COMPUTATIONAL MODELS OF ATTENTION COMPETITION

by

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Submitted to the Institute for Graduate Studies in

Science and Engineering in partial fulfillment of

the requirements for the degree of

Doctor of Philosophy

Graduate Program in Computer Engineering

Boğaziçi University

2017

ACKNOWLEDGEMENTS

I am proud of being a part of the Bogazici community. Bogazici University has a very rich academic life that helped me develop the ideas presented in this thesis. I have learned too much from my collaborators and my teachers. I would like to thank my colleagues Ahmet Emre Aladağ, Mürsel Taşgın, Melih Barsbey, Samet Atdağ, Ahmet Bombacı, Çelik Özdeş, Barış Özmen, Metin Döşlü, Övünç Çetin, İbrahim Çimentepe, Nadin Kökciyan and Ahmet Yıldırım. I want to thank Suzan Üsküdarlı and R. Murat Demirer for the inspirational scientific discussions we have made. I thank my supervisor Haluk Bingol for his guidance. I would thank the jury members, Albert Ali Salah, Adil Sarıbay and Mehmet Gençer who have different scientific backgrounds from psychology to computer science. I am grateful that they have supported my enthusiasm for pursuing an interdisciplinary research. I want to give my special thanks especially to Yaman Barlas, and his former students Gönenç Yücel and Hakan Yaşarcan, for letting me to follow their courses as a visitor and introducing me to the systems thinking. I also want to thank Gerd Gigerenzer, who accepted me as a visitor to his summer institute on “Bounded Rationality” in Max Planck Institute for Human Development. That helped me to make a glimpse to the bounded nature of human mind and showed me, once again, the importance of an interdisciplinary platform for making a better scientific research. That is something we highly need in Turkey. I think, I owe special thanks to Mehmet Ufuk Çağlayan, for providing me the financial support within the TAM Project. Thanks to the TAM Project, I had this rare privilege to solely focus on my scientific research, during the last two years of my PhD.

My heart is always with my mother Ümmügül Çetin, my father Enver Çetin and my brother Deniz Engin Çetin. I believe love is the only thing that transcends any kind of limitations.

ABSTRACT

COMPUTATIONAL MODELS OF ATTENTION COMPETITION

In the new digital age, information has grown increasingly abundant and immediately available. Thus it is easy to see that the scarcest resource of today is not information but rather attention. We design several complex adaptive social systems in which agents with limited attention capacity confront a wealth of information. We take rather an exploratory approach and use primarily agent-based models to study the dynamics of competitive endeavours, such as artificial markets and games. Our purpose is to study the dynamics of cooperation and competition among boundedly rational artificial agents. (i) First we built a simple model in which cultural items compete for the limited attention of agents and we investigate the impact of advertisement pressure. We observe that the market share of the advertised item improves as a result of an increase in the standard items. (ii) Secondly, we work on attention games in a specific context of Iterated Prisoners Dilemma. We find out it is best for agents to pay attention to defectors in order to achieve a higher social welfare. Hence, cooperators becomes more prudent to the defective moves. (iii) Thirdly, we investigate the evolution of cooperation. This time agents are "hard-wired" to pay attention to defectors. Agents have limited memory size and refuse to play with defectors. As opposed to what we expect, we observe that subsequent generations loose their memory and are ultimately invaded by defectors, when playing with a defector brings non-negative payoffs. We reformulate the payoff matrix structure to incorporate negative payoffs and show how threat (of receiving negative payoffs) fosters greater memory size and cooperation. We also observe how memory acts like an immune response of the subsequent generations against aggressive defection. This functionality of self-immunization has emerged as a result of the co-evolutionary process.

ÖZET

BİLGİSAYIMSAL DİKKAT REKABETİ MODELLERİ

İçinde buluduğumuz yeni çağda, bilgi miktarı çok hızlı artmakta ve bilgiye ulaşım gittikçe kolaylaşmaktadır. Bugün artık, bilgi kaynakları kısıtlı olmaktan çıkmış ve en kısıltı kaynak bilgiye harcanacak olan zaman ve dikkat haline gelmiştir. Biz de bu kapsamda, kısıtlı dikkate sahip etmenlerin yoğun bilgiye maruz kaldığı bir takım karmaşık sosyal simülasyonlar tasarladık. Yapay market, ekonomi ve oyunlardaki sosyal ve evrimsel rekabeti incelemek için ana yöntem olarak, etmen-temelli benzetim kullandık. Amacımız sınırlı mantıksal çıkarım yeteneklerine sahip yapay etmenler arasındaki işbirliği ve rekabet dinamiklerini incelemektir. (i) İlk olarak basit bir kültürel market modeliyle, bireylerin hafızasından yer kapmak için birbiriyle yarış halinde olan kültürel ürünlerin dinamiklerini inceledik. Reklam ve tavsiye dinamiklerini karşılaştırdık. Reklam edilen ürünün bilinirliliğinin (şöhretinin) markete daha fazla sayıda standart ürün koyulurak da arttırılabileceğini gözlemledik. (ii) İkinci çalışmamızda, tutsak ikilemi oyunu özelinde dikkat kıtlığının oyunların çıktısını nasıl etkileyebileceğini inceledik. Dikkatin kısıtlı olduğu durumlarda, en iyi stratejinin, dikkatin işbirliğinden kaçan ve tehlike arz eden, bencil bireylere yöneltilmesi gerektiği ortaya çıktı. (iii) Üçüncü çalışmamızda, bencil bireyler arasında işbirliğinin nasıl ortaya çıkabileceğini, evrimsel açıdan inceledik. Bencil bireylerle etkileşim negatif skor getirmediği sürece, bir sonraki nesillerin hafızalarından sıyrıldığını, ve koruyucu bir hafıza kalmadığında toplumun bencil bireylerce işgal edildiğini gözlemledik. Tutsak İkilemi oyunundaki skor matrisini, bencil bireylerle etkileşimin negatif skor getireceği şekilde yeniden düzenledik. Belli bir dozdaki tehtidinin işbirliğini arttırdığını gözlemledik. Evrimsel açıdan bakıldığında, hafızanın bencilliğe karşı bir bağışıklık kalkanı gibi çalıştığını gördük. Bu durum, evrim neticesinde, zuhur eden bir özellik olarak ortaya çıkmıştır.

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a	Advertised item
c_i	The numbers of games, player i plays with cooperators
$\mathcal{C}$	Set of cooperators
d_i	The numbers of games, player i plays with defectors
$\mathcal{D}$	Set of defectors
f	Forgotten item
F_a	Market share of the advertised item
F_{min}	Lowest market share among all items
$F_{5\%}$	Lowest market share required to be in the top 5 percent
g	Giver agent
I	Number of items
$\mathcal{I}$	Item set
l_i	Transition probability from state S_i to S_j where $j = i$
$m(i)$	Memory content of an agent i
M	Memory size
N	Number of agents
$\mathcal{N}$	Set of agents
N_d	Number of defectors
p	Advertisement pressure
p_i	Transition probability from state S_i to S_j where $j = i + 1$
$\overline{P_C}$	Average performances of the cooperators
$\overline{P_D}$	Average performances of the defectors
q_i	Transition probability from state S_i to S_j where $j = i - 1$
r	Recommended item
$R_{I=k}$	Relative market share compared to the reference item size
$R_{\nu=k}$	Relative market share compared to the reference time
(S, P, R, T)	Payoff matrix of the IPD game
S_i	State where there are i infected agents

t	Taker agent
$\mathbf{T}$	Matrix for transition probabilities
$\mathcal{T}$	Total number of iterations
t_{ij}	Transition probability from state S_i to S_j
α	Threat factor
β	Greed factor
γ	Probability of r to be already known by t
δ	Defector ratio
κ	Probability of mutation
μ	Memory ratio (or attention capacity ratio)
$\overline{\mu}$	Average memory ratio
ν	Expected number of interaction per each pair of agents
π	Stationary distribution
ρ	Defection rate
$\overline{\rho}$	Average defection rate
ω	Purchased item

LIST OF ACRONYMS/ABBREVIATIONS

ABM	Agent based model
EBM	Equation based model
IPD	Iterated prisoner's dilemma game
FAR	Strategy of forgetting without any preference
FEQ	Strategy of forgetting from the types with equal probability
FMJ	Strategy of forgetting one agent of the majority type
FOC	Strategy of forgetting only cooperators
FOD	Strategy of forgetting only defectors
SD	Systems dynamics
SIS	Susceptible infected susceptible
SIR	Susceptible infected recovered
SRM	Simple recommendation model
SRMwA	Simple recommendation model with advertisement

1. INTRODUCTION

"Suam habet fortuna rationem (Chance has its reasons)."
- Gaius Petronius [1].

"All men can see these tactics whereby I conquer, but what none can see is the strategy out of which victory is evolved."
- Sun Tzu [2].

Structure generates behavior. I am a computer scientist. So it has taken me several years to encounter with the famous system dynamics motto, *structure generates behavior* [3]. That is a very agreeable summary of the causal relationships surrounding our social life and the nature. Usually, what we observe as an outcome is a product of a vast amount of interactions among the internal sub-components of a system. A perfect example would be Adam Smith's "invisible hand" which describes the unintended order of economic systems as a result of interdependent actions of self-interested individuals. Another well known example is the ability of an ant colony to behave like an intelligent organism even if it is composed of relatively unintelligent parts. Immune systems, ecological systems even internet are other distributed systems composed of many sub-components without a central controller. A system is defined to be a meaningful collection of interacting parts. The term "meaningful" hints a pattern, some level of order which is above random [4]. In a *complex system*, parts awash in an ocean of feedbacks. Parts shape and are shaped by other parts and as a result "more comes out than was put in" in the words of John Holland, father of the genetic algorithms [5]. Because of the non-linearity within the actions of interconnected parts, aggregate properties can not be attained by a simple summation. In other words, "the whole becomes not only more than but very different from the sum of its parts" [6]. This distinctive property is called *emergence*. It is the essential requirement for calling a system "complex" [7]. And it is also the main source of motivation for the research of complex systems. In order to be able to investigate the intricate complexities we face in the real world, hitherto we had data-driven experimentations and equation-based mathematical models. Now we have a new and very powerful tool, that we call *computer*.

This thesis will be about building computer-based simulations of complex systems in which simple sub-components, with their limited abilities, interact with each other in the from of either cooperation or competition.

How to harness complexity? Robert Axelrod is a well known political scientist for his interdisciplinary work on the evolution of cooperation and for his research on complexity theory. He explains the process of *harnessing complexity*, as deliberately changing the structure of a system in order to increase some measure of performance [8]. The structure, *who interacts with whom* in either physical or conceptual space, is the key to understand and improve the "aggregate" or "macro" behavior of the systems. Better outcomes can be achieved by deliberately introducing barriers and boundaries into systems with the aim of altering rates of interaction among sub-components [8]. This is also exactly the same scientific methodology to investigate the possible scenarios that could have generated the observed phenomena which needs to be explained. We ask again and again what is the very structure that have generated the macro behavior, that we are interested in. Computer-based simulation gives us the fascinating opportunity to create alternative structures and to search for the correct one which is capable of generating the behavior being observed. Axelrod claims simulation as a new way of doing science [9]. Computer scientist Uri Wilensky, the author of free and widely used agent-based programming language NetLogo, put it as follows: *"We can now simulate to understand"* [10].

How to connect computer science and complexity? Being aware of object oriented programming and the necessary tools and algorithms for analysing data, computer scientists usually have a natural tendency to view the world as a computational system. But they often lack the generative perspective of how macro arises from micro. From the generativist's point of view, each sub-component has different internal states and follows different rules-of-thumb to interact with each other. The repeated interactions between the sub-components, we call them *agents* from now on, give rise to the observed macroscopic outcome of interest. Indeed, this approach is very close to so called object oriented programming. An object is a software entity that encapsulates both data and methods acting on these data. Data and methods can be public (i.e. accessible to all agents) or private (i.e. specific to the agent). Although objects have state (data) and behavior (method), they fail to encapsulate the autonomy over their choice of action. Hence, the key distinction between an agent and an object, lies on the autonomy of the corresponding software entity [11]. The encapsulation of state (data), behaviour (method) and autonomy (choice of action) give rise to more realistic representation of systems composed of interacting distributed entities having limited information and computational capabilities [12].

Social dilemmas occur where macro-behavior and micro-motives fail to align. Agent-based modeling, with its bottom-up generativist approach, fits naturally to the study of social dilemmas. We will revisit agent-based modeling in Section 1.1. Game theory is the main theoretical reference for the investigation of social dilemmas. It allows also to make contributions to model and analyze problems within various disciplines. Evolutionary game theory imported biological ideas into game theory and results from game theory help us to understand better evolutionary biology [13]. It was invented by John Von Neumann and Oskar Morgenstern to formulate the decisions and behaviors of people playing games with each other [14]. It was developed during the Cold War to bring balance to the two competing mega powers dominating the world at that time [15]. In order to make it possible to have an analytical treatment, traditional game theory assumes that the players are rational. In traditional game theory, a "rational man" is an agent who can evaluate all the alternatives for a given problem and choose the best option [16].

The problems of inferring proper lessons based on limited experience occur in almost every sphere of human activity [8]. From managerial systems to ecology we face with our limitations. The "perfect rational man" paradigm is not adequate for the real world. Critical events occur or important dynamics emerge as a result of a huge number of interactions reaching threshold levels over time [17]. From the time of Heraclitus, we know that everything changes and nothing stands still. Mountains were not lifted up in one day, species are not static, nations and their language vary little by little, social norms even taboos which appear to be unchanging, become extinct or prosper slowly over long time horizons. On the other hand sometimes very quick life-and-death decisions must be made under extreme time pressures. That is not the case for only soldiers or doctors, but also for ordinary people [18]. The overwhelming complexity of the real world, is almost always beyond our cognitive capacities.

Bounded rationality refers to the limited knowledge and limited information-processing capabilities of our minds. We owe the notion of bounded rationality to Herbert Simon. He said [19]

> "*The capacity of the human mind for formulating and solving complex problems is very small compared with the size of the problem whose solution is required for objectively rational behavior in the real world or even for a reasonable approximation to such objective rationality.*"

Thus in order to deal with the inherit complexity of the real world, we need simplified representations of it. Simplifications should be done in a such way that we keep the essential properties of reality but ignore the the rest. (By the way, this is not so different from building scientific models, explained in Section 1.2.) Overhelming complexity of reality forces us to construct fast and frugal representations and behave accordingly [18]. Interestingly, our limitations make us smart. Just to give an example, with an unlimited memory that can track every details there won't be any need for thinking with abstractions [20]. *Heuristics* are simple but useful rules-of-thumb that are acquired through natural selections and take advantage of evolution. Gigerenzer points out that there is a clear distinction between known risks and unknown risks [21]. In the world of known risks, all alternatives and their probabilities can be calculated. Thus analytical thinking turns out to be the only requirement for good decision making. In the world of uncertainty (or unknown risks), that is not the case. This is where simple heuristics (intuitions, rules-of-thumb) often can do better. For sure, we live in an uncertain world. The real research question is to understand why and when an heuristic does better. This is referred as the study of ecological rationality which brings environmental structure back into the bounded rationality [18].

In this thesis we will attack the very heart of the *bounded rationality* using three different games that we have developed during my PhD. (Please see Figure 1.5.) We think that complex adaptive social systems in which agents with limited cognitive capacities (i.e., memory size, attention capacity, reasoning etc.) confront a wealth of information will provide a fertile background for scientific investigation. And for a computer scientist, one of the best ways is to use agent based programming to investigate this kind of social dynamics.

1.1. Agent Based Modeling

"Modeling for insights, not numbers."

- Hillard G. Huntington [22].

"What I can not create, I do not understand."

- Richard Feynman [23].

Agent-based modeling (ABM) is considered as a natural and a very intuitive way of representing real world systems in silico. It is essentially a new mindset to explore systems dynamics from bottom-up. From this perspective, the dynamics of a system emerges from the characteristics and interactions of its individuals. It fits very well to the study of many areas of science. Ecology has a long tradition of bottom-up modeling [4]. ABM even has a special name, *individual-based modeling*, in Ecology [24]. ABM is also being accepted as a well-suited mindset especially for social science [25]. Today, the main modeling tool used in computational social science is agent-based modeling [26]. ABM allows us to take into account the multilayered reality of our social world. The most fascinating things related to the ABM lies within the micro-macro linkage.

Agents are the building blocks of complex adaptive system [27]. An agent is an autonomous computational unit with particular properties (state) and simple rules (action) to follow. As it is shown in Figure 1.1, a simple agent can be characterized by its state and actions. The state of an agent is given by "what agent is" and "what agent knows". Usually agents are simple entities and their actions are determined by a simple set of if-then rules. These rules correspond to heuristics of satisfying behavior. Agents are so simple yet their interactions can give rise to complex dynamics, since

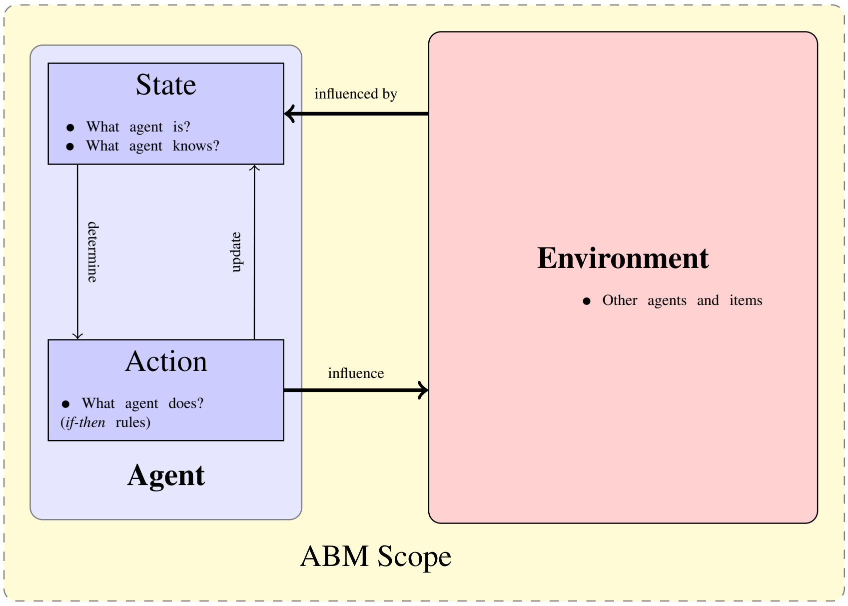


Figure 1.1. A simple agent and its environment.

they are awash in an ocean of feedbacks. To concretize the feedbacks in ABM, consider the following.

- Internal feedbacks - The state of an agent determines its action and, at the same time, its action determines its state.
- External feedbacks - Agents have an effect on their environment and environment also has an effect on the agents. They influence other agents and are also influenced by them.

The properties of agents can differ from one ABM to another. But, the most distinctive property of an agent is its autonomy and it is a must have in ABM. Agents are *autonomous*, in the sense that they take actions independent of an external controller. Agents are *reactive* in the sense that they follow simple condition-action rules. When a stimulus arrives, they react. An agent is *unique*. That is, they differ in their properties. Agents are *social*. A society of agents, usually consists of heterogeneous agents, communicate information with each other. They are *boundedly rational*. Their rationality is bounded because they lack complete information and their cognitive capacities are limited. Nonetheless they are rational in the sense that their choices are consistent with their internal preferences and their goals. Moreover, they are able to use pre-defined short-cuts or heuristics for decision making. They adapt to their social and physical environment. They shape their environment and also shaped by it. This circular causality can give rise to emergent properties, such as new heuristics for decision making that are not pre-defined.

What clearly distinguishes ABM from other modeling approaches, is the ability to show how the hard-to-predict macro-level regularities arise from a large amount of interactions among the boundedly rational and heterogeneous individuals. They are situated in conceptual or physical spaces and are following a set of simple *if-then* conditional rules. In Uri Wilensky's words, agent-based models provide us a "glass box" as opposed to a block box through which we can observe and test whether or not the hypothesized micro-specifications of agents and their interactions are sufficient to generate the macro-structure of interest [10]. This is also known as Generativist's approach [28]. Epstein explains *The Generativist's Experiment* as follows:

> "*Given some macroscopic explanandum - a regularity to be explained - situate an initial population of autonomous heterogeneous agents in a relevant spatial environment; allow them to interact according to simple local rules, and thereby generate—or "grow"—the macroscopic regularity from the bottom up.*"

Epstein's generativist motto is the following,

> *"if you don't grow it, you didn't explain it".*

The micro-to-macro linkage constitutes the most interesting part of this Generativist's approach. We can get macro-surprises despite the complete micro-knowledge. Following Epstein's Generativist approach, depicted in Figure 1.2, we ask *Are the given micro-specifications sufficient to generate a macrostructure of interest?* What is the effect of the independent variable on the macrostructure of interest under the given scenario (control variables)?

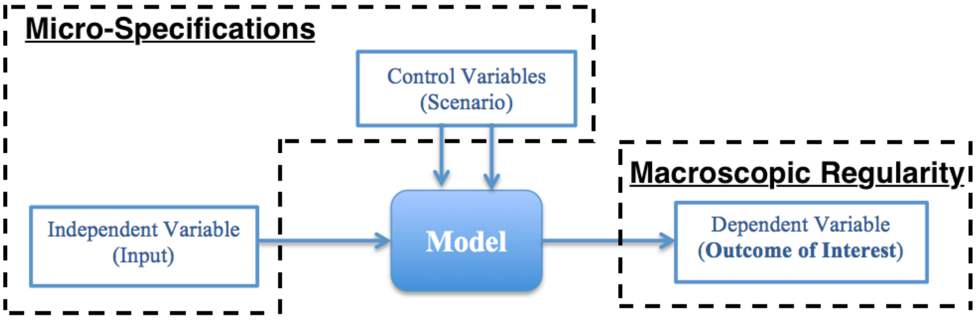


Figure 1.2. Generativist's approach emphasizing the linkage from micro to macro. This figure and Figure 1.1 is partly inspired from Gönenç Yücel's IE-48F course.

Agents, as it is shown in Figure 1.1, follows a set of simple if-then rules. These conditional rules are simple (e.g. neuron fires if received signal reaches a threshold or one can start to smoke if his rate of interaction with people who smoke increases and so on). But their effects are hard to grasp. Conditional rules are very typical of complex systems. Each conditional rule corresponds to a sharp change in behavior which makes it impossible to approximate with traditional mathematical tools (differential equations and so on). Real social phenomena are in general too complicated to be solved by analytical treatment. *Equation based models* (EBMs) employ difference/differential equations to study the dynamics of a model where sub-components can be interdependent and create causal feedback loops. This is different than standard differential equations, hence called *Systems Dynamics* (SD). ABM and SD approaches share the same simple cause-effect relationships in modeling but from different levels. That is bottom-up vs top-down. ABMs are especially useful where interactions take place in a non-random way due to environmental constraints or social preferences. We do not have an explicit mathematical procedure even for third order non-linear differential equations. That is why SDs also requires simulations to solve problems. ABM can not replace EBM. Because ABM takes so much time and computational effort while EBM can provide direct analytical solutions. Thus they complement each other. ABM is not a discipline-specific method, so it can cross the disciplinary boundaries. The view of Robert Axelrod on ABM is as follows [29].

> "From my perspective, agent-based modeling is not only a valuable technique for exploring models that are not mathematically tractable; it is also a wonderful

way to study problems that bridge disciplinary boundaries."

ABM offers a new way of doing science: by conducting computer-based experiments [10]. It gives the opportunity for understanding natural and social phenomena by recreating them in computer simulations. Any object-oriented programming language will be useful to code ABMs. Modeling environments such as Netlogo (or swarm, repast, Mason etc.) are also helpful especially for an easy start in ABM. In this thesis, we have used Java for implementing our models.

As a final note, we think that literacy in agent-based modeling will become extremely important in our increasingly complex world. Literacy - the ability to read and write - is critical to understand our world and make it a better place. For example, Turkish alphabet has a high degree of accuracy and specificity in pronunciation, unlike French. Each letter in Turkish corresponds accurately to the phonetic requirement. This is also true for ABM. It is possible to build agent based models where actual individuals and their interactions are directly represented [30]. Agent-based modeling is a powerful tool because its basic ontology corresponds accurately to the ontology of the real world [10].

1.2. Model Design and Validation

"All models are wrong, but some are useful."

- George Box [31]

"Art is a lie that helps us see the truth."

- Picasso [32]

"The first consequence of the principle of bounded rationality is that the intended rationality of an actor requires him to construct a simplified model of the real situation in order to deal with it."

- Herbert Simon [19]

A real system is something that can not be fully identified or known because of its infinitely many aspects. Since a real system is unknown, we come up with abstractions. A *model* is defined to be a simplified representation of some selected aspects of a real system with respect to a clearly stated problem. The purpose of modeling is to ease thinking by filtering out irrelevant parts of the real world phenomena under investigation. Modeling is more art than science. Like stone carving, we remove what we regard as unnecessary [33] and like cartooning, we exaggerate some distinctive features that we believe they are important to the problem of concern [34]. The scope (what to include) and necessary level of details (how to include) are determined relatively according to the purpose at hand. For example, in an evolutionary game we can use asexual reproduction, if we think the details of the reproduction are irrelevant [10]. For an individual to reproduce, a certain age is required. For the sake of scope reduction, we can ignore this requirement, too. There are two major categories of modeling in ABM [10].

- Phenomena-based modeling - It is the most common design approach across disciplines. Modeling starts with an observation of a target macro phenomena in the real world. Then, a research question is formalised in order to explain its hidden cause. Modeler takes into account only some selected aspects of the reality, that he believes to be important. Modeler determines the properties of the agents and the physical or conceptual structure that they are embedded in (who interacts with whom) at the bottom level. If the model turns out to be capable of capturing the essential properties of the target phenomena, then the model can be accepted as one way of an explanation for the target phenomena.
- Exploratory modeling - This *bottom-up* approach is more or less unique to ABM. Instead of starting with a target macro phenomena, one starts with determining the properties of the agents and the physical or conceptual structure that they are embedded in (who interacts with whom). Then he or she explores the patterns that emerge. Of course not all the emerging outputs are interesting. To count this approach as modelling, there must be some degree of correspondence between the model output and some real-world phenomena.

The design of an ABM is usually a mixture of the two major approaches shown in Figure 1.3. The approach of phenomena-based modeling is like reverse-engineering

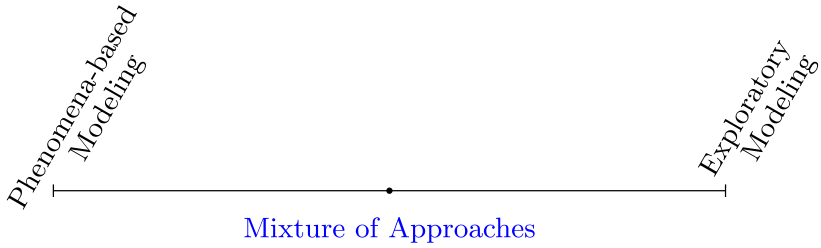


Figure 1.3. The two major model design approaches for ABM.

the micro-rules that generate a macro-level phenomena. Thus it can be a very difficult process [35]. So starting with exploratory approach and gaining experience with modeling is more common in ABM. To emphasize the importance of exploratory modeling, we can make a glimpse to the words of Arthur S. Eddington [36].

> "*The contemplation in natural science of a wider domain than the actual leads to a far better understanding of the actual.*"

<u>1.2.0.1. Verification and Validation.</u> A model needs verification (correctness) and validation (usefulness) in order to improve its credibility. *Verification* deals with correctly implementing the conceptual model. By verification, we make sure that the conceptual model is transformed into computer model (code) without error. *Validation* is a much more difficult process, because of its relative nature. Is the model an acceptable, good-enough, adequate description of a real phenomena based on the purpose, or objectives, at hand? Note the fact that when the purpose at hand changes, the degree of validity also changes. There is no such thing as general validity [37]. Since all models are simplifications of the reality, all models are incorrect. A model can not be either invalid or valid but its validity lies in-between. As the mapping between the model and the real process gets closer, we obtain a more valid model. ABM deals with micro-validation and macro-validation at both levels whereas EBM deals with only macro-validation.

Validation is the process of testing whether a model is sufficiently accurate for the purpose at hand [37]. There can be two types of validation.

- Face validation - is the process of showing that the model appears to be a reasonable imitation of a real phenomena, looking from the surface.
- Empirical validation - uses statistics to demonstrate the resemblance between the data generated by the model and empirical data gathered from the real world.

Face validation is a form of qualitative agreement between the model output and the real-world phenomena, whereas empirical validation is the quantitative one, thus it is a relatively stronger criterion. But empirical validation is not always possible, since real-world data can be missing or poorly defined. Thus, in general, face validation turns out to be only option. A model has face validity if it appears to be a reasonable imitation of a real phenomena to people who have knowledge about the real phenomena.

1.2.0.2. Modeling Example. The purpose of a model is often to explain specific patterns which has any display of order above random variation [4]. Modeling is a process of decoding the underlying mechanisms that generates the observed pattern. Suppose we observed a population of fireflies which synchronise their flashing and we want to understand how they manage this synchronisation. What causes this pattern to exist? We form our hypothesis according to the ABM perspective (bottom-up). We think that they synchronise without any central coordinator, just by using local interactions.

Agents are fireflies in Figure 1.4. Each agent has its own clock which is reset to zero when clock reaches to its maximum value. An agent flashes during its flashing-time (while clock has a value from 0 to flashing-time) and awaits for a while (while clock has a value from flashing-time to maximum value). Then flashes again. We also think that each agent occupies a location in a lattice-like environment. They can perceive other agents if they are close enough. By close enough, we mean that only local information is available to agents. Agents can adapt their clock according to their neighbors. When they perceive a sufficient number of flashing agents, they reset their own clocks. As a result, the state of an agent (flashing or not-flashing) is determined directly by its own clock and indirectly by the existence of sufficient number of local neighbors that are flashing. To understand, if this micro-specification are sufficient to

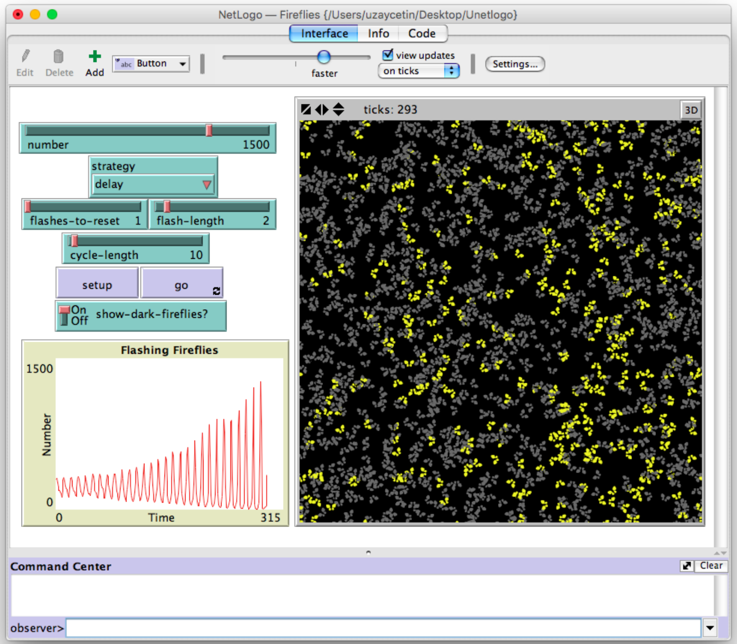


Figure 1.4. NetLogo Fireflies model.

generate the macro-structure of interest (synchronisation), we can run a simulation. That can easily be done by NetLogo Fireflies model [38].

To assess the credibility of this model, we can make a face-validation. If synchronisation can be achieved by the given micro-specifications then it is a valid model. As an analogy, we can think a model as a map. As long as it takes you where you want to go, it is a valid model.

1.3. Models of Attention Competition

1.3.0.1. Cooperation and Competition. How life has begun? We do not know it. Maybe, in the early days of the universe, resources were plenty and there was no need for neither cooperation nor competition. Once resources become limiting, things changed. We think the very cause of both type of interaction is the limited resources that threaten one's survival. Some single-celled organisms cooperated with each other when food is scarce (like slime-molds do even today) whereas some competed with one another for the limited resources to survive.

1.3.0.2. Limited Attention. Today, with the help of our high technology, resources have become plenty. An ocean of information is in front of us. It is like an open buffet, where we can eat anything but not everything. The challenge lies in what to choose. Thus, we encounter with a new form of scarcity. Let Nobel laureate Herbert Simon explain it to us [39],

> "*In an information-rich world, the wealth of information means a dearth of something else: a scarcity of whatever it is that information consumes. What information consumes is rather obvious: it consumes the attention of its recipients. Hence a wealth of information creates a poverty of attention and a need to allocate that attention efficiently among the over-abundance of information sources that might consume it.*"

An important research topic of this century is the attention scarcity of the decision maker. We perceive the world through our filters. We can deal with only a limited fraction of the information we receive. And they are often imperfect. Thus our cognitive capacities are overwhelmed by the bombardment of information. We discuss bounded rationality in terms of limited attention, memory and information processing capabilities. In this thesis, we combined hot topic of this century "scarcity of attention" with the twin hot topic of all centuries "competition" and "cooperation".

1.3.1. Computational Models of Attention Competition

Cooperation and competiton are the two main phenomena we continually face in the real world. One of the fascinating research questions is how is it possible for self-interested agents to cooperate? We construct agent-based models in order to study first competition then cooperation among agents with limited memory and attention. Our models will be a combination of phenomena-based and exploratory modeling. They are exploratory in the sense that we start with determining the properties of agents from a bottom-up approach. They have limited memory and attention. Our models are also

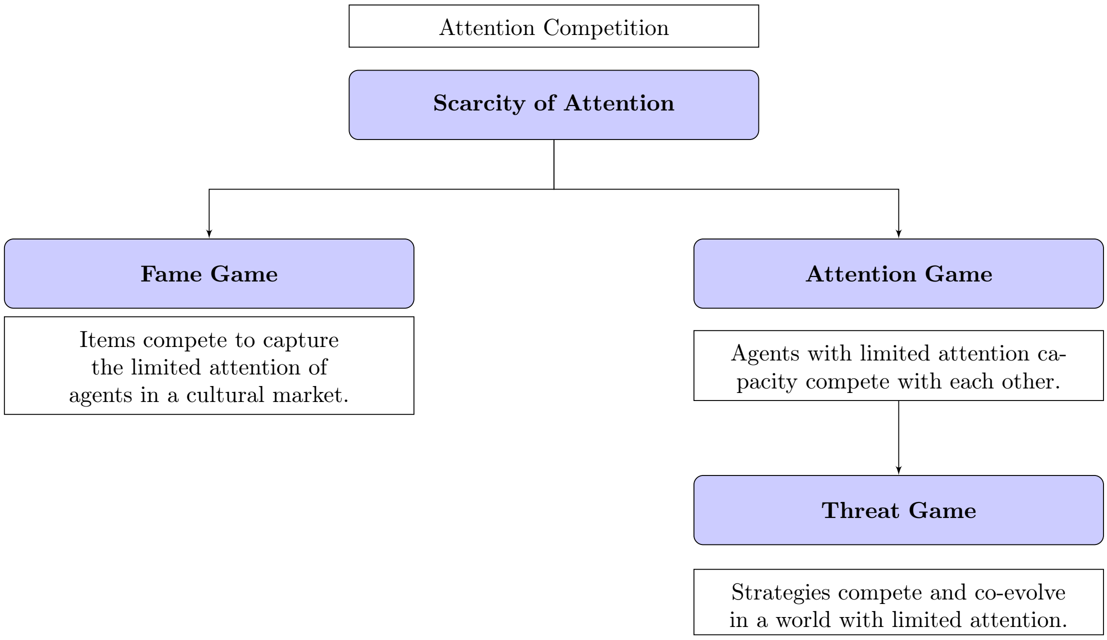


Figure 1.5. Computational Models of Attention Competition.

phenomena-based because we look first for the dynamics of competition in an artifical market then secondly we look for the dynamics of cooperation among self-interested agents. Our assumption is the following, the spotlight of our attention determines the content of our memory, which in turn determines with whom and how we interact, and this structure ultimately determines our behavior. The study of human attention and human memory is a big topic in Cognitive Psychology. We are not claiming that we are using them in their deep true meaning. We are only conceptualizing them, in a very simplified way. We rely on the fact that a model with more details does not necessarily mean a better model. And we consider our models as starting points for the investigation of the impact of limited attention on social dynamics.

Our Research Questions, as opposed to the traditional mathematical investigations which assume perfect rationality and unlimited cognitive capacity, is the following. *What is the effect of limited memory and limited attention capabilities on the dynamics of competition and cooperation?* We have investigated attention competition from different view points, as seen in Figure 1.5. Starting from the problem of attention scarcity, first, we worked on cultural markets where items compete for the limited attention of agents [40]. As items capture the attention of more agents, they become popular and get a higher market penetration. We used the term *Fame Game* for this setting. Second, we worked on games where agents with limited attention capacity compete with each other [41]. Each encounter with an opponent corresponds to a new information that has to be stored in memory. We assume that the memory is not unbounded. This raises the question of deciding *what to know* and *what to ignore*. In other words, how attention should be spared? We used the term *Attention Game* for this setting. And lastly, we investigated the evolutionary dynamics of attention competition. In this case, strategies compete and co-evolve in a world with limited attention. We changed the payoff structure of the game, such that it includes negative payoffs. We interpreted negative payoffs as threat and explored its effect on the dynamics of cooperation. Thus, we used the term *Threat Game* for this setting. We only made face validation of our models. The contributions of this thesis as follows:

- Mathematical traceability of an ABM - In the fame game, we show how an ABM, which has a low complexity level, can be traced by analytical tools. We reformulated the problem of finding the market share of an advertised item as a social contagion and represented it as a Markov Process. Our analytical solution and simulation results have confirmed each other. ABMs are known for not accepting an analytical treatment. In this setting, homogeneous agents, that are subjected to an unchanging advertisement pressure, interact without any preference. Hence, the complexity level of fame game is somewhat low. That is why, it can accept analytical treatment. Nevertheless, even for this model we are required to use approximations.
- The root of heuristics - Gigerenzer claims that our minds learned an "Adaptive toolbox" of rules to deal with the complexity of the real world [18]. That notion of "Adaptive toolbox", which is composed of rules-of-thumbs, resembles the simple if-then conditional rules, shown in Figure 1.1. We think one of the greatest questions is "how do these rules-of-thumb come to form". Our models provide some insights for the following heuristics.

(i) The Attraction-Effect heuristics - This refers to an inferior option's ability to increase the attractiveness of another alternative [42, 43]. In the fame game, we find out a surprising result. Initially an increase in the number of standard items, fosters the market share of an advertised item. This is a reminiscent of the attraction-effect heuristics, known in the Economy literature. But this needs further investigation. (Actually this study provides us a new research question on what causes one inferior option to increase the attractiveness of another.)

(ii) The negativity bias - The attention-grabbing power of negative information is much more greater than the positive one [44, 45]. In our second work, we find out that the attention must be directed towards the defectors and towards their defective moves for the success of cooperators. This is known as "negativity bias" in the psychology literature. Attention defines what to know and what to ignore. The negative information about defectors should be kept in memory and the positive information about cooperators should be ignored in order to make room for more negative information.

- Study of cooperation with memory (mindscape barriers) - Cooperation involves a cost to benefit others. The cost makes cooperation vulnerable to defection. As a result, natural selection favors defection in well-mixed populations. The common scientific approach is to use spatially structured populations instead of well-mixed populations [46]. Spatial structure provides landscape barriers that prevent interactions among cooperators and defectors. Isolated defectors are selected away and cooperation succeeds. In this thesis, we have introduced the concept of memory to tailor the interaction structure. Memory gives agents the ability to determine with whom to interact. It is a sort of mindscape barriers to avoid defectors.
- Reformulation of the Prisoner's Dilemma game - In the threat game, we change the payoff matrix structure of the Prisoner's Dilemma game, and show the impact of threat (for receiving negative payoffs as a result of being defected) on the emergence of cooperation. Interestingly an appropriate level of threat has a positive effect on memory and therefore cooperation.

(i) Threat, of receiving negative payoffs, fosters cooperation. But traditional Prisoner's Dilemma game payoff structure does not let us to incorporate negative values in a systematic way. Researchers, in general, use arbitrary payoff values for the study of cooperation. We provide a systematic way to reformulate the payoff matrix structure by two main factors of threat and greed. We think that as an important contribution to the study of cooperation.

2. COMPETITION FOR THE LIMITED ATTENTION

"I am I and my circumstances."

-José Ortega y Gasset [47].

"Most people are other people. Their thoughts are someone else's opinions, their lives a mimicry, their passions a quotation."

- Oscar Wilde [48].

In this chapter, we will extend Bingol's Fame game [49] and formalize it as an attention competition of cultural items in an artificial market. Let's first think about a simple model of economy in which agents exchange money at random with each other. Say initially each agent has a wealth of 500 dollars. Simulation runs over discrete time steps. At each tick, agents with non-zero wealth give one dollar to a random agent. What would be the resulting wealth distribution? There is a NetLogo model for this setting [50]. If we run it, we interestingly obtain a power-law distribution which corresponds to a great inequality in wealth. Actually, one can modify the model such that agents with negative wealth can also give money to others. By doing so, you remove one type of barrier for interaction and then power-law distribution disappears. Again we witness how interaction structure generates the behavior.

We considered economy as a simple process of spreading money within a society. For sure, when an agent gives money to another, the giver loses its money. This is the case for material things. What if agents exchange non-material things such as information, beliefs or cultural features? How we can model the dynamics of a culture? Another model by Robert Axelrod who has known for his influential work on cooperation can help us. In his work on the dissemination of culture [51], each agent is represented by a vector of features and is located on a 2D grid. Agents follow the homophily principle for interaction. The interaction probability is a function of similarity. As an example, consider two agents $(1, 2, 3, 4, 5)$ and $(8, 7, 6, 4, 5)$. They are similar only in the last two positions out of five positions in their representations. Their interaction probability is then $2/5 = 0.4$. If the interaction probability is not zero between two local neighbors, one of the agent adopts a non-matching feature of the other. As a result, more similar agents interact more and then become even more similar. As a side effect, less similar agents become even more dissimilar and as simulation iterates impassable group boundaries come to form. Regions with different cultures emerge. Interestingly interaction structure depending on similarity, creates global polarization on the macro level. Once again we witness how micro-motives and macro-behavior fail to align, in an agent-based model.

2.1. Fame Game

Bingol's fame game consists of a simple recommendation model (SRM). SRM investigates how individuals become popular among agents with limited memory size. Each agent knows a subset of other agents. They hold information about others, in their memory. Memory size is equal for each agent. And it is held constant. In Figure 2.1, state of the system is shown as a matrix. The number of rows equals to population size, and the number of columns equals to memory size. The row entries correspond to memory content of corresponding agents.

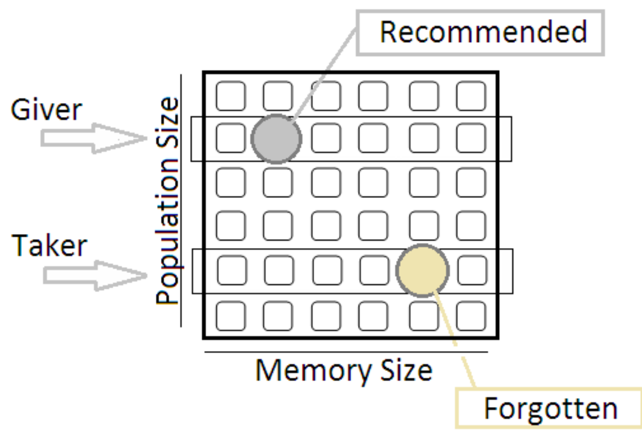


Figure 2.1. Simple Recommendation Model.

Any pair of agent can interact without any condition. Interaction is described as a recommendation process. A giver and a taker agent is selected uniformly at random. Giver agent recommends an agent from his memory at random to the taker.

Taker learns a new agent at the expense of previously known agent. That is, taker forgets, uniformly at random, an old agent from his memory. As simulation iterates, some agents become completely forgotten and some agents become extremely popular. Simulation stops when memory content of each agent becomes identical. In other words, only a very limited subset of agents known by everyone while a great majority of agents is forgotten. We will revisit SRM more formally in Section 2.2.2.

For a useful remark, consider Axelrod's culture model as a setting where imitation of similars yields to polarization. In Bingol's fame model, imitation of others without any preference yields to unanimity. To see how Axelrod's culture model can be transformed to the study of engineering problems, see [52]. In the last case, imitation of betters yields to optimization. Modeling the exchange of social information can provide very fertile scientific opportunities.

2.2. Attention Competition with Advertisement

In the new digital age, information is available in large quantities. Since information consumes primarily the attention of its recipients, the scarcity of attention is becoming the main limiting factor. In this study, we investigate the impact of advertisement pressure on a cultural market where consumers have a limited attention capacity. A model of competition for attention is developed and investigated analytically and by simulation. Advertisement is found to be much more effective when attention capacity of agents is extremely scarce. We have observed that the market share of the advertised item improves if dummy items are introduced to the market while the strength of the advertisement is kept constant.

2.2.1. Markets

Traditionally every product or service has a price tag. In order to get it, one has to pay the price. Nowadays, the price of items in some markets becomes so low, even to the point of free-of-charge, that this concept of "pay-to-get" is challenged, especially in the era of Internet. It is quite a common fact that one can get many products and services paying absolutely nothing. Among these are internet search (Google, Yahoo), email (Gmail, Hotmail), storage (DropBox, Google, Yahoo), social networks (Facebook, Twitter, LinkedIn), movie storage (Youtube), communication (Skype, WhatsApp), document formats (PDF, RTF, HTML), various software platforms (Linux, LaTeX, eclipse, Java) and recent trend in education (open course materials and massive open online courses (MOOC)).

Companies providing services, where their users pay no money at all, is difficult to explain in Economics. Even if these products are free to its user, there is still a sound business plan behind them. To obtain a large market share is the key in their business plan as in the cases of Google, Facebook, LinkedIn, or Skype. Once they become widely used, the company starts to use its customer base to create money.

2.2.1.1. New Market Concepts. In order to understand such markets new concepts such as two-sided markets and attention economy are developed. In a *two-sided market*, a company acts as a bridge between two different type of consumers [53]. It provides two products: one is free and the other with a price. Free products are used to capture the attention. Products with price are used to monetize this attention. A set of very interesting examples of two-sided markets including credit cards, operating systems, computer games, stock exchanges, can be found in [53].

Davenport defines attention as a focused mental engagement on a particular item of information [54]. When we give our attention to something, we are always taking it away from something else. Suppose there are many competing products at the free side of a two-sided market. In theory, a customer can get all the products available. In practice, this is hardly the case. Abundance of immediately available products can easily exceed customers capacity to consume them. One way to look at this phenomenon is that products compete for the attention of the users, which is referred as *attention economy* in the literature [55–57]. There is another new discipline, whose approach is very similar: *Memetics*. The word *meme*, analogous to a gene, was first popularly used by Richard Dawkins in his book, *The Selfish Gene* [58] . A *meme* is a unit of information, hosted in the minds of individuals which can reproduce itself and can be passed onto others. According to Richard Brodie, the world is full of memes all competing for a share of our minds [59], our perception, our attention. Cultural products, like memes or any other units of information, compete for a share of our minds to gain a broader popularity.

2.2.1.2. Compulsive Markets. Attention scarcity due to the vast amount of immediately available products is also the case for cultural markets. In a *cultural market*, it is assumed to have an infinite supply for cultural products and it is assumed that individual consumption behaviour is not independent of other's consumption decisions [60,61]. We focus on markets, that are slightly different, where customer compulsively purchases the item once he is aware of it. Clearly, this kind of compulsive buying behavior cannot happen for high priced items such as cars or houses. On the other hand, it could be the case for relatively low priced items such as movie DVDs or music CDs. This pattern of "compulsive purchasing" behavior becomes clearly acceptable, if the items become free as in the case of web sites, video clips, music files, and free softwares, especially free mobile applications. There are a number of services that provide such items including Youtube, Sourceforge, AppStore.

We will call such markets as *compulsive markets* and we consider the dynamics of the consumers rather then the economics of it. These new kind of markets call for new models. In this work, the Simple Recommendation Model, which is studied in [49, 62] is extended to such a model. We use the extended model to answer the following questions: Under which conditions advertisement mechanism outperforms the recommendation process? How much advertisement is enough to obtain certain market share? We first present our analytic approach and then compare it with simulation results.

2.2.2. Background

A compulsive buyer becomes aware of a product in two ways: (i) By local interactions within his social network, i.e. by means of word-of-mouth. (ii) By global interactions, i.e. by means of advertisement.

Word-of-mouth recommendations by friends make products socially contagious. Research on social contagion can provide answers to the question of how things become popular. Gladwell states, "Ideas, products, messages and behaviours spread like viruses do" [63]. He claims that the best way to understand the emergence of fashion trends is to think of them as epidemics. Infectious disease modeling is also useful for understanding opinion formation dynamics. Specifically, the transmission of ideas within a population is treated as if it were the transmission of an infectious disease. Various models have been proposed to examine this relationship [49, 61, 64–68]. There exist recent works whose essential assumption is the fact that an old idea is never repeated once abandoned [69, 70]. In other words, agents become immune to older ideas like in the susceptible-infected-recovered (SIR) model. However, behaviors, trends, etc, can occur many times over and over again. In this case it can be modeled as susceptible-infected-susceptible (SIS) model. In completely different context, limited attention and its relation to income distribution is investigated [71].

2.2.2.1. Epidemic Spreading. The study of how ideas spread is often referred to as social contagion [72]. Opinions can spread from one person to another like diseases. An agent is called *infected* iff it has the virus. It is called *susceptible* iff it does not have the virus.

Using the SIS model of epidemics, the system can be modeled as a Markov chain. Consider a population of N agents. Let S_i be the state in which the number of infected agents is i. The state space is composed of $N+1$ states, $\{S_0, S_1, \dots, S_N\}$ with S_0 and S_N being the reflecting boundaries. The system starts with the state S_0 where nobody is infected. The corresponding Markov Chain can be seen in Figure 2.2.

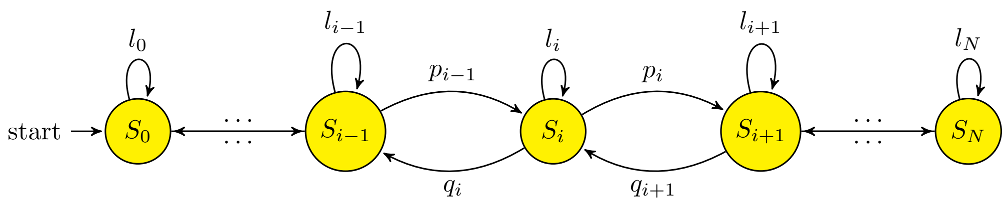


Figure 2.2. Markov Chain where states are characterized by the number of infected agents.

Let $\mathbf{T} = [t_{ij}]$ be the $(N+1) \times (N+1)$ transition matrix of the Markov chain where t_{ij} is the transition probability from state S_i to state S_j. As a result of a single recommendation, there are three possible state transitions: The number of infected agents can increase or decrease by one or stay unchanged. Such a system is called birth death process [73]. Hence, $\mathbf{T}$ is a tridiagonal matrix with entries given as

$$t_{ij} = \begin{cases} p_i, & j = i+1, \\ l_i, & j = i, \\ q_i, & j = i-1, \\ 0, & \text{otherwise} \end{cases}$$

where p_i, l_i and q_i are the transition probabilities. Then the stationary distribution $\boldsymbol{\pi} = [\pi_0 \cdots \pi_N]^\top$ of the Markov chain can be obtained from its transition matrix [73] which satisfies

$$\pi_i = \prod_{k=1}^{i} \frac{p_{k-1}}{q_k} \pi_0 \quad \text{and} \quad \sum_{i=0}^{N} \pi_i = 1. \tag{2.1}$$

<u>2.2.2.2. Simple Recommendation Model.</u> The *Simple Recommendation Model* (*SRM*) reveals the relation between the fame and the memory size of the agents [49, 62]. The SRM investigates how individuals become popular among agents with limited memory size and analyzes the word-of-mouth effect in its simplest form. The SRM

differs from many previous models by its emphasis on the scarcity of memory. In the SRM, agents, that have a strictly constant memory size M, learn each other solely via recommendations.

A *giver* agent selects an agent, that he knows, and *recommends* to a *taker* agent. Since memory space is restricted to M, the taker *forgets* an agent to make space for the *recommended* one. This dynamics is called a *recommendation* which is given more formally in Section 2.2.3.3. Note that (i) The selections have no sophisticated mechanisms. All selections are made uniformly at random. (ii) Any agent can recommend to any other agent. Therefore underlining network of interactions is a complete graph. (iii) Taker has to accept the recommended, that is, he has no options to reject.

In the SRM, no agent initially is different than the other. So the initial fames of agents are set to be the same where *fame* of an agent is defined as the ratio of the population that knows the agent. Recommendations break the symmetry of equal fames. As recommendations proceed, a few agents get very high fames while the majority of the agents get extremely low fames, even to the level of no fame at all. Once an agent's fame becomes 0, that is, *completely forgotten*, there is no way for it to come back. In the limit, the system reaches an *absorbing state* where exactly M agents are known by every one, i.e. fame of 1, and the rest becomes completely forgotten, i.e. fame of 0. The SRM offers many possibilities for extension. It is applied to minority communities living in a majority [74]. A recent work extends forgetting mechanism by introducing familiarity [75].

2.2.3. Proposed Model

In SRM, (i) the spread of information through out the system is managed by recommendation only and (ii) the results are obtained by simulations [49, 62]. In this work, we propose *Simple Recommendation Model with Advertisement* (*SRMwA*) that extends SRM in the following ways: (i) In addition to recommendation, advertisement pressure as new dynamic is introduced. (ii) Moreover, an analytical approach is developed as well as simulations. Distinctively, by SRMwA, we investigate the conditions under which social manipulation by advertisement overcomes pure recommendation.

2.2.3.1. New Interpretation for SRM. In the original model of SRM, agents recommend other agents and the term of memory size is used for the number of agents one can remember [49, 62]. As one agent is known more and more by other agents, his fame increases. In the extended model of SRMwA, agents recommend items rather than agents. Since items consume the limited attention of agents, there is a competition among items for attention. The focus of the work is no longer the fame of the agents but the attention competition among items. For these reasons, we prefer to use the term of "attention capacity" in spite of the term memory size for the number of information an agent can handle.

Note that the proposed model allows us to consider items in a wider sense. Rather than a unique object such as Mona Lisa of Leonardo, we consider items that are easily reproduced so that there are enough of them for everybody to have, if they wanted to. Therefore items are not only products and services but also as political ideas, fashion trends, or cultural products as in [61].

2.2.3.2. Advertisement. We extend the SRM to answer the following question: What happens if some items are deliberately promoted? Suppose a new item, denoted by a, is *advertised* to the over-all population. At each recommendation, the taker has to select between the *recommended* item r and the advertised one a. The item that is selected by the taker is called the *purchased item*, denoted by ω.

2.2.3.3. Model. Adapting the terminology of SRM [49] to SRMwA, a *giver* agent g recommends an item, that she already owns, to an individual. The item and the individual are called the *recommended* r and the *taker* t, respectively. The taker pays attention to, that is, *purchases*, either the recommended or the advertised item. When the attention capacity becomes exhausted, in order to get space for the purchased item, an item f that is already owned by the taker is *discarded*. The *market share* of an item is defined to be the ratio of population that owns the item.

The SRMwA is formally defined as follows. Let $\mathcal{N} = \{1, 2, \ldots, N\}$ and $\mathcal{I} = \{1, 2, \ldots, I\}$ be the sets of agents and items, respectively. Let $g, t \in \mathcal{N}$ and $r, f, \omega \in \mathcal{I} \cup \{a\}$ represent the *giver* and the *taker* agents, the *recommended*, the *discarded* and the *purchased* items, respectively.

The attention "stock" of an agent i, denoted by $m(i)$, is the set of distinct items that i owns. We say agent $i \in \mathcal{N}$ *owns* item $j \in \mathcal{I}$ iff $j \in m(i)$. For the sake of simplicity, we assume that all agents have the same *attention capacity* M, that is, $|m(i)| = M$ for all $i \in \mathcal{N}$. The attention capacity of an agent is limited in the sense that no one can pay attention to the entire set of items but to a small fraction of it, that is, $M \ll I$. Instead of directly using M, we relate M to I by means of *attention capacity ratio*, defined as $\mu = M/I$. Since $0 \leq M \leq I$, we have $0 \leq \mu \leq 1$.

The recommendation and advertisement dynamics compete. The taker agent select either the recommended or the advertised item as the purchased one. Let the *advertisement pressure*, p, be the probability of selecting the advertised item as the purchased item.

The modified recommendation is composed of the following steps:

(i) g is selected.
(ii) t is selected.
(iii) $r \in m(g)$ is selected by g for recommendation.
(iv) t selects ω where ω is set to a with probability p, and to r with probability $1 - p$.
(v) The recommendation stops if ω is already owned by t.
(vi) Otherwise, $f \in m(t)$ is selected by t for discarding and ω is put to the space emptied by f.

Note that all selections are uniformly at random. With these changes, the SRMwA becomes a model for compulsive markets with advertisement.

2.2.3.4. Some Special Cases. In general, one expects that the market share of the advertised item increases as advertisement get stronger. Depending the strength of advertisement, there are a number of special cases, the dynamics of which can be explained without any further investigation.

(i) *No advertisement.* Note that in the case of no advertisement, the original SRM is obtained since the purchased item is always the recommended item, i.e. $\omega = r$. In this case, the advertised item has no chance and its the market share is 0.
(ii) *Pure advertisement.* When the taker has no choice but get the advertised one, i.e. $\omega = a$, recommendation has no effect. In this cases after every agent becomes a taker once, the market share of the advertised is 1. Note that in this case the system will stop evolving any further. The remaining $M - 1$ entries are left unchanged, thus empty. Interestingly, this is a different state than the absorbing states of the SRM.
(iii) *Strong advertisement.* In the case of very strong advertisement, the taker almost always select the advertised item. Once all agents have the advertised item, the market share of the advertised item is 1 and the system becomes the SRM but with attention capacity of $M - 1$.

2.2.4. Analytical Approach

Note that SRMwA resembles epidemic spreading. We explore epidemic spreading to explain SRMwA as far as we can. Consider the advertised item as a virus. Agent j is called *infected* iff it has the advertised item in its attention stock, that is, $a \in m(j)$ otherwise it is called *susceptible* that is $a \notin m(j)$. Then the stationary distribution $\boldsymbol{\pi}$ provides the probability of the number of agents owning the advertised item when the system operates infinitely long durations. Hence, the mean value of the stationary distribution $\boldsymbol{\pi}$ reveals our prediction for the number of infected agents. In other words, the expected number of agents that adopted the advertised item is the mean value of this distribution. That is, using Equation 2.1, one obtains

$$<\boldsymbol{\pi}>=\sum_{i=0}^{N} i\pi_i = \pi_0 \sum_{i=0}^{N} i \prod_{k=1}^{i} \frac{p_{k-1}}{q_k}.$$

Hence, the expected market share of the advertised item becomes

$$<F_a>=\frac{<\boldsymbol{\pi}>}{N}$$

where F_a is the market share of the advertised item.

2.2.4.1. Calculation of Transition Probabilities. In order to obtain the expected market share of the advertised item, we need to figure out the stationary distribution $\boldsymbol{\pi}$, which, in turn, calls for transition probabilities p_i, l_i and q_i.

Suppose the system is in S_i and follow the steps of recommendation process given in Section 2.2.3.3. The possible selections can be represented by a tree given in Figure 2.3. A path starting from the root S_i to a leaf in the tree corresponds to a recommendation. The paths that increase the number of infected agents are marked by a $\oplus$ sign at the leaf. Similarly, recommendations resulting a transition of $S_i \to S_{i-1}$ are marked by a $\ominus$. The remaining paths that correspond to no state change are marked by a $\odot$.

Note that there three $\oplus$ and two $\ominus$ paths. Note also that the correspondence between the levels in the tree and the steps of recommendation given in Section 2.2.3.3. At each level one particular selection is made and the corresponding probability is assigned.

(i) $a \in m(g)$ level. The first level branching in Figure 2.3 corresponds to the selection of infected or susceptible giver. There are N possible agents to be selected as g. If system is in state S_i, then the probability of selecting an infected giver is i/N.

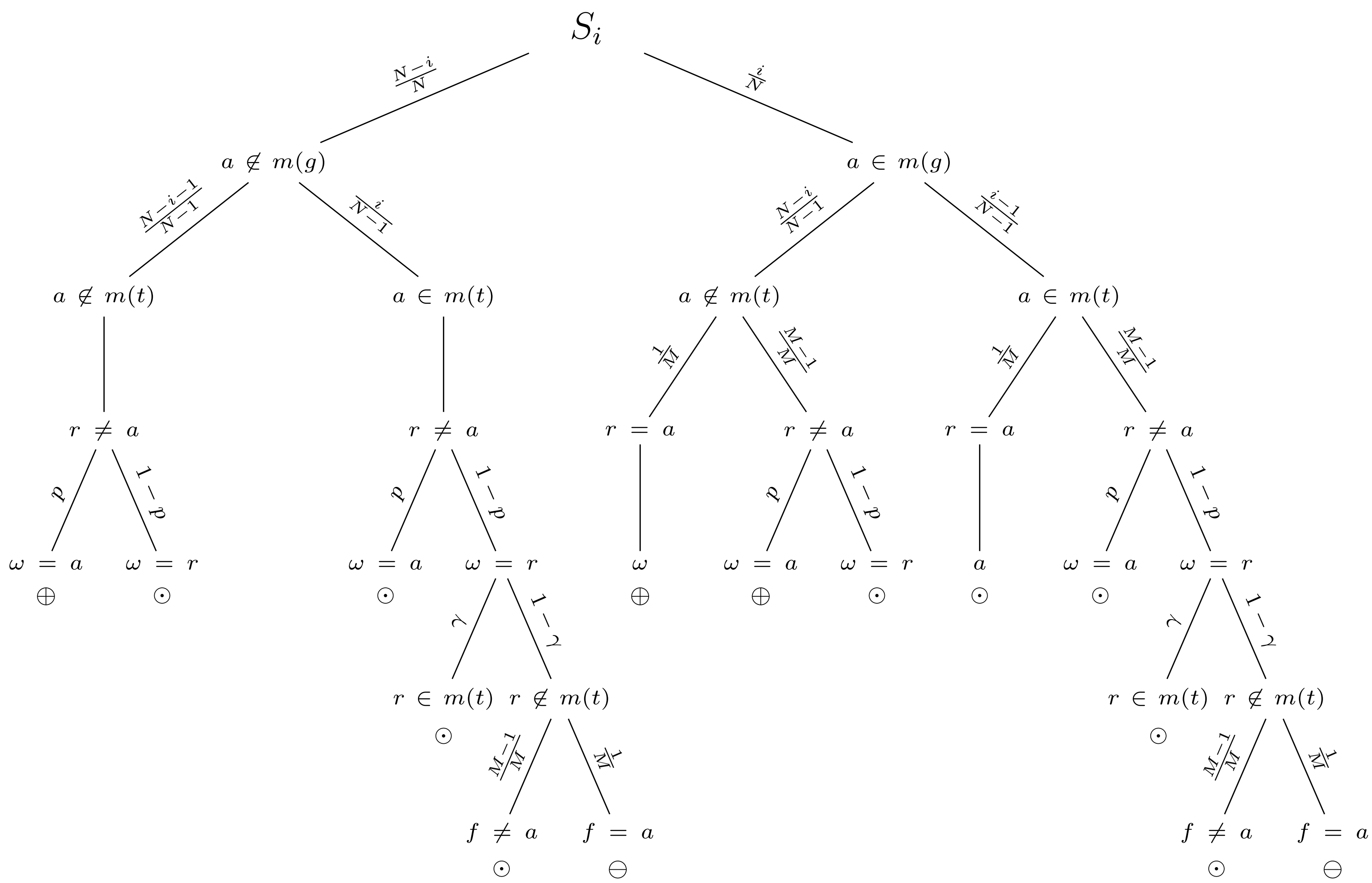


Figure 2.3. Tree diagram for possible selections.

(ii) $a \in m(t)$ level. The second level branching is due to the selection of infected or susceptible taker. Once g is selected, there are $N-1$ candidates left for t. The probability of selecting an infected taker depends on whether the selected giver is infected or not. For example, in the right most path, g is infected. So, the probability of selecting an infected taker for this case is as $(i-1)/(N-1)$.

(iii) $r = a$ level. Now consider what the giver recommends. Depending on the path, the giver could be infected and could recommend the advertised item. Then the probability of an infected giver recommending a is $1/M$, since there are M items in its stock.

(iv) $\omega = a$ level. The fourth level illustrates the taker's purchase decision. The taker agent either follows the advertisement with probability p or he accepts the recommended item with probability $1-p$.

(v) $r \in m(t)$ level. Let γ be the probability of r being already owned by the taker agent. In this case, the taker agent does not do any changes in her stock.

(vi) $f = a$ level. It is possible that a can be chosen to be the forgotten.

The transition probabilities can be obtained from Figure 2.3 as

$$p_i = \frac{N-i}{N(N-1)}\left[\left(N-1-\frac{i}{M}\right)p + \frac{i}{M}\right], \tag{2.2}$$

$$q_i = \frac{i(1-p)(1-\gamma)}{N(N-1)M}\left[N-i+\frac{(i-1)(M-1)}{M}\right], \tag{2.3}$$

$$l_i = 1-(p_i+q_i). \tag{2.4}$$

Note that (i) These equations satisfy the boundary conditions $q_0 = 0$, and $p_N = 0$. (ii) $p_i > 0$ for all $i = 0, \cdots, N-1$. (iii) $q_i = 0$ for all i when $p = 1$ or $\gamma = 1$. Therefore, for $p = 1$ or $\gamma = 1$, the system drifts to S_N and stays there forever.

2.2.4.2. Discussion on γ. The stationary distribution can be obtained by means of Equation 2.1, Equation 2.2 and Equation 2.3. The only unknown in these equations is γ, which is introduced in the fifth step of recommendation given in Section 2.2.3.3. γ is defined as the probability of recommended item to be already owned by the taker agent. Unfortunately, γ cannot be obtained analytically except for the extreme case of

$M = 1$. Therefore, we should find ways to approximate its value.

A first order estimate for γ could be $\mu = M/I$, since taker owns M item out of I in total. γ is close to 1, when M is in the range of I, since every agent owns almost all the items. The situation is quite different for $M \ll I$. Since every item initially has the same market share, γ starts with a small value at the beginning. As recommendations proceeds, we know that some items becomes completely forgotten [49]. Therefore γ increases as the number of recommendations increase and becomes 1 when the systems reaches one of its absorbing state. In this respect, γ can be interpreted as the degree of closeness to an absorbing state. In order to investigate near absorbing state behavior, we set $\gamma = \max\{0.5, M/I\}$ in our analytic results given in Figure 2.4 (b) where 0.5 is arbitrarily selected.

<u>2.2.4.3. Extremely Scarce Attention Capacity.</u> For the extremely scarce attention capacity of $M = 1$, γ can be evaluated. Consider the paths in Figure 2.3. For $M = 1$, the paths which contain a $(M-1)/M$ edge become paths with zero probabilities. The only non-zero probability path, involving γ, is the one terminating at the left $\ominus$ leaf. In this path the giver does not know the advertised item, $a \notin m(g)$, while the taker does, $a \in m(t)$. Since attention capacity is limited to 1, the giver and the taker do own different items. Therefore, the recommended item by the giver cannot be owned by the taker. Hence, $\gamma = 0$.

For $M = 1$ and $\gamma = 0$, Equation 2.2 and Equation 2.3 lead to

$$\frac{p_i}{q_i} = 1 + \frac{N-1}{i}\frac{p}{1-p}$$

for $0 \leq i < N$. For $p \neq 0$, $p_i/q_i > 1$. That means for even very small positive advertisement, the system inevitably drifts to the state S_N and once S_N is reached, the system stays there forever since $q_N = 0$. Note that S_N, which corresponds to the state where all agents own the advertised item, is the unique absorbing state for this particular case.

2.2.5. Simulation Approach

In order to simulate the model, a number of decision have to be made. The simulations start in such configurations that all I items have the same market share and no agent knows the advertised item. So that system is initially symmetric with respect to non-advertised items. When to terminate the simulation is a critical issue. We set the average number of interactions $\nu = 10^3$. Since there are N^2 pairwise interactions among agents in both directions, the total number of recommendations is set to be νN^2.

(i) We run our simulation for a population size of $N = 100$ and an item size of $I = 100$.
(ii) The behavior of the system strongly depends on the attention capacity ratio μ. We take μ as a model parameter and run simulation for various values of μ.
(iii) The advertisement pressure p is another model parameter. We use $10^{-1}, 10^{-2}$, 10^{-3} and 10^{-4} for p.

2.2.6. Observations and Discussion

We investigate the effect of the advertisement pressure p and the capacity ratio μ to market share F_a of the advertised item. In order to make a quantitative comparison of the simulation results, being in the top 5 percent is arbitrarily set as our criteria. Let $F_{5\%}$ denote the lowest market share for an item to be in the top 5 percent. Then, the advertised item is in the top 5 percent whenever $F_a > F_{5\%}$. Let F_{min} be the minimum market share among all the items.

In Figure 2.4, the simulation results of F_a, averaged over 20 realisations and versus the analytical results of $< F_a >$ can be seen for each value of $p \in \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}\}$ as functions of μ. A number of observations can be made:

(i) The analytic results given in Figure 2.4 (b) are in agreement with the simulation results in Figure 2.4 (a). Model predictions on $< F_a >$ can quantitatively

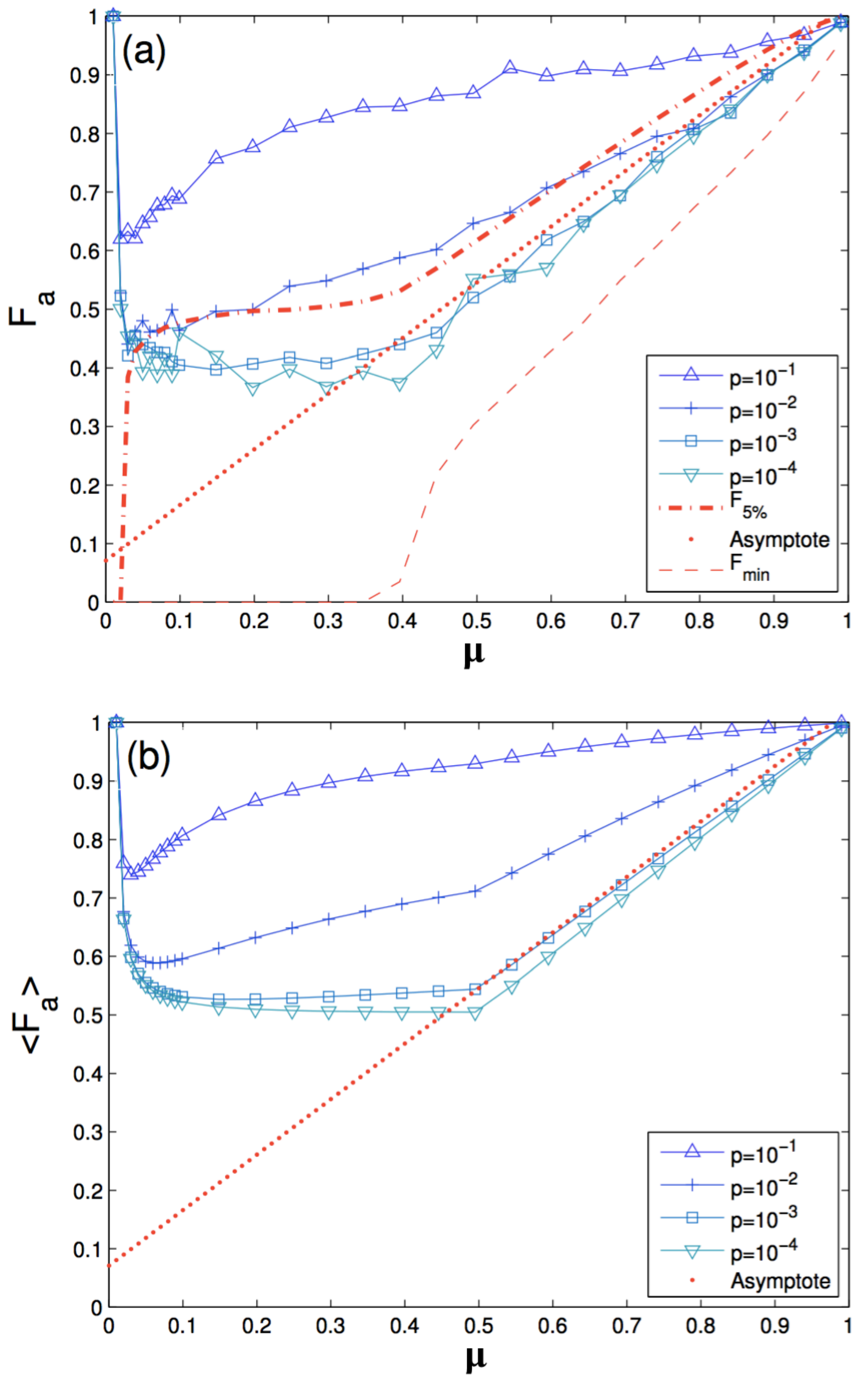


Figure 2.4. The market share of advertised item as a function of attention capacity ratio by (a) simulation and (b) analytic approaches. $F_{5\%}$, F_{min} and the asymptote line of reference [49] are given for comparison.

reproduce the simulation results of F_a although we use an approximated value for γ. We observe that for larger ν, the similarity between analytical and simulation results gets even better.

(ii) The curves of $F_{5\%}$ in Figure 2.4 (a) resemble that of in [49], although advertisement is not the case for the latter. Line $y = 0.95x + 0.071$, which is given as an asymptote for $F_{5\%}$ for large values of N in [49], is also plotted in Figure 2.4 (a) for comparison purposes.

(iii) Note that for $\mu < 0.05$, all F_a curves approaches to 1 and $F_{5\%}$ becomes 0. This is due to finite size effect. At an absorbing state, there would be exactly the same M items purchased by all the agents and the remaining items are completely forgotten. For $I = 100$, $\mu < 0.05$ means that $M < 5$. That is, there is no space left for the fifth item. Hence, in near absorbing state, the market share of the fifth item, $F_{5\%}$, approaches to 0. On the other hand, any promotion, i.e. $p > 0$, is enough to push the advertised item into the top M items.

(iv) The minimum market share F_{min} becomes 0, when at least one item is completely forgotten. This occurs for $\mu < 0.35$ in Figure 2.4 (a) which is consistent with reference [49]. We also observe that for larger ν, the advertised item leaves smaller share of attention to others, that forces the zero crossing of F_{min} to occur at an higher level of μ.

(v) As expected, a strong advertisement, i.e. $p = 10^{-1}$, easily gets the advertised item into the top 5 percent since F_a curve for $p = 10^{-1}$ is always higher than that of $F_{5\%}$ in Figure 2.4 (a) while a weak promotion such as $p = 10^{-3}$ or 10^{-4} cannot. The case of $p = 10^{-2} \approx 1/(I + 1)$ for $I = 100$ is interesting. For small and moderate values of μ, i.e. $\mu < 0.6$, the advertised item is in the top 5 percent except for one point. For the large values of μ, this is not the case.

(vi) How agents allocate their attention, when the attention capacity becomes a limiting factor? This is the critical question for markets of attention economy. Consider

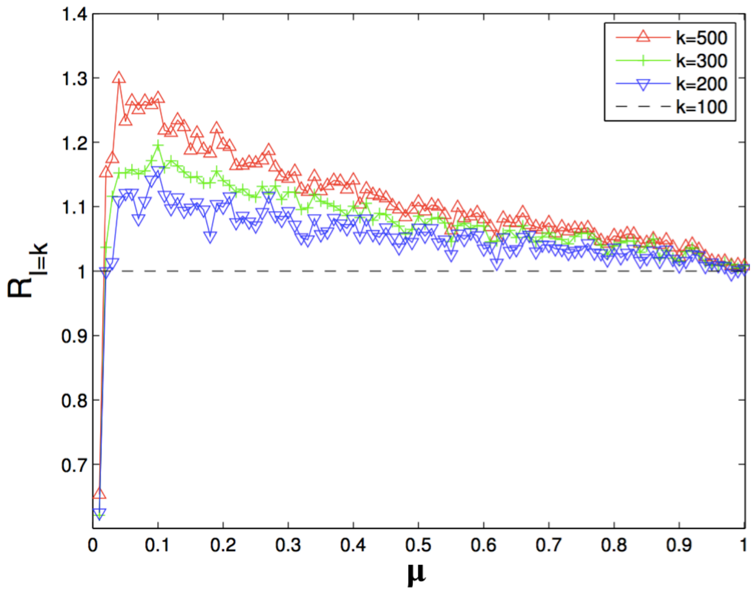


Figure 2.5. Effect of item size to the market share of the advertised item for $p = 10^{-1}$ is invesitgated as a function of attention capacity ratio.

the extreme case of attention capacity $M = 1$, which corresponds to $\mu = 0.01$ in Figure 2.4 . In this case, surprisingly, even a very small positive value of p is enough for the entire population to get the advertised item, i.e. $F_a = 1$, when $M = 1$. This observation is analytically investigated in Section 2.2.4.3.

Item size effect. We run new simulations with different item sizes of I when N is fixed to 100. Let $F_a(I = k)$ denote the market share of the advertised item when $I = k$. Then we accept $F_a(I = 100)$ as the reference market share and define relative market share $R_{I=k}$ with respect to $I = 100$ as follows

$$R_{I=k} = \frac{F_a(I = k)}{F_a(I = 100)}.$$

In Figure 2.5, we observe that for all $k \in \{100, 200, 300, 500\}$, $R_{I=k} \geq 1$ when p is fixed to 10^{-1} except for $\mu = 0.01$. The case of $\mu = 0.01$ corresponds to $M = 1$ for $I = 100$. As explained in Section 2.2.4.3, F_a gets its maximum value of 1, for $M = 1$. That is why, $R_{I=k} \leq 1$ for $\mu = 0.01$.

We have observed that the market share of the advertised item improves while the number of items are increased even if the advertisement pressure is kept constant. In order to push market share up, increasing the advertisement pressure, is not usually an option in practical life. This can be an interesting interpretation. If one cannot increase the intensity of advertisement, i.e p, it is better to have higher number of items, i.e. I. When that happens, the advertised item have better chances to get into the top 5 percent. In order to obtain this operating point, one may purposefully introduce some dummy items. This unexpected prediction of the model needs to be further investigated.

2.2.6.1. Closeness to the Absorbing State. The system gets closer to one of its absorbing states as the number of recommendations increases which is controlled by simulation parameter ν. Let $F_a(\nu = k)$ be the market share of the advertised item after νN^2 recommendations. We define relative market share $R_{\nu=k}$ at $\nu = k$ with respect to $\nu = 10^2$ as

$$R_{\nu=k} = \frac{F_a(\nu = k)}{F_a(\nu = 10^2)}.$$

The relative market share at $\nu = 10^3$ is given in Figure 2.6 for different values of $p \in \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}\}$ when $N = I = 100$.

We consider the system stationary if $R_{\nu=k}$ becomes 1, that is, the system stops changing with ν. We observe in Figure 2.6 that as the attention capacity or the advertisement pressure gets higher, model becomes closer to the stationarity. More advertisement pressure is not so different than increasing the number of iterations. Both are favorable for the market share of the advertised item.

2.2.7. Conclusions

The SRM as a model for pure word-of-mouth marketing is studied in ref [49, 62]. We extend the SRM to attention markets with advertisement. This model constructs

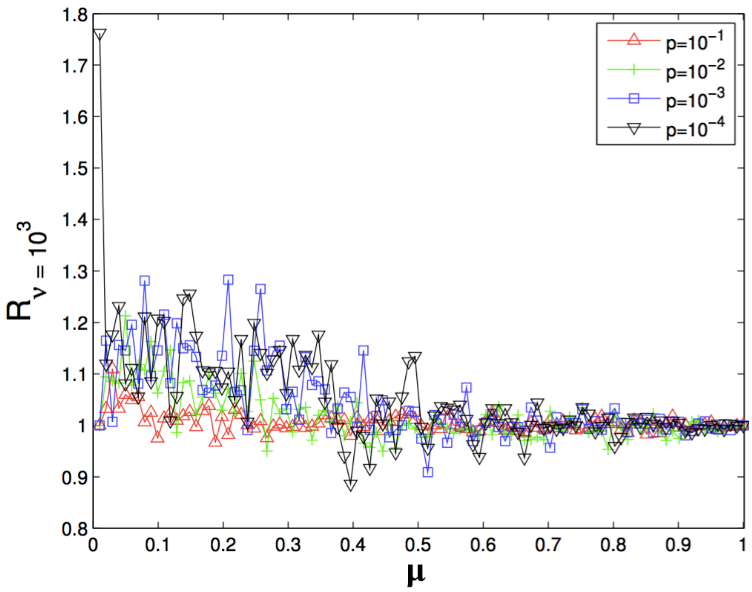


Figure 2.6. The relative market share of $R_{\nu=k}$ at $\nu = 10^3$ is investigated as a function of attention capacity ratio.

a theoretical framework for not only items but studying the propagation of any phenomena such as ideas or trends under limited attention.

The model is investigated analytically and by simulation. The analytical results agree with the simulations. As expected, strong advertisement forces every one to get the advertised item in all conditions.

Interestingly, when the attention capacity is small compared to the number of items, even a very weak advertisement can do the job. This behavior is analytically shown for the case of $M = 1$ and observed in the results of both simulations and analytic calculations as μ approaches to 0. This can be interpreted as when individuals have limited attention capacity, they tend to adopt what is promoted globally rather than recommended locally. We have also found that introducing more standard items to the market, is good for the market share of the advertised item. This observation may lead to interesting political consequences in terms of public attention and political administration. For example, public opinion can be kept under control by means of increasing the number of issues, possibly by means of artificial ones, so that the promoted idea is easily accepted by large audiences. This prediction calls for further investigation.

In this current work, there is a unique advertised item. The model can be extended to cover more than one promoted items. All selections are uniformly at random. One may investigate the effects of some other selection mechanism as in [75]. We have a complete graph as the graph of interactions. One can investigate other graphs of interactions such as Scale-Free, Small-World, regular or random graphs. The structure of interactions can also be improved by introducing a radius of influence. One may extend the model by introducing the concept of quality for items or letting agents prefer some items intrinsically as in [61].

3. COOPERATION UNDER LIMITED ATTENTION

"Evolutionary progress, the construction of new features, often requires the cooperation of simpler parts that are already available. For example, replicating molecules had to cooperate to form the first cells. Single cells had to cooperate to form the first multicellular organisms. Animals cooperate to form social structures, groups, and societies. Humans cooperate on a large scale, giving rise to cities, states, and countries. Cooperation allows specialization. Nobody needs to know everything. But cooperation is always vulnerable to exploitation by defectors."

- Martin A. Nowak [76].

These words are borrowed from Nowak's remarkable book on evolutionary dynamics [76]. They do not only emphasize the importance of cooperation in the emergence of higher-level features but also how emergent higher-level features "grow" from bottom-up. I don't think this is what Nowak has intented since he is best known for successfully using mathematics, the top-down approach, for modeling.

We think cooperation and competition as two sides of a very old coin. In this chapter and in the forthcoming chapter, we ask how cooperation can be maintained in a competitive environment. Unlike Nowak, we will use ABMs to investigate cooperation. Nevertheless, we appreciate the main motto of systems thinking, *structure generates the behavior* [3]. If the structure - that defines who interacts with whom - changes, we believe cooperation can arise in lieu of competition. When doing agent-based modeling, the spatial environment is the first thing that comes to mind to adjust the rate of interactions among individuals [77]. For example, cooperation can flourish if some landscape barriers are placed into the environment to avoid interaction with defectors. We think of a more sophisticated version of landscape barriers. In our setting, the information stored in the memory of agents will play the role of mindscape barriers. We assume that agents have the ability to choice and refusal of their partners. That is, they have a preference for not playing with defectors. In this chapter, we investigate how agents should manage their attention in order to foster cooperation. We find out that attention to defectors fosters more cooperation. That finding supports the evidence on the negativity bias [44].

3.1. Iterated Prisoners Dilemma with Limited Attention

"Keep your friends close and your enemies closer"
- Michael Corleone, Godfather II

"Know your enemy and know yourself and you will always be victorious."
- Sun Tzu [2].

How attention scarcity effects the outcomes of a game? We present our findings on a version of the Iterated Prisoners Dilemma (IPD) game in which players can accept or refuse to play with their partner. We study the memory size effect on determining the right partner to interact with. We investigate the conditions under which the cooperators are more likely to be advantageous than the defectors. This work demonstrates that, in order to beat defection, players do not need a full memorization of each action of all opponents. There exists a critical attention capacity threshold to beat defectors. This threshold depends not only on the ratio of the defectors in the population but also the attention allocation strategy of the players.

3.1.1. Introduction

Games and economic models are more related than one can imagine [78]. This is also the case for social interactions. A simplistic virtual setting for simulating trust in an e-commerce setting, would be the Iterated Prisoners Dilemma game which is, by its nature, very related to the evolution of trust [79, 80]. Each transaction in an e-commerce setting can be viewed as a round in a iterated prisoner's dilemma game. Adherence to electronic contracts or providing services with good quality can be considered as cooperation while the temptation to act deceptively for immediate gain can be considered as deception.

Economy is the study of how to allocate scarce resources. According to Davenport, the scarcest resource of today is nothing but attention [55]. Attention scarcity is first stated by Herbert Simon. He says that, "What information consumes is rather obvious: it consumes the attention of its recipients" [39]. The new digital age has come with its vast amount of immediately available information that exceeds our information processing power. Thus, attention scarcity is a natural consequence of huge amount of information. Attention is very critical to any kind of interaction, especially in the era of digital age. Conventional Economy has been transforming itself to the Attention Economy [55, 81–83]. Games should do the same. Little work has been done on games with limited attention. How attention scarcity effects the game? We will discuss attention games on the specific context of Iterated Prisoners Dilemma.

3.1.1.1. Iterated Prisoners Dilemma Game. Prisoners Dilemma game is one of the commonly studied social experiment [80, 84–87]. Two players should simultaneously select one of the two actions: cooperation or defection, and play accordingly with each other. Dependent on their choices, they receive different payoffs as seen in Figure 3.1.

Table 3.1. Standard IPD payoff matrix.

	Cooperate	Defect
Cooperate	R, R	S, T
Defect	T, S	P, P

Payoff matrix can be described by the following simple rules. In the case of mutual cooperation, both players receive the *reward* payoff, R. If one cooperates, while the other defects, cooperator gets the *sucker's* payoff, S while the defector gets *temptation* payoff, T. In the case of mutual defection, both get the *punishment* payoff P. Payoff matrix should satisfy the inequality $S < P < R < T$ and the additional constraint $T + S < 2R$ for repeated interactions. Rationality leads to defection, because $R < T$ and $S < P$ makes defection better than cooperation. But, at the same time, $P < R$ implies that mutual cooperation is superior to mutual defection. So, rationality fails and this situation is referred as a dilemma.

It is well known that the defection is the individually reasonable behavior that leads to a situation in which everyone is worse off [84]. On the other hand, cooperation results in the maximization of the joint outcomes [87].

If two players play prisoners dilemma more than once and they remember previous actions of their opponent and change their strategy accordingly, the game is called *Iterated Prisoners Dilemma* (*IPD*) [86]. Despite its level of abstraction, a large variety of situation from daily life (i.e. stop or go on when the red light is on?) to socio-economic relations (i.e. fulfill or renege on trade obligations?) may be represented as an IPD game. It is shown that repeated encounters between the same individuals foster cooperation. This is often referred as the *shadow of the future*. If individuals are likely to interact again in the future, this allows for the return of an altruistic act [84].

<u>3.1.1.2. Attention in Games.</u> Generally, a player is not capable of knowing all the players in an interacting environment and usually act based on limited information. One reason could be the huge number of players, or another could be that the players may have a very limited memory size to be informed of all the others [49, 62]. For example, in real life, a market has a few market leaders and many small brands whose number, in general, simply too big for an consumer to remember all. Therefore, a consumer can only have access to a limited number of service providers. The essence of any game is to interact with other players and get a chance to improve the payoff one gets. To interact with others, one should first capture their attention in a positive manner. When we give our attention to something, we are always taking it away from something else. We can think of having attention as owning a kind of property. This property is located in the memory of player.

<u>3.1.1.3. IPD Game Under Limited Attention.</u> In many studies related to IPD game, it is assumed that there exists enough memory to remember all the previously encountered players and their actions. Memory is an important aspect, because knowing the identity and history of an opponent allows one to respond in an appropriate manner. We use the term *limited attention* to indicate the existence of an upper bound on how many distinct encounters are remembered by a player. We ask the following reasonable question, as in [49], what if the memory size is limited? Same question can be reformulated as follows: what if attention capacity is limited? In this study, we introduce attention capacity as an important parameter to investigate the dynamics of the mentioned game.

3.1.2. Model

Tesfatsion introduced choice and refusal to IPD games [80]. In order to choose or to refuse an opponent, players should be able to remember the identity of each player and their past behaviors. It is known that choice helps players to find cooperation and refusal lets them escape from defection [80]. In our very simplistic model, we consider that there exist two type of players: *cooperators*, which always cooperate, and *defectors*, which always defect. We combine these pure strategies with a simple choice-and-refusal rule: If player knows that the opponent is a defector, then she refuses to play. Otherwise she plays.

Each round of the IPD game consumes the limited attention of its players. We assume that every player has the same *attention capacity* M. When a player encounters with an opponent, it stores the necessary information related to the opponent's action in its memory. After playing with M different opponent, the attention capacity fills up. As the player encounters with more opponents, he will have the problem of attention scarcity. He has to forget previously encountered ones. For the efficient usage of memory, one needs to decide whom to forget? In this respect, we will discuss 5 different attention allocation strategies in Section 3.1.5. Like the rest of the literature, we focus on the conditions under which "cooperative move" becomes more favorable. But our research distinctively considers that the game takes place in a world with limited attention.

The personality of a player (cooperator or defector) is randomly set. Remember that once it is set, it never changes. In each iteration, two individuals are randomly chosen to play the game. In this respect, there is no spatial pattern. One considers the underlying interaction graph is a complete graph.

Let $\mathcal{C}$ and $\mathcal{D}$ denote the sets of cooperator and defector players, respectively. Let $\mathcal{N}$ denote the set of all players, that is, $\mathcal{N} = \mathcal{C} \cup \mathcal{D}$. The number of defectors is denoted by $|\mathcal{D}|$. Thus the remaining $|\mathcal{C}| = N - |\mathcal{D}|$ players are the cooperators where $N = |\mathcal{N}|$. We define our model parameters *attention capacity ratio* and *defector ratio* as $\mu = \frac{M}{N}$

and $\delta = \frac{|D|}{N}$, respectively. Hence, we have $0 \leq \mu \leq 1$ and $0 \leq \delta \leq 1$.

We use the de facto payoff values of $(S, P, R, T) = (0, 1, 3, 5)$ throughout in this study.

3.1.3. Evaluation Metrics

Social welfare can be measured by the average payoff of players. The payoffs of all the encounters are added up to have the final outcome of each player. To make a comparison between the defectors and the cooperators, we take the average outcome of each. Let c_i and d_i be the numbers of games, player i plays with cooperators and defectors, respectively. We use the payoff matrix given in Figure 3.1 to calculate the total payoff of player i as follows:

$$\text{payoff}(i) = \begin{cases} Rc_i + Sd_i, & i \in \mathcal{C}, \\ Tc_i + Pd_i, & \text{otherwise}. \end{cases}$$

We evaluated our results by a comparison between the average performances of the cooperators and the average performances of the defectors. Our performance metrics are as follows:

$$\overline{P_C} = \frac{1}{|C|} \sum_{i \in \mathcal{C}} \text{payoff}(i) \quad \text{and} \quad \overline{P_D} = \frac{1}{|D|} \sum_{i \in \mathcal{D}} \text{payoff}(i).$$

Although further investigations calls for simulations, some analytical investigation of average performances is possible as follows.

<u>3.1.3.1. Cooperator's Average Performance.</u> Cooperator's average performance of $\overline{P_C}$ can be analytically found. For a cooperator, playing with a defector means no gain, since sucker's payoff is equal to zero, that is, $S = 0$. $\overline{P_C}$ can only increase if two cooperators play a round with each other. When two cooperators are selected to play with each other, each cooperator gets $R = 3$ points. The probability of matching two

cooperators is equal to $(1-\delta)^2$ for large N. Among $\mathcal{T} = \nu\frac{N^2}{2}$ rounds, only $(1-\delta)^2\mathcal{T}$ of them are expected to occur between two cooperators, and each gain R. As a result, $|C| = (1-\delta)N$ cooperators share $(2R)(1-\delta)^2\nu\frac{N^2}{2}$ payoffs. In other words,

$$\overline{P_C} = \frac{2R(1-\delta)^2\nu\frac{N^2}{2}}{(1-\delta)N} = R(1-\delta)\nu N.$$

Without any further investigation, we can conclude that increasing ν, N and R is favorable for $\overline{P_C}$ while increasing δ is not. Note that, neither attention capacity M nor any attention allocation strategy has no effect in this setting. If the population is composed of only cooperators, that is $|C| = N$ and $\delta = 0$, $\overline{P_C}$ would be $R\nu N$.

3.1.3.2. Defector's Average Performance. Because of the choice and refusal rule, if an opponent is known to be a defector, no player plays with him. Therefore in order to obtain the defector's average performance of $\overline{P_D}$, we need the probability of defector $j \in D$ to be unknown by player $i \in \mathcal{N}$. This probability can not be analytically found except for the special cases of players without memory and players with unlimited memory.

- *Players without memory.* When players have no memory, i.e. attention capacity is zero, they are totally forgetful and remember nothing. Note that this case actually corresponds to player playing prisoners dilemma without realising that they are playing repeatedly. As a result, players continue to play with defectors in spite of the choice and refusal rule. The probability of matching a defector with a cooperator is equal to $2\delta(1-\delta)$ and matching two defectors is equal to δ^2. Therefore for the special case of $\mu = 0$, we have

$$\overline{P_D} = \frac{\big(T\ 2\delta(1-\delta) + 2P\ \delta^2\big)\nu\frac{N^2}{2}}{\delta N} = \big(T\ (1-\delta) + P\ \delta\big)\nu N$$

Then for $T = 5$ and $P = 1$, we have $\overline{P_D} = (5-4\delta)\nu N$. We observe that increasing the number of defectors is not favorable even for defectors. Nevertheless, it is easy to verify that for $\mu = 0$, $\overline{P_D}$ is always greater than $\overline{P_C}$ which can be stated as *defection is the favorable action against the players with no memory.*

- *Players with unlimited memory.* For the special case of $M \geq N$, the players are no longer forgetful and they are able to remember each opponent's last action. Because of the choice and refusal system, any defector can play at most $|C|$ rounds with cooperators and $|D|-1$ rounds with defectors. Therefore for sufficiently large ν, we have

$$\overline{P_D} = T|C| + P(|D| - 1) = (P - T)|D| + TN - P.$$

We can conclude that as we increase the number of defectors in this setting, the average payoff of the defectors again decreases.

3.1.4. Simulations

The dynamics of the system is further investigated by simulation as the attention capacity ratio μ and the defectors ratio δ vary. The model is simulated for every possible attention capacity values of M (from 0 to N) and for every possible number of defectors (from 0 to N). We study a population of $N = 100$.

The number of iterations, $\mathcal{T}$, is another critical issue. It is set to $\mathcal{T} = \nu\frac{N^2}{2}$ since there are $\binom{N}{2}$ pairs, where ν being the third model parameter, is the number of plays for a pair of players. Note that, when $\nu = 1$, no two players are expected to meet again during the simulation. This situation corresponds to non-iterated version of the game. In order to see the effect of time ν is set to 2 and 5. The results were averaged over 20 independent realisations for every combination of parameter values.

3.1.4.1. Attention Allocation Strategies. Some people are positive and remember only the good memories. On the opposite, some remember the bad events and live to get his revenge. Motivated by these, we make comparison of 5 simple attention allocation strategies based on forget mechanisms:

(i) Players that prefer to forget only cooperators, denoted by *FOC*.

(ii) Players that prefer to forget only defectors, denoted by *FOD*.

(iii) When players have no preference, they can select someone, uniformly at random, to forget. We call this strategy as *FAR*.

(iv) Players may also prefer to use coin flips to decide which type, namely, cooperators or defectors, of player to forget. Once the type is decided, someone among that type is randomly selected and forgotten. Let *FEQ* denote this "equal probability" to types approach.

(v) If the knowledge of which type has the majority is available, this extra information can be used in devising a strategy. One possibly effective strategy could be to assume the opponent be of the type of majority, hence, pay attention to the minorities only. That is, one prefers to forget majority which we call *FMJ* strategy.

We investigate the average performances of cooperators and defectors when they use the same strategy.

3.1.5. Observations

In this section, for a more general view, we present our observations based on our simulation data. With our essential parameters of μ, δ, and ν along with the different attention allocation strategies, we can determine the conditions under which cooperation is more favorable than defection.

Simulation results for various attention capacity ratio μ and defector ratio δ values are given in Figure 3.1. Columns of Figure 3.1 correspond to five strategies. Within a column, the top plot provides the average performance of cooperators, $\overline{P_C}$, as a function of μ and δ. Similarly, the middle gives the average performance of defectors. The bottom plot is the difference of the averages. Note that being a cooperator is better when $\overline{P_C} - \overline{P_D} > 0$. For comparison purposes, $\overline{P_C} - \overline{P_D} = 0$ curves for different attention allocation strategies are superposed in Figure 3.2(b).

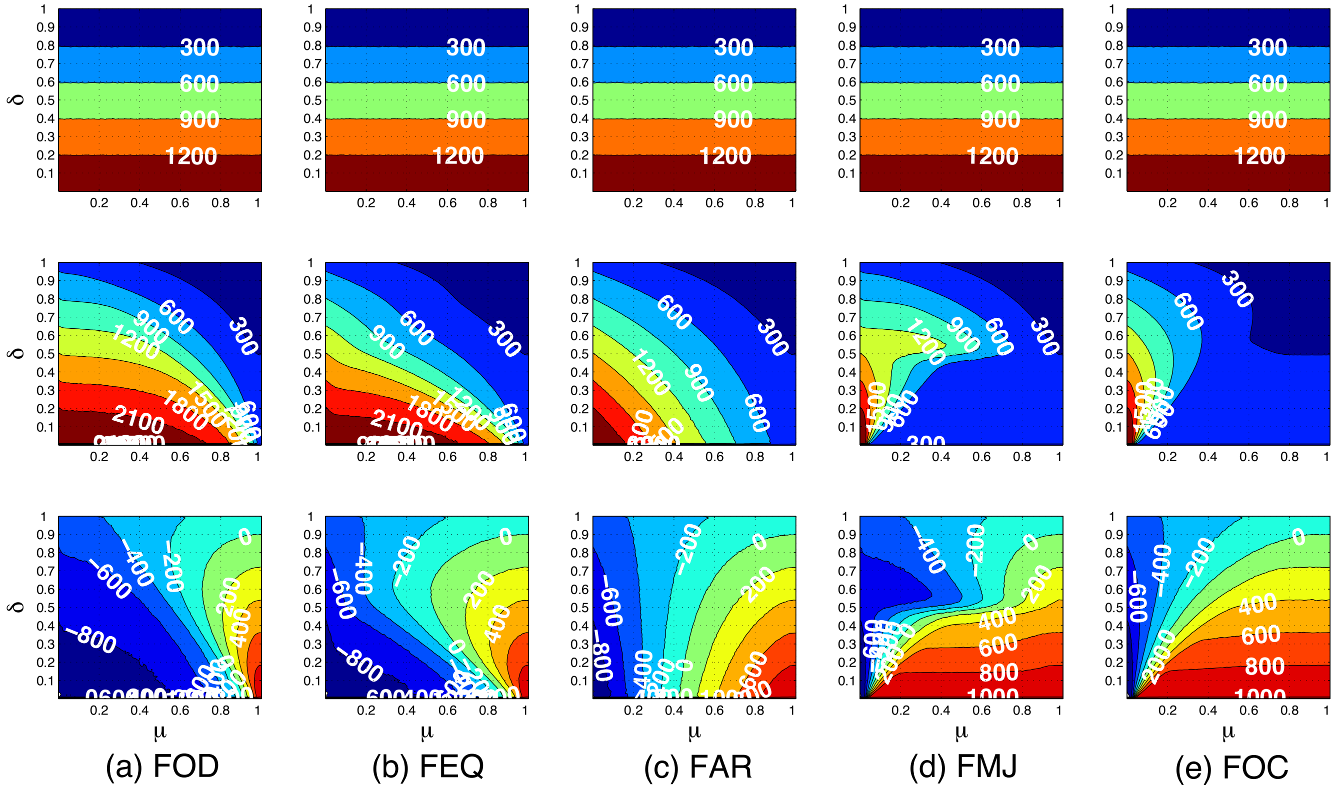


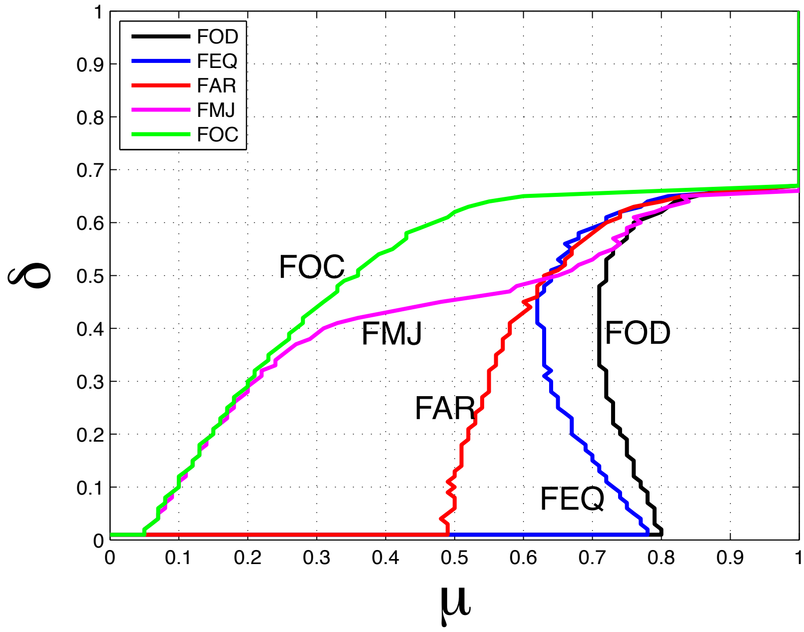
Figure 3.1. Average performances as a function of attention capacity ratio μ and defector ratio δ. The columns represent five strategies. The rows represent $\overline{P_C}$, $\overline{P_D}$ and $\overline{P_C} - \overline{P_D}$ values, respectively.

<u>3.1.5.1. Average Performance of Cooperators.</u> Findings from the first row of Figure 3.1 are as follows: (i) Interestingly, cooperator's average payoff does not change significantly neither by attention capacity ratio nor by attention allocation strategy. (ii) But the defector ratio has a negative effect on the average performances of cooperators. Our analytical explanation given in Section 3.1.3.1 is in agreement with these findings. For any δ values, $\overline{P_C} = R(1-\delta)\nu N$ gives exactly the same results seen the first row of Figure 3.1.

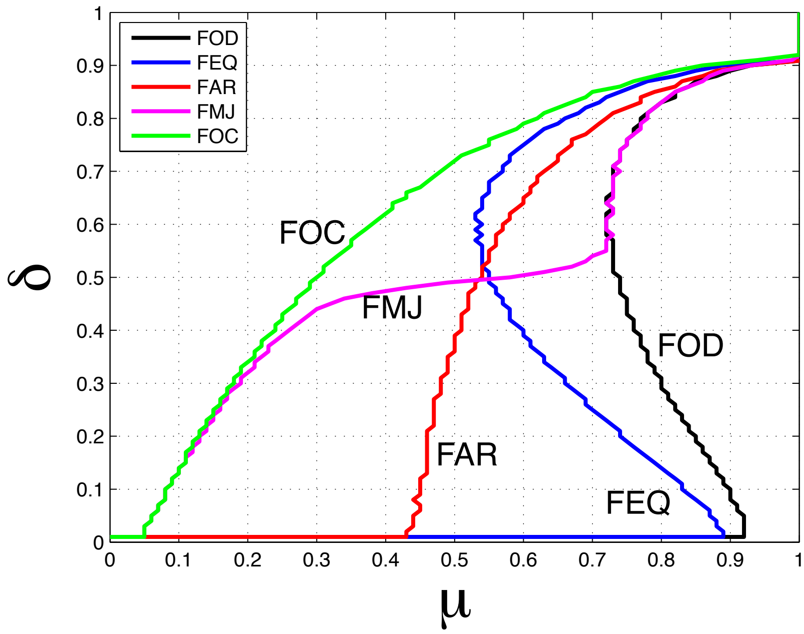
<u>3.1.5.2. Average Performance of Defectors.</u> The second row of Figure 3.1 can be interpreted as follows: (i) Greater attention capacity, i.e. increase in μ, helps players to remember the defectors. As a result, defectors experience social isolation and their average payoff severely diminishes. (ii) Increase in the number of defectors, i.e. increase in δ, leads competition among them. Thus, defectors average payoff again diminishes. (iii) Note that all five plots are in agreement with our discussion in Section 3.1.3.2 and Section 3.1.3.2 for the special cases of $\mu = 0$ and $\mu = 1$.

<u>3.1.5.3. Attention Boundaries.</u> We call the $\overline{P_C} - \overline{P_D} = 0$ contour lines, seen in the third row of Figure 3.1, as the *attention boundaries.* An attention boundary determines the favorable action. If a pair of (μ, δ) remains inside the attention boundary, it means $\overline{P_C} - \overline{P_D} > 0$ and cooperation is the favorable action, otherwise defection is the favorable action. Attention boundaries for five different attention allocation strategies seen in Figure 3.1, are visually superposed in Figure 3.2(b) for comparison purposes.

For a given defector ratio, we observe that there is a critical threshold for attention capacity, below which defection is advantageous, and above which cooperation becomes the favorable action. With lesser attention capacity, defectors can be easily overlooked. Greater attention capacity along with the choice-and-refusal rule do not let defectors to improve their payoffs. Due to the degrading of defector's performance, the average payoff of cooperators manages to exceed that of defectors when players have a greater attention capacity.



(a) $\nu = 2$



(b) $\nu = 5$

Figure 3.2. Attention boundaries of different allocation strategies are visualized in the same figure for comparison purposes.

We consider a strategy better if it has a larger area, where cooperators are doing better than defectors, in the μ, δ plain. That is, a better strategy has more (μ, δ) pairs below its attention boundary. From this perspective, the best strategy is FOC, and the worst one is FOD. All the remaining strategies are situated in between these two strategies.

The forget majority, FMJ, is a mixed strategy. When $0.5 < \delta$, defectors are the majority and FMJ acts as if forget only defectors, i.e. FOD. When $\delta < 0.5$, cooperators are the majority. Thus FMJ switches to forget only cooperator, i.e. FOC. Therefore its plot is similar to that of FOD for $0 < \delta < 0.5$ and that of FOC for $0.5 < \delta < 1$. FMJ strategy can be put differently as allocation of the minority. One can think this strategy is better than the rest, since scarcity, in general, triggers the perception of greater importance. Nevertheless, Figure 3.2(b) is against this intuition. The optimal strategy is to forget only cooperators, i.e. FOC. By doing so, players achieve to allocate their memories for only defectors. In other words, they keep their enemies closer. Thus, they become more prudent to the defectors. On the other hand, forgetting defectors seems to be the most wasteful and carefree attention consuming habit. We observe that the necessary information for refusing the defectors is dismissed while applying the FOD strategy.

The critical value of $\delta = 0.5$ determines which strategy is superior, except for the two extreme strategies of FOD and FOC. FEQ does better than FAR when $0.5 < \delta$ and FAR does better than FEQ when $\delta < 0.5$. Even if FAR strategy seems identical to FEQ strategy, there exists a slight difference between them. Notice that, forgetting at random depends on the content of the memory, while forgetting with equal probability does not. Higher defector's ratio, that is $0.5 < \delta$, causes one to encounter with more defectors. In that case, memories of the players would be plentiful with defective experiences. Thus, forgetting at random would be more biased towards to FOD. Similarly, forgetting at random would be more biased towards to FOC when $\delta < 0.5$.

3.1.5.4. Effect of Time. Literature on IPD game suggests that as number of iterations increases cooperative behavior also increases among players [84]. This is also verified by our simulations. The shadow of the future can be quantified by the parameter of ν. A short shadow of the future (lesser ν), hinders the detection of the defectors. When future of the shadow is longer, lesser attention capacity would be sufficient for cooperators to beat defectors. As ν increases, defector's performance gets worse in comparison to cooperators. Attention boundaries obtained by setting $\nu = 2$ and $\nu = 5$ are given in Figure 3.2(a) and Figure 3.2(b), respectively. The area inside the attention boundaries is much larger in Figure 3.2(b) than Figure 3.2(a). This finding suggests that the shadow of the future fosters cooperation.

3.1.6. Conclusions

We observe that as the proportion of the defectors increases, the average payoff for any player decreases. On the other hand, increase in the attention capacity has different outcomes for cooperators and defectors. As attention capacity increases, the change in the cooperators overall performance is almost negligible, but defectors performance significantly diminishes. The rule of choice-and-refusal plays an important role in this situation. Nevertheless, it is worth to point out that, even the choice-and-refusal alone, can not fulfill the desired goal without passing some threshold value of attention capacity. As attention capacity increases, or the shadow of the future gets longer, the detection of the defectors gets feasible, consequently defectors face with social isolation due to the rule of choice-and-refusal. As a result, cooperators performance, exceeds the defectors performance. Thus, cooperation becomes the favorable action. This work demonstrates that, in order to beat defection players do not need a full memorization of each action of all opponents. This finding is really important especially in the world of limited attention. We also investigate five different attention allocation strategies and we find out that the best strategy is “forgetting only cooperators”. By applying this strategy, one becomes more prudent to the deceptive actions. To our conclusion, attention must be selective, and it should be directed towards the defectors and their defective moves.

In this present work, player are pure cooperators or pure defectors. They never change their character. Various forgetting strategies are investigated but both the cooperators and the defectors use the very same strategy in the game. Cooperators using one strategy while defectors using another is left as future work. It would be also interesting to study the effect of the biased payoff matrix. As a future work, we plan to investigate other means for fostering cooperation, even in the conditions of attention scarcity. To achieve this goal, we can make use of other's experiences by taking recommendations to determine with whom to play. But from whom to take advice is very critical and must be well studied to clarify which collaboration strategy is better. We will also extend our work to the mixed strategies for interaction, such as mostly defect and mostly cooperate.

4. CO-EVOLUTION OF MEMORY AND COOPERATION

In this chapter, we highly increase the complexity of our previous model, presented in Chapter 3, and investigate the co-evolutionary dynamics of memory and cooperation among agents that are "hard-wired" to direct their attention to defectors. That is, as long as agents have enough memory size they will preferentially remember defectors and will be able to refuse to play with defectors. What happened then, is at first sight was shocking. In lieu of having a high memory size and refusing defectors, subsequent generations get rid of their memories in order to be able to play with defectors. Refusing a defector that does not cause any harm turns out to be an evolutionarily wrong attitude. From the perspective of systems science, to clear off selfish behavior from a population of individuals, it is useless to put the blame on some selfish guy. Instead, we should be able to create an ecology in which selfish behavior turns out to be unnecessary [88]. So time has come to revise our hypothesis. Solely, paying attention to defectors doesn't help cooperation for the standard non-negative payoffs of the game. We reformulate the payoff matrix structure in order to incorporate negative payoffs for receiving a defection. We have shown that cooperation is a by-product of ecological rationality. Where there is an appropriate level of threat for receiving a defection (in terms of negative payoffs) agents are nudged to cooperate and when there is none/extreme threat, agents tend to defect.

When it comes to evolution, surprises are plenty. We observe that subsequent generations develop an emergent ability for adjusting their average memory size according to the abundance of defectors in the previous population. An increase in the number of defectors, causes an increase in memory size. And a decrease in the number of defectors, causes the average memory size to shrink. Some kind of immunity against harmful defection has emerged. As a scientist in complex systems modeling, when we see something different from the micro-level emerging at the macro level, we feel that all our efforts have paid off. Maybe it is a weakness but we can't stop constructing analogies. Think about how an increase in the abundance of harmful pathogens causes an increase in the abundance of immune cells recognizing that pathogens. Literally, to deal with the astronomical number of possible pathogens, our actual immune system takes advantage of randomness and diversity in the recognition of threatening pathogens. Recognition occurs if an immune cell binds well to the pathogen. The immune cells that are able to recognize threats, reproduce more and others are eliminated. An evolutionary race between our immune cells and pathogens, keeps going on within our body [89].

4.1. The Dose of the Threat Makes the Resistance for Cooperation

> *"All things are poison and nothing is without poison; only the dose makes a thing not a poison."*
>
> - Paracelsus, Father of Toxicology

> *"When the water rises, the fish eat the ants; when the water falls, the ants eat the fish."*
>
> - Lao Proverb

We propose to reformulate the payoff matrix structure of Prisoner's Dilemma Game, by introducing threat and greed factors, and show their effect on the co-evolution of memory and cooperation. Our findings are as follows. (i) Memory protects cooperation. (ii) To our surprise, greater memory size is unfavorable to evolutionary success when there is no threat. In the absence of threat, subsequent generations lose their memory and are consequently invaded by defectors. (iii) In contrast, the presence of an appropriate level of threat triggers the emergence of a self-protection mechanism for cooperation, which manifests itself as an increase in memory size within subsequent generations. On the evolutionary level, memory size acts like an immune response of the generations against aggressive defection. (iv) Even more extreme threat results again in defection. Our findings boil down to the following: The dose of the threat makes the resistance for cooperation.

4.1.1. Introduction

Taking cooperative actions against a common threat, is frequently seen in nature and in history as well. Herbert Spencer puts it as follows, "Only by imperative need for combination in war were primitive men led into cooperation" [90]. Individuals, as a response to what they perceive as threat, bind together and tend to move as a unit. Similar collective spirit, can also be seen in fish swimming in schools or birds flying in flocks. The waves of agitation in schools or flocks are nothing but an escape maneuver from an attack of a predator [91]. Kin selection, direct reciprocity, indirect reciprocity, group selection and limited local interactions are shown to be five powerful determinants of cooperation [92, 93]. Yet, explaining cooperation still remains one of the greatest challenges across disciplines [94]. Here, we discuss the dose of the threat imposed by environment as another way to obtain cooperation.

In [95], Robert Wright says, "interaction among individual genes, or cells, or animals, among interest groups, or nations, or corporations, can be viewed through the lenses of game theory". Nevertheless, the amount of information stemming from the huge number of interactions, can easily exceed the processing capabilities of the interacting parties. This is also referred as attention scarcity problem in the literature [40, 83]. In our previous work, we coined the term *Attention Game* to define an interacting environment where players can only pay attention to a portion of the information they receive [41]. We worked on attention games in a specific context of Iterated Prisoner's Dilemma (*IPD*).

Evolutionary game theory applies mathematical and computational techniques to study the evolution of cooperation. For an important review on co-evolutionary processes, see [46]. It is shown that choice and refusal of partners accelerates the emergence of cooperation [96]. Memory is a prerequisite for engaging in reciprocity and also for partner selection on the basis of past encounters. In memory-based Prisoner's Dilemma Games, each player can keep track of only a limited number of the previous rounds for all of its partners [97]. This limited number is defined as the *memory length* [98, 99]. Tit-for-Tat, the winner of the Axelrod's tournament, is a memory-one strategy. It starts with cooperation and afterwards imitates the last action of its partners [79]. Thus the memory length of agents using Tit-for-Tat is one, even though they keep track of all of their partners. Dunbar's number indicates a cognitive limit to the number of individuals with whom one can maintain stable relationships [100].

We think that the ability to keep track of all potential game partners is not always possible. This may be thought as a natural consequence of huge amount of game partners or a very limited memory size to be informed of all. The concept of memory in Prisoner's Dilemma, is generally explored in terms of historical time-dependency of previous rounds [101–103]. Differently, from our perspective, the term *memory size* indicates the number of potential game partners one can keep track of. In our model, agents store a very brief information about the general behavior of a limited number of their partners. This information will be used to distinguish defectors from cooperators.

Evolutionary psychologists demonstrated that social exchange in a group requires the existence of some mechanisms for detecting cheaters, but do not require any mechanisms for detecting altruists [104]. Similarly, in our previous work we found that it is crucial for attention to be focused on defectors in order to foster cooperation when agents have insufficient memory size [41]. That is, attention should be allocated in such a way that agents should keep remembering defectors, and forget preferentially cooperators whenever memory is exhausted. In [41], memory size does not differ from one player to another and there exists only two type of players such as pure cooperators and pure defectors. We will use a similar attention mechanism focusing on defectors. In this study, we will introduce heterogeneity to our work by allowing agents to have different memory sizes and strategies. We will investigate how the characteristics of agents evolve from generation to generation.

The essence of how selfish beings manage to cooperate is captured by the payoff matrix of the IPD game. Axelrod used the fixed payoff matrix given in Figure 4.1(c) for his tournament [79]. A natural extension would be to investigate the impact of different payoff entries. Many studies use payoff matrices with positive payoffs. Some works on negative payoffs are also done. To this end, some researchers prefer to fix the two selected entries of the payoff matrix and explore the effect of the change in the other two entries [77, 92, 105]. In the so-called *Donation game*, given in Figure 4.1(d), cooperation corresponds to offering the other player a benefit b at a personal cost c and defection corresponds to offering nothing. Nowak has investigated the effect of these two essential parameters for various situations [92]. This is a very agreeable representation, but it requires $P = 0$. So it does not allow to study the dynamics of cooperation where the punishment payoff P is positive or negative. In [77], Epstein investigated the case of negative sucker and punishment payoffs for a special case of $S = -T$ and $P = -R$, given in Figure 4.1(e). We also want to study the case where receiving a defection leads to negative payoffs. Differently, we interpret the case of $S < P < 0$ as the presence of threat. For $S < P < 0$, the decrease in the negative values of S and P corresponds to the increasing level of threat. From our perspective, non-negative values of S and P corresponds to the absence of threat. To investigate the effect of threat, we propose to use a more general parametric payoff matrices of the form Figure 4.1(b) which covers the family of payoff matrices of the form given in Figure 4.1(d) and Figure 4.1(e).

	C	**D**
C	R	S
D	T	P

(a) Generic PD

	C	**D**	
C	$1-\beta$	$1-\alpha-\beta$	β
D	1	$1-\alpha$	
	α		

(b) Threat Game

	C	**D**
C	3	0
D	5	1

(c) Axelrod

	C	**D**	
C	$b-c$	$-c$	c
D	b	0	
	b		

(d) Donation Game

	C	**D**	
C	R	$-T$	$T-R$
D	T	$-R$	
	$T+R$		

(e) Epstein

Figure 4.1. Payoff matrices including threat game.

This work is structured as follows: In the next section we explain our motivation and in Section 4.1.3 we present our agent-based model and give the technical details of the simulations for a generic payoff matrix. In Section 4.1.4, we provide the results for two specific payoff matrices: (i) one with all non-negative entries, and (ii) the other with negative entries for sucker and punishment payoffs. In Section 4.2, we generalized the payoff matrices with two parameters. Finally, in Section 4.3 we summarize our findings and construct some analogies with various disciplines.

4.1.2. Motivation

Consider selfish agents playing evolutionary IPD game. Assume that the fitness of agent is correlated with its accumulated payoffs. Then, in order to increase its fitness, a selfish agent tries to maximize its gain at every single round of the game. Suppose agents have the right to choose or refuse to play. If all the entries of the payoff matrix are non-negative, should an agent choose to play with every opponent whether it is a defector or not?

Agents with myopic view may prefer immediate positive outcomes in the short-term at the expense of longer-term outcomes. When interacting with an opponent brings relatively low payoff, the agent will accumulate less payoffs compared to that opponent and at the end, the agent will have a lower chance to reproduce. This is the case of cooperator playing against defector. If only cooperators could have find a way to distinguish defectors from cooperators and refuse to play with the defectors, then the cooperators can outcompete the isolated defectors. So the macro-level dynamics of the population depends on the mixture of agents: how cooperative and with whom willing to play they are.

It is not possible for cooperation to flourish in a well-mixed population without any mechanism that give cooperators the ability to quarantine defectors. Spatial structure can promote cooperation by introducing physical barriers against interaction with defectors. Static networks lack the ability for modeling the dynamical interactions [106]. So, recent advances make emphasis on the co-evolution of strategy and environment [46]. In this study, we follow a different path in order to promote cooperation. In lieu of considering spatially structured population in physical space, we will consider conceptually structured populations. Agents will have mental representations of other agents and they will have the ability to choose with whom to interact. Our proposition fits nicely to the research line of conditional strategies [107–109]. In our model, agents interact with all except the ones that they perceive and remember as defectors. Thus, memory plays the role of conceptual barriers for interaction with defectors. If we consider payoff matrices with negative values, for S and P, then the dynamics may become more complicated but the need for the refusal of defectors becomes more clear. For a cooperator, there was a risk of not gaining ($S = 0$) but now losing points becomes also a possibility ($S < 0$). Whenever P also becomes negative, defectors also face the risk of losing points. To identify the characteristic of the opponent and if it is defector not to play with it becomes an essential asset especially when receiving a defection leads to negative payoffs. We will consider risk of losing points as *threat*. Hence payoff matrices with negative entries, for sucker and punishment payoffs, are considered to be games with threat.

Our main research question, in this study, will be the following. What is the effect of increasing level of threat on the co-evolutionary dynamics of memory and cooperation?

4.1.3. Model

We propose an evolutionary game, where generations do not overlap. In our model, there are N agents playing IPD game within the generation. At the end of certain number of rounds the generation is terminated, and all the agents of the old generation are removed. Before the old agents are removed, they reproduce according to their fitnesses. Roulette wheel selection is applied N times to pick agents that will reproduce. Hence, the population size is kept constant at N. We set $N = 100$.

<u>4.1.3.1. Rounds.</u> Agents interact and try to increase their accumulated payoffs by means of playing a modified IPD [79]. In each round, two agents are selected uniformly at random and given the chance to play. Each selected agent has to decide whether to choose or to refuse to play with the given opponent. If at least one of them refuses to play, no playing takes place and the round is completed, hence their scores do not change. If both agents choose to play, then they play the usual Prisoner's Dilemma game. Each agent selects its action of either cooperate or defect. According to their joint actions, each collects its payoff based on the generic payoff matrix given in Figure 4.1(a). The payoff collected is added to the cumulative payoff, called *score*,

and the round is completed.

We want every pair of agents to be selected on average ν times. Therefore each generation lasts $\nu\binom{N}{2}$ rounds. We use $\nu = 30$.

<u>4.1.3.2. Choice and Refusal of Partners..</u> Agents have probabilistic behavior. An agent i simply chooses to defect with probability ρ_i, called *defection rate*, or to cooperate with $1 - \rho_i$. The defection rate of an agent is a property that never changes.

Choice and refusal to play is based on the agent's subjective perception of the opponent as a defector or a cooperator. The agent refuses to play with an opponent if the opponent is perceived as a defector. In order to decide whether the opponent is defector, agent uses its memory. The agent keeps track of the previous rounds with the opponents. That is, for every opponent, it keeps two numbers, namely, the total number of rounds played with the opponent and in how many of them the opponent has defected. The ratio of the number of received defections to the number of total rounds is called *perceived defection rate*. The opponent is perceived as *defector* if its perceived defection rate is fifty percent or more. Otherwise, the opponent is perceived as *cooperator*. As a third case, if there is no history about the opponent, it is considered as if it is cooperator. Namely, the default decision is to play.

We should give some intuition about agent's possible misperceptions due to the small number of interactions, at this point. Different agents can perceive the same agent differently, at the same time. Suppose agent i has a low defection rate which is greater than zero. Then, in most of the games it plays, it will cooperate and in a very few of them it will defect. Therefore it is expected that most of the agents consider it as "cooperator". But it is still possible that some agents can perceive it as a "defector" and refuse to play with it again. This may happen due to the small number of interactions with i, in which i happened to defect more than cooperate. In statistics, it is known that the small sample size may not be a good representative of a probability distribution. But our agents do not know it, like most of the people [110].

4.1.3.3. Memory. Remembering the history of the opponents calls for memory for each agent. Every agent has different size of memory. Let M_i denotes the number of opponents agent i can keep track of. The ratio $\mu_i = M_i/N$ is called the *memory ratio* of agent i. If $M_i \geq N$, then agents have enough space to recall every agent. Since this case is not interesting, we investigate the case where $M_i \leq N$ for all i. That is, agents do not have enough memory space to store the history of all opponents. Hence $\mu_i \in [0, 1]$.

If the number of rounds in a generation is big enough, an agent encounters with almost all agents and in order to keep the history of each opponent, it requires memory size of N. Suppose agents have limited memory size, i.e., $M < N$. Then after M different opponents, there is no room left for the $M + 1^{th}$ opponent. This requires a selective attention mechanism. Agents should decide which agents to keep in memory and which agent to forget. Our previous study indicates that it is a better strategy to focus on defectors rather then cooperators [41]. So if there are memory spaces reserved for cooperators, select a cooperator. If there is no cooperator left, then select a defector. Then forget the selected opponent and use this reclaimed space for the new opponent. Both cooperator or defector selections are done by uniformly at random.

4.1.3.4. Fitness. In evolutionary games, agents reproduce proportional to their fitness. We define *fitness* as a function of scores. Agents of a new generation start with zero scores. As they play, the payoffs obtained are added to the score. In the traditional Prisoner's Dilemma game, the payoffs are all non-negative such as $(S, P, R, T) = (0, 1, 3, 5)$. Hence playing will not decreases the score. In this study we also consider payoff matrices with negative entries, too. In the case of negative payoffs, the scores of agents may decrease and negative scores are possible. Therefore, using scores directly as fitnesses will not work. The mapping the scores to fitness values requires attention. We adjust the scores by subtracting the minimum score from all. After this, adjusted scores become all non-negative. Then obtain the normalize score by dividing the adjusted scores to the sum of all adjusted scores. Then, use the normalized scores as the fitness for reproduction. Note that the agent with the minimum score has the normalized score of zero. Hence it does not reproduce.

4.1.3.5. Reproduction. The reproduction is asexual. Each child has exactly one parent. The *genotype* of an agent i, is the pair (μ_i, ρ_i). A child gets the exact copy of the genotype of its parent if there is no mutation. With probability of κ, called *mutation rate*, there is a mutation. When there is a mutation, only one of the entries, selected at random, in the genotype is replaced with a new number drawed from a uniform distribution of $[0, 1]$. We use $\kappa = 0.05$.

4.1.3.6. Visualization. Note that there is a useful visualization for genotype (μ_i, ρ_i). Agents can be represented by points on the unit square of the μ-ρ plane where x-axis is the memory ratio μ and y-axis is the defection rate ρ. The point (μ_i, ρ_i) displays the genotype of the i'th agent. The *average defection rate* of the current population is given by

$$\overline{\mu} = \frac{1}{N}\sum_{i=1}^{N}\mu_i$$

and the *average memory ratio* of the current population is given by

$$\overline{\rho} = \frac{1}{N}\sum_{i=1}^{N}\rho_i$$

We can picture the *average genotype* $(\overline{\mu},\overline{\rho})$, as a point on that phase plane as in Figure 4.2.

4.1.3.7. Initialization and Termination. Once an initial generation is formed, system runs from one generation to the next with the given dynamics. The parameters of the agents of the initial generation are set randomly using uniform distributions. That is, for each agent i the values for the genes μ_i and ρ_i are set using a uniform distribution over $[0, 1]$. The number of generations, before the simulations are terminated, is another model parameter. We terminate our simulations after 500 generations.

4.1.4. Results

We run simulations using various payoff matrices. Initial population starts with an average genotype close to $(0.5, 0.5)$. Tracking the values of $(\overline{\mu}, \overline{\rho})$ pairs from generation to generation will make us see the co-evolution of cooperation and memory, as in Figure 4.2.

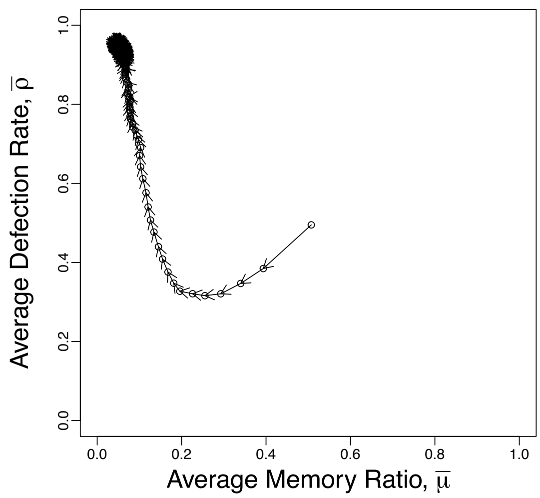


Figure 4.2. Co-evolution of average memory ratio $\bar{\mu}$ and average defection rate $\bar{\rho}$ through generations. $(S, P, R, T) = (0, 1, 3, 5)$.

<u>4.1.4.1. Absence of Threat.</u> We have discussed that payoff matrices with negative entries cause threat to the agents. We refer the case of non-negative payoffs, i.e., $0 \leq S < P < R < T$, as *absence of threat.* In the first set of simulations, we used the standard payoff values of $(S, P, R, T) = (0, 1, 3, 5)$. An averaged trajectory over 50 different realisations of the same initial population can be seen in Figure 4.2. Two dynamics are observed. (i) Average memory size tends to decrease independent of the average defection rate of the population. Neither cooperation nor defection favor greater memory size when there is no threat. (ii) Average defection rate decreases if memory size is high and increases if it is low. Average defection rate $\overline{\rho}$ decreases at the beginning since initial value of $\overline{\mu} = 0.5$ is relatively high. Interestingly, there is an unconditional decrease for $\overline{\mu}$. Once average memory ratio $\overline{\mu}$, becomes small enough, av-

erage defection rate $\overline{\rho}$ starts to increase. Memory size has a negative effect on defection rate, in other words, memory protects cooperation. Without memory, i.e., ($\overline{\mu} \to 0$), cooperation becomes vulnerable and defection succeeds, i.e., ($\overline{\rho} \to 1$). The average genotype of the population gradually reaches to a point very close to $(0, 1)$. That is, agents become memoryless and defective when there is no threat.

It is known that evolution may lead to unexpected paths. The observation of the unconditional decrease of the memory size is totally unexpected. To understand it, first consider a population that is composed of defectors only. Is it better for defectors to have greater memory size? The answer is no, as long as punishment payoff P is greater than zero. The reason is as follows: defectors with high memory size lose punishment payoff $P > 0$, just because they remember and refuse other defectors. Thus they end up with lower fitness and they are eliminated throughout the evolution. Consider the second extreme case where the population is composed of cooperators only. This case is a bit trickier. Previously we determined how agents perceive the world. Perception is open to mistakes as it is the case for real life. A cooperator with a low defection rate can be perceived as a defector, just because it is happened to defect more than cooperate within a small number of interactions. As a result of this misperception, high memory size can cause to avoid engaging rounds with agents whose intention is mostly cooperate. Cooperators with high memory size end up with lower fitnesses. The relative abundance of the cooperators with high memory in the subsequent generations decreases, and the relative abundance of the cooperators with low memory increases. This manifests itself as a reduction in the average memory size, $\overline{\mu}$.

The surprising downside of having a greater memory size is isolation, which leads to a deficient fitness. Thus, by means of mutations, subsequent generations get rid of their memory in the absence of threat. Without memory, defectors invade the subsequent generations.

4.1.4.2. Presence of Threat. We investigate the outcomes of an alternative formulation of negative payoffs, as in [77]. For $S = 0$, refusing or playing with a defector is apparently indifferent for a cooperator. Once S becomes negative, picture changes. From the perspective of defectors, it is still better to play whatever the opponent type is, as long as $S < 0 \leq P$. When P becomes negative, defectors have to be careful, too. It is known that evolution is about the survival of the most suited organisms for the current environmental conditions. When we use a different payoff matrix, environment differs and dynamics dramatically change. Let's define *aggressive defection* as an harmful act that reduces the score of the agents that are subjected to it. In the PD context, aggressive defection can be given with an additional constraint of $S < P < 0$. Now, receiving a defection results in negative values and it hurts. Thus having a greater memory size may become advantageous, in contrast to the case of non-negative payoff matrix.

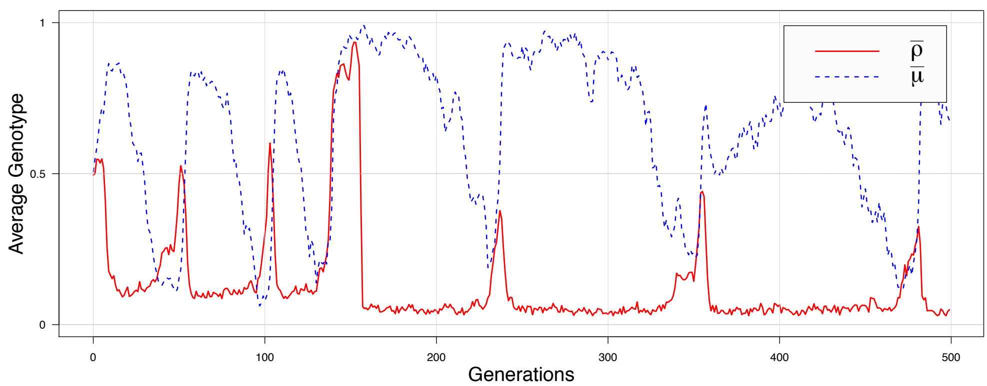


Figure 4.3. Co-evolution under aggressive defection for a single realisation. x-axis represents the generation steps and y-axis represents the average genotype of the population. $(S, P, R, T) = (-7, -6, 4, 5)$.

In the presence of threat, two dynamics begin to compete at the evolutionary level. (i) Tendency to increase memory size, in order to maintain self-protection when average defection rate gets higher. (ii) Tendency to decrease memory size, to avoid self-isolation when average defection rate gets lower. These two dynamics can give rise to oscillatory behaviours.

In Figure 4.3, we display the dynamics of a single realisation for a biased payoff matrix of $(S, P, R, T) = (-7, -6, 4, 5)$. At generation 0, simulation starts with a randomly generated initial population whose average memory ratio is relatively high, $\overline{\mu} = 0.5$. Agents with high memory size can protect themselves from defection. Thus defectors incur isolation and their fitness diminishes, $\overline{\rho} \to 0$. Eventually, cooperators with high memory size fill the population. When almost all agents turn out to be cooperator, around generation 20, misperception becomes an issue. High memory size may block interactions among cooperators and this is the reason why evolution prefers cooperators with smaller memory size, $\overline{\mu} \to 0$. Population without a valuable memory provides an excellent opportunity not to be missed by mutant defectors. Thus population starts to be filled by defectors and the average defection rate of the population increases around generation 50. Only cooperators with high memory size can resist to defectors, let's call them *skeptic cooperators*. If there exists still some critical number of skeptic cooperators, resistance can take place and defectors can be outcompeted. That is, cooperators with high memory size again fill the population, as it is seen around generation 60. This cyclic behavior repeats itself until relative abundance of the skeptic cooperators becomes inadequate to resist defectors. In that case, defectors can invade the population as it is seen around generation 150.

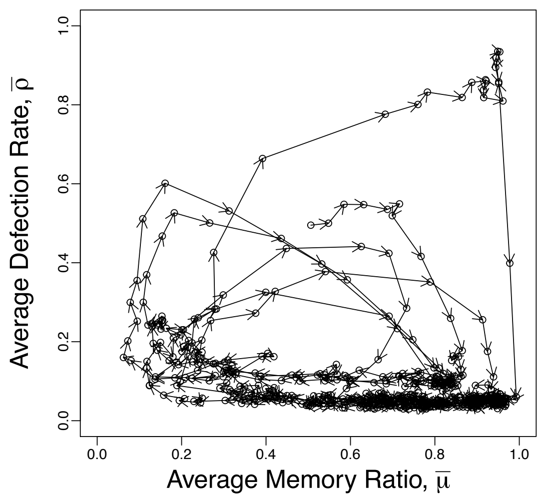


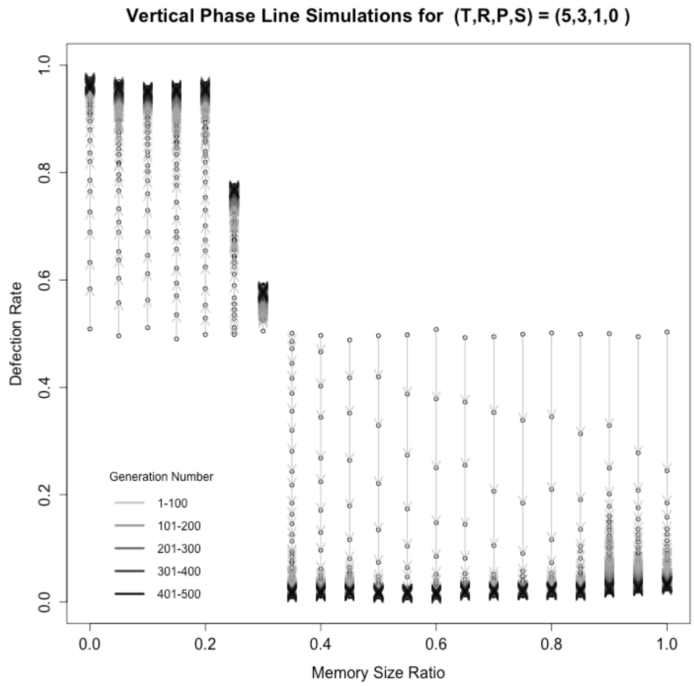
Figure 4.4. Co-evolution under aggressive defection for a single realisation. The same data, displayed in Figure 4.3, graphed on a phase plane. $(S, P, R, T) = (-7, -6, 4, 5)$.

We displayed the same data on the phase plane in Figure 4.4. Under aggressive defection, defectors with lower memory size, have no chance to survive. Thus, the average genotype of the population moves towards a point close to ($\overline{\mu}$,$\overline{\rho}$)$\rightarrow$ $(1, 1)$ on the phase plane. That point corresponds to a defective population with high memory size. Population genotype can stuck around this point or it can escape to continue its trending cyclic behavior. That depends not only the payoff matrix but also the dynamic composition of the heterogeneous population at any given time. In the next section, we will try to explore the effect of payoff matrix structure on the co-evolution of memory size and cooperation.

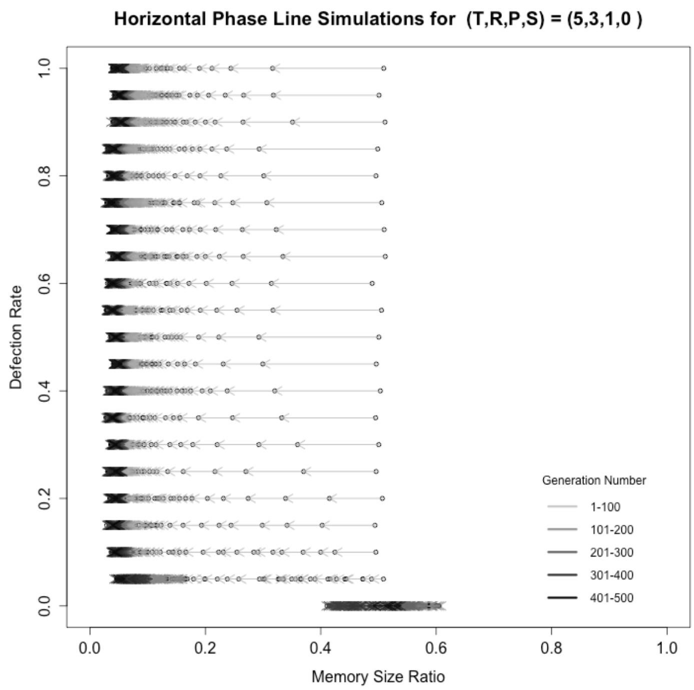
4.1.5. Phase Line Simulations

To gain a clearer guidance about the behavior of our model, we made further simulations in which one gene of the population genotype is fixed while the other varies throughout the generations. We can call this as *phase line simulations*. Varying gene of the population is again generated uniformly at random in the range $[0, 1]$ while the other is fixed one by one to the 21 distinct values from 0 to 1 by increments of 0.05. This will help us to better understand the following questions. (i) What is the impact of limited memory size on the evolutionary dynamics of cooperation? and (ii) What is the impact of low or high defection rate on the evolutionary dynamics of memory size?

In Figure 4.5(a), we investigate the relation of memory size to cooperation for $(S, P, R, T) = (0, 1, 3, 5)$. We refer this as *vertical phase line simulations*. Here, memory size is kept constant for all agents from generation to generation and the evolution of average defection rate is investigated. Each vertical line represents a different initial population with a different fixed memory size. Those 21 distinct populations evolve separately. We see that average defection rate decreases if memory size is high ($\mu > 0.3$) and increases if it is low ($\mu \leq 0.2$). In Figure 4.5(b), we investigate the relation of defection rate to memory size for $(S, P, R, T) = (0, 1, 3, 5)$. We refer this as *horizontal phase line simulations*. Here, defection rate is kept constant for all agents from generation to generation and the evolution of average memory size is investigated. Each horizontal line represents a different initial population with a different fixed defection rate. Those 21 distinct populations evolve separately. We see that average memory size tends to decrease independent of the average defection rate of the population.

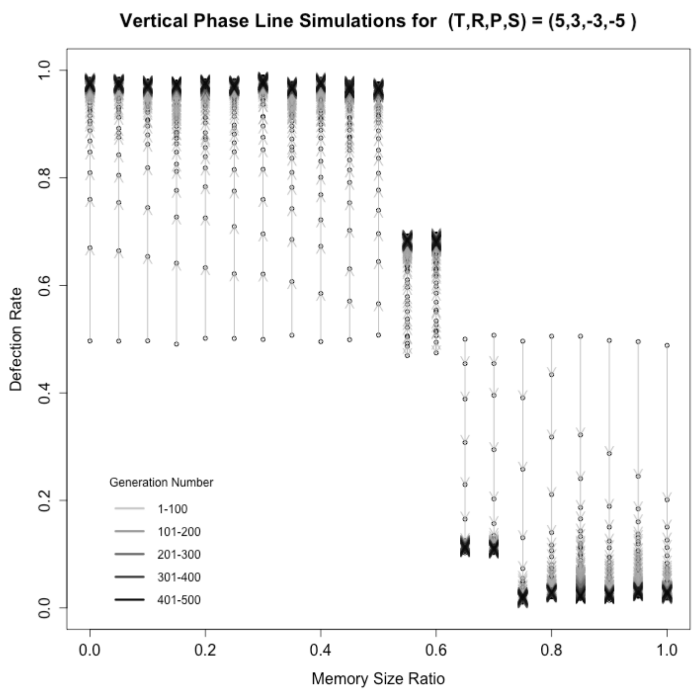


(a) The impact of memory on the evolution of cooperation.

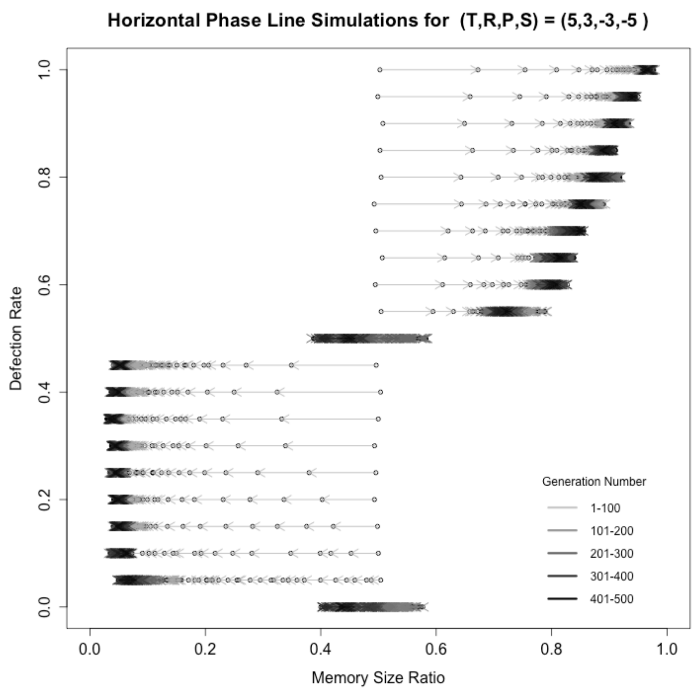


(b) The impact of defection rate to the evolution of memory.

Figure 4.5. Phase line simulations for $(S, P, R, T) = (0, 1, 3, 5)$. x-axis represents the memory ratio μ and y-axis represents the defection rate ρ. Results are obtained by taking the average of 10 runs.

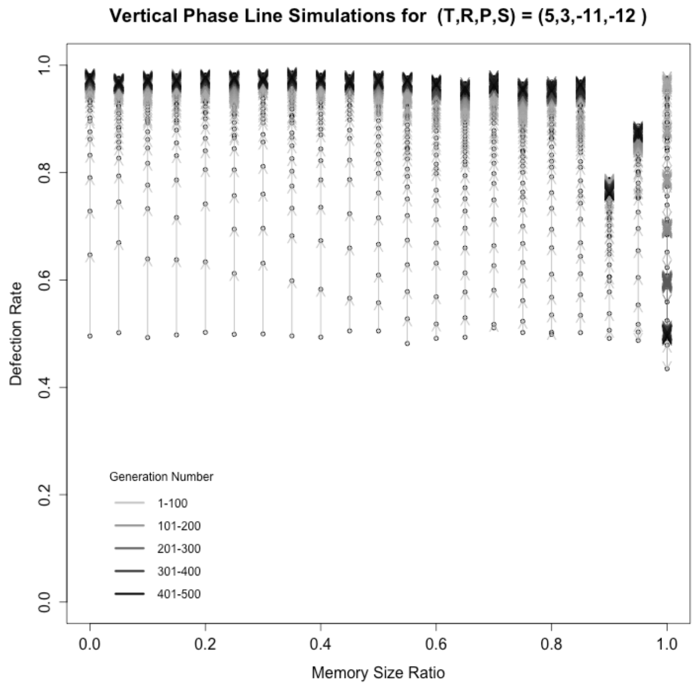


(a) The impact of memory on the evolution of cooperation.

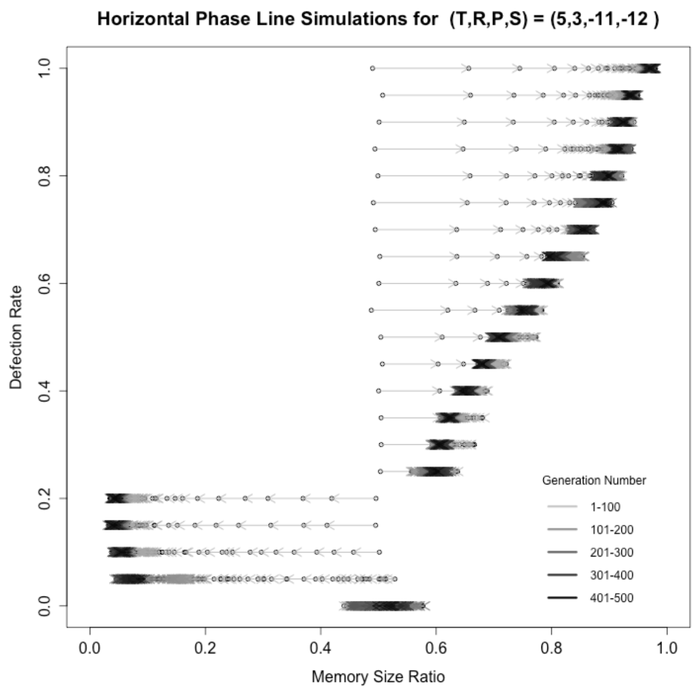


(b) The impact of defection rate to the evolution of memory.

Figure 4.6. Phase line simulations for $(S, P, R, T) = (-5, -3, 3, 5)$. x-axis represents the memory ratio μ and y-axis represents the defection rate ρ. Results are obtained by taking the average of 10 runs.

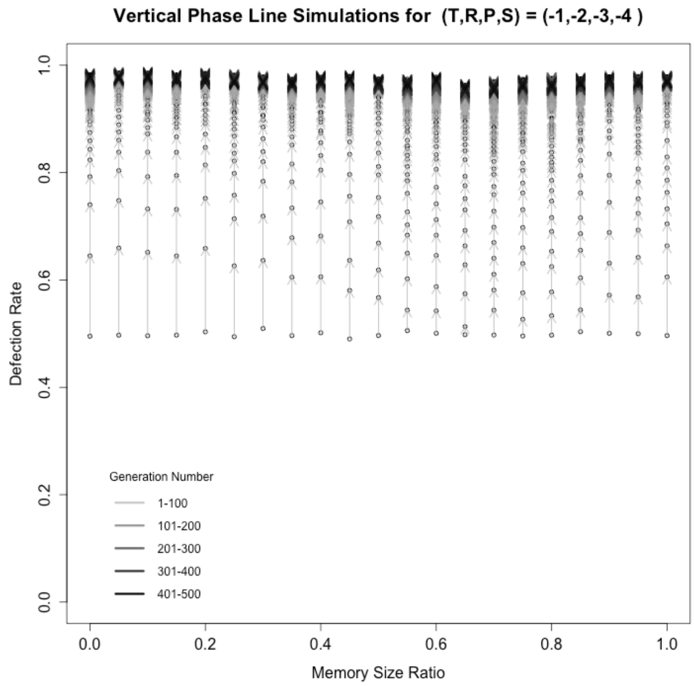


(a) The impact of memory on the evolution of cooperation.

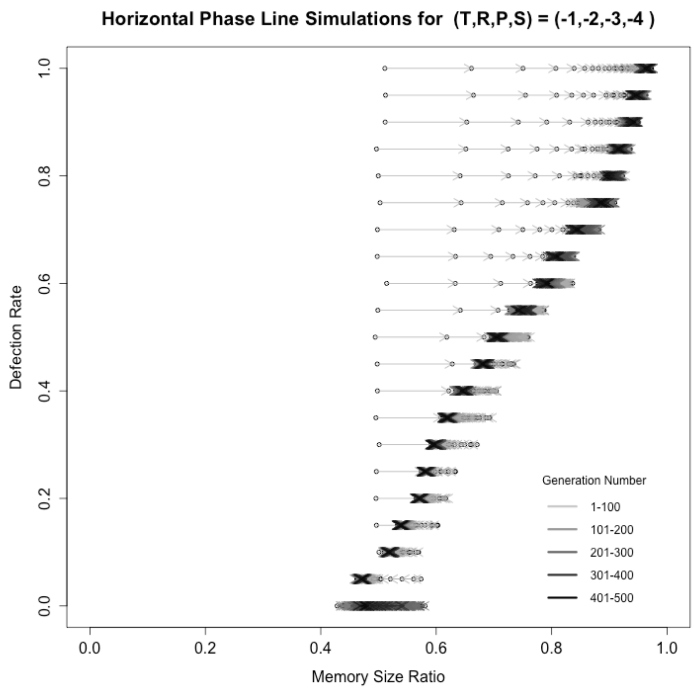


(b) The impact of defection rate to the evolution of memory.

Figure 4.7. Phase line simulations for $(S, P, R, T) = (-12, -11, 3, 5)$. x-axis represents the memory ratio μ and y-axis represents the defection rate ρ. Results are obtained by taking the average of 10 runs.



(a) The impact of memory on the evolution of cooperation.



(b) The impact of defection rate to the evolution of memory.

Figure 4.8. Phase line simulations for $(S, P, R, T) = (-4, -3, -2, -1)$. x-axis represents the memory ratio μ and y-axis represents the defection rate ρ. Results are obtained by taking the average of 10 runs.

Thus, memory size has a negative effect on defection rate but defection rate has no influence on memory for the given scenario of $(S, P, R, T) = (0, 1, 3, 5)$. These results are obtained by an average of 10 realisations and they verify the outcome shown in Figure 4.2. Here, if one imagines any arbitrary point for the average genotype of the initial population on the phase planes given in Figure 4.5, it is easy to see that the point, representing the average genotype of the subsequent generations, tends to evolve towards the attracting point $(0, 1)$. This result is in agreement with Figure 4.2.

We will discuss the phase line simulation for different payoff matrix. The average outcome over 10 realizations can be seen in Figure 4.6, Figure 4.7 and Figure 4.8 for different (S, P, R, T) values. We picture the average genotype as a point moving along a vertical line for 21 different fixed memory ratio in Figure 4.6(a), Figure 4.7(a) and Figure 4.8(a) for respective payoff values $(-5, -3, 3, 5)$, $(-12, -11, 3, 5)$ and $(1, 2, 3, 4)$. We picture the average genotype as a point moving along a horizontal line for 21 different fixed defection rate in Figure 4.6(b), Figure 4.7(b) and Figure 4.8(b) for the same payoff values respectively. We observe, by comparing Figure 4.6(a), Figure 4.7(a) and Figure 4.8(a), that there exists a critical μ band below which defectors outcompete the cooperators and above which cooperators outcompete the defectors and within the band, they are bistable. We also observe that this band approaches to $\mu = 1$, as the damage caused by the aggressive defection gets worse (lower S and P). In other words, memory protects cooperation but much more greater memory size is needed under aggressive defection. Even full memory size might be insufficient for an extremely threatening defection. We observe, by comparing Figure 4.6(b), Figure 4.7(b) and Figure 4.8(b), that there exists a critical ρ value below which agents get rid of their memory and above which agents are contented with a slightly bigger memory ratio than the average defection rate of the population. We also observe that this critical value approaches to $\rho = 0$, as the aggressive defection gets worse. Each payoff matrix has its own dynamics and it is really hard to make generalizations. We have observed that the average genotype of the population has a tendency to move towards $(0, 1)$ for Figure 4.5 and $(1, 1)$ for Figure 4.8 and Figure 4.7. We also observe that the average genotype is capable of forming a cyclic behavior for Figure 4.6.

4.2. The Effect of Payoff Matrix Structure

Each payoff matrix has its own dynamics and it is hard to make generalizations. We need to identify correctly the principal driving forces in our model, and how they will affect the co-evolution of memory and cooperation. To this end, we will reformulate the payoff matrix. Note that the payoff matrices given in Figure 4.1(d) and Figure 4.1(e) have some common properties. Within both matrices the column differences are equal, i.e., $R - S = T - P$. The row differences are also kept equal, i.e., $T - R = P - S$. If we generalize these differences we obtain two factors that are critical in the dynamics: (i) how much it is dangerous to receive a defection, i.e. the column differences, and (ii) how much it is tempting to defect, i.e., the row differences. Thus we introduce the following two principal factors of threat and greed.

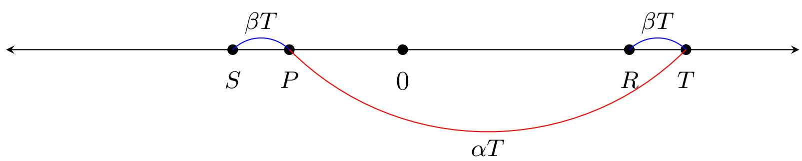


Figure 4.9. The visual representation of payoffs on a number line. Note the fact that $\alpha > 1$ makes $S < P < 0$ and $\beta > 1$ makes $R < 0$.

- *Threat factor, α.* How to measure the difference between receiving a cooperation and receiving a defection? The answer can be found in the payoff matrix seen in Figure 4.1(a). For an agent that chooses to cooperate, the difference between receiving a cooperation and receiving a defection is given by $R - S$. For an agent that chooses to defect, it is given by $T - P$. For simplification, consider the case of $R - S = T - P = \alpha T$ where $T > 0$ and $\alpha > 0$, as it is shown in Figure 4.9. Now irrespective of the chosen action, receiving a defection causes an extra cost of αT in terms of payoffs when it is compared to receiving a cooperation. It is worth to emphasize that $\alpha = 1$ is a critical value. For $\alpha < 1$, P is positive. At $\alpha = 1$, P becomes equal to 0 and for $\alpha > 1$, we have $P < 0$. Thus $\alpha > 1$ corresponds to the case of aggressive defection ($S < P < 0$). Hence we call α as the *threat factor*. For $\alpha > 1$, increasing α means increasing level of threat. On the other hand, $\alpha \leq 1$ means absence of threat.

- *Greed factor, β.* How to measure the difference between taking the two actions of defection and cooperation? When the opponent is cooperating, the difference between defecting and cooperating is given by $T - R$. When the opponent is defecting, the same difference is given by $P - S$. Again for simplification consider the case of $T - R = P - S = \beta T$ where $T > 0$ and $\beta > 0$, as it is also shown in Figure 4.9. Now irrespective of the opponent's action, choosing to defect makes an extra benefit of βT in terms of payoffs when it is compared to choosing to cooperate. Thus, we call β as the *greed factor*. When $\beta = 0$, playing cooperation or defection makes no difference. But whenever β gets larger, defection becomes more tempting. Since the case of $\beta > 1$ makes $R < 0$, it turns out to be uninteresting. It is clear that when mutual cooperation payoff R is also negative, there will be no motivation for choosing to cooperate.

As an interpretation, choosing to defect brings an extra benefit of βT (the row differences in the payoff matrix) and receiving a defection causes an extra cost of αT (the column differences).

4.2.1. Threat Game

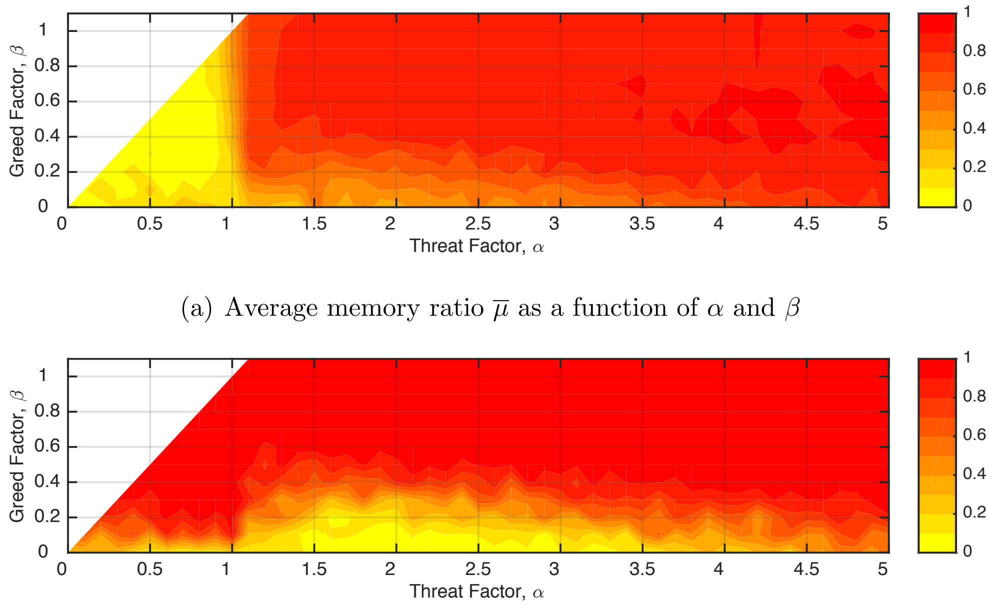
Starting with a fixed positive value for T, we can rewrite S, P, and R in terms α, β and T. That makes $S = (1 - \alpha - \beta)T$, $P = (1 - \alpha)T$, and $R = (1 - \beta)T$. IPD condition of $S < P < R < T$ implies that $0 < \beta < \alpha$. Since all payoff values are multiples of T, the score of any agent will be also a factor of T. When the normalized score is calculated, T factor cancels out and we have expression in terms of α and β only. Therefore, without loss of generality, we can take $T = 1$. Our extensive simulations for different values of $T \in \{5, 50, 100\}$ have confirmed that the dynamics are not dependent on T. Finally, we have the normalized payoff matrix given in Figure 4.1(b), which has only two parameters, namely, the threat factor α and the greed factor β. We call this special form of the IPD game as *Threat game*.

This family of payoff matrices is a special case of all possible payoff matrices, yet its is an important generalization which covers the donation game and also matrices that Epstein used. (i) The payoff matrix structure of the Donation game, given in Figure 4.1(d), can be thought as a subset of Threat game for b and c as column and row differences, respectively. The Donation game lies on the line segment of $\alpha = 1$ and $0 < \beta = \frac{c}{b} < 1$ in the α-β plain in Figure 4.10. (ii) Likewise, the payoff matrix structures, given in Figure 4.1(e), used by Epstein in [77], can be represented by $T + R$ and $T - R$ as column and row differences, respectively. Hence they correspond to the points on the line segment of $\alpha + \beta = 2$ again for $0 < \beta < 1$.

4.2.2. Observations

We investigate the effect of increasing level of threat and greed factors on the co-evolutionary dynamics of memory size and cooperation for $T = 1$. Figure 4.10 visualizes how $\overline{\mu}$ and $\overline{\rho}$ change as a function of (α, β) pairs. In both figures of Figure 4.10(a) and Figure 4.10(b), x-axis is the threat factor $\alpha \in [0, 5]$ and y-axis is the greed factor $\beta \in [0, 1.1]$. Both α and β have incremental steps of 0.1 in their given ranges. We omitted the case of $\alpha < \beta$ in Figure 4.10 since IPD condition of $S < P < R < T$ implies that $0 < \beta < \alpha$. But we showed the results for $\alpha = \beta$ and $\beta = 0$ to see the limiting conditions. The values of $\overline{\mu}$ and $\overline{\rho}$ are the averages over 20 realizations. We have only considered the average genotypes of the last 100 subsequent generations. That is from generation number 400 to 500.

<u>4.2.2.1. Evolution of Memory.</u> In Figure 4.10(a), we show the effect of threat and greed factors on the evolution of memory size. For the evolution of memory, $\alpha = 1$ is critical. P becomes negative for $\alpha > 1$, as it is shown in Figure 4.9. So the change of α value from smaller than 1 to greater than 1 corresponds to the change from absence of threat to presence of threat. In Figure 4.10(a), we observe that the average memory size exhibits a major transformation from its lowest values to its highest values when α becomes greater than 1. This shows clearly how threat fosters greater memory size. We observe no direct impact of greed factor on the memory size.



(a) Average memory ratio $\overline{\mu}$ as a function of α and β

(b) Average defection rate $\overline{\rho}$ as a function of α and β

Figure 4.10. The effect of increasing level of threat and greed factors on the co-evolutionary dynamics of memory size and cooperation. The cases for $\alpha < \beta$ are omitted since they do not fulfill the conditions of the IPD game.

4.2.2.2. Evolution of Cooperation. Memory has a positive effect on cooperation, but cooperation has a negative effect on memory.

- The role of memory is to block interactions with agents that are perceived as defectors. The increase in memory size can be thought as an introduction of (conceptual) barriers against interaction with defectors. When memory size grows, defectors incur isolation. Defectors can not gather enough fitness values for reproduction and they are eliminated.
- High memory size surprisingly raises the risk of self-isolation, especially in a population mostly composed of cooperators. Cooperators can be perceived as defectors due to the small number of interactions in which they happened to defect more than cooperate. As a result of misperceptions, cooperators with high memory size can refuse to interact with other cooperators, hence they end up with lower fitness values. In the subsequent generations, cooperators get rid of their memory.

Threat calls for high memory size.

- In the absence of threat, $\alpha \leq 1$, receiving a defection brings non-negative payoffs, $0 \leq S < P$. In this case, refusing a defector, due to the high memory size, turns out to be disadvantageous. So agents with high memory size are again eliminated throughout the evolution.
- In the presence of threat, $\alpha > 1$, receiving a defection brings negative payoffs, $S < P < 0$. This time, refusing a defector and having a high memory size become advantageous. So average memory size has a tendency to increase in the presence of threat.

Increasing level of threat, primarily increases the memory size, which, in turn, fosters cooperation. But further increase in the level of threat, can not help cooperation. In the next section we discuss the limits to cooperation.

4.2.2.3. Boundaries of Cooperation. We have made further simulations for greater number of generations. We have observed that an increase in the number of generations, up to a certain point, has a positive effect on cooperation. This raises the question whether there is a limit to cooperation. So we decided to make an analytical effort to draw the boundaries of cooperation, by simplifying our model as much as possible for the benefit of cooperators. Let's think that there exists only pure cooperators, that always cooperate, and pure defectors, that always defect, in the population. In order to favor cooperators, suppose memory ratio $\mu_i = 1$ for all agents. So agents will remember the past actions of all their opponents. Suppose also mutation is prohibited. Hence, both cooperators and defectors will always be cautious against other defectors, throughout the evolutionary process. Since agents have enough memory they will play utmost one round with one particular defector and refuse to play with it afterwards.

Suppose there are $N_d \in [0, N]$ defectors in the population. In our model, each pair of agents are matched ν times on the average. So a cooperator will play ν times with $(N - N_d - 1)$ other cooperators (and receive the reward payoff $1 - \beta$) but it will play only once with N_d defectors (and receive the sucker payoff $1-\alpha-\beta$). Hence, the average performance of a particular cooperator equals to the following.

$$P_C = \nu(N - N_d - 1)(1-\beta) + N_d(1-\alpha-\beta)$$

The average performance of a particular defector can be obtained in a similar fashion. A defector will play only once with $(N - N_d)$ cooperators (and receive the temptation payoff 1) and will again play only once with $(N_d - 1)$ other defectors (and receive the punishment payoff $1-\alpha$).

$$P_D = (N - N_d) + (N_d - 1)(1-\alpha).$$

In order to have $P_C > P_D$, α should satisfy

$$\alpha < (N - N_d - 1)(\nu(1-\beta) - 1) - N_d\beta. \tag{4.1}$$

Extremely beneficial conditions for cooperation can be achieved by setting $\beta = 0$. Even in this condition, population can resist to a single ($N_d = 1$) defector, up to a certain point. By setting $N_d = 1$ and $\beta = 0$ in Equation 4.1, we obtain α_1 and α_2 given as

$$\alpha < \alpha_1 = (N-2)(\nu - 1) < N\nu = \alpha_2.$$

For greater values of $\alpha > \alpha_2$, irrespective of the generation number and the greed factor β, it becomes impossible to resist defectors. Absence of greed ($\beta = 0$) makes cooperation and defection indifferent in terms of payoffs, see Figure 4.1(b). Then why pure cooperators ($\rho = 0$) fail and pure defectors ($\rho = 1$) rise for $\alpha > \alpha_2$? This is simply because a defector can not receive a defection from itself. So cooperators receive 1 more defection than defectors does. This difference becomes impossible to compensate when the level of threat α is greater than the average number of rounds ν with each opponent, multiplied by number of agents N, multiplied by the maximum payoff per round $T = 1$. This was an excessive simplification to show that there exists a limit to cooperation.

Our bounds for α are not tight. The α_2 value is 3000 for $\nu = 30$ and $N = 100$. We test the bounds by simulations, in which populations are not composed of only pure cooperators and pure defectors with memory ratio of 1 but of heterogenous agents with various defection rates and memory ratios. We obtained much more modest values of α_2. We have obtained $\alpha_2 = 10$ for generation number 500, and $\alpha_2 = 20$ for generation number 2500. Above these α_2 values we do not observe any cooperation.

4.2.2.4. Co-evolution of Memory and Cooperation. Necessity is the mother of invention. We see a positive function of threat in having a greater memory size. On the other hand, greater memory size raises the risk of self-isolation. Threat and misperceptions among cooperators surprisingly cause a second source of dilemma on the memory size. Thus not only cooperation but also memory size constitutes a dilemma in our model. We can summarize the resulting dynamics of our model in three predominant categories.

(i) In the absence of threat ($\alpha \leq 1$), greater memory size is unfavorable to evolutionary success. And cooperation collapses without memory. Thus, the average genotype of the population moves to a point close to $(\overline{\mu}, \overline{\rho}) = (0, 1)$. Figure 4.2 shows the triumph of memoryless defection over time, when there is no threat.

(ii) In the presence of an appropriate level of threat ($1 < \alpha < \alpha_2$) and under low greed factor ($\beta < 0.5$), the trajectory of the evolving population can exhibit emergent oscillatory dynamics. Memory size acts like an immune response of the subsequent generations. Figure 4.3 and Figure 4.4 help us to visualize the emerging oscillatory dynamics for a single realisation, in the presence of an appropriate level of threat. Memory crashes when defection rate is low, but spikes up as a response to growing defection rate in the generation, then crashes again and recover again depending on the average defection rate of the subsequent generations. Not individual agents, but populations from generations to generations evolved to develop some kind of protection mechanism against aggressive defection. This is an un-programmed functionality that emerged via evolutionary dynamics.

(iii) In the presence of threat ($\alpha > 1$) and for high greed factor ($\beta > 0.5$) the average genotype of the population moves towards a point close to $(\overline{\mu}, \overline{\rho}) = (1, 1)$ which corresponds to a defective population with high memory size. This is also true in the presence of an extreme threat ($\alpha > \alpha_2$). As α gets larger, defection starts to cause an extreme damage and it gets harder for cooperators to resist.

To our conclusion, the dose of the threat makes the resistance for cooperation, especially when the greed factor is low. Lastly, to understand the impact of attention, we compared the dynamics under selective attention (forget preferentially cooperators) and the dynamics without attention (forget at random without preference). It seems like attention favors cooperation and disfavors defection, especially for moderate values of threat and greed factors. Attention can only make a difference when memory size is limited. For higher threat and greed factors memory size gets very close to its maximum value and memory becomes sufficient to remember all opponents that, in turn, makes attention less critical. Nevertheless, we think that the effect of attention on the co-evolutionary dynamics of memory and cooperation requires a research of its own. We leave it as a future work.

4.3. Conclusion

In the research of cooperation, the effect of negative mutual punishment payoff, $P < 0$, is usually omitted, as in the case of the Donation Game. Yet there are some studies with negative payoffs for receiving a defection. Our model shares with Epstein, the effect of negative payoffs in the emergence of cooperation [77]. But it differs in many other respects. First, our model allows agents to have varying memory size and defection rate, whereas Epstein's model allows only pure cooperators and pure defectors with zero-memory. The structure of a system, determines *who interacts with whom* and causes its dynamic behavior. In our model, agents can select their partners and memory has a critical role, as it is used to hinder interactions with defectors. In Ref [77], it is only the spatial aspects of the environment which hinders cooperators from interacting with defectors. In other words, Epstein used physical barriers to avoid interactions with defectors, while we have used conceptual barriers for it. Instead of studying cooperation under fixed payoff matrix structure, we proposed to reformulate the payoff matrix structure of IPD game by introducing threat and greed factors, and showed their effect on the co-evolution of memory and cooperation.

We observe that the greater memory size is unfavorable to evolutionary success when there is no threat. One can find this, initially, deeply counterintuitive and not realistic. But there are cases where species lost their brains as a result of evolution. According to Frank Hirth, in their ancient evolutionary past, sea sponges did have neurons [111]. Some extremely simple animals, such as sea squirts, simplify their brains during their lifetimes. Sea squirt has a nervous system in order to navigate in the sea. Its only goal is to find a suitable rock to live on for the rest of its life. When it implants on a rock, the first thing it does is to digest its nervous system. Without a problem to solve, there is no need to waste energy on a brain. In order for evolution to promote increased brain size, its benefits, e.g. against predation threat, must outweigh the high energetic costs [112]. One of the most striking example related to the evolution of brain size belongs to humans. In the past 20.000 years, the human brain has shrunk by about the size of a tennis ball [113]. Nobody knows exactly why. According to a leading theory, the incredible decline in human brain size is a by-product of domesticity. The shift from the threatening lifestyle of hunter-gatherers to the highly cooperative and more secure lifestyle of agricultural society has led to the reduction in brain size. Our results support this theory.

We have shown the positive effect of an appropriate level of threat in having a greater memory size which, in turn, favors cooperation. This finding is in parallel with other forms of delicate balance (for the level of environmental harshness [114] and for the level of punishment fines [115]) within which cooperation thrives best in spatial evolutionary games. It is possible to make analogies with two different scientific results from immunology and experimental psychology. It is thought that the immune system functions by making distinction between self and non-self. This viewpoint is renewed with the idea that the immune system is more concerned with entities that do damage than with those that are foreign [116]. Actually, threat calls for taking countermeasures against the would-be-exploiter. Experimental evidence from psychology has shown that the cooperation typically collapses in the absence of sanctioning possibilities [117]. The threat of punishment is the key to maintain and promote cooperation [118, 119]. To conclude, in order for cooperation to emerge, selfish beings need to be exposed to an appropriate level of threat. When defection is harmless agents tend to defect and when defection cause an extreme damage, cooperators have no chance to survive. We observe that the conditions for the emergence of cooperation are very subtle. To increase the immunity of cooperation, differentiation of cooperators or some kind of collective memory can be incorporated to our model as a future work.

5. CONCLUSION

"To develop a complete mind: Study the science of art; Study the art of science. Learn how to see. Realize that everything connects to everything else."
- Leonardo da Vinci [120]

"Great artists study the masters; so too must great modelers."
- John H. Miller, Scott E. Page [33]

"I think the next century will be the century of complexity."
- Stephen Hawking [121]

The eye of the scientist should promenade on the previous great scientist's works, to be a part of the forthcoming revolution in science. There are striking parallels among virtually every scientific field. Ludwig von Bertalanffy's *General Systems Theory*, is one of the first attempts to formulate common laws and principles that apply to widely diverse disciplines [122]. That was a preeminently mathematical attempt to unify systems science based on the abstractions from various disciplines. Agent-based modeling shares the very same hunch that the boundaries between disciplines are somewhat arbitrary. Distinctively, ABM is preeminently a computational attempt to achieve unity across disciplines [29]. The aim of gathering the unifying principles of various disciplines and creating a new way of doing science, is a very old and a very sweet dream. We think every great scientists have seen it. ABM will gain more power as we learn how to integrate data and equations into this new tool. That would require inter-disciplinary teams of researchers to focus on the emergence of macro-level properties from micro-specifications. Uri Wilensky says that the emergence is so endemic to the social and natural world that using an emergent lens to make sense of complex patterns is a vital need in the 21 century [10].

Let's turn back from big dreams to our modest attempts. In this thesis, we worked on social dynamics of competition and cooperation using computational models. We primarily used agent-based modeling and then we tried to validate our findings via equation-based models, such as Markov Chains and phase plane analysis. We have not taken any data-driven approach, but we made comparisons between our findings and other experimental findings from various scientific disciplines.

Primary contribution of this thesis, is the systematic reformulation of the Prisoner's Dilemma payoff matrix with two principal driving factors of threat and greed, see Figure 4.1(b). We have shown their effects on the co-evolution of cooperation and memory in Chapter 4. Another important contribution of this thesis, is the use of "memory" as the conceptual structure, see Chapter 3 and Chapter 4. It is known that the spatial structure can promote cooperation by introducing physical barriers against interaction with defectors. In lieu of considering spatially structured population in physical space, we made used of conceptually structured populations. Memory plays the role of conceptual barriers for interaction with defectors. Agents have mental representations of other agents in their memory and they have the ability to choose with whom to interact. Hence, in our model memory defines the structure, who interacts with whom, and generates the behaviour. Another contribution of this thesis, is the emergence of immune response against defection. Co-evolution of memory and cooperation gives rise to the emergent behaviour. We observed that average memory ratio increases when average defection rate increases and decreases when average defection rate decreases. See Figure 4.3. Subsequent generations act like they have the ability to self-regulate their average memory ratio depending on the average defection rate. This functionality is not explicitly encoded in the program code of our model, but has emerged as a result of the evolutionary dynamics.

Our secondary contributions, that we have started but not finished, can be thought as the root of heuristics and the calculus of simulations. The study of the root of heuristics can bring together agent-based modeling and real data from social sciences such as social psychology and economy. We have shown some findings that support attraction-effect heuristics from economy, in Chapter 2 and negativity bias from psychology in Chapter 3. We need further investigation and more importantly we think, we need to collaborate with researchers from other disciplines to do a better research. By the calculus of simulations, we mean developing mathematical tools to track the outcomes of a simulation. This is not an easy task, especially for models with high complexity. But nevertheless, we think that it is necessary and doable at least for testing the extreme conditions. We have used Markov Chains and phase plane analysis as mathematical treatments for simplified agent-based models in Chapter 2 and Chapter 4, respectively. We think equation-based models and agent-based models complement each other. As an example, we have observed that the change in the number of generations has an effect on the outcomes of our evolutionary simulation. We could make much more simulations with greater number of generation numbers. (Actually, we did from 500 up to 2500 generations.) But a more effortless solution can be found with a little help from mathematics. Hence, at the end of Section 4.2, we introduced an analytical treatment to investigate the boundaries of cooperation. We have shown that, irrespective of the generation number, there exists an upper limit for threat factor above which cooperation can not flourish and can not be maintained. This shows how equation-based and agent-based models complement each other.

Our models more-or-less have an exploratory nature and they are somehow abstract. Nevertheless one can think them as a starting point for modeling boundedly rational agents in a competitive world. We think the question of how selfish beings manage to cooperate is as old as the universe and moreover, it seems to be ever-lasting. (When robots with extremely high artificial intelligence emerge, will humans and robots cooperate or compete?) In Chapter 4, we have shown that an appropriate level threat can help to foster cooperation. How individuals should spare their limited attention? In Chapter 3, we have found that it is better for their survival to direct their attention to defectors. How competing items can capture the limited attention of agents? Beside the trivial way of increasing advertisement pressure, in Chapter 2, we have found that one can achieve a higher market penetration by introducing dummy items to the market. No model is perfect. When a model is proven to be no longer useful, we should immediately seek for a better one. Here is why, science is a never ending story.

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