MARKET BASKET ANALYSIS FOR DATA MINING

by

Mehmet Aydın Ulaş

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ABSTRACT

MARKET BASKET ANALYSIS FOR DATA MINING

Most of the established companies have accumulated masses of data from their customers for decades. With the e-commerce applications growing rapidly, the companies will have a significant amount of data in months not in years. Data Mining, also known as Knowledge Discovery in Databases (KDD), is to find trends, patterns, correlations, anomalies in these databases which can help us to make accurate future decisions.

Mining Association Rules is one of the main application areas of Data Mining. Given a set of customer transactions on items, the aim is to find correlations between the sales of items.

We consider Association Mining in large database of customer transactions. We give an overview of the problem and explain approaches that have been used to attack this problem. We then give the description of the Apriori Algorithm and show results that are taken from Gima Türk A.Ş. a large Turkish supermarket chain. We also use two statistical methods: Principal Component Analysis and k-means to detect correlations between sets of items.

ÖZET

VERİ MADENCİLİĞİ İÇİN SEPET ANALİZİ

Birçok gelişmiş firma yıllar boyunca müşterilerinden çok fazla miktarda veri topladılar. Elektronik ticaret uygulamalarının da çoğalmasıyla şirketler artık çok fazla veriyi yıllar değil aylarla ölçülebilecek bir zamanda bir araya getirebiliyorlar. Veritabanlarında Bilgi Keşfi olarak da bilinen Veri Madenciliğinin amacı ilerki aşamalardaki kararlara yardımcı olması için veri içerisinde yönsemeler, örüntüler, ilintiler ve sapaklıklar bulmaktır.

İlişki Kuralları Bulma Veri Madenciliğinin ana uygulama alanlarından bir tanesidir. Sepet analizinin amacı verilen bir satış raporları üzerinden mallar arasında ilintiler bulmaktır.

Bu tezde geniş bir mal satış veri tabanı üzerinde İlişki Madenciliği çalışması yaptık. İlk kısımda problemin genel hatlarla tanımını ve bu problemi çözmek için kullanılan yaklaşımları anlattık. Bu konuda ilk kullanılan algoritmalardan birisi olan "Apriori Algoritması" nı detaylı olarak inceleyerek bu algoritmanın büyük bir süpermarket zinciri olan Gima Türk A.Ş.'nin verileri üzerinde uygulanmasıyla ortaya çıkan sonuçları verdik. Ayrıca mal satışları arasında ilintiler bulmak için iki istatistiksel method kullandık: Ana Bileşen Analizi ve k-Ortalama Öbeklemesi.

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LIST OF SYMBOLS/ABBREVIATIONS

b_j^t	1 if x^t belongs to cluster j, 0 otherwise
D	The whole database
DB_i	The database at the i^{th} distributed site
$\|D\|$	Size of the database
$E[x]$	Expected value of x
v_j	The j^{th} cluster center
x^t	The t^{th} input vector
η	Learning Factor
λ_i	i^{th} eigenvalue

AI	Artificial Intelligence
APM	Adaptive Parallel Mining
CRM	Customer Relationship Management
FDM	Fast Distributed Mining of association rules
KDD	Knowledge Discovery in Databases
OS	Operating System
PCA	Principal Component Analysis

1. INTRODUCTION

Data Mining, also known as *Knowledge Discovery in Databases* (KDD), is to find trends, patterns, correlations, anomalies in these databases which can help us to make accurate future decisions. However data mining is not magic. No one can guarantee that the decision will lead to good results. Data Mining only helps experts to understand the data and lead to good decisions. Data Mining is an intersection of the fields Databases, Artificial Intelligence and Machine Learning. Some examples of Data Mining applications are:

- **Market Basket Analysis (Association Mining).** It is the main focus of this thesis. Market Basket Analysis is the discovery of relations or correlations among a set of items.
- **Classification.** Classification analyzes a training set of objects with known labels and tries to form a model for each class based on the features in the data.
- **Regression.** Regression is to predict values of some missing data or to build a model for some attributes using other attributes of the data.
- **Time Series Analysis.** Time Series Analysis is to analyze time series data to find certain regularities and interestingness in data.
- **Clustering.** Clustering is to identify clusters embedded in the data. The task is to find clusters for which inter-cluster similarity is low and intra-cluster similarity is high.
- **Outlier Analysis.** Outlier analysis is to find outliers in the data, namely detect data which are very far away from average behaviour of the data.

Recent work focuses mostly on scaling Data Mining algorithms. An algorithm is said to be *scalable* if its runtime increases linearly with the number of records in the input database. In general, a Knowledge Discovery Process consists of the following steps:

- *Data cleaning*, which handles noisy, erroneous, missing or irrelevant data

- *Data integration*, where data sources are integrated into one
- *Data selection*, where relevant is selected from the database
- *Data transformation*, where data is formed into appropriate format for mining
- *Data mining*, which is the essential process where intelligent methods are applied
- *Pattern evaluation*, which identifies patterns using some *interestingness measures*
- *Knowledge presentation*, where visualization techniques are used to present the mined data to the user.

Lately, *mining association rules*, also called *market basket analysis*, is one of the application areas of Data Mining. *Mining Association Rules* has been first introduced in [1]. Consider a market with a collection of huge customer transactions. An association rule is $X \Rightarrow Y$ where X is called the *antecedent* and Y is the *consequent*. X and Y are sets of items and the rule means that customers who buy X are likely to buy Y with probability $\%c$ where c is called the *confidence*. Such a rule may be: "Eighty percent of people who buy cigarettes also buy matches". Such rules allows us to answer questions of the form "What is Coca Cola sold with?" or if we are interested in checking the dependency between two items A and B we can find rules that have A in the consequent and B in the antecedent.

The aim is to generate such rules given a customer transaction database. The algorithms generally try to optimize the speed since the transaction databases are huge in size. This type of information can be used in catalog design, store layout, product placement, target marketing, etc. Basket Analysis is related to, but different from Customer Relationship Management (CRM) systems where the aim is to find the dependencies between customers' demographic data, e.g., age, marital status, gender, and the products.

1.1. Formal Description of the Problem

Let $I = (i_1, i_2, \ldots, i_m)$ be a set of transactions. Each i is called an *item*. D is the set of transactions where each *transaction* T is a set of items (itemset) such that $T \subseteq I$. Every transaction has a unique identifier called the TID. An itemset having k items is called a *k-itemset.* Let X and Y be distinct itemsets. The *support* of an itemset X is the ratio of the itemsets containing X to the number of all itemsets. Let us define $|X|$ as the number of itemsets containing X and $|D|$ as the number of all items, $|X.Y|$ as the number of itemsets containing both X and Y. The support of itemset X is defined as follows:

$$support(X) = \frac{|X|}{|D|} \cdot 100 \tag{1.1}$$

The rule $X \Rightarrow Y$ has *support s* if $\%s$ of the transactions in D contain X and Y together.

$$support(X \Rightarrow Y) = \frac{|X.Y|}{|D|} \cdot 100 \tag{1.2}$$

Support measures how common the itemsets are in the database and *confidence* measures the strength of the rule. A rule is said to have *confidence c* if $\%c$ of the transactions that contains X also contains Y.

$$confidence(X \Rightarrow Y) = \frac{support(X.Y)}{support(X)} \tag{1.3}$$

Given a set of transactions D the task of *Association Rule Mining* is to find rules $X \Rightarrow Y$ such that the support of the rule is greater than a user specified minimum support called *minsupp* and the confidence is greater than a user specified minimum called *minconf.* An itemset is called *large* if its support is greater than *minsupp.*

The task of *Association Rule Mining* can be divided into two: In the first phase, the large itemsets are found using *minsupp*, and in the second phase, the rules are generated using *minconf.*

The algorithms that implement *Association Mining* make multiple passes over the data. Most algorithms first find the large itemsets and then generate the rules accordingly. They find the large itemsets incrementally increasing itemset sizes and then counting the itemsets to see if they are large or not. Since finding the large itemsets is the hard part, research mostly focused on this topic.

The problem has been approached from different perspectives with several algorithms. In [1], Agrawal, Imielinski and Swami define the concepts of *support* and *confidence* and give an algorithm which has only one item in the consequent. The algorithm makes multiple passes over the database. The *frontier set* for a pass consists of those itemsets that are extended during a pass. At each pass, *support* for certain itemsets is measured. These itemsets called *candidate itemsets*, are derived from the tuples in the database and the itemsets contained in the frontier set. The frontier sets are created using the 1-*extensions* of the candidate itemsets in the current pass. Figure 1.1 shows the pseudocode for the algorithm.

In [2], Agrawal and Srikant define the algorihms *Apriori* and *AprioriTid* which can handle multiple items in the consequent. We will explain *Aprori* in detail in Section 2.1 and give real life results taken from data obtained from Gima Türk A.Ş. that are obtained using this algorithm in Chapter 3.

An efficient way to calculate the queries that are called iceberg queries is given by Fang *et al.* [3]. The queries are used in association mining. An iceberg query performs an aggregate function over an attribute (or set of attributes) for finding the aggregate values above some threshold. A typical iceberg query is performed on a relation $R(t_1, t_2, \ldots, t_k, ID)$. An example of an iceberg query can be : SELECT $t_1, t_2, \ldots, t_k, count(ID)$ FROM R GROUPBY $t_1, t_2, \ldots, t_k$ HAVING count$(ID) > T$

In our case $t_1, t_2, \ldots, t_k$ correspond to the items and ID corresponds to the ID of the receipt. They are called iceberg queries because the data we are trying to search is huge like the iceberg but the results we are going to obtain which are above some specified threshold are usually very small like the tip of the iceberg. The techniques are applied if the results are very small compared to the whole dataset.

Toivonen [4] chooses a sample from the database which is smaller than the database itself and calculates the rules in this sample. Then these rules are tested on the whole database. The algorithm first chooses a random sample from the whole database. Then the frequent itemsets are generated on this sample but with using lower threshold than the given threshold. After finding the itemsets the itemsets are tested over the whole dataset and the results are obtained. Choosing

$$|s| \geq \frac{1}{2\varepsilon^2} \ln \frac{2}{\delta} \tag{1.4}$$

guarantees that given an itemset X, $e(X, s) > \varepsilon$ is at most δ where $e(X, s)$ is the absolute value of the error between the frequencies of X in real database and in sample s. The algorithm is given in Figure 1.2

The database is partitioned into n partitions in [5] by Sarasere *et al.* and the local large itemsets are calculated and tested if they are global. Then the association rules are generated accordingly. The algorithm is known as *Partition*. First, the original database is logically partitioned into n nonoverlapping partitions, and in each partition, the large itemsets are found and the large itemsets in the partitions are merged to find the potential global large itemsets. Then these potential itemsets are counted in the whole database to see if they are really large itemsets on the whole database. The idea depends on the following: It is hard to find large itemsets on the whole database but if we have potentially large itemsets then it is easy to count them on the whole database. The pseudocode of the algorithm can be seen in Figure 1.3.

Support constraints are not used in [6] but instead top N association rules are found. It puts a restriction only on the number of rules. At each step of this algorithm, *N-most Interesting k-itemsets* are chosen. Top N itemsets are chosen at each step with the largest support values. Then we add this set potential itemsets which can form a large itemset in the next step of the algorithm. The algorithm can be seen in Figure 1.4. The functions in the algorithm works as follows: *Find_potential_1_itemset(D, N)* finds the N-most interesting 1-*itemsets* and returns them with their support. The itemsets are sorted in descending order according to their supports. *Gen_candidate(P_k)* uses *Apriori_Gen* algorithm in [2] to generate candidate $(k + 1)$-*itemsets*.
Find_N_potential_k_itemset(C_k , N , k) finds the N-most interesting *k-itemsets*.
Reduce(newsupport) reduces the value of the threshold if there are not enough potential *k-itemsets*.

Hu, Chin and Takeichi [7] use functional languages for solving this problem. The itemsets are stored in a list structure and the set of itemsets is a list of lists. The threshold is specified not by giving a probability but giving a minimum number of items that an itemset should contain.

Hipp *et al.* [8] use lattices and graphs for solving the problem of *Association Rule Mining.*

Another way of mining association rules is to use distributed and parallel algorithms. Suppose DB is a database with $|D|$ transactions. Assume there are n sites. Assume that the database is distributed into n parts $DB_1, DB_2, \ldots DB_n$. Let the size of the partitions of DB_i be D_i, $X.sup, X.sup_i$ be the global and local support counts of an itemset X. An itemset is said to be *globally large* if $X.sup \geq s \times D$ for a given *support* s. If $X.sup_i \geq s \times D_i$ then X is called *globally large.* L denotes the globally large itemsets in DB. The task is to find L.

Adaptive Parallel Mining algorithm is implemented on a shared memory parallel machine in [9] by David Cheung Kan. The algorithm is given in Figure 1.5.

Parallel algorithms for *Mining Association Rules* are defined in [10].

In [11] Cheung *et al.* implement a distributed algorithm for *Mining Association Rules.* The algorithm is called FDM (Fast Distributed Mining of association rules). The algorithm is given in Figure 1.6. The algorithm is iterated distributively at each site S_i until $L_{(k)} = \emptyset$ or the set of candidate sets $CG_{(k)} = \emptyset$

Zaki [12] makes a survey on parallel and distributed mining of association rules.

Ganti *et al.* [13] focus on three basic problems of Data Mining. They define and give references to various algorithms for solving problems of type *market basket analysis*, *clustering* and *classification.*

In [14] Hipp *et al.* consider several Association Mining algorithms and compares them.

1.2. Outline of Thesis

The rest of the thesis is organized as follows. In Section 2 we define the algorithms used in the thesis. We give a detailed description of the algorithm *Apriori* and we describe two statistical methods, Principal Component Analysis and k-Means Clustering, to find correlations between sales of items. In Section 3 we give results obtained using these methods and in the last chapter we conclude and discuss future work.

```
procedure LargeItemsets {
    let Large set L = Ø;
    let Frontier set F = {Ø};
    while (F ≠ Ø) {
        let Candidate set C = Ø;
        forall database tuples t {
            forall itemsets f in F {
                if (t contains f) {
                    let C_f = candidate itemsets that are extensions
                    of f and contained in t
                    forall itemsets c_f in C_f {
                        if(C_f ∈ C)
                          c_f.count++
                        else {
                          c_f.count = 0
                          C = C + c_f
                        }
                    }
                }
            }
        }
        let F = Ø
        forall itemsets c in C {
            if count(c)/|D| > minsupp
                L = L + c
            if c should be used as a frontier in the next pass
                F = F + c
        }
    }
}
```

Figure 1.1. Finding large itemsets [1]

s = Draw a Random Sample from database D

S = Large_Itemsets in s

F = Itemsets having support $\geq minSupp$ in S

Report if Error

return(F)

Figure 1.2. Sampling Algorithm [4]

$P = Partition\_Database(n)$

n = Number of Partitions

for $(i = 1 \; ; i \leq n \; ; i++)$

{

  $Read\_from\_Partition(p_i \in P)$

  $L^i = gen\_large\_itemsets(p_i)$

}

  /* Merge Phase */

for $(i = 2 \; ; L^i_j \neq \emptyset \; , j = 1, 2, \ldots, n \; ; i++)$

{

  $C^G_i = \cup_{j=1,2,\ldots,n} L^i_j$

}

  /* 2nd Phase */

for $(i = 1 \; ; i \leq n \; ; i++)$

{

  $Read\_from\_Partition(p_i \in P)$

  forall candidates $c \in C^G gen\_count(c, p_i)$

} $L^G = \{c \in C^G | c.count \geq minSupp$

return(L^G)

Figure 1.3. Partition Algorithm [5]

$Itemset\_Loop(support_k, lastsupport_k, N, C_k, P_k, D)$ {

$(P_1, support_1, lassupport_1) = find\_potential\_1\_itemset(D, N)$

$C_2 = gen\_candidate(P_1)$

for(k= 2 ; $k < m$; k++){

$(P_k, support_k, lastsupport_k) = Find\_N\_potential\_k\_itemset(C_k, N, k)$

if $(k < m)$ $C_{k+1} = Gen\_Candidate(P_k)$ }

$I_k = N$-most Interesting k-itemsets in P_k

$I = \cup_k I_k$

return(I)

}

$Find\_N\_potential\_k\_itemset(C_k, N, k)$ {

$(P_k, support_k, lassupport_k) = find\_potential\_k\_itemset(C_k, N)$

$newsupport = support_k$

for $(i = 2 ; i \leq k ; i++)$ $updated_i$ = FALSE

for ($i = 2 ; i < m ; i++$){

if (i== 1) {

if ($newsupport \leq lastsupport_i$) {

$(P_i = find\_potential\_1\_itemsets\_with\_support(D, newsupport))$

if $(i < k)$ $C_{i+1} = gen_candidate(P_i)$

if (C_{i+1} is updated)$updated_{i+1}$ = TRUE } }

else {

if (($newsupport \leq lastsupport_i$) || $updated_i$ = TRUE) {

$(P_i = find\_potential\_k\_itemsets\_with\_support(C_i, newsupport))$

if $(i < k)$ $C_{i+1} = gen_candidate(P_i)$

if (C_{i+1} is updated $updated_{i+1}$ = TRUE) } }

if ((number of k-items $< N$) && $(i == k)$ && $(k == m)$) {

$newsupport = reduce(newsupport)$

for $(j = 2 ; j \leq k ; j++$ $updated_j$ = FALSE)

$i = 1$ } }

return$(P_k, support_k, lastsupport_k)$

}

```
/* Preprocessing Step */
Every processors scan their partitions to find local supports for locally large 1-itemsets.
Compute L_1 and calculate C_2 = Apriori_Gen(L_1)
Virtually prune C_2
Initialize the shared part with the remaining candidate 2-itemsets
Perform configurations to to prepare a homogeneous distribution.
/* Parallel Step :Every processor i runs this on its partition D_i*/
while (some processor has not finished counting the items on the shared part)
{
  while (processor i has not finished counting the itemsets in the shared part)
  {
    Scan the next interval on D_i and count the itemsets in the shared part
    Find the locally large itemsets among the ones in the shared part
    Generate new candidates from these locally large itemsets
    Perform virtual partition pruning and put the survivors in the shared part
    remove globally small itemsets in the shared part
  }
}
```

Figure 1.5. Adaptive Parallel Mining [9]

if (k ==1)
$T_{i(1)} = get\_local\_count(DB_i, \emptyset, 1)$
else {
$CG_{(k)} = \cup_{i=1}^{n} CG_{i(k)}$
$CG_{(k)} = \cup_{i=1}^{n} Apriori\_Gen(GL_{i(k-1)})$
$T_{i(k)} = get\_local\_count(DB_i, CG_{(i)}, i)$ }
forall $X \in T_{i(k)}$
if ($X.sup_i \geq s \times D_i$)
for ($j = 1 ; j \leq n ; j++$)
if ($polling\_site(X) = S_j$) insert $< X, X.sup_i >$ into $LL_{i,j(k)}$
for ($j = 1 ; j \leq n ; j++$) send $LL_{i,j(k)}$ to site S_j
for ($j = 1 ; j \leq n ; j++$) {
receive $LL_{j,i(k)}$
forall $X \in LL_{j,i(k)}$ {
if ($X \notin LP_{i(k)}$)
insert X into $LP_{i(k)}$
update $X.largesites$ } }
forall $X \in LP_{i(k)}$
$send\_polling\_request(X)$
$reply\_polling\_request(T_{i(k)})$ **forall** $X \in LP_{i(k)}$ {
receive $X.sup_j$ from the sites S_j where $S_j \notin X.largesites$
$X.sup = \sum_{i=1}^{n} X.sup_i$
if ($X.sup \geq s \times D$)
insert X into $G_{i(k)}$ }
broadcast $G_{i(k)}$
receive G_j from all other sites S_j $(i \neq j)$
$L_k = \cup_{i=1}^{n} G_{i(k)}$
divide L_k into $GL_{i(k)}$ $(i = 1, 2, \ldots, n)$
return(L_k)

Figure 1.6. Fast Distributed Mining of association rules with local pruning [11]

2. METHODOLOGY

In our tests we used Apriori Algorithm for finding the association rules in the input sets and we used Principal Component Analysis and k-Means algorithms for clustering customers according to their buying habits.

2.1. Algorithm Apriori

2.1.1. Finding Large Itemsets

The algorithm *Apriori* works as follows: It first generates the 1-*itemsets* that have support greater than a prespecified minimum support, *minsupp*. This task is done for each item counting the number of occurances and selecting those whose support is greater than *minsupp*. Then, the procedure generates 2-*itemsets* using these large 1-*itemsets* with the procedure *Apriori_Gen*. There is a pruning step which prunes the generated 2-*itemsets*. The algorithm goes on generating the next itemsets and pruning until no large itemset is left. The itemsets are sorted lexicographically. The algorithm is given in Figure 2.1.

2.1.1.1. Itemset Generation. If we add m items to a *k-itemset* we call this newly generated itemset an *m-extension* of the first itemset. We generate a $k+1$-*itemset* using *k-itemset* with *Apriori_Gen as* follows: We consider all *k-itemsets* two by two. If two *k-itemsets* have the first $k-1$ elements matching each other we generate the $k+1$-*itemsets* by taking the first itemset and generating 1-*extension* of this itemset with the last element of the second itemset. The algorithm is given in Figure 2.2.

An example can be seen in Table 2.1. The example was adapted from [2]. Suppose that support is 0.5. First we generate and count all the large 1-*itemsets*. The itemsets that are below the support are pruned and not included in L_1. Then using L_1 we generate C_2. Again we count the number of itemsets in the database. As can be seen,

```
L_1 = Large 1-itemsets
for (k = 2 ; L_k ≠ Ø ; k++)
{
  C_k = Apriori_Gen(L_{k-1})
  forall transactions t ∈ D
  {
    C_t = subset(C_k, t)
    forall candidates c ∈ C_t
    {
      c.count++
    }
    L_k = {c ∈ C_k | c.count ≥ minsupp * |D|}
  }
}
return ∪_k L_k
```

Figure 2.1. Algorithm Apriori [2]

the itemsets {1, 2} and {1, 5} are pruned because they do not meet the minimum support. The only itemset that can be generated is { 2, 3, 5} and since its support is 0.5, we can count it in L_3. The itemset generation algorithm finishes here because there are not any itemsets left to produce.

2.1.1.2. Pruning. The pruning idea is the following. Let X be an itemset. If X is a large itemset then any itemset Y where $Y \subset X$ is also large. Using this idea after generating $k+1$-*itemsets* we test each $k+1$-*itemset* whether its all *k-itemset* subsets exist or not. If any one of the subsets is missing then we delete it because if a *k-itemset* is not large then its *m-extensions* cannot be large. The *Pruning* algorithm is given in Figure 2.3.

Table 2.1. Example of Apriori

D		C_1		L_1	
TID	Items	Itemset	Support	Itemset	Support
100	1, 3, 4	1	0.5	1	0.5
200	2, 3, 5	2	0.75	2	0.75
300	1, 2, 3, 5	3	0.75	3	0.75
400	2, 5	4	0.25	5	0.75
		5	0.75		

C_2		L_2		C_3	
Itemset	Support	Itemset	Support	Itemset	Support
1, 2	0.25	1, 3	0.5	2, 3, 5	0.5
1, 3	0.5	2, 3	0.5		
1, 5	0.25	2, 5	0.75	L_3	
2, 3	0.5	3, 5	0.5	Itemset	Support
2, 5	0.75			2, 3, 5	0.5
3, 5	0.5				

2.1.2. Generating Rules

2.1.2.1. Rule Generation. After all large itemsets have been generated they are used to generate rules using *minconf*. In [2] there are two algorithms for this problem. In our tests we used the following algorithm.

We know that if $a \Rightarrow (l - a)$ does not hold then $\tilde{a} \Rightarrow (l - \tilde{a})$ cannot hold where $\tilde{a} \subset a$ (If a smaller set is not large then its *m-extensions* cannot be large). Suppose $X = \{ABC\}$ and $Y = \{D\}$. If the rule $\{ABC\} \Rightarrow \{D\}$ does not hold then the rule $\{AB\} \Rightarrow \{CD\}$ cannot hold because its confidence is always smaller than the first one. Rewriting this as for $(l - r) \Rightarrow r$ to hold all rules of the form $(l - \tilde{r}) \Rightarrow \tilde{r}$ must hold where $r \subset \tilde{r}$ and $\tilde{r} \neq \emptyset$. Returning to the above example for the rule $\{AB\} \Rightarrow \{CD\}$

```
forall p, q ∈ L_k
{
  if (p.item_1 = q.item_1, p.item_2 = q.item_2 ... p.item_{k-1} = q.item_{k-1})
  {
    insert into C_{k+1}{p.item_1, p.item_2, ..., p.item_k, q.item_k}
  }
}
```

Figure 2.2. Candidate generation [2]

to hold the two rules $\{ABC\} \Rightarrow \{D\}$ and $\{ABD\} \Rightarrow \{C\}$ must hold. The algorithm in Figure 2.4 uses this idea [2]. Coming back to our example in Table 2.1, let us generate the rules for confidence = 1.0. Generation of rules from large 2-itemsets is trivial. For each itemset we put one of the items in the consequent and we calculate the rules. So we have $1 \Rightarrow 3$, $2 \Rightarrow 3$, $2 \Rightarrow 5$, $5 \Rightarrow 2$, $5 \Rightarrow 3$. The other rules do not have confidence equal to 1.0. For generating rules with 3-*itemsets* we first find the 1-*item* consequents in the itemset. For example let us consider the itemset $s = \{2, 3, 5\}$. First we generate all 1-*items* in s. We have $\{2\}$, $\{3\}$ and $\{5\}$. Then we use the *Apriori_Gen* function to generate 2-*itemsets* from these three items. Now we have {2, 3} {2, 5} and {3, 5}. Then for each of these we test: $support(\{2,3,5\})/support(\{2,3\}) = 1.0$ so we generate the rule $2,3 \Rightarrow 5$. Then we test $support(\{2,3,5\})/support(\{3,5\}) = 1.0$ so we generate the rule $3,5 \Rightarrow 2$. We then test $support(\{2,3,5\})/support(\{2,5\}) < 1.0$ so we do not consider it in the second phase.

2.2. Principal Component Analysis

If we look at the transactions by each customer, then we can represent each customer with t with an n-dimensional vector x^t where x_i^t represents the amount of item i bought by customer t. The n items are chosen as the n items sold most. By looking at the correlations between x_i over all customers, we find all dependencies between items. The method we use is called Principal Component Analysis (PCA) [15]. Suppose we have a dataset which consists of items that have n attributes. We are looking for a set

forall itemsets $c \in C_k$
{
  forall $(k-1)$ subsets s of c
  {
    if $(s \notin L_{k-1})$
    {
      delete c from C_k
    }
  }
}

Figure 2.3. Pruning [2]

of d orthogonal directions which best explains the data. Thus what we want is a linear transformation of the form

$$z_i = A(x_i - \bar{x}) \tag{2.1}$$

such that $E[x] = \bar{x}$ and $\mathrm{Cov}(x) = S$. We have $E[z] = 0$ and $\mathrm{Cov}(z) = D$ where D is a diagonal matrix. This means after the transformation z_i are zero mean and correlations are eliminated. We have if $z = Ax$ then $\mathrm{Cov}(z) = ASA'$ and we require $\mathrm{Cov}(z)$ to be a diagonal matrix. So if we take $a_1, a_2, \ldots, a_n$ as eigenvectors of S then new dimensions $z_1, z_2, \ldots, z_d$ can be defined as

$$z_i = a_i'x \tag{2.2}$$

having $\mathrm{Var}(z_i) = \lambda_i$. We can use the largest of the eigenvalues discarding the rest. The amount of variance explained by the eigenvectors can be found by

$$\frac{\lambda_1 + \lambda_2 + \cdots + \lambda_d}{\lambda_1 + \lambda_2 + \cdots + \lambda_n} \tag{2.3}$$

If the first few dimensions can explain the variance upto a percentage threshold that can be tolerated then we can discard the rest. If we can reduce the number of dimensions to two we can then plot the data. Because in our application, x_i have different units; grams, pieces, etc, we work with the correlation matrix R rather than the covariance matrix S.

2.3. k-Means Clustering

In k-means clustering [16] the idea is to unsupervisedly cluster the data into k subsets. In our application this corresponds to groups of customers with the same buying behaviour. First we choose k centers $v_1, v_2, \ldots, v_k$, preferably random, from the data. We then update these centers so that the centers best represent the density of the data. In k-means clustering a tuple in the dataset belongs to one of the centers only. We have

$$b_j^t = \begin{cases} 1 \text{ if } ||x^t - v_j|| = min_l ||x^t - v_l|| \\ 0 \text{ otherwise} \end{cases} \tag{2.4}$$

The center is the mean of the tuples that belongs to that center.

$$v_j = \frac{\sum_t b_j^t x^t}{\sum_t b_j^t} \tag{2.5}$$

When we update the centers the b_j values may change namely a tuple may now belong to another center. An iterative procedure is used to update the cluster centers.

$$\Delta v_j = \eta \sum_t b_j^t (x^t - v_j) \tag{2.6}$$

η is called the learning factor which is decreased to approach to zero as the number of iterations increase. In Figure 2.5 we see the initial random vectors chosen and in Figure 2.6 we can see how the final means are constructed using an example with two dimensional data and four means.

```
forall large k - itemsets l_k, k >= 2
{
  H_1 = { Set of 1 - item consequents in l_k}
  call Ap_Genrules(l_k, H_1)
}


function Ap_Genrules(l_k : large k-itemset , H_m : set of m - item consequents)
{
  if (k > m + 1)
  {
    H_{m+1} = Apriori_Gen(H_m)
    forall h_{m+1} ∈ H_{m+1}
    {
      conf = support(l_k)/support(l_k - h_{m+1})
      if (conf >= minconf)
      {
        output rule l_k - h_{m+1} => h_{m+1} with support support(l_k) and confidence conf
      }
      else
      {
        delete h_{m+1} from H_{m+1}
      }
    }
    call Ap_Genrules(l_k, H_{m+1})
  }
}
```

Figure 2.4. Generating rules [2]

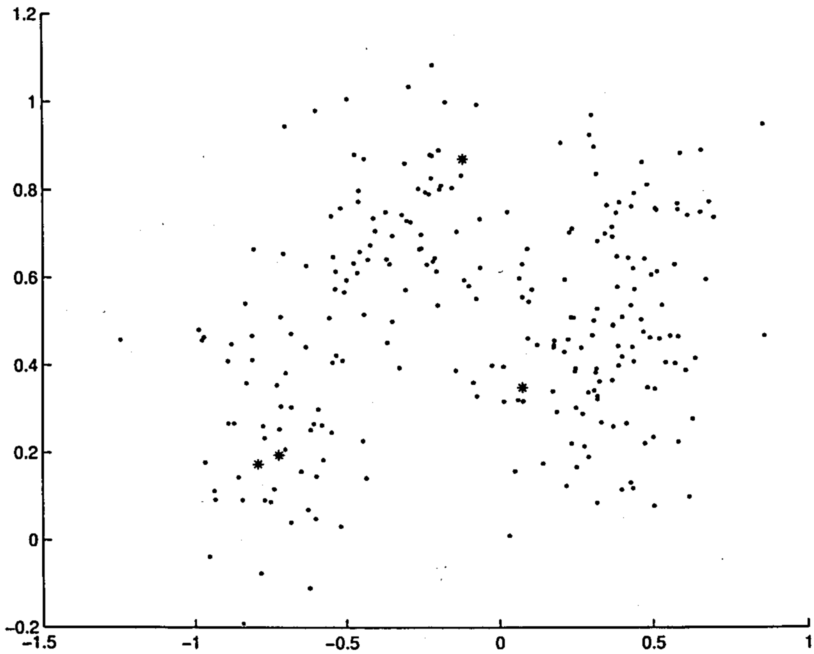


Figure 2.5. Random initial start vectors for $k = 4$

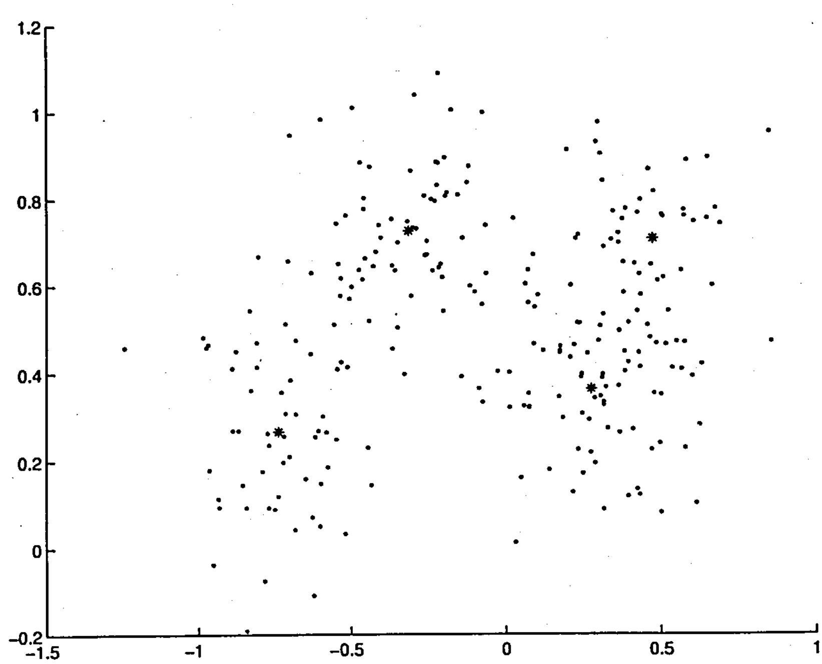


Figure 2.6. Final means for $k = 4$

3. RESULTS

3.1. Description of Datasets

The datasets we used for testing Apriori are from Gima Türk A.Ş. The test datasets are collected in June, July, August 2000 in one of the markets of the chain. The datasets contain 756,868 transactions. It contains of 140,610 itemsets. There are 7,237 types of items. Each item on the average appears in 105 itemsets. There are 5,909 items that are below average and 1,328 above average. There are 9,985 customers in the database.

The datasets we have are in the format of a relational database. We first converted the data into the format $< TID, \{item_1, item_2, \ldots, item_n\} >$ for the algorithm *Apriori*. Table 3.1 and 3.2 contain the description of the database tables used. We used the concatenation of the fields TARIH, KASA_NO, FIS_NO as the TID of the itemset.

3.2. Large Items

Since the large itemsets that have small number of items in them are large in size here we show only 30 of them. Results in Tables 3.3–3.6 are calculated using $minsupp = 0.001$ and $minconf = 0.5$. These results are obtained using the first dataset. The number in the Tables are the codes of the items in the supermarket. The

Table 3.1. Table Fis_Baslik used in the datasets

Table FIS_BASLIK

Field Name	Field Description
TARIH	Date of transaction
KASA_NO	ID of the cashier
FIS_NO	ID of the receipt
MUSTERI_NO	ID of the customer

Table 3.2. Table Fis_Detay used in the datasets

Table FIS_DETAY

Field Name	Field Description
TARIH	Date of transaction
KASA_NO	ID of the cashier
FIS_NO	ID of the receipt
FIS_SIRANO	The place of the item in that receipt
MAL_NO	ID of the item
MIKTAR	The amount of the item

data is confidential so it may not be used without the permission from the owner. We give some meaningful examples of real life results in the next section.

3.3. Rules Generated

According to the large itemsets in Tables 3.3–3.6, using *minconf*, the rules in Tables 3.7–3.9 are generated.

3.4. Finding Correlations Using PCA

There are twelve datasets from twelve stores of the chain. For each of the stores we found the items that are sold the most and the customers that have bought the most number of items (We chose 1000 customers from each store). Then we merged the stores for new items and customers. We wanted to reduce the number of dimensions on items to see if we could cluster the customers with two dimensions on items. The results in Figure 3.1 are obtained by taking the number of items as 25 on store 102. The Table 3.10 shows how much of the variances are explained using the specified number of dimensions. Energy denotes the percentage of variance explained of the original data reducing the number of dimensions to that number. The data in the graphics are sorted lexicogaphically. Similar results using different datasets and different number of dimensions can be seen in Appendix A.

Table 3.3. Large 1-itemsets

Item1	Support
4309020094	0.147998
9917010003	0.080164
9917010001	0.077057
4309020010	0.076054
4308020094	0.072918
4309020001	0.045203
4308020019	0.043553
5808010069	0.043503
4308020071	0.040366
4308020052	0.040359
4309020083	0.039321
4909020002	0.037557
4309020076	0.036995
4309020075	0.034563
4308020080	0.032799
4309020012	0.030837
4909020090	0.028340
4909010007	0.025823
4309020008	0.024564
4309020078	0.024080
4309020013	0.023924
4308020089	0.023895
4309020014	0.023853
4309020098	0.023476
4308020098	0.023070
4309020016	0.022366
4309020095	0.022245
5101010005	0.020667
4308020070	0.020638
5308020087	0.020624

Table 3.4. Large 2-itemsets

Item1	Item2	Support
4309020010	4309020094	0.051255
4308020094	4309020094	0.028952
4309020001	4309020094	0.021599
4309020012	4309020094	0.020546
4309020094	9917010001	0.019913
4309020076	4309020094	0.019166
4309020083	4309020094	0.017723
4308020094	4309020010	0.017559
4308020052	4309020094	0.017396
4309020075	4309020094	0.017040
4309020094	9917010003	0.015419
4309020014	4309020094	0.014615
4309020094	5808010069	0.014501
4309020001	4309020010	0.013029
4309020013	4309020094	0.012624
4308020019	4309020094	0.012417
4308020080	4309020094	0.012396
4309020016	4309020094	0.012261
9917010001	9917010003	0.012140
4309020001	4309020008	0.011678
4308020071	4309020094	0.011550
4308020052	4309020010	0.011436
4309020094	4309020098	0.011329
4309020010	4309020083	0.011329
4308020080	4308020094	0.010924
4309020008	4309020094	0.010774
4309020010	4309020076	0.010760
4309020010	4309020012	0.010689
4309020078	4309020094	0.010412
4308020019	4308020094	0.010149

Table 3.5. Large 3-itemsets

Item1	Item2	Item3	Support
4308020094	4309020010	4309020094	0.012368
4309020001	4309020010	4309020094	0.009885
4309020010	4309020012	4309020094	0.008712
4308020052	4309020010	4309020094	0.008506
4309020010	4309020083	4309020094	0.008193
4309020010	4309020076	4309020094	0.007958
4309020010	4309020075	4309020094	0.007460
4309020010	4309020094	9917010001	0.006756
4309020010	4309020014	4309020094	0.006379
4309020001	4309020008	4309020094	0.006024
4309020010	4309020094	5808010069	0.005839
4309020075	4309020076	4309020094	0.005725
4308020094	4309020001	4309020094	0.005597
4309020010	4309020094	9917010003	0.005547
4308020080	4309020010	4309020094	0.005398
4309020001	4309020076	4309020094	0.005355
4308020094	4309020076	4309020094	0.005291
4308020071	4309020010	4309020094	0.005249
4308020052	4308020094	4309020094	0.005199
4309020010	4309020016	4309020094	0.005156
4309020010	4309020013	4309020094	0.005028
4308020094	4309020075	4309020094	0.004993
4308020019	4309020010	4309020094	0.004971
4308020080	4308020094	4309020094	0.004943
4308020094	4309020083	4309020094	0.004850
4309020001	4309020075	4309020094	0.004793
4308020052	4309020001	4309020094	0.004765
4309020010	4309020094	4309020098	0.004751
4309020008	4309020010	4309020094	0.004694
4309020010	4309020078	4309020094	0.004644

Table 3.6. Large 4-itemsets

Item1	Item2	Item3	Item4	Support
4308020094	4309020001	4309020010	4309020094	0.002937
4309020010	4309020075	4309020076	4309020094	0.002895
4309020001	4309020008	4309020010	4309020094	0.002880
4308020052	4309020001	4309020010	4309020094	0.002795
4308020052	4308020094	4309020010	4309020094	0.002795
4309020001	4309020010	4309020076	4309020094	0.002752
4309020001	4309020010	4309020012	4309020094	0.002731
4308020094	4309020010	4309020076	4309020094	0.002710
4308020094	4309020010	4309020075	4309020094	0.002624
4308020094	4309020010	4309020083	4309020094	0.002574
4309020001	4309020010	4309020075	4309020094	0.002539
4308020080	4308020094	4309020010	4309020094	0.002525
4308020052	4309020010	4309020083	4309020094	0.002382
4309020001	4309020010	4309020083	4309020094	0.002240
4309020001	4309020008	4309020075	4309020094	0.002240
4308020089	4308020094	4309020010	4309020094	0.002155
4308020019	4308020094	4309020010	4309020094	0.002141
4309020010	4309020012	4309020076	4309020094	0.002070
4308020052	4309020010	4309020076	4309020094	0.002020
4309020001	4309020075	4309020076	4309020094	0.002006
4309020010	4309020075	4309020083	4309020094	0.001991
4309020010	4309020014	4309020076	4309020094	0.001977
4309020010	4309020013	4309020075	4309020094	0.001977
4308020094	4309020010	4309020012	4309020094	0.001977
4309020008	4309020010	4309020075	4309020094	0.001949
4308020094	4309020010	4309020014	4309020094	0.001949
4309020001	4309020010	4309020014	4309020094	0.001878
4309020001	4309020010	4309020013	4309020094	0.001870
4309020010	4309020076	4309020083	4309020094	0.001856
4308020094	4309020010	4309020016	4309020094	0.001856

Table 3.7. Rules generated using large 2-itemsets

			Confidence	Support
4309020010	⇒	4309020094	0.673929	0.051255
4309020012	⇒	4309020094	0.666282	0.020546
4309020076	⇒	4309020094	0.518069	0.019166
4309020014	⇒	4309020094	0.612701	0.014614
4309020013	⇒	4309020094	0.527645	0.012623
4309020016	⇒	4309020094	0.548171	0.012260

3.5. Clustering Customers Using k-Means

Among the twelve stores we found the items that are sold the most in all the stores. We chose 25, 46, 100 items and we ran k-Means algorithm on them. We wanted to group the customers according to their buying habits. We clustered the customers in 4, 8, 12, 16, 20 groups and from these we tried to guess what the customers buy belonging to that cluster. The results in Table 3.11 are taken from the store 102 with choosing the number of items 25 and number of clusters as 4. The numbers above the means show the percentage of the customers belonging to that cluster. We first normalized the data acording to the items to disable the effect of difference variances and means in the data. Similar results with different datasets and different means can be seen in Appendix B. For example if we look at Table 3.11 we can say the people in this group buy the items '4101020081', '4101040001', '4102010002', '4102020001', '4103030095', '4104020082', '4109010096', '4109010100', '4109010100', '4109010177' more than other customers. Such results can be derived for other stores and items.

Table 3.8. Rules generated using large 3-itemsets

				Confidence	Support
4308020094	4309020010	⇒	4309020094	0.704333	0.012367
4309020001	4309020010	⇒	4309020094	0.758733	0.009885
4309020010	4309020012	⇒	4309020094	0.815036	0.008712
4308020052	4309020010	⇒	4309020094	0.743781	0.008505
4309020010	4309020083	⇒	4309020094	0.723163	0.008192
4309020010	4309020076	⇒	4309020094	0.739590	0.007958
4309020010	4309020075	⇒	4309020094	0.747150	0.007460
4309020010	9917010001	⇒	4309020094	0.779967	0.006756
4309020010	4309020014	⇒	4309020094	0.770618	0.006379
4309020008	4309020094	⇒	4309020001	0.559075	0.006023
4309020001	4309020008	⇒	4309020094	0.515834	0.006023
4309020010	5808010069	⇒	4309020094	0.740974	0.005838
4309020075	4309020076	⇒	4309020094	0.594534	0.005725
4308020094	4309020001	⇒	4309020094	0.641401	0.005597
4309020010	9917010003	⇒	4309020094	0.737240	0.005547
4308020080	4309020010	⇒	4309020094	0.726315	0.005397
4309020001	4309020076	⇒	4309020094	0.643040	0.005355
4308020094	4309020076	⇒	4309020094	0.619483	0.005291
4308020071	4309020010	⇒	4309020094	0.746208	0.005248
4308020052	4308020094	⇒	4309020094	0.581081	0.005198
4309020010	4309020016	⇒	4309020094	0.782937	0.005156
4309020010	4309020013	⇒	4309020094	0.793490	0.005028
4308020094	4309020075	⇒	4309020094	0.617957	0.004992
4308020019	4309020010	⇒	4309020094	0.680623	0.004971
4308020094	4309020083	⇒	4309020094	0.603539	0.004850
4309020001	4309020075	⇒	4309020094	0.601785	0.004793
4308020052	4309020001	⇒	4309020094	0.638095	0.004764
4309020010	4309020098	⇒	4309020094	0.820638	0.004750
4309020008	4309020010	⇒	4309020094	0.718954	0.004693
4309020010	4309020078	⇒	4309020094	0.702906	0.004644

Table 3.9. Rules generated using large 4-itemsets

					Confidence	Support
4308020094	4309020001	4309020094	⇒	4309020010	0.524777	0.002937
4308020094	4309020001	4309020010	⇒	4309020094	0.783681	0.002937
4309020075	4309020076	4309020094	⇒	4309020010	0.505590	0.002894
4309020010	4309020075	4309020076	⇒	4309020094	0.785714	0.002894
4309020008	4309020010	4309020094	⇒	4309020001	0.613636	0.002880
4309020001	4309020008	4309020010	⇒	4309020094	0.758426	0.002880
4308020052	4308020094	4309020094	⇒	4309020010	0.537619	0.002794
4308020052	4308020094	4309020010	⇒	4309020094	0.770588	0.002794
4308020052	4309020001	4309020094	⇒	4309020010	0.586567	0.002794
4308020052	4309020001	4309020010	⇒	4309020094	0.806981	0.002794
4309020001	4309020076	4309020094	⇒	4309020010	0.513944	0.002752
4309020001	4309020010	4309020076	⇒	4309020094	0.804573	0.002752
4309020001	4309020012	4309020094	⇒	4309020010	0.592592	0.002730
4309020001	4309020010	4309020012	⇒	4309020094	0.860986	0.002730
4308020094	4309020076	4309020094	⇒	4309020010	0.512096	0.002709
4308020094	4309020010	4309020076	⇒	4309020094	0.765060	0.002709
4308020094	4309020075	4309020094	⇒	4309020010	0.525641	0.002624
4308020094	4309020010	4309020075	⇒	4309020094	0.771966	0.002624
4308020094	4309020083	4309020094	⇒	4309020010	0.530791	0.002574
4308020094	4309020010	4309020083	⇒	4309020094	0.763713	0.002574
4309020001	4309020075	4309020094	⇒	4309020010	0.529673	0.002538
4309020001	4309020010	4309020075	⇒	4309020094	0.805869	0.002538
4308020080	4308020094	4309020094	⇒	4309020010	0.510791	0.002524
4308020080	4308020094	4309020010	⇒	4309020094	0.748945	0.002524
4308020052	4309020083	4309020094	⇒	4309020010	0.563025	0.002382
4308020052	4309020010	4309020083	⇒	4309020094	0.801435	0.002382
4309020008	4309020075	4309020094	⇒	4309020001	0.552631	0.002240
4309020001	4309020008	4309020075	⇒	4309020094	0.621301	0.002240
4309020001	4309020083	4309020094	⇒	4309020010	0.535714	0.002240
4309020001	4309020010	4309020083	⇒	4309020094	0.775862	0.002240

Table 3.10. Energies in 25 dimensions

Dimensions	Energy 102	Energy 221	Energy 310	Energy All
1	8.7208	6.9129	10.1739	8.1142
2	15.6119	12.7856	16.1699	13.7807
3	20.8453	18.5378	21.4567	19.0488
4	25.9380	23.6384	26.5827	23.7955
5	30.6403	28.6230	31.2361	28.2390
6	35.0867	33.5463	35.8104	32.4682
7	39.4677	38.1337	40.2429	36.6690
8	43.8048	42.5995	44.5389	40.7784
9	47.8813	47.0094	48.7484	44.8294
10	51.9463	51.1029	52.8931	48.7786
11	55.9833	55.1006	56.8524	52.6275
12	59.9066	58.8579	60.7106	56.4191
13	63.7028	62.5834	64.5277	60.1386
14	67.3764	66.2440	68.0754	63.8380
15	70.9363	69.7447	71.5919	67.5117
16	74.4453	73.2220	75.0060	71.1116
17	77.8364	76.6103	78.3337	74.6617
18	81.1350	79.8747	81.5901	78.1426
19	84.3484	82.9969	84.7261	81.5400
20	87.2908	86.0823	87.8119	84.8643
21	90.1221	89.0275	90.8644	88.0972
22	92.7572	91.9632	93.8624	91.2881
23	95.3390	94.8034	96.4892	94.4452
24	97.7911	97.4242	98.3662	97.3642
25	100.0000	100.0000	100.0000	100.0000

Table 3.11. Store 102 with 4 means

4-MEANS				
ITEMS	14.5%	7.7%	74.2%	3.6%
0809010004	-0.0032	0.0708	-0.0021	-0.0943
4101020081	**0.7886**	0.0955	-0.1680	0.0818
4101040001	**0.9433**	-0.0560	-0.1809	0.0490
4102010002	**0.7653**	-0.2170	-0.1311	0.0829
4102020001	**0.4336**	0.1444	-0.1258	**0.5366**
4103030095	**1.0121**	-0.0917	-0.1948	0.1351
4104020082	**1.2247**	-0.1571	-0.2115	-0.2366
4104020088	-0.1241	0.0859	-0.0583	**1.5189**
4107020002	0.1045	0.2944	-0.0822	**0.6440**
4109010005	-0.0826	1.7914	-0.1796	0.2035
4109010022	-0.1053	-0.0933	-0.1489	**3.6935**
4109010032	-0.0298	0.1768	-0.0162	0.0765
4109010036	0.0761	0.0463	-0.0189	-0.0155
4109010047	-0.1351	**1.5145**	-0.1443	0.2790
4109010048	0.1592	0.2156	-0.1066	**1.0938**
4109010082	-0.0301	0.2896	-0.0190	-0.1065
4109010096	**0.8351**	-0.0977	-0.1425	-0.2171
4109010100	**0.5603**	0.2499	-0.1385	0.0636
4109010103	-0.0804	0.1851	-0.0698	**1.3661**
4109010114	**0.4763**	0.1087	-0.1131	0.1804
4109010115	-0.0377	-0.1005	0.0130	0.0999
4109010177	**1.0103**	-0.1649	-0.1723	-0.1652
4109010178	0.2307	0.2406	-0.0675	-0.0523
4109010179	-0.0408	**1.1408**	-0.1069	-0.0730
4110040081	-0.0927	**1.0201**	-0.1077	**0.4110**

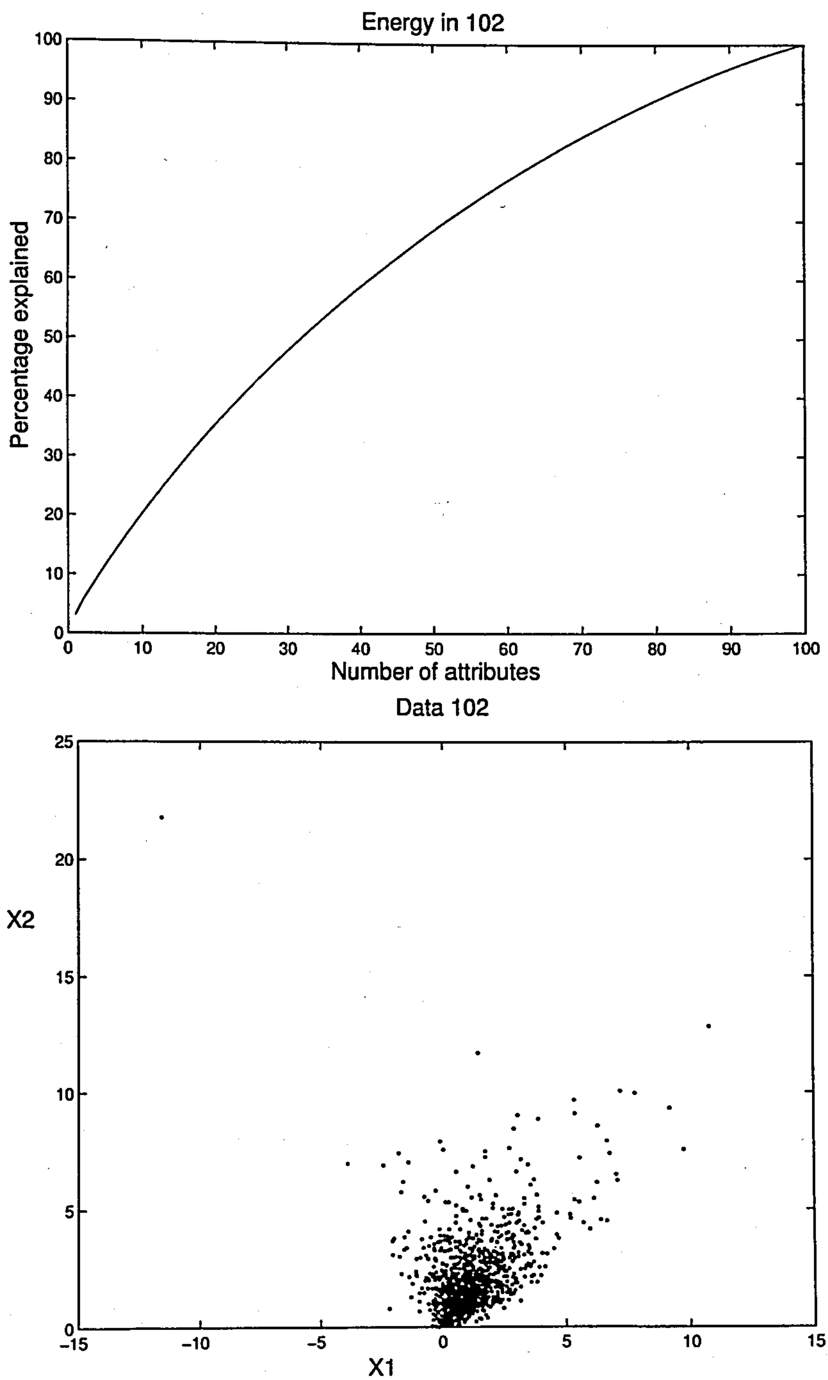


Figure 3.1. Energy and data reduced to 2 dimensions for 25 items for store number 102

4. CONCLUSIONS AND FUTURE WORK

Apriori Algorithm [2] is one of the fastest and earliest tools for *Association Mining*. We used the apriori algorithm for mining *Association Rules* in the large database of Gima Türk A.Ş. The data used in the thesis was confidential so we mined data blindly i.e. not knowing what we mined. As seen in the results in the previous section what we knew about the data is the ID's of the items. We tried to cluster the customers according to their buying habits according to these items. Our first data set was taken in summer and we had the following results. The item which is sold the most came out to be tomato. Tomato was present in 14 per cent of the transactions. The second most sold item was bread which is not surprising for our country, then came the cucumbers. The top remaining items also turned out to be items that are mostly used in making salads and Coke. As a result we can conclude that our people eat salad very much in summer time. Let me give examples of rules derived from these large itemsets. The people who bought cucumbers also bought tomato with confidence 67 per cent. By saying 67 per cent confidence we mean that 67 per cent of people who bought cucumbers also bought tomatoes. Note that we do not have a rule saying that 67 per cent of people who bought tomatoes also bought cucumbers because although the itemset {tomato, cucumber} is large the confidence of the rule tomato $\Rightarrow$ cucumber does not have enough confidence. Another such rule may be 66 per cent of people who bought green pepper also bought tomatoes. If we give examples of rules with three items we have 55per cent of people who bought dill and tomato also bought parsley.

We tried to cluster the customers and items using PCA. We wanted to reduce dimensions to two and to see the plotted data. Since the variances in the data were near each other reducing the number of dimensions to two did not cluster the data. We then used k-Means clustering with $k = 4, 8, 12, 16, 20$ to cluster the data.

Since the data was confidential we did not have the chance to make comments on the data. We only could blindly mine and find association rules. If we had more knowledge on the data then we could derive more results. The data consisted of items and the amount of items sold at each transaction. Since we did not know what the item was the amount did not mean anything to us. Also some of the amounts were in fractions i.e in kilogram and some of them were in numbers so we did not have a way of using the amount of the item in the mining of the data. We had to take the data as binary i.e. it exists in this transaction or not. Also since the data was confidential we did not have information on the prices of the items to prioritize the items. A future work could be to use the information in the amount of the items and their prices for deriving more meaningful rules.

The dataset we used did not contain the hierarchical information about the items. This means we have wine of brand X, wine of brand Y as different itemsets and the ID's of these items do not have a hierarchy saying that these items are *Wines*. If the data is supplied with hierarchical information then we can find relations such as "People who buy Wines also buy Cheese" but now we can only derive rules such as "People who buy Wine brand X also buy Cheese of brand Z."

We implemented the apriori algorithm which is one of the first and fastest methods. Most of the methods for *Mining Association Rules* depend on this algorithm. We implemented a sequential algorithm but there are parallel and distributed algorithms for this problem which can be implemented for speeding up computations as a future work.

APPENDIX A: RESULTS OF PRINCIPAL COMPONENT ANALYSIS

The results in Figures A.1–A.3 are obtained by taking the number of items as 25 on stores with numbers 221, 310 and the whole dataset respectively. The results in Figures A.4–A.7 are obtained by taking the number of items as 46 on stores with numbers 102, 221, 310 and the whole dataset respectively. The results in Figures A.8–A.11 are obtained by taking the number of items as 100 on stores with numbers 102, 221, 310 and the whole dataset respectively. The Tables A.1–A.2 shows how much of the variances are explained using the specified number of dimensions. Energy denotes the percentage of variance explained of the original data reducing the number of dimensions to that number. The data in the graphics are sorted lexicogaphically.

The Tables A.3–A.6 show the most important six eigenvalues on twenty five dimensional data. The values in the left column are the item codes i.e. ID's of items. The numbers in bold are the important values on the vectors.

Table A.1. Energies in 46 dimensions

Dimensions	Energy 102	Energy 221	Energy 310	Energy All
1	6.8250	4.7815	9.9395	6.6745
2	11.3923	8.8548	14.0537	10.5869
3	15.0288	12.6433	17.9653	14.2449
4	18.4215	16.2711	21.4087	17.4557
5	21.7830	19.5894	24.7100	20.4615
6	24.9176	22.7066	27.7381	23.3674
7	27.8139	25.7044	30.6874	26.1074
8	30.6485	28.5862	33.5558	28.7711
9	33.4230	31.4120	36.3564	31.3873
10	36.1070	34.1257	39.0208	33.8498
15	48.4428	46.7840	51.1320	45.2401
20	59.4534	58.0126	62.0026	55.9112
25	69.3741	68.2429	71.7639	66.0638
30	78.3538	77.3849	80.6022	75.4826
35	86.3052	85.4905	88.4495	84.2216
40	93.2828	92.7304	95.1932	92.3182
41	94.5777	94.0756	96.3892	93.8481
42	95.7872	95.4009	97.3896	95.2761
43	96.9350	96.6519	98.2534	96.6769
44	98.0012	97.8744	99.0330	98.0049
45	99.0314	98.9860	99.7608	99.0768
46	100.0000	100.0000	100.0000	100.0000

Table A.2. Energies in 100 dimensions

Dimensions	Energy 102	Energy 221	Energy 310	Energy All
1	3.0472	2.8192	3.5550	2.8218
2	5.5569	5.1050	6.3912	5.4010
3	7.6125	7.0773	8.4756	7.3737
4	9.5986	9.0138	10.4676	9.1752
5	11.5088	10.8593	12.3956	10.7975
10	20.2175	19.3763	20.8872	17.9335
15	28.0977	27.0103	28.3342	24.2317
20	35.3155	34.1314	35.1939	30.0427
25	41.8668	40.6784	41.5633	35.5507
30	47.9009	46.7490	47.4852	40.8553
35	53.5549	52.3916	53.0675	45.9895
40	58.8126	57.7134	58.2697	51.0123
45	63.7631	62.7319	63.2102	55.8776
50	68.4620	67.4049	67.8531	60.5977
55	72.8663	71.7863	72.2717	65.1961
60	76.9672	75.9198	76.4717	69.6623
65	80.7847	79.7959	80.3553	74.0010
70	84.3274	83.4129	83.9803	78.2028
75	87.5847	86.7863	87.3854	82.2913
80	90.5923	89.9398	90.5325	86.2489
85	93.3902	92.8366	93.4252	90.0764
90	95.9369	95.5280	96.0421	93.7192
95	98.1790	97.9272	98.3514	97.1279
96	98.5790	98.3802	98.7734	97.7799
97	98.9682	98.8093	99.1325	98.4061
98	99.3459	99.2288	99.4800	99.0178
99	99.7052	99.6245	99.7890	99.5669
100	100.0000	100.0000	100.0000	100.0000

Table A.3. Eigenvectors for 25 dimensions reduced to 6 dimensions for store 102

0809010004	-0.0356	**0.3571**	0.0649	0.1260	-0.0676	-0.0102
4101020081	-0.0337	**0.2071**	0.0987	0.0851	-0.0393	**0.3866**
4101040001	-0.0191	0.0665	0.0381	-0.1547	-0.0995	**0.3238**
4102010002	0.1057	0.0614	0.0747	-0.1180	**-0.2009**	**0.3017**
4102020001	-0.0205	**-0.2236**	0.1397	-0.1499	0.0430	**0.2676**
4103030095	0.0213	-0.1011	0.0904	-0.0236	0.0135	**0.3512**
4104020082	-0.0436	-0.1146	0.0427	-0.1403	-0.1649	**0.3978**
4104020088	**0.3273**	-0.1918	**0.2116**	**0.3185**	**0.3385**	0.0703
4107020002	-0.0793	0.1921	**-0.3283**	**-0.3110**	**0.3728**	0.0507
4109010005	**-0.4179**	-0.0636	**0.3019**	0.1280	**0.2624**	0.0820
4109010022	0.1068	**0.2819**	-0.0922	**-0.2611**	**0.3131**	0.0723
4109010032	**-0.2245**	0.0093	0.0920	-0.0986	0.0765	-0.0048
4109010036	-0.1723	**0.2131**	**0.2413**	-0.0588	0.0068	0.0920
4109010047	-0.1659	0.0460	**0.2639**	0.1457	**0.2457**	0.0410
4109010048	**0.2500**	-0.0624	-0.1517	**0.4123**	**0.2161**	0.1482
4109010082	-0.0489	-0.1063	0.0362	0.1491	-0.0329	-0.0262
4109010096	0.0802	-0.1628	**-0.3531**	-0.0762	-0.0733	**0.2162**
4109010100	-0.0714	-0.1517	**-0.3228**	**0.4248**	0.0239	**0.2354**
4109010103	**0.2011**	-0.1016	**0.2553**	-0.1355	**0.4285**	0.0857
4109010114	0.0500	**0.2385**	0.1133	0.0556	-0.0800	**0.3127**
4109010115	**-0.2422**	**-0.4142**	-0.0372	**-0.3109**	0.1364	0.0007
4109010177	-0.0647	**-0.2196**	**-0.2756**	-0.0135	-0.0469	0.1708
4109010178	-0.1637	**0.4041**	-0.1807	0.1818	0.0179	0.0786
4109010179	**-0.6036**	-0.0650	-0.1593	**0.2057**	0.0918	0.0333
4110040081	0.0541	0.1546	**-0.3072**	-0.0839	**0.3931**	0.0728

Table A.4. Eigenvectors for 25 dimensions reduced to 6 dimensions for store 221

0809010004	0.0522	**0.2041**	**-0.2540**	0.0997	0.0763	0.0092
4101020081	-0.0122	**0.3561**	-0.0954	0.1659	**-0.3487**	0.1340
4101040001	0.0073	**0.3047**	0.1160	-0.0397	**-0.3316**	**0.2231**
4102010002	**0.2350**	-0.1166	0.1795	**-0.2081**	-0.1268	**0.3669**
4102020001	0.0108	0.0260	**-0.2267**	0.0436	-0.1485	**0.4471**
4103030095	0.1092	**-0.3670**	0.1197	-0.0828	0.0026	**0.2873**
4104020082	0.0951	-0.1739	-0.1054	**-0.2616**	0.0669	**0.3606**
4104020088	-0.0031	**0.5058**	-0.0157	-0.1491	**0.2689**	0.0849
4107020002	**-0.2922**	-0.0040	0.1243	**0.2999**	**0.2835**	**0.2627**
4109010005	0.0260	**0.2853**	-0.1895	-0.1949	**0.3078**	0.0448
4109010022	**0.2744**	**0.2550**	**0.2362**	0.0167	0.1689	0.1668
4109010032	-0.0154	-0.0653	**0.2391**	0.0514	0.0922	**0.2233**
4109010036	0.0998	0.0687	0.0567	**0.4335**	0.1694	0.1333
4109010047	0.1169	0.0992	**0.3399**	-0.0992	**0.2445**	0.0519
4109010048	-0.0394	-0.1302	**-0.2396**	**0.4578**	-0.0937	0.1001
4109010082	**0.2922**	0.0735	0.1266	0.0780	0.1273	0.0861
4109010096	**-0.4006**	0.1533	0.1007	-0.1573	-0.1016	0.1766
4109010100	**-0.2139**	0.1231	**-0.2391**	-0.1003	**-0.2252**	0.1681
4109010103	-0.0073	0.0125	**-0.3603**	0.0183	**0.2901**	0.0981
4109010114	0.0971	0.0435	0.0999	**0.4729**	0.0047	0.1205
4109010115	0.0197	-0.1712	**-0.3646**	-0.0516	**0.3661**	0.0694
4109010177	-0.0728	-0.1589	-0.1398	-0.0807	-0.0111	**0.3104**
4109010178	**-0.3709**	-0.0506	**0.2297**	0.0481	0.0802	0.0128
4109010179	**-0.4913**	0.0397	0.1014	-0.0467	0.1786	0.0735
4110040081	**-0.2195**	-0.1291	0.1565	0.0398	0.0660	0.0293

Table A.5. Eigenvectors for 25 dimensions reduced to 6 dimensions for store 310

0809010004	0.0300	**-0.3150**	0.0768	-0.1450	0.1500	-0.1935
4101020081	0.0160	-0.0347	-0.0870	-0.0059	0.0382	**-0.4789**
4101040001	-0.1066	**0.3317**	0.0555	0.0573	0.0547	**-0.2064**
4102010002	0.0633	-0.1883	-0.1212	-0.1172	0.1308	**-0.4310**
4102020001	-0.0113	-0.1182	-0.0459	-0.0570	0.0215	**-0.4895**
4103030095	-0.0518	-0.1184	**-0.2678**	0.1297	**-0.2573**	-0.1607
4104020082	0.1670	0.1059	-0.1592	-0.1435	**-0.2643**	-0.1663
4104020088	0.0864	-0.1109	-0.1271	**0.3406**	**0.2484**	0.0101
4107020002	**-0.3316**	-0.0455	-0.0884	0.0346	-0.1826	-0.0241
4109010005	0.1072	0.0612	0.0026	**0.2934**	**0.3778**	-0.0249
4109010022	-0.1299	0.0224	-0.1444	**0.3017**	-0.1161	-0.0241
4109010032	-0.0817	**-0.2689**	-0.0685	0.1406	**-0.3440**	-0.0287
4109010036	**0.2779**	-0.0514	-0.1048	**0.3362**	**-0.2903**	-0.0580
4109010047	-0.0862	0.1898	**-0.2770**	0.0490	-0.0712	-0.0097
4109010048	0.1740	0.1981	**-0.2266**	**0.3045**	**-0.2224**	0.0022
4109010082	**0.3273**	-0.1854	**0.2383**	**0.3601**	0.0352	0.0014
4109010096	-0.0902	**0.5086**	0.0045	-0.1730	-0.1819	-0.1300
4109010100	0.1357	**0.2161**	**0.3071**	0.1027	-0.0694	-0.1256
4109010103	**-0.4117**	**-0.3499**	**-0.2533**	-0.0036	0.0162	0.0716
4109010114	-0.1084	-0.0831	**0.3713**	0.1690	**-0.2681**	-0.1966
4109010115	-0.0945	0.1851	**-0.4394**	0.1844	**0.2079**	0.0768
4109010177	-0.0706	0.1489	-0.0317	-0.0214	0.1371	**-0.2857**
4109010178	-0.1250	0.1303	0.0074	**0.2509**	**0.3371**	-0.1860
4109010179	**-0.2921**	0.0363	**0.2839**	**0.2452**	-0.1357	-0.0809
4110040081	**-0.5055**	0.0470	**0.2308**	**0.2070**	0.0752	0.0215

Table A.6. Eigenvectors for 25 dimensions reduced to 6 dimensions for all data

0809010004	**-0.4305**	**-0.2499**	-0.0917	0.0559	-0.1146	0.0276
4101020081	-0.1435	-0.1217	0.1124	0.0109	-0.1299	**0.3447**
4101040001	0.0137	0.0145	0.1558	0.0666	-0.1662	**0.2964**
4102010002	**-0.2319**	-0.0459	-0.0024	**-0.3591**	0.0828	**0.3473**
4102020001	**-0.2088**	-0.0219	-0.0689	0.0371	-0.0302	**0.4304**
4103030095	0.0731	0.0117	-0.1155	**-0.2621**	0.1100	**0.2799**
4104020082	0.0543	0.0424	-0.0648	**-0.4079**	0.1269	**0.2936**
4104020088	-0.0496	**-0.2446**	-0.0495	**0.2270**	**0.3046**	0.0431
4107020002	-0.0582	**0.3983**	-0.0285	**0.3401**	0.0147	**0.2055**
4109010005	0.0829	**-0.3544**	0.0385	0.0411	**0.4733**	0.0678
4109010022	**0.2736**	**0.2555**	-0.1282	0.1088	**0.2084**	0.0596
4109010032	**0.2310**	**0.4371**	-0.1284	0.0805	0.0365	0.1009
4109010036	0.0509	-0.1789	**-0.4959**	0.1414	-0.1148	0.1205
4109010047	-0.0239	-0.0342	**-0.3421**	0.0581	0.0305	0.1243
4109010048	0.1109	-0.0329	**-0.2260**	0.1390	-0.1140	0.1118
4109010082	**0.4583**	**-0.2952**	-0.0477	0.0140	0.1025	-0.0017
4109010096	0.0857	**0.2106**	**0.3547**	-0.0535	0.0013	**0.2417**
4109010100	**0.2335**	**-0.2387**	**0.3365**	0.1665	0.0052	0.1984
4109010103	**-0.3366**	**0.2186**	-0.1511	0.0951	**0.4618**	0.0062
4109010114	0.1619	-0.1133	**-0.2855**	**0.2632**	-0.1140	**0.2302**
4109010115	0.0625	-0.0180	0.0137	-0.1700	**0.4531**	-0.0420
4109010177	**0.2946**	0.0304	0.0991	-0.0623	-0.0280	**0.2165**
4109010178	-0.1333	-0.0651	**0.3010**	**0.3152**	0.0212	0.0976
4109010179	-0.0071	-0.1060	0.1642	**0.2851**	0.0799	0.0812
4110040081	-0.1090	0.1327	0.0861	**0.2731**	**0.2603**	0.0407

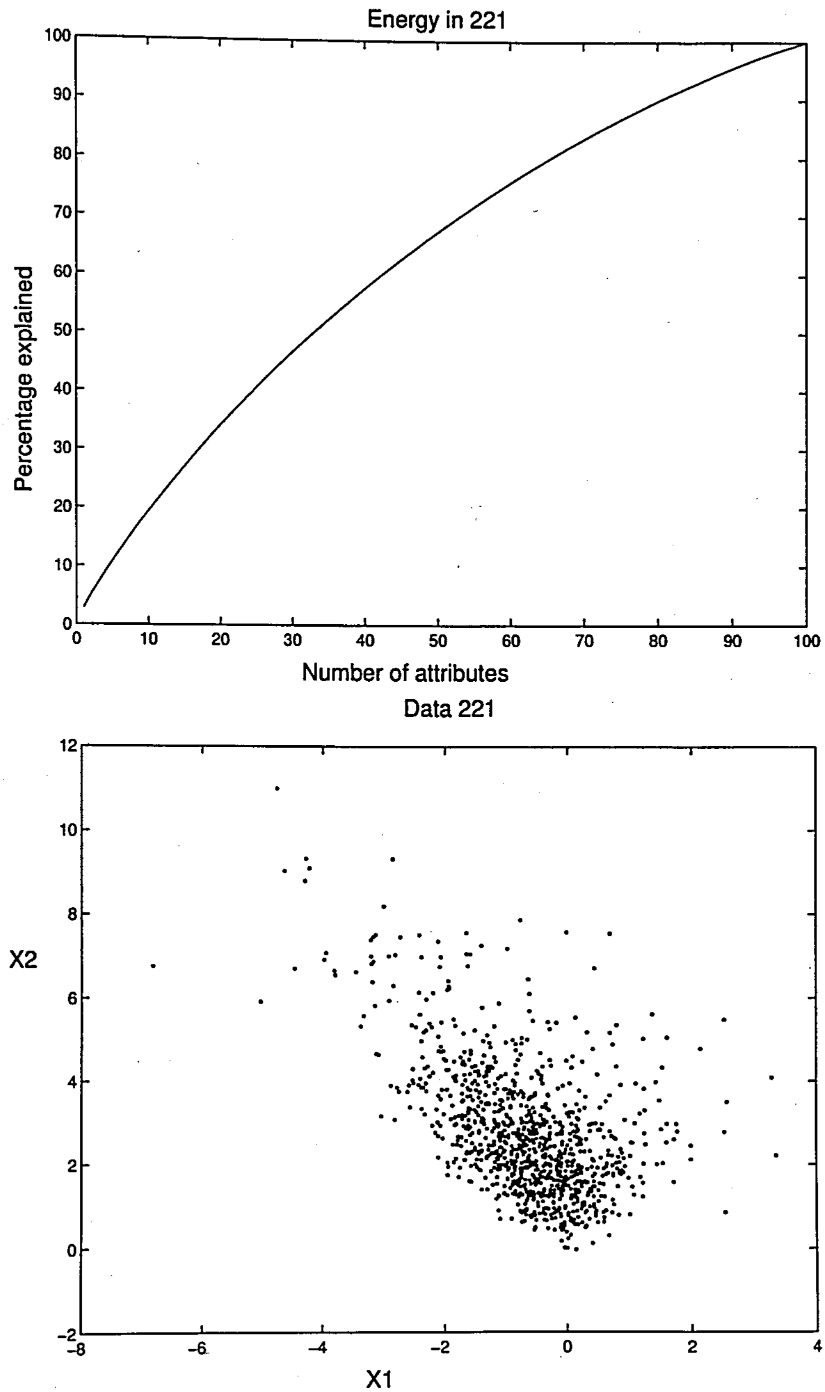


Figure A.1. Energy and data reduced to 2 dimensions for 25 items for store number 221

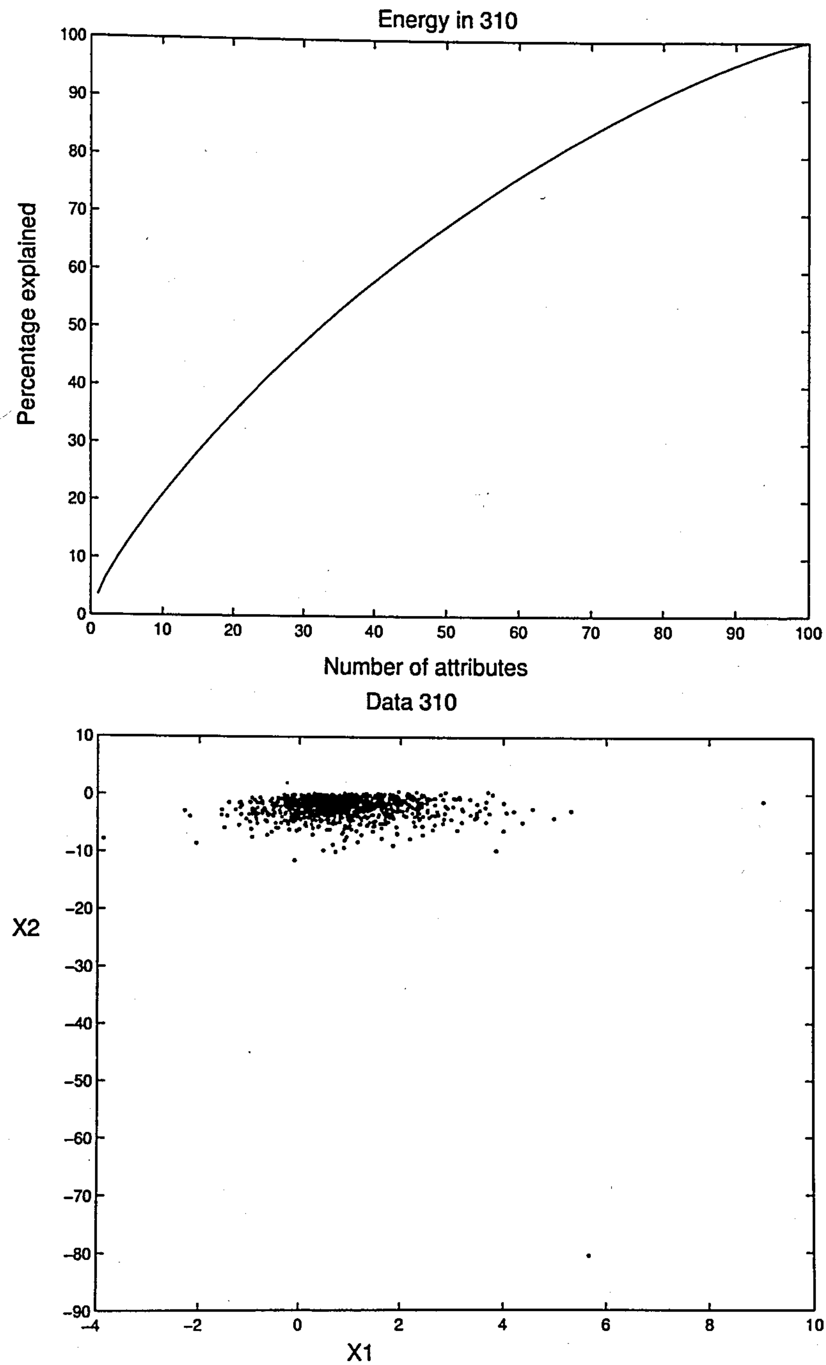


Figure A.2. Energy and data reduced to 2 dimensions for 25 items for store number 310

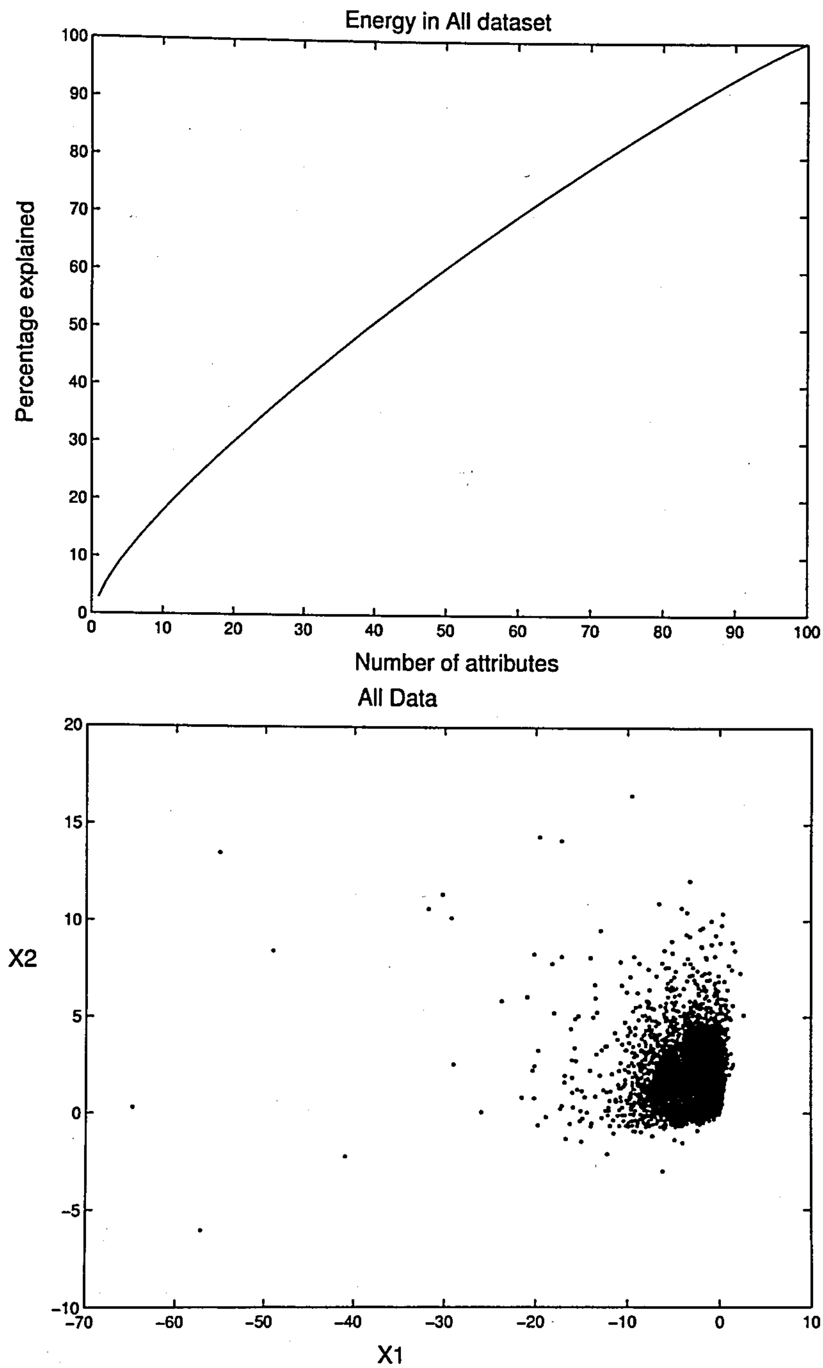


Figure A.3. Energy and data reduced to 2 dimensions for 25 items for whole data

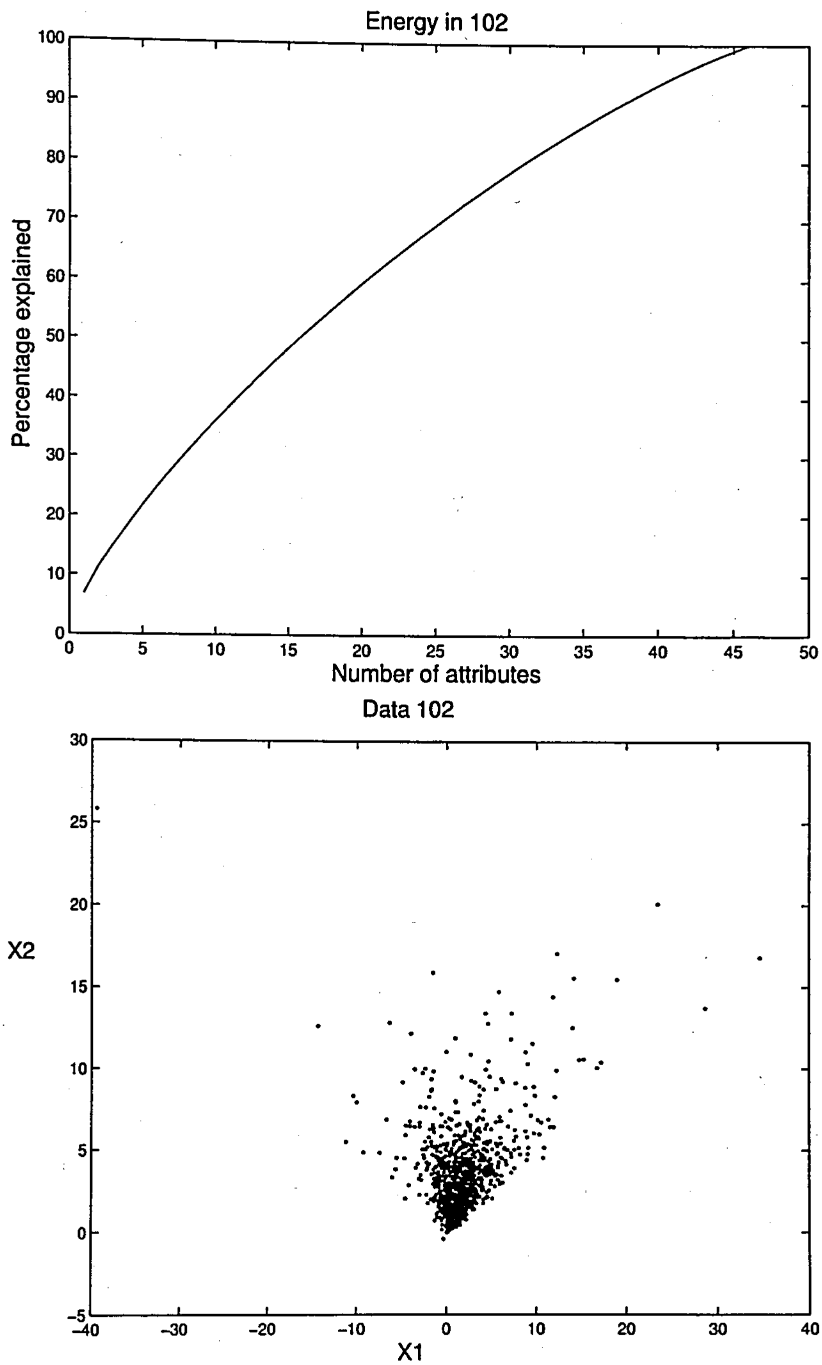


Figure A.4. Energy and data reduced to 2 dimensions for 46 items for store number 102

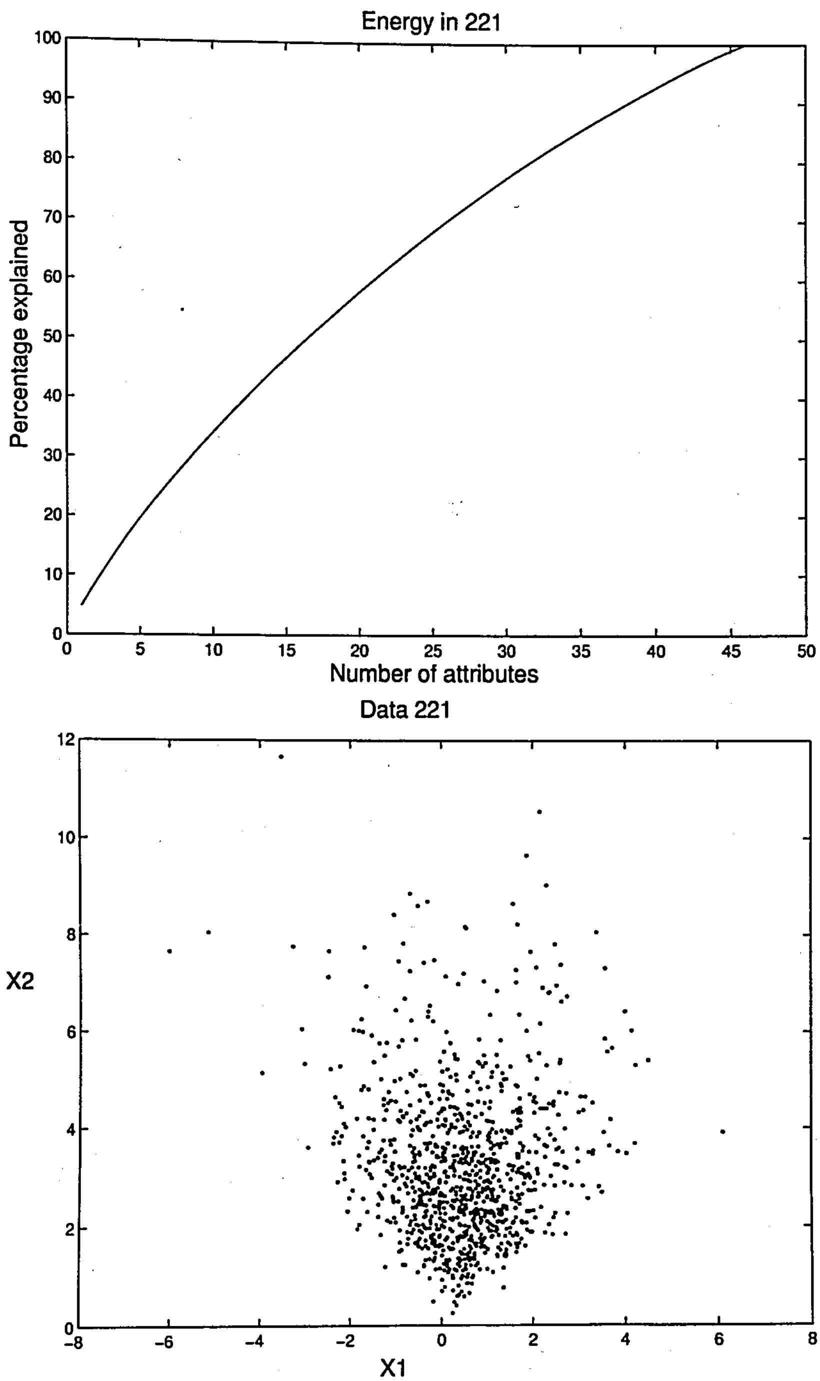


Figure A.5. Energy and data reduced to 2 dimensions for 46 items for store number 221

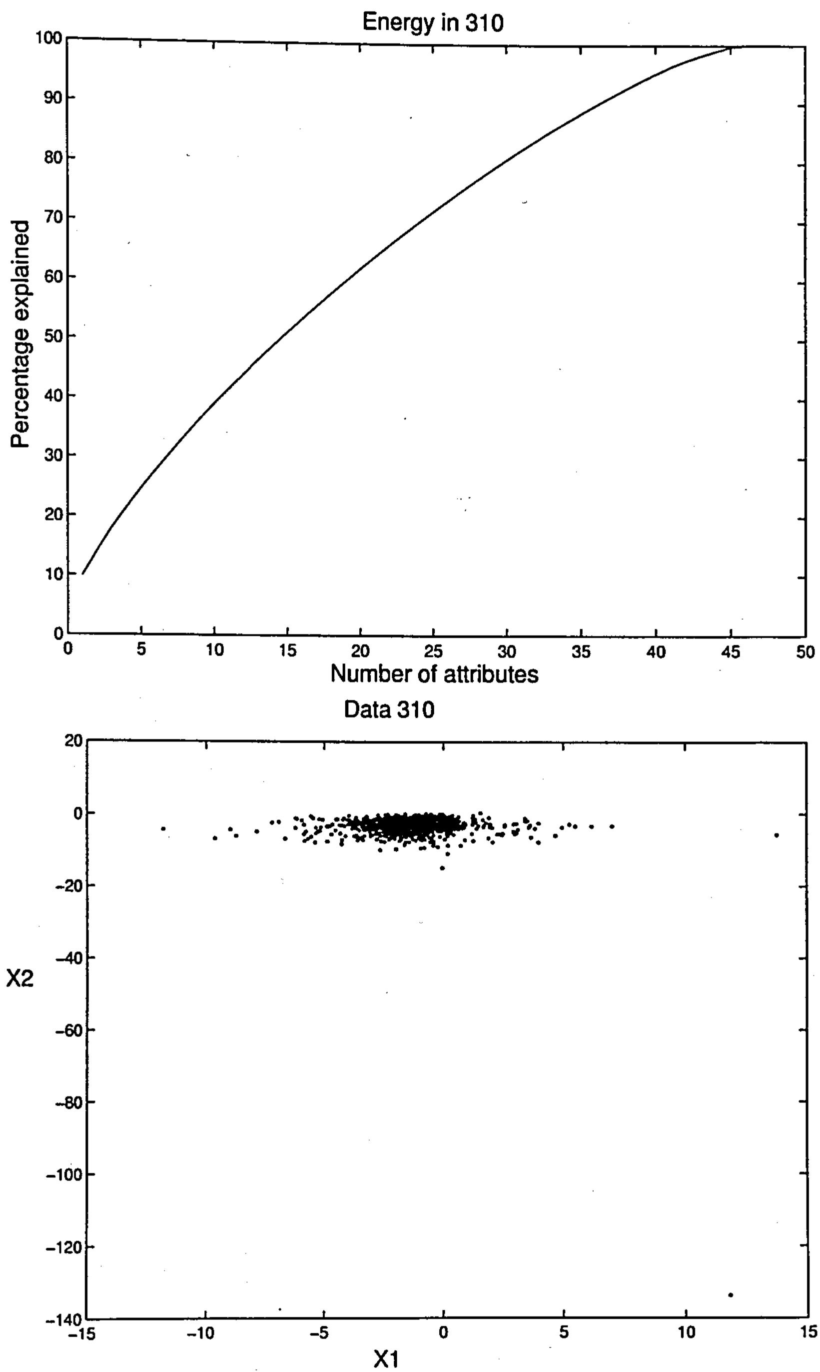


Figure A.6. Energy and data reduced to 2 dimensions for 46 items for store number 310 .

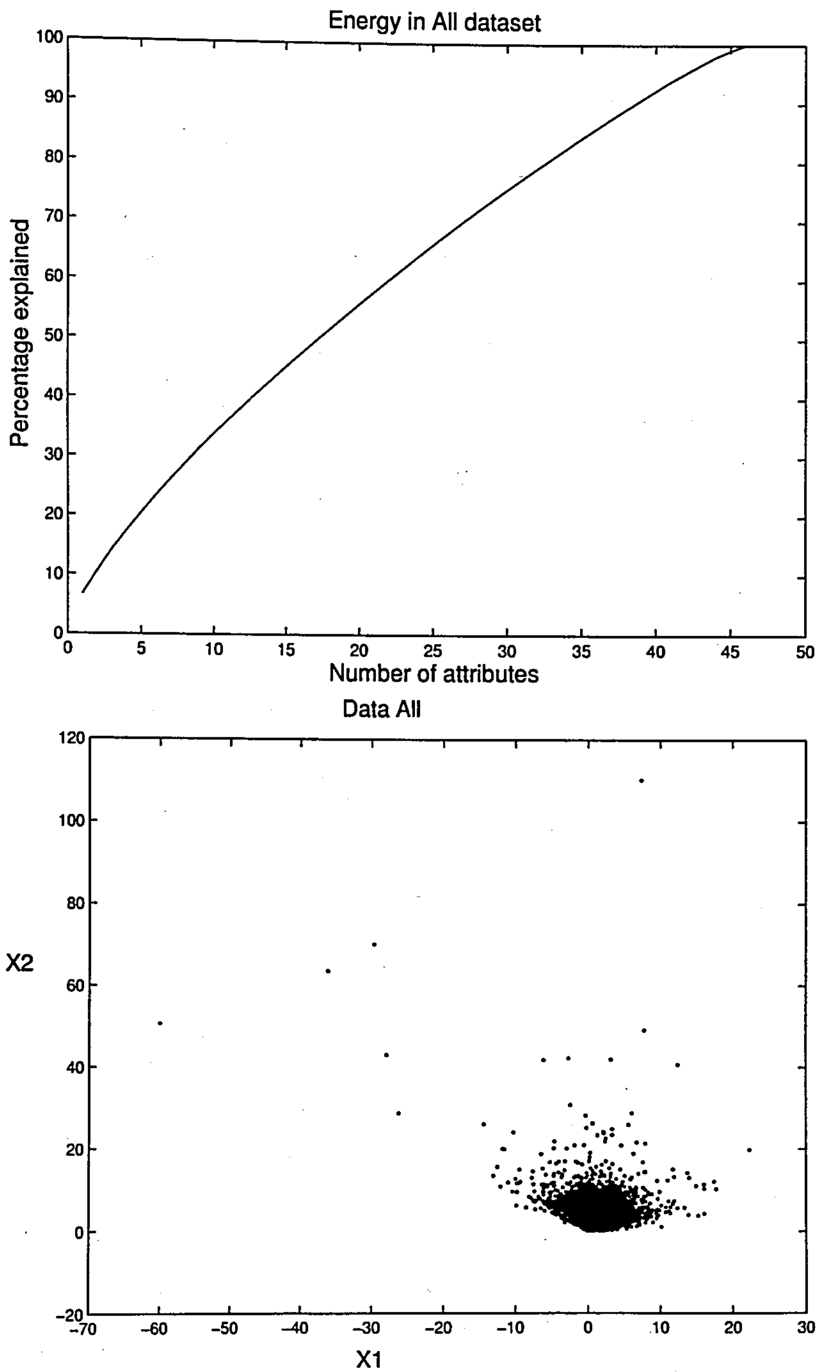


Figure A.7. Energy and data reduced to 2 dimensions for 46 items for whole data

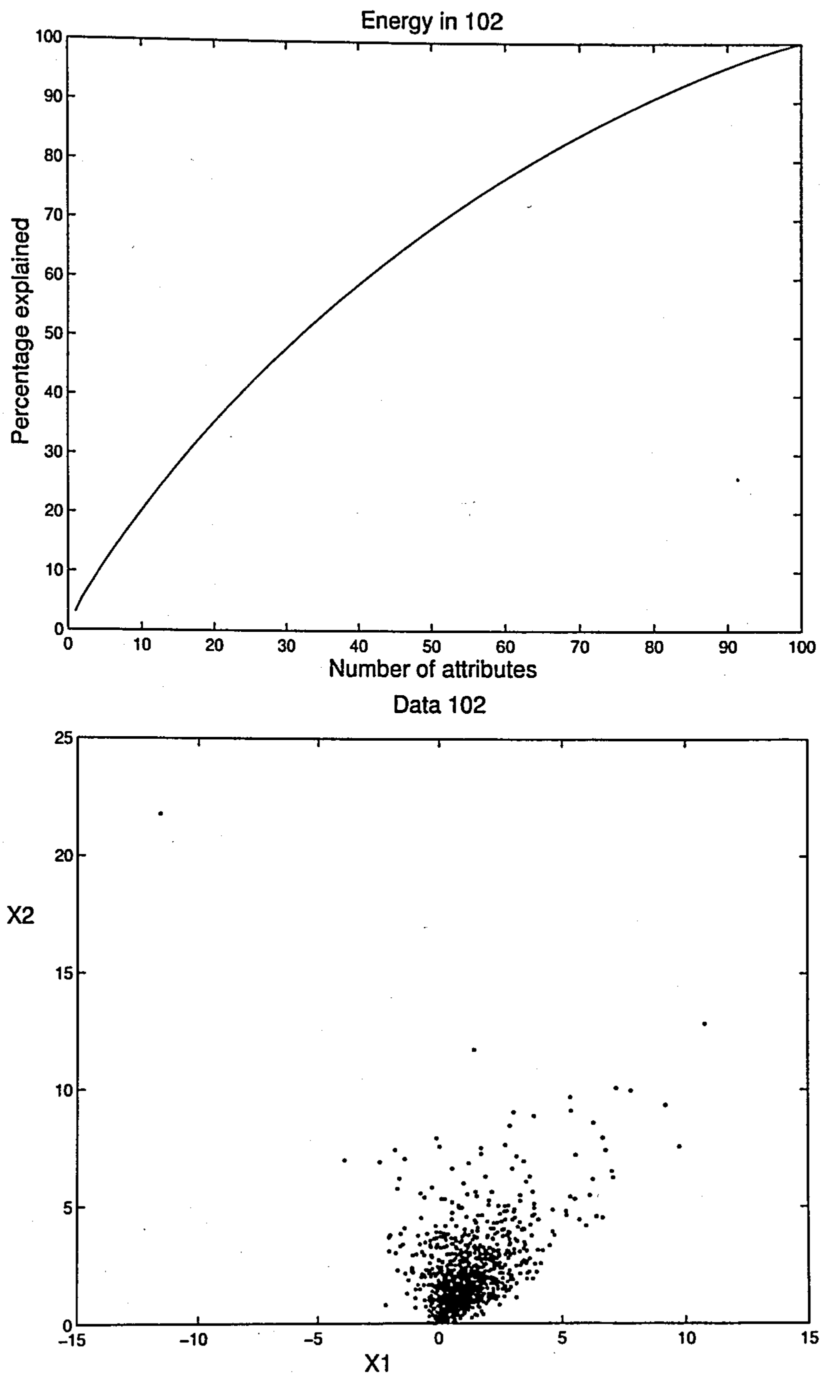
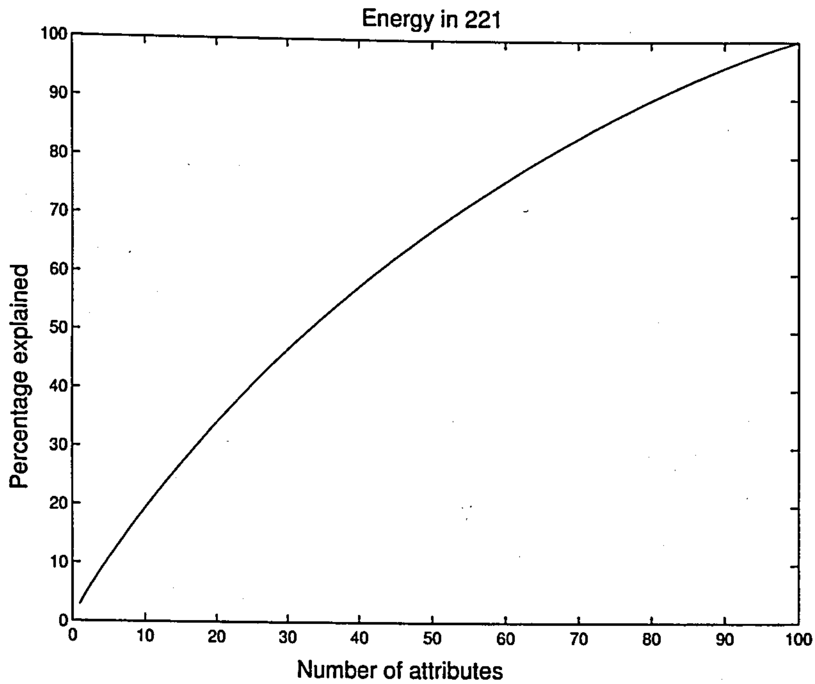


Figure A.8. Energy and data reduced to 2 dimensions for 100 items for store number 102



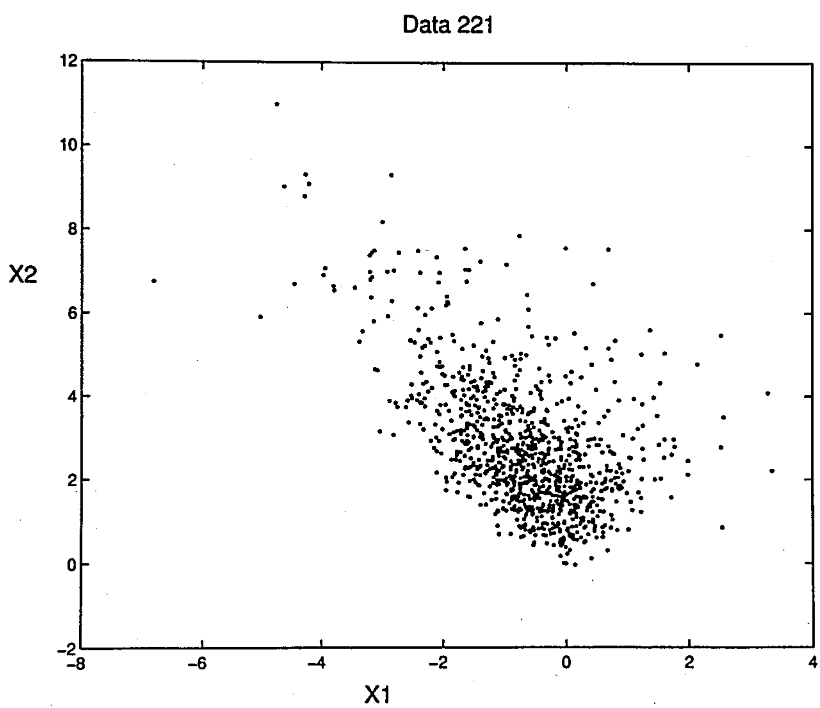


Figure A.9. Energy and data reduced to 2 dimensions for 100 items for store number 221

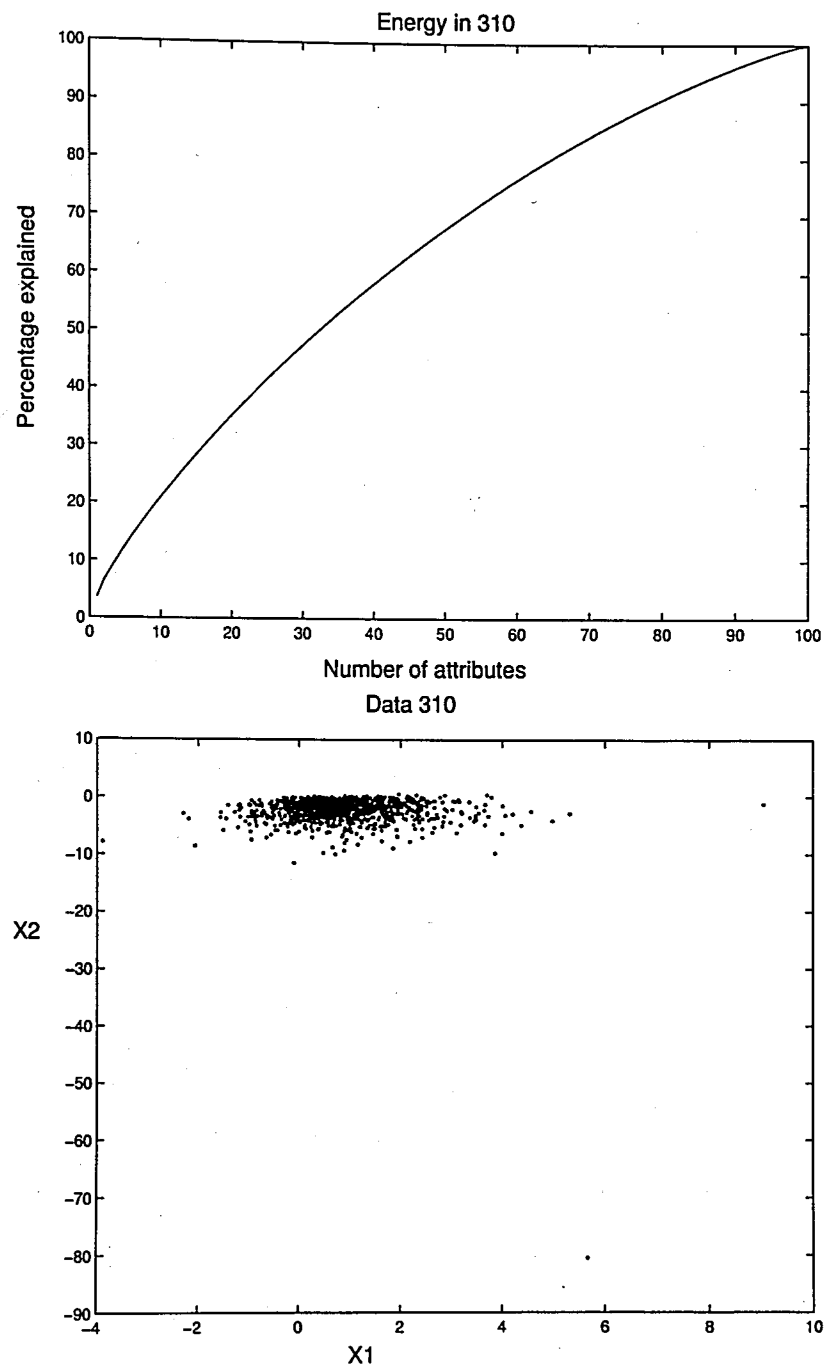


Figure A.10. Energy and data reduced to 2 dimensions for 100 items for store number 310

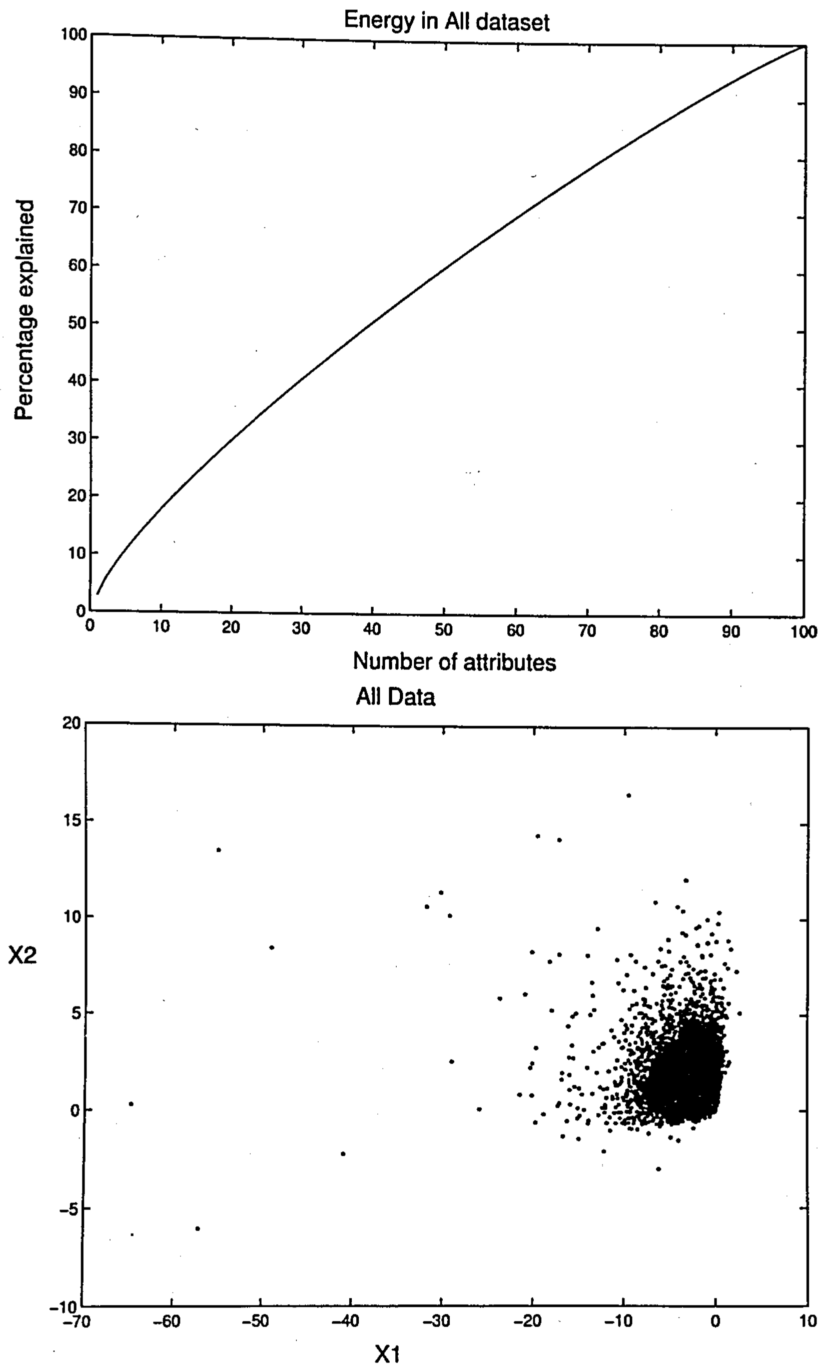


Figure A.11. Energy and data reduced to 2 dimensions for 100 items for whole data

APPENDIX B: RESULTS OF K-MEANS CLUSTERING

The results in Tables B.1–B.7 are taken from the stores numbered 102, 221, 310 and the whole database and with choosing the number of items 25 and number of clusters as 4 and 8. The numbers above the means show the percentage of the customers belonging to that cluster. We first normalized the data acording to the items to disable the effect of difference variances and means in the data.

Table B.1. Store 102 with 8 means

8-MEANS								
ITEMS	54.5%	15.5%	1.6%	12.1%	1.8%	9.3%	4.2%	1%
0809010004	0.043	-0.189	-0.217	0.039	0.029	-0.026	0.099	0.226
4101020081	-0.185	-0.139	0.178	0.374	0.371	0.022	**1.445**	**0.525**
4101040001	-0.188	-0.177	-0.097	-0.083	0.014	-0.156	**3.651**	0.289
4102010002	-0.148	-0.108	-0.302	**0.673**	-0.215	-0.048	**0.694**	0.033
4102020001	-0.267	-0.130	0.007	0.313	0.054	**1.252**	0.289	-0.139
4103030095	-0.246	-0.093	0.755	**0.701**	-0.137	-0.083	**1.320**	**0.669**
4104020082	-0.263	-0.007	-0.293	**0.855**	0.090	-0.073	**1.155**	0.262
4104020088	-0.118	-0.030	**5.655**	-0.166	0.093	-0.019	-0.075	0.221
4107020002	-0.111	0.093	-0.182	0.064	**1.674**	0.023	0.098	**0.501**
4109010005	-0.233	-0.164	0.626	-0.145	0.102	**1.643**	0.070	0.274
4109010022	-0.054	-0.072	**1.764**	-0.184	**0.770**	-0.024	**0.414**	**0.578**
4109010032	-0.066	-0.004	-0.121	-0.136	0.099	-0.055	-0.146	**6.490**
4109010036	-0.045	-0.111	-0.078	0.081	-0.182	0.218	0.235	0.636
4109010047	-0.099	-0.137	0.588	-0.115	0.301	**0.654**	-0.084	**1.723**
4109010048	-0.095	-0.139	**1.154**	0.322	**0.821**	0.025	-0.001	-0.100
4109010082	-0.034	0.024	0.112	-0.096	-0.177	0.341	-0.048	-0.154
4109010096	-0.212	-0.091	-0.265	**1.004**	**0.772**	-0.183	0.349	0.108
4109010100	-0.145	-0.209	**0.620**	**0.686**	0.376	0.062	0.201	-0.230
4109010103	-0.127	0.086	**0.930**	-0.207	**0.632**	**0.482**	0.153	0.384
4109010114	-0.088	-0.197	-0.006	0.249	0.101	0.195	**0.710**	-0.119
4109010115	**-0.413**	**1.706**	-0.165	-0.253	-0.150	-0.100	0.038	**0.474**
4109010177	-0.217	-0.149	-0.223	**1.282**	-0.049	-0.166	0.150	0.006
4109010178	-0.096	-0.014	-0.072	-0.068	**3.532**	0.063	-0.086	-0.131
4109010179	-0.107	-0.065	-0.127	-0.065	0.014	**0.848**	0.027	-0.160
4110040081	-0.097	-0.139	-0.013	-0.086	**5.119**	-0.001	-0.132	-0.113

Table B.2. Store 221 with 4 means

4-MEANS				
ITEMS	14, 10%	20, 70%	58, 40%	6, 80%
0809010004	-0.0728	0.0876	0.0024	-0.1364
4101020081	-0.0797	0.2355	-0.0514	-0.1099
4101040001	-0.0090	0.2740	-0.0730	-0.1884
4102010002	0.0248	**0.4537**	-0.1728	0.0512
4102020001	-0.1996	**0.9619**	-0.2735	-0.1655
4103030095	0.0697	**0.4890**	-0.1701	-0.1722
4104020082	0.0572	**0.4369**	-0.1887	0.1714
4104020088	-0.0158	-0.1275	-0.1569	**1.7685**
4107020002	0.0365	**0.6335**	-0.2564	0.1974
4109010005	0.0686	-0.0912	-0.1740	**1.6295**
4109010022	0.0834	0.0970	-0.1249	0.6047
4109010032	0.2018	**0.4150**	-0.1595	-0.3116
4109010036	0.0246	**0.4751**	-0.1743	0.0002
4109010047	**2.0486**	-0.2862	-0.3804	-0.1093
4109010048	**-0.4792**	**0.6312**	-0.0939	-0.1217
4109010082	0.0222	0.0454	-0.0836	**0.5338**
4109010096	-0.1274	0.0600	-0.0436	**0.4563**
4109010100	-0.1232	0.2837	-0.0725	0.0142
4109010103	-0.0321	0.2303	-0.0812	0.0631
4109010114	-0.1015	**0.4981**	-0.1253	-0.2297
4109010115	0.0512	0.1610	-0.0864	0.1456
4109010177	0.0792	**0.5415**	-0.1854	-0.2204
4109010178	-0.1140	-0.0616	-0.0794	**1.1056**
4109010179	0.0287	0.0719	-0.0691	0.3153
4110040081	-0.1086	-0.0611	-0.1304	**1.5313**

Table B.3. Store 221 with 8 means

8-MEANS								
ITEMS	0.7%	41,9%	11,3%	10,7%	5,2%	11,4%	4,1%	14,7%
0809010004	**0.460**	-0.204	-0.204	-0.204	-0.204	-0.204	**4.679**	-0.204
4101020081	0.225	-0.185	-0.144	-0.137	-0.256	0.393	0.242	**0.447**
4101040001	0.049	-0.349	-0.168	-0.165	-0.300	**2.024**	-0.015	-0.217
4102010002	-0.064	-0.195	-0.018	**0.393**	-0.079	**0.607**	0.076	-0.176
4102020001	-0.040	-0.306	-0.250	**0.693**	-0.119	**0.460**	0.028	0.239
4103030095	**-0.405**	-0.174	0.078	**0.599**	-0.134	0.306	-0.147	-0.129
4104020082	**0.611**	-0.289	-0.105	**1.721**	0.187	-0.159	-0.285	-0.239
4104020088	**0.905**	-0.244	-0.196	-0.219	**3.368**	-0.079	0.083	-0.189
4107020002	-0.200	-0.161	0.081	0.233	0.037	-0.223	-0.115	**0.430**
4109010005	**8.236**	-0.135	0.107	0.008	0.263	-0.135	-0.105	-0.054
4109010022	-0.173	-0.082	0.005	-0.027	**0.712**	-0.087	-0.062	0.094
4109010032	0.026	-0.206	0.174	**0.647**	-0.089	-0.192	0.038	0.151
4109010036	0.097	-0.240	0.020	-0.123	-0.110	-0.221	0.094	**0.939**
4109010047	**-0.458**	-0.372	**2.149**	-0.259	0.148	-0.150	-0.033	-0.307
4109010048	**-0.519**	-0.250	**-0.486**	-0.048	-0.303	-0.100	0.013	**1.329**
4109010082	-0.165	-0.108	0.068	-0.029	0.139	-0.083	-0.004	0.301
4109010096	-0.085	-0.085	-0.178	**0.519**	-0.029	0.221	-0.109	-0.124
4109010100	0.008	-0.115	-0.105	**0.821**	-0.084	-0.057	0.013	-0.116
4109010103	**0.405**	-0.063	-0.068	0.128	0.110	-0.045	**0.618**	-0.056
4109010114	-0.322	-0.143	-0.123	-0.232	-0.288	0.052	0.167	**0.702**
4109010115	**2.271**	-0.100	0.094	0.157	-0.023	-0.183	0.099	0.113
4109010177	-0.130	-0.136	0.113	**0.673**	-0.231	-0.085	-0.022	-0.028
4109010178	-0.164	0.068	-0.076	-0.137	0.135	-0.027	-0.129	-0.019
4109010179	**1.133**	-0.034	0.038	0.236	0.090	-0.049	0.048	-0.162
4110040081	0.191	0.071	0.008	-0.076	-0.122	-0.116	-0.109	0.001

Table B.4. Store 310 with 4 means

4-MEANS				
ITEMS	9, 50%	16, 40%	12%	62, 10%
0809010004	-0.0355	0.0611	-0.0922	0.0071
4101020081	**0.4820**	**0.5717**	-0.1341	-0.1988
4101040001	**1.8980**	0.0086	-0.0499	-0.2830
4102010002	0.0820	**0.4668**	-0.0987	-0.1167
4102020001	**0.4242**	**0.4993**	0.0309	-0.2027
4103030095	0.0806	**0.6665**	0.1195	-0.2114
4104020082	-0.1163	**0.9089**	-0.0343	-0.2156
4104020088	**0.5748**	-0.1454	-0.0160	-0.0464
4107020002	-0.1970	-0.2390	**1.9815**	-0.2897
4109010005	0.1017	-0.2403	-0.2343	0.0932
4109010022	**0.7294**	-0.0991	0.2456	-0.1329
4109010032	-0.2033	0.2017	**0.6385**	-0.1455
4109010036	-0.1437	**0.7113**	-0.0794	-0.1505
4109010047	-0.2237	0.1102	0.2748	-0.0480
4109010048	0.0976	0.2892	-0.1018	-0.0716
4109010082	0.1801	0.0735	-0.1175	-0.0243
4109010096	0.0178	0.3803	0.0878	-0.1201
4109010100	0.3539	0.2475	-0.0994	-0.1003
4109010103	-0.2473	-0.0290	0.1207	0.0222
4109010114	0.3457	**0.6871**	0.0106	-0.2364
4109010115	-0.0267	-0.2329	-0.1165	0.0881
4109010177	0.1009	**0.9333**	-0.1230	-0.2381
4109010178	**0.9877**	-0.0909	-0.1074	-0.1064
4109010179	0.1428	0.2164	0.0309	-0.0850
4110040081	0.3288	-0.1682	0.0370	-0.0130

Table B.5. Store 310 with 8 means

8-MEANS								
ITEMS	2,4%	5,7%	2,7%	12,3%	9,9%	2,9%	55,3%	8,8%
0809010004	-0.129	-0.129	-0.129	-0.093	0.141	0.024	0.023	-0.028
4101020081	-0.253	-0.287	0.152	0.345	0.221	0.233	-0.186	**0.570**
4101040001	**-0.470**	-0.105	0.080	**1.897**	-0.156	**0.461**	-0.364	-0.167
4102010002	-0.055	-0.025	-0.247	0.000	-0.137	-0.002	-0.166	**1.308**
4102020001	-0.149	-0.154	0.263	0.275	0.109	0.160	-0.179	**0.627**
4103030095	-0.174	-0.246	-0.169	0.039	0.781	0.239	-0.195	**0.476**
4104020082	-0.377	-0.266	-0.025	0.189	0.102	**-0.368**	-0.214	**1.374**
4104020088	**5.031**	-0.087	-0.019	-0.117	-0.051	-0.035	-0.144	-0.171
4107020002	-0.235	0.044	0.144	-0.067	**0.782**	-0.240	-0.088	-0.156
4109010005	0.319	0.074	0.250	-0.156	-0.179	0.016	0.053	-0.133
4109010022	0.108	0.074	-0.040	-0.110	**0.520**	**0.793**	-0.110	-0.061
4109010032	-0.030	-0.082	0.005	-0.208	**1.212**	-0.349	-0.168	0.159
4109010036	-0.217	-0.174	0.160	-0.070	**1.162**	-0.245	-0.173	0.084
4109010047	-0.060	-0.064	0.125	-0.029	**0.461**	0.081	-0.079	0.017
4109010048	0.004	-0.141	-0.080	-0.198	**1.330**	-0.095	-0.142	-0.178
4109010082	0.060	-0.033	0.186	-0.169	0.166	0.106	0.000	-0.037
4109010096	-0.369	-0.120	0.043	0.292	0.013	-0.055	-0.091	0.332
4109010100	-0.052	-0.184	**5.134**	-0.034	-0.187	-0.210	-0.157	-0.122
4109010103	0.157	**2.012**	-0.311	-0.150	0.059	-0.288	-0.155	-0.036
4109010114	-0.013	0.034	**0.578**	0.236	0.331	0.294	-0.159	0.006
4109010115	0.166	0.174	-0.335	-0.118	0.194	-0.004	0.012	-0.187
4109010177	-0.234	-0.151	0.345	0.036	-0.268	**0.876**	-0.229	**1.461**
4109010178	0.043	-0.034	0.076	-0.100	-0.135	**3.803**	-0.138	-0.102
4109010179	-0.199	0.056	0.333	-0.139	0.171	**2.051**	-0.102	-0.111
4110040081	-0.116	**2.719**	-0.190	-0.095	-0.167	-0.038	-0.181	-0.197

Table B.6. All stores with 4 means

4-MEANS				
ITEMS	21, 56%	5, 13%	72, 15%	1, 15%
0809010004	-0.0499	0.1245	0.0050	0.0681
4101020081	**0.5018**	0.1164	-0.1626	0.2713
4101040001	**0.7895**	0.0791	-0.2482	**0.4162**
4102010002	**0.5878**	-0.0677	-0.1697	-0.0779
4102020001	**0.6752**	0.1637	-0.2169	0.2126
4103030095	**0.5968**	0.0408	-0.1791	-0.1394
4104020082	**0.7020**	0.0108	-0.2072	-0.2137
4104020088	0.0067	0.0071	-0.0088	0.3931
4107020002	0.3047	0.1844	-0.1107	**0.4090**
4109010005	0.0835	-0.1507	-0.0158	0.0999
4109010022	0.1904	0.0614	-0.0638	0.1564
4109010032	0.3169	0.0506	-0.0983	-0.0012
4109010036	0.0718	**0.7613**	-0.0746	-0.0613
4109010047	0.1681	-0.1175	-0.0416	-0.0154
4109010048	-0.1406	**3.2039**	-0.1865	0.0354
4109010082	-0.0086	-0.0538	0.0050	0.0895
4109010096	**0.6093**	-0.1259	-0.1764	0.1990
4109010100	0.3239	-0.0621	-0.1044	**0.7520**
4109010103	0.0487	-0.1383	-0.0041	-0.0368
4109010114	0.3064	0.3489	-0.1181	0.1035
4109010115	-0.0096	-0.1236	0.0139	-0.1415
4109010177	**0.4884**	0.1367	-0.1573	0.1019
4109010178	-0.0621	-0.0833	-0.0937	**7.4143**
4109010179	0.1420	-0.0533	-0.0446	0.3723
4110040081	0.0561	-0.0132	-0.0245	**0.5461**

Table B.7. All stores with 8 means

8-MEANS								
ITEMS	8,6%	3,85%	13,0%	4,6%	2,1%	1,0%	7,7%	58,8%
0809010004	-0.129	0.052	-0.001	0.000	-0.120	0.057	-0.028	0.022
4101020081	-0.192	0.120	**0.593**	0.142	-0.143	0.212	0.299	-0.160
4101040001	-0.286	0.058	**1.534**	0.077	-0.117	**0.412**	-0.021	-0.309
4102010002	-0.086	-0.106	-0.003	0.053	-0.047	-0.075	**1.509**	-0.179
4102020001	-0.152	0.155	**0.632**	0.143	-0.046	0.148	**0.516**	-0.208
4103030095	-0.007	0.021	0.105	0.256	-0.067	-0.124	**0.932**	-0.161
4104020082	-0.133	-0.068	-0.095	0.158	-0.072	-0.196	**2.082**	-0.234
4104020088	**0.410**	0.013	-0.037	-0.008	0.075	0.378	-0.041	-0.056
4107020002	-0.086	0.177	**0.452**	0.002	0.008	**0.418**	-0.018	-0.105
4109010005	**1.179**	-0.183	-0.139	-0.063	0.222	0.098	0.058	-0.142
4109010022	0.229	0.055	0.002	0.197	0.105	0.174	0.061	-0.068
4109010032	-0.007	-0.009	0.253	0.141	0.067	0.012	0.141	-0.087
4109010036	-0.076	**0.765**	0.047	0.040	0.041	-0.046	0.049	-0.059
4109010047	0.051	-0.099	0.056	0.134	-0.034	-0.037	0.081	-0.033
4109010048	-0.199	**3.656**	-0.128	-0.041	-0.105	0.056	-0.091	-0.163
4109010082	-0.106	-0.120	-0.101	-0.103	**5.418**	-0.085	-0.106	-0.126
4109010096	-0.112	-0.116	**0.571**	**0.405**	-0.274	0.213	0.260	-0.163
4109010100	-0.118	-0.007	**0.518**	0.156	-0.058	0.193	0.037	-0.116
4109010103	**1.473**	-0.147	-0.099	-0.138	-0.208	-0.030	-0.047	-0.158
4109010114	-0.128	0.307	**0.393**	0.136	-0.044	0.100	0.032	-0.104
4109010115	**1.832**	-0.173	-0.249	-0.074	0.159	-0.130	0.030	-0.202
4109010177	-0.212	-0.028	-0.102	**3.341**	-0.088	0.100	-0.031	-0.204
4109010178	-0.104	-0.079	-0.006	-0.091	-0.092	**7.637**	-0.115	-0.092
4109010179	0.026	-0.061	0.221	-0.017	0.032	0.399	-0.064	-0.047
4110040081	-0.116	**2.719**	-0.190	-0.095	-0.167	-0.038	-0.181	-0.197

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