A CULTURAL MARKET MODEL

by

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BSc, in Computer Engineering, Bogazici University, 2004

Submitted to the Institute for Graduate Studies in

Science and Engineering in partial fulfillment of

the requirements for the degree of

Master of Science

Graduate Program in Computer Engineering

Boğaziçi University

2007

ACKNOWLEDGEMENTS

First and foremost, I thank my advisor Dr. Haluk Bingöl, for his constant support during my studies and his incredible insight for knowing when to push me for more or let me take a break to put things in an order. Under his supervision, I enjoyed both the freedom and responsibility of conducting an independent research.

I am indebted to Eser Aygün for the numerous mind stimulating conversations we had together. Many critical ideas about my thesis were sprouted during these conversations. His recommendations and criticisms made my arguments much more solid and richer.

I thank all members of the SOSLAB for the insightful discussions we had and for their feedback during my study. I also acknowledge the productive environment that I enjoyed in the Department of Computer Engineering and I thank my colleagues there for their valuable support.

The Department of Foreign Language Education provided me an enjoyable and peaceful workplace where I could keep my graduate studies ongoing without any conflicts with my work. I thank all my colleagues and professors working there for their understanding and kind help.

Dilek Şentürk has always been there when I needed her most and she never withheld her moral support and love. My warmest thanks are for you Dilek. I honestly do not know what I would have done without you.

Most important of all, I would like to express my deepest gratitude to my family for their unlimited love, support and encouragement. I thank my parents for their inspiration and trust in me. They taught me a free mind is the most precious treasure a man can posses. I thank my lovely sister Duygu, for acknowledging me as an older brother who can visit home and her only for a few days a year and for welcoming me warmly each and every time.

ABSTRACT

A CULTURAL MARKET MODEL

Social interactions and personal tastes shape our consumption behaviors of cultural products. The behavior of an individual is driven by many factors but at the simplest level we can model his behavior as the outcome of an interaction between a personal component, which represents his personal tastes, and a social component, which represents the effect of his peers, family, society and etc. on his decisions. Identifying the social and personal components and studying their interaction at an emergent level is an active area of research which borrows methods and techniques from various disciplines such as computer science, sociology, cognitive science, economics, and physics.

Constructing computational models and analyzing them is one part of the research and physicists have already come up with some models which help us to deal with simple decision models where agents are required to pick one of two alternatives. This type of problems is called binary decision problems and can be applied to a variety of real world situations like voting for or against a legislation and buying or not buying a product. However, in real cultural markets, where many products compete with each other, the consumption decision is not a binary one because the people are limited in budget, time, etc. And the agents have to consider all options before they come to a decision. Therefore, it is not possible to view the decision of consuming a product or not as a simple binary decision because it is not independent of other products.

In this thesis, we present a computational model of a cultural market and we aim to analyze the behavior of the consumer population as an emergent phenomena. We conceptualize a *cultural market* as a set of consumers and a set of cultural items such as movies, songs, or books where the consumers make decisions to consume the items or not. The consumers are in social interaction with each other and they make decisions based on their personal opinions and social pressure together.

Our results suggest that the final market shares of the cultural products dramatically depend on the consumer heterogeneity and social interaction pressure. The inequality of the resulting market and the correlation between the initial attractiveness and final market share of a product exhibits sudden increases and decreases depending on the values of social interaction pressure.

We also extend our simulations to test the robustness of the observed phenomena with respect to the topology of the social interactions between agents and other model parameters. Our findings suggest that the relation between the resulting market shares and the social interaction does not depend on the actual values of the parameters such as the number of agents or the properties of the topology but qualitatively same for a wide range of parameter settings.

ÖZET

KÜLTÜREL BİR PAZAR MODELİ

Toplumsal etkileşim ve kişisel beğeniler tüketim davranışlarımızı derinden etkiler. Bireyin davranışlarını yönlendiren pek çok etken olmakla beraber, en temel seviyede bu davranışları kişisel ve toplumsal iki bileşenin etkileşiminin bir sonucu olarak modelleyebiliriz. Kişisel bileşen bireyin beğenilerini, toplumsal bileşen ise çevresiyle (arkadaşları, ailesi, toplumun geri kalanı) girdiği etkileşimin davranışları üzerindeki etkisini temsil eder. Bu bileşenleri tanımlamak ve aralarındaki etkileşimin doğasını incelemek bilgisayar bilimleri, toplum bilim, bilişsel bilimler, ekonomi ve fizik gibi pek çok disiplinin katkıda bulunduğu aktif bir araştırma alanı olmuştur.

Hesaba dayalı modeller kurmak ve incelemek bu araştırmanın bir parçasıdır ve fizikçiler uzun zamandır bir etmenin iki alternatiften birisini seçmesi gerektiği durumları yansıtan modelleri sunmakta ve incelemektedirler. Gerçek dünyada da uygulama alanı bulabilecek bu karar verme problemlerine iki alternatiften birisinin seçimi söz konusu olduğu için ikili karar verme problemleri adı verilmiştir. Referanduma sunulmuş bir konuda evet ya da hayır oyu kullanmak, bir ürünü almaya ya da almamaya karar vermek bu modeller kullanılarak incelenebilen durumlardır. Bununla beraber kültürel pazarlarda gözlediğimiz bir durum etmenlerin kararlarının ikili karar verme problemi olarak ele alınamayacağını göstermektedir. Bu da kısıtlı bütçe ve benzeri sebeplerden dolayı etmenlerin pazardaki her ürünü tek tek değil topluca ele almaları ve ürünlerin kendi aralarında rekabet ediyor olmalarıdır. Bir ürün hakkında verilen tüketim kararı diğer ürünler hakkında verilen karardan bağımsız değildir.

Bu tezde kültürel pazarlar için hesaba dayalı bir model sunuyoruz. Amacımız bu model yardımıyla makro seviyede gerçekleşen tüketim davranışlarını incelemektir. Kültürel pazardan kastımız tüketicileri temsil eden bir etmen kümesi ve ürünleri (ör. kitaplar, filmler, müzik albümleri) temsil eden bir ürün kümesinden oluşan bir sistemdir. Tüketiciler birbirleri arasında toplumsal etkileşim içindedirler ve bu etkileşim ile kişisel beğeniler tüketmeye karar verecekleri ürünleri belirler.

Sonuçlar, ürünlerin pazar paylarının tüketicilerin kişisel beğenileri arasındaki benzerliğe ve toplumsal etkileşimin şiddetine aşırı hassas olduğunu göstermektedir. Oluşan pazar payları arasındaki adaletsizlik ve bir ürünün başlangıçtaki çekiciliği ile sahip olduğu pazar payı arasındaki korrelasyon değişen toplumsal etkileşim şiddeti değerlerine göre ani iniş ve çıkışlar göstermektedir.

Elde ettiğimiz sonuçların belirli parametre değerlerine bağımlı olmadığını göstermek için simulasyonlarımızı değişik parametre değerleri (ör. etmenler arası ilişkileri belirleyen topolojik yapılar, etmen sayısı, vb.) kullanarak tekrarladık. Sayısal sonuçların değerleri değişmekle beraber modelimizin davranışlarının çok geniş bir parametre değer yelpazesinde niteliksel olarak değişmeden tekrarlandığını gördük.

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LIST OF SYMBOLS/ABBREVIATIONS

$\langle c_\alpha \rangle$	Market share of item α
$\boldsymbol{C}$	The consumption matrix
I	Market inequality value
$\boldsymbol{G}$	The friendship matrix
i	Used to index an agent
k	Average number of neighbors in a community
$\langle l_\alpha \rangle$	Quality of item α
$\boldsymbol{L}$	The liking matrix
M	Number of items
N	Number of agents
$\boldsymbol{O}$	The opinion matrix
Q	Quartile difference value
T	Simulation length
t	Denotes time
α	Used to index an item
Γ	A configuration of the model

1. INTRODUCTION

1.1. Motivation

Social interaction is an inevitable aspect of our lives and a very strong ingredient of our decision processes. It would not be an exaggeration if one were to say that most of our decisions at least partially depend on what other people think and how they behave. Family, friends, colleagues and other social groups may effect our behaviors significantly. The extent of the society's influence on our behaviors may range from daily decisions such as what to wear at work to political decisions such as which party to vote for in the elections. The importance of the social interaction has been reflected in the social sciences for many decades and a growing body of research continues on the intersection of various disciplines including but not limited to sociology, cognitive sciences, physics and economics (Farrell and Saloner, 1985; Goldstone and Janssen, 2005; Gordon et al., 2005; Granovetter, 1978; Krauth, 2006, 2005; Markose, 2005, 2006; Schelling, 1973, 1978).

A single individual is already very complex to analyze on its own and the inclusion of social effects introduce even more complexity to the theoretical and empirical studies. Statistical physics has a long history of dealing with interacting particles and emergent phenomena. The methods of statistical physics provide convenient means of abstracting the dynamics in a complex system and avoiding irrelevant details. Interestingly, some of those techniques are applied to human populations successfully and offer us new ways to explore the dynamics in social systems (Durlauf, 1999, 2005; Galam, 1997; Galam et al., 1982; Holyst et al., 2000; Michard and Bouchaud, 2005; Phan et al., 2004; Phan and Pajot, 2006).

Economic systems and markets are interesting areas to apply the insights we get from complex systems approach. We conceptualize a *cultural market* as a set of consumers and a set of cultural items such as movies, songs, or books where the consumers make decisions to consume the items or not. The consumers are in social interaction with each other and they make decisions based on their personal opinions and social pressure together. The movie market is an excellent example of such a system and throughout this text we will develop our arguments based on this concrete example for pedagogical reasons.

1.2. A Cultural Market

In this section, we will cover some important and defining aspects of a cultural market and hence outline the scope of our model that will be introduced later in the text. Two major components of a cultural market are the set of consumers and the set of cultural items. In our example market that we will use throughout this study (i.e. the movie market), consumers are the viewers or the customers of the movie theaters and items are the movies on the market.

1.2.1. Limitations in demand and supply

Consumers have limited budgets to pick for the items they will consume. This limitation may be due to money (one can not afford to buy tickets for all movies), time (one can not view all movies on the market in a short period) or some other reason. We are not interested in the actual reason of the limitation but assume that it exists.

On the other hand, the supply of the items is unlimited. A movie stays on the theaters as long as there are customers who wish to view it or a book can be re-printed if its sales continues. There is no scarcity on the supply side of the market thanks to the "cultural" nature of the items. They can be reproduced as needed.

1.2.2. Social dimension

The decision of a consumer whether to consume an item or not may depend on various factors which are irrelevant to us for the moment. An important assumption about a cultural market is that one of the factors is the social pressure exerted on the individual. The decision of others effect the decision of the individual. Theoretically, the effect may be both negative or positive but we focus on the case where it is positive (i.e. if an item is consumed more by the others than it will have a greater chance to be consumed by the individual). The literature of psychology and economics have a large set of findings and different reasons for such effects. For instance, in certain economical settings, the value of an item to the customers increases as the number of people who have consumed the item increases. These effects are called network externalities and they are well known and studied in economics (Farrell and Saloner, 1985). A typical example is the fax machine: As the number of other people who have bought a fax machine increased, it became more sensible to buy one because of the possible use of the machine. Conformity or peer pressure can be listed as other potential sources of social pressure. Again, the real reason behind the social pressure is irrelevant to us. Our focus will be on the markets which incorporate a social pressure dimension in some way or other. Consider our example market, the movie market. Viewing a movie is a social act. Try to remember the last time you went to a movie alone; usually we do not. Our decision of which movies we will see depend both on our personal tastes and the outcome of our interaction with our friends and the rest of the society by means of media and advertisements.

1.3. Relevant Studies

Most of the relevant work in computational sociology and statistical physics (i.e. sociophysics) has focused on a very simple set of decision problems which we can call *binary decision problems*. In this set of problems, the agents in a community are faced with a binary decision such as to vote for or against a legislation or to buy a particular product or not. (Galam, 1997; Gordon et al., 2005; Granovetter, 1978; Michard and Bouchaud, 2005). The *Random Field Ising Model* (RFIM) is a commonly investigated model to analyze situations where heterogeneous individuals base their decisions on both an idiosyncratic component and a social component (Sethna2001, Galam1997, Sethna2005, Phan2004, Phan2006, Michard2005). The *idiosyncratic component* represents the agent's tendency to choose one answer over another without considering the effect of all social interactions. These values are assumed to be independent identically distributed (IID) random variables. *Social component* represents the effect of the society on the agent and is a measure calculated for each agent based on its neighborhood (i.e. other agents that it interacts).

In this section, we will adopt the notation of Michard and Bouchaud (2005) to summarize the RFIM model and the key findings relevant to our case. In RFIM, each agent i is faced with a binary decision problem. The decisions of the agents are denoted by $S_i = \pm 1$. The decision of an agent depends on three factors:

- Idiosyncratic, personal opinion $\phi_i \in \mathbb{R}$. Higher ϕ values correspond to stronger a priori tendencies to decide $S_i = +1$.
- Global and time-dependent value $F(t) \in \mathbb{R}$ which represents a global informa-

tion (e.g. price of the product) available to all agents affecting the decision process. In physical literature this value is called the *polarization field* because of the application of RFIM to magnetization phenomena.

- Social factor which represents the effect of the decisions of other agents on an agent. It is calculated as $\sum_{j \in \nu_i} J_{ij} S_j$ where ν_i is the set of all neighbors of agent i. The value J_{ij} is a measure of the influence of agent j on agent i and it is assumed that $J_{ij} > 0$ for all i and j reflecting the conformist nature of the agents. This setting tells us that if an agent j decides $S_j = +1$ then it reinforces the agent i to have the same decision (i.e. $S_i = +1$) if agent j is in the neighborhood of agent i and vice versa.

An agent i choose its decision $S_i(t)$ at time t as follows.

$$S_i(t) = \begin{cases} 0 & \text{if } \left(\phi_i + F(t) + \sum_{j \in \nu_i} J_{ij} S_j\right) \leq 0, \\ 1 & \text{otherwise.} \end{cases}$$

The average value of the all agent's decisions is called the *average opinion*, $\langle S \rangle$, and calculated as follows.

$$\langle S \rangle = \sum_{i=1}^{N} S_i / N$$

where N is the total number of agents.

In case of no social interaction (i.e. $J_{ij} = 0$ for all i and j), the effect of increasing the polarization field $F(t)$ from $-\infty$ to $+\infty$ manifests itself as a gradual change in the average opinion $\langle S \rangle$ (see Fig. 1.1). Introduction of social pressure changes the situation dramatically. A mean-field approximation (i.e. setting $J_{ij} = J = 1/N$ for all i and j) reveals that there exists a critical point J_c and for values J greater than J_c, a gradual change in polarization field $F(t)$ results in abrupt changes in average opinion. As we increase the polarization field slowly, the average opinion remains at levels close to $\langle S \rangle = -1$ for even high values of $F(t)$ but after a certain point abruptly jumps to the level $\langle S \rangle = +1$. The same behavior is also shown with decreasing polarization field.

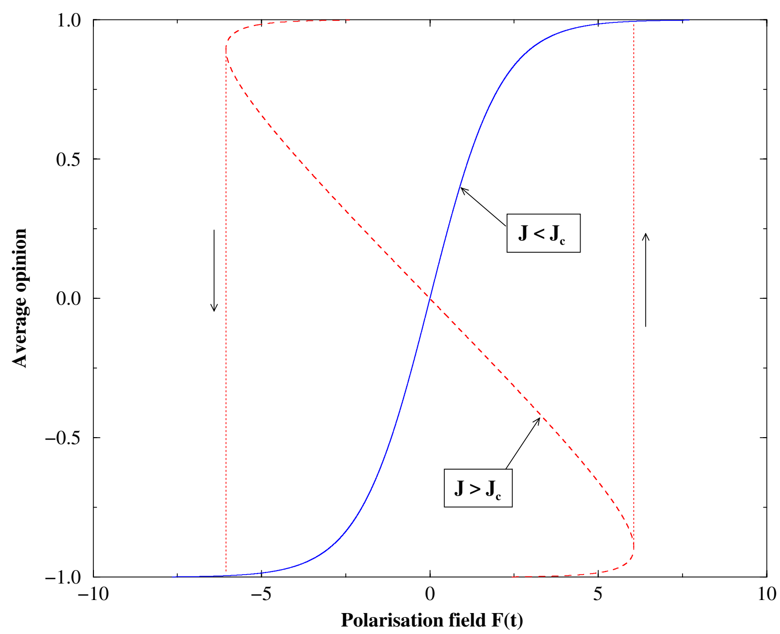


Figure 1.1. Average opinion as a function of external polarization field. Figure taken from Michard and Bouchaud (2005).

Around the proximity of the critical value J_c, the number of agents changing decisions at a given time period shows a scaling behavior. Most of the time few agents change their decisions but once in a while a large number of agents change their behaviors leading to abrupt changes in the average opinion value.

How can we extend these findings to cases where more than one decision is to be made? Specifically, the cultural markets we intend to focus on can hardly be conceptualized as involving binary decision problems. One may propose that consuming an item or not is a binary decision on its own and carry out the analyses. But this view excludes the fact that consumption of an item is not independent of the decisions about the other items. All items are in competition with each other (because of limited demand) and due to this fact we simply can not view the decision process for each item independent from each other.

Salganik et al. (2006) presents an insightful experimental study which provides us empirical evidence that social influence has an effect on the consumption decisions of people. In the experiment, the subjects are faced with a web based application and they can listen to and rate as many songs as they like among 48 songs of previously unknown bands. After they listen to and rate a song they are offered the opportunity to download the song. The study reports the results of two different experimental conditions. The first one is called the independent condition in which the subjects only see the names of the songs without any other information and make their decisions independently from the other subjects. The second condition is called the social influence condition and in this case, the number of people who have downloaded each song previously is also given to the subjects and they can make use of this information in their decisions. Any significant difference between the number of downloads of the songs (which the authors call the *success* of a song) between the two cases can be attributed to the availability of social information since there is no other experimental difference between the two settings.

The key finding of the study is that the availability of the social information significantly affects the way people behave. In the social influence condition, the final variation in the success outcomes of the songs is found to be higher than it is in the independent condition. This suggests that in the social influence case, some songs are downloaded many more times than they are in the independent case. Another measure they report is called the *unpredictability* of a song and is calculated by the calculating the average difference between the success values of a song over different realizations of the same condition (i.e. the experimental condition is repeated several times with different subjects). If a song tends to get the same outcomes over different realizations then its unpredictability value is low. As a result, the social influence condition leads to higher unpredictability values for the songs.

It is natural to ask if it is possible to come up with a computational model that will reflect the effect of social influence on the behaviors of people. The RFIM seems like a good starting point but it is only directly applicable to binary decision problems. To our knowledge, only one study has attempted to generalize the RFIM to multiple decision problems (Borghesi and Bouchaud, 2006). The authors of this study report some analytical results for their generalized model but their analysis was not publicly available during the writing of this thesis.

2. METHODOLOGY

2.1. The Model

Our model intends to capture the essential dynamics in a community of consumers and the effect of social pressure on the consumption of cultural items. Consumers are the people in the community, items are the cultural products that the people consume (e.g. movies, albums, books). Consumption of an item corresponds to paying for and enjoying that item (e.g. buying a book to read or a ticket to see a movie).

The model consists of N agents and M items. We index the agents by Roman labels (i, j) and the items by Greek labels (α, β) unless noted otherwise. An agent i represents a consumer in the community and an item α represents a cultural item on the market.

Liking Matrix: Each agent has a predetermined (and time independent) liking value for each item. These values are stored in a $N \times M$ *liking matrix* denoted by $\boldsymbol{L} = (l_{i\alpha})$ where $l_{i\alpha} \in \mathbb{R}$. The value $l_{i\alpha}$ corresponds the idiosyncratic personal taste of agent i for the item α. A positive liking value $l_{i\alpha}$ indicates that, without considering any other factor, agent i is inclined towards consuming item α.

Consumption Matrix: We assume that an item can be consumed by an agent only once (i.e. no one sees a movie twice or buys a second copy of a book). The information of which agent consumed which item so far is held in a $N \times M$ *consumption* *matrix* denoted by $\boldsymbol{C} = (c_{i\alpha})$ where

$$c_{i\alpha} = \begin{cases} 1 & \text{if agent } i \text{ has consumed item } \alpha, \\ 0 & \text{otherwise.} \end{cases}$$

Friendship Matrix: Agents are connected to each other by directed friendship links. The topology of the friendship network is kept in a $N \times N$ adjacency matrix called the *friendship matrix* and denoted by $\boldsymbol{G} = (g_{ij})$ where

$$g_{ij} = \begin{cases} 1 & \text{if agent } j \text{ is connected to agent } i, \\ 0 & \text{otherwise.} \end{cases}$$

An agent j is said to be a *friend* of agent i if $g_{ij} = 1$.

Social Component Function: *Social pressure* is defined as the effect of the behavior of others on an agent's consumption decision. We decide to model the social pressure by the ratio of the number of friends of agent i who consumed item α to the total number of friends of agent i. An agent i will be more inclined towards consuming an item α if its friends have already consumed the item. This view of social pressure is in accordance with our previous discussions in Sec. 1.2. We calculate the *social component function* $f(i, \alpha)$ to represent the effect of the social interactions on the perception of item α by agent i. The formal definition of the social component function is given below.

$$f(i, \alpha) = \frac{\sum_{j=1}^{N} g_{ij} c_{j\alpha}}{\sum_{j=1}^{N} g_{ij}} \tag{2.1}$$

Opinion Matrix: It is a common assumption that an agent's decision to consume (or not to consume) an item can be modeled by two distinct components. One of them is the idiosyncratic (personal) component and the other is the social component (Borghesi and Bouchaud, 2006; Brock and Durlauf, 2001; Galam, 1997; Michard and Bouchaud, 2005). The final decision of whether to consume an item or not depends on the combination of these two components. We have already presented and defined the idiosyncratic and social components represented by the liking matrix and social component function correspondingly. The final opinion about an item α that an agent i uses in its consumption decision is calculated by integrating these two components. We can represent the opinions of all agents about all items in a $N \times M$ *opinion matrix* denoted by $\boldsymbol{O} = (o_{i\alpha})$. It is possible to view an opinion about an item as the perceived attractiveness of that item. For any item α that has not been consumed by agent i the opinion is computed as follows:

$$o_{i\alpha} = \gamma f(i, \alpha) + (1 - \gamma) l_{i\alpha} \tag{2.2}$$

where the value $\gamma \in [0, 1]$ is the *social pressure parameter* which determines the strength of the social pressure on the decision process. The case $\gamma = 0$ corresponds to a pure-individualistic community where no agent cares about what others are doing (hence basing their decisions solely on their idiosyncratic liking values), $\gamma = 1$ corresponds to a pure-social environment where all decisions are based on the behaviors of others. As stated previously, an agent is inclined towards consuming items that are consumed by others in a social environment.

2.2. Dynamics

The model advances in discrete time periods. During each period, agents form their opinions and at the end of the period they decide on which items they will consume synchronously. Consumption of item α by agent i is reflected by updating the consumption matrix (i.e. $c_{i\alpha} = 1$). Once the period is over, the simulation clock ticks and next period is run by using the updated consumption and opinion matrices.

We further simplify the model and assume that the agents' consumption rates are constant and same for all set to one item per period (e.g. each person goes to exactly one movie per week). The decision of which item will be consumed by agent i is defined in a straightforward manner:

$$\alpha_{consumed} = \max_{\alpha} o_{i\alpha}, \text{ with the constraint that } c_{i\alpha} = 0. \tag{2.3}$$

The constraint $c_{i\alpha} = 0$ is required in order to make sure that an item is consumed only once by an agent. In case of several items with the same maximal opinion values, one of them is picked randomly.

2.3. Initial Configuration

A particular configuration of our model at a given time t can be described fully by the triplet $\Gamma_t = (\boldsymbol{G}, \boldsymbol{L}, \boldsymbol{C_t})$. Note that $\boldsymbol{G}$ and $\boldsymbol{L}$ do not have the subscript t because they are time independent and set to their initial configurations as will be explained above. The social component function f and the opinion matrix $\boldsymbol{O}$ are not considered as part of the configuration because their values can be computed by using the former three matrices as needed. The initial configuration Γ_0 is the configuration at $t = 0$ and it is configured as follows.

- $\boldsymbol{G} = (g_{ij})$ such that the underlying topology is a bidirectional ring lattice with a coordination number of k.

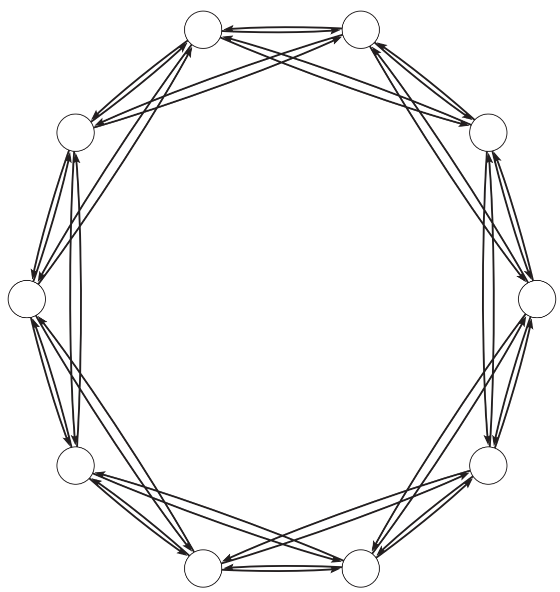


Figure 2.1. A ring lattice topology with 10 vertices and $k = 2$. Figure generated by Pajek (Batagelj and Mrvar, 2002).

- $\boldsymbol{L} = (l_{i\alpha})$ such that $l_{i\alpha}$ is a random variable that comes from a normal distribution with mean μ_α and standard deviation σ_α.

  As a simplification of the model we assume that the overall preference of items does not change from one item to another on the average hence no item has any intrinsic superiority in terms of liking values over another item a priori. In order to reflect this assumption we set the $\mu_\alpha = 0$ for all α. Since the consumption decisions depend on the relative ordering of the opinion values, any other non-zero μ_α values will lead to exactly the same results (as long as they are equal to each other for all α).

The value σ_α is called the *intra-item liking deviation* of item α and denotes the standard deviation in the liking values of agents for the same item. As a simplification for the model, we assume all items have the same intra-item deviation and let $\sigma_\alpha = \sigma_{intra}$ for all α where σ_{intra} is a model parameter.

- $\boldsymbol{C_0} = (c_{i\alpha})$ such that $c_{i\alpha} = 0$ for all i, α. Initially, no items are consumed.

2.4. Remarks

2.4.1. Constant rate of consumption assumption

Because of the constant and uniform consumption rate, exactly N items are consumed during each period. It is trivial to see that the simulation will come to an end when all agents consume all items (i.e. $c_{i\alpha} = 1$ for all i and α). With the aforementioned initial configuration, we see that the simulation can run at most M steps. The problem of when to decide to stop the simulation is addressed in the next subsection.

2.4.2. Terminating condition

In our model, a cultural item is available on the market until the end of the simulation. In real world however, not all items stay available in the market for an indefinite time period. Considering the movie market example, not all movies are shown in the theaters long enough to allow all consumers to see them. There is a continuous entrance and exit of movies. In order to keep our model simple, we ignore this fact but decide to terminate the simulations after a definite number of iterations (T). In other words, our model is limited to a simplified version of the real world where M items are put on the market at the same time and none of them leaves the market until the simulation ends.

2.4.3. Dispersion of quality values of items

We have seen that the liking values $l_{i\alpha}$ are chosen from a normal distribution with mean 0 ($\mu_\alpha = 0$ for all α) and standard deviation σ_{intra}. The fact that the expected average value of the liking values of an item over all agents is 0 does not imply that the sample mean of its liking values will be zero for a particular realization of the model. Let $\langle l_\alpha \rangle$ denote the sample mean of the liking values of item α over all agents. We call $\langle l_\alpha \rangle$ as the *quality* of item α and calculate it as follows.

$$\langle l_\alpha \rangle = \sum_{i=1}^{N} l_{i\alpha}/N \tag{2.4}$$

The Central Limit Theorem states that the distribution of the quality values will follow a normal distribution with mean 0 and standard deviation $\sigma_{intra}/\sqrt{N}$ (Ross, 2002). In the limit $N \to \infty$ the deviation of the quality values approaches to 0. Although the model assumes no a priori superiority of an item over another, we should expect to see different item quality values for particular realizations of the model for finite and small N.

Relaxing the assumption of $\mu_\alpha = 0$ and assigning nonzero values to the means can introduce significant differences between the quality values of the items even in the case of large N. If the dispersion of the μ_α values is large enough the a priori ordering of items can dominate the idiosyncratic and social components. Following the work of Borghesi and Bouchaud (2006), we keep the assumption of equal a priori average liking values leading to a normal distribution of quality values with mean 0 and standard deviation $\sigma_{intra}/\sqrt{N}$ as explained above. The next subsection discusses the effect of increasing σ_{intra} and hence the dispersion of the quality values of the items.

2.4.4. Interpreting the model parameters

The model parameters γ and σ_{intra} regulate the characteristics of the community in terms of susceptibility to collective behavior and herding effect. The social component $f(i, \alpha)$ takes values between 0 and 1 and the magnitude of its effect on the final consumption decision is regulated by the social pressure γ. Idiosyncratic component $l_{i\alpha}$ is a normally distributed random variable and in practice the range of values it can take is regulated by the standard deviation of the distribution σ_{intra}. One may think of γ as a measure of the collective nature of the community. Smaller γ corresponds to individualistic communities and bigger γ corresponds to more collectivistic communities. The standard deviation σ_{intra} on the other hand is a measure of the heterogeneity of the community. Higher values of σ_{intra} corresponds to more heterogeneous communities where the liking values of the agents are widely dispersed for the same items. Lower values of the standard deviation correspond to the more homogeneous communities where the liking values of the agents vary less for the same items. However, we should note that, neither the idiosyncratic nor the social component has an intrinsic scale: After all, they are just real numbers on their own and obtain a meaning when contrasted to each other to reach a decision. The relative magnitudes of the social pressure γ and the standard deviation σ_{intra} to each other determine the extent of the social effect on the individual decision and we will further dwell on this issue in our simulations.

2.5. Quantities of Interest

Market Share: We define the market share of an item α as the ratio of consumers who have consumed that item so far. It is denoted by $\langle c_\alpha \rangle$ and calculated as follows.

$$\langle c_\alpha \rangle = \sum_{i=1}^{N} c_{i\alpha}/N \tag{2.5}$$

This quantity is a monotonically non-decreasing function of time for all items. At any time step, either one or more agents consume the item α and $\langle c_\alpha \rangle$ increases, or no agent consumes the item and $\langle c_\alpha \rangle$ remains the same. As argued above, at the end of simulation all $\langle c_\alpha \rangle$ values will be equal to 1. By observing the quantity intermediate steps, we can gain insight about how the simulation advances.

Item quality: Item quality is already defined as the average liking value of an item over all agents and denoted by $\langle l_\alpha \rangle$ for an item α. It's formal definition is given in Eq. 2.4.

Total consumption: We can calculate the *total consumption* (i.e. number of total consumptions) easily as:

$$\sum_{i=1}^{N} \sum_{\alpha=1}^{M} c_{i\alpha} = TN$$

where N is the number of agents and T is the number of steps that the model has run so far.

Market Inequality: Inequality of a market represents the difference between the market shares of the cultural items. As a measure of the inequality, we calculate the Gini index of the market shares of all items. The *market inequality* is denoted by I and defined as follows.

$$I = \frac{(\sum_{\alpha=1}^{M} \sum_{\beta=1}^{M} |\langle c_\alpha \rangle - \langle c_\beta \rangle|)/M^2}{(2\sum_{\alpha=1}^{M} \langle c_\alpha \rangle)/M} \tag{2.6}$$

which can be interpreted as the expected difference between the market shares of two randomly chosen items normalized to scale between 0 and 1 (Salganik et al., 2006). A perfectly equal market where all items have the same market shares will have an inequality value of 0; that is the Gini index of a perfectly equal market.

Quartile difference: The market inequality value is a symmetrical measure according to the quality values of the items. It does not differentiate whether it is the low quality items that receive unfairly high market shares or the high quality items. An important question that we would like to answer is that whether there is a general trend favoring the high quality items in terms of market shares or vice versa. We employ a very simple measure that is used in economics which we call the *quartile difference*. First we divide the items into four quartiles according to their quality values. The items with the top 25% quality values are placed in the upper quartile $\mathcal{U}$ and the items with the bottom 25% quality values are placed in the lower quartile $\mathcal{L}$. Quartile difference Q is simply the difference between average market shares of the two items in these two quartiles.

$$Q = \sum_{\alpha \in \mathcal{U}} \langle c_\alpha \rangle - \sum_{\alpha \in \mathcal{L}} \langle c_\alpha \rangle \tag{2.7}$$

Since the quartile difference is a signed value, the minimal value it can take is -1 and this occurs in the extreme case in which all items in the lower quartile have market shares of 1 and all the items in the upper quartile have market shares of 0. Similarly, the maximal value it can assume can be computed as +1.

3. RESULTS AND DISCUSSIONS

All results reported in this chapter are based on values obtained by averaging over 100 independent runs of the simulation with the same parameters unless noted otherwise.

3.1. Social Interaction

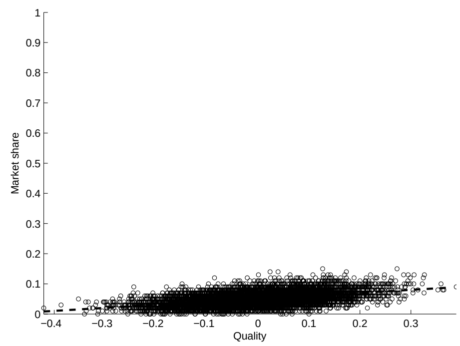
For pedagogic reasons, we first introduce a special case of our cultural model with the social pressure γ set to zero. In this section, we aim to introduce our techniques of analyses by applying them to a relatively simple example of the model and to provide the no-social interaction case as a base model to which we will contrast our future findings.

It is natural to ask if the quality of an item determines its market share at the end or not. A reasonable expectation about a cultural market is that items with high quality values should get higher market shares on the average. In other words, one might expect the quality to be a reliable indicator of the final market share of an item. A scatter plot of the market shares versus qualities will answer whether this expectation holds or not. As a starter, we let $\gamma = 0$ and $\sigma_{intra} = 1$ which corresponds to a pure-individualistic community where all agents base their consumption decisions solely on their personal tastes. We keep the number of agents N and the number of items M fixed to 100 in this set of experiments. The sensitivity of the results on these parameters will be investigated separately. Since there is no social interaction, the underlying topology has no effect on the results because its possible effects are ruled out with the zero social pressure parameter (i.e. $\gamma = 0$).

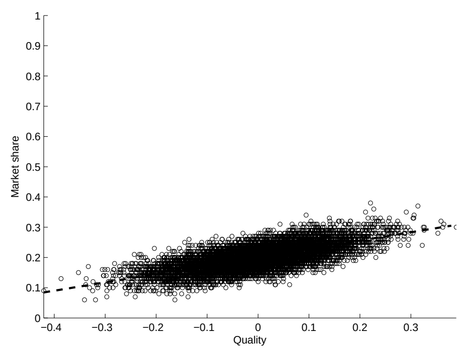
We let the model run for 5, 20 and 50 steps (i.e. $T \in \{5, 20, 50\}$). In Fig. 3.1, we see the scatter plot of quality versus market shares of the items for three cases.

Let us first consider the case with $T = 5$ given in Fig. 3.1(a). The relation between the quality and market share has a linear form. A quadratic equation does not provide a better fit than a linear equation (i.e. both fits have the same r-square $r^2 = 0.21$). A linear relation also holds for the other two cases with $T = 20$ and $T = 50$ given in Fig. 3.1(b) and 3.1(c) correspondingly.

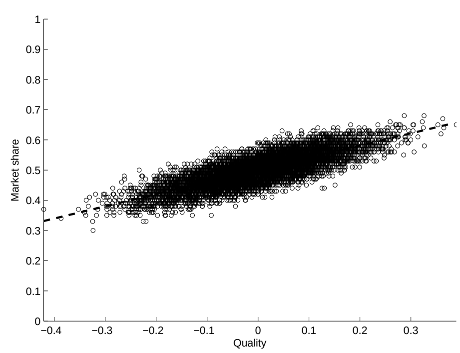
As dictated by the dynamics of the model, prolonging the simulation leads to an increased amount of total consumption and market shares. However, not all items increase their shares by the same amount. The slope of the best fitting lines differ significantly between the three cases (0.10, 0.28, and 0.40 for $T = 5$, $T = 20$, and $T = 50$ correspondingly). A steeper slope indicates that items with higher quality values increase their market shares at higher rates when compared to others. The quartile difference Q helps us to quantify this trend. In Fig. 3.2(a), quartile difference values are plotted against different simulation lengths. Until the point $T = 50$, we observe a gradual increase supporting our findings. After peaking somewhere around $T = 50$ (which happens to be $T = M/2$), Q starts to decrease and terminates at 0 when $T = M = 100$ as anticipated. We can cross check this finding by comparing the rate of increase in the market shares of high quality and low quality items. In Fig. 3.2(b), we see the market share of two items increasing as the simulation continues. The solid and dashed lines belong to the items with the highest and lowest quality values correspondingly. The series are obtained by averaging the data of highest quality and lowest quality items separately over 100 runs. The rate of the market share increase is clearly higher for the highest quality items until a certain point around $T = 50$ as we expected. But after that point



(a) Simulation length T is 5 steps



(b) Simulation length T is 20 steps



(c) Simulation length T is 50 steps

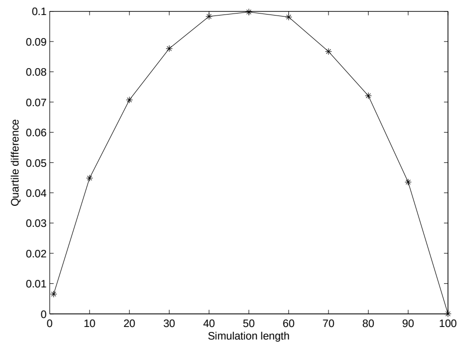
Figure 3.1. Relation between quality $\langle l_\alpha \rangle$ and market share $\langle c_\alpha \rangle$ for different simulation lengths. Observed data (circles), and linear fit (dashed line).

the rate of increase for the highest quality items starts to decline while the rate of increase for lowest quality items starts to rise. This is an indication of a ceiling effect: As the high quality items are consumed by more and more agents, their market share increase rate gets lower because a smaller number of agents have the opportunity to consume them.

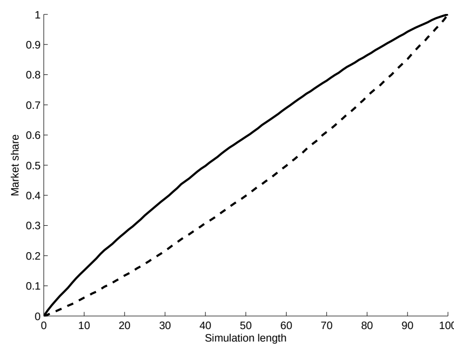
As we already noted before, the simulation length (i.e. number of steps that the simulation will run for) is an important parameter and should be set carefully with keeping in my mind the actual market to be modeled. As Fig. 3.2(c) suggests, the market inequality does not change abruptly for differing values of simulation length but rather decrease gradually. Similarly quartile difference Q shows a gradual increase for the region $0 \ll T \ll M$. We have no reason to expect that different values of T will cause any qualitative changes in our results for the region $0 \ll T \ll M$. Therefore, we set $T = 20$ during the next set of experiments for simplicity of analyses.

3.2. Introducing Social Pressure

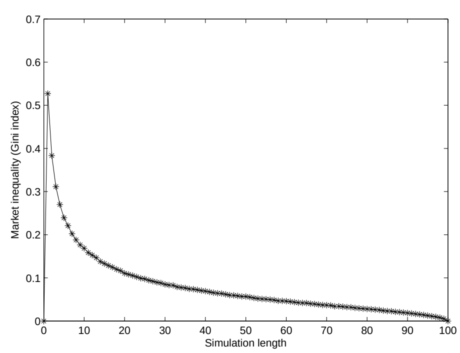
In the previous subsection, we looked at the dynamics of a community without any social interaction (i.e. $\gamma = 0$). An interesting extension will be introducing social pressure (i.e. $\gamma > 0$). To keep things simple for the moment, we create two parameter settings: One with $\gamma = 0.3$ corresponding to a low social pressure environment and one with $\gamma = 0.7$ corresponding to a high social pressure environment. The underlying topology is set to a fully connected graph which corresponds to a ring lattice with a coordination number $k = N - 1$. We can see the scatter plot of the market share versus qualities at the end of the simulations (T is set to 20) in Fig. 3.3(a) and Fig. 3.3(c). Another way to look at the effect of social pressure is to



(a) Quartile difference Q as a function of simulation length T.



(b) Market shares $\langle c_\alpha \rangle$ of highest (solid) and lowest (dashed) quality items as a function of simulation length T.



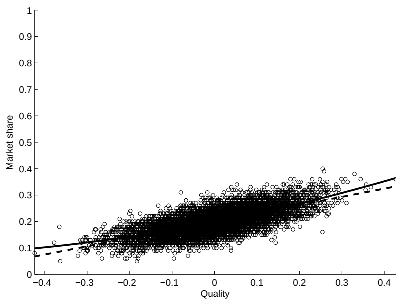
(c) Market inequality I as a function of simulation length T.

Figure 3.2. Effect of simulation length T on (a) Quartile difference Q, (b) Market share increase, and (c) Market inequality ($N = 100$, $M = 100$, $k = 99$, $\sigma_{intra} = 1$, $\gamma = 0$ for all cases).

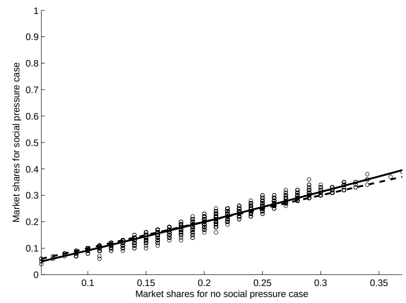
compare the market shares obtained in low and high social pressure environments to the ones obtained in Community with no no social pressure environment which is our base model. Using the same liking matrix $\boldsymbol{L}$ at the start of the both simulations allows us to compare the market shares of the same item that it obtains under different social pressure environments. Figures 3.3(b) and 3.3(d) provide us the scatter plots of the market shares obtained in low and high social pressure environments (i.e. $\gamma = 0.3$ and $\gamma = 0.7$) versus market shares obtained in our base case (i.e. $\gamma = 0$) correspondingly.

Low social pressure environment has similar results with the no social pressure environment. The best fitting quadratic and linear fits are similar for the market share versus quality in Fig. 3.3(a) and the data points are scattered around the $y = x$ line in Fig. 3.3(b) indicating that the effect of setting $\gamma = 0.3$ has no or limited effect on the final market shares of the items. In low social pressure environment the consumption decisions remain fairly intact compared to the no social pressure environment.

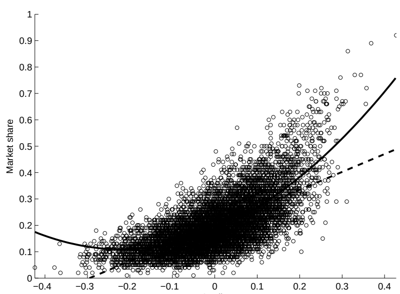
High social environment leads to a significant change in the consumption decisions. The relation between the quality and market share of an item is no longer linear as it can be seen in Fig. 3.3(c). A quadratic fit deviates significantly from the linear fit and suggests that items with high quality values obtain improportionally higher market shares at the cost of items with lower quality. The same effect is also visible in the scatter plot of market shares obtained in no social and high social environments in Fig. 3.3(d). The data points are not scattered around the $y = x$ line. The items which obtain high market shares in the no social environment obtain even higher market share in the high social pressure environment and the items which obtain low market shares in the no social pressure environment obtain even lower



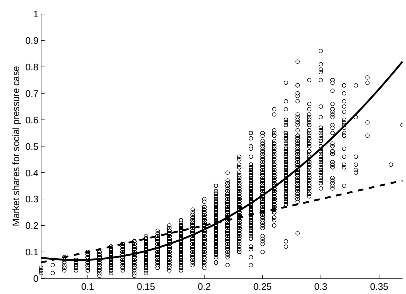
(a) Market share versus quality for low social pressure ($\gamma = 0.3$).



(b) Low social pressure ($\gamma = 0.3$) shares versus no social pressure ($\gamma = 0$) shares.



(c) Market share versus quality for high social pressure ($\gamma = 0.7$).



(d) High social pressure ($\gamma = 0.7$) shares versus no social pressure ($\gamma = 0$) shares

Figure 3.3. The effect of introducing social pressure on market shares. Circles are the data points in all cases. For (a) and (c), solid line is the best quadratic fit and dashed line is the best linear fit. For (b) and (d), solid line is the best quadratic fit and dashed line is the $y = x$ line.

market shares in high social environment.

3.3. Interaction between Intra-Item Deviation and Social Pressure

For the moment, we know that for two different values of social pressure (i.e. $\gamma = 0.3$ and $\gamma = 0.7$) we get two different market dynamics. Now it is time to ask the two obvious questions: What is the response of our model to varying degrees of social pressure? How can we extend our findings for communities with varying degrees of heterogeneity? The model parameter γ regulates the extent of social pressure and intra-item liking deviance σ_{intra} regulates the heterogeneity of the agents in the community. Until now, we set $\sigma_{intra} = 1$ but this choice is arbitrary. Smaller choices for σ_{intra} will lead to more homogeneous communities in the sense that the liking values of the agents for the same items will be closer to each other. Higher choices on the other hand, will correspond to more heterogeneous communities because the liking values of the agents for the same items will deviate more.

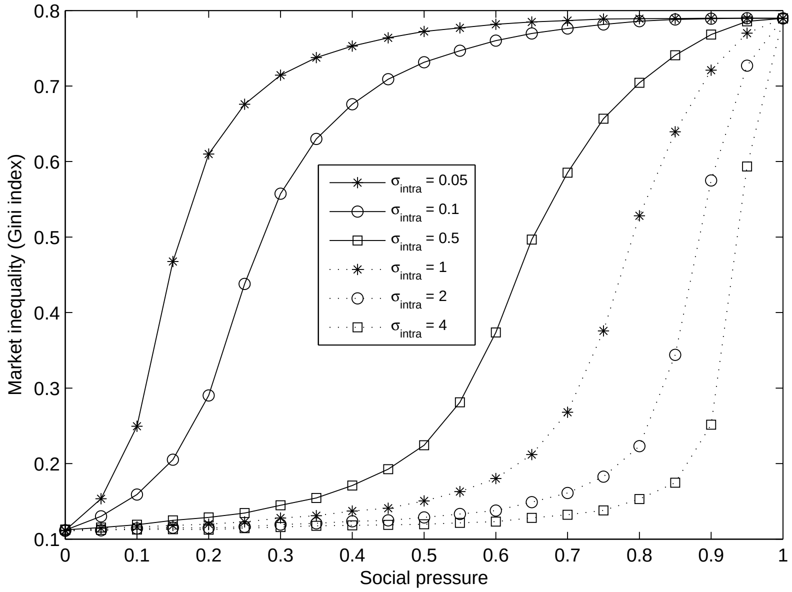
In order to come up with answers to the two questions we ask, we calculated the market inequality I and the quartile difference Q of the markets at the end of 20 steps for different pairs of γ and σ_{intra} values. Fig. 3.4 and Fig. 3.5 visualize the differing values of I and Q correspondingly.

In Fig. 3.4(a), we see the effect of increasing social pressure (γ) on the market inequality (I) for different values of intra-item liking deviation (σ_{intra}). As we have already seen previously, higher γ values lead to higher I values but the characteristic of the effect depends on intra-item liking deviation σ_{intra}. The interaction between the γ and σ_{intra} is given as contour plot in Fig. 3.4(b). Each point on the plane has a color associated with the resulting market inequality value for the corresponding

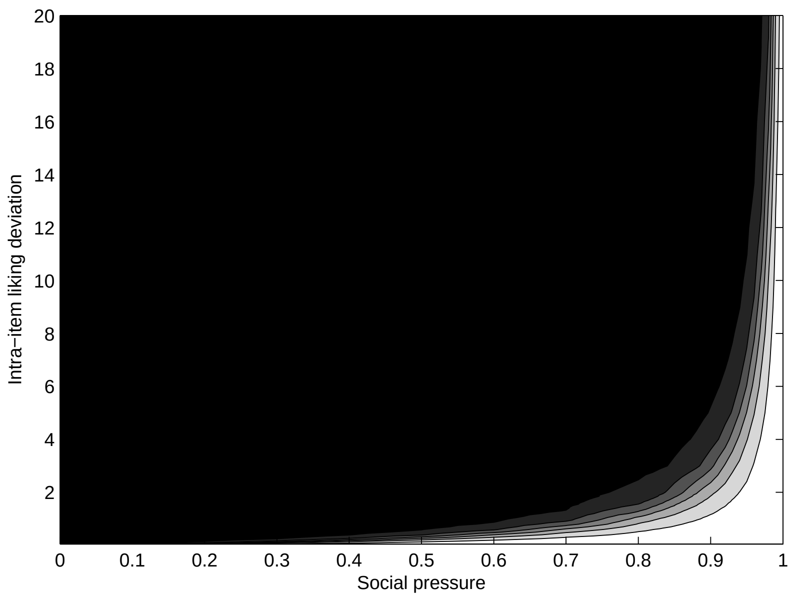
σ_{intra} γ pair. Darker colors represent low I, brighter colors represent high I values. In both figures, we see that the I values show a sharp increase from low values (e.g. $I = 0.2$) to high values (e.g. $I = 0.8$) at critical values of γ. However the specific value of the critical γ value depends on σ_{intra}.

The market inequality I is a measure of how varied the final market shares are. It does not tell us anything about the dependence of market shares on quality. In Sec. 3.2, we have seen that higher social pressure γ result in increased market shares for the high quality items. In Fig. 3.5, we can see the details of the effect of γ on the quality-market share relation. Figure 3.5(a) visualizes the effect of increasing social pressure γ on the quartile difference Q for different values of intra-item liking deviation. Figure 3.5(b) presents the contour plot of quartile difference values as a function of γ and σ_{intra}. Similar to the inequality value, the quartile difference also shows a sharp increase at a critical value of γ and the specific value of γ depends on σ_{intra}. However, at higher levels of social pressure the quartile difference starts to decrease. The turning point for the quartile difference (i.e. the γ value that it starts to decrease) is again dependent on σ_{intra}. This is an interesting observation because after the turning point the inequality value continues to increase while the quartile difference starts to decrease.

Interpreting the results for inequality and quartile difference values together we conclude that there exists a critical region of γ and for values less then the critical region the inequality and quartile difference values are in accordance with each other. High quality items gain higher market shares as we increase the social pressure and this fact is reflected in increased inequality and quartile difference values. But once the critical turning point in social pressure is passed, the relation between the quality and market share starts to weaken (hence quartile difference



(a) Effect of intra-item liking deviation σ_{intra} (line series) and social pressure γ on the inequality of the market shares I.



(b) Contour plot of market inequality value I as a function of intra-item liking deviation σ_{intra} and social pressure γ. The peak value of I is 0.8 (correponds to the white area) and each contour line corresponds to a step of 0.1.

Figure 3.4. Market inequality as a function of social pressure γ and intra-item liking deviation σ_{intra}.

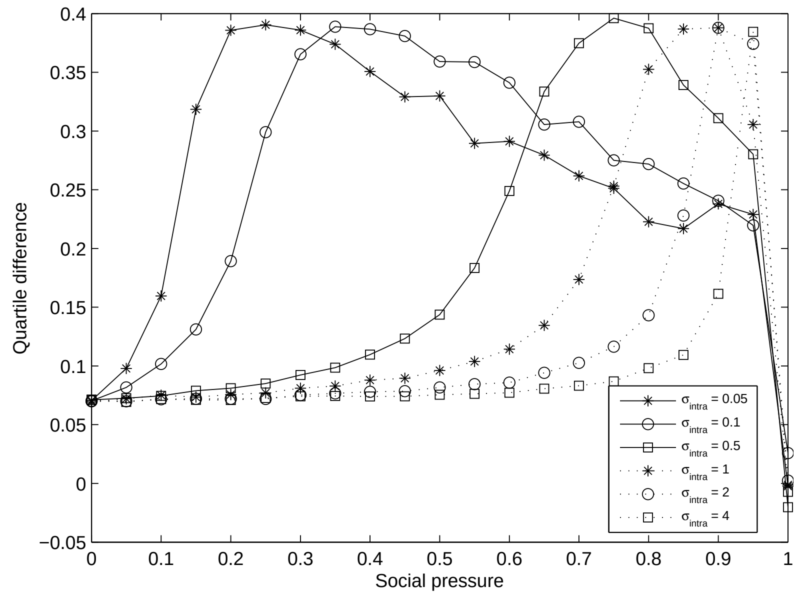
value starts to decrease), but some items still continue to arbitrarily high market shares at the cost of others (hence the inequality value continues to increase). The positive correlation between the quality and market shares disappears suddenly as reflected by the decrease in quartile difference value.

3.4. Effect of Number of Agents

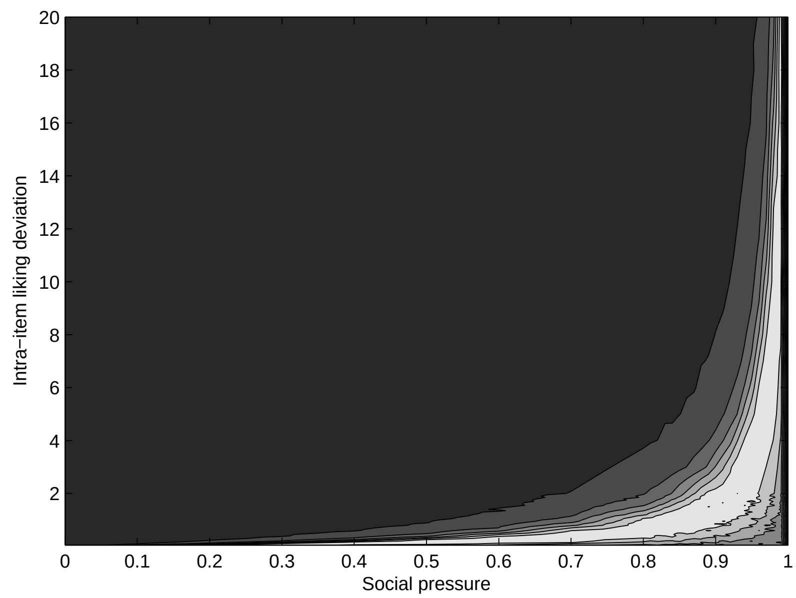
We assumed, in the limit $N \rightarrow \infty$, there are no a priori advantages of items over others in terms of quality values and let the sample mean of the liking values $l_{i\alpha}$ determine the actual quality values for realization with finite N. One possible consequence of this assumption is that as we increase the number of agents in the system the standard deviation of the realized quality values of the items will have narrower distribution as the standard deviation is shown to be $\sigma_{intra}/\sqrt{N}$ due to the Central Limit Theorem. In order to see the effect of increasing the number of agents we set N four different values (i.e. $N \in \{100, 500, 1000, 5000\}$) and run the simulations for these set of N. The inequality and quartile difference values obtained are plotted in Fig. 3.6.

3.5. Effect of Topology and Local Interactions

So far our choice of topology has been a ring lattice with a coordination number of $N - 1$ which corresponds to a fully connected graph. Translation of this setting to the real world is that every agent knows every other agent and in statistical physics this corresponds to the mean-field approximation (Phan and Pajot, 2006). Obviously, this is a very strong assumption. Phan and Pajot (2006), stresses the importance of local effects on related binary decision models and it is possible that different network topologies can have different effects on the resulting market shares.

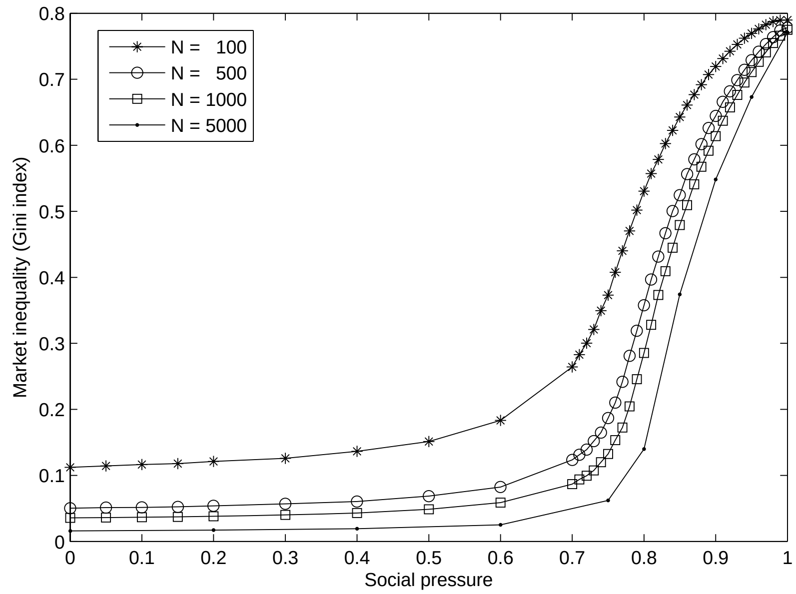


(a) Effect of intra-item liking deviation σ_{intra} (line series) and social pressure γ on the quartile difference Q.

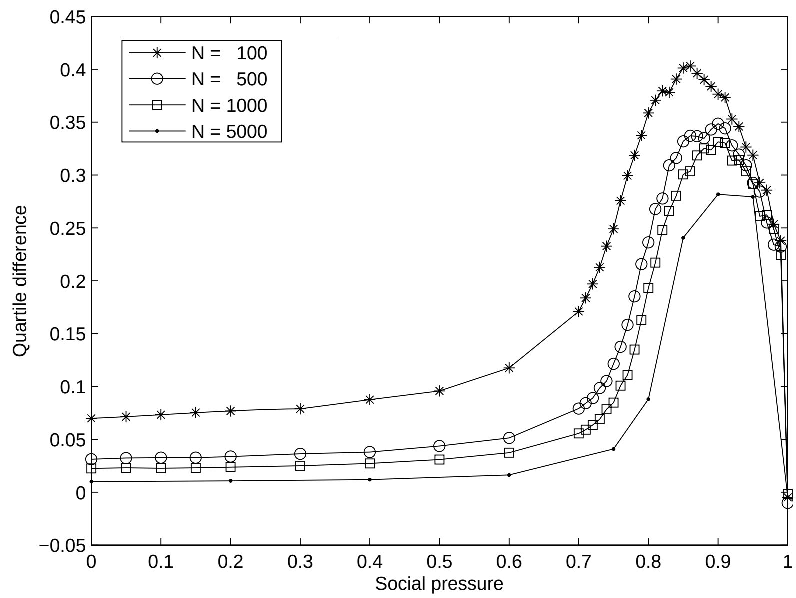


(b) Contour plot of quartile difference Q as a function of intra-item liking deviation σ_{intra} and social pressure γ. The peak value of Q is 0.4 and each contour line corresponds to a step of 0.05.

Figure 3.5. Quartile difference Q as a function of social pressure γ and intra-item liking deviation σ_{intra}.



(a) Inequality as a function of social pressure for different number of agents.



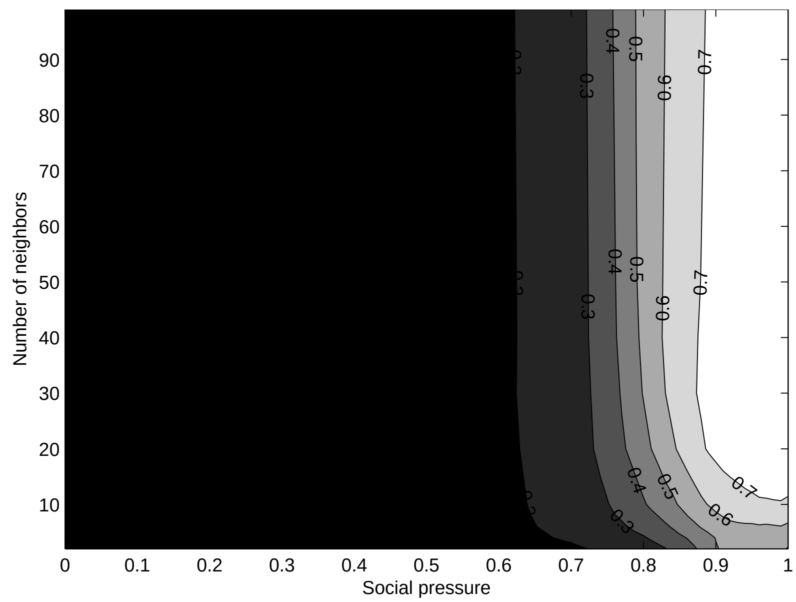
(b) Quartile difference as a function of social pressure for different number of agents.

Figure 3.6. Effect of number of agents N (line series) on (a) the inequality, (b) quartile difference ($k = N - 1$ for all N and $\sigma_{intra} = 1$ for both cases).

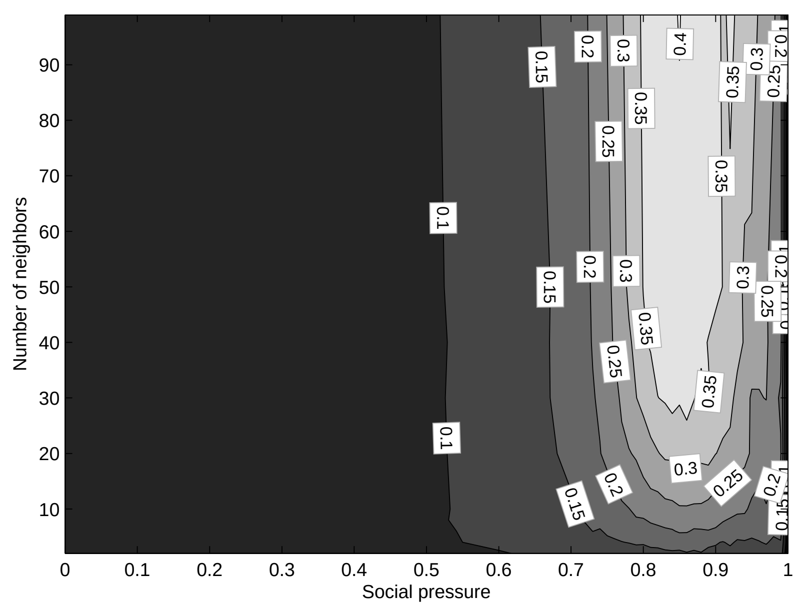
Ring topology with varying k: A trivial extension of the model along the topology dimension is to keep the ring topology intact but let the number of neighbors k have different values ranging from 2 to $N-1$. In Fig. 3.7, we can see interactions between the social pressure and number of neighbors on the inequality and quartile difference values for $N = 100$ and $\sigma_{intra} = 1$. For this set of simulations number of independent runs is set to 20 for time limitations. Apparently, the mean field approximation (i.e. setting $k = N - 1$) provides good estimates for $k > 30$. Even for lower values of k, the interaction pattern between the number of neighbors and social pressure remains qualitatively same but results in relatively lower values of inequality and quartile difference.

In real social communities we expect the number of neighbors of an agent to be very smaller than the total number of agents (i.e. $k \ll N$). In order to analyze this situation, we run another set of simulations in which we set the number of agents to a bigger value (i.e. $N = 1000$) and keep the number of neighbors at lower levels (i.e. $k < 100$) (Again, the results are averaged over 20 independent runs). The results of those simulations are given in Fig. 3.8. Note that the maximal value of k is still 100 but this time this corresponds to only 10% of all community. Again the actual values of inequality and quartile difference are lower compared to the case with $N = 100$ but the pattern of the interaction between k and γ seems to remain qualitatively intact.

When the community is fully connected ($k = N - 1$), the social component of the decision process acts as a signal indicating the percentage of agents who have consumed a particular item. If at any time step, an item has been consumed by many agents then the probability that it will increase it market share in the next time step is high. When we relax the assumption that the community is fully

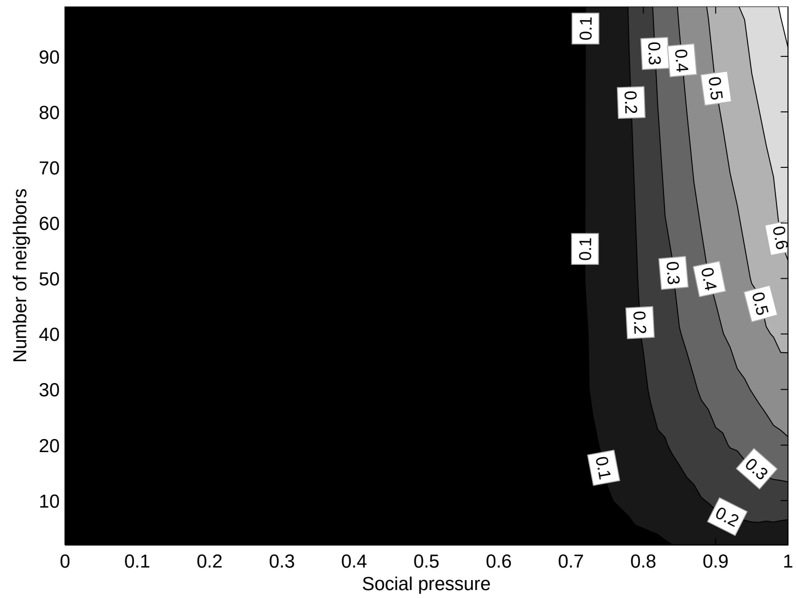


(a) Inequality as a function of number of neighbors k and social pressure γ.

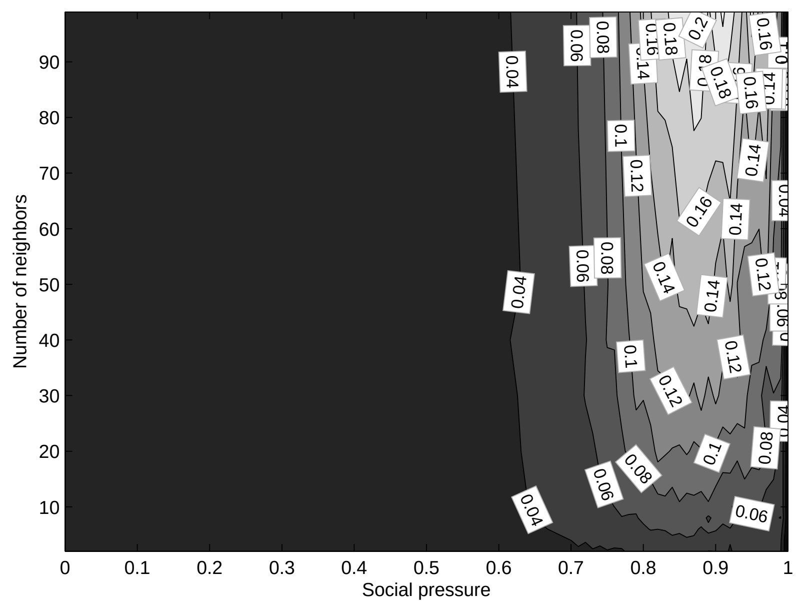


(b) Quartile difference as a function of number of neighbors k and social pressure γ.

Figure 3.7. Inequality I and quartile difference in ring topology for differing values of social pressure and average number of neighbors ($N = 100$ and $\sigma_{intra} = 1$ for both cases).



(a) Inequality as a function of number of neighbors k and social pressure γ.



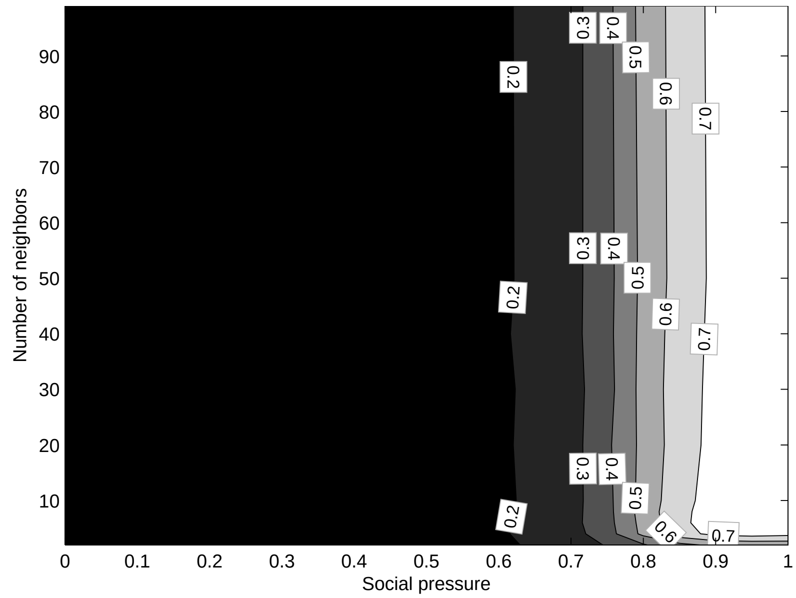
(b) Quartile difference as a function of number of neighbors k and social pressure γ.

Figure 3.8. Inequality I and quartile difference in ring topology for differing values of social pressure and average number of neighbors ($N = 1000$ and $\sigma_{intra} = 1$ for both cases).

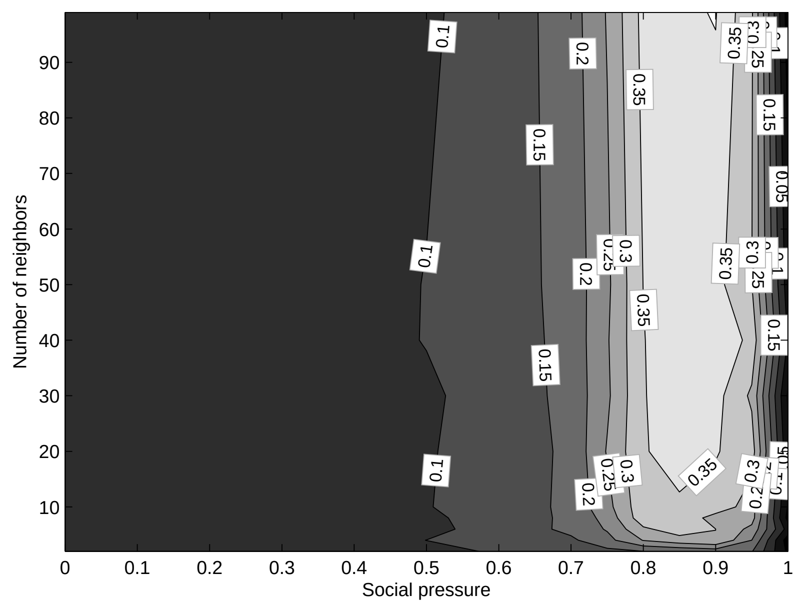
connected, then the social component acts as an imperfect signal and indicates the percentage of agents who have consumed a particular item in the neighborhood of a particular agent. Local differences reduce the impact of the positive feedback due to social interaction but we have not observed a critical point in k where the effect of social interaction abruptly changes.

Random topology with varying k: Note that the above simulations are run ring lattice topology which has not any topological heterogeneity (i.e. all agents are lined up in a regular lattice with equal number of neighbors). In order to introduce some heterogeneity in the network structure, we repeated our simulations on random graphs and interpreted k as the average number of neighbors. The results are given in Fig. 3.9 for $N = 100$ and Fig. 3.10 for $N = 1000$.

Although we do not observe a qualitative difference in the interaction between the average number of neighbors k and the social pressure γ, the actual inequality and quartile difference values are higher for the random topology for the same number of neighbors. In order to provide a comparative plot of the inequality and quartile difference values we set the number of neighbors to 20 ($k = 20$), intra-item liking deviation to 1 ($\sigma_{intra} = 1$) and run the model for two different number of agent values (i.e. $N \in \{100, 1000\}$). The results are given in Fig. 3.11 for $N = 100$ and Fig. 3.12 for $N = 1000$. The inequality and quartile difference values are almost same for the two different topologies when the number of agents is 100 (i.e. graph is closer to full connectivity because $k = 20$). Increasing the number of agents from 100 to 1000 is reflected with a decrease in both inequality and quartile difference values in ring topology. Random topology seems to be resilient to such an increase: The inequality and quartile difference values remain almost intact when switching from 100 agents to 1000 agents.

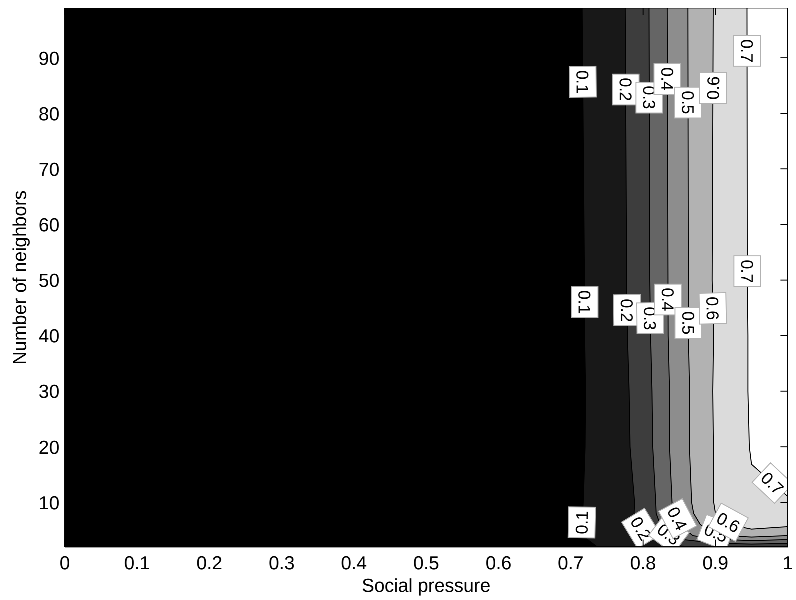


(a) Inequality as a function of number of neighbors k and social pressure γ.

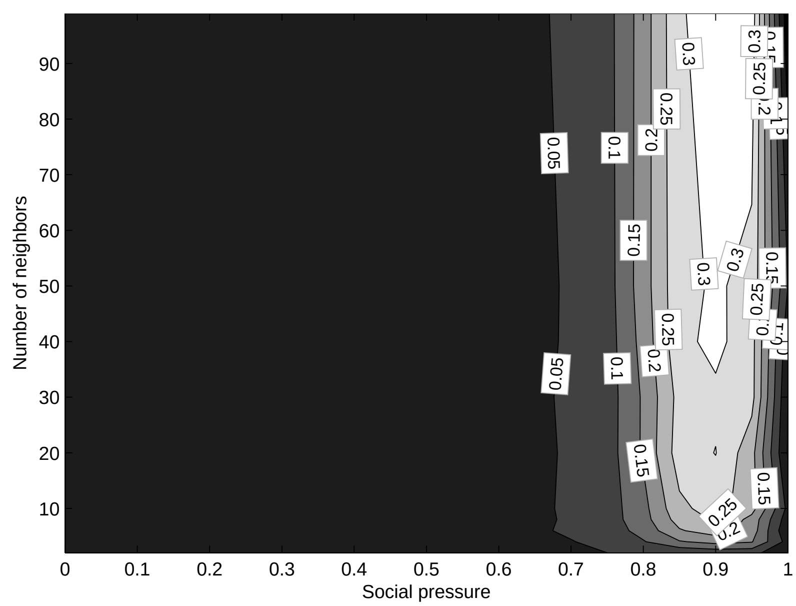


(b) Quartile difference as a function of number of neighbors k and social pressure γ.

Figure 3.9. Inequality I and quartile difference in random topology for differing values of social pressure and average number of neighbors ($N = 100$ and $\sigma_{intra} = 1$ for both cases.

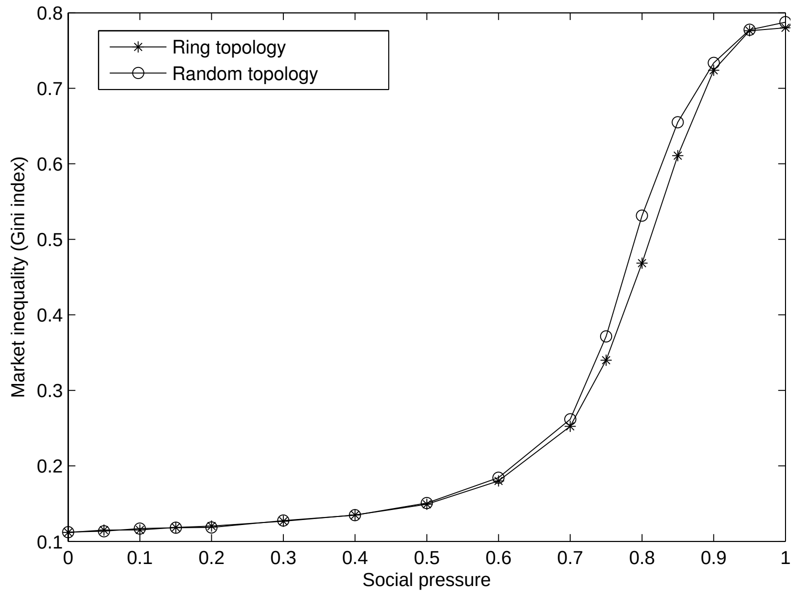


(a) Inequality as a function of number of neighbors k and social pressure γ.

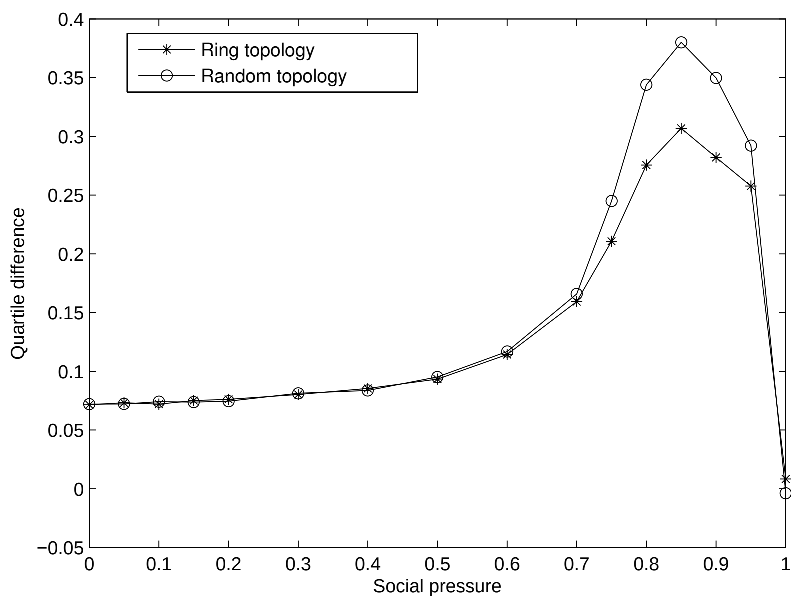


(b) Quartile difference as a function of number of neighbors k and social pressure γ.

Figure 3.10. Inequality I and quartile difference in random topology for differing values of social pressure and average number of neighbors ($N = 1000$ and $\sigma_{intra} = 1$ for both cases.

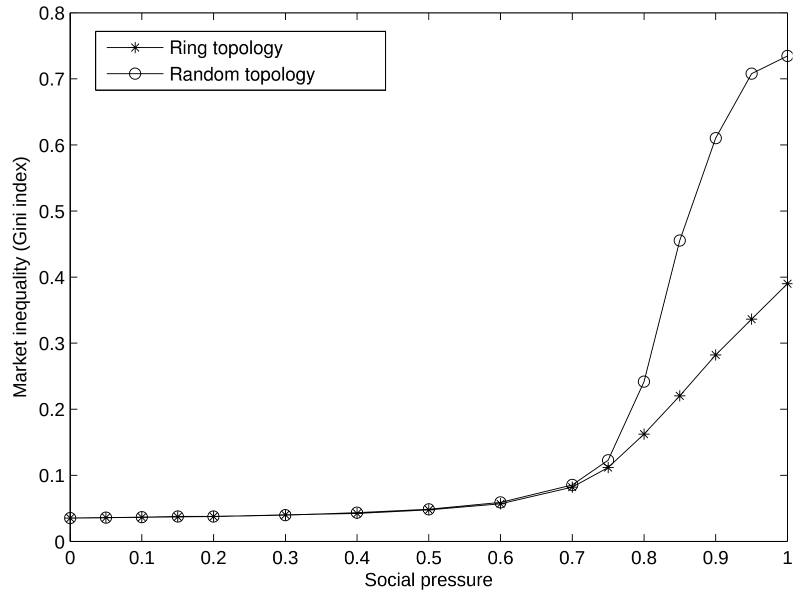


(a) Inequality values for ring and random topologies.

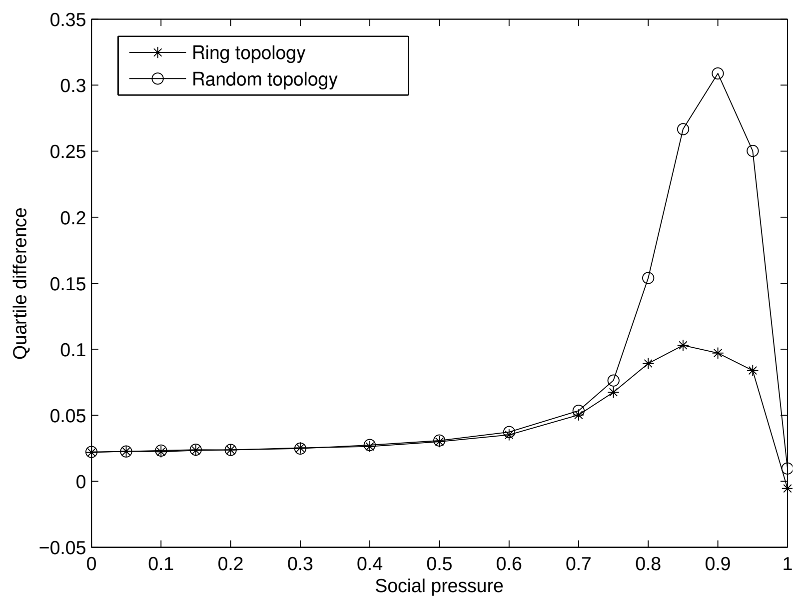


(b) Quartile difference values for ring and random topologies.

Figure 3.11. Comparison of ring and random topologies for $N = 100$, $k = 20$, $\sigma_{intra} = 1$.



(a) Inequality values for ring and random topologies, $N = 1000$.



(b) Quartile difference values for ring and random topologies.

Figure 3.12. Comparison of ring and random topologies for $N = 1000$, $k = 20$, $\sigma_{intra} = 1$.

4. CONCLUSIONS

We introduced a computational model for a cultural market and analyzed the effect of social pressure on the consumption decisions by extended simulations. The case with no-social interaction (i.e. $\gamma = 0$) served us as a base model and we compared our findings with different values of γ with the no-social pressure case. We observed that in the no-social interaction case the final market share of an item is largely determined by its quality and the relation between the two is a linear one.

Empirical findings suggested that the introduction of social interaction has a profound effect on the relation between the quality and market share of an item. For low values of social pressure (i.e. $\gamma = 0.3$), we observed the linear relation between the quality and market shares remains almost intact. As we increase the social pressure parameter ($\gamma = 0.7$), the linearity was disrupted and we observed more nonlinear relation in the favor of high quality items.

An interesting finding is that the social parameter γ does not regulate the collective nature of the model but interacts with the intra-item liking deviation σ_{intra} which represents the deviation among the liking values of the agents for the same items. We found out that a given market inequality value I can be reproduced by a set of (γ, σ_{intra}) pairs and the interaction between the two parameters remain qualitatively intact for different number of agents N and average number of neighbors k.

We also introduced another measure we call the quartile difference (Q) to analyze the relation between the quality and market shares of items. To our surprise while the inequality in the market shares increase with increasing social pressure γ, the quartile difference value first increase and than decrease with a peak at a critical value of γ dependent on σ_{intra}. The initial increase is consistent with the market inequality values: As we increase social pressure, the high quality items gain more and more market shares at the cost of low quality items and this is reflected in the increased inequality and quartile difference values. But after the critical γ value is passed, the positive correlation between the quality and market share deteriorates. Low quality items starts to gain higher market shares and Q shows a decrease. Note that we cannot observe this trend just by looking at the market inequality I which continues to increase as we increase γ. These findings suggest that for higher values of γ the dynamics of the model gets history dependent and the resulting market share of an item is determined by the initial conditions of the system rather than the quality of the item.

The parameter space is wide because there are many parameters that may affect the results of the simulations. Some of them are limited by our assumptions about the cultural markets (i.e. $T \ll M$) and some should be determined by the actual market in question: topology (i.e. ring or random), number of agents N, number of items M, and the average number of agents k. We carried out extended simulations to see if our results depend on specific values of the parameters or robust to different values of the parameters. We concluded that the qualitative nature of the simulations are robust with respect to different number of agents (i.e. $N \in \{100, 500, 1000, 5000\}$) and varying degrees of network connectivity (i.e. for different values of $k < N$). The topology on the other hand seems to be affecting the outcome of the cultural market. The sharp increases in the inequality and quartile difference were more resilient to varying number of agents in random topology than the ring topology.

An exact analytic solution of the model is very hard to find and we did not present one in this study. Actually, that is the very reason that most of the previous studies focused on binary decision problems rather than the more realistic ones like the cultural market. Nonetheless, an analytical solution for the cultural market is required in the future.

Studying more realistic topologies for the agent friendship network is also another interesting dimension to extend the cultural market. We studied the ring topology as a base model and introduced the heterogeneity by using random topology. How the model will behave if another type of network (e.g. scale free, small world) is introduced is definitely an interesting and non-trivial question and needs to be addressed in the future studies.

Another important point is to study the accordance between the real world data and the model output. Unfortunately, during the writing of this thesis no relevant dataset was publicly available. The only dataset known to exist is expected to become public in the summer of 2007 (Salganik et al., 2006). Nevertheless, the results of the model are still valuable given the costs of conducting a real world experiment is many times higher than building and analyzing a computational model. The results and the dynamics of the cultural market model can be of use in determining the settings of a real world experiment in the future.

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