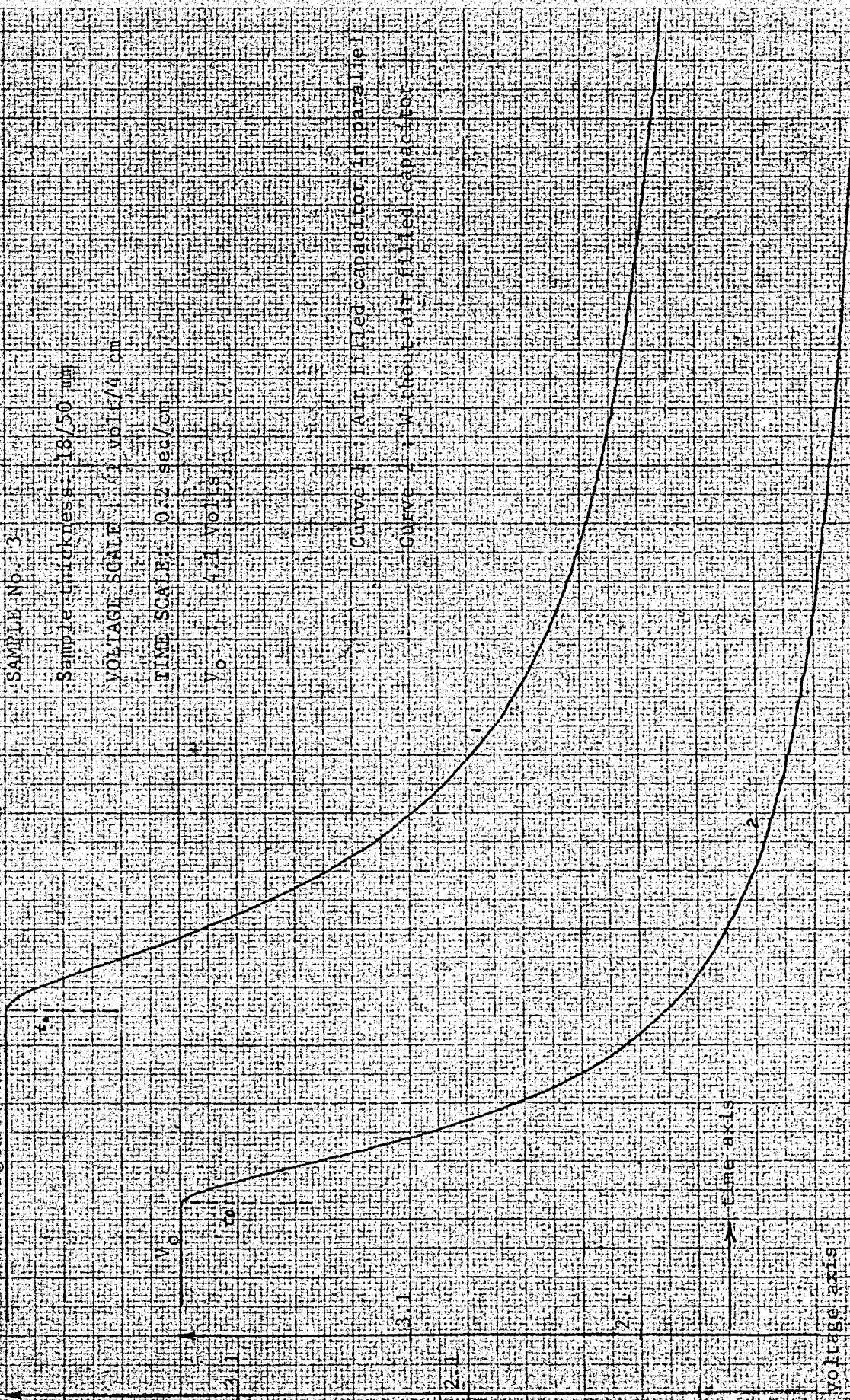
VOLTAGE SCALE: 1.2 volt/cm

TIME SCALE: 0.2 sec/cm

V_0 : 7.1 Volts

Curve 1: Air filled capacitor in parallel

Curve 2: Without air filled capacitor



Voltage Discharge Curves of the Charged
Test Condenser

SAMPLE NO. 3

Sample thickness: 18/50 mm

VOLTAGE SCALE: 1 volt/4 cm

TIME SCALE: 0.2 sec/cm

V_0 3.7 Volts

Curve 1 Air-filled capacitor in parallel

Curve 2 Without air-filled capacitor

Voltage Discharge Curves of the Charged Test Condenser

V_0

V_0

V_0

1.7

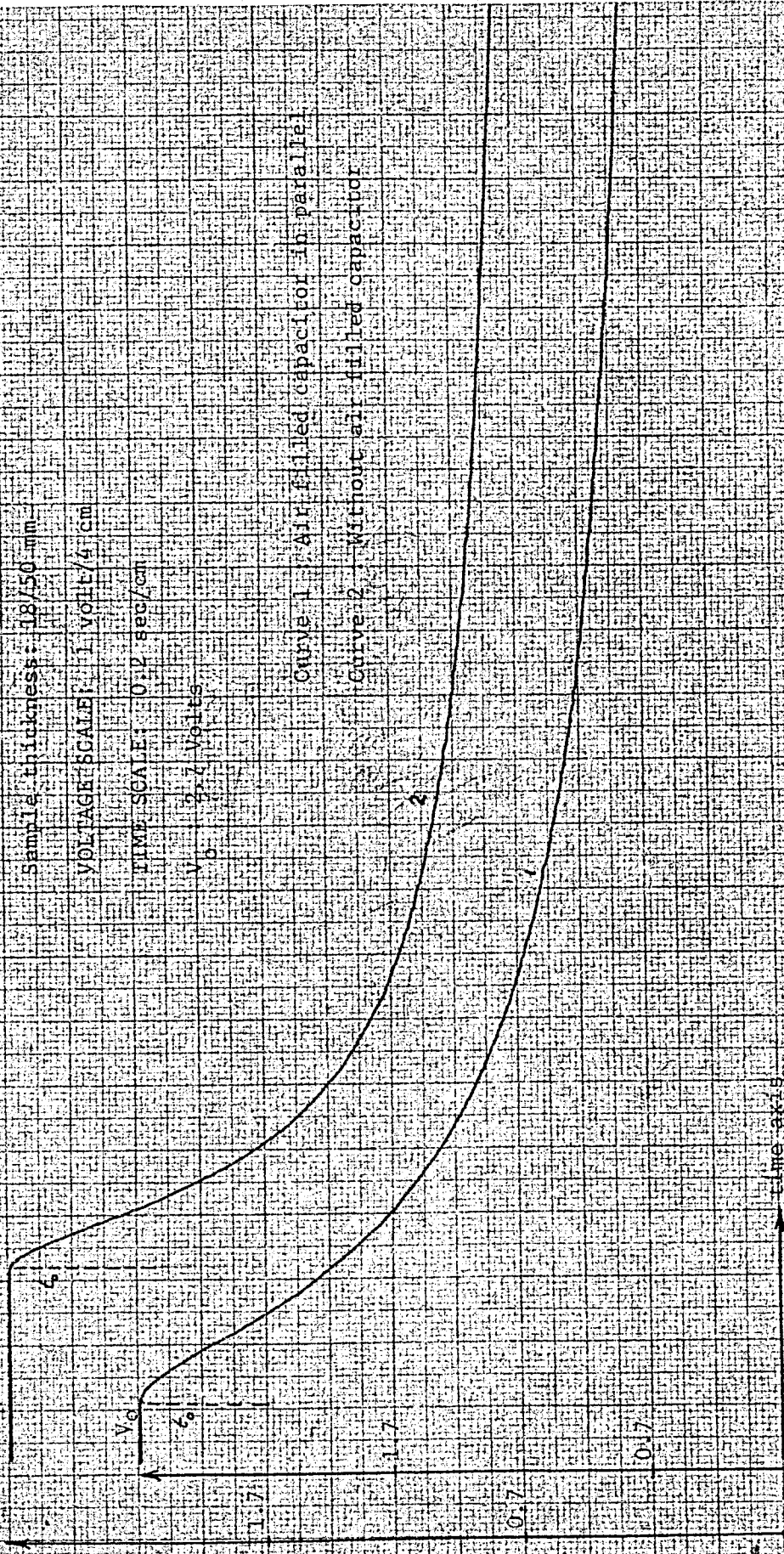
1.7

0.7

0.7

time axis

voltage axis



Voltage Discharge Curves of the Charged Test Condenser

No dielectric specimen between the electrodes of the

Test condenser

Space between electrodes: 18/50 mm

$V_0 = 5.4 \text{ V}$

t_0

$V_0 = 4.1 \text{ V}$

t_0

$V_0 = 2.7 \text{ V}$

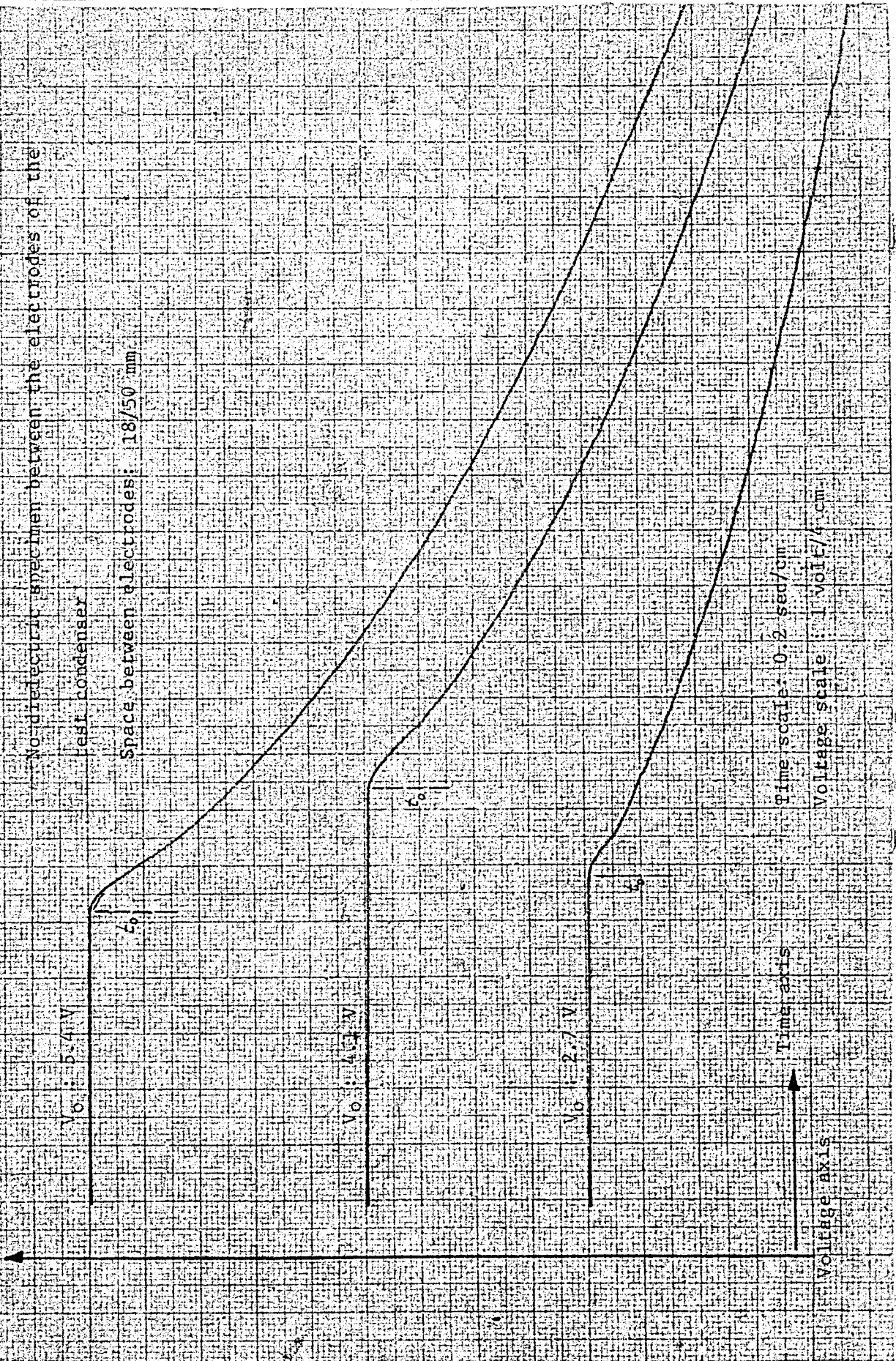
t_0

Time scale: 0.2 sec/cm

Voltage scale: 1 volt/cm

Time axis

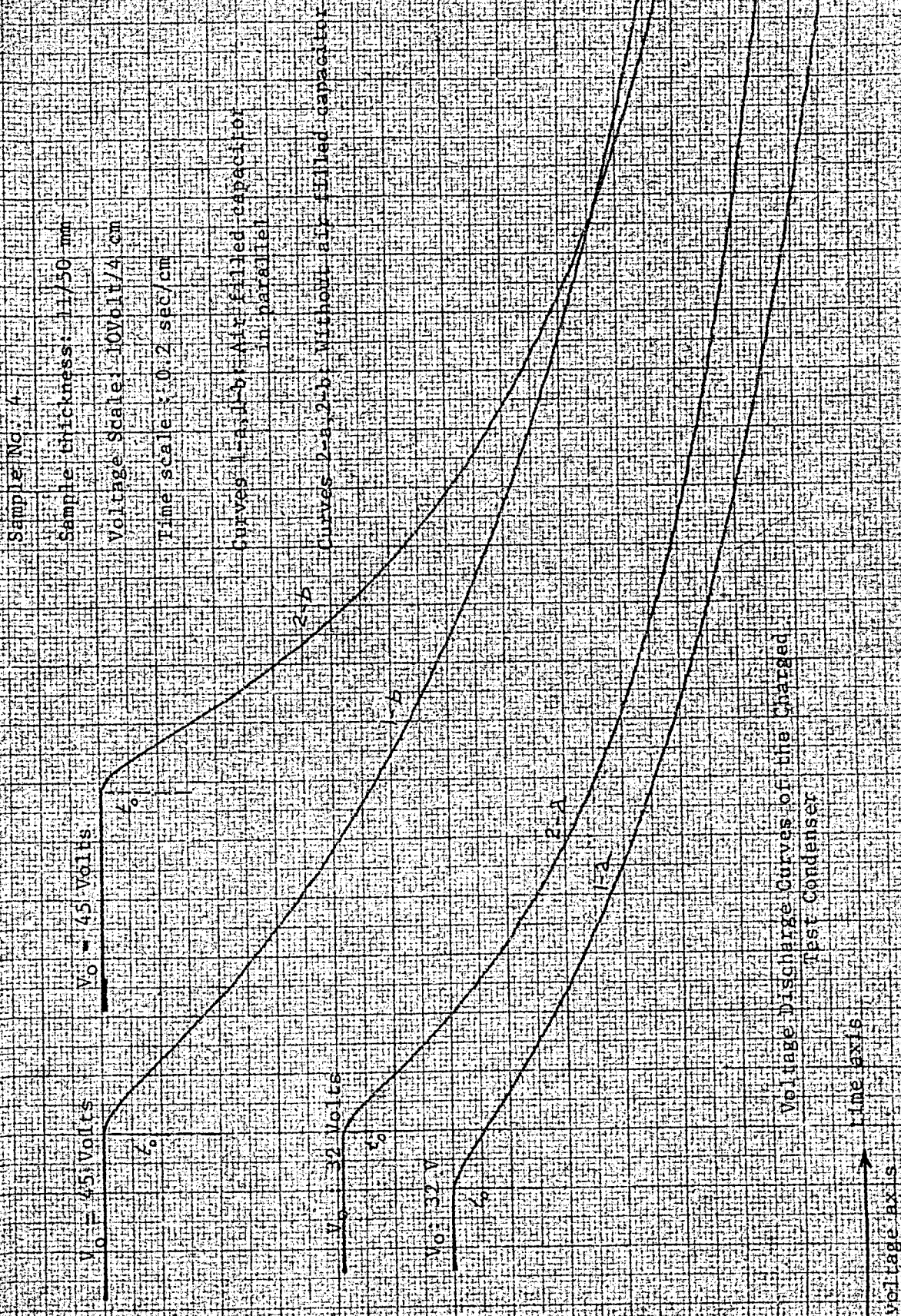
Voltage axis



THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

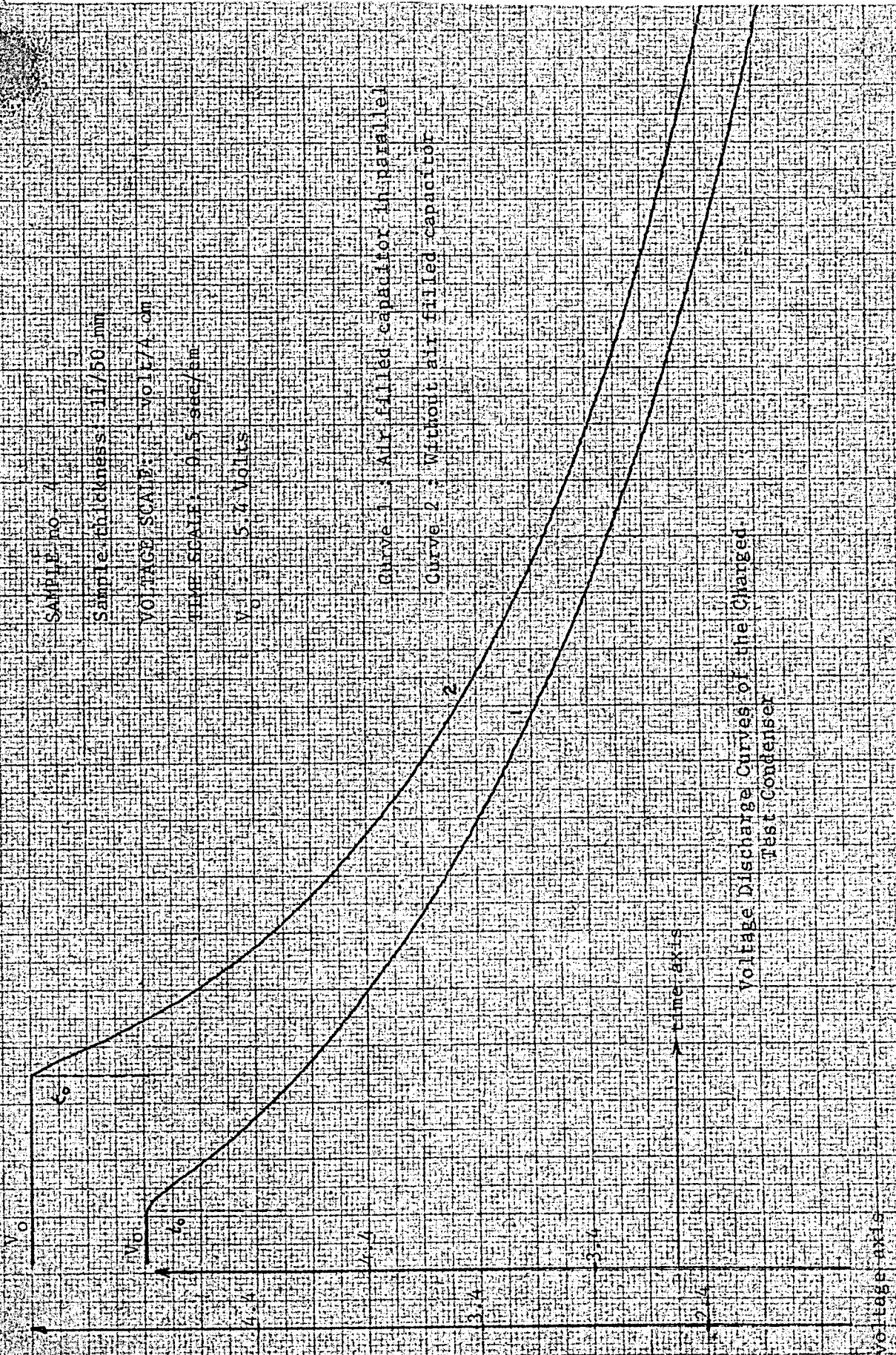
PAGE B-23



THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE B-2



SAMPLE NO. 7

SAMPLE thickness: 11/50 mm

VOLTAGE SCALE: 1 volt/cm

TIME SCALE: 0.5 sec/cm

V₀ = 5.0 Volts

Curve 1: Air filled capacitor in parallel

Curve 2: Without air filled capacitor

Voltage Discharge Curves of the Charged

Test Condenser

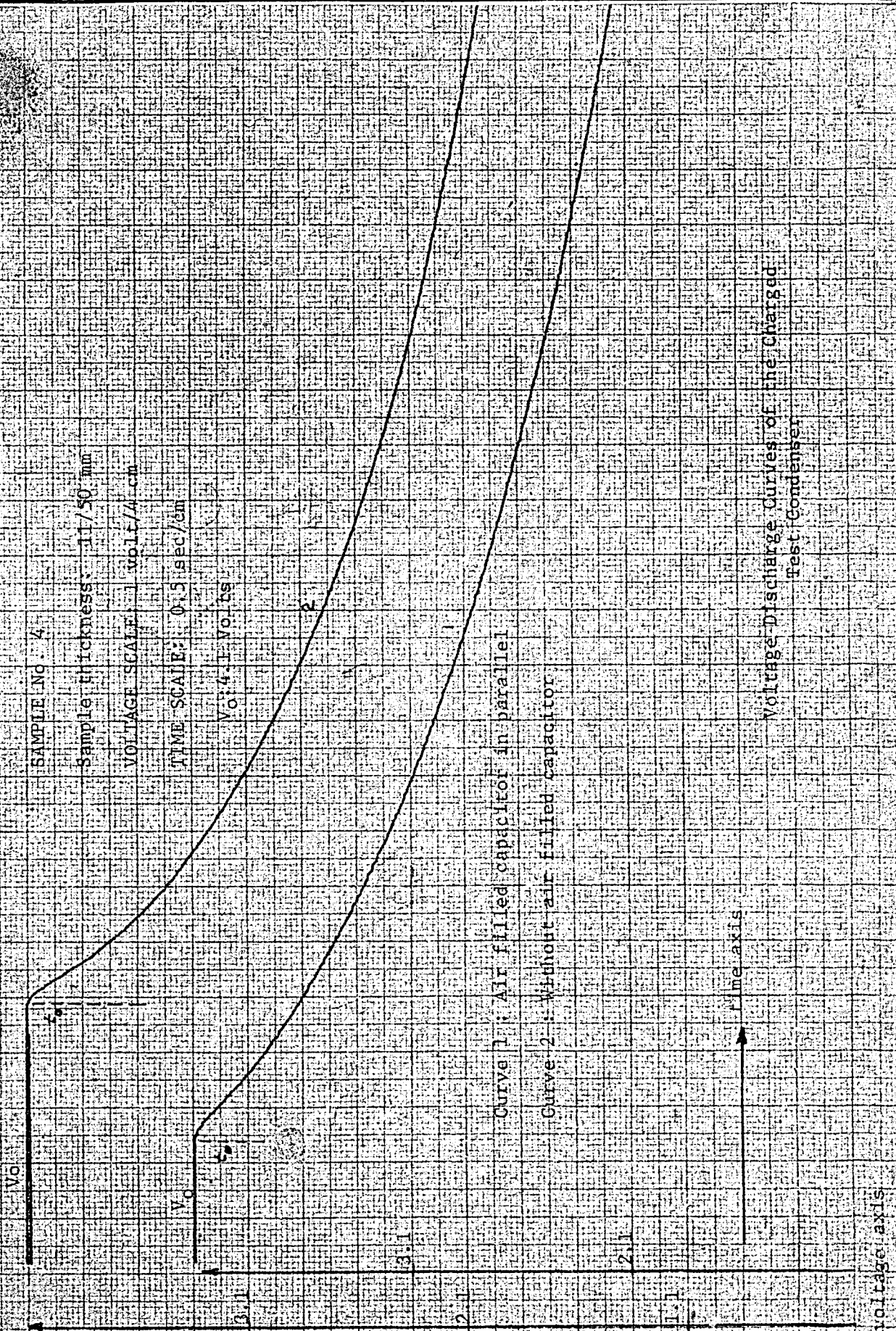
Voltage axis

time axis

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE B-2



THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE B-

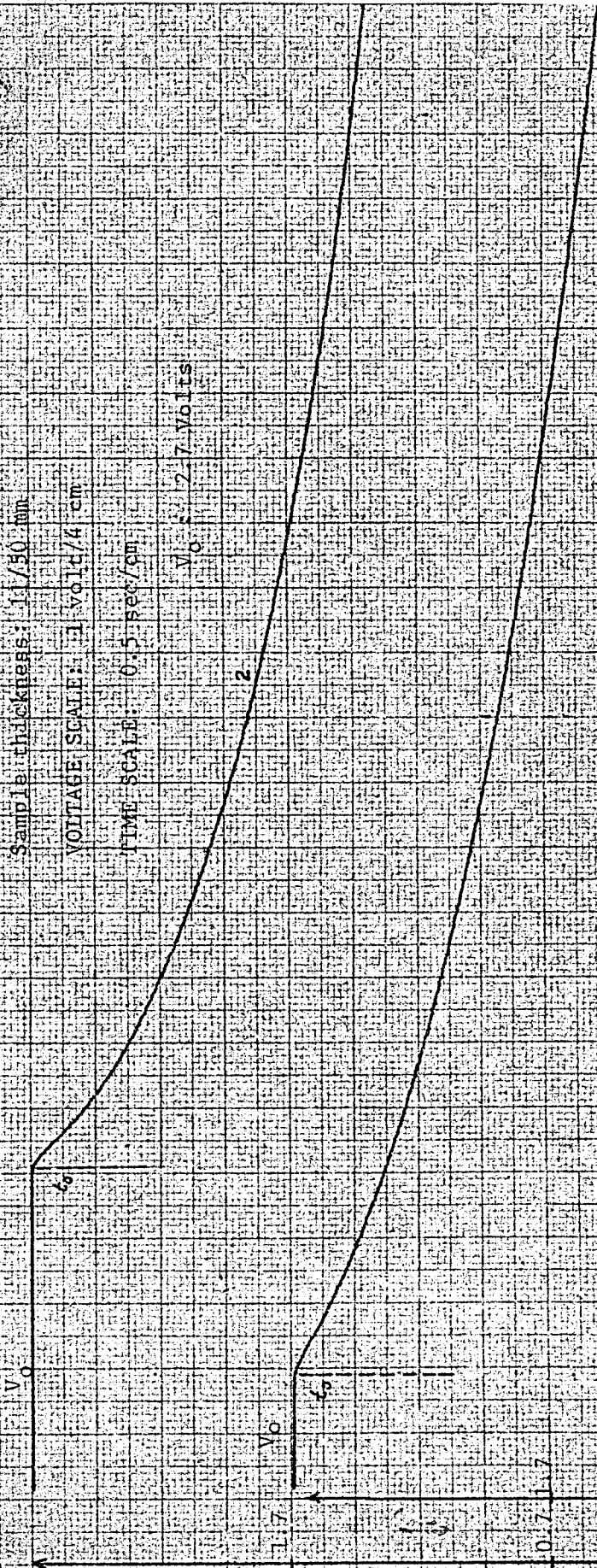
SAMPLE No. 4

Sample thickness: 1.1/50 mm

VOLTAGE SCALE: 1 Volt/cm

TIME SCALE: 0.5 sec/cm

$V_0 = 2.7$ Volts



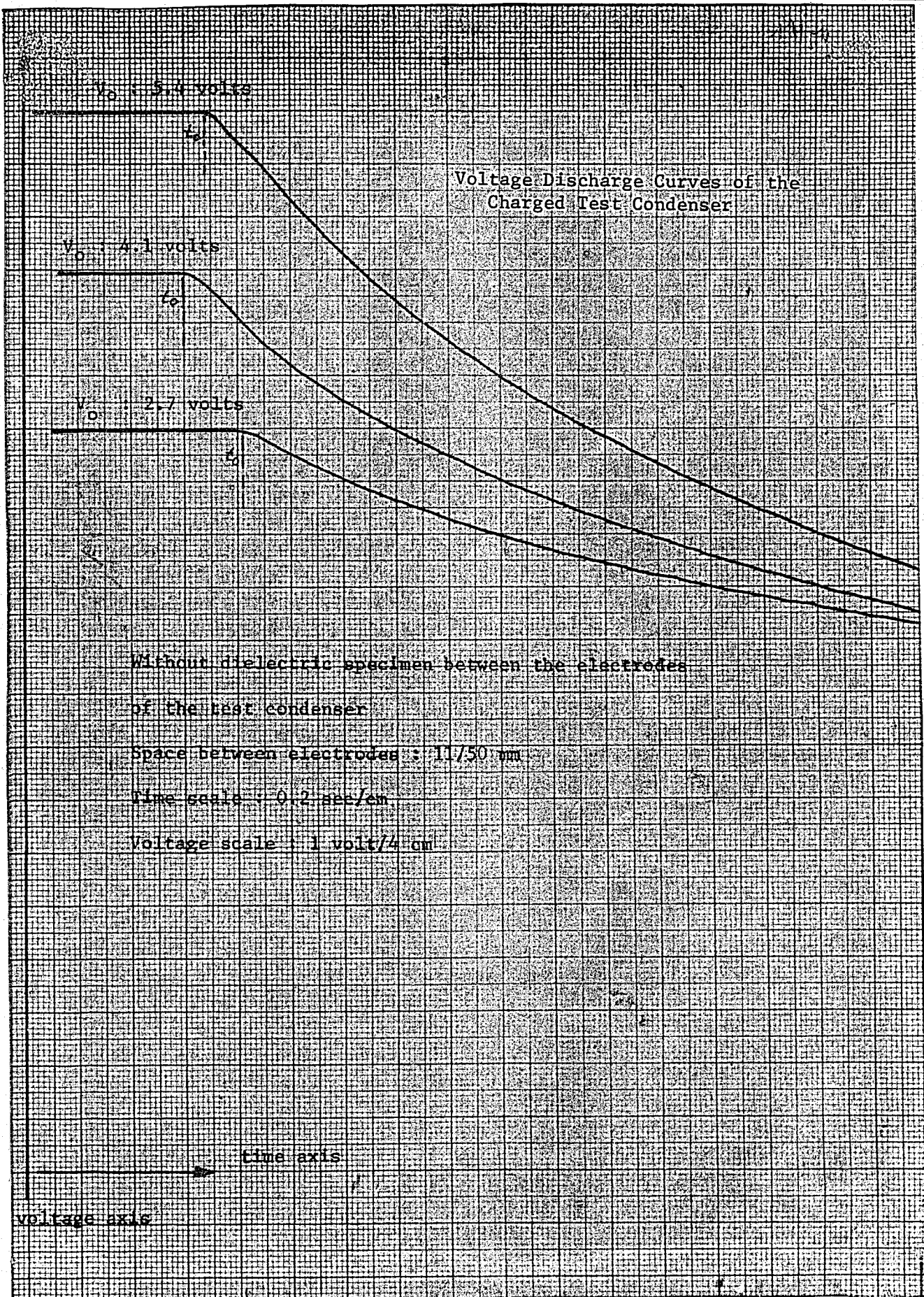
Curve 1 : Air filled capacitor in parallel

Curve 2 : without air-filled capacitor

Voltage Discharge curves of the Charged
Test Condenser

voltage axis

time axis



THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE B-2

V₀: 45 V V₀: 45 V Voltage Discharge Curves of
the Charged Test Condenser

Sample No. 5

Sample thickness: 3/50 mm

Voltage scale: 10 Volts/4 cm

Time scale: 0.2 sec/cm

V₀: 32 V

t₀

V₀: 32 V

t₀

1-b

2-b

2-a

1-a

Curves 1-a, 1-b: Air filled capacitor in parallel

Curves 2-a, 2-b: Without air filled capacitor

time axis

voltage axis

SAMPLE No. 5

Sample thickness: 5/50 mm

VOLTAGE SCALE: 1 volt/7.5 cm

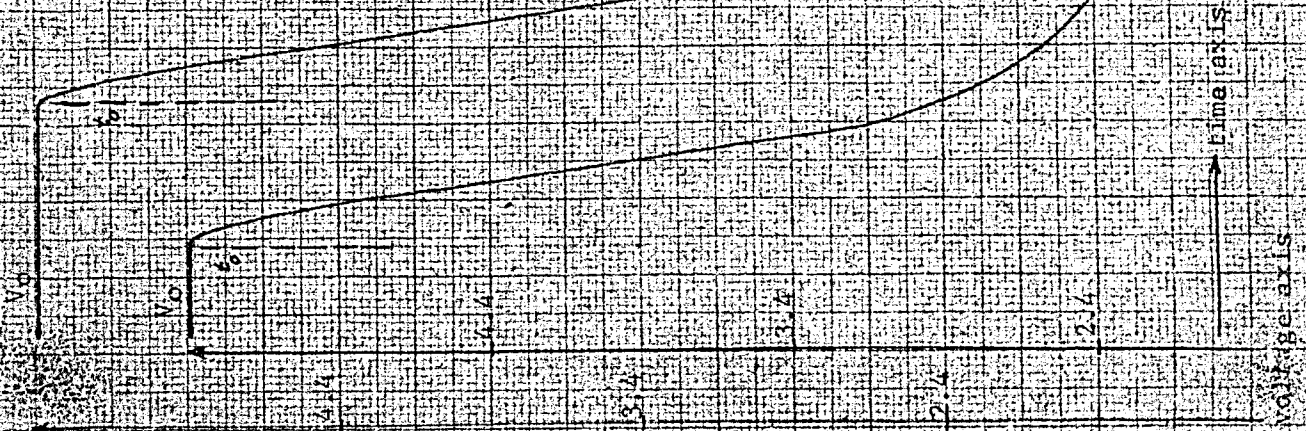
TIME SCALE: 0.2 sec/cm

V_0 5.4 Volts

Curve 1 : Air filled capacitor in parallel

Curve 2 : Without air filled capacitor

Voltage Discharge Curves of the Charged
Test Condenser

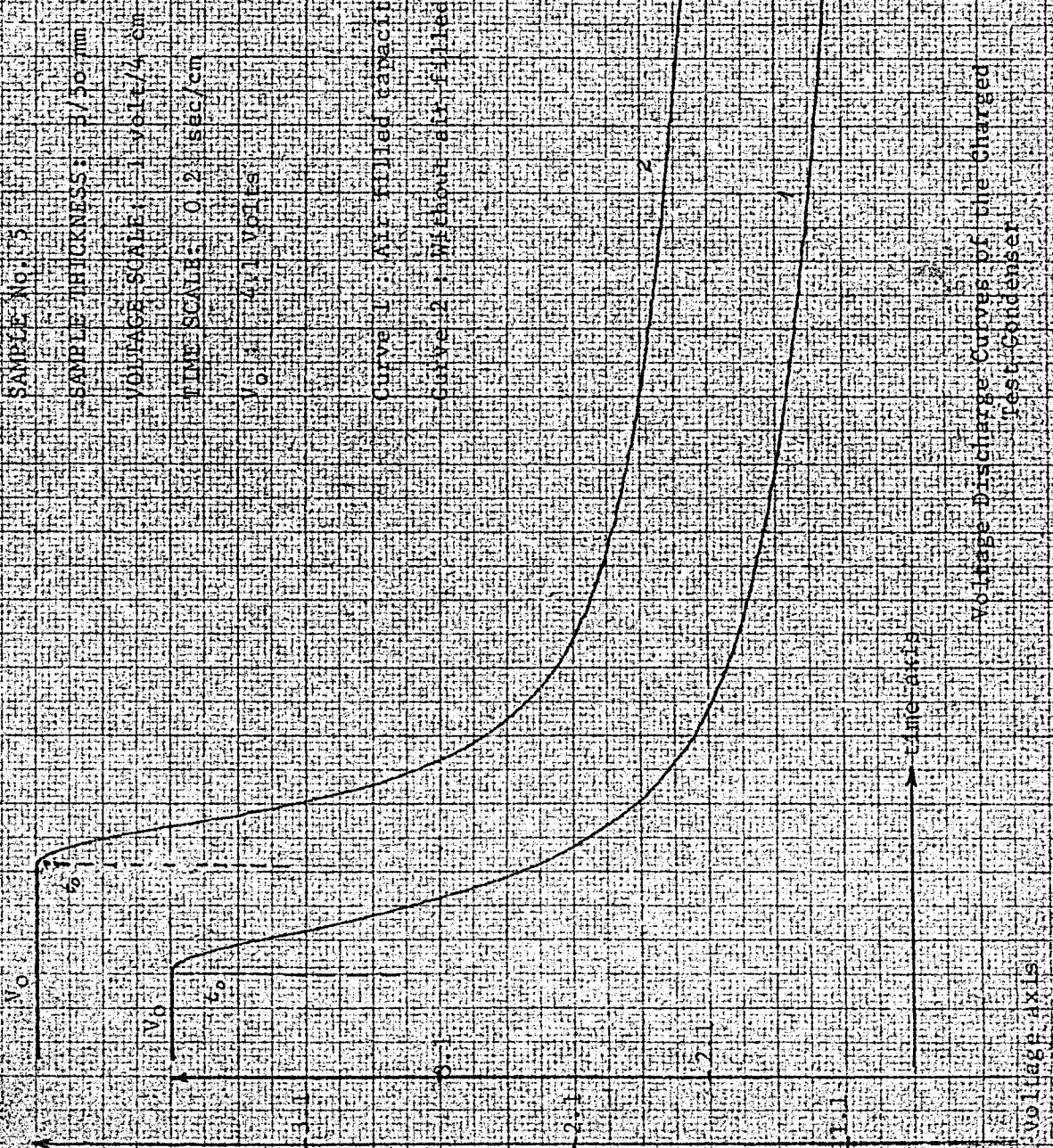


THESIS

ROBERT COLLEGE GRADUATE SCHOOL

BEBEK, ISTANBUL

PAGE B-30



SAMPLE No: 15

SAMPLE THICKNESS: 5/50 mm

VOLTAGE SCALE: 1 volt/4 cm

TIME SCALE: 0.2 sec/cm

$V_0 = 4.1$ Volts

Curve 1: Air filled capacitor in parallel

Curve 2: Without air-filled capacitor

Voltage Discharge Curves of the Charged Test Condenser

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE B-3

SAMPLE No. 5
Sample Thickness: 0.50 mm
VOLTAGE SCALE: 1 volt/4 cm

TIME SCALE: 0.2 sec/cm

V_0 2.7 Volts

Curve 1: Air filled capacitor in parallel

Curve 2: Without air filled capacitor

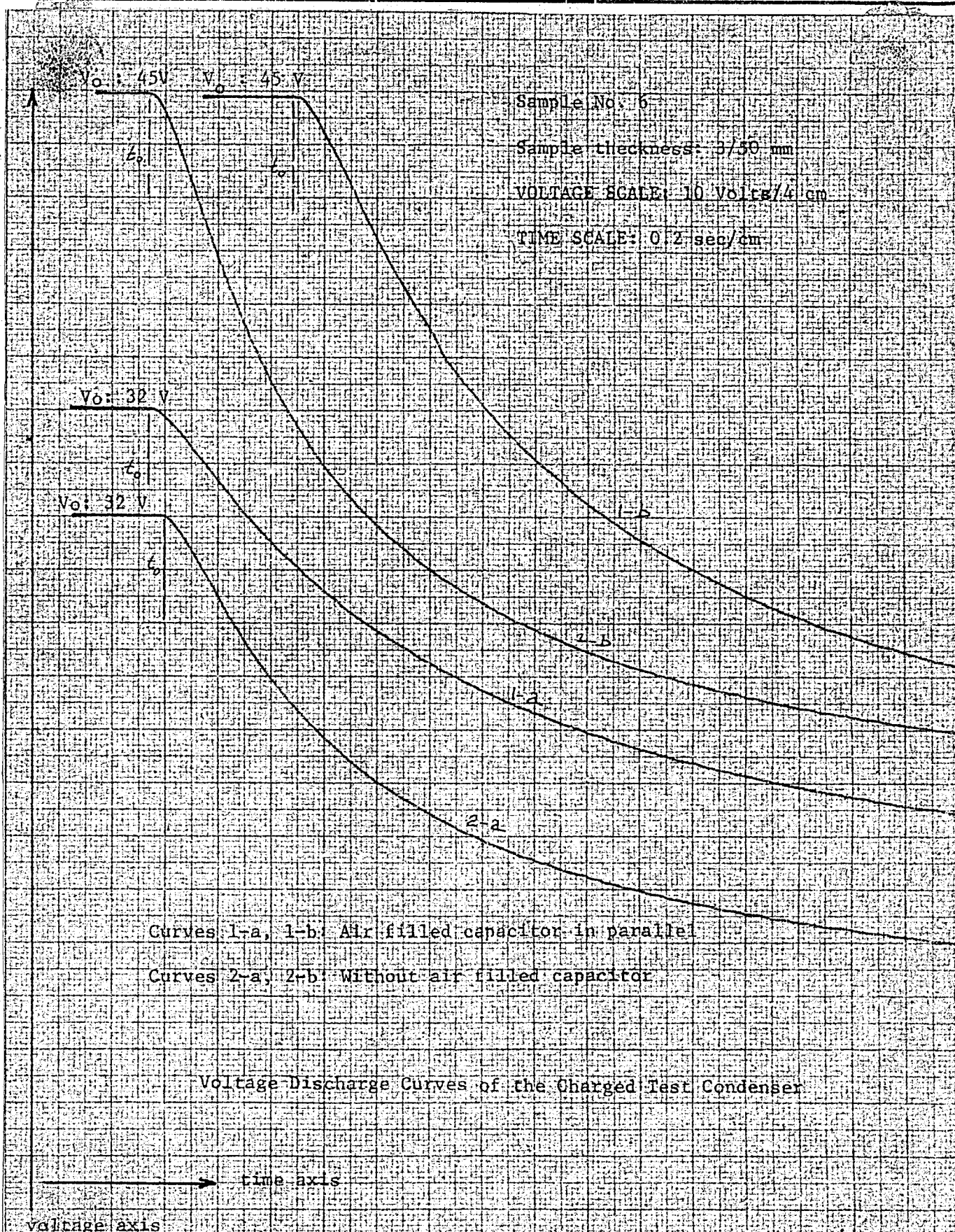
Voltage Discharge Curves of the Charged
Test Condenser



THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE B-3



THESIS

ROBERT COLLEGE GRADUATE SCHOOL

BEBEK, ISTANBUL

PAGE B-33

SAMPLE No. 6

Sample Thickness: 3/50 mm

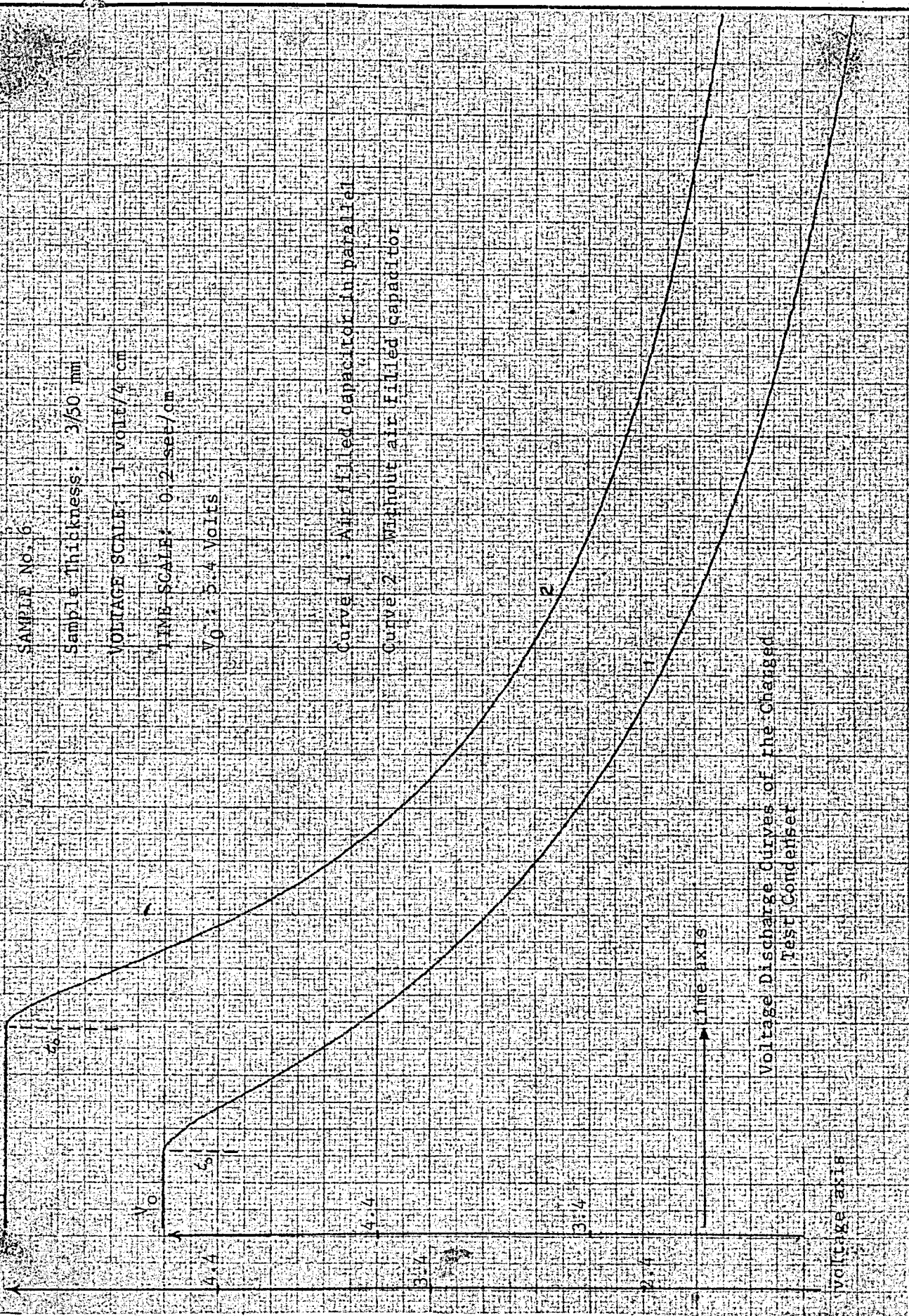
VOLTAGE SCALE: 1 volt/4 cm

TIME SCALE: 0.2 sec/cm

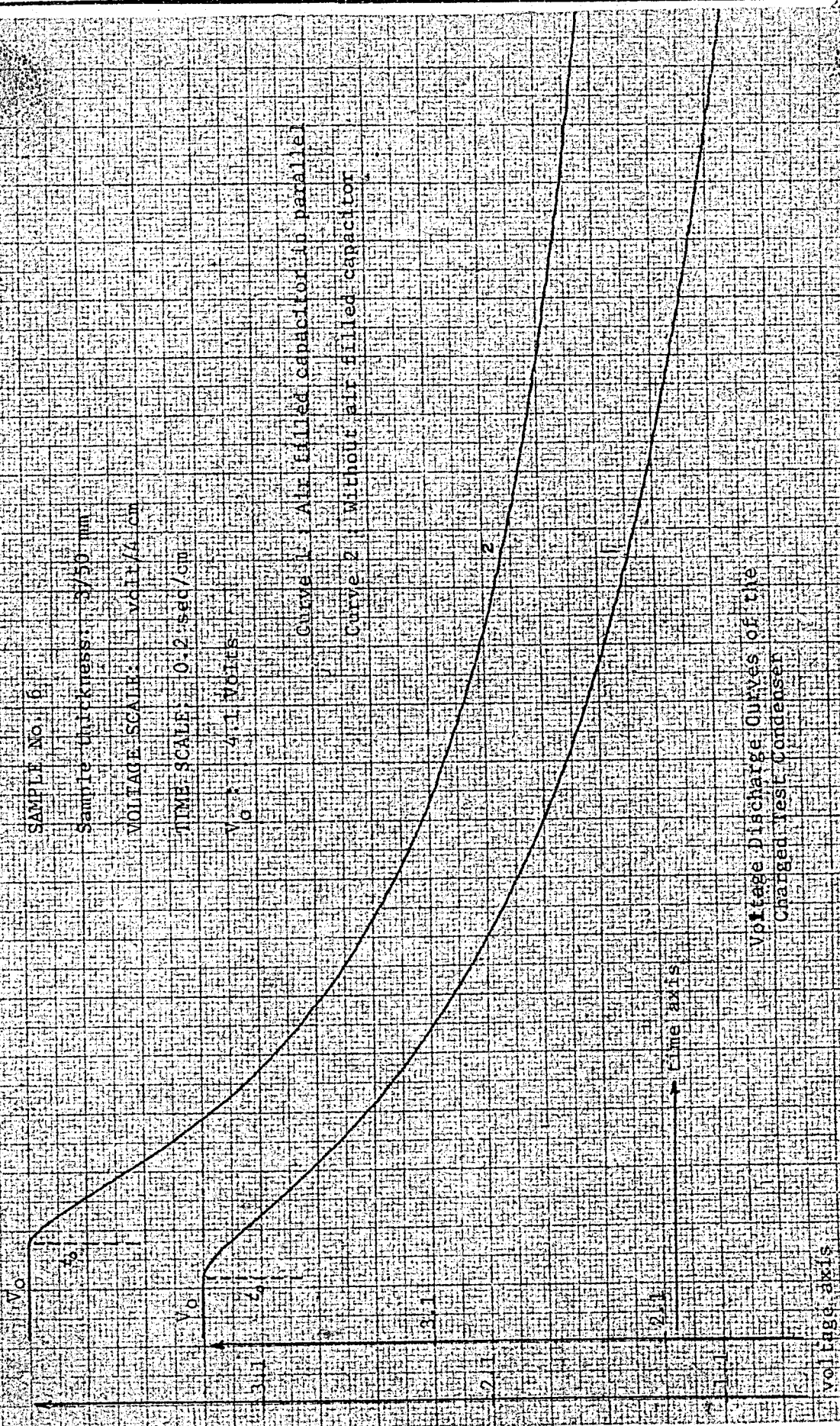
$V_0 = 5.4$ Volts

Curve 1: Air filled capacitor in parallel

Curve 2: Without air filled capacitor



Voltage Discharge Curves of the Charged Test Condenser



Voltage Discharge Curves of the Charged Test Condenser

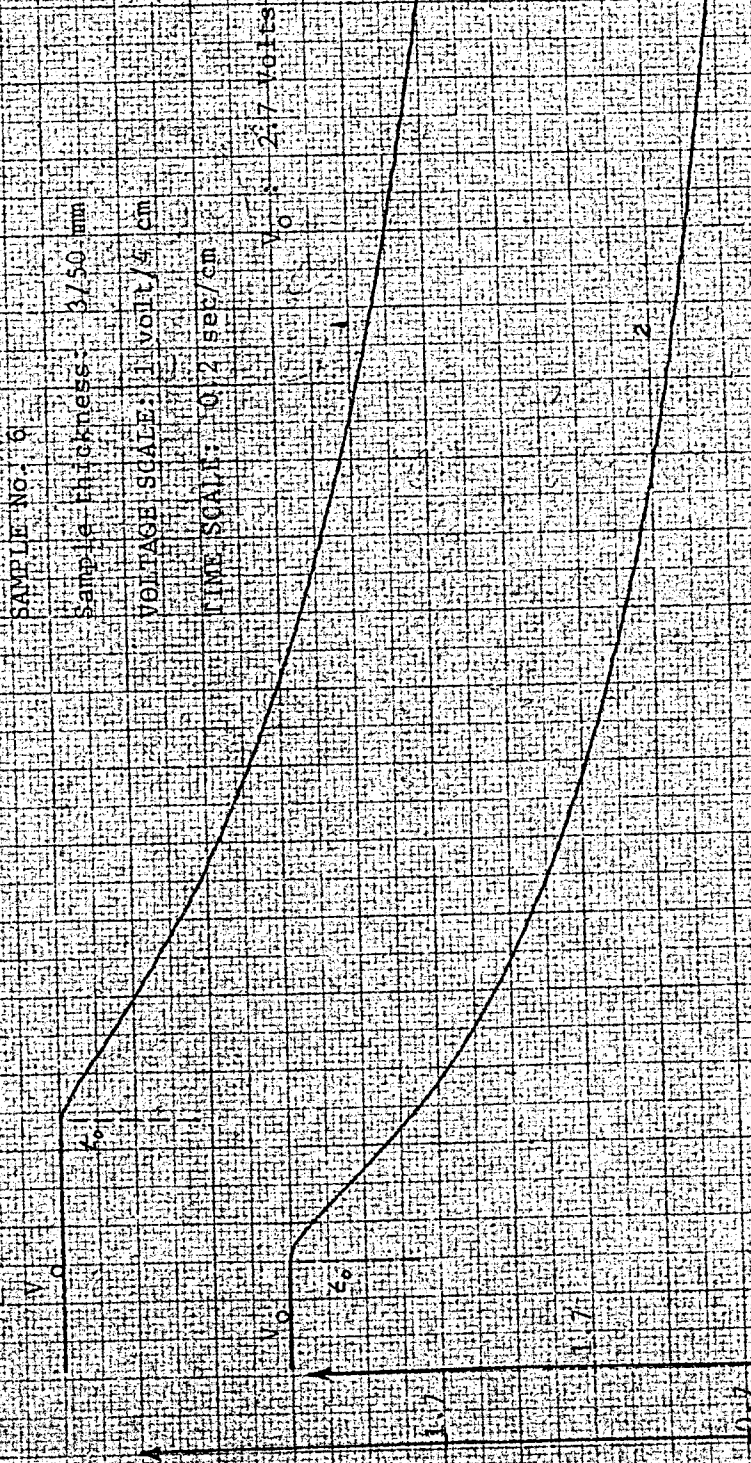
SAMPLE No. 6

Sample thickness: 3/50 mm

VOLTAGE SCALE: 1 volt/cm

TIME SCALE: 0.2 sec/cm

$V_0 = 2.7$ Volts



Curve 1: Air filled capacitor in parallel

Curve 2: Without air filled capacitor

time axis

Voltage axis

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE B-36

$V_0 = 5.4$ volts

L_3

$V_0 = 4.1$ volts

L_4

$V_0 = 2.7$ volts

L_6

time axis

voltage axis

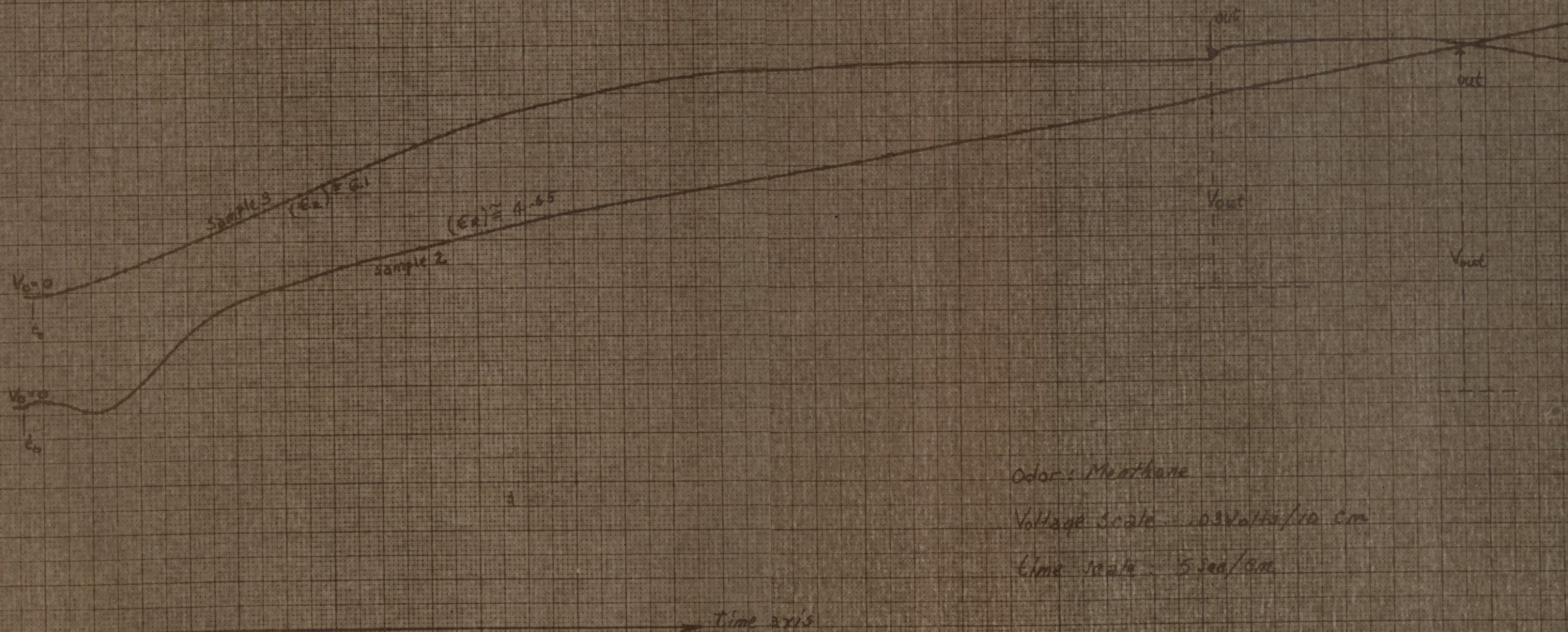
Without dielectric specimen between
the electrodes of the test condenser
Space between electrodes: 3/50 mm

Time scale: 0.2 sec/cm

Voltage scale: 1 volt/1 cm

Voltage Discharge Curves of the Charged
Test Condenser

The response of Electro-Odecell



THESIS

ROBERT COLLEGE GRADUATE SCHOOL

BEBEK, ISTANBUL

PAGE D-1.

APPENDIX D

Relative Dielectric Constants for Several Common Dielectrics.

(Values apply to normal temperature and humidity conditions and to very low frequencies.)

Material	ϵ_r
Vacuum	1
Glass(pyrex)	5
Mica	6
Nylon	3.7
Polyethylene	2.26
Teflon	2.1

Material	ϵ_r
Bakelite	4.9
Glycerine	45
Mineral Oil	2.4
Plexiglass	3.4
Polystyrene	2.53
Water(distilled)	80

These values are only representative for each material.

Depolarization Factors

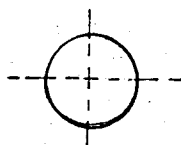
Shape:

Axis:

N

Depolarization factor:

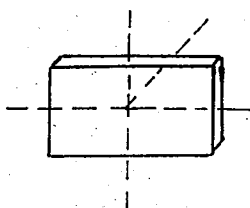
Sphere



Any

$$\frac{1}{3\epsilon_0}$$

Thin slab



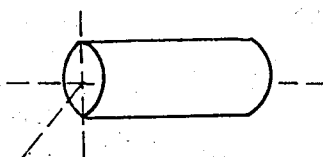
Normal

$$\frac{1}{\epsilon_0}$$

In plane

$$0$$

Infinitely long
circular cylinder



Longitudinal

$$0$$

Transverse

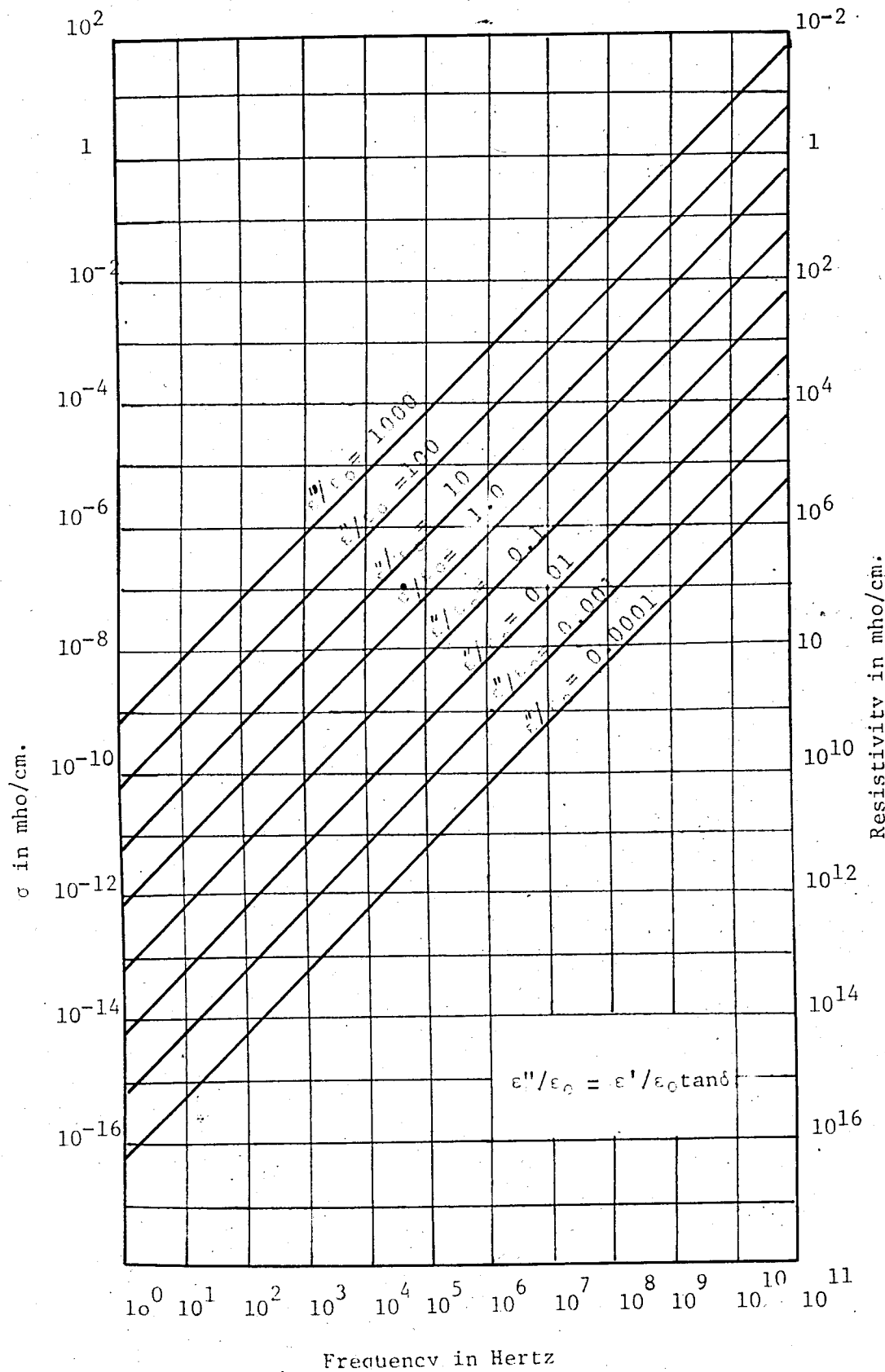
$$\frac{1}{2\epsilon_0}$$

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE D-2

Conductivity-Resistivity as Function of ϵ''/ϵ_0 and Frequency ^x



THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE D-3

Some Dielectric Data ^x

Dielectric	T °C		F R E Q U E N C I E S IN HERTZ					
			10 ²	10 ³	10 ⁴	10 ⁵	10 ⁶	10 ⁷
Sodium Chloride	25	ϵ'/ϵ_0	5.90	5.90	5.90	5.90	5.90	5.90
		Tan δ	< 1	< 1	< 1	< 2	< 2	< 2
	85	ϵ'/ϵ_0	6.35	6.11	6.00	5.98	5.98	5.98
		Tan δ	170	240	70	6	<2	<2
Ice	-12	ϵ'/ϵ_0	-	-	-	4.80	4.15	3.70
		Tan δ	-	-	-	8000	1200	180
Porcelain wet process	25	ϵ'/ϵ_0	6.47	6.24	6.08	5.98	5.87	5.82
		Tan δ	280	180	130	105	90	115
Porcelain dry process	25	ϵ'/ϵ_0	5.50	5.36	5.23	5.14	5.08	5.04
		Tan δ	220	140	105	85	75	70
Foam glass (Soda-Lime)	23	ϵ'/ϵ_0	90	82.5	68	44	17.5	9
		Tan δ	1500	1600	2380	3200	3180	1960
Fused Quartz SiO ₂	25	ϵ'/ϵ_0	3.78	3.78	3.78	3.78	3.78	3.78
		Tan δ	8.5	7.5	6	4	2	1
Alkali-Silica glass 12.8% Na ₂ O	25	ϵ'/ϵ_0	8.09	6.61	6.00	5.80	5.66	5.57
		Tan δ	3050	1370	450	240	199	126
Mica,glass, TiO ₂	24	ϵ'/ϵ_0	9.5	9.2	9.2	9.1	9.0	9.0
		Tan δ	170	125	76	42	26	21
Mykroy Grade 38 (Mica,glass)	25	ϵ'/ϵ_0	7.71	7.69	7.64	7.61	7.61	7.61
		Tan δ	43	33	27	24	21	14
Sandy soil,dry	25	ϵ'/ϵ_0	3.42	2.91	2.75	2.65	2.59	2.55
		Tan δ	.196	.08	.034	.020	.017	.016

Values for Tan δ are multiplied by 10⁴.

T H E S I S

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE D-4

	T °C		F R E Q U E N C I E S IN HERTZ					
			10 ²	10 ³	10 ⁴	10 ⁵	10 ⁶	10 ⁷
Sandy soil 2-18% moisture	25	ϵ'/ϵ_0	3.23	2.72	2.50	2.50	2.50	2.50
		Tan δ	.64	.13	.056	.030	.025	.025
Napthalene	25	ϵ'/ϵ_0	-	2.85	2.85	2.85	2.85	2.85
		Tan δ		19	9	5	3	2
Laminated fiberglass BK-174	24	ϵ'/ϵ_0	14.2	9.8	7.2	5.9	5.3	5.0
		Tan δ	2500	2600	1600	880	460	340
Amber	25	ϵ'/ϵ_0	2.7	2.7	2.7	2.7	2.65	2.65
		Tan δ	12.5	18	31	43	56	68
Beeswax, white	23	ϵ'/ϵ_0	2.65	2.63	2.56	2.48	2.43	2.41
		Tan δ	140	118	266	190	84	68
Water conductivity	25	ϵ'/ϵ_0	-	-	-	78.2	78.2	78
		Tan δ				4000	400	46
Bakalite, plastics Bm 120 Dry	25	ϵ'/ϵ_0	4.87	4.74	4.62	4.50	4.36	4.16
		Tan δ	300	220	200	210	280	350
Bakalite, 90% Rel. Humidity after 19 days	25	ϵ'/ϵ_0	7.4	5.4	4.4	-	-	-
		Tan δ	320	190	110	-	-	-
Bakalite, 90% Rel. Humidity after 18 Months.	25	ϵ'/ϵ_0	11.1	7.8	6.3	5.4	4.54	-
		Tan δ	370	200	120	78	56	-

x

Values are taken from: A. Von Hippel, Dielectric Materials and

Applications, The M.I.T. Press, Massachusetts,

1961 .

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE F-1

SELECTED BIBLIOGRAPHY

- VON HIPPEL, A., Dielectric Materials and Applications, The M.I.T. Press, Massachussetts, 1961
- FRÖHLICH, H., Theory of Dielectrics, Oxford University Press, London, 1958
- SMYTH, C. P., Dielectric Behavior and Structure, McGraw-Hill Book Company, New York, 1955
- O'DWYER, J.J., The theory of Dielectric Breakedown of Solids, Oxford Uni. Press, London, 1964
- BÖTTCHER, C. J., Theory of Electric Polarization, Elsevier Publishing Company, Amsterdam, 1952
- ANDERSON, J. C., Dielectrics, Reinhold Publishing Company, New York, 1964
- WANG, S., Solid-State Electronics, McGraw-Hill Book Company, Inc., New York, 1966
- BEAM, W. R., Electronics of Solids, McGraw-Hill Book Company, Inc., New York, 1965
- KITTEL, C., Introduction to Solid State Physics, 3rd ed. John Wiley and Sons, Inc., New York, 1966
- LAMB, D.R., Electrical Conduction Mechanisms in Thin Insulating Films, Methuen and Co Ltd., London, 1967
- VAIL, R. C., "Molecular Behavior of Composite Electrical Insulation" Electro-Technology, Feb. 1962
- VAN DER ZEIL, A., Solid State Physical Electronics, Prentice Hall, Inc., New Jersey 1968
- BECKER, SAUTER, Electromagnetic Fields and Interactions, Blaisdell Publishing Company, New York, 1964
- STRATTON, J. A., Electromagnetic Theory, McGraw-Hill Book Company, Inc., New York, 1941
- NUSSBAUM, A., Electronic and Magnetic Behavior of Matedials, Prentice-Hall, Inc., New Jersey, 1967
- NUSSBAUM, A., Electromagnetic Theory for Engineeris and Scientists, Prentice-Hall Inc., New Jersey, 1965